CEE110
Homework #7

You must show all work for full credit and submit it through Gradescope by the deadline.

1. The following shows the distribution of *X* that is the number of mails being delivered to a randomly selected faculty member at UCLA.:

x	1	2	3	4
P(x)	.4	.3	.2.	.1

a. Consider a random sample of size $n = 2$ (two faculty members), and let $\bar{X}$ be the sample mean number of mails delivered. Obtain the probability distribution of $\bar{X}$.

b. Refer to part (a) and calculate $P(\bar{X} \leq 2.5)$.

c. If a random sample of size n=4 is selected, what *is* $P(\bar{X} \leq 1.5)$?

d. Consider a random sample of size $n = 2$, but now focus on the statistic R = the sample range (difference between the largest and smallest values in the sample). Obtain the distribution of *R.*

2. Suppose that specific stiffness for a particular glass has mean value and standard deviation of 70 GPa and 1.6 GPa, respectively.

a. For a random sample of $n = 16$, where is the sampling distribution of the sample mean specific stiffness, $\bar{X}$, centered, and what is the standard deviation of the $\bar{X}$ distribution?

b. For a random sample of $n = 64$, where is the sampling distribution of the sample mean specific stiffness, $\bar{X}$, centered, and what is the standard deviation of the $\bar{X}$ distribution?

c. For which of the two random samples, the one of part (a) or the one of part (b), is *X* more likely to be within 1 GPa of 70 GPa? Explain your reasoning.

d. Suppose the distribution of specific stiffness is normal. What is $P(69 \leq \bar{X} \leq 71)$ when n=16?

e. Suppose the distribution of specific stiffness is normal. How likely is it that the sample mean exceeds 71 when n = 25?

3. Assume that the iron loss of medium-sized high voltage motor is normally distributed with standard deviation of 3.0.

a. Compute a 95% confidence interval for μ when the number of samples is 25 and the sample mean is 58.3.

b. Compute a 95% confidence interval for μ when the number of samples is 100 and the sample mean is 58.3.

c. Compute a 99% confidence interval for μ when the number of samples is 100 and the sample mean is 58.3.

d. How large must the number of samples be if the width of the 99% interval for μ is to be 1.0?

4. The following data are adhesion measurements for certain material to water:

107.42 107.11 106.60 108.58 109.20

a. Calculate a two-sided 95% confidence interval for the true adhesion.

b. Referring to a, does the interval suggest that 107 is a plausible value for the true adhesion?

c. Calculate an upper confidence interval for the true adhesion with a confidence level of 95%.

d. Referring to c, does the interval suggest that 110 is a plausible value for the true adhesion?

5. Zebra mussel was originally native to the lakes of Russia. However, it has been accidentally introduced in many other areas and becomes an invasive species in North America. Zebra mussels feed by straining suspended matter from water. The following samples show measurements of the rates (mL/mg/h) at which mussels filter suspended matters in water.

12.19 16.03 18.45 20.54 22.63 25.05 28.89

a. Find the quartiles of the data and evaluate whether the distribution is normal.

b. What is the sample mean and standard deviation? Round a number to two decimal places.

c. Compare the CIs of 95% and 99% levels. Suppose the mean and standard deviation derived from b) is the same as those for the sample mean.

6. We are interested in the half-life of cesium-137 in the Pacific Ocean to determine its mean half-life is greater than 30 years with a random sample of size 15. The sample mean is 30.15 years and the sample standard deviation is 0.2 years.
   a. What are the appropriate hypotheses?

   b. What is the rejection region for 1%?

   c. Test hypothesis with the significance level of 1% and explain your conclusion.

   d. What is the p-value? Compare the conclusion with previous hypothesis testing.

   e. Find a 99% confidence bound for the half-life and compare the conclusion with previous hypothesis testing and p-value.