COSC 3340/6309
Examination 3
Wednesday, June 24, 2009, 2 – 3:45 pm
Open Book and Notes

1. Prove that the following language L is not contextfree:

$L = \{ a^k b^j a^i \mid i>j>k\geq 0 \}$.

2. Construct a pda $\mathbb{P}$ for the following language:

$L = \{ 0^{2i} 1^i \mid i\geq 0 \}$ where $L = L_f(\mathbb{P})$ (acceptance **by final state**).

State on which side you write the top of the stack, left: ____ or right ____ .

3. Construct a pda $\mathbb{P}$ that accepts the following language **by empty stack**:

$L = L(G)$ where $G = (T, N, P, E)$ with $T = \{ id,+,*,(,) \}$,

$N = \{E\}$, and $P = \{ E \to E+E \mid E*E \mid (E) \mid id \}$.

State on which side you write the top of the stack, left: ____ or right ____ .

Note: You <u>must</u> use the construction "cfg → pda" given in class. Get G into GNF first!

4. Construct a grammar for L(G) for the language N($\mathbb{P}$):

$\mathbb{P} = (\{p,q\}, \{a,b\}, Z,X\}, \delta, p, Z, \varnothing)$ where the move function δ is given by

$\delta(p,b,Z) = \{(p,XZ)\}$ $\quad\delta(q,\varepsilon,Z) = \{(q,\varepsilon)\}$ $\quad\delta(p,b,X) = \{(p,XX)\}$

$\delta(q,b,Z) = \{(p, XZ)\}$ $\quad\delta(q,a,X) = \{(q, \varepsilon)\}$ $\quad\delta(p,a,X) = \{(p,\varepsilon)\}$.

Here, the top of the stack is on the left.

5. Construct a Turing machine for the language in Question 1,

$L = \{ a^k b^j a^i \mid i>j>k\geq 0 \}$.

Describe first in words what you are doing, then formulate the formal Turing machine.

Points:	1: 20	2: 12	3: 18	4: 30	5: 20

① Prove $L = \{a^k b^j a^i \mid i > j > k \geq 0\}$ is not context free

Assume that L is context free and there is a $G = (N, T, P, S)$ in CNF form such that $L = L(G)$

Let $k = 2^n$

$j > k \therefore j = 2^n + 1$

$i > j \therefore i = 2^n + 2$

$z = a^{2^n} b^{2^n+1} a^{2^n+2}$ ✓, $|z| = |2^n + 2^n + 1 + 2^n + 2| = 3 \cdot 2^n + 3$

$|z| > 2^n$

$z \in (L)$ by the pumping lemma for context free language z can be rewritten in the form $z = uvwxy$, where $|vx| \geq 1$

the pumping lemma tells us that $uv^s wx^s y \in L \ \forall s \geq 0$

① Consider v and x to be all a's one the left if we let $s = 2$, we increase the number of a's while the number of b's remain the same. number of a's is more than number of b's

$uv^s wx^s y \notin L$ but $\in L(G)$

therefore we have a contradiction. ✓

continued.

(2) v & x are all b's
for $s=2$ ✓ $z \in L(G)$, $z \notin L$ why

(3) v & x are all a's on the right
for $s=0$ ✓ $z \in L(G)$, $z \notin L$

(4) No a's on the right in v or x
for $s=2$ $z \in L(G)$, $z \notin L$ why?
✓

(5) No b's in v or x
for $s=2$ ✓ $z \in L(G)$, $z \notin L$

(6) No a's on the left
for $s=0$ $z \in L(G)$, $z \notin L$
✓

(7) at least one a and b in v or x
for $s>1$ the pattern will change in z
where a's and b's alternate multiple times
that pattern is not in L
✓
$z \in L(G)$, $z \notin L$

∴ we have proved that L is not context free

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② $\dashv$

		0	1	ε
q	Z	(p, XZ)	/	/
	X	/	/	/
	X_2	(p, XX_2)	(m, ε)	/
p	Z	(q, X_2)	/	/
	X	(p, ε)	/	/
	X_2	(q, X_2)	/	/
m	Z	/	/	(q_f, ε)
	X	/	/	/
	X_2	/	(m, ε)	/
q_f	Z			
	X		accepting	
	X_2			
$\rightarrow q_0$	Z	(p, XZ)	/	(q_f, ε)
	X	/	/	/
	X_2	/	/	/

X 4

more like $0^i 1^i$

1	18
2	4
3	18
4	30
5	20
	90

$P = (\{q_0, q, p, m, q_f\}, \{0, 1\}, \{Z, X, X_2\}, d, q_0, Z, \{q_f\})$

top of the stack is on the left $\dashv$

③ $E \to E+E \mid E*E \mid (E) \mid id$

Get G into GNF

remove left recursion

$E \to (E) \mid id \mid (E)E' \mid idE'$

$E' \to +E \mid *E \mid +EE' \mid *EE'$ ✓

$E \to (EX_1 \mid id \mid (EX_1E' \mid idE'$

$E' \to +E \mid *E \mid +EE' \mid *EE'$ ✓

$X_1 \to)$

		id	+	*	(	)	ε
q	E	(q, ε) (q, E')	/	/	(q, EX₁) (q, EX₁E')	/	/
	E'	/	(q, E) (q, EE')	(q, E) (q, EE')	/	/	/
	X_1	/	/	/	/	(q, ε)	/

✓

$P_N = (\{q\}, \{id, +, *, (,)\}, \{E, E', X_1\}, \delta, q, E, \emptyset\}$

the top of stack is on the left

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④

		a	b	ε
p	Z	/	(p, XZ)	/
	X	(p, ε)	(p, XX)	/
q	Z	/	(p, XZ)	(q, ε)
	X	(q, ε)	/	/

① $S \to [p, Z, p] \mid [p, Z, q]$

② $(p, XZ) \in \delta(p, b, Z)$

$[p, Z, \ ] \to b[p, X, \ ][\ , Z, \ ]$

$[p, Z, p] \to b[p, X, p][p, Z, p] \mid b[p, X, q][q, Z, p]$

$[p, Z, q] \to b[p, X, p][p, Z, q] \mid b[p, X, q][q, Z, q]$

$(p, XZ) \in \delta(q, b, Z)$

$[q, Z, \ ] \to b[p, X, \ ][\ , Z, \ ]$

$[q, Z, p] \to b[p, X, p][p, Z, p] \mid b[p, X, q][q, Z, p]$

$[q, Z, q] \to b[p, X, p][p, Z, q] \mid b[p, X, q][q, Z, q]$

$(q, \varepsilon) \in \delta(q, \varepsilon, Z)$	$(q, \varepsilon) \in \delta(q, a, X)$	$(p, \varepsilon) \in \delta(p, a, X)$
$[q, Z, q] \to \varepsilon$	$[q, X, q] \to a$	$[p, X, p] \to a$

continued ~~on~~ back