# Stabilization Achieved by Optimal Stimulus Spending

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## Optimal public expenditure w/ initial gap $u_0 - u^\#$:

$$\frac{g/c - g/c^\#}{g/c^\#} = \frac{2 \cdot \varepsilon \cdot m}{1 + z \cdot \varepsilon \cdot m^2} \cdot \frac{u_0 - u^\#}{u^*}$$

## resulting unemployment gap, after public expenditure:

$$\frac{u - u^\#}{u^*} = \frac{u_0 - u^\#}{u^*} - \frac{m}{2} \cdot z \cdot \frac{g/c - g/c^\#}{g/c^\#}$$

$$z = \left[ \frac{1}{u^*} \cdot \frac{2}{\frac{1}{g^*} + \frac{1}{c^*}} \right]$$

## Combine optimal stimulus w/ effect of stimulus on unemployment:

$$\frac{u - u^\#}{u^*} = \frac{u_0 - u^\#}{u^*} - \frac{z \, m}{2} \left( \frac{2 \cdot \varepsilon \, m}{1 + z \varepsilon \, m^2} \right) \frac{u_0 - u^\#}{u^*}$$

$$\frac{u - u^\#}{u^*} = \left[ 1 - \frac{z \varepsilon \, m^2}{1 + z \varepsilon \, m^2} \right] \frac{u_0 - u^\#}{u^*}$$

$$\boxed{\frac{u - u^\#}{u^*} = \frac{1}{1 + z \varepsilon \, m^2} \cdot \frac{u_0 - u^\#}{u^*}}$$

initial gap

final unemployment gap ( w/ optimal stimulus)

Final unemployment gap $(u - u^*)$ is a share

$$\boxed{\dfrac{1}{1 + \varepsilon \xi m^2}}$$

of the initial unemployment gap.

- share $\neq 0$ : stimulus spending never eliminates unemployment gap

- share $\in (0, 1)$ : stimulus spending always reduces the initial unemployment gap

- share is ↓ w/ $\varepsilon$ : final unemployment gap is smaller when public goods are more substitutable w/ private goods.

- share is ↓ w/ $m$ : final unemployment gap is smaller when multiplier is larger

- final unemployment gap $\to 0$ when $\begin{cases} \varepsilon \to \infty \\ m \to \infty \end{cases}$