

# Traffic Signal Coordination

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Our signatures below attest that this submission is our original work.

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## Executive Summary

As worldwide populations increase, urban areas continue to struggle with the mitigation of traffic congestion and the associated effects; the stop-and-go nature of traffic negatively impacts travel times, fuel consumption, and pollution. The key metrics in city development, each of which are directly influenced by traffic flow efficiency, are urban mobility, productivity, and environmental sustainability. Addressing congestion is a top priority as metropolises expand and urban land value increases, and a potential solution proposed by this project is that of a real-time traffic signal consensus algorithm, changing traffic rerouting strategies as it receives information from the real world.

Current static or semi controlled traffic management algorithms lack the sophistication to handle the demands of Toronto's commuter habits. The subpar methods currently in use perpetuate high vehicle volumes and inefficient traffic signal timing. Extended idling due to stop and go traffic contributes negatively to public health and air quality. By reducing stop time and frequency, the quality of life for commuters and the general population will increase. Inefficient traffic management is costly to industry and taxpayers. Wasted fuel, increased vehicle maintenance, and lost productivity are symptomatic of an ineffective system and impose costs on urban society.

Key stakeholders desire higher quality of life. Commuters stand to benefit from continuous driving experiences with shorter commute times and lower fuel consumption. City authorities desire efficient traffic management without costly and ineffective road expansions. Environmental health advocates desire reduced stop and go traffic and less idling cars, each of which reduces pollution and improves air quality.

Obtaining reliable data on traffic volume, speed, and flow serves as a challenge because this data is both valuable and difficult to record; however, intelligent traffic management systems are critical for areas with limited capacity for physical modification of road infrastructure. The problem of mitigating traffic congestion proves to be an important topic in urban development, with healthier living conditions being achieved by employing efficient traffic management systems without significant economic responsibility.

The proposed solution maximizes sustainability and quality of life while meeting the strict constraints of the urban environment. By employing a consensus algorithm that adjusts signal timings based on real-time traffic information, congestion will be significantly reduced, resulting in a more efficient traffic management system and subsequently, improved societal operations.

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# 1. Introduction

Most Canadians encounter traffic during the commute to and from work, especially in urban areas. While in the short term this can be frustrating, there are more serious long-term consequences of this congestion. These include significant air pollution produced by idling vehicles at intersections, which is both harmful to the planet and people, especially those with respiratory illnesses, such as the elderly and young children [1]. Worldwide urbanization is increasing at a rapid rate with the need for intelligent urban planning to mirror the growth.

This report outlines a proposal for the implementation of a novel traffic signal coordination method in the city of Toronto. It will focus specifically on a grid of downtown Toronto with high traffic density. Toronto is one of the most populated cities in North America, with 3.026 million inhabitants as of 2022, and the home of Ontario's provincial government [2]. These factors make it incredibly congested, and therefore, a prime location for the implementation of a traffic signal coordination algorithm. Existing solutions have proven inadequate in handling the daily congestion of urban centers. A solution must handle data and adapt its strategy in real-time, features that many legacy technologies do not provide.

The creation of a smart traffic management system would both improve citizens' daily routines and provide benefits for public health and Toronto's environmental footprint. Advocates pose sustainability goals regarding the environment and urban development, each of which is aimed at improving societal quality of life. Local and national commitments in these areas align with the premise of this project.

The objective of this project is to develop a multi-agent group formation algorithm, specifically a consensus algorithm, which coordinates traffic signals to distribute congestion and increase traffic flow based on real-time traffic data. The problem of traffic congestion in downtown Toronto significantly impacts urban mobility, commuter experience, and air quality. Current traffic management systems are inadequate and unable to keep up with the growing population. Addressing these inefficiencies requires a novel approach that works in real-time and is adaptive to the variable traffic conditions.

## 2. Problem Definition

In this section, the considerations and challenges of creating a consensus algorithm and database for the management of traffic are thoroughly explored and defined. This is explored with the purpose of guiding design choices throughout the project.

### 2.1. Background Information

Traffic is a major problem in Toronto. When polled, 86% of residents of the Greater Toronto Area and Hamilton Area said there is a traffic and congestion crisis in the region [3]. There are many causes, and many potential solutions, one of which is improved traffic signal coordination. In Toronto, most major intersections are controlled by a 'Fixed Time' signal [4].

The City of Toronto uses three different traffic control systems; TransSuite, Split Cycle Offset Optimization Technique (SCOOT), and Sydney Coordinated Adaptive Traffic System (SCATS). TransSuite is by far the most common signal control system, with 92% of the city's traffic lights under its control [5]. Under the TransSuite system, traffic lights are programmed with fixed timings that do not adapt to any unforeseen circumstances. Each signal usually has 5 different timing plans for morning peak, afternoon peak, midday off peak, night, and weekend. Optimal timings are calculated using traffic data measured at each intersection [6].

SCOOT and SCATS are both adaptive traffic control systems; however, they only account for between 3% and 5% of traffic lights respectively [6]. These systems are installed on major roads, outside of the downtown core, such as on Steeles Avenue and Don Mills Road. They usually prioritize the flow of traffic in a single direction and can lead to long stop times for cross traffic. The system is not in use in downtown because of the increased pedestrian and cyclist presence, as well as the large volume of traffic in multiple directions [7].

Approximately 85% of traffic lights in Toronto are semi-actuated, which means that they are equipped with some form of vehicle detecting device [5]. This information, combined with open-source traffic data collected by the City of Toronto, is available to help the team design an adaptive traffic signal coordination plan.

## 2.2. Stakeholder Needs

Traffic signal coordination plays a crucial role in the management of Toronto's traffic crisis. There are many stakeholders with distinct needs and expectations for an improved system. These stakeholders can be defined as any individual or organization who has an interest or influence over the effects or development of traffic related projects.

As end users, commuters and residents are major stakeholders. They desire shorter travel times, less delays, and more predictable commutes. They would benefit from an adaptive traffic system that can respond dynamically to surges in traffic due to events, collisions, and road closures.

Government agencies and political officials of federal, provincial, and municipal levels have influence over the development and interest in the effects of the project. The success of the project is directly related to the government, as the agencies should have a vested interest in the quality of life in their jurisdiction.

Another crucial stakeholder group is public transit users and operators. The TTC is an integral part of what keeps Toronto running, so ensuring a new traffic signal system improves the operation of buses and streetcars that pass through the downtown core is very important. While not in the scope of the project, transit operators would want transit signal priority to ensure they stay on schedule.

Coordination for streetcars, which have wide and slow turns, is also important.

Environmental advocates are a growing stakeholder group. They desire a traffic control system that accommodates pedestrian and bike traffic, to decrease the use of cars and minimize idling at red lights. Similarly, city officials and urban planners seek this design to be cost-effective, sustainable, and easy to integrate into existing infrastructure to minimize disruption in implementation.

