Test III Overview

(1) Prove the following language L is not context free

$L = \{ a^k b^j a^i \mid i > j > k \geq 1 \}$

a	b	a

Assume is c.f. $\Rightarrow \exists$ CNG $G = (N, T, P, S)$

Let m be the number of variables in G.

2^m $\quad Z = a^{2^m} + b^{2^m} + a^{2^m+2} \in L$

$\in L(G)$

$Z = uvwxy \qquad |vx| \geq 1$

$|vwx| < 2^{m-1}$

$uv^i w x^i y \in L$

$\forall_i \geq 0$

vwx

vx all a's: first group of a's $i = 2$ *

second group of a's $i = 0$ *

vx all b's either one of them should work

$i = 0$, *

$i = 2$, *

vx at least 1 a and at least 1 b: $i = 2$ *

vx at least 1 b and at least one a: $i = 0$ *

contradictions in All cases, L is <u>Not</u> <u>Context</u> <u>Free</u>.

TEST III OVERVIEW

(2) construct a PDA P for the following language

$\{0^i 1^{2i} \mid i \geq 0\}$

		0		
q_0	Z_0	(q_0, XXZ_0)	/	(q_f, Z_0)
	X	(q_0, XXX)	(q_1, ε)	/
q_1	Z_0	/	/	(q_f, Z_0)
	X	/	(q_1, ε)	/
q_f	Z_0	/	/	/
	X	/	/	/

final state

TEST III OVERVIEW

③ Construct a pda P that accepts the following language by empty stack:

$L = L(G)$ where $G = (T, N, P, E)$ with $T = \{id, +, *, (,)\}$

$N =$

$E \to E+E \mid E*E \mid (E) \mid id$

$E \to (E) \mid id \mid (E)E' \mid id\,E'$

$E' \to +E \mid *E \mid +EE' \mid *EE'$

$E \to (EX_) \mid id \mid (EX_)E' \mid id\,E'$

$E' \to +E \mid *E \mid +EE' \mid *EE'$

$X_) \to)$

		+	*	id	(	)	ε
	E	/	/	$\{(q,\varepsilon), (q,E')\}$	$\{(q,EX_)), (q,EX_)E')\}$	/	/
q	E'	$\{(q,E), (q,EE')\}$	$\{(q,E), (q,EE')\}$	/	/	/	/
	$X_)$	/	/	/	/	(q,ε)	/

$\vdash (q,\varepsilon) \quad (p, XZ) \quad (q,\varepsilon)$

(4) construct a grammar for $L(G)$ for the language $N(P)$

$P = (\{$

$\delta(p,b,Z) = \{(p,XZ)\}$ $\quad \delta(q,\varepsilon,Z) = \{(q,\varepsilon)\}$ $\quad \delta(p,b,X) = \{(p,XX)\}$

$\delta(q,b,Z) = \{(p,XZ)\}$ $\quad \delta(q,a,X) = \{(q,\varepsilon)\}$ $\quad \delta(p,a,X) = \{(p,\varepsilon)\}$

$S \to [p,Z,p] \mid [p,Z,\varepsilon]$

$[q,a,p] \to a\underbrace{[q_1,B_1,q_2] \cdots [q_m,B_m,q_{m+1}]}$

$p = q_{m+1}$ $\qquad \delta(q,a,A)$ contains $(q_1, B_1, \dots, B_m)$

$\delta(q,a,A)$ contains (p,ε) : $[q,A,p] \to a$

$(p,XZ) = \delta(p,b,Z) \qquad [p,Z,\ ] \to b[p,X,\ ][\ ,Z$

$[p,Z,p] \to b[p,X,p][p,Z,p] \mid b[p,X,q][q,Z,p]$

$(p,X,X) \in \delta(p,b,X)$

$[p,X,p] \to b[p,X,p][p,X,p] \mid b[p,X,q][q,X,p]$

$[p,X,q] \to b[p,X,p][p,X,p] \mid b[p,X,p][q,X,p]$

$(p,XZ) \in \delta(q,b,Z)$

$[q,Z,\ ] \to b[p,X,\ ][\ ,Z,\ ]$

$[q,Z,p] \to b[p,X,p][p,Z,p] \mid b[p,X,q][q,Z,p]$

$[q,Z,q] \to b[p,X,p][p,Z,p] \mid b[p,X,q][q,Z,p]$

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TEST III OVERVIEW

(4) $(p, \varepsilon) \in \delta(p, a, X)$

$[p, X, p] \rightarrow a$

$(q, \varepsilon) \in \delta(q, \varepsilon, Z)$:

$[q, a, q] \rightarrow \varepsilon$

$(q, \varepsilon) \in \delta(q, a, X)$:

$[q, X, q] \rightarrow a$

$\boxed{\text{15 productions}}$

TEST III OVERVIEW

(5) Construct a turing machine for the language in Question 1.

$L = \{ a^k b^j a^i \mid i > j > k \geq 1 \}$

	a	b	a'	b'	$\not{b}$
q_0	(q_1, a', R)	/	/	/	/
q_1	(q_1, a, R)	(q, b')	/	(q_0, b', R)	/
q_2	(q_2, a, L)	/	(q_3, a', R)	(q_2, b', L)	/
q_3	(q_1, a', R)	/	/	(q_4, b, R)	/
q_4	/	(q_5, b, L)	/	(q_4, b', R)	/
q_5	/	/	(q_6, a', R)	(q_5, b, L)	/
q_6	/	(q_7, b', R)	/	/	/
q_7	(q_8, a', L)	(q_8, b, L)	(q_7, a', R)	/	/
q_8	/	(q_8, b, L)	(q_8, a', L)	(q_9, b', R)	/
q_9	/	(q_7, b', R)	(q_{10}, a', R)	/	/
q_{10}	(q_{11}, a, R)	/	(q_{10}, a', R)	/	/
q_{11}	(q_{11}, a, R)	/	/	/	/
q_f	A c c e p t i n g				