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COSC 3340/6309

Examination 3

Wednesday, June 24, 2015, 10 am – 12 noon
Open Book and Notes

20 1. Prove that the following language L is not contextfree:

$$L = \{ a^i b^j a^i \mid j > i \geq 1 \}.$$

12 2. Construct a pda \mathbb{P} for the following language:

$$L = \{ 0^{3i} 1^i \mid i \geq 0 \} \text{ where } L = L(\mathbb{P}) \text{ (acceptance by final state).}$$

(18) 3. Construct a pda \mathbb{P} that accepts the following language by empty stack:

$$L = L(G) \text{ where } G = (T, N, P, E) \text{ with } T = \{ \text{id}, *, /, (,) \},$$

$$N = \{ E \}, \text{ and } P = \{ E \rightarrow E^* E \mid E/E \mid (E) \mid \text{id} \}.$$

Note: You must use the construction "cfg \rightarrow pda" given in class. Get G into GNF first!

(22) 4. Construct a grammar for $L(G)$ for the language $N(\mathbb{P})$:

$$\mathbb{P} = (\{ p, q \}, \{ a, b \}, \{ Z, X \}, \delta, p, Z, \emptyset) \text{ where the move function } \delta \text{ is given by}$$
$$\begin{array}{lll} \delta(p, b, Z) = \{ (p, XZ) \} & \delta(q, \epsilon, Z) = \{ (q, \epsilon) \} & \delta(p, b, X) = \{ (p, XX) \} \\ \delta(q, b, Z) = \{ (q, XZ) \} & \delta(q, a, X) = \{ (q, \epsilon) \} & \delta(p, a, X) = \{ (p, \epsilon) \}. \end{array}$$

20 5. Construct a Turing machine for the language in Question 1,

$$L = \{ a^i b^j a^i \mid j > i \geq 1 \}.$$

Describe first in words what you are doing, then formulate the formal Turing machine.

Points:

1: 20

2: 12

3: 18

4: 30

5: 20



99

Exam 3

Prove

1.) $L = \{a^i b^j a^i \mid j \geq i \geq 1\}$ is not a CFLProof: Indirect. Assume L is CFL. $\Rightarrow \exists G = (n, T, P, S)$ in Chomsky Normal Form
s.t. $L = L(G)$ Let $z \in L(G)$, by Pumping Lemma $z = uvwxy$ where
 $|v| \geq 1$, so then $uv^s w x^s y \in L(G) \forall s \geq 0$ Consider $z = a^{2^n} b^{2^n} \cdot a^{2^n}$ Case 1: v and x are only in a 's on the left side.
if $s=2$, then there are too many a 's on leftCase 2: v, x are for a 's on the right side.
if $s=2$, then right side a 's have too manyCase 3: v, x are for only b 's.
if $s=0$, then there are not enough b 'sCase 4: v, x are for left a 's and b 's.
if $s=2$, then there are too many left a 'sCase 5: v, x are for right a 's and b 's.
if $s=2$, then there are too many right a 'sCase 6: v, x are for both left and right a 's.
if $s=2$, then there are too many a 's, not enoughContradiction: $z \in L(G)$ but $\notin L$

Each case above results in a contradiction

2.) $L = \{0^3; 1^i | i \geq 0\} \quad L = L(P)$ (accepted by final state)

		0	1	ϵ
		$(q_0, z_0 z_0)$	(q_1, ϵ)	(q_1, ϵ)
		$(q_0, z_0 z_0)$	(q_1, ϵ)	(q_1, ϵ)
z_0	z_0	$/$	$/$	(q_1, ϵ)
	z_0	$/$	$/$	(q_1, ϵ)
z_1	z_1	$/$	$/$	(q_1, ϵ)
	z_1	$/$	$/$	(q_1, ϵ)

δ'

		0	1	ϵ
		$/$	$/$	$(q_0, z_0 z_0)$
		$/$	$/$	(q_1, ϵ)
z_0	z_0	$/$	$/$	(q_1, ϵ)
	z_0	$/$	$/$	(q_1, ϵ)
z_1	z_1	$/$	$/$	(q_1, ϵ)
	z_1	$/$	$/$	(q_1, ϵ)

