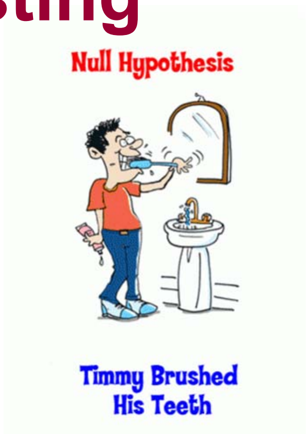
Hypothesis Testing



Today's Class

- Null vs. alternative hypothesis
- Hypothesis Testing
- Type of Errors
- *P*-Values

Bottled Water

- UCLA bottled water has the water volume of 20 Fl Oz on the label. We assumed it is true.
- But is it?
- Assumption
  - Quantity of Water = 20 Oz

Null vs. Alternative Hypotheses

- We always test two contradictory hypotheses:
  - Null hypothesis (H_0) is the belief that is initially assumed to be true (prior belief)
  - Alternative hypothesis (H_a) is the assertion that is contradictory to H_0

Hypothesis Testing

- The claim is the alternative hypothesis, H_a
- The counterclaim is stated as the null hypothesis, H_0
  - supposed to be true unless proven otherwise
- The hypotheses test assesses how probable the observable differences are assuming H_0

Hypothesis Testing

- The result of a hypotheses test is either:
  - The null hypothesis is rejected
    - This is a strong result
    - It indicates that your alternative hypothesis has convincing data behind it
  - The null hypothesis fails to be rejected
    - This is a weak result
    - It DOES NOT imply that the null hypothesis is true
    - Only that there is not a convincing amount of data to support the alternative

Errors in Hypothesis Testing

- H_0: $\mu = 20$ Oz
- H_a: $\mu \neq 20$ Oz

	Actual Condition	
	$\mu = 20$ Oz	$\mu \neq 20$ Oz
Do not reject H_0	Correct	Type II error
Reject H_0	Type I error	Correct

Errors in Hypothesis Testing

	H_0 is True	H_0 is False
Do not reject H_0	Correct	Type II error
Reject H_0	Type I error	Correct

Type I and II Errors: Court Example

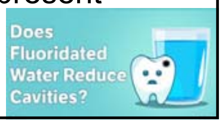
- In the prosecution of an accused person,
  - H_0: the person is innocent
  - H_a: the person is guilty
- Which of the following is Type I error?
  - A: the error of convicting an innocent person
  - B: the error of not convicting a guilty person

Type I and II Errors: Medical Example

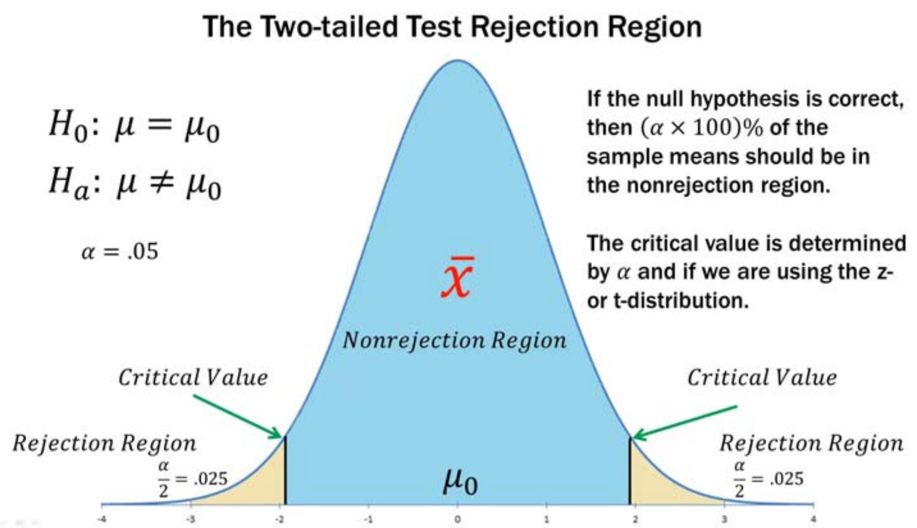
- In diagnostic testing for corona virus,
  - H_0: the tested person is corona virus-free
  - H_a: the person is infected
- Which of the following is Type I error?
  - A: the test gives a false positive result
  - B: the test gives a false negative result

Type I and II Errors: Public Health Example

- In the developing public policy,
  - H_0: Adding fluoride to water/toothpaste has no effect on cavities
  - H_a: Adding fluoride to water/toothpaste protects against cavities
- Which of the following is Type I error?
  - A: detecting an effect (adding fluoride protects against cavities) that is not present.
  - B: failing to detect an effect that is present



Rejection Region



Rejection Region

The One-tailed (Lower) Test Rejection Region

In a one-tailed hypothesis test, all of the α is in one tail or the other depending on the alternative hypothesis.

