Q6: $L = \{0^n 1^n 0^{n+1} \mid n \geq 1\}$

- Assume that is a context free language, then $\exists$ a $G(N, T, P, S)$ in CNF such that $L = L(G)$.

- Consider the case where $n = 2^m$.

- So, $Z = 0^{2^m} 1^{2^m} 1^{2m+1}$, $|Z| > 2^m$ and $Z \in L$, then, by Pumping lemma for CFL, we have, $Z = uvwxy$, and $|vx| \geq 1$ and $u v^i w x^i y \in L$, $\forall i \geq 0$.

Case1: v and x has only left 0's:

- we increase the number of left 0's, while the number right 0's remains the same.
- for $i = 2$, # left 0's $\geq$ # right 0's. (contradiction)

Case 2: v and x has only 1's.

- for $i = 2$, we increase the number of 1's, while The number of right 0's remains the same.
- # 1's $\geq$ # right 0's. (contradiction)

Case 3: v and x has only right 0's:

- for $i = 0$, we decrease the number of right 0's, while of 1's remains the same.
- # right 0's $\leq$ # 1's (contradiction)

Case 4: v or x has 1's and right 0's, but no left 0's:

- for $s=0$, we decrease the number of right 0's, while, the number of left 0's remains the same.
- # left 0's $\geq$ # right 0's. (contradiction)

Case 5: v or x has NO 1's

- for $s=0$, we decrease the number of right 0's, while the number of 1's remains the same.
- # right 0's $\leq$ # 1's (contradiction)

Case 6: v or x has No right 0's:

- for $s=2$, we increase the number of 1's, while the number of right 0's remains the same.
- # 1's $\geq$ # right 0's. (contradiction)

Case 7: v contains more than one left 0's, 1's, right 0's.
- for $s=2$, we get the 0 following 0. (contradiction)

Case 8: x contains more than one left 0's, 1's, right 0's.
- for $s=2$, we get the 0 following 0.

Case 4: v or x has 1's and right 0's, but no left 0's:

- for $s=0$, we decrease the number of right 0's, while, the number of left 0's remains the same.
- # left 0's $\geq$ # right 0's. (contradiction)

Case 5: v or x has NO 1's

- for $s=0$, we decrease the number of right 0's, while the number of 1's remains the same.
- # right 0's $\leq$ # 1's (contradiction)

Case 6: v or x has NO right 0's:

- for $s=2$, we increase the number of 1's, while the number of right 0's remains the same.
- # 1's $\geq$ # right 0's. (contradiction)

Case 7: v contains more than one left 0's, 1's, right 0's.
- for $s=2$, we get the 0 following 0. (contradiction)

Case 8: x contains more than one left 0's, 1's, right 0's.
- for $s=2$, we get the 0 following 0.

	0	1	
0	1	—	1
1	—	2	0
2	1	—	1

U

	0	1	
0	4	3	1
4	—	5	0
3	4	3	1
—	—	—	1
5	—	5	0

Assume δ = —

	0	1	
→ 0	14	3	1
1	8	2	0
2	1	8	1
4	8	5	0
3	4	3	1
8	8	8	1
5	8	5	0

sig →

		0	1	
A	0	14	3	1
B	14	8	25	0
C	3	4	3	1
D	8	8	8	1
E	25	18	58	1
F	4	8	5	0
G	18	28	28	1
H	58	8	58	1
I	5	8	5	0
J	28	18	8	1

[illegible] →

	0	1	
A	B	C	0
B	D	E	1
C	F	C	0
D	D	D	0
E	G	H	0
F	D	I	1
G	D	J	0
H	D	H	0
I	D	I	1
J	G	D	0

0			1
ACDEGHJ			BFI
ADEGHJ		C	
A	DEGHJ	C	
A	DEGHJ	C	(4)
(1)	(2)	(3)	

reduced δ =

	0	1
1	4	3
2	2	2
3	4	3
4	2	2

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Y

Hadeel Al-Mubaireek

1371070

Final Exam 2016

Q1:

	a	b	c	
→ A	—	B,C	—	0
B	B	—	C	0
C	—	A,B	—	1

$L_A = bL_B \cup bL_C$

$L_B = aL_B \cup cL_C$

$L_C = bL_A \cup bL_B \cup \varepsilon$

1) Subsitute L_C in L_B:

$L_B = aL_B \cup c(bL_A \cup bL_B \cup \varepsilon)$

$L_B = aL_B \cup cbL_A \cup cbL_B \cup c$

$L_B = (a \cup cb)L_B \cup (cbL_A \cup c)$ $\quad (L^* \cup M) = L^*M$

$\therefore L_B = (a \cup cb)^* (cbL_A \cup c) = (a \cup cb)^* cbL_A \cup (a \cup cb)^* c$

2) Subsitute L_C in L_A:

$L_A = bL_B \cup b(bL_A \cup bL_B \cup \varepsilon) = bL_B \cup bbL_A \cup bbL_B \cup b$

3) Subsitute L_B in L_A:

$L_A = b((a \cup cb)^* cbL_A \cup (a \cup cb)^* c) \cup bbL_A \cup bb((a \cup cb)^* cbL_A \cup (a \cup cb)^* c$ $\cup b$

$= b(a \cup cb)^* cbL_A \cup b(a \cup cb)^* c \cup \underline{bbL_A} \cup \underline{bb(a \cup cb)^* cbL_A} \cup bb(a \cup cb)^* c$ $\cup b$

$= (b(a \cup cb)^* cb \cup bb \cup bb(a \cup cb)^* cb)^* (b(a \cup cb)^* c \cup bb(a \cup cb)^* c)$

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Q5:

S → AS | A

A → SSS | aba

1) Eliminate S → A (replace S with A)

S → AS | AA

A → SSS | ASS | SAS | SSA | AAS | SAA | ASA | AAA | aba

2) replace S with AS | AA in A

A → ASSS | AASS | ASS | ASASS | AAAS | ASSA | AASA | AAS |
ASAA | AAAA | ASA | AAA | aba

3) Eliminate left recursion:

A → aba | abaA'

A' → SSS | ASS | SS | SAS | AAS | SSA | ASA | AS | SAA | AAA | SA | AA |
SSSA' | ASSA' | SSA' | SASA' | AASA' | SSAA' | ASAA' | ASA' | SAAA' |
AAAA' | SAA' | AAA'

4) Find the production of S:

S → abaS | abaA'S | abaA | abaA'A

VG

Q2:

$S \to Aaa \mid B$

$A \to aS$

$B \to b$

$S \to Aaa \to aSaa \to aAaaaa \to aaSaaaa$

$S \to B \to b$

$aSaa \to aBaa \to abaa$

$aaSaaaa \to aaBaaaa \to aabaaaa$

$\therefore L(G) = \{ a^i b a^j \mid j \geq i \geq 0 \}$

- To prove that $L(G)$ is <u>not</u> regular, let's assume that $L(G)$ is a regular language.
- Hence, there exists a dfa $\underset{\sim}{D}$ that has n states that accepts the language $L(G)$. $L(G) = L(\underset{\sim}{D})$
- Consider $X = w.v = a^i b a^j$, where $w = a^i$, and $v = b a^j$
- Since $|w| = n$, we can apply the pumping lemma:

  $w = w_1 w_2 w_3$, such that $|w_2| \geq 1$
- So, $\tau(q_0, w) = \tau(q_0, w_1 (w_2)^s w_3)$, where $s \geq 0$ is $\in L(G)$
- let's assume $s = 0$:
  - we have $\tau(q_0, w) = \tau(q_0, w_1 w_3)$, but $|w_1 w_3| = n - |w_2| < n$
  - Therefore $\tau(q_0, w_1 w_3) \notin L(G)$

. Since $\tau(q_0, w) \in L(G)$, and $\tau(q_0, w_1 w_3) \notin L(G)$, therefore, we have a contradiction.

Thus, $L(G)$ is <u>NOT</u> regular.

