Problem Set III

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1 Problem 1

Tayler expansion of $e^{-iqr}V(r)$

$$\frac{df}{dr'} = -iqe^{-iqr'}V(r') + \frac{dV}{dr'}(r')e^{-iqr'} \tag{1}$$

$$\frac{d^2f}{dr'^2} = (-iq)^2 e^{-iqr'}V(r') + \frac{dV}{dr'}(r')e^{-iqr'} - (iq)\frac{dV}{dr'}(r')e^{-iqr'} + \frac{d^2V}{dr'^2}(r')e^{-iqr'} \tag{2}$$

$$f_{Taylor}(r, r' = 0) \approx f(0) + f'(0)r = V(0) + [V(0) + \frac{dv}{dr}(0)]r \tag{3}$$

When we substiture the given equation for $V(r)$ into the above

$$f(\theta, \phi) = -\frac{m}{2\pi\hbar}[V_0 2\pi r_0(1 - iq - \frac{r_0^2}{4})] \tag{4}$$

$$|f(\theta, \phi)|^2 = \frac{m^2 r_0^2 V_0^2}{\hbar^2 16}(16q^2 + r^2 - 4) \tag{5}$$

2 Problem 2

$$k + p = k' + p' \tag{6}$$

$$k + k' = p' - p \tag{7}$$

$$(k + k')^2 = (p' - p)^2 \tag{8}$$

$$\cancelto{0}{k^2} + \cancelto{0}{k'^2} - 2k_\mu k^\mu = \cancelto{M}{p'^2} + \cancelto{M}{p^2} - 2p_\mu p'^\mu \tag{9}$$

$$-2k_\mu k^\mu = 2M - 2p_\mu p'^\mu \tag{10}$$

$$k_\mu k^\mu = p_\mu p'^\mu - M \tag{11}$$

$$k = \langle E, 0, 0, E\rangle \tag{12}$$

$$k' = \langle E', 0, E'\sin\theta, E'\cos\theta\rangle \tag{13}$$

$$p = \langle M, 0, 0, 0\rangle \tag{14}$$

$$p' = \langle M, 0, -E\sin\theta, E - E\cos\theta\rangle \tag{15}$$

$$EE' + EE'\cos\theta = M^2 - M \tag{16}$$

$$E' = \frac{M^2 - M}{E(1+\cos\theta)} \tag{17}$$

3 Problem 3

$$N = \frac{d\sigma}{d\Omega} \cdot I \cdot \Omega \cdot \alpha \cdot t \tag{18}$$

$$t = \frac{N}{\frac{\alpha^2\cos^2\frac{\theta}{2}}{E^2\sin^2\frac{\theta}{2}} \cdot I \cdot \Omega \cdot \alpha} \tag{19}$$

When all values are subbed in I found:

$$t_{10^\circ} = 3.783251e + 11 \text{ seconds}$$

$$t_{30^\circ} = 3.129442e + 13 \text{ seconds}$$

$$t_{45^\circ} = 1.634924e + 14 \text{ seconds}$$

$$t_{60^\circ} = 5.422360e + 14 \text{ seconds}$$

$$t_{90^\circ} = 3.253416e + 15 \text{ seconds}$$

$$t_{170^\circ} = 8.436348e + 17 \text{ seconds}$$

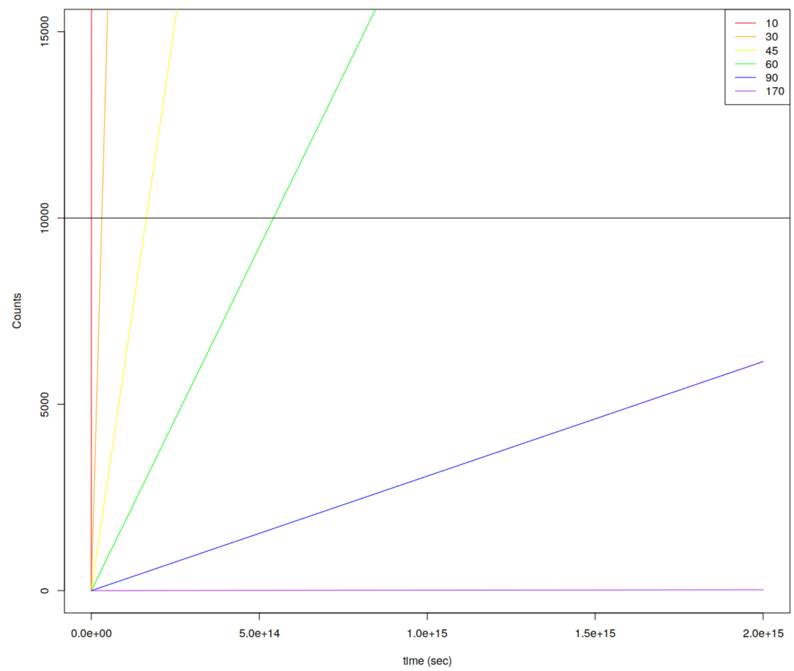

Figure 1: Time to 10000 Counts for various scattering angles

4 Problem 4

$$F(q) = \int e^{-iqr} \rho(r) d^3 r \tag{20}$$

$$F(q) = \int\int\int_0^f e^{-iqr} \rho_0 r^2 dr d\cos\theta d\phi + \cancelto{0}{\int\int\int_r^\infty e^{-iqr} \rho(0) r^2 dr d\cos\theta d\phi} \tag{21}$$

$$F(q) = \rho_0 2\pi \int r^2 \int e^{-iqr} d\cos\theta dr \tag{22}$$

$$F(q) = \rho_0 2\pi \int r * \frac{e^{-iqr\cos\theta}}{-iq} |_{-1}^{1} dr \tag{23}$$

$$F(q) = \rho_0 2\pi \int \frac{2r}{q} \sin qr dr = \frac{4\pi\rho_0(\sin qr - qr\cos qr)}{q^3} \tag{24}$$