All these stakeholder categories can be more broadly classified as taxpayers, and taxpayers want to feel that the tax dollars they pay are used effectively. They prefer a solution that is least intrusive, does not require high construction costs, and can be implemented quickly.

## 2.3. Scope and Constraints

The purpose of this project is to optimize the flow of traffic in downtown Toronto by developing an algorithm for dynamic traffic signal timing. The targeted grid is bounded by Bathurst Street on the west, Church Street on the east, Dundas Street to the North, and King Street to the south (see Figure 2: Initial



target grid in downtown Toronto). This is a region of extremely high traffic density which makes it an ideal location for optimization. For simplification, the target grid was condensed to the connective roads as shown in Figure 3: Simplified target grid in downtown Toronto with specified road segments.

The scope of this project involves determining a measure of traffic pressure and density at a given intersection using variables such as the volume of cars in each direction, number of lanes, time interval, and distance between adjacent intersections. An optimized traffic signal timing system would be achieved using a consensus algorithm. The algorithm will use existing traffic data from the Toronto and Ontario Open Data platforms which offer 8-hour datasets with 15-minute intervals since 2020 [5]. These datasets include the vehicle volumes by type, direction, turning, and signal activation type. This algorithm will focus solely on vehicular traffic and not pedestrian, bike, or motorcycles.

There exist several constraints for this project such as lack of data granularity, missing or sporadic dataset information, and technological limitations related to the existing infrastructure of intersection cameras and sensors. The data is only available in 15-minute intervals by volume which does not give a measure of traffic congestion or pressure. The data is also limited to major roads and intersections, and therefore, the model cannot precisely account for traffic entering unmapped streets both within the grid and entering or exiting the grid. This may impact the overall accuracy of the model's baseline dataset. The accuracy and responsiveness of existing cameras and sensors located at these intersections may also restrict the level of control that implementing this algorithm would have in practice.

## 2.4. Assumptions

Due to the scope and constraints of the traffic signal timing system, several assumptions will be made with respect to the datasets, driver behaviour, existing traffic signal infrastructure in Toronto, and interactions between vehicular and non-vehicular modes. The primary assumption is that all traffic occurs within the defined grid and with negligible effect from traffic on residential roads within, entering, or exiting the grid. The collected data from cameras and sensors are also assumed to be perfectly accurate and will be used to create a baseline measure of traffic congestion.

The project also assumes that pedestrian, bicycle, and motorcycle traffic are negligible, and therefore are excluded from analysis. It is, however, acknowledged that this limits the applicability of the algorithm in real-world implementation where these modes would have significant impact on traffic congestion. Another assumption is that drivers have consistent reaction time, vehicle acceleration, and deceleration rates, and do not behave unlawfully.

Similarly, the duration of yellow lights, transition interval between signal colors, and time to stop and start at each light will be assumed to be consistent among all intersections in the grid and accounted for in the calculations. While the basis data source is in 15-minute intervals, the algorithm will operate in much more frequent intervals so that it can adapt to the fluctuation of traffic trends.

## 2.5. Specifications and Functional Requirements

The most crucial function of the design is to minimize congestion within the specified grid. Congestion is not an easily quantifiable measure, so it must be broken down into the elements of traffic that can be modeled by volumes and time intervals. As mentioned above in 2.4, congestion must be measured in a quantifiable and comparable way. This must be done using a pressure function that considers volume of vehicles in each direction, number of lanes, and road segment length. Pressure will be modeled using the available traffic volumes from public databases, then portrayed into the model. By implementing a measurement of congestion and pressure, it will create a way to compare the results from the model to real-world datasets. This comparison will ultimately determine the project's success.

While the pressure function will express the congestion with a single value, other measures will be used to expose the nuances of traffic and how the design solution affects it. Traffic light wait time, stops made, car density, and average speed could be measured with the model. These averages can then be compared with different mechanisms to rank their performance. Subsequently, this would allow the team to create a weighted evaluation matrix and a success matrix that could set baseline averages for what the team determines a successful signal timing mechanism. While all this data might not all be available through public databases for all the selected intersections, it will aid with decision making and evaluation at the end of the project, determining the best solution in an empirical way.

The solution must adapt to the real-time traffic data instead of a one size fits all solution. All the data produced by the model will also be averaged across specific time intervals such as morning rush (8-9am), afternoon rush (5-6pm), and evening and weekend traffic. Traffic flow will be deemed optimal once congestion and pressure measures are evenly distributed across the entire grid.

## 2.6. Ethical Considerations

The traffic patterns of downtown Toronto affect thousands of people every day, and as a result, any changes to the system made must be thoroughly evaluated through several lenses. As traffic light

maintenance and systems are regulated and managed by the municipal government, it would be funded by tax money, and therefore, must align with the interests of the Toronto taxpayers.

From a triple bottom line perspective, there are both gains and losses for the municipality. There is no direct profit for the government or the citizens, though over time, the increased efficiency should contribute to indirect economic benefits. A new system, reducing traffic, would benefit the planet significantly. Car emissions are one of the biggest contributors to air pollution in urban areas, and air pollution is a significant contributor to the climate change crisis. By improving the traffic flow, this system would decrease Toronto's daily emissions; therefore, supporting the preservation of the planet. A decrease in traffic would primarily benefit the inhabitants of Toronto commuting to work every day, saving them time and improving their quality of life.

The topics of ethics, diversity, inclusion, and indigeneity (EDI), also bring up several key considerations for the design. As the subject of the project is public and largely involved in civilians' lives, public safety must be the highest priority throughout the entire design process. The roadways can be incredibly dangerous for both people in vehicles and pedestrians, and although the purpose of this project is to decrease traffic, the system must not endanger human lives. This is in accordance with the Professional Engineers Ontario Code of Ethics, which emphasizes an engineer's duty to public welfare [8].

While the scope of the project does not include optimizing foot and bicycle traffic, it will be accommodating of the needs of diverse road users. Future iterations of this design should integrate considerations for pedestrians, cyclists, and those with disabilities, to align with the principles of equity, diversity, inclusion, and indigenization.