Q4:

$S \to Sb \mid Ba$

$B \to cBBd \mid S \mid \varepsilon$

1) Eliminate $B \to \varepsilon$

$S \to Sb \mid Ba$

$B \to cBBd \mid cBd \mid cd \mid S$

2) Eliminate $B \to S$ (replace every B with S)

$S \to Sb \mid Ba \mid Sa$

$B \to cBBd \mid cSBd \mid cBSd \mid cSSd \mid cBd \mid cSd \mid cd$

$S \to S X_b \mid B X_a \mid S X_a$

$B \to X_c B B X_d \mid X_c S B X_d \mid X_c B S X_d \mid X_c S S X_d \mid X_c B X_d \mid X_c S X_d \mid X_c X$

$\to S \to S X_b \mid B X_a \mid S X_a$

$B \to X_c B_1 \mid X_c B_3 \mid X_c B_4 \mid X_c B_6 \mid X_c B_2 \mid X_c B_5 \mid X_c X_d$

$B_1 \to B B_2$

$B_2 \to B X_d$

$B_3 \to S B_2$

$B_4 \to B B_5$

$B_5 \to S X_d$

$B_6 \to S B_5$

$X_a \to a$

$X_b \to b$

$X_c \to c$

$X_d \to d$

Q3: $[\overline{(01^*)^*} \cap 1^*01^*] \equiv [(01^*)^* \cup (\overline{1^*01^*})]$

	0	1	
0	1	–	0
1	–	–	1

	0	1	
0	–	2	0
2	–	–	1

	0	1	
0	–	3	0
3	–	–	1

	0	1	
0	4	–	0
4	–	–	1

	0	1	
0	–	5	0
5	–	–	1

$\downarrow *$

	0	1	
0	–	2	1
2	–	2	1

$\downarrow *$

	0	1	
0	–	3	1
3	–	3	1

$\downarrow *$

	0	1	
0	–	5	1
5	–	5	1

	0	1	
0	1	–	0
1	–	2	0
2	–	–	1

$\downarrow *$

	0	1	
0	1	–	1
1	–	2	0
2	–	–	1

	0	1	
→0	4	3	0
3	4	3	0
4	–	5	1
5	–	5	1

$\downarrow$ dfa

	0	1		
→0	4	3	0	1
4	–	5	1	0
3	4	3	0	1
–	–	–	0	1
5	–	5	1	0

flip

Q7:

a) is CF_A Countable?

- Since CF_A is a context free language, we know that there must be at least one Pda that accepts CF_A. And we know that pda is a finite automaton. Hence, $\boxed{CF_A \text{ is Countable.}}$ (1)

b) is $NOTCF_A$ countable?

- We know that CF_A is a context free language, We also know that $NOTCF_A$ consists of all languages that are not context free.
- Therefore, we don't know if $NOT\,CF_A$ is context free or not. It may be very complex, or it may be a problem or infinite.
- Also, let A^* indicates all possible combinations of the fixed alphabet:

$\because A^*$ is countable infinite

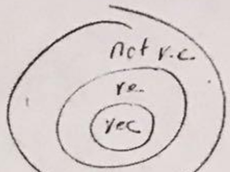
$\therefore 2^{A^*}$ is also countable infinite.

$\therefore CF_A$ is countable

$\therefore 2^{A^*} - CF_A = NOTCF_A$, which is infinite.

Thus, we cannot account for the languages represented by $NOTCF_A$, So $\boxed{\text{It's NOT countable.}}$ (2)

Q9:



i) L_1 and L_2 are both recursive: (i) ONLY!!

Q1: $L_1 - L_2$ contains a given fixed word w?

$L_1 - L_2$ is recursive because recursive languages are closed under set difference. A TM for deciding if a word is in a recursive language is recursive. which means always halt for a given fixed word w belongs L_1 but not L_2.

Q2: $L_1 - L_2$ empty? $L_1 - L_2 = \emptyset$?

$L_1 - L_2 = \emptyset$ if $L_1 = L_2$. To decide $L_1 - L_2 = \emptyset$, the TM has to run for ever to enumerate all the possibilities with no guarantee to stop. | then, $L_1 - L_2$ is not re |

Q3: $L_1 \cap L_2$ contains a given fixed word w?

Since both L_1 and L_2 is recursive, TM for $L_1 \cap L_2$ can always answer yes for a fixed word w in $L_1 \cap L_2$ and say no when $w \notin (L_1 \cap L_2)$. Also, recursive languages are closed under intersection. therefore | $L_1 \cap L_2$ is recursive |

Q4: $L_1 \cap L_2$ empty? $L_1 \cap L_2 = \emptyset$?

