

Linear Regression Exercises

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Predicting Life Expectancy in the United States

A) We want to explore the data of different factors within the United States.

First, we want to import StateData.csv.

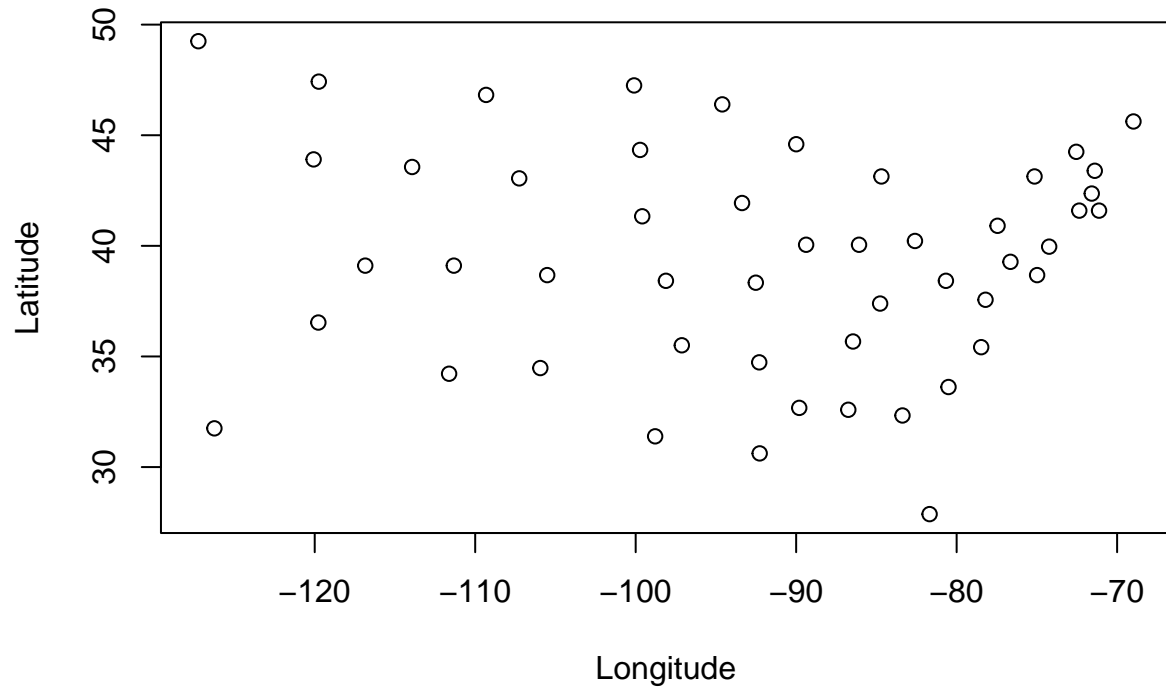
```
StateData <- read_csv("StateData.csv")
```

```
## Rows: 50 Columns: 11
## -- Column specification -----
## Delimiter: ","
## chr (1): Region
## dbl (10): Population, Income, Illiteracy, LifeExp, Murder, HighSchoolGrad, F...
##
## i Use `spec()` to retrieve the full column specification for this data.
## i Specify the column types or set `show_col_types = FALSE` to quiet this message.
summary(StateData)
```

```
##      Population      Income      Illiteracy      LifeExp
## Min.   : 365      Min.   :3098      Min.   :0.500      Min.   :67.96
## 1st Qu.: 1080      1st Qu.:3993      1st Qu.:0.625      1st Qu.:70.12
## Median : 2838      Median :4519      Median :0.950      Median :70.67
## Mean   : 4246      Mean   :4436      Mean   :1.170      Mean   :70.88
## 3rd Qu.: 4968      3rd Qu.:4814      3rd Qu.:1.575      3rd Qu.:71.89
## Max.   :21198      Max.   :6315      Max.   :2.800      Max.   :73.60
##      Murder      HighSchoolGrad      Frost      Area
## Min.   : 1.400      Min.   :37.80      Min.   : 0.00      Min.   : 1049
## 1st Qu.: 4.350      1st Qu.:48.05      1st Qu.: 66.25      1st Qu.: 36985
## Median : 6.850      Median :53.25      Median :114.50      Median : 54277
## Mean   : 7.378      Mean   :53.11      Mean   :104.46      Mean   : 70736
## 3rd Qu.:10.675      3rd Qu.:59.15      3rd Qu.:139.75      3rd Qu.: 81162
## Max.   :15.100      Max.   :67.30      Max.   :188.00      Max.   :566432
##      Longitude      Latitude      Region
## Min.   : -127.25      Min.   :27.87      Length:50
## 1st Qu.: -104.16      1st Qu.:35.55      Class :character
## Median : -89.90      Median :39.62      Mode  :character
## Mean   : -92.46      Mean   :39.41
## 3rd Qu.: -78.98      3rd Qu.:43.14
## Max.   : -68.98      Max.   :49.25
```

```
plot(StateData$Longitude, StateData$Latitude, main="United States", xlab = "Longitude", ylab = "Latitude")
```

i) First, let's create a scatterplot of all of the states by putting Longitude on the x-axis and Latitude on the y-axis.



This scatterplot was generated via a built-in R function, and used factors in the StateData dataset.

```
regiongrad <- split(StateData$HighSchoolGrad, StateData$Region)
sapply(regiongrad, mean)
```

ii) We want to see which region of the United States (West, North Central, South, Northeast) has the highest average high graduation rate.

