Q1

Pumping Lemma (For CFL)

Pumping Lemma (for CFL) is used to prove that a language is NOT context free.

If A is a context free language, then, A has a pumping length 'P' such that any string 'S', where $|S| \geq N$ may be divided into 5 pieces $S = uvxyz$ such that the following conditions must be true.

(1) uv^iwx^iy is in A for every $i \geq 0$
(2) $vz \neq \epsilon$. At least one of the outer symbols cannot be empty
(3) $|vxy| \leq N$

Step 1: Assume L is a context-free language, then $\exists G = \{N, T, P, S\}$ such that $L = L(G)$

Step 2: Define variable values in terms of n so it satisfies L

Step 3: So $Z = \{...\}$ we have $|Z| > \{...\}$ and $Z \in L$ then by pumping Lemma for CFL, we have that $Z = uvwxy$ and $|vx| \geq 1$ and $uv^iwx^iy \in L \; \forall i \geq 0$

Step 4: Consider every possible way for 'S' to be divided into 'uvwxy'. Show that none of these combinations satisfy all three pumping conditions above.

Step 5: If none of the above combinations satisfy all three pumping condions, then we have a CONTRADICTION, thus L is NOT context free.

Spring 2008

$L = \{0^k 1^j 2^i \mid i > j > k \geq 0\}$

Assume L is a context free language, then $\exists\, G(N, T, P, S)$ is in chomsky normal form such that $L = L(G)$

The String $S = 0^{2P} 1^{2P+1} 2^{2P+2}$ satisfies L such that P is the pumping length such that $|S| > 2^P$

By pumping lemma $S = uvwxy$ where $|vy| \geq 1$ and $uv^i w x^i y \in L(G)$ for all $i \geq 0$

Case 1: V and X are all zeros $\Rightarrow$

Note: When we choose how to partision S into $uvwxy$, make sure it satisfies $|vwx| \geq P$ and $|vx| \geq 1$

* We can choose any P value as long as *
$|S| \geq P$ $|vwx| \leq P$ $|vx| \geq 1$

for $i = 0$,

If we choose an arbitrary $P = 2$ then

$S = \underset{u}{0}\,\underset{v}{0}\,\underset{x}{0}\,\underbrace{0\,11111}_{w}\,\underbrace{222222}_{y}$

$uv^0wx^0y = \underset{u}{0}\ \underset{v}{0^0}\ \underbrace{011111}_{w}\ \underset{x}{0^0}\underbrace{222222}_{y}$

order $\rightarrow$ 0 0 11111 222222

We cannot choose $i=0$ because this string satisfies L such that $2 > 5 > 6 \geq 0$

for $i = 1$

If we choose an arbitrary $P = 2$ then

$S = \underset{u}{0}\,\underset{v}{0}\,\underset{x}{0}\,\underbrace{0\,11111}_{w}\,\underbrace{222222}_{y}$

$uv^1wx^1y = \underset{u}{0}\ \underset{v}{0^1}\ \underbrace{011111}_{w}\ \underset{x}{0^1}\underbrace{222222}_{y}$

order $\rightarrow$ 0000 11111 222222

We cannot choose $i=0$ because this string satisfies L such that $4 > 5 > 6 \geq 0$

for $i = 2$ uv^2wx^2y

If we choose an arbitrary $P = 2$ then

$S = \underbrace{0}_{u}\underbrace{0}_{v}\underbrace{0}_{x}\underbrace{0\,11111}_{w}\underbrace{222222}_{y}$

$uv^2wx^2y = \underbrace{0}_{u}\ \underbrace{0^2}_{v}\ \underbrace{1\,1\,1\,1\,1}_{w}\ \underbrace{0^2}_{x}\ \underbrace{2\,2\,2\,2\,2\,2}_{y}$

$= 000\,11111\,00\,222222$

order it $\rightarrow$ $\underbrace{00000}_{0^k}\ \underbrace{11111}_{1^j}\ \underbrace{222222}_{2^i}$

This does not satisfy L's conditions
$i > j > k \geq 0$ because $5 > 5 > 6 \geq 0$ is false

