COSC 3340/6309
Examination 3
Thursday, June 29, 2017, 10 am – 12 noon
Open Book and Notes

1. Prove that the following language L is not contextfree:

$L = \{ a^i b^j a^i \mid j \geq i \geq 1 \}$.

2. Construct a pda $\mathbb{P}$ for the following language:

$L = \{ 0^{5i} 1^i \mid i \geq 0 \}$ where $L = L(\mathbb{P})$ (acceptance **by final state**).

3. Construct a pda $\mathbb{P}$ that accepts the following language **by empty stack**:

$L = L(G)$ where $G = (T, N, P, E)$ with $T = \{ id, *, /, (,) \}$,
$N = \{E\}$, and $P = \{ E \to E*E \mid E/E \mid (E) \mid id \}$.

Note: You <u>must</u> use the construction "cfg → pda" given in class. Get G into GNF first!

4. Construct a grammar for L(G) for the language $N(\mathbb{P})$:

$\mathbb{P} = (\{p,q\}, \{a,b\}, \{Z,X\}, \delta, p, Z, \varnothing)$ where the move function δ is given by

$\delta(p,b,Z) = \{(p,XZ)\}$ $\quad \delta(q,\varepsilon,Z) = \{(q,\varepsilon)\}$ $\quad \delta(p,b,X) = \{(p,XX)\}$
$\delta(q,b,Z) = \{(p, XZ)\}$ $\quad \delta(q,a,X) = \{(q, \varepsilon)\}$ $\quad \delta(p,a,X) = \{(p,\varepsilon)\}$.

5. Construct a Turing machine for the language in Question 1,

$L = \{ a^i b^j a^i \mid j \geq i \geq 1 \}$.

Describe first in words what you are doing, then formulate the formal Turing machine.

Points:	**1: 20**	**2: 12**	**3: 18**	**4: 30**	**5: 20**

Exam 3

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1) $L = \{a^i b^j a^i \mid j \geq i \geq 1\}$

Assume L is a CFL, then $\exists\ G = (N,T,P,S)$ in CNF such that $L = L(G)$

let $z \in L(G)$ it can be shown as $z = uvwxy$

with $|vx| \geq 1$ then by $uv^s wx^s y \in L(G)\ \forall s \geq 0$

consider $z = a^{2^n} b^{2^n} a^{2^n}$

case 1: v and x are a's on the right, for $s=2$ there are too many a's on the right hand side

case 2: v and x are a's on the left, for $s=2$ there are too many a's on the left hand side

case 3: v and x are a's right and b's, for $s=2$ there are too many a's on the right

case 4: v and x are a's on the left and b's, for $s=2$ there are too many a's on the left

case 5: v and x are b's, for $s=0$ there are too few b's

case 6: v and x are left and right a's. for $s=2$ there are too many a's

$z \in L(G)$ but $\notin L$ as it causes contradictions which we found for every case, Proving L is not context free

$L = \{ 0^{5i} 1^{i} \mid i \geq 0 \}$ where $L = L(P)$

first step is to construct a Pda that accepts by empty stack then change it to one accepting of the final state

$P_e = (\{0,1\}, \{q_0, q\}, \{Z_0, Z\}, \delta, q_0, \emptyset, Z_0)$

		0	1	ε
q_0	Z_0	—	$(q_0, ZZZZZZ_0)$	(q, ε)
	Z	(q, ε)	$(q_0, ZZZZZZ)$	—
q_1	Z_0	—	—	(q, ε)
	Z	—	(q, ε)	—

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$P_f = (\{0,1\}, \{q_0', q_0, q_1, q_f\}, \{Z_0', Z_0, Z\}, \delta', q_0', q_f, Z_0')$

final state on back

↓

		0	1	ε
q_0'	Z_0	/	/	$(q_0, Z_0 Z_0')$
	Z_0	/	/	/
	Z	/	/	/
q_0	Z_0'	/	/	/
	Z_0	/	$(q_0, ZZZZZZ_0)$	(q_0, ε)
	Z	(q_0, ε)	$(q_0, ZZZZZZ)$	/
q_1	Z_0'	/	/	(q_f, ε)
	Z_0	/	/	(q_1, ε)
	Z	/	(q_1, ε)	/
q_f	Z_0'	/	/	/
	Z_0	/	/	/
	Z	/	/	/

