

The content on this final exam is mostly placed in chronological order within each section, from Unit 1-Unit 8. Answer all non-multiple choice questions to the best of your ability in order to receive the greatest amount of credit.

This final exam has 102 questions (100 exam questions and 2 bonus questions), worth a total of 100 points in addition to 13 bonus points. Good luck!

Multiple Choice

1. (1 point) If a non-horizontal line has a slope of $\sin(\theta)$, it will be perpendicular to a line with slope...
 A. $\cos \theta$ B. $-\cos \theta$ C. $\csc \theta$ D. $-\csc \theta$ E. $-\sin \theta$

1. _____

2. (1 point) Which quadrant is the angle 75° in?
 A. QI B. QII C. QIII D. QIV

2. _____

3. (1 point) In how many different ways can the letters in MISSISSIPPI be arranged?
 A. 7560 B. 34650 C. 75600 D. 151200 E. 302400

3. _____

4. (1 point) Given the angle 92° , find the reference angle.
 A. 90° B. 8° C. 272° D. 180° E. 2°

4. _____

5. (1 point) Choose the answer that best describes the transformation of $f(\frac{1}{3}x)$.
 A. vertical stretch B. vertical compression C. horizontal stretch D. horizontal compression

5. _____

6. (1 point) Choose the answer that best describes the transformation of $f(x+5)$.
 A. vertical stretch B. vertical compression C. horizontal stretch D. horizontal translation

6. _____

7. (1 point) Given $f(x) = x^2 - 8$ and $g(x) = -5 - x$, find $f(g(x))$.
 A. $x^2 + 10x + 17$ B. $-x^2 - 13$ C. $-x^2 + 3$ D. $x^2 - 10x + 3$

7. _____

8. (1 point) Order the functions from the widest to narrowest graph, given $f(x) = \frac{1}{3}x^2$, $g(x) = -9x^2$ and $h(x) = \frac{-1}{2}x^2$.
 A. $f(x)$, $g(x)$ and $h(x)$ B. $h(x)$, $f(x)$ and $g(x)$ C. $f(x)$, $h(x)$ and $g(x)$ D. $g(x)$, $h(x)$ and $f(x)$
 E. $g(x)$, $f(x)$ and $h(x)$

8. _____

9. (1 point) What is the multiplicity of the zero $x = -4$ in $f(x) = (x + 1)^2(x + 4)^3(x - 6)^5(x - 4)^6$?
A. 2 B. 1 C. 3 D. 5 E. 6

9. _____

10. (1 point) Given the point $(-3, -3)$, determine the sign of $\tan(\theta)$.
A. Positive B. Negative C. Neutral D. QIV E. None of the above

10. _____

11. (1 point) The growth factor for $f(x) = \frac{5}{6} \cdot (4)^x$ is
A. $\frac{6}{5}$ B. $\frac{1}{4}$ C. $\frac{5}{6}$ D. 5 E. 4

11. _____

12. (1 point) What is the constant percentage decay rate of $P(t) = 2.609 \cdot 0.093^t$?
A. 93% B. 0.093% C. 26.09% D. 90.7% E. 260.9%

12. _____

13. (1 point) What is the approximate value of the common log of 5?
A. 0.69897 B. 1.60943 C. 5 D. 3.30103 E. Undefined

13. _____

14. (1 point) The graph of $y = \csc(x)$ never intersects the graph of $y =$
A. x B. x^2 C. $\cos(x)$ D. $\csc(x)$ E. $\sin(x)$

14. _____

15. (1 point) If θ is the smallest angle in a 3-4-5 triangle, then what is the $\cos(\theta)$?
A. $\frac{3}{5}$ B. $\frac{5}{4}$ C. $\frac{4}{3}$ D. $\frac{3}{4}$ E. $\frac{4}{5}$

15. _____

16. (1 point) A sinusoid with an amplitude of 3 has a minimum value of -2. Its maximum value is
A. 10 B. 6 C. 4 D. 2 E. -2

16. _____

17. (1 point) Simplify $\cos(-x)$
A. $\sec x$ B. $\csc -x$ C. $-\cos x$ D. $\cos x$ E. $\cos \frac{1}{x}$

17. _____

18. (1 point) A simpler expression for $(1 - \sin \theta)(1 + \sin \theta)$ is
A. $\sin^2 \theta + \cos^2 \theta$ B. $\sin^2 \theta$ C. $-\sin^2 \theta$ D. $\cos^2 \theta$ E. $1 - \sin^2 \theta$

18. _____

19. (1 point) Write the expression $\sin 48^\circ \cos 15^\circ - \cos 48^\circ \sin 15^\circ$ as a single sine or cosine of an angle.

