Q1

Pumping Lemma (For CFL)

Pumping Lemma (for CFL) is used to prove that a language is NOT context free.

If A is a context free language, then, A has a pumping length 'P' such that any string 'S', where $|S| \geq N$ may be divided into 5 pieces $S = uvxyz$ such that the following conditions must be true.

(1) uv^iwx^iy is in A for every $i \geq 0$
(2) $uz \neq \epsilon$. At least one of the outer symbols cannot be empty
(3) $|vxy| \leq N$

Step 1: Assume L is a context-free language, then $\exists\, G = \{N, T, P, S\}$ such that $L = L(G)$

Step 2: Define variable values in terms of n so it satisfies L

Step 3: So $Z = \{\ldots\}$ we have $|Z| > \{\ldots\}$ and $Z \in L$ then by pumping Lemma for CFL, we have that $Z = uvwxy$ and $|vx| \geq 1$ and $uv^iwx^iy \in L \ \forall i \geq 0$

Step 4: Consider every possible way for 'S' to be divided into 'uvwxy' Show that none of these combinations satisfy all three pumping conditions above.

Step 5: If none of the above combinations satisfy all three pumping condions, then we have a CONTRADICTION, thus L is NOT context free.

Spring 2008 $L = \{0^k 1^j 2^i \mid i > j > k \geq 0\}$

Assume L is a context free language, then $\exists\, G(N, T, P, S)$ is in chomsky normal form such that $L = L(G)$

The String $S = 0^{2P} 1^{2P+1} 2^{2P+2}$ satisfies L such that P is the pumping length such that $|S| > 2^P$

By pumping lemma $S = uvwxy$ where $|uy| \geq 1$ and $uv^iwx^iy \in L(G)$ for all $i \geq 0$

Case 1: V and X are all zeros $\Rightarrow$

Note: When we choose how to partition S into $uvwxy$, make sure it satisfies $|vwx| \geq P$ and $|vx| \geq 1$

* We can choose any P value as long as *
$|S| \geq P$ $|vwx| \leq P$ $|vx| \geq 1$

for $i = 0$,

If we choose an arbitrary $P = 2$ then

$S = \underbrace{0}_{u}\underbrace{0}_{v}\underbrace{0}_{x}\underbrace{0\,11111}_{w}\underbrace{222222}_{y}$

$uv^0wx^0y = \underbrace{0}_{u}\ \underbrace{0^0}_{v}\ \underbrace{011111}_{w}\ \underbrace{0^0}_{x}\underbrace{222222}_{y}$

order $\rightarrow$ $00\ 11111\ 222222$

We cannot choose $i = 0$ because this string satisfies L such that $2 > 5 > 6 \geq 0$

for $i = 1$

If we choose an arbitrary $P = 2$ then

$S = \underbrace{0}_{u}\underbrace{0}_{v}\underbrace{0}_{x}\underbrace{0\,11111}_{w}\underbrace{222222}_{y}$

$uv^1wx^1y = \underbrace{0}_{u}\ \underbrace{0^1}_{v}\ \underbrace{011111}_{w}\ \underbrace{0^1}_{x}\underbrace{222222}_{y}$

order $\rightarrow$ $0000\ 11111\ 222222$

We cannot choose $i = 0$ because this string satisfies L such that $4 > 5 > 6 \geq 0$

for $i = 2$ $\quad uv^2wx^2y$

If we choose an arbitrary $P = 2$ then

$S = \underbrace{0}_{u}\underbrace{0}_{v}\underbrace{0}_{x}\underbrace{0\,11111}_{w}\underbrace{222222}_{y}$

$uv^2wx^2y = \underbrace{0}_{u}\ \underbrace{0^2}_{v}\ \underbrace{1\,1\,1\,1\,1}_{w}\ \underbrace{0^2}_{x}\ \underbrace{2\,2\,2\,2\,2\,2}_{y}$

$= 000111110022222\,2$

order it $\rightarrow \underbrace{00000}_{0^k}\ \underbrace{11111}_{1^j}\ \underbrace{222222}_{2^i}$

This does not satisfy L's conditions $i > j > k \geq 0$ because $5 > 5 > 6 \geq 0$ is false

