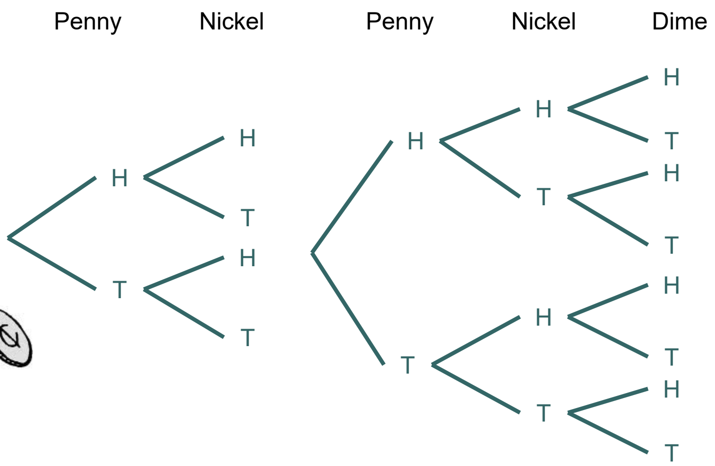
Counting

The statistics student tossed a coin throughout the Boston Marathon to study the behavior of a chance process in the long run.

Today's Class

- Tree Diagram
- Permutations
- Combinations

Tree Diagram: Coin Toss



Tree Diagram

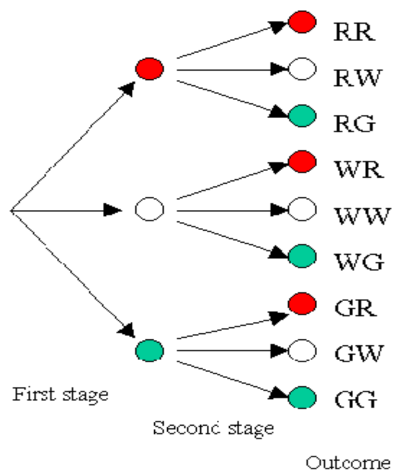
- Pictorial representation of all possibilities
- Two options
  - First part of branch is first option
  - Second branching represents section options

Example: Tree Diagram

- Draw a tree diagram for drawing two balls out of three colors (red, white and green).
  - What is the probability of each outcome?

http://math.youngzones.org/tree.html

Tree Diagram Example



http://math.youngzones.org/tree.html

Example: Tree Diagram

- Draw a tree diagram for drawing two balls out of three colors (red, white and green).
  - What is the probability of getting two balls of the same color?

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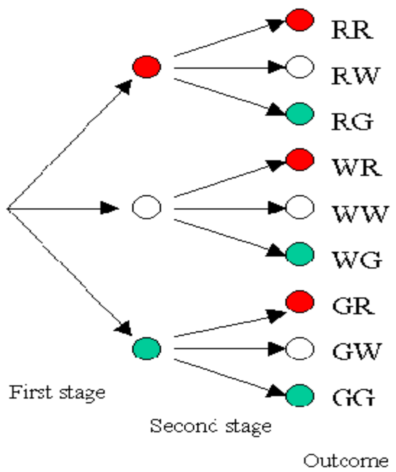
Tree Diagram Example



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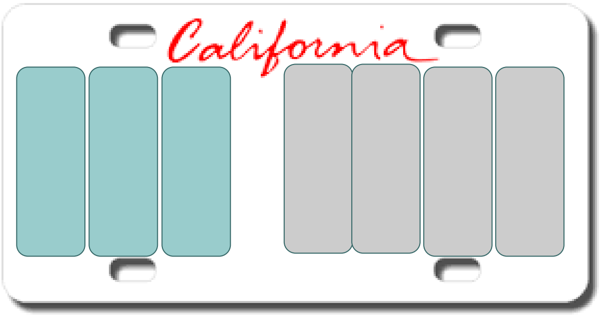
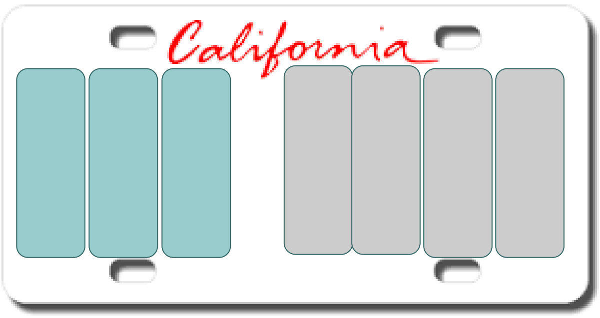
Ordered Sampling with Replacement Example

- How many different 7-place license plates are possible if the first 3 places are to be occupied by letters and the final 4 by numbers?