This means for $i = 2$ $\in L(G)$ $\notin L$

Case 2: V and X are all ones

for $i = 0$,

If we choose an arbitrary $P = 2$ then

$S = 0000\,11111\,222222$

$uv^0wx^0y =$

order $\rightarrow$

We cannot choose $i=0$ because this falsifies L such that $2 > 5 > 6 >$

Q2

Anita R Video: PDA Example $a^n b^{2n}$

$\{ abb, aabbbb, \ldots \}$

Give a PDA to accept the following language $L = \{a^n b^{2n} \mid n \geq 1\}$ by final state

δ(current state, current input, top of stack) = (current state, operation representation)

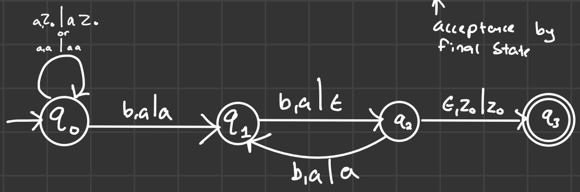
1. $\delta(q_0, a, Z_0) = (q_0, aZ_0)$
2. $\delta(q_0, a, a) = (q_0, aa)$
3. $\delta(q_0, b, a) = (q_1, a)$ If state changes Reset Operation
4. $\delta(q_1, b, a) = (q_2, \epsilon)$
5. $\delta(q_2, b, a) = (q_1, a)$

Repeat 4

6. $\delta(q_2, \epsilon, Z_0) = (q_3, Z_0)$

acceptance by final state

aabbbbϵ	empty stack Z_0
aabbbbϵ	a Z_0
aabbbbϵ	a a Z_0
aabbbbϵ	a a Z_0
aabbbbϵ	$\cancel{a}$ a Z_0
aabbbbϵ	a Z_0
aabbbbϵ	$\cancel{a}$ Z_0
aabbbbϵ	Z_0



Spring 2009

Construct a PDA $\mathbb{P}$ for the following language.

$L = \{0^i 1^{4i} \mid i \geq 0\}$ where $L = L_F(P)$
(Acceptance by final state)

State on which side you write the top of stack, Left] or right [.

Note: Put 4 markers on the stack
for every 0

Notations used:

- $Z_0 \rightarrow$ initial stack symbol
- $q_0 \rightarrow$ initial state
- $q_1 \rightarrow$ final state
- $Z \rightarrow$ stack symbol

Left] side will be used for the top of the stack

δ(current state, current input, top of stack) = (current state, operation representation)

$\boxed{\text{Concept?}}$

$001111111 1\epsilon$ $\qquad Z^{4i}$

$0011111111\epsilon \quad \delta(q_0, 0, Z_0) = (q_0, ZZZZZ_0)$

$0011111111\epsilon \quad \delta(q_0, 0, Z) = (q_0, ZZZZZ)$

$0011111111\epsilon \quad \delta(q_0, 1, Z) = (q_1, \epsilon)$

$0011111111\epsilon \quad \delta(q_0, 1, Z) = (q_1, \epsilon)$

$0011111111\epsilon \quad \delta(q_1, \epsilon, Z_0) = (q_0, \epsilon)$

$0011111111\epsilon \quad \delta(q_1, \epsilon, Z_0) = (q_0, \epsilon)$

$P = (\{0,1\}, \{q_0, q_1\}, \{Z_0, Z\}, \delta, q_0, Z_0, \emptyset\}$

		0	1	ϵ
q_0	Z_0	$(q_0, ZZZZZ_0)$	—	(q_1, ϵ)
	Z	$(q_0, ZZZZZ)$	(q_1, ϵ)	—
q_1	Z_0	—	—	(q_1, ϵ)
	Z	—	(q_1, ϵ)	—

$\longrightarrow$

		0	1	ϵ
q_0	Z_0	$\{(q_0, ZZZZZ_0)\}$	/	$\{(q_1, \epsilon)\}$
	Z	$\{(q_0, ZZZZZ)\}$	$\{(q_1, \epsilon)\}$	/
q_1	Z_0	/	/	$\{(q_1, \epsilon)\}$
	Z	/	$\{(q_1, \epsilon)\}$	/

