COSC 3340
Examination 3
Wednesday, April 22, 2009, 1 – 2:30 pm
Open Book and Notes

1. Prove that the following language L is not contextfree:

$L = \{ 0^k 1^j 0^i \mid i>j>k\geq 0 \}$.

2. Construct a pda $\mathbb{P}$ for the following language:

$L = \{ 0^i 1^{4i} \mid i\geq 0 \}$ where $L = L_f(\mathbb{P})$ (acceptance **by final state**).

State on which side you write the top of the stack, left: $\boxed{}$ or right $\boxed{}$.

Hint: Put four markers on the stack for every 0.

3. Construct a pda $\mathbb{P}$ that accepts the following language **by empty stack**:

$L = L(G)$ where $G = (T, N, P, S)$ with $T = \{<,>,[,]\}$,

$N = \{S,A\}$, and $P = \{ S \to <S>A \mid [A]A\ ,\ A \to [A]S \mid <S>S \mid \varepsilon \}$.

State on which side you write the top of the stack, left: $\boxed{}$ or right $\boxed{}$.

Note: You <u>must</u> use the construction "cfg → pda" given in class. Get G into GNF first!

4. Construct a grammar for L(G) for the language $N(\mathbb{P})$:

$\mathbb{P} = (\{p,q\}, \{a,b\}, Z,X\}, \delta, p, Z, \varnothing)$ where the move function δ is given by

$\delta(p,a,Z) = \{(p,XZ)\}$ $\quad\delta(p,\varepsilon,Z) = \{(p,\varepsilon)\}$ $\quad\delta(p,a,X) = \{(p,XX)\}$

$\delta(q,a,Z) = \{(q,\varepsilon)\}$ $\quad\delta(p,b,X) = \{(q,X)\}$ $\quad\delta(q,b,Z) = \{(p,Z)\}$.

Here, the top of the stack is on the left.

Steven's copy

5. Construct a Turing machine for the language in Question 1,

$L = \{ 0^k 1^j 0^i \mid i>j>k\geq 0 \}$.

Describe first in words what you are doing, then formulate the formal Turing machine.

Points: **1: 20** **2: 12** **3: 18** **4: 30** **5: 20**

1

$L = \{0^k 1^j 0^i \mid i > j > k \geq 0\}$

Proof:

Assume that L is a context free language, then $\exists$ a $G = (N, T, P, S)$ in CNF st $L = L(G)$

1	20
2	12
3	18
4	30
5	20
	100

Consider the case when $k = 2^n$

We have that $j > k = 2^n$

let $j = 2^n + 1$

We also have $i > j = 2^n + 1$

let $i = 2^n + 2$

So $z = 0^{2^n} 1^{2^n+1} 0^{2^n+2}$, we have $|z| > 2^n$ and $z \in L$, then by pumping lemma for cfl, we have that $z = uvwxy$ and $|vx| \geq 1$ and $uv^i w x^i y \in L \quad \forall\, i \geq 0$

From now, we call 0's before 1's be "left 0", 0's after 1's be "right 0"

Case

1(i) consider v and x to be all left 0's

we have $|vx| \geq 1$ and take $i = 2$

Thus we increase the No. of left 0's while the No. of 1's remains the same

$\Rightarrow$ (No. of left 0's) $\geq$ (No. of 1's)

Hence, $uv^i w x^i y \notin L$ with $i = 2$

We have a contradiction

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(ii) consider v and x be all 1's
We have $|vx| \geq 1$ and taking $i=2$
We increase the No. of 1's while the No. of right 0's remains the same
Thus $\Rightarrow$ (No. of 1's) $\geq$ (No. of right 0's)
Hence, $uv^iwx^iy \notin L$ with $i=2$
We have a contradiction

(iii) Consider v and x be all right 0's
We have $|vx| \geq 1$ and taking $i=0$
we decrease the No. of right 0's while the No. of 1's remains the same
Thus $\Rightarrow$ (No. of 1's) $\geq$ (No. of right 0's)
Hence, $uv^iwx^iy \notin L$ with $i=0$
We have a contradiction

(iv) Consider when v or x has 1's and right 0's, but no left 0's
We have $|vx| \geq 1$ and taking $i=0$
so, we diminish the No. of 1's while No. of left 0's remains the same
Thus (No. of 1's) $\leq$ (No. of left 0's)
Hence, $uv^iwx^iy \notin L$ with $i=0$
We have a contradiction

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(v) Consider when there is no 1's in v or x
We have $|vx| \geq 1$ and taking $i=2$
we increase the No. of left 0's while the No. of 1's remain the same
Thus (No. of left 0's) $\geq$ (No. of 1's)
Hence $uv^iwx^iy \notin L$ with $i=2$
We have a contradiction

(vi) Consider when there is no right 0's in v or x, we have $|vx| \geq 1$ and taking $i=2$
we increase the No. of 1's, while the No. of right 0's remains the same
Thus (No. of 1's) $\geq$ (No. of right 0's)
Hence $uv^iwx^iy \notin L$ with $i=2$
We have a contradiction

Case 2 Consider when v or x has more than one left 0's, 1's right 0's

(i) v contains more than one left 0's, 1's, right 0's; for $i=2$
we get the 0 follows 0, hence $uv^iwx^iy \notin L$ with $i=2$
We have a contradiction

(ii) x contains more than one left 0's, 1's, right 0's; for $i=2$
we get the 0 follows 0, hence $uv^iwx^iy \notin L$ with $i=2$
We have a contradiction

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Hence, our assumption is not correct, L is not context free language.

