

OPTIMAL PUBLIC EXPENDITURE WITH INEFFICIENT UNEMPLOYMENT

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Paper available at <https://www.pascalmichailat.org/6.html>

MAIN STABILIZATION POLICY: MONETARY POLICY

- policymakers rely primarily on monetary policy for stabilization
 - accordingly: extensive research on optimal monetary policy
- but monetary policy is sometimes constrained
 - zero lower bound (Japan, USA, EU)
 - monetary union (EU, USA)
 - ⇒ high unemployment
- then other stabilization policies are needed
 - but: very little is known about these alternative policies

THIS PAPER: OPTIMAL PUBLIC EXPENDITURE

- public expenditure is commonly used for stabilization
 - US: Great Depression (New Deal), Great Recession (ARRA)
- framework: matching model from Michaillat & Saez (2015)
- outcome: formula linking optimal stimulus spending to 3 sufficient statistics
 1. unemployment gap
 2. unemployment multiplier
 3. elasticity of substitution between public consumption & private consumption

OPTIMAL PUBLIC EXPENDITURE: EXISTING RESULTS

- Samuelson (1954):
 - public goods financed by lump-sum taxation
 - efficient level of production
 - rule: spend until marginal utilities are equalized
 - but: what if production is inefficient?
- Keynes (1936):
 - no tradeoffs between public consumption & private consumption (multiplier > 1)
 - rule: spend to fill output gap
 - but: what if there is a tradeoff?
- our theory blends the theories of Samuelson & Keynes

INFORMAL DESCRIPTION OF THE MODEL

A SERVICE ECONOMY, WITHOUT FIRMS



A SERVICE ECONOMY, WITHOUT FIRMS



AN ASSET FOR SAVING



PRIVATE SERVICES (c) & PUBLIC SERVICES (g)



PRIVATE SERVICES (c) & PUBLIC SERVICES (g)



MATCHING: NOT ALL SERVICES ARE SOLD



MATCHING: NOT ALL SERVICES ARE SOLD



MATCHING: COSTLY TO PURCHASE SERVICES



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- Cooks/Housekeepers
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MATCHING: COSTLY TO PURCHASE SERVICES

FEE SCHEDULE:

Fee for Long Term Services:

CHILDCARE

- ♦ All **Full Time** Nanny, Parent Helper, Family Assistant, Governess
15% of annual Gross Compensation (minimum fee = \$3000)
- ♦ All **Part Time** Nanny, Parent Helper, Family Assistant, Governess
15% of annual Gross Compensation (minimum fee = \$1500)

HOUSEHOLD

- ♦ All **Full Time** Housekeeper, Executive Housekeeper, Cook, Handyman, Companion
15% of annual Gross Compensation (minimum fee = \$3000)
- ♦ All **Part Time** Housekeeper, Executive Housekeeper, Cook, Handyman, Companion
15% of annual Gross Compensation (minimum fee = \$1500)

ESTATE/ PRIVATE OFFICE

- ♦ All **Full Time and Part Time** Estate Managers, Household Managers, Chefs, Valets, Butlers, Master Gardeners, Security Body Guards, Chauffeurs, Couples, Personal Assistants, Executive Assistant Candidates
20% of annual Gross Compensation (minimum fee = \$3000)

Fee for On-Call & Temporary Services

- ♦ All On-Call and Temporary Work Assignments except for Baby Nurses, Newborn Specialists and Doulas
35% of ongoing Gross Compensation (minimum fee = \$35 a day)
- ♦ All Baby Nurses, Newborn Specialists & Doulas
20% of ongoing Gross Compensation (minimum fee = \$50 a day)

SOCIALLY EFFICIENT RATE OF UNEMPLOYMENT

- too much unemployment is bad
 - too many services are idle
 - too little unemployment is bad
 - too many services are devoted to recruiting
 - there is a socially efficient rate of unemployment (u^*)
 - number of services enjoyed ($y = g + c$) is maximized
- ⇒ when unemployment is efficient, Samuelson rule holds

FORMAL DESCRIPTION OF THE MODEL

STRUCTURE

- dynamic matching model
 - building on Michaillat & Saez (2015)
- identical, self-employed households
- government
- 2 consumption goods traded on a matching market
 - public services & private services
- 1 asset for saving

MATCHING MARKET

- capacity of each household: k services
- household purchases: $C(t)$ private services
- government purchases: $G(t)$ public services
- output: $Y(t) = C(t) + G(t) < k$
- unemployment rate: $u(t) = 1 - Y(t)/k$
- price of services: $p(t)$

