COSC 3340 — Final Examination

Wednesday, July 2, 2008, 2 – 5 pm

Open Book and Notes Final grades only through PeopleSoft

YOU MUST USE THE CONSTRUCTIONS GIVEN IN CLASS

1. Construct a regular expression over {a,b,c} for the language accepted by this nfa:

	a	b	c	
→ A	/	B	/	0
B	B	/	A,C	1
C	/	B,C	/	1

2. Prove that the language L(G) is not regular where G is the following cfg:

G = ({S,A,B,C}, {a,b,c}, {S→Aa|B|C, A→aS, B→a, C→b}, S).

Note: You must first determine L(G).

3. Construct a reduced dfa for the following extended regular expression over {0,1,2}:

$[(100^*)^* \cap \overline{1^*}\,]$

Note: You must first determine nfas for (100*)* and 1*, then do the intersection. The answer must then be reduced.

4. Construct a Chomsky normal form grammar for L(G) for the following cfg G:

G = ({S,B}, {a,b,c,d}, {S→SaSbc|Ba, B→cBd|S|ε}, S).

Note: You must first remove all ε- and all unit productions.

5. Construct a Greibach normal form grammar for L(G) for the following CNF G:

G = ({S,A}, {a,d}, {S→AS|A|d, A→SS|a}, S).

Note: You must first remove all unit productions. You must derive all the productions for S and A; indicate how the result looks for S' and A' as applicable.

6. Prove that the following language L is not contextfree: $L = \{ 0^{n+1}1^{n-1}0^n \mid n>0 \}$.

7. Consider the class $\mathcal{L}_A$ of all contextfree languages over the fixed alphabet A.

(a) Is $\mathcal{L}_A$ countable?

(b) Is the class $\mathcal{M}_A$ countable where $\mathcal{M}_A$ consists of all languages over A that are not in $\mathcal{L}_A$?

(c) Is the class $\mathcal{L}_A \cap \mathcal{M}_A$ countable?

For each question, you must give a precise argument substantiating your answer.

8. Construct a Turing machine for the language in Question 6, $L = \{ 0^{n+1}1^{n-1}0^n \mid n>0 \}$.

Note: Describe first the process in English; then translate this into moves of the Turing machine.

9. Let L_1 and L_2 be arbitrary languages, subject to the specification in either (i) or (ii). Consider the following four general questions:

(Q1) Does $\overline{L_1} - L_2$ contain a given fixed word w? (Q2) Is $\overline{L_1} - L_2$ non-empty?

(Q3) Does $L_1 \cap L_2$ contain a given fixed word w? (Q4) Is $L_1 \cap L_2$ non-empty?

For each of these four questions **explain with reasons** whether the problem is recursive, not recursive but r. e., or non-r. e., provided

(i) Both L_1 and L_2 are recursive. (ii) Both L_1 and L_2 are r. e., but not recursive.

Points: 1: 6 2: 8 3: 14 4: 12 5: 12 6: 12 7: 13 8: 8 9:15

1 $A \to bB$

$B \to aB \vee cA \vee cC \vee \varepsilon$

$\to a^*c(A \vee C) \vee a^*$

$C \to bB \vee bC \vee \varepsilon$

$\to b^*bB \vee b^*$

$B \to a^*cA \vee a^*cbB \vee a^*cb^*bB \vee a^*cb^* \vee a^*$

$\to (a^*cb \vee a^*cb^*b)^*a^*cA \vee (a^*cb \vee a^*cb^*b)^*(a^*cb^* \vee a^*)$

$A \to b(a^*cb \vee a^*cb^*b)^*a^*cA \vee b(a^*cb \vee a^*cb^*b)^*(a^*cb^* \vee a^*)$

$\to [b(a^*cb \vee a^*cb^*b)^*a^*c]^* b(a^*cb \vee a^*cb^*b)^*(a^*cb^* \vee a^*)$

2 $S \to Aa \mid B \mid C$

$A \to aS$

$B \to a$

$C \to b$

$S \to aSa \mid a \mid b$ $L(G) = \{ a^i(a \cdot b)a^i \mid a \cdot 0 \}$

Assume that L_6 is regular: it is accepted by some DFA $\mathcal{A}$.

