

COSC 3340
Examination 3
Wednesday, April 22, 2009, 1 – 2:30 pm
Open Book and Notes

- 1.** Prove that the following language L is not contextfree:

$$L = \{ 0^k 1^j 0^i \mid i > j > k \geq 0 \}.$$

- 2.** Construct a pda \mathbb{P} for the following language:

$$L = \{ 0^i 1^{4i} \mid i \geq 0 \} \text{ where } L = L_f(\mathbb{P}) \text{ (acceptance by final state).}$$

State on which side you write the top of the stack, left: or right
Hint: Put four markers on the stack for every 0.

- 3.** Construct a pda \mathbb{P} that accepts the following language by empty stack:

$$\begin{aligned} L &= L(G) \text{ where } G = (T, N, P, S) \text{ with } T = \{<,>, [,]\}, \\ N &= \{S, A\}, \text{ and } P = \{ S \rightarrow <S>A \mid [A]A, A \rightarrow [A]S \mid <S>S \mid \varepsilon \}. \end{aligned}$$

State on which side you write the top of the stack, left: or right
Note: You must use the construction "cfg \rightarrow pda" given in class. Get G into GNF first!

- 4.** Construct a grammar for $L(G)$ for the language $N(\mathbb{P})$:

$\mathbb{P} = (\{p, q\}, \{a, b\}, Z, X), \delta, p, Z, \emptyset$) where the move function δ is given by
 $\delta(p, a, Z) = \{(p, XZ)\} \quad \delta(p, \varepsilon, Z) = \{(p, \varepsilon)\} \quad \delta(p, a, X) = \{(p, XX)\}$
 $\delta(q, a, Z) = \{(q, \varepsilon)\} \quad \delta(p, b, X) = \{(q, X)\} \quad \delta(q, b, Z) = \{(p, Z)\}.$

Here, the top of the stack is on the left.

- 5.** Construct a Turing machine for the language in Question 1,

$$L = \{ 0^k 1^j 0^i \mid i > j > k \geq 0 \}.$$

Describe first in words what you are doing, then formulate the formal Turing machine.

Points: 1: 20 2: 12 3: 18 4: 30 5: 20

Examination - 3

1	17
2	12
3	18
4	30
3	21
5	15
	92

$$1) L = \{0^k 1^j 0^i \mid j \geq i \geq k \geq 0\}$$

, 2 3

assume L is CFL

then $\exists G(N, T, P, S)$ in CNF such that $L = L(G)$

assuming $t = \text{no. of variables}$, $k = 2t$, $j = 2t+1$, $i = 2t+2$

take a ^{work} word $z = 0^{2t} 1^{2t+1} 0^{2t+2}$ $|z| > 2t$

by pumping lemma $z = uv^wxy$ where $|ux| \neq 0$
and $uv^iwx^iy \in L(G)$
for all $i \geq 0$

Case 1

$v \neq x$ are all 0's, for $i=0$, no of zeroes can be ~~more~~ than no. of 1, $L(G) \neq L$

Case 2

$v \neq x$ are all 1's for $i=2$, $L(G) \neq L$, as no. of 1 can be equal to 0.

Case 3 and 4

~~V and x are both~~ ^{only one} ^{one} do not ~~fix~~ in this case if ~~i > 1~~
 $L(G) \neq L$ as the sequence of 0 and 1 might change. If ~~V or x contains 2 symbols~~ for both "01" and "10".
basically V or x contains 2 symbols for ~~i > 1~~.

Case 5

$v x$ contains 0's before 1 than no. of 0's before 1 can be greater than no. of zeroes after 1 $\therefore L(G) \neq L$

Case 6

No. of 0's after $i \in V$ and x , then $L(Q_i) \neq L$

for $i=0$ obs no. of zeroes after i can
be ~~less~~ than no. of 0's before i .

there is contradiction in this case.

