Guvenen (2009)

1 Variables

1. The set of 2 endogenous state variables has the following elements:

   K_t, B^n_t

   CODE: Kx, Bnx (*code variables: states end with 'x', controls with 'y'*)

2. The set of 1 exogenous state variables has the following elements:

   Z_t

   CODE: Zx

3. The set of 17 control variables has the following elements:

   Core: $c_{h,t}$, $c_{n,t}$, I_t, B^n_{t+1}, $\lambda_{h,t}$, $\lambda_{n,t}$, P^s_t, P^f_t

   CODE: chy, cny, Iy, Bny, lambdahy, lambdany, Psy, Pfy

   Additional: q_t, W_t, $\frac{\Lambda_{h,t+1}}{\Lambda_{h,t}}$, $\frac{\Lambda_{n,t+1}}{\Lambda_{n,t}}$, $U_{h,t}$, $U_{n,t}$, D_t, $b_{h,t+1}$, $b_{n,t+1}$

   CODE: qy, Wy, Lambdahy, Lambdany, Uhy, Uny, Dy, bhy, bny

2 Equations

legend: states in green (given), controls in orange (given by policy guess from current states), next period's exogenous states in magenta (to be integrated over), and next period's controls in blue (given by policy guess at next period's states), parameters are black

Core Equations

$$P^f_t = \beta(1+\lambda_{h,t})\mathbf{E_t}\left(\frac{\Lambda_{h,t+1}}{\Lambda_{h,t}}\right) \tag{1}$$

$$P^f_t = \beta(1+\lambda_{n,t})\mathbf{E_t}\left(\frac{\Lambda_{n,t+1}}{\Lambda_{n,t}}\right) \tag{2}$$

$$P^s_t = \beta\mathbf{E_t}\left[\frac{\Lambda_{h,t+1}}{\Lambda_{h,t}}(P^s_{t+1}+D_{t+1})\right] \tag{3}$$

$$\lambda_{h,t}(b_{h,t+1}+\underline{B}) = 0 \tag{4}$$

$$\lambda_{n,t}(b_{n,t+1} + \underline{B}) = 0 \tag{5}$$

$$c_{h,t} + P_t^f b_{h,t+1} + \frac{P_t^s}{\mu} = P_t^s + D_t + \frac{\chi\bar{K} - B_t^n}{\mu} + W_t \tag{6}$$

$$c_{n,t} + P_t^f b_{h,t+1} = P_t^s + \frac{B_t^n}{1-\mu} + W_t \tag{7}$$

$$q_t = \beta \mathbf{E_t} \left\{ \frac{\Lambda_{h,t+1}}{\Lambda_{h,t}} \left[\theta Z_t {K_t}^{\theta-1} - \frac{I_{t+1}}{K_{t+1}} + q_{t+1} \left(1 - \delta + \Phi\left(\frac{I_{t+1}}{K_{t+1}} \right) \right) \right] \right\} \tag{8}$$

Additional Equations

$$q_t \Phi' \left(\frac{I_{t+1}}{K_{t+1}} \right) = 1 \tag{9}$$

$$w_t = (1-\theta) Z_t \left(\frac{K_t}{L_t} \right)^{\theta}, \quad L_t = 1 \tag{10}$$

$$w_t = (1-\theta) Z_t \left(\frac{K_t}{L_t} \right)^{\theta}, \quad L_t = 1 \tag{11}$$

$$\beta \frac{\Lambda_{h,t+1}}{\Lambda_{h,t}} = \beta^{\frac{1-\alpha}{1-\rho^h}} \left(\frac{c_{h,t+1}}{c_{h,t}} \right)^{-\rho^h} \left[\frac{\frac{U_{h,t+1}}{c_{h,t}}}{\left[\left(\frac{U_{h,t}}{c_{h,t}} \right)^{1-\rho^h} - (1-\beta) \right]^{1/(1-\rho^h)}} \right] \tag{12}$$

$$\beta \frac{\Lambda_{n,t+1}}{\Lambda_{n,t}} = \beta^{\frac{1-\alpha}{1-\rho^n}} \left(\frac{c_{n,t+1}}{c_{n,t}} \right)^{-\rho^n} \left[\frac{\frac{U_{n,t+1}}{c_{n,t}}}{\left[\left(\frac{U_{n,t}}{c_{n,t}} \right)^{1-\rho^n} - (1-\beta) \right]^{1/(1-\rho^n)}} \right] \tag{13}$$

$$U_{h,t} = \left\{ (1-\beta) {c_{h,t}}^{1-\rho^h} + \beta \left[\mathbf{E_t}({\mathbf{U_{h,t+1}}}^{\mathbf{1}-\alpha}) \right]^{\frac{1-\rho^h}{1-\alpha}} \right\}^{1/(1-\rho^h)} \tag{14}$$

