1552 Unofficial Exam 4 Practice

- I recommend you study the other material first, then once you feel ready, try this practice exam out. You should time yourself and give yourself 50 minutes.
- Even though this is just a practice, you should still be practicing good notation. You can always email me at ethanphan@gatech.edu if you want me to check your work.
- Solutions will be posted `https://ethanphan.me/f24-exam4-sol.pdf` on Saturday evening.
- Made by Ethan Phan. Good luck!

Question	Points	Score
1	8	
2	30	
3	30	
4	32	
Total:	100	

1. (8 points) True/False. [NOTE: For practice, you should go back after the exam and determine *why* each of the statements is true/false.]

   (a) Let $f(x) = xe^x$. Then $f^{(5)}(0) = 4$
   A. True B. False

   (b) $e^2 = 1 + 2 + \frac{2^2}{2!} + \frac{2^3}{3!} + \frac{2^4}{4!} + \ldots$
   A. True B. False

   (c) If the power series $\sum_{n=1}^{\infty} a_n(x-2)^n$ diverges at $x = 4$, then the power series diverges at $x = -1$
   A. True B. False

   (d) Let $\sum_{n=1}^{\infty} a_n(x+2)^n$ be a power series and suppose $\lim_{n\to\infty} \left|\frac{a_n}{a_{n+1}}\right| = 2$. Then the power series converges at $x = 4$.
   A. True B. False

2. (30 points) (a) Find the radius of convergence of the power series. [NOTE: On the actual exam, you should make your final answer very clear - write something like $\boxed{\text{radius} = ...}$ and circle it.]

$$\sum_{n=0}^{\infty} \frac{3^n(2x-1)^n}{2^{2n+1}\sqrt{n+2}}$$

(b) What is the *open* interval of convergence of the power series?

(c) Check the endpoints for convergence

$$\sum_{n=0}^{\infty} \frac{3^n(2x-1)^n}{2^{2n+1}\sqrt{n+2}}$$

(d) What is the interval of convergence of the power series?

3. (30 points) (a) Find a second degree Taylor polynomial for $f(x) = \ln(x)$ centered at $x = 1$. Show all your work (i.e., do not use the Taylor polynomial you memorized).

(b) Approximate $\ln(1.5)$ using part (a).

(c) Using Taylor's Remainder Theorem, estimate the error of your answer in part (b).

$$|R_n(x)| \leq \max \left| f^{(n+1)}(c) \right| \frac{|x-a|^{n+1}}{(n+1)!}$$

4. (32 points) (a) Find a series expansion for $g(t)$. Write your series in sigma notation and simplify exponents as far as possible.

$$g(t) = \sin\left(\frac{t^2}{2}\right)$$

(b) Integrate your series in part (a) to find a series expansion for $f(x)$. Write your series in sigma notation.

$$f(x) = \int_0^x \sin\left(\frac{t^2}{2}\right)\, dt$$

(c) Using part (b), find a series expansion for $f(1)$.

$$f(1) = \int_0^1 \sin\left(\frac{t^2}{2}\right)\,dt$$

(d) Using the Alternating Series Error Theorem, estimate the value of $f(1)$ within an error of at most 0.05. [NOTE: You should simplify your answer as far as possible, even if the exam instructions don't specify to!]