

Que's Compendium of Memorization and Reference Items for the Hard Working Physicist

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This list of physics knowledge bits was originally intended to be a study guide for the physics GRE.

As such, the sections are organized in the order of study topics listed by good ol' ETS.

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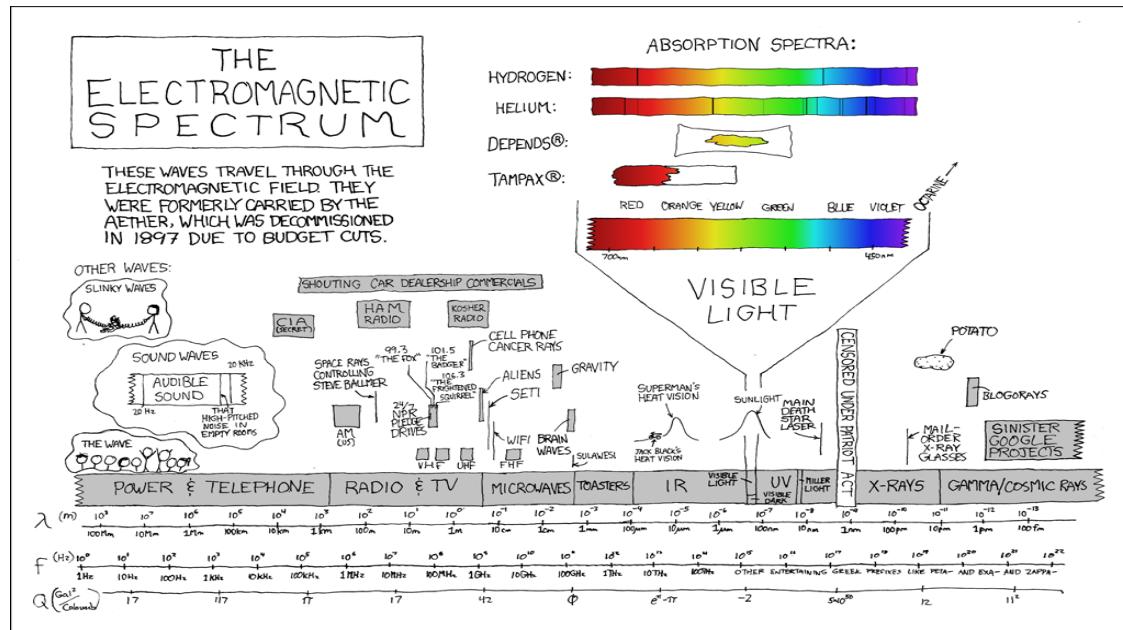
1 Good numbers to know

Mass of proton	1.67×10^{-27} kg
Diameter of proton	about a femtometer (10^{-15})
Diameter of an atom	about 0.1 nanometers = 1 Å
Mass of electron	9.1×10^{-31} kg
Elementary charge	1.6×10^{-19} C
Mass of the Earth	6×10^{24} kg
Radius of the Earth	6×10^6 m
Mass of the Sun	2×10^{30} kg
Radius of the Sun	10,000 km = 7×10^8 m
The astronomical unit	150 million km = 1.5×10^{11} m
Density of water	1000 kg/m ³

Wavelength frequency ranges of visible light.

Color	Wavelength interval	Frequency interval
violet	~ 430 to 380 nm	~ 700 to 790 THz
blue	~ 500 to 430 nm	~ 600 to 700 THz
cyan	~ 520 to 500 nm	~ 580 to 600 THz
green	~ 565 to 520 nm	~ 530 to 580 THz
yellow	~ 590 to 565 nm	~ 510 to 530 THz
orange	~ 625 to 590 nm	~ 480 to 510 THz
red	~ 740 to 625 nm	~ 405 to 480 THz

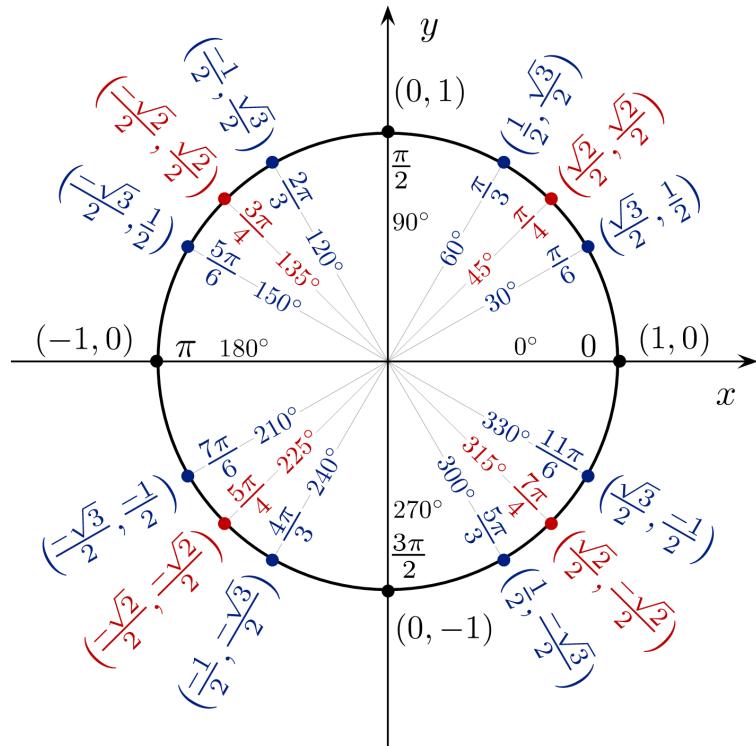
http://reikiservices.cfsites.org/files/color_wavelength_frequency.png



2 Math

Normal Gaussian Distribution	$f(x) = \frac{1}{\sqrt{2\sigma^2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}},$ <p>where μ is expectation, σ is standard deviation, σ^2 is variance.</p>
Empirical Rule	<p>• Stolen without permission from the internet.</p>

2.1 Trigonometry



$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{2}}{2} \approx \frac{1.4}{2} = 0.7$
$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{3}}{2} \approx \frac{1.7}{2} = .85$

First three pythagorean triples	(3, 4, 5), (5, 12, 13), (8, 15, 17)
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$\sin(\theta \pm \phi) =$	$\sin(\theta) \cos(\phi) \pm \cos(\theta) \sin(\phi)$
$\cos(\theta \pm \phi) =$	$\cos(\theta) \cos(\phi) \mp \sin(\theta) \sin(\phi)$
$\cos(\theta) \cos(\phi) =$	$\frac{1}{2}[\cos(\theta + \phi) + \cos(\theta - \phi)]$
$\sin(\theta) \sin(\phi) =$	$\frac{1}{2}[\cos(\theta - \phi) - \cos(\theta + \phi)]$
$\sin(\theta) \cos(\phi) =$	$\frac{1}{2}[\sin(\theta + \phi) + \sin(\theta - \phi)]$

$\sin(\theta) + \sin(\phi) =$	$2 \sin\left(\frac{\theta + \phi}{2}\right) \cos\left(\frac{\theta - \phi}{2}\right)$
$\sin(\theta) - \sin(\phi) =$	$2 \sin\left(\frac{\theta - \phi}{2}\right) \cos\left(\frac{\theta + \phi}{2}\right)$
$\cos(\theta) + \cos(\phi) =$	$2 \cos\left(\frac{\theta - \phi}{2}\right) \cos\left(\frac{\theta + \phi}{2}\right)$
$\cos(\theta) - \cos(\phi) =$	$-2 \sin\left(\frac{\theta - \phi}{2}\right) \sin\left(\frac{\theta + \phi}{2}\right)$

$\cos^2(\theta) =$	$\frac{1}{2}[1 + \cos(2\theta)]$
$\sin^2(\theta) =$	$\frac{1}{2}[1 - \cos(2\theta)]$
$\cos^2(\theta) + \sin^2(\theta) =$	1
$e^{i\theta} =$	$\cos \theta + i \sin \theta$
$\cos \theta =$ (Euler's)	$\frac{1}{2}(e^{i\theta} + e^{-i\theta})$
$\sin \theta =$ (Euler's)	$\frac{1}{2i}(e^{i\theta} - e^{-i\theta})$

2.2 Calculus

The first fundamental theorem of calculus...	... states that if f is continuous on the closed interval $[a, b]$ and F is the indefinite integral of f on $[a, b]$ then $\int_a^b f(x)dx = F(b) - F(a)$.
The second fundamental theorem of calculus holds for f , a continuous function on an open interval l , and a , any point in l , and states that if F is defined by the integral (antiderivative) $F(x) = \int_a^x f(t)dt$, then $F'(x) = f(x)$ at each point in l , where $F'(x)$ is the derivative of $F(x)$.
If $f = f(x, y)$ then $df =$	$\frac{\partial f}{\partial x}dx + \frac{\partial f}{\partial y}dy$

Chain rule	If we have $z(y)$ and $y(x)$, then $\frac{dz}{dx} = \frac{dz}{dy} \cdot \frac{dy}{dx}$
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2.3 Taylor and Maclaurin series

$f(x) =$ Expand about the point a .	$f(a) + f'(a)(x - a) + \frac{1}{2!}f''(a)(x - a)^2 + \frac{1}{3!}f'''(a)(x - a)^3 \dots$
$e^x =$	$1 + x + \frac{1}{2!}x^2 + \frac{1}{3!}x^3 + \dots$
$(1 + x)^n =$	$1 + nx + \frac{n(n - 1)}{2!}x^2 + \dots [x < 1]$ [binomial series]
$\cos(x) =$	$1 - \frac{1}{2!}x^2 + \frac{1}{4!}x^4 + \dots$
$\cosh(x) =$	$1 + \frac{1}{2!}x^2 + \frac{1}{4!}x^4 + \dots$

$\sin(x) =$	$x - \frac{1}{3!}x^3 + \frac{1}{5!}x^5 - \dots$ Also, for small angles, $\sin \theta \approx \tan \theta$.
$\sinh(x) =$	$x + \frac{1}{3!}x^3 + \frac{1}{5!}x^5 + \dots$
$\tan(x) =$	$x + \frac{1}{3!}x^3 + \frac{1}{15!}x^5 + \dots [x < \pi/2]$
$\tanh(x) =$	$x - \frac{1}{3!}x^3 + \frac{1}{15!}x^5 - \dots [x < \pi/2]$
$\ln(1 + x) =$	$x - \frac{1}{2}x^2 + \frac{1}{3}x^3 - \dots [x < 1]$

2.4 Vector Calculus

Laplacian	$\nabla^2 T = \vec{\nabla} \cdot (\vec{\nabla} T) = \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2}$ $\nabla^2 \vec{v} = \nabla^2 v_x \hat{x} + \nabla^2 v_y \hat{y} + \nabla^2 v_z \hat{z}$
What are the two zero vector derivatives?	$\vec{\nabla} \times (\vec{\nabla} T) = 0$ $\vec{\nabla} \cdot (\vec{\nabla} \times \vec{v}) = 0$
$\vec{\nabla} \times (\vec{\nabla} \times \vec{v}) =$	$\vec{\nabla}(\vec{\nabla} \cdot \vec{v}) - \nabla^2 \vec{v}$
Gradient Theorem:	$\int_a^b (\vec{\nabla} f) \cdot d\vec{l} = f(b) - f(a)$ Independent of Path. $\oint (\vec{\nabla} f) \cdot d\vec{l} = 0$
Stoke's Theorem:	$\int_{surface} (\vec{\nabla} \times \vec{v}) \cdot d\vec{a} = \oint_{path} \vec{v} \cdot d\vec{l}$
Green's (Divergence) Theorem:	$\int_{volume} (\vec{\nabla} \cdot \vec{v}) d\tau = \oint_{surface} \vec{v} \cdot d\vec{a}$

2.5 Matrices

Typical eigenvalue problem

$$\hat{T} |\alpha\rangle = \lambda |\alpha\rangle$$

$$\mathbf{T}\mathbf{a} = \lambda\mathbf{a}$$

$$(\mathbf{T} - \lambda\mathbf{I})\mathbf{a} = \mathbf{0}$$

By assumption, $\mathbf{a} \neq \mathbf{0}$, so $(\mathbf{T} - \lambda\mathbf{I})$ is singular: $\det(\mathbf{T} - \lambda\mathbf{I}) = 0$.

This gives an algebraic equation for λ , which is called the characteristic equation for the matrix; its solutions determine the eigenvalues:

$$C_n\lambda^n + C_{n-1}\lambda^{n-1} + \dots + C_1\lambda + C_0 = 0,$$

where n is the dimensionality of the vector space. By what's apparently called the 'fundamental theorem of algebra', there are n complex roots (the eigenvalues). Well, for an $n \times n$ matrix, there is at least one and at most n distinct eigenvalues. The collection of the eigenvalues of a matrix is called its spectrum. If two or more linearly independent eigenvectors share the same eigenvalue, the spectrum is degenerate. To construct the eigenvectors, plug each λ back into $\mathbf{T}\mathbf{a} = \lambda\mathbf{a}$ and solve by one-by-one.

This entry paraphrased and largely quoted from Griffiths, *Introduction to Quantum Mechanics* Appendix 5.

	<p>If the eigenvectors span the space, they can be used as a basis.</p> $\hat{T} f_n\rangle = \lambda f_n\rangle,$ $\mathbf{T} = \begin{pmatrix} \lambda_1 & 0 & \dots & 0 \\ 0 & \lambda_2 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & \lambda_n \end{pmatrix}$ <p>and the normalized eigenvectors are</p> $\begin{pmatrix} 1 \\ 0 \\ \vdots \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \\ \vdots \\ 0 \end{pmatrix}, \dots, \begin{pmatrix} 0 \\ 0 \\ \vdots \\ 1 \end{pmatrix}.$ <p>A matrix that can be brought to diagonal form by a change of basis is said to be diagonalizable. Also note that the trace of a matrix (sum of the diagonal components) is invariant under transformation.</p> <p><small>This entry paraphrased and largely quoted from Griffiths, <i>Introduction to Quantum Mechanics</i> Appendix 5.</small></p>
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3 Classical Mechanics

(such as kinematics, Newton's laws, work and energy, oscillatory motion, rotational motion about a fixed axis, dynamics of systems of particles, central forces and celestial mechanics, three-dimensional particle dynamics, Lagrangian and Hamiltonian formalism, noninertial reference frames, elementary topics in fluid dynamics)

3.1 Kinematics

$v(t) =$	$v(t) = v_0 + at$
$x(t) =$	$x(t) = x_0 + vt + \frac{1}{2}at^2$
$v(x)$ in uniform acceleration	$KE = KE_0 + W$ $\frac{1}{2}mv^2 = \frac{1}{2}mv_0^2 + ma(x - x_0)$ $v(x) = \sqrt{v_0^2 + 2a(x - x_0)}$

Projectile motion: $v_{xi} =$	$v_{xi} = v_i \cos \theta_i$
Projectile motion: $v_{yi} =$	$v_{yi} = v_i \sin \theta_i$
Acceleration (tangential) of a particle on a fixed track, $y(x)$, in a gravitational field	Position vector for the track: $\vec{f}(x) = (x, y(x))$. Tangential vector: $\vec{T}(x) = \frac{d\vec{f}}{dx} = (1, \frac{dy}{dx})$. Tangential acceleration: $a(x) = \vec{g} \cdot \frac{\vec{T}}{ \vec{T} }$. Supposing \vec{g} is in the y -direction, $a(x) = \frac{g T_y}{ \vec{T} }$.

3.2 Newton's Laws, implications, common forces

Newton's Law I	Law of Inertia: Every body persists in its state of being at rest or of moving uniformly straight forward, except insofar as it is compelled to change its state by force impressed.
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Newton's Law II	Force Law: The alteration of motion is ever proportional to the motive force impress'd; and is made in the direction of the right line in which that force is impress'd.
	$\mathbf{F}_{net} = \frac{d\mathbf{p}}{dt} = m\mathbf{a}$

Newton's Law III	Action-Reaction Law: To every action there is always opposed an equal reaction: or the mutual actions of two bodies upon each other are always equal, and directed to contrary parts.
	$\mathbf{F}_{12} = -\mathbf{F}_{21}$

Law of gravitation:	$F_g = G \frac{m_1 m_2}{r^2}$
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Force of static friction	$f_s \leq \mu_s n$
Force of kinetic friction	$f_k = \mu_k n$

Resistive force at low speed:	$\mathbf{R} = -b\mathbf{v}$
Resistive force at high speed:	$\mathbf{R} = -c\mathbf{v}^2$

Differential for object falling through resistive medium.	$\frac{dv}{dt} = g - \frac{b}{m}v \quad v(t) = \frac{mg}{b}(1 - e^{-bt/m})$
Terminal speed:	$v_T = \frac{mg}{b}, \text{ or } v_T = \sqrt{\frac{mg}{c}}, \text{ or } v_T = \sqrt{\frac{2mg}{D\rho A}}$

3.3 Work and Energy

Potential from a given force	$U = - \int_{ref}^r \mathbf{F} \cdot d\mathbf{r}$
Force from a given potential	$\mathbf{F}(x) = -\frac{\partial U}{\partial x} \hat{x}$ or, rather, $\mathbf{F} = -\nabla U$
Work $W =$	$W = \int_{x_i}^{x_f} \mathbf{F}_x \cdot d\mathbf{x} = Fd \cos \theta$
Kinetic Energy, friction, work	$\Delta KE = -f_k \Delta x + W_{net}$

Power:	$P = \frac{dE}{dt} = \frac{dW}{dt} = \mathbf{F} \cdot \mathbf{v}$
Average power:	$\bar{P} = \frac{W}{\Delta t}$
Gravitational potential energy:	$U_g = mgh = -G \frac{m_1 m_2}{r}$
Conservation of energy:	$KE + U + E_{internal} = constant$
Impulse	$\mathbf{I} = \int_{t_i}^{t_f} \sum \mathbf{F}_{\text{ext}} dt = \Delta \mathbf{p}_{tot}$

3.4 Oscillatory motion

Hooke's law:	$F_s = -kx$
$x(t) =$	$x(t) = A \cos(\omega t + \phi)$ or $x(t) = c_1 \cos(\omega t) + c_2 \sin(\omega t)$
Resonant frequency	$\omega = \sqrt{\frac{k}{m}}$
Period	$T = 2\pi \sqrt{\frac{m}{k}}$

$a(x)$	$a(x) = -\omega^2 x$
Kinetic Energy	$T = \frac{1}{2}k\dot{x}^2 = \frac{1}{2}kA^2 \sin^2(\omega t + \phi)$
Elastic potential	$U_s = \frac{1}{2}kx^2 = \frac{1}{2}kA^2 \cos^2(\omega t + \phi)$
Total energy	$E = \frac{1}{2}kA^2$
Effective spring coefficient of multiple springs	Sum in parallel. Reciprocals sum in series.
Reduced mass and resonant frequency of two masses connected by a spring.	$\mu = \frac{m_1 m_2}{m_1 + m_2}, \quad \omega = \sqrt{\frac{k}{\mu}}$
Solution to the damped harmonic oscillator	<p>Homogeneous second order D.E.: $m\ddot{x} + c\dot{x} + kx = 0$.</p> <p>Solution of the form $x(t) = e^{\lambda t}$.</p> <p>Auxiliary equation: $m\lambda^2 + c\lambda + k = 0$.</p> <p>Roots: $\lambda = \frac{-c \pm \sqrt{c^2 - 4mk}}{2m}$.</p> <p>Resulting behavior:</p> <ul style="list-style-type: none"> $c^2 - 4mk > 0 \rightarrow$ Overdamped (Slower exponential decay) $c^2 - 4mk = 0 \rightarrow$ Critically damped (Fastest exponential decay) $c^2 - 4mk < 0 \rightarrow$ Underdamped (Exp. decaying sinusoid)
Damping coefficient	If $F_{damping} = -cv$ then the damping coefficient is $\gamma = \frac{c}{2m}$.

Damping ratio and damping

Using $\omega_0 = \sqrt{k/m}$, rewrite: $\ddot{x} + 2\zeta\omega_0\dot{x} + \omega_0^2x = 0$, where

$\zeta = \frac{c}{2\sqrt{mk}}$ is the ‘damping ratio’.

Solution of the form $x(t) = e^{\lambda t}$.

Auxiliary equation: $\lambda^2 + 2\zeta\omega_0\lambda + \omega_0^2 = 0$.

Roots: $\lambda = \frac{-2\zeta\omega_0 \pm \sqrt{4\zeta^2\omega_0^4 - 4\omega_0^2}}{2}$.

$$\zeta = \frac{c}{2m\omega_0}$$

$\zeta > 1 \rightarrow$ Overdamped

$\zeta = 1 \rightarrow$ Critically damped

$\zeta < 1 \rightarrow$ Underdamped

Underdamped oscillator solution

$x(t) = e^{-\gamma t}a \cos(\omega_1^2 t - \alpha)$, where $\omega_1 = \sqrt{\omega_0^2 - \gamma^2} = \omega_0\sqrt{1 - \zeta^2}$, using $\gamma = \omega_0\zeta$. This is an exponentially decaying sinusoid. In full glory: $x(t) = e^{-\frac{c}{2m}t}a \cos\left(\omega_0^2\left(1 - \frac{c^2}{4mk}\right)t - \alpha\right)$.

