COSC 3340/6309

Examination 3

Monday, June 28, 2010, 2 – 3:45 pm

Open Book and Notes

1. Prove that the following language L is not contextfree:

$L = \{ a^k b^j a^i \mid i>j>k\geq 1 \}$.

2. Construct a pda $\mathbb{P}$ for the following language:

$L = \{ 0^i 1^{2i} \mid i \geq 0 \}$ where $L = L_f(\mathbb{P})$ (acceptance **by final state**).

State on which side you write the top of the stack, left: ____ or right ____ .

3. Construct a pda $\mathbb{P}$ that accepts the following language **by empty stack**:

$L = L(G)$ where $G = (T, N, P, E)$ with $T = \{ id,+,*,(,) \}$,

$N = \{E\}$, and $P = \{ E \to E+E \mid E*E \mid (E) \mid id \}$.

State on which side you write the top of the stack, left: ____ or right ____ .

Note: You <u>must</u> use the construction "cfg → pda" given in class. Get G into GNF first!

4. Construct a grammar for L(G) for the language $N(\mathbb{P})$:

$\mathbb{P} = (\{p,q\}, \{a,b\}, Z,X\}, \delta, p, Z, \varnothing)$ where the move function δ is given by

$\delta(p,b,Z) = \{(p,XZ)\}$ $\quad$ $\delta(q,\varepsilon,Z) = \{(q,\varepsilon)\}$ $\quad$ $\delta(p,b,X) = \{(p,XX)\}$

$\delta(q,b,Z) = \{(p, XZ)\}$ $\quad$ $\delta(q,a,X) = \{(q, \varepsilon)\}$ $\quad$ $\delta(p,a,X) = \{(p,\varepsilon)\}$.

Here, the top of the stack is on the left.

5. Construct a Turing machine for the language in Question 1,

$L = \{ a^k b^j a^i \mid i>j>k\geq 1 \}$.

Describe first in words what you are doing, then formulate the formal Turing machine.

Points:	1: 20	2: 12	3: 18	4: 30	5: 20

① $L = \{a^k b^j a^i \mid i > j > k \geq 1\}$

Assume L is a context free language, $\exists$ a and $G = (N, T, P, S)$ in CNF such that $L = L(G)$

when $k = 2^n$

$\because\ i > j > k \geq 1$

$j = 2^n + 1$ & $i = 2^n + 2$

$\therefore\ z = a^{2^n} b^{2^n+1} a^{2^n+2}$, $\Rightarrow |z| > 2^n$, $z \in L$

we can apply pumping Lemma, for context free language

$z = uvwxy$ & $|vx| \geq 1$ & $uv^iwx^iy \in L$ for every $i \geq 0$

$\boxed{\text{notation}}$

a $\rightarrow a_\ell$ (left) b $\rightarrow b_m$ (middle) a to the right $\rightarrow a_r$

Case ① let v, x be a_ℓ

$|vx| \geq 1$ and when $i = 2$ we increase a_ℓ and keep b_m the same,

of $a_\ell \geq$ # of b_m

$uv^2wx^2y \notin L \longrightarrow$ contradiction

② let v, x be b_m

$|vx| \geq 1$ and when $i = 2$ we increase b_m and keep a_r the same

of $b_m \geq$ # of a_r

$uv^2wx^2y \notin L \rightarrow$ contradiction

③ let v & x be a_r, we pump the a_r while b_m stays the same.

we have # of $b_m \geq$ # of a_r

$\Rightarrow uv^0wx^0y \notin L$, $\Rightarrow$ contradiction

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④ let v OR x be $v \Rightarrow b_m$ OR $x \Rightarrow a_r$ we ~~dont have~~ a_l

$\Rightarrow |vx| \geq 1$ and we take $i = 0$

$\therefore$ decrease the number of b_m and a_l be the same

of $b_m \leq$ # of a_l

$\Rightarrow uv^0wx^0y \notin L \rightarrow$ contradiction

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⑤ When there is no b_m in v or x

$|vx| \geq 1$ and $i = 2$, we can increase a_l and keep b_m to be the same.

$\therefore$ # of $a_l \geq$ # of b_m

$\Rightarrow uv^2wx^2y \notin L$, $\Rightarrow$ contradiction

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⑥ If there is no a_r in v or x,

$|vx| \geq 1$ and $i=2$, we can increase b_m and ~~keep~~ a_r the same

$\therefore$ # of $b_m \geq$ # of a_r

$\Rightarrow uv^2wx^2y \notin L \Rightarrow$ Contradiction v & x are more than One

When there are more than One a_l, b_m and a_r

>1 we have consecutive a and a, thus,

$uv^2wx^2y \notin L \rightarrow$ contriadiction

⑧ x is more than One, a_l, b_m and a_r

we can get another a follows a, thus

$x>1$ $uv^2wx^2y \notin$ with $i=2$

$\Rightarrow$ again we have contriadiction.

