

Practice Integrals

(S): U-sub

1. $\int \frac{x}{1+x^4} dx$

2. $\int \frac{1}{\sqrt{x} + \sqrt[3]{x}} dx$

3. $\int \frac{\sqrt{x}}{1+x} dx$

(P): Integration by Parts

1. $\int x \ln x dx$

3. $\int x^3 \cos(x^2) dx$

5. $\int \arctan x dx$

2. $\int e^{2x} \cos(3x) dx$

4. $\int \frac{\ln x}{x^2} dx$

6. $\int \arctan\left(\frac{1}{x}\right) dx$

(I): Trig Identities

1. $\int \sin^3 x \cos^2 x dx$

3. $\int \tan^4 x dx$

5. $\int \csc^9 x \cot^3 x dx$

2. $\int \tan^3 x dx$

4. $\int \tan^5 x dx$

6. $\int \frac{\sec^3 x}{\tan x} dx$

(T): Trig Sub

1. $\int \frac{\sqrt{9-x^2}}{x^2} dx$

4. $\int \sqrt{1-4x^2} dx$

7. $\int \frac{\sqrt{25x^2+1}}{x} dx$

2. $\int \frac{1}{x^2 \sqrt{x^2+4}} dx$

5. $\int \frac{\sqrt{9x^2-4}}{3x} dx$

8. $\int \sqrt{e^{2x}+1} dx$

3. $\int \sqrt{9-x^2} dx$

6. $\int \sqrt{25x^2+4} dx$

9. $\int \frac{1}{e^{2x} \sqrt{1-e^{4x}}} dx$

(F): Partial Fractions

1. $\int \frac{3x+11}{x^2-x-6} dx$

3. $\int \frac{3x+1}{(x-1)^2(x+2)} dx$

5. $\int \frac{x^2-x+6}{x^3+3x} dx$

2. $\int \frac{x^2+4}{3x^3+4x^2-4x} dx$

4. $\int \frac{x^2}{(x+1)(x^2+1)} dx$

6. $\int \frac{1}{x^3+8} dx$

(C): Challenge

1. $\int \sec^3 x dx$

3. $\int \frac{x^2}{x^4+1} dx$

5. $\int \frac{x}{x^{3/2}+8} dx$

2. $\int \arctan(\sqrt{x}) dx$

4. $\int \sqrt{\tan x} dx$

6. $\int \sqrt{1+\sin x} dx$

Hints

(S): U-sub

1. $u = x^2$ (and $x^4 = (x^2)^2$)
2. $u = x^{1/6}$
3. $u = \sqrt{x}$

(P): Integration by Parts

1. $u = \ln x, dv = x dx$
2. do IBP twice
3. substitute $t = x^2$ first
4. $u = \ln x, dv = \frac{dx}{x^2}$
5. $u = \arctan x, dv = dx$
6. $u = \arctan(1/x), dv = dx$

(I): Trig Identities

1. break off $\sin^2 x = 1 - \cos^2 x$
2. $\tan^2 x = \sec^2 x - 1$
3. $\tan^2 x = \sec^2 x - 1$ once
4. $\tan^2 x = \sec^2 x - 1$ once
5. $\cot^2 x = \csc^2 x - 1$
6. Simplify to $\frac{\sec^2 x}{\sin x}$ then $\sec^2 x = 1 + \tan^2 x$

(T): Trig Sub

1. $x = 3 \sin \theta$
2. $x = 2 \tan \theta$
3. $x = 3 \sin \theta$
4. $2x = \sin \theta$
5. $3x = 2 \sec \theta$
6. $5x = 2 \tan \theta$
7. $5x = \tan \theta$, see I6
8. $u = e^x, u = \tan \theta$, see I6
9. $u = e^x, u = \sin \theta$

(F): Partial Fractions

1. $\frac{A}{x-3} + \frac{B}{x+2}$
2. $\frac{A}{x} + \frac{B}{3x-2} + \frac{C}{x+2}$
3. $\frac{A}{x-1} + \frac{B}{(x-1)^2} + \frac{C}{x+2}$
4. $\frac{A}{x+1} + \frac{Bx+C}{x^2+1}$
5. $\frac{A}{x} + \frac{Bx+C}{x^2+3}$
6. $\frac{A}{x+2} + \frac{Bx+C}{x^2-2x+4}$

(C): Challenge

1. IBP $u = \sec x, dv = \sec^2 x dx$
2. IBP $u = \arctan \sqrt{x}, dv = dx$, then see S3
3. Partial fractions $x^4 + 1 = (x^2 + \sqrt{2}x + 1)(x^2 - \sqrt{2}x + 1)$
4. $u = \sqrt{\tan x}$, then see C3
5. $u = \sqrt{x}$, then see F6
6. Multiply by the conjugate

This file: <https://ethanphan.me/sp25-integrals.pdf>

Solutions: <https://ethanphan.me/sp25-integrals-ans.pdf>