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COSC 3340/6309
Examination 3
Thursday, June 28, 2012, 4 – 6 pm
Open Book and Notes

1. Prove that the following language L is not contextfree:

20 $L = \{ a^i b^j a^i \mid j \geq i \geq 1 \}$.

2. Construct a pda $\mathbb{P}$ for the following language:

12 $L = \{ 0^i 1^{4i} \mid i \geq 0 \}$ where $L = L(\mathbb{P})$ (acceptance **by final state**).

3. Construct a pda $\mathbb{P}$ that accepts the following language **by empty stack**:

18 $L = L(G)$ where $G = (T, N, P, E)$ with $T = \{ id,+,-,(,) \}$,

$N = \{E\}$, and $P = \{ E \rightarrow E+E \mid E-E \mid (E) \mid id \}$.

Note: You <u>must</u> use the construction "cfg → pda" given in class. Get G into GNF first!

4. Construct a grammar for L(G) for the language $N(\mathbb{P})$:

30 $\mathbb{P} = (\{p,q\}, \{a,b\}, \{Z,X\}, \delta, p, Z, \varnothing)$ where the move function δ is given by

$\delta(p,b,Z) = \{(p,XZ)\}$ $\quad \delta(q,\varepsilon,Z) = \{(q,\varepsilon)\}$ $\quad \delta(p,b,X) = \{(p,XX)\}$

$\delta(q,b,Z) = \{(p, XZ)\}$ $\quad \delta(q,a,X) = \{(q, \varepsilon)\}$ $\quad \delta(p,a,X) = \{(p,\varepsilon)\}$.

5. Construct a Turing machine for the language in Question 1,

20 $L = \{ a^i b^j a^i \mid j \geq i \geq 1 \}$.

Describe first in words what you are doing, then formulate the formal Turing machine.

Points: 1: 20 2: 12 3: 18 4: 30 5: 20

1) $L = \{a^i b^j a^i \mid j \geq i \geq 1\}$

Assume L is CFL, then $\exists\ G = (M, T, P, S)$ in CNF s.t. $L = L(G)$

With $z \in L(G)$, it can be expressed as $z = uvwxy$ with $|vx| \geq 1$, then by $uv^s wx^s y \in L(G)\ \forall s \geq 0$

$\Rightarrow$ Consider $z = a^{2^n} b^{2^n} a^{2^n}$ with $j \neq i$ to prove $L \neq$ CFL

Case 1: v and x only a's on left
for $s = 2$ a's on the left is too many

Case 2: v and x only a's on right
for $s = 2$ a's on the right is too many

Case 3: v and x only b's
for $s = 0$ b's are too few

Case 4: v and x are a's left and b's
for $s = 2$ a's on the left is too many

Case 5: v and x are a's right and b's
for $s = 2$ a's on the right is too many

Case 6: v and X are left and right a's
for $s = 2$ Too few of b's

$z \in L(G)$ but $\notin L \Rightarrow$ Contradiction

For every case, we found contradictions, which proves that L is not context free

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$(q_0, 0, z_0) \rightarrow (q_0, zzzzz_0)$
$(q_0, 0, z) \rightarrow (q_0, zzzzz)$
$(q_0, 1, z) \rightarrow (q_1, \varepsilon)$
$(q_1, 1, z) \rightarrow (q_1, \varepsilon)$

$(q_1, 1, z_0) \rightarrow (q_1, \varepsilon)$
$(q_0, \varepsilon, z_0) \rightarrow (q_1, \varepsilon)$

2) $L = \{ 0^i 1^{4i} \mid i \geq 0 \}$

A pda is constructed for empty stack ac...

$P_e = (\{0,1\}, \{q_0, q_1\}, \{z_0, z\}, \delta, q_0, \emptyset, z_0)$

Top of stack is on left.

δ		0	1	ε
q_0	z_0	$\{(q_0, zzzzz_0)\}$	/	$\{(q_1, \varepsilon)\}$
	z	$\{(q_0, zzzzz)\}$	$\{(q_1, \varepsilon)\}$	/
q_1	z_0	/	/	$\{(q_1, \varepsilon)\}$
	z	/	$\{(q_1, \varepsilon)\}$	/

Final state acception

$P_f = (\{0,1\}, \{(q_0', q_0, q_1, q_f)\}, (z_0', z_0, z), \delta', q_0', q_f, z_0')$