		0	1	ϵ
		$(q_0, z_0 z_0)$	(q_1, ϵ)	(q_1, ϵ)
		$(q_0, z_0 z_0)$	(q_1, ϵ)	(q_1, ϵ)
z_0	z_0	$/$	$/$	(q_1, ϵ)
	z_0	$/$	$/$	(q_1, ϵ)
z_1	z_1	$/$	$/$	(q_1, ϵ)
	z_1	$/$	$/$	(q_1, ϵ)

		0	1	ϵ
		$/$	$/$	(q_1, ϵ)
		$/$	$/$	(q_1, ϵ)
z_0	z_0	$/$	$/$	(q_1, ϵ)
	z_0	$/$	$/$	(q_1, ϵ)
z_1	z_1	$/$	$/$	(q_1, ϵ)
	z_1	$/$	$/$	(q_1, ϵ)

Accepting
State

Exam 3 cont'd

- $L = L(G)$ where $G = (T, N, P, E)$ with $T = \{\text{id}, *, /, (\), \}$
- 3.) $N = \{E\}$, and $P = \{E \rightarrow E * E \mid E / E \mid (E) \mid \text{id}\}$

$$E \rightarrow E * E \mid E / E \mid (E) \mid \text{id}$$

GNF

remove left recursion

$$E \rightarrow (E) \mid \text{id} \mid (E) E' \mid \text{id} E'$$

$$E' \rightarrow * E \mid / E \mid * E E' \mid / E E'$$

GNF

$$E \rightarrow (EX) \mid \text{id} \mid (EX, E') \mid \text{id} E'$$

$$E' \rightarrow * E \mid / E \mid * E E' \mid / E E'$$

✓

Let X, \rightarrow)

	id	*	/	())	GE
E	$(q, \epsilon)(q, E')$	-	-	-	$(q, EX)(q, EX, E')$	-	ϵ
E'	-	$(q, E)(q, EE')$	$(q, E)(q, EE)$	-	-	-	ϵ
anything X,	-	-	-	-	-	(q, ϵ)	-

$$P_N = \{q\}, \{\text{id}, *, /, (\), \}, \{E, E'\}, \{X\}, \delta, \epsilon, E, \emptyset\}$$

Top of the stack is the left ✓

18

$$P = (\{p, q\}, \{a, b\}, \{z, x\}, \delta, p, z, \emptyset)$$

4)

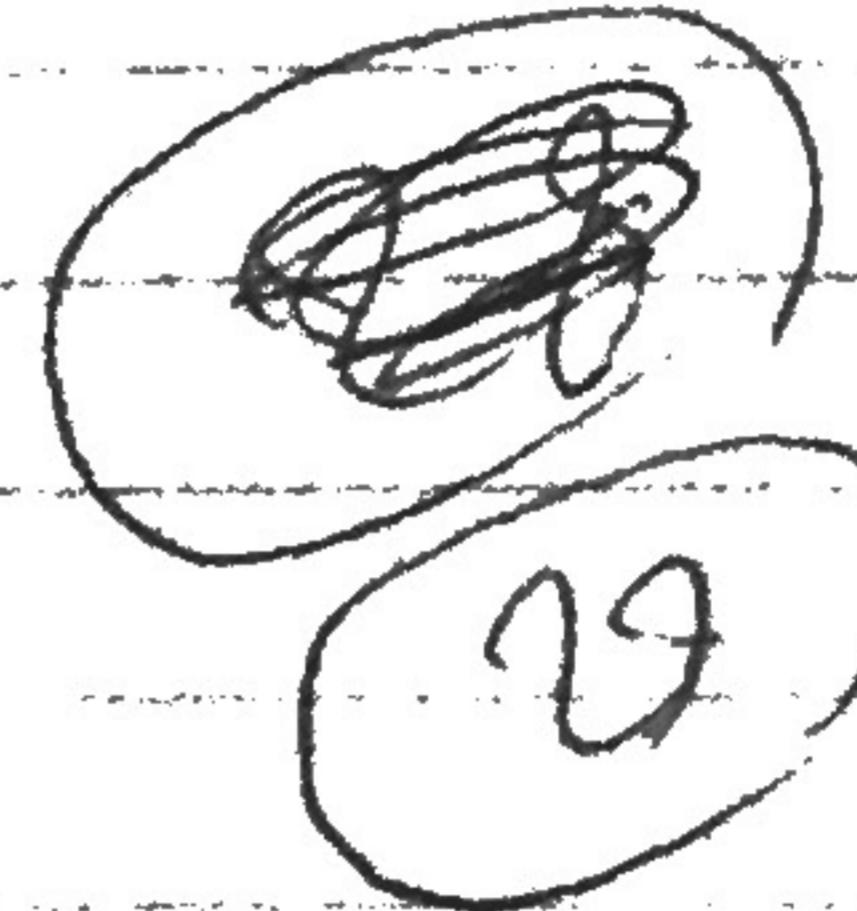
	a	b	ϵ
p	(p, z)	(p, x)	(p, ϵ)
x	(p, ϵ)	(p, x)	(p, ϵ)
z	(p, x)	(p, ϵ)	(q, ϵ)
q	(q, ϵ)	(q, ϵ)	(q, ϵ)

