On my honor, I have neither received nor given any unauthorized assistance on this examination.

Agreed By Ethan Wong
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CEE 110 Midterm

1. 119 83 5 74 33 58 22 40 37 30 5

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a) Mean : $(5+5+22+30+33+37+40+58+74+83+119) \div 11 = 46$

Median : 37

The sample mean is 46 mg/L ; the sample median is 37 mg/L. They're different because the average of a data set is not necessarily the same as the middle value of the sorted data set.

b) Water quality standard : 100 mg/L

The quality of the effluent on average is 46 mg/L. This is significantly lower than the standard of 100 mg/L, so that means the quality of effluent is much better than required. Overall the data shows good-quality effluent.

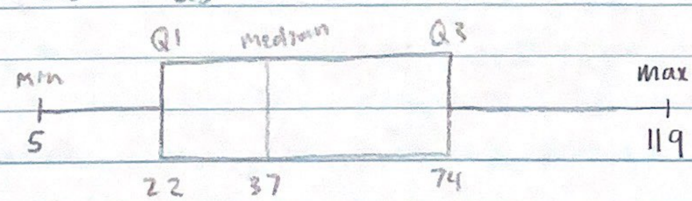
c) 5 5 22 30 33 37 40 58 74 83 119

Q1 (22) Q2 (37) Q3 (74)

The lower quartile is the median of the first half.
The upper quartile is the median of the second half.
The Fourths Spread is Q3 - Q1.

Lower Quartile (Q1) = 22 Upper Quartile (Q3) = 74
Fourths Spread = 52

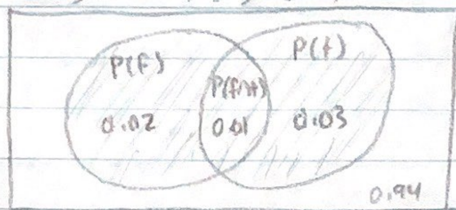
d) Min whisker : 1.5IQR + Q1 = 56 → Min = 5
Max whisker : 1.5IQR + Q3 = 152 → Max = 119
Q1 = 22 Q3 = 74 Median : 37



There are no outliers because no values are less than the minimum whisker or greater than the maximum whisker.

2. flow $P(f) = 0.03$ turbid $P(t) = 0.04$ both $P(f \cap t) = 0.01$

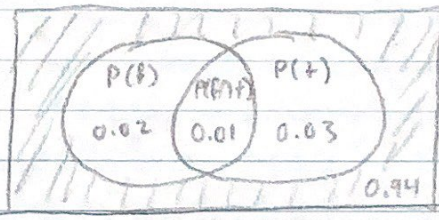
a)



$0.02 + 0.01 + 0.03 = 0.06$

The probability of an operational problem is 0.06

b)



$1 - P(\text{Problem}) = P(\text{No problem})$

$1 - 0.06 = 0.94$

The probability of no operational problem is 0.94.

c) $P(f \mid t)$

$$\frac{P(f \cap t)}{P(t)} = \frac{0.01}{0.04} = 0.25$$

The probability that a plant will have an operational problem because of the flow variation given that it has a variation in turbidity is 0.25

3. $P(C) = 0.2$ $\quad P(N) = 0.45$ $\quad P(T) = 0.35$

$P(A|C) = 0.55$ $\quad P(A|N) = 0.3$ $\quad P(A|T) = 0.8$

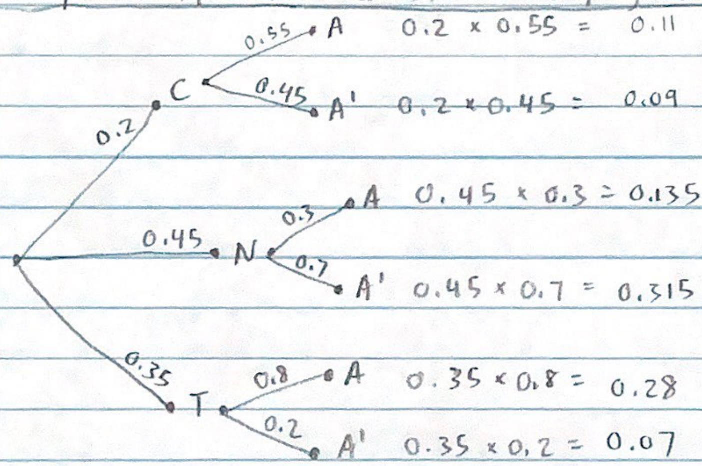
a) Apple: $P(A) = P(A \cap C) + P(A \cap N) + P(A \cap T)$