MATCHING FUNCTION

- number of vacancies: $v(t)$
- matching function: $h(t) = \omega \cdot [k - Y(t)]^\eta \cdot v(t)^{1-\eta}$
- market tightness: $x(t) = v(t)/(k - Y(t))$
- selling rate & buying rate:

$$f(x(t)) = \frac{h(t)}{k - Y(t)} = \omega \cdot x(t)^{1-\eta}$$

$$q(x(t)) = \frac{h(t)}{v(t)} = \omega \cdot x(t)^{-\eta}$$

MARKET FLOWS

- relationships separate at rate s
- given x , output and unemployment converge to

$$Y(x, k) = \frac{f(x)}{s + f(x)} \cdot k, \quad u(x) = \frac{s}{s + f(x)}$$

- convergence to steady state is extremely fast, so we assume:
 - $Y(t) = Y(x(t), k)$
 - $u(t) = u(x(t))$
 - see Hall (2005)

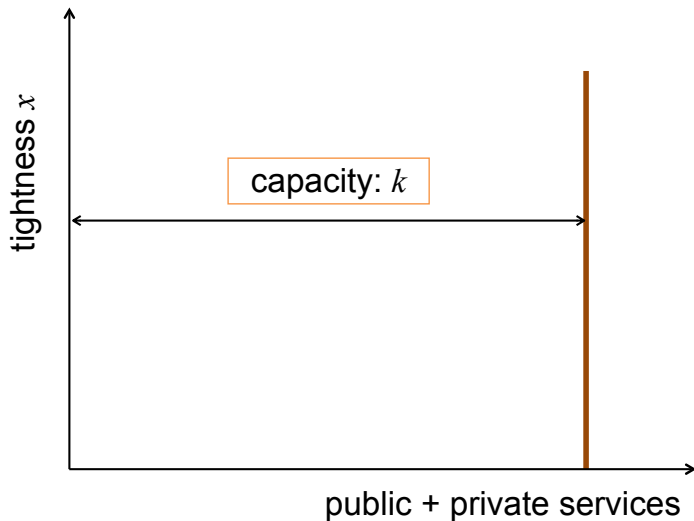
MATCHING COST: ρ SERVICES PER VACANCY

- output (Y) = consumption (y) + matching cost

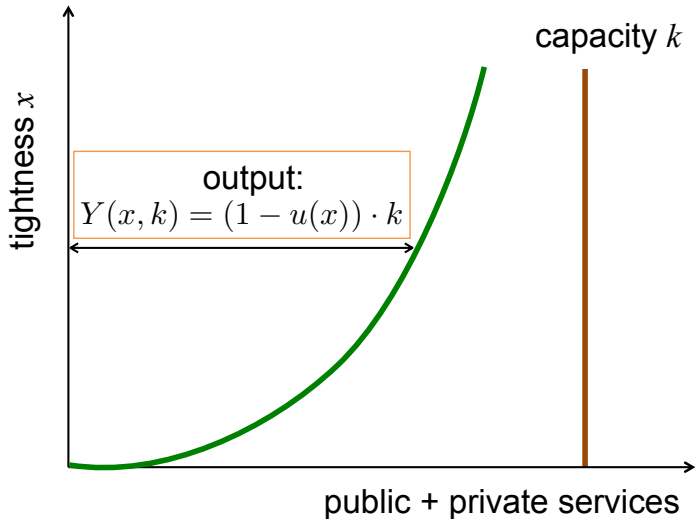
$$Y = y + \rho \cdot v = y + s \cdot Y \cdot \frac{\rho}{q(x)}$$

- matching wedge: $\tau_+(x) = s \cdot \rho / [q(x) - s \cdot \rho]$
- total consumption: $y = Y / [1 + \tau(x)]$
- private consumption: $c = C / [1 + \tau(x)]$
- public consumption: $g = G / [1 + \tau(x)]$