3 masses and 2 springs, find modes and frequencies

<http://scienceworld.wolfram.com/physics/SpringsThreeMasses.html>

3.5 Rotational motion about a fixed axis

Centripetal acceleration	$a_c = \frac{v^2}{r} = \frac{\left(\frac{2\pi r}{T}\right)^2}{r} = \frac{4\pi^2 r}{T^2} = \omega^2 r$ <p style="text-align: center;">http://hyperphysics.phy-astr.gsu.edu/hbase/cf.html</p>
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Period: $T =$	$T = \frac{2\pi r}{v}$
Tangential acceleration: $a_t =$	$a_t = \frac{d \mathbf{v} }{dt}$

Radial acceleration: $a_r =$	$a_r = -a_c = \frac{-v^2}{r}$
Total acceleration $\mathbf{a} =$	$\mathbf{a}_r + \mathbf{a}_t$

Angular position	$\theta = \frac{s}{r}$
Angular velocity	$\omega = \frac{d\theta}{dt} = \frac{v}{r}$
Angular acceleration	$\alpha = \frac{d^2\theta}{dt^2} = \frac{a}{r}$

Angular position a.a.f.o. time	$\theta(t) = \frac{1}{2}\alpha t^2 + \omega t + \theta_0$
Angular velocity a.a.f.o. time	$\omega(t) = \alpha t + \omega_0$

Torque	$\tau = \mathbf{r} \times \mathbf{F} = I\alpha$
Angular momentum	$\mathbf{L} = \mathbf{r} \times \mathbf{p} \quad \mathbf{L}_{body} = I\omega$

Moment of inertia	$I = \sum_i m_i r_i^2 = \int_0^M r^2 dm = \int_V r^2 \rho dV$
Rotational Kinetic Energy	$E_{Rot} = \frac{1}{2} I \omega^2 = \frac{L^2}{2I}$

Radius of gyration	$R_g = \sqrt{I/M}$
Frequency of pendulum of arbitrary shape	$\omega = \sqrt{\frac{MgR_{com}}{I}}$

Angular frequency gyroscopic precession	<p>Consider the old bicycle tire on a string trick. The tire is connected to the string by a rod of length r. Consider a change to the tire's angular momentum vector \mathbf{L} pointing out to the side.</p> $d\mathbf{L} = \boldsymbol{\tau} dt$ $dL = Mgr dt$ <p>Also, $dL = L d\theta$, and we are interested in $d\theta$, so</p> $d\theta = \frac{dL}{L} = \frac{Mgr}{L} dt$ $\omega_{\text{precession}} = \frac{Mgr}{L}$
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Moment of inertia and frequency of a mass on a massless pendulum of length l .	$I = ml^2$ $I \vec{\dot{\theta}} = \boldsymbol{\tau} = \mathbf{r} \times \mathbf{F} \rightarrow I \ddot{\theta} = -rmg \sin \theta \approx -rmg\theta$ <p>Think Hooke's law: $m\ddot{x} = -kx \rightarrow \omega = \sqrt{\frac{k}{m}}$</p> $\omega = \sqrt{\frac{g}{l}}$
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Parallel Axis Theorem	$I_{new} = I_{cm} + md^2$
Angular frequency of hoop on a nail	$\sqrt{\frac{g}{2r}}$

3.6 Dynamics of systems of particles

Center of mass	System of particles: $\mathbf{R} = \frac{1}{M} \sum_{i=1}^n m_i \mathbf{r}_i$ Continuous volume: $\mathbf{R} = \frac{1}{M} \int_V \rho(\mathbf{r}) \mathbf{r} dV$
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Elastic collision	Total energy and total momentum are both conserved.
Elastic collision with velocities along the x-direction	$v_{i1} - v_{i2} = v_{f1} - v_{f2}$??? is x axis the axis of approach?
Momentum of center of mass in elastic collision	$MV_{CM,i} = MV_{CM,f}$

Inelastic collision	Kinetic energy is not conserved, but total momentum is. e.g. sticking globs.
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Rocket equation	$m \frac{dV}{dt} = -v_e \frac{dm}{dt}$ where m is the mass of the rocket, V is the velocity according to an outside observer, and $-v_e$ is the exhaust speed in the rocket frame. Solving this yields $\Delta V = v_e \ln \frac{m_0}{m_1}$
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3.7 Central forces and celestial mechanics

Kepler's Law I	<p>The orbit of every planet is an ellipse with the Sun at one of the two foci:</p> $r(\theta) = \frac{p}{1 + \mathcal{E} \cos \theta},$ <p>where $\theta = 0$ indicates the direction toward the periapsis.</p>
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peri- vs. apo-	peri- = closest ... apo- = farthest
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Semi-major axis	Arithmetic mean between r_p and r_a . $r_a - a = a - r_p$ $a = \frac{p}{1 - \mathcal{E}^2}$
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Semi-minor axis	Geometric mean between r_p and r_a . $\frac{r_a}{b} = \frac{b}{r_p}$ $b = \frac{p}{\sqrt{1 - \mathcal{E}^2}}$
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Semi-latus rectum p	Harmonic mean between r_p and r_a . $\frac{1}{r_p} - \frac{1}{p} = \frac{1}{p} - \frac{1}{r_a}$ $pa = r_a r_p = b^2$
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Eccentricity calculation	A.K.A. coefficient of variation between r_p and r_a . $\mathcal{E} = \frac{r_a - r_p}{r_a + r_p}$
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Eccentricity values	$\mathcal{E} = 0 \rightarrow \text{circle}$
	$0 < \mathcal{E} < 1 \rightarrow \text{ellipse}$
	$\mathcal{E} = 1 \rightarrow \text{parabola}$
	$\mathcal{E} > 1 \rightarrow \text{hyperbola}$

Area of the ellipse	$A = \pi ab$
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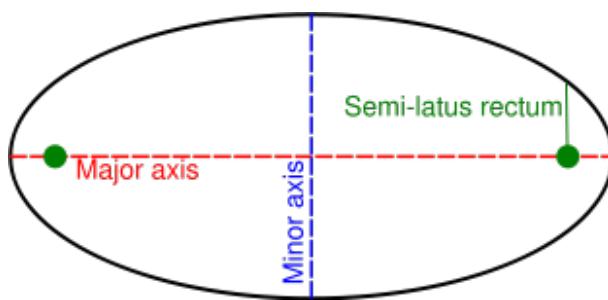


Figure 1: Ellipse notation. When an r for an orbiting object coincides with the semi-latus rectum, $\theta = \pi/2$.

Kepler's Law II	A line joining a planet and the Sun sweeps out equal areas during equal intervals of time. $\frac{dA}{dt} = \frac{1}{2} r^2 \frac{d\theta}{dt}$
------------------------	--

Period	$P \cdot \frac{1}{2} r^2 \frac{d\theta}{dt} = \pi ab$
Mean motion	$n = 2\pi/P$ $r^2 d\theta = abn dt$

Kepler's Law III	<p>The square of the orbital period of a planet is directly proportional to the cube of the semi-major axis of its orbit.</p> $\frac{T^2}{a^3} = \frac{4\pi^2}{G(M+m)},$ <p>(Importantly) $T^2 \propto a^3$</p>
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Derive T for circular orbit of planet about fixed mass M .	$\frac{GMm}{r^2} = \frac{mv^2}{r} = m\omega^2 r = \frac{mr(2\pi)^2}{T^2}$ $T = \sqrt{\frac{4\pi^2 r^3}{GM}}$
--	--

3.8 Three-dimensional particle dynamics

3.9 Lagrangian and Hamiltonian formalism

Lagrangian	$L = L(q, \dot{q}, t) = T - V$
Euler-Lagrange equation	$\frac{\partial L}{\partial q} = \frac{d}{dt} \frac{\partial L}{\partial \dot{q}}$
Action	$S = \int_{t_0}^{t_1} L(q(t), \dot{q}(t), t) dt$

Principal of least action	<p>The path taken by the system between times t_0 and t_1 is the one for which the action is stationary (no change) to first order.</p> $\delta S = 0$
Conjugate momentum	$p = \frac{\partial L}{\partial \dot{q}}$

Hamiltonian	$H = p\dot{q} - L = T + V$
Canonical Hamiltonian equations of motion	$\frac{\partial H}{\partial q} = -\dot{p}, \quad \frac{\partial H}{\partial p} = \dot{q}, \quad \frac{\partial H}{\partial t} = -\frac{\partial L}{\partial t}$

3.10 Noninertial reference frames

Coriolis Force	$\mathbf{F}_{coriolis} = -2m\boldsymbol{\omega} \times (\mathbf{v}_{in\ rotating\ frame})$ Mnemonic: Coriolis makes it hard 2 mov straight. (Steven Byrnes)
Centrifugal Force	$\mathbf{F}_{centrifugal} = -m\boldsymbol{\omega} \times (\boldsymbol{\omega} \times \mathbf{r})$

3.11 Elementary topics in fluid dynamics

Bernoulli's Equation for fluid flow (Incompressible, nonviscous, laminar flow)	$p + \rho gy + \frac{1}{2}\rho v^2 = constant$
Incompressible flow	$\nabla \cdot \mathbf{u} = 0$, where \mathbf{u} is the flow velocity. For a pipe this infers $\mathbf{A} \cdot \mathbf{u}_\perp = (\text{cross-sectional area}) \times (\text{velocity}) = constant.$

Buoyancy force	$B = \rho_{fluid}V_{disp}g$
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4 Electromagnetism

(such as electrostatics, currents and DC circuits, magnetic fields in free space, Lorentz force, induction, Maxwell's equations and their applications, electromagnetic waves, AC circuits, magnetic and electric fields in matter)

4.1 Electrostatics

ϵ is called...	permittivity
μ is called	permeability
Gauss's Law	Flux $\Phi \equiv \oint \mathbf{E} \cdot d\mathbf{a} = Q_{enc}/\epsilon_0 \Leftrightarrow$ M.E. I

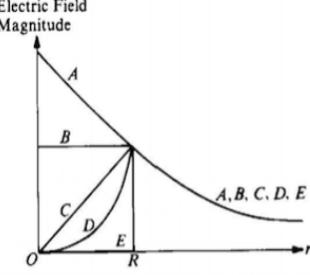
E-field for sphere of radius R with uniformly distributed charge Q	$E_{inside} = \frac{Q}{4\pi\epsilon_0} \frac{r}{R^3}$ $E_{outside} = \frac{Q}{4\pi\epsilon_0} \frac{1}{r^2}$
Electric Potential	$V(\mathbf{a}) = - \int_0^a \mathbf{E}(\mathbf{r}) \cdot d\mathbf{l}$ $V(a) - V(b) = \int_a^b \mathbf{E}(\mathbf{r}) \cdot d\mathbf{l}$
Energy of a point charge	$W = QV$

Energy of a collection of point charges	$U = \frac{1}{8\pi\epsilon_0} \sum_{j=0}^n \sum_{i=0, i \neq j}^n \frac{q_i q_j}{r_{if}}$ $U = \frac{1}{2} \sum_{all} V_{due to others}(r_i) q_i.$
Energy of a continuous distribution of charge	$W = \frac{1}{2} \int_V \rho V d\tau = \frac{\epsilon_0}{2} \int_{allspace} E^2 d\tau.$
Energy density of E field	$\eta_E = \frac{1}{2} \epsilon E^2$

Conductor: $\mathbf{E}_{inside}, \rho_{inside}$	$\mathbf{E}_{inside} = \mathbf{0}$	$\rho_{inside} = 0$	$V(r) = constant$	$\hat{E}_{surface} =$
$V(r) =, \hat{E}_{surface}$	$\perp_{surface}$			

Coulomb's Law	$\mathbf{E}(\mathbf{r}) = \frac{1}{4\pi\epsilon_0} \iiint_{volume} \frac{1}{R^2} \rho(\mathbf{r}') d\tau' \hat{R}$ $\mathbf{E}(\mathbf{r}) = \frac{1}{4\pi\epsilon_0} \iint_{surface} \frac{1}{R^2} \sigma(\mathbf{r}') da' \hat{R}$ $\mathbf{E}(\mathbf{r}) = \frac{1}{4\pi\epsilon_0} \int_{line} \frac{1}{R^2} \lambda(\mathbf{r}') dl' \hat{R}$
Coulomb Potential	$V(\mathbf{r}) = \frac{1}{4\pi\epsilon_0} \int \frac{1}{R} \rho(\mathbf{r}') d\tau'$

Poisson's Equation	$\nabla^2 V = -\rho/\epsilon_0$
Laplace's Equation	$\nabla^2 V = 0$

 <p>Electric Field Magnitude</p> <p>O R</p> <p>A, B, C, D, E</p>	C
<p>10. An isolated sphere of radius R contains a uniform volume distribution of positive charge. Which of the curves on the graph above correctly illustrates the dependence of the magnitude of the electric field of the sphere as a function of the distance r from its center?</p> <p>(A) A (B) B (C) C (D) D (E) E</p> <p>GR8677</p>	
<p>Given R and either Q or ρ find</p> <p>E-field inside sphere and</p> <p>outside sphere</p>	$E(r < R) = \frac{Qr}{4\pi\epsilon_0 R^3} = \frac{\rho r}{3\epsilon_0}$ $E(r > R) = \frac{Q}{4\pi\epsilon_0 r^2} = \frac{\rho R^3}{3\epsilon_0 r^2}$

Griffith's triangle of ρ , V & \mathbf{E}

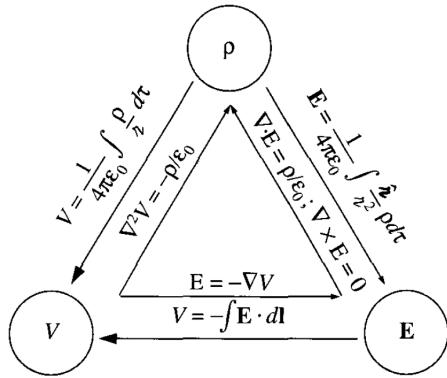


Figure 2.35

Introduction to Electrodynamics, Griffiths

Electric potential for multipole expansion (far field)

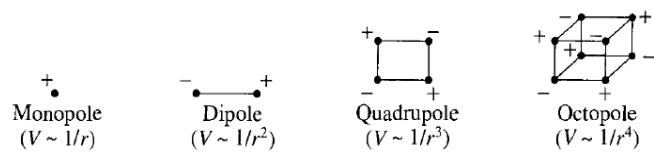


Fig. 3.27 in Griffiths, *Introduction to Electrodynamics*

Electric dipole moment

$$\mathbf{p} \equiv \int \mathbf{r}' \rho \mathbf{r}' d\tau'$$

For point charges, $\mathbf{p} = \sum_{i=1}^n q_i \mathbf{r}'_i$.

Electric potential for dipole

$$V_{\text{dip}}(\mathbf{r}) = \frac{1}{4\pi\epsilon_0} \frac{\mathbf{p} \cdot \hat{\mathbf{r}}}{r^2} = \frac{p \cos \theta}{4\pi\epsilon_0 r^2}.$$

Electric field of a dipole

$$\mathbf{E}_{\text{dip}}(\mathbf{r}) = \frac{1}{4\pi\epsilon_0} \frac{1}{r^3} [3(\mathbf{p} \cdot \hat{\mathbf{r}})\hat{\mathbf{r}} - \mathbf{p}].$$

$$\mathbf{E}_{\text{dip}}(r, \theta) = \frac{p}{4\pi\epsilon_0 r^3} (2 \cos \theta \hat{\mathbf{r}} + \sin \theta \hat{\boldsymbol{\theta}}).$$

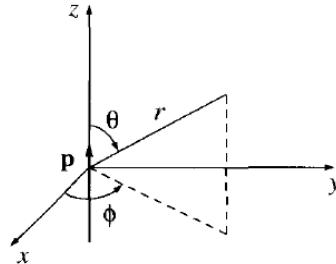


Fig. 3.36 from Griffiths, *Introduction to Electrodynamics*

4.2 Capacitors

Capacitance	$C = \frac{q}{V}$
Voltage/current relationship	$I(t) = C \frac{dV(t)}{dt}$
Charging energy	$W_{\text{charging}} = \frac{1}{2} CV^2$

Capacitance of a capacitor	Mutual capacitance between two conducting plates, i.e. a regular capacitor, is given by $C = \epsilon_r \epsilon_0 \frac{A}{d}$
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4.3 Currents and DC

Amperes =	Coulombs / second
Ohm's Law	$V = IR$

Current in a wire of area A ,

velocity of charges v ,

charge density ρ

$$I = v\rho A$$

Kirchoff's current law

The algebraic sum of currents in a network of conductors meeting at a point is zero.

Kirchoff's voltage law

The directed sum of the electrical potential differences (voltage) around any closed network is zero.

RC circuit $V(t)$ where $V(0) = V_0$

Start with Kirchoff's current law for a circuit

consisting of just a capacitor and a resistor.

$$C \frac{dV}{dt} + \frac{V}{R} = 0$$

Solve the DE.

$$V(t) = V_0 e^{-\frac{t}{RC}}$$

RL circuit voltages across R and L

$$V_L(t) = V e^{-t \frac{R}{L}}$$

$$V_R(t) = V(1 - e^{-t \frac{R}{L}})$$

4.4 Magnetic fields in free space

Biot-Savart Law

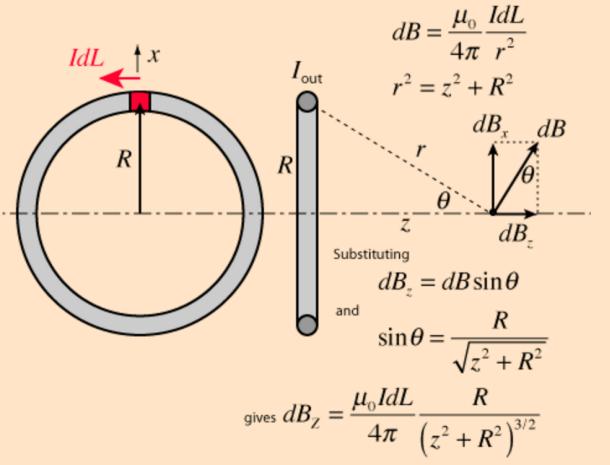
$$\mathbf{B} = \frac{\mu_0}{4\pi} \int \frac{\mathbf{I} \times \hat{R}}{R^2} d\mathbf{l}' = \frac{\mu_0 I}{4\pi} \int \frac{d\mathbf{l}' \times \hat{R}}{R^2}$$

Ampere's law

$$\oint \mathbf{B} \cdot d\mathbf{l} = \mu_0 I_{enc}$$

B-field for solenoid with n turns per unit length and current I .	$\mathbf{B} = \mu_0 n I \hat{z}$
---	----------------------------------

B-field for infinite wire	$B = \frac{\mu_0 I}{2\pi r}$
---------------------------	------------------------------

Field on axis of current loop. General, $z = 0$, and $z \gg R$	 $dB = \frac{\mu_0}{4\pi} \frac{IdL}{r^2}$ $r^2 = z^2 + R^2$ <p>Substituting and</p> $dB_z = dB \sin \theta$ $\sin \theta = \frac{R}{\sqrt{z^2 + R^2}}$ $\text{gives } dB_z = \frac{\mu_0 IdL}{4\pi} \frac{R}{(z^2 + R^2)^{3/2}}$
--	--

Magnetic vector potential	$\mathbf{B} = \nabla \times \mathbf{A}$
	$\mathbf{A} \parallel \mathbf{J}$
	$\mathbf{A}(\mathbf{r}) = \frac{\mu_0}{4\pi} \int \frac{\mathbf{J}(\mathbf{r}')}{R} d\tau'$
	$\oint \mathbf{A} \cdot d\mathbf{l} = \Phi_B$

Griffiths triangle for magnetostatics

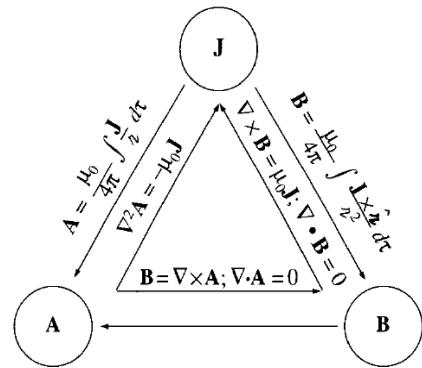


Figure 5.48

Griffiths, *Introduction to Electrodynamics*

Magnetic vector potential
for magnetic dipole

$$\mathbf{A}_{\text{dip}}(\mathbf{r}) = \frac{\mu_0}{4\pi} \frac{\mathbf{m} \times \hat{\mathbf{r}}}{r^2}$$

$$\mathbf{A}_{\text{dip}}(\mathbf{r}) = \frac{\mu_0 m \sin \theta}{4\pi r^2} \hat{\phi}.$$

Magnetic dipole moment

$$\mathbf{m} \equiv I \int d\mathbf{a} = I \mathbf{a}$$

Magnetic field of magnetic
dipole

$$\mathbf{B}_{\text{dip}}(\mathbf{r}) = \frac{\mu_0}{4\pi} \frac{1}{r^3} [3(\mathbf{m} \cdot \hat{\mathbf{r}}) \hat{\mathbf{r}} - \mathbf{m}].$$

$$\mathbf{B}_{\text{dip}}(\mathbf{r}) = \frac{\mu_0 m}{4\pi r^3} (2 \cos \theta \hat{\mathbf{r}} + \sin \theta \hat{\theta}).$$

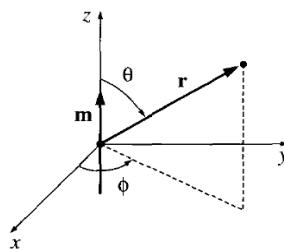


Fig. 5.54 from Griffiths, *Introduction to Electrodynamics*

Potential energy associated with magnetic moment

$$U(\theta) = -\boldsymbol{\mu} \cdot \mathbf{B}$$

Larmor precession

$\tau = \mu \times \mathbf{B}$. This causes centripetal acceleration (think of the poles of the magnet).

$$\frac{dL}{dt} = \tau$$

$$\frac{L \sin \theta d\phi}{dt} = |\mu B \sin \theta|$$

$$\omega_{\text{precession}} = \frac{\mu B}{L}$$

4.5 Lorentz force

The Lorentz Force

$$\mathbf{F} = q(\mathbf{E} + \mathbf{v} \times \mathbf{B})$$

Magnetic force on a current carrying wire of length l and current I in a field \mathbf{B}

$$\mathbf{F} = q\mathbf{v} \times \mathbf{B} = qvB(\hat{v} \times \hat{B}) = q\frac{l}{t}B(\hat{v} \times \hat{B}) = \frac{q}{t}lB(\hat{v} \times \hat{B}) = I�B(\hat{v} \times \hat{B})$$

Larmor radius, cyclotron or gyroradius rather

A charged particle traveling perpendicular to a magnetic field will gyrate in a circle whose area vector is parallel to the direction of travel resulting in a corkscrew-like trajectory. The radius of gyration is determined by considering the Lorentz force on such a particle.

$$\frac{mv_\perp^2}{r_g} = |q|v_\perp B \Rightarrow r_g = \frac{mv_\perp}{|q|B}$$

	$T_g = \frac{2\pi r_g}{v_\perp} = \frac{2\pi}{v_\perp} \frac{mv_\perp}{ q B} = \frac{2\pi m}{ q B}$
Cyclotron frequency	$f_g = \frac{ q B}{2\pi m}$
	$\omega_g = \frac{ q B}{m}$

4.6 Induction

Faraday's/Lenz's law	If an induced current flows, its direction is always such that it will oppose the change which produced it.
	$\varepsilon = -\frac{d\Phi_B}{dt}$

EMF for tightly wound coil of wire	$\varepsilon = -N \frac{d\Phi_B}{dt}$
Inductance	$L \equiv \frac{\Phi_B}{I}$

Voltage, given inductance	$V = \frac{d}{dt}(LI) = L \frac{dI}{dt}$
Energy in an inductor	$W = \int P dt = \int IV dt = \int IL \frac{dI}{dt} dt = L \int I dI = \left[\frac{1}{2} LI^2 \right].$

Energy density in magnetic field	$\eta_B = \frac{W}{V} = \frac{\frac{1}{2} LI^2}{Al} = \frac{1}{2} \frac{\frac{\Phi_B}{I} I^2}{Al} = \frac{1}{2} \frac{BAI}{Al} = \frac{1}{2} \frac{BI}{l}.$ But in a solenoid, we have $\mathbf{B} = \mu n I \hat{z} = \frac{\mu I}{l}$, so $\frac{I}{l} = \frac{B}{\mu}$. Therefore, $\eta_B = \frac{1}{2} \frac{B^2}{\mu}.$
----------------------------------	---

4.7 Maxwell's Equations

Maxwell's equations in vacuum.	$\nabla \cdot \mathbf{E} = \rho/\epsilon_0$ (Gauss)
Differential form	$\nabla \cdot \mathbf{B} = 0$
	$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$ (Faraday)

Maxwell's equations in vacuum.	$\oint_{\partial\Omega} \mathbf{E} \cdot d\mathbf{S} = \frac{1}{\epsilon_0} \iiint_{\Omega} \rho dV$ (Gauss)
Integral form	$\oint_{\partial\Omega} \mathbf{B} \cdot d\mathbf{S} = 0$
	$\oint_{\partial\Sigma} \mathbf{E} \cdot d\mathbf{l} = -\frac{d}{dt} \iint_{\Sigma} \mathbf{B} \cdot d\mathbf{S}$ (Faraday)

Maxwell's equations in matter. Differential form	$\nabla \cdot \mathbf{D} = \rho_{free}$ (Gauss), $D = \epsilon_0 E + P$, $D = \epsilon E$ $\nabla \cdot \mathbf{B} = 0$ $\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$ (Faraday) $\nabla \times \mathbf{H} = \mathbf{J}_{free} + \frac{\partial \mathbf{D}}{\partial t}$ (Ampere), $B = \mu_0(H + M)$, $B = \mu H$
---	---

Maxwell's equations in matter.	$\oint_{\partial\Omega} \mathbf{D} \cdot d\mathbf{S} = \iiint_{\Omega} \rho_{free} dV$ (Gauss)
Integral form	$\oint_{\partial\Omega} \mathbf{B} \cdot d\mathbf{S} = 0$
	$\oint_{\partial\Sigma} \mathbf{E} \cdot d\mathbf{l} = -\frac{d}{dt} \iint_{\Sigma} \mathbf{B} \cdot d\mathbf{S}$ (Faraday)

	(i) $D_1^\perp - D_2^\perp = \sigma_f$
Boundary conditions	(ii) $B_1^\perp - B_2^\perp = 0$
	(iii) $\mathbf{E}_1^{\parallel\parallel} - \mathbf{E}_2^{\parallel\parallel} = 0$
	(iv) $\mathbf{H}_1^{\parallel\parallel} - \mathbf{H}_2^{\parallel\parallel} = \mathbf{K}_f \times \hat{\mathbf{n}}$