$P(A) = [P(A|C) \cdot P(C)] + [P(A|N) \cdot P(N)] + [P(A|T) \cdot P(T)]$

$P(A) = [0.55 \times 0.2] + [0.3 \times 0.45] + [0.8 \times 0.35]$

$P(A) = 0.525$

$\boxed{\text{The probability of the next student buying an Apple product is } 0.525}$



$0.11 + 0.135 + 0.28$

$= 0.525$

b) $P(C|A) = \frac{P(C \cap A)}{P(A)} = \frac{P(A|C) \cdot P(C)}{0.525} = \frac{0.55 \times 0.2}{0.525} = 0.2095 \approx 0.21$

$P(N|A) = \frac{P(N \cap A)}{P(A)} = \frac{P(A|N) \cdot P(N)}{0.525} = \frac{0.3 \times 0.45}{0.525} = 0.2571 \approx 0.26$

$P(T|A) = \frac{P(T \cap A)}{P(A)} = \frac{P(A|T) \cdot P(T)}{0.525} = \frac{0.8 \times 0.35}{0.525} = 0.533 \approx 0.53$

$\boxed{P(C|A) = 0.21 \quad P(N|A) = 0.26 \quad P(T|A) = 0.53}$

c) $P(C|A') = \frac{P(C \cap A')}{P(A')}$

$P(A') = 1 - P(A) \rightarrow 1 - 0.525 = 0.475$

$P(C) = P(C \cap A) + P(C \cap A')$

$0.2 = [P(A|C) \cdot P(C)] + P(C \cap A')$

$0.2 = [0.55 \cdot 0.2] + P(C \cap A')$

$0.09 = P(C \cap A')$

$P(C|A') = \frac{P(C \cap A')}{P(A')} = \frac{0.09}{0.475} = 0.1895 \approx 0.19$ $\quad \boxed{P(C|A') = 0.19}$

Green light

4. 4 traffic lights $P(G) = 0.7$ Lights are independent

a) Because the lights are independent:

$P(\text{All 4 green}) = P(G) \times P(G) \times P(G) \times P(G)$

$= 0.7 \times 0.7 \times 0.7 \times 0.7 = 0.240$

$\boxed{\text{The probability that all four lights are green is } 0.240}$

b) This is a binomial distribution.

$b(x; n, p)$

X : number of times (out of 5) that all 4 lights are green

n : total number of times bus visits campus

p : probability of all four green lights

$x = \{0, 1, 2, 3, 4, 5\}$ $n = 5$ $p = 0.240$

c) If $x = 4$ then:

$b(x; 5, 0.240) \longrightarrow b(\overset{x}{4}; \overset{n}{5}, \overset{p}{0.240})$

$= \binom{5}{4} 0.240^4 (1 - 0.240)^{5-4}$

$= 5 \times 0.00332 \times 0.76$

$= 0.12616$

≈ 0.126

$\binom{5}{4} = \dfrac{5!}{4!(5-4)!} = \dfrac{120}{24 \times 1}$

$\binom{5}{4} = 5$

$\boxed{\text{If } x = 4, \text{ the probability is } 0.126}$

5. 17 total students 11 first-timers

4 of 17 assigned to set A X: number of first-timers among them

a) This is a hypergeometric distribution.

$P(x) = h(x; n, M, N)$

X: # of first-timers in set A $X = \{0, 1, 2, 3, 4\}$

M: # of first-timers $M = 11$

n: # of students in set A $n = 4$

N: # of students (total) $N = 17$

$$P(x) = \frac{\binom{M}{x}\binom{N-M}{n-x}}{\binom{N}{n}} = \frac{\binom{11}{x}\binom{17-11}{4-x}}{\binom{17}{4}} = h(x; 4, 11, 17)$$

b) $P(X=2) = h(2; 4, 11, 17)$

$$P(x) = \frac{\binom{11}{2}\binom{17-11}{4-2}}{\binom{17}{4}} = \frac{\binom{11}{2}\binom{6}{2}}{\binom{17}{4}} = \frac{55 \times 15}{2380} = 0.35$$