$\therefore$ This L is not context free language due to the inconsistancy of uv^iwx^iy by pumping Lemma

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* **Left** side write top of stack $\sqsupset$

$L = \{0^i 1^{2i} \mid i \geq 0\}$ where $L = L_f(P)$

accepted by final state.

We have $Z_0 \rightarrow$ initial stack $Z \rightarrow$ stack

$q_0 \rightarrow$ initial state

$q_f \rightarrow$ final state

Push down$_1$ = $(\{0, 1\}, \{Z_0, Z\}\ \{q_0, q_1\}, \delta, q_0, \phi, Z_0)$

δ		0	1	ε
q_0	Z_0	$\{(q_0, ZZZ_0)\}$	/	$\{(q_1, \varepsilon)\}$
	Z	$\{(q_0, ZZZ)\}$	$\{(q_1, \varepsilon)\}$	/
q_1	Z_0	/	/	$\{(q_1, \varepsilon)\}$
	Z	/	$\{(q_1, \varepsilon)\}$	/

$\Downarrow$

We get final state acception

Push down final = $(\{0, 1\}, \{(q_0', q_0, q_1, q_f)\}, \{Z_0', Z_0, Z\}$
$\delta', q_0', q_f, Z_0')$

next page $\rightarrow$

		0	1	ε
q_0'	Z_0'	/	/	$\{(q_0, Z_0 Z_0')\}$
	Z_0	/	/	/
	Z	/	/	/
q_0	Z_0'	/	/	/
	Z_0	$\{(q_0, ZZZ_0)\}$	/	$\{(q_1, \varepsilon)\}$
	Z	$\{(q_0, ZZZ)\}$	$\{(q_1, \varepsilon)\}$	/
q_1	Z_0'	/	/	$\{(q_f, \varepsilon)\}$
	Z_0	/	/	$\{(q_1, \varepsilon)\}$
	Z	/	$\{(q_1, \varepsilon)\}$	/
q_f	Z_0'	/	/	/
	Z_0	/	/	/ → Final state which we accept.
	Z	/	/	/

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$L = L(G) \quad G = (T, N, P, E)$

$T = \{id, +, *, (,)\}$

$N = \{E\} \quad P = \{E \to E+E \mid E*E \mid (E) \mid id\}$

GNF → no useless
no ε
no unit

$E \to E+E \mid E*E \mid (E) \mid id$

* immidiate left reccursion detected.

$E \to E\underbrace{+E}_{\alpha} \mid E\underbrace{*E}_{\alpha} \mid \underbrace{(E)}_{\beta} \mid \underbrace{id}_{\beta}$

$E \to \beta \mid \beta E'$

$E' \to \alpha \mid \alpha E'$

$E \to (E) \mid id \mid (E)E' \mid idE'$

$E' \to +E \mid *E \mid +EE' \mid *EE'$

$E \to (EX_1 \mid id \mid (EX_1E' \mid idE'$

$E' \to +E \mid *E \mid +EE' \mid *EE'$

$X_1 \to)$

left side stack used.

next page →

$\sqsupset$

		id	+	*	(	)	ε
q	E	(q, ε) (q, E′)	/	/	(q, EX₁) (q, EX₁E′)	/	/
	E′	/	(q, E) (q, EE′)	(q, E) (q, EE′)	/	/	/
	X₁	/	/	/	/	(q, ε)	/

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Push down $N = \{\{q\}, \{id, +, *, (,)\}, \{E, E', X_1\}, \delta, q, E, \phi\}$

The top of stack is left $\sqsupset$

#

$P = (\{p, q\}, \{a, b\}, \{Z, X\}, \delta, p, Z, \emptyset)$

δ		a	b	ε
p	Z	/	(p, XZ)	/
p	X	(p, ε)	(p, XX)	/
q	Z	/	(p, XZ)	(q, ε)
q	X	(q, ε)	/	/

