

Wednesday, July 2, 2008, 2 – 5 pm

Open Book and Notes

Final grades only through PeopleSoft

YOU MUST USE THE CONSTRUCTIONS GIVEN IN CLASS

1. Construct a regular expression over $\{a,b,c\}$ for the language accepted by this nfa:

	a	b	c	
$\rightarrow A$	/	B	/	0
B	B	/	A,C	1
C	/	B,C	/	1

2. Prove that the language $L(G)$ is not regular where G is the following cfg:

$G = (\{S,A,B,C\}, \{a,b,c\}, \{S \rightarrow Aa|B|C, A \rightarrow aS, B \rightarrow a, C \rightarrow b\}, S)$.

Note: You must first determine $L(G)$.

3. Construct a reduced dfa for the following extended regular expression over $\{0,1,2\}$:

$$[(100^*)^* \cap \overline{1^*}]$$

Note: You must first determine nfes for $(100^*)^*$ and 1^* , then do the intersection. The answer must then be reduced.

4. Construct a Chomsky normal form grammar for $L(G)$ for the following cfg G :

$G = (\{S,B\}, \{a,b,c,d\}, \{S \rightarrow SaSbc|Ba, B \rightarrow cBd|S|\epsilon\}, S)$.

Note: You must first remove all ϵ - and all unit productions.

5. Construct a Greibach normal form grammar for $L(G)$ for the following CNF G :

$G = (\{S,A\}, \{a,d\}, \{S \rightarrow AS|A|d, A \rightarrow SS|a\}, S)$.

Note: You must first remove all unit productions. You must derive all the productions for S and A ; indicate how the result looks for S' and A' as applicable.

6. Prove that the following language L is not contextfree: $L = \{0^{n+1}1^{n-1}0^n \mid n > 0\}$.

7. Consider the class \mathcal{L}_A of all contextfree languages over the fixed alphabet A .

(a) Is \mathcal{L}_A countable?

(b) Is the class \mathcal{M}_A countable where \mathcal{M}_A consists of all languages over A that are not in \mathcal{L}_A ?

(c) Is the class $\mathcal{L}_A \cap \mathcal{M}_A$ countable?

For each question, you must give a precise argument substantiating your answer.

8. Construct a Turing machine for the language in Question 6, $L = \{0^{n+1}1^{n-1}0^n \mid n > 0\}$.

Note: Describe first the process in English; then translate this into moves of the Turing machine.

9. Let L_1 and L_2 be arbitrary languages, subject to the specification in either (i) or (ii).

Consider the following four general questions:

(Q1) Does $\overline{L_1 - L_2}$ contain a given fixed word w ? (Q2) Is $\overline{L_1 - L_2}$ non-empty?

(Q3) Does $L_1 \cap L_2$ contain a given fixed word w ? (Q4) Is $L_1 \cap L_2$ non-empty?

For each of these four questions explain with reasons whether the problem is recursive, not recursive but r. e., or non-r. e., provided

(i) Both L_1 and L_2 are recursive. (ii) Both L_1 and L_2 are r. e., but not recursive.

Points: 1: 6 2: 8 3: 14 4: 12 5: 12 6: 12 7: 13 8: 8 9: 15

$$1 \quad A \rightarrow bB$$

$$B \rightarrow aB \vee cA \vee cC \vee \epsilon$$

$$\rightarrow a^*c(A \vee C) \vee a^*$$

$$C \rightarrow bB \vee bC \vee \epsilon$$

$$\rightarrow b^*bB \vee b^*$$

$$B \rightarrow a^*cA \vee a^*cbB \vee a^*cb^*bB \vee a^*cb^* \vee a^*$$

$$\rightarrow (a^*cb \vee a^*cb^*b)^* a^*cA \vee (a^*cb \vee a^*cb^*b)^* (a^*cb^* \vee a^*)$$

$$A \rightarrow b(a^*cb \vee a^*cb^*b)^* a^*cA \vee b(a^*cb \vee a^*cb^*b)^* (a^*cb^* \vee a^*)$$

$$\rightarrow [b(a^*cb \vee a^*cb^*b)^* a^*c]^* b(a^*cb \vee a^*cb^*b)^* (a^*cb^* \vee a^*)$$

$$2 \quad S \rightarrow Aa | B | C$$

$$A \rightarrow aS$$

$$B \rightarrow a$$

$$C \rightarrow b$$

$$S \rightarrow aSa | a | b \quad L(G) = \{ a^i (a^*b) a^i \mid i \geq 0 \}$$

Assume that L_G is regular \therefore it is accepted by some DFA R .

