Design and Analysis of SCMA Codebook Based on Star-QAM Signaling Constellations

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***Abstract*—In this paper, a novel codebook design method for sparse code multiple access (SCMA) is proposed and an analytical framework to evaluate the bit error rate (BER) performance is developed. In particular, to meet the codebook design criteria based on the pairwise error probability, a novel codebook with large minimum Euclidean distance employing the star quadrature amplitude modulation signal constellations is designed. In addition, with the aid of the phase distribution of the presented SCMA constellations, BER of SCMA system over downlink Rayleigh fading channel is obtained in closed-form expression. The simulation and analytical results show that the presented SCMA codebook outperforms the existing codebooks and low-density signature, and the proposed design is more efficient for the SCMA codebook with large size and/or high dimension. Moreover, the derived theoretical BER results match well the simulation results, especially in the high signal-to-noise ratio regimes.**

***Index Terms*—SCMA, codebook design, Star-QAM constellation, phase distribution, BER.**

I. Introduction

In ORDER to meet the challenging requirements of the next generation wireless communication [1], such as low latency, massive connectivity and high spectral efficiency (SE), the innovative concept of non-orthogonal multiple access (NOMA) has been proposed [2]. The basic idea of NOMA is to serve multiple users at the same time/frequency resource via different power levels or/and codebooks. Recently, several NOMA schemes have been proposed, which can be mainly divided into two categories: single-carrier NOMA and multi-carrier NOMA. In single-carrier NOMA, multiple users with different power levels share the same resource blocks, while it employs successive interference cancellation (SIC) to eliminate interference between different users at the receiver [3], [4]. As a result, NOMA outperforms conventional orthogonal multiple access (OMA), e.g., time division multiple access (TDMA), in terms of SE and connections.

In order to further improve the performance of NOMA, a multi-carrier NOMA, namely sparse code multiple access (SCMA), is recently proposed, in which multiple users can be served simultaneously by employing different codebooks [5]. Different from single-carrier NOMA, a multi-user detector based on message passing algorithm (MPA) [18] is adopted at the receiver to eliminate inter-user interference in SCMA. Taking advantage of the sparsity of codebook, the MPA detector has much lower complexity compared to the joint maximum likelihood (ML) detector. Unfortunately, complexity of the receiver is still very high for the hardware of downlink system. Therefore, it is a practical way to divide all the users into several groups with a small number of users in each group [6]. In each group, every layer or user has its dedicated codebook. Therefore, the design of codebooks is particularly important to the success of SCMA systems [7].

Various research works about SCMA codebook design have been presented in the open literatures. In [14], [15], codebook design on the network layer has been studied, where network control and resource management are investigated based on SCMA codebook. Specially, a repetitive coding, namely low density signature (LDS), is introduced in [8]. Furthermore, a novel method of SCMA codebook design based on the design principles of lattice constellations has been proposed in [7], and it showed that the performance of SCMA outperforms LDS due to the shaping gain of multi-dimensional codebook. Inspired by this, SCMA codebooks based on star quadrature amplitude modulation (Star-QAM) constellations and constellation rotation have been proposed in [9] and [11], respectively. [12] shows a capacity-based codebook design method for SCMA systems. Performance evaluation results presented in [9], [11] and [12] have shown that the bit error rate (BER) performance of using these new codebooks is much better than that of using the codebooks proposed in [7] and the LDS. However, the systematic approach to design SCMA codebooks is not given yet, and these performance evaluation results have been obtained exclusively by means of computer simulations which can be very time consuming. In [13], a joint design of multiuser codebooks for uplink SCMA systems over Rayleigh fading channels was proposed. However, here the method to design SCMA codebooks is limited to the small size and/or low dimension codebook.

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On the other hand, the performance analysis of SCMA has drawn a significant amount of attention. In [16], the upper bound of symbol error rate (SER) of SCMA system was derived based on the pairwise error probability (PEP). Furthermore, a lower bound of SER has been obtained in [17] when the overloading of the system is small. In [22], a theoretical expression for the BER performance over additive white Gaussian noise (AWGN) channel has been derived, based on the distribution of the phase angle in constellations. However, most of the existing analytical works did not give the closed-form expressions of BER based on the proposed codebooks for Rayleigh fading channels. Motivated by these, in this paper, a downlink SCMA system is studied, focusing mainly on the codebook design and BER performance analysis.

The main contributions of this paper are summarized as follows.

- A multi-dimensional SCMA codebook based on the Star-QAM constellations is first introduced for both Gaussian and Rayleigh fading channels. Unlike the codebook design in [9], the proposed SCMA codebook in this paper has a larger jointly minimum Euclidean distance and the parameters of codebooks are optimized with the proposed algorithm, to meet the codebook design criteria based on pairwise error probability (PEP). Compared with the codebook based on Star-QAM constellations in [9], the performance of the newly proposed codebook is much better when the codebook size M is large. This can be explained by the fact that the amplitude of mother codebook (MC) in [9] increases exponentially, while increases linearly in the new codebook. Meanwhile, Monte Carlo simulation results demonstrate that the BER performance of the newly proposed SCMA codebook outperforms the existing SCMA codebooks in [7], [12] and LDS.
- The novel SCMA codebook design is flexible and can meet the needs of different overloaded SCMA systems while ensure the BER performance. Unlike the existing codebook design method in [7], [12], [16], which focus on the computer search for small size and/or low dimension codebook, the newly proposed method is systematic and particularly efficient for large size and/or high dimension codebook.
- An analytical approach to obtain the theoretical BER performance over Rayleigh fading channel is proposed. The exact expressions and closed-form approximations of SCMA's BER are obtained based on the the newly cumulative distribution function (CDF) of the phase angle, φ, in SCMA constellation. The analytical results match well the Monte Carlo simulation results even if the number of user or the size of SCMA codebook is large, especially in high signal-to-noise ratio (SNR) regions.

The rest of the paper is organized as follows. The SCMA system model is presented in Section II. Section III is devoted to the new design of the SCMA codebooks based on the Star-QAM constellations. Section IV gives the BER analysis of the SCMA system over Rayleigh fading channel in detail. Numerical and computer simulation results of BER performance evaluation are presented in Section V. The conclusions of the paper are summarized in Section VI.

Notations: Throughout the paper, we use $\mathbb{B}$ and $\mathbb{C}$ to denote the sets of binary and complex numbers, respectively. A scalar, a vector and a matrix are denoted by x, $\mathbf{x}$ and $\mathbf{X}$, respectively. $\mathbf{x}_j$ is jth column of matrix $\mathbf{X}$. Similarly, $(\mathbf{F})_{kj}$ is the element of kth row and jth column of matrix $\mathbf{F}$, and $\mathbf{x}(k)$ is the kth element of the vector $\mathbf{x}$. The matrix $\mathbf{F}$ with m rows and n columns is denoted by $\mathbf{F}_{m\times n}$. For a matrix or vector, we use the operations $(\cdot)^T$ and $\|\cdot\|$ to represent transpose and Euclidean norm, respectively. Also, diag$(\mathbf{h})$ is a diagonal matrix. The probability density function (PDF) of a complex Gaussian random variable (RV) with mean μ and variance σ^2 is denoted by $\mathcal{CN}(\mu, \sigma^2)$.

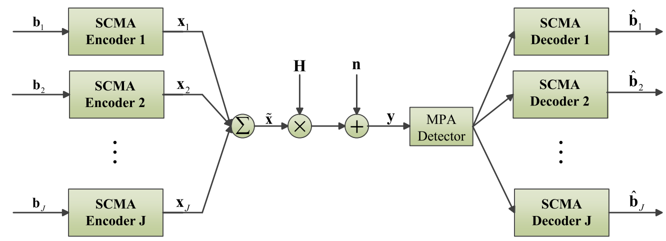


Fig. 1. The system model diagram of downlink SCMA.

II. System Model

A downlink wireless communication system with multiple users and one base station (BS) is considered, where both users and BS are equipped with a single antenna. Due to the complexity of MPA receiver, a hybrid multiple access [6] is adopted here, in which every J users are grouped to perform SCMA technique, as shown in Fig. 1. Specially, the SCMA scheme with K resource elements (REs) can support $J (J > K)$ users simultaneously leading to an overloading factor $\lambda = J/K > 1$ [5]. However, in the orthogonal scenarios, e.g., TDMA, OFDMA, in order to make sure each user can communicate on an orthogonal RE, J is less than or equal to K.

