

# 101b\_hw\_03\_Daren\_Sathasivam

Daren Sathasivam

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## Problem 1: 4.6

Consider the single-factor completely randomized experiment shown in Problem 3.7. Suppose that this experiment had been conducted in a randomized complete block design and that the sum of squares for blocks was 80.00. Modify the ANOVA for this experiment to show the correct analysis for the randomized complete block experiment.

##	Source	DF	SS	MS	F	P
## 1	Factor	4	987.71	246.9300	33.09488	1.184819e-09
## 2	Error	25	186.53	7.4612	NA	NA
## 3	Total	29	1174.24	NA	NA	NA

### Table for Block Design:

$$DF \sim \text{Block} = DF_{total} = DF_{factor} + DF_{block} + DF_{error}$$

$$DF_{total} = 29$$

$$29 = 4 + DF_{block} + 20$$

$$DF_{block} = 5 \text{ (also } 29 - 4 - 20 = 5 \text{)}$$

$$SS \sim \text{Total} = SS_{total} = SS_{factor} + SS_{block} + SS_{error}$$

$$SS_{total} = 987.71 + 80.00 + 5.3265$$

$$SS_{total} = 1174.24$$

$$MS \sim \text{Block} = MS_{block} = \frac{SS_{block}}{DF_{block}}$$

$$MS_{block} = \frac{80}{5}$$

$$MS_{block} = 16$$

```
source <- c("Factor", "Block", "Error", "Total")
df <- c(4, 5, 20, 29)
ss <- c(987.71, 80, 106.53, 1174.24)
ms <- c(246.9275, 16, 5.3265, NA) # MS for Total is typically not calculated
f <- c(46.3583, NA, NA, NA) # F and P-values for Error and Total are not applicable
p <- c(7.669035e-10, NA, NA, NA)

# Create a data frame for the ANOVA table
block_table <- data.frame(Source = source, DF = df, SS = ss,
  MS = ms, F = f, P = p)

# Print the table
print(block_table)
```

##	Source	DF	SS	MS	F	P
## 1	Factor	4	987.71	246.9275	46.3583	7.669035e-10
## 2	Block	5	80.00	16.0000	NA	NA
## 3	Error	20	106.53	5.3265	NA	NA
## 4	Total	29	1174.24	NA	NA	NA

- The experiment design effectively controlled for specific block effects, which can be supported by the significant F-statistic value of 46.36. Additionally, the p-value is very low with a value of 7.669035e-10, indicating that the influence of the treatment on the outcome variable being independent of the block difference.

## Problem 2: 4.13

```
oil <- c(rep(1, 5), rep(2, 5), rep(3, 5))
truck <- rep(c(1:5), 3)
fuel <- c(0.5, 0.634, 0.487, 0.329, 0.512, 0.535, 0.675, 0.52,
          0.435, 0.54, 0.513, 0.595, 0.488, 0.4, 0.51)
```

