COSC 3340
Examination 3
Wednesday, April 9, 2008, 1 – 2:30 pm
Open Book and Notes

1. Prove that the following language L is not contextfree:

   $L = \{ 0^k 1^j 2^i \mid i > j > k \geq 0 \}$.

2. Construct a pda $\mathbb{P}$ for the following language:

   $L = \{ 0^i 1^{3i} \mid i \geq 0 \}$ where $L = L_f(\mathbb{P})$ (acceptance **by final state**).

   State on which side you write the top of the stack, left: ____| or right |____ .

   Hint: Put three markers on the stack for every 0.

3. Construct a pda $\mathbb{P}$ that accepts the following language **by empty stack**:

   $L = L(G)$ where $G = (T, N, P, S)$ with $T = \{<,>,[,]\}$,

   $N = \{S,A\}$, and $P = \{ S \to \text{<S>A} \mid \text{[A]A} ,\ A \to \text{[A]S} \mid \text{<S>S} \mid \varepsilon \}$.

   State on which side you write the top of the stack, left: ____| or right |____ .

   Note: You <u>must</u> use the construction "cfg → pda" given in class. Get G into GNF first!

4. Construct a grammar for L(G) for the language $N(\mathbb{P})$:

   $\mathbb{P} = (\{p,q\}, \{a,b\}, \{Z,X\}, \delta, p, Z, \varnothing)$ where the move function δ is given by

   $\delta(p,a,Z) = \{(p,XZ)\}$ $\quad \delta(p,\varepsilon,Z) = \{(p,\varepsilon)\}$ $\quad \delta(p,a,X) = \{(p,XX)\}$

   $\delta(q,a,Z) = \{(q,\varepsilon)\}$ $\quad \delta(p,b,X) = \{(q,X)\}$ $\quad \delta(q,b,Z) = \{(p,Z)\}$.

   Here, the top of the stack is on the left.

5. Construct a Turing machine for the language in Question 1,

   $L = \{ 0^k 1^j 2^i \mid i > j > k \geq 0 \}$.

   Describe first in words what you are doing, then formulate the formal Turing machine.

Points:	**1: 20**	**2: 12**	**3: 18**	**4: 30**	**5: 20**

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$L = \{0^k 1^j 2^i \mid i > j > k \geq 0\}$

1) assume L is cfL

∴ then $\exists\, G(N,T,P,S)$ in CNF s.t. $L = L(G)$

assume t = no. of variables, assume a word $z = 0^{2t} 1^{2t+1} 2^{2t+2}$ $|z| > 2^t$

1	20
2	12
3	18
4	30
5	20
	100

by pumping Lemma $z = uvwxy$ where $|vx| \geq 1$
& $uv^{i'}wx^{i'}y \in L(G)$
for all $i' \geq 0$

case 1:- V & X are all 0's $\Rightarrow$ for $i' = 2$ no. of zero can be equal no. of 1 $\in L(G), \notin L$

case 2:- V & X are all 1's $\Rightarrow$ for $i' = 2$ $\in L(G)$, $\notin L$ why

case 3:- V & X are all 2's $\Rightarrow$ for $i' = 0$ $\in L(G)$, $\notin L$

case 4:- No 2's in V or X $\Rightarrow$ for $i' = 2$ $\in L(G)$, $\notin L$
because No of 1's can be equal to no. of 2's

case 5:- No 1's in V or X $\Rightarrow$ for $i' = 2$ $\in L(G)$, $\notin L$ why

case 6:- No 0's in V or X $\Rightarrow$ for $i' = 0$ $\in L(G)$, $\notin L$
because No of 1's decrease to be equal No of zeros

case 7:- at least one 0, one 1, one 2 in V or X
∴ for $i' > 1$ the pattern will change
ie:- zero's can follow 2's which is not acceptable by L