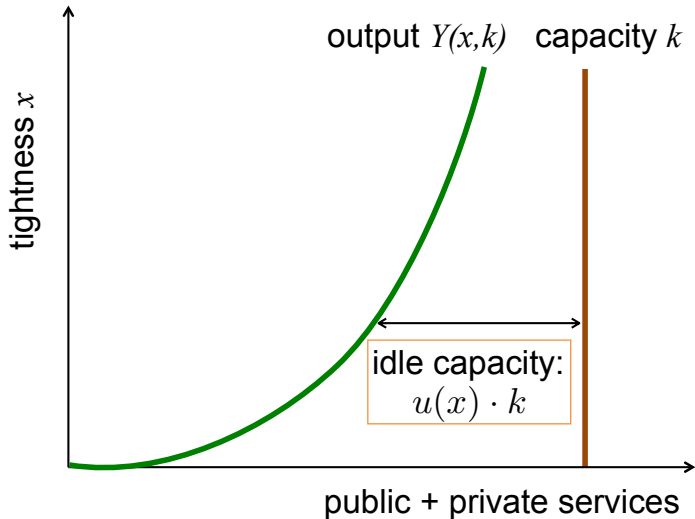
SUPPLY STRUCTURE: SUMMARY



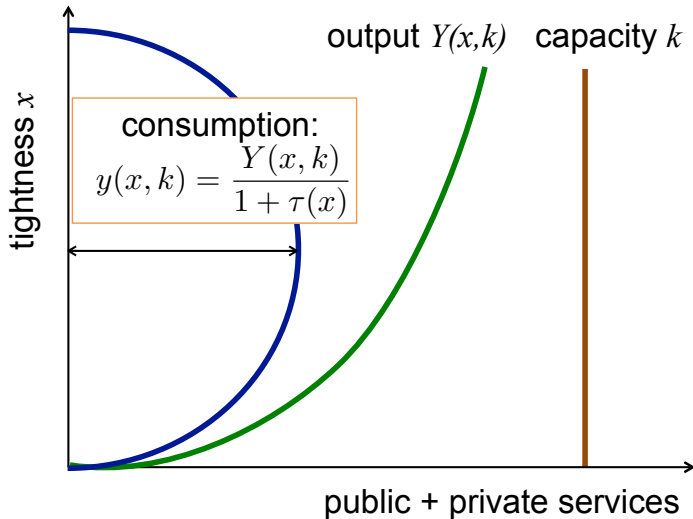
SUPPLY STRUCTURE: SUMMARY



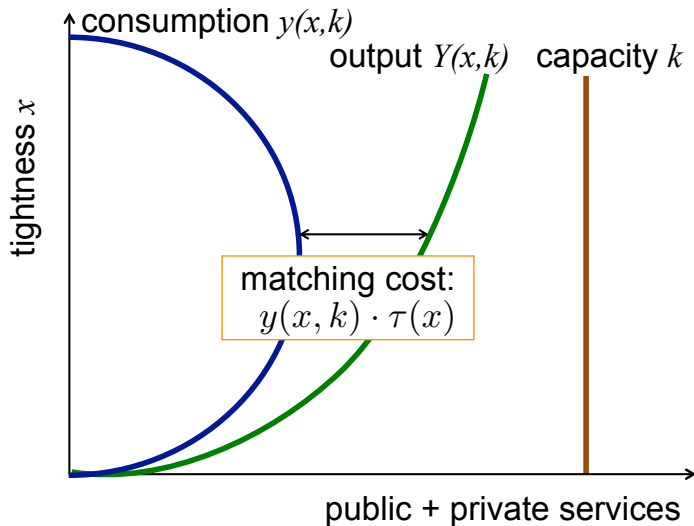
SUPPLY STRUCTURE: SUMMARY



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SUPPLY STRUCTURE: SUMMARY



DEMAND STRUCTURE: EXAMPLE

- asset: land $l(t)$ in fixed supply l_0
 - traded on a competitive market
 - Iacoviello (2005) and Liu, Wang, Zha (2013)
- households choose $c(t)$ and $l(t)$ to maximize utility

$$\int_0^{+\infty} e^{-\delta \cdot t} \cdot [\mathcal{U}(c, g) + \mathcal{V}(l)] dt$$

- subject to flow budget constraint

$$\dot{l} = p \cdot [1 - u(x)] \cdot k - p \cdot [1 + \tau(x)] \cdot c - T$$

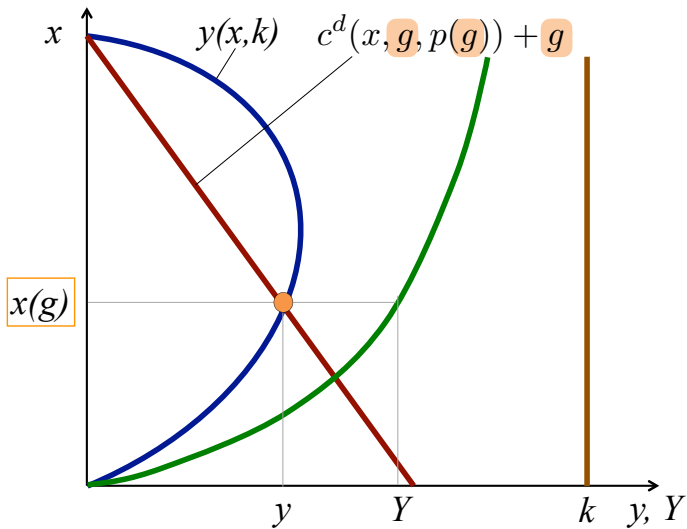
AGGREGATE DEMAND IN THE EXAMPLE

- market clearing on housing market: $l = l_0$
- private demand $c^d(x, g, p)$ is solution to Euler equation:

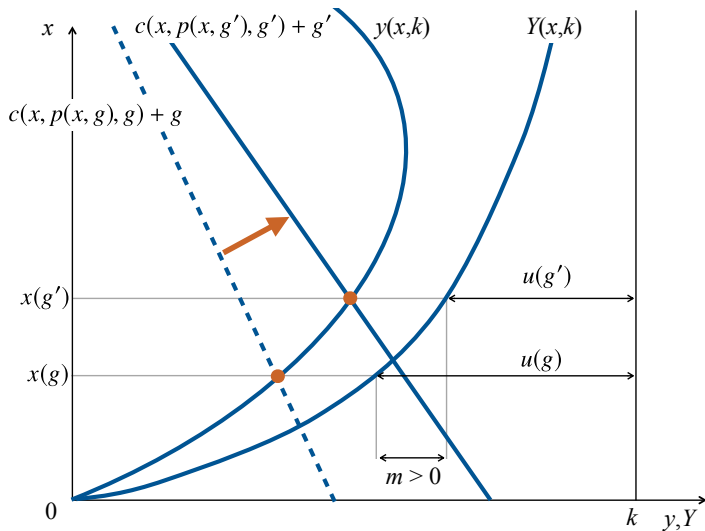
$$\frac{\partial \mathcal{U}}{\partial c}(c, g) = \frac{p \cdot (1 + \tau(x)) \cdot \mathcal{V}'(l_0)}{\delta}$$

- price of services relative to housing: $p = p(x, g)$
 - general price mechanism
 - (assumption required in matching model)

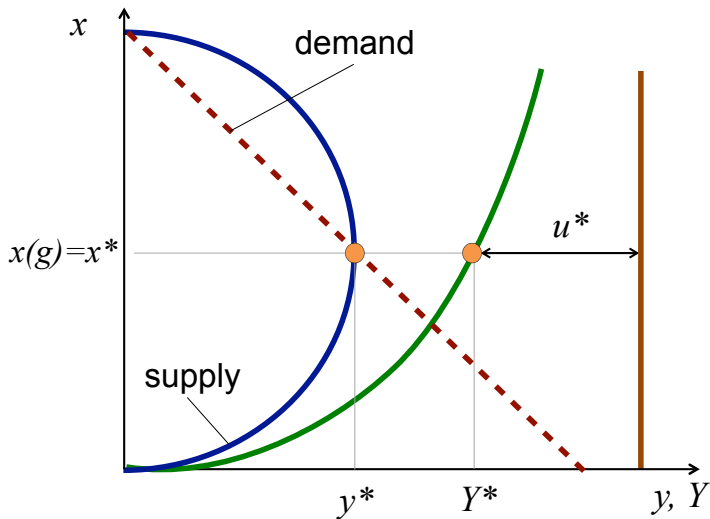
EQUILIBRIUM TIGHTNESS $x(g)$



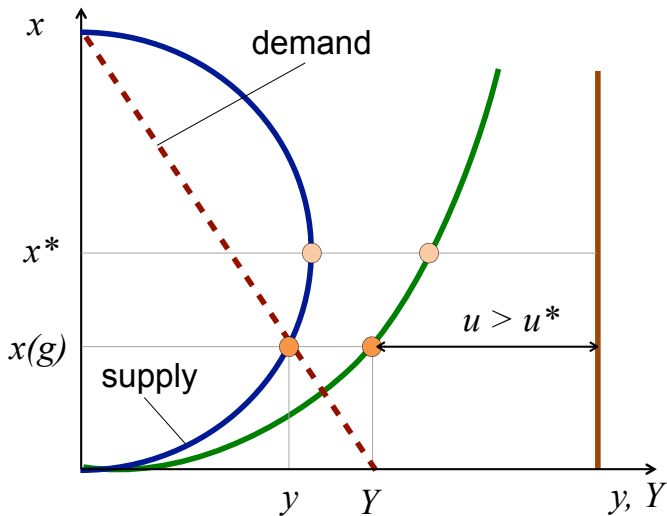
UNEMPLOYMENT MULTIPLIER m



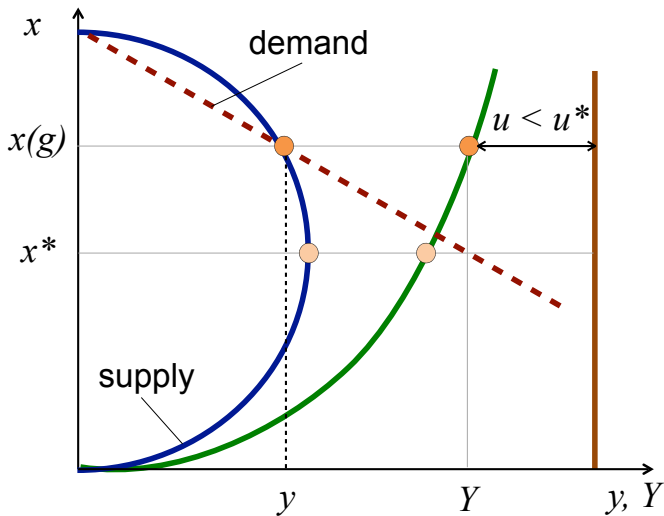
SOCIALLY EFFICIENT UNEMPLOYMENT RATE u^*



