

COSC 3340/6309

Examination 3

Wednesday, June 26, 2013, 10 am – 12 noon

Open Book and Notes

95 92

20/20

1. Prove that the following language L is not contextfree:

$$L = \{ a^i b^j a^i \mid j > i \geq 1 \}.$$

10 7/12

2. Construct a pda P for the following language:

$$L = \{ 0^i 1^{3i} \mid i \geq 0 \} \text{ where } L = L(P) \text{ (acceptance by final state).}$$

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3. Construct a pda P that accepts the following language **by empty stack**:

$$L = L(G) \text{ where } G = (T, N, P, E) \text{ with } T = \{ id, *, /, (,) \},$$

$$N = \{ E \}, \text{ and } P = \{ E \rightarrow E * E \mid E / E \mid (E) \mid id \}.$$

Note: You must use the construction "cfg \rightarrow pda" given in class. Get G into GNF first!

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4. Construct a grammar for $L(G)$ for the language $N(P)$:

$$P = (\{ p, q \}, \{ a, b \}, \{ Z, X \}, \delta, p, Z, \emptyset) \text{ where the move function } \delta \text{ is given by}$$

$$\delta(p, b, Z) = \{ (p, XZ) \} \quad \delta(q, \epsilon, Z) = \{ (q, \epsilon) \} \quad \delta(p, b, X) = \{ (p, XX) \}$$

$$\delta(q, b, Z) = \{ (p, XZ) \} \quad \delta(q, a, X) = \{ (q, \epsilon) \} \quad \delta(p, a, X) = \{ (p, \epsilon) \}.$$

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5. Construct a Turing machine for the language in Question 1,

$$L = \{ a^i b^j a^i \mid j > i \geq 1 \}.$$

Describe first in words what you are doing, then formulate the formal Turing machine.

Points: 1: 20 2: 12 3: 18 4: 30 5: 20

① CLAIM: L IS NOT CONTEXT FREE

PROOF: ASSUME L IS CFL, THEN \exists CNF G GENERATING

L , SO THAT: $Z = \overset{2^n}{a} \overset{2^{n+1}}{b} \overset{2^n}{a} \in L(G)$

* THEN, ACCORDING TO PUMPING LEMMA WE HAVE:

$Z = UVWXY$ AND $|VX| \geq 1$ AND $|VWX| \leq 2^{n-1}$, SO

$UV^sWX^sY \in L(G) \forall s \geq 0$

* THUS WE HAVE THESE SCENARIOS:

① VWX HAS ONLY LEFT a 's, THEN NUMBER OF LEFT a 's WILL BE BIGGER THAN b 's (CONTRADICTION)

② VWX HAS ONLY b 's, THEN $|VX| \geq 1$ AND TAKING $s = 0$, WE DECREASE THE NUMBER OF RIGHT a 's WHILE LEAVING THE OTHERS INTACT (CONTRADICTION)

③ VWX HAS ONLY RIGHT a 's, THEN NUMBER OF RIGHT a 's WILL BE BIGGER THAN b (CONTRADICTION)

④ V HAS b 's AND X HAS a 's, THEN $|\text{LEFT } a\text{'s}| \leq |\text{RIGHT } a\text{'s}|$ (CONTRADICTION)

⑤ V HAS LEFT a 's AND X HAS RIGHT a 's, THEN NUMBER OF a 's WILL BE BIGGER THAN b 's, ($s = 2$) (CONTRADICTION)

① V HAS LEFT a 'S AND X HAS b 'S , THEN

$$| \text{LEFT } a\text{'s} | \neq | \text{RIGHT } a\text{'s} | \quad (\text{CONTRADICTION})$$

SINCE ALL SCENARIOS ARE CONTRADICTIONS, L IS NOT
A CONTEXT FREE LANGUAGE

② $L = \{0^i 1^{3i} \mid i \geq 0\}$ & $L = L_f(P)$

* LEFT STACK \square

		0	1	ϵ
q_0	z_0	$(q_0, zzzz_0)$	/	(q_1, z_0)
	z	$(q_0, zzzz)$	(q_1, ϵ)	/
q_1	z_0	/	/	(q_1, z_0)
	z	/	(q_1, ϵ)	/
q_f	z_0	ACCEPTING		
	z			

③ * LEFT STACK.]

$$P = E \rightarrow E * E \mid E / E \mid (E) \mid id$$

$$G = (T, N, P, E)$$

$$T = \{id, *, /, (,)\}$$

$$N = \{E\}$$

$$E \rightarrow E * E \mid E / E \mid (E) \mid id$$

* ELIMINATED LEFT RECURSION, THEN I CONVERTED) TO X, TO GET GNF

$$E \rightarrow (EX) \mid id \mid (EX)E' \mid idE'$$

$$E' \rightarrow *E \mid /E \mid *EE' \mid /EE'$$

$$X) \rightarrow)$$

	id	*	/	()	ϵ
E	$(q, \epsilon), (q, E')$	/	/	$(q, EX), (q, EX, E')$	/	/
E'	/	$(q, E), (q, EE')$	$(q, E), (q, EE')$	/	/	/
$X)$	/	/	/	/	(q, ϵ)	/