$$U_{n,t} = \left\{ (1-\beta) {c_{n,t}}^{1-\rho^n} + \beta \left[\mathbf{E_t}({\mathbf{U_{n,t+1}}}^{\mathbf{1}-\alpha}) \right]^{\frac{1-\rho^n}{1-\alpha}} \right\}^{1/(1-\rho^n)} \tag{15}$$

$$D_t = Z_t {K_t}^{\theta} L_t^{1-\theta} - W_t L_t - I_t - (1 - P_t^f)\chi \bar{K} \tag{16}$$

$$B_{t+1}^n = (1 - \mu) b_{n,t+1} \tag{17}$$

$$\Phi(I/K) = \frac{\phi_k}{2}\left(\frac{I}{K}\right)^2$$

Low of Motion

$$lnZ_{t+1} = \phi lnZ_t + \epsilon_{t+1}, \quad \epsilon \sim N(0, \sigma_\epsilon^2) \tag{18}$$

(Ky, Bny) of today become (Kx, Bnx) of tomorrow.

We approximte the normally distributed shocks (in the laws of motion for the exogenous states) by Gauss-Hermite quadrature with only three quadrature points. Accordingly, the three realizations of these five shocks are as follows:

1.
$$\varepsilon_{A,t}(i) = \sqrt{2} \cdot q_p(i) \cdot s_A, i = 1, ..., 3$$

   where

$$q_p(1) = -1.224744871, q_p(2) = 0.000000000, q_p(3) = 1.224744871$$

   The probabilities/weights of these realizations are given by:

$$q_w(1) = 0.2954089751/\sqrt{\pi}, q_w(2) = 1.181635900/\sqrt{\pi}, q_w(3) = 0.2954089751/\sqrt{\pi}$$

3 Solving the model with DNN

Let us denote the state of the economy by x, the policy by y, and the policy function by θ, thus:

1.
$$x = \left[K_t, B_t^n, Z_t\right] \in \mathbb{R}^3$$

2.
$$y = \left[c_{h,t}, c_{n,t}, I_t, B_{t+1}^n, \lambda_{h,t}, \lambda_{n,t}, P_t^s, P_t^f, q_t, W_t, \frac{\Lambda_{h,t+1}}{\Lambda_{h,t}}, \frac{\Lambda_{n,t+1}}{\Lambda_{n,t}}, U_{h,t}, U_{n,t}, D_t, b_{h,t+1}, b_{n,t+1}\right] \in \mathbb{R}^{17}$$

3.

$$\theta : x \Rightarrow y, \quad \theta : \mathbb{R}^3 \to \mathbb{R}^{17}$$

We approximate the policy function θ using a DNN with trainable prameters ρ:

1.

$$\mathcal{N}(\rho, x) \cong \theta(x) = \left[c_{h,t}, c_{n,t}, I_t, B^n_{t+1}, \lambda_{h,t}, \lambda_{n,t}, P^s_t, P^f_t, q_t, W_t, \frac{\Lambda_{h,t+1}}{\Lambda_{h,t}}, \frac{\Lambda_{n,t+1}}{\Lambda_{n,t}}, U_{h,t}, U_{n,t}, D_t, b_{h,t+1}, b_{n,t+1} \right](x)$$

2.

$$\mathcal{L}(\rho, \mathcal{D}) = \frac{1}{|\mathcal{D}|} \sum_{x \in \mathcal{D}} \omega_1 (e^1_x(\rho))^2 + \ldots + \omega_{17} (e^{17}_x(\rho))^2$$

where $\mathcal{D}$ is the training set and ρ are the trainable prameters, and $e^i_x(\rho)$, $i = 1, \ldots, 17$ are the residuals from the equilibrium equations, that is the right hand side of equations 1 to 17 when evaluated at x using the policy function $\mathcal{N}(\rho, x)$. The weight vector ω can initially be set equal to the unit vector, yet for scaling reasons we might want to play around with it.

4 Parameters

Symbol (Code)	Parameter	Value
α (alpha)	Risk aversion	6
β (beta)	Discount rate	0.9966
$b\theta$ (theta)	Cap. share	0.30
ρ^h (rho_h)	inv IES stock	$1/0.3$
ρ^h (rho_n)	inv IES nonstock	$1/0.1$
δ (delta)	depreciation rate	0.0066
μ (mu)	participation rate	0.2
ϕ_k (phi_k)	adjustment cost	0.4
χ (chi)	leverage ratio	0.005
$\bar{K}$ (Kbar)	steadysate capital	$((1/\beta - 1 + \delta)/\theta)^(1/(\theta - 1))$
$\bar{B}$ (Bbar)	borrowing constraint	$-0.1 * (1 - \theta) * Kss^{\theta}$