PHYS4175 - Nuclear Physics

Problem Set 3

Due Friday, September 25, at 17:00 EDT

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1. In 3D quantum mechanical scattering, we can use the Born approximation to write down an approximate solution to the Schrödinger equation in the limit that the potential is weak:

$$\psi(\vec{x}) = e^{ikz} + \frac{e^{ikr}}{r} f(\theta, \phi),$$
$$f(\theta, \phi) = -\frac{m}{2\pi\hbar} \int e^{-i\vec{q}\cdot\vec{r}'} V(\vec{r}')d^3r',$$

where $\vec{q}$ is the momentum transfer vector, i.e. the vector difference bewteen the incoming wave momentum vector $\vec{k} = k\hat{z}$ and the outgoing momentum vector $\vec{k}'$ pointing in the direction of θ and ϕ.

Let's imagine scattering from a spherically symmetric potential defined by:

$$V(r) = \begin{cases} \frac{V_0}{r_0}(r_0 - r) & r < r_0 \\ 0 & r \geq r_0 \end{cases}$$

in the limit where the incoming particle has **very low energy**.

   (a) Taylor expand the exponential inside the integral to find an integral expression for $f(\theta, \phi)$ for any generic potential $V(r)$. How do you interpret the lowest-order result?

   (b) Recalling that $\frac{d\sigma}{d\Omega} = |f(\theta, \phi)|^2$, find the differential cross section for the specific potential listed above, to lowest order in the Taylor expansion.

   (c) Extra credit: What is the first order correction term to the differential cross section?

2. Using the conservation of energy and momentum in relativistic kinematics, derive the expression for the final energy, E' of an electron that scatters elastically from a stationary target nucleus with mass M. The electron has initial energy E. Treat the electron as ultra-relativistic, i.e., its mass is negligible compared to E and E', and thus its energy and momentum are equal. Write the final result in terms of E, M, and the electron scattering angle θ.

There are many ways to go about this, but one of the easier ways uses Lorentz four-vectors. If k and k' are the initial and final energy-momentum four vectors of the electron, and p and p' are the initial and final four-vectors of the proton, then conservation laws require:

$$k + p = k' + p'.$$

By re-arranging and then squaring both sides, the answer can be found without much mess.

A quick reminder that when you square any energy-momentum four-vector v, you get

$$v^2 = E_v^2 - p_v^2.$$

If v is the energy-momentum four-vector of a single particle, this is simply $v^2 = E_v^2 - p_v^2 = M_v^2$.

3. You are conducting an elastic electron scattering experiment. You can shoot 100 MeV electrons with a beam current of 5 μA (i.e., 5 μC/s). You have a target with a density of 10^{18} nuclei per cm^2 (or 10^{-8} nuclei per fm^2, if you prefer). You have detector that covers 0.1 sr of solid angle. Assume the Mott scattering cross section:

$$\frac{d\sigma}{d\Omega} = \frac{\alpha^2 \cos^2 \frac{\theta}{2}}{4E^2 \sin^4 \frac{\theta}{2}}.$$

   How long would you have to collect data to achieve 1% statistical precision (i.e. 10,000 detections) if your detector is positioned at 10°, 30°, 45°, 60°, 90°, and 170°?

   Show work for full credit.

   Extra credit: make a nice computer graph.

4. Derive the form factor, $F(q)$, for a uniform sphere of charge, with total charge Q and radius R? Make a graph of your result.

   You may find the following relation useful:

$$\sin(x) = \frac{1}{2i}\left[e^{ix} - e^{-ix}\right]$$

5. **Programming Problem**
   You have conducted electron scattering experiments and measured $|F(q)|^2$ over regular intervals in q (See provided data file.) Write a program to analyze the data and visualize what the charge distribution $\rho(r)$ might look like. You may assume that $\rho(r)$ is spherically symmetric, and so is $F(q)$.

   For full credit, describe in words, with equations when necessary, how your program works. Include a graph of your results.