COSC 3340 — Final Examination

Friday, May 8, 2009, 2 – 5 pm

Open Book and Notes — Final grades only through PeopleSoft

YOU MUST USE THE CONSTRUCTIONS GIVEN IN CLASS

1. Construct a regular expression over {a,b,c} for the language accepted by this nfa:

	a	b	c	
→ A	/	B	/	1
B	B	/	A,B,C	0
C	/	B,C	/	0

2. Prove that the language L(G) is not regular where G is the following cfg:

$G = (\{S,A,B,C\}, \{a,b,c\}, \{S \rightarrow aA|B|C,\ A \rightarrow Sa,\ B \rightarrow b,\ C \rightarrow a\}, S)$.

Note: You must first determine L(G).

3. Construct a reduced dfa for the following extended regular expression over {0,1,2}:

$$[(10^*)^* \cap \overline{1^*}\,]$$

Note: You must first determine nfas for $(10^*)^*$ and 1^*, then do the intersection. The answer must then be reduced.

4. Construct a **Chomsky** normal form grammar for L(G) for the following cfg G:

$G = (\{S,B\}, \{a,b,c,d\}, \{S \rightarrow SSbS|Ba,\ B \rightarrow cBd|S|\varepsilon\}, S)$.

Note: You must first remove all ε- and all unit productions.

5. Construct a **Greibach** normal form grammar for L(G) for the following CNF G:

$G = (\{S,A\}, \{a,b\}, \{S \rightarrow AS|A,\ A \rightarrow SS|ab\}, S)$.

Note: You must first remove all unit productions. You must derive all the productions for S and A; indicate how the result looks for S' and A'.

6. Prove that the following language L is not contextfree: $L = \{\ 0^n 1^{n+2} 0^n \mid n>0\ \}$.

7. Consider the class CFL_A of all contextfree languages over the fixed alphabet A.
   (a) Is CFL_A countable?
   (b) Is the class $NOTCFL_A$ countable where $NOTCFL_A$ consists of all languages over A that are not contextfree?
   (c) Is the class $CFL_A \cap NOTCFL_A$ countable?

For each question, you must give a precise argument substantiating your answer.

8. Construct a Turing machine for the language in Question 6, $L = \{\ 0^n 1^{n+2} 0^n \mid n>0\ \}$.

Note: Describe first the process in English; then translate this into moves of the Turing machine.

9. Let L_1 and L_2 be arbitrary languages, subject to the specification in either (i) or (ii). Consider the following four questions:

(Q1) Does $L_1 - L_2$ contain a given fixed word w? (Q2) Is $L_1 - L_2$ empty?

(Q3) Does $L_1 \cap L_2$ contain a given fixed word w? (Q4) Is $L_1 \cap L_2$ empty?

For each of these four questions **explain with reasons** whether the problem is recursive, not recursive but r. e., or non-r. e., provided

(i) Both L_1 and L_2 are recursive. (ii) Both L_1 and L_2 are r. e., but not recursive.

Note that there are **eight** different questions to be answered.

Points: 1: 6 2: 8 3: 14 4: 12 5: 12 6: 12 7: 13 8: 8 9:15

1. $L_1 \rightarrow bL_2 \cup \varepsilon$

$L_2 \rightarrow aL_2 \cup cL_1 \cup cL_2 \cup cL_3$

$L_3 \rightarrow bL_2 \cup bL_3$

$L_3 \rightarrow b^* bL_2$

plus in L_3 and L_1 into L_2

$L_2 \rightarrow aL_2 \cup c(bL_2 \cup \varepsilon) \cup cL_2 \cup c(b^* bL_2)$

$L_2 \rightarrow aL_2 \cup cbL_2 (\cup c) \cup cL_2 \cup cb^* bL_2$

$\rightarrow$ pull out L_2

$L_2 \rightarrow L_2(a \cup cb \cup c \cup cb^* b) \cup c$

$\rightarrow$ apply theory

$L_2 \rightarrow (a \cup cb \cup c \cup cb^* b)^* c$

$L_1 \rightarrow b[(a \cup cb \cup c \cup cb^* b)^* c] \cup \varepsilon$

② 2009

$S \to aA \mid B \mid C$
$A \to Sa$
$B \to b$
$C \to a$

we can get $L(G)$ from above.

