PHY 180 - Computational Physics - Spring 2023

Project 5: The Solar System
Due: Friday, March 10th
Trevin Detwiler

1 Introduction

Once again, we have a project that deals with a seemingly simple system in an introductory physics course. Here, students learn that orbit of a planet around the Sun (or a satellite around the earth) is one of the cases of centripetal motion, and so the speed of the planet, the radius of orbit, and the period of the orbit are all fixed by the given conditions of the problem. If you are a particularly keen student, or an astronomy student, you would be aware that this is only a special case of the many Kepler orbits. You might also be aware that the Sun is not necessarily be fixed in place. Both the Sun and the planet should orbit around their center of mass (COM), which is especially important when considering a binary star system.

This is not the end of the story quite yet. Our solar system is not an isolated system of a single planet and the Sun. There are seven other planets that have gone unmentioned. Each of these planets not only pulls on the Sun, but will also pull on the first planet. This is a true N-body problem, which is a very complicated system with no general solution. Luckily for us, we can leave all the calculations to the computer and analyze the results to study the effect these planets have on the first.

The objective of this project is to first explore the Kepler problem in which the Sun is stationary, and then the three-body problem using the Sun-Earth-Jupiter system. Starting with the Kepler problem, the orbits of some planets are calculated and plotted to ensure that the code produces reasonable results. Following this, the Euler and Euler-Cromer methods will be analyzed by giving the code "bad" initial conditions for a particular planet. Then, the $\frac{1}{r^2}$ nature of the gravitational force will be explored by looking at how deviations from 2 effect the orbit of Mercury. This concludes the Kepler problem and brings us to the three-body problem. The effect of Jupiter on the orbit of the Earth is explored as the mass of Jupiter becomes comparable to the mass of the Sun. This is done first with the assumption that the Sun is stationary, and second by dropping this assumption and performing calculations in the COM rest frame.

2 Modifications to the Code

We were given two programs, `kepler-demo.f90` and `threebody-final.f90` to modify for this project.

2.1 Kepler Problem

Several aesthetic changes were made to the `kepler-demo.f90` program. To start, the code was split into two subroutines, `initialize` and `calculate`. The `constants` module is used for the constants `pi`, `pp` $= 4\pi^2$, and the input file unit `u` $= 100$ to avoid redefining them in these subroutines.

The initialize subroutine takes care of reading all of the input values from the input_kep.dat file and setting initial values for the planet's position and velocity. This includes the introduction of several new variables to help with the investigation. The eccentricity e, and the semimajor axis a are read in to allow non-circular Kepler orbits. This changes how the initial position and velocity are defined to be

$$x_0 = a(1 \mp e) \qquad\qquad v_{y,0} = 2\pi\sqrt{\frac{(1 \pm e)}{a(1 \mp e)}} \tag{1}$$

where the top sign initializes the planet at perihelion and the bottom sign at aphelion. The initial values for y and vx are maintained as 0. Note that the circular orbit initial conditions are maintained when $e = 0$. Next, the variable b is read in to allow for the gravitational force to be proportional to r^{-b} instead of r^{-2}. When not investigating this relation, b is set to 2. The euler variable is a toggle that allows the switch between the Euler-Cromer method (euler set to 0) and the Euler method (euler set to 1). For this reason, the vx and vy are changed to arrays of dimension 2, the first index to store the "old" velocity (Euler) and the second index to store the "new" velocity (Euler-Cromer). Finally, the toggle variable manual is used to allow the initial x and vy to be read in from the input file. This is useful in the investigation of "bad" initial conditions.

Some minimal changes were made to the calculate subroutine. The new velocity is calculated first and stored into vx(2) and vy(2). Here, the r**3 is replaced with r**(b+1) to allow for deviations from the r^{-2} law. An if-else statement is used for the calculation of the new position using either the Euler method or the Euler-Cromer method. Following this, the old velocity vx(1) and vy(1) is updated with the new velocity.

2.2 Three-Body Problem

The threebody-final.f90 program was modified into 3Body.f90, which was further modified into 3BodyCOM.f90. Starting with 3Body.f90, the entire code is once again split between two subroutines, initialize and calculate, and the same constants module is utilized. Also, many of the variables are either renamed or redefined for better clarity or utility. The position and velocities of Jupiter and Saturn are handled by arrays of dimension 2 for the x and y-components.

