COSC 3340/6309
Examination 2
Monday, March 27, 2023, 2:30-4 pm
Open Book and Notes

1. Prove that the language L(G) is not regular where G is the following context-free grammar: $G = (\{S,A\}, \{a,b\}, \{ S \to aSa \mid bAb,\ A \to bAb \mid a \}, S)$.
Note: You <u>must</u> first determine L(G).

2. Eliminate all ε-productions in the following cfg G:

$$G = (\{S,A,B\}, \{a,b,c\}, \{S\to Aa|BB,\ A\to bS|\varepsilon,\ B\to cSSc|\varepsilon\}, S).$$

3. Construct a **reduced** dfa for the following extended regular expression over the alphabet {a,b,c}:

$$\overline{\overline{a^*}\cdot\overline{b^*}\cdot\overline{b^*}}$$

Note: You <u>must</u> first determine nfas for a* and b* over {a,b,c}, then handle the complementation, then do the concatenations, and then deal with the complementation. Finally reduce the resulting dfa.

4. Construct a Chomsky normal form grammar for L(G) for the following cfg G:

$$G = (\{S,B\}, \{a,b,c,d\}, \{S\to SdSaBaS|SBS|b,\ B\to Sda|S|dcb\}, S).$$

Note: You <u>must</u> first remove all unit productions.

5. Construct a Greibach normal form grammar for L(G) for the following CNF G:

$$G = (\{S,A,B\}, \{a,b\}, \{S\to AS|a,\ A\to BS,\ B\to SB|b\}, S).$$

Note: First derive <u>all</u> the productions for S, A, and B. Then do the primed variables.

Points: 1: 20 2: 10 3: 30 4: 15 5: 25

13.5 24

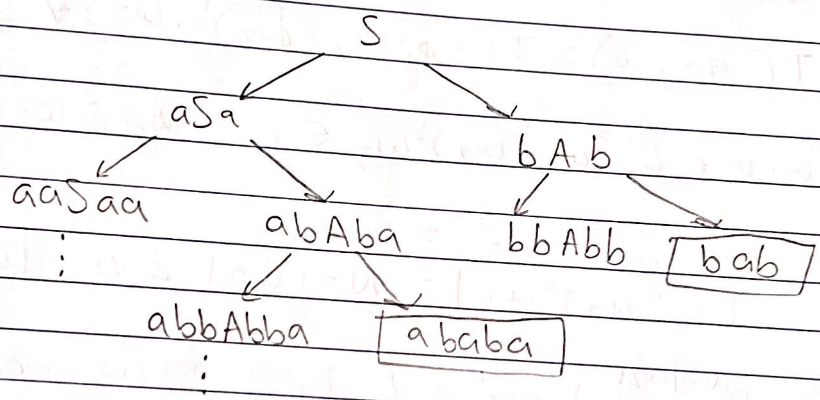
19

Q3 30

#1

$S \to aSa \mid bAb$

$A \to bAb \mid a$



bab

a bab a

19

-1

$\boxed{L(G) = \{a^n b^M a b^r a^n \mid n \geq 0\}}$ $M \geq 1$

Claim that L is not regular $\exists$ DFA ℓ accepting L

N is the number of states in ℓ

Consider $x = a^n baba^n$ with $|x| = N$

$x = w \cdot v$, $w = a^N$ and $v = baba^N$ $|w| = N$

Continued next page $\longrightarrow$

Pumping Lemma

$w = w_1 \cdot w_2 \cdot w_3$ such that $|w_2| \geq 1$ and

$$T(q_0, w) = T(q_0, w_1 (w_2)^s \cdot w_3) \; \forall \, s \geq 0$$

where $w \cdot v \in L$, $w_1 (w_2)^s w_3 \in L \quad \forall_s \geq 0$

if $s = 0$ $\quad w_1 (w_2)^0 w_3 = w_1 w_3$

$|w_1 (w_2)^0 w_3| = N - |w_2| < N \quad |w_2| \geq 1$

$\Rightarrow a^{N-|w_2|} baba^N \notin L$, but it is accepted by automata

$\rightarrow$ we can see a contradiction

$\rightarrow$ $L(G)$ is not regular

#2 $S \to Aa \mid BB$
$A \to bS \mid \varepsilon$
$B \to cSSc \mid \varepsilon$

Remove ε from A
$S \to Aa \mid a \mid BB$
$A \to bS$
$B \to cSSc \mid \varepsilon$

Remove ε from B
$S \to Aa \mid a \mid BB \mid B \mid \varepsilon$
$A \to bS$
$B \to cSSc$

Remove ε from S
$S \to Aa \mid a \mid BB \mid B$
$A \to bS \mid b$
$B \to cSSc \mid cSc \mid cc$

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#3 $\overline{a^*} \cdot \overline{b^*} \cdot \overline{b^*}$

