Today's Class

- Sample Space
- Events
- Venn Diagram
- Set Theory

Sample Space

- **Outcome**: Each possible result of such an experiment that we perform
- **Sample space (S)**: the set of all possible outcomes of an experiment
  - Discrete
  - Continuous

Sample Space for the Toss of a Single Coin

- S={head, tail}

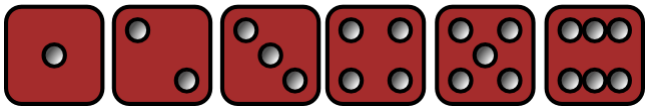
Sample Space for the Toss of Two Coins

- S={(H,H), (H,T), (T,H), (T,T)}

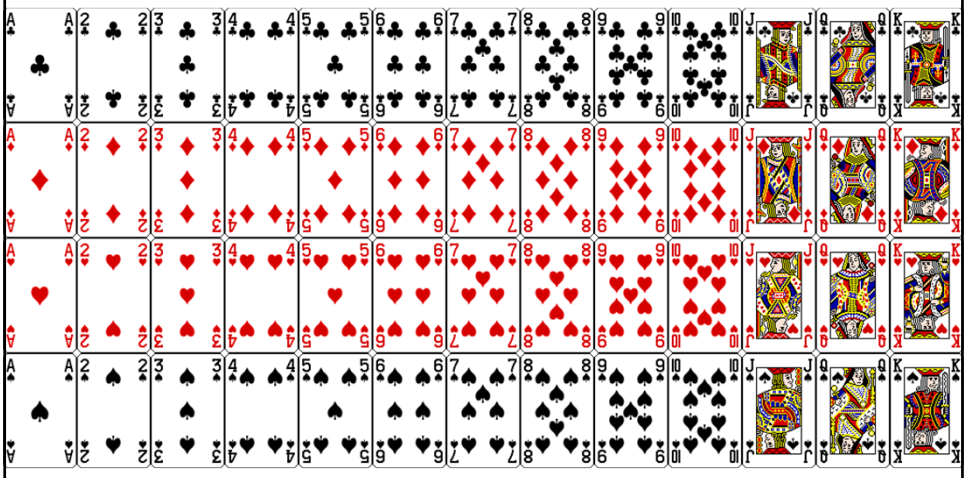
Sample Space for Rolling One Die

- Suppose a die is rolled
  What are the outcomes?
  - The outcomes are 1, 2, 3, 4, 5, 6

- What is the sample space?
  - S={1, 2, 3, 4, 5, 6}



Sample Space for Choosing One Card

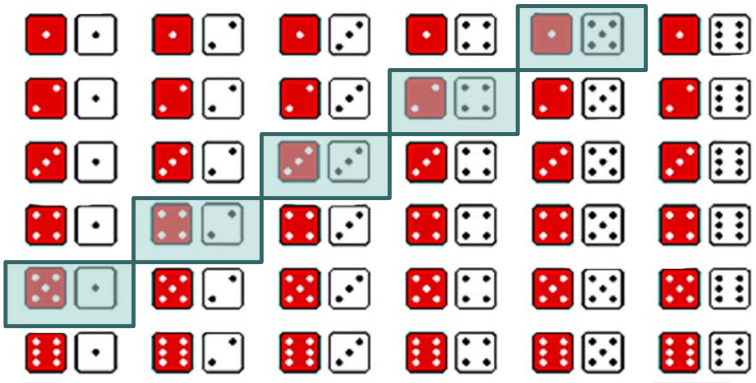


Events

- Event: any subset of outcomes contained in the sample space, S
- What is the event that an even score when rolling a die?
  - Even= {2, 4, 6}

Events Example

- What is the event that the sum of the scores of two dice is equal to 6?

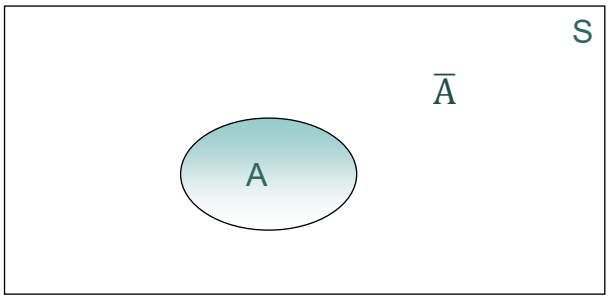


Combination of Events

- Union: event consisting of all outcomes of both events, e.g. $A \cup B$
- Intersection: event of outcomes that are part of both events, e.g. $A \cap B$
- Complement: event of all outcomes not part of the event, e.g. A', A^c or $\overline{A}$
- Contained: one event is a subset of another event, e.g. $A \subset B$
- Mutually exclusive or disjoint: events have no common outcomes, e.g. $A \not\subset C, C \not\subset A$

