

### Properties of Expected Value

$$E(X_1+X_2) = E(X_1) + E(X_2)$$

Proof:

$$E(X_1+X_2) = \sum [(x_{1i}+x_{2i}) p(x_i)] = \sum [x_{1i}p(x_i)] + \sum [x_{2i}p(x_i)] = E(X_1) + E(X_2)$$

$$E(aX+b) = aE(X) + b$$

Proof:

$$E(aX+b) = \sum [(ax_i+b) p(x_i)] = a\sum [x_i p(x_i)] + b\sum [p(x_i)] = aE(X) + b$$

### Variances

$$V(X) = E(X^2) - (E(X))^2$$

Proof:

$$\begin{aligned} V(X) &= E((X-\mu)^2) \\ &= E(X^2 - 2\mu X + \mu^2) \\ &= E(X^2) - 2\mu E(X) + \mu^2 \\ &= E(X^2) - \mu^2 \quad \leftarrow \mu = E(X) \\ &= E(X^2) - (E(X))^2 \end{aligned}$$

### Properties of Variance

$$V(aX+b) = a^2 V(X)$$

Proof:

$$\begin{aligned} V(aX+b) &= E((aX+b)^2) - (E(aX+b))^2 \\ &= E(a^2 X^2 + 2abX + b^2) - (aE(X) + b)^2 \\ &= E(a^2 X^2 + 2abX + b^2) - (a^2 (E(X))^2 + 2abE(X) + b^2) \\ &= a^2 E(X^2) + 2abE(X) + b^2 - a^2 (E(X))^2 - 2abE(X) - b^2 \\ &= a^2 E(X^2) - a^2 (E(X))^2 \\ &= a^2 (E(X^2) - (E(X))^2) \\ &= a^2 V(X) \end{aligned}$$