```
function Np1BodyProblem(N)

% System Properties:
S.G = 6.6742*10^(-11); % Gravitational Parameter[Nm^2/kg^2]
S.N = N; % Number of Bodies
S.mN = rand([N,1])*5*10^24+5*10^24; % Mass of Bodies[kg]
S.m = rand*5*10^23+5*10^23; % Mass of N+1 body[kg]

% Initial Conditions:
xN = (rand([3*N,1])-.5)*5*10^13; % Initial Position of N bodies
vN = (rand([3*N,1])-.5)*15; % Initial Velocity of N bodies
x1 = (rand([3,1])-.5)*5*10^13; % Initial Position of N+1 body
v1 = (rand([3,1])-.5)*1; % Initial Velocity of N+1 body

xbary = reshape(xN,3,N)*S.mN/sum(S.mN); % bary center of system
vbary = reshape(vN,3,N)*S.mN/sum(S.mN); % bary center velocity
xNbary = reshape(reshape(xN,3,N)-xbary,3*N,1); % move barycenter to
 origin
x1bary = x1-xbary;
vNbary = reshape(reshape(vN,3,N)-vbary,3*N,1);
v1bary = v1-vbary;

% Barycentric System ICs
x0bary = [xNbary;
     vNbary;
     x1bary;
     v1bary];

xbody = xN(3*N-2:3*N); % body center of system
vbody = vN(3*N-2:3*N);
xNbody = reshape(reshape(xN,3,N)-xbody,3*N,1); % move body center to
 origin
x1body = x1-xbody;
vNbody = reshape(reshape(vN,3,N)-vbody,3*N,1);
v1body = v1-vbody;

% Bodycentric system ICs
x0body = [xNbody;
     vNbody;
     x1body;
     v1body];

% Simulate:
tf = 10^13;
[tbary,xbary] = ode45(@(t,x) xdotbary(S,x),[0
 tf],x0bary,odeset('AbsTol',1e-12,'RelTol',1e-9));
[tbody,xbody] = ode45(@(t,x) xdotbody(S,x),[0
 tf],x0body,odeset('AbsTol',1e-12,'RelTol',1e-9));

% Plot:
plotSim(S,xbary',tbary);
plotSim(S,xbody',tbody);
```

```
disp('Final Position of (N+1)st body relative to Nth body from
 barycentric formulation:');
xbary(end,6*S.N+1:6*S.N+3)-xbary(end,3*S.N-2:3*S.N)
%xbary(end,1:3)-xbary(end,3*S.N-2:3*S.N)

disp('Final Position of (N+1)st body relative to Nth body from
 bodycentric formulation:');
xbody(end,6*S.N+1:6*S.N+3)
%xbody(end,1:3)

end

function xdot = xdotbary(S,x)

xdot = zeros(size(x));

% N-bodies:
for i = 1:S.N
    ri = x(3*i-2:3*i);
    xdot(3*i-2:3*i) = x(3*i-2+S.N*3:3*i+S.N*3);
    for j = 1:S.N
        if i == j
            continue;
        end
        rj = x(3*j-2:3*j);
        rji = ri-rj;
        xdot(3*i-2+S.N*3:3*i+S.N*3) = xdot(3*i-2+S.N*3:3*i+S.N*3) -
 S.G*S.mN(j)*rji/norm(rji)^3;
    end
end

% N+1 body:
ri = x(6*S.N+1:6*S.N+3);
xdot(6*S.N+1:6*S.N+3) = x(6*S.N+4:6*S.N+6);
for j = 1:S.N
    rj = x(3*j-2:3*j);
    rji = ri-rj;
    xdot(6*S.N+4:6*S.N+6) = xdot(6*S.N+4:6*S.N+6) - S.G*S.mN(j)*rji/
norm(rji)^3;