INEFFICIENTLY HIGH UNEMPLOYMENT RATE



INEFFICIENTLY LOW UNEMPLOYMENT RATE



OPTIMAL PUBLIC EXPENDITURE

GOVERNMENT'S PROBLEM

- households' flow utility is $\mathcal{U}(c, g)$
- public expenditure is financed by a lump-sum tax to maintain a balanced budget
- given $x(g)$, the government chooses g to maximize

$$\mathcal{U}\left(\underbrace{y(x(g), k) - g}_c, g\right)$$

CORRECTING THE SAMUELSON FORMULA

- first-order condition of government's problem is

$$0 = \frac{\partial \mathcal{U}}{\partial g} - \frac{\partial \mathcal{U}}{\partial c} + \frac{\partial \mathcal{U}}{\partial c} \cdot \frac{\partial y}{\partial x} \cdot \frac{dx}{dg}$$

- optimal public expenditure satisfies

$$\underbrace{1 = MRS_{gc}}_{\text{Samuelson formula}} + \underbrace{\frac{\partial y}{\partial x} \cdot \frac{dx}{dg}}_{\text{correction}}$$

- $MRS_{gc} = (\partial \mathcal{U} / \partial g) / (\partial \mathcal{U} / \partial c)$
- correction due to effect of public expenditure on welfare through tightness

INTRODUCING ESTIMABLE STATISTICS

- $(g/c)^*$: Samuelson spending
- elasticity of substitution between g and c :

$$1 - MRS_{gc} \approx \frac{1}{\epsilon} \cdot \frac{g/c - (g/c)^*}{(g/c)^*}$$

- unemployment gap:

$$\frac{\partial y}{\partial x} \propto u - u^*$$

- unemployment multiplier:

$$\frac{dx}{dg} \propto m = -\frac{y}{1-u} \cdot \frac{du}{dg}$$

IMPLICIT FORMULA FOR OPTIMAL STIMULUS

$$\frac{g/c - (g/c)^*}{(g/c)^*} \approx z_0 \epsilon m \cdot \frac{u - u^*}{u^*}$$

- $g/c - (g/c)^*$: stimulus spending
- ϵ : elasticity of substitution between g and c
 - = marginal social value of public spending
- m : unemployment multiplier
 - decrease in u when g increases by 1% of y
- $u - u^*$: unemployment gap
 - = productive inefficiency
- z_0 : constant of the parameters η, u^*

DEPARTURES FROM SAMUELSON RULE

	$m < 0$	$m = 0$	$m > 0$
$u > u^*$	$g/c < (g/c)^*$	$g/c = (g/c)^*$	$g/c > (g/c)^*$
$u = u^*$	$g/c = (g/c)^*$	$g/c = (g/c)^*$	$g/c = (g/c)^*$
$u < u^*$	$g/c > (g/c)^*$	$g/c = (g/c)^*$	$g/c < (g/c)^*$

MARGINAL VALUE OF PUBLIC SERVICES

- $\epsilon = 0$: digging holes or building pyramids
 - $g/c = (g/c)^*$: Samuelson rule holds, no stimulus spending
- $\epsilon \rightarrow +\infty$: perfect substitution
 - $u = u^*$: entirely fill unemployment gap, as in Keynes
- $\epsilon \in (0, +\infty)$: medium substitution
 - medium stabilization: $g/c \neq (g/c)^*$ but $u \neq u^*$
 \rightsquigarrow partially fill unemployment gap

MAKING THE FORMULA EXPLICIT

- implicit formula: not useful for quantitative results because u in RHS responds to g/c in LHS
- starting from $(g/c)^*$ and $u_0 \neq u^*$:

$$\frac{g/c - (g/c)^*}{(g/c)^*} \approx z_0 \epsilon m \cdot \frac{u(g/c) - u^*}{u^*}$$

- first-order Taylor expansion of u at $u((g/c)^*) = u_0$:

$$\frac{u - u^*}{u^*} \approx \frac{u_0 - u^*}{u^*} - z_1 m \cdot \frac{g/c - (g/c)^*}{(g/c)^*}$$

- z_1 : constant of the parameters $u^*, (g/c)^*$

EXPLICIT FORMULA

- optimal g/c depends on fixed quantities:

$$\frac{g/c - (g/c)^*}{(g/c)^*} \approx \frac{z_0 \epsilon m}{1 + z_1 z_0 \epsilon m^2} \cdot \frac{u_0 - u^*}{u^*}$$

