COSC 3340 — Final Examination

Wednesday, July 2, 2008, 2 – 5 pm

Open Book and Notes — **Final grades only through PeopleSoft**

<u>YOU MUST USE THE CONSTRUCTIONS GIVEN IN CLASS</u>

1. Construct a regular expression over {a,b,c} for the language accepted by this nfa:

	a	b	c	
→ A	/	B	/	0
B	B	/	A,C	1
C	/	B,C	/	1

2. Prove that the language L(G) is not regular where G is the following cfg:

   $G = (\{S,A,B,C\}, \{a,b,c\}, \{S \to Aa|B|C,\ A \to aS,\ B \to a,\ C \to b\}, S)$.

Note: You <u>must</u> first determine L(G).

3. Construct a reduced dfa for the following extended regular expression over {0,1,2}:

$$[(100^*)^* \cap \overline{1^*}\ ]$$

Note: You <u>must</u> first determine nfas for (100*)* and 1*, then do the intersection. The answer must then be reduced.

4. Construct a Chomsky normal form grammar for L(G) for the following cfg G:

   $G = (\{S,B\}, \{a,b,c,d\}, \{S \to SaSbc|Ba,\ B \to cBd|S|\varepsilon\}, S)$.

Note: You <u>must</u> first remove all ε- and all unit productions.

5. Construct a Greibach normal form grammar for L(G) for the following CNF G:

   $G = (\{S,A\}, \{a,d\}, \{S \to AS|A|d,\ A \to SS|a\}, S)$.

Note: You <u>must</u> first remove all unit productions. You <u>must</u> derive all the productions for S and A; indicate how the result looks for S' and A' as applicable.

6. Prove that the following language L is not contextfree: $L = \{\ 0^{n+1}1^{n-1}0^n \mid n>0\ \}$.

7. Consider the class $\mathcal{L}_A$ of all contextfree languages over the fixed alphabet A.

   (a) Is $\mathcal{L}_A$ countable?

   (b) Is the class $\mathcal{M}_A$ countable where $\mathcal{M}_A$ consists of all languages over A that are not in $\mathcal{L}_A$?

   (c) Is the class $\mathcal{L}_A \cap \mathcal{M}_A$ countable?

For each question, you must give a precise argument substantiating your answer.

8. Construct a Turing machine for the language in Question 6, $L = \{\ 0^{n+1}1^{n-1}0^n \mid n>0\ \}$.

Note: Describe first the process in English; then translate this into moves of the Turing machine.

9. Let L_1 and L_2 be arbitrary languages, subject to the specification in either (i) or (ii). Consider the following four general questions:

$\overline{L_1} - L_2 = \emptyset$ if $L_1 = \overline{L_2}$

(Q1) Does $\overline{L_1} - L_2$ contain a given fixed word w? (Q2) Is $\overline{L_1} - L_2$ non-empty? $\overline{L_1 \cup L_2}$

(Q3) Does $L_1 \cap L_2$ contain a given fixed word w? (Q4) Is $L_1 \cap L_2$ non-empty? $L_1 \cup L_2 \neq S$

For each of these four questions **<u>explain with reasons</u>** whether the problem is <u>recursive</u>, <u>not recursive but r. e.</u>, or <u>non-r. e.</u>, provided

(i) Both L_1 and L_2 are <u>recursive</u>. (ii) Both L_1 and L_2 are <u>r. e., but not recursive</u>.

Points: 1: 6 2: 8 3: 14 4: 12 5: 12 6: 12 7: 13 8: 8 9:15

1. $A \to bB$

$B \to aB \vee cA \vee cC \vee \varepsilon$

$\to a^*c(A \vee C) \vee a^*$

$C \to bB \vee bC \vee \varepsilon$

$\to b^*bB \vee b^*$

$B \to a^*cA \vee a^*cbB \vee a^*cb^*bB \vee a^*cb^* \vee a^*$

$\to (a^*cb \vee a^*cb^*b)^* a^*cA \vee (a^*cb \vee a^*cb^*b)^*(a^*cb^* \vee a^*)$

$A \to b(a^*cb \vee a^*cb^*b)^* a^*cA \vee b(a^*cb \vee a^*cb^*b)^*(a^*cb^* \vee a^*)$

$\to [b(a^*cb \vee a^*cb^*b)^* a^*c]^* b(a^*cb \vee a^*cb^*b)^*(a^*cb^* \vee a^*)$

2. $S \to Aa \mid B \mid C$

$A \to aS$

$B \to a$

$C \to b$

$S \to aSa \mid a \mid b$ $\quad L(G) = \{ a^i(a \vee b)a^i \mid a \geq 0 \}$

Assume that L_G is regular $\therefore$ it is accepted by some DFA R.

R has n states. Choose $x = a^n b a^n \in L_G$

x is long enough $\therefore$ it can be pumped $\quad w = a^n \quad u = ba^n$

$w = w_1 w_2 w_3$ & $|w_2| > 0$ & $\forall s \geq 0$ st $w_1 w_2^s w_3 u \in L_G$

Choose $s = 2$ $\quad a^{n+|w_2|} b a^n \in L_G$ contradiction $\therefore L_G$ is not regular

$3\ [(100^*)^* \cap \overline{1^*}] = [\overline{\underbrace{\overline{(100^*)^*}}_{1,2,3} \cup \underbrace{1^*}_{4}}]$

