Trajectory Optimization for an Inertia Wheel Cube

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Inspiration

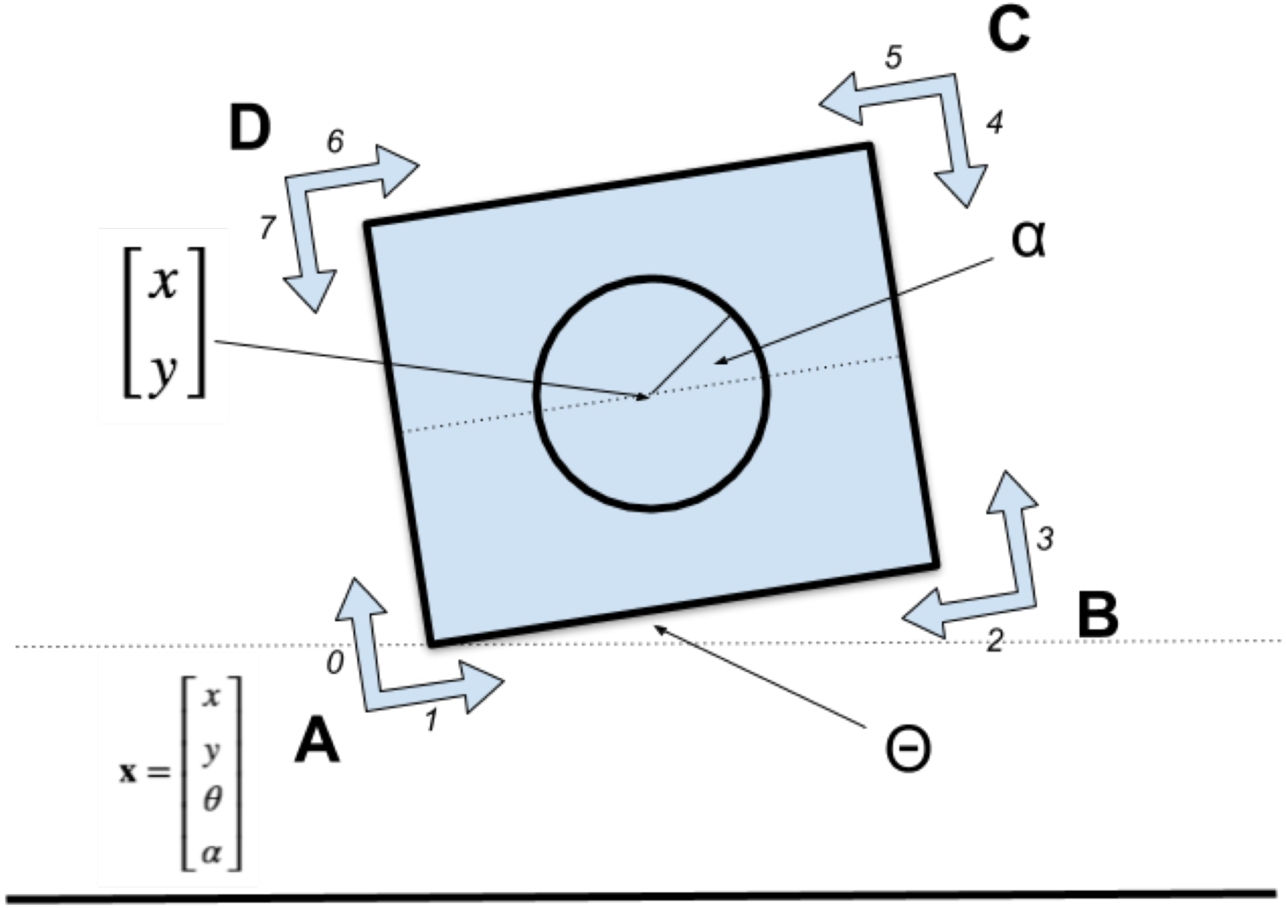
- M-Blocks
- Cubli

http://www.idsc.ethz.ch

http://hplusmagazine.com/wp-content/uploads/2014/05/m-blocks.jpg

Formulation

- Floating-base coordinates



$$\ddot{x} = (f_1 - f_2 + f_6 - f_5)\cos\theta - (f_0 + f_3 - f_4 - f_7)\sin\theta \tag{3}$$

$$\ddot{y} = (f_1 - f_2 + f_6 - f_5)\sin\theta + (f_0 + f_3 - f_4 - f_7)\cos\theta - g \tag{4}$$