- optimal u depends on fixed quantities:

$$u \approx u^* + \frac{u_0 - u^*}{1 + z_1 z_0 \epsilon m^2}$$

- approximations valid up to 2nd-order terms

RESULTS WITH DISTORTIONARY TAXATION

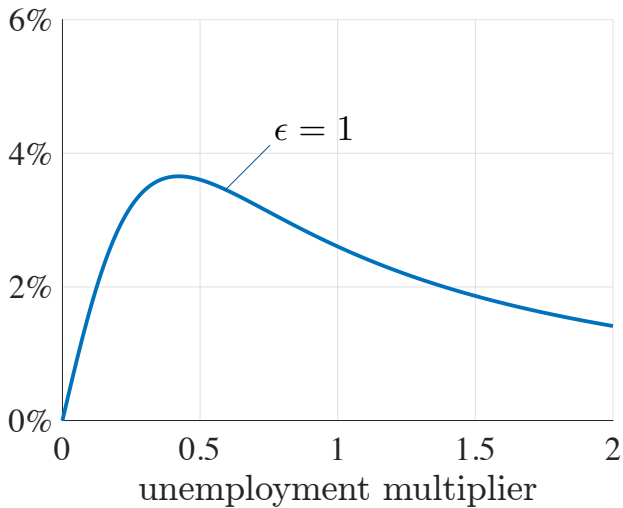
- endogenous capacity: $\mathcal{U}(c, g, k)$ with $\partial \mathcal{U} / \partial k < 0$
- linear income tax: $T = \tau^L \cdot (1 - u(x)) \cdot k$
- everything remains valid
 - but $(g/c)^*$ is lower because of tax distortions
- however: link between multipliers changes
 - no tax distortions: $m = dY/dG$
 - tax distortions: $m > dY/dG$
 - with taxes, we may have $dY/dG < 0$ but $m > 0$

NUMERICAL ILLUSTRATION: GREAT RECESSION IN THE US

STARTING POINT: WINTER 2008–2009

- unemployment = 6% and public spending = 16.5% of GDP
 - for illustration: we take these values as efficient
- unemployment is forecast to increase to 9%
 - initial unemployment gap = $9\% - 6\% = 3\%$
- we compute optimal stimulus for various elasticities of substitution and unemployment multipliers

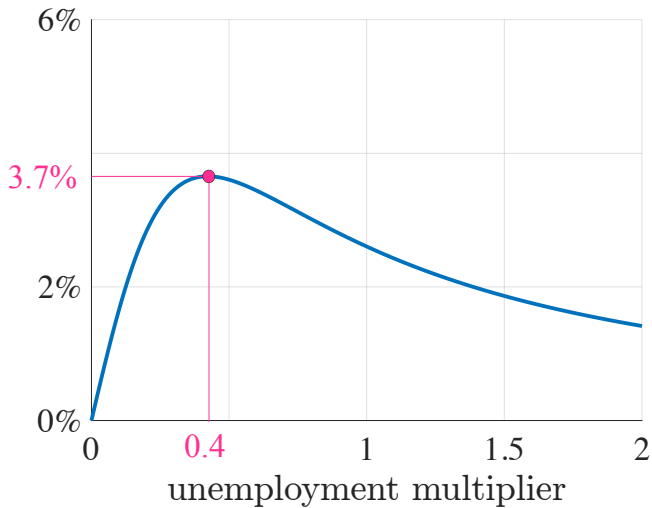
OPTIMAL STIMULUS SPENDING (% OF GDP)



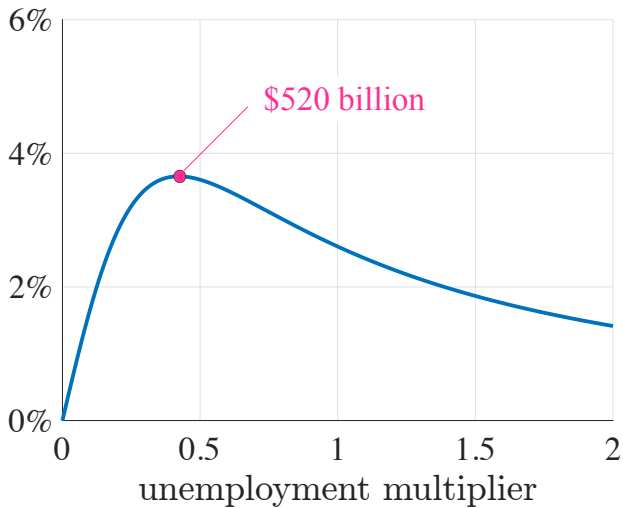
OPTIMAL STIMULUS SPENDING (% OF GDP)