	North Central	Northeast	South	West
##	54.51667	53.96667	44.34375	62.00000

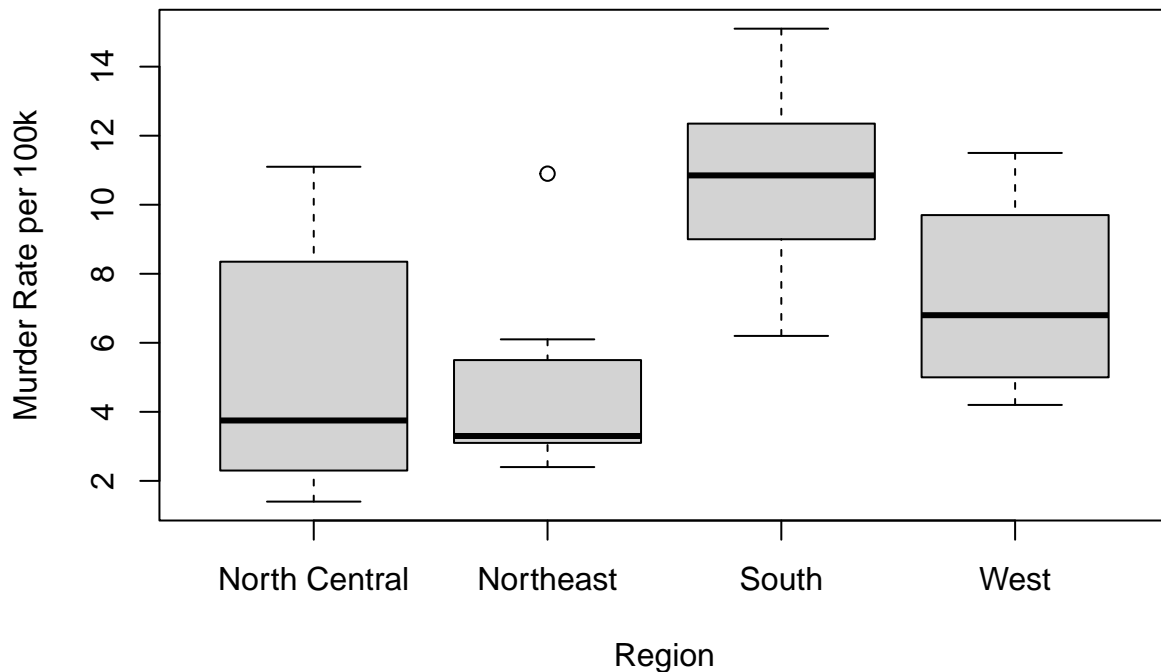
With this in mind, the highest average graduation rate in 1970 is 62% in the West.

iii) Create a box plot of the variable Murder for each Region (four box plots total)

1. Describe the statistical distribution of the murder rate for each region.
2. Which region has the highest median murder rate?

3. The largest range of values?

```
regionmurder <- split(StateData$Murder, StateData$Region)
boxplot(regionmurder, xlab = "Region", ylab = "Murder Rate per 100k")
```



1. Statistical Distribution:

- North Central has a wide range in Murder Rate, with the median low at 4. IQR is wide and corresponds to range.
- Northeast has a small range and a low Murder Rate, with the median below North Central's 4. Interestingly, there is an outlier at 11. IQR is narrow.
- South has a wide range and higher median Murder Rate than the other regions, at ~11. IQR is relatively narrow, but it has a wide range between Max and Min.
- West has a medium range with a median murder rate of ~7 per 100k. IQR is somewhat narrow, and Max and Min are close to their quartiles.

2. The south has the highest median at ~11 murders per 100k.

3. North Central has the largest range with the min at 1 and the max at 11 (10 units).

B) Build a linear regression model to predict life expectancy (LifeExp) using the following variables as independent variables: Population, Income, Illiteracy, Murder, HighSchoolGrad, Frost, and Area.

```
LifeExpPredict = lm(LifeExp ~ Population + Income + Illiteracy + Murder + HighSchoolGrad + Frost + Area)
summary(LifeExpPredict)
```

```
##
## Call:
## lm(formula = LifeExp ~ Population + Income + Illiteracy + Murder +
##     HighSchoolGrad + Frost + Area, data = StateData)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -1.48895 -0.51232 -0.02747  0.57002  1.49447
##
```

```
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept)   7.094e+01  1.748e+00  40.586 < 2e-16 ***
## Population    5.180e-05  2.919e-05   1.775  0.0832 .
## Income        -2.180e-05  2.444e-04  -0.089  0.9293
## Illiteracy     3.382e-02  3.663e-01   0.092  0.9269
## Murder        -3.011e-01  4.662e-02  -6.459 8.68e-08 ***
## HighSchoolGrad 4.893e-02  2.332e-02   2.098  0.0420 *
## Frost         -5.735e-03  3.143e-03  -1.825  0.0752 .
## Area          -7.383e-08  1.668e-06  -0.044  0.9649
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.7448 on 42 degrees of freedom
## Multiple R-squared:  0.7362, Adjusted R-squared:  0.6922
## F-statistic: 16.74 on 7 and 42 DF,  p-value: 2.534e-10
```

```
summary(LifeExpPredict)$coefficient
```

```
##              Estimate Std. Error      t value      Pr(>|t|)
## (Intercept)   7.094322e+01 1.747975e+00 40.58594017 2.510609e-35
## Population    5.180036e-05 2.918703e-05  1.77477309 8.318351e-02
## Income        -2.180424e-05 2.444256e-04 -0.08920603 9.293422e-01
## Illiteracy     3.382032e-02 3.662799e-01  0.09233464 9.268712e-01
## Murder        -3.011232e-01 4.662073e-02 -6.45899735 8.679582e-08
## HighSchoolGrad 4.892948e-02 2.332328e-02  2.09788176 4.197175e-02
## Frost         -5.735001e-03 3.143230e-03 -1.82455682 7.518682e-02
## Area          -7.383166e-08 1.668163e-06 -0.04425927 9.649075e-01
```

i) What is the regression equation produced by your model? Include all of the coefficients and independent variables they correspond to.

