Automata Exam #3 #Q2

2) $L=\{0^{3i}1^{i} \mid i\geq 0\}$ where $L=L_f(P)$

notations used

$Z_0 \rightarrow$ initial stack symbol

$q_0 \rightarrow$ initial state

$q_f \rightarrow$ final state

$Z \rightarrow$ Stack symbol

Empty Set

$P=(\{0,1\},\{q_0,q_1,q_2,q_3\},\{Z_0,Z\},\delta,q_0,\emptyset,Z_0)$

δ		0	1	ε
q_0	Z_0	(q_1,Z_0)	/	(q_0,ε)
	Z	(q_1,Z)	(q_3,ε)	/
q_1	Z_0	(q_2,Z_0)	/	/
	Z	(q_2,Z)	/	/
q_2	Z_0	(q_0,ZZ_0)	/	/
	Z	(q_0,ZZ)	/	/
q_3	Z_0	/	/	(q_3,ε)
	Z	/	(q_3,ε)	/

Final State acception

$P_f=(\{0,1\},\{q_0',q_0,q_1,q_2,q_3,q_f\},\{Z_0',Z_0,Z\},\delta',q_0',q_f,Z_0')$

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δ'		0	1	ε
q_0'	Z_0'	/	/	$(q_0, Z_0 Z_0')$
	Z_0	/	/	/
	Z	/	/	/
q_0	Z_0'	/	/	(q_f, ε)
	Z_0	(q_1, Z_0)	/	(q_0, ε)
	Z	(q_1, Z)	(q_3, ε)	/
q_1	Z_0'	/	/	/
	Z_0	(q_2, Z_0)	/	/
	Z	(q_2, Z)	/	/
q_2	Z_0'	/	/	/
	Z_0	(q_0, ZZ_0)	/	/
	Z	(q_0, ZZ)	/	/
q_3	Z_0'	/	/	(q_f, ε)
	Z_0	/	/	(q_3, ε)
	Z	/	(q_3, ε)	/
q_f	Z_0'	/	/	/
	Z_0	/	/	/
	Z	/	/	/

2

$L = \{0^i 1^{4i} \mid i \geq 0\}$ where $L = L_f(P)$

Notations used:

$Z_0 \rightarrow$ initial stack symbol

$q_0 \rightarrow$ initial state

$q_f \rightarrow$ final state

$Z \rightarrow$ stack symbol

left ⊐ side will be used to write the top of the stack

		0	1	ε
q_0	Z_0	$(q_0, ZZZZZ_0)$	—	(q_f, ε)
	Z	$(q_0, ZZZZZ)$	(q_1, ε)	—
q_1	Z_0	—	—	(q_f, ε)
	Z	—	(q_1, ε)	—
q_f		—	—	—

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Accepting

00111111111

$(q_0, 0, Z_0) \rightarrow (q_0, ZZZZZ_0)$
$(q_0, 0, Z) \rightarrow (q_0, ZZZZZ)$
$(q_0, 1, Z) \rightarrow (q_1, \varepsilon)$
$(q_1, 1, Z) \rightarrow (q_1, \varepsilon)$

$(q_1, 1, Z_0) \rightarrow (q_1, \varepsilon)$
$(q_0, \varepsilon, Z_0) \rightarrow (q_1, \varepsilon)$

2) $L = \{0^i 1^{4i} \mid i \geq 0\}$

$P_e = (\{0,1\}, \{q_0, q_1\}, \{Z_0, Z\}, \delta, q_0, \emptyset, Z_0)$

Top of stack is on left.

δ		0	1	ε
q_0	Z_0	$\{(q_0, ZZZZZ_0)\}$	/	$\{(q_1, \varepsilon)\}$
	Z	$\{(q_0, ZZZZZ)\}$	$\{(q_1, \varepsilon)\}$	/
q_1	Z_0	/	/	$\{(q_1, \varepsilon)\}$
	Z	/	$\{(q_1, \varepsilon)\}$	/

Final state acception

$P_f = (\{0,1\}, \{(q_0', q_0, q_1, q_f)\}, (Z_0', Z_0, Z), \delta', q_0', q_f, Z_0')$