1 2 3

	a	b	c	
0	1	/	/	0
1	/	/	/	1

	a	b	c	
0	/	2	/	0
2	/	/	/	1

	a	b	c	
0	/	3	/	0
3	/	/	/	1

$*$

	a	b	c	
0	1	/	/	1
1	1	/	/	1

$*$

	a	b	c	
0	/	2	/	1
2	/	2	/	1

	a	b	c	
0	/	3	/	1
3	/	3	/	1

⇓ bar

	a	b	c	
0	1	/	/	1
1	1	/	/	1
/	/	/	/	0

⇓ bar

	a	b	c	
0	/	2	/	1
/	/	/	/	0
2	/	2	/	1

⇓ bar

	a	b	c	
0	/	3	/	1
/	/	/	/	0
3	/	3	/	1

	a	b	c	
0	1	4	4	0
1	1	4	4	0
4	4	4	4	1

	a	b	c	
0	5	2	5	0
5	5	5	5	1
2	5	2	5	0

	a	b	c	
0	6	3	6	0
6	6	6	6	1
3	6	3	6	0

c

	a	b	c	
0	1	4	4	0
1	1	4	4	0
4	4,5	4,2	4,5	0
5	5	5	5	1
2	5	2	5	0

c

	a	b	c	
0	1	4	4	0
1	1	4	4	0
4	4,5	4,2	4,5	0
5	5,6	5,3	5,3	0
2	5	2	5	0
6	6	6	6	1
3	6	3	6	0

next page →

#13 continued

dfa

	a	b	c	
0	1	4	4	0
1	1	4	4	0
4	4,5	4,2	4,5	0
4,5	4,5,6	4,2,5,3	4,5,6	0
4,2	4,5	4,2	4,5	0
4,5,6	4,5,6	4,2,5,3,6	4,5,6	1
4,2,5,3	4,5,6	4,2,5,3	4,5,6	0
4,2,5,3,6	4,5,6	4,2,5,3,6	4,5,6	1

$\xrightarrow{\text{bar}}$

		a	b	c	
A	0	1	4	4	1
B	1	1	4	4	1
C	4	4,5	2,4	4,5	1
D	4,5	4,5,6	2,3,4,5	4,5,6	1
E	4,2	4,5	2,4	4,5	1
F	4,5,6	4,5,6	2,3,4,5,6	4,5,6	0
G	4,2,5,3	4,5,6	2,3,4,5	4,5,6	1
H	4,2,5,3,6	4,5,6	2,3,4,5,6	4,5,6	0

	a	b	c	
A	B	C	C	1
B	B	C	C	1
C	D	E	D	1
D	F	G	F	1
E	D	E	D	1
F	F	H	F	0
G	F	G	F	1
H	F	H	F	0

$\longrightarrow$

0	1		
F H	A, B, C, D, E, G		
FH	AB	CE	DG
1	2	3	4

	a	b	c	
2	2	3	3	1
3	4	3	4	1
4	1	4	1	1
1	1	1	1	0

#4. $S \to SdSaBaS \mid SBS \mid b$
$B \to Sda \mid \cancel{S} \mid dcb$

$\boxed{13.5}$

$B \to S$
$S \to SdSaBaS \mid SdS\,aSaS \mid SBS \mid SSS \mid b$
$B \to Sda \mid dcb$

$S \to SX_d\,SX_a\,B\,X_aS \mid SX_dSX_aSX_aS \mid SBS \mid SSS \mid X_b$ b
$B \to SX_d\,X_a \mid X_aX_cX_b$

How about SSS?

$S \to SS_1 \mid SS_6 \mid SS_{11} \mid X_b$
$B \to SB_1 \mid X_dB_2$

$S_1 = X_dS_2$
$S_2 = SS_3$
$S_3 = X_aS_4$
$S_4 = BS_5$
$S_5 = X_aS$
$S_6 = X_aS_7$
$S_7 = SS_8$

$S_8 = X_aS_9$
$S_9 = SS_{10}$
$S_{10} = X_aS$
$S_{11} = BS$

$X_a = a$
$X_c = c$
$X_b = b$
$X_d = d$

$B_1 = X_dX_a$ $B_2 = X_cX_b$

Scanned with CamScanner

rule : $i < j$

#5

$S_1 \to AS \mid a$

$A_2 \to BS$

$3 \not< 1$ $\to$ $B_3 \to SB \mid b$

$B \not< S$

$3 \not< 2$ $\quad B \to ASB \mid aB \mid b$

$B \not< A$

$B \to \underbrace{BSSB}_{\alpha} \mid \underbrace{aB}_{b_1} \mid \underbrace{b}_{b_2}$

$B' \to SSB \mid SSBB'$

$B \to aB \mid b \mid aBB' \mid bB'$

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$A \to abS \mid bS \mid aBB'S \mid bB'S$

$S \to abSS \mid bSS \mid aBB'SS \mid bB'SS \mid a$

$S \to B'$

$B' \to aBSSSSB \mid bSSSB \mid aBB'SSSB \mid bB'SSSB \mid c \mid aBSSSSBB' \mid bSSSBB' \mid aBB'SSSBB' \mid bB'SSSBB'$

$S \to B$

$B \to ASB \mid aB \mid b$

$B' \to SSB \mid SSBB'$

$B \to AB \mid b \mid aBB' \mid bB'$

$B \to A$

$A \to aBS \mid bS \mid aBB'S \mid bB'S$

$A \to S$

$S \to aBSS \mid bSS \mid aBB'SS \mid bB'SS \mid a$