YOU MUST USE THE CONSTRUCTIONS GIVEN IN CLASS

1. Construct a regular expression over {a,b,c} for the language accepted by this nfa: ✓

	a	b	c	
→ A	/	B	/	0
B	B	/	A,C	1
C	/	B,C	/	1

$L_1 = b(L_2)$
$L_2 = a(L_2) \cup c(L_1 \cup L_3) \cup \varepsilon$
$L_3 = b(L_2 \cup L_3)$

2. Prove that the language L(G) is not regular where G is the following cfg: ✓

G = ({S,A,B,C}, {a,b,c}, {S→Aa|B|C, A→aS, B→a, C→b}, S).

Note: You <u>must</u> first determine L(G).

3. Construct a reduced dfa for the following extended regular expression over {0,1,2}: ✓

$[(100^*)^* \cap \overline{1^*}]$

Note: You <u>must</u> first determine nfas for (100*)* and 1*, then do the intersection. The answer must then be reduced.

4. Construct a Chomsky normal form grammar for L(G) for the following cfg G:

G = ({S,B}, {a,b,c,d}, {S→SaSbc|Ba, B→cBd|S|ε}, S).

Note: You <u>must</u> first remove all ε- and all unit productions.

5. Construct a Greibach normal form grammar for L(G) for the following CNF G:

G = ({S,A}, {a,d}, {S→AS|A|d, A→SS|a}, S).

Note: You <u>must</u> first remove all unit productions. You <u>must</u> derive all the productions for S and A; indicate how the result looks for S' and A' as applicable.

6. Prove that the following language L is not contextfree: $L = \{ 0^{n+1}1^{n-1}0^{n} \mid n>0 \}$. $\{0^{n+2}1^{n}0^{n+1} \mid n \geq 0\}$

7. Consider the class $\mathcal{L}_A$ of all contextfree languages over the fixed alphabet A. ✓

(a) Is $\mathcal{L}_A$ countable? Yes

(b) Is the class $\mathcal{M}_A$ countable where $\mathcal{M}_A$ consists of all languages over A that are not in $\mathcal{L}_A$? No.

(c) Is the class $\mathcal{L}_A \cap \mathcal{M}_A$ countable? Yes

For each question, you must give a precise argument substantiating your answer.

8. Construct a Turing machine for the language in Question 6, $L = \{ 0^{n+1}1^{n-1}0^{n} \mid n>0 \}$.

Note: Describe first the process in English; then translate this into moves of the Turing machine.

9. Let L_1 and L_2 be arbitrary languages, subject to the specification in either (i) or (ii). Consider the following four general questions:

/not re

$\alpha - \beta = \alpha \cap \overline{\beta}$

(Q1) Does $\overline{L_1} - L_2$ contain a given fixed word w? rec (Q2) Is $\overline{L_1} - L_2$ non-empty? r.e. but not rec/not

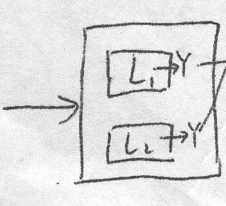
(Q3) Does $L_1 \cap L_2$ contain a given fixed word w? rec (Q4) Is $L_1 \cap L_2$ non-empty? r.e. but not rec. / re but not rec.

For each of these four questions **<u>explain with reasons</u>** whether the problem is <u>recursive</u>, <u>not recursive but r. e.</u>, or <u>non-r. e.</u>, provided / r.e. but not rec.

(i) Both L_1 and L_2 are <u>recursive</u>. (ii) Both L_1 and L_2 are <u>r. e., but not recursive</u>.