### a. Analyze the data from this experiment.

```
model1 <- aov(fuel ~ factor(oil) + factor(truck))
summary(model1)
```

```
##              Df Sum Sq Mean Sq F value    Pr(>F)
## factor(oil)    2  0.00671  0.003353    6.353  0.0223 *
## factor(truck)  4  0.09210  0.023025   43.626 1.78e-05 ***
## Residuals      8  0.00422  0.000528
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

- Factor(oil) has an F-value of 6.353 and p-value of 0.0223. This indicates that there are statistically significant differences in fuel consumption efficiencies among different oils. Factor(truck) has an F-value of 43.626 and a p-value of 1.78e-05. This suggests that the differences in truck engines have a statistically significant impact on fuel consumption, thus supporting the choice of using a block design to control this variability.

### b. Use the Fisher LSD method to make comparisons among the three lubricating oils to determine specifically which oils differ in brake-specific fuel consumption.

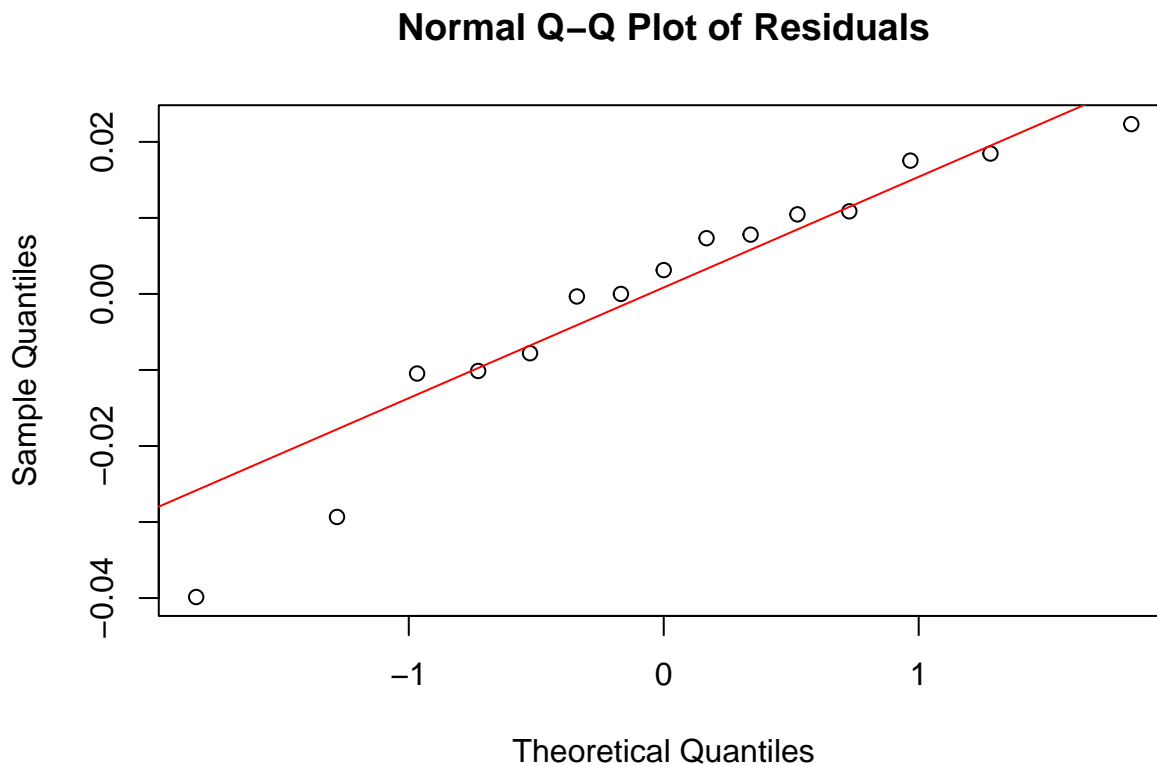
```
library(DescTools)
PostHocTest(model1, method = "lsd")$"factor(oil)"
```

```
##      diff      lwr.ci      upr.ci      pval
## 2-1  0.0486  0.01509436  0.08210564 0.01015749
## 3-1  0.0088 -0.02470564  0.04230564 0.56152936
## 3-2 -0.0398 -0.07330564 -0.00629436 0.02548036
```

- The oils that specifically differ in brake-specific fuel consumption are oil 2 in comparison to oil 1 and oil 3 as it has the highest difference amongst the three. The difference between oil 1 and 3 is relatively small, suggesting that using oil 1 or oil 3 will result in increased fuel efficiency in comparison to oil 2.

c. Analyze the residuals from this experiment.

```
res1 <- model1$residuals
# Normal Q-Q plot of residuals
qqnorm(res1, main = "Normal Q-Q Plot of Residuals")
qqline(res1, col = "red")
```



```
summary(res1)
```

```
##      Min.   1st Qu.   Median     Mean   3rd Qu.    Max.
## -0.039867 -0.008967  0.003133  0.000000  0.010667  0.022333
```

- A linear pattern can be observed in the qqplot of residuals, indicating that the residuals closely follow a normal distribution. There is no S-shaped curve therefore no indication of heavy tails or outliers within the residuals.

### Problem 3: 4.24

Effect of five different ingredients on the reaction time of a chemical process is being studied.

```
# Data
batch <- c(rep(1, 5), rep(2, 5), rep(3, 5), rep(4, 5), rep(5,
5))
day <- rep(1:5, 5)
ingredients <- c("A", "B", "D", "C", "E", "C", "E", "A", "D",
"B", "B", "A", "C", "E", "D", "D", "C", "E", "B", "A", "E",
"D", "B", "A", "C")
time <- c(8, 7, 1, 7, 3, 11, 2, 7, 3, 8, 4, 9, 10, 1, 5, 6, 8,
6, 6, 10, 4, 2, 3, 8, 8)
```

```
# Model
model2 <- aov(time ~ factor(ingredients) + factor(batch) + factor(day))
summary(model2)
```

