COSC 3340
Examination 3
Wednesday, April 9, 2008, 1 – 2:30 pm
Open Book and Notes

1. Prove that the following language L is not contextfree:

$L = \{ 0^k 1^j 2^i \mid i>j>k\geq 0 \}$.

2. Construct a pda P for the following language:

$L = \{ 0^i 1^{3i} \mid i\geq 0 \}$ where $L = L_f(P)$ (acceptance by final state).

State on which side you write the top of the stack, left: ____| or right |____.

Hint: Put three markers on the stack for every 0.

3. Construct a pda P that accepts the following language by empty stack:

$L = L(G)$ where $G = (T, N, P, S)$ with $T = \{<,>,[,]\}$,

$N = \{S,A\}$, and $P = \{ S \to <S>A \mid [A]A\ ,\ A \to [A]S \mid <S>S \mid \varepsilon \}$.

State on which side you write the top of the stack, left: ____| or right |____.

Note: You must use the construction "cfg → pda" given in class. Get G into GNF first!

4. Construct a grammar for L(G) for the language N(P):

$P = (\{p,q\}, \{a,b\}, \{Z,X\}, \delta, p, Z, \varnothing)$ where the move function δ is given by

$\delta(p,a,Z) = \{(p,XZ)\}$ $\quad \delta(p,\varepsilon,Z) = \{(p,\varepsilon)\}$ $\quad \delta(p,a,X) = \{(p,XX)\}$

$\delta(q,a,Z) = \{(q,\varepsilon)\}$ $\quad \delta(p,b,X) = \{(q,X)\}$ $\quad \delta(q,b,Z) = \{(p,Z)\}$.

Here, the top of the stack is on the left.

5. Construct a Turing machine for the language in Question 1,

$L = \{ 0^k 1^j 2^i \mid i>j>k\geq 0 \}$.

Describe first in words what you are doing, then formulate the formal Turing machine.

Points: 1: 20 2: 12 3: 18 4: 30 5: 20

$L = \{0^k 1^j 2^i \mid i > j > k \geq 0\}$

1) assume L is cfL

∴ then $\exists$ G(N,T,P,S) in CNF s.t. L = L(G)

assume t = no. of variables, assume a word $z = 0^{2t} 1^{2t+1} 2^{2t+2}$ ✓ $|z| \geq 2^t$

1	20
2	12
3	18
4	30
5	20
	100

by pumping Lemma $z = uv^i wx^i y$ where $|vx| \geq 1$ & $uv^i wx^i y \in L(G)$ for all $i \geq 0$

case 1:- V & X are all 0's ⇒ for i = 2 no. of zero can be equal no. of 1 $\in L(G)$, $\notin L$

case 2:- V & X are all 1's ⇒ For i = 2 ✓ $\notin L(G)$, $\notin L$

case 3:- V & X are all 2's ⇒ for i = 0 ✓ $\in L(G)$, $\notin L$

case 4:- No 2's in vorx ⇒ for i = 2 $\in L(G)$, $\notin L$
because No of 1's can be equal to no. of 2's

case 5:- No 1's in Vorx ⇒ for i = 2 $\in L(G)$, $\notin L$

case 6:- No 0's in Vorx ⇒ for i = 0 $\in L(G)$, $\notin L$
because No of 1's decrease to be equal No of zeros

case 7:- at least one 0, one 1, one 2 in vorx

∴ for i > 1 the pattern will change

i.e. zero's can follow 2's which is not acceptable by L

∴ there is contradiction in each case ✓ 20

∴ the language L is not Context free

④

		a	b	∈
p	Z	{(p, xz)}		{(p, ∈)}
	X	{(p, xx)}	{(q, x)}	
q	Z	{(q, ∈)}	{(p, z)}	
	X			

30

⑥ S → [p, z, p] | [p, z, q] → ①

* (p, xz) ∈ δ(p, a, z)
  $[p, z, \begin{smallmatrix} p \\ q \end{smallmatrix}] \rightarrow a[p, x, \begin{smallmatrix} p \\ q \end{smallmatrix}][\begin{smallmatrix} p \\ q \end{smallmatrix}, z, \begin{smallmatrix} p \\ q \end{smallmatrix}]$

② ← $[p, z, p] \rightarrow a[p, x, p][p, z, p] | a[p, x, q][q, z, p]$

③ ← and $[p, z, q] \rightarrow a[p, x, p][p, z, q] | a[p, x, q][q, z, q]$

* (p, xx) ∈ δ(p, a, x)
  $[p, x, \begin{smallmatrix} p \\ q \end{smallmatrix}] \rightarrow a[p, x, \begin{smallmatrix} p \\ q \end{smallmatrix}][\begin{smallmatrix} p \\ q \end{smallmatrix}, x, \begin{smallmatrix} p \\ q \end{smallmatrix}]$