Points: 1: 6 2: 8 3: 14 4: 12 5: 12 6: 12 7: 13 8: 8 9:15

L_1 rec $\Rightarrow \overline{L_1}$ rec



Y yes ⟺ ≠ ∅

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1.

	a	b	c	
A	–	B	–	0
B	B	–	A,C	1
C	–	B,C	–	1

$L_1 = bL_2$

$L_2 = aL_2 \sqcup c(L_1 \sqcup L_3) \sqcup \varepsilon$

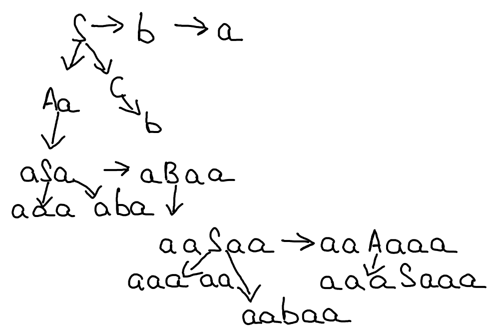
$L_3 = b(L_2 \sqcup L_3) \sqcup \varepsilon$

$L_2 = aL_2 \sqcup cbL_2 \sqcup cL_3 \sqcup \varepsilon$
$= (a \sqcup cb)L_2 \sqcup cL_3 \sqcup \varepsilon$
$= (a \sqcup cb)^* (cL_3 \sqcup \varepsilon)$

$L_3 = b(a \sqcup cb)^*(cL_3 \sqcup \varepsilon) \sqcup b(L_3) \sqcup \varepsilon$
$= b(a \sqcup cb)^* cL_3 \sqcup b(a \sqcup cb)^* \sqcup bL_3 \sqcup \varepsilon$
$= [[b(a \sqcup cb)^* c] \sqcup b] L_3 \sqcup b(a \sqcup cb)^* \sqcup \varepsilon$
$= [(b(a \sqcup cb)^* c) \sqcup b]^* (b(a \sqcup cb)^* \sqcup \varepsilon)$

2. Prove language $L(G)$ is not regular:

$S \to Aa \mid B \mid C$
$A \to aS$
$B \to a$
$C \to b$



So $\{a^n \mid n \geq 0\}$
$\{a^n b a^n \mid n \geq 0\}$

Assume $\{a^n b a^n \mid n \geq 0\}$ is regular language so $\exists$ dfa which has n states. Consider $x = a^n b a^n = wu$, & $w = a^n$, $u = ba^n$. Since $|w| = n$, we can apply pumping lemma.
$\Rightarrow w = w_1 w_2 w_3$ such that $|w_2| \geq 1$

So, $\tau(q_0, w) = \tau(q_0, w_1 (w_2)^s w_3)$ $s \geq 0$
Consider $s = 0$:
We have $\tau(q_0, w) = \tau(q_0, w_1 w_3)$
But $|w_1 w_3| = n - |w_2| \leq n$, so $\tau(q_0, w_1 w_3) \notin L_2$ (*)
Since $\tau(q_0, w) \notin L_2$ (") but $\tau(q_0, w_1 w_3) \notin L_2$ (*)
We have contradiction, so $a^n b^n$ is not regular

3. $((100^*)^* \cap \overline{1^*}) \Rightarrow \overline{(\overline{(100^*)^*} \sqcup 1^*)}$

	0	1	2	
0	–	1	–	0
1	–	–	–	1

	0	1	2	
0	2	–	–	0
2	–	–	–	1

	0	1	2	
0	3	–	–	0
3	–	–	–	1

	0	1	2	
0	–	4	–	0
4	–	–	–	1

$\cdot$

	0	1	2	
0	–	1	–	0
1	2	–	–	0
2	–	–	–	1

$\downarrow *$

	0	1	2	
0	3	–	–	1
3	3	–	–	1

$\downarrow *$

	0	1	2	
0	–	4	–	1
4	–	4	–	1

0	-	1	-	0
1	2	-	-	0
2	-	-	-	1

0	3	-	-	1
3	3	-	-	1

0	-	4	-	1
4	-	4	-	1

·

	0	1	2	
0	-	1	-	0
1	2	-	-	0
2	3	-	-	1
3	3	-	-	1

↓ DFA

	0	1	2	
0	-	1	-	0
-	-	-	-	0
1	2	-	-	0
2	3	-	-	1
3	3	-	-	1

↓ Rename & Flip

	0	1	2	
A	B	C	B	1
B	B	B	B	1
C	D	B	B	1
D	E	B	B	0
E	E	B	B	0

∪

	0	1	2	
A	B	C4	B	1
B	B	B	B	1
C	D	B	B	1
D	E	B	B	0
E	E	B	B	0
4	-	4	-	1

↓ DFA

	0	1	2	
A	B	C4	B	1
C4	D	B4	B	1
B	B	B	B	1
D	E	B	B	0
B4	B	B4	B	1
E	E	B	B	0

← Rename & Flip

	0	1	2	
1	3	2	3	0
2	4	5	3	0
3	3	3	3	0
4	6	3	3	1
5	3	5	3	0
6	6	3	3	1

Reduce →

[2 3 5] [4 6]

