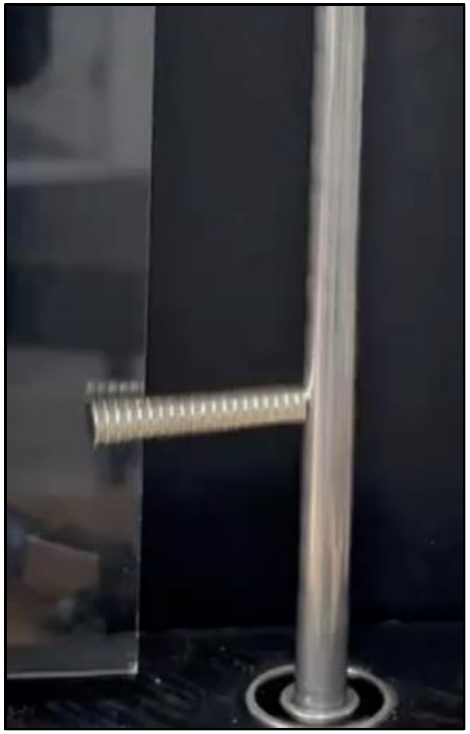
4. Climbing Magnets

Bailin Wang| Team Canada | Reporter

"Attach a rod assembled from cylindrical neodymium magnets horizontally to a vertical ferromagnetic rod. Limit the motion of the magnets to the vertical direction. When the ferromagnetic rod is spun around its axis of symmetry, the magnetic rod begins to climb up. Explain this phenomenon and investigate how the rate of climbing depends on relevant parameters."



Problem Statement

"Attach a rod assembled from cylindrical neodymium magnets horizontally to a vertical ferromagnetic rod. Limit the motion of the magnets to the vertical direction. When the ferromagnetic rod is spun around its axis of symmetry, the magnetic rod begins to climb up. Explain this phenomenon and investigate how the rate of climbing depends on relevant parameters."

Parameters Accounted

Magnets

- *Number of Magnets*
- *Radius of Magnets*
- *Magnet Magnetization*

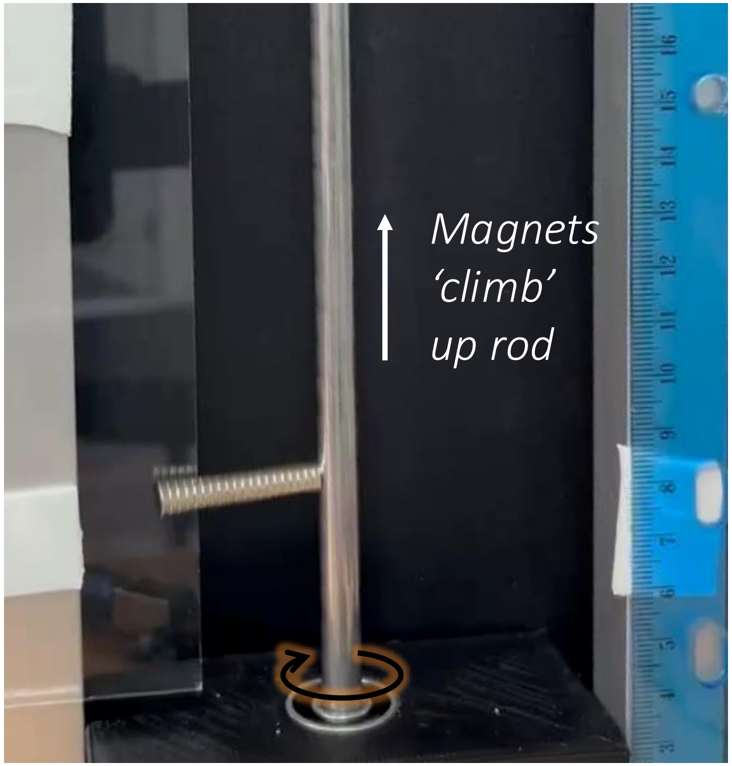
Rod

- *Angular Velocity*
- *Rod Permeability*
- *Rod Radius*
- *Rod-on-Magnet Friction*