$S \to aA \to aSa \to aaAa \to aaSaa \to aaaAaa \to a^3 \ldots$

$S \to B \to b$
$S \to C \to a$
$aSa \to aBa \to aba$
$aSa \to aCa \to aaa$
$\cancel{aa}$
$aaSaa \to aaBaa \to aabaa$
$aaSaa \to aaCaa \to aaaaa$.

the language of Grammar G is
$L(G) = \{ a^n b a^n \mid n \geq 0 \}$
Assume $L(G)$ is a regular language, for $\exists$
dfa which has n states that:
Consider $x = a^n b a^n = wu$ and $w = a^n$, $u = b$
since $|w| = n$, the pumping lemma states
that $w = w_1 w_2 w_2$ st $|w_2| \geq 1$
so, $t(q_0, w) = t(q_0, w_1 (w_2) w_3)$
consider $s = 0$
we have $t(q_0, w) = t(q_0, w_1 w_3)$
but $|w_1 w_3| = n - |w_2| < n$, so $t(q_0, w_1, w$
$\notin L(G)$
since $t(q_0, w) \in L(G)$, but $(q_0, w_1 w_3)$
$\notin L(G)$
$\to$ contradiction
$\to$ $L(G)$ is not regular

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question 3:

$[(10^*)^* \wedge \overline{1^*}]$ over $\{0,1,2\}$

$\Rightarrow [\overline{\underset{1\ 2}{(10^*)^*} \vee \underset{3}{1^*}}]$

	0	1	2	
→0	–	1	–	0
1	–	–	–	1

	0	1	2	
→0	2	–	–	0
2	–	–	–	1

	0	1	2	
→0	–	3	–	0
3	–	–	–	1

↓*

	0	1	2	
→0	2	–	–	1
2	2	–	–	1

↓*

	0	1	2
→0	–	3	–
3	–	3	–

	0	1	2	
→0	~~2~~ –	1	–	0
1	2	–	–	1
2	2	–	–	1

↓*

	0	1	2	
→0	–	1	–	1
1	2	1	–	1
2	2	1	–	1

dfa →

	0	1	2	
→0	–	1	–	1
1	2	1	–	1
2	2	1	–	1
–	–	–	–	0

rename →

	0	1	
→A	D	B	D
B	C	B	D
C	C	B	D
D	D	D	D

complement:

	0	1	2	
→A	D	B	D	0
B	C	B	D	0
C	C	B	D	0
D	D	D	D	1

$\xrightarrow{\vee}$

	0	1	2	
→A	D	3, B	D	1
B	C	B	D	0
C	C	B	D	0
D	D	D	D	1
3	–	3	–	1

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dfa:

	0	1	2	
→A	D	3,B	D	1
D	D	D	D	1
3,B	C	3,B	D	1
C	C	~~C~~	D	0

Rename →

	0	1	
→I	J	K	J
J	J	J	J
K	L	K	J
L	L	L	J

complement

	0	1	2	
→ I	J	K	J	0
J	J	J	J	0
K	L	K	J	0
L	L	L	J	1

Accept	Reject
L	I J K
L	I J K
$\boxed{L}$	$\boxed{I}$ $\boxed{J}$ $\boxed{K}$
1	2 3 4

	0	1	2	
1	1	1	J	1
2	3	4	3	0
3	3	3	3	0
4	1	4	3	0

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4) eliminate: useless variables, **ε** productions, unit productions

$S \rightarrow SSbS|Ba$
$B \rightarrow cBd|S|\varepsilon$

$B \rightarrow \varepsilon$
$S \rightarrow SSbS|Ba|a$
$B \rightarrow cBd|cd|S$

$B \rightarrow S$
$S \rightarrow SSbS|Ba|Sa|a$
$B \rightarrow cBd|cSd|cd$

$X_a \rightarrow a$ $X_b \rightarrow b$ $X_c \rightarrow c$ $X_d \rightarrow d$
Rewrite
$S \rightarrow SSX_bS|BX_a|SX_a|X_a$
$B \rightarrow X_cBX_d|X_cSX_d|X_cX_d$

