

COSC 3340/6309
Examination 3
Wednesday, June 24, 2009, 2 – 3:45 pm
Open Book and Notes

- 1.** Prove that the following language L is not contextfree:

$$L = \{ a^k b^j a^i \mid i > j > k \geq 0 \}.$$

- 2.** Construct a pda \mathbb{P} for the following language:

$$L = \{ 0^{2i} 1^i \mid i \geq 0 \} \text{ where } L = L_f(\mathbb{P}) \text{ (acceptance by final state).}$$

State on which side you write the top of the stack, left: or right .

- 3.** Construct a pda \mathbb{P} that accepts the following language by empty stack:

$$\begin{aligned} L &= L(G) \text{ where } G = (T, N, P, E) \text{ with } T = \{ \text{id}, +, *, (,) \}, \\ N &= \{ E \}, \text{ and } P = \{ E \rightarrow E+E \mid E^*E \mid (E) \mid \text{id} \}. \end{aligned}$$

State on which side you write the top of the stack, left: or right .

Note: You must use the construction “cfg \rightarrow pda” given in class. Get G into GNF first!

- 4.** Construct a grammar for $L(G)$ for the language $N(\mathbb{P})$:

$\mathbb{P} = (\{p, q\}, \{a, b\}, Z, X \}, \delta, p, Z, \emptyset)$ where the move function δ is given by

$$\begin{array}{lll} \delta(p, b, Z) = \{(p, XZ)\} & \delta(q, \epsilon, Z) = \{(q, \epsilon)\} & \delta(p, b, X) = \{(p, XX)\} \\ \delta(q, b, Z) = \{(p, XZ)\} & \delta(q, a, X) = \{(q, \epsilon)\} & \delta(p, a, X) = \{(p, \epsilon)\}. \end{array}$$

Here, the top of the stack is on the left.

- 5.** Construct a Turing machine for the language in Question 1,

$$L = \{ a^k b^j a^i \mid i > j > k \geq 0 \}.$$

Describe first in words what you are doing, then formulate the formal Turing machine.

Points:

1: 20

2: 12

3: 18

4: 30

5: 20

① Prove: $L = \{a^i b^j a^k \mid i > j > k \geq 0\}$ is not context free

Assume that L is context free and there is a grammar $G = (N, T, P, S)$ in CNF form such that $L = L(G)$

Let $k = 2^n$

$$\begin{array}{l} j > k \\ i > j \end{array} \Rightarrow \begin{array}{l} j = 2^n + 1 \\ i = 2^n + 2 \end{array}$$

$$z = a^{2^n} b^{2^n+1} a^{2^n+2}, |z| = |2^n + 2^n + 1 + 2^n + 2| = 3 \cdot 2^n + 3 \\ |z| > 2^n$$

$z \in L$ by the pumping lemma for context free language
 z can be rewritten in the form $z = uvwxy$,
where $|vxy| \leq |z|$

the pumping lemma tells us that $uv^s w x^s y \in L \forall s \geq 0$

① Consider v and x to be all 'a's on the left
if we let $s = 2$, we increase the number of
a's while the number of b's remain the same.
number of a's is more than number of b's

$uv^s w x^s y \notin L$ but $\in L(G)$
therefore we have a contradiction

→ continued.

② V & X are all Vs

for $s=2 \quad \checkmark, z \in L(a), z \notin L$ why

③ V & X are all 2's on the right

for $s=0 \quad \checkmark, z \in L(G), z \notin L$

④ No a's on the right in V or X

for $s=2 \quad \checkmark, z \in L(G), z \notin L$

⑤ No b's in V or X

for $s=2 \quad \checkmark, z \in L(a), z \notin L$

⑥ No a's on the left

for $s=0 \quad \checkmark, z \in L(b), z \notin L$

⑦ at least one a and b in V or X

for $s>1$ the pattern will change in z
where a's and b's alternate multiple times
that pattern is not in L

$\checkmark, z \in L(a), z \notin L$

∴ we have proved that L is not context free

Carlos Varegas

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		\emptyset	$\{Z\}$	$\{X\}$	$\{E\}$				
q _a	Z	<u>(P₁, XZ)</u>	/	/	/				
	X	/	/	/					
	X ₂	(P ₁ , XX ₂)	<u>(M₁, E)</u>	/					
p	Z	(q _a , X ₂)	/	/	/				
	X	(P ₁ , E)	/	/	/	x 4			
	X ₂	(q _a , X ₂)	/	/	/				
m	Z	/	/	(q _{f1} , E)					
	X	/	/						
	X ₂	/	<u>(M₁, E)</u>	/					
q _f	Z								
	X								
	X ₂								
$\rightarrow q_0$	Z	<u>(P₁, XZ)</u>	/	(q _{f1} , E)					
	X	/	/	/					
	X ₂	/	/	/					

$R = \{g_{d,191PM943}, \{0,13\}Z, X, Z, d, g_{0,171SW3}\}$

top of the stack is on the left

$$③ E \rightarrow E+E | E*E | (E) | id$$

Get G into GNF

remove left recursion

$$E \rightarrow C(E) | id | (E)E' | idE'$$

$$E' \rightarrow +E | *E | +EE' | *EE'$$

$$E \rightarrow (EX) | id | (EX)E' | idE'$$

$$E' \rightarrow +E | *E | +EE' | *EE'$$

~~QX₃~~ >)

		id	+	*	()	ε	
E		(q ₁ , ε)	,	,	(q ₁ , EX ₁)	,	,	
		(q ₁ , E')	,	,	(q ₁ , EX, E)	,	,	
q ₁	E'	,	(q ₁ , E)	(q ₁ , E)	,	,	,	
			(q ₁ , EE)	(q ₁ , EE)	,	,	,	
X ₁		/	/	/	/	/	(q ₁ , ε)	/

$$P_N = \{q_3, \{id, +, *\}, \{(), \}, \{E, E'\}, X_3, \delta, q_1, E, \emptyset\}$$

the top of stack is on the left

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	α	b	ϵ	
P	Z	(P, XZ)	(P, XX)	
	X	(P, ϵ)		
	Z		(P, XZ)	(Q, ϵ)
Q	X	(Q, ϵ)		

① $S \rightarrow [P, Z, P] \mid [P, Z, Q]$

② $(P, XZ) \in S(P, b, Z)$

$$[P, Z,] \rightarrow b[P, X,] [, Z,]$$

$$[P, Z, P] \rightarrow b[P, X, P] [P, Z, P] \mid b[P, X, Q] [Q, Z, P]$$

$$[P, Z, Q] \rightarrow b[P, X, P] [P, Z, Q] \mid b[P, X, Q] [Q, Z, Q]$$

$$(P, XZ) \in S(Q, b, Z)$$

$$[Q, Z,] \rightarrow b[P, X,] [, Z,]$$

$$[Q, Z, P] \rightarrow b[P, X, P] [P, Z, P] \mid b[P, X, Q] [Q, Z, P]$$

$$[Q, Z, Q] \rightarrow b[P, X, P] [P, Z, Q] \mid b[P, X, Q] [Q, Z, Q]$$

$$(Q, \epsilon) \in S(Q, \epsilon, Z) \quad | \quad (Q, \epsilon) \in S(Q, a, X) \quad | \quad (P, \epsilon) \in S(P, Z, X)$$

$$[Q, Z, Q] \rightarrow \epsilon$$

$$[Q, X, Q] \rightarrow a$$

$$[P, X, P] \rightarrow z$$

continued
on
back