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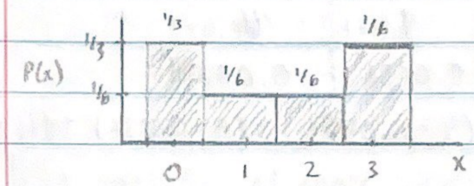
CEE 110 HW 3

1.
- At most 3 computers at a time
- $P(2) = P(1)$ • $P(0) = P(3)$
- $P(2)$ or $P(1)$ = $\frac{1}{2}[P(0) \text{ or } P(3)]$

a) $\boxed{S = \{0, 1, 2, 3\} \text{ computers}}$

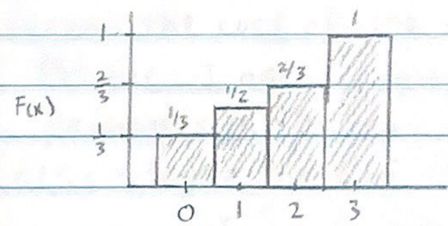
b)

X	0	1	2	3
P(x) = pmf	$\frac{1}{3}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{3}$



c)

x	0	1	2	3
F(x) = cdf	$\frac{1}{3}$	$\frac{1}{2}$	$\frac{2}{3}$	$\frac{3}{3} = 1$



d) $E(x) = (0 \times \frac{1}{3}) + (1 \times \frac{1}{6}) + (2 \times \frac{1}{6}) + (3 \times \frac{1}{3})$

$E(x) = 0 + \frac{1}{6} + \frac{1}{3} + 1$

$E(x) = 1.5$

$\boxed{\text{The expected number of computers being repaired is } 1.5}$

2.

X	0	1	2	3	4
$P(X)$	0.6561	0.2916	0.0486	0.0036	0.0001

a) $E = (0 \times 0.6561) + (1 \times 0.2916) + (2 \times 0.0486) + (3 \times 0.0036) + (4 \times 0.0001)$

$E = 0.4$

The expected number of bits in error is 0.4

b)

X^2	0	1	4	9	16
$P(X)$	0.6561	0.2916	0.0486	0.0036	0.0001

$E = (0 \times 0.6561) + (1 \times 0.2916) + (4 \times 0.0486) + (9 \times 0.0036) + (16 \times 0.0001)$

$E = 0.52$

The expected value of the square of the number of bits in error is 0.52

c) $V(x) = E(x^2) - (E[x])^2$

$E[x] \rightarrow$ from part a

$E[x^2] \rightarrow$ from part b

$V(x) = 0.52 - (0.4)^2$

$V(x) = 0.36$

The variance of the number of bits in error is 0.36

3.

Passengers / Car	1	2	3	4	5	6	Total
Cars	35	18	12	21	4	10	100

a) $E(x) = (1\times 0.35) + (2\times 0.18) + (3\times 0.12) + (4\times 0.21) + (5\times 0.04) + (6\times 0.10)$

$E(x) = 2.71$

$\boxed{\text{The expected value of their sales is } 2.71}$

b) $E(x^2) = (1\times 0.35) + (4\times 0.18) + (9\times 0.12) + (16\times 0.21) + (25\times 0.04) + (36\times 0.10)$

$E(x)^2 = 10.11 \qquad [E(x)]^2 = 7.3441$

$V(x) = E(x)^2 - [E(x)]^2 = 10.11 - 7.3441 = 2.7659$

$\boxed{\text{The variance of their sales is } 2.7659}$

c) $E(aX + b) = aE(x) + b$

$a = 5 \qquad b = 50$

Because the cost of the driver is included within the \$50 fee, I am assuming the \$5 fee is only for passengers in the car OTHER THAN the driver

$E(5x + 50) = 5 \times E(x-1) + 50$

$= 5 \times E(2.71 - 1) + 50$

$= 58.55$

$\boxed{\text{The expected revenue is } \$58.55}$

If the driver must pay \$5 on top of the \$50 fee:

$E(5x + 50) = 5 \times E(x) + 50$

$= [5 \times 2.71] + 50$

$= 63.55$

$\boxed{\text{The expected revenue is } \$63.55}$

The wording is a bit confusing so I wrote out both possibilities

4. 10% of vehicles violate   12 vehicles selected at random

a) $p = 0.1 \quad x = 3 \quad n = 12$

$b(x; n, p) \longrightarrow b(3; 12, 0.1)$

$\binom{12}{3} \times 0.1^3 \times (1-0.1)^{12-3}$

$\binom{12}{3} \times 0.1^3 \times (0.9)^9$

$220 \times 0.001 \times 0.38742 = 0.0852$

$\boxed{\text{The probability that exactly 3 violate the standard is } 0.0852}$

b) Fewer than 3 : 0 violations, 1 violation, 2 violations

0 violations : $b(0; 12, 0.1)$

$\binom{12}{0} \times 0.1^0 \times (0.9)^{12}$

$1 \times 1 \times 0.2824 = 0.2824$

1 violation : $b(1; 12, 0.1)$

$\binom{12}{1} \times 0.1^1 \times (0.9)^{11}$

$12 \times 0.1 \times 0.3138 = 0.3766$

2 violations : $b(2; 12, 0.1)$

$\binom{12}{2} \times 0.1^2 \times (0.9)^{10}$

$66 \times 0.01 \times 0.3487 = 0.2301$

P(0 violations) + P(1 violation) + P(2 violations)