- A. $\cos 63^\circ$ B. $\sin 63^\circ$ C. $\sin 15^\circ$ D. $\cos 15^\circ$ E. $\sin 33^\circ$

19. _____

20. (1 point) Let $\mathbf{a} = \langle 3, 4 \rangle$ and $\mathbf{b} = \langle -5, 6 \rangle$. Which of the following is equal to $\mathbf{a} - \mathbf{b}$?

- A. $\langle -1, -11 \rangle$ B. $\langle 8, -2 \rangle$ C. $\langle -2, 10 \rangle$ D. $\langle 3, 4 \rangle - \langle -5, 6 \rangle$ E. $\langle -15, 20 \rangle$

20. _____

21. (1 point) Let $\mathbf{a} = \langle 2, 4 \rangle$ and $\mathbf{b} = \langle 3, 5 \rangle$. Which of the following is equal to $\mathbf{a} \cdot \mathbf{b}$?

- A. 90 B. 23 C. 26 D. 120 E. $\langle 6, 20 \rangle$

21. _____

22. (1 point) Which of the following gives the number of petals of the rose curve $r = 5 \cos 4\theta$?

- A. 2 B. 4 C. 6 D. 8 E. 10

22. _____

23. (1 point) Let $\mathbf{a} = \langle 1, 4 \rangle$ and $\mathbf{b} = \langle 2, 6 \rangle$. Find the component form of the vector given $3\mathbf{b}$.

- A. $\langle 6, 18 \rangle$ B. $\langle 6, 8 \rangle$ C. $\langle 9b, 15b \rangle$ D. $\langle 6, 12 \rangle$ E. $\langle 6, 20 \rangle$

23. _____

24. (1 point) Let $\mathbf{a} = \langle 3, 7 \rangle$ and $\mathbf{b} = \langle 2, 9 \rangle$. Find the component form of the vector given $32\mathbf{a}$.

- A. $\langle 48, 144 \rangle$ B. $\langle 96a, 224b \rangle$ C. $\langle 64, 169 \rangle$ D. $\langle 96, 160 \rangle$ E. $\langle 96, 224 \rangle$

24. _____

25. (1 point) Let $\mathbf{a} = \langle 1, 3 \rangle$ and $\mathbf{b} = \langle 4, 6 \rangle$. Find the component form of the vector given $3\mathbf{a} - 4\mathbf{b}$.

- A. $\langle 19, 33 \rangle$ B. $\langle -13, -15 \rangle$ C. $\langle 12, 40 \rangle$ D. $\langle 12a, 40b \rangle$ E. $\langle 27, 384 \rangle$

25. _____

Short Answer (Open Ended)

26. (1 point) Use this table of frequencies for colors of Skittles in a bag to solve the following problems.

red	orange	yellow	green	blue	brown
0.1	0.2	0.1	0.2	0.1	0.3

- (a) What is the probability of selecting one orange or green Skittle out of the bag?
- (b) What is the probability of **not** selecting one yellow or one blue Skittle out of the bag?
- (c) Suppose there are two bags of Skittles. What is the probability of picking one brown from one bag and one red from the other bag?
27. (1 point) You are rolling two dice and are taking the sum of the numbers rolled.
- (a) State the probability of rolling a 6 or 11.
- (b) State the probability of rolling a number divisible by 2.
- (c) State the probability of rolling a double-digit number.

28. (1 point) Given 129° , express one positive coterminal angle and one negative coterminal angle.
29. (1 point) From among 8 projects for consideration, the mayor must put together a prioritized list of 3 projects to submit to the city council for funding. How many lists can be formed?
30. (1 point) Convert $\frac{3\pi}{2}$ from radians to degrees **AND** state which quadrant the angle is in. The answer should be simplified to the tenth.
31. (1 point) Determine the domain of $f(x) = \frac{5}{x^2-4}$ algebraically.
32. (1 point) Determine the domain of $f(x) = \sqrt{x^2 + 13}$ algebraically.
33. (1 point) Given $f(x) = 3x - 5$ and $g(x) = x - 2$, find $f(g(x))$.
34. (1 point) Given $f(x) = x^2 + 1$ and $g(x) = \frac{1}{x^2+4}$, find $f(g(x))$.
35. (1 point) Determine the slant asymptote of $f(x) = \frac{x^2-6x+7}{x+5}$ algebraically.
36. (1 point) Determine the slant asymptote of $f(x) = \frac{3x^3-2x}{-2x^2+4}$ algebraically.