④ ← $[p, x, p] \rightarrow a[p, x, p][p, x, p] | a[p, x, q][q, x, p]$

⑤ ← $[p, x, q] \rightarrow a[p, x, p][p, x, q] | a[p, x, q][q, x, q]$

* $(p, \epsilon) \in \delta(p, \epsilon, z)$

  $[p, z, p] \to \epsilon$ ✓ → (6)

* $(q, x) \in \delta(p, b, x)$

  $[p, x, \begin{matrix} p \\ q \end{matrix}] \to b\,[q, x, \begin{matrix} p \\ q \end{matrix}]$

  $\Rightarrow$ $[p, x, p] \to b\,[q, x, p]$ ✓ → (7)

  $[p, x, q] \to b\,[q, x, q]$ ✓ → (8)

* $(q, \epsilon) \in \delta(q, a, z)$

  $[q, z, q] \to a$ ✓ → (9)

* $(p, z) \in \delta(q, b, z)$

  $[q, z, \begin{matrix} p \\ q \end{matrix}] \to b\,[p, z, \begin{matrix} p \\ q \end{matrix}]$

  $\Rightarrow$ $[q, z, p] \to b\,[p, z, p]$ ✓ → (10)

  $[q, z, q] \to b\,[p, z, q]$ ✓ → (11)

The $L(G)$ is defined with ~~productions~~ productions 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11

(2) First I'll construct a pda that accepts by empty stack then change it to one that accepts by final state. $L = \{0^i 3^j \mid i \geq j\}$

$P_e = (\{0,1\}, \{q_0, q_1\}, \{z_0, z\}, \delta, q_0, \emptyset, z_0)$

(the top of the stack is on the left)

δ:

		0	1	$\in$
	z_0	$\{(q_0, zzzz_0)\}$	/	$\{(q_1, \in)\}$
q_0	z	$\{(q_0, zzzz)\}$	$\{(q_1, \in)\}$	/
	z_0	/	/	$\{(q_1, \in)\}$
q_1	z	/	$\{(q_1, \in)\}$	/

$P_f = (\{0,1\}, \{q_i', q_0, q_1, q_f\}, \{z_0', z_0, z\}, \delta', q_0', q_f, z_0')$

δ':

		0	1	$\in$
	z_0'	/	/	$\{(q_0, zz_0)\}$
q_0'	z_0	/	/	/
	z	/	/	/
	z_0'	/	/	/
	z_0	$\{(q_0, zzzz_0)\}$	/	$\{(q_1, \in)\}$
q_0	z	$\{(q_0, zzzz)\}$	$\{(q_1, \in)\}$	/
	z_0'	/	/	$\{(q_f, \in)\}$
	z_0	/	/	$\{(q_1, \in)\}$
q_1	z	/	$\{(q_1, \in)\}$	/
	z_0'	/	/	/
	z_0	/	/	/
q_f	z	/	/	/

final state

(3) $S \to \langle S \rangle A \mid [A]A$

$A \to [A]S \mid \langle S \rangle S \mid \epsilon$

eliminate ϵ

$S \to \langle S \rangle A \mid \langle S \rangle \mid [A]A \mid [A] \mid [\,]A \mid [\,]$

$A \to [A]S \mid [\,]S \mid \langle S \rangle S$

to GNF

$S \to \langle S X_2 A \mid \langle S X_2 \mid [A X_3 A \mid [A X_3 \mid [\,X_3 A \mid [X_3]$

$A \to [A X_3 S \mid [X_3 S \mid \langle S X_2 S$

$X_2 \to \rangle$

$X_3 \to]$

18 $P_N = (\{\langle, \rangle, [,]\}, \{q_0\}, \{S, A, X_2, X_3\}, \delta, q_0, \emptyset, S)$