This means for $i = 2$ $\in L(G)$ $\notin L$

Case 2: V and X are all ones

for $i=0$,

If we choose an arbitrary $P=2$ then

$S = 0000\,11111\,222222$

$uv^0wx^0y =$

order $\rightarrow$

We cannot choose $i=0$ because this l'sfies L such that $2>5>6>$

Q4

Notes on Leiss Lecture

PPA Format

$= \{$states, input symbols, stack symbols, move function, initial state, initial stack, final$\}$

$= \{Q, T, P, \delta, q_0, Z_0, \emptyset\}$

$G = (N, T, P, S)$

$N = \{S\} \cup \{[q, A, p] \mid p, q \in Q, A \in T\}$

P: 1. $S \to [\underbrace{q_0, Z_0}_{\text{Final States}}, q] \ \forall q \in Q$

Free Variable

q_0 and q_1

		0	1	ϵ
q_0	Z	(q_0, XZ)	/	/
	X	(q_0, XX)	(q_1, ϵ)	/
q_1	Z	/	/	(q_1, ϵ)
	X	/	(q_1, ϵ)	(q_1, ϵ)

rule 1. $S \to [q_0, Z, \begin{smallmatrix} q_0 \\ q_1 \end{smallmatrix}]$

Spring 2008/2009

Construct a grammar for L(G) for the following language N(P)

$P = (\{p,q\}, \{a,b\}, \{Z, X\}, \delta, p, Z, \emptyset)$

Where the move function δ is given by:

$\delta(p,a,Z) = \{(p,XZ)\}$ $\delta(p,\epsilon,Z) = \{(p,\epsilon)\}$ $\delta(p,a,X) = \{(p,XX)\}$

$\delta(q,a,Z) = \{(q,\epsilon)\}$ $\delta(p,b,X) = \{(q,X)\}$ $\delta(q,b,Z)\} = \{(p,Z)\}$

		a=1	b=0	ϵ
p	Z	(p, XZ)	/	(p, ϵ)
	X	(p, XX)	(q, X)	/
q	Z	(q, ϵ)	(p, Z)	/
	X	/	/	/

1) $S \rightarrow$ [p, initial stack symbol, all state symbols]

$S \rightarrow [p, Z, {}^{p}_{q}]$

Therefor we have two productions

$S \rightarrow [p, Z, p] [p, Z, q]$

2) $(p, XZ) \in \delta(p, a, Z)$

$[p, Z, \substack{p \\ q}] \rightarrow a[p, X, \substack{p \\ q}][\substack{p \\ q}, Z, \substack{p \\ q}]$

We will have 4 productions.

$[p,Z,p] \rightarrow a[p,X,p][p,Z,p] \quad a[p,X,q][q,Z,p]$

$[p,Z,q] \rightarrow a[p,X,p][p,Z,q] \quad a[p,X,q][q,Z,q]$

$(p, XX) \in \delta(p, a, X)$ $\qquad XX \cdot pq = 4$

$[p, X, \substack{p \\ q}] \rightarrow a[p, X, \substack{p \\ q}][\substack{p \\ q}, X, \substack{p \\ q}]$

We will have 4 productions.

$[p,X,p] \rightarrow a[p,X,p][p,X,p] \quad a[p,X,q][q,X,p]$

$[p,X,q] \rightarrow a[p,X,p][p,X,q] \quad a[p,X,q][q,X,q]$

$(q, X) \in \delta(p, b, X)$ $\qquad X \cdot pq = 2$

We will have two productions

$[p, X, \substack{p \\ q}] \rightarrow b[q, X, \substack{p \\ q}]$

$[p,X,p] \rightarrow b[q,X,p]$

$[p,X,q] \rightarrow b[q,X,q]$

$(p, Z) \in \delta(q, b, Z)$ $\qquad Z \cdot qz = 2$

We will have two productions

$[q, Z, \substack{p \\ q}] \rightarrow b[p, Z, \substack{p \\ q}]$

$[q,Z,p] \rightarrow b[p,Z,p]$

$[q,Z,q] \rightarrow b[p,Z,q]$

$(q, \epsilon) \in \delta(q, a, Z)$

We will have 1 production

$[q, Z, q] \to a$

$(p, \epsilon) \in \delta(p, \epsilon, Z)$

We will have one production.