At the transmitter, an SCMA encoder performs a mapping from $q = \log_2 M$ channel coded bits to a K-dimension complex codebook with size M, which can be defined as

$$f : \mathbb{B}^{\log_2 M} \rightarrow \mathcal{X}, \ \mathbf{x} = f(\mathbf{b}), \tag{1}$$

where $\mathbf{b}$ denotes the incoming channel coded bits, $M = 2^q$, $\{q = 1, 2, 3, \cdots\}$, and $\mathbf{x} \in \mathcal{X} \subset \mathbb{C}^K$ with $\|\mathcal{X}\| = M$. An SCMA encoder includes several separate layers. Without loss of generality, here it is assumed that each user is assigned one layer, i.e., user and layer are used interchangeably in this paper. Unlike LDS, the SCMA encoder combines symbol mapping and spreading together, i.e., the incoming bits are mapped into multi-dimensional codewords directly based on the SCMA codebook sets. As we know, it is still an open problem to design an optimal multi-dimensional SCMA encoder, so a sub-optimal SCMA codebook construction is obtained by a multi-stage approach [7].

In order to illustrate the structure of sub-optimal SCMA encoder clearly, let $\mathbf{c}$ denote an N-dimensional complex constellation point defined based on the original constellation (OC) set $\mathcal{C} \subset \mathbb{C}^N$ with $\|\mathcal{C}\| = M$, where $N < K$. A mapping from $\mathbb{B}^{\log_2 M}$ to $\mathcal{C}$ is defined as:

$$g : \mathbb{B}^{\log_2 M} \rightarrow \mathcal{C}, \ \mathbf{c} = g(\mathbf{b}). \tag{2}$$

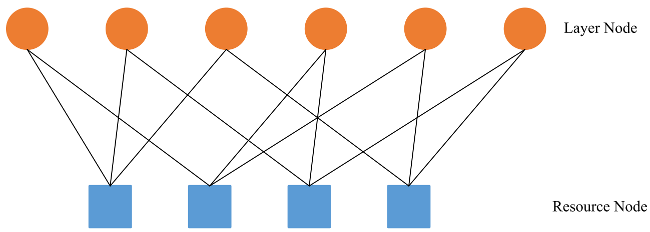


Fig. 2. Factor graph representation of an SCMA system with $J = 6$, $K = 4$, $N = 2$ and $d_f = 3$.

Hence, the SCMA encoder in (1) can be mathematically expressed as:

$$f :\equiv \mathbf{V}g,\ \mathbf{x} = \mathbf{V}g(\mathbf{b}). \tag{3}$$

In the above formula, the matrix, $\mathbf{V} \in \mathbb{B}^{K\times N}$, is a spreading matrix, which simply denotes the mapping from the N-dimensional complex codeword of OC to a K-dimensional codeword of the designed SCMA codebook. In order to control the size of d_f conflicting over the same REs and reduce the complexity of the receiver, the spreading matrix $\mathbf{V}$ is designed as a sparse matrix, which includes $K - N$ all-zero rows, i.e., the all-zero elements of one SCMA codebook are in the same $K - N$ dimensions. Thus, the sparse matrix $\mathbf{V}$ can be obtained by inserting $K - N$ rows of zero elements into $\mathbf{I}_N$. In addition, the whole structure of SCMA codeword can be represented by a factor graph matrix defined as $\mathbf{F} = (\mathbf{f}_1, \mathbf{f}_2, \ldots, \mathbf{f}_J)$, where $\mathbf{f}_j = \text{diag}(\mathbf{V}_j\mathbf{V}_j^T)$, $\mathbf{V}_j$ is the sparse matrix for the jth user. The layer node (LN) j and resource node (RN) k are connected if and only if $(\mathbf{F})_{kj} = 1$. Fig. 2 shows an example of a factor graph representation with 6 LNs and 4 RNs, where each LN is connected with $N = 2$ RNs, and each RNs is shared with $d_f = 3$ LNs. Specially, the factor graph and sparse matrices are given as follows.

$$\mathbf{F} = \begin{bmatrix} 1&1&1&0&0&0\\ 1&0&0&1&1&0\\ 0&1&0&1&0&1\\ 0&0&1&0&1&1 \end{bmatrix},$$

$$\mathbf{V}_1 = \begin{bmatrix} 1&0\\0&1\\0&0\\0&0 \end{bmatrix}, \mathbf{V}_2 = \begin{bmatrix} 1&0\\0&0\\0&1\\0&0 \end{bmatrix}, \mathbf{V}_3 = \begin{bmatrix} 1&0\\0&0\\0&0\\0&1 \end{bmatrix},$$

$$\mathbf{V}_4 = \begin{bmatrix} 0&0\\1&0\\0&1\\0&0 \end{bmatrix}, \mathbf{V}_5 = \begin{bmatrix} 0&0\\1&0\\0&0\\0&1 \end{bmatrix}, \mathbf{V}_6 = \begin{bmatrix} 0&0\\0&0\\1&0\\0&1 \end{bmatrix}.$$

Based on (1) and (3), a specific SCMA codebook, $\mathbf{C}_j$, for user j can be given by:

$$\mathbf{C}_j = \mathbf{V}_j(\boldsymbol{\Delta}_j)\mathbf{A}_{\text{MC}}, \tag{4}$$

where $\boldsymbol{\Delta}_j$ is the constellation operator for user j, which can refer to [9], [10], mainly defined by the following two typical operators, and $\mathbf{A}_{\text{MC}}$ is the mother codebook to be designed.

i) Phase operator

$$(\circ : \phi)\mathbf{Z} := e^{i\phi}\mathbf{Z};$$

ii) Matrix permutation

$$(\otimes : \boldsymbol{\Pi})\mathbf{Z} := \boldsymbol{\Pi}\mathbf{Z},$$

where $\boldsymbol{\Pi}$ is a permutation matrix. Therefore, the operator can be expressed as

$$\begin{aligned} \boldsymbol{\Delta} &= (\otimes\circ : \boldsymbol{\Pi}\phi)\mathbf{Z} = (\otimes : \boldsymbol{\Pi})(\circ : \phi)\mathbf{Z} \\ &= \boldsymbol{\Pi}\text{diag}(e^{i\phi}) : \mathbf{Z}. \end{aligned} \tag{5}$$

In an SCMA downlink, the SCMA codewords from different users are multiplexed over the same orthogonal REs, e.g. OFDMA tones. After synchronous user multiplexing, the received signal can be mathematically expressed as [5]

$$\mathbf{y} = \sum_{j=1}^{J}\mathbf{H}\mathbf{x}_j + \mathbf{n} = \sum_{j=1}^{J}\mathbf{H}\mathbf{V}_j g(\mathbf{b}_j) + \mathbf{n}, \tag{6}$$

where $\mathbf{x}_j = (x_{1j}, x_{2j}, \ldots, x_{Kj})^T$ is a K-dimensional SCMA codeword for user j from SCMA codebooks, $\mathbf{H} = \text{diag}(\mathbf{h})$, $\mathbf{h} = (h_1, h_2, \ldots, h_K)^T$ denotes the channel vector, $h_k \sim \mathcal{CN}(0, 1)$ and $\mathbf{n} = (n_1, n_2, \ldots n_K)^T$ is the additive white Gaussian noise with zero mean and $N_0\mathbf{I}_K$ variance.

At the receiver, the MPA [18] detector is adopted to eliminate interference between users based on the sparsity of the codewords [5], which takes advantage of the iterative exchange of messages between the resource nodes and layer nodes. The MPA algorithm used in SCMA can be explained as follows in detail [18], [19].