Overview

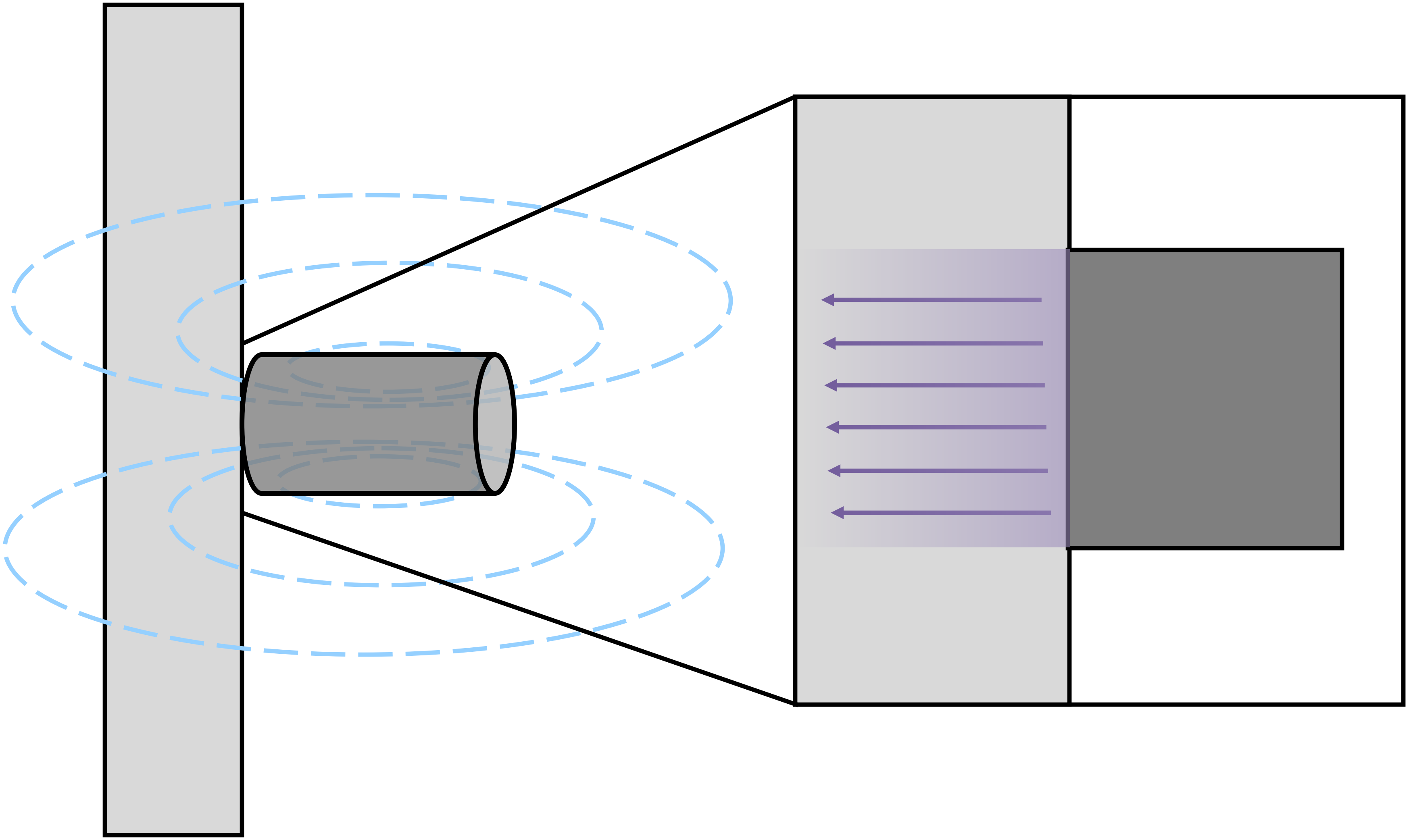
Phenomenon

Phenomenon Demonstration

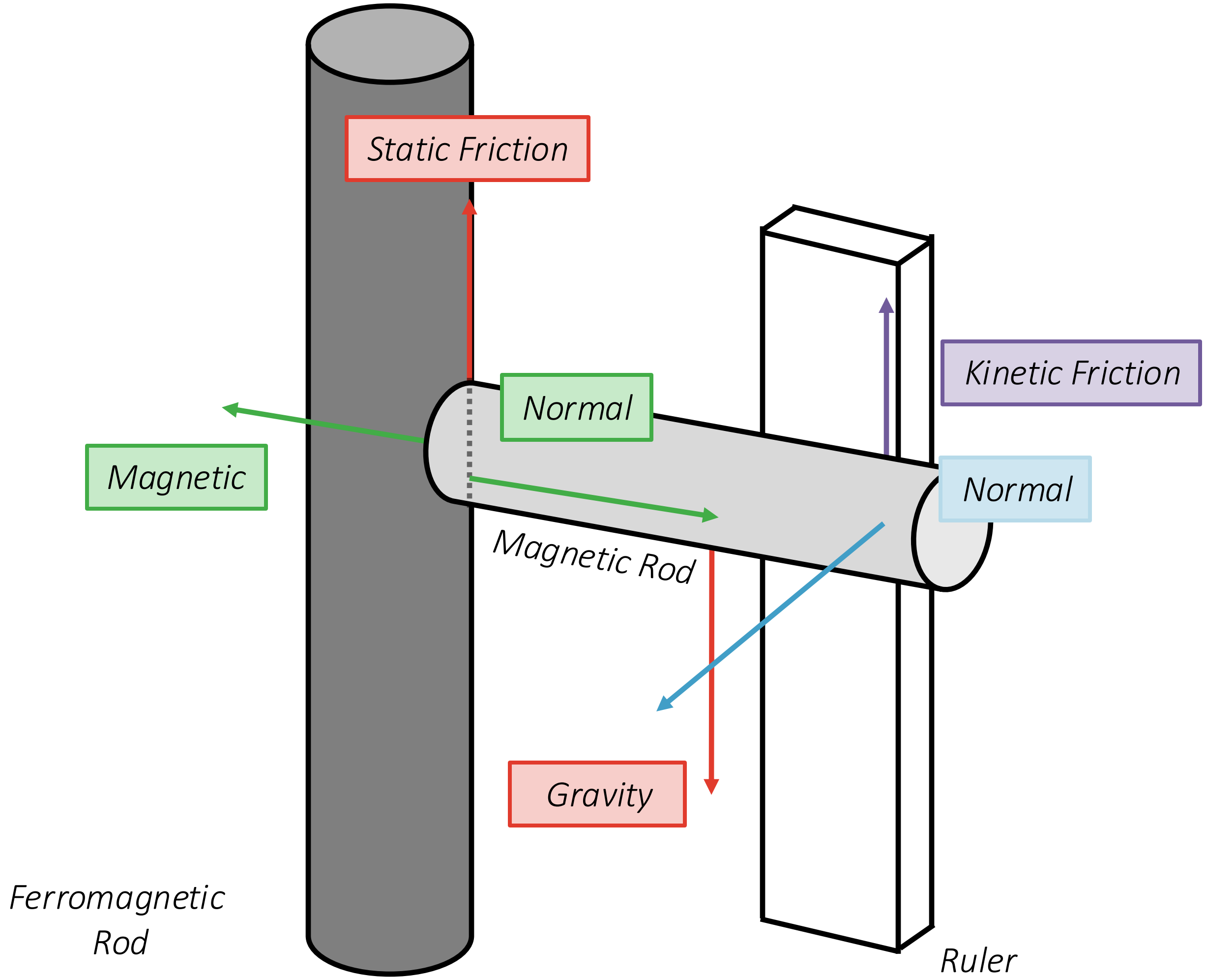


Qualitative

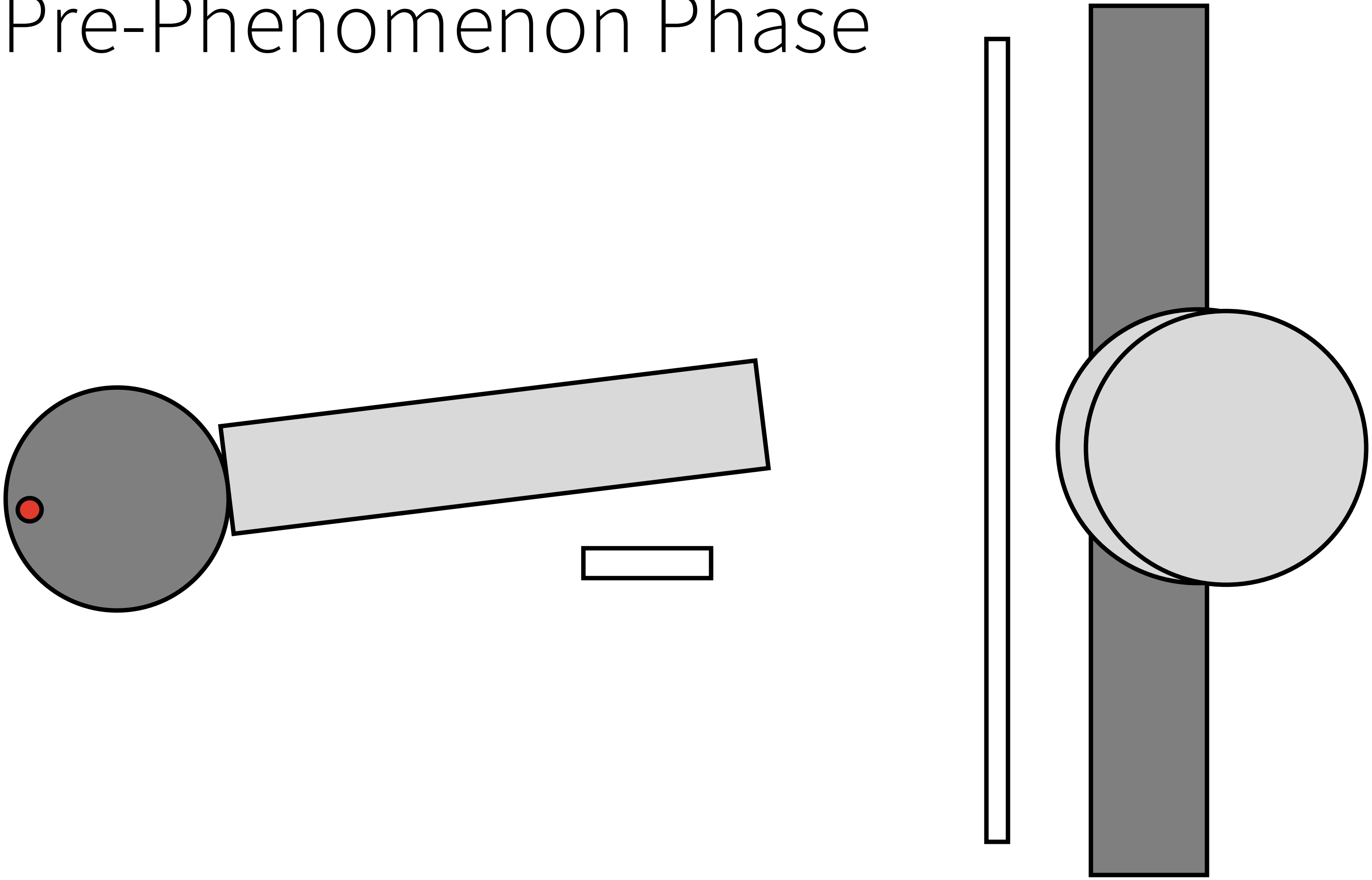
Magnet Force



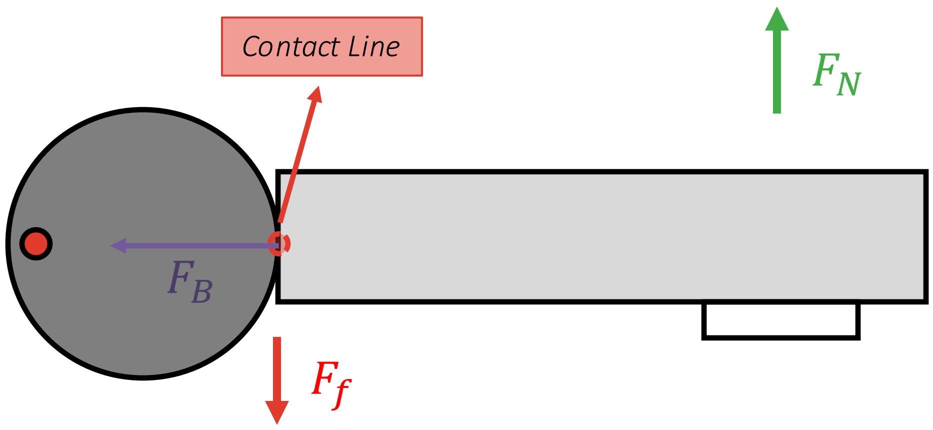
Force Diagram

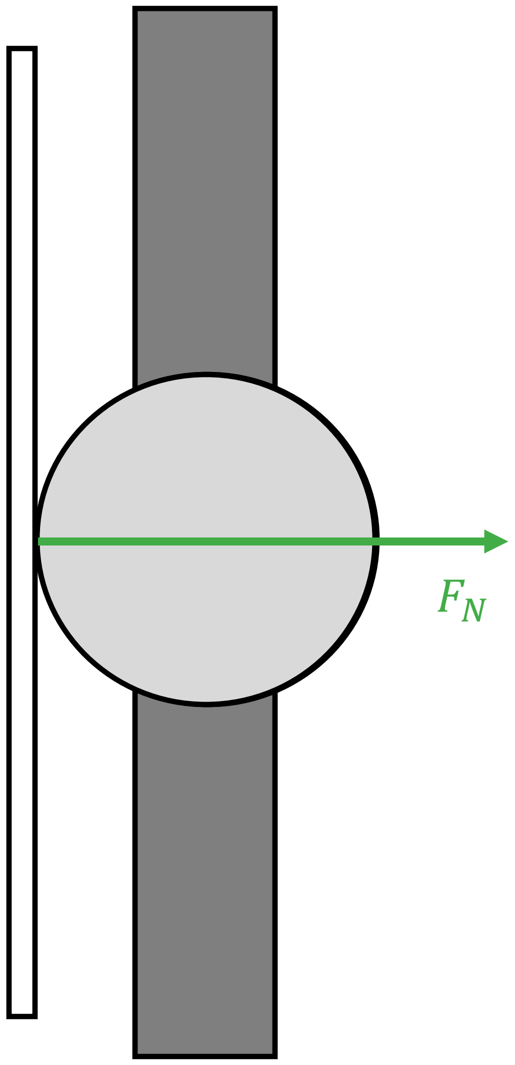


Pre-Phenomenon Phase

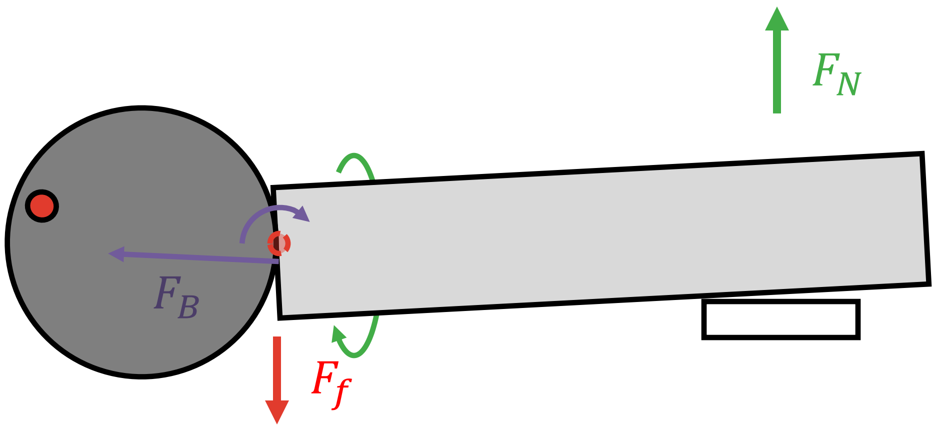


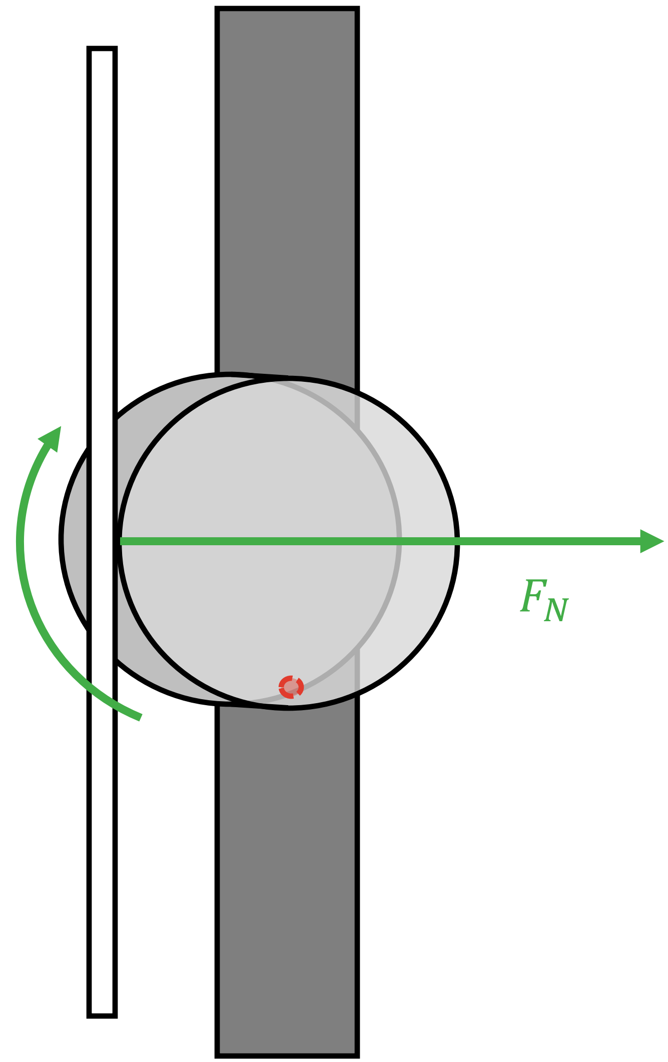
Stick Phase



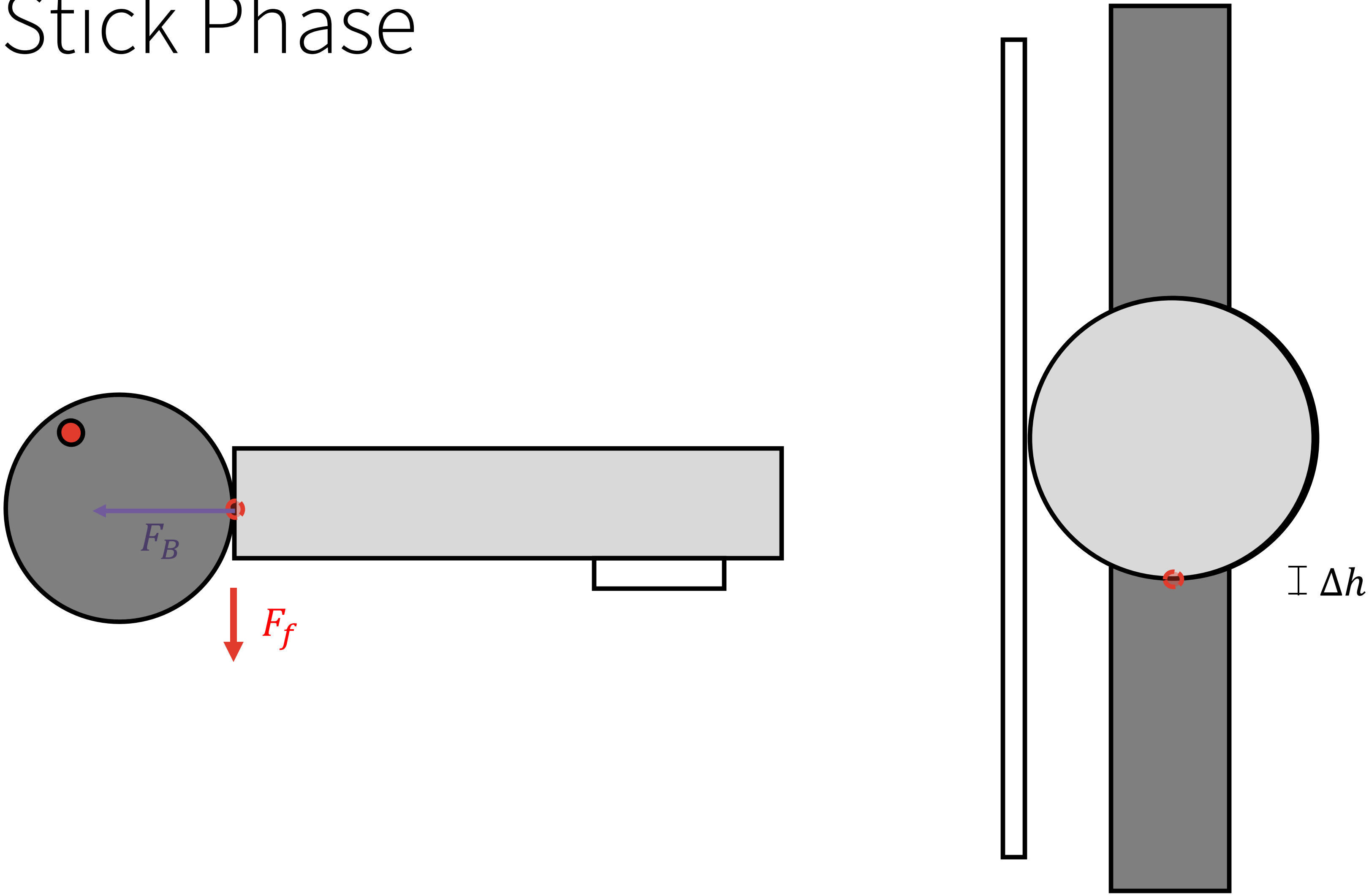


Slip Phase



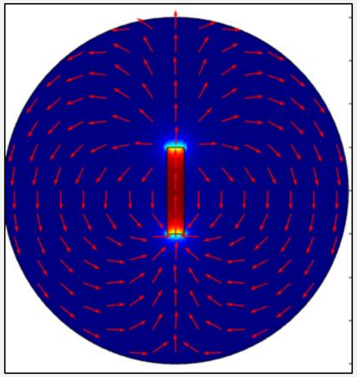


Stick Phase



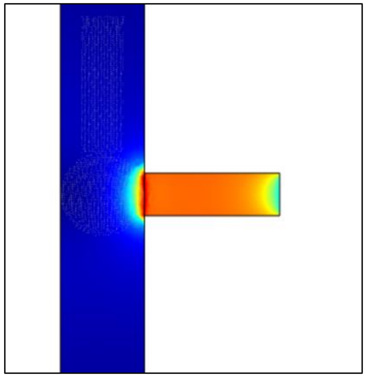
Quantitative

Magnetic Force Model



Permanent Magnet Model

Solving the Permanent Magnet field



Magnetization & Magnet Interaction

Solving for the magnetization and the induced magnetic force

Assumption

1 *There is no Free Current Induced in the magnet and the rod*

2 *Magnetic Insulation applied on the surrounding air boundaries such that*

$$\hat{n} \times A = 0$$

3 *Continuity Boundary Condition at the material Boundaries*

$$\hat{n} \cdot (\boldsymbol{B_1} - \boldsymbol{B_2}) = \boldsymbol{0} \qquad \hat{n} \times (\boldsymbol{H_1} - \boldsymbol{H_2}) = \boldsymbol{0}$$

Magnetic Field Model

Maxwell-Ampere's Law

$$\nabla \times \left(\frac{1}{\mu_0}\boldsymbol{B} - \boldsymbol{M}\right) = \nabla \times \boldsymbol{H} = 0$$

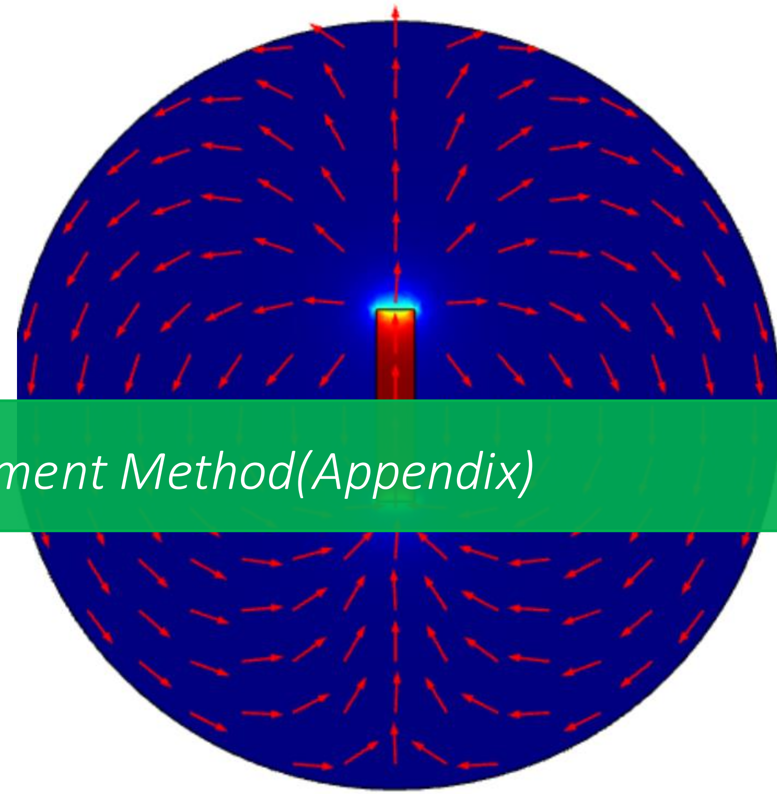
$$\boldsymbol{H} = -\nabla V_m$$

$$\boldsymbol{B} = \mu_r \mu_0 (\boldsymbol{H} + \boldsymbol{M})$$

Solve for V_m using Finite Element Method(Appendix)

Magnetostatic Equation

$$-\nabla \cdot \left(\mu_0(\nabla V_m - \boldsymbol{M})\right) = 0$$



$\boldsymbol{H}$= Magnetic Field intensity

$\boldsymbol{M}$ = Magnetization per unit volume

$\boldsymbol{B}$= Magnetic Flux intensity

$\boldsymbol{V_m}$ = Magnetic Scalar Potential

$\boldsymbol{\mu_r}$ = Relative Permeability

$\boldsymbol{\mu_0}$ = Permeability of Free Space

Magnetic Field Model

Remanent Magnetic Flux Density

$$\boldsymbol{B} = \mu_0 \mu_r \boldsymbol{H} + \boldsymbol{B_r}$$

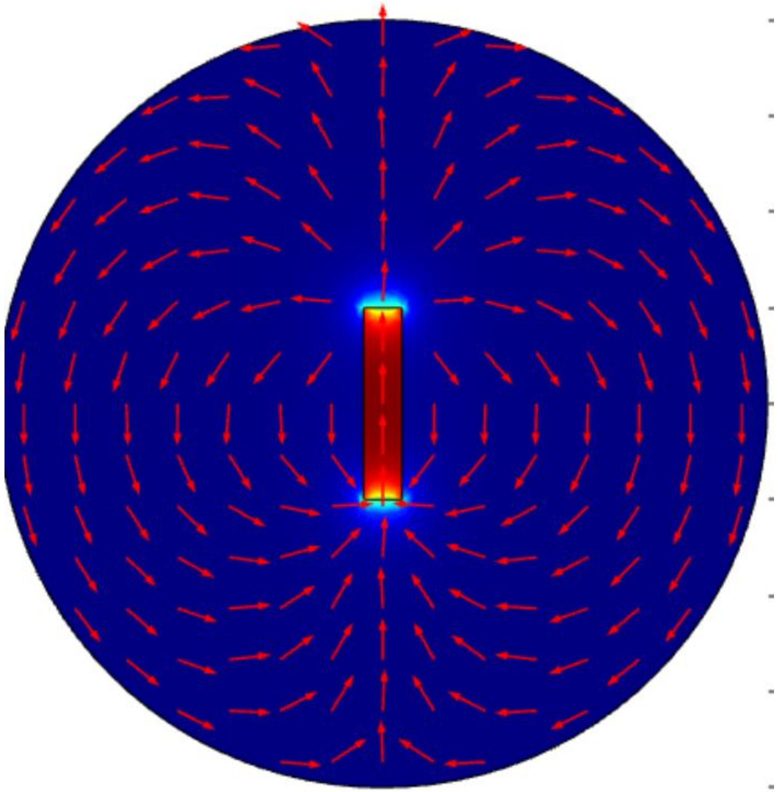
$\boldsymbol{H}$= Magnetic Field intensity

$\boldsymbol{B}$= Magnetic Flux intensity

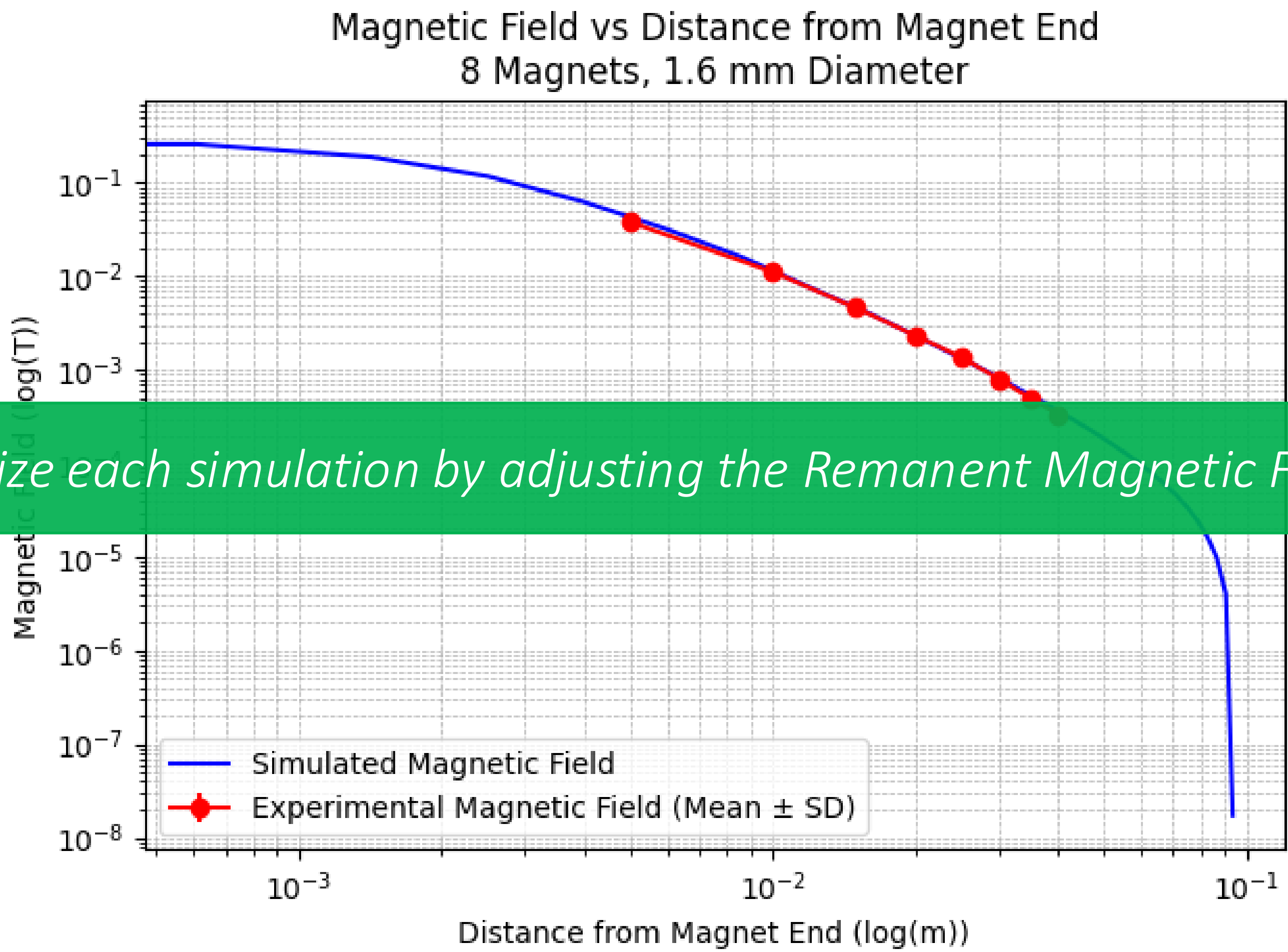
$\boldsymbol{\mu_r}$ = Relative Permeability

$\boldsymbol{\mu_0}$ = Permeability of Free Space

$\boldsymbol{B_r}$= Remanent Flux Density

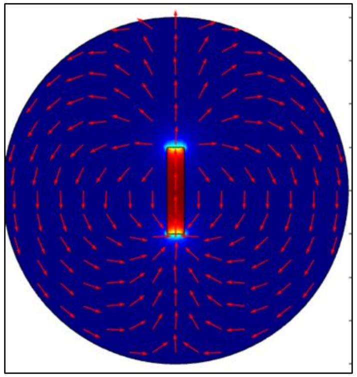


Permanent Magnet Simulation



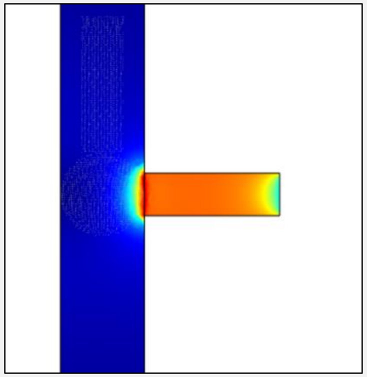
Characterize each simulation by adjusting the Remanent Magnetic Flux Density

Magnetic Force Model



Permanent Magnet Model

Solving the Permanent Magnet field



Magnetization & Magnet Interaction

Solving for the magnetization and the induced magnetic force

Magnetization Model

B-H Curve Characterization

$$\boldsymbol{B} = \boxed{f(\boldsymbol{H})}\boxed{\frac{\boldsymbol{H}}{|\boldsymbol{H}|}}$$