The initialize subroutine reads in the eccentricities e_e and e_j and calculates the initial positions and velocities is the same manner as in kepler.f90. The variables pe and pj store the mass ratios of the Earth and Jupiter with the Sun. The header output file is also set up here. In the calculate subroutine, the same variable names are read in rather than using new variable names.

For the 3BodyCOM.f90 program, the position and velocity of the Sun is included as arrays of dimension 2 and are initialized to 0. The initial velocities for the Earth and Jupiter are changed to now include the mass ratio. For example, the earth's initial velocity is

$$v_{y,0} = 2\pi\sqrt{\frac{(1 \pm e_e)}{x_e(1 \mp e_e)}\left(1 + \frac{m_e}{m_s}\right)} \tag{2}$$

The position and velocity of the COM is then calculated and subtracted from the initial x and v_y for each of the celestial bodies. Finally, in the calculate subroutine, the position and velocity of the Sun is calculated in a similar manner.

3 Results

For part (b), the following plots were produced:

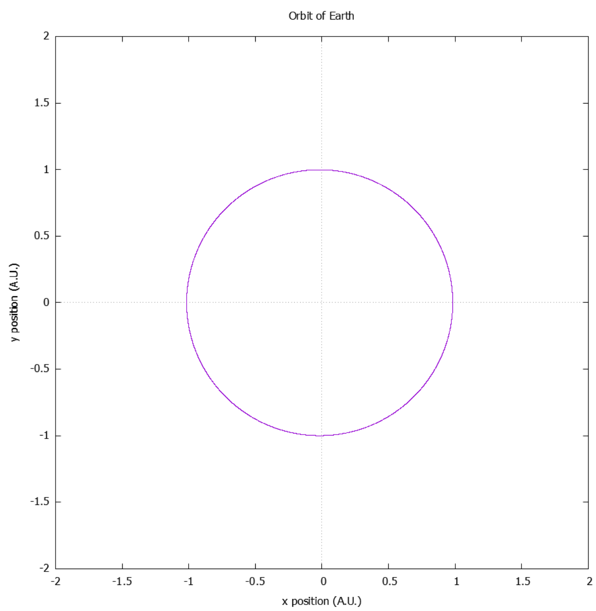

Figure 1: Orbit of the Earth

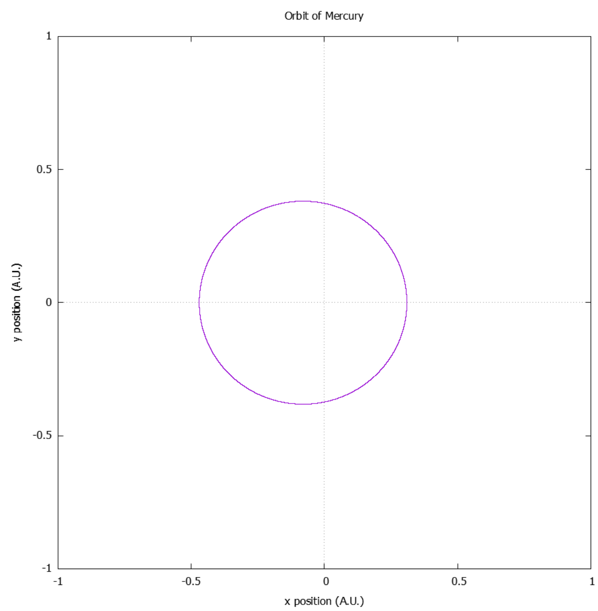

Figure 2: Orbit of Mercury

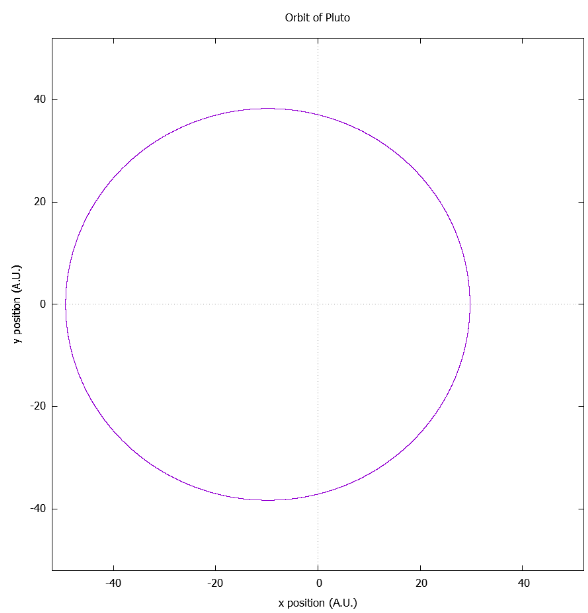

Figure 3: Orbit of Pluto

For part(c), the following plots were produced:

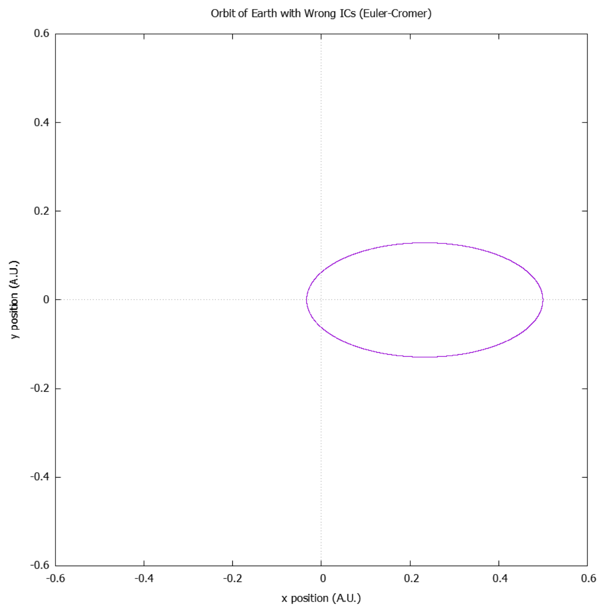

Figure 4: Orbit of the Earth using the Euler-Cromer method with the "bad" initial conditions $x_0 = 0.5$, $v_{y,0} = 3.14$.

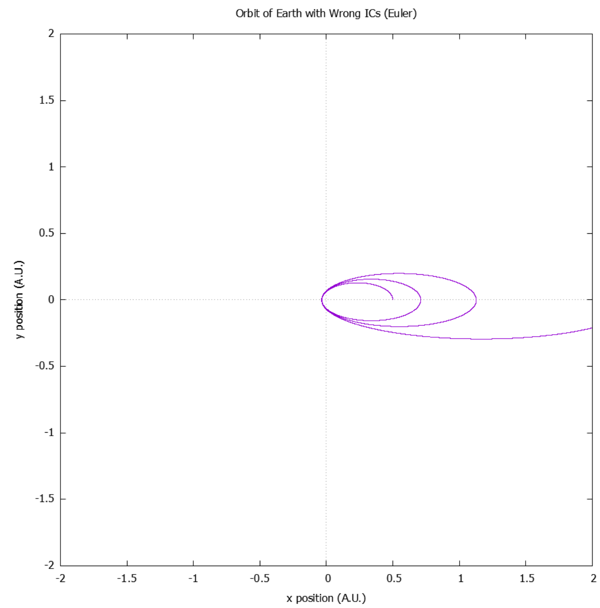

Figure 5: Orbit of the Earth using the Euler method with the "bad" initial conditions $x_0 = 0.5$, $v_{y,0} = 3.14$.

For part (d), the following plots were produced:

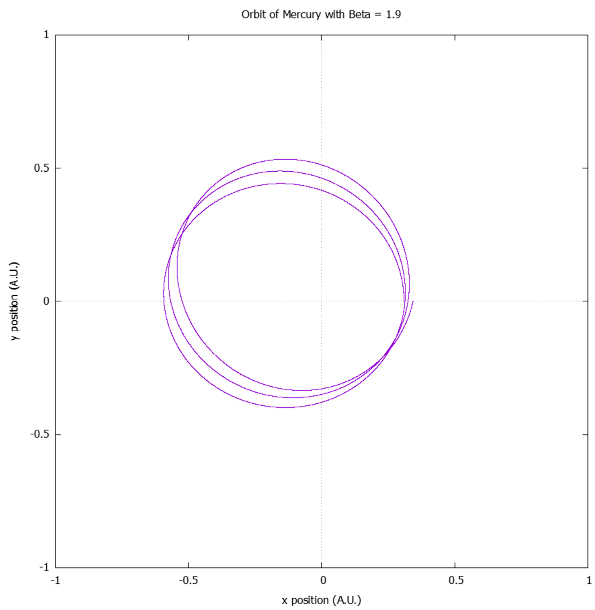

Figure 6: Orbit of Mercury with the gravitational force proportional to $r^{-1.9}$.

