# COSC 3340/6309 Final Examination

**Saturday, July 9, 2016, 11 am – 2 pm**

**Open Book and Notes    Final grades only through PeopleSoft**

**<u>YOU MUST USE THE CONSTRUCTIONS GIVEN IN CLASS</u>**

**1**. Construct a regular expression over {a,b,c} for the language accepted by this nfa:

| | a | b | c | |
|---|---|---|---|---|
| → A | / | B,C | / | 1 |
| B | B | / | C | 0 |
| C | / | A,B | / | 0 |

**2**. Prove that the language L(G) is not regular where G is the following cfg:

G = ({S,A,B}, {a,b}, {S→Aaa|B, A→aS, B→b}, S).

**Note: You <u>must</u> first determine L(G).**

**3**. Construct a reduced dfa for the following extended regular expression over {0,1}:

[(10*)* ∩ 0*10* ]

**Note: You <u>must</u> first determine nfas for (10*)* and 0*10*, then do the intersection. The answer must then be reduced.**

**4**. Construct a **Chomsky** normal form grammar for L(G) for the following cfg G:

G = ({S,B}, {a,b,c,d}, {S→Sb|Ba, B→cBBd|S|ε}, S).

**Note: You <u>must</u> first remove all ε- and all unit productions.**

**5**. Construct a **Greibach** normal form grammar for L(G) for the following CNF G:

G = ({S,A}, {a,b}, {S→ASS|A, A→SSS|bab}, S).

**Note: You <u>must</u> first remove all unit productions. You <u>must</u> derive all the productions for S and A; indicate how the result looks for S' and A'.**

**6**. Prove that the following language L is not contextfree: L = { $0^n1^n0^{n+1}$ | n≥1 }.

**7**. Consider the class $CF_A$ of all context free languages over the fixed alphabet A.

(a) Is $CF_A$ countable?

(b) Is the class $NOTCF_A$ countable where $NOTCF_A$ consists of all languages over A that are not context free?

(c) Is the class $CF_A \cap NOTCF_A$ countable?

For each question, you must give a precise argument substantiating your answer.

**8**. Construct a Turing machine for the language in Question 6, L = { $0^n1^n0^{n+1}$ | n≥1 }.

**Note: Describe first the process in English; then translate this into moves of the Turing machine.**

**9**. Let $L_1$ and $L_2$ be arbitrary languages, subject to the specification in either (i) or (ii). Consider the following four questions:

(Q1) Does $L_1-L_2$ contain a given fixed word w?    (Q2) Is $L_1-L_2$ empty?

(Q3) Does $L_1 \cap L_2$ contain a given fixed word w?    (Q4) Is $L_1 \cap L_2$ empty?

For each of these four questions **<u>explain with reasons</u>** whether the general problem is <u>recursive</u>, <u>not recursive but r. e.</u>, or <u>non-r. e.</u>, provided

(i) Both $L_1$ and $L_2$ are <u>recursive</u>.

(ii) $L_1$ is <u>r. e., but not recursive</u> and $L_2$ is <u>recursive.</u>

Note that there are **eight** different questions to be answered.

**Points: 1: 6 2: 8 3: 15 4: 11 5: 12 6: 12 7: 13 8: 8 9:15**

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Final Exam 2016

Q1:

| | a | b | c | |
|---|---|---|---|---|
| → A | – | B, C | – | 0 |
| B | B | – | C | 0 |
| C | – | A, B | – | 1 |

$L_A = bL_B \cup bL_C$

$L_B = aL_B \cup cL_C$

$L_C = bL_A \cup bL_B \cup \varepsilon$

1) Substitute $L_C$ in $L_B$:

$L_B = aL_B \cup c(bL_A \cup bL_B \cup \varepsilon)$

$L_B = aL_B \cup cbL_A \cup cbL_B \cup c$

$L_B = (a \cup cb)L_B \cup (cbL_A \cup c)$  $(L^* \cup M) = L^*M$

$\therefore L_B = (a \cup cb)^*(cbL_A \cup c) = (a \cup cb)^* cbL_A \cup (a \cup cb)^* c$

2) Substitute $L_C$ in $L_A$:

$L_A = bL_B \cup b(bL_A \cup bL_B \cup \varepsilon) = bL_B \cup bbL_A \cup bbL_B \cup b$

3) Substitute $L_B$ in $L_A$:

$L_A = b((a \cup cb)^* cbL_A \cup (a \cup cb)^* c) \cup bbL_A \cup bb((a \cup cb)^* cbL_A \cup (a \cup cb)^* c) \cup b$

$= b(a \cup cb)^* cbL_A \cup b(a \cup cb)^* c \cup bbL_A \cup bb(a \cup cb)^* cbL_A \cup bb(a \cup cb)^* c \cup b$

$= (b(a \cup cb)^* cb \cup bb \cup bb(a \cup cb)^* cb)^* (b(a \cup cb)^* c \cup bb(a \cup cb)^* c \cup b)$

Q2:

$S \to Aaa \mid B$
$A \to aS$
$B \to b$

$S \to Aaa \to aSaa \to aAaaaa \to aaSaaaa$

$S \to B \to b$

$aSaa \to aBaa \to abaa$

$aaSaaaa \to aaBaaaa \to aabaaaa$

$\therefore L(G) = \{ a^i b a^j \mid j \geq i \geq 0 \}$

- To prove that $L(G)$ is <u>not</u> regular, let's assume that $L(G)$ is a regular language.
- Hence, there exists a dfa $D$ that has $n$ states that accepts the language $L(G)$. $L(G) = L(D)$
- Consider $X = w.v = a^i b a^j$, where $w = a^i$, and $v = ba^j$
- Since $|w| = n$, we can apply the pumping lemma:
  $w = w_1 w_2 w_3$, such that $|w_2| \geq 1$
- So, $\tau(q_0, w) = \tau(q_0, w_1 (w_2)^s w_3)$, where $s \geq 0$ is $\in L(G)$
- let's assume $s = 0$:
  - we have $\tau(q_0, w) = \tau(q_0, w_1 w_3)$, but $|w_1 w_3| = n - |w_2| < n$
  - Therefore, $\tau(q_0, w_1 w_3) \notin L(G)$
- Since $\tau(q_0, w) \in L(G)$, and $\tau(q_0, w_1 w_3) \notin L(G)$, therefore, we have a contradiction.

Thus, $L(G)$ is **NOT** regular.

$$Q3: \left[\overline{(01^*)^*} \cap 1^*01^*\right] \equiv \left[\overline{(01^*)^* \cup (\overline{1^*01^*})}\right]$$

$$\begin{array}{c|cc|c} & 0 & 1 & \\ \hline 0 & 1 & - & 0 \\ 1 & - & - & 1 \end{array} \quad \begin{array}{c|cc|c} & 0 & 1 & \\ \hline 0 & - & 2 & 0 \\ 2 & - & - & 1 \end{array} \quad \begin{array}{c|cc|c} & 0 & 1 & \\ \hline 0 & - & 3 & 0 \\ 3 & - & - & 1 \end{array} \quad \begin{array}{c|cc|c} & 0 & 1 & \\ \hline 0 & 4 & - & 0 \\ 4 & - & - & 1 \end{array} \quad \begin{array}{c|cc|c} & 0 & 1 & \\ \hline 0 & - & 5 & 0 \\ 5 & - & - & 1 \end{array}$$

$\downarrow *$

$$\begin{array}{c|cc|c} & 0 & 1 & \\ \hline 0 & - & 2 & 1 \\ 2 & - & 2 & 1 \end{array} \quad \begin{array}{c|cc|c} & 0 & 1 & \\ \hline 0 & - & 3 & 1 \\ 3 & - & 3 & 1 \end{array} \quad \begin{array}{c|cc|c} & 0 & 1 & \\ \hline 0 & - & 5 & 1 \\ 5 & - & 5 & 1 \end{array}$$

$$\begin{array}{c|cc|c} & 0 & 1 & \\ \hline 0 & 1 & - & 0 \\ 1 & - & 2 & 0 \\ 2 & - & - & 1 \end{array}$$

$\downarrow +$

$$\begin{array}{c|cc|c} & 0 & 1 & \\ \hline 0 & - & - & 1 \\ 1 & - & 2 & 0 \\ 2 & - & - & 1 \end{array}$$

$$\begin{array}{c|cc|c} & 0 & 1 & \\ \hline \to 0 & 4 & 3 & 0 \\ 3 & 4 & 3 & 0 \\ 4 & - & 5 & 1 \\ 5 & - & 5 & 1 \end{array}$$