4.8 EM Waves

The whole EM wave spiel	<p>For E-wave, curl Faraday.</p> $\nabla \times \nabla \times \mathbf{E} = \nabla \times \left(-\frac{\partial \mathbf{B}}{\partial t} \right)$ $\nabla(\nabla \cdot \mathbf{E}) - \nabla^2 \mathbf{E} = -\frac{\partial}{\partial t}(\nabla \times \mathbf{B})$ <p>Substitute Gaussian and Amperian terms.</p> $\nabla\left(\frac{\rho}{\epsilon_0}\right) - \nabla^2 \mathbf{E} = -\frac{\partial}{\partial t}\left(\mu_0 \mathbf{J} + \epsilon_0 \mu_0 \frac{\partial \mathbf{E}}{\partial t}\right)$ $\rho = 0 \text{ and } \mathbf{J} = 0, \text{ therefore } \boxed{\nabla^2 \mathbf{E} = \epsilon_0 \mu_0 \frac{\partial^2 \mathbf{E}}{\partial t^2} = \frac{1}{c^2} \frac{\partial^2 \mathbf{E}}{\partial t^2}}$
	For B-wave, similar, but start with curling Ampere's Law.

speed of light in vacuum	$c = \frac{1}{\sqrt{\mu_0 \epsilon_0}} = 3 \times 10^8 \text{ m/s}$
speed of light in medium, $v =$	$\frac{1}{\sqrt{\mu \epsilon}} = \frac{c}{\sqrt{\epsilon_R \mu_R}}$, where $\mu = \mu_R \mu_0$ and likewise for ϵ
Plane wave solutions	$E = E_m \sin(kx - \omega t)$ and $B = B_m \sin(kx - \omega t)$

Relation of E_m and B_m magnitudes	Plug plane wave solutions into Ampere or Faraday and find that $\frac{E_m}{B_m} = \frac{\omega}{k} = c$
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Energy in the E and B fields for plane wave	Recall the energy densities $\eta_E = \frac{1}{2}\epsilon_0 E^2$ and $\eta_B = \frac{1}{2}\frac{B^2}{\mu_0}$. $\frac{\eta_E}{\eta_B} = \frac{\mu_0 \epsilon_0 E^2}{B^2} = \frac{E^2}{c^2 B^2}.$ From above, $E = cB$. Therefore $\eta_E = \eta_B$ for an EM wave; the energy carried in the E-field is equal to the energy carried in the B-field.
--	--

Poynting Vector	$\mathbf{S} = \frac{1}{\mu_0} \mathbf{E} \times \mathbf{B}$ $S = \frac{1}{\mu_0} EB = \frac{c}{\mu_0} B^2 = \frac{1}{c\mu_0} E^2$
-----------------	--

Average intensity	$\bar{S} = \frac{1}{c\mu_0} E_m^2 \overline{\sin^2(kx - \omega t)} = \frac{1}{c\mu_0} \frac{E_m^2}{2}$
Power radiated by a non relativistic point charge as it accelerates	Larmor formula $P = \frac{2}{3} \frac{q^2 a^2}{4\pi\epsilon_0 c^3} = \frac{q^2 a^2}{6\pi\epsilon_0 c^3}.$

Application of boundary conditions
for reflection of a wave incident to a
conducting surface

(i) Incident wave, $E^\perp = 0$ on both sides $\rightarrow \sigma_f = 0$

(ii) Automatically satisfied because $B^\perp = 0$

$$(iii) \tilde{E}_{0I} + \tilde{E}_{0R} = \tilde{E}_{0T}$$

(iv)

and (iv) (with $\mathbf{K}_f = 0$) says

$$\frac{1}{\mu_1 v_1} (\tilde{E}_{0I} - \tilde{E}_{0R}) - \frac{\tilde{k}_2}{\mu_2 \omega} \tilde{E}_{0T} = 0, \quad (9.144)$$

or

$$\tilde{E}_{0I} - \tilde{E}_{0R} = \tilde{\beta} \tilde{E}_{0T}, \quad (9.145)$$

where

$$\tilde{\beta} = \frac{\mu_1 v_1}{\mu_2 \omega} \tilde{k}_2. \quad (9.146)$$

It follows that

$$\tilde{E}_{0R} = \left(\frac{1 - \tilde{\beta}}{1 + \tilde{\beta}} \right) \tilde{E}_{0I}, \quad \tilde{E}_{0T} = \left(\frac{2}{1 + \tilde{\beta}} \right) \tilde{E}_{0I}. \quad (9.147)$$

These results are formally identical to the ones that apply at the boundary between *nonconductors* (Eq. 9.82), but the resemblance is deceptive since $\tilde{\beta}$ is now a complex number.

For a *perfect conductor* ($\sigma = \infty$), $k_2 = \infty$ (Eq. 9.126), so $\tilde{\beta}$ is infinite, and

$$\tilde{E}_{0R} = -\tilde{E}_{0I}, \quad \tilde{E}_{0T} = 0. \quad (9.148)$$

Wave is reflected with π phase shift. Also, from the last equation above, $\tilde{B}_{0R} = -\tilde{B}_{0I}$, $\tilde{B}_{0T} = 0$. From the first expression it follows that the magnetic field at the surface is twice the magnitude of either incident or reflected.

Introduction to Electrodynamics, Griffiths

Oscillating electric dipole	$q(t) = q_0 \cos(\omega t) \quad \mathbf{p}(t) = p_0 \cos(\omega t) \hat{\mathbf{z}} \quad p_0 \equiv q_0 d.$
-----------------------------	---

Approximations for oscillating electric dipole far from the source (such that it can be approximated as a plane wave)

Approximation 1: $d \ll r$.

Approximation 2: $d \ll \frac{c}{\omega} = \frac{\lambda}{2\pi}$.

Approximation 3: $r \gg \frac{c}{\omega} = \frac{\lambda}{2\pi}$.

Electric potential far from oscillating electric dipole

$$V(r, \theta, t) = -\frac{p_0 \omega}{4\pi \epsilon_0 c} \left(\frac{\cos \theta}{r} \right) \sin[\omega(t - r/c)]$$

Magnetic vector potential far from oscillating electric dipole on the z-axis

$$\mathbf{A}(r, \theta, t) = -\frac{\mu_0 p_0 \omega}{4\pi r} \sin[\omega(t - r/c)] \hat{\mathbf{z}}$$

Electric field far from oscillating electric dipole

$$\mathbf{E} = -\nabla V - \frac{\partial \mathbf{A}}{\partial t} = -\frac{\mu_0 p_0 \omega^2}{4\pi} \left(\frac{\sin \theta}{r} \right) \cos[\omega(t - r/c)] \hat{\theta}$$

Magnetic field far from oscillating electric dipole

$$\mathbf{B} = \nabla \times \mathbf{A} = -\frac{\mu_0 p_0 \omega^2}{4\pi c} \left(\frac{\sin \theta}{r} \right) \cos[\omega(t - r/c)] \hat{\phi}$$

4.9 AC circuits

Second-order DE for RLC circuits

Kirchoff's voltage law:

$$V_R + V_L + V_C = V(t)$$

$$RI(t) + L \frac{dI}{dt} + \frac{1}{C} \int_{-\infty}^{\tau=1} I(\tau) d\tau = V(t)$$

In case of $\frac{dV}{dt} = 0$, we differentiate and rearrange

$$L \frac{d^2 I(t)}{dt^2} + R \frac{dI(t)}{dt} + \frac{1}{C} I(t) = 0$$

Resonant frequency RLC

$$\omega_0 = \frac{1}{\sqrt{LC}}$$

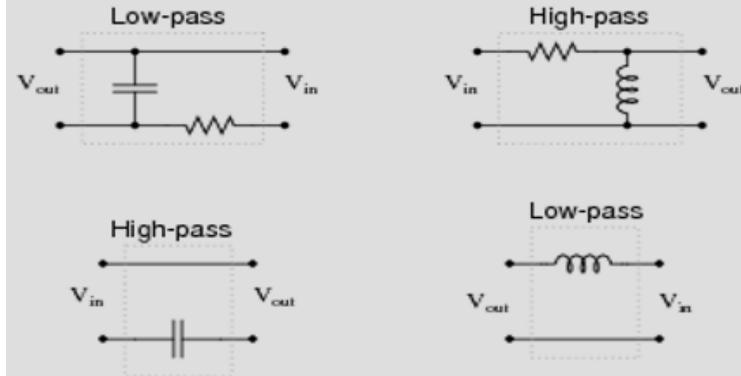
Impedance matching

To maximize power transfer, $Z_S = Z_L^*$, where Z_S is the complex source impedance, and Z_L is the load impedance.

To minimize reflection in a transmission line where Z_S is the characteristic impedance of the line, $Z_S = Z_L$.

High- and Low-pass filters.

One of each with a capacitor. One of each with an inductor.



<http://www.learningelectronics.net/images/quiz/00615x02.png>

4.10 Magnetic and electric fields in matter

Dipole moment	$\mathbf{p} = \alpha \mathbf{E}$, where α is atomic polarizability.
Polarization	$\mathbf{P} = \mathbf{p} / \text{unit volume}$
Bound charge	$\sigma_b = \mathbf{P} \cdot \hat{n}, \quad \rho_b = -\nabla \cdot \mathbf{P}$
Electric displacement field	$\mathbf{D} = \epsilon_0 \mathbf{E} + \mathbf{P} = \epsilon \mathbf{E}$

Gauss's Law in L.D.

$$\epsilon_0 \nabla \cdot \mathbf{E} = \rho = \rho_b + \rho_f = -\nabla \cdot \mathbf{P} + \rho_f$$

$$\nabla \cdot (\epsilon_0 \mathbf{E} + \mathbf{P}) = \nabla \cdot \mathbf{D} = \rho_f$$

Polarization in linear dielectrics	$\mathbf{P} = \epsilon_0 \chi_e \mathbf{E}$, where χ_e is the electric susceptibility.
\mathbf{D} in terms of χ_e	$\mathbf{D} = \epsilon_0(1 + \chi_e)\mathbf{E} = \epsilon\mathbf{E}$, where permittivity $\epsilon \equiv \epsilon_0(1 + \chi_e)$
Dielectric constant	$\epsilon_r \equiv 1 + \chi_e \equiv \epsilon/\epsilon_0$, (a.k.a. relative permittivity)

\mathbf{E} in dielectric filled space	$\frac{1}{\epsilon_r} \mathbf{E}_{vac}$
E-field in capacitor with dielectric	$E = \Delta V/d$
Capacitance with linear dielectric	$C = \epsilon_r C_{vac}$

Boundary conditions	$\epsilon_a E_a^\perp - \epsilon_b E_b^\perp = \sigma_f$ $V(a) = V(b)$
---------------------	---

Magnetic dipole moment	$\mathbf{m} = I\mathbf{a}$
Magnetization	$\mathbf{M} \equiv \mathbf{m}/\text{unit volume}$

H field	$\mathbf{H} \equiv \frac{1}{\mu_0} \mathbf{B} - \mathbf{M}$ $\nabla \times \mathbf{H} = \mathbf{J}_f$ $\nabla \cdot \mathbf{H} = -\nabla \cdot \mathbf{M}$ Generally, $\mathbf{H} \parallel \mathbf{B} \parallel \mathbf{M}$.
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Linear magnetic materials	$\mathbf{M} = \chi_m \mathbf{H}$ where χ_m = magnetic susceptibility.
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B-field in linear mag. material

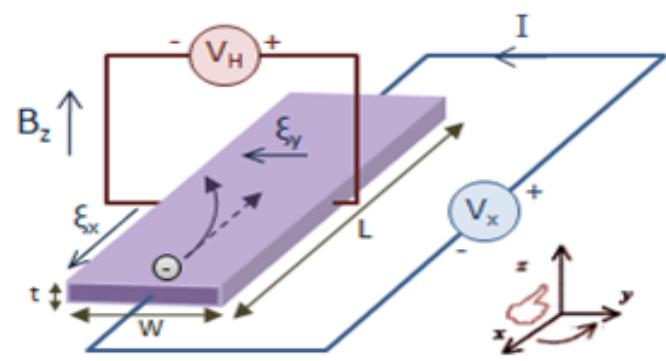
$$\mathbf{B} = \mu_0(1 + \chi_m)\mathbf{H} = \mu\mathbf{H}$$

where $\mu \equiv \mu_0(1 + \chi_m)$ = magnetic permeability.

$$B_{material} = \frac{\mu}{\mu_0} B_{vacuum} = \mu_r B_{vacuum}$$

where $\mu_r \equiv \mu/\mu_0$ = relative permeability.

Hall effect



https://en.wikipedia.org/wiki/Hall_effect

Hall voltage for electrons

$$V_H = v_x B_z w \text{ and } I_x = ntw(-vx)(-e)$$

$$V_H = \frac{I_x B_z}{n t e}$$

Hall coefficient for electrons

$$R_H = \frac{E_y}{j_x B} = \frac{V_H t}{IB} = -\frac{1}{ne}$$

$$[R_H] = \text{m}^3/\text{C}$$

5 Optics and Wave Phenomena

(such as wave properties, superposition, interference, diffraction, geometrical optics, polarization, Doppler effect)

5.1 Wave properties

The wave equation	$\frac{\partial^2 u}{\partial t^2} = c^2 \nabla^2 u.$
General solution	$u(x, t) = A \sin(kx - \omega t) + B \cos(dx - \omega t).$
c	$c = \omega/k$

General wave function $f(x, t, k, \omega) =$	$A \sin(kx \pm \omega t)$
wave function $f(x, t, \lambda, T) =$	$A \sin 2\pi(\frac{x}{\lambda} \pm \frac{t}{T})$

Amplitude	A
Period	T

Direction	The sin function preserves a sine wave form as it traverses x and t . The argument must remain constant: $kx \pm \omega t = \text{constant}$. In order to preserve this, must x increase or decrease as t increases? Depends on the \pm
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$k =$	$k = 2\pi/\lambda$
$\omega =$	$= \omega = 2\pi f = 2\pi/T$

Phase velocity	$v_{ph} = \omega/k = \lambda/T$
Group velocity	$v_g = \frac{\partial \omega}{\partial k}$

5.2 Waves on a string

Characteristic impedance of a string	$Z = \mu c$ where μ is the mass per unit length & c is the wave speed.
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If you want to be super in the know about transverse waves on strings, check this out:

<http://www.people.fas.harvard.edu/~djmorin/waves/transverse.pdf>

5.3 Interference

Michelson Interferometer equation relating distance (moved perhaps), number of fringes, and wavelength.	$2d = m\lambda$, where m is the number of fringes. The factor of 2 comes from the fact that the light crosses the same distance twice. See GR9277 #96 for an interesting problem.
---	--

5.3.1 Two waves at the same speed

Sine waves in same/opp. direction	Same \rightarrow interference. Opposite \rightarrow standing.
Constructive if	$\phi_1 = \phi_2$
Destructive if	$ \phi_1 - \phi_2 = \pi$

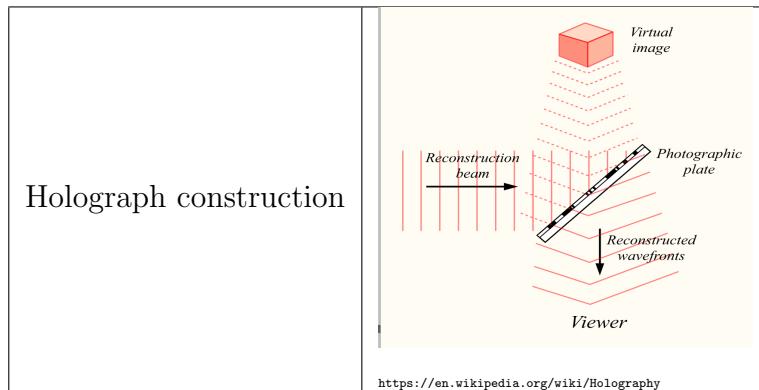
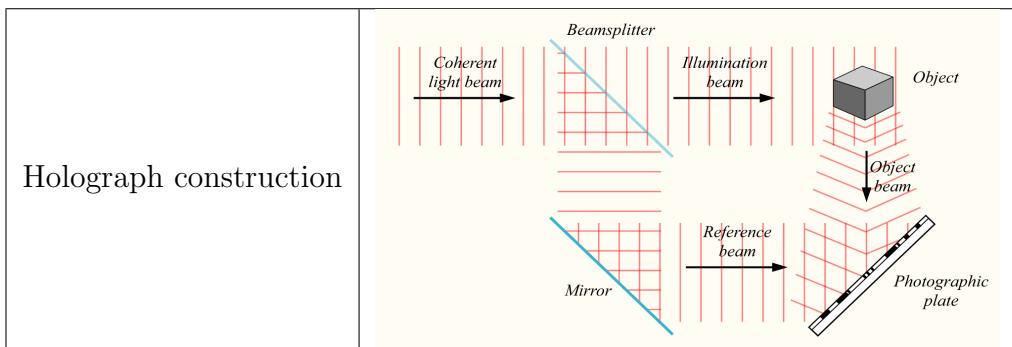
Standing wave equation	$\begin{aligned} u(x, t) &= A \sin(kx - \omega t) + A \sin(kx + \omega t) \\ &= 2A \sin(kx) \cos(\omega t). \end{aligned}$
-------------------------------	--

Condition for nodes	$u(x, t) = 0 \Rightarrow kx = n\pi$
Condition for antinodes	$u(x, t) = \text{max.} = 2A \Rightarrow kx = \pi/2 + n\pi$

Condition for beats	Same direction, different frequency
Equation for beats	$u(x, t) = A \sin(k_1 x - \omega_1 t) + A \sin(k_2 x + \omega_2 t)$ $= 2A \sin\left[\frac{(k_1 + k_2)x}{2} - \frac{(\omega_1 + \omega_2)t}{2}\right] \cos\left[\frac{(k_1 - k_2)x}{2} - \frac{(\omega_1 - \omega_2)t}{2}\right]$
Beat frequency	$\omega_{beat} = \omega_1 - \omega_2 $

5.3.2 Holography

Holographs	encode the light field as an interference pattern, recording phase and amplitude.
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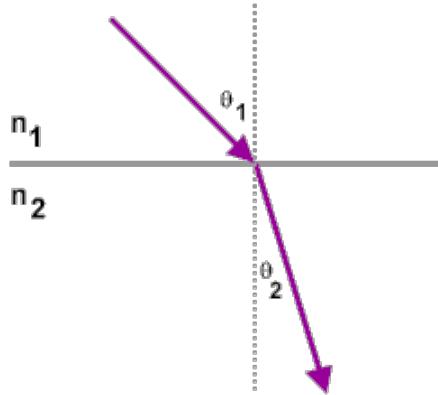


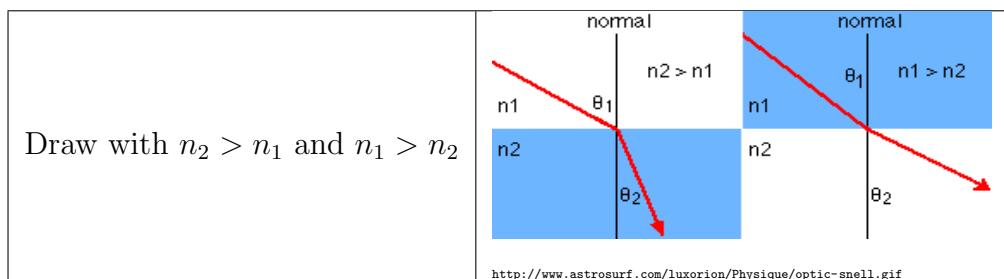
Figure 2: <http://vignette2.wikia.nocookie.net/schools/images/4/4c/097b8946-3139-4ea1-8573-05128eb66b29.gif/revision/latest?cb=20060612224020>

5.4 Refraction

Index of refraction	$n \equiv \frac{c}{v}$
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I.o.R. and wavelengths	$n = \frac{\lambda_0}{\lambda}$ <p>Where λ_0 is the wavelength in vacuum and λ is the wavelength in medium.</p>
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n can be < 1 if	the phase velocity is greater than c.
Snell's law	$n_i \sin \theta_i = n_f \sin \theta_f$



Condition for total internal reflection	$\frac{n_i \sin \theta_i}{n_f} > 1$
Critical angle	$\theta_c = \theta_i = \arcsin \left(\frac{n_2}{n_1} \right)$

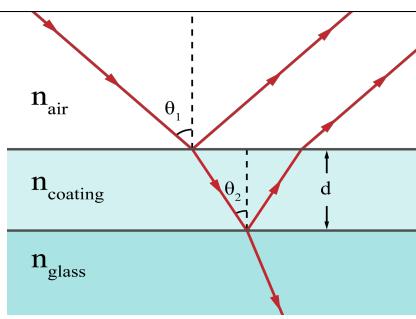
Dispersion and general trend	$n(\lambda)$, as $\lambda \uparrow, n \downarrow$
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Fraction of light reflected R	$R = \left(\frac{n_1 - n_2}{n_1 + n_2} \right)^2$
Fraction of light transmitted	$T = 1 - R = \frac{4n_1 n_2}{(n_1 - n_2)^2}$

5.4.1 Thin Films

<p>Soap/oil film constructive and destructive interference</p>	<p>The diagram illustrates light rays interacting with a thin film of thickness d. The film has refractive index n_{film} and is situated between two layers of air with refractive index n_{air}. A red ray enters from the left at an angle θ_1 relative to the normal. It reflects off the top surface at an angle θ_1' and the bottom surface at an angle θ_2'. A second red ray enters from the right at an angle θ_2 relative to the normal, reflects off the bottom surface at an angle θ_2', and reflects back up at an angle θ_2. The path difference between these two rays is $2d \cos(\theta_2')$. The diagram is labeled "interference" and includes a URL: https://en.wikipedia.org/wiki/Thin-film_interference</p>
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Coating film constructive
and destructive interference



https://en.wikipedia.org/wiki/Thin-film_interference

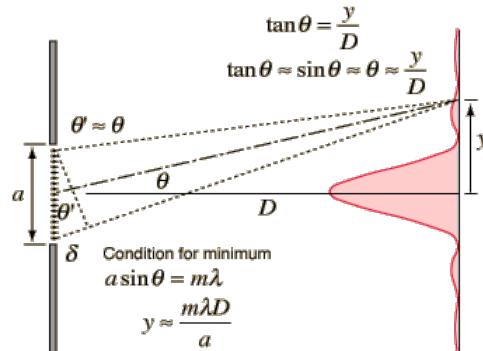
$$n_{glass} > n_{film} > n_{air}$$

$$\text{Constructive: } 2n_{coating}d \cos(\theta_2) = m\lambda$$

$$\text{Destructive: } 2n_{coating}d \cos(\theta_2) = (m - \frac{1}{2})\lambda$$

5.5 Diffraction

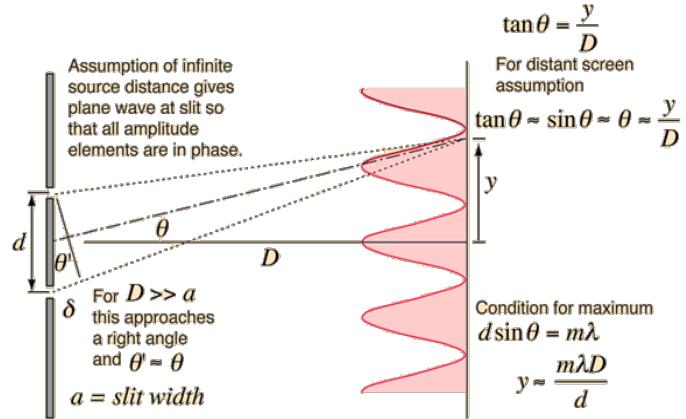
Single slit diffraction



Fraunhofer Single Slit

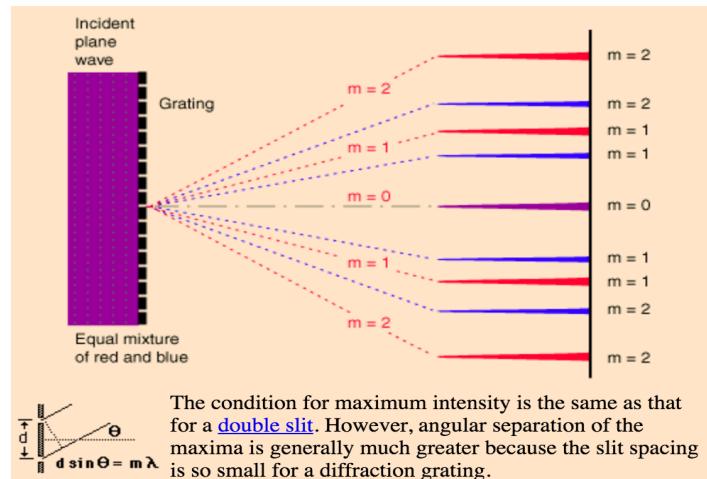
<http://hyperphysics.phy-astr.gsu.edu/hbase/phyopt/imgpho/sinslit.gif>

Double slit interference



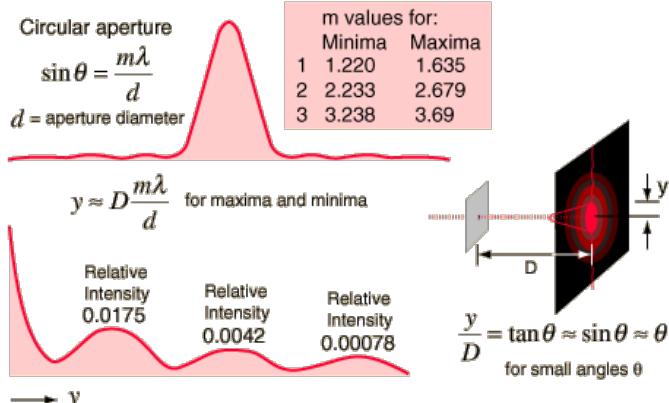
<http://hyperphysics.phy-astr.gsu.edu/hbase/phyopt/imgpho/doubsli.gif>

Diffraction grating



<http://hyperphysics.phy-astr.gsu.edu/hbase/phyopt/grating.html>

Circular Aperture diffraction



Use for resolving power of telescopes.