The edge that connects resource node k to layer node j with $\widetilde{m}$th message is represented by $e_k(x_{j,\widetilde{m}})$. Let $Q^i_{k,\leftarrow}(x_{j,\widetilde{m}})$ be the message sent along the edge $e_k(x_{j,\widetilde{m}})$ at ith iteration, from layer node j to resource node k. Similarly, the message sent from the resource node to the layer node is given by $Q^i_{k,\rightarrow}(x_{j,\widetilde{m}})$. It is assumed that there are no *a priori* probabilities available, and the initial messages $(i = 0)$ are set to zeros: $Q^0_{k,\rightarrow}(x_{j,\widetilde{m}}) = 0$, $Q^0_{k,\leftarrow}(x_{j,\widetilde{m}}) = 1/M$, $\forall\widetilde{m} \in \{1, 2, \ldots, M\}, k \in \{1, 2, \ldots, K\}, j \in \{1, 2, \ldots, J\}$. Specifically, the messages between kth resource node and jth layer node are updated using the following rules [20]

$$\begin{aligned} &Q^i_{k,\rightarrow}(x_{j,\widetilde{m}}) \propto \\ &f\left(x_{j,\widetilde{m}}\,|y_k\,, Q^{i-1}_{k,\leftarrow}(x_{j',\widetilde{m}'}), \forall(j', \widetilde{m}') \in \mathcal{J}_k\backslash(j, \widetilde{m})\right), \end{aligned} \tag{7}$$

$$Q^i_{k,\leftarrow}(x_{j,\widetilde{m}}) = \prod_{\ell\in\varsigma_{j,\widetilde{m}}\backslash k} Q^i_{\ell,\rightarrow}(x_{j,\widetilde{m}}), \tag{8}$$

where $\mathcal{J}_k = \{(j, \widetilde{m})/(\mathbf{F})_{kj} = 1\}$ is the set of data symbols (which may be assigned to different users) that interfere on RE k. Also, $\varsigma_{j,\widetilde{m}} = \{k/(\mathbf{F})_{kj} = 1\}$ is the set of different REs that the symbol of user j is spread on. It can be seen that all messages are updated by the extrinsic information from (7) and (8). The marginalize product of function (MPF) [21] in (7) can be written as (9), shown at the bottom of the next page, where $\widetilde{\mathbf{x}}$ is a point of the combination constellation for all users' possible codewords.

If the message-passing has reached the maximum number of iterations I, *a posteriori* probability of the transmitted symbol

$x_{j,\tilde{m}}$ can be represented as

$$p(x_{j,\tilde{m}}) = \prod_{\ell \in \varsigma_{j,\tilde{m}}} Q^{I}_{\ell,\rightarrow}(x_{j,\tilde{m}}). \tag{10}$$

After employing the hard decision, the estimated value of $x_{j,\tilde{m}}$ is given by

$$\hat{x}_{j,\tilde{m}} = \arg\max_{x_{j,\tilde{m}} \in \mathcal{X}} p(x_{j,\tilde{m}}). \tag{11}$$

III. SCMA Codebook Design and Discussions

In this section, a design method of MC, $\mathbf{A}_{\text{MC}}$, based on the Star-QAM signaling constellation (SQ-SCMA codebook) is introduced and discussed. In addition, an SCMA codebook design criteria is given to measure the SQ-SCMA codebook performance.

A. SQ-SCMA Codebook Design

The steps for the previously described SCMA codebooks are summarized as follows.

- Determine the sparse matrix $\mathbf{V}_j$ of user j by inserting $K - N$ all-zero row vectors into the rows of $\mathbf{I}_N$ based on the factor graph representation $\mathbf{F}$[7].
- Generate the MC, $\mathbf{A}_{\text{MC}}$.
- Determine the operator $\boldsymbol{\Delta}_j$ and the codebook $\mathbf{C}_j$ can be expressed as $\mathbf{C}_j = \mathbf{V}_j(\boldsymbol{\Delta}_j)\mathbf{A}_{\text{MC}}$.

Clearly, when the mother codebook, $\mathbf{A}_{\text{MC}}$, of SCMA codebook is simply designed by the traditional modulation constellations (e.g., PSK), the SCMA codebook is the same as the LDS codebook [8]. In addition, the SCMA codebook can outperform the LDS codebook, when we carefully design the mother codebook, which can be explained that the existence of shaping gains of mother codebook [5]. Therefore, we focus mainly on how to design the mother codebook of SQ-SCMA codebooks. The constellation operators for each user can be obtained based on the combination of typical operators in [9], [10].

From [7], [10], it can be noticed that the main advantage of SCMA over LDS is its shaping gain due to the fact that SCMA enjoys additional degrees of freedom in the multi-dimensional constellations design. In addition, power variation in each codebook is another benefit to help MPA receiver operate more efficiently to cancel inter-layer interferences [7]. Also, we know that the minimum Euclidean distance of codewords is an important factor to determine the designed codebook performance. Considering these, we design the $\mathbf{A}_{\text{MC}}$ of SQ-SCMA codebook based on the Star-QAM constellation, to obtain a combination constellation with large minimum Euclidean distance. The steps to design SQ-SCMA mother codebook in detail are illustrated as follows:

Step 1: Design four vectors as

$$\mathbf{t}_1 = \left[r_1, r_2, \ldots, r_{\frac{M}{4}}\right]^T, \mathbf{t}_2 = \left[r_{\frac{M}{4}+1}, r_{\frac{M}{4}+2}, \ldots, r_{\frac{M}{2}}\right]^T,$$

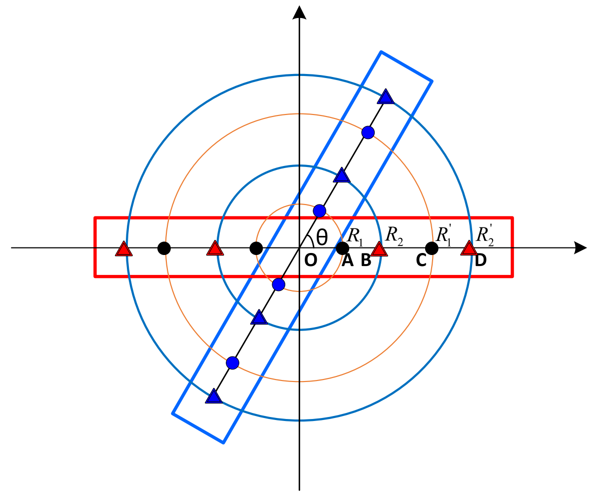


Fig. 3. Four-rings Star-QAM constellation. The radii of the four rings are R_1, R_2, R_1', R_2', respectively.

$$\bar{\mathbf{t}_1} = \left[r_{\frac{M}{4}}, r_{\frac{M}{4}-1}, \ldots, r_1\right]^T, \bar{\mathbf{t}_2} = \left[r_{\frac{M}{2}}, r_{\frac{M}{2}-1}, \ldots, r_{\frac{M}{4}+1}\right]^T,$$

where we have $r_l = ((l-1)(\alpha-1)+1), l = 1, 2, \ldots, \frac{M}{2}$, and $\alpha \in (1, +\infty)$ is a parameter to be designed later.

Step 2: According to step 1, we define a mapping from the four vectors to the elements of the n-th dimension of SQ-SCMA codebook as following, $n = 1, 2, \ldots, N$:

$$\mathbf{g}_n = \begin{cases} \left[\bar{\mathbf{t}_2}^T, \bar{\mathbf{t}_1}^T, -\mathbf{t}_1^T, -\mathbf{t}_2^T\right]^T, & n \text{ is odd} \\ \left[-\mathbf{t}_1^T, \mathbf{t}_2^T, -\bar{\mathbf{t}_2}^T, \bar{\mathbf{t}_1}^T\right]^T, & n \text{ is even} \end{cases} \tag{12}$$

Step 3: According to step 2, the mother codebook, $\mathbf{A}_{\text{MC}}$, of an N-dimensional SQ-SCMA codebook with size M can be given by

$$\mathbf{A}_{\text{MC}}^{N\times M} = ae^{i\theta}\left[\mathbf{g}_1, \beta\mathbf{g}_2, \ldots, \beta^{N-1}\mathbf{g}_N\right]^T, \tag{13}$$

where a is a predetermined constant decided by the average energy, $\theta \in [0, 2\pi]$, and $\beta \in (1, +\infty)$ is also a parameter to be designed later.