δ

	$\langle$	$\rangle$	$]$	$[$	ϵ
S	$\{(q_0, SX_2A), (q_0, SX_2)\}$			$\{(q_0, AX_3A), (q_0, AX_3), (q_0, X_3A), (q_0, X_3)\}$	
A	$\{(q_0, SX_2S)\}$			$\{(q_0, AX_3S), (q_0, X_3S)\}$	
X_2		$\{(q_0, \epsilon)\}$			
X_3			$\{(q_0, \epsilon)\}$		

q_0

The top of stack on the left

(5) $\{0^k 1^j 2^i \mid i > j > k \geq 0\}$

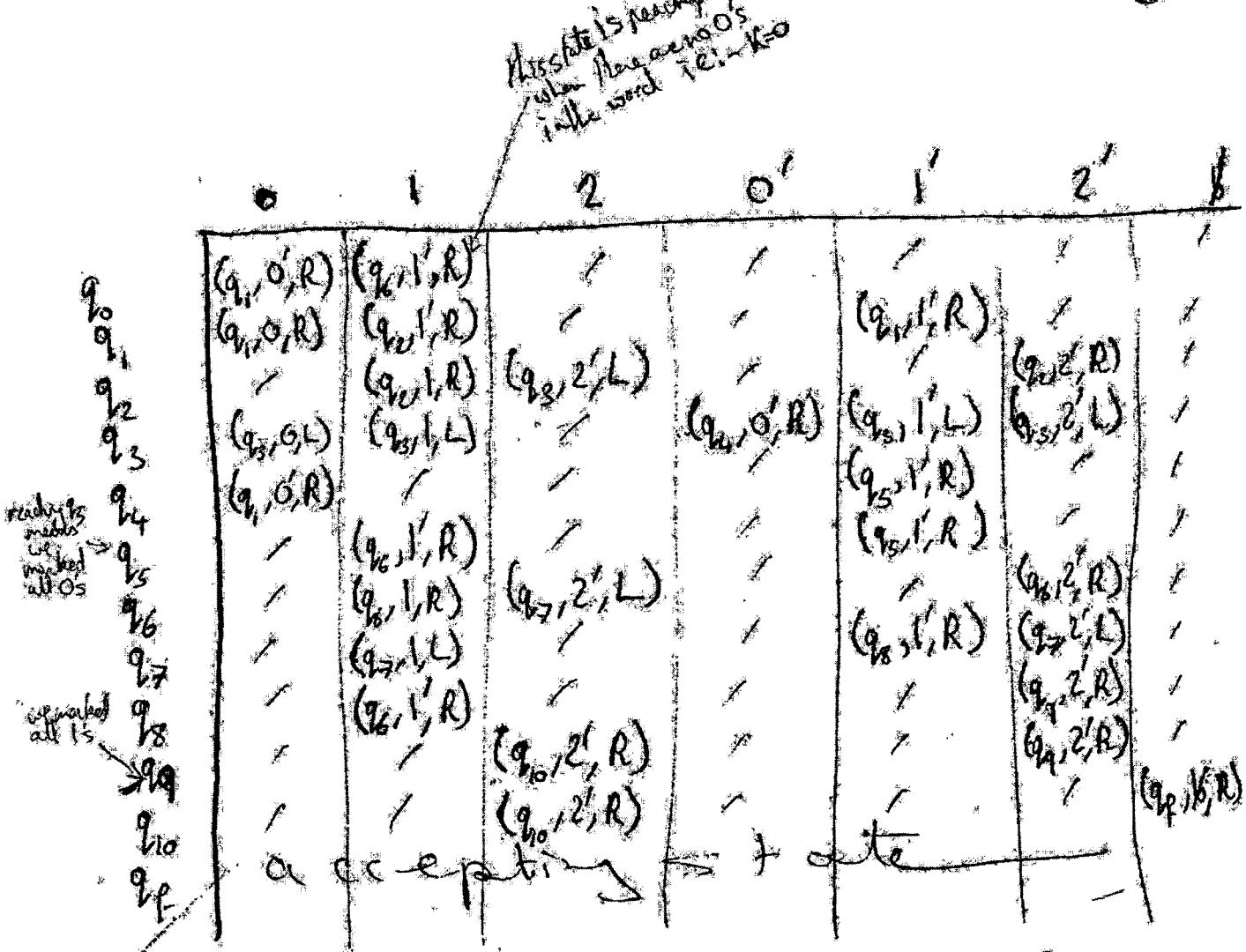
ex. of accepted words 122 , 011222

00111 2222

- first we ~~mark~~ start reading from the left if a zero is found then we mark it and skip all zeros till we find a 1 then we mark it and skip 1's till we find a two then we mark it then we start move back to the left skiping everything till we find a 0' then we turn to move right and repeat the whole process till all zeros are marked.

Then we repeat the same process for only the 1's and 2's till all 1's are marked then we make sure that there is at least a 2's left (or more) then we mark all 2's till we reach b

- if at the start no zero's are found we proceed as if all zero's were marked and this is the case when K=0



The Turning Machine for the language $L = \{0^k 1^j 2^i \mid i > j > k > 0\}$