### 3. Project Plan

Table 2: Project Plan

Task	Task Description	Expected Days	Actual Days	Activity Leader
<b>1.</b>	<b>Pitch</b>	<b>7</b>	<b>7</b>	
1.1	Reflect on problems that could be solved using algorithms presented in class	5	5	All
1.2	Discuss as a group on possible options and decide on two options	1	1	All
1.3	Pitch presentation	1	1	Ethan
<b>2.</b>	<b>Proposal</b>	<b>12</b>	<b>12</b>	
2.1	Background Research on the problem	3	3	Ethan
2.2	Identify which algorithm is best suited	1	1	All
2.3	Identify stakeholder needs	2	3	Varun
2.4	Discuss scope and objectives	2	3	Emerson
2.5	Design criteria and specifications based on research	2	3	Ethan
2.6	Create project plan	1	1	Sebastien
2.7	Write executive summary	1	1	Ridley
2.8	Review and edit the proposal	3	4	All
<b>3.</b>	<b>Progress Report</b>	<b>14</b>		
3.1	Improve problem statement based on feedback	1	0	All
3.2	Discuss research collected & group idea generation	1	1	All
3.3	Find traffic datasets and merge into one large dataset	4	6	Ethan
3.4	Develop consensus algorithm to work for the problem using arbitrary data	5	6	Varun
3.5	Create pressure function	1	1	Ethan
3.6	Visualize the dataset	1	1	Ethan
3.7	Add all research and design concepts to progress report	6	4	All
<b>4.</b>	<b>Defining design solution</b>	<b>4</b>	<b>4</b>	
4.1	Improve prob stat and design sol based on feedback	1	1	All
4.2	Implement dataset into consensus algorithm	2	4	Varun
4.3	Implement dataset into MATLAB to create a functioning simulation	3	0	Ethan
<b>5.</b>	<b>Preparing final product</b>	<b>5</b>	<b>4</b>	
5.1	Describe concepts and alignment with success metrics	2	4	All
5.2	Outline decision making process & how solution found	2	3	Ethan
5.3	Define the design solution with specific detail	2	2	Seb and Ethan
5.4	Evaluate solution's adherence to success metrics	2	2	Ridley & Varun
5.5	Discuss next steps and the future of the project	1	2	Sébastien
5.6	Prepare final presentation of design solution	2	2	Emerson

The Gantt chart for the project plan can be found in Appendix I – Figures as Figure 1: Gantt Chart.

## 4. Creative Thinking

Upon receiving the assignment of solving an issue using group formation algorithms for a multi agent system, it was the team's goal to implement something new and interesting. Most of the examples given focused on physical moving agents, like birds or search formations, which is the obvious use of group algorithms, especially the consensus algorithm. As the team wanted to do something out of the box, the "Post-it Note Method" was used to generate unique problems that could be handled with the given algorithms. These were then discussed and evaluated for actual feasibility before the team chose to challenge the issue of road congestion.

Once the task was identified, the team fleshed it out by using mind mapping to split it into subsections, it was at this point that the idea of using real traffic data emerged. Multiple locations were considered for the target area, including a section of downtown Kingston, however downtown Toronto was eventually chosen. Some factors that were considered in this decision were the necessity, the team members' familiarity, and the availability of real traffic data. Initially the team discussed simply using the number of cars waiting at an intersection as that agent's state, however that would be an extreme simplification of how real traffic congestion works, so the team set out to develop a metric that could represent traffic at any intersection. It was at this point that the combined dataset was conceptualized, as the values used to calculate the traffic pressure could not all be found in the same places, therefore making the new function difficult to implement. By using publicly available information the team was able to construct a database that facilitates both the use of the traffic pressure function, and the comparison of the results with real data.

## 5. Concept Selection and Preliminary Work

In this section, the process of preparing a baseline dataset and selecting a consensus algorithm are outlined. This is explained in a technical manner, walking through exact steps and relevant lines of code.

### 5.1 Dataset Preparation and Manipulation

To establish a baseline for measuring traffic flow improvements resulting from the consensus algorithm implementation, a comprehensive dataset representing existing traffic conditions in Toronto was prepared. This involved integrating multiple open data sources from the Toronto Open Data platform and Ontario Open Data [5], [9]. However, inconsistencies such as sporadic data points and mismatched

identifiers posed significant challenges in merging datasets—for example, aligning street names and geometries with the number of lanes per road was non-trivial due to incompatible formats and missing attributes.

### 5.1.1 Coding Libraries and Datasets

The data preparation process leveraged a combination of software tools and Python libraries optimized for geospatial data handling, spatial queries, and data manipulation. Key libraries consisted of Pandas for data frame manipulation, NumPy for numerical operations like KDTree construction, SciPy for nearest-neighbour searching, Geopy for geodesic distance validations, Shapely to construct geometric objects, and Pyproj for coordinate transformations. Additional software included the Quantum Geographic Information System (QGIS) which was used for merging spatial layers of varying data types, defining bounding regions, and calculating the length and position of intersection adjacent roads.

Due to the mismatched identifiers and coordinates between datasets, these tools were essential in creating a cohesive final data frame with accurate traffic signal, traffic volume, and geospatial information. This ultimately enabled the team to measure the theoretical improvement of traffic flow in Toronto should the algorithm be implemented, notwithstanding any assumptions.

The input datasets consisted of traffic data, including geographic coordinates and aggregated statistics such as traffic density and speed, and traffic signal data, containing positions of traffic lights with attributes like signal type and operational status.

These datasets came in various formats such as shapefiles and CSVs. These were imported into QGIS and unified under a common Coordinate Reference System (CRS) for spatial compatibility. Polygon boundaries were then created to define analysis zones, facilitating the segmentation of traffic data for adjacent road analysis.

### 5.1.2 Coordinate Thresholds and Spatial Indexing

To prepare the coordinates for spatial indexing, the latitude and longitude were extracted into tuple pairs from the line string geometric format. Since the locations of the traffic signal and road intersection were not identical, a distance buffer or threshold of 0.00009 degrees, equivalent to about 10 meters, was implemented to merge the features together.

As previously mentioned, KDTree structures were constructed to facilitate efficient proximity searches. A k-dimensional tree is a space-partitioning data structure used for organizing points in a k -dimensional space. These were created in Python using the script:

```
traffic_tree = cKDTree(traffic_coords)
```

```
signal_tree = cKDTree(signal_coords).
```

To then query for matches, a *query\_ball\_tree* was used to identify all traffic points within the threshold distance of any traffic light:

```
matched_indices = traffic_tree.query(ball_tree(signal_tree,distance_threshold_degrees)).
```

### 5.1.3 Merging and Adjacency

An iterative matching process was used to merge attributes of these traffic data points:

```
for traffic_index,signal_indices in enumerate(matched_indices):
    for signal_index in signal_indices:
        matched_row = {
            ** grid_traffic_data_averaged.iloc[traffic_index],
            ** grid_traffic_lights.iloc[signal_index]
            }
        matched_data.append(matched_row)
```

Using the library, shapely.geometry, bounding boxes were defined in QGIS to ensure matches occurred only within specified regions. Furthermore, to calculate the length of road segments between adjacent connected roads, the road network layer was intersected with the bounding polygons of the traffic lights. Midpoints were then placed on each of the adjacent line segments and a cardinality calculation was used to identify the northmost, southmost, eastmost, and westmost road segment.