17 and language L is not context free

how about $\{x^0y^0z^0 \mid x, y, z \in \Sigma\}$

Q-2

2) First a pda is constructed for empty stack acceptance and then change it to being accepted by final state.

$$P_e = (\{0, 1\}, \{q_0, q_1\}, \{z_0, z\}, \delta, q_0, \emptyset, z_0)$$

Top of stack is on left.

		0	1	ϵ
q_0	z_0	$\{(q_0, zzzz z_0)\}$	/	$\{(q_1, \epsilon)\}$
	z	$\{(q_0, zzzz z)\}$	$\{(q_1, \epsilon)\}$	/
	z_0	/	/	$\{(q_1, \epsilon)\}$
q_1	z	/	$\{(q_1, \epsilon)\}$	/

Now for final state acceptance.

$$S' \quad P_f = (\{0, 1\}, \{(q_0', q_0, q_1, q_f)\}, (z_0', z_0, z), \delta', q_0', q_f, z_0')$$

	0	1	ϵ
z_0'	/	/	$\{(q_0, z_0 z_0)\}$
q_0'	z_0	/	/
z_0	/	/	/
z	/	/	/
z_0'	/	/	/
q_0	z_0	$\{(q_0, zzzzz_0)\}$	$\{(q_1, \epsilon)\}$
z	$\{(q_0, zzzzz)\}$	$\{(q_1, \epsilon)\}$	/
z_0'	/	/	$\{(q_f, \epsilon)\}$
q_1	z_0	/	$\{(q_1, \epsilon)\}$
z	/	$\{(q_1, \epsilon)\}$	/
z_0'	/	/	/
q_f	z_0	/	/
z	/	/	/

final state

==

$$3) S \rightarrow \langle s \rangle A \mid [A] A$$

$$A \rightarrow [A] S \mid \langle s \rangle S \mid \epsilon$$

Eliminating ϵ .

$$\begin{aligned} S &\rightarrow \langle s \rangle A \mid \langle s \rangle \mid [A] A \mid [A] \mid [E] A \mid [E] \\ A &\rightarrow [A] S \mid [E] S \mid \langle s \rangle S \end{aligned}$$

To GNF.