Venn Diagrams: Complements

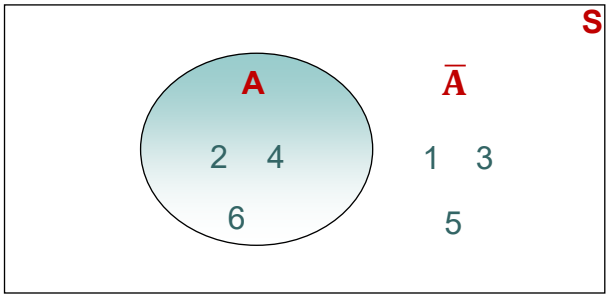
- Complementary event, $\overline{A}$: Not A



Sample space S

Example: Complements

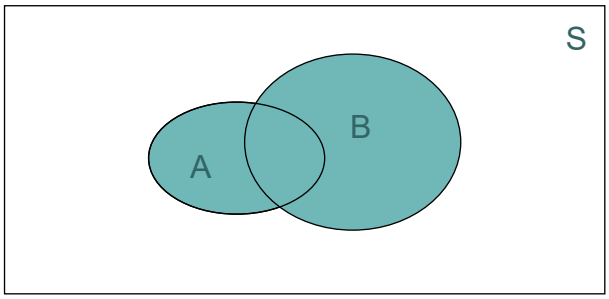
- Throw one die
  - A = die is even
  - $\bar{A}$?



Sample space S

Venn Diagrams: Union

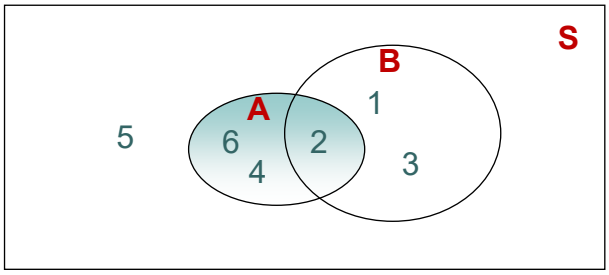
- The union of two events, A ∪ B
  - includes all sample points from A and B
  - Union means “OR”



Example: Union

- Suppose we throw one die

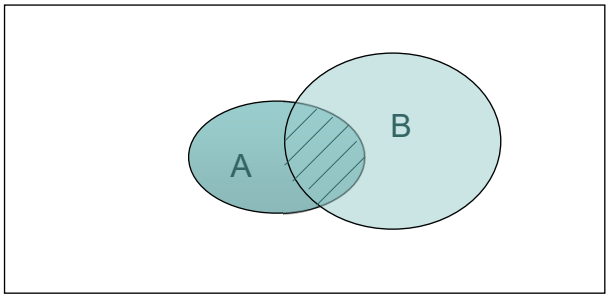
  - A = die is even
  - B = die is less than 4
  - A∪B?



  - A∪B ={1, 2, 3, 4, 6}

Venn Diagrams: Intersection

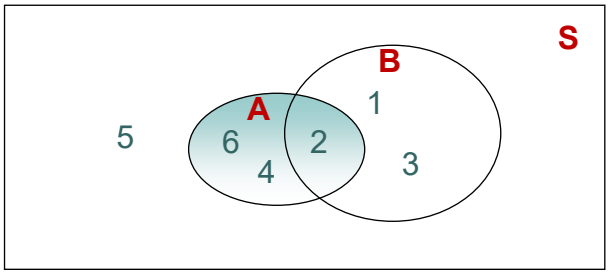
- The intersection of two events, A ∩ B
  - includes all sample points that are in both A and B
  - Intersection means “AND”



Example: Intersection

- Suppose we throw one die

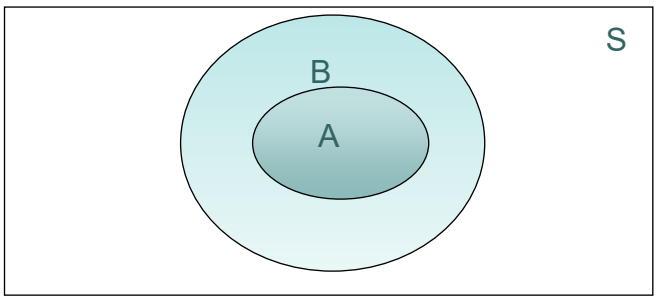
  - A = die is even
  - B = die is less than 4
  - $A \cap B$?