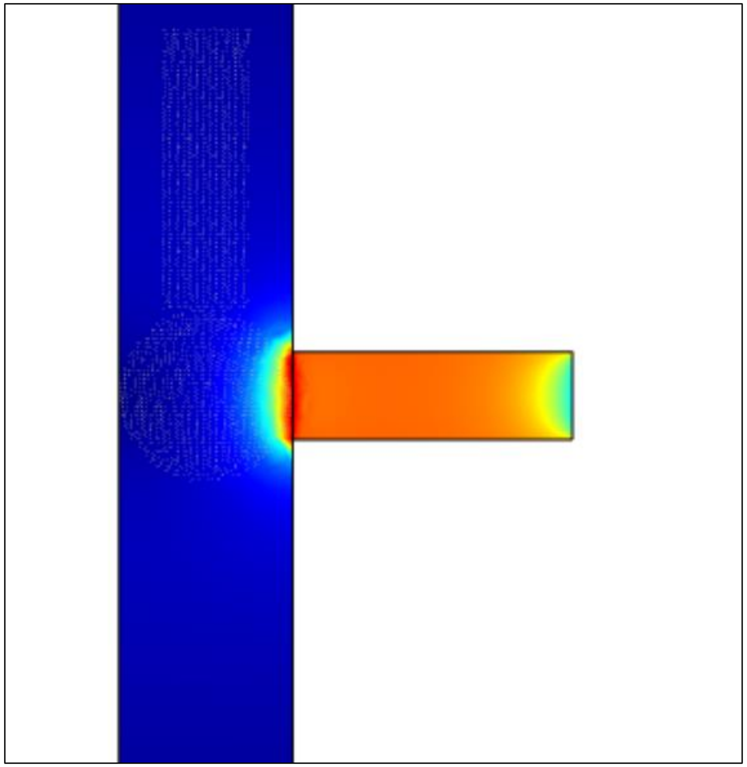
B-H Curve

Direction of H

$\boldsymbol{H}$= Magnetic Field intensity

$\boldsymbol{B}$= Magnetic Flux intensity

$f(\boldsymbol{H})$= Non-linear B-H Response Curve



Electrostatic Force

Electrostatic Force

$$\boldsymbol{F} = \int_{\partial\Omega} \boldsymbol{nT}dS$$

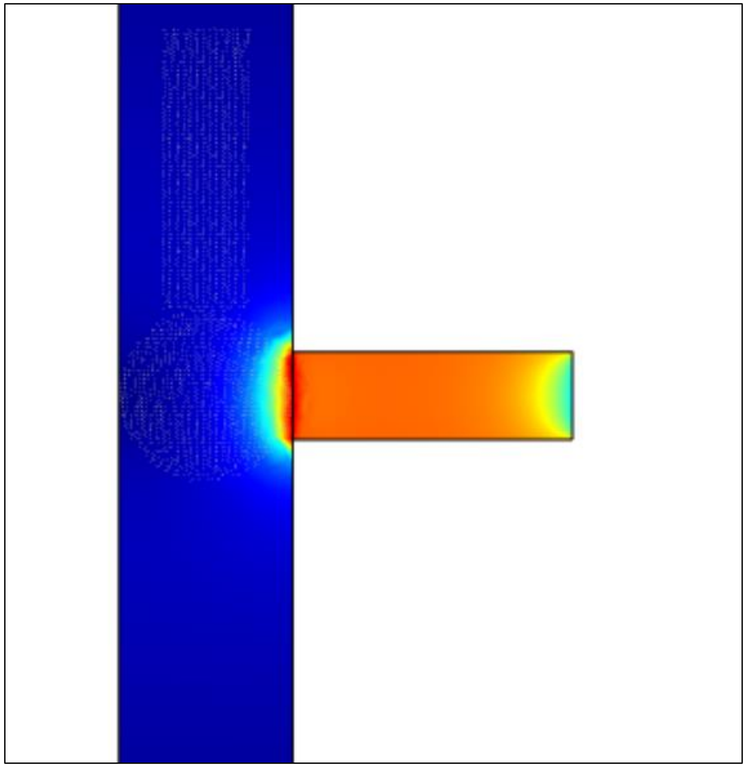
$\boldsymbol{T}$= Maxwell Stress Tensor

$\boldsymbol{S}$= Surface

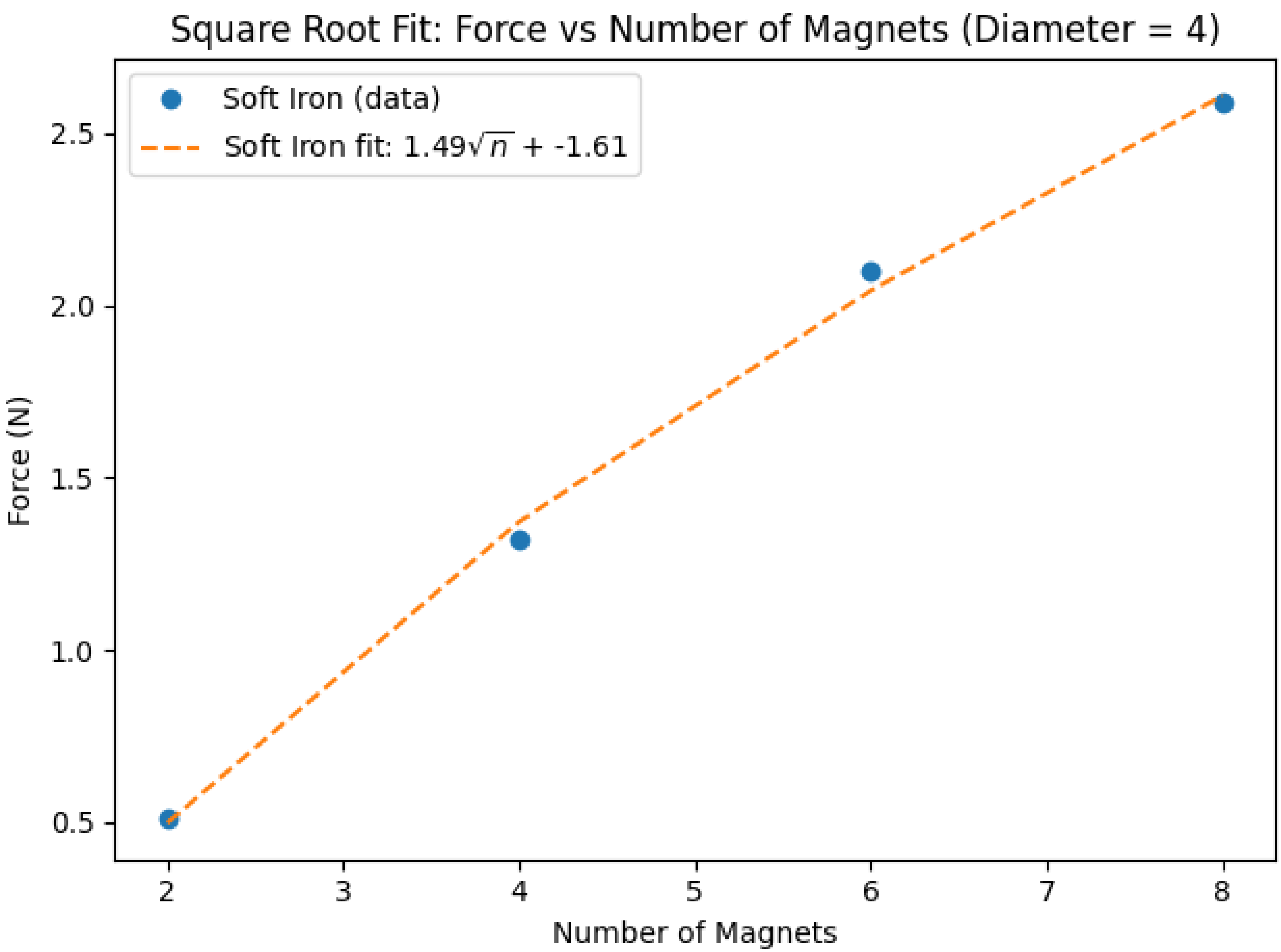
$\boldsymbol{n}$= unit normal vector

Maxwell Stress Tensor

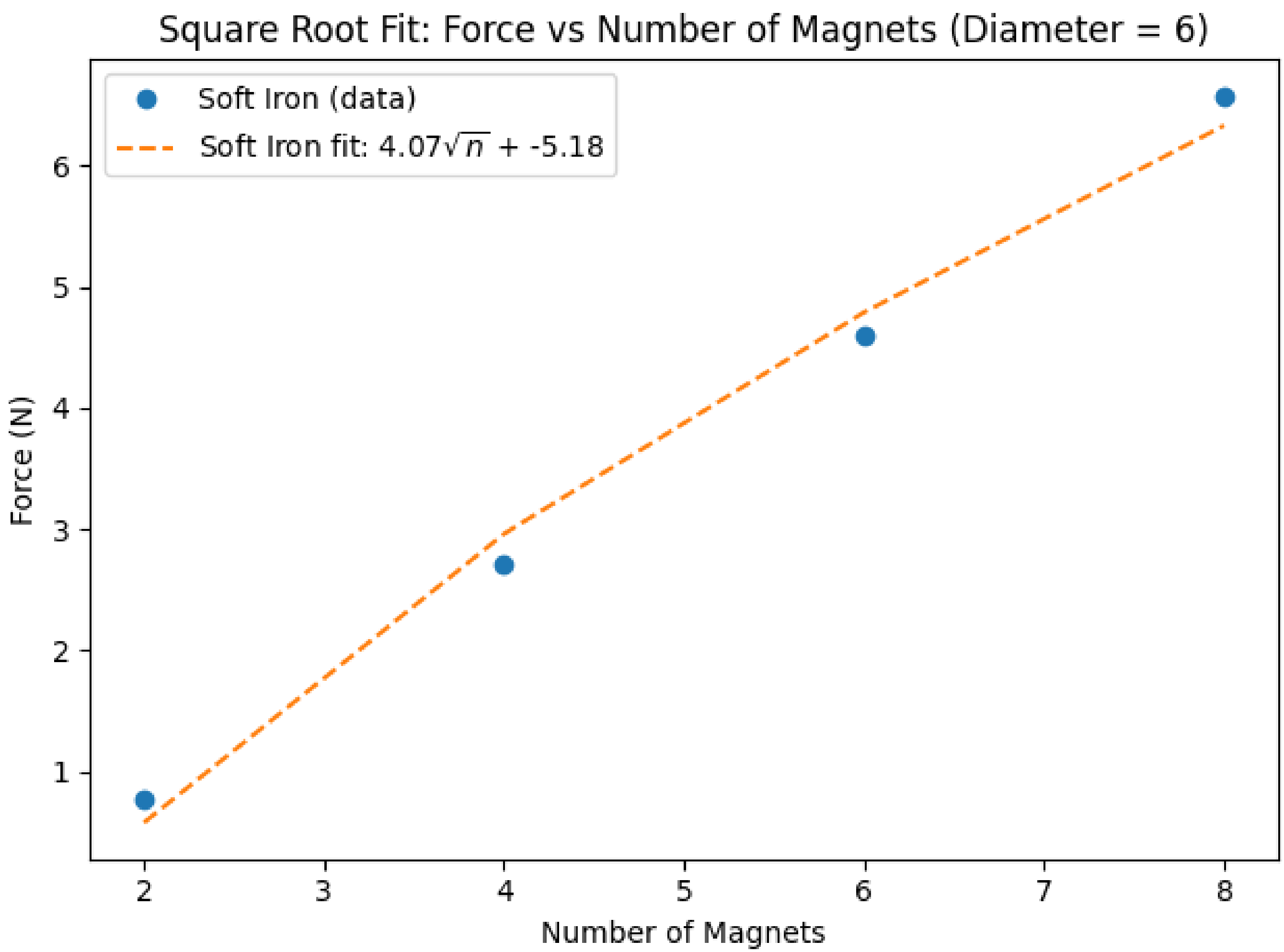
$$T_{ij} = \frac{1}{\mu_0}(B_iB_j - \frac{1}{2}\delta_{ij}B^2)$$



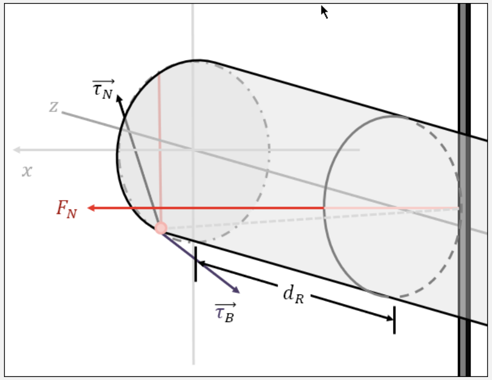
Simulation Results (D=4mm)



Simulation Results (D=6mm)

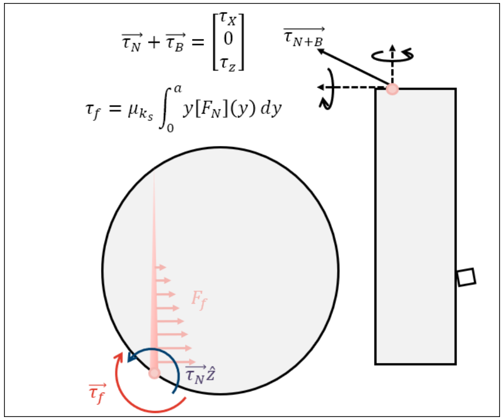


Dynamics Model



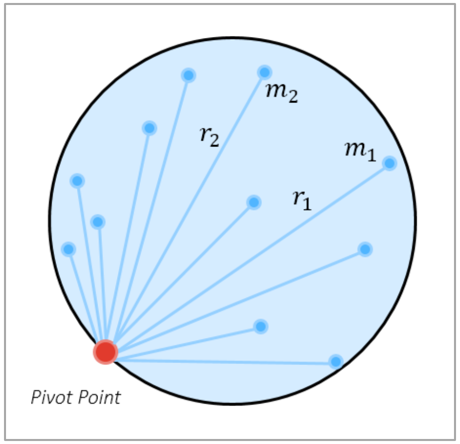
Torque Model

Solving for the relevant torques of the system



Critical Slip Point Model

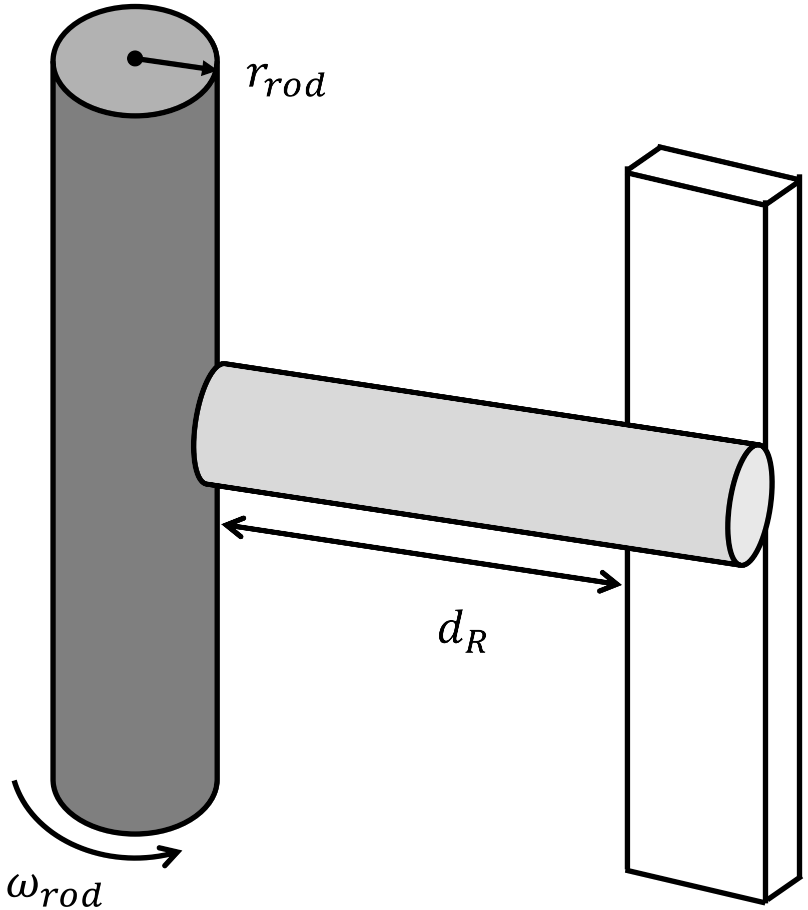
Solving for the stick-slip transition via Euler's method



Rotational (Slip) Model

Solving for the motion in the slip phase

Geometry



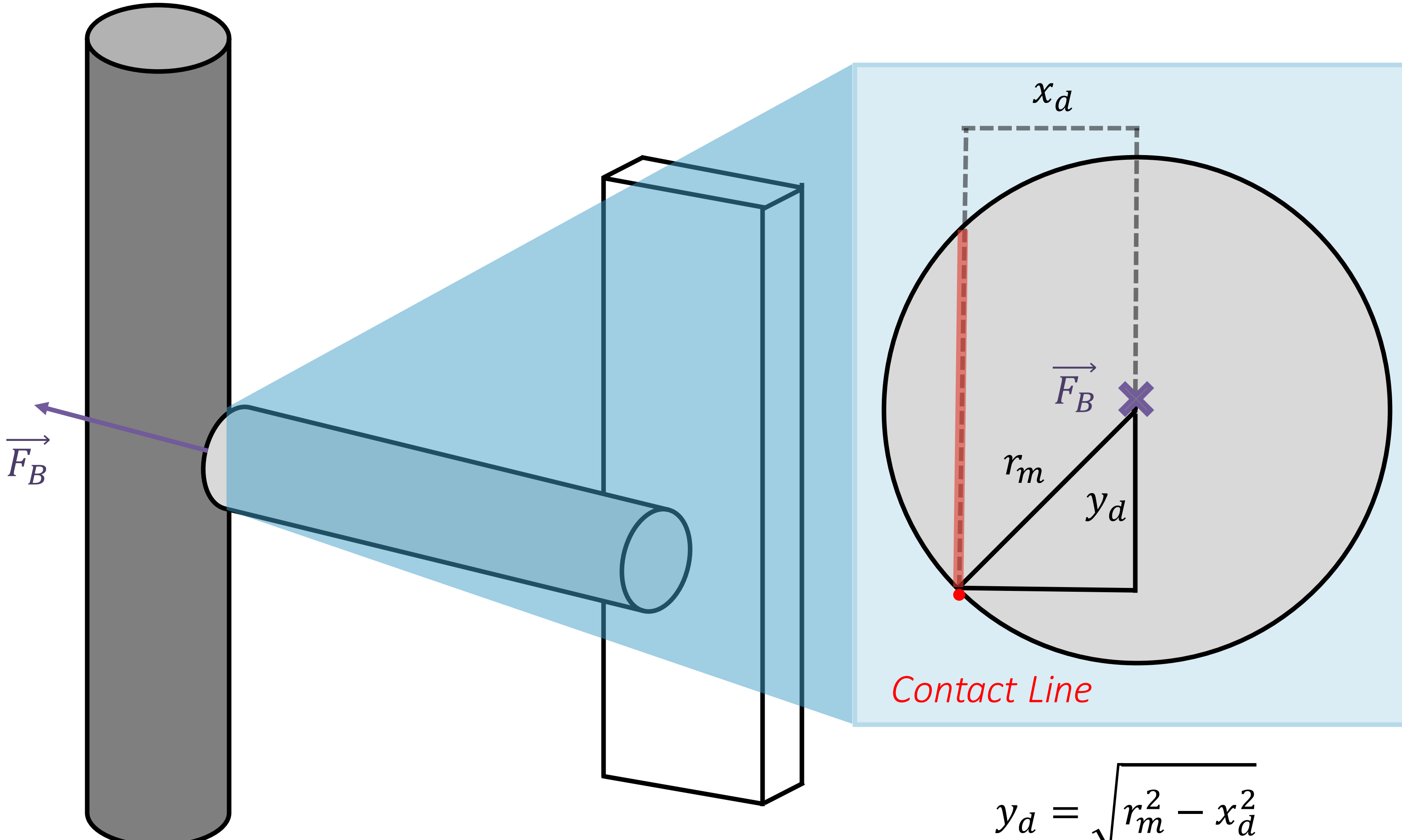
Assumptions

Ruler is a rigid body

Magnetic rod is a rigid body

Magnetic force is between center of rod and magnet

Geometry



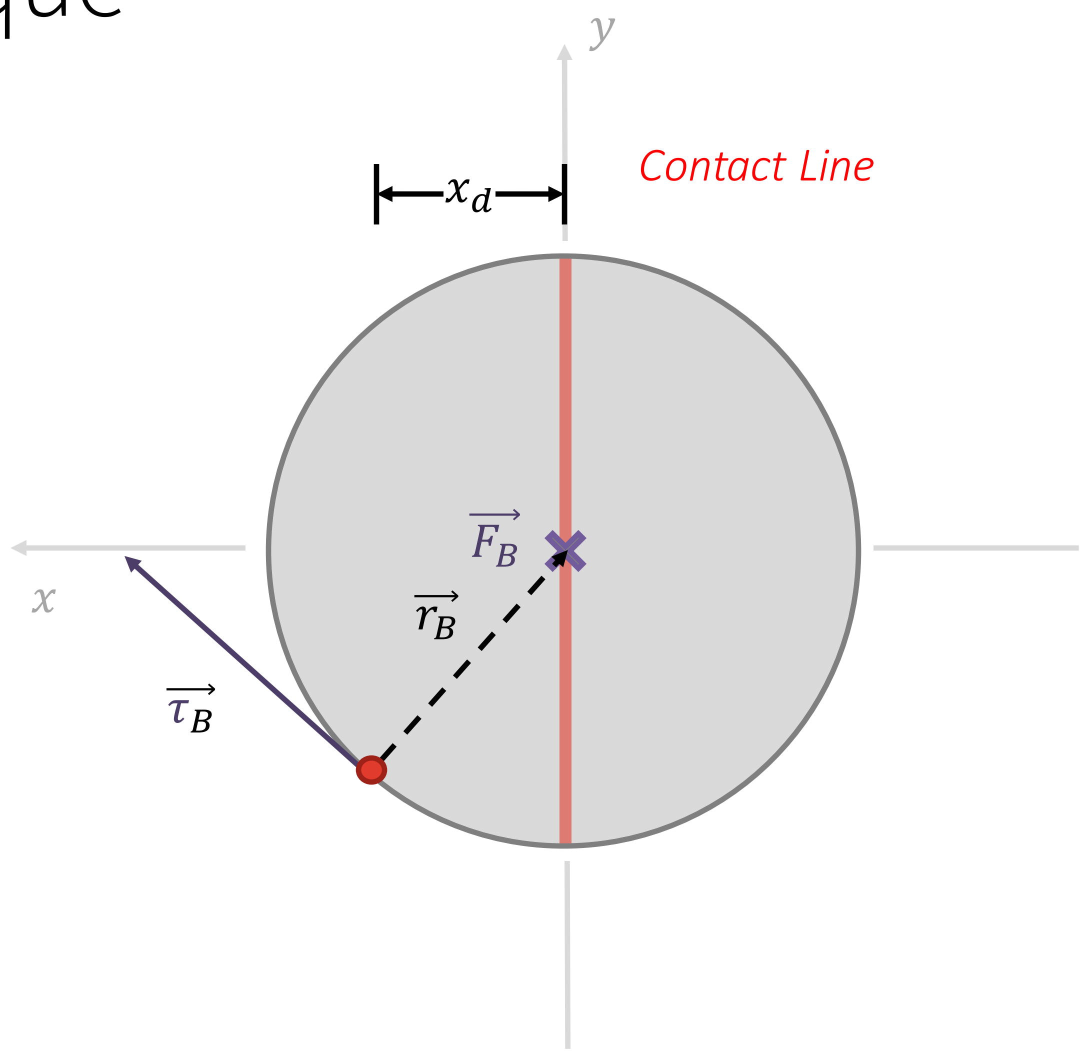
$$y_d = \sqrt{r_m^2 - x_d^2}$$

Magnetic Torque

$$\overrightarrow{F_B} = \begin{bmatrix} \widehat{x_d} \\ 0 \\ r_{rod} \end{bmatrix} ||F_B||$$