$\downarrow$ dfa

$$\begin{array}{c|cc|c|c} & 0 & 1 & & \\ \hline \to 0 & 4 & 3 & 0 & 1 \\ 4 & - & 5 & 1 & 0 \\ 3 & 4 & 3 & 0 & 1 \\ - & - & - & 0 & 1 \\ 5 & - & 5 & 1 & 0 \end{array}$$

flip

| | 0 | 1 | |
|---|---|---|---|
| 0 | 1 | — | 1 |
| 1 | — | 2 | 0 |
| 2 | 1 | — | 1 |

U

| | 0 | 1 | |
|---|---|---|---|
| 0 | 4 | 3 | 1 |
| 4 | — | 5 | 0 |
| 3 | 4 | 3 | 1 |
| — | — | — | 1 |
| 5 | — | 5 | 0 |

Assume $\Delta$ = —

=

| | 0 | 1 | |
|---|---|---|---|
| → 0 | 14 | 3 | 1 |
| 1 | 8 | 2 | 0 |
| 2 | 1 | 8 | 1 |
| 4 | 8 | 5 | 0 |
| 3 | 4 | 3 | 1 |
| 5 | 8 | 8 | 1 |
| 5 | 8 | 5 | 0 |

Ha →

| | | 0 | 1 | |
|---|---|---|---|---|
| A | 0 | 14 | 3 | 1 |
| B | 14 | 8 | 25 | 0 |
| C | 3 | 4 | 3 | 1 |
| D | 8 | 8 | 8 | 1 |
| E | 25 | 18 | 58 | 1 |
| F | 4 | 8 | 5 | 0 |
| G | 18 | 8 | 28 | 1 |
| H | 58 | 8 | 58 | 1 |
| I | 5 | 8 | 5 | 0 |
| J | 28 | 18 | 8 | 1 |

| | 0 | 1 | |
|---|---|---|---|
| A | B | C | 0 |
| B | D | E | 1 |
| C | F | C | 0 |
| D | D | D | 0 |
| E | G | H | 0 |
| F | D | I | 1 |
| G | D | J | 0 |
| H | D | H | 0 |
| I | D | I | 1 |
| J | G | D | 0 |

| 0 | | | 1 |
|---|---|---|---|
| ACDEGHJ | | | BFI |
| ADEGHJ | | C | |
| A | DEGHJ | C | |
| A | DEGHJ | C | |
| (1) | (2) | (3) | (4) |

reduced $\Delta$ =

| | 0 | 1 |
|---|---|---|
| | 4 | 3 |
| 2 | 2 | 2 |
| 3 | 4 | 3 |
| 4 | 2 | 2 |

Scanned by CamScanner

Case 4: v or x has 1's and right 0's, but no left 0's:

- for $\delta = 0$, we decrease the number of right 0's, while, the number of left 0's remains the same.
- # left 0's $\geq$ # right 0's. (contradiction)

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Case 5. v or x has NO 1's

- for $\delta = 0$, we decrease the number of right 0's, while the number of 1's remains the same.
- # right 0's $\leq$ # 1's (contradiction)

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Case 6: v or x has No right 0's:

- for $\delta = 2$, we increase the number of 1's, while the number of right 0's remains the same.
- # 1's $\geq$ # right 0's. (contradiction)

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Case 7: v contains more than one Left 0's, 1's, right 0's.
- for $\delta = 2$, we get the 0 following 0. (contradiction)

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Case 8: x contains more than one left 0's, 1's, right 0's.
- for $\delta = 2$, we get the 0 following 0.

Q6: $L = \{0^n 1^n 0^{n+1} \mid n \geq 1\}$

- Assume that is a context free language, then $\exists$ a $G(N, T, P, S)$ in CNF such that $L = L(G)$.

- Consider the case where $n = 2^m$.

- So, $Z = 0^{2^m} 1^{2^m} 1^{2m+1}$, $|Z| > 2^m$ and $Z \in L$, then, by Pumping lemma for CFL, we have, $Z = uvwxy$, and $|zv| \geq 1$ and $uv^i w z^i y \in L$, $\forall i \geq 0$.

Case 1: v and x has only left 0's:

- we increase the number of left 0's, while the number right 0's remains the same.
- for $s = 2$, # left 0's $\geq$ # right 0's. (Contradiction)

---

Case 2: v and x has only 1's.