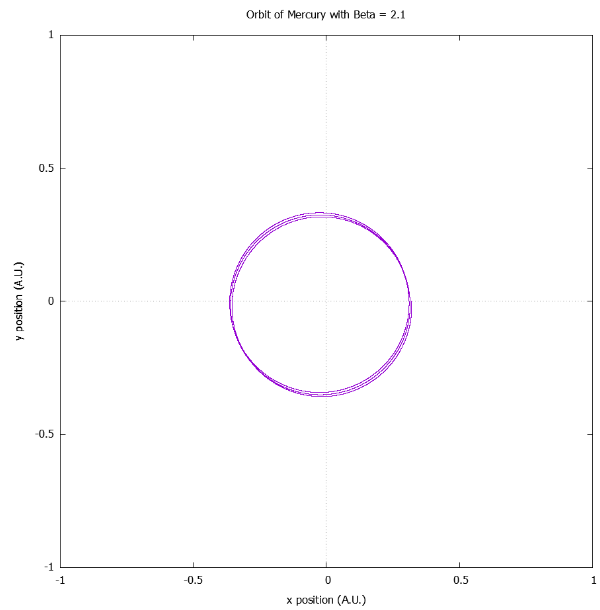

Figure 7: Orbit of Mercury with the gravitational force proportional to $r^{-2.1}$.

For part (e), the following plots were produced:

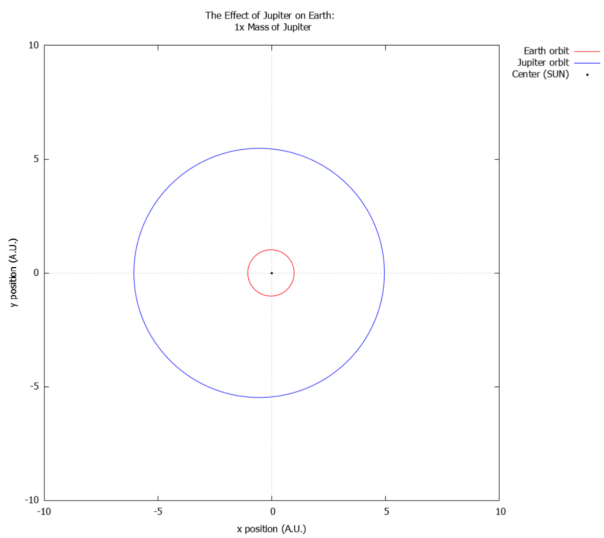

Figure 8: Earth and Jupiter in orbit around the Sun. The mass of Jupiter has been multiplied by 1.

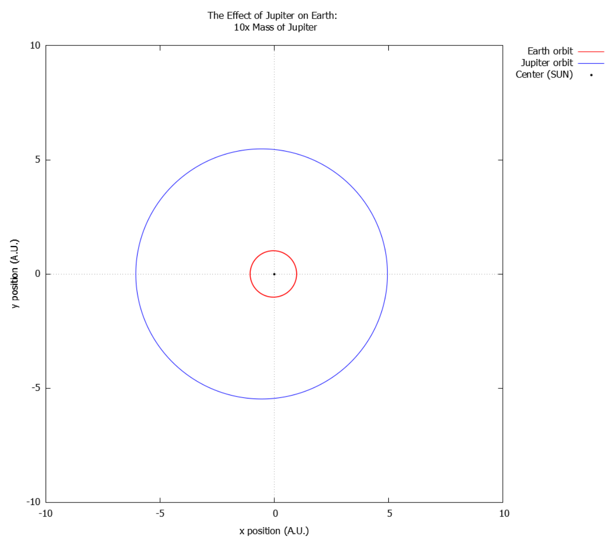

Figure 9: Earth and Jupiter in orbit around the Sun. The mass of Jupiter has been multiplied by 10.

