

Hypothesis Testing

Null Hypothesis



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His Teeth

TM

Today's Class

- Null vs. alternative hypothesis
- Hypothesis Testing
- Type of Errors
- *P*-Values





Bottled Water

- UCLA bottled water has the water volume of 20 Fl Oz on the label. We assumed it is true.
- But is it?
- Assumption
 - Quantity of Water = 20 Oz



Null vs. Alternative Hypotheses

- We always test two contradictory hypotheses:
 - Null hypothesis (H_0) is the belief that is initially assumed to be true (prior belief)
 - Alternative hypothesis (H_a) is the assertion that is contradictory to H_0



Hypothesis Testing

- The claim is the alternative hypothesis, H_a
- The counterclaim is stated as the null hypothesis, H_0
 - Supposed to be true unless proven otherwise
- The hypotheses test assesses how probable the observable differences are assuming H_0



Null vs. Alternative Hypotheses Example







- $H_0 \mu = 20 \text{ Oz}$
- $H_a \mu \neq 20 \text{ Oz}$
- If the data indicates the bottles are being filled properly, then we fail to reject the null; fail to reject our assumption
- If the data indicates the bottles are not being filled properly, then we reject the null; reject our assumption

Errors in Hypothesis Testing



- $H_0: \mu = 20 \text{ Oz}$
- $H_a: \mu \neq 20 \text{ Oz}$

	Actual Condition	
	$\mu = 20 \text{ Oz}$	$\mu \neq 20 \text{ Oz}$
Do not reject H_0	Correct 	Type II error 
Reject H_0	Type I error 	Correct 

Type I and II Errors Example

- In diagnostic testing for corona virus,
 - H_0 : the tested person is corona virus-free
 - H_a : the person is infected
- Type I error is that the test gives a false positive result
- Type II error is that the test gives a false negative result



Type I and II Errors Example

- In the prosecution of an accused person,
 - H_0 : the person is innocent
 - H_a : the person is guilty
- Which of the following is Type I error?
 - A: the error of convicting an innocent person
 - B: the error of not convicting a guilty person

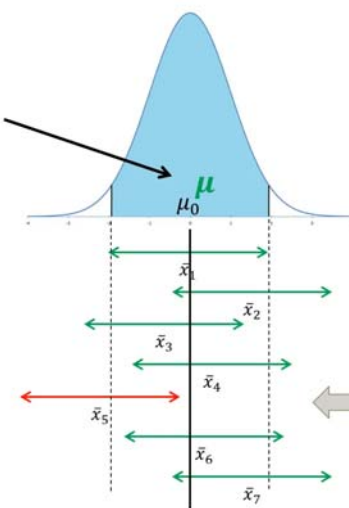


Visualizing Type I Error

$$\alpha = .05$$

95% of all sample means (\bar{x}) are hypothesized to be in this region.

Fail to reject null hypothesis
 Fail to reject null hypothesis
 Fail to reject null hypothesis
 Fail to reject null hypothesis
Reject null hypothesis
 Fail to reject null hypothesis
 Fail to reject null hypothesis



$$H_0: \mu = \mu_0$$

$$H_a: \mu \neq \mu_0$$

If we took a sample and it was by chance like \bar{x}_5 , we would incorrectly reject the null hypothesis.

Type I Error

α is the "level of significance" or our tolerance for making a Type I error.



Visualizing Type II Error

95% of all sample means (\bar{x}) are hypothesized to be in this region.
 $\alpha = .05$

Reject null hypothesis

Reject null hypothesis

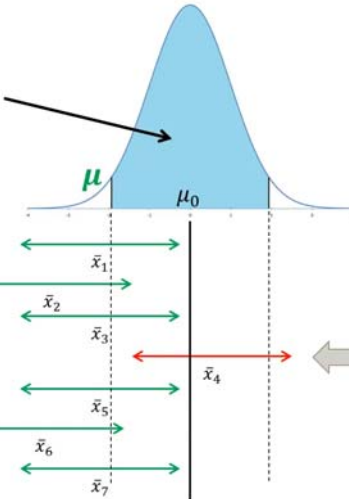
Reject null hypothesis

Fail to reject null hypothesis

Reject null hypothesis

Reject null hypothesis

Reject null hypothesis



$$H_0: \mu = \mu_0$$

$$H_a: \mu \neq \mu_0$$

If we took a sample and it was by chance like \bar{x}_4 , we would incorrectly "accept" the null hypothesis.

Type II Error

Beta (β) is the probability of committing a Type II error. The value of β varies with certain experimental factors.



Rejection Region

The Two-tailed Test Rejection Region

$$H_0: \mu = \mu_0$$

$$H_a: \mu \neq \mu_0$$

$$\alpha = .05$$

If the null hypothesis is correct, then $(\alpha \times 100)\%$ of the sample means should be in the nonrejection region.

The critical value is determined by α and if we are using the z- or t-distribution.





Rejection Region

The One-tailed (Lower) Test Rejection Region

In a one-tailed hypothesis test, all of the α is in one tail or the other depending on the alternative hypothesis.

The trick is that H_a "points" to the tail where the critical value and rejection region are.