- for $s = 2$, we increase the number of 1's, while The number of right 0's remains the same.
- # 1's $\geq$ # right 0's. (Contradiction)

---

Case 3: v and x has only right 0's:

- for $s = 0$, we decrease the number of right 0's, while of 1's remains the same.
- # right 0's $\leq$ # 1's (Contradiction)

Q5:

S → AS | A

A → SSS | aba

1) Eliminate S → A (replace S with A)

S → AS | AA

A → SSS | ASS | SAS | SSA | AAS | SAA | ASA | AAA | aba

2) replace S with AS | AA in A

A → ASSS | AASS | ASS | ASAS | AAAS | ASSA | AASA | AAS | ASAA | AAAA | ASA | AAA | aba

3) Eliminate left recursion:

A → aba | abaA'

A' → SSS | ASS | SS | SAS | AAS | SSA | ASA | AS | SAA | AAA | SA | AA |
SSSA' | ASSA' | SSA' | SASA' | AASA' | SSAA' | ASAA' | ASA' | SAAA' |
AAAA' | SAA' | AAA'

4) Find the production of S:

S → abaS | abaA'S | abaA | abaA'A

Q4:

$S \to Sb \mid Ba$

$B \to cBBd \mid S \mid \varepsilon$

1) Eliminate $B \to \varepsilon$

$S \to Sb \mid Ba$

$B \to cBBd \mid cBd \mid cd \mid S$

2) Eliminate $B \to S$ (replace every B with S)

$S \to Sb \mid Ba \mid Sa$

$B \to cBBd \mid cSBd \mid cBSd \mid cSSd \mid cBd \mid cSd \mid cd$

$S \to S X_b \mid B X_a \mid S X_a$

$B \to X_c B B X_d \mid X_c S B X_d \mid X_c B S X_d \mid X_c S S X_d \mid X_c B X_d \mid X_c S X_d \mid X_c X$

$\to S \to S X_b \mid B X_a \mid S X_a$

$B \to X_c B_1 \mid X_c B_3 \mid X_c B_4 \mid X_c B_6 \mid X_c B_2 \mid X_c B_5 \mid X_c X_d$

$B_1 \to B B_2$

$B_2 \to B X_d$

$B_3 \to S B_2$

$B_4 \to B B_5$

$B_5 \to S X_d$

$B_6 \to S B_5$

$X_a \to a$

$X_b \to b$

$X_c \to c$

$X_d \to d$

| | 0 | 1 | 0' | 1' | b |
|---|---|---|---|---|---|
| $q_0$ | $(q_1, 0', R)$ | – | – | – | – |
| $q_1$ | $(q_1, 0, R)$ | $(q_2, 1', R)$ | – | – | – |
| $q_2$ | – | $(q_3, 1', L)$ | – | – | – |
| $q_3$ | $(q_3, 0, L)$ | – | $(q_4, 0', R)$ | $(q_3, 1', L)$ | – |
| $q_4$ | $(q_4, 0, R)$ | $(q_5, 1', R)$ | – | $(q_4, 1', R)$ | – |
| $q_5$ | $(q_6, 0', L)$ | $(q_5, 1, R)$ | $(q_5, 0', R)$ | – | $q_f$ |
| $q_6$ | – | $(q_6, 1, L)$ | $(q_6, 0', L)$ | $(q_7, 1', L)$ | – |
| $q_7$ | $(q_7, 0, L)$ | – | $(q_8, 0', R)$ | $(q_7, 1', L)$ | – |
| $q_8$ | $(q_4, 0', R)$ | – | – | – | – |
| $q_f$ | Accepting State! | | | | |

Q8: $L = \{0^n 1^n 0^{n+1} \mid n \geq 1\}$

left 0's    right 0's.

- Number left 0's is equal to number of 1's, and less than the number of right 0's.

1) Configure the Turing Machine from the left, find the first left 0 and change it to 0'.

2) Then, move right until you find a 1, and change it to 1'.

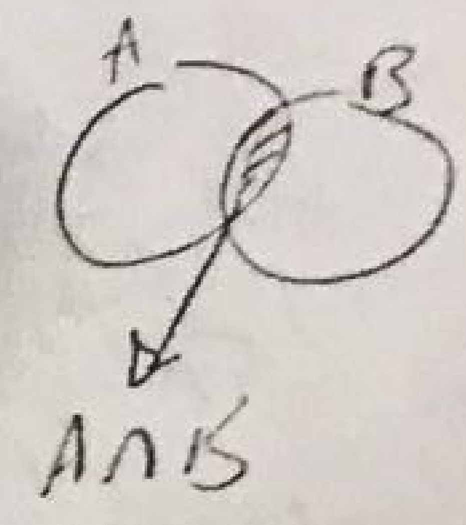
3) Then move left until you find the first 0', then stop and right until you find the first 0.