2

$L = \{0^i 1^{4i} \mid i \geq 0\}$ where $L = L_f(P)$

Notations used :

$Z_0 \rightarrow$ initial stack symbol

$q_0 \rightarrow$ initial state

$q_f \rightarrow$ final state

$Z \rightarrow$ stack symbol

left $\sqsupset$ side will be used to write the top of the stack

		0	1	ε
q_0	Z_0	$(q_0, ZZZZZ_0)$	—	(q_f, ε)
	Z	$(q_0, ZZZZZ)$	(q_1, ε)	—
q_1	Z_0	—	—	(q_f, ε)
	Z	—	(q_1, ε)	—
q_f		—	—	—

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Accepting

3

Answer:

$$S \to < S > A \mid [A]A$$
$$A \to [A]S \mid < S > S \mid \varepsilon$$

Eliminate $A \to \varepsilon$

$$S \to < S > A \mid < S > \mid [A]A \mid [\,]A \mid [A] \mid [\,]$$
$$A \to [A]S \mid [\,]S \mid < S > S$$

$$S \to < S S_1 A \mid < S S_1 \mid [A S_2 A \mid [S_2 A \mid [A S_2 \mid [S_2$$
$$A \to [A S_2 S \mid [S_2 S \mid < S S_1 S$$
$$S_1 \to >$$
$$S_2 \to]$$

left ⊐ side will be used

		<	>	[	]	ε
	S	$\{(q, SS_1A)(q, SS_1)\}$	—	$\{(q, AS_2A)(q, S_2A)$ $(q, AS_2)(q, S_2)\}$	—	—
q	A	(q, SS_1S)	—	$\{(q, AS_2S)(q, S_2S)\}$	—	—
	S_1	—	(q, ε)	—	—	—
	S_2	—	—	—	(q, ε)	—

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4

		$a=1$	$b=0$	ε
p	Z	(p, XZ)	—	(p, ε)
	X	(p, XX)	(q, X)	—
q	Z	(q, ε)	(p, Z)	—
	X	—	—	—

Answer:

1) $S \to [p, Z, {}^{p}_{q}]$

So we have 2 productions

$S \to [p, Z, p] \mid [p, Z, q]$

2) $(p, XZ) \in \delta(p, a, Z)$

$[p, Z, {}^{p}_{q}] \to a[p, X, {}^{p}_{q}][{}^{p}_{q}, Z, {}^{p}_{q}]$

We will have 4 productions

$[p, Z, p] \to a[p, X, p][p, Z, p] \mid a[p, X, q][q, Z, p]$

$[p, Z, q] \to a[p, X, p][p, Z, q] \mid a[p, X, q][q, Z, q]$

$(p, XX) \in \delta(p, a, X)$

$[p, X, {}^{p}_{q}] \to a[p, X, {}^{p}_{q}][{}^{p}_{q}, X, {}^{p}_{q}]$

We will have 4 productions

$[p, X, p] \Rightarrow a[p, X, p][p, X, p] \mid a[p, X, q][q, X, p]$

$[p, X, q] \Rightarrow a[p, X, p][p, X, q] \mid a[p, X, q][q, X, q]$

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$(q, X) \in \delta(p, b, X)$

We will have 2 productions

$[p, X, {p \atop q}] \to b[q, X, {p \atop q}]$

$[p, X, p] \to b[q, X, p]$

$[p, X, q] \to b[q, X, q]$

$(p, Z) \in \delta(q, b, Z)$

We will have 2 productions

$[q, Z, {p \atop q}] \to b[p, Z, {p \atop q}]$

$[q, Z, p] \to b[p, Z, p]$

$[q, Z, q] \to b[p, Z, q]$

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$(q, \varepsilon) \in \delta(q, a, Z)$

We will have 1 production

$[q, Z, q] \to a$

$(p, \varepsilon) \in \delta(p, \varepsilon, Z)$

We will have 1 production

$[p, Z, p] \to \varepsilon$

Total we have 16 productions

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$L = \{0^k 1^j 0^i \mid i > j > k \geq 0\}$

We know that number of left 0's is less than number of 1's which is less than number of right 0's

In my Turing machine, I'll traverse through the tape and prime 0 for corresponding left 0's, prime 1 for 1's, specially, \$ for right 0's.

Keep on doing that until left 0's exaucted, continue to mark 1's and right 0's until 1's exausted. We will mark the remaining right 0's until right 0's exaucted.

My Turing machice is on the next page.

	0	0'	1	1'	\$	$\cancel{b}$
q_0	$(q_1, 0', R)$	—	$(q_6, 1', R)$	—	—	—
q_1	$(q_1, 0, R)$	—	$(q_2, 1', R)$	$(q_1, 1', R)$	—	—
q_2	$(q_3, \$, R)$	—	$(q_2, 1, R)$	—	$(q_2, \$, R)$	—
q_3	$(q_3, 0, L)$	$(q_4, 0', R)$	$(q_3, 1, L)$	$(q_3, 1', L)$	$(q_3, \$, L)$	—
q_4	$(q_1, 0', R)$	—	—	$(q_5, 1', R)$	—	—
q_5	—	—	$(q_6, 1', R)$	$(q_5, 1', R)$	—	—
q_6	$(q_7, \$, R)$	—	$(q_6, 1, R)$	—	$(q_6, \$, R)$	—
q_7	$(q_7, \$, L)$	—	$(q_7, 1, L)$	$(q_8, 1', R)$	—	—
q_8	—	—	$(q_6, 1', R)$	—	$(q_9, \$, L)$	—
q_9	$(q_{10}, \$, R)$	—	—	—	$(q_9, \$, R)$	—
q_{10}	$(q_{10}, \$, R)$	—	—	—	—	$(q_f, \cancel{b}, R)$
q_f	—	—	—	—	—	—

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