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CEE 110 HW7

1.

X_1	X_2	$p(x_1, x_2)$	$\bar{X}$	R
1	1	0.16	1	0
1	2	0.12	1.5	1
1	3	0.08	2	2
1	4	0.04	2.5	3
2	1	0.12	1.5	1
2	2	0.09	2	0
2	3	0.06	2.5	1
2	4	0.03	3	2
3	1	0.08	2	2
3	2	0.06	2.5	1
3	3	0.04	3	0
3	4	0.02	3.5	1
4	1	0.04	2.5	3
4	2	0.03	3	2
4	3	0.02	3.5	1
4	4	0.01	4	0

x	P(x)
1	0.4
2	0.3
3	0.2
4	0.1

a) $\bar{X} = 1 \rightarrow 0.16$

$\bar{X} = 1.5 \rightarrow 0.12 + 0.12 = 0.24$

$\bar{X} = 2.0 \rightarrow 0.08 + 0.09 + 0.08 = 0.25$

$\bar{X} = 2.5 \rightarrow 0.04 + 0.06 + 0.06 + 0.04 = 0.20$

$\bar{X} = 3.0 \rightarrow 0.03 + 0.04 + 0.03 = 0.10$

$\bar{X} = 3.5 \rightarrow 0.02 + 0.02 = 0.04$

$\bar{X} = 4.0 \rightarrow 0.01$

$\bar{X}$	1.0	1.5	2.0	2.5	3.0	3.5	4.0
$P_x(\bar{x})$	0.16	0.24	0.25	0.20	0.10	0.04	0.01

b) $P(\bar{X} \leq 2.5) = P(\bar{X} = 1) + P(\bar{X} = 1.5) + P(\bar{x} = 2.0) + P(\bar{X} = 2.5)$

$P(\bar{x} \leq 2.5) = 0.16 + 0.24 + 0.25 + 0.20$

$\boxed{P(\bar{x} \leq 2.5) = 0.85}$

X	1	2	3	4
	0.4	0.3	0.2	0.1

c) $n = 4$ $\quad P(\bar{X} \le 1.5)$

X_1	X_2	X_3	X_4	$P(X_1, X_2, X_3, X_4)$	$\bar{X}$	
1	1	1	1	0.0256	1	
1	1	1	2	0.0192	1.25	1
1	1	1	3	0.0128	1.5	1
1	1	2	1	0.0192	1.25	2
1	1	3	1	0.0128	1.5	2
1	2	1	1	0.0192	1.25	3
1	3	1	1	0.0128	1.5	3
2	1	1	1	0.0192	1.25	4
3	1	1	1	0.0128	1.5	4
1	2	1	2	0.0144	1.5	1
2	2	1	1	0.0144	1.5	2
2	1	2	1	0.0144	1.5	3
2	1	1	2	0.0144	1.5	4
1	2	2	1	0.0144	1.5	5
1	1	2	2	0.0144	1.5	6

These are all possible ways that $\bar{X} \le 1.5$

Add all these probabilities to get $P(\bar{X} \le 1.5)$

$\boxed{P(\bar{X} \le 1.5) = 0.24}$

d) $R = 0 \rightarrow 0.16 + 0.09 + 0.04 + 0.01 = 0.30$

$R = 1 \rightarrow 0.12 + 0.12 + 0.06 + 0.06 + 0.02 + 0.02 = 0.40$

$R = 2 \rightarrow 0.08 + 0.03 + 0.08 + 0.03 = 0.22$

$R = 3 \rightarrow 0.04 + 0.04 = 0.08$

R	0	1	2	3
$P_R(R)$	0.30	0.40	0.22	0.08

2. $\bar{X} = 70$ GPa   STD $= 1.6$ GPa

a) $\bar{X}$ is centered at the population mean : 70 GPa

$\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} = \frac{1.6}{\sqrt{16}} = \frac{1.6}{4} = 0.4$ GPa

$\boxed{\text{Sampling distribution of sample mean, centered} = 70\text{ GPa} \quad \text{Standard deviation of } \bar{X} \text{ distribution} : 0.4\text{ GPa}}$

b) $\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} = \frac{1.6}{\sqrt{100}} = \frac{1.6}{10} = 0.16$