37. (1 point) Solve the equation. $\frac{4x}{x-1} + \frac{6}{x+2} = \frac{24}{x^2+x-2}$

38. (1 point) Expand the following (rewrite as an expanded logarithm): $\ln(7xy)$.

39. (1 point) Condense the following (rewrite as a single, condensed logarithm): $\log 12 + \log x + \log y$.

40. (1 point) The number B of a flesh-eating bacteria in a petri dish culture after t hours is given by

$$B = 68e^{0.867t}$$

(a) What is the initial amount of bacteria present?

(b) What is the rate that the bacteria are increasing at?

(c) How many bacteria are present after $8\frac{1}{2}$ hours?

(d) How many bacteria are present after 3 days?

41. (1 point) Find the exact value of $\sin^{-1}(\frac{\sqrt{2}}{2})$ using the unit circle.

42. (1 point) Find the exact value of $\cos^{-1}(\frac{1}{2})$ using the unit circle.

43. (1 point) Using the unit circle, find the exact value of $\cos(150^\circ)$.

44. (1 point) Using the unit circle, find the exact value of $\csc(60^\circ)$.

45. (1 point) State the amplitude, period, phase shift and vertical shift of

$$y = 7 \cos(x - \pi) + 6$$

Amplitude: _____

Period: _____

Phase Shift: _____

Vertical Shift: _____

46. (1 point) Simplify $\cot \theta \csc \theta$.

47. (1 point) Verify that $\cos x + \sin x + \tan x$ is $\sec x$

48. (1 point) Verify that $\frac{1}{1-\sin \theta} + \frac{1}{1+\sin \theta}$ is $2 \sec^2 \theta$

49. (1 point) Find all values of x in the interval $[0, 2\pi)$ that solves $2 \cos x + 2 = 0$. The answer can be provided in either radians or degrees.

50. (1 point) Solve $\triangle ABC$ given that $\angle A = 60^\circ$, $b = 15$ and $c = 22$.

51. (1 point) Solve $\triangle ABC$ given that $a = 20$, $b = 30$ and $\angle B = 140^\circ$.

52. (1 point) Solve the non-linear system:

$$x^2 + y = 6$$

$$x + y = 14$$

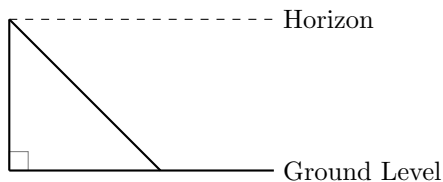
53. (1 point) Convert $y = 2x + 1$ from rectangular to polar form.

54. (1 point) Convert $r(2 \cos \theta + \sin \theta) = 4$ from polar to rectangular form.

55. (1 point) Perform the following matrix operation:
$$\begin{bmatrix} 1 & -7 \\ 0 & 3 \\ 5 & -10 \end{bmatrix} + \begin{bmatrix} 11 & 17 \\ -12 & 8 \\ 4 & 9 \end{bmatrix}$$

Word Problems

56. (1 point) You are about to go base jumping and are second guessing if the cliff you are standing on is tall enough. You ask your friend to help you measure the height so she goes down to the bottom of the cliff to help you measure the height. She starts at the base of the cliff and walks out 100 meters. You measure an angle of depression of 44° from the horizon to her. When you base jump you prefer to be at least 100 meters above the ground to give yourself time to safely deploy your parachute. Should you attempt this base jump? Give a mathematical reason for your decision.



57. (1 point) You are given an $8.5\text{in} \times 11.5\text{in}$ piece of paper. After using it, you decide to cut out a square of the same size from each corner. After you cut out the squares, you fold the paper into an open box.
- (a) Draw a diagram for the piece of paper and for the open top box. Include variables and known values.
- (b) What is the interval for plausible x values, where x is the side length of the square you cut out?
- (c) What is the equation for volume, with respect to x ?
- (d) What is the x value that maximizes the volume?
- (e) What is the maximum volume?