Consider an $N = 2$-dimensional SQ-SCMA codebook with size $M = 4$. According to step 1 and 2, we can design a four-rings Star-QAM constellation as shown in Fig. 3, where the radii are given by R_1, R_2, R_1', R_2', respectively, and we have $R_1' = \alpha R_1, R_2' = \alpha R_2, R_2 = \beta R_1$. Thus, the mother codebook, $\mathbf{A}_{\text{MC}}$, can be expressed as

$$\mathbf{A}_{\text{MC}} = \begin{bmatrix} \alpha R_1 & R_1 & -R_1 & -\alpha R_1 \\ -R_2 & \alpha R_2 & -\alpha R_2 & R_2 \end{bmatrix}, \tag{14}$$

$$f\left(x_{j,\tilde{m}} | y_k, Q^{i-1}_{k,\leftarrow}(x_{j',\tilde{m}'}), \forall(j', \tilde{m}') \in \mathcal{J}_k \backslash (j, \tilde{m})\right) = \sum_{x_{j,\tilde{m}}} \left(\exp\left(-\frac{1}{2\sigma^2}\left\|y_k - \mathbf{h}^T_{[k]}\tilde{\mathbf{x}}\right\|^2\right) \prod_{(j',\tilde{m}') \in \mathcal{J}_k \backslash (j,\tilde{m})} Q^{i-1}_{k,\leftarrow}(x_{j',\tilde{m}'})\right). \tag{9}$$

in which $R_1 = \sqrt{\frac{1}{2(\alpha^2+\beta^2+\alpha^2\beta^2+1)}}$, assumed that each user's codebook energy is $E_s = 1$.

B. SCMA Codebook Design Criteria

Firstly, we define that the combination constellation is $\mathfrak{M}$, $\mathbf{s}^{(1)} \in \mathfrak{M}$ and $\mathbf{s}^{(2)} \in \mathfrak{M}$. From (6), the pairwise error probability (PEP) between the transmitted vector $\mathbf{s}^{(1)}$ and detected vector $\mathbf{s}^{(2)}$ can be obtained as [23]

$$P\{\mathbf{s}^{(1)} \rightarrow \mathbf{s}^{(2)}|\mathbf{H}\} = Q\left(\sqrt{\frac{||\mathbf{H}(\mathbf{s}^{(1)} - \mathbf{s}^{(2)})||^2}{2N_0}}\right). \tag{15}$$

Then the error rate when $\mathbf{s}^{(1)}$ is transmitted can be given by

$$\begin{aligned} P\{\mathbf{s}^{(1)}|\mathbf{H}\} &= \sum_{c=1}^{M^J-1} P\{\mathbf{s}^{(1)} \rightarrow \mathbf{s}^{(c)}|\mathbf{H}\} \\ &= \sum_{c=1}^{M^J-1} Q\left(\sqrt{\frac{\sum_{k=1}^{K} |h_k|^2 d_{1,c}^2(k)}{2N_0}}\right), \end{aligned} \tag{16}$$

where $\mathbf{s}^{(1)} = \sum_{j=1}^{J} \mathbf{x}_j$ and $d_{1,c}^2(k) = |\mathbf{s}^{(1)}(k) - \mathbf{s}^{(c)}(k)|^2$. We define $d_{\min}^{\{1\}} = \min_{k,c}\{d_{1,c}^2(k)|k = 1, 2, ..., K, c = 1, 2, ..., M^J - 1\}$ and $d_{\min}^2$ to be the minimum square Euclidean distance of the combination constellation $\mathfrak{M}$. The $P\{\mathbf{s}^{(1)}|\mathbf{H}\}$ in (16) can be upper bounded by

$$P\{\mathbf{s}^{(1)}|\mathbf{H}\} \leq (M^J - 1)Q\left(\sqrt{\frac{\sum_{k=1}^{K} |h_k|^2}{2N_0} d_{\min}^{\{1\}}}\right). \tag{17}$$

Moreover, for the additive white Gaussian noise (AWGN) channel, the error rate in (17) can be simplified to

$$P\{\mathbf{s}^{(1)}\} \leq (M^J - 1)Q\left(\frac{d_{\min}}{\sqrt{2N_0}}\right). \tag{18}$$

Thus, the SCMA codebook design criteria is to minimize the value of $(M^J - 1)Q(\frac{d_{\min}}{\sqrt{2N_0}})$.

It can be noted that the parameters α and β in the steps for SCMA codebook design is very important. Thus, to meet the codebook design criteria, we proposed a numerical search algorithm to obtain the optimal values of α and β. The detailed algorithm is introduced in Algorithm 1.

From (13), it can be seen that the symbol energy of each dimension of the mother codebook, $\mathbf{A}_{\text{MC}}$, is different. Specially,

$$\begin{aligned} \eta &= \frac{E_{N-1}}{E_N} \\ &= \frac{2R_{N-1}^2\left(\left((\frac{M}{2}-1)(\alpha-1)+1\right)^2 + \cdots + \alpha^2 + 1\right)}{2R_N^2\left(1 + \alpha^2 + \cdots + \left((\frac{M}{2}-1)(\alpha-1)+1\right)^2\right)} \\ &= \frac{R_1^2}{R_2^2} = \frac{1}{\beta^2}. \end{aligned} \tag{19}$$

Also, the $\eta = 1$ for LDS [8] and the existing SCMA codebook in [7], [12]. On the one hand, the degrees of freedom of SQ-SCMA codebook are fully utilized, and the performance of MPA detection can be improved [5] due to the dimensional dependency and symbol energy variation. On the other hand, the proposed mother codebook can be guaranteed to get a large minimum Euclidean distance of the combination constellation.

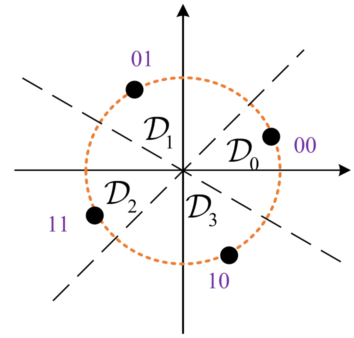


Fig. 4. An example of decision region for 4-point SCMA codebook.

Algorithm 1: SQ-SCMA Codebook Parameters α and β Design Algorithm.

Input: SCMA system parameters: $J, M, N, K, \mathbf{F}$
Initialization: $\alpha = 1$, $\beta = 1$
$\mathbf{V}_j$ is obtained from $\mathbf{f}_j = \text{diag}(\mathbf{V}_j\mathbf{V}_j^T)$
$\mathbf{\Delta}_j$ is from [9]
$\mathbf{A}_{\text{MC}}$ is from (13)
The codebook $\mathbf{C}_j$ is $\mathbf{C}_j = \mathbf{V}_j(\mathbf{\Delta}_j)\mathbf{A}_{\text{MC}}$
while $\alpha \leq B_\alpha$ **do**
  $\alpha = \alpha + w_\alpha$
  while $\beta \leq D_\beta$ **do**
    $\beta = \beta + u_\beta$
    Calculate $d_{\min}^2$, i.e., the minimum square Euclidean distance of the combination constellation $\mathfrak{M}$, according to $d_{1,c}^2(k) = |\mathbf{s}^{(1)}(k) - \mathbf{s}^{(c)}(k)|^2$, and $d_{\min}^2 = \min_{k,c}\{d_{1,c}^2(k)|k = 1, 2, ..., K, c = 1, 2, ..., M^J - 1\}$
    Calculate $(M^J - 1)Q(\frac{d_{\min}}{\sqrt{2N_0}})$ according to (18)
    Judge whether it is the minimum value currently. If so, save this value and save current $d_{\min}$, α and β
  end while
end while
Output $d_{\min}$, α and β

Note: B_α and D_β are the maximum search values of α and β, respectively. w_α and u_β are the sizes of every step in search. In this paper, we use $B_\alpha = 5$, $w_\alpha = 0.1$, $D_\beta = 3$ and $u_\beta = 0.1$ for simulations.