$\boxed{\text{Sampling distribution of sample mean, centered} = 70\text{ GPa} \quad \text{Standard deviation of } \bar{x} \text{ distribution} : 0.16\text{ GPa}}$

c) X is more likely to be within 1 GPa of 70 GPa for the situation in part b ($n = 100$, STD $= 0.16$). This is because the standard deviation is less than that of part a. There will be less variation from the sample mean of 70 in part b, which means a higher chance for X to be within 1 GPa of 70 GPa.

d) $P(69 \le \bar{X} \le 71)$ $n = 16$ [normal distribution]

$P(69 \le \bar{X} \le 71) = P\left(\frac{69-\mu}{\sigma} \le Z \le \frac{71-\mu}{\sigma}\right)$

$= P\left(\frac{69-70}{0.4} \le Z \le \frac{71-70}{0.4}\right)$

$= P(-2.5 \le Z \le 2.5)$

$= P(Z = 2.5) - P(Z = -2.5)$

$= 0.9938 - 0.0062 = 0.9876$

$\boxed{P(69 \le \bar{x} \le 71) = 0.9876}$

e) $P(\bar{X} > 71) = 1 - P(X < 71)$ $n = 25$ $\sigma_{\bar{x}} = \frac{1.6}{\sqrt{25}} = \frac{1.6}{5} = 0.32$

$= 1 - P\left(Z < \frac{71-70}{0.32}\right)$

$= 1 - P(Z < 3.125)$

$\approx 1 - P(Z < 3.13)$

$\approx 1 - 0.9991$

≈ 0.0009

$\boxed{P(\bar{X} > 71) = 0.0009}$

3. normally distributed, STD = 3.0

a) 95% confidence interval $n = 25$ $\bar{x} = 58.3$ deg. of freedom = 24

$\alpha = 0.05$ (two-tailed) $\rightarrow$ $\frac{\alpha}{2} = 0.025$

$n < 40$, use t-distribution

$t_{0.025,\,24} = 2.064$

$\bar{x} \pm (2.064) \cdot \frac{3.0}{\sqrt{25}} \rightarrow 58.3 \pm (2.064) \cdot \frac{3}{5}$

$\boxed{(57.06,\ 59.54)}$

b) 95% confidence interval $n = 100$ $\bar{x} = 58.3$ deg. of freedom = 99

$\alpha = 0.05 \rightarrow \alpha = 0.025$ $n > 40$, use z-table

$Z_{0.025} = 1.96$

$x \pm (1.96) \cdot \frac{3.0}{\sqrt{100}} \rightarrow 58.3 \pm (1.96) \cdot \frac{3}{10}$

$\boxed{(57.7,\ 58.9)}$

c) 99% confifence interval $n = 100$ $\bar{x} = 58.3$

$\alpha = 0.01 \rightarrow \alpha = 0.005$

$Z_{0.005} = 2.58$

$x \pm 2.58 \cdot \left(\frac{3.0}{\sqrt{100}}\right) \rightarrow 58.3 \pm 2.58 \cdot \frac{3}{10}$

$\boxed{(57.5,\ 59.1)}$

d) width of the 99% interval is 1.0

Interval : (57.8, 58.8)

$57.8 = \bar{x} - Z_{0.005}\left(\frac{3.0}{\sqrt{n}}\right)$

$57.8 = 58.3 - (2.58)\left(\frac{3.0}{\sqrt{n}}\right)$

$n = 239.6$

(round up, want whole number)

$n = 240$

$\boxed{\text{240 samples would be needed for this to happen.}}$

4. 107.42 107.11 106.60 108.58 109.20

$\bar{X} = (107.42 + 107.11 + 106.60 + 108.58 + 109.20) \div 5 = 107.782$

$STD = \sqrt{\dfrac{(107.42-107.782)^2 + (107.11-107.782)^2 + \ldots + (109.20-107.782)^2}{5-1}} = 1.0756$

a) $\alpha = 0.05$ $\quad \frac{\alpha}{2} = 0.025$ $\quad$ degrees of freedom $= 4$ $\quad n=5$

$n < 40$, use t-distribution

$t_{0.025,4} = 2.776$

$\bar{X} \pm 2.776 \cdot \frac{1.0756}{\sqrt{5}} \rightarrow 107.782 \pm 2.776 \cdot \frac{1.0756}{\sqrt{5}}$

$\boxed{(106.45, 109.12)}$

b) 107 is within the interval (106.45, 109.12) so it is a plausible value for the true adhesion.

c) $\alpha = 0.05$ $\quad$ degrees of freedom $= 4$

$t_{0.05,4} = 2.132$

$\mu < \bar{X} + 2.132 \cdot \frac{1.0756}{\sqrt{5}}$

$\boxed{\mu < 108.81}$

d) No, 110 is not a plausible value for the true adhesion because it exceeds the upper confidence interval.