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58. (1 point) How much pure acid must be added to 75 mL of a 50% acid solution in order to produce a mixture that is 80% acid?
59. (1 point) Find the dimensions of the rectangle with minimum perimeter if its area is 400 square feet. Find this least perimeter.
60. (1 point) Given $A = Pe^{rt}$, suppose Rob invests \$150 at a 12% interest rate compounded continuously. Find the value of his investment at the end of 6 years.
61. (1 point) Assume Harper invests \$1200 at an 8% interest rate compounded quarterly. Given $A = P(1 + \frac{r}{n})^{(nt)}$, how much money will she have in 3 years?
62. (1 point) Suppose Jim invests \$800 at an 9% interest rate compounded quarterly. Given $A = P(1 + \frac{r}{n})^{(nt)}$, how much will he have **earned** in 4 years?
63. (1 point) Warren Buffett invested \$20,000 twice into two separate bank accounts (his total investment was \$40,000). One account pays 2% in interest monthly and the other pays 6% in interest annually. Given $A = P(1 + \frac{r}{n})^{(nt)}$, determine how much money, in total, he **earned** over the course of two years.
64. (1 point) The angle of depression from the top of a lighthouse 250ft above sea level to a sailboat is 12° . How far is the boat from the base of the lighthouse?
65. (1 point) A wire stretches from the top of a tower to the ground 40ft away from the base of the tower. The wire makes an angle of 70° with the ground. Find the height of the tower and the length of the wire.

66. (1 point) Two points, A and B , are on the same side of a canyon and are 70ft apart. A hiker is located across the rim at point C . A surveyor determines that $\angle BAC$ is 60° and $\angle ABC$ is 65° .

(a) Sketch a diagram of this situation.

(b) What is the distance between the hiker and point A ?

67. (1 point) A boat leaves Miami and heads due east for 35km. At the same time, a second boat also travels from Miami for 25km. The two boats are 15.2km apart when they reach their final destination.

(a) Sketch a diagram of this situation.

(b) What was the angle between the two boats when they left Kingston?

68. (1 point) A ship is heading due north at 16mph. The current is flowing southwest at 5mph.

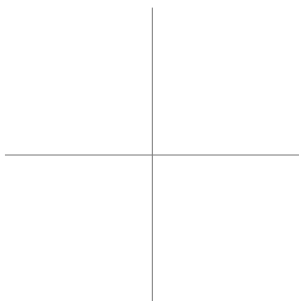
(a) Sketch a diagram of this scenario.

(b) Find the actual bearing and speed of the ship.

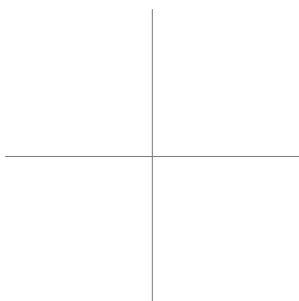
69. (1 point) The admission fee at the Iowa state fair is \$1.50 for children and \$4.00 for adults. On a certain day, 2200 people enter the fair and \$5,050 is collected. How many children and how many adults attended?
70. (1 point) At a movie premiere, 351 tickets were sold. The cost for child admission was \$1.50 per ticket and the cost for adult admission was \$2.00. What was the total price of the adult tickets?
71. (1 point) An exam worth 145 points contains 50 questions. Some of the questions are worth two points and some are worth five points. How many two point questions are on the test? How many five point questions are on the test?

Graphing

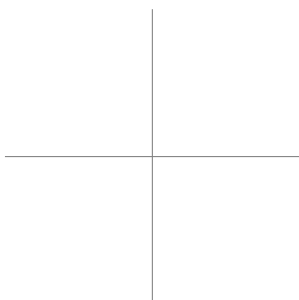
72. (1 point) Given an angle in standard position with a terminal ray that passes through the point $(4, 8)$, find the values of the six trigonometric functions. Graph this and label the point and the angle.



73. (1 point) Given an angle in standard position with a terminal ray that passes through the point $(-2, -5)$, find the values of the six trigonometric functions. Graph this and label the point and the angle.



74. (1 point) Sketch the graph of $f(x) = (x - 3)^2 + 4$ and state its domain, range and parent function.

