# COSC 3340/6309
## Examination 3
## Wednesday, June 26, 2013, 10 am – 12 noon
## Open Book and Notes

**1**. Prove that the following language L is not contextfree:

$$L = \{ a^i b^j a^i \mid j > i \geq 1 \}.$$

**2**. Construct a pda $\mathbb{P}$ for the following language:

$$L = \{ 0^i 1^{3i} \mid i \geq 0 \} \text{ where } L = L(\mathbb{P}) \text{ (acceptance \textbf{by final state}).}$$

**3**. Construct a pda $\mathbb{P}$ that accepts the following language **by empty stack**:

$$L = L(G) \text{ where } G = (T, N, P, E) \text{ with } T = \{ id, *, /, (,) \},$$
$$N = \{E\}, \text{ and } P = \{ E \rightarrow E*E \mid E/E \mid (E) \mid id \}.$$

**Note: You <u>must</u> use the construction "cfg → pda" given in class. Get G into GNF first!**

**4**. Construct a grammar for L(G) for the language N($\mathbb{P}$):

$$\mathbb{P} = ( \{p,q\}, \{a,b\}, \{Z,X\}, \delta, p, Z, \varnothing ) \text{ where the move function } \delta \text{ is given by}$$

| | | |
|---|---|---|
| $\delta(p,b,Z) = \{(p,XZ)\}$ | $\delta(q,\varepsilon,Z) = \{(q,\varepsilon)\}$ | $\delta(p,b,X) = \{(p,XX)\}$ |
| $\delta(q,b,Z) = \{(p, XZ)\}$ | $\delta(q,a,X) = \{(q, \varepsilon)\}$ | $\delta(p,a,X) = \{(p,\varepsilon)\}.$ |

**5**. Construct a Turing machine for the language in Question 1,

$$L = \{ a^i b^j a^i \mid j > i \geq 1 \}.$$

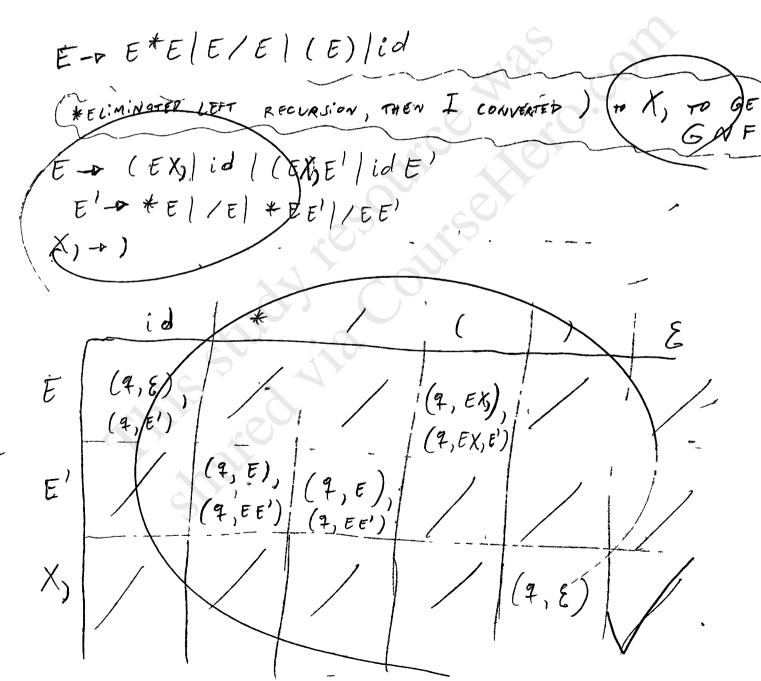Describe first in words what you are doing, then formulate the formal Turing machine.

| **Points:** | **1: 20** | **2: 12** | **3: 18** | **4: 30** | **5: 20** |
|---|---|---|---|---|---|

