

## I. CCSD

Defining some quantities  
ERIs, which notation?

$$v_{qs}^{pr} = (pq|rs) \quad (1)$$

$$w_{ab}^{ij} = 2v_{ab}^{ij} - v_{ba}^{ij} \quad (2)$$

CCSD energy equation

$$E = \langle \Phi_0 | H e^{T_1 + T_2} | \Phi_0 \rangle \quad (3)$$

$$E_{\text{CCSD}} = 2 \sum_{i,a} f_i^a t_i^a + \sum_{i,j,a,b} w_{ab}^{ij} t_i^a t_j^b + \sum_{i,j,a,b} w_{ab}^{ij} t_{ij}^{ab} \quad (4)$$

Defining the  $T_1$  transformed operators

$$\hat{O} = e^{-T_1} O e^{T_1} \quad (5)$$

The CCSD amplitude equations with transformed operators

$$\langle \Phi_i^a | \hat{H} + [\hat{H}, T_2] | \Phi_0 \rangle = 0 \quad (6)$$

$$\langle \Phi_{ij}^{ab} | \hat{H} + [\hat{H}, T_2] + \frac{1}{2} [[\hat{H}, T_2], T_2] | \Phi_0 \rangle = 0 \quad (7)$$

$T_1$  amplitude equation

$$\begin{aligned} 0 = & f_i^a - 2 \sum_{k,c} f_c^k t_k^a t_i^c + \sum_c \kappa_c^a t_i^c - \sum_k \kappa_i^k t_k^a \\ & + \sum_{k,c} \kappa_c^k (2t_{ki}^{ca} - t_{ik}^{ca}) + \sum_{k,c} \kappa_c^k t_i^c t_k^a + \sum_{k,c} w_{ic}^{ak} t_k^c \\ & + \sum_{k,c,d} w_{cd}^{ak} t_{ik}^{cd} + \sum_{k,c,d} w_{cd}^{ak} t_i^c t_k^d - \sum_{k,l,c} w_{ic}^{kl} t_{kl}^{ac} \\ & - \sum_{k,l,c} w_{ic}^{kl} t_k^a t_l^c \end{aligned} \quad (8)$$

$T_2$  Amplitude Equations

$$\begin{aligned} 0 = & v_{ab}^{ij} + \sum_{k,l} \chi_{ij}^{kl} t_{kl}^{ab} + \sum_{k,l} \chi_{ij}^{kl} t_k^a t_l^b + \sum_{c,d} \chi_{cd}^{ab} t_{ij}^{cd} \\ & + \sum_{c,d} \chi_{cd}^{ab} t_i^c t_j^d + P \sum_c \lambda_c^a t_{ij}^{cb} - P \sum_k \lambda_i^k t_{kj}^{ab} \\ & + P \sum_c \left( v_{ic}^{ab} - \sum_k v_{ic}^{kb} t_k^a \right) t_j^c - P \sum_k \left( v_{ij}^{ak} + \sum_c v_{ic}^{ak} t_j^c \right) t_k^b \\ & + P \sum_{k,c} \left( 2\chi_{ic}^{ak} - \chi_{ci}^{ak} \right) t_{kj}^{cb} - P \sum_{k,c} \chi_{ic}^{ak} t_{kj}^{bc} \\ & - P \sum_{k,c} \chi_{ci}^{bk} t_{kj}^{ac} \end{aligned} \quad (9)$$

Intermediates

$$\kappa_i^k = f_i^k + \sum_{l,c,d} w_{cd}^{kl} t_{il}^{cd} + \sum_{l,c,d} w_{cd}^{kl} t_i^c t_l^d \quad (10)$$

$$\kappa_c^a = f_c^a - \sum_{k,l,d} w_{cd}^{kl} t_{kl}^{ad} - \sum_{k,l,d} w_{cd}^{kl} t_{kl}^{ad} \quad (11)$$

$$\kappa_c^k = f_c^k + \sum_{l,d} w_{cd}^{kl} t_l^d \quad (12)$$

$$\lambda_i^k = \kappa_i^k + \sum_c f_c^k t_i^c + \sum_{l,c} w_{ic}^{kl} t_l^c \quad (13)$$

$$\lambda_c^a = \kappa_c^a - \sum_k f_c^k t_k^a + \sum_{k,d} w_{cd}^{ak} t_k^d \quad (14)$$