### 5.1.4 Outputs, Applications, and Limitations

Points that were found to be unmatched with a road segment were recorded for further analysis using the script:

```

unmatched_traffic_indices
    = set(range(len(grid_traffic_data_averaged)))
    - {i for i, matches in enumerate(matched_indices) if matches}

unmatched_(signal_indices)
    = set(range(len(grid_traffic_lights)))
    - {j for matches in matched_indices for j in matches}

```

This process was crucial in identifying intersections that required infrastructural upgrades to be able to track traffic volumes. This process could also expose data inconsistencies or under-utilized signals.

Due to the curvature of the Earth, angular distances introduce errors in large-scale or non-planar datasets [10]. These were assumed to be negligible during any distance threshold calculations and coordinate conversions.

## 5.2 Consensus Algorithm and MATLAB Implementation

The team developed a multi-agent consensus algorithm in MATLAB to calculate the optimal signal timing adjustments for a given amount of traffic pressure at a certain time. An agent is placed at each intersection and can communicate with the next intersection in each cardinal direction. A rough template of the code was provided however, the code needed to be changed so much that a complete re-write was needed.

### 5.2.1 Consensus Algorithm

In a consensus algorithm there are various agents, each with an estimated consensus state that reach a common consensus state by communicating with some, but not necessarily all other agents [11]. In this implementation, the consensus space is represented by the pressure on each intersection. This consensus space is 4-dimensional, as the pressure is calculated for each cardinal direction. The consensus algorithm equalizes the pressure throughout the network, and the signal timings are updated to reflect those changes. The algorithm runs 1,000 iterations for each timestep to reach a consensus state with optimum signal timings.



### 5.2.2 Adjacency and Laplacian Matrices

As the positions of the agents were constant, there was no need to calculate the adjacency matrix and resulting Laplacian matrix during each iteration of the algorithm. Instead, the adjacency matrix was constructed by hand and stored in a csv file, to then be referenced by the program at the runtime. The Laplacian matrix was then calculated using the following script, where  $G$  is the adjacency matrix and  $L$  is the resulting Laplacian:

$$D = \text{diag}(\text{sum}(G));$$

$$L = D - G;$$

Since the adjacency matrix and Laplacian were not recalculated, the signal timings were altered each iteration to reflect the difference from the average pressure of the intersection's neighbours.

### 5.2.3 Consensus Filter

In each iteration, the updated consensus states were calculated using the following equation:

*Equation 1: Equation for Consensus State Updates*

$$x_1 = x_0 - Lx_0\Delta t \quad (1)$$

In which  $x_1$  represents the new consensus states of every agent as a 62x4 matrix,  $x_0$  represents the old consensus states in the same manner,  $L$  represents the 62x62 Laplacian Matrix, and  $\Delta t$  is the change in time between iterations. It can be proven that through repeated iterations the consensus states will converge to a common value.

### 5.2.4 Signal Timings

During each iteration, the signal timings were updated using the following script:

```

for i = 1:n
    neighbours = find(L(i,:) < 0);
    avg = mean(x1(neighbours,:));
    difference = x1(i,:) - avg;
    adjustment = 5 * [max(difference(1),difference(2)), max(difference(3),difference(4))];
    y1(i,:) = y0(i,:) + adjustment * dt;
end

```

In this script, the average pressures of each intersection's neighbours were calculated. The adjustments were calculated to be the difference in pressure between the intersection and the average pressures, multiplied by a constant. These adjustments were added to the initial signal times, multiplied by the time interval.

This process means that intersections with a high-pressure build-up will have their signal length increased, while intersections with low pressure will have their signal length decreased. This would lead to better traffic flow by limiting the amount of time a light is green, but no cars are present to pass.

### 5.2.5 Code Development

Initially, the signal timings generated by the algorithms tended to drift. While the consensus states converged, they never reached exact equilibrium, and through many iterations, it significantly impacted the signal timings. This was corrected by multiplying the adjustment by the time interval to prevent the number of iterations from affecting the final output.

In the code, all references to energy, or velocity were removed. This was done because the application has no movement, so there is no need for such constraints. The consensus space originally represented two directions (north/south and east/west), which was modified to represent the pressure of each road segment leaving each intersection. Subsequently, the consensus space was changed from 2-dimensional to 4-dimensional, with each dimension representing the cardinal pressure. This required the *run\_consensus.m* file to be rewritten, and the generated plots altered to fit the new consensus space.

During code development, the team faced issues with erratic outputs. The algorithm worked as expected with 2-7 agents; however, the consensus states overshot the equilibrium point when the agent count was greater than 7. The results when the equilibrium point was missed resembled a cloud, rather than a convergence. This was resolved by increasing the number of iterations.

## 6. Final Design

Building upon the data preparation and algorithm development outlined in 5. Concept Selection and Preliminary Work, the final design integrates the consensus algorithm with the processed traffic dataset to optimize signal timings across downtown Toronto's intersections. This section showcases the theoretical improvements of implementing this algorithm to urban traffic management.

Below, in Table 3, is a sample of the final dataset from the intersection of Dundas Street at Yonge Street.

*Table 3: Sample Data from Finalized Traffic Database*

Coord	System	Lanes	Time	North Pressure	South Pressure	East Pressure	West Pressure
(-79.38, 43.65)	TransSuite	3	7:30	0.2976	0.3426	0.2735	0.1623
(-79.38, 43.65)	TransSuite	3	7:45	0.2542	0.3891	0.3457	0.1858
(-79.38, 43.65)	TransSuite	3	8:00	0.3427	0.4430	0.4160	0.2102

The system column refers to the current type of signal management system used at the intersection. In this instance, it is TransSuite, as discussed in section 2.1. Background Information.

The traffic pressure (P) for a given road segment is calculated as:

*Equation 2: Derived Equation for Traffic Pressure*

$$P = \min\left(\frac{\rho}{\rho_{\max}}, 1.0\right) \quad (2)$$

Where the traffic density is given by:

*Equation 3: Equation for Traffic Density*

$$\rho = \frac{V}{L * S} \quad (3)$$

- $\rho$ : Traffic density (vehicles / kilometer / lane; directional)
- V: Traffic volume of vehicles (directional)
- L: Number of lanes (default = 2; if missing in dataset)
- S: Segment length (directional kilometers)
- $\rho_{\max} = 150$ : Maximum density (vehicles / kilometer / lane)

The resulting dataset is a 1500 row database with 15-minute time intervals for each of the selected intersections in the designated grid. These intervals are from 7:30 am to 2:45 pm every single day. The northbound, southbound, eastbound, and westbound traffic pressures are the pressure of traffic going into the intersection from the respective cardinal direction.

The consensus algorithm attempts to equalize the pressure of traffic throughout the network and optimizes the signal timings to do so as described in Section 5.2. Bathurst St. and King St. is a busy intersection in both directions, so the algorithms increase the signal timings (Table 4). This is done to lower the amount of time stopping and going at red lights. Bathurst St. and Adelaide St. is busy in the North/South direction, but calmer in the East/West direction. The algorithm increases the green light time going North/South and decreases the green light time going East/West (Table 5).

