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Z^0 Boson Mass Calculation

I. Introduction

The following report overviews my team's calculation of the Z^0 boson mass, an important part of the ATLAS experiment at CERN. The Large Hadron Collider (LHC) at CERN was designed to force proton-proton collisions. These collisions "break" the protons and form fundamental particles, including Z^0 bosons. These Z^0 bosons then decay, occasionally into a pair of leptons. These interactions are denoted as $Z^0 \to \ell\bar{\ell}$. My team has used the ATLAS measurements of lepton momentum and energy in $Z^0 \to \ell\bar{\ell}$ interactions to calculate the mass of the Z^0 boson. All graphs and calculations were made in Python using numpy, matplotlib.pyplot, scipy.stats, and scipy.optimize.curve_fit.

II. Invariant Mass Distribution

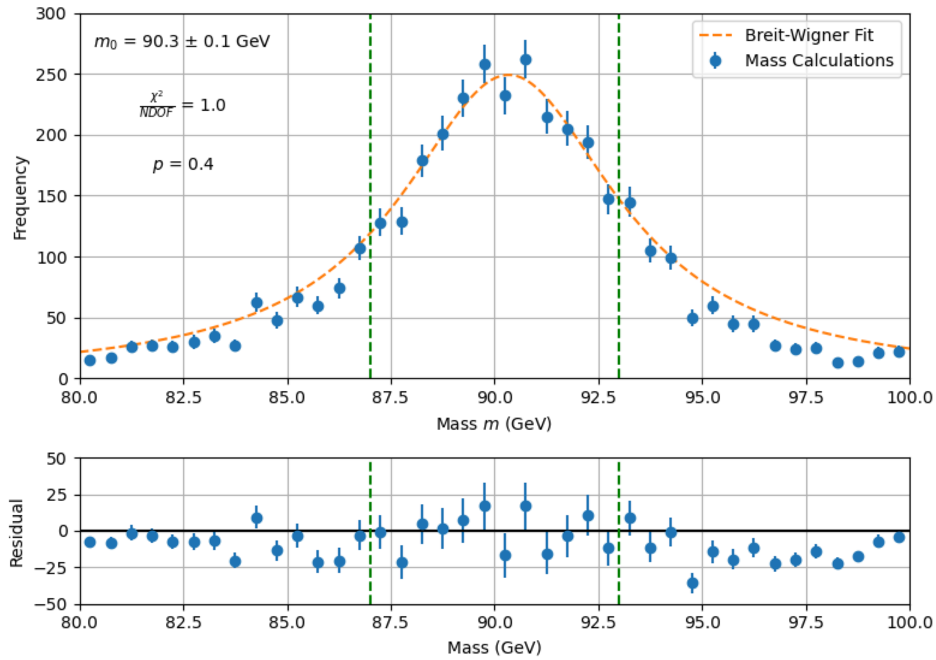
From conservation of energy, we know that the total combined energy of each lepton pair sums to the total energy of the particle from which they had decayed, the Z^0 boson. The total energy of a fundamental particle is the sum of the rest energy, which is associated with the particle's mass, and the kinetic energy, which is associated with the particle's motion. Since the momentum of a particle is associated with its motion, the mass of the Z^0 boson is related to the difference between the leptons' total energy and their combined momentum:

$$M = \sqrt{E^2 - (p_x^2 + p_y^2 + p_z^2)}$$

where E is the combined total energy of the two leptons, and p_x, p_y, and p_z are the combined momentum components of the two leptons.

My team has used 5,000 ATLAS data events to make 5,000 calculations for the Z^0 boson mass. We then histogrammed these calculations from 80 GeV to 100 GeV with 41 bins. The number of data events in each bin is shown in Figure 1 below.