- $y = 70.94 + .00005X_1 + -.000022X_2 + .0338X_3 + -.3011X_4 + .0489X_5 + -.0057X_6 + -.0000007X_7$

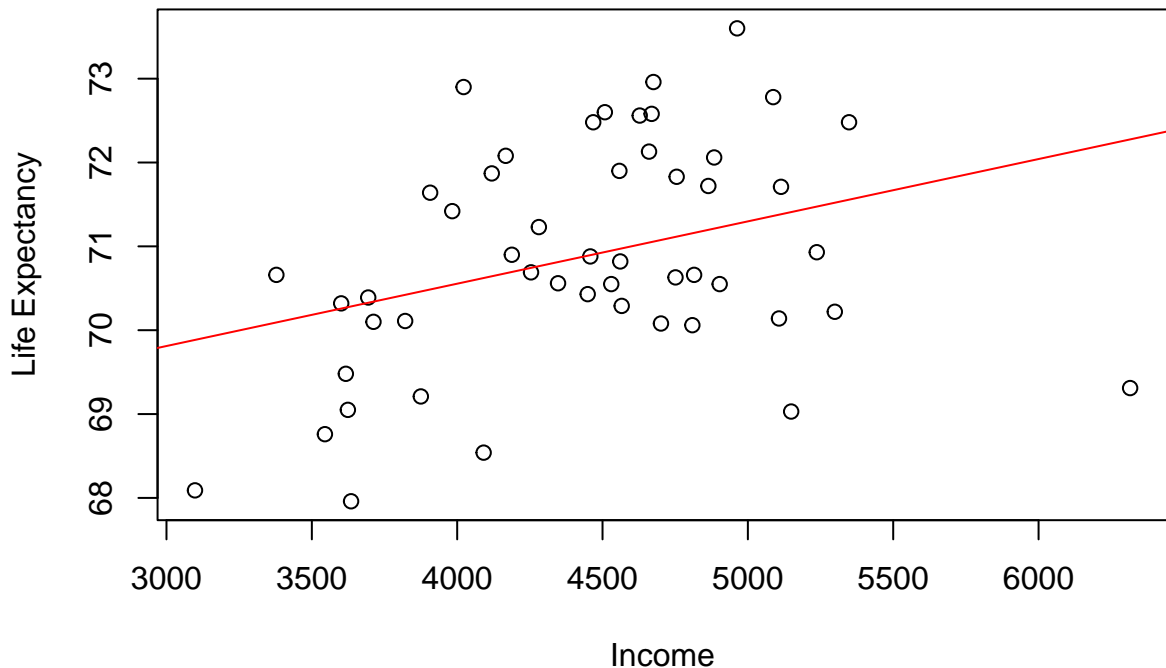
ii) What is the interpretation of the coefficient for Income?

- For an increase in income, life expectancy decreases slightly. Income is not a statistically significant determinant for life expectancy.

```
plot(StateData$Income, StateData$LifeExp, main = "Life Expectancy vs Income", xlab = "Income", ylab = "Life Expectancy")
abline(lm(StateData$LifeExp ~ StateData$Income), col="red")
```

iii) Create a scatterplot with Income on the x-axis, and LifeExp on the y-axis. Does this relationship agree with the coefficient for Income in your linear regression model? Why or why

Life Expectancy vs Income



not?

- It generally agrees with the coefficient, with the relationship between the two variables being weak.

C) Rebuild the linear regression model, using the set of independent variables you think is the best for predicting LifeExp. This means any subset of the 7 independent variables previously used. Use the significance of the coefficients, the R^2 of the model, and the interpretability of the model when selecting the final set of variables.

```
revismodel = lm(StateData$LifeExp ~ StateData$Murder + StateData$HighSchoolGrad + StateData$Population + StateData$Frost)
summary(revismodel)
```

```
##
## Call:
## lm(formula = StateData$LifeExp ~ StateData$Murder + StateData$HighSchoolGrad +
##     StateData$Population + StateData$Frost)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -1.47095 -0.53464 -0.03701  0.57621  1.50683
##
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept)    7.103e+01  9.529e-01  74.542 < 2e-16 ***
## StateData$Murder -3.001e-01  3.661e-02  -8.199 1.77e-10 ***
## StateData$HighSchoolGrad  4.658e-02  1.483e-02   3.142  0.00297 **
## StateData$Population    5.014e-05  2.512e-05   1.996  0.05201 .
## StateData$Frost    -5.943e-03  2.421e-03  -2.455  0.01802 *
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.7197 on 45 degrees of freedom
```

```
## Multiple R-squared:  0.736, Adjusted R-squared:  0.7126
## F-statistic: 31.37 on 4 and 45 DF,  p-value: 1.696e-12
```

```
summary(revismodel)$coefficients
```

```
##              Estimate Std. Error  t value    Pr(>|t|)
## (Intercept)    7.102713e+01 9.528530e-01  74.541541 8.612596e-49
## StateData$Murder    -3.001488e-01 3.660946e-02 -8.198669 1.774520e-10
## StateData$HighSchoolGrad  4.658225e-02 1.482706e-02  3.141704 2.968091e-03
## StateData$Population    5.013998e-05 2.512002e-05  1.996017 5.200514e-02
## StateData$Frost      -5.943290e-03 2.420875e-03 -2.455017 1.801778e-02
```

i) What is your new linear regression equation?