$E \to E*E \mid E/E \mid (E) \mid id$

end left recursion

$E \to (E) \mid id \mid (E)E' \mid idE'$

$E' \to *E \mid /E \mid *EE' \mid /EE'$

$E \to (EX_1 \mid id \mid (EX_1E' \mid idE'$

$E' \to *E \mid /E \mid *EE' \mid /EE'$

$X_1 \to)$

	*	/	(	)	id	ε
E	/	/	(q, EX_1) (q, EX_1E')	/	(q, ε) (q, E')	/
E'	(q, E) (q, EE')	(q, E) (q, EE')	/	/		/
X_1	/	/	/	(q, ε)		/

q

$P_n = (\{q\}, \{*, /, (,), id, \varepsilon\}, \{E, E', X_1\}, \delta, q, E, \emptyset\}$

		a	b	ε
p	z	—	(p, xz)	—
	x	(p, ε)	(p, xx)	—
q	z	—	(p, xz)	(q, ε)
	x	(q, ε)	—	—

1) $S \to [p, z, p] \mid [p, z, q]$

2) $(p, xz) \in \delta(p, b, z)$

4 $\begin{cases} [p, z, p] \to b[p, x, p][p, z, p] \mid b[p, x, q][q, z, p] \\ [p, z, q] \to b[p, x, p][p, z, q] \mid b[p, x, q][q, z, q] \end{cases}$

$(p, xz) \in \delta(q, b, z)$

4 $\begin{cases} [q, z, p] \Rightarrow b[p, x, p][p, z, p] \mid b[p, x, q][q, z, p] \\ [q, z, q] \Rightarrow b[p, x, p][p, z, q] \mid b[p, x, q][q, z, q] \end{cases}$

✓

$(p, xx) \in \delta(p, b, x)$

4 $\begin{cases} [p, x, p] \Rightarrow b[p, x, p][p, x, p] \mid b[p, x, q][q, x, p] \\ [p, x, q] \Rightarrow b[p, x, p][p, x, q] \mid b[p, x, q][q, x, q] \end{cases}$

$(p, \varepsilon) \in \delta(p, a, x)$ $(q, \varepsilon) \in \delta(q, \varepsilon, z)$ $(q, \varepsilon) \in \delta(q, a, x)$

1 $[p, x, p] \Rightarrow a$ 1 $[q, z, q] \to \varepsilon$ 1 $[q, x, q] \Rightarrow a$

15 total Productions

✓

$L = \{ a^i b^j a^i \mid j \geq i \geq 1 \}$

let right a's be renamed as a^+ and ß is blank
we know the number of a's on the left is equal to the number of a's on the right and b is strictly greater than or equal to the number of a's

for the turing machine, start from the left hand side. Prime a for left a's and then continue right. once a b is reached prime it. move right until ß is reached. now start moving left till a^+ is reached, mark it as \$. move left until a'. now repeat process above unless a' is within the current state and b' is the next state. find and mark remaining b's. all characters should be marked and moved to final accepting state if the first condition is met

✓

for $i=j=1$ (aba^+)

move from left to right while marking left a and b pass a^+ and reach ß. turn right and mark a^+ as \$ and continue moving until a', move left to b' and go to final accepting state

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	a	b	a^+	a'	b'	$	␢
q_0	(q_1, a', R)	/	/	/	/	/	/
q_1	(q_1, a, R)	(q_2, b', R)	/	/	/	/	$(q_3, ␢, L)$
q_2	/	(q_2, b, R)	(q_2, a^+, R)	/	/	/	/
q_3	/	/	$(q_4, \$, L)$	/	/	/	/
q_4	(q_4, a, L)	(q_4, b, L)	(q_4, a^+, L)	(q_5, a', R)	(q_4, b', L)	/	/
character b' → q_5	(q_6, a', R)	/	/	/	(q_f, b', R)	/	/
q_6	(q_6, a, R)	(q_7, b', R)	/	/	(q_6, b', R)	/	/
q_7	/	(q_7, b, R)	(q_7, a^+, R)	/	/	$(q_8, \$, L)$	/
q_8	/	/	$(q_9, \$, L)$	/	/	/	/
q_9	(q_9, a, L)	(q_9, b, L)	(q_9, a^+, L)	(q_{10}, a', R)	(q_9, b', L)	/	/
q_{10}	(q_{11}, a', R)	/	/	/	(q_{11}, b', R)	/	/
q_{11}	(q_6, a, R)	(q_{12}, b', R)	/	/	/	$(q_f, \$, R)$	/
q_{12}	/	(q_{12}, b', R)	/	/	/	/	/
accepting state q_f	/	/	/	/	/	/	/