R has n states. Choose $x = a^n b a^n \in L_G$

x is long enough \therefore it can be pumped. $w = a^n \quad u = b a^n$

$$w = w_1 w_2 w_3 \quad + \quad |w_2| > 0 \quad + \quad \forall s \geq 0 \quad \text{st} \quad w_1 w_2^s w_3 u \in L_G$$

Choose $s = 2 \quad a^{n+|w_2|} b a^n \in L_G \quad \rightarrow \text{contradiction: } L_G \text{ is not regular}$

$$3 \ [(100^*)^* \wedge 1^*] = [(100^*)^* \vee 1^*]$$

	0	1	2
→ 0	/	1	/
1	2	/	/
2	2	/	/

	0	1	2
→ 0	/	1	/
1	2	/	/
2	2	1	/

	0	1	2
→ 0	/	4	/
4	/	4	/

	0	1	2
→ 0	/	4	/
1	2	/	/
2	2	1	/
4	/	4	/

	0	1	2	
→ 0	/	4	/	0
/	/	/	/	0
4	2	4	/	0
2	2	1	/	1
4	/	4	/	0
1	2	/	/	0

A	B	C	E	F	D
A	B	E	C	F	D
A	B	E	C	F	D
A	B	E	C	F	D

	0	1	2
→ A	B	C	B
B	B	B	B
C	D	B	B
D	D	C	B

$$4 \quad S \rightarrow S_a S_b c \mid B a$$

$$B \rightarrow c B d \mid S \mid \epsilon$$

$$S \rightarrow S_a S_b c \mid B a \mid a$$

$$B \rightarrow c B d \mid c d \mid S$$

$$S \rightarrow S_a S_b c \mid B a \mid S_a \mid a$$

$$B \rightarrow c B d \mid c S d \mid c d$$

$$S \rightarrow S S_1 \mid B X_a \mid S X_a \mid a$$

$$S_1 \rightarrow X_a S_2$$

$$S_2 \rightarrow S S_3$$

$$S_3 \rightarrow X_b X_c$$

$$B \rightarrow X_c B_1 \mid X_c B_2 \mid X_c X_d$$

$$B_1 \rightarrow B X_d$$

$$B_2 \rightarrow S X_d$$

$$X_a \rightarrow a$$

$$X_b \rightarrow b$$

$$X_c \rightarrow c$$

$$X_d \rightarrow d$$

$$5 \quad S \rightarrow AS | A | d$$

$$A \rightarrow SS | a$$

$$S \rightarrow AS | AA | d$$

$$A \rightarrow SS | AS | SA | AA | a$$

$$A \rightarrow a | aA'$$

$$A' \rightarrow SS | AS | SA | AA$$

$$SS | SA$$

$$S \rightarrow aS | aA'S | aA | aA'A | d$$

$$A \rightarrow a | aA'$$

$$A' \rightarrow aSS | aA'SS | aAS | aA'AS | dS | aSA | aAA | aAA' | aA'A | dA$$

$$| aS | aA'S | aA | aA'A$$

7a Yes. All CFLs can be described and are unique so they are countable.

b No. There are languages in M_A that cannot be described so they cannot be given a value. This makes the set uncountable.

c $L_A \cap M_A = \emptyset$ so it is countable because empty set is countable.

8 Erase 2 O_i from front of string & 1 from end of string.
 If tape is empty, accept. Return to front of string. Check
 if there are equal number first O_i , I_i , & second O_i by
 marking off first unmarked of each until reach end of string.
 When finished marking first set of O_i , check that rest of string has been marked.

	O	I	B	\bar{O}	T
q_0	(B, q_1, R)	/	/	/	/
q_1	(B, q_2, R)	/	/	/	/
q_2	(O, q_2, R)	(I, q_2, R)	(B, q_3, L)	/	/
q_3	(O, q_3, L)	/	(B, q_4, R)	/	/
q_4	(O, q_4, L)	(I, q_4, L)	(B, q_5, R)	/	/
q_5	(\bar{O}, q_5, R)	/	/	/	/
q_6	(O, q_6, R)	(T, q_7, R)	/	/	(\bar{T}, q_8, R)
q_7	(\bar{O}, q_7, L)	(I, q_7, R)	/	(\bar{O}, q_7, R)	/
q_8	/	(I, q_8, L)	/	(\bar{O}, q_8, L)	(\bar{T}, q_9, L)
q_9	(O, q_9, L)	/	/	(\bar{O}, q_9, R)	(\bar{T}, q_9, L)
q_{10}	(\bar{O}, q_{10}, R)	/	/	/	(\bar{T}, q_{11}, R)
q_{11}	/	/	/	(\bar{O}, q_{12}, R)	(\bar{T}, q_{11}, R)
q_{12}	/	/	(B, q_{12}, R)	(\bar{O}, q_{12}, R)	/
q_f	accept				

all even here

note: I got notation backwards. Should be (state, symbol, direction)
 not (symbol, state, direction)

1: $\bar{L}_1 - L_2$ is recursive because recursive languages are closed under complement & set difference. A TM for deciding if a word is in a recursive language is recursive.

ii $\bar{L}_1 - L_2$ is not r.e. because r.e. languages that are not recursive are not closed under complement or set difference. The problem is non-r.e.

2: ~~Recursive~~ $\bar{L}_1 - L_2 \neq \emptyset$ if $\bar{L}_1 \neq L_2$. To prove that two languages are different a word must be found that is in one but not the other. This is r.e.

ii \bar{L}_1 is non-r.e. Because there is no way to prove that the language contains a word, the problem is non-r.e.

3: $L_1 \cap L_2$ is recursive. Recursive languages are closed under intersection. A TM for the language would be recursive.

ii $L_1 \cap L_2$ is r.e. because r.e. languages are closed under intersection. A TM for this language would be r.e.

4: $L_1 \cap L_2 \neq \emptyset$ Same logic as 2i. r.e.

$$\bar{L}_1 \cap \bar{L}_2 \neq \emptyset$$

$$L_1 - \bar{L}_2 \neq \emptyset$$

$$L_1 \neq \bar{L}_2$$

ii Same logic as 2ii. non-r.e.