2.1 Logical Form and Logical Equivalence

An **argument** is a sequence of **statements** used to prove an assertion called the **conclusion**.

The **argument form** focuses on the logical structure.

A **statement** is a sentence that is either true or false. Example:

- $1 + 3 = 7$ (false)

Non-example:

- $x^2 + y^2 = 4$ (unknown x, y)

Logical Operators

- $\sim$: simation, "not"
- $\vee$: Disjunction, "or"
- $\wedge$: Conjunction, "and"

Logical Equivalence

Two statement forms are **logically equivalent** if they have identical truth values for all inputs. Proven using truth tables.

De Morgan's Laws:

- $\sim (p \wedge q) \Leftrightarrow \sim p \vee \sim q$
- $\sim (p \vee q) \Leftrightarrow \sim p \wedge \sim q$

A **tautology** is a statement form that is always true, regardless of the inputs. A **contradiction** is a statement form that is always false.

Law of excluded middle: $p \vee \sim p$ is a tautology.

Law of non-contradiction: $p \wedge \sim p$ is a contradiction.

Common Logical Equivalences

- **1. Commutative Laws:** $P \wedge Q \equiv Q \wedge P$, $P \vee Q \equiv Q \vee P$
- **2. Associative Laws:** $(P \wedge Q) \wedge R \equiv P \wedge (Q \wedge R)$, $(P \vee Q) \vee R \equiv P \vee (Q \vee R)$
- **3. Identity Laws:** $P \wedge \mathrm{T} \equiv P$, $P \vee \mathrm{F} \equiv P$
- **4. Double simation:** $\sim\sim P \equiv P$
- **5. De Morgan's Laws:** $\sim (P \wedge Q) \equiv \sim P \vee \sim Q$, $\sim (P \vee Q) \equiv \sim P \wedge \sim Q$
- **6. Absorption Laws:** $P \wedge (P \vee Q) \equiv P$, $P \vee (P \wedge Q) \equiv P$

2.2 Conditional Statements

Conditionals

A conditional statement has the form:

$$P \to Q$$

where P is called the hypothesis and Q is called the conclusion.

- The only time $P \to Q$ is false is when P is true and Q is false.
- $P \to Q$ is logically equivalent to $\sim P \vee Q$. This can be proven using a truth table.
- The contrapositive of $P \to Q$ is $\sim Q \to \sim P$. An implication is logically equivalent to its contrapositive.
- The converse of $P \to Q$ is $Q \to P$. The converse is not logically equivalent to the original implication.
- The inverse of $P \to Q$ is $\sim P \to \sim Q$. The inverse is not logically equivalent to the original.

Biconditionals

A biconditional statement has the form:

$$P \leftrightarrow Q$$

which can also be written as P if and only if Q.

- $P \leftrightarrow Q$ is true when P and Q have the same truth value.
- $P \leftrightarrow Q$ is false when P and Q have different truth values.
- $P \leftrightarrow Q$ is logically equivalent to $(P \to Q) \wedge (Q \to P)$.

Examples

- Let P be "It is raining" and let Q be "The grass is wet." The statement "If it is raining, then the grass is wet" can be written as $P \to Q$.
- The contrapositive of the previous statement is "If the grass is not wet, then it is not raining" or $\sim Q \to \sim P$.
- The converse is "If the grass is wet, then it is raining" or $Q \to P$.
- The biconditional "The grass is wet if and only if it is raining" is $P \leftrightarrow Q$.

2.3 Valid and Invalid Arguments

Validity
An argument form is **valid** if when the premises are true, the conclusion must also be true.

To show validity:

- Construct a truth table for the argument form
- Identify **critical rows** where all premises are true
- Check that the conclusion is true in all critical rows

If the conclusion is false in any critical row, the argument form is invalid.

Common Valid Argument Forms

- **Modus Ponens**: $P \to Q, P \therefore Q$
- **Modus Tollens**: $P \to Q, \sim Q \therefore \sim P$
- **Hypothetical Syllogism**: $P \to Q, Q \to R \therefore P \to R$

Fallacies
Common invalid argument forms (fallacies):

- **Converse error**: $P \to Q, Q \therefore P$
- **Inverse error**: $P \to Q, \sim P \therefore \sim Q$

Validity vs. Truth
Validity is different from truth. An argument can be:

- Valid with false premises/conclusion
- Invalid with true premises/conclusion

A **sound** argument is valid and has true premises. An **unsound** argument is invalid or has false premises.

Contradiction Rule
The contradiction rule states:

$$\text{If } \sim P \to C \text{ where } C \text{ is a contradiction, then } P \text{ must be true.}$$

This can be used to prove arguments valid.

2.4 Circuits

A circuit can be **open** (no current flows) or **closed** (current flows).

- **Series** - Light bulb turns on if all switches closed.
- **Parallel** - Light bulb turns on if at least one switch closed.

Parallel circuit truth table is like OR. Series circuit truth table is like AND.

Basic Gates

- **NOT** - Inverts input
- **AND** - Output 1 if all inputs 1
- **OR** - Output 1 if any input 1

Combinatorial Circuits

More complicated circuits created by combining basic gates:

- Can split wires but not combine input wires
- Output of one gate can be input of another
- No feedback loops allowed

Boolean expression: Composed of variables taking values 0 or 1 and operations like NOT, AND, OR.

Boolean expression can be derived from circuit by tracing signals.

Recognizers

A **recognizer** circuit outputs 1 for only one input combination. Useful building blocks.

Equivalent Circuits

Two circuits are **equivalent** if input-output tables are identical. Can show equivalence by deriving boolean expressions and using logical equivalences.

NAND and NOR Gates

NAND = AND followed by NOT.

NOR = OR followed by NOT.

NAND and NOR gates can simplify circuit designs.