<http://hyperphysics.phy-astr.gsu.edu/hbase/phyopt/cirapp2.html>

5.6 Geometrical Optics

Optical Path Length	$OPL = \int_a^b n(s) ds$
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5.6.1 Mirrors

For concave and convex spherical mirrors, the focal length is	... half the radius of curvature of the mirror
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Spherical mirror eqn.

$$\frac{1}{f} = \frac{1}{d_O} + \frac{1}{d_i}$$

Spherical magnification

$$M = \frac{h_i}{h_O} = -\frac{d_i}{d_O}$$

d_O and object real/virtual

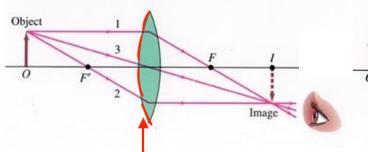
$d_O > 0 \rightarrow$ object in front (real).

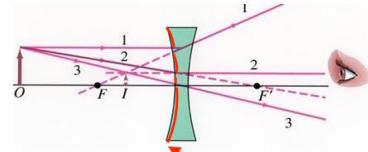
object in back (virtual) $\leftarrow d_O < 0$

d_i and image real/virtual	$d_i > 0 \rightarrow$ image in front (real). image in back (virtual) $\leftarrow d_i < 0$
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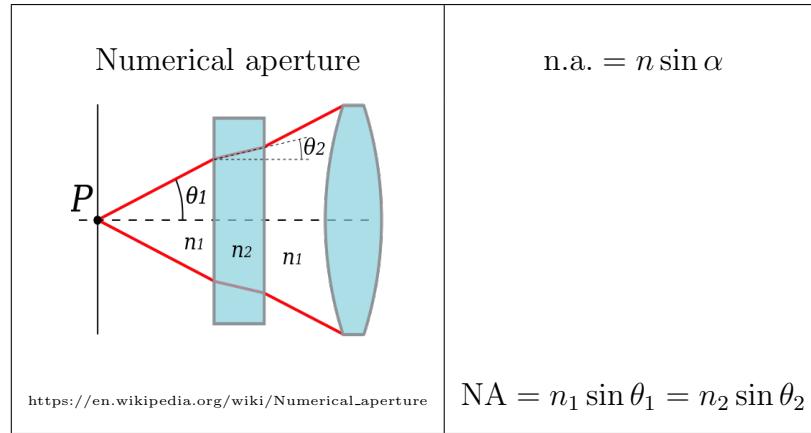
f and concavexity	$f > 0 \rightarrow$ concave.convex $\leftarrow f < 0$
Magnification image orientation	$M > 0 \rightarrow$ upright.inverted $\leftarrow M < 0$

5.6.2 Lenses

Converging lens - mirror Relations for real, virtual, larger, smaller	 <ul style="list-style-type: none"> • Converging lens, concave mirror • $d_o > 2f$ – (real, inverted), smaller • $2f > d_o > f$ – (real, inverted), larger • $d_o < f$ – (virtual, upright), larger
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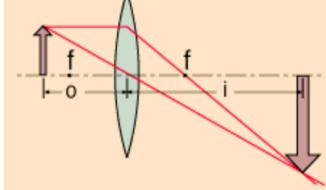
Diverging lens - mirror Relations for real, virtual, larger, smaller	 <ul style="list-style-type: none"> • Diverging lens, convex mirror • Image is always (virtual, upright), smaller. <p>http://images.slideplayer.com/13/3858792/slides/slide_13.jpg</p>
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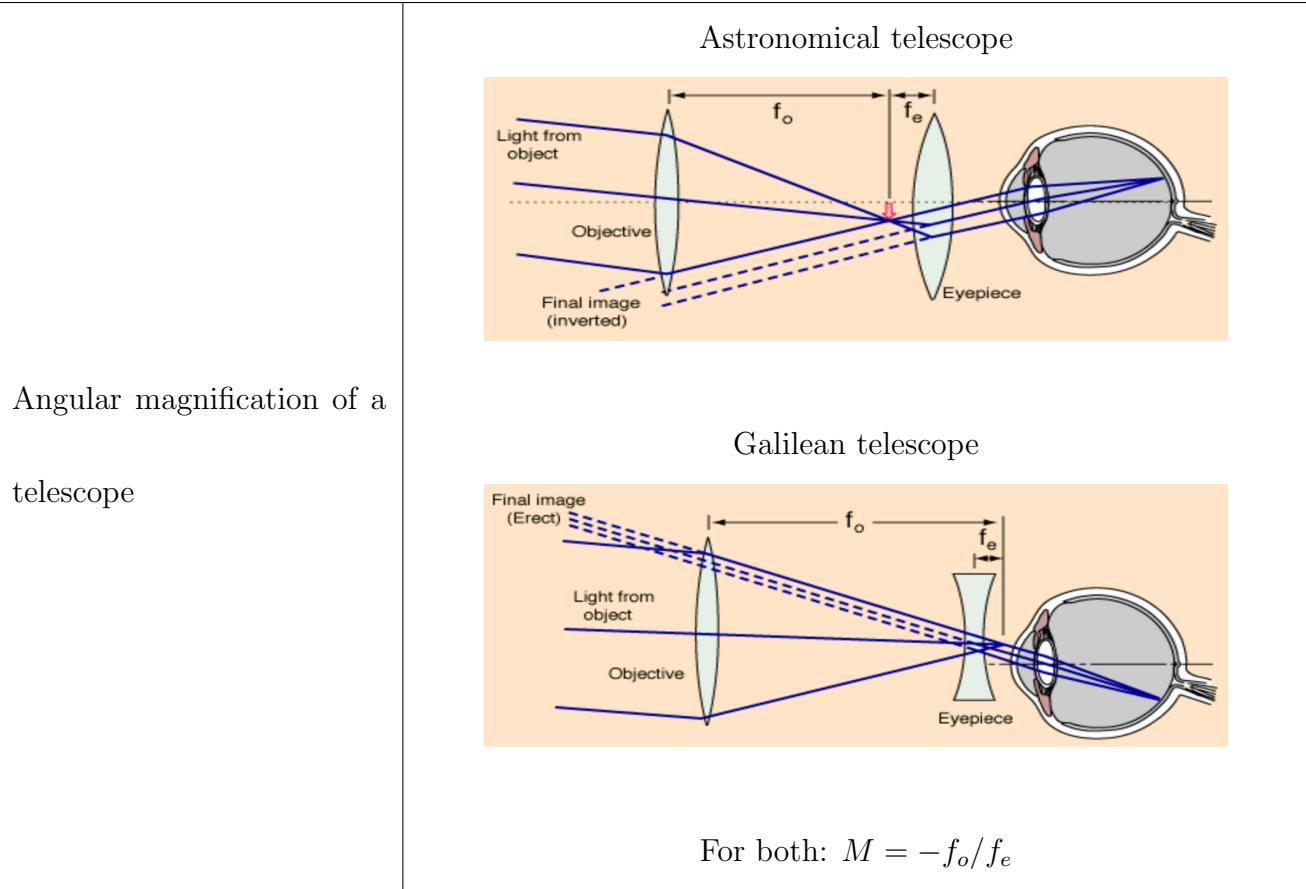
f-number or relative aperture	$f\text{-number} = N = \frac{f}{D}$
	where f if focal length, D is diameter
Increased f	increased light-gathering power.



Relate NA to N (i.e. f-num)	$\text{NA}_i = n \sin \theta = n \sin[\arctan(\frac{D}{2f})] \approx \frac{nD}{2f}$ $\Rightarrow N \approx \frac{1}{2\text{NA}_i} \text{ assuming } n = 1.$
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Formula for focal length (spherical)	$\frac{1}{f} = (n - 1) \left(\frac{1}{R_1} - \frac{1}{R_2} \right),$ <p>where R_1 is the radius of curvature for the lens nearest the object and R_2 is the r.o.c. for the farther</p>
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Thin lens equation (a.k.a. 'Lens-maker's formula')	$\frac{1}{f} = \frac{1}{d_O} + \frac{1}{d_i}$
d_O and object real/virtual	$d_O > 0 \rightarrow$ object in front (real). $\text{object in back (virtual)} \leftarrow d_O < 0$
d_i and image real/virtual	$d_i > 0 \rightarrow$ image in back (real). $\text{image in } \mathbf{\text{front}} \text{ (virtual)} \leftarrow d_i < 0$
R_n and center of lens	$R_n > 0$ if center of lens in back $R_n < 0$ if center of lens in front
f and concavexity	$f > 0 \rightarrow$ concave.convex $\leftarrow f < 0$
resulting f for two close lenses	$\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2}$
Magnification	 $\frac{1}{o} + \frac{1}{i} = \frac{1}{f}$ Lens equation Linear magnification: $M = \frac{-i}{o} = \frac{h'}{h}$ <p style="text-align: center;">http://hyperphysics.phy-astr.gsu.edu/hbase/geoopt/lensdet.html</p>



5.7 Polarization

Malus' law	First, consider electric field vector $I_{transmitted} = I_0 \cos^2 \theta_i$
Intensity of polarized from unpolarized	$I = I_0 \int_0^\pi \cos^2 \theta d\theta = \frac{I_0}{2}$

Brewster's Angle

First, read Ch. 9 of Griffith's *EM*. It falls out of the Fresnel

equations that there is 0 reflectance when

$$\theta_1 + \theta_2 = 90^\circ$$

Use Snell's law:

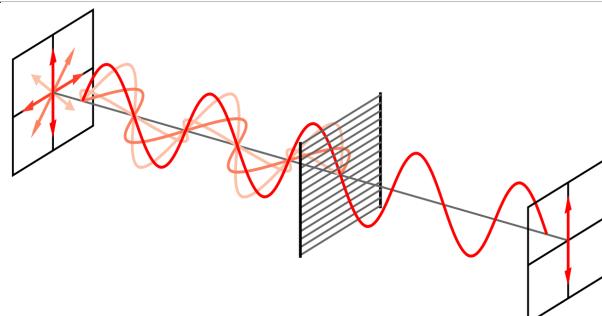
$$n_1 \sin \theta_1 = n_2 \sin \theta_2.$$

Let θ_B be the Brewster angle

$$n_1 \sin \theta_B = n_2 \sin(90^\circ - \theta_B) = n_2 \cos \theta_B,$$

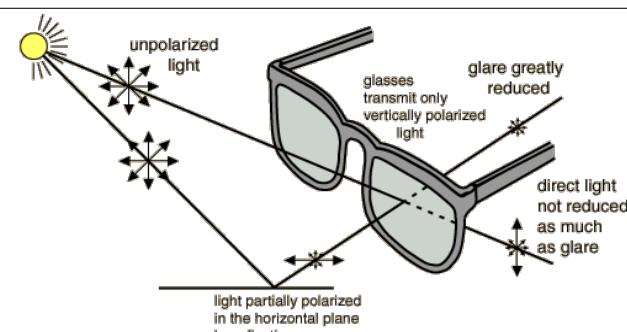
Finally, $\boxed{\theta_B = \arctan\left(\frac{n_2}{n_1}\right)}$

Wire grid polarizer

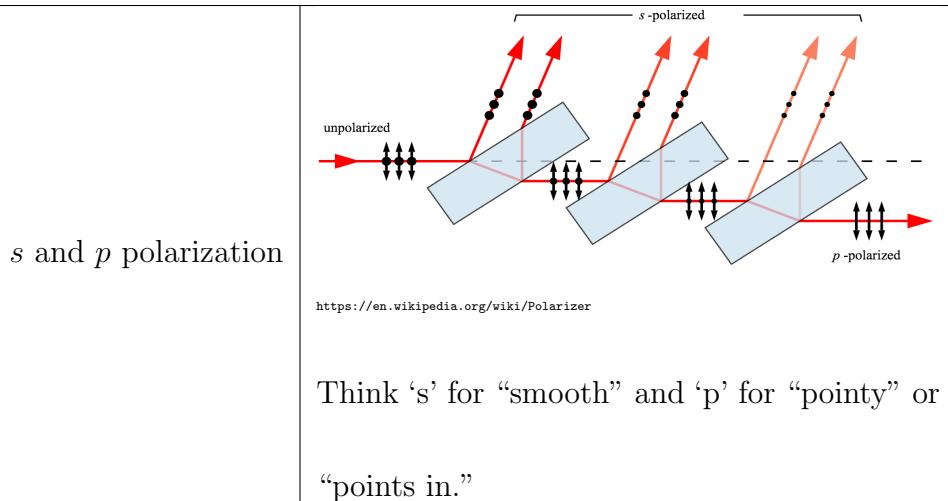


<https://en.wikipedia.org/wiki/Polarizer>

Polarized Sunglasses



<http://assets.openstudy.com/updates/attachments/4f175a76e4b0aeb795f5672a-jamesj-1326931194153-sunglass.gif>



5.8 Doppler effect

<p>Derivation Wavelengths and Frequencies</p>	<p>c = velocity of waves in medium, v_s = velocity of source relative to medium, v_o = velocity of observer relative to medium.</p> <p>Waves travel from source to observer in the positive whatever direction: O....source))))waves))))observer.....>+x</p> <p>Observer moving, source still: $c + v_o = \frac{\lambda_s}{T}$</p> <p>Source moving, observer still: $c - v_s = \frac{\lambda_o}{T}$</p> $T = \frac{\lambda_o}{c - v_s} = \frac{\lambda_s}{c + v_o}$ <div style="border: 1px solid black; padding: 5px; display: inline-block;"> $\lambda_o = \frac{c - v_s}{c + v_o} \lambda_s$ </div> <div style="border: 1px solid black; padding: 5px; display: inline-block;"> $f_o = \frac{c + v_o}{c - v_s} f_s$ </div>
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6 Thermodynamics and Statistical Mechanics

(such as the laws of thermodynamics, thermodynamic processes, equations of state, ideal gases, kinetic theory, ensembles, statistical concepts and calculation of thermodynamic quantities, thermal expansion and heat transfer)

6.1 Laws of Thermodynamics

Zeroth Law	If two systems are in thermal equilibrium independently with a third system, they must be in thermal equilibrium with each other. This law helps define the notion of temperature.
First Law	When energy passes, as work, as heat, or with matter, into or out from a system, its internal energy changes in accord with the law of conservation of energy. Equivalently, perpetual motion machines of the first kind are impossible. $\Delta U = Q + W$

Second Law	In a natural thermodynamic process, the sum of the entropies of the interacting thermodynamic systems increases. Equivalently, perpetual motion machines of the second kind are impossible.
Third Law	The entropy of a system approaches a constant value as the temperature approaches absolute zero. With the exception of non-crystalline solids (glasses) the entropy of a system at absolute zero is typically close to zero, and is equal to the logarithm of the multiplicity of the quantum ground states.

Fundamental Assumption	In an isolated system, all accessible microstates are equally probable.
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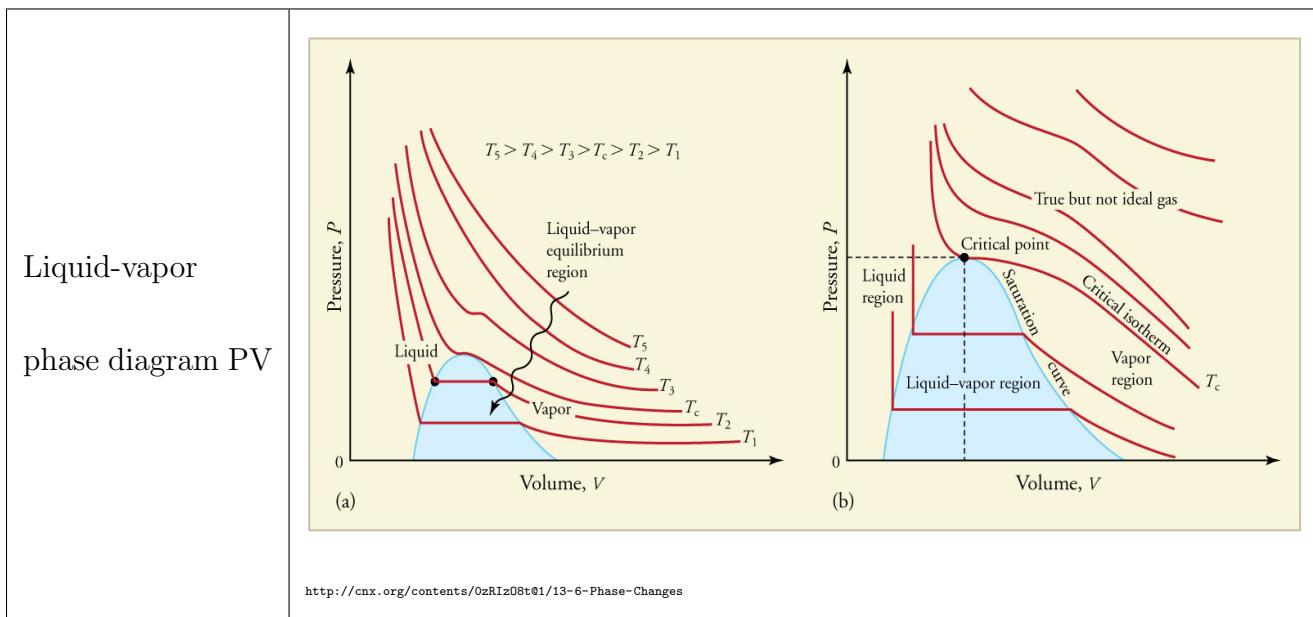
6.2 Thermodynamic Processes

W for an ideal gas	$W = - \int_{V_i}^{V_f} p dV$
W in a PV diagram is area under the curve.

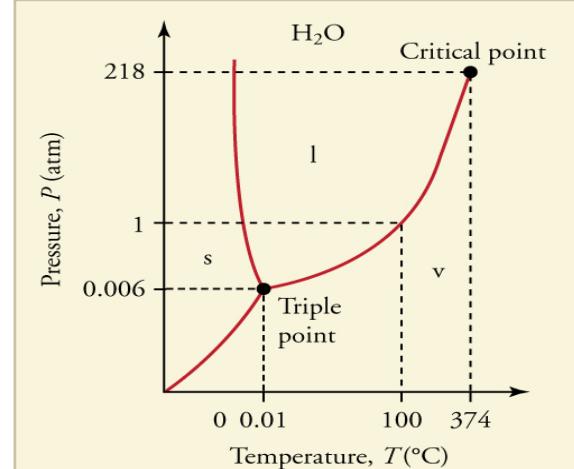
Isothermal	Constant temperature!
$constant =$	$PV = constant$
$W =$	$W = nRT \ln \left[\frac{V_f}{V_i} \right]$
Internal energy does not change. (Equipartition Theorem)

Adiabatic	No heat gained or lost by the system.
$constant =$	$PV^\gamma = constant$
$\gamma =$	$\frac{C_P}{C_V} = \frac{f+2}{f}$
W	$W = PV^\gamma \frac{V_f^{1-\gamma} - V_i^{1-\gamma}}{1-\gamma}$

Adiabat or isotherm steeper in PV ?	Adiabatic
$W, Q, \Delta U$ for adiabatic free expansion	$W = 0, Q = 0, \Delta U = 0$
Critical point	$\left(\frac{\partial p}{\partial V} \right)_T = \left(\frac{\partial^2 p}{\partial V^2} \right)_T = 0$



H_2O phase diagram PT



<http://cnx.org/contents/0zRIZ08t@1/13-6-Phase-Changes>

Fourier's law

$$\vec{q} = -k\nabla T, \text{ where}$$

\vec{q} is the local heat flux density, $\text{W}\cdot\text{m}^{-2}$,

k is the material's conductivity, $\text{W}\cdot\text{m}^{-1}\text{K}^{-1}$

∇T is the temperature gradient, $\text{K}\cdot\text{m}^{-1}$.

6.3 Thermal expansion and heat transfer

Heat capacity	$C = \frac{Q}{\Delta T}$
Specific heat capacity	$c = \frac{C}{m} \rightarrow Q = mc\Delta T$
Latent heat	$L = Q/m$

6.4 Engines & Refrigerators

CW and CCW	Engines: CW. Fridges: CCW. Think at const. V what happens, $P \uparrow\downarrow$?
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Engine efficiency	$\eta = \frac{W}{Q_H} = 1 - \frac{Q_C}{Q_H}$
Refrigerator Coefficient of performance	$CP_R = \eta_R = \frac{Q_C}{W} = \frac{Q_C}{Q_H - Q_C}$
Heat pump efficiency	$CP_H = \eta_H = \frac{Q_H}{W} = \frac{Q_H}{Q_H - Q_C}$

Carnot cycle is ...	Adiabat, Isotherm, Adiabat, Isotherm
Carnot entropy	$dS = \frac{dQ}{T} = 0$ (Carnot)

Heat entering system along isotherm	$Q = nRT \ln \left(\frac{V_{end}}{V_{start}} \right)$
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Carnot engine efficiency	$\eta = 1 - \frac{T_C}{T_H}$ (Carnot)
Carnot refrigerator efficiency	$CP_R = \eta_R = \frac{T_C}{T_H - T_C}$ (Carnot)
Heat pump efficiency	$CP_H = \eta_H = \frac{T_H}{T_H - T_C}$ (Carnot)

6.5 Equations of state

Thermodynamic identity	$dU = T dS - p dV + \mu dN$
Change in entropy	$\Delta S \geq \frac{Q}{T}$, const. T , no spontaneous ΔS

Helmholtz Free Energy	$F = U - TS$ @ const. T
	$F_{sys} = \min$ when $S_{universe} = \max.$
Gibb's Free Energy	$G = U - TS + PV$

Definition of Entropy	$S = k \ln \Omega$
Definition of Temperature	$\frac{1}{T} = \left(\frac{\partial S}{\partial U} \right)_{V,N}$

Chemical potential	$\mu = -T \left(\frac{\partial S}{\partial N} \right)_{U,V}$
	also $\mu = \left(\frac{\partial U}{\partial N} \right)_{S,V}$
	also $\mu = \left(\frac{\partial F}{\partial N} \right)_{T,V}$

For systems A and B in diffusive contact with $\mu_A > \mu_B$, which way do particles flow?	$A \rightarrow B$
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Heat capacity, constant volume	$C_V = \left(\frac{\partial E}{\partial T} \right)_V$
Heat capacity, constant pressure	$C_P = \left(\frac{dE}{dT} \right)_P + P \left(\frac{dV}{dT} \right)_P$

6.6 Ideal gases

Ideal gas law	$PV = nRT = NkT$
Relation between n, N, k and R	nR is the number of moles times the Gas Constant. Nk is the number of molecules times the Boltzmann Constant.