4) Repeat the process until 0' is the next state from 1' and then find all 0 and change them to 0'.

$\longrightarrow$

c) Is $REG_A \cap NOTREG_A$ Countable?

- Intersection states that an element is in $A \cap B$ iff:
  1) The element is in A AND
  2) The element is in B




- But since $NOTCF_A$ consists of all languages that are not in $CF_A$, no element can be in both A and B.

- And by the difinition of intersection, $REG_A \cap NOTREG_A$ will yield to an empty set. The empty set is considered to be a finite set with cardinality of zero. $|\{\emptyset\}| = 0$ which is a countable number.

- Therefore $REG_A \cap NOTREG_A$ is Countable

3

Q7:

a) is $CF_A$ Countable?

- Since $CF_A$ is a context free language, we know that there must be at least one pda that accepts $CF_A$. And we know that pda is a finite automaton. Hence, $\boxed{CF_A \text{ is Countable.}}$ (1)

b) is $NOTCF_A$ countable?

- We know that $CF_A$ is a context free language, We also know that $NOTCF_A$ consists of all languages that are not context free.
- Therefore, we don't know if $NOT\ CF_A$ is context free or not. It may be very complex, or it may be a problem or infinite.
- Also, let $A^*$ indicates all possible combinations of the fixed alphabet:

$\because A^*$ is countable infinite

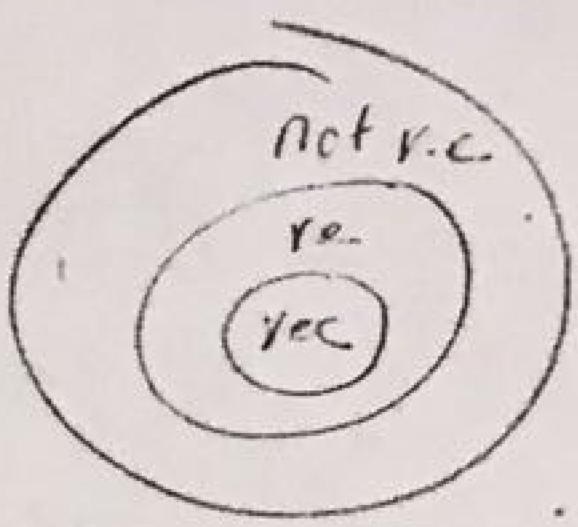
$\therefore 2^{A^*}$ is also countable infinite.

$\therefore CF_A$ is countable

$\therefore 2^{A^*} - CF_A = NOTCF_A$, which is infinite.

Thus, we cannot account for the languages represented by $NOTCF_A$, So $\boxed{\text{it's NOT countable.}}$ (2)

Q9:




i) $L_1$ and $L_2$ are both recursive: (i) ONLY!!

Q1: $L_1 - L_2$ contains a given fixed word $w$?

$L_1 - L_2$ is recursive because recursive languages are closed under set difference. A TM for deciding if a word is in a recursive language is $\boxed{\text{recursive.}}$ which means always halt for a given fixed word $w$ belongs $L_1$ but not $L_2$.

Q2: $L_1 - L_2$ empty? $L_1 - L_2 = \emptyset$?

$L_1 - L_2 = \emptyset$ if $L_1 = L_2$. To decide $L_1 - L_2 = \emptyset$, the TM has to run for ever to enumerate all the possibilities with no guarantee to stop. $\boxed{\text{Then, } L_1 - L_2 \text{ is not re}}$

Q3: $L_1 \cap L_2$ contains a given fixed word $w$?

Since both $L_1$ and $L_2$ is recursive, TM for $L_1 \cap L_2$ can always answer yes for a fixed word $w$ in $L_1 \cap L_2$ and say no when $w \notin (L_1 \cap L_2)$. Also, recursive languages are closed under intersection, therefore $\boxed{L_1 \cap L_2 \text{ is recursive}}$

Q4: $L_1 \cap L_2$ empty? $L_1 \cap L_2 = \emptyset$?

TM for $L_1 \cap L_2$, simulate input word $w$ for $L_1 \cap L_2$ when $w \in L_1 \cap L_2$, and $w \notin \emptyset$, the TM halts and answer yes, but for $L_1 \cap L_2 = \emptyset$, The TM has to go through all cases for $L_1 \cap L_2$ and runs for ever and say no

$\models L_1 \cap L_2 = \emptyset$ is not