[1 3 5] [2] [4 6]

[1] [3] [5] [2] [4 6]

[1] [3] [5] [2] [4 6]

A B C D E

	0	1	2	
A	B	D	B	0
B	B	B	B	0
C	B	C	B	0
D	E	C	B	0
E	E	B	B	1

4 $S \rightarrow SaSbc \mid Ba$

$B \rightarrow cBd \mid S \mid \varepsilon$

ε production $B \rightarrow \varepsilon$

$S \rightarrow SaSbc \mid Ba \mid a$

$B \rightarrow cBd \mid S \mid cd$

Unit production $B \to S$

$S \to SaSbc \mid Ba \mid Sa \mid a$
$B \to cBd \mid cSd \mid cd$

$S \to SX_aSX_bX_c \mid BX_a \mid SX_a \mid X_a$
$B \to X_cBX_d \mid X_cSX_d \mid X_cX_d$

$S \to SS_1 \mid BX_a \mid SX_a \mid X_a$
$S_1 \to X_aS_2$
$S_2 \to SS_3$
$S_3 \to X_bX_c$
$B \to X_cB_1 \mid X_cB_2 \mid X_cX_d$
$B_1 \to BX_d$
$B_2 \to SX_d$
$X_a \to a$, $X_b \to b$, $X_c \to c$, $X_d \to d$

5. $S \to AS \mid A \mid d$
$A \to SS \mid a$

Unit production $S \to A$
$S \to AS \mid AA \mid d$ $\quad i=1$
$A \to SS \mid SA \mid AS \mid AA \mid a$ $\quad i=2$

$i=2, j=1$: $A \to ASS \mid AAS \mid S \mid ASA \mid AAA \mid AA \mid AS \mid AA \mid a$

$A \to dS \mid dA \mid a \mid dSA' \mid dAA' \mid aA'$
$A' \to SS \mid AS \mid SA \mid AA \mid A \mid S \mid SSA' \mid ASA' \mid SAA' \mid AAA' \mid AA' \mid SA'$
$S \to dSS \mid dAS \mid aS \mid dSA'S \mid dAA'S \mid aA'S \mid dS \mid dA \mid a \mid dSA' \mid AAA' \mid aA' \mid d$

7 a) L_A is context free language $\to$ there must be a dfa accept L_A. Dfa is finite automaton.
$\therefore L_A$ is countable

b) L_A is context free & countable. M_A is not in L_A $\to$ we can't determine. M_A is context free or not $\to$ we don't know. $\therefore$ the language of M_A is not countable.

c) $L_A \cap M_A$ consists of alphabets that are in both L_A & M_A. Since M_A is not L_A, there is no element that can be in both L_A & M_A
$\to L_A \cap M_A$ will give an empty set & an empty set by definition is finite ; $\therefore L_A \cap M_A$ is countable.

9. 1) $w \in \bar{L}_1 - L_2$

2) $L_1 - L_2 \neq \phi$

3) $w \in L_1 \wedge L_2$

i	ii
✓ $\bar{L}_1 - L_2 = \bar{L}_1 \wedge \bar{L}_2$	X
✓ $\bar{L}_1 - L_2 = \bar{L}_1 \wedge \bar{L}_2$	X
✓	✓
✓	✓

4) $L_1 \wedge L_2 \neq \phi$

	Recursive	r.e not recursive	context free
$\cup$ union	✓	✓	✓
$\cdot$ Concatenation	✓	✓	✓
$\cap$ Intersection	✓	✓	X
$-$ Complement	✓	X	X

1 i) $\overline{L}_1$ & $\overline{L}_2$ are closed under recursion; Intersection is closed under recursion; $w \in \overline{L}_1 - L_2$ is also closed under recursion.