- $y = 71.02 + -.3001X_1 + .0466X_2 + .0005X_3 + -.0059X_4$

ii) Compare and contrast this model to the original model, paying special attention to the R^2 of the model and significance of the coefficients.

- The multiple R^2 of the model is slightly worse than the original. However, all of the coefficients here are statistically significant.

```
predict_vector <- predict(revismodel)

vector_frame <- data.frame(predict_vector)

coordinates <- data.frame(StateData$Latitude, StateData$Longitude)

est_lifexp <- cbind(coordinates, vector_frame)

print(est_lifexp %>% arrange(desc(vector_frame)))
```

iii) Using your simplified model, create a vector of predictions for the dataset StateData.

```
##      StateData.Latitude StateData.Longitude predict_vector
## 47          47.4231         -119.7460          72.68272
## 21          42.3645         -71.5800          72.44105
## 37          43.9078        -120.0680          72.41445
## 15          41.9358         -93.3714          72.39653
## 23          46.3943         -94.6043          72.26560
## 27          41.3356         -99.5898          72.17032
## 11          31.7500        -126.2500          72.09317
## 44          39.1063        -111.3300          72.05753
## 7           41.5928         -72.3573          72.03459
## 41          44.3365         -99.7238          72.01161
## 49          44.5937         -89.9941          72.00996
## 16          38.4204         -98.1156          71.90352
## 34          47.2517        -100.0990          71.87649
## 19          45.6226         -68.9801          71.86095
## 5           36.5341        -119.7730          71.79565
## 39          41.5928         -71.1244          71.76007
## 29          43.3934         -71.3924          71.72636
## 30          39.9637         -74.2336          71.59612
## 12          43.5648        -113.9300          71.49989
## 3           34.2192        -111.6250          71.41416
```

## 26	46.8230	-109.3200	71.40025
## 38	40.9069	-77.4500	71.38046
## 36	35.5053	-97.1239	71.15860
## 8	38.6777	-74.9841	71.12647
## 6	38.6777	-105.5130	71.10354
## 35	40.2210	-82.5963	71.08549
## 45	44.2508	-72.5450	71.06135
## 14	40.0495	-86.0808	70.90159
## 50	43.0504	-107.2560	70.87679
## 32	43.1361	-75.1449	70.62937
## 9	27.8744	-81.6850	70.61539
## 20	39.2778	-76.6459	70.51852
## 48	38.4204	-80.6665	70.44983
## 13	40.0495	-89.3776	70.19244
## 46	37.5630	-78.2005	70.14691
## 25	38.3347	-92.5137	70.10610
## 31	34.4764	-105.9420	70.03119
## 43	31.3897	-98.7857	69.97886
## 22	43.1361	-84.6870	69.86893
## 2	49.2500	-127.2500	69.85740
## 4	34.7336	-92.2992	69.57374
## 28	39.1063	-116.8510	69.52482
## 42	35.6767	-86.4560	69.46583
## 33	35.4195	-78.4686	69.28624
## 17	37.3915	-84.7674	69.24418
## 18	30.6181	-92.2724	69.15045
## 40	33.6190	-80.5056	69.06109
## 24	32.6758	-89.8065	69.00535
## 10	32.3329	-83.3736	68.63694
## 1	32.5901	-86.7509	68.48112

- Which state does your model predict to have the lowest life expectancy? **Alabama**
- Which state actually has the lowest life expectancy? **Mississippi**
- Which state does your model predict to have the highest life expectancy? **Washington**
- Which state actually has the highest life expectancy? **Hawaii**

Climate Change

Studying the relationship between average global temperature and several other factors.

A) Start by splitting the dataset into a training set (observations ≤ 2006) and a testing set (observations > 2006). This will build the model and evaluate the predictive ability of the model. Build a linear regression model to predict Temp using all of the other variables as independent variables, using the training set.

```
climate <- read_csv("ClimateChange.csv")
```

```
## Rows: 308 Columns: 11
## -- Column specification -----
## Delimiter: ","
## db1 (11): Year, Month, MEI, CO2, CH4, N2O, CFC.11, CFC.12, TSI, Aerosols, Temp
##
## i Use `spec()` to retrieve the full column specification for this data.
```

```
## i Specify the column types or set `show_col_types = FALSE` to quiet this message.
train_data <- subset(climate, Year <= 2006)
test_data <- subset(climate, Year > 2006)

climatemodel = lm(Temp ~ CFC.11 + CFC.12 + CO2 + N2O + CH4 + Aerosols + TSI + MEI, data = train_data)

summary(climatemodel)

##
## Call:
## lm(formula = Temp ~ CFC.11 + CFC.12 + CO2 + N2O + CH4 + Aerosols +
##     TSI + MEI, data = train_data)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -0.25888 -0.05913 -0.00082  0.05649  0.32433
##
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept) -1.246e+02  1.989e+01  -6.265 1.43e-09 ***
## CFC.11       -6.631e-03  1.626e-03  -4.078 5.96e-05 ***
## CFC.12        3.808e-03  1.014e-03   3.757 0.00021 ***
## CO2          6.457e-03  2.285e-03   2.826 0.00505 **
## N2O         -1.653e-02  8.565e-03  -1.930 0.05467 .
## CH4          1.240e-04  5.158e-04   0.240 0.81015
## Aerosols    -1.538e+00  2.133e-01  -7.210 5.41e-12 ***
## TSI          9.314e-02  1.475e-02   6.313 1.10e-09 ***
## MEI          6.421e-02  6.470e-03   9.923 < 2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.09171 on 275 degrees of freedom
## Multiple R-squared:  0.7509, Adjusted R-squared:  0.7436
## F-statistic: 103.6 on 8 and 275 DF,  p-value: < 2.2e-16

summary(climatemodel)$coefficients

##              Estimate Std. Error t value Pr(>|t|)
## (Intercept) -1.245943e+02 1.988680e+01 -6.2651739 1.431046e-09
## CFC.11       -6.630489e-03 1.625983e-03 -4.0778339 5.957288e-05
## CFC.12        3.808103e-03 1.013523e-03  3.7572927 2.097199e-04
## CO2          6.457359e-03 2.284643e-03  2.8264197 5.052521e-03
## N2O         -1.652800e-02 8.564948e-03 -1.9297260 5.466931e-02
## CH4          1.240419e-04 5.158324e-04  0.2404694 8.101456e-01
## Aerosols    -1.537613e+00 2.132523e-01 -7.2103008 5.411273e-12
## TSI          9.314108e-02 1.475488e-02  6.3125609 1.095945e-09
## MEI          6.420531e-02 6.470206e-03  9.9232260 4.898887e-20
```