$\mathcal{A}$ has n states. Choose $x = a^n ba^n \in L_6$

x is long enough: it can be pumped $w = a^n$ $u = ba^n$

$w = w_1w_2w_3$: $|w_2| > 0$, $\forall s \geq 0$ st $ww_1^sw_3 \in L_6$

Choose $s = 2$ $a^{n+1}ba^n \in L_6$ ⟵ contradiction: L_6 is not regular

3 $[((00^*)^* \cap 1^*)] = [\overline{((00^*)^*} \cup \overline{1^*})]$

	0	1	2	
→0	/	1	/	0
1	2	/	/	0
2	2	/	/	1

	0	1	2	
→0	/	1	/	0
1	2	/	/	1
2	2	1	/	0

	0	1	2	
→0	/	4	/	1
4	/	4	/	1

	0	1	2	
→0	/	1,4	/	0
1	2	/	/	0
2	2	1	/	1
4	/	4	/	0

$\cup \ \varepsilon$

	0	1	2		
→0	/	14	/	0	A
/	/	/	/	0	B
1,4	2	4	/	0	C
2	2	1	/	1	D
4	/	4	/	0	E
1	2	/	/	0	F

A B C E F		D
A B E	C F	D
A	B E	C F	D
A	B E	C F	D

	0	1	2	
→A	B	C	B	0
B	B	B	B	0
C	D	B	B	0
D	D	C	B	1

4 $S \to SaSbc \mid Ba$
$B \to cBd \mid S \mid \varepsilon$

$S \to SaSbc \mid Ba \mid a$
$B \to cBd \mid cd \mid S$

$S \to SaSbc \mid Ba \mid Sa \mid a$
$B \to cBd \mid cSd \mid cd$

$S \to SS_1 \mid BX_a \mid SX_a \mid a$ $\qquad X_a \to a$
$S_1 \to X_a S_2$ $\qquad X_b \to b$
$S_2 \to S S_3$ $\qquad X_c \to c$
$S_3 \to X_b X_c$ $\qquad X_d \to d$
$B \to X_c B_1 \mid X_c B_2 \mid X_c X_d$
$B_1 \to BX_d$
$B_2 \to SX_d$

5. $S \to AS \mid \widehat{AA} \mid d$
$A \to SS \mid a$

$S \to AS \mid AA \mid d$
$A \to SS \mid AS \mid SA \mid AA \mid a$

$A \to a \mid aA'$
$A' \to SS \mid AS \mid SA \mid AA$

$S \to aS \mid aA'S \mid aA \mid aA' \mid d$
$A \to a \mid aA'$
$A' \to aSS \mid aA'SS \mid aAS \mid aA'AS \mid dS \mid aSA \mid aASA \mid aAA \mid aA'AA \mid dA$
$\quad \mid aS \mid aA'S \mid aA \mid aA'A$

7. a. Yes. All CFLs can be described and are unique so they are countable.
b. No. There are languages in M_0 that cannot be described so they cannot be given a value. This makes the set uncountable.
c. $L_0 \cap M_0 = \emptyset$ so it is countable because empty set is countable

8. Erase 2 0's from front of string & 1 from end of string.
If type is empty accept. Return to front of string. Check if there are equal number first 0's, ie, a second 0 by marking off first unmarked of each until reach end of string.
When finished marking first set of 0's, check that rest of string has been marked.

	0	1	B	Ø	T	
q_0	(B, q_1, R)	/	/	/	/	
q_1	(B, q_2, R)	/	/	/	/	
q_2	$(0, q_3, R)$	$(1, q_2, R)$	(B, q_4, L)	/	/	
q_3	$(0, q_3, L)$	/	(B, q_3, R)	/	/	
q_4	$(0, q_5, L)$	$(1, q_4, L)$	(B, q_5, R)	/	/	
q_5	$(0, q_6, R)$	/	/	/	/	all even here
q_6	$(0, q_6, R)$	(T, q_6, R)	/	/	(T, q_6, R)	
q_7	$(0, q_7, L)$	$(1, q_8, R)$	/	$(0, q_8, R)$	/	
q_8	/	$(1, q_8, L)$	/	$(0, q_9, L)$	(T, q_9, L)	
q_9	$(0, q_9, L)$	/	/	$(0, q_{10}, R)$	(T, q_9, L)	
q_{10}	$(0, q_{10}, R)$	/	/	/	(T, q_{11}, R)	
q_{11}	/	/	/	(B, q_{12}, R)	(T, q_{11}, R)	
q_{12}	/	/	(B, q_f, R)	$(0, q_{12}, R)$	/	
q_f			accept			

note: I got notation backwards. should be (state, symbol, direction) not (symbol, state, direction

9 i: $L_1 - L_2$ is recursive because recursive languages are closed under complement & set difference. A TM for deciding if a word is in a recursive language is recursive

ii $L_1 - L_2$ is not r.e. because r.e. languages that are not recursive are not closed under complement or set difference. The problem is non-r.e.

2. i: Recursive. $L_1 - L_2 \neq \emptyset$ if $L_1 \neq L_2$. To prove that two languages are different a word must be found that is in one but not the other. This is r.e.

ii: $\Sigma^* L_1$ is non-r.e. Because there is no way to prove that the language contains a word, the problem is non-r.e.

3. i: $L_1 \cap L_2$ is recursive. Recursive languages are closed under intersection. A TM for the language would be recursive.

ii: $L_1 \cap L_2$ is r.e. because r.e. languages are closed under intersection. A TM for the language would be r.e.

4. i: $L_1 \cap L_2 \neq \emptyset$ Same logic as 2i. r.e.
$\overline{L_1 \cap L_2 \neq \emptyset}$
$L_1 - L_2 \neq \emptyset$
$L_1 \neq L_2$

ii: Same logic as 2ii. non r.e.