$$\overrightarrow{r_B} = \begin{bmatrix} x_d \\ \sqrt{r^2 - x_d^2} \\ 0 \end{bmatrix}$$

$$\boxed{\overrightarrow{\tau_B} = \overrightarrow{r_B} \times \overrightarrow{F_B}}$$



Normal Torque

$$\overrightarrow{F_B} = \begin{bmatrix} 0 \\ 0 \\ ||F_B|| \end{bmatrix} \qquad \overrightarrow{r_B} = \begin{bmatrix} x_d \\ \sqrt{r^2 - x_d^2} \\ 0 \end{bmatrix}$$

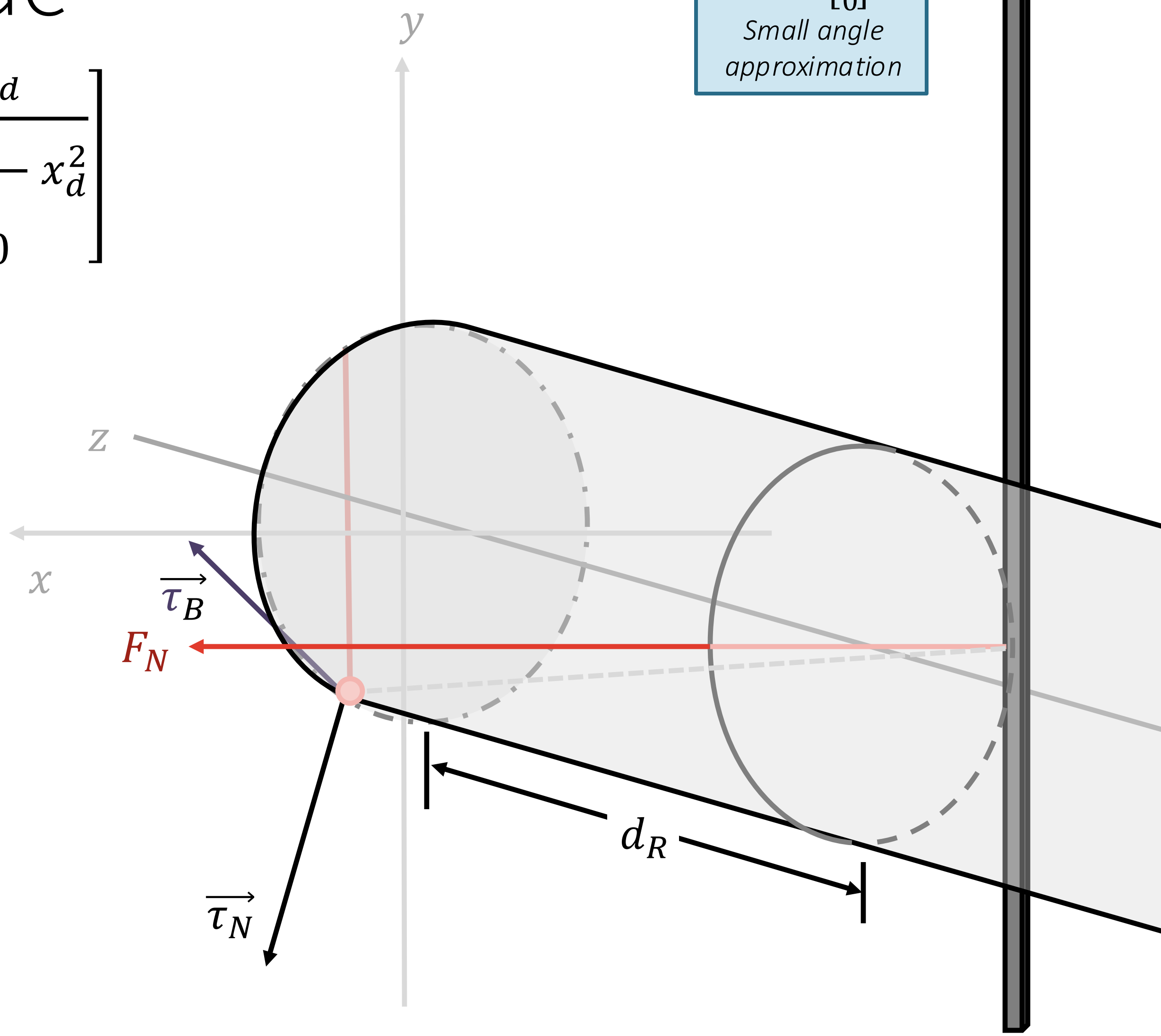
$$\boxed{\overrightarrow{\tau_B} = \overrightarrow{r_B} \times \overrightarrow{F_B}}$$

$$\widehat{F_N} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$

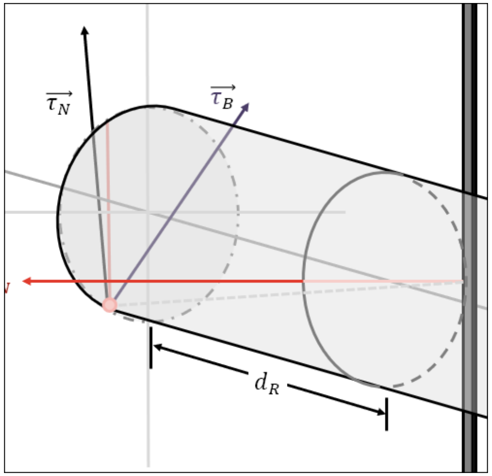
Small angle approximation

$$\overrightarrow{\tau_B}\,\hat{y} + \overrightarrow{\tau_N}\,\hat{y} = 0$$

$$\frac{\overrightarrow{\tau_N}}{||F_N||} = \begin{bmatrix} r + x_d \\ \sqrt{r^2 - x_d^2} \\ d_R \end{bmatrix} \times \widehat{F_N}$$

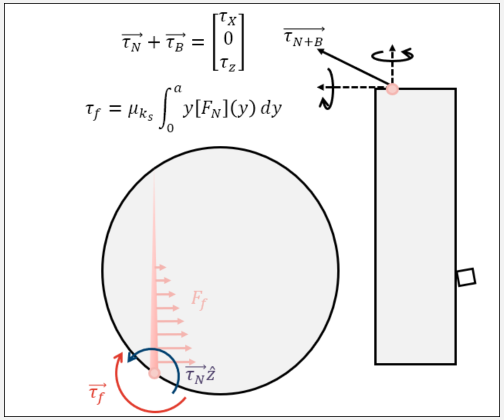


Dynamics Model



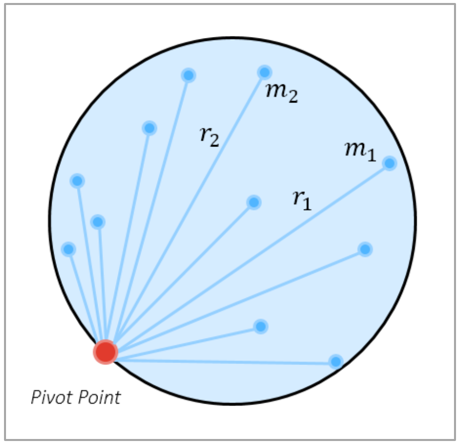
Torque Model

Solving for the relevant torques of the system



Critical Slip Point Model

Solving for the stick-slip transition via Euler's method



Rotational (Slip) Model

Solving for the motion in the slip phase

Relevant Net Torque

$$\overrightarrow{\tau_N} + \overrightarrow{\tau_B} = \begin{bmatrix} \tau_X \\ 0 \\ \tau_z \end{bmatrix}$$

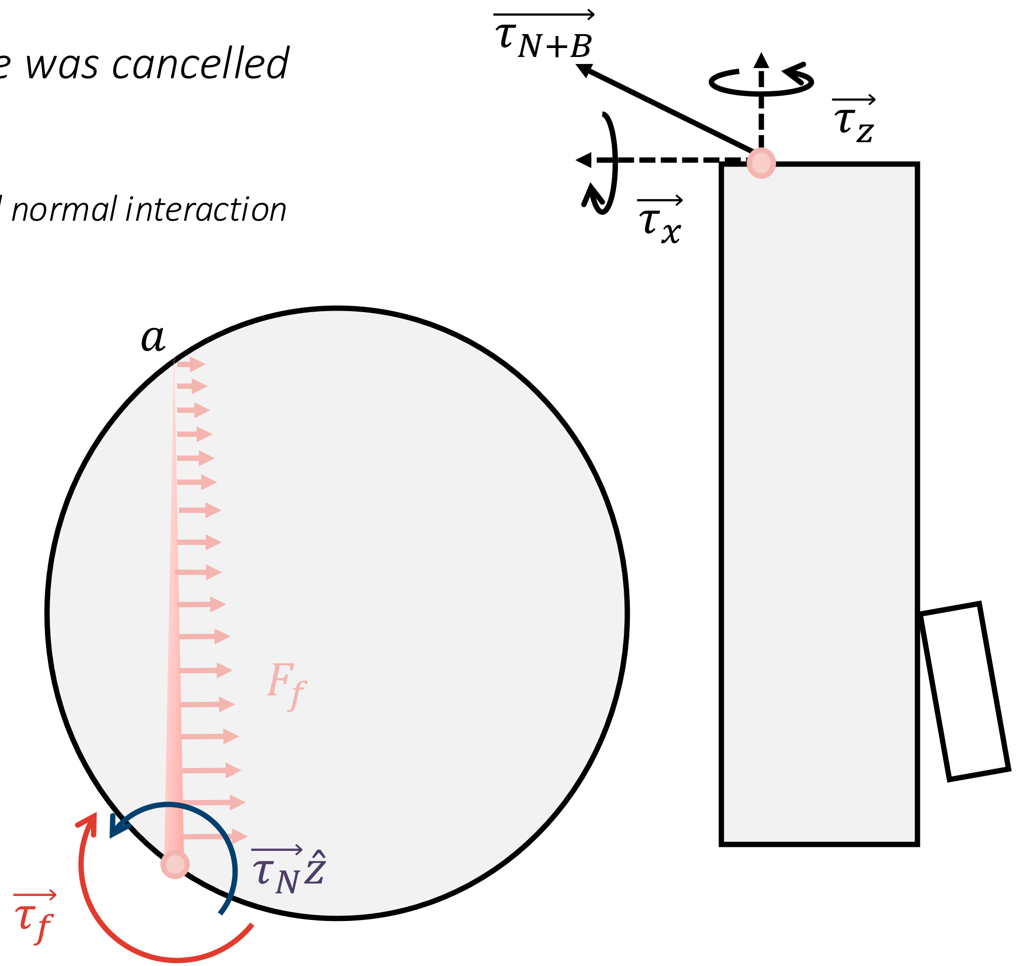
Y-axis of torque was cancelled

X-axis of torque cancelled by magnet-rod normal interaction

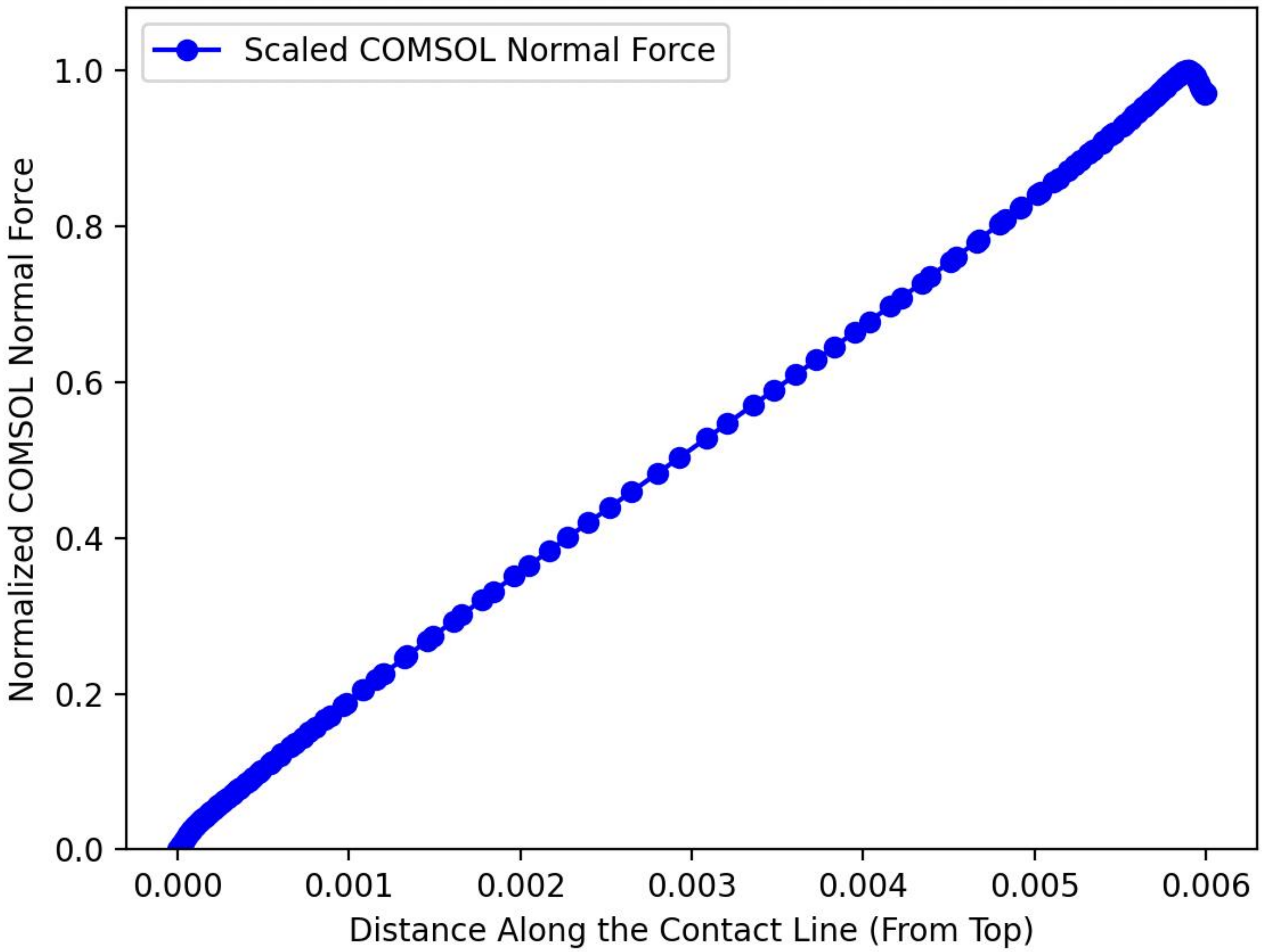
$$\tau_{fk} = \mu_k \int_0^a y[F_N](y)\,dy$$

Slip occurs when

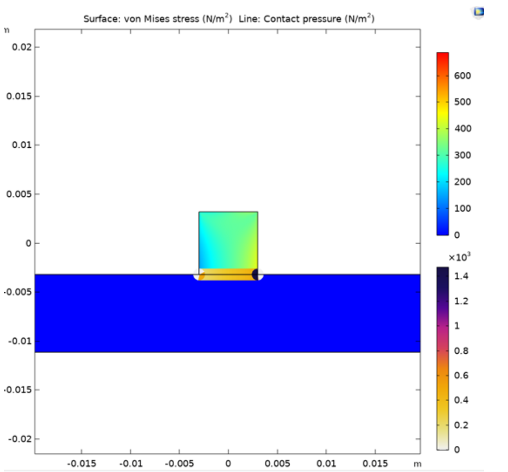
$$\overrightarrow{\tau_N}\hat{z} > \tau_{fk}\hat{z}$$