i) What is the linear regression equation produced by your model?

- $y = -124.6 + 0.006X_1 + .0038X_2 + .0064X_3 + -.0165X_4 + .00012X_5 + -1.537X_6 + .0931X_7 + .0642X_8$

ii) Evaluate the quality of the model. What is the R^2 value? Which independent variables are significant?

- The model does a good job, with most independent variables related significantly to Temp. The multiple R-squared value is 0.7509. The significant independent variables are: CFC.11, CFC.12, CO2, Aerosols, TSI, and MEI.

iii) What is the simplest explanation for this contradiction (N20 and CFC-11 associated with high temperatures, but not clear in model)

- The model as a whole reflects recent industrialization, and while there is a negative correlation for the two variables, it does not reflect real world values.

```
cor(train_data)
```

iv) Compute the correlations between all independent variables in the training set. Which independent variables is N20 highly correlated with (>0.7)? Which independent variables is CFC.11 highly correlated with (>0.7)?

```
##          Year      Month      MEI      CO2      CH4
## Year      1.00000000 -0.0279419602 -0.0369876842  0.98274939  0.91565945
## Month     -0.02794196  1.0000000000  0.0008846905 -0.10673246  0.01856866
## MEI       -0.03698768  0.0008846905  1.0000000000 -0.04114717 -0.03341930
## CO2       0.98274939 -0.1067324607 -0.0411471651  1.00000000  0.87727963
## CH4       0.91565945  0.0185686624 -0.0334193014  0.87727963  1.00000000
## N20       0.99384523  0.0136315303 -0.0508197755  0.97671982  0.89983864
## CFC.11    0.56910643 -0.0131112236  0.0690004387  0.51405975  0.77990402
## CFC.12    0.89701166  0.0006751102  0.0082855443  0.85268963  0.96361625
## TSI       0.17030201 -0.0346061935 -0.1544919227  0.17742893  0.24552844
## Aerosols -0.34524670  0.0148895406  0.3402377871 -0.35615480 -0.26780919
## Temp      0.78679714 -0.0998567411  0.1724707512  0.78852921  0.70325502
##          N20      CFC.11      CFC.12      TSI      Aerosols
## Year      0.99384523  0.56910643  0.8970116635  0.17030201 -0.34524670
## Month     0.01363153 -0.01311122  0.0006751102 -0.03460619  0.01488954
## MEI      -0.05081978  0.06900044  0.0082855443 -0.15449192  0.34023779
## CO2       0.97671982  0.51405975  0.8526896272  0.17742893 -0.35615480
## CH4       0.89983864  0.77990402  0.9636162478  0.24552844 -0.26780919
## N20       1.00000000  0.52247732  0.8679307757  0.19975668 -0.33705457
## CFC.11    0.52247732  1.00000000  0.8689851828  0.27204596 -0.04392120
## CFC.12    0.86793078  0.86898518  1.0000000000  0.25530281 -0.22513124
## TSI       0.19975668  0.27204596  0.2553028138  1.00000000  0.05211651
## Aerosols -0.33705457 -0.04392120 -0.2251312440  0.05211651  1.00000000
## Temp      0.77863893  0.40771029  0.6875575483  0.24338269 -0.38491375
##          Temp
## Year      0.78679714
## Month     -0.09985674
## MEI       0.17247075
## CO2       0.78852921
## CH4       0.70325502
## N20       0.77863893
## CFC.11    0.40771029
## CFC.12    0.68755755
## TSI       0.24338269
## Aerosols -0.38491375
## Temp      1.00000000
```

- N20 correlations: Year, CO2, CH4, CFC.12
- CFC.11 correlations: CH4, CFC.12

B) Build a new linear regression model, this time only using MEI, TSI, Aerosols, and N20 as the independent variables. Use the training data set.

```
revised_climate = lm(Temp ~ N20 + MEI + TSI + Aerosols, data = train_data)
summary(revised_climate)
```

```
##
## Call:
## lm(formula = Temp ~ N20 + MEI + TSI + Aerosols, data = train_data)
##
## Residuals:
```

	Min	1Q	Median	3Q	Max
	-0.27916	-0.05975	-0.00595	0.05672	0.34195

```
##
## Coefficients:
```