$$\chi_{ij}^{kl} = v_{ij}^{kl} + \sum_c v_{ic}^{kl} t_j^c + \sum_c v_{cj}^{kl} t_i^c + \sum_{c,d} v_{cd}^{kl} t_{ij}^{cd} + \sum_{l,d} v_{dc}^{lk} t_i^d t_l^a \quad (15)$$

$$\chi_{cd}^{ab} = v_{cd}^{ab} - \sum_k v_{cd}^{ak} t_k^b - \sum_k v_{cd}^{kb} t_k^a \quad (16)$$

$$\chi_{ic}^{ak} = v_{ic}^{ak} - \sum_l v_{ic}^{lk} t_l^a + \sum_d v_{dc}^{ak} t_i^d - \frac{1}{2} \sum_{l,d} v_{dc}^{lk} t_{il}^{da} \quad (17)$$

$$\begin{aligned} & - \sum_{l,d} v_{dc}^{lk} t_i^d t_l^a + \frac{1}{2} \sum_{l,d} w_{dc}^{lk} t_{il}^{ad} \\ \chi_{ci}^{ak} = & v_{ci}^{aj} - \sum_l v_{ci}^{lk} t_l^a + \sum_d v_{cd}^{ak} t_i^d - \frac{1}{2} \sum_{l,d} v_{cd}^{lk} t_{il}^{da} \\ & - \sum_{l,d} v_{cd}^{lk} t_i^d t_l^a \end{aligned} \quad (18)$$

## II. CC2

The doubles equations are approximated to first order.

The CC2 doubles amplitude equations with transformed operators

$$\langle \Phi_{ij}^{ab} | [F, T_2] + \hat{H} | \Phi_0 \rangle = 0 \quad (19)$$

Note that  $F$  doesn't have a hat!

Current implementation of  $T_2$  amplitude equations

$$\begin{aligned} 0 = & v_{ab}^{ij} + \sum_{k,l} \chi_{ij}^{kl} t_k^a t_l^b + \sum_{c,d} \chi_{cd}^{ab} t_i^c t_j^d + P \sum_c \lambda_c^a t_{ij}^{cb} - P \sum_k \lambda_i^k t_{kj}^{ab} \\ & + P \sum_c \left( v_{ic}^{ab} - \sum_k v_{ic}^{kb} t_k^a \right) t_j^c - P \sum_k \left( v_{ij}^{ak} + \sum_c v_{ic}^{ak} t_j^c \right) t_k^b \end{aligned} \quad (20)$$

Intermediates

$$\kappa_i^k = f_i^k \quad (21)$$

$$\kappa_c^a = f_c^a \quad (22)$$

$$\lambda_i^k = \kappa_i^k + \sum_c f_c^k t_i^c \quad (23)$$

$$\lambda_c^a = \kappa_c^a - \sum_k f_c^k t_k^a \quad (24)$$

$$\chi_{ij}^{kl} = v_{ij}^{kl} + \sum_c v_{ic}^{kl} t_j^c + \sum_c v_{cj}^{kl} t_i^c + \sum_{l,d} v_{dc}^{lk} t_i^d t_l^a \quad (25)$$

$$\chi_{cd}^{ab} = v_{cd}^{ab} - \sum_k v_{cd}^{ak} t_k^b - \sum_k v_{cd}^{kb} t_k^a \quad (26)$$

$$(27)$$

$T_1$  transform

$$X_{ai} = C_{ai} \quad (28)$$

$$X_{\alpha a} = C_{\alpha a} - \sum_i C_{\alpha i} t_i^a \quad (29)$$

$$Y_{\alpha i} = C_{\alpha i} + \sum_a C_{\alpha a} t_i^a \quad (30)$$

$$Y_{\alpha a} = C_{\alpha a} \quad (31)$$

$T_1$  transformed two-electron integral

$$(pq|rs) = \sum_{\mu\nu\lambda\sigma} X_{\mu p} Y_{\nu q} X_{\lambda r} Y_{\sigma s} (\mu\nu|\lambda\sigma) \quad (32)$$

Doubles amplitude

$$t_{ij}^{ab} = \frac{(ai|bj)}{\varepsilon_i - \varepsilon_a + \varepsilon_j - \varepsilon_b} \quad (33)$$

### III. EOMEE

Original Barlett Paper

$$\langle a^\dagger b \rangle = \rho_{ab} = r_0 \left[ t_m^b l_a^m + \frac{1}{2} t_{mn}^{eb} l_{ea}^{mn} \right] + r_m^b l_a^m + \frac{1}{2} r_{mn}^{eb} l_{ea}^{mn} + t_n^b r_m^e l_{ea}^{mn} \quad (34)$$