Figure 1



The distribution D of the decay masses m can be predicted using a Breit-Wigner fit:

$$D(m, m_0, \Gamma) = \frac{1}{\pi}\frac{\Gamma/2}{(m-m_0)^2+(\Gamma/2)^2}$$

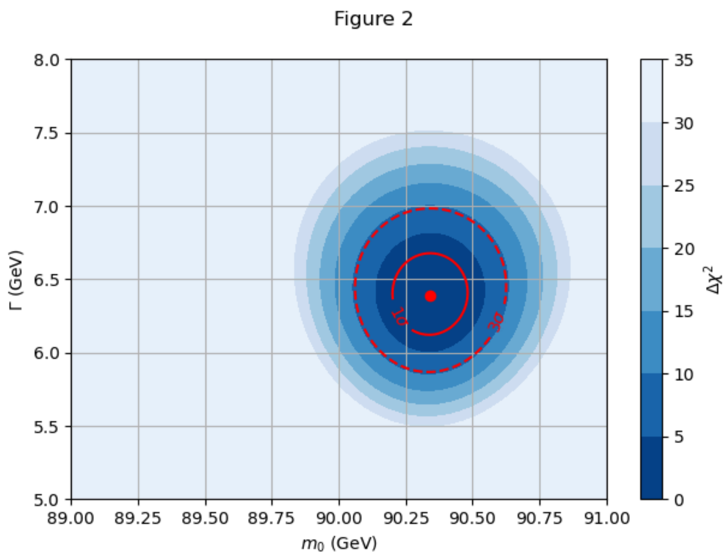
where m_0 is the true rest mass of the Z^0 boson and Γ is the "width" parameter. Using scipy.optimize.curve_fit for the twelve data points between 87 GeV and 93 GeV, my team fitted this model to the data, as shown in Figure 1. This best-fit model yields a Z^0 boson mass of 90.3 ± 0.1 GeV.

My team found that, within the fitting range, χ^2 = 10.0. The number of degrees of freedom (*NDOF*) is calculated by subtracting the number of fitting parameters from the number of data points. Since the fit was made with twelve data points and two parameters, *NDOF* = 10. Using *NDOF* = 10 yields *reduced* χ^2 = 1.0.

Using χ^2 = 10.0 and *NDOF* = 10 yields p = 0.4. The p-value is a measure of the probability, assuming the model is correct, that one would collect data at least as extreme as my team's data. p = 0.4 means that there is a 40% chance one would collect data at least as far from the model as our data, and there is a 60% chance one would collect data closer to the model than our data. Using significance level α = 0.05, our p-value of 0.4 indicates that our data is compatible with the best-fit model.

III. χ^2 for Models with Different Parameters

Since we do not know with absolute certainty that the best-fit model above is fitted around the most accurate m_0 and Γ values, it is important to consider how well the data would fit the model if said model were fitted slightly differently. Figure 2 shows the difference between the χ^2 value calculated above and the χ^2 value for a model that was fitted around different m_0 and Γ values. This difference is denoted as $\Delta\chi^2$.



The solid red line is drawn at $\Delta\chi^2 = 2.3$, the $\Delta\chi^2$-value corresponding to 1 standard deviation for a two-parameter fit. The dotted red line is drawn at $\Delta\chi^2 = 9.2$, the $\Delta\chi^2$-value corresponding to 3 standard deviations for a two-parameter fit.

IV. Discussion and Future Work

My team analyzed 5,000 ATLAS data events of $Z^0 \rightarrow \ell\bar{\ell}$ interactions, yielding a Z^0 boson mass of 90.3 ± 0.1 GeV. However, the most commonly accepted Z^0 boson mass, attained by the Particle Data Group (PDG) in 2024, is 91.2 GeV. This discrepancy is likely due to the assumption of no uncertainty on the original ATLAS data. The detector in the LHC might not have been calibrated perfectly, and its energy resolution might have been too low. Because these uncertainties were not taken into account, the errors on the mass distribution (Figure 1) are smaller than they should be. Perhaps if the uncertainties on the original data were taken into account, my team's calculated Z^0 boson mass would be compatible with the mass attained by the PDG.