$k =$	$1.38 \times 10^{-23} \text{ J/K}$
$R =$	$kN_A = 1.38 \times 10^{-23} \text{ J/K} \times 6.02 \times 10^{23} \text{ mol}^{-1} = 8.31 \text{ J mol}^{-1} \text{ K}^{-1}$

Multiplicity of an IG	$\Omega(N, V, U) = f(N) V^N U^{fN/2}$
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Standard Temperature and Pressure	Standard Temperature = $0^\circ \text{ C} = 273.15 \text{ K}$ Standard Pressure 1 Atm = 101.3 kPa . 1 mole of gas occupies 22.4 L.
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6.7 Kinetic theory

Equipartition Theorem	$U = \frac{f}{2} N k T$
v_{rms}	$= \sqrt{\bar{v}^2} = \sqrt{\frac{v_1^2 + v_2^2 + \dots + v_N^2}{N}}$
$\frac{1}{2} m \bar{v}^2 =$	$\frac{3}{2} k T$

Mean free path	$l = \frac{1}{n\sigma} \text{ where } n \text{ is number density } \frac{N}{V}.$
L_{rms}	$\sqrt{N} l$
Distance between particles	$d = \left(\frac{V}{N}\right)^{1/3}$

Degrees of freedom	<p>Monoatomic ideal gas [edit]</p> <p>For a monoatomic ideal gas of N particles you have $6N$ degrees of freedom (all are translational). In the classical limit, a micro-state is given by the positions and the impusions of all particles. In 3D, that's 6 d.o.f.s for each atom, or $6N$ in total.</p> <p>For each atom, the three degree of freedom corresponding to position have no energy attached to them (this is an ideal gas). They do not contribute.</p> <p>For each atom, the three degrees of freedom corresponding to momentum have a quadratic energy attached to them: that's the kinetic energy $p^2/2m$.</p> <p>We thus have $3N$ quadratic d.o.f.s.</p> <p>In the classical limit, the internal energy of an ideal gas is thus:</p> $U = \frac{3}{2} N k_B T$ <p>diatomic ideal gas [edit]</p> <p>We still have 3 translational d.o.f.s per molecule, like for the monoatomic ideal gas.</p> <p>However we also have:</p> <ul style="list-style-type: none"> • For each molecule, 4 degrees of freedom corresponding to the rotational modes of the molecule. We can describe the orientation of the molecule by 2 angles, which are the equivalent of a position, and we can describe the rotational speeds of the molecule by two angular momentums, which are the equivalent of momentums. Molecules are free to rotate in space, so that no energy is attached to the angular d.o.f.s. The rotational kinetic energy attached to each angular momentum is proportional to the square of of the 2 angular momentums, which means we have 2 quadratic rotational d.o.f.s per molecule. • For each molecule, the vibrational mode is linked to 2 degrees of freedom. One describes the linear distance between the two atoms (=position), the other describes the speed of vibration (=momentum). This time, all 2 are quadratic d.o.f.s, because kinetic energy is quadratic as usual, and the valence link between the two atoms can be quite adequately described as having a linear elastic response, which means a quadratic potential energy. This means 2 vibrational quadratic rotational d.o.f per molecule. <p>The total number of quadratic d.o.f.s per molecule is $3+2+2 = 7$.</p> <p>In the classical limit, we have:</p> $U = \frac{7}{2} N k_B T$ <p>Now this works only in the classical limit, which explains why you have $3/2$ which becomes $5/2$ at some temperature which then only becomes $7/2$ at a higher temperature. The reason is that translational d.o.f.s most generally reach the classical limit at a lower temperature than rotational degrees of freedom, which in turn reach the classical limit at a lower temperature than vibrational degrees of freedom. I will go back to this in more details later.</p> <p>This decisive explanation was brought to you by ThorinMuglindir 13:34, 31 October 2005 (UTC)</p> <p><small>https://en.wikipedia.org/wiki/Talk%3ADegrees_of_freedom_(physics_and_chemistry)</small></p> <p><small>http://demonstrations.wolfram.com/TheSixDegreesOfFreedomOfADiatom/</small></p>
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6.8 Ensembles

	the energy difference between the highest and lowest occupied single-particle states in a quantum system of non-interacting fermions at absolute zero temperature
Fermi Energy of infinite square well with N atoms	$E_F = \frac{\hbar^2\pi^2}{2mL^2}(N/2)^2$ for even N , and for odd N substitute $N - 1$ for N in the expression.
Fermi energy of a metal with N electrons per volume V	<p>Metal \approx 3-D ISW</p> $E_{n_x,n_y,n_z} = E_0 + \frac{\hbar^2\pi^2}{2mL^2}(n_x^2 + n_y^2 + n_z^2),$ <p>but let $\vec{n} = \{n_x, n_y, n_z\}$ so that</p> $E_{\vec{n}} = E_0 + \frac{\hbar^2\pi^2}{2mL^2} \vec{n} ^2.$ <p>In the ground state, the number of fermions is</p> $N = 2 \times \frac{1}{8} \times \frac{4}{3}\pi n_F^3$ <p>where n_F is the radius of the Fermi-sphere in n-space. Thus,</p> $n_F = \left(\frac{3N}{\pi}\right)^{1/3}.$ <p>So, the Fermi energy is</p> $E_F = \frac{\hbar^2\pi^2}{2mL^2}n_F^2 = \frac{\hbar^2\pi^2}{2mL^2}\left(\frac{3N}{\pi}\right)^{2/3}$ <div style="border: 1px solid black; padding: 5px; display: inline-block;"> $E_F = \frac{\hbar^2}{2m}\left(\frac{3\pi^2 N}{V}\right)^{2/3}$ </div>

Fermi temperature, momentum, velocity and wave vector	$T_F = \frac{E_F}{k_B}; \quad p_F = \sqrt{2m_e E_F}; \quad v_F = \frac{p_F}{m_e}; \quad k_F = \frac{p_F}{\hbar}$
typical Fermi energy for metal	$10^{28} \text{ to } 10^{29} \text{ e/m}^3 \rightarrow E_F \approx 2 \text{ to } 10 \text{ eV}$

Number of independent oscillators in crystal with N atoms	$3N$ according to Einstein and Debye
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Multiplicity of an Einstein Solid	$\Omega = \binom{q+N-1}{q} = \frac{(q+N-1)!}{q! (1+N-1-q)!}$ $= \frac{(q+N-1)(q+N-2)...(q+N-q)(N-1)(N-2)...}{q!(N-1)(N-2)...}$ $= \frac{(q+N-1)(q+N-2)...N}{q!}$
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Entropy of an Einstein Solid

$$S = k(\ln(1 + N)! - \ln q! - \ln N!)$$

Stirling's →

$$\ln \Omega \approx (q + N) \ln(q + N) - (q + N) - q \ln q + q - N \ln N + N$$

$$= (q + N) \ln(q + N) - q \ln q - N \ln N$$

... and ...

$$\ln(q + N) = \ln \left[q \left(1 + \frac{N}{q} \right) \right]$$

$$= \ln q + \ln \left(1 + \frac{N}{q} \right)$$

$$\approx \ln q + \frac{N}{q} \text{ assuming } q \gg N$$

... algebra ...

$$S = k \ln \Omega = k \left(N \ln \frac{q}{N} + N + \frac{N^2}{q} \right)$$

drop the last term

$$S \approx Nk \left(\ln \frac{q}{N} + 1 \right)$$

if energy unit $hf = \epsilon$ and tot. int. energy $U = q\epsilon$ then

$$S = Nk \left(\ln \frac{U}{q\epsilon} + 1 \right)$$

Temperature Einstein Solid	$T = \left(\frac{\partial S}{\partial U} \right)^{-1} = \frac{U}{Nk}$
Energy of an Einstein Solid	$U = NkT$

6.9 Statistical concepts and calculation of thermodynamics quantities

Stirling's Approximation	$\ln(N!) = N \ln(N) - N$
Boltzmann Factor	Any term of the form $e^{-E/kT}$

For a thermodynamic system with two accessible states, what is the ratio of probability that it's in state 1 to the probability that it is in state 2?	$\frac{P(1)}{P(2)} = \frac{e^{-E_1/kT}}{e^{-E_2/kT}}$
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Probability of E_i	$P(i) = \frac{e^{-E_i/kT}}{Z}$
Partition function	$Z = \sum_{\text{all } \mu \text{states } i} e^{-E_i/kT} = \sum_{\text{all energies } E} g(E) e^{-E/kT}$

Average energy	$\langle E \rangle = \sum_i E_i p_i = \frac{\sum_i E_i e^{-E_i/k_B T}}{\sum_j e^{-E_j/k_B T}}$ Conveniently, $\langle E \rangle = -\frac{1}{Z} \frac{\partial Z}{\partial \beta}_{N,V}$ or $\langle E \rangle = -\frac{\partial \ln Z}{\partial \beta}$ where $\beta \equiv \frac{1}{k_B T}$.
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Statistical entropy	$S = k \sum_i p_i \ln p_i$ if high T then $S = Nk \ln Z$.
Gibbs factor system in diffusive eq. and Thermal contact with reservoir	$P(N, i) = \frac{1}{Z} e^{(N\mu - E_i)/kT}$ $e^{(N\mu - E_i)/kT}$ is the Gibbs factor.
Thermal diffusive partition func.	$Z_{N,i} = \sum_i e^{(N\mu - E_i)/kT}$

	<p>We are concerned with the number of particles in a given microstate i.</p> $\frac{N_i}{N} = \frac{e^{-E_i/kT}}{\sum_j e^{-E_j/kT}}$ <p>From Wikipedia: "The assumptions of this equation are that the particles do not interact, and that they are classical; this means that each particle's state can be considered independently from the other particles' states. Additionally, the particles are assumed to be in thermal equilibrium."</p>
Maxwell-Boltzmann distribution for momentum vector	$\frac{N_i}{N} = \frac{1}{Z} \exp \left[-\frac{p_{i,x}^2 + p_{i,y}^2 + p_{i,z}^2}{2mkT} \right]$ <p>Probability density function for finding a molecule with a particular momentum vector:</p> $f_{\mathbf{p}}(p_x, p_y, p_z) = \frac{c}{Z} \exp \left[-\frac{p_x^2 + p_y^2 + p_z^2}{2mkT} \right].$ <p>Normalize using the Gaussian integral ($\int_{-\infty}^{\infty} e^{-ax^2} dx = \sqrt{\pi/a}$) to find that $c = Z(2\pi mkT)^{-3/2}$.</p> <div style="border: 1px solid black; padding: 5px; width: fit-content;"> $f_{\mathbf{p}}(p_x, p_y, p_z) = (2\pi mkT)^{-3/2} \exp \left[-\frac{p_x^2 + p_y^2 + p_z^2}{2mkT} \right]$ </div>

Maxwell-Boltzmann energy distribution

Consider $d^3\mathbf{p}$ as an infinitesimal phase-space volume of momenta corresponding to the interval dE . Specifically, the interval dE coincides with a spherical shell of thickness $d|\mathbf{p}|$ in momentum space. Use this and the energy-momentum dispersion relation to find

$$d^3\mathbf{p} = 4\pi|\mathbf{p}^2|d|\mathbf{p}| = 4\pi m\sqrt{2mE} dE$$

Now impose the following:

$$\begin{aligned} f_E(E) dE &= f_{\mathbf{p}}(\mathbf{p}) d^3\mathbf{p} \\ &= (2\pi mkT)^{-3/2} e^{-E/kT} 4\pi m\sqrt{2mE} dE \\ f_E(E) &= \boxed{2\sqrt{\frac{E}{\pi}} \left(\frac{1}{kT}\right)^{3/2} e^{-E/kT}} \end{aligned}$$

Maxwell-Boltzmann velocity distribution

Use $f_{\mathbf{v}} d^3v = f_{\mathbf{p}} \left(\frac{dp}{dv}\right)^3 d^3v$ and $\mathbf{p} = m\mathbf{v}$ and the momentum distribution above to get

$$f_{\mathbf{v}}(v_x, v_y, v_z) = \left(\frac{m}{2\pi kT}\right)^{3/2} \exp\left[-\frac{m(v_x^2 + v_y^2 + v_z^2)}{2kT}\right].$$

This can be expressed as the product of three speed distributions for the three directions:

$$f_{\mathbf{v}}(v_x, v_y, v_z) = f_v(v_x)f_v(v_y)f_v(v_z), \text{ where}$$

$$f_v(v_i) = \sqrt{\frac{m}{2\pi kT}} \exp\left[-\frac{mv_i^2}{2kT}\right]$$

Note: $\mu_i = 0$, $\mu_{\mathbf{v}} = \mathbf{0}$, $\sigma_i = \sqrt{kT/m}$, and $\sigma_{\mathbf{v}} = \sqrt{3kT/m}$.

6.10 Black-body radiation

<p>Planck's law</p>	<p>The spectral radiance of a body at absolute temperature T is given by</p> $B_\nu(\nu, T) = \frac{2h\nu^3}{c^2} \frac{1}{e^{\frac{h\nu}{k_B T}} - 1}$ $B_\lambda(\lambda, T) = \frac{2hc^2}{\lambda^5} \frac{1}{e^{\frac{hc}{\lambda k_B T}} - 1}$ <p>Note: $[B_\nu] = \text{W}\cdot\text{sr}^{-1}\cdot\text{m}^{-2}\cdot\text{Hz}^{-1}$ and $[B_\lambda] = \text{W}\cdot\text{sr}^{-1}\cdot\text{m}^{-3}$.</p>
<p>Stefan-Boltzmann Law</p>	<p>Radiant exitance or emissive power, $\frac{P}{A} = \varepsilon\sigma(T^4 - T_C^4)$, where ε is the emissivity ($\varepsilon = 1$ for blackbody, < 1 for 'gray' body), $\sigma \approx 5.67\text{E}-8$ is the S-B constant, and T_C is the temperature of cooler surroundings.</p>
<p>Wien's displacement law</p>	<p>The spectral radiance of black body radiation per unit wavelength, peaks at $\lambda_{max} = \frac{b}{T}$, where Wien's displacement constant $b = 2.9 \times 10^{-3}\text{m K}$.</p>

7 Quantum Mechanics

(such as fundamental concepts, solutions of the Schrodinger equation (including square wells, harmonic oscillators, and hydrogenic atoms), spin, angular momentum, wave function symmetry, elementary perturbation theory)

7.1 Fundamental concepts

7.1.1 Some Common Integrals and Probability Theory

$\operatorname{erf}(\infty), \operatorname{erf}(-\infty)$	$\operatorname{erf}(\infty) = 1, \operatorname{erf}(-\infty) = -1$
<p>The Gaussian Integral</p> $\int_{-\infty}^{+\infty} e^{-ax^2} dx =$	<p>Option 1: Know the error function.</p> $\frac{\sqrt{\pi}}{2\sqrt{a}} \operatorname{erf}(x\sqrt{a}) \Big _{-\infty}^{+\infty} = \sqrt{\frac{\pi}{a}}$ <p>Option 2: Use polar coordinate trick.</p> $\begin{aligned} I^2 &= \left(\int_{-\infty}^{\infty} e^{-ax^2} dx \right)^2 \\ &= \int \int_{\mathbf{R}^2} e^{-a(x^2+y^2)} d(x,y) \\ &= \int_0^{2\pi} \int_0^{\infty} e^{-ar^2} r dr d\theta \\ &= 2\pi \int_0^{\infty} r e^{-ar^2} dr \end{aligned}$ <p>Let $s = -ar^2$ and then $ds = -2ar dr$.</p> $\begin{aligned} I^2 &= \frac{\pi}{a} \int_{-\infty}^0 e^s ds \\ &= \frac{\pi}{a} \\ I &= \sqrt{\frac{\pi}{a}} \end{aligned}$

$\langle f(x) \rangle =$	$\int_{-\infty}^{+\infty} f(x)p(x) dx$
Q is an operator corresponding to physical observable x . One may obtain the expectation value of x by using $\langle Q \rangle = \langle \psi^* Q \psi \rangle = \int \psi^* Q \psi$, from which one may obtain the expectation value of x by knowing how Q corresponds to x .

Standard deviation	$\sigma = \sqrt{\langle (\Delta x)^2 \rangle}$ $= \sqrt{\langle x^2 \rangle - \langle x \rangle^2}$ <p>For a measurement Q corresponding to hermitian operator \hat{Q},</p> $\sigma^2 = \langle (\hat{Q} - \langle Q \rangle)^2 \rangle.$
Normalize $\psi(\phi) = A e^{im\phi}$	$1 = \int \psi(\phi) ^2 d\phi = A^2 \int_0^{2\pi} e^{im\phi} e^{-im\phi} d\phi = 2\pi A^2$ $A = \frac{1}{\sqrt{2\pi}}$

7.1.2 Formalism and Linear Algebra

This section could be called 'Griffiths runs the voodoo down' because it is a list of the cardinal equations of QM as explained in Chapter 3 of, *Introduction to Quantum Mechanics*.

vector representation	$ \alpha\rangle \rightarrow \mathbf{a} = \begin{pmatrix} a_1 \\ a_2 \\ \vdots \\ a_N \end{pmatrix}$
Inner product of two vectors	$\langle \alpha \beta \rangle = a_1^* b_1 + a_2^* b_2 + \dots a_N^* b_N.$
Properties of an inner product	$\langle \beta \alpha \rangle = \langle \alpha \beta \rangle^*$ $\langle \alpha \alpha \rangle \geq 0$, and $\langle \alpha \alpha \rangle = 0 \iff \alpha\rangle = 0,$ $\langle \alpha (b \beta \rangle + c \gamma \rangle) = b \langle \alpha \beta \rangle + c \langle \alpha \gamma \rangle. .$
Linear transformations	Linear transformations T are represented by matrices. $ \beta\rangle = T \alpha\rangle \rightarrow \mathbf{b} = \mathbf{T}\mathbf{a} = \begin{pmatrix} t_{11} & t_{12} & \dots & t_{1N} \\ t_{21} & t_{22} & \dots & t_{2N} \\ \vdots & \vdots & \ddots & \vdots \\ t_{N1} & t_{N2} & \dots & t_{NN} \end{pmatrix} \begin{pmatrix} a_1 \\ a_2 \\ \vdots \\ a_N \end{pmatrix}.$

Hilbert space	Hilbert space comprises the set of all square-integrable functions, on a specified interval, $f(x)$ such that $\int_a^b f(x) ^2 dx < \infty$. Wave functions live in Hilbert space.
Inner product of two functions $f(x)$ and $g(x)$.	$\langle f g \rangle \equiv \int_a^b f(x) * g(x) dx.$
Orthonormality	A set of functions is orthonormal if they are normalized and mutually orthogonal: $\langle f_m f_n \rangle = \delta_{mn}.$

Completeness	A set of functions is complete if any other function (in Hilbert space) can be expressed as a linear combination of them: $f(x) = \sum_{n=1}^{\infty} c_n f_n(x)$
Coefficients	If the functions $\{f_n(x)\}$ are orthonormal, the coefficients are $c_n = \langle f_n f \rangle = \int f_n(x)^* f(x, t) dx.$
Normalization condition	$\int_{-\infty}^{\infty} dx \Psi(x, t) ^2 = 1$ $\sum_n c_n ^2 = 1.$
Expectation value	The expectation value of an observable $Q(x, p)$: $\langle Q \rangle = \int \Psi^* \hat{Q} \Psi dx = \langle \Psi \hat{Q} \Psi \rangle$ The expectation value may not be the eigenvalue (measured value) of a Hermitian operator (observable).
Probability of getting eigenvalue q_n associated with eigenfunction $f_n(x)$	Probability for discrete is $ c_n ^2$. Probability for continuous in range dz is $ c(z) ^2 dz$.

	Mode not mean! Find max of $ \psi ^2$. Ex: Most probable r for radially dependent, spherically symmetrical wave function $\psi(r)$.
Most probable value	$ \psi ^2 dV = \psi ^2 4\pi r^2 dr$ <p>For max, $\frac{d}{dr} \psi ^2 4\pi r^2 = 0$. Solve for r to find most probable radial coordinate.</p>

Hermitian operators as measurements	<p>The outcome of a measurement is real, so we must have $\langle Q \rangle = \langle Q \rangle^*$. Therefore, $\langle \Psi \hat{Q} \Psi \rangle = \langle \hat{Q} \Psi \Psi \rangle$. The operators representing observables are hermitian: $\langle f \hat{Q} f \rangle = \langle \hat{Q} f f \rangle$ for all $f(x)$. The last expression implies $\langle f \hat{Q} g \rangle = \langle \hat{Q} f g \rangle$ for all $f(x)$ and $g(x)$.</p>
Hermitian operators as linear transformations	<p>Hermitian operators are linear transformations: $\beta\rangle = \hat{Q} \alpha\rangle$. Just as vectors are represented, with respect to a particular basis ($e_n\rangle$), by their components,</p> $ \alpha\rangle = \sum_n a_n e_n\rangle, \text{ with } a_n = \langle e_n \alpha \rangle,$ <p>operators are represented by their matrix elements $\langle e_m \hat{Q} e_n \rangle \equiv Q_{mn}$. In this notation, the linear transformation looks like this: $\sum_n b_n e_n\rangle = \sum_n a_n \hat{Q} e_n\rangle$. Take the inner product with $e_m\rangle$, i.e. $\sum_n b_n \langle e_m e_n \rangle = \sum_n a_n \langle e_m \hat{Q} e_n \rangle$, and hence $b_m = \sum_n Q_{mn} a_n$.</p> <p>Check out this way dope list of QM operators https://en.wikipedia.org/wiki/Operator_(physics)#Table_of_QM_operators.</p>

	<p>Ket is a vector, but what is bra?</p> <p>In function space it can be thought of as an instruction to braket integrate: $\langle f = \int f^*[some\ ket]dx.$</p>
Projection operator	<p>In finite-dimensional vector space, bra is a row vector:</p> $\langle \alpha = (a_1^* \ a_2^* \ \dots \ a_n^*)$ $\hat{P} \equiv \alpha\rangle \langle \alpha $ $\hat{P} \beta\rangle = \langle \alpha \beta\rangle \alpha\rangle.$
Identity operator	<p>If $\{ e_n\rangle\}$ is a discrete orthonormal basis, then $\sum_n e_n\rangle \langle e_n = 1$ (the identity operator).</p> $\sum_n e_n\rangle \langle e_n \alpha\rangle = \alpha\rangle.$ <p>For orthonormalized continuous basis, $\int e_z\rangle \langle e_z dz = 1.$</p>
Inverse of an operator	\hat{A}^{-1} such that $\hat{A}\hat{A}^{-1} = \hat{A}^{-1}\hat{A} = \hat{I}$

Determinate states	Determinate states are the physical meaning of the eigenfunctions of hermitian operators. If the eigenfunction spectrum is discrete, the eigenfunctions lie in the Hilbert space and constitute physically realizable states. If the spectrum is continuous, they are non-normalizable and do not represent possible wave functions, though linear combinations of them may be normalizable.
Properties of hermitian operators	<p>I. Their eigenvalues are real.</p> $\hat{Q}f = qf$ <p>II. Eigenfunctions belonging to distinct eigenvalues are orthogonal.</p> <p>If $\hat{Q}f = qf$ and $\hat{Q}g = q'g$, then $\langle f g \rangle = 0$.</p>
Hermitian conjugate or adjoint	\hat{Q}^\dagger such that $\langle f \hat{Q}g \rangle = \langle \hat{Q}^\dagger f g \rangle$ for all f and g . A Hermitian operator is equal to its hermitian conjugate: $\hat{Q} = \hat{Q}^\dagger$. Also, $(\hat{Q}\hat{R})^\dagger = \hat{R}^\dagger\hat{Q}^\dagger$.

Compatibility Theorem

Let us have two observables, A and B , represented by \hat{A} and \hat{B} .