① **Claim:** L is not context free

**Proof:** Assume L: CFL, then $\exists$ CNF G generating L, so that:

$$Z = a^{2^n} b^{2^{n+1}} a^{2^n} \in L(G)$$

\* then, according to pumping lemma we have:

$$Z = UVWXY \text{ and } |VX| \geq 1 \text{ and } |VWX| \leq 2^{n-1}, \text{ so}$$

$$UV^s W X^s Y \in L(G) \ \forall \ s \geq 0$$

\* Thus we have these scenarios:

Ⓐ VWX has only left a's, then number of left a's will be bigger than b's (contradiction)

Ⓑ VWX has only b's, then $|VX| \geq 1$ and taking $s = 0$, we decrease the number of right a's while leaving the others intact (contradiction)

Ⓒ VWX has only right a's, then number of right a's will be bigger than b (contradiction)

Ⓓ V has b's and X has a's, then $|\text{left a's}| \leq |\text{right a's}|$ (contradiction)

Ⓔ V has left a's and X has right a's, then number of a's will be bigger than b's, ($s = 2$) (contradiction)

(F) $V$ HAS LEFT $a$'s AND $X$ HAS $b$'s, THEN

| LEFT $a$'s | $\neq$ | RIGHT $a$'s | (CONTRADICTION)

SINCE ALL SCENARIOS ARE CONTRADICTIONS, $L$ IS NOT A CONTEXT FREE LANGUAGE

② $L = \{0^i 1^{3i} \mid i \geq 0\}$ && $L = L_f(P)$

↓ LEFT STACK ⌐

|  |  | 0 | 1 | $\varepsilon$ |
|---|---|---|---|---|
| $q_0$ | $z_0$ | $(q_0, ZZZz_0)$ |  | $(q_f, z_0)$ |
|  | $Z$ | $(q_0, ZZZZ)$ | $(q_1, \varepsilon)$ |  |
| $q_1$ | $z_0$ |  |  | $(q_f, z_0)$ |
|  | $Z$ |  | $(q_1, \varepsilon)$ |  |
| $q_f$ | $z_0$ | ACCEPTING |  |  |
|  | $Z$ |  |  |  |

③ * LEFT STACK · ]

$P = E \to E*E \mid E / E \mid (E) \mid id$

$G = (T, N, P, E)$

$T = \{id, *, /, (, )\}$

$N = \{E\}$

$E \to E*E \mid E / E \mid (E) \mid id$

(*ELIMINATED LEFT RECURSION, THEN I CONVERTED ) to $X_)$ TO GET GNF

$E \to (E X_) \mid id \mid (E X_) E' \mid id E'$

$E' \to *E \mid /E \mid *E E' \mid /E E'$

$X_) \to )$

| $q$ | id | * | / | ( | ) | $\varepsilon$ |
|---|---|---|---|---|---|---|
| $E$ | $(q, \varepsilon)$, $(q, E')$ | | | $(q, E X_))$, $(q, E X_) E')$ | | |
| $E'$ | $(q, E)$, $(q, E E')$ | $(q, E)$, $(q, E E')$ | | | | |
| $X_)$ | | | | | $(q, \varepsilon)$ | ✓ |

④ ① **RULE:**

$$S \to [P, Z, P] \mid [P, Z, q] \qquad \boxed{* 2 \text{ PRODUCTIONS}}$$



② **RULE:**

• $(P, XZ) \to \delta(P, b, Z) \qquad \boxed{* 4 \text{ PRODS.}}$

$$[P, Z, \begin{smallmatrix}P\\q\end{smallmatrix}] \to b[P, X, \begin{smallmatrix}P\\q\end{smallmatrix}][\begin{smallmatrix}P\\q\end{smallmatrix}, Z, \begin{smallmatrix}P\\q\end{smallmatrix}]$$