\bar{x} from population with a mean less than μ_0

Reject H_0

\bar{x}

.05

μ_0

$$H_0: \mu \geq \mu_0$$

$$H_a: \mu < \mu_0$$

$$\alpha = .05$$

\bar{x} from a population with a mean greater than or equal to μ_0

\bar{x} *Fail to Reject H_0*



Rejection Region

The One-tailed (Upper) Test Rejection Region

In a one-tailed hypothesis test, all of the α is in one tail or the other depending on the alternative hypothesis.

The trick is that H_a "points" to the tail where the critical value and rejection region are.

\bar{x} from population with mean less than or equal to μ_0

Fail to Reject H_0

\bar{x}

μ_0

.05

$$H_0: \mu \leq \mu_0$$

$$H_a: \mu > \mu_0$$

$$\alpha = .05$$

\bar{x} from a population with mean greater than μ_0

\bar{x} *Reject H_0*



Hypothesis Testing Procedure

1. Establish hypotheses: null & alternative
2. Determine appropriate statistical test and sampling distribution
3. Choose the Type I error rate (significance level, α)
4. State the decision rule
5. Gather sample data
6. Calculate test statistics
7. State statistical conclusion: Decide whether H_0 should be rejected



Example Hypothesis Testing



- The mean water volume is expected to be 20 Oz. Determine the mean water volume differs from 20 Oz assuming that the population STD to be 2 Oz
- A sample of size 36 finds the sample mean water volume to be 19 Oz
- Is this difference statistically significant at a significance level of .01?

Hypothesis Testing: Normal with Known STD

- $H_0: \mu = \mu_0$
- Test statistic: $z = \frac{\bar{X} - \mu_0}{\sigma / \sqrt{n}}$

Alternative Hypothesis	Rejection region for level α
$H_a: \mu > \mu_0$	$z > z_\alpha$
$H_a: \mu < \mu_0$	$z < -z_\alpha$
$H_a: \mu \neq \mu_0$	$z > z_{\alpha/2}$ or $z < -z_{\alpha/2}$

Example

- The mean water volume is expected to be 20 Oz. Determine the mean water volume differs from 20 Oz
- A sample of size 36 finds the sample mean water volume to be 19 Oz and the sample STD to be 2 Oz
- Is this difference statistically significant at a significance level of .01?



Hypothesis Testing: Normal with Unknown STD

- $H_0: \mu = \mu_0$
- Our test statistic is: $t = \frac{\bar{x} - \mu_0}{s/\sqrt{n}}$

Alternative Hypothesis	Rejection region for level α test
$H_a: \mu > \mu_0$	$t \geq t_{\alpha, n-1}$
$H_a: \mu < \mu_0$	$t \leq -t_{\alpha, n-1}$
$H_a: \mu \neq \mu_0$	$t \geq t_{\alpha/2, n-1}$ or $t \leq -t_{\alpha/2, n-1}$

P-values

- The p-value is the smallest level of significance at which the H_0 would be rejected
 - $p(\text{data}|H_0) = \text{p-value}$
- If $\text{p-value} \leq \alpha$, then reject H_0 at level α
 If $\text{p-value} > \alpha$, then do not reject H_0 at level α
- The lower the p-value, the stronger your evidence in support of alternative hypothesis





Example: p-Value

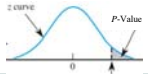
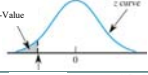
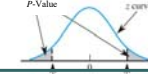


- The mean water volume is expected to be 20 Oz. Determine the mean water volume differs from 20 Oz assuming that the population STD to be 2 Oz
- A sample of size 36 finds the sample mean water volume to be 19 Oz
- What is the p-value?



P-values

- $H_0: \mu = \mu_0$
- Our test statistic is: $z = \frac{\bar{X} - \mu_0}{\sigma / \sqrt{n}}$

Alternative Hypothesis	Rejection region for level α test
$H_a: \mu > \mu_0$ 	$P = p(Z > z) = 1 - \Phi(z)$
$H_a: \mu < \mu_0$ 	$P = p(Z < z) = \Phi(z)$
$H_a: \mu \neq \mu_0$ 	$P = 2 (1 - \Phi(z))$



Example: p-Value



- The mean water volume is expected to be 20 Oz. Determine the mean water volume differs from 20 Oz
- A sample of size 36 finds the sample mean water volume to be 19 Oz and the sample STD to be 2 Oz
- What is the p-value?



P-value

- $H_0: \mu = \mu_0$
- Our test statistic is: $t = \frac{\bar{x} - \mu_0}{s/\sqrt{n}}$

Alternative Hypothesis	Rejection region for level α test
$H_a: \mu > \mu_0$	$P = p(X > t)$
$H_a: \mu < \mu_0$	$P = p(X < t)$
$H_a: \mu \neq \mu_0$	$P = 2p(X > t)$



Relationship between CI and Hypothesis Test



- The mean water volume is expected to be 20 Oz. Determine the mean water volume differs from 20 Oz
- A sample of size 36 finds the sample mean water volume to be 19 Oz and the sample STD to be 2 Oz
 - What is 99% two-sided CI?
 - Compare the result with the p-Value