1 ii) $\overline{L}_1$ & $\overline{L}_2$ are not closed under r.e. without recursion $\therefore$ $w \in \overline{L}_1 - L_2$ is not closed under recursion.

3 i) L_1 & L_2 are closed under recursion; Intersection is closed under recursion. $\therefore$ $w \in L_1 \cap L_2$ is closed under recursion.

2 ii) $\overline{L}_1$ is not closed under r.e. without recursion. $\therefore$ $\overline{L}_1 - L_2 \neq \phi$ is not closed under r.e. without recursion

* For question # 6 & # 8, see the other file

6. L={$\mathbf{0^{n+1}1^{n-1}0^{n}}$ | $\boldsymbol{n > 0}$}

Proof:

Assume that L is a context free language, then ∃ a G = (N, T, P, S) in CNF such that L = L(G).
Consider the case when $n = 2^m + 1$
So, $Z = 0^{2^m+2}1^{2^m}0^{2^m+1}$, we have $|Z| > 2^m$ and $Z \in L$, then by pumping the lemma for cfl, we have that $Z = uvwxy$ and $|xv| \geq 1 and$ $uv^iwx^iy \in L \quad \forall\, i \geq 0$.

*** From now, we call 0's before to be "left 0", and 0's after to be "right 0"

Case: 1(i)
Consider v and x to be all left 0's
We have $|vx| \geq 1$ and take $i = 0$.
Thus we decrease the number of left 0's while the number of 1's remains the same.
⟹ (number of left 0's) ≤ (number of 1's)
Hence, $uv^iwx^iy \notin L \quad with \quad i = 0$
We have a contradiction!

Case: 1(ii)
Consider v and x to be all 1's
We have $|vx| \geq 1$ and take $i = 2$.
Thus we increase the number of 1's while the number of right 0's remains the same.
⟹ (number of 1's) ≥ (number of right 0's)
Hence, $uv^iwx^iy \notin L \quad with \quad i = 2$
We have a contradiction!

Case: 1(iii)
Consider v and x to be all right 0's
We have $|vx| \geq 1$ and take $i = 0$.
Thus we decrease the number of right 0's while the number of 1's remains the same.
⟹ (number of 1's) ≥ (number of right 0's)
Hence, $uv^iwx^iy \notin L \quad with \quad i = 0$
We have a contradiction!

Case: 1(iv)
Consider when v or x has 1's and right 0's, but no left 0's.
We have $|vx| \geq 1$ and take $i = 2$.
Thus we increase the number of 1's while the number of left 0's remains the same.
⟹ (number of 1's) ≥ (number of left 0's)
Hence, $uv^iwx^iy \notin L \quad with \quad i = 2$
We have a contradiction!

Case: 1(v)
Consider when there is no 1's in v or x.
We have $|vx| \geq 1$ and take $i = 0$.
Thus we decrease the number of left 0's while the number of 1's remains the same.
⟹ (number of left 0's) ≤ (number of 1's)
Hence, $uv^iwx^iy \notin L \quad with \quad i = 0$
We have a contradiction!

Case: 1(vi)

Consider when there is no right 0's in v or x.
We have $|vx| \geq 1$ and take $i = 2$.
Thus we increase the number of 1's while the number of right 0's remains the same.
$\Longrightarrow$ (number of 1's) $\geq$ (number of right 0's)
Hence, $uv^iwx^iy \notin L \quad with \quad i = 2$
We have a contradiction!

Case: 2 Consider when v or x has more than one left 0's, 1's, right 0's.

Case: 2(i)
v contains more than one left 0's, 1's, right 0's;
for i=2
we get the 0 following 0.
Hence, $uv^iwx^iy \notin L \quad with \quad i = 2$
We have a contradiction!

Case: 2(ii)
x contains more than one left 0's, 1's, right 0's;
for i=2
we get the 0 following 0.
Hence, $uv^iwx^iy \notin L \quad with \quad i = 2$
We have a contradiction!

Hence, our assumption is NOT correct, L is NOT context free language.