The trick is that H_a "points" to the tail where the critical value and rejection region are.

H_0: $\mu \geq \mu_0$

H_a: $\mu < \mu_0$

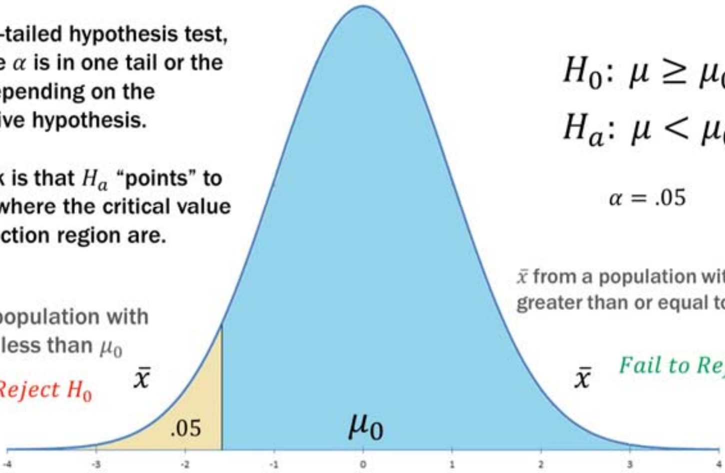
$\alpha = .05$

$\bar{x}$ from population with a mean less than μ_0

Reject H_0 $\bar{x}$

$\bar{x}$ from a population with a mean greater than or equal to μ_0

$\bar{x}$ *Fail to Reject H_0*



Rejection Region

The One-tailed (Upper) Test Rejection Region

In a one-tailed hypothesis test, all of the α is in one tail or the other depending on the alternative hypothesis.

The trick is that H_a "points" to the tail where the critical value and rejection region are.

H_0: $\mu \leq \mu_0$

H_a: $\mu > \mu_0$

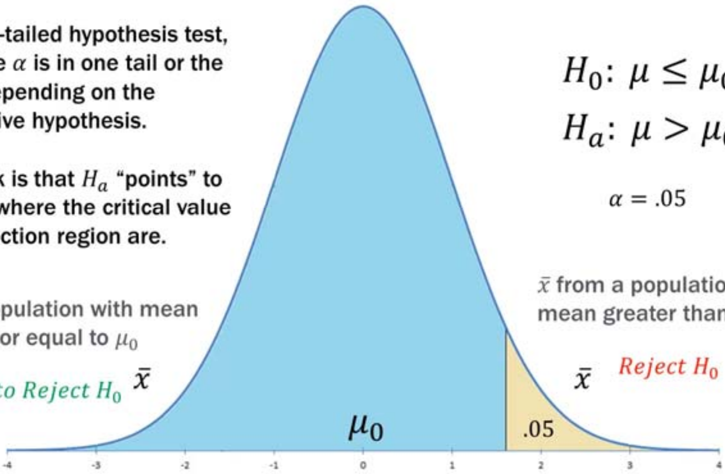
$\alpha = .05$

$\bar{x}$ from population with mean less than or equal to μ_0

Fail to Reject H_0 $\bar{x}$

$\bar{x}$ from a population with mean greater than μ_0

$\bar{x}$ *Reject H_0*



Hypothesis Testing Procedure

1. Establish hypotheses: null & alternative
2. Determine appropriate statistical test and sampling distribution
3. Choose the Type I error rate (significance level, α)
4. State the decision rule (rejection region)
5. Gather sample data
6. Calculate test statistics
7. State statistical conclusion: Decide whether H_0 should be rejected

Example Hypothesis Testing

- The mean water volume is expected to be 20 Oz. Determine the mean water volume differs from 20 Oz assuming that the population STD to be 2 Oz
- A sample of size 36 finds the sample mean water volume to be 19 Oz
- Is this difference statistically significant at a significance level of .01?

Table 4.1 Standard Normal Percentiles and Critical Values

Percentile	90	95	97.5	99	99.5	99.9	99.95
α (tail area)	.1	.05	.025	.01	.005	.001	.0005
$z_\alpha = 100(1-\alpha)$th percentile	1.28	1.645	1.96	2.33	2.58	3.08	3.27

Solution

- Step 1: Establish hypothesis
  - H_0: μ = 20 Oz
  - H_a: $\mu \neq$ 20 Oz
- Step 2: Determine appropriate statistical test and sampling distribution
  - a two-tailed test
  - σ is known: use z-distribution
- Step 3: Specify the Type I error rate (significance level)
  - α = 0.01