$$\frac{11!}{2!\,9!} = 55 \qquad \frac{6!}{2!\,4!} = 15 \qquad \frac{17!}{4!\,13!} = 2380$$

$$\boxed{P(X=2) = 0.35}$$

c) $P(X<2) = P(X=0) + P(X=1)$

$X=0$ $$P(x) = \frac{\binom{11}{0}\binom{17-11}{4-0}}{\binom{17}{4}} = \frac{\binom{11}{0}\binom{6}{4}}{\binom{17}{4}} = \frac{1 \times 15}{2380} = 0.0063025$$

$$\frac{11!}{0!\,11!} = 1 \qquad \frac{6!}{4!\,2!} = 15$$

$X=1$ $$P(x) = \frac{\binom{11}{1}\binom{17-11}{4-1}}{\binom{17}{4}} = \frac{\binom{11}{1}\binom{6}{3}}{\binom{17}{4}} = \frac{11 \times 20}{2380} = 0.092436$$

$$\frac{11!}{1!\,11!} = 11 \qquad \frac{6!}{3!\,3!}\ 20$$

$0.0063025 + 0.092436 = 0.0987 \approx 0.10$

$$\boxed{P(X < 2) = 0.10}$$

c) $P(X \geq 2) = 1 - P(X<2)$

$= 1 - 0.1$

$= 0.9$

$$\boxed{P(X \geq 2) = 0.90}$$

CEE110 Midterm Reference Sheet

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Min whisker - lowest value in category, or Q1 - 1.5 IQR (minimum value in range)
Max whisker - largest value that is less than or equal to 1.5 IQR + Q3

Lower Quartile - median of lower half

Upper Quartile - median of upper half

Fourths Spread (Interquartile Range) - $Q_3 - Q_1$

Union $\cup$ - OR

Intersection $\cap$ - AND

Mutually Exclusive - two events that can't happen at the same time

Collectively Exhaustive - $A \cup B$ covers all events in sample space

- $P(A) = 1 - P(A')$
- $P(A \cup B) = P(A) + P(B) - P(A \cap B)$ // $P(A \cap B) = P(A) + P(B) - P(A \cup B)$

Odds - Earnings from win (W) - Losings for loss (L)   probability to win : $\frac{L}{W+L}$

$\boxed{\text{Tree Diagram}}$ - Good for drawing out all possibilities

Permutations : order matters

# of permutations of size k from n objects is $P_{k,n} = \frac{n!}{(n-k)!}$

Combination : order does not matter

# of combinations of size k from n objects is $C_{k,n} = \frac{n!}{(n-k)!\,k!}$

X chose Y - "randomly select X objects from population of Y things"

$\boxed{\text{Conditional probability}}$ $(A \mid B)$ - Probability of A given B

$$P(A \mid B) = \frac{P(A \cap B)}{P(B)}$$

Independence - events are independent if $P(A \mid B) = P(A)$

↳ implies that $P(A \cap B) = P(A) \times P(B)$

$\boxed{\text{Bayes' Theorem}}$ - $P(A \mid B) = \frac{P(B \mid A)P(A)}{P(B)}$

$P(A) = P(B \cap A) + P(B' \cap A)$ $P(B) = P(B \cap A) + P(B \cap A')$

$\boxed{pmf = P(x)}$ - probability of an event, written in a table

$\boxed{cdf = F(x)}$ - adding up pmf over time, final cdf = 1

$\boxed{\text{Expected Value} = E(x)}$ - long run expected mean $E[x] = \mu_x = \sum x \cdot p(x)$

$$E(aX + b) = a \cdot E(x) + b$$

$\boxed{\text{Variance} = V(x)}$ - $V(x) = \sigma^2 = E(x^2) - [E(x)]^2$

$\sqrt{V(x)} = \sigma$ = standard deviation

$$V(aX + b) = a^2 V(x) = a^2 \sigma^2{}_x$$

$$\sigma_{ax+b} = |a|\,\sigma_x$$

$\boxed{\text{Binomial Distribution}}$ - $b(x; n, p) = \binom{n}{x} p^x (1-p)^{n-x}$

X : number of successes in n trials   p: probability

$\binom{n}{x} = \frac{n!}{x!(n-x)!}$

Expected Value : $E(x) = n \times p$

Variance : $V(x) = np(1-p)$