6. L={$\mathbf{0^n 1^{n+2} 0^n}$ | $\boldsymbol{n > 0}$}

Proof:

Assume that L is a context free language, then ∃ a G = (N, T, P, S) in CNF such that L = L(G).
Consider the case when $n = 2^m$
So, $Z = 0^{2^m} 1^{2^m+2} 0^{2^m}$, we have $|Z| > 2^m$ and $Z \in L$, then by pumping the lemma for cfl, we have that $Z = uvwxy$ and $|xv| \geq 1 and$ $uv^i wx^i y \in L \quad \forall\, i \geq 0$.

*** From now, we call 0's before to be "left 0", and 0's after to be "right 0"

Case: 1(i)
Consider v and x to be all left 0's
We have $|vx| \geq 1$ and take $i = 2$.
Thus we increase the number of left 0's while the number of 1's remains the same.
⟹ (number of left 0's) ≥ (number of 1's)
Hence, $uv^i wx^i y \notin L \quad with \quad i = 2$
We have a contradiction!
Case: 1(ii)
Consider v and x to be all 1's
We have $|vx| \geq 1$ and take $i = 0$.
Thus we decrease the number of 1's while the number of right 0's remains the same.
⟹ (number of 1's) ≤ (number of right 0's)
Hence, $uv^i wx^i y \notin L \quad with \quad i = 0$
We have a contradiction!
Case: 1(iii)
Consider v and x to be all right 0's
We have $|vx| \geq 1$ and take $i = 2$.
Thus we increase the number of right 0's while the number of 1's remains the same.
⟹ (number of 1's) ≤ (number of right 0's)
Hence, $uv^i wx^i y \notin L \quad with \quad i = 2$
We have a contradiction!
Case: 1(iv)
Consider when v or x has 1's and right 0's, but no left 0's.
We have $|vx| \geq 1$ and take $i = 0$.
Thus we decrease the number of 1's while the number of left 0's remains the same.
⟹ (number of 1's) ≤ (number of left 0's)
Hence, $uv^i wx^i y \notin L \quad with \quad i = 0$
We have a contradiction!
Case: 1(v)
Consider when there is no 1's in v or x.
We have $|vx| \geq 1$ and take $i = 2$.
Thus we increase the number of left 0's while the number of 1's remains the same.
⟹ (number of left 0's) ≥ (number of 1's)
Hence, $uv^i wx^i y \notin L \quad with \quad i = 2$
We have a contradiction!
Case: 1(vi)

Consider when there is no right 0's in v or x.
We have $|vx| \geq 1$ and take $i = 0$.
Thus we decrease the number of 1's while the number of right 0's remains the same.
$\Longrightarrow$ (number of 1's) $\leq$ (number of right 0's)
Hence, $uv^iwx^iy \notin L \quad with \quad i = 0$
We have a contradiction!

Case: 2 Consider when v or x has more than one left 0's, 1's, right 0's.

Case: 2(i)

v contains more than one left 0's, 1's, right 0's;
for i=2
we get the 0 following 0.
Hence, $uv^iwx^iy \notin L \quad with \quad i = 2$
We have a contradiction!

Case: 2(ii)

x contains more than one left 0's, 1's, right 0's;
for i=2
we get the 0 following 0.
Hence, $uv^iwx^iy \notin L \quad with \quad i = 2$
We have a contradiction!

Hence, our assumption is NOT correct, L is NOT context free language.

Number 8

	0	**1**	**0'**	**1'**	**b**
q0	(q1, 0', R)	/	/	/	/
q1	(q2, 0', R)	/	/	/	/
q2	(q3, 0', R)	/	/	/	/
q3	(q4, 0', R)	(q8, 1', R)	/	/	(qf, b, R)
q4	(q4, 0, R)	(q5, 1', R)	/	(q4, 1', R)	/
q5	(q6, 0', L)	(q5, 1, R)	(q5, 0', R)	/	(qf, b, R)
q6	/	(q6, 1, L)	(q6, 0', L)	(q7, 1', L)	/
q7	(q7, 0, L)	/	(q3, 0', R)	/	/
q8	/	(q9, 1', R)	/	/	/
q9	(q10, 0', R)	/	(q9, 0', R)	(q9, 1', R)	/
q10	(qf, 0', R)	/	/	/	/
qf	Accepting				