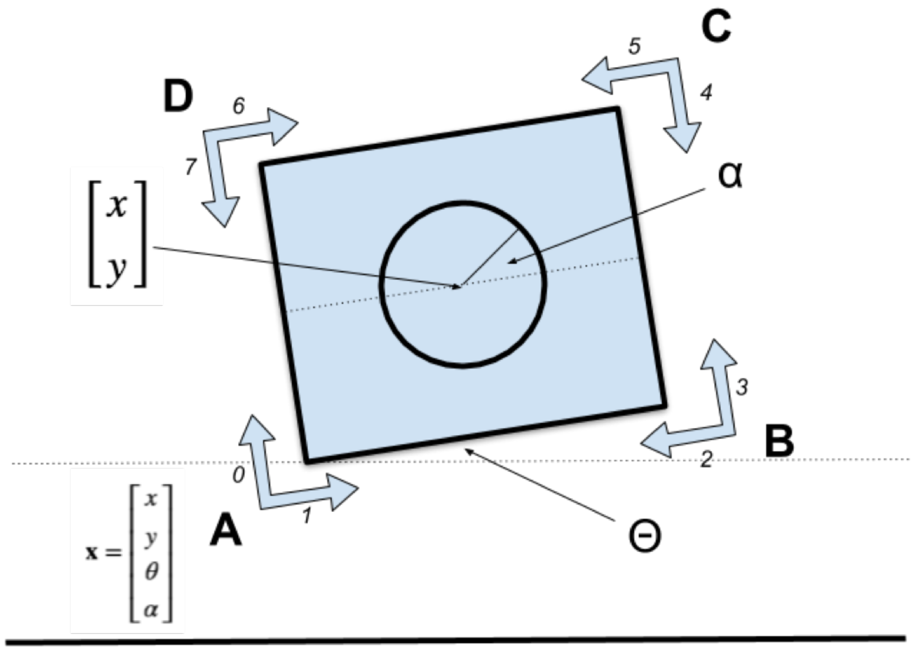
$$\ddot{\theta} = \frac{-u + b_w\dot{\alpha} - b_c\dot{\theta}}{I_c} + \frac{1}{2}\left(\sum_{n\in 1,3,5,7} f_n - \sum_{n\in 0,2,4,6} f_n\right) \tag{5}$$

$$\ddot{\alpha} = \frac{u(I_c + I_w) + b_c I_w\dot{\theta} - b_w\frac{I_c+I_w}{2}\dot{\alpha}}{I_w I_c} \tag{6}$$

$$\mathbf{x} = \begin{bmatrix} x & y & \theta & \alpha & \dot{x} & \dot{y} & \dot{\theta} & \dot{\alpha} \end{bmatrix}^T$$

$$\dot{\boldsymbol{x}} = \begin{bmatrix} \dot{x} & \dot{y} & \dot{\theta} & \dot{\alpha} & \ddot{x} & \ddot{y} & \ddot{\theta} & \ddot{\alpha} \end{bmatrix}^T$$