```
##              Df Sum Sq Mean Sq F value    Pr(>F)
## factor(ingredients)  4 141.44   35.36  11.309 0.000488 ***
## factor(batch)        4  15.44    3.86   1.235 0.347618
## factor(day)          4  12.24    3.06   0.979 0.455014
## Residuals           12  37.52    3.13
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

- For factor(ingredients), it can be observed that the F-value is 11.309 and p-value is 0.000488. This suggests that the type of ingredient used in the process has a statistically significant effect on the reaction time. Both factor(batch) and factor(day) have a relatively high p-value, respectively 0.347618 and 0.455014. This indicates that both of these factors do not have a statistically significant effect on the reaction time. This suggests that future experiments should focus on the ingredients to control and optimize reaction time. Additionally, the insignificance of the batch and day effects validates the effectiveness of the Latin square design in controlling potential variability allowing for a clear assesment of the ingredients factor.

## Problem 4: 4.37

Graeco-Latin square. Analyze the data from this experiment (use  $\alpha = 0.05$ ) and draw conclusions

```
# Data
batch <- c(rep(1, 5), rep(2, 5), rep(3, 5), rep(4, 5), rep(5,
5))
acid <- rep(1:5, 5)
times <- c("A", "B", "C", "D", "E", "B", "C", "D", "E", "A",
"C", "D", "E", "A", "B", "D", "E", "A", "B", "C", "E", "A",
"B", "C", "D")
catalyst <- c("a", "b", "g", "d", "e", "g", "d", "e", "a", "b",
"e", "a", "b", "g", "d", "b", "g", "d", "e", "a", "d", "e",
"a", "b", "g")
yield <- c(26, 16, 19, 16, 13, 18, 21, 18, 11, 21, 20, 12, 16,
25, 13, 15, 15, 22, 14, 17, 10, 24, 17, 17, 14)

# Model
model3 <- aov(yield ~ factor(times) + factor(catalyst) + factor(batch) +
factor(acid))
summary(model3)
```

```
##              Df Sum Sq Mean Sq F value    Pr(>F)
## factor(times)  4  342.8   85.70  14.650 0.000941 ***
## factor(catalyst) 4   12.0    3.00   0.513 0.728900
## factor(batch)   4   10.0    2.50   0.427 0.785447
## factor(acid)    4   24.4    6.10   1.043 0.442543
## Residuals       8   46.8    5.85
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

- From the model summary, we can observe that p-values of the catalyst(0.728900), batch(0.785447),

and acid(0.442543) are larger than  $\alpha = 0.05$ . This suggests that the catalyst, batch, and acid factors do not have a statistically significant effect on the yield of the chemical process. The standing time has a p-value of 0.000941, indicating that the standing time factor does have a statistically significant effect on the yield of the chemical process. By having the single significant factor, this supports the effectiveness of the Graeco-Latin square design in controlling the variability from the other factors.

## Problem 5: 4.44

**BIBD.** Analyze the data from this experiment (use  $\alpha = 0.05$ ) and draw conclusions.

```
# Data
hardwood <- c(rep(2, 7), rep(4, 7), rep(6, 7), rep(8, 7), rep(10,
7), rep(12, 7), rep(14, 7))
days <- rep(1:7, 7)
strength <- c(114, NA, NA, NA, 120, NA, 117, 126, 120, NA, NA,
NA, 119, NA, NA, 137, 117, NA, NA, NA, 134, 141, NA, 129,
149, NA, NA, NA, NA, 145, NA, 150, 143, NA, NA, NA, NA, 120,
NA, 118, 123, NA, NA, NA, NA, 136, NA, 130, 127)

# Model
model4 <- aov(strength ~ factor(days) + factor(hardwood))
summary(model4)

##              Df Sum Sq Mean Sq F value    Pr(>F)
## factor(days)    6 1114.3   185.71     8.814 0.00358 **
## factor(hardwood) 6 1317.4   219.57    10.420 0.00205 **
## Residuals       8  168.6     21.07
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
## 28 observations deleted due to missingness
```

- From the balanced incomplete block design experiment, it can be observed that the days factor has an F-value of 8.814 and a p-value of 0.00358. This indicates that the variation in days has a statistically significant impact on the outcome of strength of the paper produced. Additionally, the hardwood concentration factor has an F-value of 10.420 and a p-value of 0.00205, indicating that it has a statistically significant effect on the strength of paper produced. Through the BIBD, the balanced design does not affect the impact of the factors and therefore it can be concluded that the hardwood concentration and day-to-day variability have an effect on the strength of paper produced.