Gamma Distribution

If X has a gamma distribution with parameters α and β, then the mean of X is

$E(X) = \int_0^\infty x \frac{x^{\alpha-1}e^{-x/\beta}}{\beta^\alpha \Gamma(\alpha)} dx = \alpha\beta$

Proof. $\int_0^\infty x \frac{x^{\alpha-1}e^{-x/\beta}}{\beta^\alpha \Gamma(\alpha)} dx = \int_0^\infty \frac{x^{\alpha}e^{-x/\beta}}{\beta^\alpha \Gamma(\alpha)} dx$

$= \alpha\beta \int_0^\infty \frac{x^{\alpha}e^{-x/\beta}}{\beta^{\alpha+1} \Gamma(\alpha+1)} dx \qquad \leftarrow \Gamma(\alpha+1) = \alpha\Gamma(\alpha)$

$= \alpha\beta \leftarrow \int_0^\infty \frac{x^{\alpha-1}e^{-x/\beta}}{\beta^\alpha \Gamma(\alpha)} dx = \int_0^\infty f(x, \alpha+1, \beta)\, dx = 1$

If X has a gamma distribution with parameters α and β, then the variance of X is

$V(X) = \alpha\beta^2$

Proof. $E(X^2) = \int_0^\infty x^2 \frac{x^{\alpha-1}e^{-x/\beta}}{\beta^\alpha \Gamma(\alpha)} dx = \int_0^\infty \frac{x^{\alpha+1}e^{-x/\beta}}{\beta^\alpha \Gamma(\alpha)} dx$

$= \alpha(\alpha+1)\beta^2 \int_0^\infty \frac{x^{\alpha+1}e^{-x/\beta}}{\beta^{\alpha+2} \Gamma(\alpha+2)} dx \qquad \leftarrow \Gamma(\alpha+2) = \alpha(\alpha+1)\Gamma(\alpha)$

$= \alpha(\alpha+1)\beta^2 \int_0^\infty f(x, \alpha+2, \beta)\, dx$

$= \alpha(\alpha+1)\beta^2$

$V(X) = E(X^2) - \mu_X^2 = \alpha(\alpha+1)\beta^2 - (\alpha\beta)^2 = \alpha\beta^2$