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CEE 110 HW2

1. 5 chips selected randomly out of 140, 10 bad chips

a) different samples → combination

$$\binom{140}{5} = \frac{140!}{135!5!} = \boxed{416,465,528 \text{ samples}}$$

b) exactly one bad chip, 4 good chips

$$\binom{10}{1} \cdot \binom{130}{4} = \frac{10!}{9!1!} \cdot \frac{130!}{126!4!} = \boxed{113,588,800 \text{ samples}}$$

c) at least one nonconforming chip

$$\binom{10}{1}\binom{130}{4} + \binom{10}{2}\binom{130}{3} + \binom{10}{3}\binom{130}{2} + \binom{10}{4}\binom{130}{1} + \binom{10}{5}\binom{130}{0}$$

$$= \boxed{130,721,752}$$

2. 5 - 1TB 6 - 2TB 4 - 3TB 3 selected

a) 2 3TB SSD

$$\frac{\overset{3TB \rightarrow}{\binom{4}{2}}\binom{11}{1}^{\leftarrow \text{other}}}{\binom{15}{3}^{\leftarrow \text{total}}} \quad \frac{6 \cdot 11}{455} = \frac{66}{455} = 0.14$$

$$\boxed{0.14 \text{ probability}}$$

b) All three same storage

$$\frac{\binom{4}{3}}{\binom{15}{3}} + \frac{\binom{6}{3}}{\binom{15}{3}} + \frac{\binom{5}{3}}{\binom{15}{3}} \rightarrow \frac{4}{455} + \frac{20}{455} + \frac{10}{455} = \frac{34}{455}$$

$$\boxed{0.75 \text{ probability}}$$

c) One of each

$$\frac{\binom{4}{1}\binom{5}{1}\binom{6}{1}}{\binom{15}{3}} \quad \frac{4 \cdot 5 \cdot 6}{455} = 0.2637$$

$$\boxed{0.264 \text{ probability}}$$

3. 24 plants 6 exceeds 4 selected

a) one exceeds standards

$$\frac{\binom{6}{1}\cdot\binom{18}{3}}{\binom{24}{4}} = \frac{6 \cdot 816}{10626} = 0.4608$$

$\boxed{0.461 \text{ probability}}$

b) At least one plant exceeds standards

1 - (Probability of none exceeding)

$$1 - \frac{\binom{18}{4}}{\binom{24}{4}} \rightarrow 1 - \frac{3060}{10626}$$

$\boxed{0.71 \text{ probability}}$

c) 6 exceed standards 4 high turbidity

1 exceeds , 1 high turbidity

$$\frac{\binom{6}{1}\binom{4}{1}\binom{14}{2}}{\binom{24}{4}} = \frac{6 \cdot 4 \cdot 91}{10626} = 0.206$$

$\boxed{0.206 \text{ probability}}$

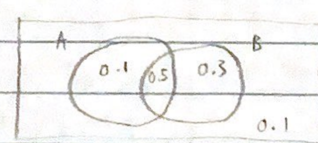
4. A : 60% B : 80% At least one of the two works 90% of time

a) $P(A \cap B) = P(A) + P(B) - P(A \cup B)$

$P(A \cap B) = 0.6 + 0.8 - 0.9$

$P(A \cap B) = 0.5$ $\boxed{0.5 \text{ probability}}$

b) $P(B \mid A') = \frac{P(B \cap A')}{P(A')} = \frac{0.3}{0.4}$



$P(B \mid A') = 0.75$

$\boxed{0.75 \text{ probability}}$

c) Independent if $P(A \mid B) = P(A)$

$$P(A \mid B) = \frac{P(A \cap B)}{P(B)} = \frac{0.5}{0.8} = 0.625$$

$0.625 \neq 0.6$

$\boxed{\text{Not independent}}$

d) V_A and V_B will be mutually exclusive only in the case that $A \cap B = \emptyset$. However, we found that in part A that this was not the case because $A \cap B = 0.5$. Therefore they are not mutually exclusive.

e) V_A and V_B are collectively exhaustive only in the case that their union covers the entire sample space $S = A \cup B$. The problem stated that $P(A \cup B) = 0.9$. This means that there is 0.1 of the sample space that is not encapsulated by $A \cup B$. It is important to note that $A \cup B$ only covers the case that V_A and V_B are successful, or at least one of them is. It doesn't factor in if the vaccine fails to produce antibodies. They are not collectively exhaustive.