  - $A \cap B = \{2\}$

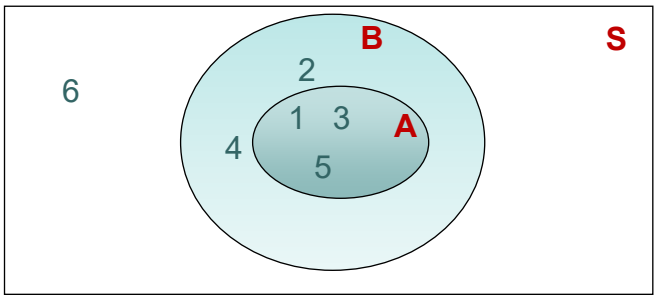
Venn Diagrams: Contained

- A is a subset of B, $A \subset B$



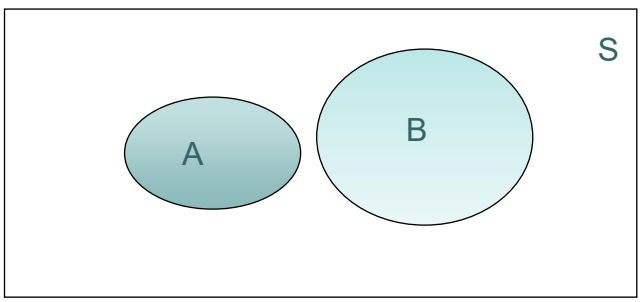
Venn Diagram Example: Contained

- Throw one die
  - A = die is odd
  - B = die is less than 6
  - Is A⊂B?



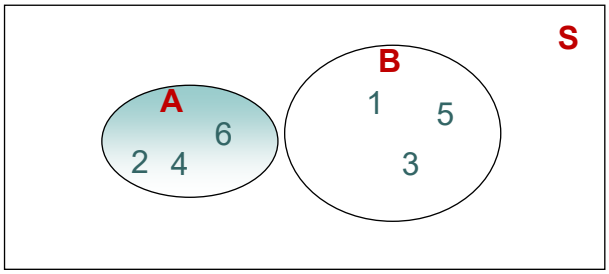
Venn Diagrams: Mutually Exclusive Events

- A and B are mutually exclusive if there is no overlap: $A \cap B = \varnothing$



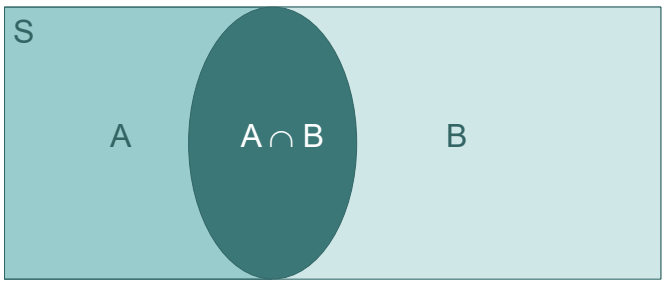
Example: Mutually Exclusive

- Suppose we throw one die
  - A = die is even
  - B = die is odd
  - Are A and B mutually exclusive?



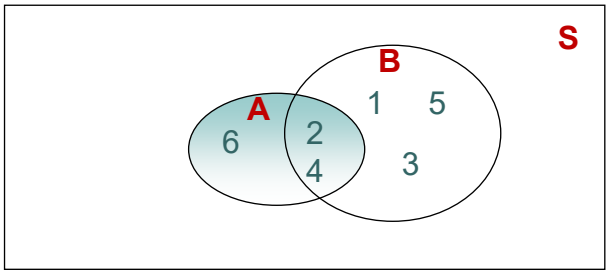
Venn Diagrams: Collectively exhaustive

- A and B are collectively exhaustive if their union must cover all the events within the entire sample space: A∪B = S



Example: Collectively exhaustive

- Suppose we throw one die
  - A = die is even
  - B = die is not 6
  - Are A and B collectively exhaustive?