δ'		0	1	ε
q_0'	Z_0'	/	/	$\{(q_0, Z_0 Z_0')\}$
	Z_0	/	/	/
	Z	/	/	/
q_0	Z_0'	/	/	/
	Z_0	$\{(q_0, ZZZZZ_0)\}$	/	$\{(q_1, \varepsilon)\}$
	Z	$\{(q_0, ZZZZZ)\}$	$\{(q_1, \varepsilon)\}$	/
q_1	Z_0'	/	/	/
	Z_0	/	/	$\{(q_f, \varepsilon)\}$
	Z	/	$\{(q_1, \varepsilon)\}$	$\{(q_1, \varepsilon)\}$
q_f	Z_0'	/	/	/
	Z_0	/	/	/
	Z	/	/	/

final state

1

PDA → General Way

Transition (move) Function of the PDA

$(q, b, X) = (p, y)$

current state — q; reading input symbol — b; current top of the stack — X; new state — p; stack symbol that replaces X at the top of the stack — y

if: $\boxed{y == \varepsilon}$ — when you pop of the top of the stack, put an epsilon there

* it means stack is popped

$y == X$

* nothing changed (neither popped/pushed)

. What does $L = \{0^n 1^n \mid n \geq 0\}$ mean?

* The number of zeros (n) should exactly be equal to the ones (n)
* If we have n # of zeros, we also should have n # of ones
* Whenever we see a '0', push it onto the stack
* Whenever we see a '1', pop the corresponding 0 from the stack
* When input is consumed, if the stack is empty, ACCEPT

EX $n = 3$

"000111" ε

push push push pop pop pop — at the end we have an epsilon (empty state)

Assume bottom of the stack (initial stack is Z_0)

1) $(q_0, 0, Z_0) = (q_0, 0Z_0)$ — top of the stack is now 0

current state — q_0; reading input (we want to PUSH it) — 0; currently top of the stack — Z_0; pushed — 0

we are still at q_0 state cuz we need to get 0 n times, which means we need to stay same until getting a new input.

2) $(q_0, 0, 0) = (q_0, 00)$ — input: 0; top of the stack: 0

3) $(q_0, 1, 0) = (q_1, \varepsilon)$

when there is a pop operation, it means the top of the stack is ε

* now we need to pop the corresponding 0
* because ε remove a 0 from the stack

4) $(q_1, 1, 0) = (q_1, \varepsilon)$

* top of the stack is still 0,

5) $(q_1, \varepsilon, Z_0) = (q_f, Z_0)$

2

PDA

→ Leiss's way

Example

Construct a pda P for:

$L = \{0^i 1^{4i} \mid i \geq 0\}$

* The number of ones (4i) should be 4x the number of zeros (i)

* If we have i # of zeros, we also should have (4i) # of ones.

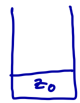
* Whenever we see a '0', push 4 states (Z) into the stack

* Whenever we see a '1', pop the corresponding 0 (4 states) from the stack

Ex// i=2

$\underset{push}{00}\,\underbrace{11111111}_{pop}\,\underset{bottom}{\varepsilon}$

\# Assume the bottom of the stack - initial stack symbol is Z_0



1) $(p_0, 0, Z_0)$ → in general PDA way → $(p_0, 0Z_0)$

input, top of the stack

* but in Leiss way, we can just write four state instead of 0

→ in Leiss way → $(p_0, ZZZZZ_0)$ new top of the stack

2) $(p_0, 0, Z) = (p_0, \underbrace{ZZZZ}_{\text{new 4x set}}Z)$ old top of the stack

3) $(p_0, 1, Z) = (p_1, \varepsilon)$ * new state (q_1) → when you switch the input character, also switch the state

↳ remember: when you see a 1 → pop the corresponding 4Z states

... and when you pop a symbol from the stack, transition result is ε

4) $(p_1, 1, Z) = (p_1, \varepsilon)$ we've pushed Z twice and this one the last one.