end

end

function xdot = xdotbody(S,x)

xdot = zeros(size(x));

% N-bodies:
rN = x(3*S.N-2:3*S.N);
vN = x(6*S.N-2:6*S.N);
for i = 1:S.N-1
    ri = x(3*i-2:3*i)-rN;
    xdot(3*i-2:3*i) = x(3*i-2+S.N*3:3*i+S.N*3)-vN;
```

```
    xdot(3*i-2+S.N*3:3*i+S.N*3) = -S.G*(S.mN(S.N)+S.mN(i))/
norm(ri)^3*ri;
    for j = 1:S.N-1
        if i == j
            continue;
        end
        rj = x(3*j-2:3*j)-rN;
        rji = ri-rj;
        xdot(3*i-2+S.N*3:3*i+S.N*3) = xdot(3*i-2+S.N*3:3*i+S.N*3) -
 S.G*S.mN(j)*(rji/norm(rji)^3+1/norm(rj)^3*rj);
    end
end

% N+1 body:
ri = x(6*S.N+1:6*S.N+3)-rN;
xdot(6*S.N+1:6*S.N+3) = x(6*S.N+4:6*S.N+6);
xdot(6*S.N+4:6*S.N+6) = -S.G*S.mN(S.N)/norm(ri)^3*ri;
for j = 1:S.N-1
    rj = x(3*j-2:3*j)-rN;
    rji = ri-rj;
    xdot(6*S.N+4:6*S.N+6) = xdot(6*S.N+4:6*S.N+6) - S.G*S.mN(j)*(rji/
norm(rji)^3+rj/norm(rj)^3);
end

end

function [EN,HN,EN1,HN1] = ConservedQuantities(S,t,x)

% N- Bodies
T = zeros(1,length(x));
U = zeros(1,length(x));
HN = zeros(3,length(x));
for i = 1:S.N
    ri = x(3*i-2:3*i,:);
    vi = x(3*i-2+S.N*3:3*i+S.N*3,:);
    T = T + 1/2*S.mN(i)*vecnorm(vi).^2;
    HN = HN + S.mN(i)*cross(ri,vi);
    for j = 1:S.N
        if i == j
            continue;
        end
        rj = x(3*j-2:3*j,:);
        U = U + 1/2*S.G*S.mN(i)*S.mN(j)./vecnorm(ri-rj);
    end
end

EN = T-U;

% N+1 body
rN1 = x(6*S.N+1:6*S.N+3,:);
vN1 = x(6*S.N+4:6*S.N+6,:);
HN1 = HN + S.m*cross(rN1,vN1);
T = T + 1/2*S.m*vecnorm(vN1).^2;
for j = 1:S.N
```

```
    rj = x(3*j-2:3*j,:);
    U = U + 1/2*S.G*S.m*S.mN(j)./vecnorm(rN1-rj);
end
EN1 = T-U;

end

function plotSim(S,x,t)

figure; hold on;
for i = 1:S.N
    plot3(x(3*i-2,1),x(3*i-1,1),x(3*i,1),'g*');
    plot3(x(3*i-2,end),x(3*i-1,end),x(3*i,end),'r*');
    plot3(x(3*i-2,:),x(3*i-1,:),x(3*i,:),'-k');
end
plot3(x(6*S.N+1,1),x(6*S.N+2,1),x(6*S.N+3,1),'g*');
plot3(x(6*S.N+1,end),x(6*S.N+2,end),x(6*S.N+3,end),'r*');
plot3(x(6*S.N+1,:),x(6*S.N+2,:),x(6*S.N+3,:),'-b');

[EN,HN,EN1,HN1] = ConservedQuantities(S,t,x);

figure; hold on;
subplot(2,1,1); hold on;
plot(t,EN);
plot(t,EN1);
legend('N-Body Problem Energy','(N+1)st-Body Problem Energy');
xlabel('Time[s]'); ylabel('Energy[J]');
subplot(2,1,2); hold on;
plot(t,vecnorm(HN));
plot(t,vecnorm(HN1));
legend('N-Body Problem Momentum','(N+1)st-Body Problem Momentum');
xlabel('Time[s]'); ylabel('Momentum[kgm^2/s]');


end
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