5. Boelter Fire : 0.01 False alarm : 0.1 Only one fire / year

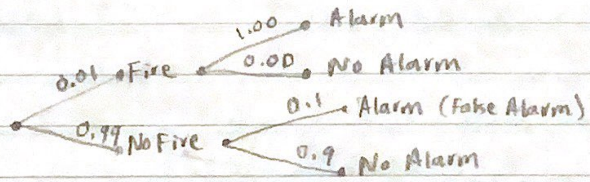
A) F = Fire F' = No Fire A = Alarm A' = No Alarm

mutually exclusive : F + A' → If there is a fire, there must be alarm

collectively exhaustive : F + A , F' + A' , F' + A

- these cover the entire sample space

B)



Probabilities

Fire + Alarm : $0.01 \cdot 1.00 = 0.01$ Fire + No Alarm : $0.01 \cdot 0.00 = 0.00$

No Fire + Alarm : $0.99 \cdot 0.1 = 0.099$ No Fire + No Alarm : $0.99 \cdot 0.9 = 0.891$

C) Add up all probabilities where alarm does go off

(Fire + Alarm) + (No Fire + Alarm) → $0.01 + 0.099 = 0.109$

$\boxed{\text{0.109 probability of alarm going off}}$

d) $P(F|A)$ → Probability of fire, given alarm goes off for sure

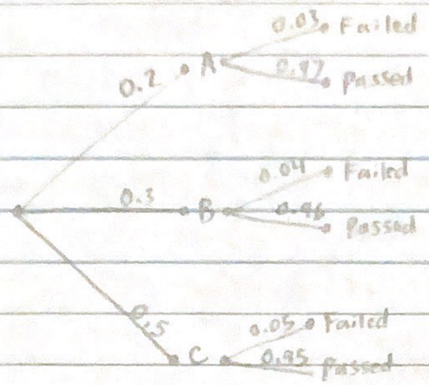
$$P(F|A) = \frac{P(F \cap A)}{P(A)} = \frac{(0.01)(1.00)}{(1.00)} = 0.01$$

$\boxed{\text{0.01 probability of an actual fire given the alarm went off}}$

6. A: 20%   B: 30%   C: 50%

Fail: 3%   Fail: 4%   Fail: 5%

a)



$0.2 \times 0.03 = 0.006$

$0.2 \times 0.97 = 0.194$

$0.3 \times 0.04 = 0.012$

$0.3 \times 0.96 = 0.288$

$0.5 \times 0.05 = 0.025$

$0.5 \times 0.95 = 0.475$

b) Probability of failure for each valve:

A: $0.2 \times 0.03 = 0.006$ B: $0.3 \times 0.04 = 0.012$

C: $0.5 \times 0.05 = 0.025$

$0.006 + 0.012 + 0.025 =$ $\boxed{\text{0.043 probability that a valve fails}}$

c) Add probability of failure for valve B and valve C (from part b)

$0.012 + 0.025 = 0.037$

Divide this number by total probability that a valve fails

$0.037 \div 0.043 = 0.86$

$\boxed{\text{0.86 probability that a failed valve has quality score B or C}}$

d) Probability of success for each valve:

A: $0.2 \times 0.97 = 0.194$ B: $0.3 \times 0.96 = 0.288$

C: $0.5 \times 0.95 = 0.475$

$0.194 + 0.288 + 0.475 = 0.957$ probability that a valve succeeds

Probability of successful A valve $\div$ total success probability

$0.194 \div 0.957 = 0.20271$

Round to nearest hundredth: 0.2

$\boxed{\text{0.2 probability of a successful valve to be quality score A}}$