∴ there is contradiction in each case 20

∴ the language L is not Context free

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		a	b	ϵ
p	Z	$\{(p,XZ)\}$	/	$\{(p,\epsilon)\}$
	X	$\{(p,XX)\}$	$\{(q,X)\}$	/
q	Z	$\{(q,\epsilon)\}$	$\{(p,Z)\}$	/
	X	/	/	/

$$S \rightarrow [p,Z,p] \mid [p,Z,q] \rightarrow 1$$

* $(p,XZ) \in \delta(p,a,Z)$

$$[p,Z,\begin{smallmatrix}p\\q\end{smallmatrix}] \rightarrow a[p,X,\begin{smallmatrix}p\\q\end{smallmatrix}][\begin{smallmatrix}p\\q\end{smallmatrix},Z,\begin{smallmatrix}p\\q\end{smallmatrix}]$$

2 $\leftarrow$ $[p,Z,p] \rightarrow a[p,X,p][p,Z,p] \mid a[p,X,q][q,Z,p]$

3 $\leftarrow$ and $[p,Z,q] \rightarrow a[p,X,p][p,Z,q] \mid a[p,X,q][q,Z,q]$

* $(p,XX) \in \delta(p,a,X)$

$$[p,X,\begin{smallmatrix}p\\q\end{smallmatrix}] \rightarrow a[p,X,\begin{smallmatrix}p\\q\end{smallmatrix}][\begin{smallmatrix}p\\q\end{smallmatrix},X,\begin{smallmatrix}p\\q\end{smallmatrix}]$$

4 $\leftarrow$ $[p,X,p] \rightarrow a[p,X,p][p,X,p] \mid a[p,X,q][q,X,p]$

5 $\leftarrow$ $[p,X,q] \rightarrow a[p,X,p][p,X,q] \mid a[p,X,q][q,X,q]$

* $(p,\epsilon) \in \delta(p,\epsilon,z)$

$[p,z,p] \to \epsilon$ $\to$ 6

* $(q,x) \in \delta(p,b,x)$

$[p,x,\substack{p\\q}] \to b\,[q,x,\substack{p\\q}]$

$\therefore$ $[p,x,p] \to b[q,x,p]$ $\to$ 7

$[p,x,q] \to b[q,x,q]$ $\to$ 8

* $(q,\epsilon) \in \delta(q,a,z)$

$[q,z,q] \to a$ $\to$ 9

* $(p,z) \in \delta(q,b,z)$

$[q,z,\substack{p\\q}] \to b\,[p,z,\substack{p\\q}]$

$\Rightarrow$ $[q,z,p] \to b[p,z,p]$ $\to$ 10

$[q,z,q] \to b[p,z,q]$ $\to$ 11

The $L(G)$ is defined with ~~equations~~ productions in 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11

[illegible]

④

② First I'll construct a pda that accepts by emptystack then change it to one that accept by final state $L = \{0^i 1^{3i} \mid i \geq 0\}$

$P_e = (\{0,1\}, \{q_0, q_1\}, \{Z_0, Z\}, \delta, q_0, \emptyset, Z_0)$

The top of the stack is on the left

δ:

		0	1	ϵ
q_0	Z_0	$\{(q_0, ZZZZ_0)\}$	/	$\{(q_1, \epsilon)\}$
	Z	$\{(q_0, ZZZZ)\}$	$\{(q_1, \epsilon)\}$	/
q_1	Z_0	/	/	$\{(q_1, \epsilon)\}$
	Z	/	$\{(q_1, \epsilon)\}$	/

$P_f = (\{0,1\}, \{q_0', q_0, q_1, q_f\}, \{Z_0', Z_0, Z\}, \delta', q_0', q_f, Z_0')$

δ'

		0	1	ϵ
q_0'	Z_0'	/	/	$\{(q_0, Z_0Z_0')\}$
	Z_0	/	/	/
	Z	/	/	/
q_0	Z_0'	/	/	/
	Z_0	$\{(q_0, ZZZZ_0)\}$	/	$\{(q_1, \epsilon)\}$
	Z	$\{(q_0, ZZZZ)\}$	$\{(q_1, \epsilon)\}$	/
q_1	Z_0'	/	/	$\{(q_f, \epsilon)\}$
	Z_0	/	/	$\{(q_1, \epsilon)\}$
	Z	/	$\{(q_1, \epsilon)\}$	/
q_f	Z_0'	/	/	/
	Z_0	/	/	/
	Z	/	/	/