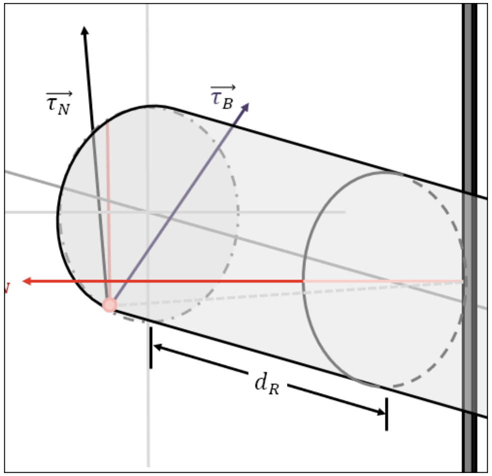
Pressure Profile Characterization



COMSOL

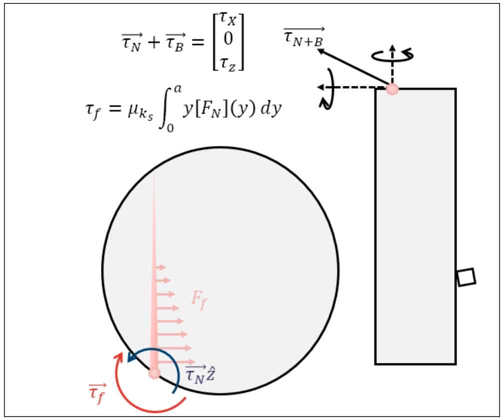


Dynamics Model



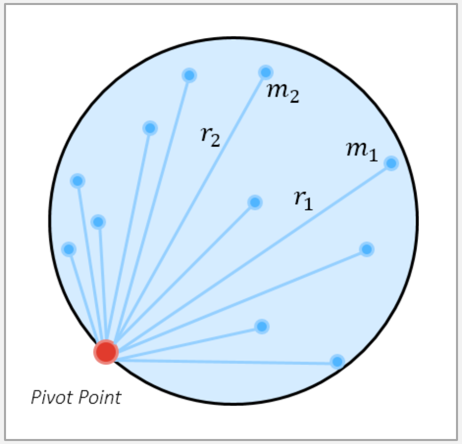
Torque Model

Solving for the relevant torques of the system



Critical Slip Point Model

Solving for the stick-slip transition via Euler's method



Rotational (Slip) Model

Solving for the motion in the slip phase

Rotational Motion

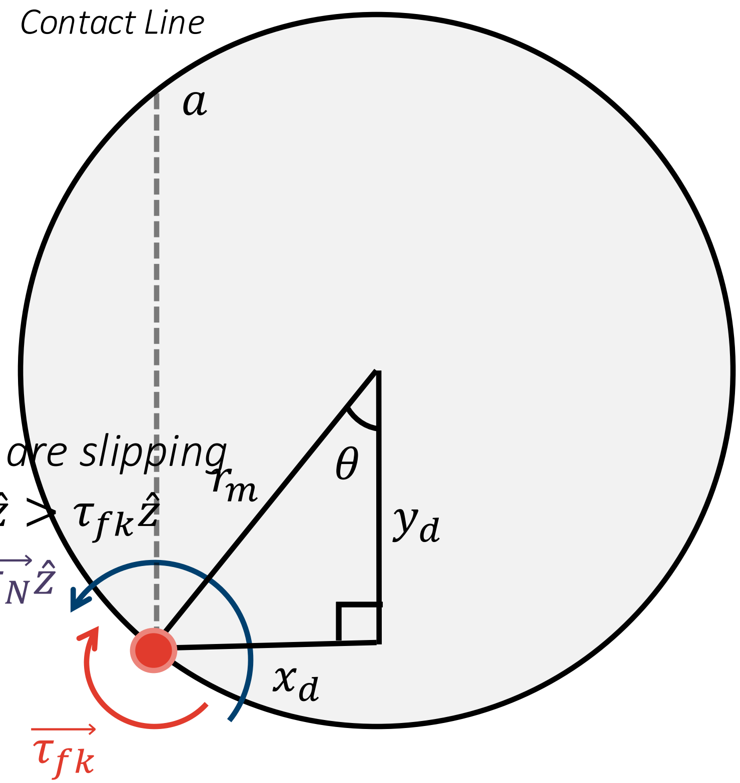
$$\overrightarrow{\tau_N} + \overrightarrow{\tau_B} = \begin{bmatrix} \tau_x \\ 0 \\ \tau_z \end{bmatrix}$$

Kinetic Friction

$$\tau_{fk} = \mu_k \int_0^a y[F_N](y)\, dy$$

Net Torque

$$\overrightarrow{\tau_{net}} = \begin{bmatrix} \tau_X \\ 0 \\ \tau_z \end{bmatrix} - sgn(\omega)\ (\tau_{fk})$$



Magnets are slipping
$\therefore \overrightarrow{\tau_N}\hat{z} > \tau_{fk}\hat{z}$

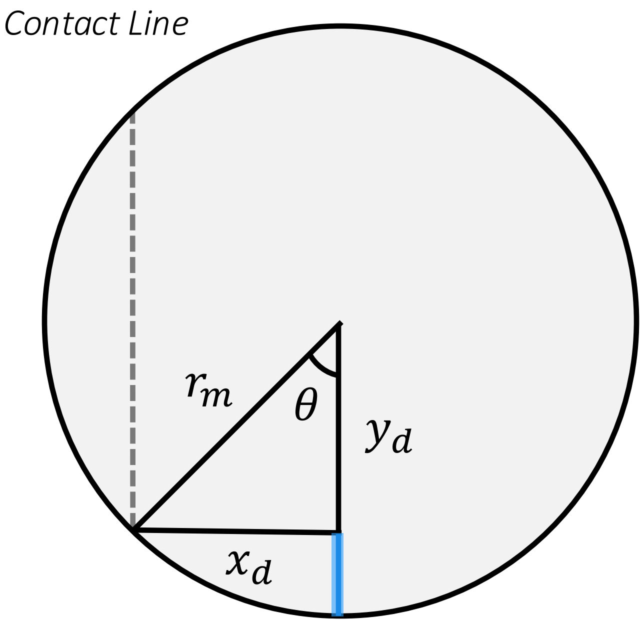
Numerical Solution Cont.

"Climb" distance per 1 cycle

$$r_m - \sqrt{r_m^2 - x_d^2}$$

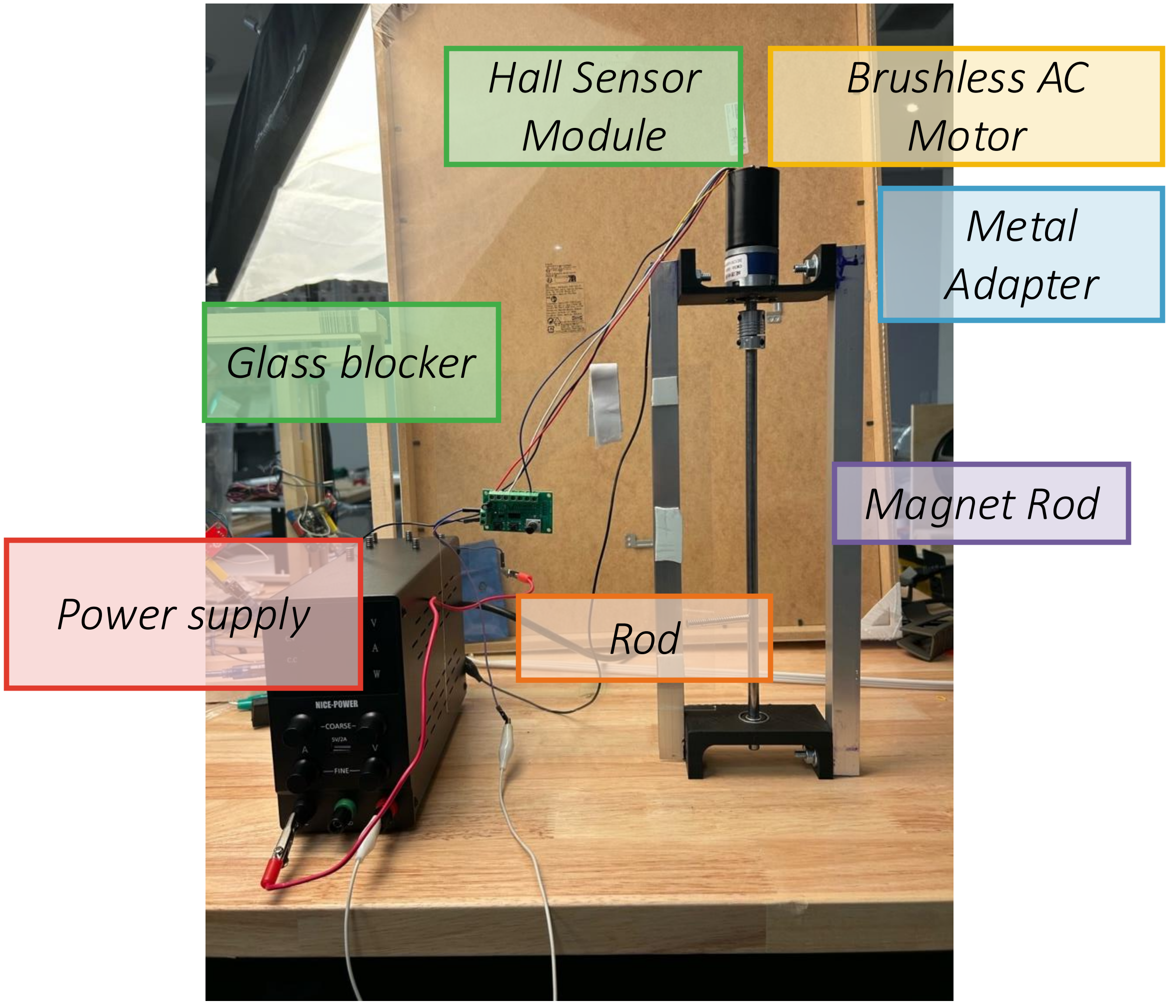
Climbing Speed

$$v = \frac{r_m - \sqrt{r_m^2 - x_d^2}}{\Delta t}$$

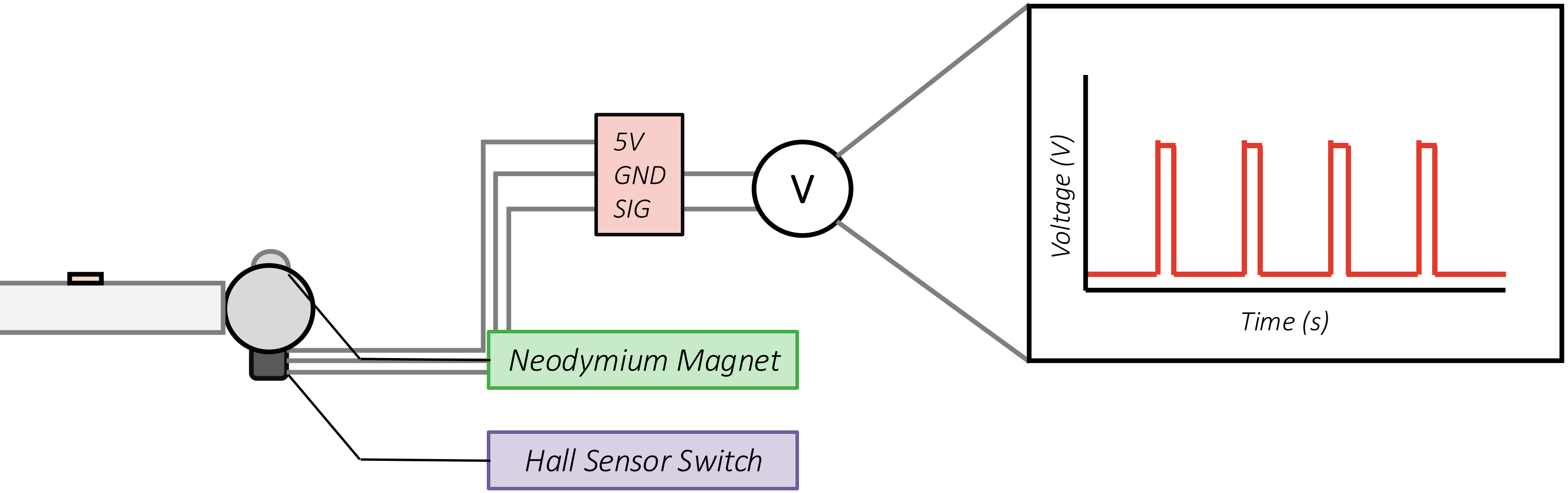


Experiments

Experimental Setup

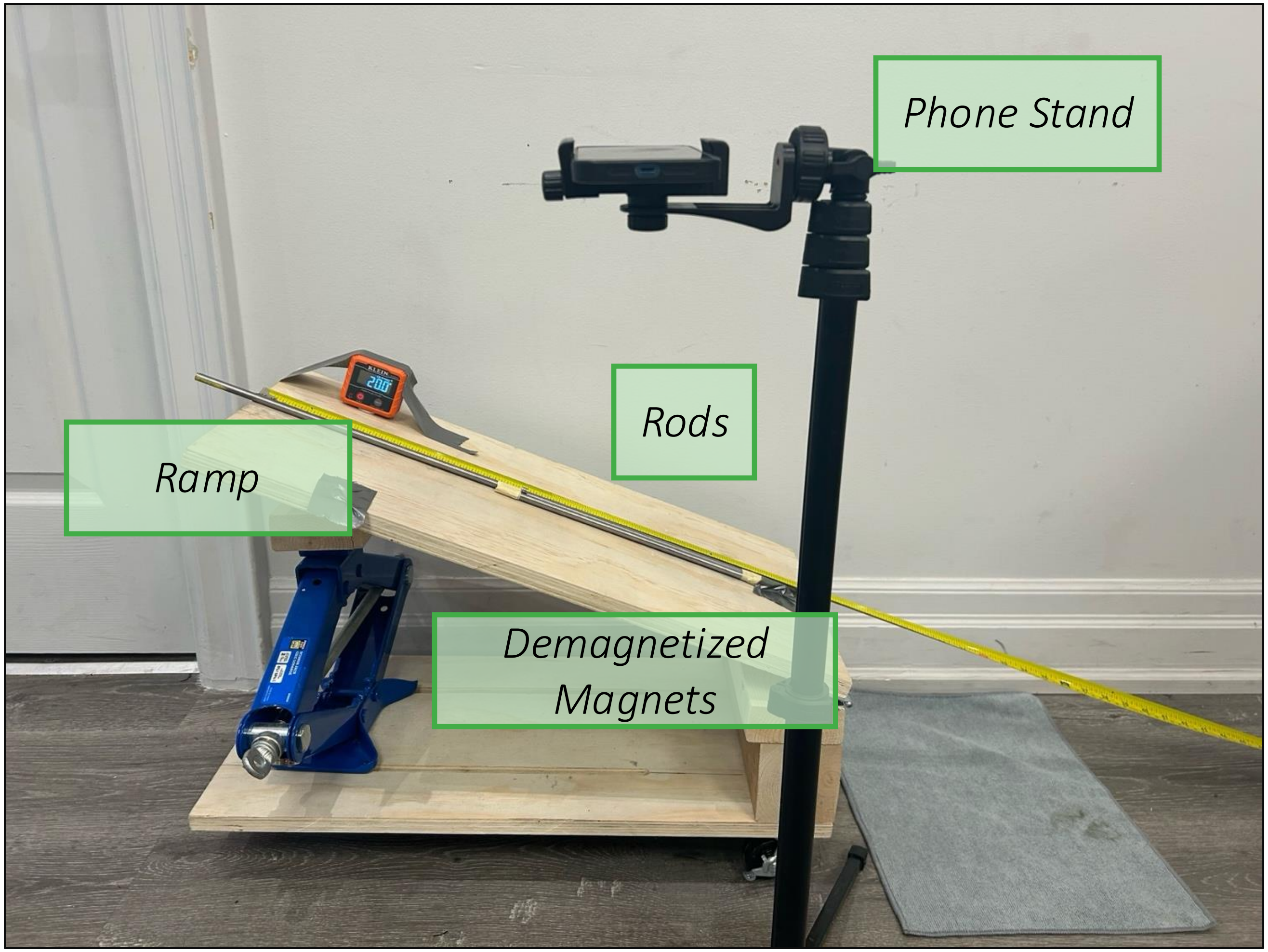


Hall Sensor Module



Characterizing Friction

Demagnetizing Magnets

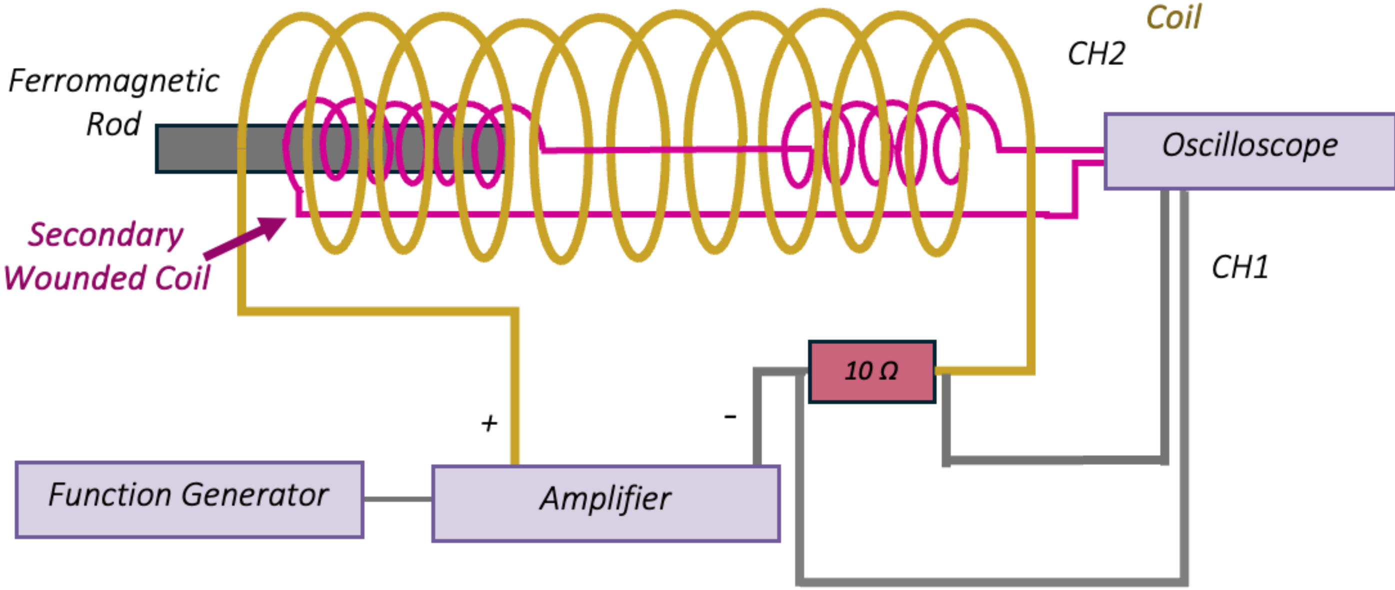


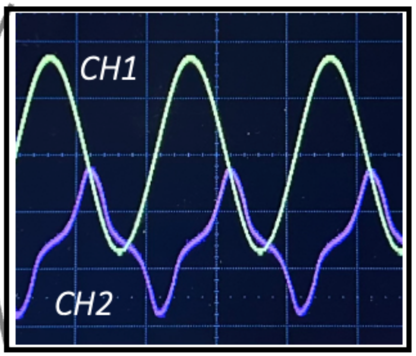
Characterizing Friction

Material	Static μ_s ($\pm$SD)	Kinetic μ_k ($\pm$SD)
Low Carbon steel rod	0.414 ± 0.049	0.25 ± 0.04
304 Stainless Steel rod	0.341 ± 0.054	0.18 ± 0.03
416 Stainless Steel rod	0.339 ± 0.040	0.19 ± 0.03
4140 Alloy Steel rod	0.328 ± 0.045	0.22 ± 0.03
1045 Carbon Steel rod	0.322 ± 0.031	0.19 ± 0.03
O1 tool steel rod	0.340 ± 0.026	0.26 ± 0.03
Glass blocker	0.329 ± 0.012	0.17 ± 0.03

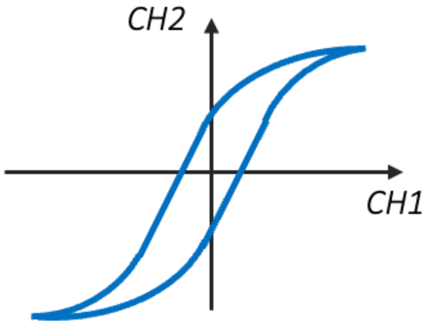
Characterizing Magnetic Force

Note: *The two portions of the secondary coil are wounded in opposite directions*