7:00 AM		7:15 AM		7:30 AM		7:45 AM	
North/South	East/West	North/South	East/West	North/South	East/West	North/South	East/West
45s	45s	46.279s	46.355s	47.466s	47.625s	48.609s	48.866s

Table 4: Sample of outputted signal times: Bathurst St. and King St.

7:00 AM		7:15 AM		7:30 AM		7:45 AM	
North/South	East/West	North/South	East/West	North/South	East/West	North/South	East/West
45s	45s	46.079s	44.595s	47.104s	44.147s	48.12s	43.659s

Table 5: Sample of outputted signal times: Bathurst St. and Adelaide St.

Figure 4 is a map of the network evolving over time. Each intersection has two arrows representing how much the signal timing has changed. An arrow in the North and East direction indicates an increase in green light time and South and West a decrease. The size of the arrow is relative to the magnitude of that change. Figure 5 depicts the changing signal times, with each line representing an intersection.

## 7. Project Economics

Economic viability must be considered for the project to be successful. Predictive financial analysis provides insight into the sustainability and budgetary constraints. Accuracy regarding capital is crucial as expenses are scrutinized intensely, especially in the public sector. Successful project financial management balances upfront costs with long-term economic efficiency. Operational savings regarding reduced traffic congestion and emissions must also outweigh the capital spent on initial installation and maintenance.

Project funds will be allocated from the municipal, provincial, and federal governments. Smart city development financial programs will serve as the majority contributions to project capital. Each level of government has a history of supporting transportation innovation projects. The Ontario Vehicle Innovation Network (OVIN) provides a program which the project will utilize and provides up to \$1,000,000 CAD of co-investment towards the development of Smart Mobility projects including Intelligent Transportation Systems (ITS) [12]. Historically, the Canadian federal government has facilitated challenges based on Smart City innovation, with the max prize being \$50,000,00 CAD [13]. While the Canadian federal government is not currently executing a challenge of this nature, contact with the organization would provide information on expected upcoming challenges of similar nature, providing guidance on project timeline to maximize funding. Project infrastructure installation costs are minimal as the MoveTO program's 'Intelligent' intersections plan has introduced systems usable for the project [14]. Annual site operations, maintenance and replacement costs will be \$2,700 - \$4,200 CAD per intersection [15], [16].

The project does not generate capital, but instead reduces economic burdens in other facets of society. Similar projects have demonstrated up to 40% quicker arterial travel times and up to 10% reduced total system delay, metric that the project will match [17]. These metrics represent economic savings in fuel cost, commuter productivity, and emission reductions. Most project economic challenges stem from initial installation. Upfront capital for system upgrades is significant and technological integration with legacy infrastructure is potentially costly. These challenges will be alleviated by allocating extra capital for unexpected expenses and integrating with the MoveTO project as it is already installing intelligent systems at intersections.

The return on investment of the project is significant, stemming from reduced congestion, reduced emissions, and increased commuter productivity, each of which proves project viability. Project financial risks are minimized by anticipating and allocating capital to related expenditures, taking advantage of government grants regarding sustainable urban development, and using existing infrastructure.

## 8. Evaluation

As described in 2.5, the most crucial function of the design is to minimize congestion within the specified grid. Congestion is not an easily quantifiable measure, so it must be broken down into the elements of traffic that can be modeled by a pressure function. The pressure function depends on data that could be found in both the real traffic database and the results from the MATLAB run of the optimized solution. This way, the improvement of congestion due to the consensus algorithm could be calculated. Many different sections of the pressure data were averaged into different categories, and then the improvement ( $I$ ) in percentage could be calculated using (4), where  $P_r$  is the real traffic pressure and  $P_o$  is the optimized pressure.

$$I = 100 \times \left(1 - \frac{P_o}{P_r}\right) \quad (4)$$

Now that the improvement of a category could be calculated and put into Table 6, there had to be a threshold that determined whether the project was successful in reducing congestion. This was discussed by the team, and it was decided that given the time and experience constraints, a 5% decrease in congestion was significant enough to be successful as a minimum viable product (MVP), while a 10% improvement would be ideal. Similar conversations were had within the team to determine Table 7, the success requirements.

Table 6: Categorical Pressure Evaluation Matrix

Category	Traffic Data	Optimized	Improvement (%)	Success?
Average pressure	0.6299	0.6036	4.168	No
Average pressure on 1 lane roads	0.9014	0.6872	23.765	Yes
Average pressure on 2 lane roads	0.6731	0.6022	10.539	Yes
Average pressure on 3 lane roads	0.6105	0.4676	23.420	Yes
Average pressure on 4 lane roads	0.4858	0.5988	-23.258	No
Average pressure on 5 lane roads	0.4864	0.5148	-5.837	No
Average morning rush (8-9am) pressure	0.6012	0.5719	4.872	No

<b>Average midday (11am-12pm) pressure</b>	0.6422	0.6142	4.366	No
<b>Average afternoon rush (2-3pm) pressure</b>	0.6854	0.6606	3.616	No

*Table 7: Success Requirements*

<b>Requirements</b>	<b>Value</b>	<b>Results</b>
<b>Significant average pressure improvement</b>	4.168%	False
<b>7/13 categories were improved significantly</b>	5/13	False
<b>Adapts to traffic fluctuation</b>	Improvement remains positive throughout the day	True

As seen in Table 6, the average pressure was improved by 4.168% between the real traffic data and the consensus algorithm. While this is not a significant enough improvement for the project to be a success, it shows that traffic was still optimized more than it is currently. This small percentage makes it hard to analyze the effectiveness of the consensus algorithm, so the pressures were also categorized and compared for more insight.

The categories with the most improvement was by far the average pressure on 1, 2, and 3 lane roads, while the average pressure on 4 and 5 lane roads was worse when using the consensus algorithm. This shows that the pressure function and model that the team created is best adjusted for use on roads with only a few lanes. It is possible that the higher lane roads are too complex for the current model, and modifications would have to be made to improve those areas. The 23% decrease in pressure for both 1 and 3 lane roads indicates the potential that this solution has when it is at its most effective.

Furthermore, the grid was chosen for high traffic density and around 75% of the roads within the grid are 1-3 lanes wide. This is not only the case in downtown Toronto as most dense cities don't have space for wide roads. The design solution is therefore the most applicable in the downtown cores of big cities, which is also the area with the most people and the most traffic. Overall, the consensus algorithm is the most effective in the regions where it will have the most impact, showing that despite its shortcomings the solution could be viable with optimization and increased testing.

