Practice Integrals

Some integrals are much harder in their group, and these are colored red.

(S): U-sub

1. $\int \frac{x}{1+x^4}dx$
2. $\int \frac{1}{\sqrt{x}+\sqrt[3]{x}}dx$
3. $\int \frac{\sqrt{x}}{1+x}dx$

(P): Integration by Parts

1. $\int x\ln x dx$
2. $\int e^{2x}\cos(3x)dx$
3. $\int x^3\cos(x^2)dx$
4. $\int \frac{\ln x}{x^2}dx$
5. $\int \arctan x dx$
6. $\int \arctan\left(\frac{1}{x}\right)dx$

(I): Trig Identities

1. $\int \sin^3 x\cos^2 x dx$
2. $\int \tan^3 x dx$
3. $\int \tan^4 x dx$
4. $\int \tan^5 x dx$
5. $\int \csc^9 x\cot^3 x dx$
6. $\int \frac{\sec^3 x}{\tan x}dx$

(T): Trig Sub

1. $\int \frac{\sqrt{9-x^2}}{x^2}dx$
2. $\int \frac{1}{x^2\sqrt{x^2+4}}dx$
3. $\int \sqrt{9-x^2}dx$
4. $\int \sqrt{1-4x^2}dx$
5. $\int \frac{\sqrt{9x^2-4}}{3x}dx$
6. $\int \sqrt{25x^2+4}dx$
7. $\int \frac{\sqrt{25x^2+1}}{x}dx$
8. $\int \sqrt{e^{2x}+1}dx$
9. $\int \frac{1}{e^{2x}\sqrt{1-e^{4x}}}dx$

(F): Partial Fractions

1. $\int \frac{3x+11}{x^2-x-6}dx$
2. $\int \frac{x^2+4}{3x^3+4x^2-4x}dx$
3. $\int \frac{3x+1}{(x-1)^2(x+2)}dx$
4. $\int \frac{x^2}{(x+1)(x^2+1)}dx$
5. $\int \frac{x^2-x+6}{x^3+3x}dx$
6. $\int \frac{1}{x^3+8}dx$

(C): Challenge

1. $\int \sec^3 x dx$
2. $\int \arctan\left(\sqrt{x}\right)dx$
3. $\int \frac{x^2}{x^4+1}dx$
4. $\int \sqrt{\tan x}dx$
5. $\int \frac{x}{x^{3/2}+8}dx$
6. $\int \sqrt{1+\sin x}dx$

Hints

(S): U-sub

1. $u = x^2$ (and $x^4 = (x^2)^2$)
2. $u = x^{1/6}$
3. $u = \sqrt{x}$

(P): Integration by Parts

1. $u = \ln x, dv = xdx$
2. do IBP twice
3. substitute $t = x^2$ first
4. $u = \ln x, dv = \frac{dx}{x^2}$
5. $u = \arctan x, dv = dx$
6. $u = \arctan(1/x), dv = dx$

(I): Trig Identities

1. break off $\sin^2 x = 1 - \cos^2 x$
2. $\tan^2 x = \sec^2 x - 1$
3. $\tan^2 x = \sec^2 x - 1$ once
4. $\tan^2 x = \sec^2 x - 1$ once
5. $\cot^2 x = \csc^2 x - 1$
6. Simplify to $\frac{\sec^2 x}{\sin x}$ then $\sec^2 x = 1 + \tan^2 x$

(T): Trig Sub

1. $x = 3\sin\theta$
2. $x = 2\tan\theta$
3. $x = 3\sin\theta$
4. $2x = \sin\theta$
5. $3x = 2\sec\theta$
6. $5x = 2\tan\theta$
7. $5x = \tan\theta$, see I6
8. $u = e^x$, $u = \tan\theta$, see I6
9. $u = e^x$, $u = \sin\theta$

(F): Partial Fractions

1. $\dfrac{A}{x-3} + \dfrac{B}{x+2}$
2. $\dfrac{A}{x} + \dfrac{B}{3x-2} + \dfrac{C}{x+2}$
3. $\dfrac{A}{x-1} + \dfrac{B}{(x-1)^2} + \dfrac{C}{x+2}$
4. $\dfrac{A}{x+1} + \dfrac{Bx+C}{x^2+1}$
5. $\dfrac{A}{x} + \dfrac{Bx+C}{x^2+3}$
6. $\dfrac{A}{x+2} + \dfrac{Bx+C}{x^2-2x+4}$

(C): Challenge

1. IBP $u = \sec x, dv = \sec^2 x dx$
2. IBP $u = \arctan\sqrt{x}, dv = dx$, then see S3
3. Partial fractions $x^4 + 1 = (x^2+\sqrt{2}x+1)(x^2-\sqrt{2}x+1)$
4. $u = \sqrt{\tan x}$, then see C3
5. $u = \sqrt{x}$, then see F6
6. Multiply by the conjugate

This file: `https://ethanphan.me/sp25-integrals.pdf`
Solutions: `https://ethanphan.me/sp25-integrals-ans.pdf`