TM for $L_1 \cap L_2$, simulate input word w for $L_1 \cap L_2$ when $w \in L_1 \cap L_2$, and $w \notin \emptyset$, the TM halts and answer yes, but for $L_1 \cap L_2 = \emptyset$, The TM has to go through all cases for $L_1 \cap L_2$ and runs for ever and say no

|= $L_1 \cap L_2 = \emptyset$ is not

	0	1	0'	1'	$\not{b}$
q_0	$(q_1, 0', R)$	–	–	–	–
q_1	$(q_1, 0, R)$	$(q_2, 1', R)$	–	–	–
q_2	–	$(q_3, 1', L)$	–	–	–
q_3	$(q_3, 0, L)$	–	$(q_4, 0', R)$	$(q_3, 1', L)$	–
q_4	$(q_4, 0, R)$	$(q_5, 1', R)$	–	$(q_4, 1', R)$	–
q_5	$(q_6, 0', L)$	$(q_5, 1, R)$	$(q_5, 0', R)$	–	q_f
q_6	–	$(q_6, 1, L)$	$(q_6, 0', L)$	$(q_7, 1', L)$	–
q_7	$(q_7, 0, L)$	–	$(q_8, 0', R)$	$(q_7, 1', L)$	–
q_8	$(q_4, 0', R)$	–	–	–	–
q_f	Accepting state!				

COSC 3340/6309 Final Examination

Saturday, July 9, 2016, 11 am – 2 pm

Open Book and Notes **Final grades only through PeopleSoft**

YOU MUST USE THE CONSTRUCTIONS GIVEN IN CLASS

1. Construct a regular expression over {a,b,c} for the language accepted by this nfa:

	a	b	c	
→ A	/	B,C	/	1
B	B	/	C	0
C	/	A,B	/	0

2. Prove that the language L(G) is not regular where G is the following cfg:

$G = (\{S,A,B\}, \{a,b\}, \{S \to Aaa|B, A \to aS, B \to b\}, S)$.

Note: You <u>must</u> first determine L(G).

3. Construct a reduced dfa for the following extended regular expression over {0,1}:

$[(10^*)^* \cap 0^*10^*]$

Note: You <u>must</u> first determine nfas for (10*)* and 0*10*, then do the intersection. The answer must then be reduced.

4. Construct a **Chomsky** normal form grammar for L(G) for the following cfg G:

$G = (\{S,B\}, \{a,b,c,d\}, \{S \to Sb|Ba, B \to cBBd|S|\varepsilon\}, S)$.

Note: You <u>must</u> first remove all ε- and all unit productions.

5. Construct a **Greibach** normal form grammar for L(G) for the following CNF G:

$G = (\{S,A\}, \{a,b\}, \{S \to ASS|A, A \to SSS|bab\}, S)$.

Note: You <u>must</u> first remove all unit productions. You <u>must</u> derive all the productions for S and A; indicate how the result looks for S' and A'.

6. Prove that the following language L is not contextfree: $L = \{ 0^n1^n0^{n+1} \mid n \geq 1 \}$.

7. Consider the class CF_A of all context free languages over the fixed alphabet A.

(a) Is CF_A countable?

(b) Is the class $NOTCF_A$ countable where $NOTCF_A$ consists of all languages over A that are not context free?

(c) Is the class $CF_A \cap NOTCF_A$ countable?

For each question, you must give a precise argument substantiating your answer.

8. Construct a Turing machine for the language in Question 6, $L = \{ 0^n1^n0^{n+1} \mid n \geq 1 \}$.

Note: Describe first the process in English; then translate this into moves of the Turing machine.

9. Let L_1 and L_2 be arbitrary languages, subject to the specification in either (i) or (ii).
Consider the following four questions:

(Q1) Does $L_1 - L_2$ contain a given fixed word w? (Q2) Is $L_1 - L_2$ empty?

(Q3) Does $L_1 \cap L_2$ contain a given fixed word w? (Q4) Is $L_1 \cap L_2$ empty?

For each of these four questions **<u>explain with reasons</u>** whether the general problem is <u>recursive</u>, <u>not recursive but r. e.</u>, or <u>non-r. e.</u>, provided

(i) Both L_1 and L_2 are <u>recursive</u>.

(ii) L_1 is <u>r. e., but not recursive</u> and L_2 is <u>recursive.</u>

Note that there are **eight** different questions to be answered.

Points: 1: 6 2: 8 3: 15 4: 11 5: 12 6: 12 7: 13 8: 8 9:15