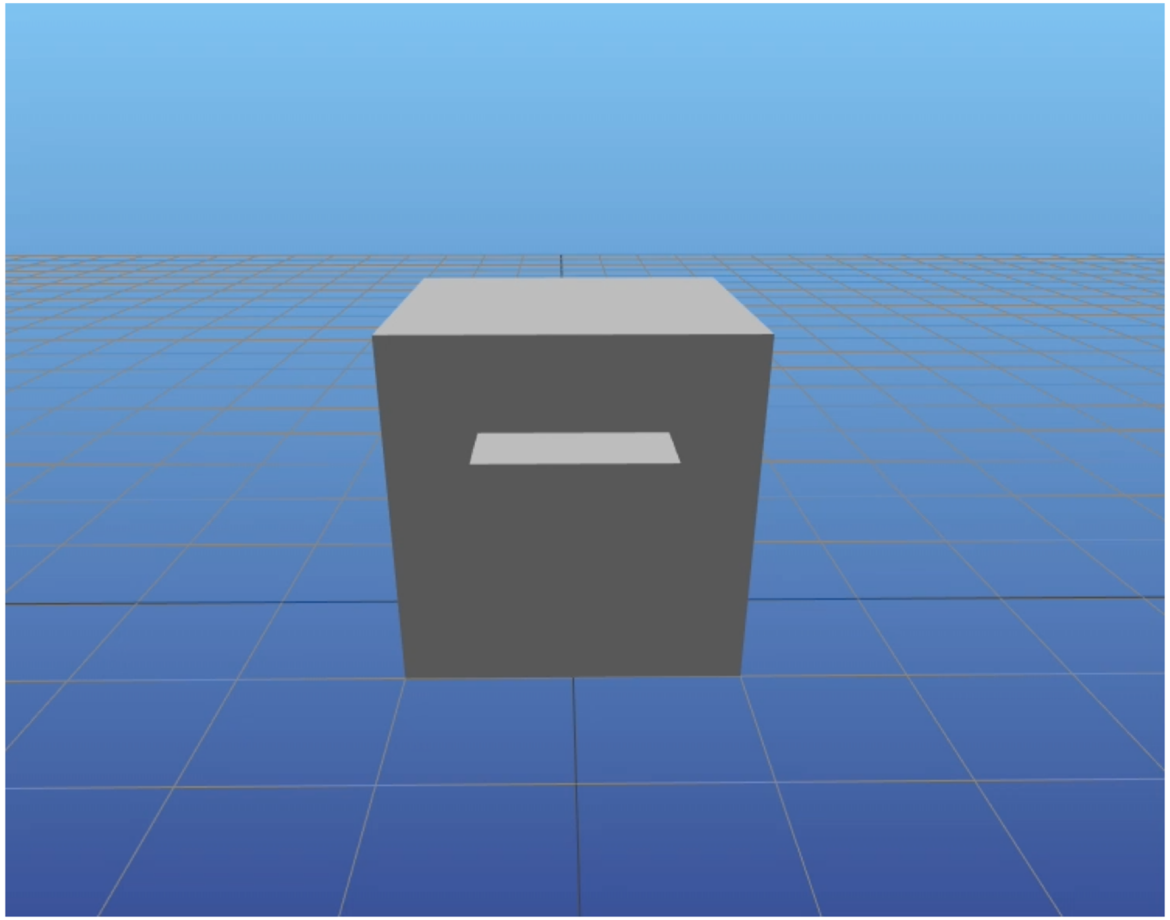
Formulation