Since the pressure data was given in 15-minute intervals, specific times of the day could be averaged together to give an idea on how the consensus algorithm reacted to the fluctuation of traffic. Since the data used did not extend past 4pm, only three one-hour time periods were chosen: the morning rush

from 8am to 9am, midday traffic from 11am to 12pm, and afternoon rush from 2pm to 3pm.

Improvement remained positive throughout the day, showing that the algorithm can adjust to rushes while still being efficient. Unfortunately, the improvement dropped by 1% throughout the day as the pressure slowly increased. This means that the design solution cannot perform as well with higher volumes of traffic, but this is expected as the higher concentration of cars complicates the process of finding the optimal traffic light timings.

Table 7 shows the three success requirements that the team determined had to be true for the project to be considered a success. First off, the average pressure overall had to have improved significantly, which unfortunately did not happen, but it was close. The majority (7/13) of the categories also had to have significant improvement, but only five of them were high enough so that was again close but not successful. Finally, the consensus algorithm had to dynamically adjust to traffic fluctuation, which was a success since the improvement remained positive throughout the day.

While the project may not be a success by the metrics predetermined by the team, it shows that use of consensus algorithms in traffic light coordination has the potential to reduce the impact of congestion in urban areas. Had the consensus algorithm only been implemented at intersections with 1,2 and 3 lane roads, the average pressure improvement would have been 12.5%, which is a promising number to reach for in future development.

## 9. Recommendations

Even though the team used about 6000 points of data to run through consensus algorithm, this is still a relatively small sample size. If this project were to be continued, the team would create a systematic way to test the algorithm with even more data. This would allow for in depth analysis of how the algorithm manages congestion, which would enable tweaks to be made to improve performance.

The pressure function is a key element of the project, but there was no testing during the design process of different pressure functions to gauge differences in performance. In the future, this could be modeled using different factors, and then run through the consensus algorithm to determine the model which yields the best improvement.

The team was able to create a simple simulation (see Figure 4), but due to time constraints it did not have as much detail as anticipated. If a vehicle simulation was implemented, it would greatly improve analysis of congestion reduction. Metrics other than pressure such as traffic light wait time, stops made,



car density, and average speed could all be measured to improve the teams understanding of the model as well as allow for a more detailed pressure equation. The data could also be represented in a much more visual way using the simulation.

## 10. Conclusion

By implementing a novel traffic signal coordination system, traffic congestion in downtown Toronto, as well as in urban areas all over the globe could be significantly reduced. This can be achieved by addressing the logistical and technological limitations of current traffic management systems. The use of real-time traffic data with a consensus algorithm aims to improve traffic flow, commuter experience, and urban sustainability.

The project's goal was to develop and test a fully implementable algorithm for traffic management, now reaching the end of the assignment's timeline it is obvious that this was too ambitious of an undertaking for just over a month of work. What the team has developed is a working framework for this system, as well as a function to evaluate traffic at each intersection, and a complex database handling information about traffic density and road capacity at intersections.

The team believes that this was overall a productive endeavor and produced proof of concept for the use of a consensus algorithm to manage traffic in real time.

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## Appendix I – Figures

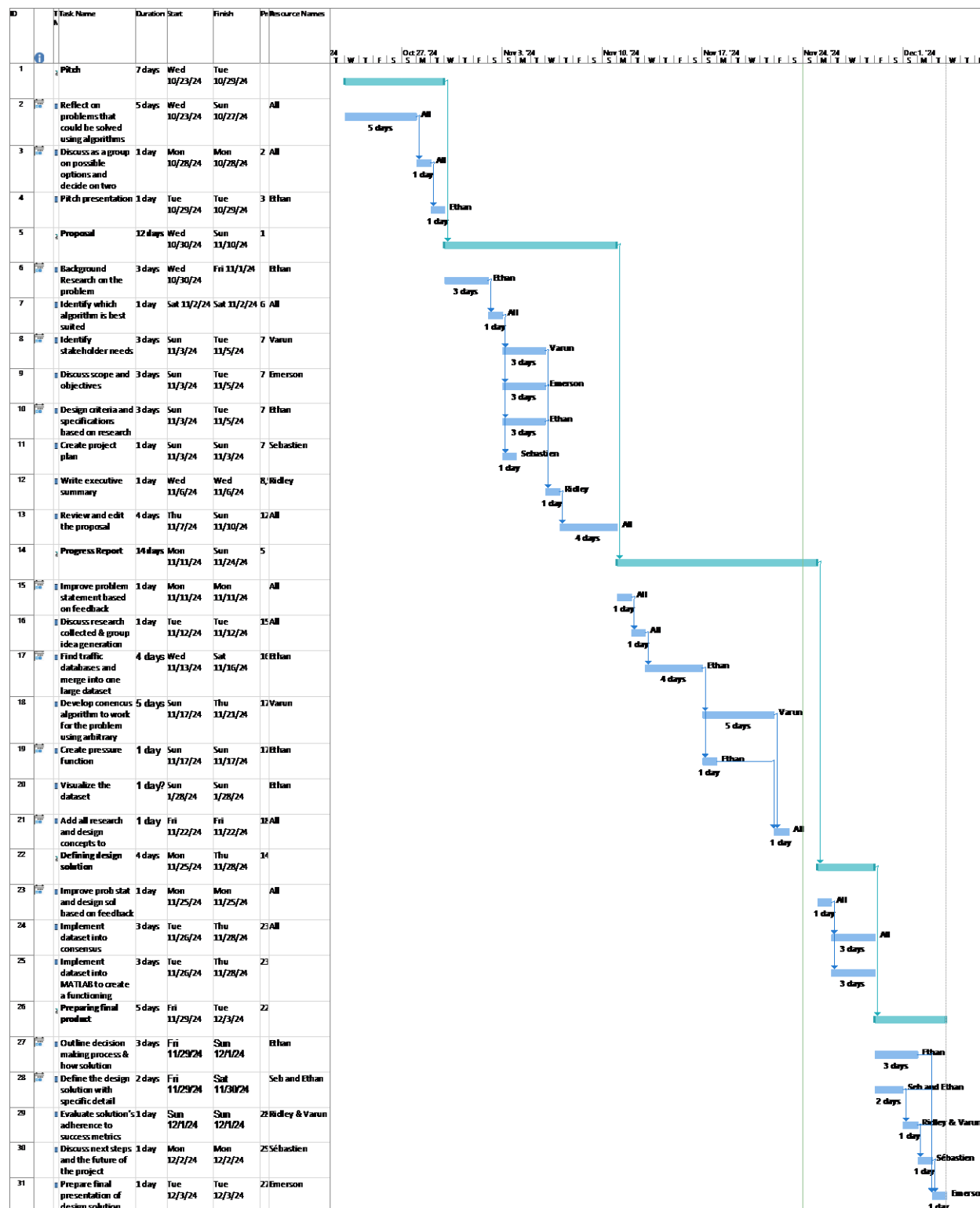


Figure 1: Gantt Chart



Figure 2: Initial target grid in downtown Toronto



Figure 3: Simplified target grid in downtown Toronto with specified road segments.

