

# Computational Physics HW1

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## 1 Introduction

A harmonic oscillator is defined by a system that experiences a restorative force proportional to its displacement from its rest location. This is known as Hook's Law and the mathematical form is seen in  $F_x = -kx$ .  $F_x$  is the force on the object.  $k$  is an arbitrary constant determined by the physical properties of a system (i.e. for a spring  $k$  would be determined by the stiffness of the material).  $x$  is the displacement of an object from its "rest" position.

This Homework deals with a special case where an object follows this law in X and Y independantly. As such it is governed by the following 1.  $A$  is the maximum amplitude reached by the system in a particular direction.  $\omega = \sqrt{k/m}$ .  $m$  is the mass of the object oscillating.  $\phi$  is the initial phase position of the system.

$$x(t) = A_x \sin(\omega_x t + \phi_x) \quad y(t) = A_y \sin(\omega_y t + \phi_y) \quad (1)$$

The first part of the homework deals with cases when  $\omega_x$  and  $\omega_y$  are equal.

For the Bonus Problems I will explore some of the possibilities when  $\omega_x$  and  $\omega_y$  are varied independently.

## 2 Results

### Question 1

Code submitted on Blackboard

### Question 2

See attached 4

### Question 3

If  $\omega_x = \omega_y$  then the range of shapes possible are rather limited. The variation is from perfectly circular to a line. As  $\Delta\phi$  grows from 0 to  $\pi$  the shape transitions from a line, of  $y = A_x/A_y x$ , through an elliptical shape to a circle, at  $\Delta\phi = \pi/2$ , then back to a line, of  $y = -A_x/A_y x$ .

### Bonus 1

For small ratios of  $\omega_x$  and  $\omega_y$  a closed loop was formed and the pattern would repeat like the center plot in 3, so long as  $\Delta\phi \neq n\pi$  where  $n$  is an integer. If  $n$  is an integer, then instead of "looping" the system will oscillate back and forth along a single path like seen in the first plot in 3.

### Bonus 2

If the ratios of  $\omega_x$  and  $\omega_y$  are irrational numbers i.e.  $\sqrt{5}$  like seen in the right most plot of 3, then the system cannot repeat. This leads to what appears to be random dots. But if you look carefully you can see a structure to the dot. The system can almost, but not quite loop back over itself and leave a series of nearly parallel paths.

### 3 Conclusions

Main challenges faced in this coding project were discovering the "-lm" flag for gcc. It also served a good refresher on how to work with c. Furthermore, I had never worked with Mathematica before, discovering some of its' features for the first time was fun.

### 4 Plots

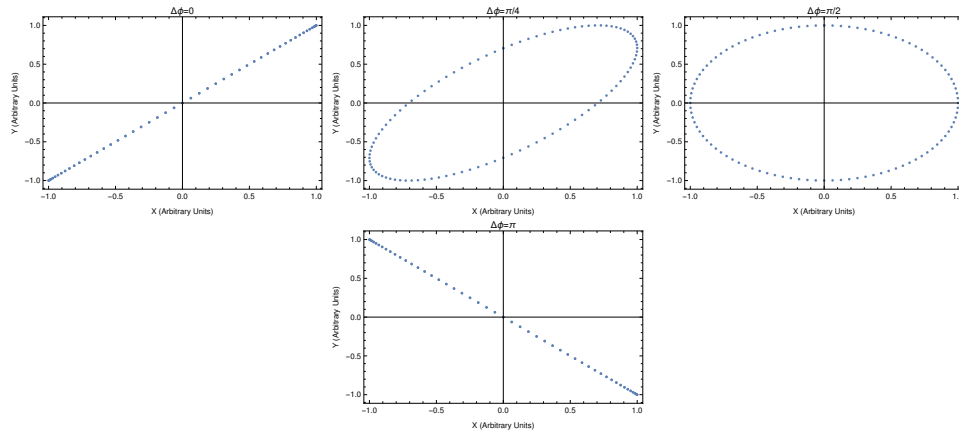


Figure 1: These plots above show the types of shapes possible to make using this system of harmonic oscillators.

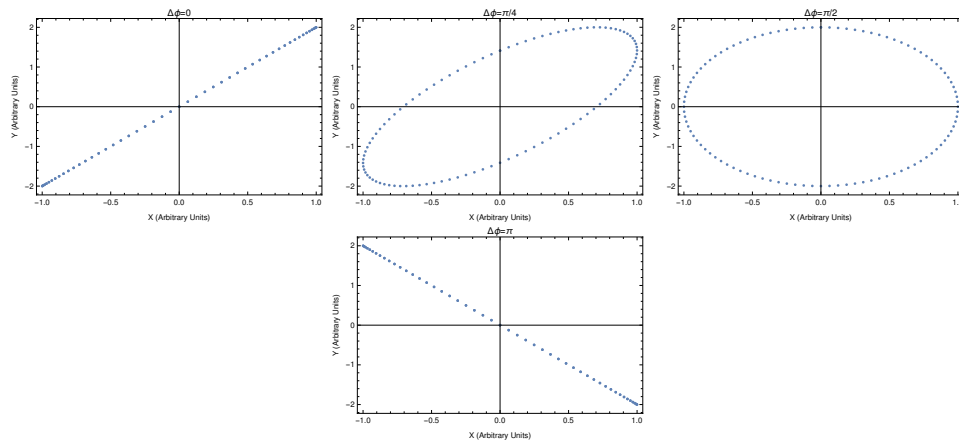


Figure 2: These plots are nearly identical to the plots above, with the exception that in the amplitude in the y direction is twice what it would be in the corresponding picture above.

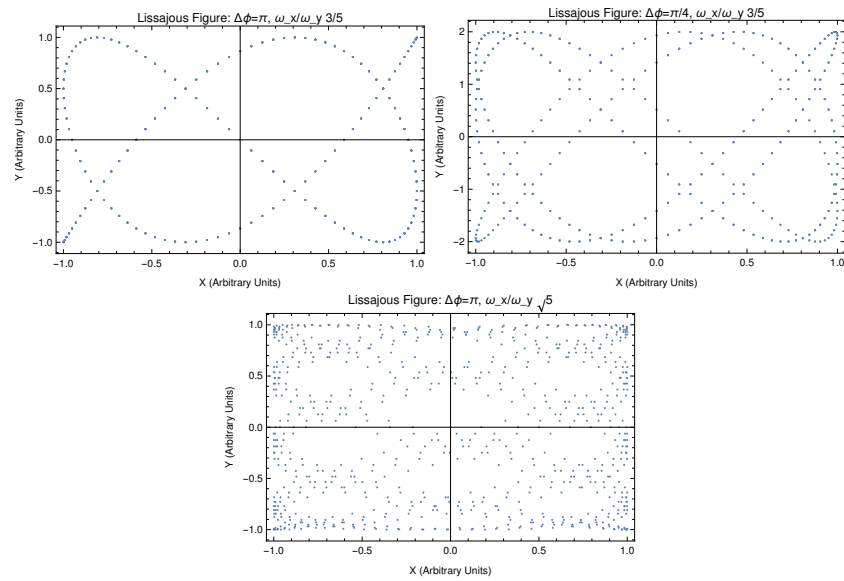


Figure 3: Seen above are some of the patterns formed by Lissajous figures