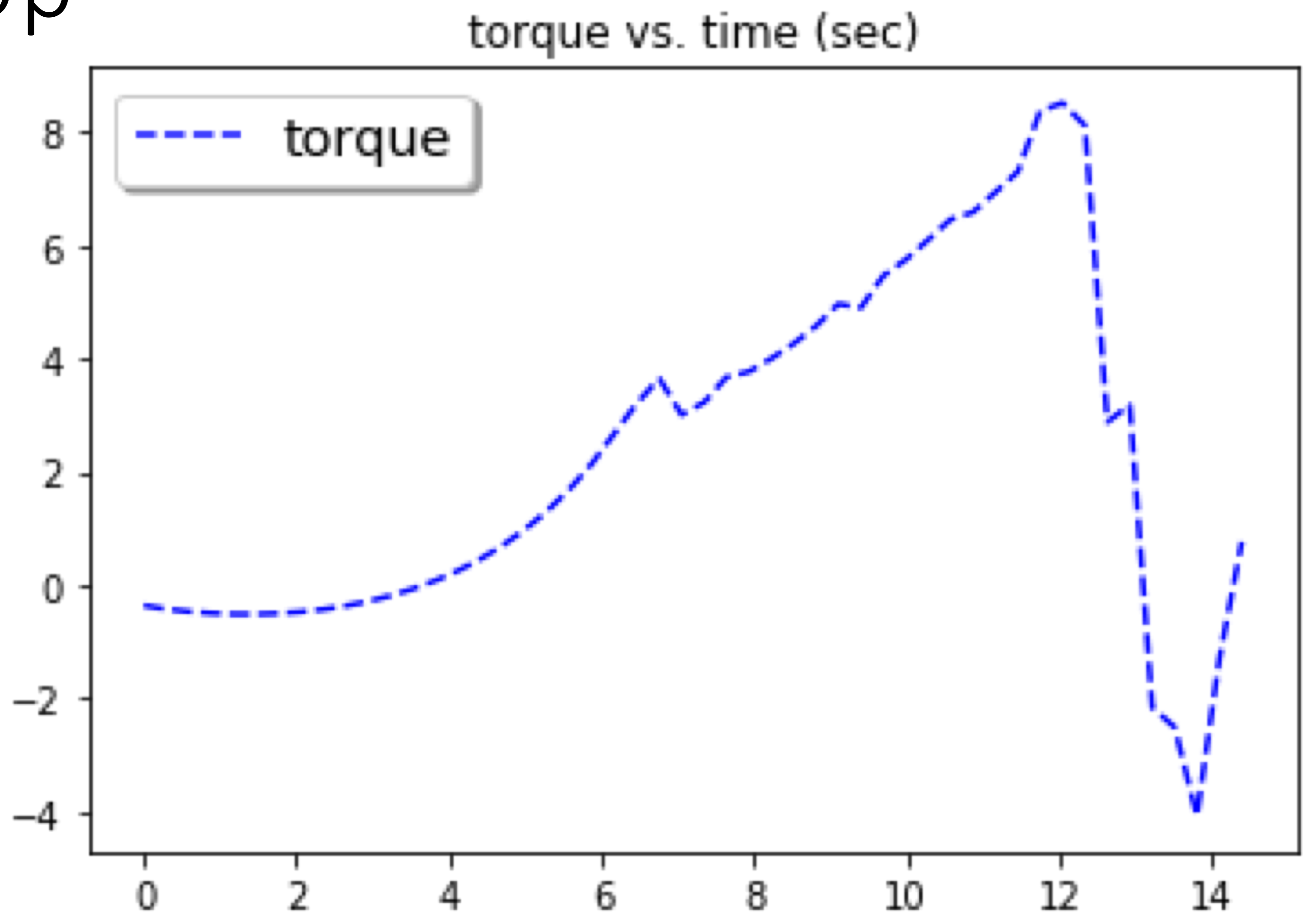
- Contact-implicit trajectory optimization

