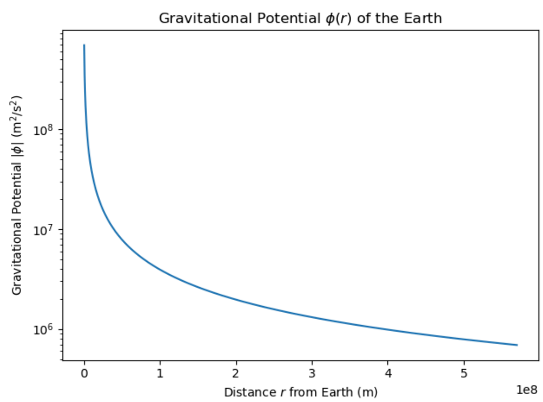
I. Introduction

This report discusses the basic understandings of gravity and rocket propulsion needed for the Apollo program. All of my graphs and calculations were made by importing numpy, matplotlib.pyplot, and scipy.integrate.quad into a JupyterLab python notebook.

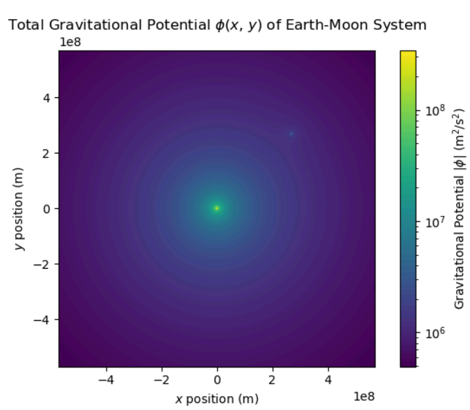
II. Gravitational Potential of the Earth-Moon System

Gravitational potential measures how much potential energy per unit mass any object would have at any given distance from a certain planet, moon, or other massive object. Gravitational potential *Φ* follows the equation $\Phi = \frac{-GM}{r}$, where *G* is the universal gravitational constant, *M* is the mass of the planetary object, and *r* is the "radius" or the distance between the centers of the two objects.

Because a falling object would not gain as much kinetic energy when "dropped" from a closer distance, the gravitational potential decreases (becomes more negative) as the radius increases:



This pattern holds when the Moon is also accounted for. The following graph shows the total gravitational potential depending on one's position in the Earth-Moon system. This graph places the Moon up and to the right of the Earth and about 1.41 times closer to the Earth than the Moon actually is.

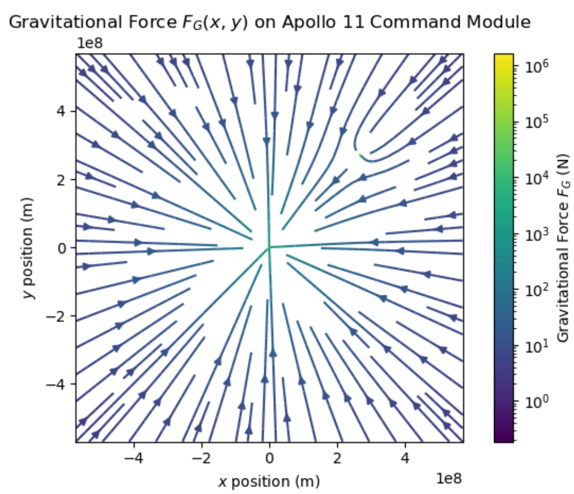


The absolute value of the gravitational potential increases rapidly as one approaches Earth and, to a lesser extent, as one approaches the Moon.

III. Gravitational Force of the Earth-Moon System

Massive objects like the Earth and Moon exert attractive gravitational forces on all nearby objects. The strength of this force F_G is given by $F_G = \frac{GM_1m_2}{r^2}$, where G is the universal gravitational constant, M_1 is the mass of the planetary object, m_2 is the mass of the smaller object, and r is the distance between the centers of the two objects.

The total gravitational force on the Apollo 11 Command Module increases as it gets closer to the Earth or Moon, following an inverse square relationship. The following graph shows the total gravitational force on the Apollo 11 Command Module in the Earth-Moon system. It too places the Moon up and to the right of the Earth and about 1.41 times closer to the Earth than the Moon actually is.



Except when very close to the moon, the total gravitational force is always in the direction of the Earth.

IV. Projected Performance of the Saturn V Stage 1

The time T it takes for the Saturn V Stage 1 rocket to burn all its fuel can be calculated by the equation $T = \frac{m_0 - m_f}{\dot{m}}$, where m_0 is the "wet mass" or the initial total rocket mass including fuel, m_f is the "dry mass" or the mass once the rocket has burned all its fuel, and $\dot{m}$ is the "burn rate" or the rate at which the rocket burns its fuel. Using a wet mass of $2.8 \cdot 10^6$ kg, a dry mass of $7.5 \cdot 10^5$ kg, and a burn rate of $1.3 \cdot 10^4$ kg/s, I have predicted that the rocket will burn all its fuel in about 160 seconds.

The Tsiolkovsky rocket equation $\Delta v = v_e ln(\frac{m_0}{m}) - gt$ gives the instantaneous change in rocket velocity Δv as a function of time t. In this equation, v_e is "exhaust velocity" or the speed the rocket will reach once it uses all its fuel, m_0 is the wet mass, m is the total mass of the rocket at time t, and g is the acceleration due to gravity near the surface of the Earth.

The height h of the rocket can be found by integrating the instantaneous change in velocity Δv with respect to the time t. To find the velocity of the rocket once it runs out of fuel, integrate from $t = 0$ to the burn time T: $h = \int_0^T \Delta v \, dt$. Using this equation, I found that the rocket will reach an altitude of approximately 74 kilometers before running out of fuel.

V. Discussion and Future Work

The above findings on gravitational potential and gravitational force provide a helpful general understanding of how the Apollo 11 Command Module will be affected by gravity, but some adjustments should be made to make calculations more accurate. Most notably, the graphs in Section II and III take the Moon to be $2.7 \cdot 10^8$ m from the Earth, but the Moon is actually $3.8 \cdot 10^8$ m from the Earth.

Calculations pertaining to the Saturn V rocket could also be made more accurate by accounting for air resistance and treating acceleration due to gravity as a function of height. These inaccuracies explain the discrepancies between my calculations and NASA's test results. The NASA team also found that the rocket took about 160 seconds to burn all its fuel, but they measured an altitude of 70 kilometers when the rocket finished burning fuel. Since the Tsiolkovsky rocket equation I used did not account for air resistance, my calculations overestimated the altitude.

Additionally, all calculations can be made more precise if they use measurements that allow for more significant figures.