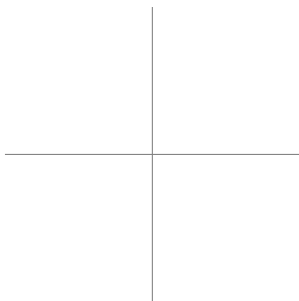


Domain: _____

Range: _____

Parent Function: _____

75. (1 point) Sketch the graph of $f(x) = -3|x + 1| - 2$ and state its domain, range and parent function.

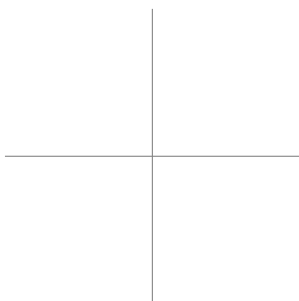


Domain: _____

Range: _____

Parent Function: _____

76. (1 point) Sketch the graph of $f(x) = \frac{3}{x-4}$ and state its domain, range and parent function.

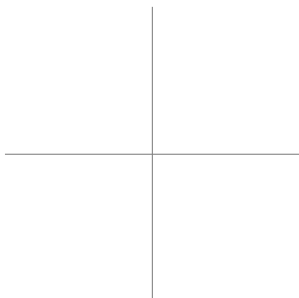


Domain: _____

Range: _____

Parent Function: _____

77. (1 point) Sketch the graph of the rational function $\frac{x}{x^2-4}$ and find its end behavior and vertical asymptote(s).

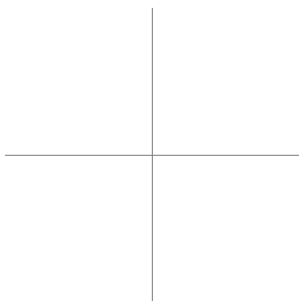


$\lim_{x \rightarrow \infty} f(x) =$

$\lim_{x \rightarrow -\infty} f(x) =$

Vertical Asymptote(s): _____

78. (1 point) Sketch the graph of the rational function $\frac{2x-2}{x-4}$ and find its end behavior and vertical asymptote(s).

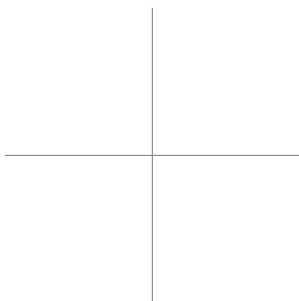


$$\lim_{x \rightarrow \infty} f(x) =$$

$$\lim_{x \rightarrow -\infty} f(x) =$$

Vertical Asymptote(s): _____

79. (1 point) Sketch the graph of the rational function $\frac{x^3}{x^2-16}$ and find its end behavior and vertical asymptote(s).

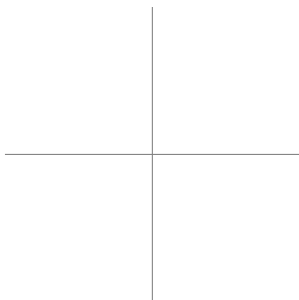


$$\lim_{x \rightarrow \infty} f(x) =$$

$$\lim_{x \rightarrow -\infty} f(x) =$$

Vertical Asymptote(s): _____

80. (1 point) Sketch the graph of the rational function $\frac{3x-3}{x-5}$ and find its end behavior and vertical asymptote(s).

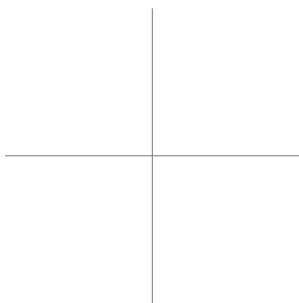


$$\lim_{x \rightarrow \infty} f(x) =$$

$$\lim_{x \rightarrow -\infty} f(x) =$$

Vertical Asymptote(s): _____

81. (1 point) Sketch the graph of the function $f(x) = \frac{x^2-5}{x^2-4}$ and state the vertical and/or horizontal asymptote(s), slant asymptote(s), domain, range and end behavior.