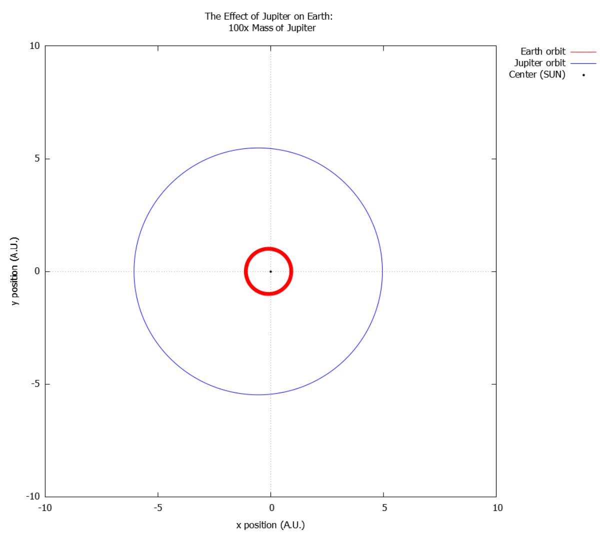

Figure 10: Earth and Jupiter in orbit around the Sun. The mass of Jupiter has been multiplied by 100.

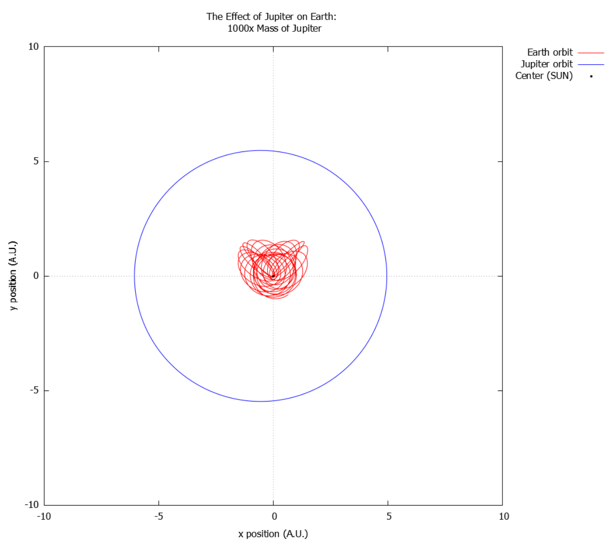

Figure 11: Earth and Jupiter in orbit around the Sun. The mass of Jupiter has been multiplied by 1000.

For part(g), the following plots were produced:

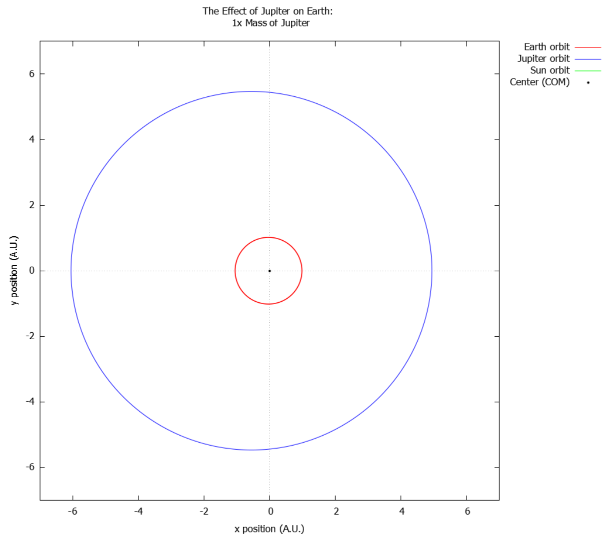

Figure 12: Earth,Jupiter, and the Sun in orbit the system COM. The mass of Jupiter has been multiplied by 1.

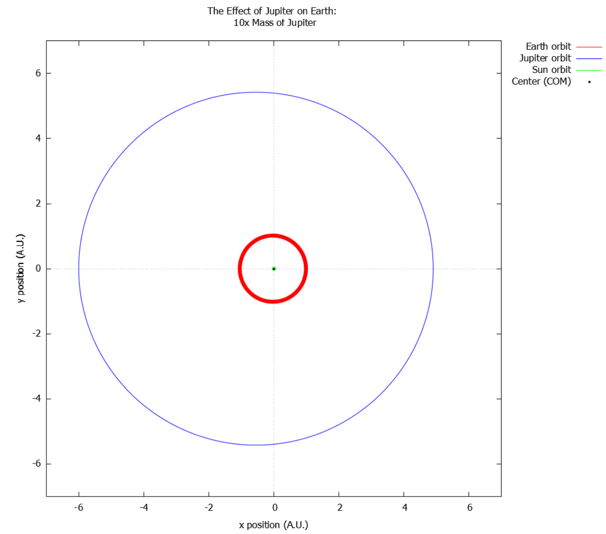

Figure 13: Earth,Jupiter, and the Sun in orbit the system COM. The mass of Jupiter has been multiplied by 10.