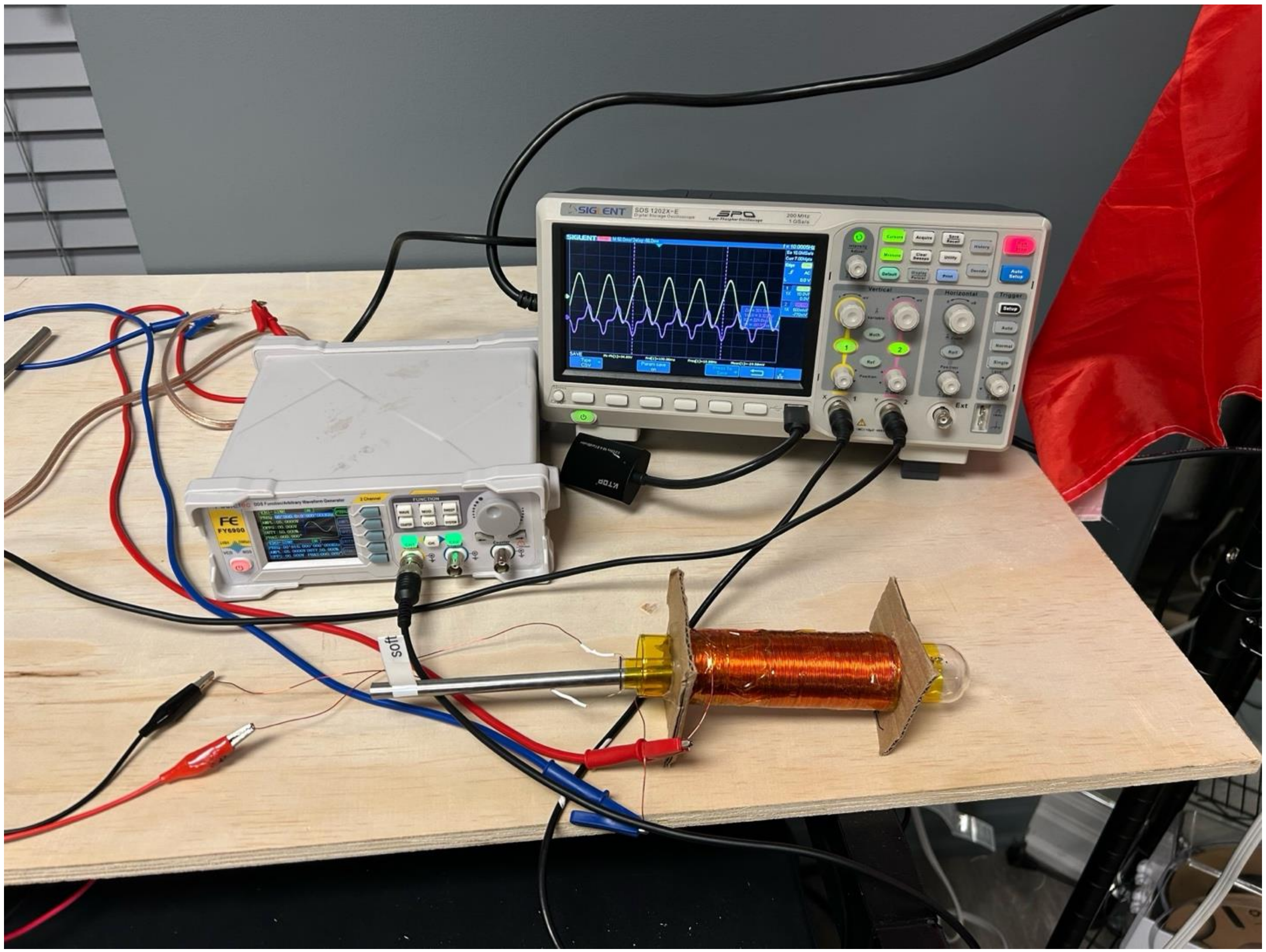




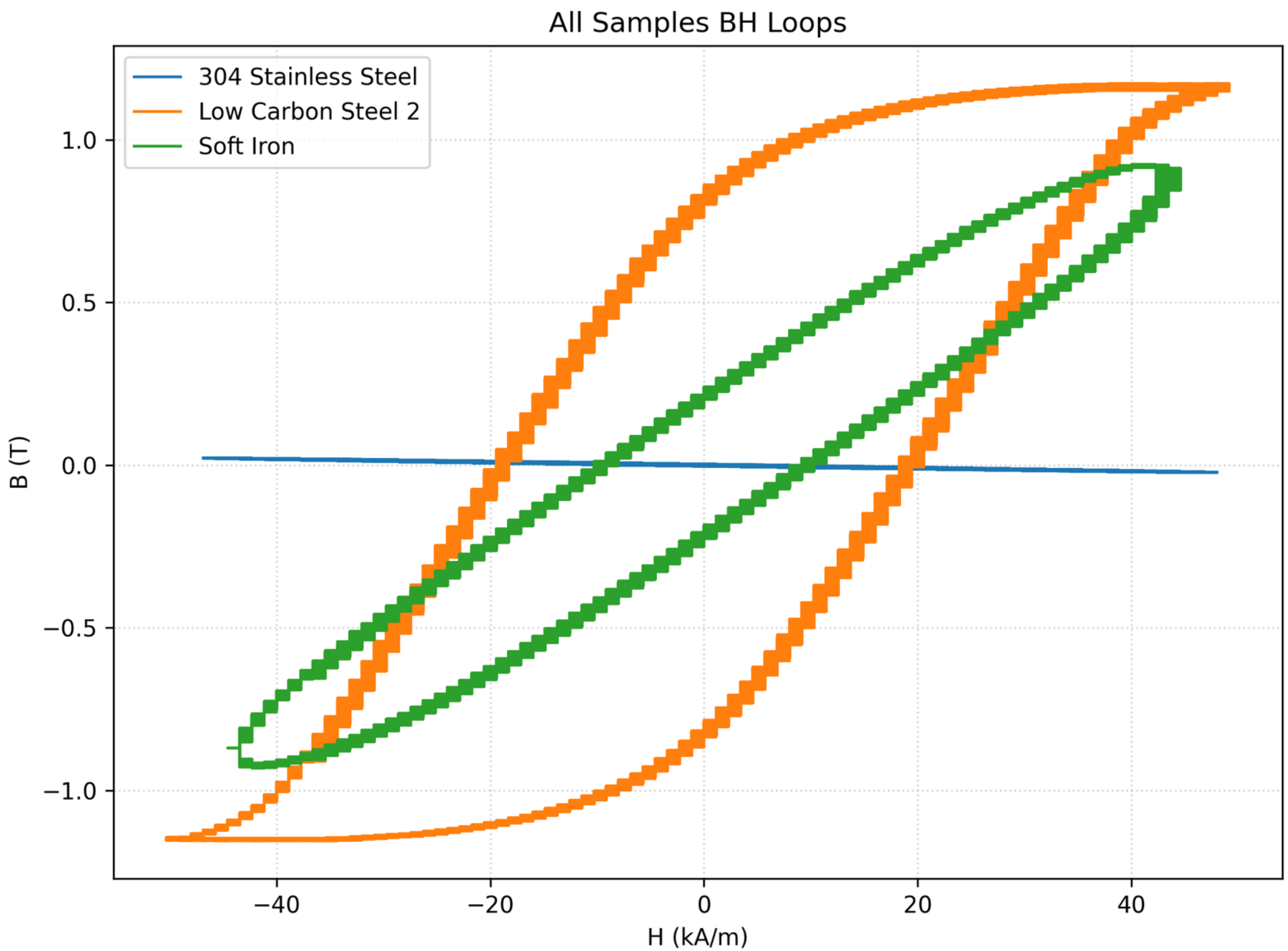
Pk-Pk Voltage ≈ 35V



Characterizing Magnetic Force



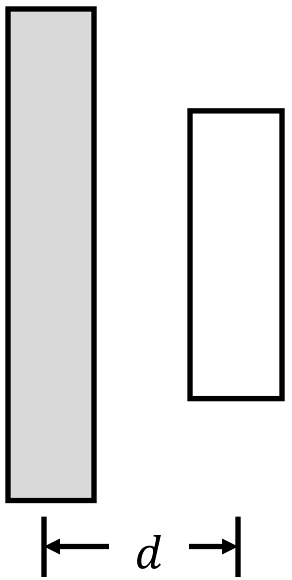
Characterizing Magnetic Force



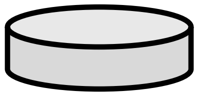
[5]. Matched with literature BH curves

Characterizing Constants

Caliper and digital scale used



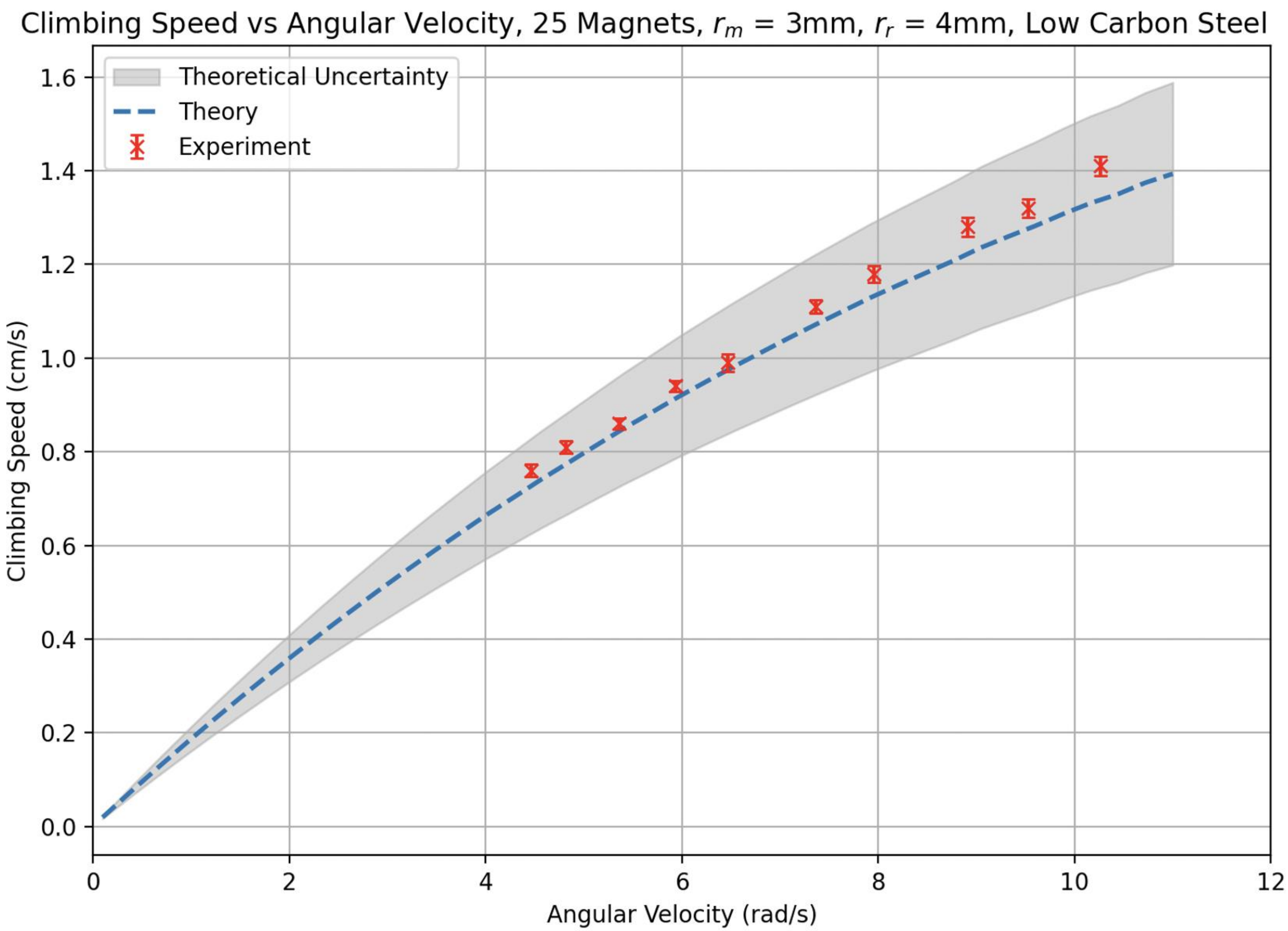
$r = 0.400cm, 0.600cm$
$d \Rightarrow, \{2.381cm, 3.49cm\}$



$m = 1.900g, 3.600g$
$r = 0.300cm, 0.500cm$

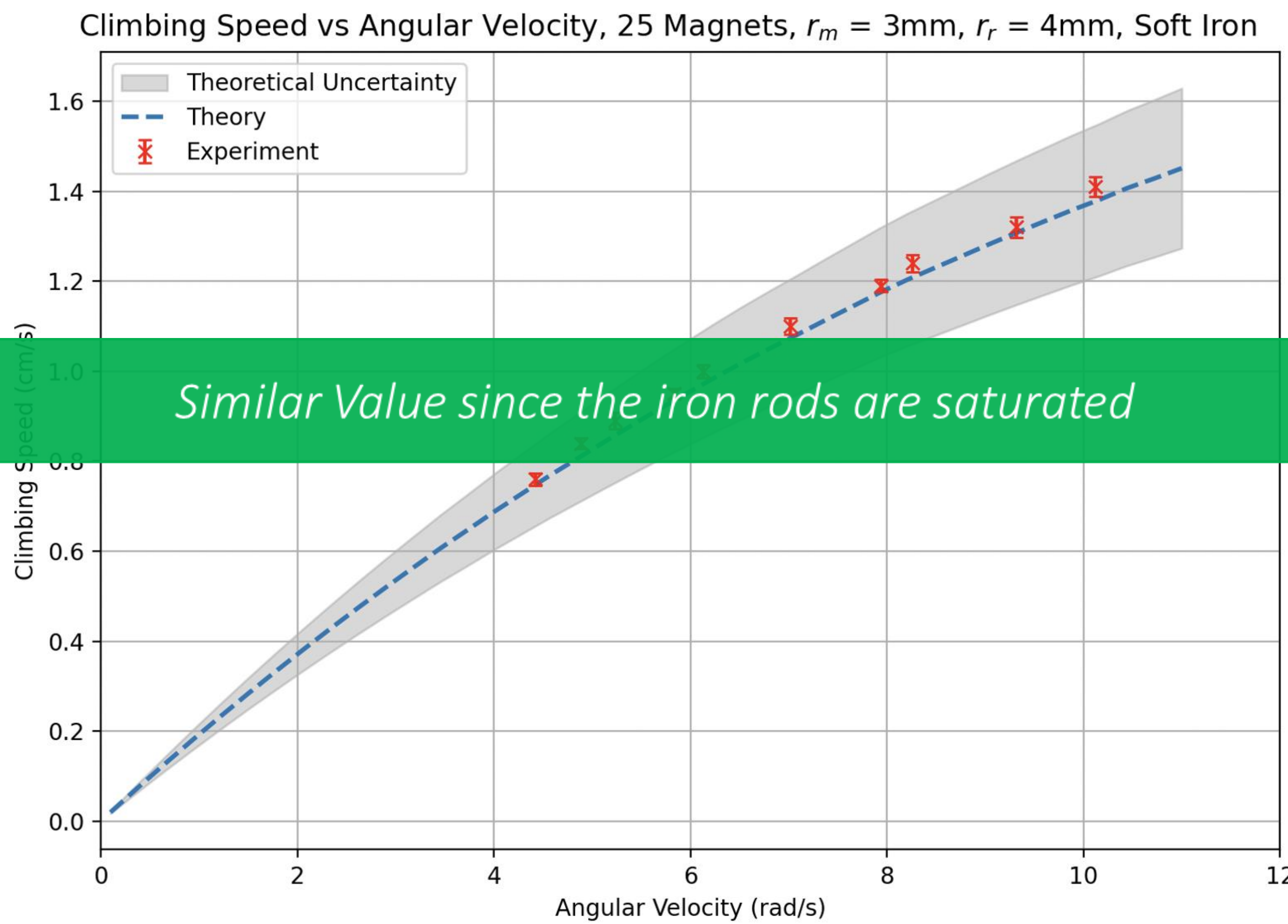
Results

Varying Rod Angular Velocity



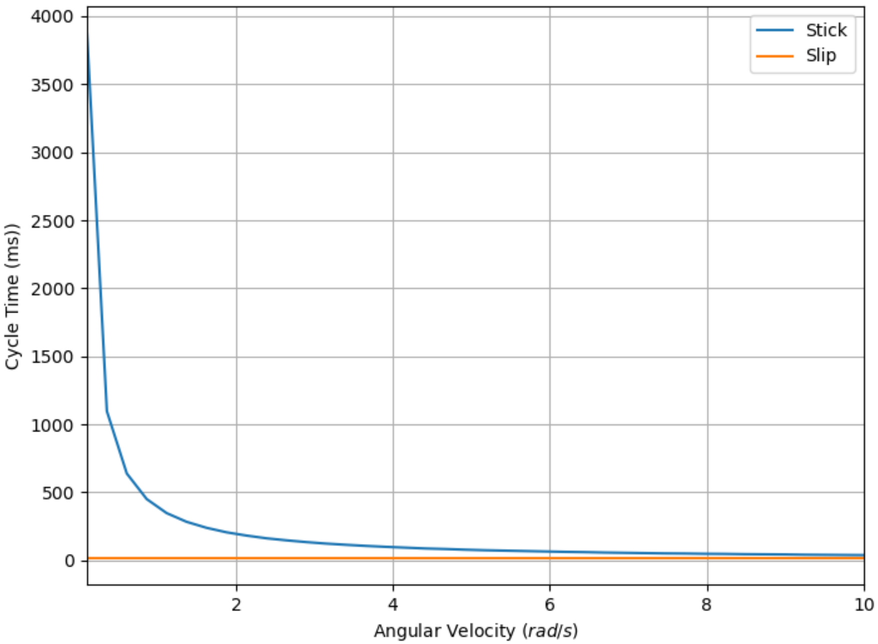
Note that we plotted out our theoretical, within the error range for the Normal force, and friction.

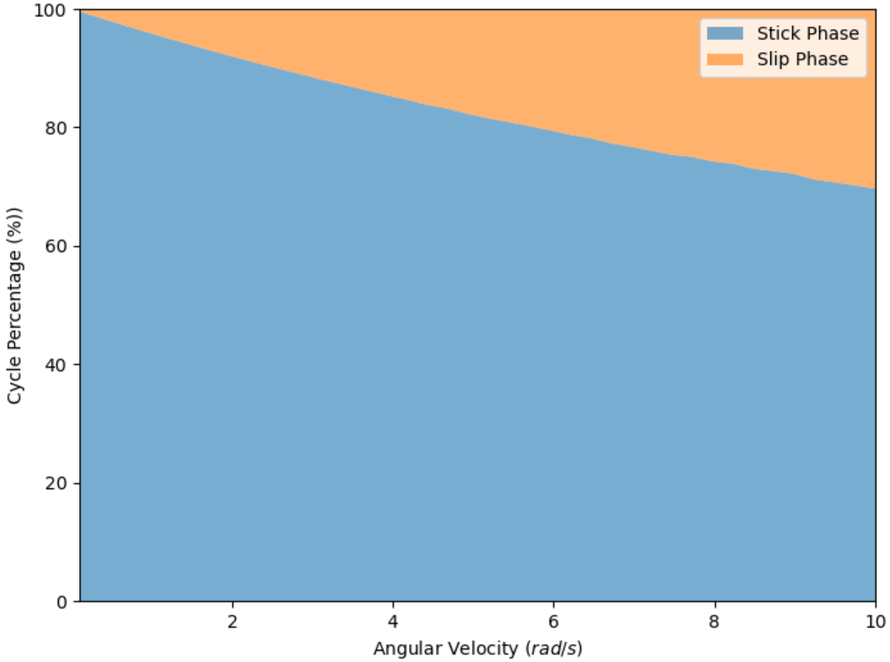
Varying Rod Angular Velocity



Note that we plotted out our theoretical, within the error range for the Normal force, and friction.