Sub stuff in:
$S \rightarrow SS_1|BX_a|SX_a|X_a$
$S_1 \rightarrow SS_2$
$S_2 \rightarrow X_bS$
$B \rightarrow X_cB_1|X_cB_2|X_cX_d$
$B_1 \rightarrow BX_d$
$B_2 \rightarrow SX_a$
$X_a \rightarrow a$ $X_b \rightarrow b$ $X_c \rightarrow c$ $X_d \rightarrow d$

So in the end you have:
$S \rightarrow SS_1|BX_a|SX_a|X_a$
$B \rightarrow X_cB_1|X_cB_2|X_cX_d$
$X_a \rightarrow a$ $X_b \rightarrow b$ $X_c \rightarrow c$ $X_d \rightarrow d$

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4/ $S \to SSbS \mid Ba$
$B \to cBd \mid S \mid \varepsilon$

Eliminate $B \to \varepsilon$
$S \to SSbS \mid Ba \mid a$
$B \to cBd \mid cd \mid S$

Eliminate $B \to S$
$S \to SSbS \mid Ba \mid Sa \mid a$
$B \to cBd \mid cSd \mid cd$

set $X_a = a$
$X_b = b$
$X_c = c$
$X_d = d$

have $S \to SSX_bS \mid BX_a \mid SX_a \mid X_a$
$B \to X_cBX_d \mid X_cSX_d \mid X_cX_d$

$S \to SS_1 \mid BX_a \mid SX_a \mid X_a$
$S_1 \to SS_2$
$S_2 \to X_bS$

$B \to BB_1 \mid X_cB_2 \mid X_cX_d$
$B_1 \to BX_d$
$B_2 \to SX_d$

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5) $S \to AS \mid A$

$A \to SS \mid ab$

Eliminate $S \to A$

$S \to AS - AA$

$A \to SS \mid AS \mid SA \mid AA \mid ab$

Plug S into A

$A \to \beta \mid \beta A'$

$A' \to \alpha \mid \alpha A'$

$A \to ASS \mid AAS \mid AS \mid ASA \mid AAA \mid AA \mid ab$

$A \to ab \mid abA'$

$A' \to SS \mid AS \mid S \mid SA \mid AA \mid A \mid SSA' \mid ASA' \mid SA' \mid SAA' \mid AAA' \mid AA'$

Plug A into S

$S \to abS \mid abA'S \mid ab \mid abA'$

⑦ 2009 version.

a) CFL_A is contextfree languages → ther
must be a dfa accept CFL_A. Dfa is
finite automaton
→ CFL_A is countable.

b) CFL_A is context free and countable,
NOT CFL_A is not in CFL_A → we can't
determine NOT CFL_A is contextfree or not
→ we don't know the language of $\cancel{M_A}$ NOT
→ NotCFL$_A$ is not countable.

c) $CFL_A \cap$ NOT CFL_A consist of the alpha
that both in CFL_A and NOT CFL_A since NOT C
is not CFL_A, there is no element that
can be in both NOT CFL_A and CFL_A.
→ $CFL_A \cap$ NOT CFL_A will give an emty set.
an emty set is finite by definition
→ NOT $CFL_A \cap CFL_A$ is countable

	0	**1**	**0'**	**1'**	**b**
q0	(q1, 0', R)	/	/	/	/
q1	(q1, 0, R)	(q2, 1', R)	/	/	/
q2	/	(q3, 1', L)	/	/	/
q3	(q3, 0, L)	/	(q4, 0', R)	(q3, 1', L)	/
q4	(q4, 0, R)	(q5, 1', R)	/	(q4, 1', R)	/
q5	(q6, 0', L)	(q5, 1, R)	(q5, 0', R)	/	/
q6	/	(q6, 1, L)	(q6, 0', L)	(q7, 1', L)	/
q7	(q7, 0, L)	/	(q8, 0', R)	(q7, 1', L)	/
q8	(q4, 0', R)	/	/	(q9, 1', R)	/
q9	/	/	(q9, 0', R)	(q9, 1', R)	qf
qf	Accepting state				