Then any one of the following statements implies the other two:

- A and B are compatible observables.
- \hat{A} and \hat{B} have a common eigenbasis.
- The operators \hat{A} and \hat{B} commute, that is, $[\hat{A}, \hat{B}] = 0$

https://en.wikipedia.org/wiki/Complete_set_of_commuting_observables

Generalized statistical interpretation	<p>If the spectrum of \hat{Q} is discrete, the probability of getting the particular eigenvalue q_n associated with the orthonormalized eigenfunction $f_n(x)$ is $c_n ^2$, where $c_n = \langle f_n \Psi \rangle$.</p> <p>If the spectrum is continuous, with real eigenvalues $q(z)$ and associated Dirac-orthonormalized eigenfunctions $f_z(x)$, the probability of getting a result in the range dz is $c(z) ^2 dz$, where $c(z) = \langle f_z \Psi \rangle$. Upon measurement, the wave function collapses to the corresponding eigenstate.</p>
Generalized uncertainty principle	$\sigma_A^2 \sigma_B^2 \geq \left(\frac{1}{2i} \langle [\hat{A}, \hat{B}] \rangle \right)^2$ <p>There is an uncertainty principle for every pair of observables whose operators do not commute, i.e. incompatible observables.</p> <p>See pages 110-111 in Griffiths for more.</p>

7.1.3 Uncertainty

Commutator	$[\hat{A}, \hat{B}] = \hat{A}\hat{B} - \hat{B}\hat{A}$
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7.1.4 Position and Momentum

de Broglie relations	$\lambda = h/p \quad \& \quad \nu = E/h \quad \text{or}$ $\mathbf{p} = \hbar\mathbf{k} \quad \& \quad E = \hbar\omega = h\nu$
One dimensional operator	Plane wave solution to SE $\psi = e^{i(kx-\omega t)}$ $\frac{\partial\psi}{\partial x} = ike^{i(kx-\omega t)} = ik\psi$ De Broglie relation: $p = \hbar k$ $\frac{\partial\psi}{\partial x} = i\frac{p}{\hbar}\psi$ $\boxed{\hat{p} = -i\hbar\frac{\partial}{\partial x}}$
Three dimensional operator	Plane wave solution to SE $\psi = e^{i(\mathbf{k}\cdot\mathbf{r}-\omega t)}$ $\boxed{\hat{\mathbf{p}} = -i\hbar\nabla}$
$\langle p \rangle$	$\langle p \rangle = m\frac{d\langle x \rangle}{dt} = -i\hbar\int \psi \frac{\partial\psi}{\partial x} dx$

Probability current density	$\mathbf{j} = \frac{\hbar}{2mi}(\Psi\nabla\Psi^* - \Psi^*\nabla\Psi) = \frac{1}{2m}(\Psi^*\hat{\mathbf{p}}\Psi - \Psi\hat{\mathbf{p}}\Psi^*)$
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Uncertainty principle	$\sigma_x \sigma_p \geq \frac{\hbar}{2}$
Canonical commutation relation	From Wikipedia: "In quantum mechanics (physics), the canonical commutation relation is the fundamental relation between canonical conjugate quantities (quantities which are related by definition such that one is the Fourier transform of another)."
Commutation of position and momentum	$[\hat{x}, \hat{p}] = i\hbar$
Position space and momentum space wave functions	Momentum: $\phi(p, t) = \frac{1}{\sqrt{2\pi\hbar}} \int_{-\infty}^{\infty} e^{-ipx/\hbar} \Psi(x, t) dx$ Position: $\Psi(x, t) = \frac{1}{\sqrt{2\pi\hbar}} \int_{-\infty}^{\infty} e^{-ipx/\hbar} \phi(p, t) dp$

	<p>Functions and operators in position space [edit]</p> <p>Suppose we have a three-dimensional wave function in position space $\psi(\mathbf{r})$, then we can write this functions as a weighted sum of orthogonal basis functions $\psi_j(\mathbf{r})$:</p> $\psi(\mathbf{r}) = \sum_j \phi_j \psi_j(\mathbf{r})$ <p>or, in the continuous case, as an integral</p> $\psi(\mathbf{r}) = \int_{\text{k-space}} \phi(\mathbf{k}) \psi_{\mathbf{k}}(\mathbf{r}) d^3\mathbf{k}$ <p>It is clear that if we specify the set of functions $\psi_j(\mathbf{r})$, say as the set of eigenfunctions of the momentum operator, the function $\phi(\mathbf{k})$ holds all the information necessary to reconstruct $\psi(\mathbf{r})$ and is therefore an alternative description for the state ψ.</p> <p>$\psi(\mathbf{r})$</p> <p>In quantum mechanics, the momentum operator is given by</p> $\hat{\mathbf{p}} = -i\hbar \frac{\partial}{\partial \mathbf{r}}$ <p>(see matrix calculus for the denominator notation) with appropriate domain. The eigenfunctions are</p> $\psi_{\mathbf{k}}(\mathbf{r}) = \frac{1}{(\sqrt{2\pi})^3} e^{i\mathbf{k}\cdot\mathbf{r}}$ <p>and eigenvalues $\hbar\mathbf{k}$. So</p> $\psi(\mathbf{r}) = \frac{1}{(\sqrt{2\pi})^3} \int_{\text{k-space}} \phi(\mathbf{k}) e^{i\mathbf{k}\cdot\mathbf{r}} d^3\mathbf{k}$ <p>and we see that the momentum representation is related to the position representation by a Fourier transform.^[5]</p> <p>Functions and operators in momentum space [edit]</p> <p>Conversely, a three-dimensional wave function in momentum space $\phi(\mathbf{k})$ as a weighted sum of orthogonal basis functions $\phi_j(\mathbf{k})$:</p> $\phi(\mathbf{k}) = \sum_j \psi_j \phi_j(\mathbf{k})$ <p>or as an integral:</p> $\phi(\mathbf{k}) = \int_{\text{r-space}} \psi(\mathbf{r}) \phi_{\mathbf{r}}(\mathbf{k}) d^3\mathbf{r}$ <p>$\phi(\mathbf{k})$</p> <p>the position operator is given by</p> $\hat{\mathbf{r}} = i\hbar \frac{\partial}{\partial \mathbf{p}} = i \frac{\partial}{\partial \mathbf{k}}$ <p>with eigenfunctions</p> $\phi_{\mathbf{r}}(\mathbf{k}) = \frac{1}{(\sqrt{2\pi})^3} e^{-i\mathbf{k}\cdot\mathbf{r}}$ <p>and eigenvalues \mathbf{r}. So a similar decomposition of $\phi(\mathbf{k})$ can be made in terms of the eigenfunctions of this operator, which turns out to be the inverse Fourier transform:^[5]</p> $\phi(\mathbf{k}) = \frac{1}{(\sqrt{2\pi})^3} \int_{\text{r-space}} \psi(\mathbf{r}) e^{-i\mathbf{k}\cdot\mathbf{r}} d^3\mathbf{r}$
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7.2 Solutions of the Schrodinger equation, square wells, harmonic oscillators, hydrogenic atoms

7.2.1 Taming of the Schrod

General Schrodinger Eqn.	$i\hbar \frac{\partial}{\partial t} \Psi(\mathbf{r}, t) = \hat{H}\Psi(\mathbf{r}, t)$
Typical SE, nonrelativistic	$i\hbar \frac{\partial}{\partial t} \Psi(\mathbf{r}, t) = \left[\frac{-\hbar^2}{2\mu} \nabla^2 + V(\mathbf{r}, t) \right] \Psi(\mathbf{r}, t)$ where μ is reduced mass of the particle.
Time-independent Schrodinger Equation and interpretation	$E\Psi = \hat{H}\Psi$ When the Hamiltonian operator acts on a certain wave function Ψ , and the result is proportional to the same wave function Ψ , then Ψ is a stationary state, and the proportionality constant, E , is the energy of the state Ψ .
Typical TISE	$E\Psi(\mathbf{r}) = \left[\frac{-\hbar^2}{2\mu} \nabla^2 + V(\mathbf{r}) \right] \Psi(\mathbf{r})$

7.2.2 Infinite Square Well

Consider a particle of mass m trapped in an infinite square well between 0 and a .

$\psi_n(x)$	$\psi_n(x) = \sqrt{\frac{2}{a}} \sin(k_n x)$, where $k = \frac{n\pi}{a}$
Energy	$E\psi = \hat{H}\psi$ $V(x) = 0$ in well, so we have $E_n = \frac{-\hbar^2}{2m} \nabla^2 \psi = \frac{\hbar^2 k_n^2}{2m} = \frac{\hbar^2 n^2 \pi^2}{2ma^2}$
ω_n	$\omega_n = \frac{E}{\hbar} = \frac{\hbar k_n^2}{2m} = \frac{\hbar n^2 \pi^2}{2ma^2}$
values of n	$n = 1, 2, 3\dots$

7.2.3 Quantum Harmonic Oscillator

$V(x)$	$V(x) = \frac{1}{2}kx^2 = \frac{1}{2}m\omega^2 x^2$
$\psi_n(x)$	https://en.wikipedia.org/wiki/Quantum_harmonic_oscillator
Energy levels	$E_n = \hbar\omega\left(n + \frac{1}{2}\right)$
values of n	$n = 0, 1, 2\dots$
Wave functions and potential	<p>Griffiths, <i>Introduction to Quantum Mechanics</i></p>

7.3 Angular momentum

Commutation relations of angular momentum operators

Consider classical angular momentum $\mathbf{L} = \mathbf{r} \times \mathbf{p}$. Do some algebra and find

$$[L_x, L_y] = i\hbar L_z; \quad [L_y, L_z] = i\hbar L_x; \quad [L_z, L_x] = i\hbar L_y.$$

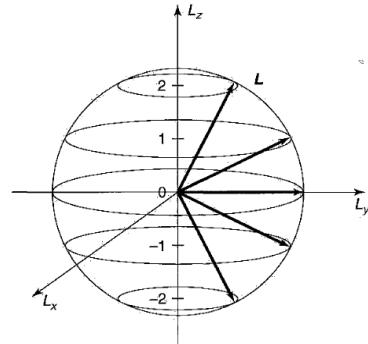
Also, $[L^2, \mathbf{L}] = 0$ where $L^2 \equiv L_x^2 + L_y^2 + L_z^2$.

Eigenvalues of angular momentum operators

$$L^2 f_l^m = \hbar^2 l(l+1) f_l^m; \quad L_z f_l^m = \hbar m f_l^m,$$

where $l = 0, 1/2, 1, 3/2, \dots$; and $m = -l, -l+1, \dots, l-1, l$.

Note that $\sqrt{l(l+1)} > l$ except trivially when $l = 0$.



Angular momentum states (for $l = 2$).

Griffiths, *Introduction to Quantum Mechanics*

Ladder operator	$L_{\pm} \equiv L_x \pm iL_y$ $L_{\pm}f_l^m = \hbar\sqrt{l(l+1) - m(m \pm 1)}f_l^{m \pm 1}.$
Eigenfunctions	$L_z = \frac{\hbar}{i} \frac{\partial}{\partial \phi}$ $L^2 = -\hbar^2 \left[\frac{1}{\sin \theta} \frac{\partial}{\partial \theta} \left(\sin \theta \frac{\partial}{\partial \theta} \right) + \frac{1}{\sin^2 \theta} \frac{\partial^2}{\partial \phi^2} \right]$ See page 168 in Griffiths for more.
Energy of a quantum rotor	Recall classical rotational energy $\frac{L^2}{2I}$. Quantum mechanically, $E_{rot} = \frac{\hbar^2 l(l+1)}{2mr^2}, \quad l = 0, 1, 2, 3, \dots$

7.4 Spin

Intrinsic (not orbital) angular momentum

Commutation relations	$[S_x, S_y] = i\hbar S_z; \quad [S_y, S_z] = i\hbar S_x; \quad [S_z, S_x] = i\hbar S_y.$
Eigenvectors and eigenvalues	$S^2 s m\rangle = \hbar^2 s(s+1) s m\rangle; \quad S_z s m\rangle = \hbar m s m\rangle$ where $s = 0, 1/2, 1, 3/2, \dots$; and $m = -s, -s+1, \dots, s-1, s.$
Ladder operator	$S_{\pm} \equiv S_x \pm iS_y$ $S_{\pm} s m\rangle = \hbar \sqrt{s(s+1) - m(m \pm 1)} s (m \pm 1)\rangle.$
Eigenstate	Up: $ \frac{1}{2} \frac{1}{2}\rangle$; Down: $ \frac{1}{2} - \frac{1}{2}\rangle$
Spinor (spin vector)	$\chi = \begin{pmatrix} a \\ b \end{pmatrix} = a \begin{pmatrix} 1 \\ 0 \end{pmatrix} + b \begin{pmatrix} 0 \\ 1 \end{pmatrix} = a\chi_+ + b\chi_-$
Spin operators	Non-hermitian: $\mathbf{S}_+ = \hbar \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}, \quad \mathbf{S}_- = \hbar \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}$ Hermitian observables: $\mathbf{S}^2 = \frac{3}{4}\hbar^2 \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}; \quad \mathbf{S} = \frac{\hbar}{2}\boldsymbol{\sigma}$ where $\boldsymbol{\sigma}$ represents the Pauli spin matrices.
Pauli Spin Matrices	$\boldsymbol{\sigma}_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \boldsymbol{\sigma}_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \quad \boldsymbol{\sigma}_z = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}.$

Eigenspinors	$\chi_+^z = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$ with eigenvalue $\frac{\hbar}{2}$ $\chi_-^z = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$ with eigenvalue $-\frac{\hbar}{2}$ $\chi_+^x = \begin{pmatrix} \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \end{pmatrix}$ with eigenvalue $\frac{\hbar}{2}$ $\chi_-^x = \begin{pmatrix} \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} \end{pmatrix}$ with eigenvalue $-\frac{\hbar}{2}$ $\chi_+^y = \begin{pmatrix} \frac{1}{\sqrt{2}} \\ i\frac{1}{\sqrt{2}} \end{pmatrix}$ with eigenvalue $\frac{\hbar}{2}$ $\chi_-^y = \begin{pmatrix} \frac{1}{\sqrt{2}} \\ -i\frac{1}{\sqrt{2}} \end{pmatrix}$ with eigenvalue $-\frac{\hbar}{2}$
	And, using $\hat{r} = \sin \theta \cos \phi \hat{i} + \sin \theta \sin \phi \hat{j} + \cos \theta \hat{k}$, we have the general expression $\chi_+^r = \begin{pmatrix} \cos(\theta/2) \\ e^{i\phi} \sin(\theta/2) \end{pmatrix}$ with eigenvalue $\frac{\hbar}{2}$ $\chi_-^r = \begin{pmatrix} e^{-i\phi} \sin(\theta/2) \\ -\cos(\theta/2) \end{pmatrix}$ with eigenvalue $-\frac{\hbar}{2}$

Electron magnetic dipole moment	$\boldsymbol{\mu} = g \frac{q}{2m} \mathbf{S} = \gamma \mathbf{S}$, where g is the g factor.
Hamiltonian of spinning charged particle at rest in a magnetic field \mathbf{B}	$H = -\gamma \mathbf{B} \cdot \mathbf{S}$

Addition of angular momentum in a two particle system, e.g. proton and electron in ground state of H

Possible combos of up and down	$\uparrow\uparrow, \uparrow\downarrow, \downarrow\uparrow, \downarrow\downarrow$
Total angular momentum	$\mathbf{S} \equiv \mathbf{S}^{(1)} + \mathbf{S}^{(2)}$ $S_z \chi_1 \chi_2 = (S_z^{(1)} + S_z^{(2)}) \chi_1 \chi_2 = \hbar(m_1 + m_2) \chi_1 \chi_2$
Triplet combination of states	Being that the quantum number for the composite system is $m = m_1 + m_2$; being the four possible combos yield 1, 0, 0, -1; and being that the possible values of m range in integer steps from $-s$ to $+s$, we infer that $s = 1$. This is enough to imply the first or last state of the triplet combo. The middle state can be found by applying S_{\pm} to one of the other two. Representing the states as $ s\ m\rangle$ $\left\{ \begin{array}{lcl} 1\ 1\rangle & = & \uparrow\uparrow \\ 1\ 0\rangle & = & \frac{1}{\sqrt{2}}(\uparrow\downarrow + \downarrow\uparrow) \\ 1\ -1\rangle & = & \downarrow\downarrow \end{array} \right\}$
Singlet state	The orthogonal state: $\left\{ 0\ 0\rangle = \frac{1}{\sqrt{2}}(\uparrow\downarrow - \downarrow\uparrow) \right\}.$
Possible values of spin for the composite system	$s = (s_1 + s_2), (s_1 + s_2 - 1), (s_1 + s_2 - 2), \dots, s_1 - s_2 $, where $s_1 \geq s_2$.

7.5 Spherical quantum mechanics

Proportionality of first six spherical harmonics	$Y_l^m(\theta, \phi) : Y_0^0 = \text{const.} \quad Y_l^0 \propto \cos \theta \quad Y_l^{\pm 1} \propto \sin \theta e^{\pm i\phi}$ $Y_2^0 \propto 3 \cos^2 \theta - 1 \quad Y_2^{\pm 1} \propto \sin \theta \cos \theta e^{\pm i\phi} \quad Y_2^{\pm 2} \propto \sin^2 \theta e^{\pm i\phi}$
First two Bessel functions	$j_n(x) : j_0 = \frac{\sin x}{x} \approx_{\text{small } x} 1 \quad j_1 = \frac{\sin x}{x^2} - \frac{\cos x}{x} \approx_{\text{small } x} \frac{x}{3}$ (soln. for radial component of spherical ISW)

7.6 Multiparticle Wave Functions and Symmetry

Hamiltonian for a two-particle system	$H = -\frac{\hbar^2}{2m_1} \nabla_1^2 - \frac{\hbar^2}{2m_2} \nabla_2^2 + V(\mathbf{r}_1, \mathbf{r}_2, t)$
Statistical interpretation	$ \Psi(\mathbf{r}_1, \mathbf{r}_2, t) ^2 d^3\mathbf{r}_1 d^3\mathbf{r}_2 = 1$
Two particle wave function	<p>If particle 1 is in state $\psi_a(\mathbf{r})$ and particle 2 is in state $\psi_b(\mathbf{r})$ then $\psi(\mathbf{r}_1, \mathbf{r}_2) = \psi_a(\mathbf{r}_1)\psi_b(\mathbf{r}_2)$</p> <p>If the particles are indistinguishable then we need a wave function that does not distinguish which particle is in which state:</p> $\psi_{\pm}(\mathbf{r}_1, \mathbf{r}_2) = A[\psi_a(\mathbf{r}_1)\psi_b(\mathbf{r}_2) \pm \psi_b(\mathbf{r}_1)\psi_a(\mathbf{r}_2)]$ <p>Bosons use the $+$, and fermions use the $-$.</p>

Bosons	<i>integer</i> spin. Statistics do not restrict the number of them that occupy a single quantum state. Ex: photons, gluons, composite particles (e.g. mesons and stable nuclei of even mass number such as deuterium), some quasiparticles (e.g. Cooper pairs, plasmons, and phonons) https://en.wikipedia.org/wiki/Boson
Fermions	<i>half integer</i> spin. Restricted by Pauli Exclusion. The Standard Model recognizes two types of elementary fermions: quarks and leptons. In all, the model distinguishes 24 different fermions. There are six quarks (up, down, strange, charm, bottom and top quarks), and six leptons (electron, electron neutrino, muon, muon neutrino, tau particle and tau neutrino), along with the corresponding antiparticle of each of these. https://en.wikipedia.org/wiki/Fermion
Pauli Exclusion principle	Two identical fermions (particles with half-integer spin) cannot occupy the same quantum state simultaneously. Consider the two-particle wave function above. If $\psi_a = \psi_b$, then $\psi_- = 0$.
Exchange operator, P	Switcheroo: $Pf(\mathbf{r}_1, \mathbf{r}_2) = f(\mathbf{r}_2, \mathbf{r}_1)$ If the particles are identical, the Hamiltonian must treat them the same: $m_1 = m_2$ and $V(\mathbf{r}_1, \mathbf{r}_2) = V(\mathbf{r}_2, \mathbf{r}_1)$. It follows that P and H are compatible observables. $[P, H] = 0$. <small>Griffiths, <i>Introduction to Quantum Mechanics</i></small>
Symmetrization requirement	For identical particles, the wave function is required to satisfy $\psi(\mathbf{r}_1, \mathbf{r}_2) = \pm\psi(\mathbf{r}_2, \mathbf{r}_1)$ with + for bosons and - for fermions. <small>Griffiths, <i>Introduction to Quantum Mechanics</i></small>

7.7 Time-independent perturbation theory

Perturbed Hamiltonian	$H = H^0 + \lambda H'$ This may also be expressed as $H = H_0 + H_1$ or $H = H^0 + \Delta H$ or $H = H_0 + \epsilon V$ or something similar. The term λ or ϵ are small numbers, rather they indicate that $\lambda H'$ is a small correction to H_0 . Later on, $\lambda = 1$ and the symbol just serves to keep track of the order of the correction.
Corrections to n th eigenfunction	$\psi_n = \psi_n^0 + \lambda \psi_n^1 + \lambda^2 \psi_n^2 + \dots$
Corrections to n th eigenvalue	$E_n = E_n^0 + \lambda E_n^1 + \lambda^2 E_n^2 + \dots$
First-Order correction to energy	$E_n^1 = \langle \psi_n^0 H' \psi_n^0 \rangle$
First-Order correction to wave function	$\psi_n^1 = \sum_{m \neq n} \frac{\langle \psi_m^0 H' \psi_n^0 \rangle}{(E_n^0 - E_m^0)} \psi_m^0$
Second-Order correction to energy	$E_n^2 = \sum_{m \neq n} \frac{ \langle \psi_m^0 H' \psi_n^0 \rangle ^2}{(E_n^0 - E_m^0)}$

8 Atomic Physics

(such as proper ties of electrons, Bohr model, energy quantization, atomic structure, atomic spectra, selection rules, black-body radiation, x-rays, atoms in electric and magnetic fields)

8.1 Properties of electrons

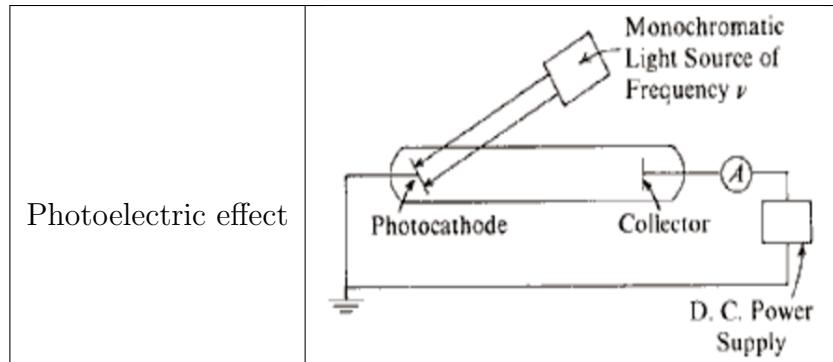
Spin	They are fermions, $s = \frac{1}{2}$.
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8.2 Bohr Model

Derive orbital radius	<p>Classical centripetal force for electron about a nucleus:</p> $\mathbf{F} = m\mathbf{a} \Rightarrow \frac{Ze^2}{4\pi\epsilon_0 r^2} = \frac{m_e v^2}{r}$ <p>Orbital momentum is quantized in units of \hbar, i.e. $mvr = n\hbar$</p> <p>Therefore, $r_n = \frac{4\pi\epsilon_0 n^2 \hbar^2}{Ze^2 m_e}$</p> $r(n, Z) = \frac{n^2 a_0}{Z}$
Bohr radius & Fine Structure Constant	$a_0 = \frac{4\pi\epsilon_0 \hbar^2}{e^2 m_e} = \frac{\hbar}{m_e c \alpha} = 0.53 \text{ \AA}$ <p>where $\alpha \approx \frac{1}{137}$ is the Fine Structure constant.</p>
Wave function ground state H	$\psi_{100}(r, \theta, \phi) = \frac{1}{\sqrt{\pi a^3}} e^{-r/a} \propto e^{-r/a}$
Binding and Rydberg Energy	$E_B = -\frac{1}{4\pi\epsilon_0} \frac{Ze^2}{2r_n} = -\frac{R_E Z^2}{n^2} \text{ where } R_E = \frac{m_e e^4}{8\epsilon_0^2 \hbar^2} = 13.6 \text{ eV}$
Positronium binding energy	<p>Positronium is a bound state of an electron and positron</p> <p>Use formulation as Hydrogen, but with electron reduced mass:</p> $m_{\text{red}} = \frac{m_e m_p}{m_e + m_p} = m_e \frac{1}{1 + m_e/m_p} = m_e/2$ <p>So, $E_n = \frac{R_E}{2n^2}$ for positronium.</p>