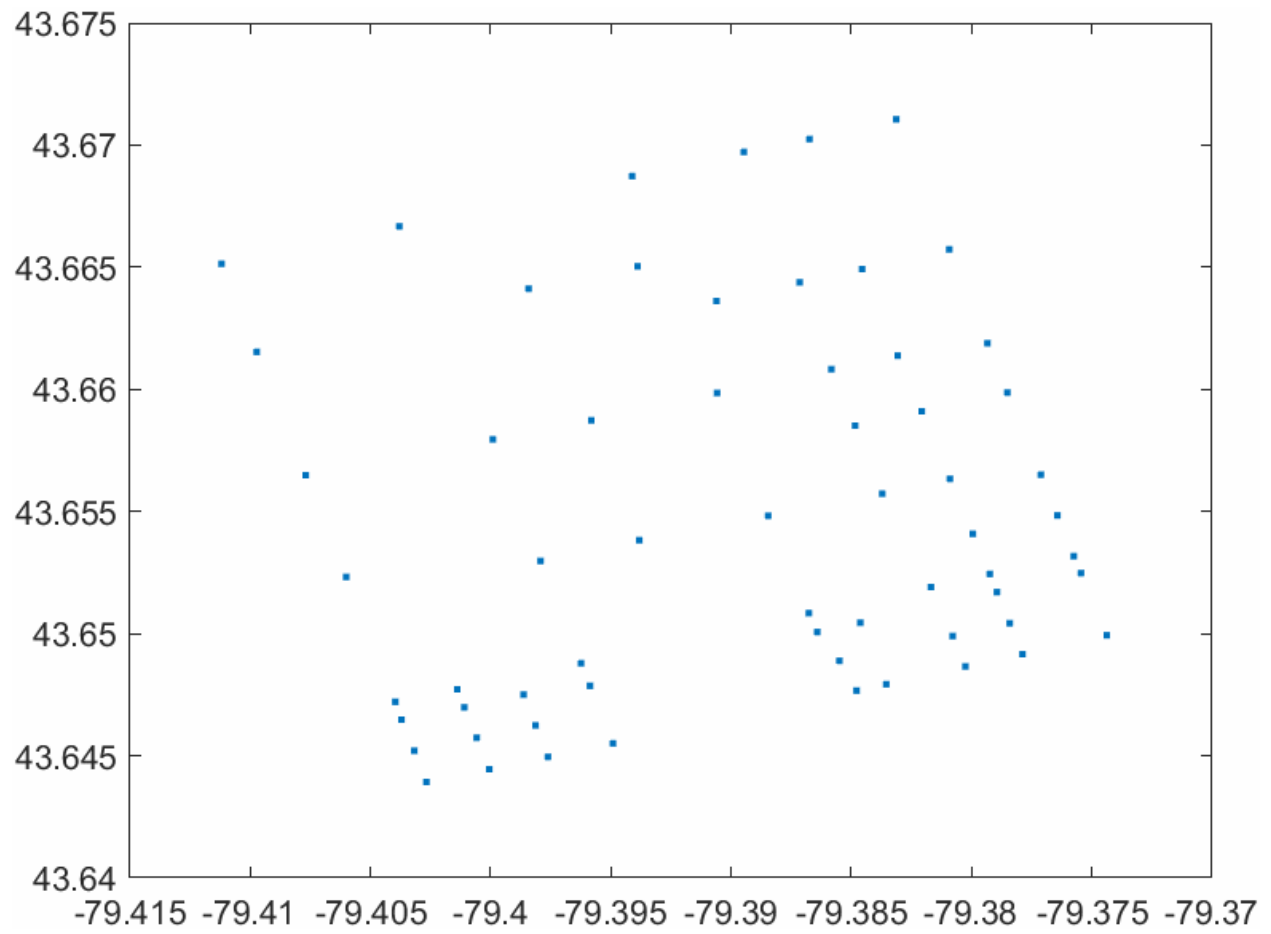


Figure 4: Updated Signal Timings at Intersections

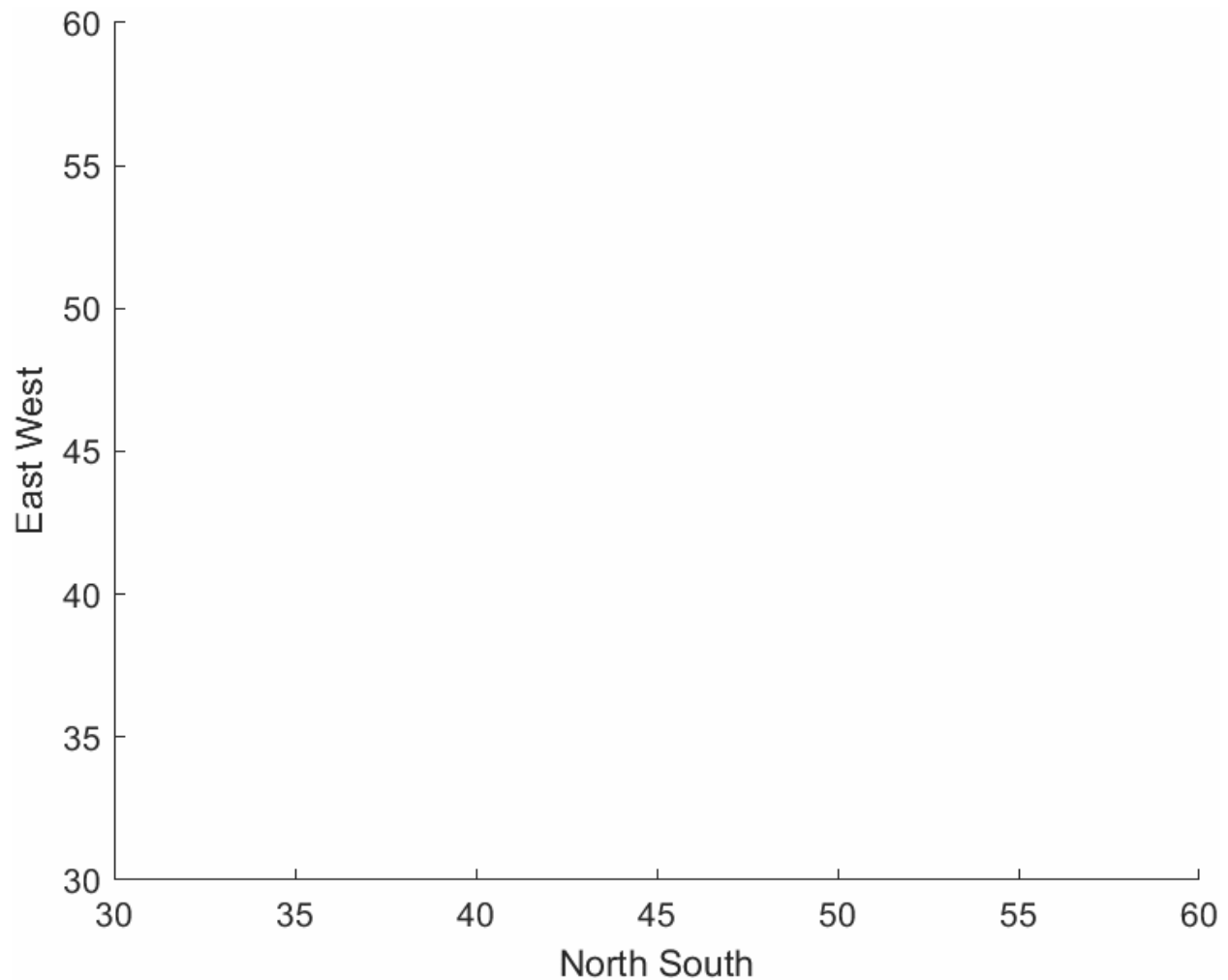


Figure 5: Signal Timings

## Appendix II – MATLAB Code

### consensus.m

```
%APSC 200 P2 - Multi-Agent Traffic Signal Optimization
%Varun Upadhye
%Ethan Solnik
%Emerson Currie
%Sebastian Van Der Voude
%Ridley Horton

%Gets initial signal timings of 45s for each intersection
lights = readmatrix("timings\output.csv");

%Initialises necessary variables
```



```

FINALT = 10;
NUMSTEPS = 1000;
data = zeros(NUMSTEPS*24,62,2);

%Iterates through the 24 timesteps
for p = 1:24

    %Sets up variables
    t = linspace(0,FINALT,NUMSTEPS);
    step = t(2);
    initialValues = readmatrix("data\data"+num2str(p-1)+".csv");
    numAgents = size(initialValues,1);
    timings = [lights(:,2*p-1) lights(:,2*p)];

    %Sets up arrays for Position, Light Timings and Consensus States

    P = nan(numAgents, 2);          % all positions
    for i = 1:2
        P(:,i) = initialValues(:,i)';
    end

    O = nan(NUMSTEPS, numAgents, 2);    % all output data
    for i = 1:2
        O(:, :, i) = ones(NUMSTEPS,1) * timings(:,i)';
    end

    C = nan(NUMSTEPS, numAgents, 4);    % all consensus data
    for i = 1:4
        C(:, :, i) = ones(NUMSTEPS,1) * initialValues(:,i+2)';
    end

    %Gets the Adjacency Matrix and Laplacian
    G = readmatrix("adjacency_matrix.csv");
    G(isnan(G)) = 0;
    %G = consensus_adjacency_matrix(P);
    D=diag(sum(G));
    L=D-G;

    %Iterates through all timesteps
    for i = 2:NUMSTEPS-1

        %Updates the Consensus States and light timings
        o0 = [O(i,:,1)' O(i,:,2)'];
        c0 = [C(i,:,1)' C(i,:,2)' C(i,:,3)' C(i,:,4)'];
        c1 = consensus_filter(c0,L,step);
        o1 = update_agents(o0,c1,step,L);
        for k = 1:numAgents
            for j = 1:2
                O(i+1,k,j) = o1(k,j)';
            end
            for j = 1:4
                C(i+1,k,j) = c1(k,j)';
            end
        end
    end
end

```

```

        end

    end

    %Saves the light timings
    data(NUMSTEPS*(p-1)+1:NUMSTEPS*p, :, :) = 0;
    lights(:, 2*p+1) = 0(NUMSTEPS-1, :, 1);
    lights(:, 2*p+2) = 0(NUMSTEPS-1, :, 2);

end

csvwrite("timings\output.csv", lights);

```

## consensus\_filter.m

```

function x1 = consensus_filter(x0, L, dt)
%{
Let "n" be the number of agents.

--- Inputs ---
