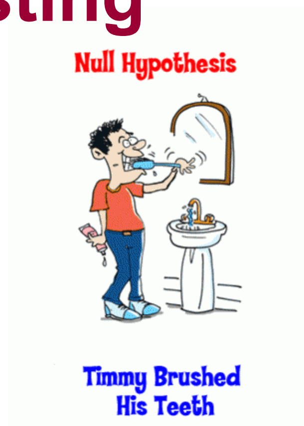
Hypothesis Testing



Today's Class

- Null vs. alternative hypothesis
- Hypothesis Testing
- Type of Errors
- *P*-Values

Bottled Water

- UCLA bottled water has the water volume of 20 Fl Oz on the label. We assumed it is true.
- But is it?
- Assumption
  - Quantity of Water = 20 Oz

Null vs. Alternative Hypotheses

- We always test two contradictory hypotheses:
  - Null hypothesis (H_0) is the belief that is initially assumed to be true (prior belief)
  - Alternative hypothesis (H_a) is the assertion that is contradictory to H_0

Hypothesis Testing

- The claim is the alternative hypothesis, H_a
- The counterclaim is stated as the null hypothesis, H_0
  - Supposed to be true unless proven otherwise
- The hypotheses test assesses how probable the observable differences are assuming H_0

Null vs. Alternative Hypotheses Example

- H_0 $\mu = 20$ Oz
- H_a $\mu \neq 20$ Oz
- If the data indicates the bottles are being filled properly, then we fail to reject the null; fail to reject our assumption
- If the data indicates the bottles are not being filled properly, then we reject the null; reject our assumption

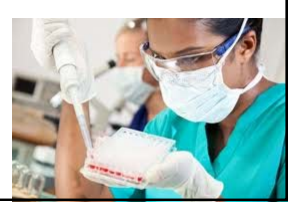
Errors in Hypothesis Testing

- H_0: μ = 20 Oz
- H_a: $\mu \neq$ 20 Oz

	Actual Condition	
	μ = 20 Oz	$\mu \neq$ 20 Oz
Do not reject H_0	Correct	Type II error
Reject H_0	Type I error	Correct

Type I and II Errors Example

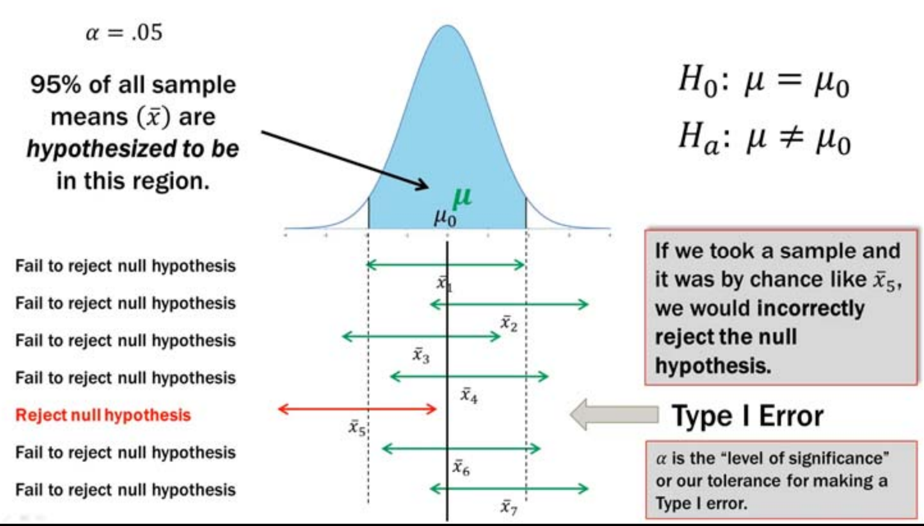
- In diagnostic testing for corona virus,
  - H_0: the tested person is corona virus-free
  - H_a: the person is infected
  - Type I error is that the test gives a false positive result
  - Type II error is that the test gives a false negative result