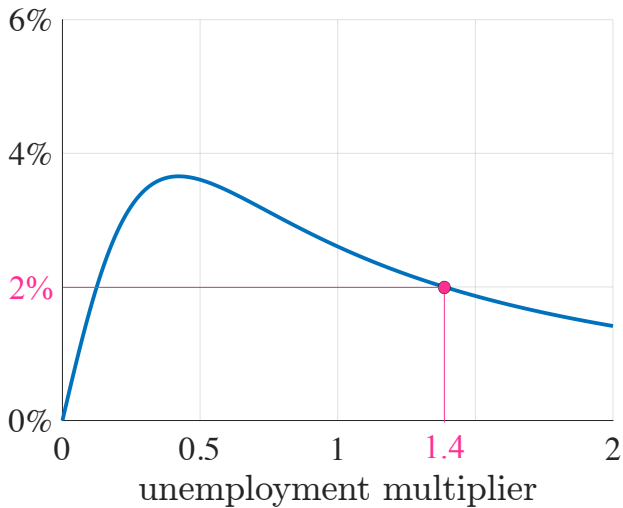
OPTIMAL STIMULUS SPENDING (% OF GDP)



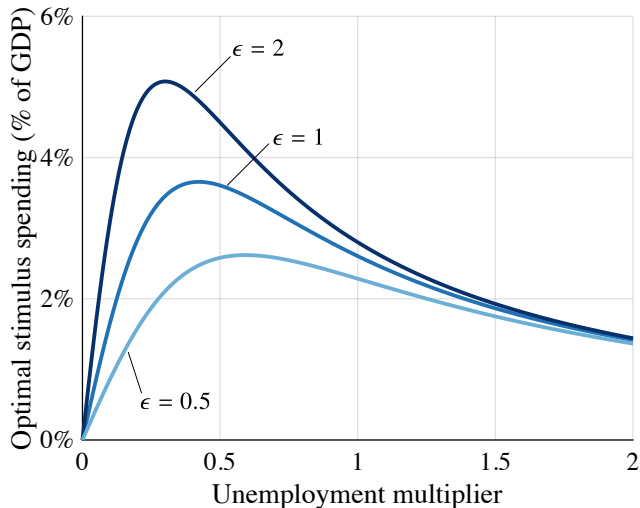
OPTIMAL STIMULUS SPENDING (% OF GDP)



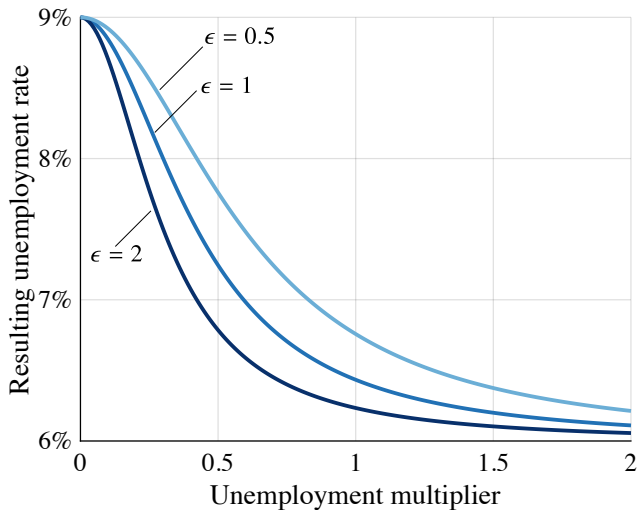
OPTIMAL STIMULUS SPENDING (% OF GDP)