Vertical Asymptote(s): _____

Horizontal Asymptote(s): _____

Slant Asymptote(s): _____

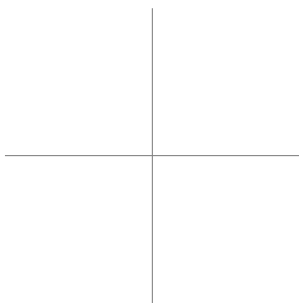
Domain: _____

Range: _____

$\lim_{x \rightarrow \infty} f(x)$: _____

$\lim_{x \rightarrow -\infty} f(x)$: _____

82. (1 point) Sketch the graph of the function $f(x) = \frac{-4}{x^2-8x+3}$ and state the vertical and/or horizontal asymptote(s), slant asymptote(s), domain, range and end behavior.



Vertical Asymptote(s): _____

Horizontal Asymptote(s): _____

Slant Asymptote(s): _____

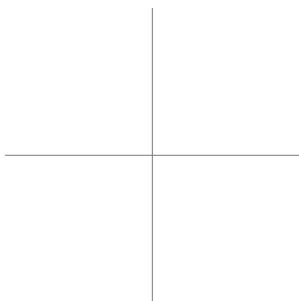
Domain: _____

Range: _____

$\lim_{x \rightarrow \infty} f(x)$: _____

$\lim_{x \rightarrow -\infty} f(x)$: _____

83. (1 point) Sketch the graph of the function $f(x) = \frac{x+4}{x+3}$ and state the vertical and/or horizontal asymptote(s), slant asymptote(s), domain, range and end behavior.



Vertical Asymptote(s): _____

Horizontal Asymptote(s): _____

Slant Asymptote(s): _____

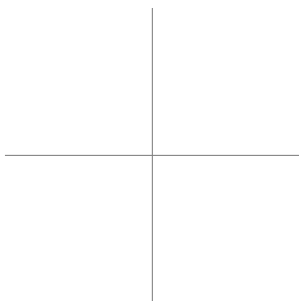
Domain: _____

Range: _____

$\lim_{x \rightarrow \infty} f(x)$: _____

$\lim_{x \rightarrow -\infty} f(x)$: _____

84. (1 point) Sketch the graph of the function $f(x) = \frac{5}{x^2-16}$ and state the vertical and/or horizontal asymptote(s), slant asymptote(s), domain, range and end behavior.



Vertical Asymptote(s): _____

Horizontal Asymptote(s): _____

Slant Asymptote(s): _____

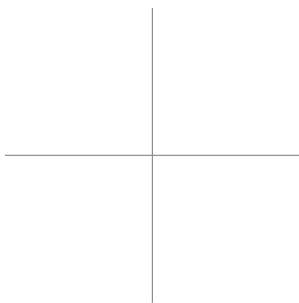
Domain: _____

Range: _____

$\lim_{x \rightarrow \infty} f(x)$: _____

$\lim_{x \rightarrow -\infty} f(x)$: _____

85. (1 point) Given the function $f(x) = e^x + 5$, draw a sketch, state the vertical asymptote (as $x =$) or horizontal asymptote (as $y =$), domain, range and end behavior.



Vertical Asymptote/Horizontal Asymptote: _____

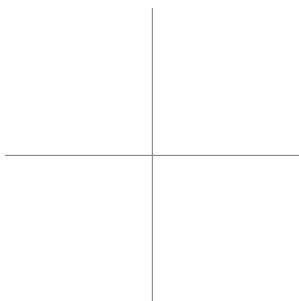
Domain: _____

Range: _____

$\lim_{x \rightarrow \infty} f(x)$: _____

$\lim_{x \rightarrow -\infty} f(x)$: _____

86. (1 point) Given the function $f(x) = 4^{x+5}$, draw a sketch, state the vertical asymptote (as $x =$) or horizontal asymptote (as $y =$), domain, range and end behavior.



Vertical Asymptote/Horizontal Asymptote: _____

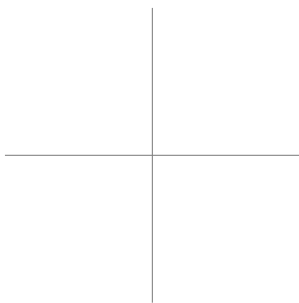
Domain: _____

Range: _____

$\lim_{x \rightarrow \infty} f(x)$: _____

$\lim_{x \rightarrow -\infty} f(x)$: _____

87. (1 point) Given the function $f(x) = (\frac{3}{4})^x + 4$, draw a sketch, state the vertical asymptote (as $x =$) or horizontal asymptote (as $y =$), domain, range and end behavior.