IV. BER Performance Analysis over Rayleigh Fading Channel

As described and discussed in the previous section, the M-point SCMA codebook can be designed based on the Star-QAM signaling constellations, and the codewords are mapped to the Gray codes one by one. Then the decision region at the receiver can be illustrated in Fig. 4, where phases of codewords are evenly distributed in circle and they depend on the codebook design and the constellation operator. Without loss of generality, we define the decision region, $\mathcal{D}_m$, as [24]

$$\mathcal{D}_m = \left\{\mathbf{u}|\mathbf{u} \in \mathbb{C}^K, ||\mathbf{u} - \widetilde{\mathbf{s}}^{(m)}||^2 < ||\mathbf{u} - \widetilde{\mathbf{s}}^{(m')}||^2, \forall m' \neq m\right\},$$

where $\widetilde{\mathbf{s}}^{(m')}$ is the nearest codeword of $\widetilde{\mathbf{s}}^{(m)}$ after Gray mapping, $m \in \{0, 1, \ldots, M-1\}$, and $m' \in \{0, 1, \ldots, M-1\}$.

With Gray mapping, the decision error probability is independent of the transmitted symbols. Thus, without any loss of generality that $U = 0$ was transmitted, the probability that the received signal, $\widetilde{\mathbf{y}}$, falls within the decision region, $\mathcal{D}_m$, of transmitted symbol, U, can be defined as

$$B_m = \Pr\{\widetilde{\mathbf{y}} \in \mathcal{D}_m \,|U = 0 \text{ is sented}\}. \tag{20}$$

Defined the phase angle between the received signal, $\widetilde{\mathbf{y}}$, and the transmitted codeword, $\widetilde{\mathbf{s}}^{(m)}$, as φ, which is perturbed by Gaussian noise [24]. Then the CDF can be given by the following lemma.

Lemma 1: The CDF[1] of phase angle, φ, which denotes the phase difference between the received signal, $\widetilde{\mathbf{y}}$, and the transmitted codeword, $\widetilde{\mathbf{s}}^{(m)}$, can be expressed as

$$F(\varphi) = -\frac{1}{4\pi}\int_0^{2\pi-2\varphi} \exp\left[-\frac{\Lambda \sin^2\varphi}{1-\cos\varphi\cos(\varphi+\phi)}\right] d\phi, \tag{21}$$

in which $\Lambda = q\frac{E_b}{N_0}$ is the average SNR, $q = \log_2 M$.

Proof: See [22]. ■

In addition, to evaluate the performance of SCMA systems over Rayleigh fading channel, an important statistical characteristic of fading channels is defined as [26]

$$M_\gamma(s) = \int_0^\infty p_\gamma(\gamma) e^{s\gamma} d\gamma, \tag{22}$$

in which $M_\gamma(s)$ is the moment generating function (MGF) of the instantaneous fading SNR γ associated with the fading PDF $p_\gamma(\gamma)$.

According to (22), the new form of CDF of phase angle, φ, over Rayleigh fading channel can be obtained as the following lemma.

Lemma 2: The new form of CDF of phase angle, φ, in SCMA constellation for Rayleigh fading channel, which denotes the phase difference between the received signal, $\widetilde{\mathbf{y}}$, and the transmitted codeword, $\widetilde{\mathbf{s}}^{(m)}$, is given by

$$\bar{F}_\varphi(\gamma) = -\frac{1}{4\pi}\int_0^{2\pi-2\varphi} \prod_{n=1}^{N} M_{\gamma_n}\left(-\frac{\sin^2\varphi}{1-\cos\varphi\cos(\varphi+\phi)}\right) d\phi, \tag{23}$$

where $M_{\gamma_1}(s) = M_{\gamma_2}(s) = \cdots = M_{\gamma_N}(s) = 1/(1-s\bar{\gamma})$ [27], and $\bar{\gamma} = (q\frac{E_b}{N_0})/N$.

Proof: With reference to lemma 1, which gives the CDF of phase angles in SCMA constellation for the AWGN channel, it can be observed that the integrand is already an exponential function of the symbol SNR. Hence, here all that we need to do is to take the place of E_s/N_0 by γ in the argument of the integrand and then average over the PDF of γ resulting in the MGF-based expression as (24).

$$\bar{F}_\varphi(\gamma) = -\frac{1}{4\pi}\int_0^{2\pi-2\varphi} M_\gamma\left(-\frac{\sin^2\varphi}{1-\cos\varphi\cos(\varphi+\phi)}\right) d\phi. \tag{24}$$

[1]The variant CDF defined here to obtain the probability is different from the classical distribution function, which is firstly proposed in [25]. For sake of simplicity, we still use CDF to be described in this paper.

TABLE I
HAMMING WEIGHTS OF DIFFERENT SCMA CODEBOOKS WITH SIZE M

m	Bits	ω_m	m	Bits	ω_m
$M=4$			$M=16$		
0	0 0	0	0	0 0 0 0	0
1	0 1	1	1	0 0 0 1	1
2	1 1	2	2	0 0 1 1	2
3	1 0	1	3	0 0 1 0	1
$M=8$			4	0 1 1 0	2
			5	0 1 1 1	3
0	0 0 0	0	6	0 1 0 1	2
1	0 0 1	1	7	0 1 0 0	1
2	0 1 1	2	8	1 1 0 0	2
3	0 1 0	1	9	1 1 0 1	3
4	1 1 0	2	10	1 1 1 1	4
5	1 1 1	3	11	1 1 1 0	3
6	1 0 1	2	12	1 0 1 0	2
7	1 0 0	1	13	1 0 1 1	3
			14	1 0 0 1	2
			15	1 0 0 0	1

In addition, considering the diversity gain of SCMA systems [10], the MGF $M_\gamma(s)$ can be related to each channel. Meanwhile, it is a practical way to assume that the channels are independent of each other, that is to say, the MGF $M_\gamma(s)$ can be expressed as the product of the MGFs related to each channel, i.e., $M_\gamma(s) = \prod_{n=1}^{N} M_{\gamma_n}(s)$ [26]. Hence, the new form of CDF, $\bar{F}_\varphi$, can be rewritten as (23), in which $M_{\gamma_1}(s) = M_{\gamma_2}(s) = \cdots = M_{\gamma_N}(s) = 1/(1-s\bar{\gamma})$ [27], and $\bar{\gamma} = (q\frac{E_b}{N_0})/N$. Then, the proof is completed. ■

According to the equation (20) and the result of lemma 2, we have [26]

$$\begin{aligned} B_m &= \int_{(2m-1)\theta_1}^{(2m+1)\theta_1} f(\varphi) d\varphi \\ &= \bar{F}\left[(2m+1)\theta_1\right] - \bar{F}\left[(2m-1)\theta_1\right], \end{aligned} \tag{25}$$

where $m = 1, 2, \ldots, \frac{M}{2}$, $f(\varphi)$ is the PDF of variable, φ, and $\theta_1 = \frac{8\pi}{11M-12}$ is the initial phase in SCMA codebook design [9], [22].

Note that the SQ-SCMA codebook designed in the previous section is symmetrical, then we have

$$B_m = B_{M-m}, \ \forall m = 1, 2, \ldots, \tfrac{M}{2}. \tag{26}$$

Hence, the average BER of the SCMA system over Rayleigh fading channel can be expressed as the following proposition.

Proposition 1: The average BER of SCMA system over Rayleigh fading channel can be expressed as

$$P_b^{\text{Rayleigh}} = \frac{1}{\log_2(M)} \sum_{m=1}^{M/2} \omega'_m B_m, \tag{27}$$

where $\omega'_m = \omega_m + \omega_{M-m}, \omega'_{M/2} = \omega_{M/2}, \omega_m$ is the Hamming weight of the bits assigned to symbol m (see Table I).