Venn Diagrams: Mutually Exclusive & Collectively Exhaustive

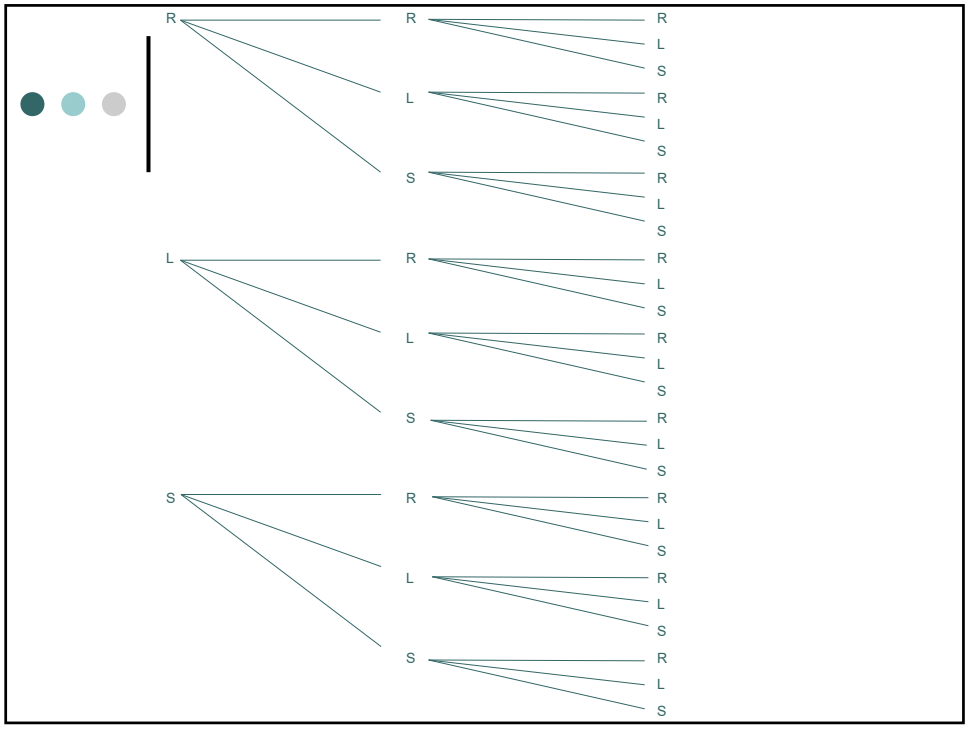
- A and B, are mutually exclusive and collectively exhaustive

Set Theory

- Associative Rule
  - $(A \cup B) \cup C = A \cup (B \cup C)$
  - $(A \cap B) \cap C = A \cap (B \cap C)$
- Distributive Rule
  - $(A \cup C) \cap (B \cup C) = (A \cap B) \cup C$
  - $(A \cap C) \cup (B \cap C) = (A \cup B) \cap C$
- De Morgan's Rule
  - $\overline{A \cup B} = \overline{A} \cap \overline{B}$

Exercises 2.2

- Suppose that vehicles taking a particular freeway exit can turn right (R), turn left (L), or go straight (S). Consider observing the direction for each of three successive vehicles.
  - List all outcomes in the event A that all three vehicles go in the same direction
  - List all outcomes in the event B that all three vehicles take different directions
  - List all outcomes in the event C that exactly two of the three vehicles turn right
  - List all outcomes in the event D that exactly two vehicles go in the same direction
  - List outcomes in D′, $C \cup D$, and $C \cap D$



Solution

- Event A = { RRR, LLL, SSS }
- Event B = { RLS, RSL, LRS, LSR, SRL, SLR }
- Event C = { RRL, RRS, RLR, RSR, LRR, SRR }
- Event D = { RRL, RRS, RLR, RSR, LRR, SRR, LLR, LLS, LRL, LSL, RLL, SLL, SSR, SSL, SRS, SLS, RSS, LSS }
- Event D′ contains outcomes where all cars go the same direction, or they all go different directions:
  - D′ = { RRR, LLL, SSS, RLS, RSL, LRS, LSR, SRL, SLR }
- Because Event C $\subset$ Event D, the compound event C$\cup$D = D
  - C$\cup$D = D = { RRL, RRS, RLR, RSR, LRR, SRR, LLR, LLS, LRL, LSL, RLL, SLL, SSR, SSL, SRS, SLS, RSS, LSS }
- Because Event C $\subset$ Event D, the compound event C$\cap$D = C
  - C$\cap$D = C = { RRL, RRS, RLR, RSR, LRR, SRR }