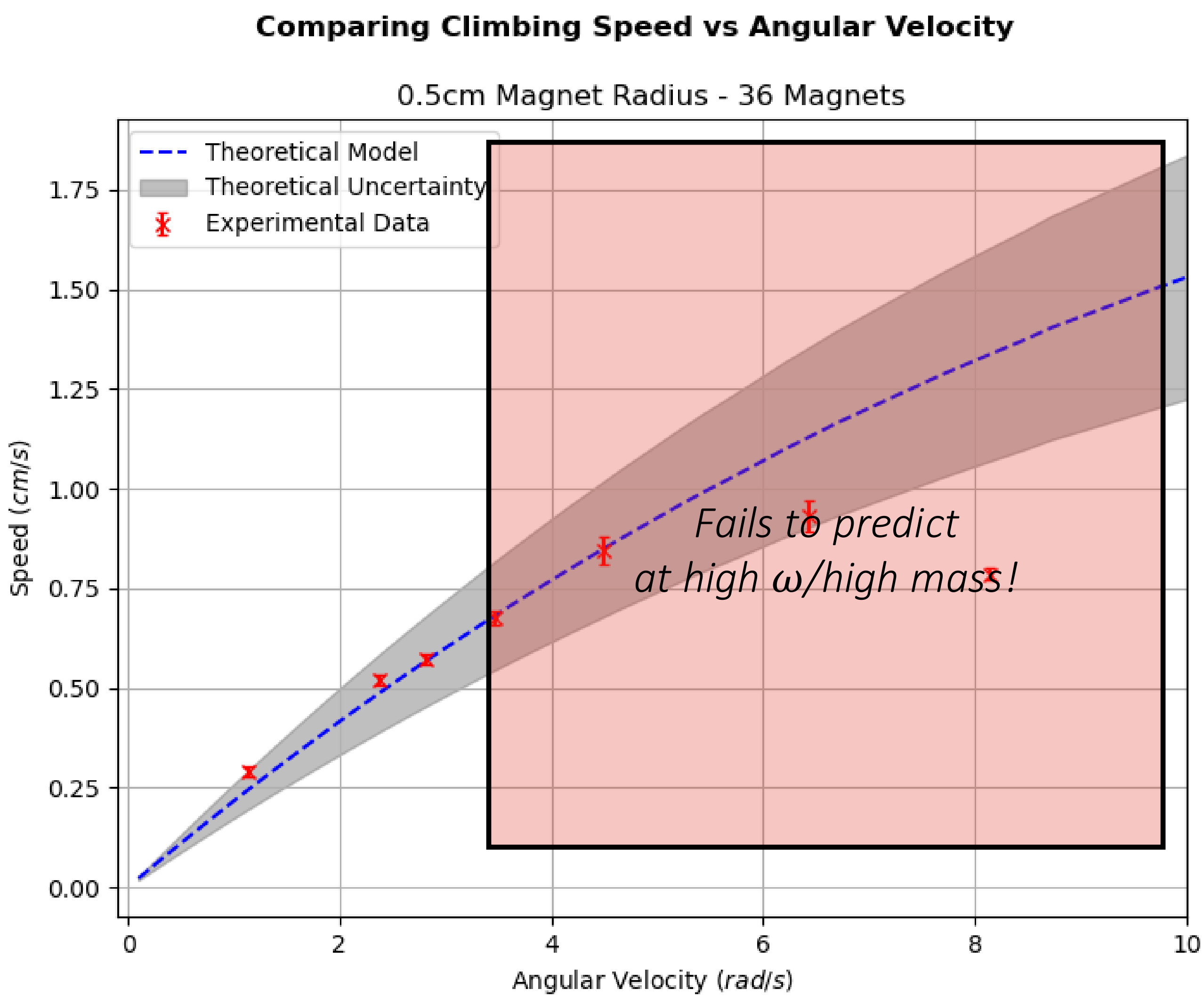
Theoretical Analysis – Time spent in Phases



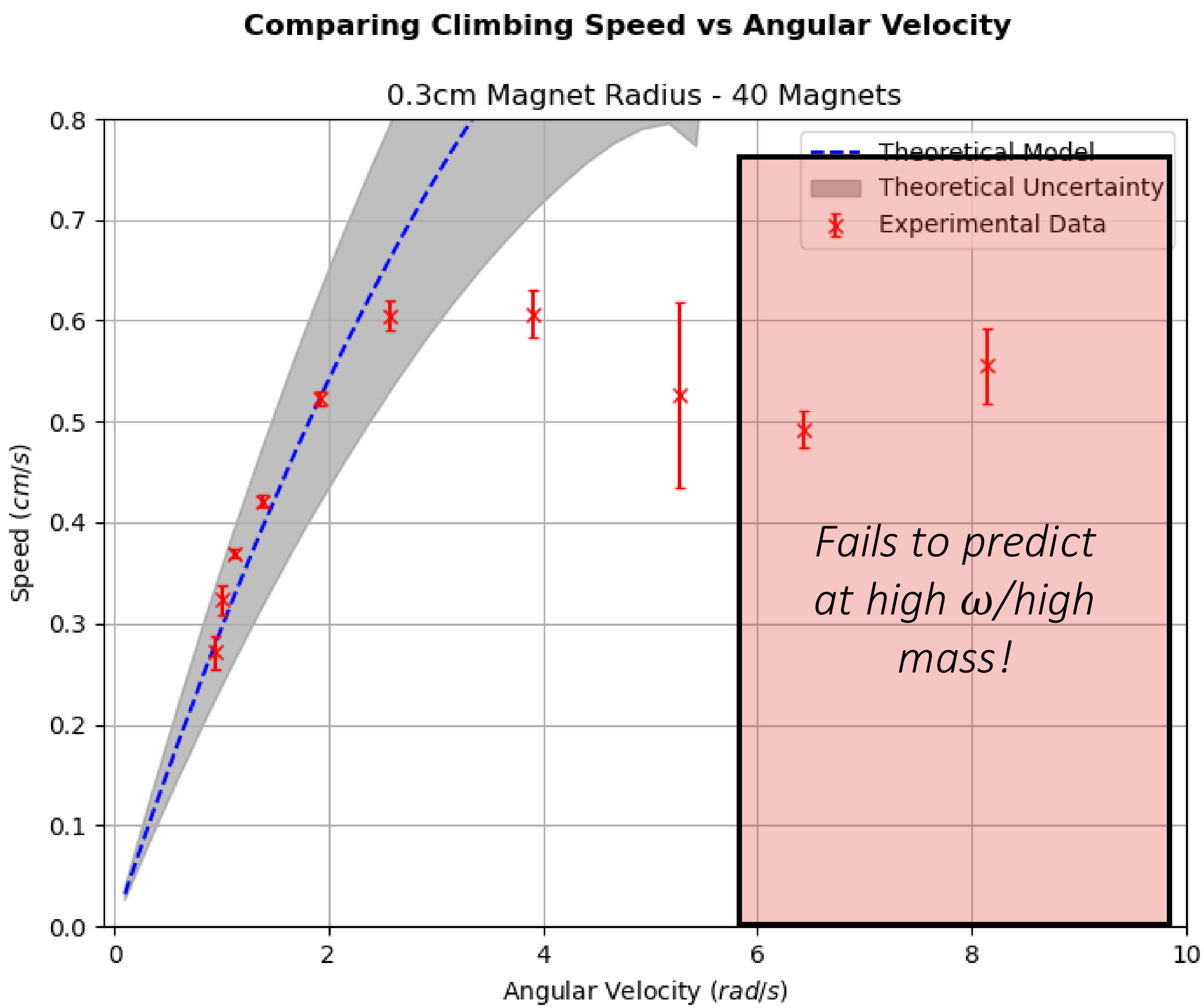


Slip phase time stays relatively constant due to low dependence on ω, this explains the flattening out curve

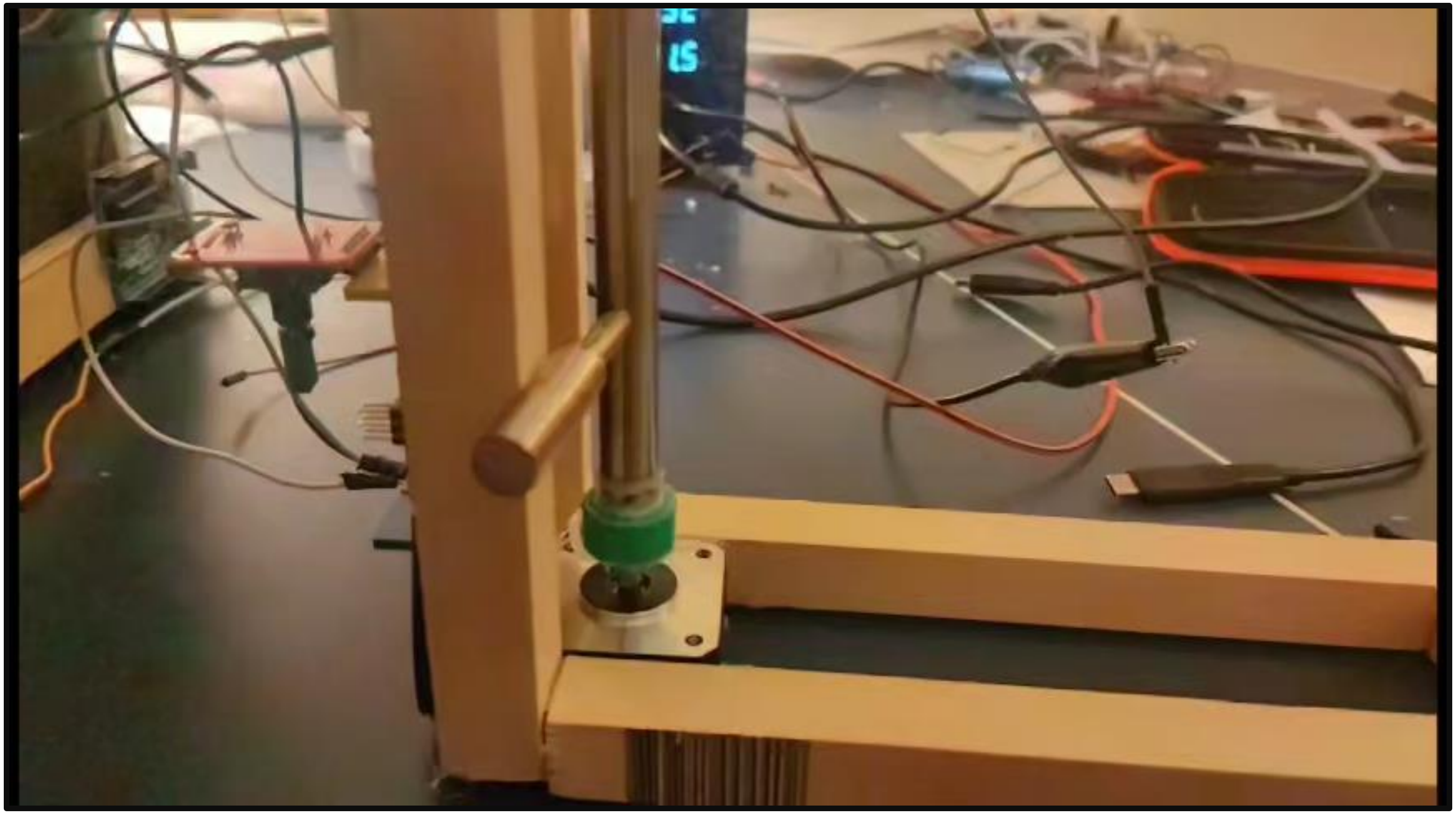
Varying Rod Angular Velocity (0.6mm rod)



Varying Rod Angular Velocity (0.6mm rod)



Magnet Instability



Increasing ω

Summary

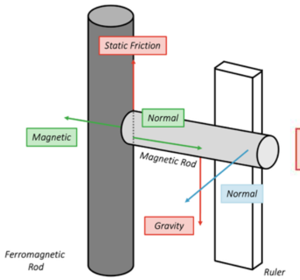
“Attach a rod assembled from cylindrical neodymium magnets horizontally to a vertical ferromagnetic rod. Limit the motion of the magnets to the vertical direction. When the ferromagnetic rod is spun around its axis of symmetry, the magnetic rod begins to climb up. Explain this phenomenon and investigate how the rate of climbing depends on relevant parameters.”

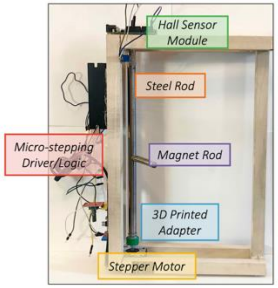
Qualitative Account

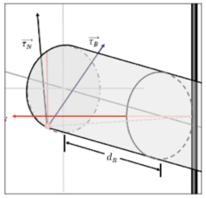
Demonstrated cases of the phenomenon

Qualitatively and Quantitatively explained cases of the phenomenon

Devised experimental set-up and collected data

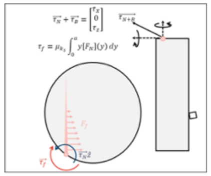






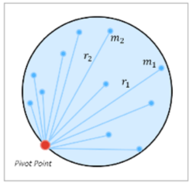
Torque Model

Solving for the relevant torques of the system



Critical Slip Point Model

Solving for the stick-slip transition via Euler’s method



Rotational (Slip) Model

Solving for the motion in the slip phase

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[4] The Engineering ToolBox (2004). Friction - Coefficients for Common Materials and Surfaces. Available at: https://www.engineeringtoolbox.com/friction-coefficients-d_778.html February 26, 2025.

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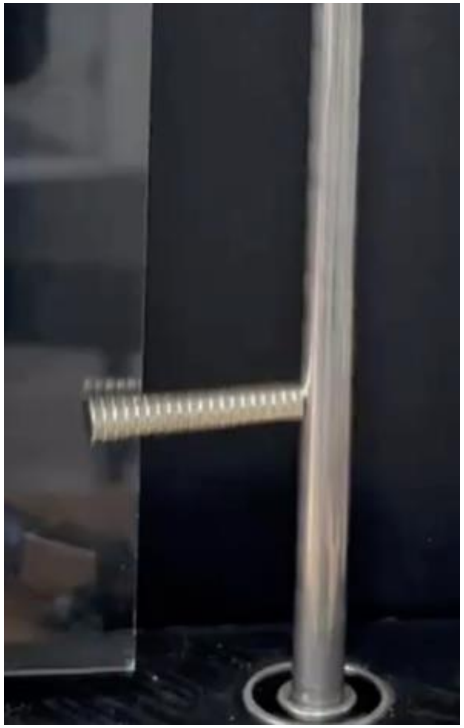
Thanks For Listening!

Climbing Magnets | Bailin Wang | Team Canada

"Attach a rod assembled from cylindrical neodymium magnets horizontally to a vertical ferromagnetic rod. Limit the motion of the magnets to the vertical direction. When the ferromagnetic rod is spun around its axis of symmetry, the magnetic rod begins to climb up. Explain this phenomenon and investigate how the rate of climbing depends on relevant parameters."



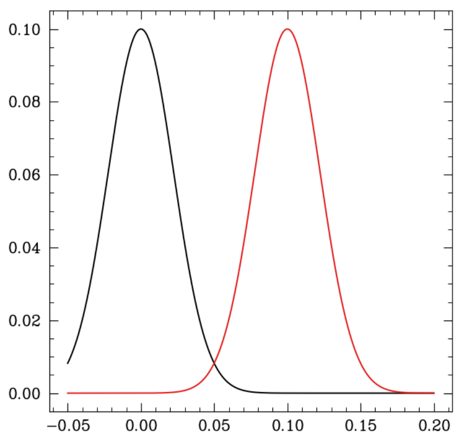
Appendix

Estimating Eddy Currents & Free Currents

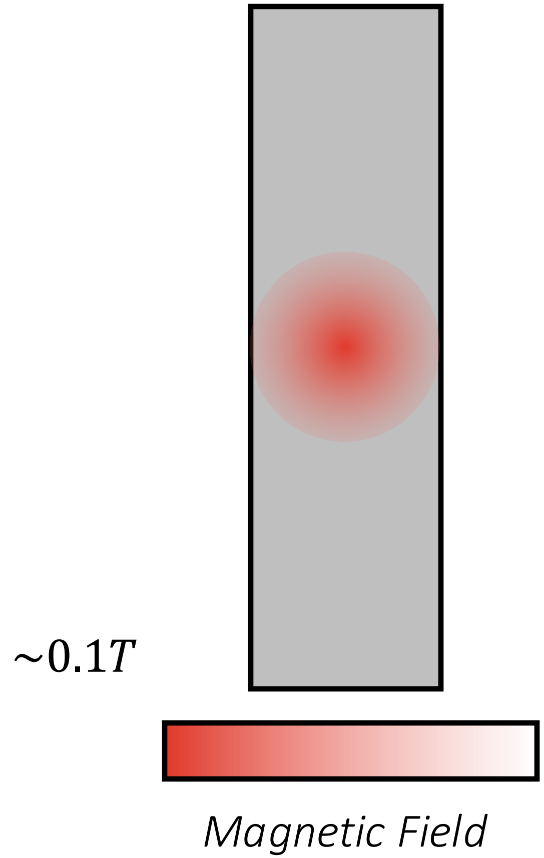
Faraday's Law

$$emf = -\frac{\Delta \phi_B}{\Delta t}$$

$$\omega = 10\, rad/s$$
$$\rho \approx 10^{-7}\ \Omega\mathrm{m}$$



$\omega r dt$



$$I \approx 0.8\, mA$$
$$P \approx 10^{-7} W$$

B-H Correction Curve

From

$$NI = H_{\mathrm{rod}}\, L_{\mathrm{rod}} \; + \; \frac{B}{\mu_0}\, \ell_{\mathrm{air}}$$

we solve for the true field H_{rod}:

$$\begin{aligned} H_{\mathrm{rod}} &= \frac{NI}{L_{\mathrm{rod}}} \; - \; \frac{\ell_{\mathrm{air}}}{\mu_0\, L_{\mathrm{rod}}}\; B \\ &= H_{\mathrm{meas}} \; - \; k_d\, B, \quad k_d = \frac{\ell_{\mathrm{air}}}{\mu_0\, L_{\mathrm{rod}}}. \end{aligned}$$

Thus for every point on your loop,

$$H_{\mathrm{corr}}(B) = H_{\mathrm{meas}}(B) - \underbrace{\frac{\ell_{\mathrm{air}}}{\mu_0\, L_{\mathrm{rod}}}}_{k_d}\; B.$$

B-H Correction Curve

Equivalently, fold both reluctances into one effective reluctance:

$$B = \frac{\Phi}{A} = \frac{\mu_0 \, N \, I}{\dfrac{L_{\mathrm{rod}}}{\mu_r} + \ell_{\mathrm{air}}} \implies H_{\mathrm{rod}} = \frac{N \, I}{L_{\mathrm{rod}} + \mu_r \, \ell_{\mathrm{air}}} = H_{\mathrm{meas}} \; \times \; f,$$

