

Friday, May 8, 2009, 2 – 5 pm

Open Book and Notes Final grades only through PeopleSoft

YOU MUST USE THE CONSTRUCTIONS GIVEN IN CLASS

1. Construct a regular expression over $\{a,b,c\}$ for the language accepted by this nfa:

	a	b	c	
$\rightarrow A$	/	B	/	1
B	B	/	A,B,C	0
C	/	B,C	/	0

2. Prove that the language $L(G)$ is not regular where G is the following cfg:

$$G = (\{S,A,B,C\}, \{a,b,c\}, \{S \rightarrow aA|B|C, A \rightarrow Sa, B \rightarrow b, C \rightarrow a\}, S).$$

Note: You must first determine $L(G)$.

3. Construct a reduced dfa for the following extended regular expression over $\{0,1,2\}$:

$$[(10^*)^* \cap \overline{1^*}]$$

Note: You must first determine nfes for $(10^*)^*$ and 1^* , then do the intersection. The answer must then be reduced.

4. Construct a Chomsky normal form grammar for $L(G)$ for the following cfg G :

$$G = (\{S,B\}, \{a,b,c,d\}, \{S \rightarrow SSbS|Ba, B \rightarrow cBd|S|e\}, S).$$

Note: You must first remove all ϵ - and all unit productions.

5. Construct a Greibach normal form grammar for $L(G)$ for the following CNF G :

$$G = (\{S,A\}, \{a,b\}, \{S \rightarrow AS|A, A \rightarrow SS|ab\}, S).$$

Note: You must first remove all unit productions. You must derive all the productions for S and A ; indicate how the result looks for S' and A' .

6. Prove that the following language L is not contextfree: $L = \{0^n 1^{n+2} 0^n \mid n > 0\}$.

7. Consider the class CFL_A of all contextfree languages over the fixed alphabet A .

- Is CFL_A countable?
- Is the class $NOTCFL_A$ countable where $NOTCFL_A$ consists of all languages over A that are not contextfree?
- Is the class $CFL_A \cap NOTCFL_A$ countable?

For each question, you must give a precise argument substantiating your answer.

8. Construct a Turing machine for the language in Question 6, $L = \{0^n 1^{n+2} 0^n \mid n > 0\}$.

Note: Describe first the process in English; then translate this into moves of the Turing machine.

9. Let L_1 and L_2 be arbitrary languages, subject to the specification in either (i) or (ii).

Consider the following four questions:

(Q1) Does $L_1 - L_2$ contain a given fixed word w ? (Q2) Is $L_1 - L_2$ empty?

(Q3) Does $L_1 \cap L_2$ contain a given fixed word w ? (Q4) Is $L_1 \cap L_2$ empty?

For each of these four questions explain with reasons whether the problem is recursive, not recursive but r. e., or non-r. e., provided

(i) Both L_1 and L_2 are recursive. (ii) Both L_1 and L_2 are r. e., but not recursive.

Note that there are **eight** different questions to be answered.