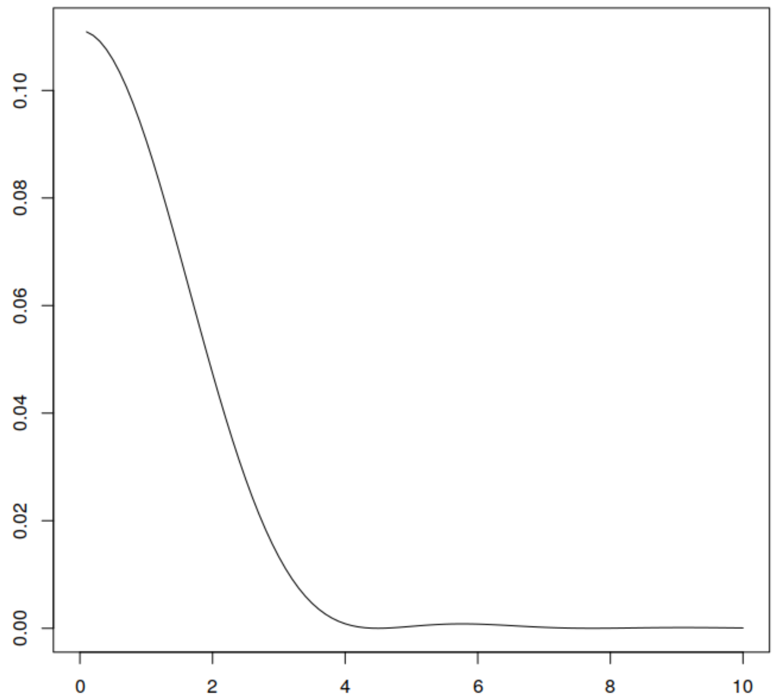

Figure 2: $|F(q)|^2$ for various momentum transfer

5 Problem 5

To do 5, I used Pythons built in fast fourier transform. This let me read in the data from the provided file, and then perform the the transform on the data.

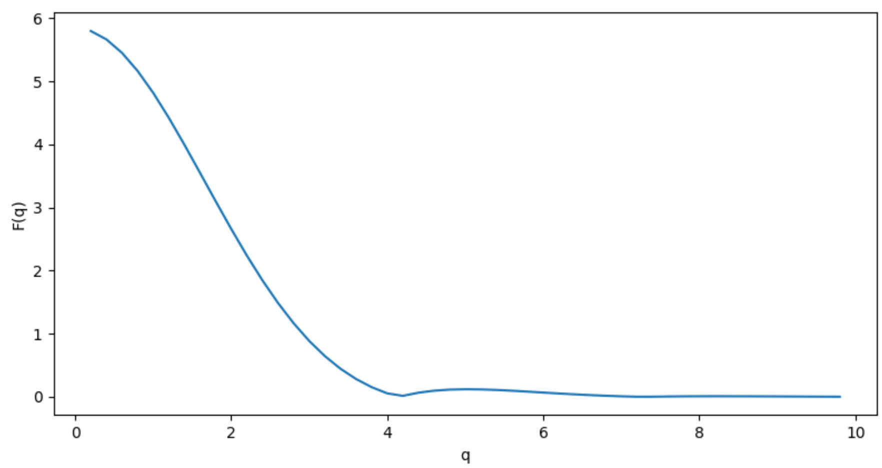

Figure 3: $F(q)$ for various momentum transfer

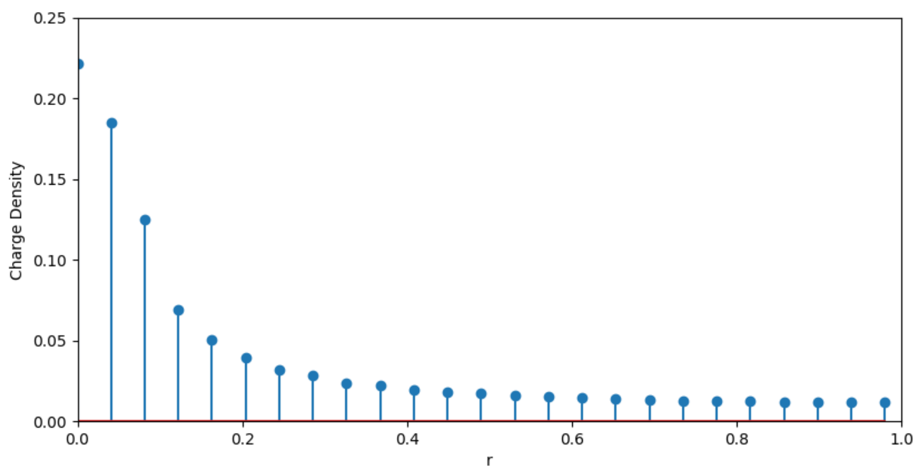

Figure 4: $F(q)$ for various momentum transfer