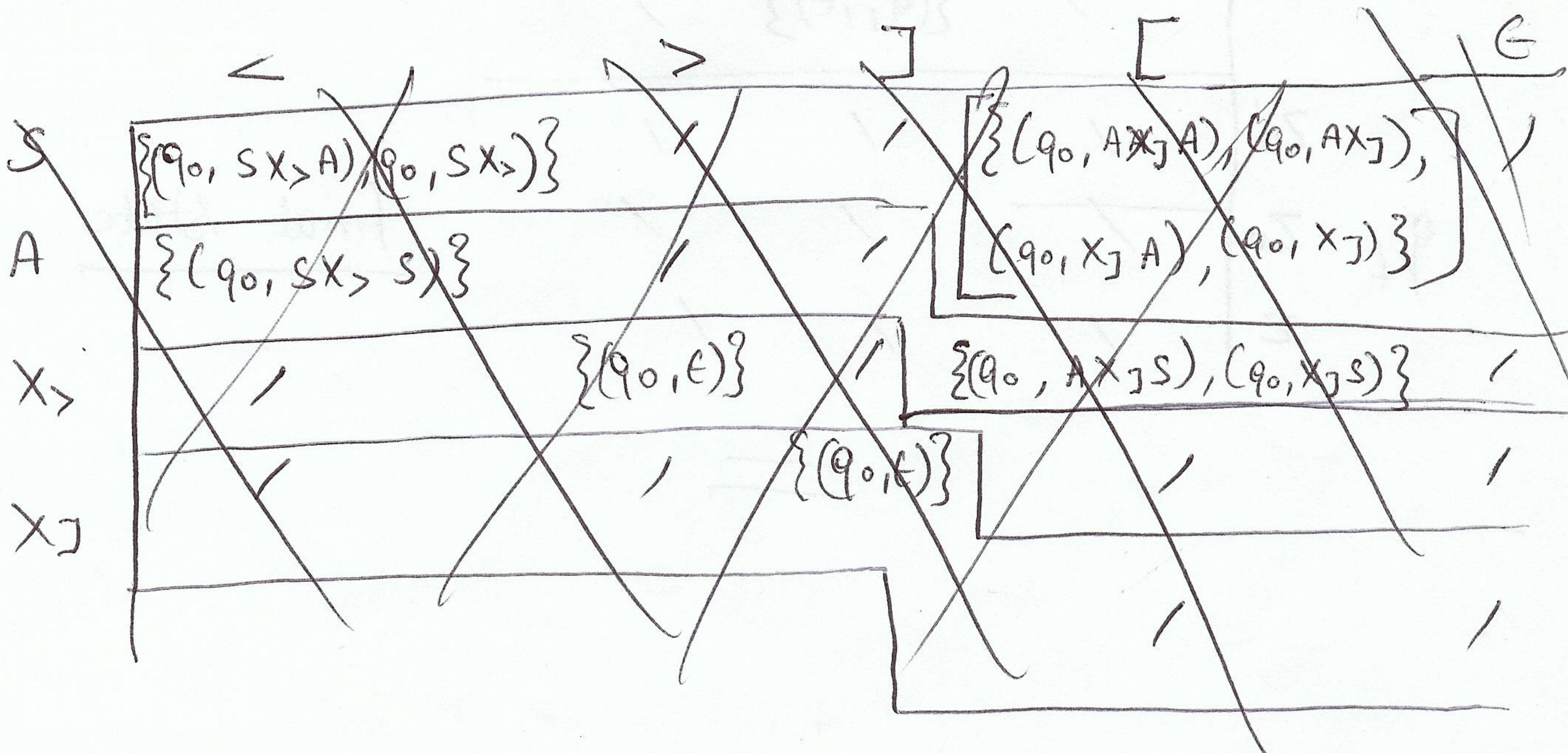
$$\begin{aligned} S &\rightarrow \langle s \rangle X_> A \mid \langle s \rangle X_> \mid [A] X_> A \mid [A] X_> \mid [E] X_> A \mid [E] X_> \\ A &\rightarrow [A] X_> S \mid [E] X_> S \mid \langle s \rangle X_> S. \end{aligned}$$

$$X_> \rightarrow >$$

$$X_> \rightarrow]$$

$$P_N = (\{\langle, \rangle, [,]\}, \{q_0\}, \{S, A, X_>, X_>S\}, S, q_0, \phi^*, S)$$

S



4). a b e

P	z	$\boxed{\{P, Xz\}}$	$/$	$\boxed{\{P, e\}}$
x	$\boxed{\{P, xx\}}$	$\boxed{\{Q, x\}}$	$/$	
q	z	$\boxed{\{Q, e\}}$	$\boxed{\{P, z\}}$	$/$
x	$/$	$/$	$/$	

$$S \rightarrow [P, z, P] / [P, z, q] \quad \checkmark \quad ①$$

for
 $(P, Xz) \in S(P, a, z)$

$$[P, z, \overset{P}{q}] \rightarrow a[P, X, \overset{P}{q}] [\overset{P}{q}, z, \overset{P}{q}]$$

$$② [P, z, P] \rightarrow a[P, X, P] [P, z, P] / a[P, X, q] [q, z, P]$$

$$③ [P, z, \overset{q}{q}] \rightarrow a[P, X, P] [P, z, \overset{q}{q}] / a[P, X, q] [q, z, q]. \quad \checkmark$$

for
 $(P, xx) \in S(P, a, x)$

$$④ [P, x, \overset{P}{q}] \rightarrow a[P, X, \overset{P}{q}] [\overset{P}{q}, x, \overset{P}{q}]$$

$$⑤ [P, x, P] \rightarrow a[P, X, P] [P, x, P] / a[P, X, q] [q, x, P]$$

$$[P, x, \overset{q}{q}] \rightarrow a[P, X, P] [P, x, q] / a[P, X, q] [q, x, q]. \quad \checkmark$$