Vertical Asymptote/Horizontal Asymptote: _____

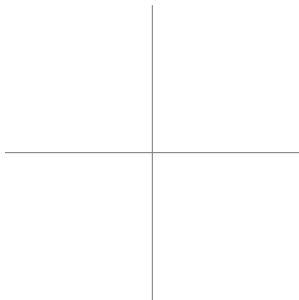
Domain: _____

Range: _____

$\lim_{x \rightarrow \infty} f(x)$: _____

$\lim_{x \rightarrow -\infty} f(x)$: _____

88. (1 point) Given the function $f(x) = \log(x - 2) + 1$, sketch a graph and state the vertical and horizontal asymptote(s), slant asymptote(s), domain, range and end behavior.



Vertical Asymptote(s): _____

Horizontal Asymptote(s): _____

Slant Asymptote(s): _____

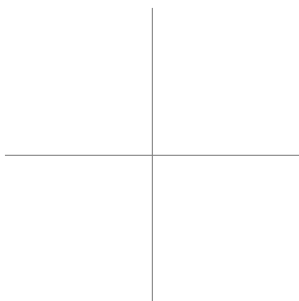
Domain: _____

Range: _____

$\lim_{x \rightarrow \infty} f(x)$: _____

$\lim_{x \rightarrow -\infty} f(x)$: _____

89. (1 point) Given the function $f(x) = \ln(x) + 3$, sketch a graph and state the vertical and horizontal asymptote(s), slant asymptote(s), domain, range and end behavior.



Vertical Asymptote(s): _____

Horizontal Asymptote(s): _____

Slant Asymptote(s): _____

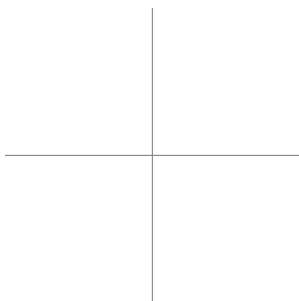
Domain: _____

Range: _____

$\lim_{x \rightarrow \infty} f(x)$: _____

$\lim_{x \rightarrow -\infty} f(x)$: _____

90. (1 point) Given the function $f(x) = 3 \log(x - 6)$, sketch a graph and state the vertical and horizontal asymptote(s), slant asymptote(s), domain, range and end behavior.



Vertical Asymptote(s): _____

Horizontal Asymptote(s): _____

Slant Asymptote(s): _____

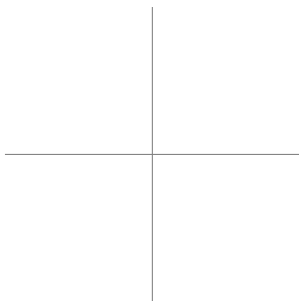
Domain: _____

Range: _____

$\lim_{x \rightarrow \infty} f(x)$: _____

$\lim_{x \rightarrow -\infty} f(x)$: _____

91. (1 point) Graph $r = 4 \sin(\theta) + 3$

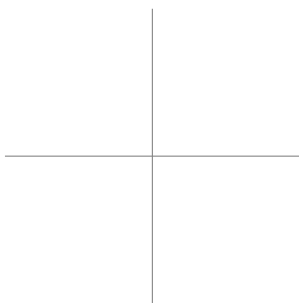


Range: _____

Domain: _____

Period: _____

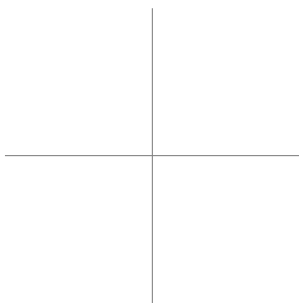
92. (1 point) Graph $r = 5 \csc(\theta) - 4$



Range: _____

Period: _____

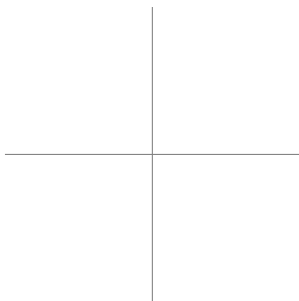
93. (1 point) Graph $r = \sec(\theta + \frac{\pi}{4}) + 3$



Range: _____

Period: _____

94. (1 point) Graph $r = 5(\sin(\theta))^2$



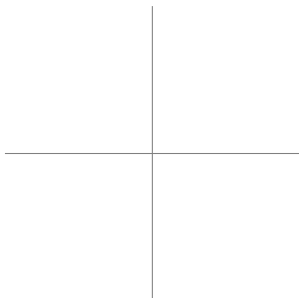
Amplitude: _____

Domain: _____

Range: _____

Period: _____

95. (1 point) Sketch the function $r = 4 \cos 2\theta$ and analyze the graph of the polar curve.