1) $S \to [p, Z, p] \mid [p, Z, q]$

2) $\boxed{(p, XZ) \in \delta(p, b, Z)}$

$[p, Z, \ ] \to b[p, X, \ ][\ , Z, \ ]$

$[p, Z, p] \to b[p, X, p][p, Z, p] \mid b[p, X, q][q, Z, p]$

$[p, Z, q] \to b[p, X, p][p, Z, q] \mid b[p, X, q][q, Z, q]$

x x x x x x x x x x x x x x x x x x x

$\boxed{(p, XZ) \in \delta(q, b, Z)}$

$[q, Z, \ ] \to b[q, X, \ ][\ , Z, \ ]$

$[q, Z, p] \to b[p, X, p][p, Z, p] \mid b[p, X, q][q, Z, p]$

$[q, Z, q] \to b[p, X, p][p, Z, q] \mid b[p, X, q][q, Z, q]$

$(q, \varepsilon) \in \delta(q, \varepsilon, Z)$

$[q, Z, q] \rightarrow \varepsilon$

~~x x x x xx xxx xxxxxxx~~

$(q, \varepsilon) \in \delta(q, a, X)$

$[q, X, q] \rightarrow a$

~~xxxxxxxxxxxxxxxxxx~~

$\boxed{(P, XX) \in \delta(P, b, X)}$

$[P, X, \ ] \rightarrow b[P, X, \ ][\ , X, \ ]$

$[P, X, P] \rightarrow b[P, X, P][P, X, P] \mid b[P, X, q][q, X, P]$

$[P, X, q] \rightarrow b[P, X, P][P, X, q] \mid b[P, X, q] \mid b[P, X, q][q, X, q]$

?

~~xxxxxxxxxxxxxxxxxxxxxx~~

$(P, \varepsilon) \in \delta(P, a, X)$

$[P, X, P] \rightarrow a$

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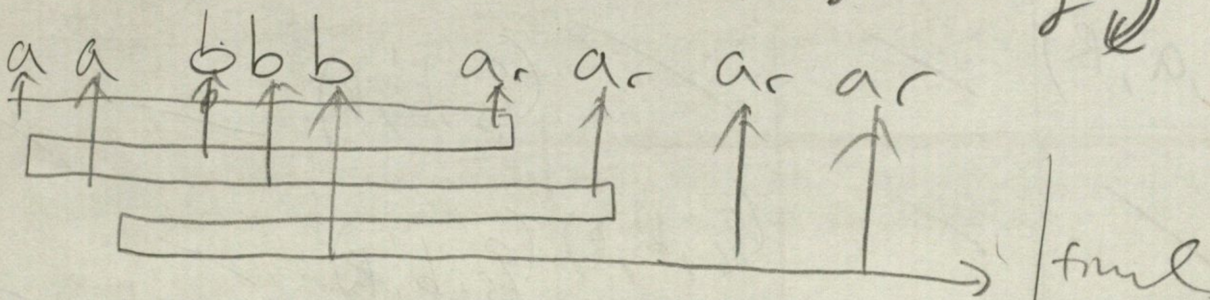
$L = \{a^k b^j a^i \mid i > j > k \geq 1\}$

We can be certain from the language given, the a's on the <u>left</u> is less than b's and is also less than a's to the <u>right</u>

We will construct a turing machine that would traverse through the string from the left a's we call it a and b's we call it b, and a's to the right we call it a_r

(accepting string)



we will traverse until the a's are primed and also b's and a_r are exacuted,

next page →

explain in words -5

	a	a'	b	b'	a_r	$\not b$
q_0	(q_1, a', R)	/	(q_6, b', R)	/	/	/
q_1	(q_1, a, R)	/	(q_2, b', R)	(q_1, b', R)	/	/
q_2	(q_3, a_r, R)	/	(q_2, b, R)	/	(q_2, a_r, R)	/
q_3	(q_3, a, R) 0	(q_4, a', R)	(q_3, b, L)	(q_3, b', L)	(q_3, a_r, L)	/
q_4	(q_1, a', R)	/	/	(q_5, b', R)	/	/
-5 q_5	/	/	(q_6, b', R)	(q_5, b', R)	/	/
q_6	(q_7, a_r, R)	/	(q_6, b, R)	/	(q_6, a_r, R)	/
q_7	(q_7, a_r, L)	/	(q_7, b, L)	(q_8, b', R)	/	/
10 q_8	/	/	(q_6, b', R)	/	q_9, a_r, L	/
q_9	(q_{10}, a_r, R)	/	/	/	q_9, a_r, R	/
q_{10}	q_{10}, a_r, R	/	/	/	/	/
q_{final}	/	/	/	/	$q_f, \not b, R$	/