$[p, Z, p] \to \epsilon$

So total we have 16 productions

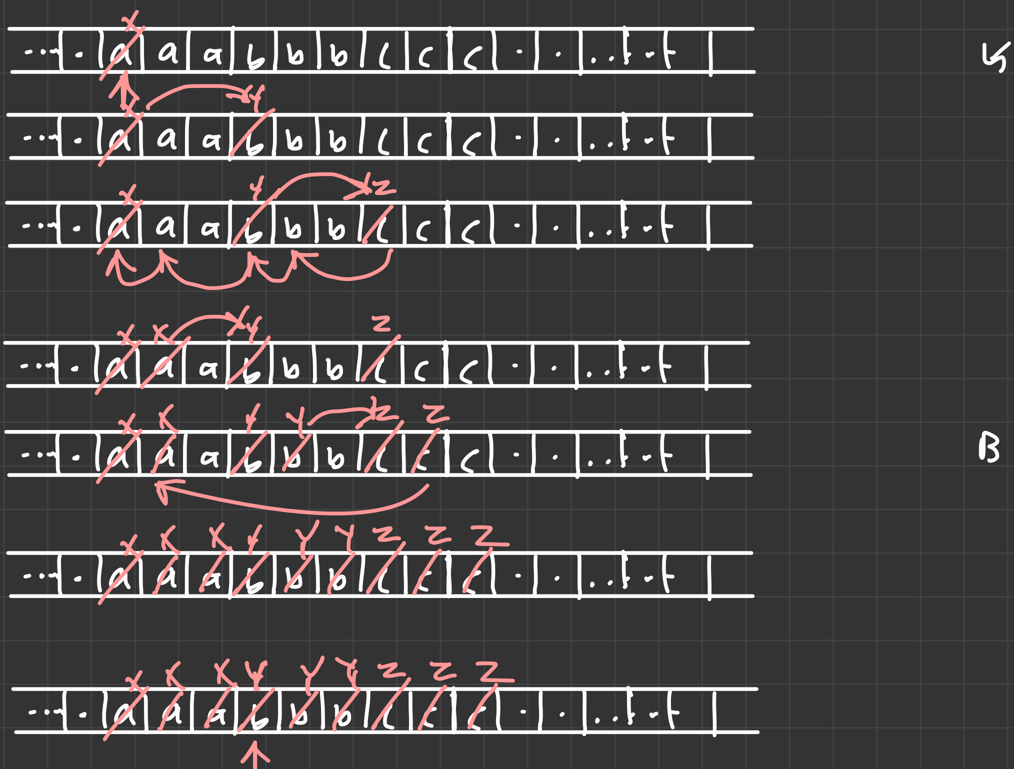
Q5

Construct a turing machine to accept

$L = \{a^n b^n c^n \mid n \geq 1\}$

$L = \{abc, aabbcc, aaabbbccc\}$

Load input string on tape



States	a	b	c	X	Y	Z	$\varnothing$
q_0	(q_1, X, R)	—	—	—	(q_4, Y, R)	—	—
q_1	(q_1, a, R)	(q_2, Y, R)	—	—	(q_1, Y, R)	—	—
q_2	—	(q_2, b, R)	(q_3, Z, L)	—	—	(q_2, Z, R)	—
q_3	(q_3, a, L)	(q_3, b, L)	—	(q_0, X, L)	(q_3, Y, L)	(q_3, Z, L)	—
q_4	—	—	—	—	(q_4, Y, R)	(q_4, Z, R)	(q_5, B, R)

TM, $M = (\{q_0, q_1, q_2, q_3, q_4, q_5\}, \{a, b, c\}, \{a, b, c, X, Y, Z, \varnothing\}, \delta, q_0, \varnothing, \{q_5\})$

Spring 2009

Construct a Turing Machine for
$L = \{0^k 1^j 0^i \mid i > j > k \geq 0\}$
Describe in words what you are doing, then formulate the process.

We know that the number of left zeros is greater than the number of 1's which is less than the number of right zeros.