OPTIMAL STIMULUS SPENDING FOR VARIOUS ϵ

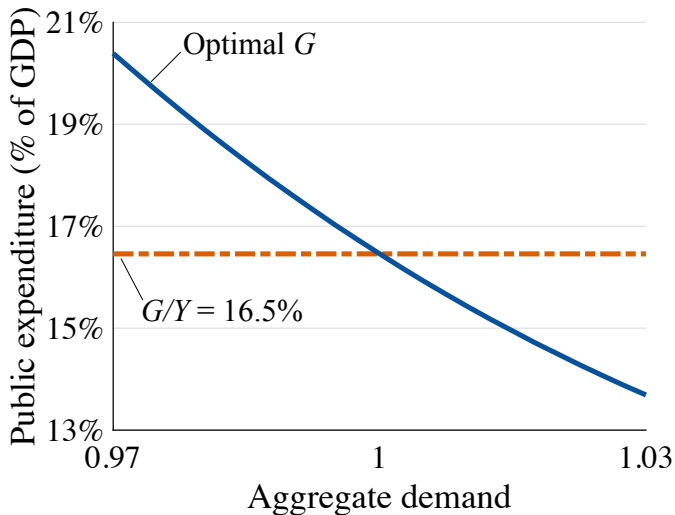


UNEMPLOYMENT UNDER OPTIMAL STIMULUS

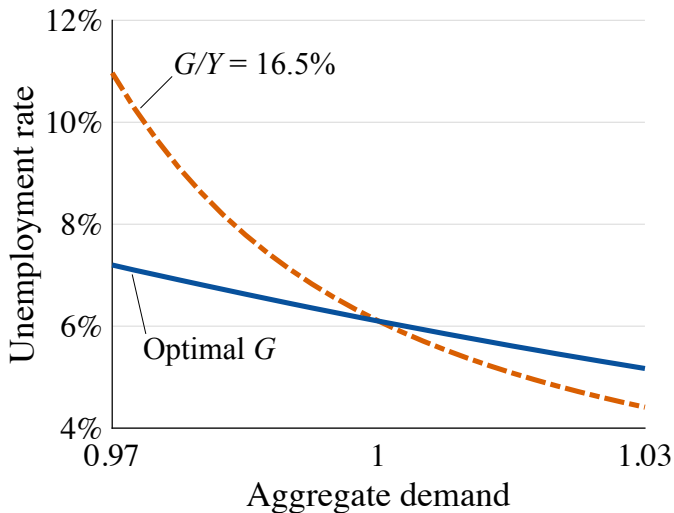


SOME SIMULATIONS

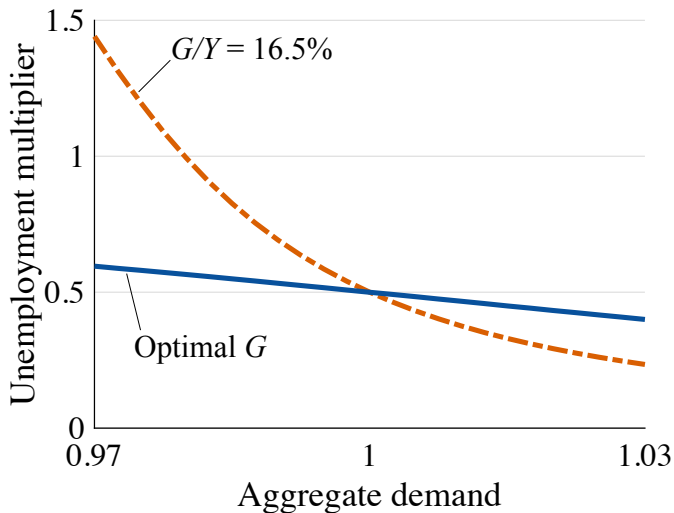
OPTIMAL STIMULUS IN CALIBRATED MODEL



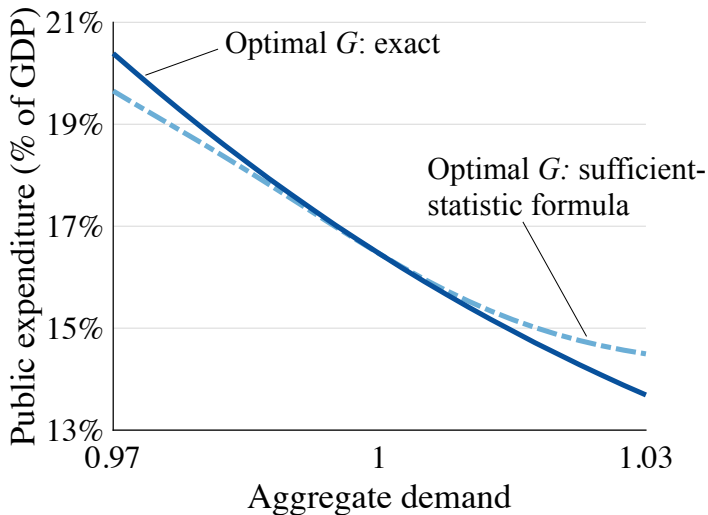
UNEMPLOYMENT RATE IN CALIBRATED MODEL



MULTIPLIER IN CALIBRATED MODEL



QUALITY OF APPROXIMATIONS IN FORMULA



SUMMARY & DISCUSSION

1. $dY/dG > 1$ is not necessary for stimulus
 - stimulus requires unemployment multiplier > 0 (as in data)
2. bang-for-the-buck logic does not hold
 - strongest stimulus for $m = 0.4$
 - same stimulus for $m = 0.1$ and $m = 1.4$
3. completely filling the unemployment gap is not optimal
 - optimal to partially fill unemployment gap
 - except if public services = private services
4. low marginal social value of g does not imply no stimulus
 - optimal to reduce unemployment gap
 - except if public services = digging holes

DISTORTIONARY TAXES \nRightarrow SMALLER STIMULUS

- formula remains valid with distortionary taxation
 - but Samuelson spending is lower
- however, dY/dG is not useful anymore because $dY/dG \neq m$
 - $dY/dG = m$ + labor-supply response to taxes
 - labor-supply distortion reduces dY/dG but not m
 - so: $m > dY/dG$
 - possibly: $dY/dG < 0$ while $m > 0$
- distortionary taxation does not imply smaller stimulus
 - only average public spending is lower