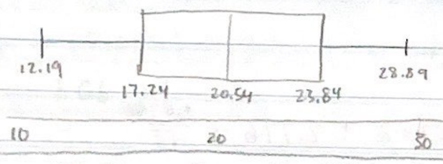
5. 12.19 16.03 18.45 20.54 22.63 25.05 28.89

Median

a) $Q_1 = \frac{16.03 + 18.45}{2} = 17.24$

$Q_2 = \text{Median} = 20.54$

$Q_3 = \frac{22.63 + 25.05}{2} = 23.84$



$\boxed{Q_1 = 17.24, \quad Q_2 = 20.54, \quad Q_3 = 20.84}$

The distribution is normal because the boxplot is pefectly symmetric. Also, $Q_2 - Q_1 = Q_3 - Q_2$ and the median = mean

b) Sample mean : $(12.19 + 16.03 + 18.45 + 20.54 + 22.63 + 25.05 + 28.89) \div 7$

$= 20.54$

$$\sigma = \sqrt{\frac{(12.19 - 20.54)^2 + (16.03 - 20.54)^2 + \dots + (28.89 - 20.54)^2}{7-1}}$$

$= 5.61$

Sample mean = 20.54 , standard deviation = 5.61

c) 95% $\alpha = 0.05$ $\frac{\alpha}{2} = 0.025$

$\bar{X} \pm t_{0.025,6} \cdot \frac{\sigma}{\sqrt{n}}$

$20.54 \pm 2.447 \cdot \frac{5.61}{\sqrt{7}}$

$(13.35, 25.73)$

99% $\alpha = 0.01$ $\frac{\alpha}{2} = 0.005$

$\bar{X} \pm t_{0.005,6} \cdot \frac{\sigma}{\sqrt{n}}$

$20.54 \pm 3.707 \cdot \frac{5.61}{\sqrt{7}}$

$(12.68, 28.40)$

95% Confidence Interval : (13.35, 25.73)
99% Confidence Interval : (12.68, 28.40)
The 99% confidence interval is wider than the 95% CI

6. $\bar{X} = 30.15$ $n = 15$ sample STD = 0.2 degree of freedom: 14

a) $\boxed{\text{Null Hypothesis } (H_0) \rightarrow \mu = 30 \text{ years}}$

$\boxed{\text{Alternative Hypothesis } (H_a) \rightarrow \mu > 30 \text{ years}}$

b) Significance level : 0.01 $\alpha = 0.01$

one-tailed test, $n < 40$: use t-distribution

$t > t_{14, 0.01} \rightarrow t > 2.624$ reject H_0

$\boxed{\text{Rejection region is where } t > 2.624}$

c) Test statistic : $t = \dfrac{\bar{X} - \mu}{\frac{s}{\sqrt{n}}} \rightarrow t = \dfrac{30.15 - 30}{\frac{0.2}{\sqrt{15}}}$

$t = 2.90473$

$2.90473 > 2.624$

Reject the null hypothesis because the value of t falls within the rejection region found in part b

d) $\alpha = 0.01$ $t = 2.90473$

$\Phi(|t|) = \Phi(2.90473) \approx \Phi(2.90) \rightarrow 0.005$

[look at t-table]

P-value = 0.005, which is less than 0.01
This means that the null hypothesis should be rejected.
This is the same conclusion we observed in part b.

e) 99% confidence interval $\rightarrow$ $\alpha = 0.01$ $\frac{\alpha}{2} = 0.005$ df: 14

Confidence interval : $\bar{X} \pm t_{\frac{\alpha}{2}, 14} \frac{s}{\sqrt{n}}$

$30.15 \pm 2.977 \frac{0.2}{\sqrt{15}}$

$(29.996, 30.304)$

30.15 falls within this range, which means we fail to reject the null hypothesis. This means there is not enough evidence to support the claim that the half-life is greater than 30. This conclusion is different than that of hypothesis testing and p-value.