S

E

>

J

L

C

S

$\{(q_0, Sx_J A), (q_0, Sx_J)\}$

,

$\{(q_0, AX_J A), (q_0, AX_J),$
 $(q_0, x_J A), (q_0, x_J)\}$

A

$\{(q_0, Sx_J S)\}$

,

$\{(q_0, AX_J S), (q_0, x_J S)\}$

x_J

$\{(q_0, \epsilon)\}$

x_J

$\{(q_0, \epsilon)\}$

Top of stack is on left

⑤ Turing Machine

$$\{0^k 1^j 0^j \mid i > j > k \geq 0\}$$

Accepted words are! - ~~000, 00000, 0000000~~
100, 011000, 001110000

First we start from left and if a 0 is found we mark it 0'. Then we skip all zeroes and if we found 1 we mark it 1' and skip all 1's and keep on moving right.

Now when we found 0 mark it 0'' and move backward towards left until we find one 0' and when we find it we again moving right and repeat whole process until we have marked all 0's.

Then we repeat the same process with 1's and 0's until all 1's are marked and when they all are marked we make sure that atleast one (or more) 0's are left. Then we mark it until we reach p.

In case k=0

If at start no zeroes are found we proceed as if all zeroes have been marked..

for

$$(P, \epsilon) \in \delta(P, \epsilon, z)$$

⑥ $[P, z, P] \rightarrow \epsilon$

for

$$(q, x) \in \delta(P, b, x)$$

$$[P, x, q] \rightarrow b[q, x, q]$$

⑦ $[P, x, P] \rightarrow b[q, x, P]$

⑧ $[P, x, q] \rightarrow b[q, x, q]$

for

$$(q, \alpha) \in \delta(q, a, z)$$

⑨ $[q, z, q] \rightarrow a$

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for

$$(P, z) \in \delta(q, b, z)$$

$$[q, z, q] \rightarrow b[P, z, q]$$

⑩ $[q, z, P] \rightarrow b[P, z, P]$

⑪ $[q, z, q] \rightarrow b[P, z, q]$

The L(G) is obtained with production $\Rightarrow [1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11]$

Now do for $k=0$ & write
 the two

	0	1	\emptyset	$0'$	1'	\emptyset''	b
q_0	$(q_1, 0'R)$	$(q_6, 1'R)$	/	/	/	/	/
q_1	$(q_1, 0R)$	$(q_2, 1'R)$	/	/	$(q_1, 1'R)$	/	/
q_2	/	$(q_2, 1R)$	$(q_3, \emptyset''L)$	/	/	$(q_2, \emptyset''R)$	/
q_3	$(q_3, 0L)$	$(q_3, 1L)$	/	$(q_4, 0'R)$	$(q_3, 1'L)$	$(q_3, \emptyset''L)$	/
q_4	$(q_1, 0'R)$	/	/	/	$(q_5, 1'R)$	/	/
q_5	/	$(q_6, 1'R)$	/	/	$(q_5, 1'R)$	/	/
q_6		$(q_6, 1R)$	$(q_7, \emptyset''L)$	/	/	$(q_6, \emptyset''R)$	/
q_7	/	$(q_7, 1L)$	/	/	$(q_8, 1'R)$	$(q_7, \emptyset''L)$	/
q_8	/	$(q_6, 1'R)$	/	/	/	$(q_9, \emptyset''R)$	/
q_9	/	/	$(q_{10}, \emptyset''R)$	/	/	$(q_9, \emptyset''R)$	/
q_{10}	/	/	$(q_{10}, \emptyset''R)$	/	/	/	(q_f, bR)
q_f	Accepting state.						

Twining Machine for language - $0^k 1^j \emptyset^i$

Twining

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0 before 1 $\longrightarrow 0'$

0 after 1 $\longrightarrow 0''$