	Estimate	Std. Error	t value	Pr(> t)
(Intercept)	-1.162e+02	2.022e+01	-5.747	2.37e-08 ***
N20	2.532e-02	1.311e-03	19.307	< 2e-16 ***
MEI	6.419e-02	6.652e-03	9.649	< 2e-16 ***
TSI	7.949e-02	1.487e-02	5.344	1.89e-07 ***
Aerosols	-1.702e+00	2.180e-01	-7.806	1.19e-13 ***

```
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.09547 on 279 degrees of freedom
## Multiple R-squared:  0.7261, Adjusted R-squared:  0.7222
## F-statistic: 184.9 on 4 and 279 DF,  p-value: < 2.2e-16
```

```
summary(revised_climate)$coefficients
```

```
##
##          Estimate   Std. Error   t value    Pr(>|t|)
## (Intercept) -116.22685815 20.223028005 -5.747253 2.373584e-08
## N20          0.02531975  0.001311434 19.306911 2.487588e-53
## MEI          0.06418576  0.006651795  9.649389 3.373572e-19
## TSI          0.07949028  0.014875381  5.343747 1.893732e-07
## Aerosols     -1.70173707  0.217995842 -7.806282 1.193197e-13
```

i) How does the coefficient for N20 in this model compare to the coefficient in the previous model?

- The N20 coefficient in this model is positively correlated with Temp, as opposed to negatively in the previous model.

ii) How does the coefficient of this model compare to the previous one? Consider the R^2 value and the significance of the independent variables when answering this question.

- The coefficient of the model is similar, but the original model has a slightly higher R^2 value. The independent variables are all highly related to each other.

C) Using the simplified model you created in part (B), calculate predictions for the testing dataset. What is the R^2 on the test set? What does this tell you about the model?

```
test_climate = lm(Temp ~ N20 + MEI + TSI + Aerosols, data = test_data)
summary(test_climate)
```

```
##
## Call:
## lm(formula = Temp ~ N2O + MEI + TSI + Aerosols, data = test_data)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -0.21741 -0.02439  0.01930  0.03430  0.17768
##
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept) 1334.70893   951.60350    1.403  0.1769
## N2O          -0.05695    0.04289   -1.328  0.2000
## MEI           0.06019    0.03111    1.934  0.0681 .
## TSI          -0.96384    0.69064   -1.396  0.1789
## Aerosols     71.32377   30.89366    2.309  0.0324 *
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.0821 on 19 degrees of freedom
## Multiple R-squared:  0.5212, Adjusted R-squared:  0.4204
## F-statistic: 5.171 on 4 and 19 DF,  p-value: 0.005444
```

```
predict(test_climate)
```

```
##           1           2           3           4           5           6           7           8
## 0.5677143 0.5205199 0.4143372 0.4391000 0.4572208 0.3580366 0.3717633 0.3571366
##           9          10          11          12          13          14          15          16
## 0.3516708 0.3101729 0.3716495 0.3252922 0.2914086 0.2330769 0.2693219 0.2600574
##          17          18          19          20          21          22          23          24
## 0.2745682 0.3449913 0.3569679 0.3747531 0.3875059 0.3746566 0.3566392 0.3434387
```

- The Multiple R^2 is 0.5212.
- The model has a low R^2 , suggesting that the independent variables do not significantly explain Temperature variance.

Hyundai Elantra

Forecasting Hyundai Elantra sales.

A) Split the dataset into training (2010, 2011, 2012) and testing (2013, 2014). Build a linear regression model to predict monthly Elantra sales (ElantraSales) using Unemployment, Queries, CPI.Energy, and CPI.All. Use the training set to build the model.

```
elantra <- read_csv("Elantra.csv")
```

```
## Rows: 50 Columns: 7
## -- Column specification -----
## Delimiter: ","
## dbl (7): Month, Year, ElantraSales, Unemployment, Queries, CPI.Energy, CPI.All
##
## i Use `spec()` to retrieve the full column specification for this data.
## i Specify the column types or set `show_col_types = FALSE` to quiet this message.

etrain_data <- subset(elantra, Year <= 2012)
etest_data <- subset(elantra, Year > 2012)
```

```
elantramodel = lm(ElantraSales ~ Unemployment + Queries + CPI.Energy + CPI.All, data = etrain_data)

summary(elantramodel)
```

```
##
## Call:
## lm(formula = ElantraSales ~ Unemployment + Queries + CPI.Energy +
##      CPI.All, data = etrain_data)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -6785.2 -2101.8  -562.5   2901.7   7021.0
##
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept)  95385.36  170663.81   0.559   0.580
## Unemployment -3179.90   3610.26  -0.881   0.385
## Queries       19.03     11.26   1.690   0.101
## CPI.Energy    38.51     109.60   0.351   0.728
## CPI.All      -297.65     704.84  -0.422   0.676
##
## Residual standard error: 3295 on 31 degrees of freedom
## Multiple R-squared:  0.4282, Adjusted R-squared:  0.3544
## F-statistic: 5.803 on 4 and 31 DF,  p-value: 0.00132
```

```
summary(elantramodel)$coefficients
```

```
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept)  95385.36360 170663.81417  0.5589080 0.5802400
## Unemployment -3179.89957   3610.26225 -0.8807946 0.3852069
## Queries       19.02968     11.25896  1.6901807 0.1010267
## CPI.Energy    38.50604     109.60117  0.3513287 0.7277185
## CPI.All      -297.64563     704.83667 -0.4222902 0.6757278
```

i) What is the linear regression equation produced by your model? Make sure to give the coefficients for each of the independent variables.