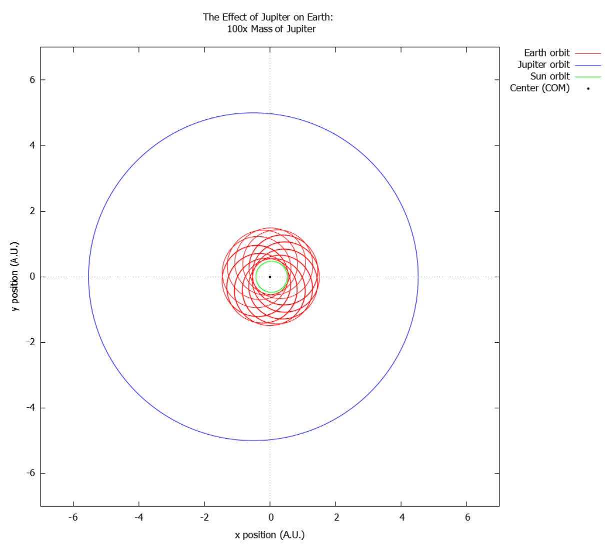

Figure 14: Earth,Jupiter, and the Sun in orbit the system COM. The mass of Jupiter has been multiplied by 100.

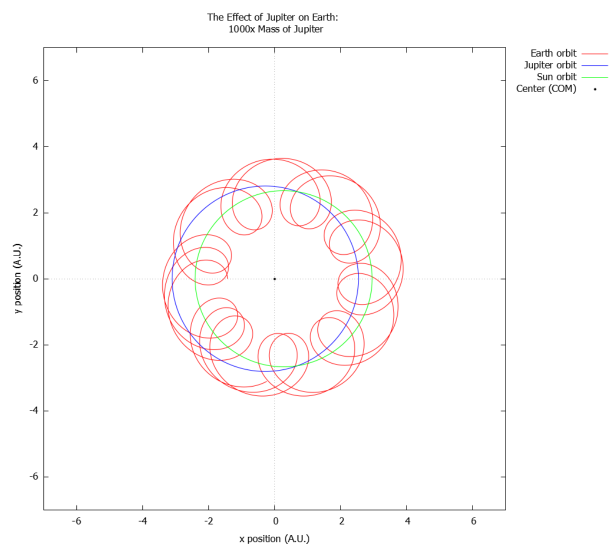

Figure 15: Earth,Jupiter, and the Sun in orbit the system COM. The mass of Jupiter has been multiplied by 1000.

4 Conclusions

From Figures 1-3, we can see that the `kepler.f90` program produces reasonable planetary orbits, including elliptical orbits of Mercury and Pluto. Thus, we can have good confidence that the code is working properly for our further investigations. Figures 4 and 5 compare the Euler-Cromer method with the Euler method when "bad" initial conditions are used. We see that the Euler-Cromer method still produces a Kepler orbit, where the Euler method spirals out of control. Thus, the Euler-Cromer method will be used solely for the remainder of the investigations.

Our first real investigation is the r^{-2} law. There is no real reason why the exponent is precisely 2. Figures 6 and 7 show the orbit of Mercury when this exponent is changed slightly from 2. As we can see, these orbits deviate from the observed elliptical orbit of Mercury, confirming that the exponent is precisely 2. It should be noted that the precession of Mercury's orbit is a result from general relativity and is not accounted for in any of these plots.

Next, we investigated the three-body problem by looking at the effect that Jupiter has on the Earth's orbit. In Figures 8-11, the mass of the Sun is assumed to be large enough that the gravitational pull of the Earth and Jupiter on it are negligible. We can see that as the mass of Jupiter gets closer to that of the Sun, the Earth's orbit deviates greatly from its typical orbit and is very irregular in Figure 11. This can be compared with Figures 12-15 in which the previous assumption is dropped. These plots are made in the COM rest frame. We can see that although the orbit of the Earth deviates from its typical orbit, the deviations are more uniform.