Proof: From Table I, we have the the Hamming weight, ω_m, of the bits assigned to symbol m. According to equation (25), the average BER of SCMA system over Rayleigh fading channel can be expressed as [26]

$$P_b^{\text{Rayleigh}} = \frac{1}{\log_2(M)} \sum_{m=1}^{M-1} \omega_m B_m. \tag{28}$$

TABLE II
THE MINIMUM EUCLIDEAN DISTANCE

Codebook	$d_{\min}$
LDS	0.4115
SCMA [7]	0.5931
SCMA [12]	0.6714
SQ-SCMA($\alpha = 2$, $\beta = 1.2$)	0.7123
SQ-SCMA($\alpha = 3$, $\beta = 1.6$)	0.8067
SQ-SCMA($\alpha = 4$, $\beta = 2$)	0.6384

Since $B_m = B_{M-m}$ in (26), the above equation can be rewritten as (27), in which $\omega'_m = \omega_m + \omega_{M-m}, \omega'_{M/2} = \omega_{M/2}$. Thus, the proof is completed. ■

It is clear from (27) that the average BER of SCMA system over Rayleigh fading channel is very complicated when the size of codebook is large. Moreover, the BER and SER of SCMA codebook with Gray mapping have the following ship [24]

$$P_b^{\text{Rayleigh}} \geq \frac{1}{\log_2(M)} P_s^{\text{Rayleigh}}. \tag{29}$$

It is more convenient to calculate the average BER form the following theorem.

Theorem 1: The average BER of SCMA system over Rayleigh fading channel can also be approximately expressed as

$$P_b^{\text{Rayleigh}} \approx \frac{1}{2q\widetilde{K}} \sum_{\widetilde{k}=1}^{\widetilde{K}} \sqrt{1 - \widetilde{x}_{\widetilde{k}}^2} \times \left(\frac{1}{1 + q\frac{E_b}{N_0} \frac{\sin^2 \frac{8\pi}{11M-12}}{(1-\cos\frac{8\pi}{11M-12}\cos(\frac{8\pi}{11M-12}+\phi_{\widetilde{k}}))N}} \right)^N, \tag{30}$$

in which $\widetilde{K}$ is a parameter for Gaussian-Chebyshev quadrature, $\widetilde{x}_{\widetilde{k}}^2 = \cos\left(\frac{2\widetilde{k}-1}{2\widetilde{K}}\right)$, and $\phi_{\widetilde{k}} = (\pi - \theta_1)(1 + \widetilde{x}_{\widetilde{k}})$.

Proof: See Appendix A. ■

Note that the closed-form result of theorem 1 is easier to calculate the average BER and also it is very tight with the result of proposition 1 when the E_b/N_0 is large.

V. NUMERICAL AND SIMULATION RESULTS

In this section, the simulation results and analytical results are presented to illustrate the BER performance of SCMA system. Firstly, after using Algorithm 1, we can obtain the optimal values of α and β, meanwhile, we can get the corresponding minimum Euclidean distance $d_{\min}$. Specially, an example with 4-point codebook to show the minimum Euclidean distance of the combination constellation with different codebooks is given in Table II. It can be seen that the minimum Euclidean distance, $d_{\min}$, of the proposed SQ-SCMA codebook with $\alpha = 3$ and $\beta = 1.6$ is the largest. Also, if the choice is $\alpha = 4$ and $\beta = 2$, the value of $d_{\min}$ is smaller.

Then, in the following Table III, we present some general comparisons for different codebooks with size $M = 4$ in terms of codeword's energy, dimensional energy, Euclidean distance

TABLE III
COMPARISONS OF DIFFERENT CODEBOOKS

	LDS	SCMA [7], [12]	SQ-SCMA
The Energy of Each Codeword in a Codebook	Same value	Same value	$\frac{M}{2}$ values
The Energy of Each Dimension of a Codebook	Same value	Same value	N values
$d_{\min}$	SQ-SCMA $>$ [12] $>$ [7] $>$ LDS		
Constellation Type	QPSK	Square-QAM	Star-QAM

TABLE IV
SIMULATION PARAMETERS FOR SCMA AND LDS

Parameter	**Value**
LDS Modulation Type	QPSK
SCMA Constellation Point	4, 8, 16
Number of Symbol	1e7
Number of Iteration	5
Number of REs K	4, 5, 6, 8
Dimension of Mother Codebook N	2, 3, 4
Channels	AWGN, Rayleigh fading
Approximation Terms	10

TABLE V
MOTHER CODEBOOKS FOR SCMA [9] AND SQ-SCMA

Codebook	$d_{\min}$	**mother codebook**[1] $M = 8, N = 2$
SCMA [9]	0.0539	[17.01,5.67,1.89,0.63,-0.63,-1.89,-5.67,-17.01; -1,-3,9,27,-27,-9,3,1]
SQ-SCMA	0.1266	[4.41,3.15,1.89,0.63,-0.63,-1.89,-3.15,-4.41; -1,-3,5,7,-7,-5,3,1]

[1]There is no normalized processing in the mother codebook.

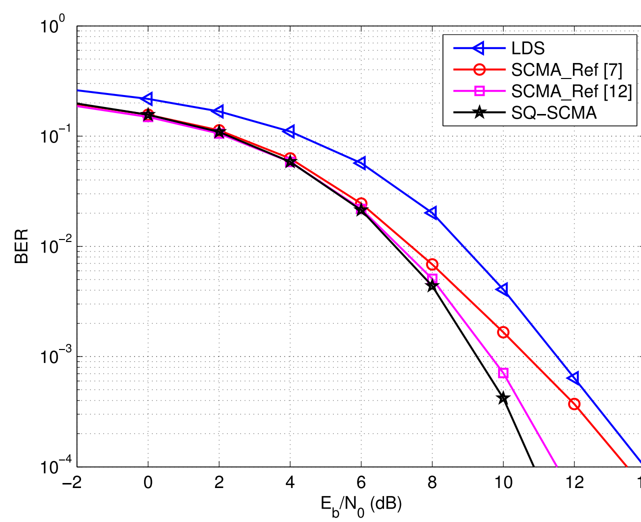
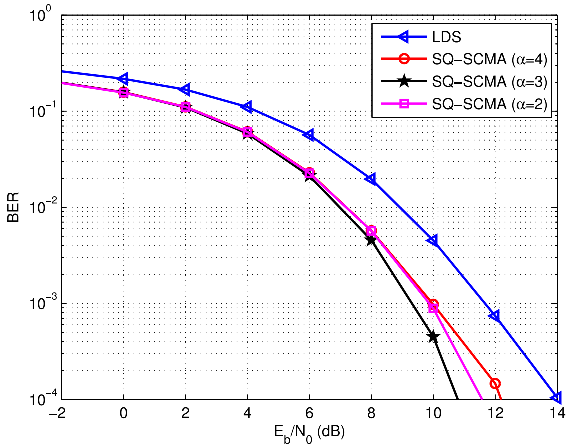
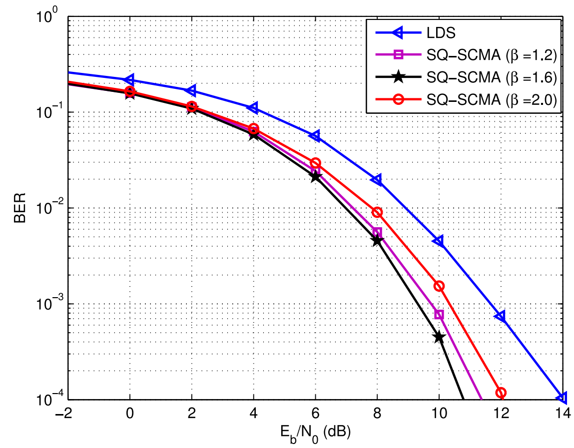


Fig. 5. Performance comparisons between SCMA_Reference [7], [12], SQ-SCMA and LDS in AWGN channel ($\alpha = 3$, $\beta = 1.6$, $M = 4$).