- $y = 95385.4 + -3179.9X_1 + 19.02X_2 + 38.51X_3 + -297.6X_4$

ii) What is the R^2 of the model?

- The multiple R^2 is .4282.

iii) Which variables are significant? What does this tell you about the model?

- None of the variables are statistically significant. This model shows that those independent variables do not significantly explain variance in elantra sales.

B) We want to incorporate seasonality into our model by using the Month variable. Build a new linear regression model, this time using the Month variable as an additional independent variable, using the training data.

```
monthmodel = lm(ElantraSales ~ Month + Unemployment + Queries + CPI.Energy + CPI.All, data = etrain_data)

summary(monthmodel)
```

```
##
## Call:
## lm(formula = ElantraSales ~ Month + Unemployment + Queries +
##     CPI.Energy + CPI.All, data = etrain_data)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -6416.6 -2068.7  -597.1  2616.3  7183.2
##
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept)  148330.49  195373.51   0.759   0.4536
## Month         110.69    191.66   0.578   0.5679
## Unemployment -4137.28   4008.56  -1.032   0.3103
## Queries        21.19     11.98   1.769   0.0871 .
## CPI.Energy     54.18     114.08   0.475   0.6382
## CPI.All       -517.99    808.26  -0.641   0.5265
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 3331 on 30 degrees of freedom
## Multiple R-squared:  0.4344, Adjusted R-squared:  0.3402
## F-statistic: 4.609 on 5 and 30 DF,  p-value: 0.003078
```

```
summary(monthmodel)$coefficients
```

```
##              Estimate Std. Error  t value  Pr(>|t|)
## (Intercept) 148330.48770 195373.50659  0.7592150 0.45364852
## Month        110.68527    191.65738  0.5775163 0.56790018
## Unemployment -4137.28256   4008.55786 -1.0321125 0.31026872
## Queries       21.18552     11.97849  1.7686295 0.08712393
## CPI.Energy    54.18332     114.07565  0.4749770 0.63824315
## CPI.All      -517.99104     808.25901 -0.6408726 0.52647121
```

i) Describe your new model. What is the regression equation? What is the R^2 ? Which variables are significant?

- $y = 148330.5 + 110.69X_1 + -4137.3X_2 + 21.19X_3 + 54.18X_4 + -518X_5$
- The multiple R^2 is .4344.
- The queries variable is statistically significant.

ii) We are currently modeling Month as a numeric variable. This causes our model to see Feburary as “larger” than January and so on. Is this the right way to model this variable? What if we made Month a categorical variable instead?

- This is the wrong way to model the variable, as “time” is not increasing over itself.
- Making month a categorical variable would be the correct way to model sales over time.

C) Create a new linear regression model, this time with Month model as a categorical variable. You can manually change the values, or in R, convert Month to a factor variable.

```
etrain_data$factormonth <- as.factor(etrain_data$Month)

emonthmodel = lm(ElantraSales ~ factormonth + Unemployment + Queries + CPI.Energy + CPI.All, data = e
```

```
summary(emonthmodel)
```

```
##
## Call:
## lm(formula = ElantraSales ~ factormonth + Unemployment + Queries +
##     CPI.Energy + CPI.All, data = etrain_data)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -3865.1 -1211.7   -77.1  1207.5  3562.2
##
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept)  312509.280 144061.867   2.169 0.042288 *
## factormonth2    2254.998   1943.249   1.160 0.259540
## factormonth3    6696.557   1991.635   3.362 0.003099 **
## factormonth4    7556.607   2038.022   3.708 0.001392 **
## factormonth5    7420.249   1950.139   3.805 0.001110 **
## factormonth6    9215.833   1995.230   4.619 0.000166 ***
## factormonth7    9929.464   2238.800   4.435 0.000254 ***
## factormonth8    7939.447   2064.629   3.845 0.001010 **
## factormonth9    5013.287   2010.745   2.493 0.021542 *
## factormonth10   2500.184   2084.057   1.200 0.244286
## factormonth11   3238.932   2397.231   1.351 0.191747
## factormonth12   5293.911   2228.310   2.376 0.027621 *
## Unemployment   -7739.381   2968.747  -2.607 0.016871 *
## Queries         -4.764     12.938  -0.368 0.716598
## CPI.Energy       288.631     97.974   2.946 0.007988 **
## CPI.All        -1343.307    592.919  -2.266 0.034732 *
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 2306 on 20 degrees of freedom
## Multiple R-squared:  0.8193, Adjusted R-squared:  0.6837
## F-statistic: 6.044 on 15 and 20 DF, p-value: 0.0001469
```

```
summary(emonthmodel)$coefficients
```

```
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept)  312509.280182 144061.86707  2.1692713 0.0422884369
## factormonth2    2254.997812   1943.24856  1.1604269 0.2595399946
## factormonth3    6696.556764   1991.63473  3.3623418 0.0030989082
## factormonth4    7556.607380   2038.02192  3.7078146 0.0013916585
## factormonth5    7420.248994   1950.13889  3.8049849 0.0011095428
## factormonth6    9215.832605   1995.22974  4.6189331 0.0001658816
## factormonth7    9929.464426   2238.80038  4.4351718 0.0002544591
## factormonth8    7939.447434   2064.62932  3.8454590 0.0010095185
## factormonth9    5013.286649   2010.74490  2.4932485 0.0215417274
## factormonth10   2500.183753   2084.05722  1.1996714 0.2442864246
## factormonth11   3238.931505   2397.23116  1.3511136 0.1917468055
## factormonth12   5293.910735   2228.30966  2.3757518 0.0276210171
## Unemployment   -7739.381433   2968.74725 -2.6069520 0.0168712350
## Queries         -4.763646     12.93793 -0.3681922 0.7165981623
## CPI.Energy       288.631413     97.97365  2.9460108 0.0079881486
```

```
## CPI.All          -1343.306829    592.91880 -2.2655831 0.0347321946
```

i) Describe your new model. What is the regression equation? What is the R^2 ? Which variables are significant?