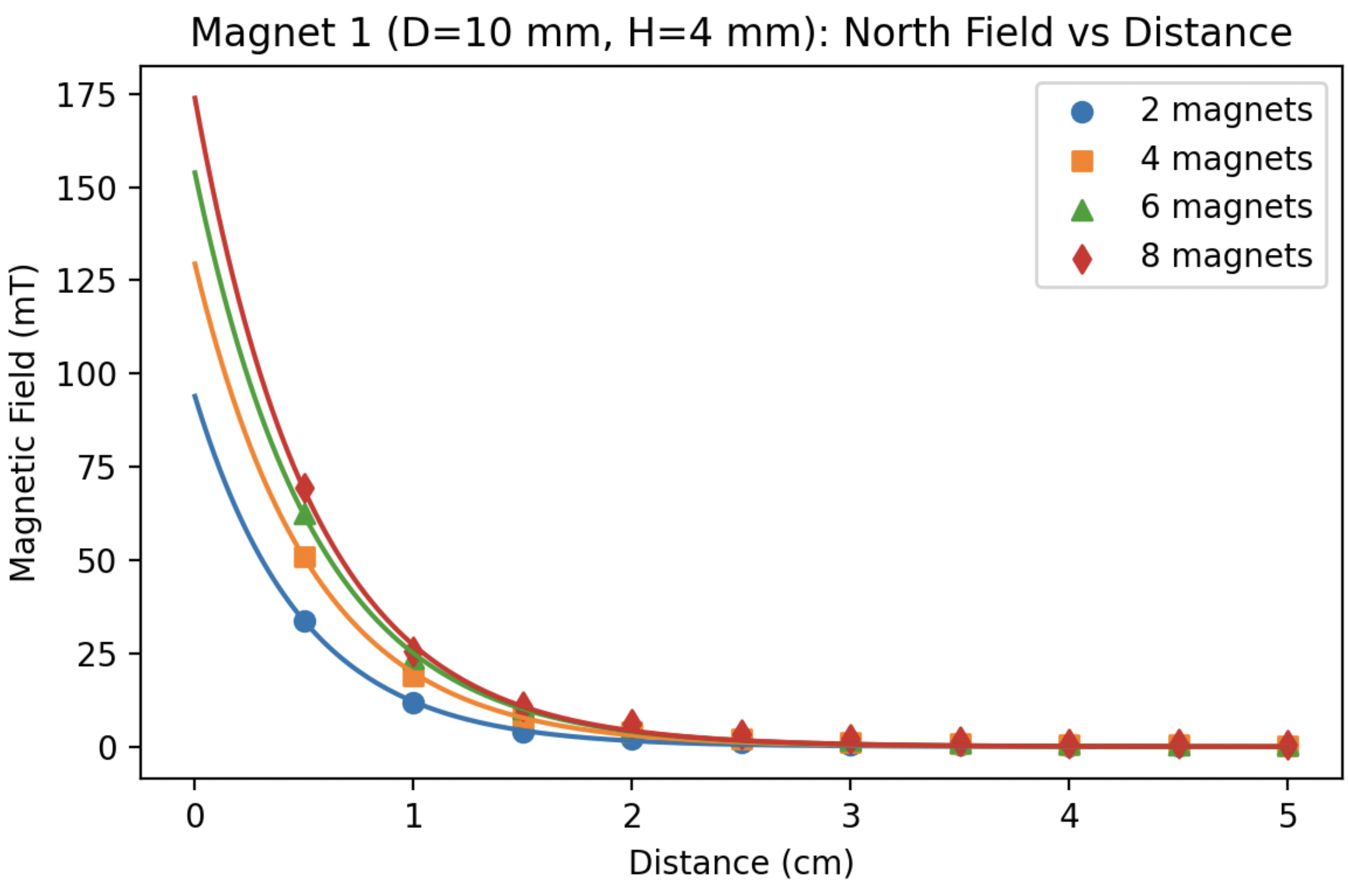
where

$$f = \frac{L_{\mathrm{rod}}}{L_{\mathrm{rod}} + \mu_r \, \ell_{\mathrm{air}}}.$$

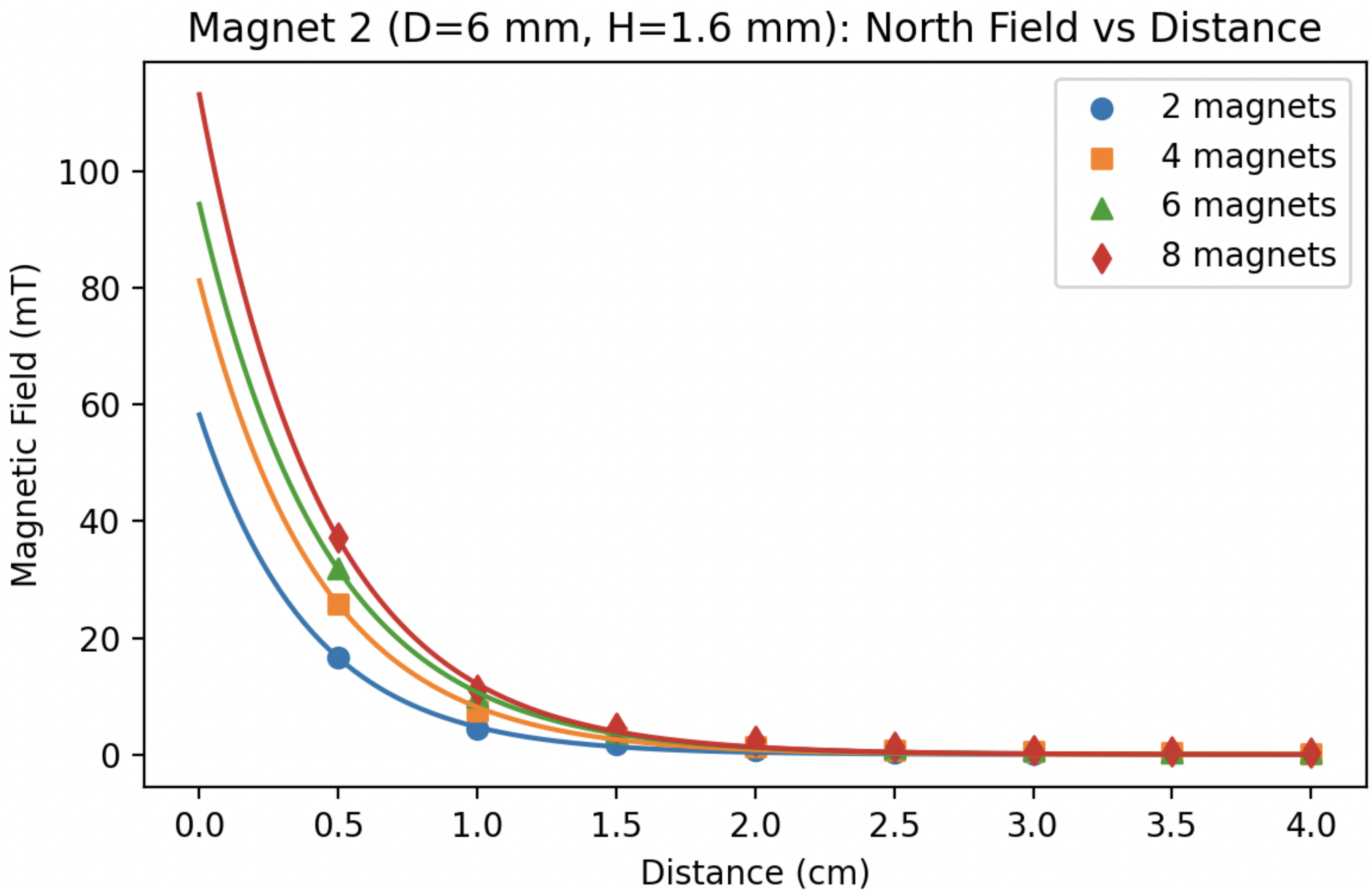
So simply multiply your measured H–axis by f:

$$H_{\mathrm{corr}} = H_{\mathrm{meas}} \, f.$$

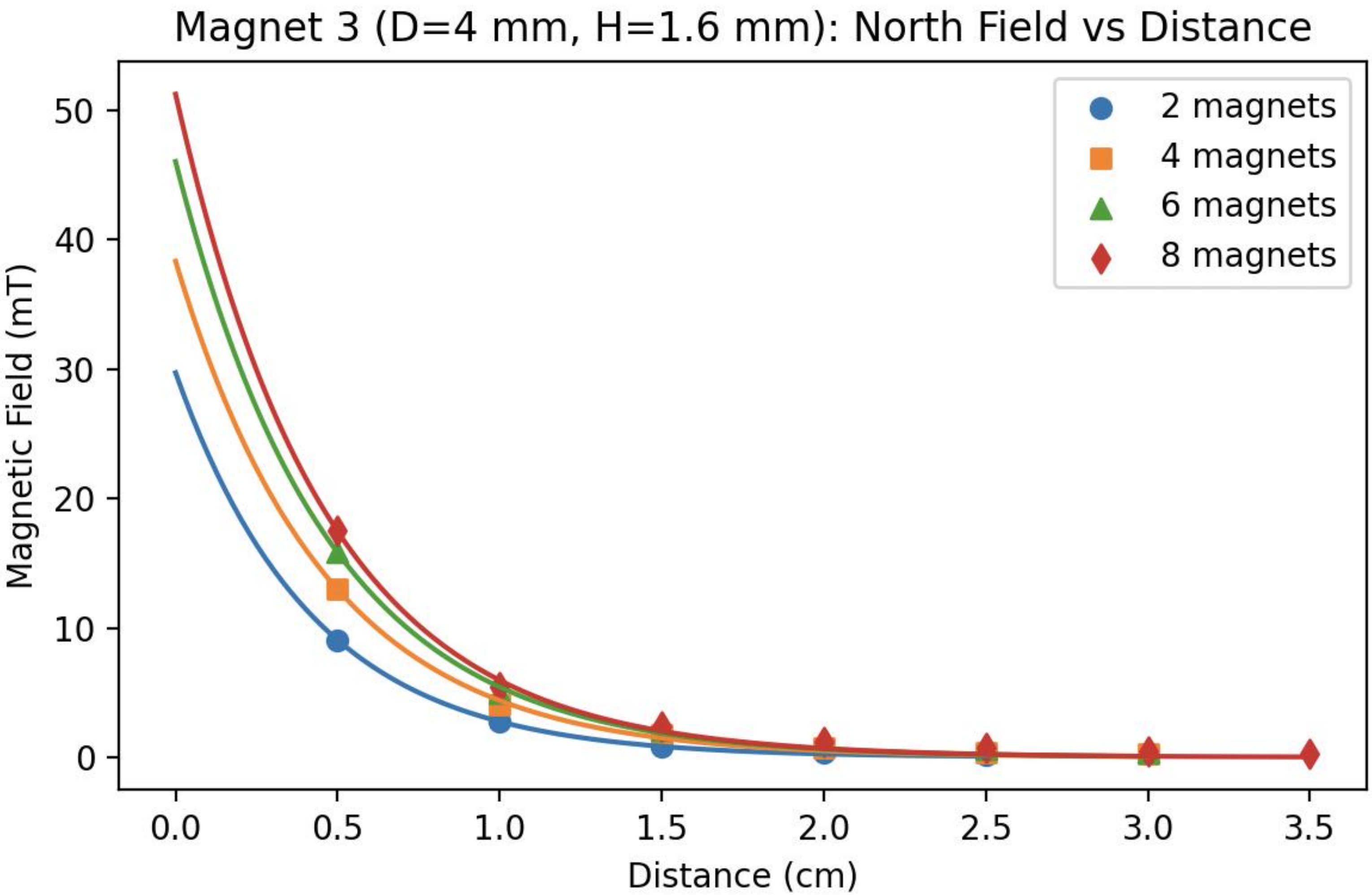
Magnet Characterization



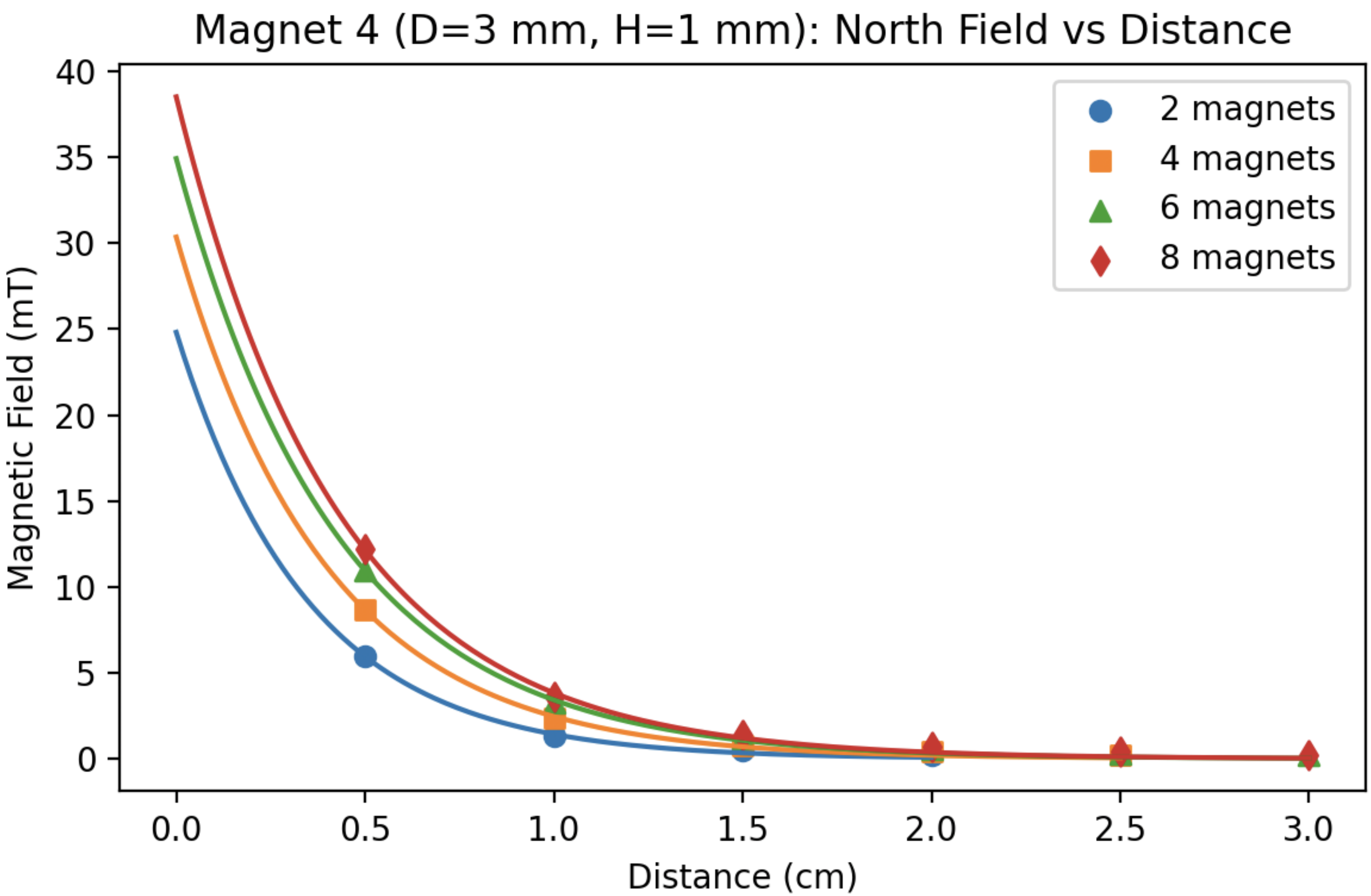
Magnet Characterization



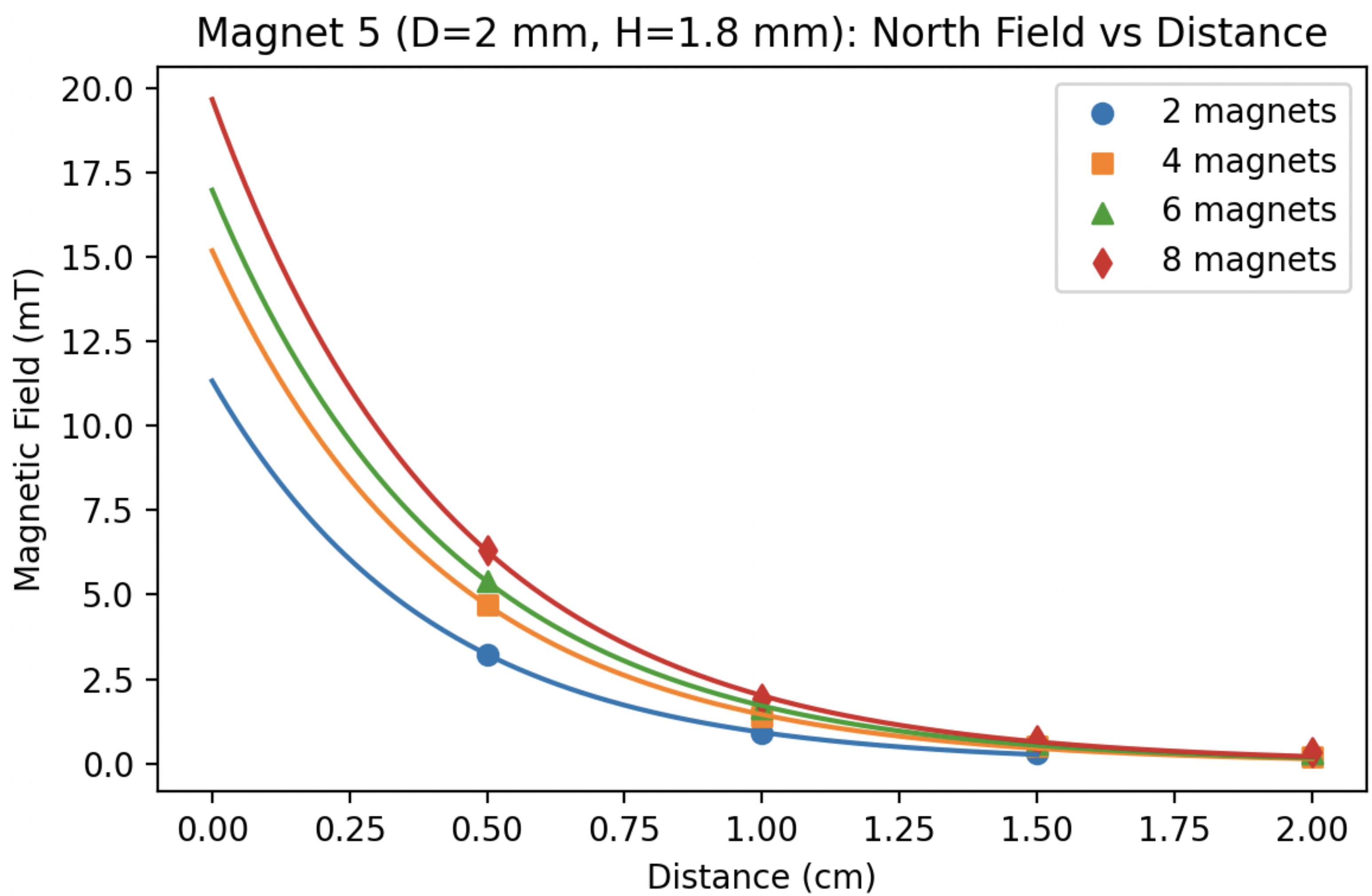
Magnet Characterization



Magnet Characterization



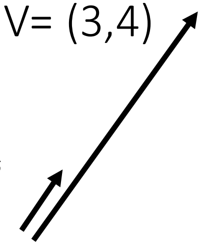
Magnet Characterization



Explaining Vectors and Cross

$||v|| = 5$

$V = (3,4)$



$\hat{v}$

$|| \hat{v} || = 1$

$$\left|\left|v\right|\right|\hat{v} = v$$

$$torque = r \times \hat{F}\left|\left|F\right|\right|$$

$$\frac{torque}{||F||} = r \times \hat{F}$$

Mass Measurements

Normal Force/Saturation

Rod Material	Magnet Diameter	Magnet Number	Magnet Rod Length	0.5cm Flux	Flux Ratio	Force(N)
Soft Iron						
	4	2	3.2	9.06667	1	-0.51217
		4	6.4	13.0333	1.4375	-1.32128
		6	9.6	16	1.764706	-2.0992
		8	12.8	17.5667	1.9375	-2.58893
	6	2	3.2	16.5	1	-0.77212
		4	6.4	25.8333	1.565657	-2.71684
		6	9.6	31.8667	1.931313	-4.59236
		8	12.8	37.2	2.254545	-6.57624
Low Carbons Ste	4	2	3.2	9.06667	1	-0.51222
		4	6.4	13.0333	1.4375	-1.32141
		6	9.6	16	1.764706	-2.09942
		8	12.8	17.5667	1.9375	-2.5892
	6	2	3.2	16.5	1	-0.7722
		4	6.4	25.8333	1.565657	-2.71715
		6	9.6	31.8667	1.931313	-4.59291
		8	12.8	37.2	2.254545	-6.57704
Stainless Steel	4	2	3.2	9.06667	1	
		4	6.4	13.0333	1.4375	
		6	9.6	16	1.764706	
		8	12.8	17.5667	1.9375	
	6	2	3.2	16.5	1	-0.02295
		4	6.4	25.8333	1.565657	-0.05938
		6	9.6	31.8667	1.931313	-0.10511
		8	12.8	37.2	2.254545	-0.15767