* PROD = PRODUCTION

④ 1st RULE:

$$S \rightarrow [P, Z, P] \mid [P, Z, q]$$

* 2 PRODUCTIONS

		a	b	ϵ
P	Z	/	(P, XZ)	/
	X	(P, ϵ)	(P, XX)	/
q	Z	/	(P, XZ)	(q, ϵ)
	X	(q, ϵ)	/	//

②nd RULE:

• $(P, XZ) \rightarrow \delta(P, b, Z)$ * 4 PRODS.

$$[P, Z, q] \rightarrow b [P, X, q] [q, Z, q]$$

$$\begin{cases} \rightarrow [P, Z, P] \rightarrow b [P, X, P] [P, Z, P] \mid b [P, Z, q] [q, Z, P] \\ \rightarrow [q, Z, q] \rightarrow b [q, X, P] [P, Z, q] \mid b [q, X, q] [q, Z, q] \end{cases}$$

• $(P, \epsilon) \rightarrow \delta(P, a, X)$ * 1 PROD.

$$[P, X, P] \rightarrow a$$

* 1 PROD

• $(q, \epsilon) \rightarrow \delta(q, \epsilon, Z)$

$$[q, Z, q] \rightarrow \epsilon$$

• $(q, \epsilon) \rightarrow \delta(q, a, X)$ * 1 PROD.

$$[q, X, q] \rightarrow a$$

* CONTINUE ON
BACK OF PAGE

• $(P, XX) \rightarrow b(P, b, X)$ * 4 PRODS.

$$\begin{cases} \rightarrow [P, X, P] \rightarrow b[P, X, P][P, X, P] \mid b[P, X, q][q, X, P] \\ \rightarrow [P, X, q] \rightarrow b[P, X, P][P, X, q] \mid b[P, X, q][q, X, q] \end{cases}$$

• $(P, XZ) \rightarrow b(q, b, z)$ * 4 PRODS.

$$\begin{cases} \rightarrow [P, z, P] \rightarrow b[P, X, P][P, z, P] \mid b[P, X, q][q, z, P] \\ \rightarrow [P, z, q] \rightarrow b[P, X, P][P, z, q] \mid b[P, X, q][q, z, q] \end{cases}$$

17 PRODUCTIONS TOTAL

(5)

$$L = \{a^i b^j a^i \mid j \geq i \geq 1\}$$

$$* | \text{LEFT } a's | = | \text{RIGHT } a's |$$

THE FOLLOWING TURING MACHINE WILL TRAVERSE THROUGH THE TAPE COMPARING LEFT a 's WITH RIGHT a 's AS A FIRST STEP. FOR EVERY COUPLED LEFT AND RIGHT a 's, a b WILL BE MARKED AS b' . WHEN ALL LEFT a 's HAVE BEEN COUPLED WITH ALL RIGHT a 's, THEN THE MACHINE WILL TRACK THE REMAINING b 's TO COMPLETE THE WORD INPUTTED.

TURING MACHINE ON BACK OF PAGE \longrightarrow

Needs more detail

	a	b	a'	b'	b
q ₀	(q ₁ , a', R)	/	/	/	/
q ₁	(q ₁ , a, R)	(q ₂ , b, R)	/	(q₁, b', L)	/
q ₂	(q ₃ , a', L)	(q ₂ , b, R)	(q ₂ , a', R)	/	/
q ₃	/	(q ₄ , b', L)	/	(q ₃ , b', L)	/
q ₄	(q ₄ , a, L)	(q ₄ , b, L)	(q ₅ , a', R)	(q ₄ , b', L)	/
q ₅	(q ₆ , a', R)	(q ₅ , b, R)	(q ₆ , a', L)	(q ₅ , b', R)	/
q ₆	(q ₆ , a, R)	(q ₆ , b, R)	/	(q ₇ , b', R)	/
q ₇	(q ₈ , a', L)	/	(q ₇ , a', R)	(q ₇ , b', R)	/
q ₈	/	(q ₇ , b', L)	(q ₈ , a', L)	(q ₈ , b', L)	/
q ₉	(q ₉ , a, L)	(q ₉ , b, L)	(q ₁₀ , a', R)	/	/
q ₁₀	/	(q ₁₀ , b', R)	(q ₁₀ , a', R)	(q ₁₀ , b', R)	(q ₁₁ , b, R)
q _f	ACCEPTING				