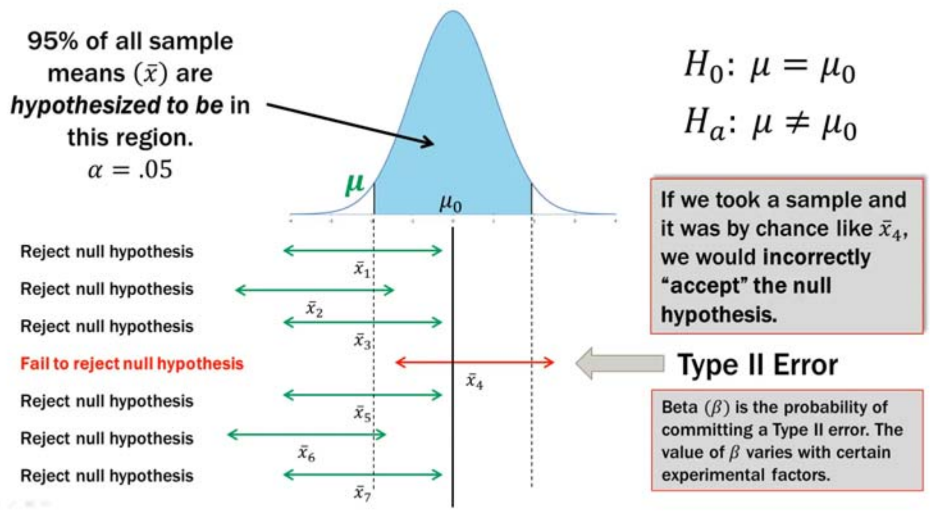
Type I and II Errors Example

- In the prosecution of an accused person,
  - H_0: the person is innocent
  - H_a: the person is guilty
- Which of the following is Type I error?
  - A: the error of convicting an innocent person
  - B: the error of not convicting a guilty person

Visualizing Type I Error

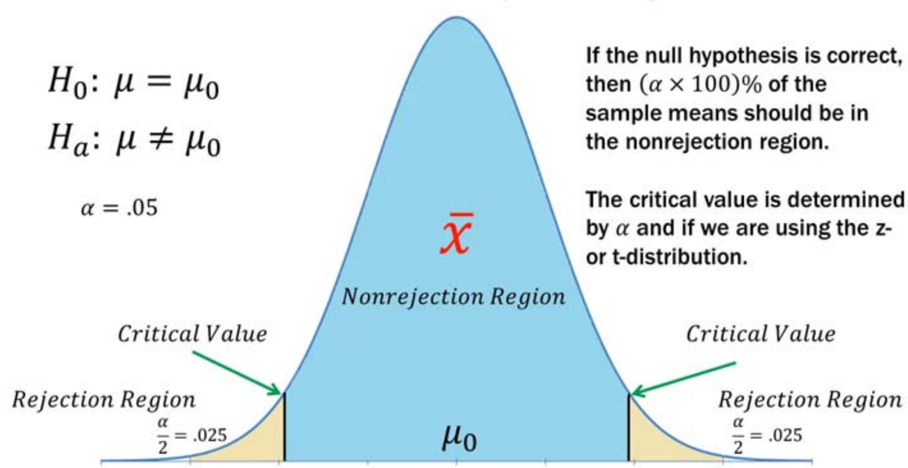


Visualizing Type II Error



Rejection Region

The Two-tailed Test Rejection Region



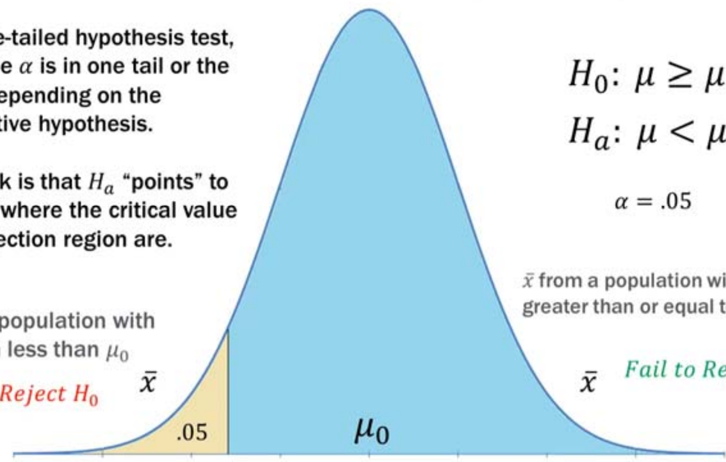
Rejection Region

The One-tailed (Lower) Test Rejection Region

In a one-tailed hypothesis test, all of the α is in one tail or the other depending on the alternative hypothesis.

The trick is that H_a "points" to the tail where the critical value and rejection region are.

$$H_0: \mu \geq \mu_0$$
$$H_a: \mu < \mu_0$$
$$\alpha = .05$$



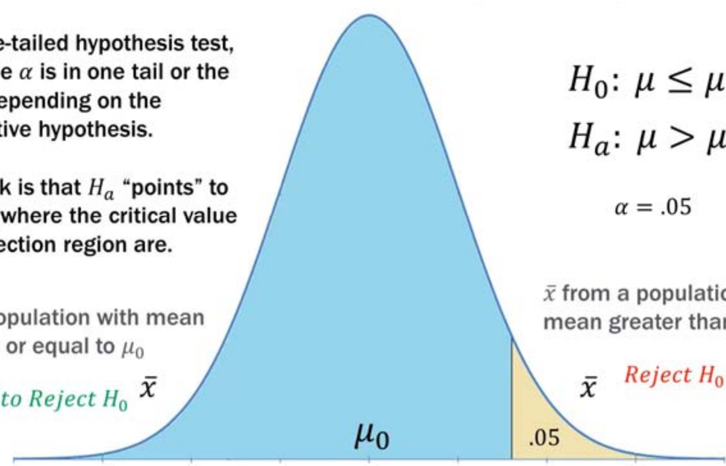
Rejection Region

The One-tailed (Upper) Test Rejection Region

In a one-tailed hypothesis test, all of the α is in one tail or the other depending on the alternative hypothesis.