δ'		0	1	ε
q_0'	z_0'	/	/	$\{(q_0, z_0 z_0')\}$
	z_0	/	/	/
	z	/	/	/
q_0	z_0'	/	/	/
	z_0	$\{(q_0, zzzzz_0)\}$	/	$\{(q_1, \varepsilon)\}$
	z	$\{(q_0, zzzzz)\}$	$\{(q_1, \varepsilon)\}$	/
q_1	z_0'	/	/	$\{(q_f, \varepsilon)\}$
	z_0	/	/	$\{(q_1, \varepsilon)\}$
	z	/	$\{(q_1, \varepsilon)\}$	/
q_f	z_0'	/	/	/
	z_0	/	/	/
	z	/	/	/

final state

3) $E \to E+E \mid E-E \mid (E) \mid id$

Convert to GNF

Remove left immediate recursion

$E \to (E) \mid id \mid (E)E' \mid idE'$

$E' \to +E \mid -E \mid +EE' \mid -EE'$

$E \to (EX_1 \mid id \mid (EX_1E' \mid idE'$

$E' \to +E \mid -E \mid +EE' \mid -EE'$

$X_1 \to)$

		id	+	−	(	)	ε
q	E	$\{(q,\varepsilon),\ (q,E')\}$	/	/	$\{(q,EX_1),\ (q,EX_1E')\}$	/	/
	E'	/	$\{(q,E),\ (q,EE')\}$	$\{(q,E),\ (q,EE')\}$	/	/	/
	X_1	/	/	/	/	(q,ε)	/

$P_N = (\{q\}, \{id, +, -, (,)\}, \{E, E', X_1\}, \delta, q, E, \varnothing\}$

Top of stack is on the left

4)

		a	b	ε
p	Z	/	(p, XZ)	/
	X	(p, ε)	(p, XX)	/
q	Z	/	(p, XZ)	(q, ε)
	X	(q, ε)	/	/

1) S → [p, Z, p] | [p, Z, q]

2) ① (p, XZ) ∈ δ(p, b, Z)

[p, Z,] → b[p, X,][, Z,]

4 Productions

[p, Z, p] → b[p, X, p][p, Z, p] | b[p, X, q][q, Z, p]

[p, Z, q] → b[p, X, p][p, Z, q] | b[p, X, q][q, Z, q]

② (p, XZ) ∈ δ(q, b, Z)

[q, Z,] → b[p, X,][, Z,]

4 Productions:

[q, Z, p] → b[p, X, p][p, Z, p] | b[p, X, q][q, Z, p]

[q, Z, q] → b[p, X, p][p, Z, q] | b[p, X, q][q, Z, q]

1 Production

③ (q, ε) ∈ δ(q, ε, Z)

[q, Z, q] → ε

1 Production

④ (q, ε) ∈ δ(q, a, X)

[q, X, q] → a

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⑤ (p, ε) ∈ δ(p, a, X) 1 Production

[p, X, p] → a

⑥ (p, XX) ∈ δ(p, b, X) ⇒ [p, X,] → b[p, X,][, X,]

4 Production =

[p, X, p] → b[p, X, p][p, X, p] | b[p, X, q][q, X, p]

[p, X, q] → b[p, X, p][p, X, q] | b[p, X, q][q, X, q] aa bbb aa

5) $L = \{ a^i b^j a^i \mid j \geq i \geq 1 \}$

Example: a a b b b a^+ a^+ ƀ   a b a

a a a b b b a a a

First condition

Let right a's be renamed as a^+ and ƀ is blank.
We know that number of left a's is equal to number of right a's but less than or equal to number of b's

In the turing machine, iniate from left side. Prime a for corresponding left a's, move right till reach b and prime b. Move right again till reach ƀ. Move Left till reach a^+(right) and mark as \$. Move left till reach a'. Then move right and repeat the process above unless the current state contains a' and the next state contains b'. Find the rest of the b's prime them. All should be mark and gone to final accepting state if the first condition is followed.

If $i = j = 1$, or aba^+
Move from Left to right while marking left a and b. Skip a^+ and reach ƀ. Turn right, mark a^+ as \$, and move till reach a'. Move left to b' and go to final accepting state (Look at state q_5 under column b').

Continue

$a' a' a \; b' b b b \; a^+ \$ \$ \not{b}$

$a' b' \$$

5)

	a	b	a^+	a'	b'	\$	$\not{b}$
q_0	(q_1, a', R)						
q_1	(q_1, a, R)	(q_2, b', R)					
q_2		(q_2, b, R)	(q_2, a^+, R)				$(q_3, \not{b}, L)$
q_3			$(q_4, \$, L)$				
q_4	(q_4, a, L)	(q_4, b, L)	(q_4, a^+, L)	(q_5, a', R)	(q_4, b', L)		
q_5	(q_6, a', R)				(q_f, b', R)		
q_6	(q_6, a, R)	(q_7, b', R)			(q_6, b', R)		
q_7		(q_7, b, R)	(q_7, a^+, R)			$(q_8, \$, L)$	
q_8			$(q_9, \$, L)$				
q_9	(q_9, a, L)	(q_9, b, L)	(q_9, a^+, L)	(q_{10}, a', R)	(q_9, b', L)		
q_{10}	(q_{11}, a', R)						
q_{11}	(q_6, a, R)	(q_{12}, b', R)			(q_{11}, b', R)		
q_{12}		(q_{12}, b', R)				$(q_f, \$, R)$	
q_f	/	/	/	/	/	/	/

Moving first a' → q_5

accepting state → q_f

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