Trajectory Optimization for an Inertial Cube

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***Abstract*—Inertially-actuated robots pose a very interesting problem for underactuated control. Here we present a trajectory opimization method to control a 2d cube with a fully-controllable flywheel mounted inside. Inspired by earlier work done by with the Cubli and M-Blocks, we use contact-implicit trajectory optimization to generate more complex behavior of the inerial cube. We use one actuator (the flywheel) to control the 8 states of the robot over time. Using Sparse Nonlinear OPTimizer (SNOPT) in Drake, the interial cube can be descriped as a floating body with contact implicit movements via linear complementarity contraints. This leads to elegant optimal trajectory control for the interial cube.**

***Index Terms*—trajectory optimization, underactuated, inertia**

I. Introduction

This document is a model and instructions for LaTeX. Please observe the conference page limits.

II. Defining the Model

A. State Space Explanation

Here is a diagram depicting the states of the cube. The cube is defined in free space, meaning there is no notion of the ground in the state description. Here is an image describing the cube states.

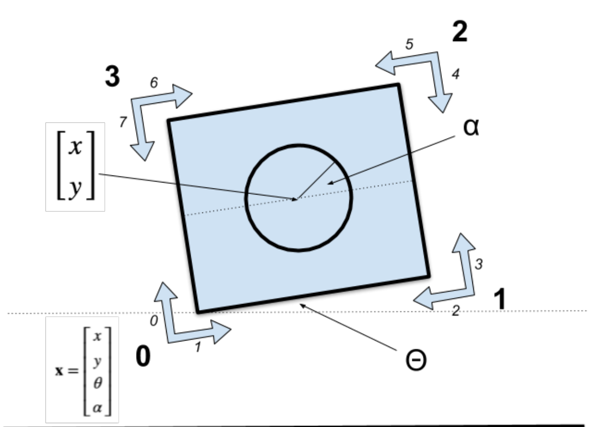

Fig. 1. The states of the floating inertial cube. The derivates of these states are also included in the total state vector **x** but are not shown here for convenience.

Following from the diagram, the states are the following.

$$\mathbf{x} = \begin{bmatrix} x \\ y \\ \theta \\ \alpha \\ \dot{x} \\ \dot{y} \\ \dot{\theta} \\ \dot{\alpha} \end{bmatrix} \qquad \dot{\boldsymbol{x}} = \begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{\theta} \\ \dot{\alpha} \\ \ddot{x} \\ \ddot{y} \\ \ddot{\theta} \\ \ddot{\alpha} \end{bmatrix} \tag{1}$$

The free body dynamics can be described in the following way.

$$\ddot{x} = (f_1 - f_2 + f_6 - f_5)\cos\theta - (f_0 + f_3 - f_4 - f_7)\sin\theta \tag{2}$$

$$\ddot{y} = (f_1 - f_2 + f_6 - f_5)\sin\theta + (f_0 + f_3 - f_4 - f_7)\cos\theta - g \tag{3}$$

$$\ddot{\theta} = \frac{-u + b_w\dot{\alpha} - b_c\dot{\theta}}{I_c} + \frac{1}{2}\Big(\sum_{x\in 1,3,5,7} f_n - \sum_{n\in 0,2,4,6} f_n\Big) \tag{4}$$

$$\ddot{\alpha} = \frac{u(I_c + Iw) + b_c I_w\dot{\theta} - b_w\frac{I_c+I_w}{2}\dot{\alpha}}{I_w I_c} \tag{5}$$

In these dynamics, we are assuming a simplified cube of unit size. These dynamics were informed by prior work done at Chalmers University of Technology [4].

III. Optimization Formulation

In this section, we explain the necessary nonlinear program formulation to solve for a cube trajectory through space. We'll start by explaining this in the context of a simple swing up of the cube.

A. The Simple Swing-Up

The swing-up is performed by getting the cube to stand on exactly one corner in a stable position. By using the contact-implicit trajectory optimization, this is possible. The computed trajectory values are showing in Fig. 2, 3, and 4.

Explain that LQR could be used to control after achieving swing up.

B. Experiments in Higher Dimensional State Space

Explain how 3D seemed to help in some cases even though some states don't enter dynamics.

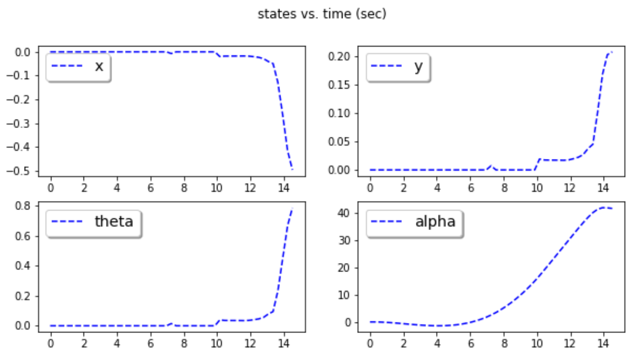

Fig. 2. Swing up trajectory states.

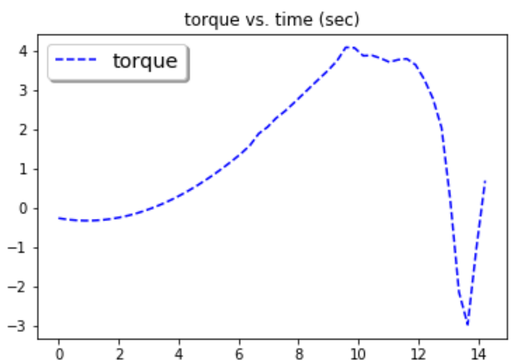

Fig. 3. Swing up trajectory torque input.

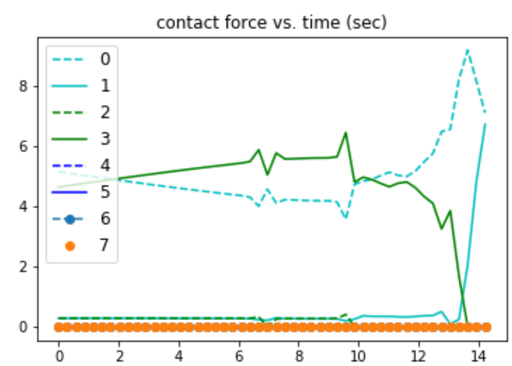

Fig. 4. Swing up trajectory ground forces.

C. Stable Walking Motion

Explain how to achieve stable walking motion.

Results

Explanation of results section. This should include more diagrams.

Future Work

Future work section.

References

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