Final state

✓ 12

Alex Fleury

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(3) $S \rightarrow \ < S > A \mid [A]A$

$A \rightarrow [A]S \mid \ < S > S \mid \varepsilon$

eliminate ϵ

$S \rightarrow \ < S > A \mid \ < S > \mid [A]A \mid [A] \mid [\,]A \mid [\,]$

$A \rightarrow [A]S \mid [\,]S \mid \ < S > S$

to GNF

$S \rightarrow \ < S X_{>} A \mid \ < S X_{>} \mid [A X_{]} A \mid [A X_{]} \mid [X_{]} A \mid [X_{]}$

$A \rightarrow [A X_{]} S \mid [X_{]} S \mid \ < S X_{>} S$

$X_{>} \rightarrow \ >$

$X_{]} \rightarrow]$

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$P_N = (\{<, >, [,]\}, \{q_0\}, \{S, A, X_{>}, X_{]}\}, \delta, q_0, \emptyset, S)$

δ		$<$	$>$	$]$	$[$	ϵ
q_0	S	$\{(q_0, S X_{>} A), (q_0, S X_{>})\}$	/	/	$\{(q_0, A X_{]} A), (q_0, A X_{]}), (q_0, X_{]} A), (q_0, X_{]})\}$	/
	A	$\{(q_0, S X_{>} S)\}$	/	/	$\{(q_0, A X_{]} S), (q_0, X_{]} S)\}$	/
	$X_{>}$	/	$\{(q_0, \epsilon)\}$	/	/	/
	$X_{]}$	/	/	$\{(q_0, \epsilon)\}$	/	/

The top of stack on the left

5) $\{0^k 1^j 2^i \mid i > j > k \geq 0\}$

ex: of accepted words 122, 011222
001112222

0' 0' 1' 1' 1 2' 2' 2 2 b̸

. first we ~~mark~~ start reading from the left right if a zero is found then we mark it and skip all zeros till we find a 1 then we mark it and skip 1's till we find a two then we mark it then we start move back to the left skipping everything till we find a 0' then we turn to move right and repeat the whole process till all zeros are marked.

then we repeat the same process for only the 1's and 2's till all 1's are marked then we make sure that there is at least a 2 left (or more) then we mark all 2's till we reach b̸

. if at the start no zero's are found we proceed as if all zero's were marked and this is the case when $K=0$

0' 0' 1' 1' 1' 2' 2' 2' 2' b̸

this state is reached when there are no 0's in the word i.e. k=0

	0	1	2	0'	1'	2'	b
q_0	$(q_1, 0', R)$	$(q_6, 1', R)$	/	/	/	/	/
q_1	$(q_1, 0, R)$	$(q_2, 1', R)$	/	/	$(q_1, 1', R)$	/	/
q_2	/	$(q_2, 1, R)$	$(q_3, 2', L)$	/	/	$(q_2, 2', R)$	/
q_3	$(q_3, 0, L)$	$(q_3, 1, L)$	/	$(q_4, 0', R)$	$(q_3, 1', L)$	$(q_3, 2', L)$	/
q_4	$(q_1, 0', R)$	/	/	/	$(q_5, 1', R)$	/	/
reaching q_5 means we marked all 0's q_5	/	$(q_6, 1', R)$	/	/	$(q_5, 1', R)$	/	/
q_6	/	$(q_6, 1, R)$	$(q_7, 2', L)$	/	/	$(q_6, 2', R)$	/
q_7	/	$(q_7, 1, L)$	/	/	$(q_8, 1', R)$	$(q_7, 2', L)$	/
q_8	/	$(q_6, 1', R)$	/	/	/	$(q_9, 2', R)$	/
if marked all 1's q_9	/	/	$(q_{10}, 2', R)$	/	/	$(q_9, 2', R)$	(q_f, b, R)
q_{10}	/	/	$(q_{10}, 2', R)$	/	/	/	/
q_f	accepting state						

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The turning Machine for the language $L = \{0^k 1^j 2^i \mid i > j > k \geq 0\}$