8.3 Energy quantization

Typical atomic/molecular energy levels by type	Electronic levels: ~ 1 eV Vibrational levels: ~ 0.1 eV Rotational levels: ~ 0.001 eV
Energy of photon emitted by H atom	$E = E_i - E_f = R_E \left(\frac{1}{n_f^2} - \frac{1}{n_i^2} \right)$
Wavelength of photon	$E = h\nu = \frac{hc}{\lambda}$ Wavelength given by $\frac{1}{\lambda} = R \left(\frac{1}{n_f^2} - \frac{1}{n_i^2} \right)$
Hydrogen Spectral Series	<p>The diagram illustrates the hydrogen atom's energy levels and the resulting spectral series. The central nucleus is surrounded by concentric elliptical orbits labeled $n = 1$, $n = 2$, $n = 3$, $n = 4$, $n = 5$, and $n = 6$. Transitions are shown as arrows between these levels. The Lyman series (red arrows) corresponds to transitions ending at $n = 1$, with wavelengths of 122 nm, 103 nm, 97 nm, 95 nm, and 94 nm. The Balmer series (green arrows) corresponds to transitions ending at $n = 2$, with wavelengths of 656 nm, 486 nm, 434 nm, and 410 nm. The Paschen series (blue arrows) corresponds to transitions ending at $n = 3$, with wavelengths of 1875 nm, 1282 nm, and 1094 nm.</p> <p>https://en.wikipedia.org/wiki/Hydrogen_spectral_series</p> <p>Lyman ($n_f = 1$), Balmer ($n_f = 2$), Paschen ($n_f = 3$)</p>



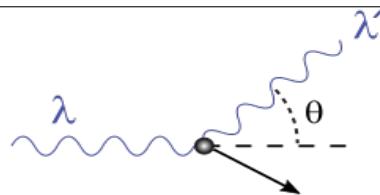
Photoelectric effect

(GRE 8677 #31-33)

Photoelectric effect

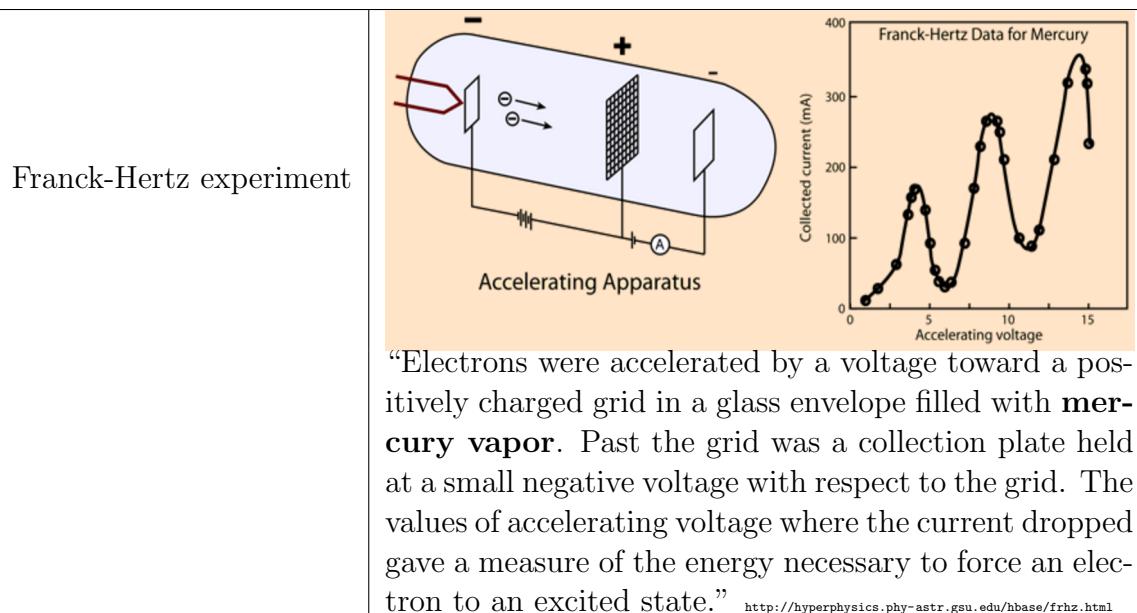
Light is absorbed in quanta: $E = h\nu$
 $|eV_0| = K_{max} = h\nu - W$ where V_0 is
 the stopping potential ($I = 0$), K_{max}
 is the maximum kinetic energy of an
 emitted electron, and W is the work
 function of the metal. $W = h\nu_0$,
 where ν_0 is the threshold frequency.

Compton effect



“inelastic scattering of a photon by a charged particle, usually an electron. It results in a decrease in energy (increase in wavelength) of the photon (which may be an X ray or gamma ray photon)” https://en.wikipedia.org/wiki/Compton_scattering

$$\lambda' - \lambda = \frac{h}{m_e c} (1 - \cos \theta)$$



	$l = 0, 1, 2, 3, 4, 5, 6, 7, 8$
Angular momentum values	$l = s, p, d, f, g, h, i, k, l$ For a given n , l can take values up to $n - 1$.
	Archaic: ‘s’harp, ‘p’rincipal, ‘d’iffuse, ‘f’undamental

8.4 Atomic structure

	$\psi(\mathbf{r})\chi(\mathbf{s})$
State of an electron	<p>Where ψ is the spatial wave function and χ is the spinor.</p> <p>Recall the composite spin states. The singlet state is antisymmetric and hence must be joined with a symmetric spatial function. Each triplet state is symmetric and requires a antisymmetric spatial function.</p>
Aufbau principle	<p>“Hypothetically, electrons orbiting one or more atoms fill the lowest available energy levels before filling higher levels.”</p> <p style="text-align: center;">Aufbau Principle for Electron Configuration: $1s^2 2s^2 2p^6 3s^2 3p^6 4s^2 3d^{10} 4p^6$</p> <p>http://study.com/cimages/multimages/16/aufbau.png</p>
Diagonal rule	<p>Orbitals are filled in order of increasing $n + l$ value. nl:</p> <p>https://en.wikipedia.org/wiki/Aufbau_principle</p>

Total angular momentum	$j = l + s$
Hund's Rules	<p>I) Consistent with Pauli exclusion, the state with the highest total spin (S) has the lowest energy: $\uparrow S \equiv \downarrow E$</p> <p>II) For a given spin, the state with the highest total orbital angular momentum (L), consistent with overall antisymmetrization, will have the lowest energy. $\uparrow L \equiv \downarrow E$</p> <p>III) If a subshell (n, l) is no more than half-filled, then the lowest energy level has $J = L - S$; if it is more than half-filled, then $J = L + S$ has the lowest energy.</p>
Hieroglyphic	$^{2S+1}L_J$

TABLE 5.1: Ground state electron configurations for the first four rows of the Periodic Table.

Z	Element	Configuration
1	H	(1s)
2	He	(1s) ²
3	Li	(He)(2s)
4	Be	(He)(2s) ²
5	B	(He)(2s) ² (2p)
6	C	(He)(2s) ² (2p) ²
7	N	(He)(2s) ² (2p) ³
8	O	(He)(2s) ² (2p) ⁴
9	F	(He)(2s) ² (2p) ⁵
10	Ne	(He)(2s) ² (2p) ⁶
11	Na	(Ne)(3s)
12	Mg	(Ne)(3s) ²
13	Al	(Ne)(3s) ² (3p)
14	Si	(Ne)(3s) ² (3p) ²
15	P	(Ne)(3s) ² (3p) ³
16	S	(Ne)(3s) ² (3p) ⁴
17	Cl	(Ne)(3s) ² (3p) ⁵
18	Ar	(Ne)(3s) ² (3p) ⁶
19	K	(Ar)(4s)
20	Ca	(Ar)(4s) ²
21	Sc	(Ar)(4s) ² (3d)
22	Ti	(Ar)(4s) ² (3d) ²
23	V	(Ar)(4s) ² (3d) ³
24	Cr	(Ar)(4s) ² (3d) ⁵
25	Mn	(Ar)(4s) ² (3d) ⁵
26	Fe	(Ar)(4s) ² (3d) ⁶
27	Co	(Ar)(4s) ² (3d) ⁷
28	Ni	(Ar)(4s) ² (3d) ⁸
29	Cu	(Ar)(4s)(3d) ¹⁰
30	Zn	(Ar)(4s) ² (3d) ¹⁰
31	Ga	(Ar)(4s) ² (3d) ¹⁰ (4p)
32	Ge	(Ar)(4s) ² (3d) ¹⁰ (4p) ²
33	As	(Ar)(4s) ² (3d) ¹⁰ (4p) ³
34	Se	(Ar)(4s) ² (3d) ¹⁰ (4p) ⁴
35	Br	(Ar)(4s) ² (3d) ¹⁰ (4p) ⁵
36	Kr	(Ar)(4s) ² (3d) ¹⁰ (4p) ⁶

First 36 configs.

Giffiths, *Quantum*

8.5 Atomic Spectra

8.6 Selection rules

Selection rule for m	No transitions occur unless $\Delta m = \pm 1$ or 0.
Selection rule for l	<p>No transitions occur unless $\Delta l = \pm 1$.</p>

FIGURE 9.6: Allowed decays for the first four Bohr levels in hydrogen.

Griffiths, *Quantum Mechanics*

8.7 X-rays

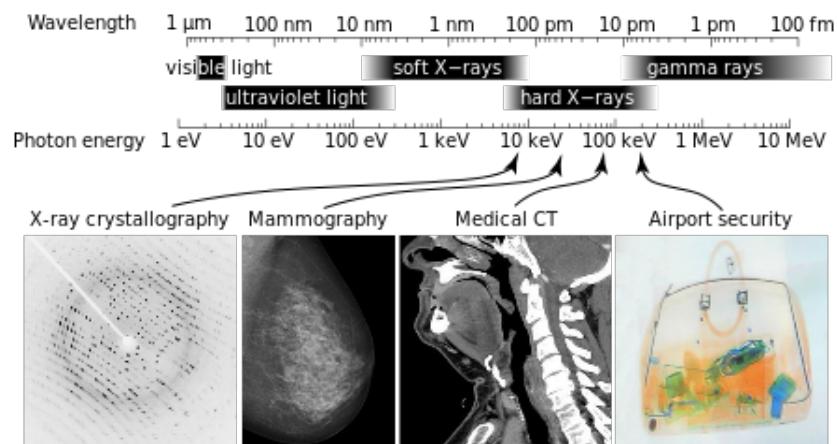


Figure 3: <https://en.wikipedia.org/wiki/X-ray>

Xray transitions	<p>The diagram illustrates the Bohr model of an atom with five concentric elliptical orbits labeled n=1 through n=5. The innermost shell is labeled K, followed by L, M, N, and the outermost by n=5. Red arrows represent X-ray transitions between different shells. Transitions from higher shells to the K shell are labeled L_γ, L_β, L_α, M_α, and M_β. Transitions within the K shell are labeled K_δ, K_γ, K_β, and K_α. A URL at the bottom of the diagram is http://hyperphysics.phy-astr.gsu.edu/hbase/quantum/xterm.html</p>
Moseley's Law & description	

8.8 Atoms in electric and magnetic fields

9 Nuclear and Particle

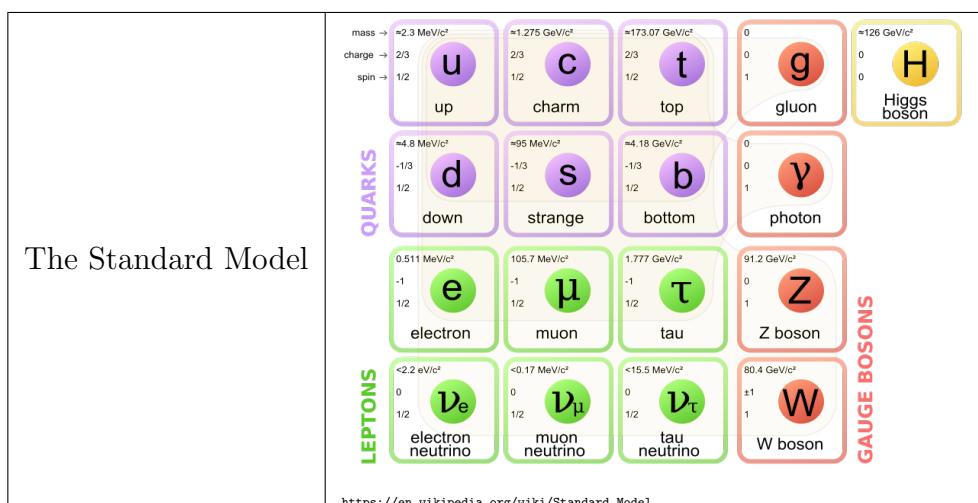
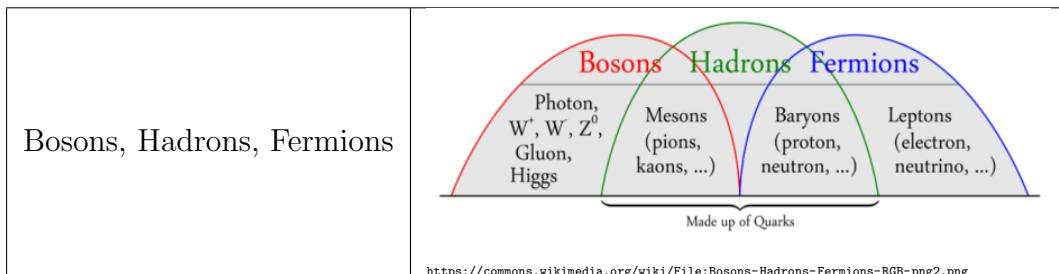
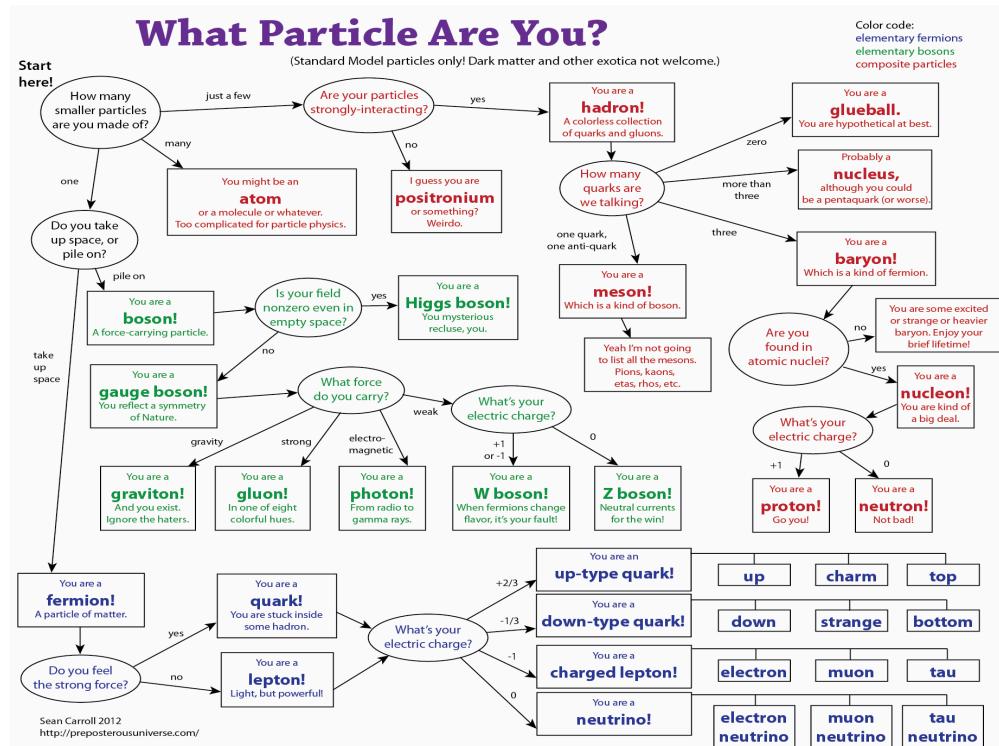
nuclear properties, radioactive decay, fission and fusion, reactions, fundamental properties of elementary particles

9.1 Light and matter interaction

Scattering Cross Section	$\sigma = \frac{\mu}{n} = \frac{1}{n\Phi} \left(-\frac{d\Phi}{dz} \right) = \frac{1}{nIA} \frac{dW}{dz} = \frac{P}{\rho l}$, where σ is the cross section of the event (m^2); μ is the attenuation coefficient due to the occurrence of this event (m^{-1}); n is the number density of the target particles (m^{-3}); Φ is the flux of the incident beam; $-d\Phi$ is the amount of flux lost due to the occurrence of the event (i.e. amount scattered); dz is the thickness of the target material; I is the particle flux(or intensity) of the incident beam ($\text{m}^{-2}\text{s}^{-1}$); A is the area of overlap between beam and target (m^2); dW is the rate at which the event occurs (s^{-1}); P is the probability that a beam particle is scattered; ρ is the number density (N/V) of target particles; l is the length of the sample (i.e. same as dz). If a scattering question is asked, you can probably use dimensional analysis assuming linear relations.
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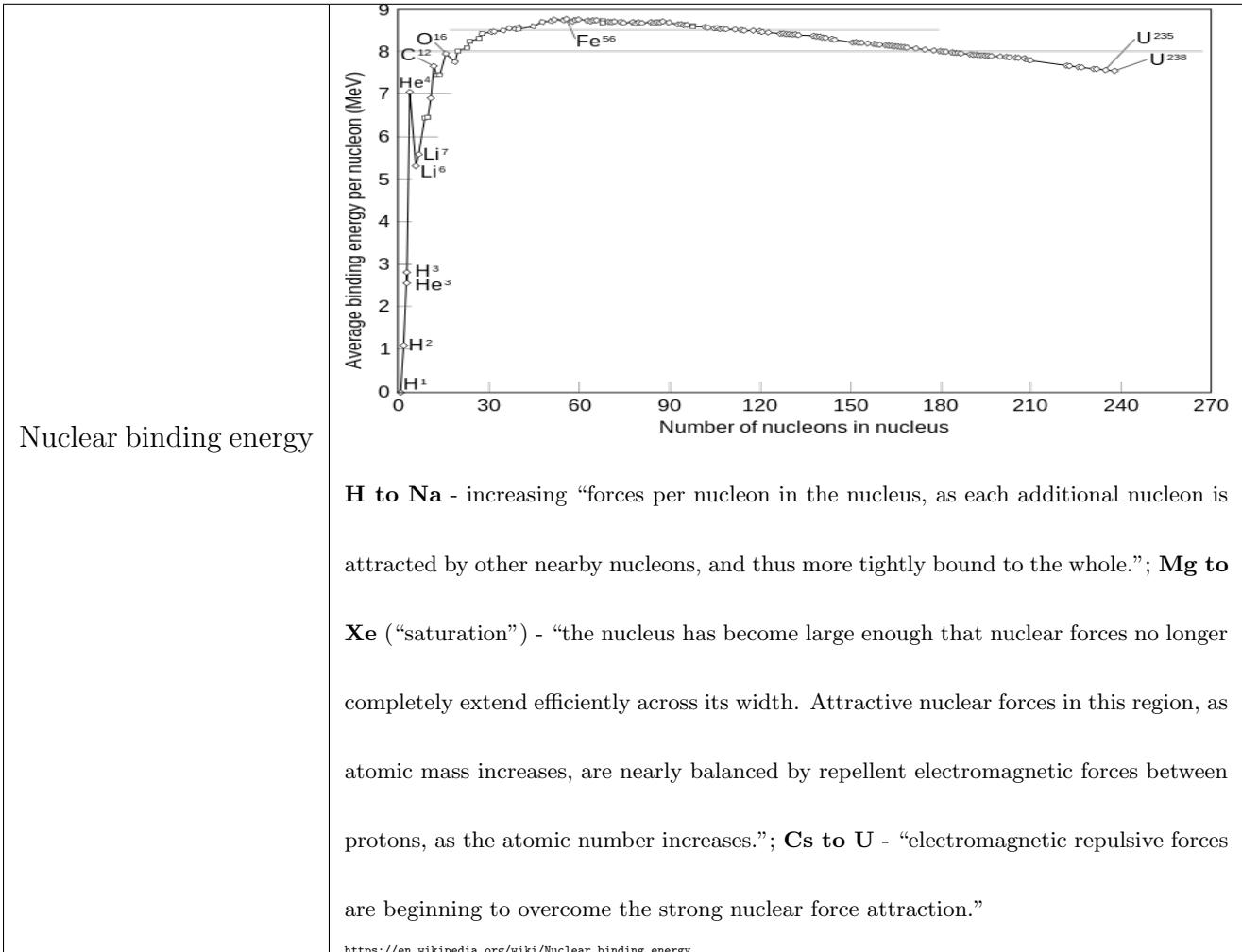
Energy ranking of the four major light/matter interactions	(low energy) Photoelectric → Thomson → Compton → Pair production (high energy)
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9.2 Particles

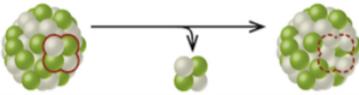
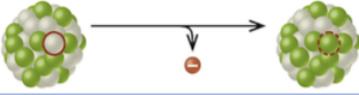
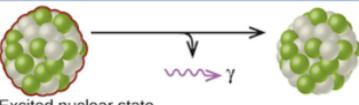
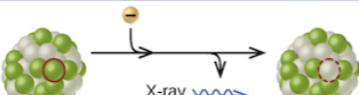


Protium	1H
Deuterium	Symbol: D or 2H . Also known as ‘heavy hydrogen’. Nucleus consists of one proton and one neutron.
Deuteron	The nucleus of deuterium. It is a boson and exists in the triplet state. The singlet state is virtual and only transiently exists during neutron-proton inelastic scattering. Duh!
Tritium	T or 3H . One proton, two neutrons. Watch out.
Triton	Yup. You guessed it.