$$\begin{aligned}
&\underset{x[0:N],u[0:N],f[0:N]}{\text{find}} \\
&\text{subject to} \quad && x[n+1] = x[n] + f(x[n], u[n], f[n])dt, \\
& && n \in [0, N-1], \\
& && -u_{max} < u < u_{max}, \\
& && 0 < f[n][i] < f_{max}, i \in [0, 7], \\
& && c[n][:] \cdot \phi[:] = 0.0, \\
& && x[0] = x_{initial}, \\
& && x[N] = x_{final},
\end{aligned}$$

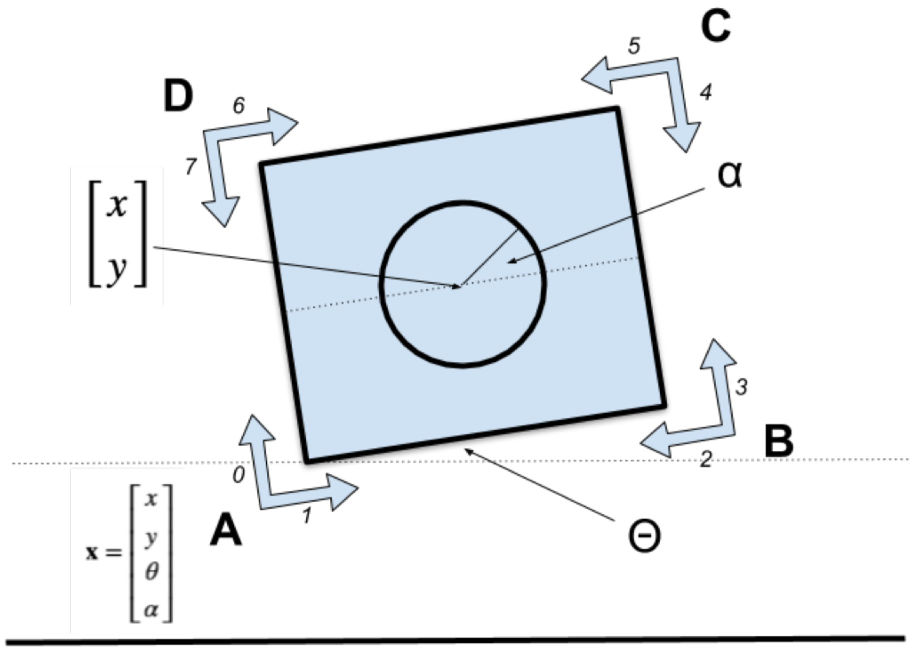
Swing Up



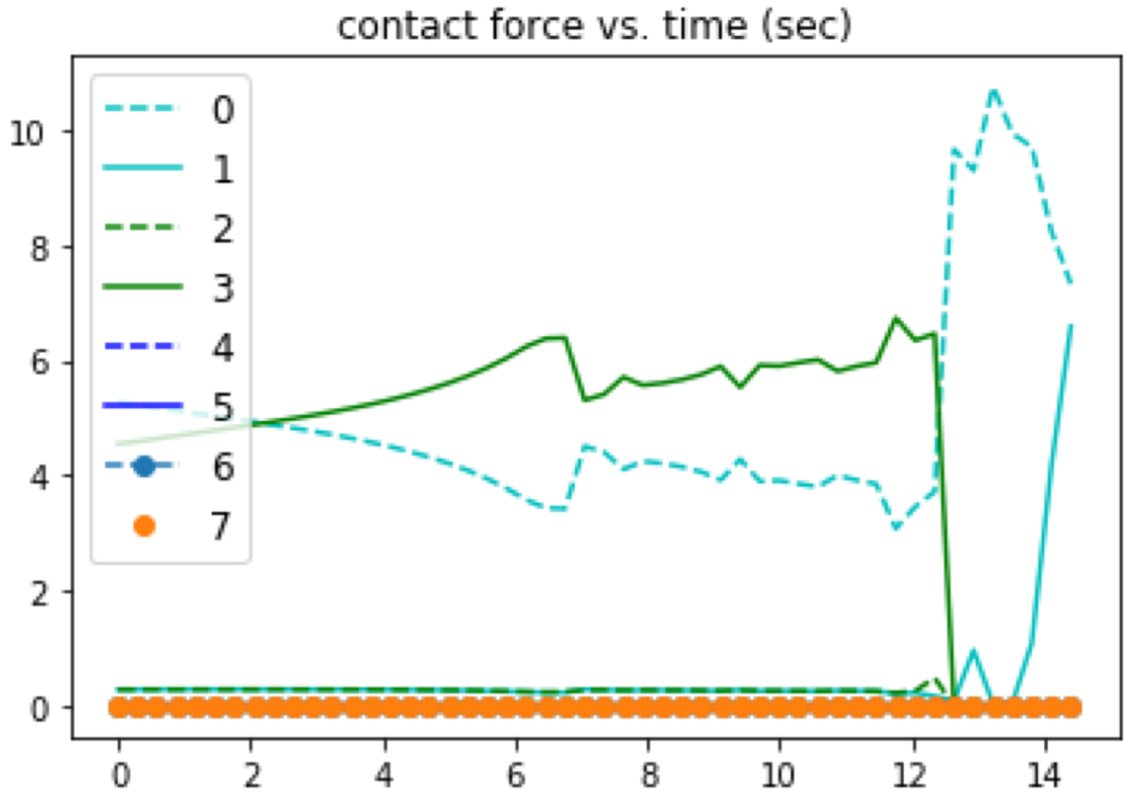
Swing Up



Swing Up

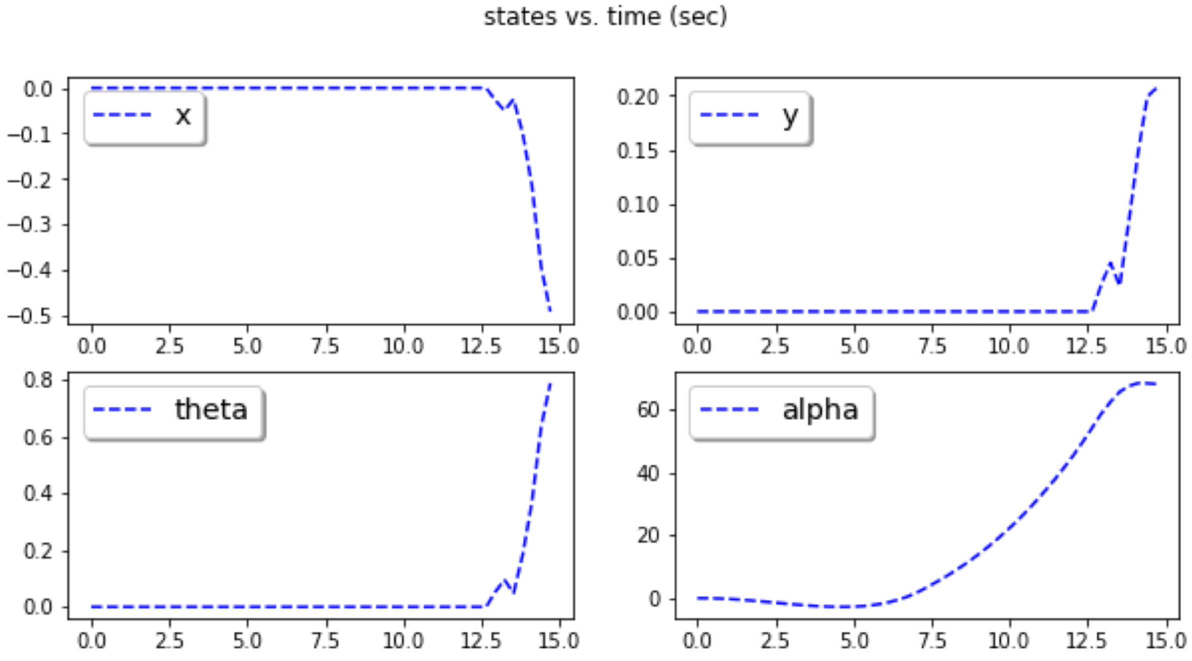


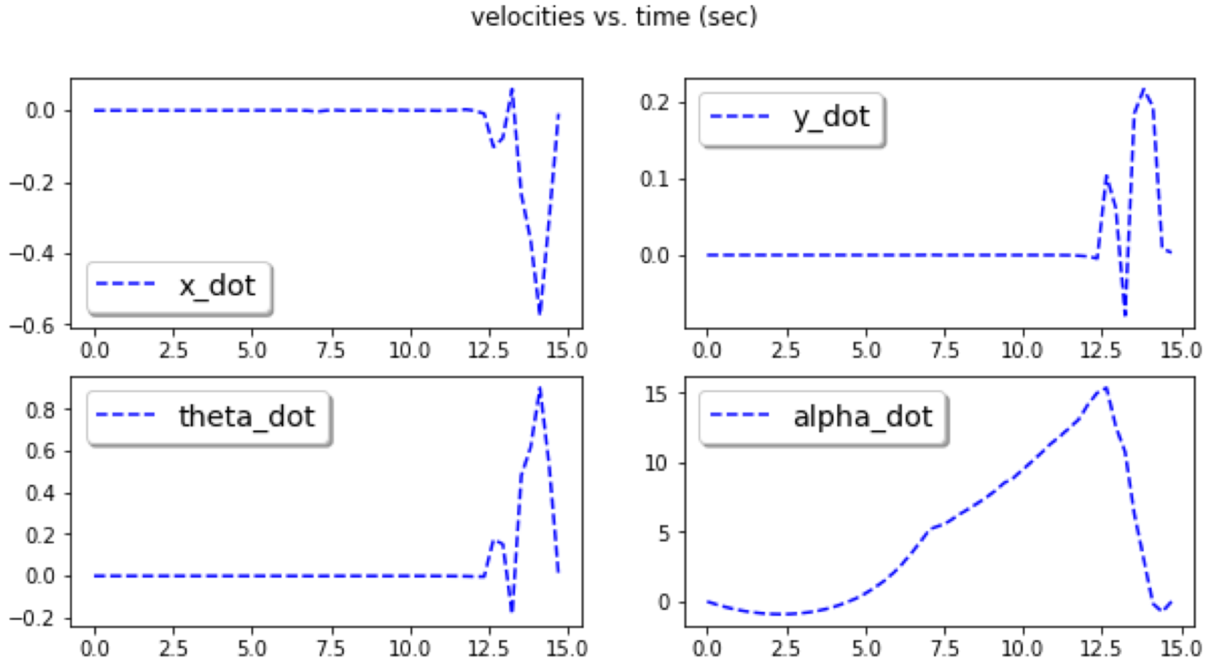
C
5
4
D
6
7
α
$\begin{bmatrix} x \\ y \end{bmatrix}$
3
B
2
0
1
A
Θ
$\mathbf{x} = \begin{bmatrix} x \\ y \\ \theta \\ \alpha \end{bmatrix}$



contact force vs. time (sec)
0
1
2
3
4
5
6
7
0
2
4
6
8
10
0
2
4
6
8
10
12
14

Swing Up


states vs. time (sec)
x
y
theta
alpha


velocities vs. time (sec)
x_dot
y_dot
theta_dot
alpha_dot

PD & QP Controller at Upright

- Quadratic cost on corner position
- PD control for torque input

Limit Cycle

- Cost functions
  - Maximize vel.
  - Minimize torque
  - Minimize time

M-Blocks Example