Exponential Distribution

Expected Value Proof

$$E[X] = \int_0^{\infty} x f_x(x) dx \quad (Integrating \;\; by \;\; parts)$$

$$= \int_0^{\infty} x\lambda exp(-\lambda x) dx \quad = [-x exp(-\lambda x)]_0^{\infty} + \int_0^{\infty} \exp(-\lambda x)\, dx$$

$$= (0-0) + [-\frac{1}{\lambda}\exp(-\lambda x)]_0^{\infty} = 0 + \left(0 + \frac{1}{\lambda}\right) = \frac{1}{\lambda}$$

Variance Proof

$$E[X] = \int_0^{\infty} x^2 \lambda \exp(-\lambda x)\, dx$$

$$= [-x^2 \exp(-\lambda x)]_0^{\infty} + \int_0^{\infty} 2x \exp(-\lambda x)\, dx \quad (integrating \;\; by \;\; parts)$$

$$= (0-0) + [-\tfrac{2}{\lambda} x exp(-\lambda x)]_0^{\infty} + \tfrac{2}{\lambda}\int_0^{\infty} \exp(-\lambda x)\, dx$$

$(integrating \;\; by \;\; parts)$

$$= (0-0) + \tfrac{2}{\lambda}[-\tfrac{1}{\lambda} exp(-\lambda x)]_0^{\infty} = \tfrac{2}{\lambda^2}$$

$$E[X]^2 = (\frac{1}{\lambda})^2 = \frac{1}{\lambda^2}$$

$$Var[X] = E[X^2] - E[X]^2 = \frac{2}{\lambda^2} - \frac{1}{\lambda^2} = \frac{1}{\lambda^2}$$