9.3 Nuclear properties

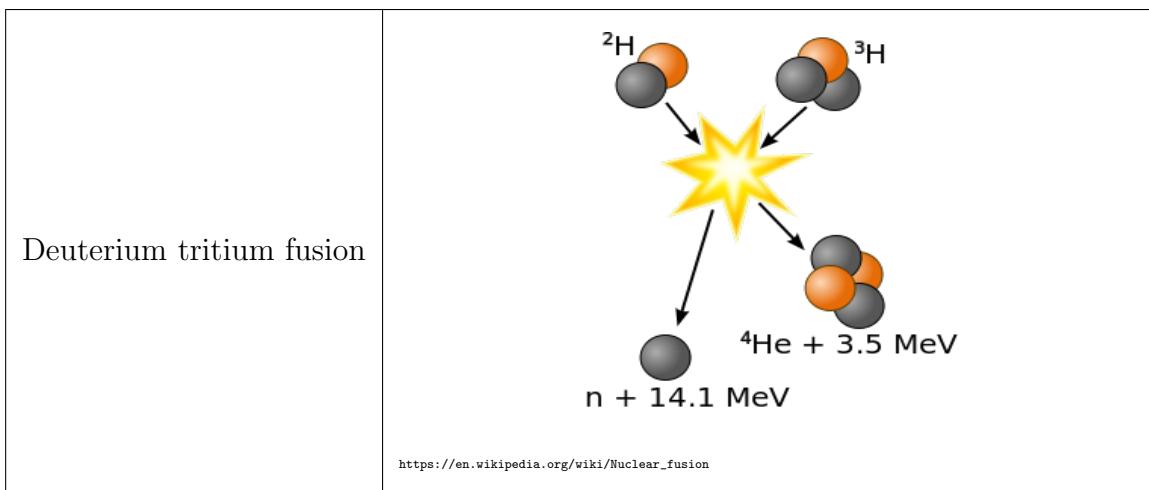


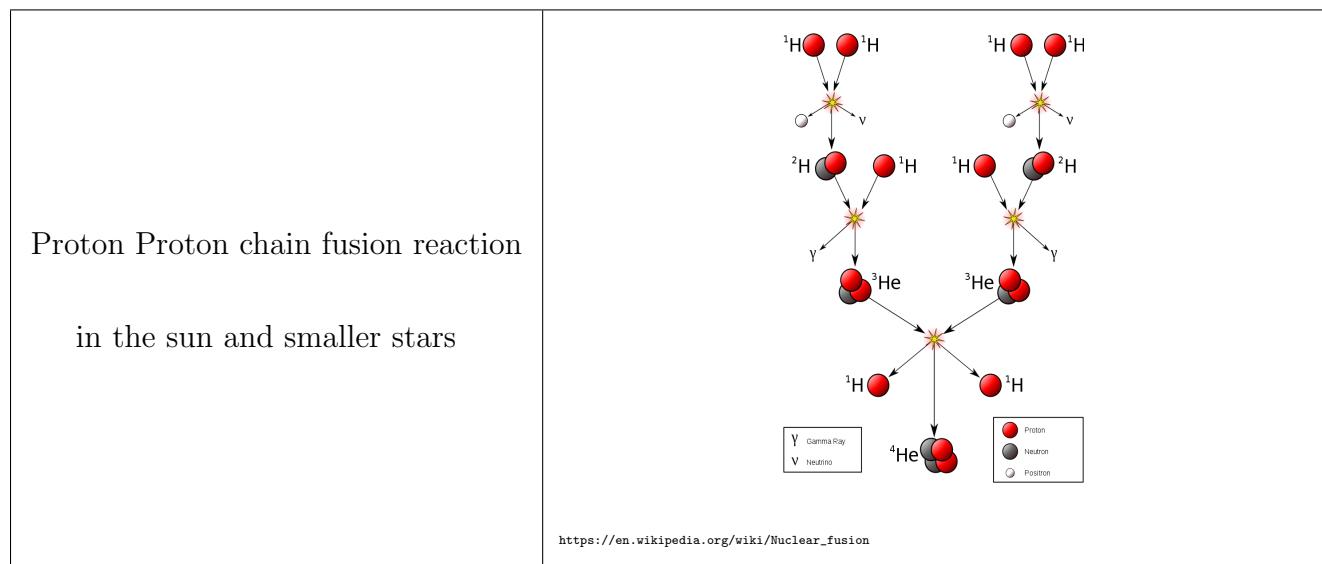
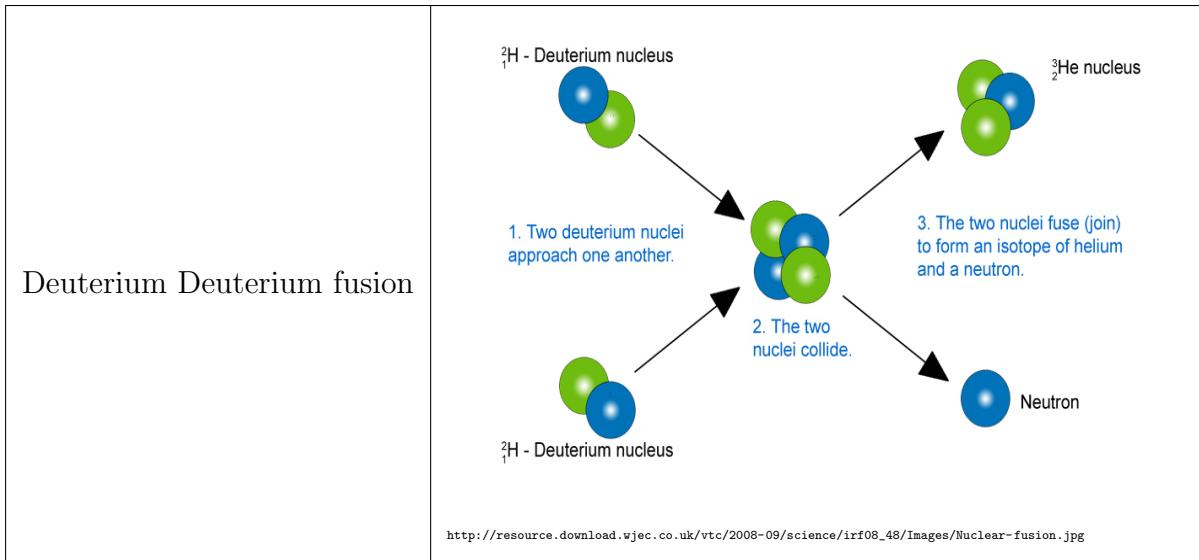
9.4 Radioactive decay

Type	Nuclear equation	Representation	Change in mass/atomic numbers
Alpha decay	${}_{Z}^{A}X \rightarrow {}_{2}^{4}\text{He} + {}_{Z-2}^{A-4}Y$		A: decrease by 4 Z: decrease by 2
Beta decay	${}_{Z}^{A}X \rightarrow {}_{-1}^{0}\text{e} + {}_{Z+1}^{A}Y$		A: unchanged Z: increase by 1
Gamma decay	${}_{Z}^{A}X \rightarrow {}_{0}^{0}\gamma + {}_{Z}^{A}Y$	 Excited nuclear state	A: unchanged Z: unchanged
Positron emission	${}_{Z}^{A}X \rightarrow {}_{+1}^{0}\text{e} + {}_{Z-1}^{A}Y$		A: unchanged Z: decrease by 1
Electron capture	${}_{Z}^{A}X + {}_{-1}^{0}\text{e} \rightarrow {}_{Z-1}^{A}Y + \text{X-ray}$		A: unchanged Z: decrease by 1

[http://chemwiki.ucdavis.edu/Textbook_Maps/General_Chemistry_Textbook_Maps/Map%3A_Chemistry_\(OpenSTAX\)/21%3A_Nuclear_Chemistry/21.3%3A_Radioactive_Decay](http://chemwiki.ucdavis.edu/Textbook_Maps/General_Chemistry_Textbook_Maps/Map%3A_Chemistry_(OpenSTAX)/21%3A_Nuclear_Chemistry/21.3%3A_Radioactive_Decay)

9.5 Fission and fusion





9.6 Reactions

Chemical symbol convention	${}^A_Z Na^C$ where $A = \text{Mass } \# \text{ of protons + neutrons},$ $Z = \text{Atomic } \# \text{ of protons},$ $C = \text{Charge, e.g. } '+2'$
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9.7 Fundamental properties of elementary particles

10 Special Relativity

(such as introductory concepts, time dilation, length contraction, simultaneity, energy and momentum, four-vectors and Lorentz transformation, velocity addition)

10.1 Introductory concepts

An <i>event</i> is a location and a time: (x, y, z, ct) .
$\beta =$	v/c generally < 1
$\gamma =$	$\frac{1}{\sqrt{1 - \beta^2}}$ generally > 1

10.2 Lorentz invariant intervals

Lorentz invariant interval	$s^2 = \Delta x^2 + \Delta y^2 + \Delta z^2 - (c\Delta t)^2$ $= \Delta x'^2 + \Delta y'^2 + \Delta z'^2 - (c\Delta t')^2$
Time-like interval, description and inequality:	Enough time passes between two events that they could be causally related. $s^2 < 0 \text{ or } c^2\Delta t^2 > \Delta r^2$
Proper time interval $\Delta\tau$	Would be measured by an observer traveling between two time-like events in an inertial frame. $\Delta\tau = \sqrt{\Delta t^2 - \frac{\Delta r^2}{c^2}}$
Light-like interval	Events which occur to or are initiated by a photon along its path. $s^2 = 0 \text{ or } c^2\Delta t^2 = \Delta r^2$
Space-like interval	Not enough time between the events for the possibility of a causal relation. There exists a ref. frame in which the two events are simultaneous, but no ref. frame in which they occur in the spatial location. $s^2 > 0 \text{ or } c^2\Delta t^2 < \Delta r^2$
Proper distance $\Delta\sigma$	The measurement of space-like separation between events. $\Delta\sigma = \sqrt{s^2} = \sqrt{\Delta r^2 - c^2 t^2}$

10.3 Time dilation

‘Moving clocks run slower.’
Proper time $\Delta\tau =$	Time read on the face of the moving clock (i.e. clock’s ref. fr.) $\frac{\Delta t}{\gamma}$ Lorentz invariant

Relativistic Doppler Effect	$\frac{\lambda_o}{\lambda_s} = \sqrt{\frac{1+\beta}{1-\beta}}$ $\beta = \frac{(\frac{\lambda_o}{\lambda_s})^2 - 1}{(\frac{\lambda_o}{\lambda_s})^2 + 1}$ Where β is v/c and $+v$ means the object is going away from the observer.
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10.4 Length contraction

‘Moving sticks are...	... shorter.’
Proper length ΔL_0	Length measured in ref. fr. of the stick. $\Delta L_0 = \gamma \Delta L$

10.5 Simultaneity

10.6 Energy and momentum

For a particle with rest mass m_0 and velocity \vec{v} $\mathbf{p} =$	$\gamma m_0 \mathbf{v}$
$E =$	$\gamma m_0 c^2 = RestE + KE$
Kinetic energy	$KE = E - RestE = mc^2 - m_0 c^2 = m_0 c^2 (\gamma - 1)$
Energy and momentum relation	$ \begin{aligned} p^2 c^2 &= \gamma^2 m_0^2 v^2 c^2 \\ &= \gamma^2 m_0^2 \frac{v^2}{c^2} c^4 \\ &= \gamma^2 \left(m_0^2 \frac{v^2}{c^2} c^4 - m_0^2 c^4 + m_0^2 c^4 \right) \\ &= \gamma^2 \left(m_0^2 c^4 \left(\frac{v^2}{c^2} - 1 \right) + m_0^2 c^4 \right) \\ &= -m_0^2 c^4 + \gamma^2 m_0^2 c^4 \end{aligned} $ <p>Rearrange: $E^2 = (pc)^2 + (m_0 c^2)^2$</p>

10.7 Lorentz transformation

Lorentz transformation		
$x' =$		$\gamma(x - \beta ct)$
$ct' =$		$\gamma(ct - \beta x)$
Inverse Lorentz Transformation		$v \rightarrow -v$
$x =$		$\gamma(x' + \beta ct')$
$ct =$		$\gamma(ct' + \beta x')$
Electromagnetic fields parallel and perpendicular to \mathbf{v}		$\mathbf{E}'_{\parallel} = \mathbf{E}_{\parallel}$ $\mathbf{B}'_{\parallel} = \mathbf{B}_{\parallel}$ $\mathbf{E}'_{\perp} = \gamma(\mathbf{E}_{\perp} + \mathbf{v} \times \mathbf{B})$ $\mathbf{B}'_{\perp} = \gamma\left(\mathbf{B}_{\perp} - \frac{1}{c^2}\mathbf{v} \times \mathbf{E}\right)$

10.8 Derivation of $E = mc^2$ using four-vectors

$x^\mu =$	(x, y, z, ct) where $\mu = 1, 2, 3, 4$
$x_\mu =$	$(x, y, z, -ct)$
$x^\mu x_\mu =$	$(x^2 + y^2 + z^2 - (ct)^2)$
$\Delta x^\mu =$	$(\Delta x, \Delta y, \Delta z, c\Delta t) = (\Delta \vec{r}, c\Delta t)$
$\frac{\Delta x^\mu}{\Delta \tau} =$	$\gamma \frac{\Delta x^\mu}{\Delta t}$ $= (\gamma v_x, \gamma v_y, \gamma v_z, \gamma c)$
four-velocity	$= (\gamma \vec{v}, \gamma c)$
$m \frac{\Delta x^\mu}{\Delta \tau} =$	$m\gamma \frac{\Delta x^\mu}{\Delta t}$
four-momentum	$= (\gamma m \vec{v}, \gamma mc) = p^\mu$
$p^\mu p_\mu =$ rest and non-rest frames	rest frame, $\gamma = 1$, so $p^\mu = (0, mc)$ non-rest, $p^\mu = (\gamma m \vec{v}, \gamma mc)$ four-vector dot product is invariant, so $p^\mu p_\mu$ is the same for rest and non-rest: $p^\mu p_\mu = (\gamma mv)^2 - (\gamma mc)^2 = -(mc)^2$
$(\gamma mv)^2 - (\gamma mc)^2 = -(mc)^2$ Rewrite, solve in terms of E .	$(p)^2 - (E/c)^2 = -(mc)^2$ Multiply by c^2 and rearrange. $E^2 = (pc)^2 + (mc^2)^2$

10.9 Four-vectors and Lorentz transformation

10.10 Relativistic Velocity Addition

$$v_{tot} = \boxed{\frac{v_1 + v_2}{1 + \frac{v_1 v_2}{c^2}}}$$

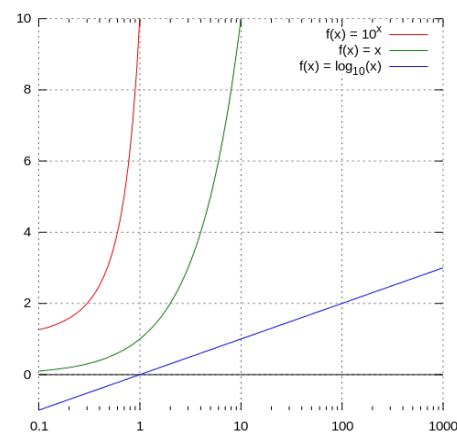
10.11 Twin astronauts problem

11 Laboratory Methods

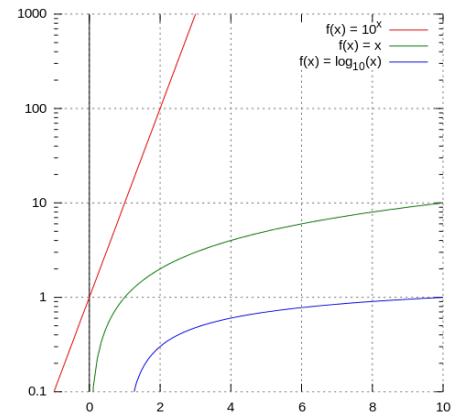
(such as data and error analysis, electronics, instrumentation, radiation detection, counting statistics, interaction of charged particles with matter, lasers and optical interferometers, dimensional analysis, fundamental applications of probability and statistics)

11.1 Data and error analysis

Semi-log (lin-log and log-lin)

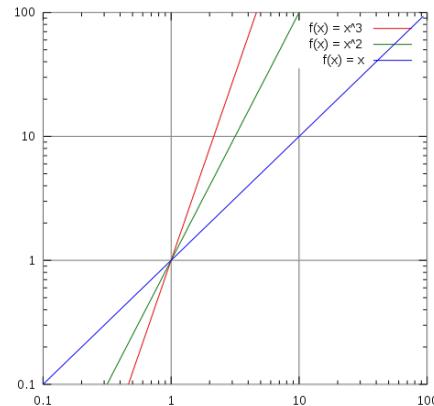


Plot 3 characteristic $f(x)$



https://en.wikipedia.org/wiki/Semi-log_plot

Plot 3 characteristic $f(x)$ for log-log



https://en.wikipedia.org/wiki/Log%E2%80%93log_plot

Uncertainty or error δR for $R(X, Y, \dots)$	$\delta R = \sqrt{\left(\frac{\partial R}{\partial X} \cdot \delta X\right)^2 + \left(\frac{\partial R}{\partial Y} \cdot \delta Y\right)^2 + \dots}$
	$R = X + Y - Z$
Addition of measured quantities	$\delta R \approx \delta X + \delta Y + \delta Z$
	$\delta R = \sqrt{(\delta X)^2 + (\delta Y)^2 + (\delta Z)^2}$
Multiplication of measured quantities	$R = \frac{X \cdot Y}{Z}$ $\frac{\delta R}{ R } \approx \frac{\delta X}{ X } + \frac{\delta Y}{ Y } + \frac{\delta Z}{ Z }$ $\delta R = R \cdot \sqrt{\left(\frac{\delta X}{ X }\right)^2 + \left(\frac{\delta Y}{ Y }\right)^2 + \left(\frac{\delta Z}{ Z }\right)^2}$
Multiplication of with a constant	$R = c \cdot X$ $\delta R = c \cdot \delta X$
Polynomial functions	$R = X^n$ $\delta R = R \cdot n \cdot \frac{\delta X}{ X }$ <p style="text-align: right;"><small>http://lectureonline.cl.msu.edu/~mmp/labs/error/e2.htm</small></p>

11.2 Electronics

11.3 Instrumentation

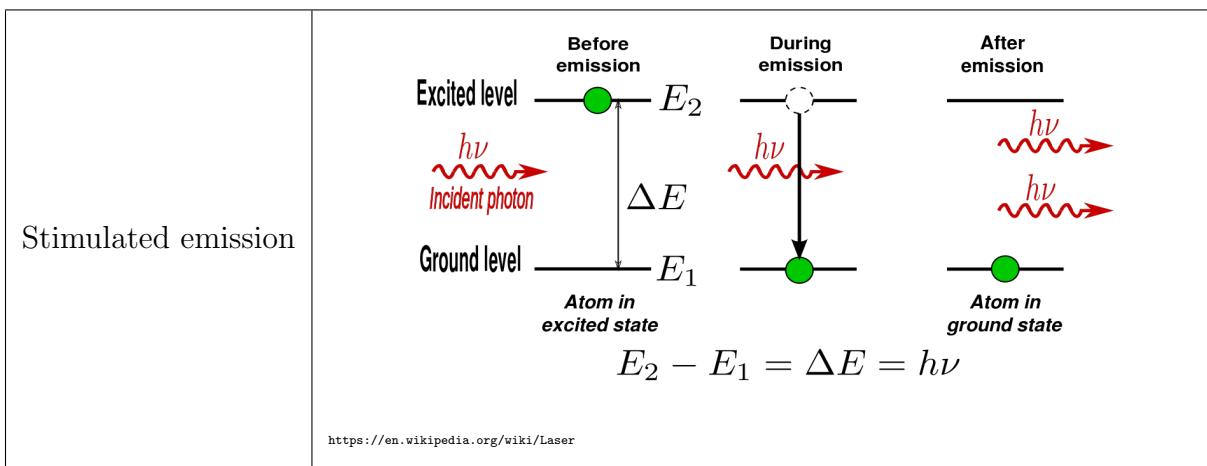
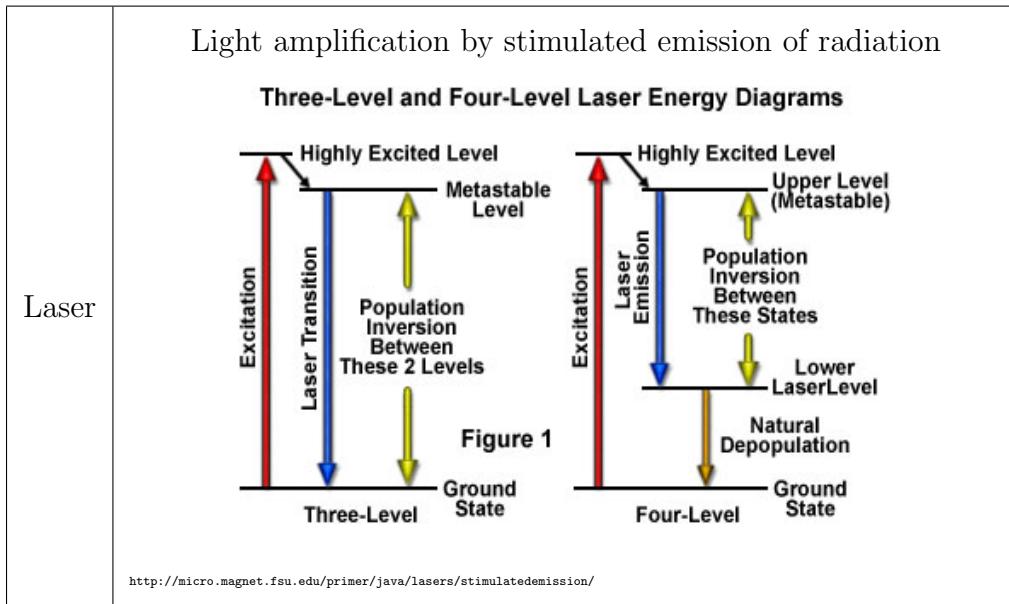
11.4 Radiation detection

11.5 Counting statistics

Poisson distribution	$P(k \text{ events in interval}) = \frac{\lambda^k e^{-\lambda}}{k!}$
When to use?	<p>λ is the avg. # of events per interval.</p> <p>k is the integer # of times an event occurs in an interval.</p> <p>Events occur independently.</p> <p>Constant rate of occurrence.</p> <p>Events cannot be identical in space and time.</p> <p>Probability of an event in an interval \propto length of interval.</p> <p>e.g. # of decays from a radioactive sample over 1 hour.</p>
Properties	<p>mean = variance = λ, so $\sigma = \sqrt{\text{var}} = \sqrt{\lambda}$</p> <p>If $\lambda >$ about 10, normal distribution is good approximation</p> <p>if continuity correction is used. See:</p> <p>https://en.wikipedia.org/wiki/Poisson_distribution</p>

11.6 Interaction of charged particles with matter

11.7 Lasers and optical interferometers



11.8 Dimensional analysis

11.9 Fundamental applications of probability and statistics

12 Miscellany

Condensed Matter (e.g., crystal structure, x-ray diffraction, thermal properties, electron theory of metals, semiconductors, superconductors) Miscellaneous (e.g., astrophysics, mathematical methods, computer applications)

12.1 Chemistry

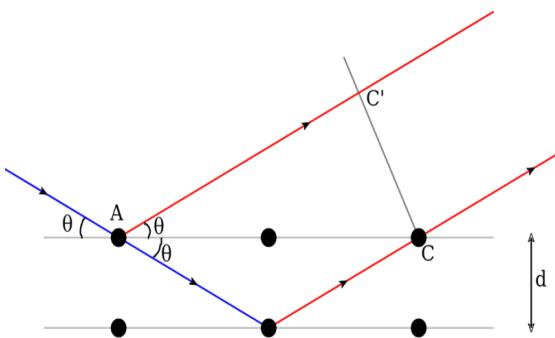
12.2 Plasma

Debye length	$\lambda_D = \sqrt{\frac{\epsilon_0 k_B T_e}{n_e q_e^2}}$
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12.3 Condensed Matter

12.4 Crystal structure

12.5 X-ray diffraction

Bragg Diffraction	$n\lambda = 2d \sin \theta$ like a thin film, but using a different angle, i.e. the angle between the beam and the surface plane.  <p>https://en.wikipedia.org/wiki/Bragg%27s_law</p>
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12.6 Thermal properties

12.7 Electron theory of metals

At the highest energies of the valence band in many semiconductors (Ge, Si, GaAs, ...), and the lowest energies of the conduction band in some semiconductors (GaAs, ...), the band structure $E(\mathbf{k})$ can be locally approximated as

$$E(\mathbf{k}) = E_0 + \frac{\hbar^2 \mathbf{k}^2}{2m^*}$$

Effective mass of electron in metals

$$\begin{aligned} E &= \frac{\hbar^2 k^2}{2m} \\ \frac{dE}{dk} &= \frac{\hbar^2 k}{m} = \hbar v_g \\ \frac{dv_g}{dt} &= \frac{1}{\hbar} \frac{d^2 E}{dt^2} \frac{dk}{dt} = \frac{1}{\hbar} \frac{d^2 E}{dt^2} \frac{F}{\hbar} \\ F &= \hbar^2 \left(\frac{d^2 E}{dt^2} \right)^{-1} \frac{dv_g}{dt} \\ m^* &= \hbar^2 \left(\frac{d^2 E}{dt^2} \right)^{-1} \end{aligned}$$

12.8 Semiconductors

<i>n</i> -doped semiconductor	<p><i>n</i> stands for negative, so <i>n</i>-type silicon is doped with negatively charged atoms (say, phosphorus) . This means that these atoms have extra electrons, and can easily part with the extra electron. Hence, they are donors.</p>
<i>p</i> -doped semiconductor	<p><i>p</i> stands for positive, so <i>p</i>-type silicon is doped with positively charged atoms (say, boron). This means that these atoms have missing electrons, so they can easily accept new electrons to fill the vacancy. Hence, they are acceptors.</p>

12.9 Superconductors

12.10 Astrophysics

Schwarzschild radius	$R = \frac{2MG}{c^2}$
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12.11 Mathematical methods

12.12 Computer applications