5) $(p_1, 1, Z_0) = (q_1, \varepsilon)$ now remove the initial stack symbol

6) $(p_1, \varepsilon, Z_0) = (q_1, \varepsilon)$ (final state)

$P = (\{0,1\}, \{q_0, q_1\}, \{Z_0, Z\}, \delta, q_0, \emptyset, Z_0)$

input symbols, states, stack symbols, transition, initial state, initial stack symbol

③

PDA

→ Creating table

input symbols

stack symbols

states

δ

		0	1	ε
q_0	Z_0	$(q_0, ZZZZZ_0)$	/	(q_1, ε)
	Z	$(q_0, ZZZZZ)$	(q_1, ε)	/
q_1	Z_0	/	/	(q_1, ε)
	Z	/	(q_1, ε)	/

in order to find these, read the transition functions, and place the result

EX// $(q_0, 0, Z_0) = (p_0, ZZZZZ_0)$

state, input, stack symbol — write the result to the corresponding cell

STEP: Final State Acception

$P_f = (\{0,1\}, \{(q_0', q_0, q_1, q_f)\}, \{Z_0', Z_0, Z\}, \delta', q_0', q_f, Z_0')$ ← updated P

*add updated P's states and stack symbols (q_0', q_f, Z_0')

δ'

		0	1	ε
q_0'	Z_0'	/	/	$(q_0, Z_0 Z_0')$
	Z_0	/	/	/
	Z	/	/	/
q_0	Z_0'	/	/	/
	Z_0	$(q_0, ZZZZZ_0)$	/	(q_1, ε)
	Z	$(q_0, ZZZZZ)$	(q, ε)	/
q_1	Z_0'	/	/	(q_f, ε)
	Z_0	/	/	(q_1, ε)
	Z	/	(q_1, ε)	/
q_f	Z_0'	/	/	/
	Z_0	/	/	/
	Z	/	/	/

always empty (at least it is in all the previous exams) it's mean we're going to have 3 states

Example

Construct a PDA that accepts:

$L = \{0^{3i} 1^i \mid i \geq 0\}$ where $L = L_f(P)$

notations used

- $Z_0 \rightarrow$ initial stack symbol
- $q_0 \rightarrow$ initial state
- $q_f \rightarrow$ final state
- $Z \rightarrow$ stack symbol

Need to find for both Z_0 and Z stack symbols

$(q_0, 0, Z_0) \longrightarrow (q_1, Z_0)$

$(q_1, 0, Z_0) \longrightarrow (q_2, Z_0)$

$(q_2, 0, Z_0) \longrightarrow (q_3, ZZ_0)$

$(q_3, \varepsilon, Z_0) \longrightarrow (q_3, \varepsilon)$

$(q_0, 0, Z) \longrightarrow (q_1, Z)$

$(q_1, 0, Z) \rightarrow (q_2, Z)$

$(q_2, 0, Z) \rightarrow (q_3, ZZ)$

$(q_3, 1, Z) \rightarrow (q_3, \varepsilon)$ popped

$(q_0, 1, Z) \rightarrow (q_3, \varepsilon)$

$(q_0, \varepsilon, Z_0) = (q_0, \varepsilon)$

$P = (\{0,1\}, \{q_0, q_1, q_2, q_3\}, \{Z_0, Z\}, \delta, q_0, \emptyset, Z_0)$

inputs — states — stack inputs — start state — initial stack

Top of the stack is on ~~RIGHT~~ LEFT

δ

		0	1	ε
q_0	Z_0	(q_1, Z_0)	/	(q_0, ε)
	Z	(q_1, Z)	(q_3, ε)	
q_1	Z_0	(q_2, Z_0)	/	/
	Z	(q_2, Z)	/	/
q_2	Z_0	(q_3, ZZ_0)	/	/
	Z	(q_3, ZZ)	/	/
q_3	Z_0	/	/	(q_3, ε)
	Z	/	(q_3, ε)	/

2 construct a PDA P for the following language

$L = \{1\,0^{2i}\,1^{i} \mid i \geq 1\}$

Empty set

		0	1	ε
q_0	Z_0	/	(q_1, Z_0)	(q_0, ε)
	Z	/	(q_1, Z)	/
q_1	Z_0	(q_2, Z_0)	/	/
	Z	(q_2, Z)	/	/
q_2	Z_0	(q_2, Z_0)	/	/
	Z	(q_2, Z)	/	/
q_3	Z_0	(q_3, ZZ_0)	/	/
	Z	(q_1, ZZ)	/	/
q_4	Z_0	/	/	(q_4, ε)
	Z	/	(q_4, ε)	/

