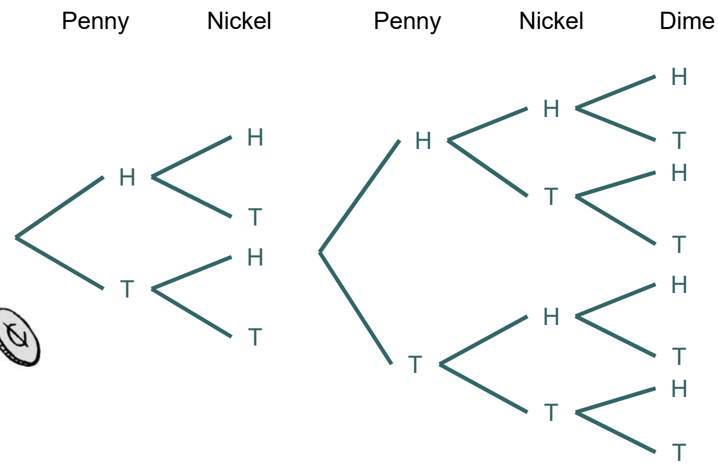
Counting

The statistics student tossed a coin throughout the Boston Marathon to study the behavior of a chance process in the long run.

Today's Class

- Tree Diagram
- Permutations
- Combinations

Tree Diagram: Coin Toss



Tree Diagram

- Pictorial representation of all possibilities
- Two options
  - First part of branch is first option
  - Second branching represents section options

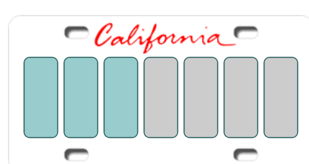
Example: Tree Diagram

- Draw a tree diagram for drawing two balls out of three colors (red, white and green).
  - What is the probability of each outcome?
  - What is the probability of getting two balls of the same color?

http://math.youngzones.org/tree.html

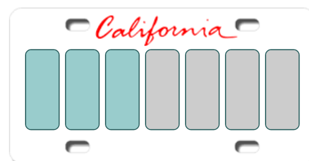
Ordered Sampling with Replacement Example

- How many different 7-place license plates are possible if the first 3 places are to be occupied by letters and the final 4 by numbers?



Ordered Sampling without Replacement Example

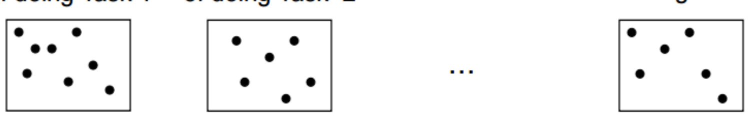
- How many license plates would be possible if repetition among letters or numbers were prohibited?



Product Rule for k-Tuples

- A k-tuples is ordered collection of k elements. e.g. (3,1,4,2)≠(1,4,2,3)

n_1	n_2		n_k
distinct ways of doing Task 1	**distinct** objects of doing Task 2		**distinct** objects of doing Task k



- How many different k-tuples can be made this way?

$$n_1 \times n_2 \times \cdots \times n_k$$

Ordered Sampling without Replacement Example

- If you want to select **three** of the billiard balls to form a lineup of speakers. In how many ways can you choose the billiard balls?

Order matters

1 2 3
1 3 2
2 1 3
2 3 1
3 1 2
3 2 1
…

Permutation

- Chosen without replacement
- Order matters
- The number of permutations of size k from n objects is denoted by $P_{k,n}$:

$$P_{k,n} = n \times (n-1) \times (n-2) \times \cdots \times (n-k+1)$$
$$= \frac{n!}{(n-k)!}$$

Factorial

- The **factorial function** (**!**) means to multiply a series of descending natural numbers
- Examples
  - 4! = 4 × 3 × 2 × 1 = 24
  - 7! = 7 × 6 × 5 × 4 × 3 × 2 × 1 = 5040
  - 1! = 1

Unordered Sampling without Replacement Example

- If you wanted to select **three** of the billiard balls and you just want to know which 3 pool balls were chosen, not the order. In how many ways can you choose the billiard balls?

Order does matter	Order doesn't matter
1 2 3 1 3 2 2 1 3 2 3 1 3 1 2 3 2 1	1 2 3

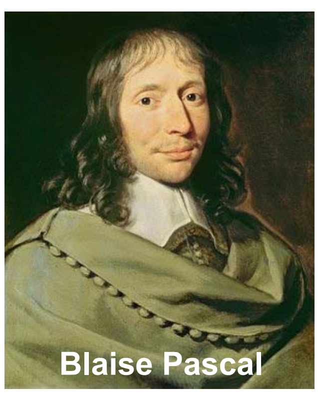
Combination

- Order does not matter
- The number of combinations of size k from n distinct objects will be denoted by $C_{k,n}$:

$$C_{k,n} = \binom{n}{k} = \frac{P_{k,n}}{k!} = \frac{n!}{k!(n-k)!}$$

$$P_{k,n} = k! \times C_{k,n}$$

Interrupted Game, Revisit



7:8

Pascal's Triangle

$$1\binom{0}{0}$$

$$1\binom{1}{0} \quad 1\binom{1}{1}$$

$$1\binom{2}{0} \quad 2\binom{2}{1} \quad 1\binom{2}{2}$$

$$1\binom{3}{0} \quad 3\binom{3}{1} \quad 3\binom{3}{2} \quad 1\binom{3}{3}$$

$$1\binom{4}{0} \quad 4\binom{4}{1} \quad 6\binom{4}{2} \quad 4\binom{4}{3} \quad 1\binom{4}{4}$$

Example: Probability

- UCLA SEAS has received a shipment of 25 printers, of which 10 are B&W laser printers and 15 are color laser printers. If 6 of these 25 are selected at random to be checked by a technician. What is the probability that exactly 3 of those selected are B&W laser printers?