Krylov derivation

$$\gamma'_{ab} = \frac{1}{2} P_+(ab) \left( \sum_i l_i^a \tilde{r}_{ib} + \tilde{l}^{ab} + \sum_i Y_{ia}^1 t_i^b \right) \quad (35)$$

$$\tilde{r}_{ib} = r_i^b + r_0 t_i^b \quad (36)$$

$$\tilde{l}^{ab} = \frac{1}{2} \sum_{ijc} l_{ij}^{ac} \tilde{r}_{ij}^{bc} \quad (37)$$

$$\tilde{r}_{ij}^{bc} = r_{ij}^{bc} + r_0 t_{ij}^{bc} \quad (38)$$

$$\gamma'_{ab} = \frac{1}{2} P_+(ab) \left( \sum_i l_i^a r_i^b + r_0 l_i^a t_i^b + \frac{1}{2} l_{ij}^{ac} r_{ij}^{bc} + \frac{1}{2} r_0 l_{ij}^{ac} t_{ij}^{bc} + \sum_i l_{ij}^{ab} r_j^b t_i^b \right) \quad (39)$$

$$\gamma'_{ab} = \frac{1}{2} \left( \sum_i l_i^a r_i^b + r_0 l_i^a t_i^b + \frac{1}{2} l_{ij}^{ac} r_{ij}^{bc} + \frac{1}{2} r_0 l_{ij}^{ac} t_{ij}^{bc} + \sum_i l_{ij}^{ab} r_j^b t_i^b \right) \quad (40)$$

$$+ \frac{1}{2} \left( \sum_i l_i^b r_i^a + r_0 l_i^b t_i^a + \frac{1}{2} l_{ij}^{bc} r_{ij}^{ac} + \frac{1}{2} r_0 l_{ij}^{bc} t_{ij}^{ac} + \sum_i l_{ij}^{ba} r_j^a t_i^a \right) \quad (41)$$

$$Y_{ia}^1 = \sum_{jb} l_{ij}^{ab} r_j^b \quad (42)$$

#### IV. EOMIP

$$\gamma'_{ab} = \frac{1}{2} P_+(ab) \left( \tilde{l}^{ab} - \sum_i Y_{ia}^1 t_i^b \right) \quad (43)$$

$$Y_{ia}^1 = \sum_j l_{ij}^a r_j \quad (44)$$

$$\tilde{l}^{ab} = \frac{1}{2} \sum_{ij} l_{ij}^a r_{ij}^b \quad (45)$$

$$\gamma'_{ab} = \frac{1}{2} P_+(ab) \left( \frac{1}{2} \sum_{ij} l_{ij}^a r_{ij}^b - \sum_j l_{ij}^a r_j t_i^b \right) \quad (46)$$

$$\gamma'_{ab} = \frac{1}{2} \left( \frac{1}{2} \sum_{ij} l_{ij}^a r_{ij}^b - \sum_{ij} l_{ij}^a r_j t_i^b \right) \quad (47)$$

$$+ \frac{1}{2} \left( \frac{1}{2} \sum_{ij} l_{ij}^b r_{ij}^a - \sum_{ij} l_{ij}^b r_j t_i^a \right) \quad (48)$$

#### V. EOMEA

$$\gamma'_{ab} = \frac{1}{2} P_+(ab) \left( l^a r^b + \tilde{l}_{ab} \sum_k Y_{ka}^1 t_k^b \right) \quad (49)$$

$$\tilde{l}_{ab} = \sum_{kc} l_k^{ac} r_k^{bc} \quad (50)$$

$$Y_{ka}^1 = \sum_c l_k^{ac} r^c \quad (51)$$

$$\gamma'_{ab} = \frac{1}{2} P_+(ab) \left( \sum_i l^a r^b + \frac{1}{2} l_i^{ac} r_i^{bc} + \sum_m l_k^{ac} r^c t_k^b \right) \quad (52)$$

$$\gamma'_{ab} = \frac{1}{2} \left( \sum_i l^a r^b + \frac{1}{2} l_i^{ac} r_i^{bc} + \sum_k l_k^{ac} r^c t_k^b \right) \quad (53)$$

$$+ \frac{1}{2} \left( \sum_i l^b r^a + \frac{1}{2} l_i^{bc} r_i^{ac} + \sum_i l_k^{bc} r^c t_k^a \right) \quad (54)$$