CEE110
Homework #2 Solution

Problem 1.

a. The number of samples of size five is $\binom{140}{5} = \frac{140!}{5!135!} = 416{,}965{,}528$

b. There are 10 ways of selecting one nonconforming chip and there are $\binom{130}{4} = \frac{130!}{4!126!} = 11{,}358{,}880$ways of selecting four conforming chips.
Therefore, the number of samples that contain exactly one nonconforming chip is
$10\times \binom{130}{4} = 113{,}588{,}800$

c. The number of samples that contain at least one nonconforming chip is the total number of samples $\binom{140}{5}$ - the number of samples that contain no nonconforming chips $\binom{130}{5}$.
That is
$\binom{140}{5} - \binom{130}{5} = \frac{140!}{5!135!} - \frac{130!}{5!125!} = 130{,}721{,}752$

Problem 2.

a. There are four 3 TB SSDs available and 5+6 = 11 non-3 TB SSDs available. The number of ways to select exactly two of the former (and, thus, exactly one of the latter) is
$\binom{4}{2}\binom{11}{1}$
Hence, the probability is $\frac{\binom{4}{2}\binom{11}{1}}{\binom{15}{3}} = \frac{6\times 11}{455} = 0.145$

b. The number of ways to select three 1 TB SSDs is $\binom{5}{3}$. Similarly, there are $\binom{6}{3}$ ways to select three 2 TB SSDs and $\binom{4}{3}$ ways to select three 3 TB SSDs. So, the probability is
$\frac{\binom{5}{3}+\binom{6}{3}+\binom{4}{3}}{\binom{15}{3}} = \frac{10+20+4}{455} = 0.075$

c. The number of ways to obtain one of each storage is $\binom{5}{1}\binom{6}{1}\binom{4}{1}$ and so the probability is $\frac{\binom{5}{1}\binom{6}{1}\binom{4}{1}}{\binom{15}{3}} = \frac{5\times 6\times 4}{455} = 0.264$

Problem 3.

a. The total number of samples possible is $\binom{24}{4} = \frac{24!}{4!20!} = 10{,}626.$

The number of samples in which exactly one plant exceeds the standard is $\binom{6}{1}\binom{18}{3} = \frac{6!}{1!5!} \times \frac{18!}{3!15!} = 4896$. Therefore, the probability is $\frac{\binom{6}{1}\binom{18}{3}}{\binom{24}{4}} = \frac{4896}{10626} = 0.461.$

b. The number of samples that contain no tank exceeding the standard is $\binom{18}{4}$.

Therefore, the requested probability is $1 - \frac{\binom{18}{4}}{\binom{24}{4}} = 1 - \frac{3060}{10626} = 0.712.$

c. The number of samples that meet the requirements is $\binom{6}{1}\binom{4}{1}\binom{14}{2}$

Therefore, the probability is $\frac{\binom{6}{1}\binom{4}{1}\binom{14}{2}}{\binom{24}{4}} = \frac{2184}{10626} = 0.206$

Problem 4.

a. P(A∩B) = P(A) + P(B) - P(A∪B)
= 0.6 + 0.8 - 0.9 = 0.5

b. $P(B|\bar{A}) = \frac{P(B\cap\bar{A})}{P(\bar{A})} = \frac{P(B)-P(A\cap B)}{1-P(A)} = \frac{0.8-0.5}{1-0.6} = 0.75$

c. $P(B|A) = \frac{P(B\cap A)}{P(A)} = \frac{0.5}{0.6} = 0.833 \neq P(B)$
Therefore A and B are not independent.

d. P(A∩B) ≠ 0 therefore An and B are not mutually exclusive.

e. P(A∪B) = 0.9, which does not generate the whole sample space (≠1), Therefore A and B are not collectively exhaustive.

Problem 5.

a. $F \cap A$, $F \cap \bar{A}$, $\bar{F} \cap A$, $\bar{F} \cap \bar{A}$

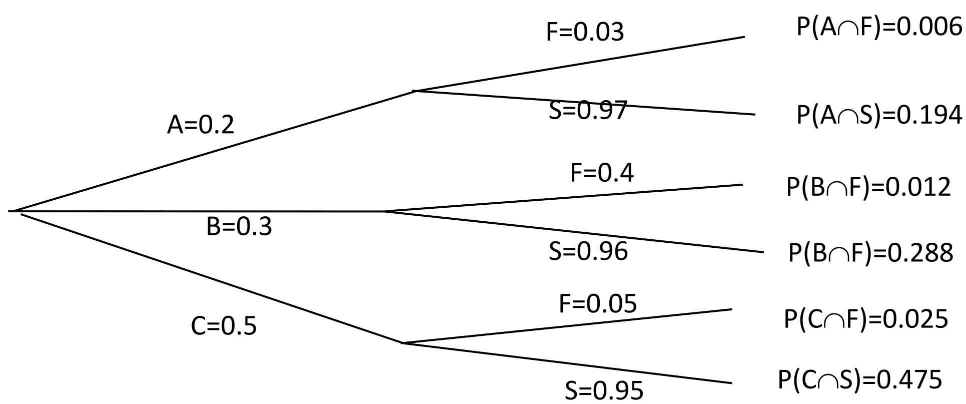
b. $P(F \cap A) = P(A|F)P(F) = 1 \times 0.01 = 0.01$
$P(F \cap \bar{A}) = P(\bar{A}|F)P(F) = 0 \times 0.01 = 0$
$P(\bar{F} \cap A) = P(A|\bar{F})P(\bar{F}) = 0.1 \times 0.99 = 0.099$
$P(\bar{F} \cap \bar{A}) = P(\bar{A}|\bar{F})P(\bar{F}) = 0.9 \times 0.99 = 0.891$

c. $P(A) = P(A|F)P(F) + P(A|\bar{F})P(\bar{F})$
$= 0.01 + 0.099 = 0.109$

d. $P(F|A) = \frac{P(A|F)P(F)}{P(A)} = \frac{0.01}{0.109} = 0.0917$

Problem 6.

a. Tree diagram



b. $P(A) = 0.2$ $P(B) = 0.3$ $P(C) = 0.5$
$P(F|A) = 0.03$ $P(F|B) = 0.04$ $P(F|C) = 0.05$
$P(F) = P(F|A) \times P(A) + P(F|B) \times P(B) + P(F|C) \times P(C)$
$= 0.03 \times 0.2 + 0.04 \times 0.3 + 0.05 \times 0.5$
$= 0.006 + 0.012 + 0.025$
$= 0.043$

c. $P(B|F) = \frac{P(F|B) \times P(B)}{P(F)} = \frac{0.04 \times 0.3}{0.043} = 0.28$

$$P(C|F) = \frac{P(F|C) \times P(C)}{P(F)} = \frac{0.05 \times 0.5}{0.043} = 0.58$$

and therefore $P(B|F) + P(C|F) = 0.28 + 0.58 = 0.86$

d. $P(A|S) = \frac{P(A) \times P(S|A)}{P(S)} = \frac{0.2 \times (1-0.03)}{(1-0.043)} = 0.2$