PHYS4175 - Nuclear Physics

Problem Set 1

Due Friday, September 11, at 12:00 EDT
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1. Proton number, neutron number, mass number

   (a) Make a table listing Z, N, and A for these nuclei with important applications: ^{14}C (carbon dating), ^{32}P (radioactive tracer in biology), ^{60}Co (useful in many applications, but a threat in "dirty bombs"), ^{99}Tc (radioactive tracer in medicine), ^{208}Pb (heaviest stable isotope), ^{226}Ra (discovered by the Curies), and ^{235}U (nuclear power).

   The interactive chart of the nuclides at `https://www.nndc.bnl.gov/nudat2/` could be a helpful resource.

   (b) Based on the number of protons and neutrons, which of these nuclear reactions is possible, and which are impossible?

   - $^{226}\text{Ra} \longrightarrow {}^{222}\text{Rn} + \alpha\ ({}^{4}\text{He})$
   - $^{235}\text{U} + n \longrightarrow {}^{138}\text{Xe} + {}^{94}\text{Sr} + 4n$
   - $^{239}\text{Pu} \longrightarrow {}^{140}\text{Cs} + {}^{98}\text{Zr} + n$

2. A type of particle accelerator called a cyclotron uses a big magnet to keep protons trapped in circular orbits, and an synchronized time-varying electric field between big electrodes to impart an additional ΔT of kinetic energy each orbit. When the protons reach the desired energy, they are ejected from the magnetic trap.

   Imagine that you have a cyclotron in which protons cross a 100 kV potential gap every orbit. The protons can escape the trap when their momentum magnitude, p, reaches 200 MeV (technically MeV/c, but we like to work in units in which $c = 1$!). Use the relativistic energy-momentum relation:

   $$E^2 = p^2 + m^2 \qquad (c = 1)$$

   to determine the terminal kinetic energy of the cyclotron ($T = E - m$), and how many orbits it takes for protons to reach that energy.

3. Derive the differential cross section in two dimensions, $db/d\Theta$ for scattering from a hard circle with radius R. By hard circle, I mean that there is no force between the projectile and the circle until the projectile hits the edge of the circle, at which point it bounces off without penetrating. This is not a problem for using Binet equations or heavy duty differential equations. This is a geometry problem. But the steps should be familiar.

   - Using geometry/trigonometry, determine the relationship between the scattering angle Θ and the impact parameter, b. Assume that the particle deflects from a surface with an equal angle from the line perpendicular to the surface.
   - Use the relation derived above to get the differential cross section, $db/d\Theta$.
   - Integrate the differential cross section to get the total cross section. How do you interpret the result?

4. Repeat the above exercise in three dimensions—scattering from a hard sphere of radius, R, solving the differential cross section, $d\sigma/d\Omega$. The steps are the same.

   - Using geometry/trigonometry, determine the relationship between the scattering angle Θ and the impact parameter, b.
   - Use the relation derived above to get the differential cross section, making use of $d\sigma = b \cdot db \cdot d\phi$ and $d\Omega = \sin\Theta \cdot d\Theta \cdot d\phi$.
   - Integrate the differential cross section to get the total cross section. How do you interpret the result?

5. **Programming Problem**
   Imagine you are scattering, in two dimensions, from a thin foil in which the atoms are hard equilateral triangles, whose sides have length 1. The orientation of these triangles, however, is completely random. Use your programming skills to make a graph of the differential cross section, $db/d\Theta$, that you would measure in the experiment.

   There are several ways you could attack this problem, and you are welcome to choose any of them. My recommendation is to use the "Monte Carlo Method", in which you draw random numbers to simulate the random orientations of different triangles.

   This problem will probably take the most time. I recommend starting early and coming to office hours to discuss it as you progress. I also recommend discussing with your classmates!

   (a) **Extra-Credit Follow-Up**
   How does the differential cross section change if the triangle is isoceles, in which one side has length 1.5, or 0.5?