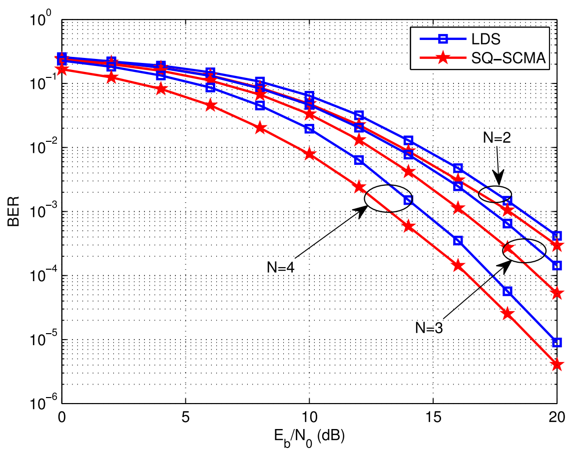
as well as modulation type. Especially, it can be noted that the minimum Euclidean distance of the proposed design is the largest. This is one of the main reason that the performance of the new codewords has better BER performance, which will be illustrated in the following figures. In addition, the simulation parameters are listed in Table IV, and an example of mother constellation with size $M = 8$ for SQ-SCMA and SCMA [9] is shown in Table V. Moreover, the factor graph matrices used are



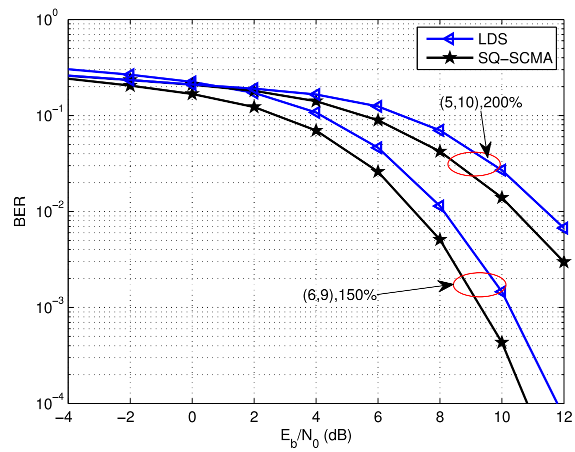
(a) Different α in AWGN channel ($\beta = 1.6$).



(b) Different β in AWGN channel ($\alpha = 3$).



(c) Different N in Rayleigh fading channel ($\alpha = 3$, $\beta = 1.6$).



(d) Different overloading factors in AWGN channel ($\alpha = 3$, $\beta = 1.6$).

Fig. 6. BER performance comparisons at different parameters of SCMA systems ($M = 4$).

specified as follows.

$$\mathbf{F}_{4\times 6} = \begin{bmatrix} 1&1&1&0&0&0 \\ 1&0&0&1&1&0 \\ 0&1&0&1&0&1 \\ 0&0&1&0&1&1 \end{bmatrix},$$

$$\mathbf{F}_{6\times 9} = \begin{bmatrix} 0&0&1&0&0&0&1&0&1 \\ 0&1&0&1&0&0&1&0&0 \\ 0&0&1&1&0&1&0&0&0 \\ 1&0&0&0&1&0&0&0&1 \\ 1&0&0&0&0&1&0&1&0 \\ 0&1&0&0&1&0&0&1&0 \end{bmatrix},$$

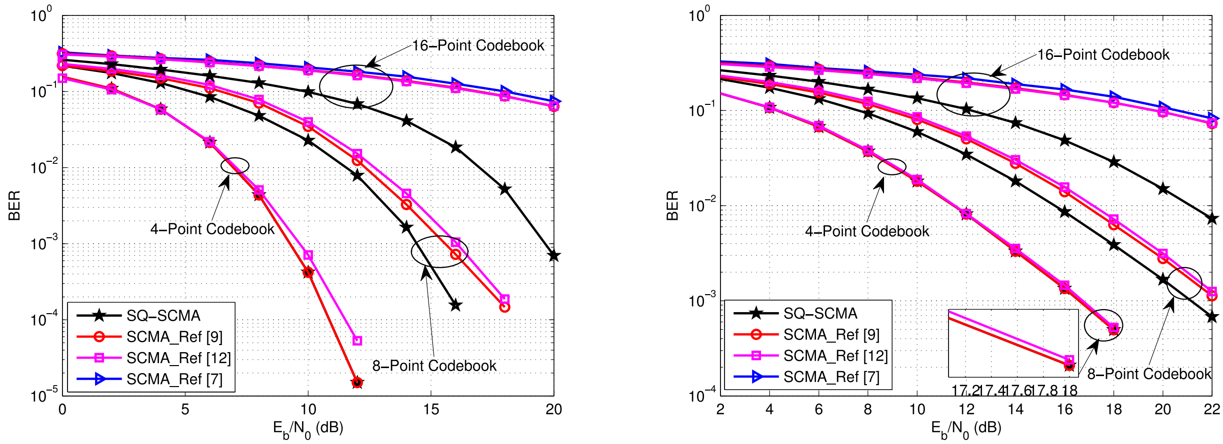
$$\mathbf{F}_{5\times 10} = \begin{bmatrix} 1&1&1&1&0&0&0&0&0&0 \\ 1&0&0&0&1&1&1&0&0&0 \\ 0&1&0&0&1&0&0&1&1&0 \\ 0&0&1&0&0&1&0&1&0&1 \\ 0&0&0&1&0&0&1&0&1&1 \end{bmatrix}.$$

Fig. 5 shows the BER performance of LDS and different SCMA codebooks with size $M = 4$. It can be seen that the new codebooks can outperform LDS and the existing codebook in [7], [12]. Clearly, the performance gain is about 3.0 dB over LDS in high SNR region. The simulation results in Fig. 5 confirm our proposal in Section III where the minimum Euclidean distance of the SQ-SCMA codebook is the largest.

Fig. 6(a) illustrates the effect of α on the system performance. One can observe that $\alpha = 3$ achieves the better performance than other values, which confirms our proposal in Section III. On the other hand, we can see that the BER performance of the new codebook with different values of α is still better than that of LDS.

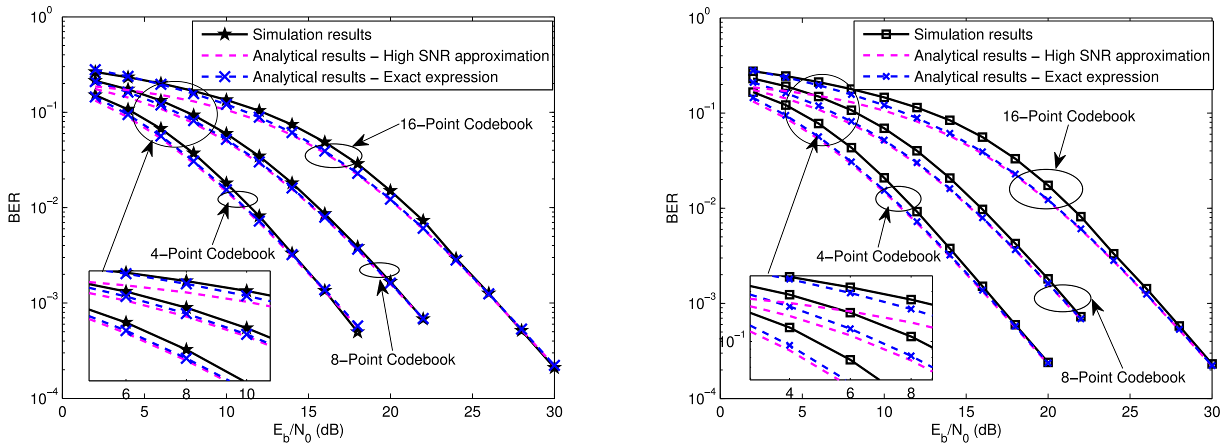
The effect of β on BER performance is presented in Fig. 6(b). The simulation results confirm the choice of $\beta = 1.6$, which can provide the best performance. Meanwhile, it can be noted that the BER performance of the new codebook with different values of β is still better than that of LDS.

As shown in Fig. 6(c), the BER performance of the SQ-SCMA codebook in Rayleigh fading channel is better than LDS with different values of N, but their diversity order is the same. Meanwhile, it can be seen that the performance of the SQ-SCMA codebook can be better with the increase of N values. Thus, the value of N is an important parameter to evaluate the performance of SCMA systems, which confirms the structured



(a) In AWGN channels. (b) In Rayleigh fading channels.

Fig. 7. BER performance comparisons with different codebook sizes between the SCMA_Reference [7], [9], [12] and the newly proposed SQ-SCMA of 4 REs and 6 users in different channels.



(a) 4 REs and 6 users. (b) 6 REs and 9 users.

Fig. 8. BER performance comparisons between simulation results and analytical results of 150% loaded SCMA systems with different REs and users in Rayleigh fading channel.