$$1. S \rightarrow [p, z, p] | [p, z, q]$$

$$2. (p, xz) \in \delta(p, b, z)$$

$2^2=4$

products $[p, z,] \rightarrow b[p, x, T, z,]$



$$[p, z, p] \rightarrow b[p, x, p] | b[p, x, q] | [q, z, p]$$

$$[p, z, q] \rightarrow b[p, x, p] | [p, z, q] | b[p, x, q] | [q, z, q]$$

$2^2=4$ $(p, xz) \in \delta(q, b, z)$

products

$$[q, z,] \rightarrow b[p, x, T, z,]$$

$$[q, z, p] \rightarrow b[p, x, p] | b[p, x, q] | [q, z, p]$$

$$[q, z, q] \rightarrow b[p, x, p] | [q, z, q] | b[p, x, q] | [q, z, q]$$

$(p, xx) \in \delta(q, b, x)$

$2^2=4$ $[p, x,] \rightarrow b[p, x, T, x,]$

products

$$[p, x, p] \rightarrow b[p, x, p] | b[p, x, q] | [q, x, p]$$

$$[p, x, q] \rightarrow b[p, x, p] | b[p, x, q] | [q, x, q]$$

$2^0=1$ $(q, \epsilon) \in \delta(q, \epsilon, z)$ $(q, \epsilon) \in \delta(q, a, x)$ $(p, \epsilon) \in \delta(p, a, x)$

products $[q, z, q] \rightarrow \epsilon$ $[q, x, q] \rightarrow a$ $[p, x, p] \rightarrow a$