$$\triangleright [P, Z, P] \to b[P, X, P][P, Z, P] \mid b[P, Z, q][q, Z, P]$$

$$\triangleright [q, Z, q] \to b[P, X, P][P, Z, q] \mid b[P, X, q][q, Z, q]$$

• $(P, \varepsilon) \to \delta(P, a, X) \qquad \boxed{* 1 \text{ PROD.}}$

$$[P, X, P] \to a$$



• $(q, \varepsilon) \to \delta(q, a, X) \qquad \boxed{* 1 \text{ PROD.}}$

$$[q, X, q] \to a$$



• $(q, \varepsilon) \to \delta(q, \varepsilon, Z) \qquad \boxed{* 1 \text{ PROD}}$

$$[q, Z, q] \to \varepsilon$$

* CONTINUE ON BACK OF PAGE

$$\bullet \ (P, XX) \to \delta(P, b, X) \qquad \boxed{* \ 4 \ \text{PRODS.}}$$

$$\left[ \begin{array}{l} \to [P, X, \dot{P}] \to b[P, X, P][P, X, P] \mid b[P, X, q][q, X, P] \\ \to [P, X, q] \to b[P, X, P][P, X, q] \mid b[P, X, q][q, X, q] \end{array} \right.$$

$$\bullet \ (P, XZ) \to b(q, b, Z) \qquad \boxed{* \ 4 \ \text{PRODS}}$$

$$\left[ \begin{array}{l} \to [P, Z, P] \to b[P, X, P][P, Z, P] \mid b[P, X, q][q, Z, P] \\ \to [P, Z, q] \to b[P, X, P][P, Z, q] \mid b[P, X, q][q, Z, q] \end{array} \right.$$

$$17 \quad \text{PRODUCTIONS TOTAL}$$

(5)  $L = \{ a^i b^j a^i \mid j \geq i \geq 1 \}$

\* $|\text{LEFT } a\text{'s}| = |\text{RIGHT } a\text{'s}|$

THE FOLLOWING TURING MACHINE WILL TRAVERSE THROUGH THE TAPE COMPARING LEFT $a$'s WITH RIGHT $a$'s AS A FIRST STEP. FOR EVERY COUPLED LEFT AND RIGHT $a$'s, A $b$ WILL BE MARKED AS $b'$. WHEN ALL LEFT $a$'s HAVE BEEN COUPLED WITH ALL RIGHT $a$'s, THEN THE MACHINE WILL TRACK THE REMAINING $b$'s TO COMPLETE THE WORD INPUTTED.

TURING MACHINE ON BACK OF PAGE ⟶

Needs more detail

| | a | b | a' | b' | b̶ |
|---|---|---|---|---|---|
| $q_0$ | $(q_1, a', R)$ | | | | |
| $q_1$ | $(q_1, a, R)$ | $(q_2, b, R)$ | | $(q_1, \ldots)$ | |
| $q_2$ | $(q_3, a', L)$ | $(q_2, b, R)$ | $(q_2, a', R)$ | | |
| $q_3$ | | $(q_4, b', L)$ | | $(q_3, b', L)$ | |
| $q_4$ | $(q_4, a, L)$ | $(q_4, b, L)$ | $(q_5, a', R)$ | $(q_4, b', L)$ | |
| $q_5$ | $(q_6, a', R)$ | $(q_5, b, R)$ | $(q_6, a', L)$ | $(q_5, b', R)$ | |
| $q_6$ | $(q_6, a, R)$ | $(q_6, b, R)$ | | $(q_7, b', R)$ | |
| $q_7$ | $(q_8, a', L)$ | | $(q_7, a', R)$ | $(q_7, b', R)$ | |
| $q_8$ | | $(q_7, b', L)$ | $(q_8, a', L)$ | $(q_8, b', L)$ | |
| $q_9$ | $(q_4, a, L)$ | $(q_9, b, L)$ | $(q_{10}, a', R)$ | | |
| $q_{10}$ | | $(q_{10}, b', R)$ | $(q_{10}, a', R)$ | $(q_{10}, b', R)$ | $(q_f, b, R)$ |
| $q_f$ | ACCEPTING | | | | |