Exercise M

To prove that (o ((curry map) f) ((curry map) g)) == ((curry map) (o f g)) by structural induction, I will show that when applied to equal arguments, both sides return equal results. That is, I will prove that for any xs, ((o ((curry map) f) ((curry map) g)) xs) == (((curry map) (o f g)) xs).

Let's start with the base case, where xs = '( ):

((o ((curry map) f) ((curry map) g)) '( ))

= { apply-compose law }

(((curry map) f) (((curry map) g) '( )))

= { apply-curried law }

(((curry map) f) (map g '( )))

= { apply-curried law }

(map f (map g '( )))

= { map-null law }

(map f '( ))

= { map-null law }

'( )

= { map null law (reverse) }

(map (o f g) '( ))

= { apply-curried law (reverse) }

(((curry map) (o f g)) '( )) == (((curry map) (o f g)) xs), when xs = '( )

Now I will prove the inductive case, where xs = (cons y ys):

((o ((curry map) f) ((curry map) g)) (cons y ys))

= { apply-compose law }

(((curry map) f) (((curry map) g) (cons y ys)))

= { apply-curried law }

(((curry map) f) (map g (cons y ys)))

= { apply-curried law }

(map f (map g (cons y ys)))

= { cons-map law }

(map f (cons (g y) (map g ys)))

= { cons-map law }

(cons (f (g y)) (map f (map g ys)))

= { apply-compose law (reverse) }

(cons ((o f g) y) (map f (map g ys)))

= { apply-curried law (reverse) }

(cons ((o f g) y) (((curry map) f) (map g ys)))

= { apply-curried law (reverse) }

(cons ((o f g) y) (((curry map) f) (((curry map) g) ys)))

= { apply-compose law (reverse) }

(cons ((o f g) y) ((o ((curry map) f) ((curry map) g)) ys))

= { induction hypothesis }

(cons ((o f g) y) ((curry map) (o f g) ys))

= { apply-curried law (reverse) }

(cons ((o f g) y) (map (o f g) ys))

= { cons-map law }

(map (o f g) (cons y ys))

= { apply-curried law (reverse) }

(((curry map) (o f g)) (cons y ys)) == (((curry map) (o f g)) xs), when xs = (cons y ys)

Our proof by structural induction is now complete after proving the base case and inductive case.