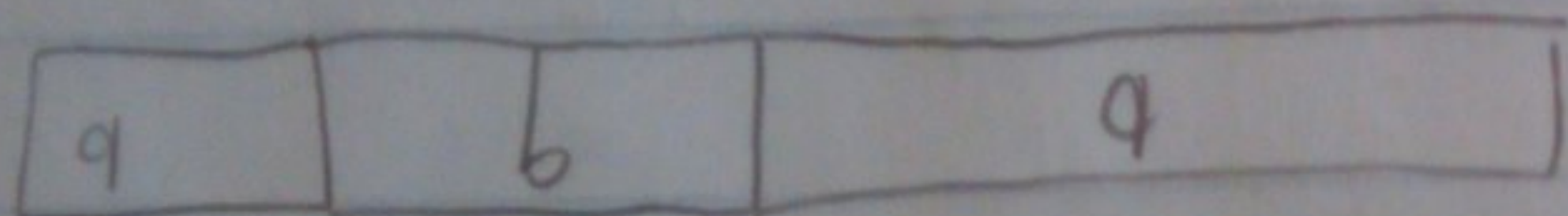


### Test III Overview

- ① Prove the following language  $L$  is not context free
- $$L = \{ a^i b^j a^i \mid i > j > k \geq 1 \}$$



Assume is c.f.  $\Rightarrow \exists$  CNF  $G = (N, T, P, S)$

Let  $m$  be the number of variables in  $G$ .

$$z^m \quad z = a^m + b^m + a^m + z \in L$$

$$\in L(G)$$

$$z = uvwx, \quad \begin{aligned} |vx| &\geq 1 \\ |vwx| &< z^{m-1} \end{aligned}$$

$$uv^i w x^i y \in L$$

$$\forall i \geq 0$$

$vwx$

$vx$  all a's: first group of a's  $i=2$  \*

second group of a's  $i=0$  \*

$vx$  all b's: either one of them should work

$i=0$ , \*

$i=2$ , \*

$vx$  at least 1 a and at least 1 b:  $i=2$  \*

$vx$  at least 1 b and at least one a:  $i=0$  \*

Contradictions in All cases,  $L$  is Not Context Free



# TEST III OVERVIEW

(2) construct a PDA  $P$  for the following language  
 $\{0^i 1^j 2^k \mid i=j\}$

		0		
$q_0$	$z_0$	$(q_1, x, z_0)$	/	$(q_f, z_0)$
	X	$(q_1, x, x)$	$(q_1, \epsilon)$	/
$q_1$	$z_0$	/	/	$(q_2, z_0)$
	X	/	$(q_1, \epsilon)$	/
$q_2$	$z_0$	/	/	/
	X	/	/	/
		final	state	



# TEST III OVERVIEW

3) Construct a pda  $P$  that accepts the following language by empty stack:

$$L = L(G) \text{ where } G = (T, N, P, E) \text{ with } T = \{id, +, *, (, )\}$$

$$N =$$

$$E \rightarrow E + E \mid E * E \mid (E) \mid id$$

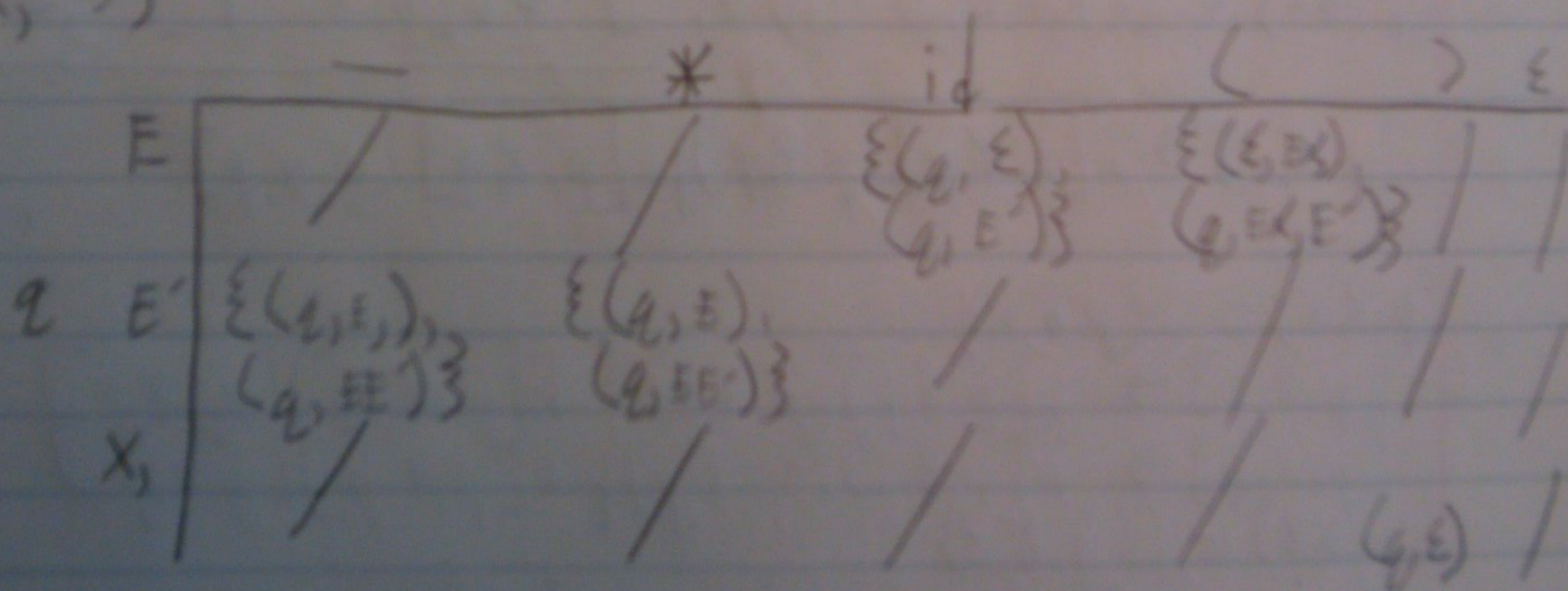
$$E \rightarrow (E) \mid id \mid (E) E' \mid id E'$$