Ordered Sampling without Replacement Example

- How many license plates would be possible if repetition among letters or numbers were prohibited?



Product Rule for k-Tuples

- A k-tuples is ordered collection of k elements. e.g. (3,1,4,2)≠(1,4,2,3)

n_1 **distinct** ways of doing Task 1 | n_2 **distinct** objects of doing Task 2 | … | n_k **distinct** objects of doing Task k

- How many different k-tuples can be made this way?

$$n_1 \times n_2 \times \cdots \times n_k$$

Ordered Sampling without Replacement Example

- If you want to select **three** of the billiard balls to form a lineup of speakers. In how many ways can you choose the billiard balls?

Order matters

1 2 3
1 3 2
2 1 3
2 3 1
3 1 2
3 2 1
…

Ordered Sampling without Replacement Ex, Cont'd

- If you wanted to select **three** of the billiard balls

$$P_{3,16} = \frac{16!}{(16-3)!} = \frac{16!}{13!} = 3{,}360$$

Permutation

- Chosen without replacement
- Order matters
- The number of permutations of size k from n objects is denoted by $P_{k,n}$:

$$P_{k,n} = n \times (n-1) \times (n-2) \times \cdots \times (n-k+1) = \frac{n!}{(n-k)!}$$

Factorial

- The **factorial function** (**!**) means to multiply a series of descending natural numbers
- Examples
  - 4! = 4 × 3 × 2 × 1 = 24
  - 7! = 7 × 6 × 5 × 4 × 3 × 2 × 1 = 5040
  - 1! = 1

Unordered Sampling without Replacement Example

- If you wanted to select **three** of the billiard balls and you just want to know which 3 pool balls were chosen, not the order. In how many ways can you choose the billiard balls?

Order does matter	Order doesn't matter
1 2 3	1 2 3
1 3 2	
2 1 3	
2 3 1	
3 1 2	
3 2 1	

Unordered Sampling without Replacement Ex, Cont'd

$$C_{3,16} = \frac{16!}{3!(16-3)!}$$
$$= \frac{16!}{3!\ 13!}$$
$$= \frac{16 \times 15 \times 14}{3 \times 2 \times 1}$$
$$= 560$$

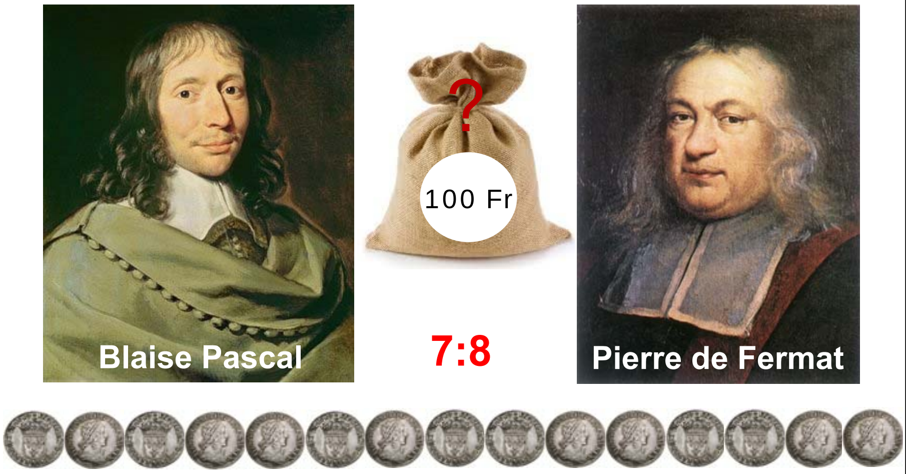
Combination

- Order does not matter
- The number of combinations of size k from n distinct objects will be denoted by $C_{k,n}$:

$$C_{k,n} = \binom{n}{k} = \frac{P_{k,n}}{k!} = \frac{n!}{k!(n-k)!}$$

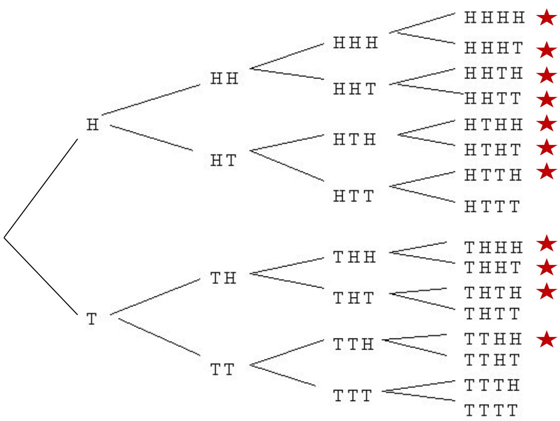
$$P_{k,n} = k! \times C_{k,n}$$

Interrupted Game, Revisit



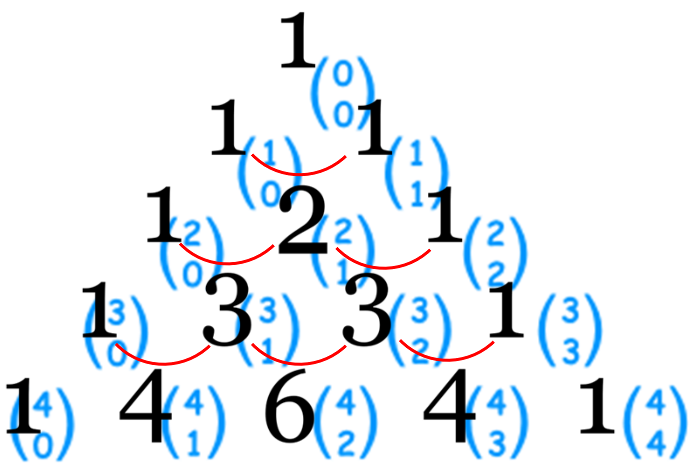
Interrupted Game Outcomes

Fermat



Pascal $\frac{{}_4C_2 + {}_4C_3 + {}_4C_4}{total\ outcomes} = \frac{\frac{4!}{2!2!}+\frac{4!}{3!1!}+\frac{4!}{4!0!}}{16} = \frac{6+4+1}{16} = \frac{11}{16}$

Pascal's Triangle



Example: Probability

- UCLA SEAS has received a shipment of 25 printers, of which 10 are B&W laser printers and 15 are color laser printers. If 6 of these 25 are selected at random to be checked by a technician. What is the probability that exactly 3 of those selected are B&W laser printers?

Example: Solution

$$P(D_3) = \frac{ND_{3B\&W} \cdot ND_{3color}}{ND_{6printers}}$$

$$= \frac{\binom{10}{3}\binom{15}{3}}{\binom{25}{6}}$$

$$= \frac{\frac{10!}{3!\,7!} \cdot \frac{15}{3!\,12!}}{\frac{25!}{6!\,19!}}$$

$$= 0.31$$

Bonus question

- Suppose you have two queen cards and four non-queen cards. You are pulling two cards out of these.
  - What are the probabilities of getting no queens?
  - What are the probabilities of getting at least a queen?

Solution

a. $P(2NQ) = \frac{ND_{2NQ} \cdot ND_{0Q}}{ND_{2cards}}$

$$= \frac{\binom{4}{2}\binom{2}{0}}{\binom{6}{2}}$$

$$= \frac{\frac{4!}{2!\,2!} \cdot \frac{2!}{0!\ 2!}}{\frac{6!}{2!\,4!}} = \frac{12}{30} = 0.4$$

b. 1- 0.4 $= 0.6$

Exercise 2.34

- Computer keyboard failures can be attributed to electrical defects or mechanical defects. A repair facility currently has 25 failed keyboards, 6 of which have electrical defects and 19 of which have mechanical defects.
  - How many ways are there to randomly select 5 of these keyboards for a thorough inspection (without regard to order)?

$$\binom{25}{5} = \frac{25!}{5!\,(25-5)!} = 53{,}130$$

Exercise 2.34

- Computer keyboard failures can be attributed to electrical defects or mechanical defects. A repair facility currently has 25 failed keyboards, 6 of which have electrical defects and 19 of which have mechanical defects.
  - In how many ways can a sample of 5 keyboards be selected so that exactly two have an electrical defect?

$$\binom{6}{2}\binom{19}{3} = \frac{6!}{2!\,(6-2)!}\frac{19!}{3!\,(19-3)!} = 15{\times}969 = 14{,}535$$

Exercise 2.34

- Computer keyboard failures can be attributed to electrical defects or mechanical defects. A repair facility currently has 25 failed keyboards, 6 of which have electrical defects and 19 of which have mechanical defects.
  - If a sample of 5 keyboards is randomly selected, what is the probability that at least 4 of these will have a mechanical defect?

$$\frac{\binom{19}{4}\binom{6}{1} + \binom{19}{5}\binom{6}{0}}{\binom{25}{5}} = \frac{3876 \times 6 + 11628 \times 1}{53130} = 0.657$$