	0	1	2	
→0	/	1	/	0
1	2	/	/	0
2	2	/	/	1

$\xrightarrow{**}$

	0	1	2	
→0	/	1	/	0
1	2	/	/	1
2	2	1	/	0

	0	1	2	
→0	/	4	/	1
4	/	4	/	1

$\xleftarrow{\cup 4}$

	0	1	2	
→0	/	1,4	/	0
1	2	/	/	0
2	2	1	/	1
4	/	4	/	0

	0	1	2		
→0	/	1,4	/	0	A
/	/	/	/	0	B
1,4	2	4	/	0	C
2	2	1	/	1	D
4	/	4	/	0	E
1	2	/	/	0	F

A B C E F			D
A B E	C F		D
A	B E	C F	D
A	B E	C F	D

	0	1	2	
→A	B	C	B	0
B	B	B	B	0
C	D	B	B	0
D	D	C	B	1

4 $S \to SaSbc \mid Ba$
$B \to cBd \mid S \mid \varepsilon$

$S \to SaSbc \mid Ba \mid a$
$B \to cBd \mid cd \mid S$

$S \to SaSbc \mid Ba \mid Sa \mid a$
$B \to cBd \mid cSd \mid cd$

$S \to SS_1 \mid BX_a \mid SX_a \mid a$ $\quad X_a \to a$
$S_1 \to X_a S_2$ $\quad X_b \to b$
$S_2 \to S S_3$ $\quad X_c \to c$
$S_3 \to X_b X_c$ $\quad X_d \to d$
$B \to X_c B_1 \mid X_c B_2 \mid X_c X_d$
$B_1 \to BX_d$
$B_2 \to SX_d$

5 $S \to AS \mid A \mid d$

$A \to SS \mid a$

$S \to AS \mid AA \mid d$

$A \to SS \mid AS \mid SA \mid AA \mid a$

$A \to a \mid aA'$

$A' \to SS \mid AS \mid SA \mid AA$

$S \to aS \mid aA'S \mid aA \mid aA'A \mid d$

$A \to a \mid aA'$

$A' \to aSS \mid aA'SS \mid aAS \mid aA'AS \mid dS \mid aSA \mid aA'SA \mid aAA \mid aA'AA \mid dA$
$\mid aS \mid aA'S \mid aA \mid aA'A$

7a Yes. All CFLs can be described and are unique so they are countable.

b No. There are languages in M_A that cannot be described so they cannot be given a value. This makes the set uncountable.

c $L_A \cap M_A = \emptyset$ so it is countable because empty set is countable.

8 Erase 2 0s from front of string + 1 from end of string. If tape is empty accept. Return to front of string. Check if there are equal number first 0s, 1s, + second 0s by marking off first unmarked of each until reach end of string. When finished marking first set of 0s, check that rest of string has been marked.

	0	1	$\not b$	$\overline{0}$	$\overline{1}$
q_0	$(\not b, q_1, R)$	/	/	/	/
q_1	$(\not b, q_2, R)$	/	/	/	/
q_2	$(0, q_2, R)$	$(1, q_2, R)$	$(\not b, q_3, L)$	/	/
q_3	$(0, q_4, L)$	/	$(\not b, q_F, R)$	/	/
q_4	$(0, q_4, L)$	$(1, q_4, L)$	$(\not b, q_5, R)$	/	/
q_5	$(\overline{0}, q_6, R)$	/	/	/	/
q_6	$(0, q_6, R)$	$(\overline{1}, q_7, R)$	/	/	$(\overline{1}, q_6, R)$
q_7	$(\overline{0}, q_8, L)$	$(1, q_7, R)$	/	$(\overline{0}, q_7, R)$	/
q_8	/	$(1, q_8, L)$	/	$(\overline{0}, q_8, L)$	$(\overline{1}, q_9, L)$
q_9	$(0, q_9, L)$	/	/	$(\overline{0}, q_{10}, R)$	$(\overline{1}, q_9, L)$
q_{10}	$(\overline{0}, q_6, R)$	/	/	/	$(\overline{1}, q_{11}, R)$
q_{11}	/	/	/	$(\overline{0}, q_{12}, R)$	$(\overline{1}, q_{11}, R)$
q_{12}	/	/	$(\not b, q_F, R)$	$(\overline{0}, q_{12}, R)$	/
q_F			accept		

all even here

note: I got notation backwards. should be (state, symbol, direction) not (symbol, state, direction

9 1: $\bar{L}_1 - L_2$ is recursive because recursive languages are closed under complement & set difference. A TM for deciding if a word is in a recursive language is <u>recursive</u>

ii $\bar{L}_1 - L_2$ is not r.e. because r.e. languages that are not recursive are not closed under complement or set difference. The problem is <u>non-r.e.</u>

2: ~~[illegible] recursive~~. $\bar{L}_1 - L_2 \neq \emptyset$ if $\bar{L}_1 \neq L_2$. To prove that two languages are different a word must be found that is in one but not the other. This is <u>r.e.</u>

ii $\bar{L}_{1}$ is non-re. Because there is no way to prove that the language contains a word, the problem is <u>non-r.e.</u>

3: $L_1 \cap L_2$ is recursive. Recursive languages are closed under intersection. A TM for the language would be <u>recursive</u>.

ii $L_1 \cap L_2$ is r.e. because r.e. languages are closed under intersection. A TM for this language would be <u>r.e.</u>

4: $\overline{L_1 \cap L_2} \neq \emptyset$ Same logic as 2i. <u>r.e.</u>

$\bar{L}_1 \cup \bar{L}_2 \neq \emptyset$

$L_1 - \bar{L}_2 \neq \emptyset$

$L_1 \neq \bar{L}_2$

ii Same logic as 2ii. non-r.e.