x0: n by 2 array holding consensus state information for timestep i
    e.g., x0(j,1) is the first consensus state for agent-j

L: n by n Laplacian matrix

dt: timestep

--- Outputs ---
x1: n by 2 array holding consensus state information for timestep i+1
    Same structure as x0.
%}

delta = L*x0*dt;
x1 = x0-delta;

end

```

## update\_agents.m

```

function y1 = update_agents(y0, x1, dt, L)
%{
Let "n" be the number of agents.

--- Inputs ---
y0: n by 2 array holding all system outputs at timestep i
    e.g., y0(j,2) is the second output for agent-j

x1: n by 2 array holding all consensus states for all agents at timestep i+1

```

```

    e.g., x1(j,2) is the second consensus state for agent-j

dt: timestep

L: n by n Laplacian matrix

--- Outputs ---
y1: n by 2 array holding all system outputs at timestep i+1
    Same structure as y0.
%}

%Sets up necessary variables
n=size(y0,1);
y1 = zeros(n,2);

%Iterates through all intersections
for i = 1:n
    %Finds the difference between the intersections pressure, and the neighbours
    neighbours = find(L(i,:) < 0);
    avg = mean(x1(neighbours,:));
    difference = x1(i,:) - avg;

    %Updates the Light Timings
    adjustments =
5*[max(difference(1),difference(2)),max(difference(3),difference(4))];
    y1(i,:) = y0(i,:) + adjustments*dt;
end

```

## diagram.m

```

g = arrayfun(@(x) animatedline("Color",[rand() rand() rand()]), 1:numAgents);
xlim([30,60]);
ylim([30,60])
xlabel("North South");
ylabel("East West");
for i = 1:240
    for ih = 1:numAgents
        x = data((i-1)*100+1,ih,1);
        y = data((i-1)*100+1,ih,2);
        addpoints(g(ih),x,y)
    end
    %drawnow;
    exportgraphics(gcf,"animation.gif",'Append',true);
end

```

## map.m

```

intersections = readlines("intersections.csv");

east = [P(2,1)-P(1,1),P(2,2)-P(1,2)];

```

```

north = [P(10,1)-P(1,1),P(10,2)-P(1,2)];

east = east/norm(east);
north = north/norm(north);

NS = ones(24,62,2);
EW = ones(24,62,2);

for i = 1:62
    for j = 1:24
        NS(j,i,:) = north*(data(1000*j,i,1)-45);
        EW(j,i,:) = east*(data(1000*j,i,2)-45);
    end
end

positions = [P;P];

directions = cat(2,NS,EW);

size(positions(:,2))
size(directions(j,:,2))

for j = 1:24

quiver(positions(:,1),positions(:,2),directions(j,:,1)',directions(j,:,2)',0.7, "Marke
r", ".");
    drawnow;
    exportgraphics(gcf, "map.gif", "Append", true);
end

```