$$E' \rightarrow + E \mid * E \mid + E E' \mid * E E'$$

$$E \rightarrow (EX \mid id \mid (EX, E' \mid id E'$$

$$E' \rightarrow + E \mid * E \mid + E E' \mid * E E'$$

$$X, \rightarrow )$$





④ construct a grammar for  $L(G)$  for the language  $N(p)$

$$P = \{ \dots \}$$

$$\delta(p, b, z) = \{ (p, xz) \} \quad \delta(q, \epsilon, z) = \{ (q, \epsilon) \} \quad \delta(p, b, x) = \{ (p, xx) \}$$

$$\delta(q, b, z) = \{ (p, xz) \} \quad \delta(q, a, x) = \{ (q, \epsilon) \} \quad \delta(p, a, x) = \{ (p, \epsilon) \}$$

$$S \rightarrow [p, z, p] [p, z, \epsilon]$$

$$[q, a, p] \rightarrow a [q_1, B_1, q_2] \cdots [q_m, B_m, q_{m+1}]$$

$$p = q_{m+1} \quad \delta(q, a, A) \text{ contains } (q_1, B_1, \dots, B_m)$$

$$\delta(q, a, A) \text{ contains } (p, \epsilon) : [q, A, p] \rightarrow a$$

$$(p, xz) \in \delta(p, b, z) \quad [p, z, ] \rightarrow b [p, x, ] [ , z ]$$

$$[p, z, p] \rightarrow b [p, x, p] [p, z, p] \mid b [p, x, q] [q, z, p]$$

$$(p, xx) \in \delta(p, b, x)$$

$$[p, x, p] \rightarrow b [p, x, p] [p, x, p] \mid b [p, x, q] [q, x, p]$$

$$[p, x, q] \rightarrow b [p, x, p] [p, x, p] \mid b [p, x, p] [q, x, p]$$

$$(p, xz) \in \delta(q, b, z)$$

$$[q, z, ] \rightarrow b [p, x, ] [ , z, ]$$

$$[q, z, p] \rightarrow b [p, x, p] [p, z, p] \mid b [p, x, q] [q, z, p]$$

$$[q, z, q] \rightarrow b [p, x, p] [p, z, p] \mid b [p, x, q] [q, z, p]$$

SEE NEXT PAGE



### TEST III OVERVIEW

(4)  $(p, \varepsilon) \in L(p, q, x)$   
 $[p, x, p] \rightarrow a$   
 $(q, \varepsilon) \in L(q, \varepsilon, z) :$   
 $[q, a, q] \rightarrow \varepsilon$   
 $(q, \varepsilon) \in L(q, a, x) :$   
 $[q, x, q] \rightarrow a$

15 productions



# TEST III OVERVIEW

- (5) Construct a Turing machine for the language in question 1.  
 $L = \{ a^k b^j a^i \mid i > j > k \geq 1 \}$

	a	b	a'	b'	b
q <sub>0</sub>	(q <sub>1</sub> , a, R)	/	/	/	/
q <sub>1</sub>	(q <sub>1</sub> , a, R)	(q <sub>2</sub> , b')	/	(q <sub>0</sub> , b', R)	/
q <sub>2</sub>	(q <sub>2</sub> , a, L)	/	(q <sub>3</sub> , a', R)	(q <sub>3</sub> , b', L)	/
q <sub>3</sub>	(q <sub>1</sub> , a', R)	/	/	(q <sub>4</sub> , b', R)	/
q <sub>4</sub>	/	(q <sub>5</sub> , b', L)	/	(q <sub>4</sub> , b', R)	/
q <sub>5</sub>	/	/	(q <sub>6</sub> , a', R)	(q <sub>5</sub> , b', L)	/
q <sub>6</sub>	/	(q <sub>7</sub> , b', R)	/	/	/
q <sub>7</sub>	(q <sub>8</sub> , a', L)	(q <sub>8</sub> , b', L)	(q <sub>7</sub> , a', R)	/	/
q <sub>8</sub>	/	(q <sub>8</sub> , b', L)	(q <sub>8</sub> , a', L)	(q <sub>9</sub> , b', R)	/
q <sub>9</sub>	/	(q <sub>7</sub> , b', R)	(q <sub>10</sub> , a', R)	/	/
q <sub>10</sub>	(q <sub>11</sub> , a, R)	/	(q <sub>10</sub> , a', R)	/	/
q <sub>11</sub>	(q <sub>11</sub> , a, R)	/	/	/	/
q <sub>f</sub>	A	c	c	e	p + i n g