Lecture 04

Friday, January 9, 2026 16:28

MSE2273_Lec_04_Fri...

Pressure variation in a fluid at rest:

Concept: For a fluid <u>at rest</u> the pressure at a point depends only on the weight of fluid <u>vertically</u> above that point ("Vertical" = parallel to gravitational vector) and any horizontal plane is an isobar (constant pressure plane).

Aim: To show that

(I) for liquids $P_1 = P_{surface} + \rho g h$

(2) for gases (isothermal) $P_1 = P_o \, e^{\left[-\frac{g(z_1 - z_0)}{RT_0}\right]}$

(3) for real atmosphere $P_1 = P_o \left(1 - \frac{\beta z_1}{T_0}\right)^{g/R\beta}$

Z k j Y i X

$\beta = 0.0065$ K/m (Kelvin/m)

T_0 = absolute temperature at sea level
P_0 = absolute pressure at sea level
R = 287 J/Kg-K

∇ → top side of the liquid..

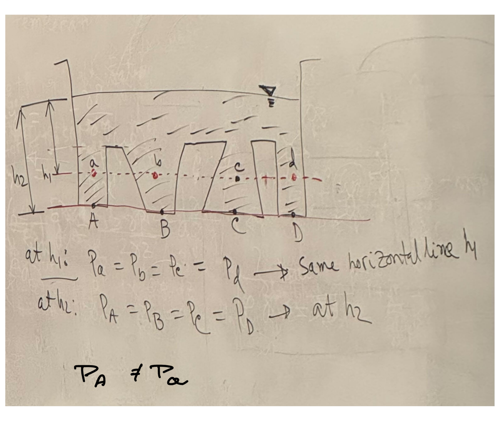
Pressure variation in a fluid at rest:

We have seen that

$$\rho a_x = \frac{\partial P}{\partial x} \quad -①$$

$$\rho a_y = \frac{\partial P}{\partial y} \quad -②$$

$$\rho a_z = -\frac{\partial P}{\partial z} - \rho g \quad -③$$



For a fluid at rest:

$a_x = a_y = a_z = 0$

① $\frac{\partial P}{\partial x} = 0$ ② $\frac{\partial P}{\partial y} = 0$

x-y plane acts as a isobar

③ $-\frac{\partial P}{\partial z} - \rho g = 0$

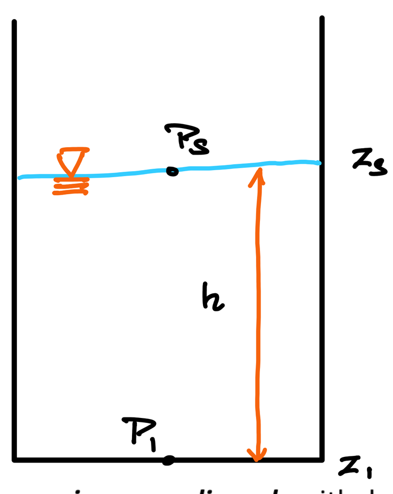
$$\boxed{\frac{dP}{dz} = -\rho g}$$

- Pressure varies only in z direction (vertically)

Z k j Y i X

(1) Liquids: Hydrostatic distribution: (assume $\rho g = \gamma$ = constant)

We have seen that



$$\frac{dP}{dz} = -\rho g \quad \| \text{ integrate}$$

$$\int_{P_1}^{P_s} dP = \int_{z_1}^{z_s} -\rho g \, dz$$

$$P_s - P_1 = -\rho g (z_2 - z_1)$$

$$P_1 = P_s + \rho g (z_2 - z_1)$$

$$\boxed{P_1 = P_s + \rho g h}$$

$P \propto h$

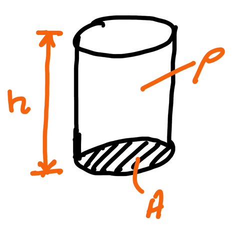
Z k j Y i X

∴ Pressure ***increases linearly*** with depth in a liquid

(1) Liquids: Hydrostatic distribution: continued

What is the term ρgh ? no idea

Consider liquid column of height h and small base area A:



weight of column $= m * g$

$= \rho V g$

$= \rho A h g$

Pressure at base $= \frac{N}{A}$

$= \frac{\rho A h g}{\cancel{A}}$

$= \rho h g$ //

∴ We can express any pressure P (Pa, N/m²) in terms of a height h (m) of the fluid of density ρ (kg/m³) i.e.,

(2) Compressible medium (isothermal gas):

$T = T_0$ = constant

- Important when height variation are large (>100 m)
- A very good approximation for the real atmosphere for heights <500 m

Assume isothermal gas ∴ $T = T_0$

$$\frac{dP}{dz} = -\rho g \qquad \rho = \frac{P}{RT_0}$$

$$\frac{dP}{dz} = -\frac{Pg}{RT_0}$$

$$\int_{P_0}^{P_1} \frac{dP}{P} = -\frac{g}{RT_0} \int_{z_0}^{z_1} dz$$

$$\ln |P| \Big|_{P_0}^{P_1} = -\frac{g}{RT_0} z \Big|_{z_0}^{z_1}$$

$$\ln\left(\frac{P_1}{P_0}\right) = -\frac{g}{RT_0} h$$

$$\boxed{\frac{P_1}{P_0} = e^{-\left(\frac{gh}{RT_0}\right)}}$$

(3) Real (Standard) atmosphere:

- Idealized or "average" representation of earth's atmosphere
- Good for design purposes
- Has linear decrease of T with Z in lower 11 km (troposphere)

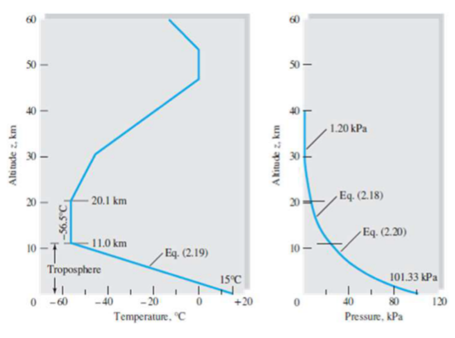


Figure 2.7 (FM White): Temperature and pressure distribution in Standard Atmosphere.

∴ $T = T_0 - \beta Z$

$$\frac{dP}{dz} = -\rho g \qquad \rho = -\frac{P}{RT}$$

$$\frac{dP}{dz} = -\frac{Pg}{RT}$$

$$\frac{dP}{dz} = \frac{-Pg}{R(T_0 - \beta z)}$$

$$\int_{P_0}^{P_1} \frac{dP}{P} = -\frac{g}{R} \int_{z_0}^{z_1} \frac{1}{T_0 - \beta z}$$

$$\ln\left(\frac{P_1}{P_0}\right) = \frac{g}{R\beta}\left[\ln(T_0 - \beta z_1) - \ln(T_0 - \beta z_0)\right]$$

$$= \frac{g}{R\beta}\left[\ln\left(\frac{T_0 - \beta z_1}{T_0 - \beta z_0}\right)\right]$$

$$\boxed{\frac{P_1}{P_0} = \left(\frac{T_0 - \beta z_1}{T_0 - \beta z_0}\right)^{g/R\beta}}$$