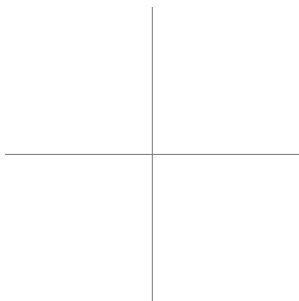
Continuous: _____

Symmetric: _____

Domain: _____

Range: _____

96. (1 point) Sketch the function $r = 5 + \sin \theta$



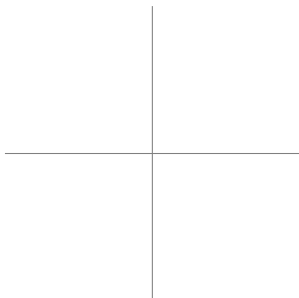
Continuous: _____

Symmetric: _____

Domain: _____

Range: _____

97. (1 point) Sketch the function $r = 7 \cos 3\theta$ and analyze the graph of the polar curve.



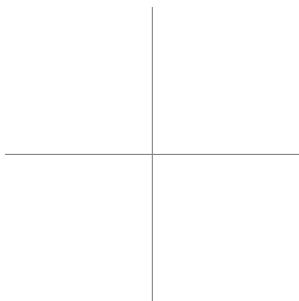
Continuous: _____

Symmetric: _____

Domain: _____

Range: _____

98. (1 point) Sketch the function $r = 4 \cos 2\theta$ and analyze the graph of the polar curve.



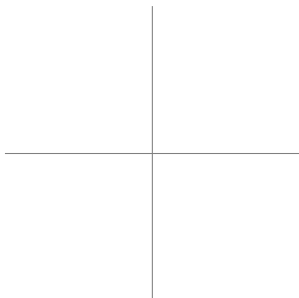
Continuous: _____

Symmetric: _____

Domain: _____

Range: _____

99. (1 point) Sketch the function $r = 8 \cos 4\theta$ and analyze the graph of the polar curve.



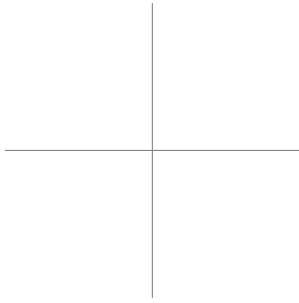
Continuous: _____

Symmetric: _____

Domain: _____

Range: _____

100. (1 point) Sketch the function $r = 5 \cos 2\theta$ and analyze the graph of the polar curve.



Continuous: _____

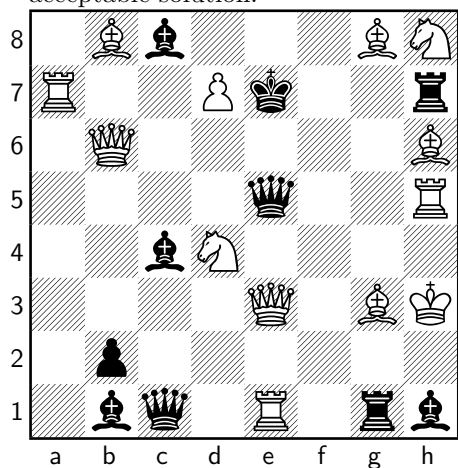
Symmetric: _____

Domain: _____

Range: _____

Extra Credit

101. (10 points (bonus)) Find checkmate in one move. Assume it is white's move and there is only one acceptable solution.



102. (3 points (bonus)) Who founded the Cartesian coordinate system?

Formulas and Equations

$a^2 = b^2 + c^2 - 2bc \cos A$, $b^2 = a^2 + c^2 - 2ac \cos B$, $c^2 = a^2 + b^2 - 2ab \cos C$, $x = r \cos \theta$, $y = r \sin \theta$, $r^2 = x^2 + y^2$,
 $\theta = \tan^{-1}\left(\frac{y}{x}\right)$