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Final State Acception

$$P_f = (\{0,1\}, \{q_0', q_0, q_1, q_f\}, (Z_0', Z_0, Z), \delta', q_0', q_f, Z_0')$$

δ'

		0	1	ϵ
q_0'	Z_0'	/	/	$\{(q_0, Z_0Z_0')\}$
	Z_0	/	/	/
	Z	/	/	/
q_0	Z_0'	/	/	/
	Z_0	$\{(q_0, ZZZZZ_0)\}$	/	$\{(q_1, \epsilon)\}$
	Z	$\{(q_0, ZZZZZ)\}$	$\{(q_1, \epsilon)\}$	/
q_1	Z_0'	/	/	$\{(q_f, \epsilon)\}$
	Z_0	/	/	$\{(q_1, \epsilon)\}$
	Z	/	$\{(q_1, \epsilon)\}$	/
q_f	Z_0'	/	/	/
	Z_0	/	/	/
	Z	/	/	/

Final State

QB

Spring 2009

Construct a PDA $\mathbb{P}$ that accepts the following language by empty stack.

$L = L(G)$ where $G = (T, N, P, S)$ with $T = \{<, >, [,]\}$, $N = \{S, A\}$ and $P = \{S \to <S>A \mid [A]A,\ A \to [A]S \mid <S>S \mid \epsilon\}$

Starting Productions

$S \to <S>A \mid [A]A$

$A \to [A]S \mid <S>S \mid \epsilon$

Eliminate $A \to \epsilon$. Add an option to replace all 'A' with 'ϵ'.

$S \to <S>A \mid <S> \mid [A]A \mid [\,]A \mid [A] \mid [\,]$

$A \to [A]S \mid [\,]S \mid <S>S$

Replace closing symbols with S_1, S_2

$S \to <SS_1A \mid <SS_1 \mid [AS_2A \mid [S_2A \mid [AS_2 \mid [S_2$

$A \to [AS_2S \mid [S_2S \mid <SS_1S$

$S_1 \to >$

$S_2 \to]$

Left $\sqsupset$ side will be used for top of stack.

Rewriting productions for clarity.

$S \to <SS_1A \mid <SS_1 \mid [AS_2A \mid [S_2A \mid [AS_2 \mid [S_2$

$A \to [AS_2S \mid [S_2S \mid <SS_1S \qquad S_1 \to > \qquad S_2 \to]$

Traverse each column and add matching productions.

		$<$	$>$	$[$	$]$	ϵ
	S	$\{(q, SS_1A)(q, SS_1)\}$	/	$\{(q, AS_2A)(q, S_2A)(q, AS_2)(q, S_2)\}$	/	/
q	A	(q, SS_1S)	/	$\{(q, AS_2S)(q, S_2S)\}$	/	/
	S_1	/	(q, ϵ)	/	/	/
	S_2	/	/	/	(q, ϵ)	/

Q4

Notes on Leiss Lecture

PDA Format

$= \{$states, input symbols, stack symbols, move function, initial state, initial stack, final$\}$

$= \{Q, T, P, \delta, q_0, Z_0, \emptyset\}$

$G = (N, T, P, S)$

$N = \{S\} \cup \{[q, A, p]\} \mid p, q \in Q, A \in T$

P: 1. $S \to [\underbrace{q_0, Z_0}_{\text{Final States}}, q] \; \forall q \in Q$

Free Variable

q_0 and q_1

		0	1	ε
q_0	Z	(q_0, XZ)	/	/
	X	(q_0, XX)	(q_1, ϵ)	/
q_1	Z	/	/	(q_1, ϵ)
	X	/	(q_1, ϵ)	(q_1, ϵ)

rule 1. $S \to [q_0, Z, \begin{smallmatrix} q_0 \\ q_1 \end{smallmatrix}]$

Spring 2008/2009

Construct a grammar for L(G) for the following language N(P)

$P = (\{p, q\}, \{a, b\}, \{Z, X\}, \delta, p, Z, \emptyset)$

Where the move function δ is given by:

$\delta(p, a, Z) = \{(p, XZ)\}$ $\delta(p, \epsilon, Z) = \{(p, \epsilon)\}$ $\delta(p, a, X) = \{(p, XX)\}$

$\delta(q, a, Z) = \{(q, \epsilon)\}$ $\delta(p, b, X) = \{(q, X)\}$ $\delta(q, b, Z)\} = \{(p, Z)\}$

		a=1	b=0	ε
p	Z	(p, XZ)	—	(p, ε)
	X	(p, XX)	(q, X)	—
q	Z	(q, ε)	(p, Z)	—
	X	—	—	—

1) $S \rightarrow$ [p, initial stack symbol, all state symbols]

$S \rightarrow [p, Z, {}^{p}_{q}]$

Therefor we have two productions

$S \rightarrow [p, Z, p][p, Z, q]$

2) $(p, XZ) \in \delta(p, a, Z)$

$[p, Z, {}^{p}_{q}] \rightarrow a[p, X, {}^{p}_{q}][{}^{p}_{q}, Z, {}^{p}_{q}]$

We will have 4 productions.

$[p, Z, p] \rightarrow a[p, X, p][p, Z, p]\ a[p, X, q][q, Z, p]$

$[p, Z, q] \rightarrow a[p, X, p][p, Z, q]\ a[p, X, q][q, Z, q]$

$(p, XX) \in \delta(p, a, X)$ $\quad XX \cdot pq = 4$

$[p, X, {}^{p}_{q}] \rightarrow a[p, X, {}^{p}_{q}][{}^{p}_{q}, X, {}^{p}_{q}]$

We will have 4 productions.

$[p, X, p] \rightarrow a[p, X, p][p, X, p]\ a[p, X, q][q, X, p]$

$[p, X, q] \rightarrow a[p, X, p][p, X, q]\ a[p, X, q][q, X, q]$

$(q, X) \in \delta(p, b, X)$ $\quad X \cdot pq = 2$

We will have two productions

$[p, X, {}^{p}_{q}] \rightarrow b[q, X, {}^{p}_{q}]$

$[p, X, p] \rightarrow b[q, X, p]$

$[p, X, q] \rightarrow b[q, X, q]$

$(p, Z) \in \delta(q, b, Z)$ $\quad Z \cdot qz = 2$

We will have two productions

$[q, Z, {}^{p}_{q}] \rightarrow b[p, Z, {}^{p}_{q}]$

$[q, Z, p] \rightarrow b[p, Z, p]$

$[q, Z, q] \rightarrow b[p, Z, q]$

$(q, \epsilon) \in \delta(q, a, Z)$

We will have 1 production

$[q, Z, q] \to a$

$(p, \epsilon) \in \delta(p, \epsilon, Z)$

We will have one production.

$[p, Z, p] \to \epsilon$

So total we have 16 productions

Q5

Construct a turing machine to accept

$L = \{a^n b^n c^n \mid n \geq 1\}$

$L = \{abc, aabbcc, aaabbbccc\}$

Load input string on tape

S

B

States	a	b	c	X	Y	Z	$\emptyset$
q_0	(q_1, X, R)	—	—	—	(q_4, Y, R)	—	—
q_1	(q_1, a, R)	(q_2, Y, R)	—	—	(q_1, Y, R)	—	—
q_2	—	(q_2, b, R)	(q_3, Z, L)	—	—	(q_2, Z, R)	—
q_3	(q_3, a, L)	(q_3, b, L)	—	(q_0, X, L)	(q_3, Y, L)	(q_3, Z, L)	—
q_4	—	—	—	—	(q_4, Y, R)	(q_4, Z, R)	(q_5, B, R)

TM, $M = (\{q_0, q_1, q_2, q_3, q_4, q_5\}, \{a, b, c\}, \{a, b, c, X, Y, Z, \emptyset\}, \delta, q_0, \emptyset, \{q_5\})$

Spring 2009

Construct a Turing Machine for
$L = \{0^k 1^j 0^i \mid i > j > k \geq 0\}$
Describe in words what you are doing, then formulate the process.

We know that the number of left zeros is greater than the number of 1's which is less than the number of right zeros.