- $y = 312509 + 2255X_1 + 6697X_2 + 7557X_3 + 7420X_4 + 9216X_5 + 9930X_6 + 7940X_7 + 5013X_8 + 2500X_9 + 3239X_{10} + 5294X_{11} + -7739X_{12} + -4.764X_{13} + 228.6X_{14} + -1343X_{15}$
- The multiple R^2 is 0.8193.
- The significant variables are factormonths3-9 (Spring and Summer), Unemployment, and both CPI stats.

```
etest2013 <- subset(etest_data, Year < 2014)

testsales = lm(ElantraSales ~ Unemployment + Queries + CPI.Energy + CPI.All + as.factor(Month), data = etest_data)

testsales2013 = lm(ElantraSales ~ Unemployment + Queries + CPI.Energy + CPI.All + as.factor(Month), data = etest2013)

summary(testsales2013)
```

ii) Using this model, make predictions on the test set. Remember to convert the Month variable to a categorical variable in the test set before making predictions. What is the R^2 of the model on the test set?

```
##
## Call:
## lm(formula = ElantraSales ~ Unemployment + Queries + CPI.Energy +
##     CPI.All + as.factor(Month), data = etest2013)
##
## Residuals:
## ALL 12 residuals are 0: no residual degrees of freedom!
##
## Coefficients: (4 not defined because of singularities)
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept)    305214.197         NaN      NaN    NaN
## Unemployment   -8203.217         NaN      NaN    NaN
## Queries         63.439          NaN      NaN    NaN
## CPI.Energy       7.237           NaN      NaN    NaN
## CPI.All        -1057.323         NaN      NaN    NaN
## as.factor(Month)2    3111.729         NaN      NaN    NaN
## as.factor(Month)3    6214.775         NaN      NaN    NaN
## as.factor(Month)4    8282.719         NaN      NaN    NaN
## as.factor(Month)5    9099.584         NaN      NaN    NaN
## as.factor(Month)6    2604.812         NaN      NaN    NaN
## as.factor(Month)7    6088.100         NaN      NaN    NaN
## as.factor(Month)8    6398.771         NaN      NaN    NaN
## as.factor(Month)9         NA          NA      NA     NA
## as.factor(Month)10         NA          NA      NA     NA
## as.factor(Month)11         NA          NA      NA     NA
## as.factor(Month)12         NA          NA      NA     NA
##
## Residual standard error: NaN on 0 degrees of freedom
## Multiple R-squared:      1, Adjusted R-squared:      NaN
## F-statistic:      NaN on 11 and 0 DF, p-value: NA
```

```
summary(testsales)
```

```
##
## Call:
## lm(formula = ElantraSales ~ Unemployment + Queries + CPI.Energy +
##     CPI.All + as.factor(Month), data = etest_data)
##
## Residuals:
## ALL 14 residuals are 0: no residual degrees of freedom!
##
## Coefficients: (2 not defined because of singularities)
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept)    -2.860e+06      NaN      NaN      NaN
## Unemployment     2.910e+04      NaN      NaN      NaN
## Queries          1.544e+02      NaN      NaN      NaN
## CPI.Energy       -1.834e+01      NaN      NaN      NaN
## CPI.All          1.129e+04      NaN      NaN      NaN
## as.factor(Month)2 -5.764e+03      NaN      NaN      NaN
## as.factor(Month)3  4.332e+03      NaN      NaN      NaN
## as.factor(Month)4  1.671e+04      NaN      NaN      NaN
## as.factor(Month)5  1.207e+04      NaN      NaN      NaN
## as.factor(Month)6 -9.584e+03      NaN      NaN      NaN
## as.factor(Month)7  7.188e+02      NaN      NaN      NaN
## as.factor(Month)8  2.773e+03      NaN      NaN      NaN
## as.factor(Month)9 -9.892e+03      NaN      NaN      NaN
## as.factor(Month)10 -3.608e+03      NaN      NaN      NaN
## as.factor(Month)11      NA        NA        NA        NA
## as.factor(Month)12      NA        NA        NA        NA
##
## Residual standard error: NaN on 0 degrees of freedom
## Multiple R-squared:      1, Adjusted R-squared:      NaN
## F-statistic:      NaN on 13 and 0 DF,  p-value: NA
```

```
predict(testsales)
```

```
##      1      2      3      4      5      6      7      8      9     10     11     12     13
## 12174 15326 16219 16393 26153 24445 25090 22163 23958 24700 19691 14876 16751
##      14
## 21692
```

- Ran into an error regarding the model and month factorization and got a multiple R-squared of 1.
- The predictions are still interesting regarding their showing of seasonality, even with the error.

D) From what you saw in the problem, what can you conclude about predicting Hyundai Elantra sales? Do you think these conclusions generalize to predicting sales for other products?

- Many of the independent variables are not useful on their own, but introducing months shows that there is a seasonality associated with Hyundai Elantra sales. You could probably predict sales for many products based on cultural patterns.

E) If you could collect additional independent variables for this problem which variables do you think would be useful for predicting sales?

- I would collect region and inflation rate. It would be interesting to see which regions have higher sales, as well as if inflation has an effect on sales overall (both in car price and loan).