CEE110
Homework #3 Solution

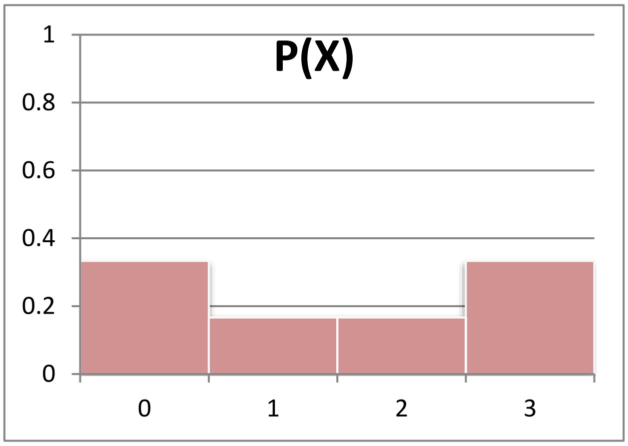
Problem 1.

a. S={0,1,2,3}

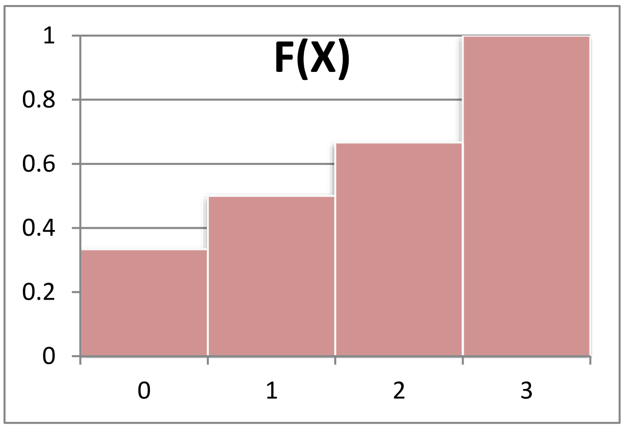
b. p(1) = p(2)
p(0) = p(3)
p(1) + p(2) =½*(p(0) + p(3))
A. = ½* (2*p(0)) = p(0)
p(1) = p(2) = ½ p(0)
Then, we have the following equation:

$$\sum_{k=0}^{3} p(x) = 1$$

p(0)+ ½p(0) +½p(0) + p(0) = 1
Therefore,
p(0) = p(3) = 1/3
p(1) = p(2) = 1/6



c. f(0)=1/3,
f(1)=1/3+1/6=1/2,
f(2)=1/3+1/6+1/6=2/3,
f(3)= 1/3+1/6+1/6+1/3=1



d. E(X) = 0×(1/3) + 1× (1/6) +
2× (1/6) + 3×(1/3)

=1.5

Problem 2.

a. $E(X) = 0\times0.6561 + 1\times0.2916 + 2\times0.0486 + 3\times0.0036 + 4\times0.0001 = 0.4$

b. $E(X^2) = 0^2\times0.6561 + 1^2\times0.2916 + 2^2\times0.0486 + 3^2\times0.0036 + 4^2\times0.001 = 0.52$

c. $V(X) = E(X^2) - (E(X))^2 = 0.52 - 0.4^2 = 0.36$

Problem 3.

a. $E[X] = 1 \times \frac{35}{100} + 2 \times \frac{18}{100} + \cdots + 6 \times \frac{10}{100} = 2.71$

b.

Passengers/car (X)	1	2	3	4	5	6	Total
X^2	1	4	9	16	25	36	
p(x)	0.35	0.18	0.12	0.21	0.04	0.1	1

$\sigma^2 = E[X^2] - \mu^2$
$E[X^2]$= $= 1 \times 0.35 + 4 \times 0.18 + 9 \times 0.12 + \cdots + 36 * 0.1 = 10.11$
Therefore variance=10.11-2.71^2=2.7659

c. h(X)= \$50+\$5(X-1)=\$45 +\$5X

Passengers/car (X)	1	2	3	4	5	6	Total
Cars	35	18	12	21	4	10	100
p(x)	0.35	0.18	0.12	0.21	0.04	0.1	1
x * p(x)	0.35	0.36	0.36	0.84	0.2	0.6	2.71
Cost/car, h(X)	50	55	60	65	70	75	375
h(x) * p(x)	17.5	9.9	7.2	13.65	2.8	7.5	58.55

