

Thursday, July 6, 2017, 11 am – 2 pm

Open Book and Notes

Final grades only through PeopleSoft

YOU MUST USE THE CONSTRUCTIONS GIVEN IN CLASS

1. Construct a regular expression over  $\{a,b,c\}$  for the language accepted by this nfa:

	a	b	c	
$\rightarrow A$	/	B,C	/	0
B	B	/	C	1
C	/	A,B	/	1

2. Prove that the language  $L(G)$  is not regular where  $G$  is the following cfg:

$$G = (\{S,A,B\}, \{a,b\}, \{S \rightarrow Abb|B, A \rightarrow aS, B \rightarrow b\}, S).$$

Note: You must first determine  $L(G)$ .

3. Construct a reduced dfa for the following extended regular expression over  $\{0,1\}$ :

$$[(10^*)^* \cap 0^*10^*]$$

Note: You must first determine nfes for  $(10^*)^*$  and  $0^*10^*$ , then do the intersection. The answer must then be reduced.

4. Construct a Chomsky normal form grammar for  $L(G)$  for the following cfg  $G$ :

$$G = (\{S,B\}, \{a,b,c,d\}, \{S \rightarrow Sb|Ba, B \rightarrow cBdB|S|\epsilon\}, S).$$

Note: You must first remove all  $\epsilon$ - and all unit productions.

5. Construct a Greibach normal form grammar for  $L(G)$  for the following CNF  $G$ :

$$G = (\{S,A\}, \{a,b\}, \{S \rightarrow ASS|A, A \rightarrow SSS|aba\}, S).$$

Note: You must first remove all unit productions. You must derive all the productions for  $S$  and  $A$ ; indicate how the result looks for  $S'$  and  $A'$ .

6. Prove that the following language  $L$  is not contextfree:  $L = \{0^n 1^n 0^{n+2} \mid n \geq 1\}$ .

7. Consider the class  $CF_A$  of all context free languages over the fixed alphabet  $A$ .

(a) Is  $CF_A$  countable?

(b) Is the class  $NOTCF_A$  countable where  $NOTCF_A$  consists of all languages over  $A$  that are not context free?

(c) Is the class  $CF_A \cap NOTCF_A$  countable?

For each question, you must give a precise argument substantiating your answer.

8. Construct a Turing machine for the language in Question 6,  $L = \{0^n 1^n 0^{n+2} \mid n \geq 1\}$ .

Note: Describe first the process in English; then translate this into moves of the Turing machine.

9. Let  $L_1$  and  $L_2$  be arbitrary languages, subject to the specification in either (i) or (ii).

Consider the following four questions:

(Q1) Does  $L_1 - L_2$  contain a given fixed word  $w$ ? (Q2) Is  $L_1 - L_2$  empty?

(Q3) Does  $L_1 \cap L_2$  contain a given fixed word  $w$ ? (Q4) Is  $L_1 \cap L_2$  empty?

For each of these four questions explain with reasons whether the general problem is recursive, not recursive but r. e., or non-r. e., provided

(i) Both  $L_1$  and  $L_2$  are recursive.

(ii)  $L_1$  is r. e., but not recursive and  $L_2$  is recursive.

Note that there are **eight** different questions to be answered.

Points: 1: 6 2: 8 3: 15 4: 11 5: 12 6: 12 7: 13 8: 8 9: 15