$0.2824 + 0.3766 + 0.2301 = 0.8891$

$\boxed{\text{The probability that fewer than three cars violate the standard is } 0.8891}$

c) 0 violations : $b(0; 12, 0.1)$

$\binom{12}{0} \times 0.1^0 \times (0.9)^{12}$

$1 \times 1 \times 0.2824 = 0.2824$

$\boxed{\text{The probability that none of the cars violate the standard is } 0.2824}$

0.264

5. Max contaminant: 10mg/L 10 days of monitoring - P(max) = 0.1

a) P(at most 8 no violations) = 1 - P(0 violations) + P(1 violations)

0 violations → $b(0; 10, 0.1)$

$$\binom{10}{0} 0.1^0 (1-0.1)^{10-0} = 0.348$$

1 violations → $b(1; 10, 0.1)$

$$\binom{10}{1} 0.1^1 (1-0.1)^{10-1} = 0.387$$

P(at most 8 no violations) = 1 - 0.348 - 0.387

$\boxed{\text{The probability that at most 8 days of monitoring will comply is } 0.265}$

b) P(at least 8 no violations) = P(8 no violations) + P(9 no violations) + P(10 no violations)

8 no violations → $b(8; 10, 0.9)$

$$\binom{10}{8} 0.9^8 (1-0.9)^{10-8} = 0.194$$

9 no violations → $b(9; 10, 0.9)$

$$\binom{10}{9} 0.9^9 (1-0.9)^{10-9} = 0.387$$

10 no violations → $b(10; 10, 0.9)$

$$\binom{10}{10} 0.9^{10} (1-0.9)^{10-10} = 0.349$$

$0.194 + 0.387 + 0.349 = 0.93$

$\boxed{\text{The probability that at least 8 days of monitoring will comply is } 0.93}$

c) P(2 violations) → $b(2; 10, 0.1)$

$$\binom{10}{2} 0.1^2 (1-0.1)^{10-2} = 0.1937$$

$\boxed{\text{The probability that exactly 2 days will violate is } 0.1937}$

d) P(no violation) = 1 - 0.1 = 0.9

$E(x) = np = 10 \times 0.9 = 9$

$V(x) = np(1-p)$

$= 9(1-0.9) = 0.9$

$\sigma = \sqrt{V(x)} = \sqrt{0.9}$

$\boxed{\text{The expected value is } 9. \text{ The standard deviation is } \sqrt{0.9} \approx 0.949}$

6. 10 basaltic 10 granite randomly select 15

a) Sample space : {5, 6, 7, 8, 9, 10}

Hypergeometric distribution

5 granite : $P(5) = h(5; 15, 10, 20)$

$$P(5) = \frac{\binom{10}{5}\binom{20-10}{15-5}}{\binom{20}{15}} = 0.0163$$

6 granite : $P(6) = h(6; 15, 10, 20)$

$$P(6) = \frac{\binom{10}{6}\binom{20-10}{15-6}}{\binom{20}{15}} = 0.1354$$

7 granite : $P(7) = h(7; 15, 10, 20)$

$$P(7) = \frac{\binom{10}{7}\binom{20-10}{15-7}}{\binom{20}{15}} = 0.3483$$

8 granite : $P(8) = h(8; 15, 10, 20)$

$$P(8) = \frac{\binom{10}{8}\binom{20-10}{15-8}}{\binom{20}{15}} = 0.3483$$

9 granite : $P(9) = h(9; 15, 10, 20)$

$$P(9) = \frac{\binom{10}{9}\binom{20-10}{15-9}}{\binom{20}{15}} = 0.1354$$

10 granite : $P(10) = h(10; 15, 10, 20)$

$$P(10) = \frac{\binom{10}{10}\binom{20-10}{15-10}}{\binom{20}{15}} = 0.0163$$

x	5	6	7	8	9	10
P(x) = pmf	0.0163	0.1354	0.3483	0.3483	0.1354	0.0163

b) $P(X=5) + P(X=10) = 0.0163 + 0.0163 = 0.0326$

There is a 0.0326 probability that this will happen.

c) mean = expected value $E(x) = n \times \frac{M}{N}$ $E(x) = 15 \times \frac{10}{20} = 7.5$

$V(x) = \left(\frac{20-15}{20-1}\right) \times 15 \times \frac{10}{20} \times \left(1 - \frac{10}{20}\right) = 0.9869$

$\sigma = \sqrt{V(x)} = \sqrt{0.9869} = 0.9934$

$P(6.5066 \le x \le 8.4934) \rightarrow P(7) + P(8) = 0.3483 + 0.3483 = 0.6966$

There is a 0.6966 probability that the number of granite specimens selected is within 1 std. deviation of its mean value