Final state acception $P_f = (\{0,1\}, \{q_0', q_0, q_1, q_2, q_3, q_4\} \{Z_0', Z_0, Z\} \delta', q_0' q_f, Z_0')$

		0	1	ε
q_0'	Z_0'	/	/	$(q_0, Z_0 Z_0')$
	Z_0	/	/	/
	Z	/	/	/
q_0	Z_0'	/	/	(q_f, ε)
	Z_0	/	(q_1, Z_0)	(q_0, ε)
	Z	(q_4, ε)	(q_1, Z)	/
q_1	Z_0'	/	/	/
	Z_0	(q_2, Z_0)	/	/
	Z	(q_2, Z)	/	/
q_2	Z_0'	/	/	/
	Z_0	(q_3, Z_0)	/	/
	Z	(q_3, Z)	/	/
q_3	Z_0'	/	/	/
	Z_0	(q_4, ZZ_0)	/	/
	Z	(q_4, ZZ)	/	/
q_4	Z_0'	/	/	(q_f, ε)
	Z_0	/	/	(q_4, ε)
	Z	/	(q_4, ε)	/
q_f	Z_0'	/	/	/
	Z_0	/	/	/

PDA for $0^n 1^m \mid n > m$

$L = \{0^{3i} 1^i \mid i \geq 0\}$

e.g: 0001

switch state

Think of these zeros as 3x loop. after reaching the third zero, go back to start again until you get a 1. Once you get 1, push it into the stack.

#Assume the bottom of the stack - initial stack symbol "Z_0"

1) $(q_0, 0, Z_0)$ → we've got our first zero, but we're not pushing anything on our stack yet! Just change states

(q_1, Z_0)

$(q_1, ZZ_0) = (q_1, 0Z_0)$

2) $(q_1, 0, Z_0) \rightarrow (q_2, Z_0)$ "second zero"

3) $(q_2, 0, Z_0) \rightarrow (q_1, ZZ_0)$ after getting our third zero, we back to the first step by adding a Z (means "a stack symbol that shows we completed 3i zeros)"

if we get a 1 in this step, it means we've reached q3.

4) $(q_3, 1, Z) \rightarrow (q_3, \varepsilon)$ once we get a 1, we can pop a Z, which means we completed 3x zeros. and when we apply a pop operation, we need to put an epsilon on the result of the transition.

top of the stack

But if we don't get a 1, we have to back to step 1 and continue from there

5) $(q_0, 0, Z) = (q_1, Z)$

started to getting 0 again

top of the stack is now Z.

nothing pushed. just switched state

6) $(q_1, 0, Z) = (q_2, Z)$

7) $(q_2, 0, Z) = (q_0, ZZ)$

same process

we've pushed our third zero and a stack symbol to show it.

PDA Table

δ

		0	1	ϵ
q_0	Z_0	(q_1, Z_0)	/	(q_0, ε)
	Z	(q_1, Z)	(q_3, ε)	/
q_1	Z_0	(q_2, Z_0)	/	/
	Z	(q_2, Z)	/	/
q_2	Z_0	(q_0, ZZ_0)	/	/
	Z	(q_0, ZZ)	/	/
q_3	Z_0	/	/	(q_3, ε)
	Z	/	(q_3, ε)	/

FINAL STATE ACCEPTION

δ'

- Add new states: q_0', q_f
- Add new stack symbol: Z_0'

		0	1	ε
q_0'	Z_0'	/	/	(q_0, Z_0Z_0')
	Z_0	/	/	/
	Z	/	/	/
q_0	Z_0'	/	/	(q_f, ε)
	Z_0	(q_1, Z_0)	/	(q_f, ε)
	Z	(q_1, Z)	(q_3, ε)	/
q_1	Z_0'	/	/	/
	Z_0	(q_2, Z_0)	/	/
	Z	(q_2, Z)	/	/
q_2	Z_0'	/	/	/
	Z_0	(q_0, ZZ_0)	/	/
	Z	(q_0, ZZ)	/	/
q_3	Z_0'	/	/	(q_f, ε)
	Z_0	/	/	(q_3, ε)
	Z	/	(q_3, ε)	/
q_f	Z_0'	/	/	/
	Z_0	/	/	/
	Z	/	/	/