Therefore, E[h(X)]=50*0.35+55*0.18+60*0.12+65*0.21+70*0.04+75*0.1=58.55

Problem 4.

a. $P(X = 3) = \frac{12!}{3!(12-3)!}(0.1)^3(1-0.1)^{12-3} = 0.0852$

b. $P(X < 3) = P(X = 0) + P(X = 1) + P(X = 2)$

$= \frac{12!}{0!\,(12-0)!}(0.1)^0(1-0.1)^{12-0} + \frac{12!}{1!\,(12-1)!}(0.1)^1(1-0.1)^{12-1} + \frac{12!}{2!\,(12-2)!}(0.1)^2(1-0.1)^{12-2}$

$= 0.2824 + 0.3766 + 0.2301 = 0.8891$

c. $P(X = 0) = \frac{12!}{0!(12-0)!}(0.1)^0(1-0.1)^{12-0} = 0.2824$

Problem 5.

a. X: number of days for compliance
   P=1-0.1
   X ~ Bin (10,0.9)
   From the cumulative Binomial Table, P(X≤8) = 0.264 (or 0.26)

b. P(X≥8) = P(X=8)+ P(X=9)+ P(X=10)

$$= \binom{10}{8} \cdot 0.9^8(0.1)^2 + \binom{10}{9} \cdot 0.9^9(0.1)^1 + \binom{10}{10} \cdot 0.9^{10}(0.1)^0$$

$$= \frac{10!}{8!\,(2)!}0.9^8(0.1)^2 + \frac{10!}{9!\,(1)!}0.9^9(0.1)^1 + \frac{10!}{10!\,(0)!}0.9^{10}(0.1)^0$$

$$= 0.194 + 0.387 + 0.349 = 0.93$$

   Alternatively, P(X≥8) = 1 – P(X<8) =1 – P(X≤7)
   From the cumulative Binomial Table, P(X≤7) = 0.07
   Therefore, 1 - 0.07 = 0.93

c. Either p(y=2)~Bin(10,0,1) for 2 days with violation or p(x=8)~Bin(10,0.9) for 8 days with compliance

$$= \binom{10}{2} \cdot 0.1^2(1-0.1)^8 = \binom{10}{8} \cdot 0.9^8(1-0.9)^2 = 0.194(0.19)$$

   Alternatively, P(X=8) = P(X≤8) – P(X≤7) = 0.264-0.07=0.194 (or 0.19)

d. E(X) = np = 10×0.9 = 9
   V(X)= np(1-p) = 10×0.9×(1-0.9) = 0.9
   STD = $\sqrt{0.9}$ = 0.95

Problem 6.

a. Possible values of X are 5, 6, 7, 8, 9, 10. (In order to have less than 5 of the granite, there would have to be more than 10 of the basaltic). X is hypergeometric, with n = 15, N = 20, and M = 10. So, the pmf of X is

$$p(x) = h(s; 15{,}10{,}20) = \frac{\binom{10}{x}\binom{10}{15-x}}{\binom{20}{15}}$$

The pmf is also provided in table form below.

x	5	6	7	8	9	10
$p(x)$	.0163	.1354	.3483	.3483	.1354	.0163

b. P(all 10 of one kind or the other) = $P(X = 5) + P(X = 10) = .0163 + .0163 = .0326$.

c. $\mu = n\frac{M}{N} = 15\frac{10}{20} = 7.5$

$$V(X) = \frac{20-15}{20-1} \times 15 \times \frac{10}{20} \times \left(1 - \frac{10}{20}\right) = 0.9868$$

$$\sigma = \sqrt{0.9868} = 0.9934$$

$\mu \pm \sigma = 7.5 \pm .9934 = (6.5066, 8.4934)$

so we want $P(6.5066 < X < 8.4934)$.

That equals P(X = 7) + P(X = 8) = .3483 + .3483 = .6966.