Exam 3 Cont'd

5) $L = \{a^i b^j a^i \mid j > i \geq 1\}$

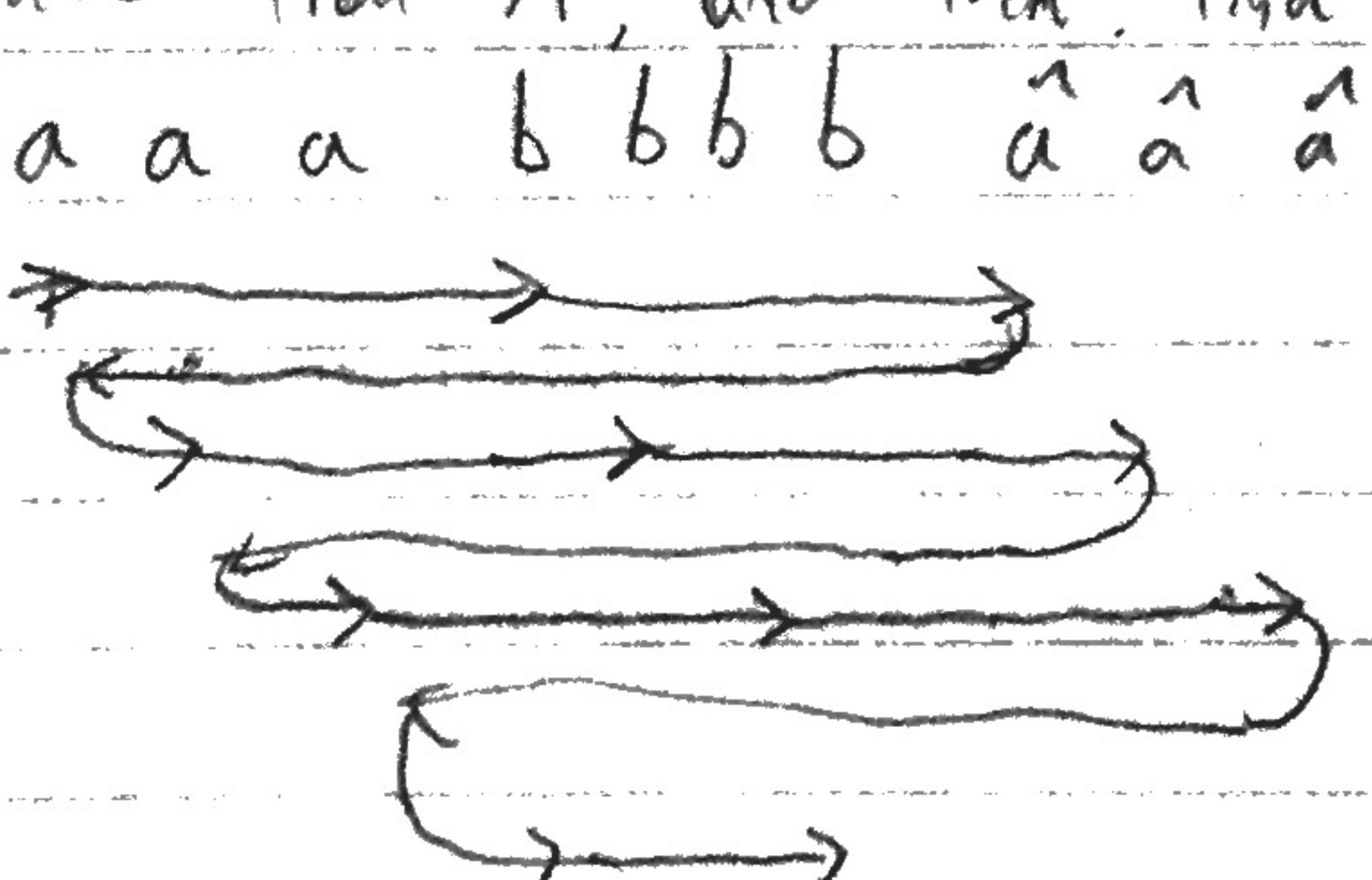
ex:

aaa bbb bbⁿ⁺¹ aⁿ λ

use A and \bar{A}
{as "visited" letters}

Let the 'a's on the right be named \hat{a} and λ be blank symbol
of a's are equal to # of \hat{a} 's and all
less than # of b's

Configure the Turing machine from the left, find the
first a and change it to \underline{A} , then move right,
until we find b , change it to \underline{B} . Move
right until we find \hat{a} , change it to $\underline{\hat{A}}$.
Now move left until we find the first a ,
then stop and move right to find the first a
and repeat the above process until B is the
next state from A , and then find all b and change them to B .



	a	b	\hat{a}	A	B	\bar{A}	X
q_0	(q_1, A, R)	-	-	-	-	-	-
q_1	(q_1, a, R)	(q_2, B, R)	-	-	-	-	-
q_2	-	(q_2, b, R)	(q_3, \bar{A}, L)	-	-	(q_2, \bar{A}, R)	-
q_3	(q_3, a, L)	(q_3, b, L)	-	(q_4, A, R)	(q_3, B, L)	(q_3, \bar{A}, L)	-
q_4	(q_1, A, R)	-	-	-	-	(q_5, B, R)	-
q_5	-	(q_6, B, R)	-	-	-	(q_5, B, R)	(q_5, A, R)
q_f	-	-	-	-	-	-	(q_f, f, L)

accepting
state \rightarrow