

CEE110**Homework #6**

You must show all work for full credit and submit it through the Gradescope by the deadline.

1. Suppose X and Y are the number of new and used batteries chosen with the accompanying joint probability table as follows:

		y		
		0	1	2
X	0	0.125	0.075	0.05
	1	0.1	0.06	0.04
	2	0.125	0.075	0.05
	3	0.15	0.09	0.06

- What is the probability that there is exactly one new battery and exactly one used battery?
 - What is the probability that there is at most one new battery and at most one used battery?
 - What is the probability that there is exactly one new battery? Exactly one used battery?
 - Are X and Y independent? Explain.
2. In a gas station in Westwood, X and Y denote the number of hoses being used at gasoline and diesel pumps at a particular time, respectively with the joint pmf table as follows:

		y		
		0	1	2
x	0	0.09	0.08	0.03
	1	0.07	0.18	0.05
	2	0.05	0.15	0.3

- Given that $X = 1$, determine the conditional pmf of Y .
- Given that two hoses are in use at the gasoline pump, what is the conditional pmf of the number of hoses in use on the diesel pump?
- Use the result of part (b) to calculate the conditional probability $P(Y \leq 1/X=2)$.
- Given that two hoses are in use at the diesel pump, what is the conditional pmf of the number in use at the gasoline pump?

3. For a particular computer in Seasnet Lab, X and Y are the lifetime of the first and second memory chip, which follow an exponential distribution with parameter $\lambda=1$.

- Find the joint pdf of X and Y .
- Find the probability that $X \leq 1$ and $Y \leq 1$.
- Find the probability that the total lifetime of the two memory chips is at most 2.
- Find the probability that the total lifetime is between 1 and 2.

4. For a student in CEE110, X and Y are the number of points earned on the midterm and final exams, respectively, with the accompanying table as follows.

		y			
		0	50	75	100
x	0	0.01	0.05	0.03	0.01
	50	0.02	0.2	0.15	0.1
	100	0.01	0.14	0.15	0.13

- If the score recorded in the gradebook is the total number of points earned on the two exams, what is the expected recorded score $E(X + Y)$?
 - If the maximum of the two scores is recorded, what is the expected recorded score?
 - Compute the covariance for X and Y .
 - Compute correlation coefficient for X and Y .
5. Each front tire on a particular vehicle is supposed to be filled to a pressure of 26 psi. Suppose that X and Y are the actual air pressure for the right tire and the left tire, respectively, with joint pdf:

$$f(x, y) = \begin{cases} K(x^2 + y^2) & 20 \leq x \leq 30, 20 \leq y \leq 30 \\ 0 & \text{otherwise} \end{cases}$$

- Determine the (marginal) distribution of air pressure in the right tire alone.
- Are X and Y independent?
- Compute the covariance between X and Y .
- Compute the correlation coefficient for this X and Y .

6. The joint probability density function of the number of hours for two mountain bike tires to be filled to a pressure of 25 psi is shown below. X is for the front tire and Y is for the rear tire

$$f(x, y) = \begin{cases} k(x + y)^2 & 0 < x < 1, \quad 0 < y < 1 \\ 0 & \text{otherwise} \end{cases}$$

- a. What is the value of k to make the joint density function legitimate?
- b. Compute the marginal density function of the number of hours for each tire.
- c. Evaluate whether X and Y are independent and make a justification.
- d. Evaluate the correlation of X and Y