The trick is that H_a "points" to the tail where the critical value and rejection region are.

$$H_0: \mu \leq \mu_0$$
$$H_a: \mu > \mu_0$$
$$\alpha = .05$$



Hypothesis Testing Procedure

1. Establish hypotheses: null & alternative
2. Determine appropriate statistical test and sampling distribution
3. Choose the Type I error rate (significance level, α)
4. State the decision rule
5. Gather sample data
6. Calculate test statistics
7. State statistical conclusion: Decide whether H_0 should be rejected

Example Hypothesis Testing

- The mean water volume is expected to be 20 Oz. Determine the mean water volume differs from 20 Oz assuming that the population STD to be 2 Oz
- A sample of size 36 finds the sample mean water volume to be 19 Oz
- Is this difference statistically significant at a significance level of .01?

Hypothesis Testing: Normal with Known STD

- H_0: $\mu = \mu_0$
- Test statistic: $z = \dfrac{\bar{x} - \mu_0}{\sigma / \sqrt{n}}$

Alternative Hypothesis	Rejection region for level α
H_a: $\mu > \mu_0$	$z > z_\alpha$
H_a: $\mu < \mu_0$	$z < - z_\alpha$
H_a: $\mu \neq \mu_0$	$z > z_{\alpha/2}$ or $z < - z_{\alpha/2}$

Example

- The mean water volume is expected to be 20 Oz. Determine the mean water volume differs from 20 Oz
- A sample of size 36 finds the sample mean water volume to be 19 Oz and the sample STD to be 2 Oz
- Is this difference statistically significant at a significance level of .01?

Hypothesis Testing: Normal with Unknown STD

- H_0: $\mu = \mu_0$
- Our test statistic is: $t = \dfrac{\bar{x} - \mu_0}{s / \sqrt{n}}$

Alternative Hypothesis	Rejection region for level α test
H_a: $\mu > \mu_0$	$t \geq t_{\alpha,\, n-1}$
H_a: $\mu < \mu_0$	$t \leq - t_{\alpha,\, n-1}$
H_a: $\mu \neq \mu_0$	$t \geq t_{\alpha/2,\, n-1}$ or $t \leq - t_{\alpha/2,\, n-1}$

P-values

Where Statisticians go to get their P-values.

- The p-value is the smallest level of significance at which the H_0 would be rejected
  - $p(\text{data}|H_0)$ = p-value
- If p-value $\leq \alpha$, then reject H_0 at level α

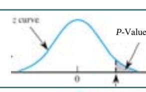
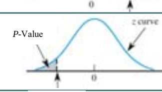
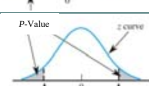
  If p-value $> \alpha$, then do not reject H_0 at level α
- The lower the p-value, the stronger your evidence in support of alternative hypothesis

Example: p-Value

- The mean water volume is expected to be 20 Oz. Determine the mean water volume differs from 20 Oz assuming that the population STD to be 2 Oz
- A sample of size 36 finds the sample mean water volume to be 19 Oz
- What is the p-value?

P-values

- H_0: $\mu = \mu_0$
- Our test statistic is: $z = \dfrac{\bar{x} - \mu_0}{\sigma / \sqrt{n}}$

Alternative Hypothesis	Rejection region for level α test
H_a: $\mu > \mu_0$ 	$P = p(Z>z) = 1-\Phi(z)$
H_a: $\mu < \mu_0$ 	$P = p(Z<z) = \Phi(z)$
H_a: $\mu \neq \mu_0$ 	$P = 2\,(1-\Phi(\lvert z \rvert))$

Example: p-Value

- The mean water volume is expected to be 20 Oz. Determine the mean water volume differs from 20 Oz
- A sample of size 36 finds the sample mean water volume to be 19 Oz and the sample STD to be 2 Oz
- What is the p-value?

P-value

- H_0: $\mu = \mu_0$
- Our test statistic is: $t = \frac{\bar{x} - \mu_0}{s / \sqrt{n}}$

Alternative Hypothesis	Rejection region for level α test
H_a: $\mu > \mu_0$	P = p(X>t)
H_a: $\mu < \mu_0$	P = p(X<t)
H_a: $\mu \neq \mu_0$	P = 2p(X>|t|)

Relationship between CI and Hypothesis Test

- The mean water volume is expected to be 20 Oz. Determine the mean water volume differs from 20 Oz
- A sample of size 36 finds the sample mean water volume to be 19 Oz and the sample STD to be 2 Oz
  - What is 99% two-sided CI?
  - Compare the result with the p-Value