SQ-SCMA codebook design method we proposed. In addition, it is particularly important to design SCMA multi-dimensional mother codebooks with different N values.

It is shown in Fig. 6(d) that our new codebooks achieve about 1.0 dB gain over QPSK LDS for 9 and 10 users. Meanwhile, it can be seen that it is worse with the increase of overloading. Therefore, the new codebooks can be applied to the SCMA systems with different numbers of users or overloading, although the performance gain becomes smaller. However, the detection complexity will be higher than the case of 6 users.

As shown in Fig. 7(a) and Fig. 7(b), the performance of the newly proposed Star-QAM based scheme over AWGN channel and Rayleigh fading channel in this paper is much better when the codebook size M is large, compared with the codebook in [7], [9] and [12], which confirms our proposed schemes in Section III. It can be explained that the amplitude of codebook in [9], [12] increases exponentially, while increases linearly in the newly proposed codebook. Meanwhile, the 16-point codebook proposed in [7] is based on a shuffling multi-dimensional constellations method, which is simple to construct an N-dimensional complex constellation from Cartesian product of two N-dimensional real constellation. Moreover, from Fig. 7(b), it is noted that, all of them have the same diversity order.

From Fig. 8(a) and Fig. 8(b), we can see that the analytical results and simulation results over Rayleigh fading channel match well even if the number of users increases. There is also a very small difference between simulation results and analytical results in low SNR region due to the existence of multi-user interference in SCMA systems. On the other hand, we can see that formula (27) is more accurate to evaluate the BER performance of SCMA system at low SNR, though with the more complicated integral expressions. At the same time, it can be seen that all of them have the same diversity order.

The analytical results and simulation results with different diversity orders in Rayleigh fading channel under the same

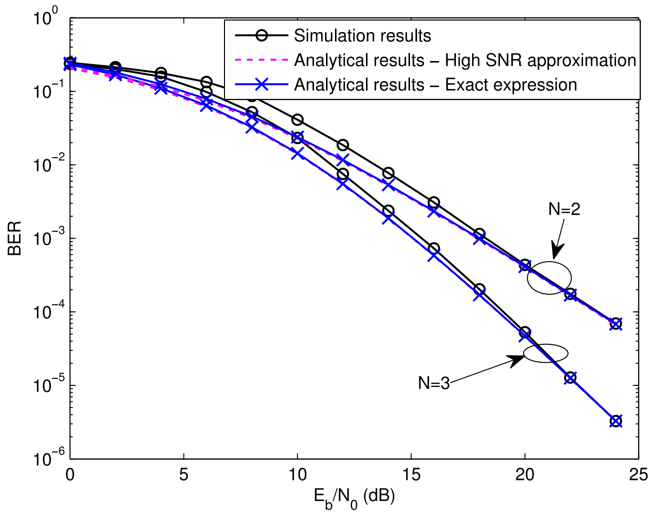


Fig. 9. BER performance comparisons between simulation results and analytical results of same loaded SCMA systems with different Ns in Rayleigh fading channel ($M = 4$).

overloading factor are shown in Fig. 9. From Fig. 9, it can be seen that the performance of our codebook with diversity order $N = 3$ is still better than that with $N = 2$. Moreover, the theoretical BER results match well the simulation results both with $N = 2$ and $N = 3$.

VI. Conclusion

A novel design of SCMA codebook based on Star-QAM signaling constellations is proposed, based on a new mother codebook with larger minimum Euclidean distance. In contrast to the MC based on Star-QAM constellations in [9] and capacity-based codebook in [12], the performance of the newly proposed MC is much better when the codebook size M is large. Simulation results have shown that the obtained SCMA codebooks can significantly enhance the BER performance without sacrificing the system implementation complexity, especially when the SNR is high. Besides, the theoretical analysis of BER performance of SCMA systems over downlink Rayleigh fading channel is conducted, using the CDF of the phase angle, φ, in SCMA constellation and MGF-based method. Numerical and simulation results show that the analytical results and simulation results are well matched, especially in high SNR region.

Appendix A
The Proof of Theorem 2

In SCMA systems, the channel input symbols of each layer are mapped to the constellation points. Hence, the symbol error rate (SER) of constellation points of SCMA system in this work is [24]

$$P_s = 2\int_{\theta_1}^{\theta_2} f(\varphi)d\varphi, \tag{31}$$

in which $f(\varphi)$ is the PDF of variable φ, and the limits of θ_1, θ_2 are determined by the phase rotations defined in SCMA codebook design. By using the same method in [22], we get $\theta_1 = \frac{8\pi}{11M-12}$ and $\theta_2 = \pi$.

In addition, we have $M_{\gamma_1}(s) = M_{\gamma_2}(s) = \cdots = M_{\gamma_N}(s) = 1/(1 - s\bar{\gamma})$ and $\bar{\gamma} = (q\frac{E_b}{N_0})/N$ [26]. Then, the average SER over Rayleigh fading channel can be obtained just as (32) based on (31) and the CDF in lemma 2.

$$\begin{aligned} P_s^{\text{Rayleigh}} &= 2(\bar{F}(\theta_2) - \bar{F}(\theta_1)) \\ &= \frac{1}{2\pi}\int_0^{2\pi-2\theta_1} \prod_{n=1}^{N} M_{\gamma_n} \\ &\quad \times \left(-\frac{\sin^2\theta_1}{1-\cos\theta_1\cos(\theta_1+\phi)}\right) d\phi, \end{aligned} \tag{32}$$

in which $\theta_1 = \frac{8\pi}{11M-12}$ and $\theta_2 = \pi$.

For the Gray mapping, we can directly obtain the BER just as (33) by using $P_b^{\text{Rayleigh}} = P_s^{\text{Rayleigh}}/\log_2 M$, especially at high SNR [24].

$$\begin{aligned} P_b^{\text{Rayleigh}} &= \frac{1}{2q\pi}\int_0^{2\pi-\frac{16\pi}{11M-12}} \prod_{n=1}^{N} M_{\gamma_n} \\ &\quad \times \left(-\frac{\sin^2\frac{8\pi}{11M-12}}{1-\cos\frac{8\pi}{11M-12}\cos\left(\frac{8\pi}{11M-12}+\phi\right)}\right) d\phi. \end{aligned} \tag{33}$$

$$P_b^{\text{Rayleigh}} = \frac{1}{2q\pi}\int_0^{2\pi-2\theta_1}\left(\frac{1}{1+q\frac{E_b}{N_0}\frac{\sin^2\theta_1}{(1-\cos\theta_1\cos(\theta_1+\phi))N}}\right)^N d\phi \tag{34}$$

$$\approx \frac{1}{2q\widetilde{K}}\sum_{\widetilde{k}=1}^{\widetilde{K}}\sqrt{1-\widetilde{x}_{\widetilde{k}}^2}\left(\frac{1}{1+q\frac{E_b}{N_0}\frac{\sin^2\theta_1}{(1-\cos\theta_1\cos(\theta_1+\phi_{\widetilde{k}}))N}}\right)^N \tag{35}$$

$$\begin{aligned} &= \frac{1}{2q\widetilde{K}}\sum_{\widetilde{k}=1}^{\widetilde{K}}\sqrt{1-\widetilde{x}_{\widetilde{k}}^2} \\ &\quad \times \left(\frac{1}{1+q\frac{E_b}{N_0}\frac{\sin^2\frac{8\pi}{11M-12}}{(1-\cos\frac{8\pi}{11M-12}\cos(\frac{8\pi}{11M-12}+\phi_{\widetilde{k}}))N}}\right)^N. \end{aligned} \tag{36}$$

It is often reasonable in practice to assume that the channels are independent of each other. Hence, the closed-form approximation of the BER performance of the downlink SCMA system with i.i.d. Rayleigh fading channels is given as (36), in which $\widetilde{K}$ is a parameter for Gaussian-Chebyshev quadrature, $\widetilde{x}_{\widetilde{k}}^2 = \cos\left(\frac{2\widetilde{k}-1}{2\widetilde{K}}\right)$, and $\phi_{\widetilde{k}} = (\pi - \theta_1)(1 + \widetilde{x}_{\widetilde{k}})$. Thus, the proof is completed.

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