Solution, Cnt'd

- Step 4: State the decision rule
  - If z > $z_{.005}$), reject H_0
  - If z < $-z_{.005}$), reject H_0
- Step 5: Gather data
  - n = 36, $\bar{X}$ = 19
- Step 6: Calculate test statistic
  - μ_0 = 20 Oz, σ = 2 Oz
  - $z = \frac{\bar{x}-\mu_0}{\frac{\sigma}{\sqrt{n}}} = \frac{19-20}{\frac{2}{\sqrt{36}}} = -3$
- Step 7: State statistical conclusion
  - Z = -3 < -2.58: reject the H_0 at the 1% level
  - It is very unlikely that the mean is actually 20 Oz

Hypothesis Testing: Normal with Known STD

- H_0: $\mu = \mu_0$
- Test statistic: $z = \frac{\bar{x} - \mu_0}{\sigma / \sqrt{n}}$

Alternative Hypothesis	Rejection region for level α
H_a: $\mu > \mu_0$	$z > z_\alpha$
H_a: $\mu < \mu_0$	$z < -z_\alpha$
H_a: $\mu \neq \mu_0$	$z > z_{\alpha/2}$ or $z < -z_{\alpha/2}$

Example

- The mean water volume is expected to be 20 Oz. Determine the mean water volume differs from 20 Oz
- A sample of size 36 finds the sample mean water volume to be 19 Oz and the sample STD to be 2 Oz
- Is this difference statistically significant at a significance level of .01?

Solution

- Step 1: Establish hypothesis
  - H_0: $\mu = 20$ Oz
  - H_a: $\mu \neq 20$ Oz
- Step 2: Determine appropriate statistical test and sampling distribution
  - a two-tailed test
  - σ is unknown, n<40: use t-distribution
- Step 3: Specify the Type I error rate (significance level)
  - $\alpha = 0.01$

Solution, Cnt'd

- Step 4: State the decision rule
  - For df=35, If t > (= $t_{35,\ 0.005}$), reject H_0
  - If t < (= $-t_{35,\ 0.005}$), reject H_0
- Step 5: Gather data
  - n = 36, $\bar{X}$ = 19
- Step 6: Calculate test statistic
  - μ_0 = 20 Oz, s = 2 Oz
  - $t = \frac{\bar{x}-\mu_0}{\frac{s}{\sqrt{n}}} = \frac{19-20}{\frac{2}{\sqrt{36}}} = -3$
- Step 7: State statistical conclusion
  - t = -3 < -2.724: reject the H_0 at the 1% level
  - It is very unlikely that the mean is actually 20 Oz

df/α =	.40	.25	.10	.05	.025	.01	.005	.001	.0005
1	0.325	1.000	3.078	6.314	12.706	31.821	63.657	318.309	636.619
2	0.289	0.816	1.886	2.920	4.303	6.965	9.925	22.327	31.599
3	0.277	0.765	1.638	2.353	3.182	4.541	5.841	10.215	12.924
4	0.271	0.741	1.533	2.132	2.776	3.747	4.604	7.173	8.610
5	0.267	0.727	1.476	2.015	2.571	3.365	4.032	5.893	6.869
6	0.265	0.718	1.440	1.943	2.447	3.143	3.707	5.208	5.959
7	0.263	0.711	1.415	1.895	2.365	2.998	3.499	4.785	5.408
8	0.262	0.706	1.397	1.860	2.306	2.896	3.355	4.501	5.041
9	0.261	0.703	1.383	1.833	2.262	2.821	3.250	4.297	4.781
10	0.260	0.700	1.372	1.812	2.228	2.764	3.169	4.144	4.587
11	0.260	0.697	1.363	1.796	2.201	2.718	3.106	4.025	4.437
12	0.259	0.695	1.356	1.782	2.179	2.681	3.055	3.930	4.318
13	0.259	0.694	1.350	1.771	2.160	2.650	3.012	3.852	4.221
14	0.258	0.692	1.345	1.761	2.145	2.624	2.977	3.787	4.140
15	0.258	0.691	1.341	1.753	2.131	2.602	2.947	3.733	4.073
16	0.258	0.690	1.337	1.746	2.120	2.583	2.921	3.686	4.015
17	0.257	0.689	1.333	1.740	2.110	2.567	2.898	3.646	3.965
18	0.257	0.688	1.330	1.734	2.101	2.552	2.878	3.610	3.922
19	0.257	0.688	1.328	1.729	2.093	2.539	2.861	3.579	3.883
20	0.257	0.687	1.325	1.725	2.086	2.528	2.845	3.552	3.850
21	0.257	0.686	1.323	1.721	2.080	2.518	2.831	3.527	3.819
22	0.256	0.686	1.321	1.717	2.074	2.508	2.819	3.505	3.792
23	0.256	0.685	1.319	1.714	2.069	2.500	2.807	3.485	3.768
24	0.256	0.685	1.318	1.711	2.064	2.492	2.797	3.467	3.745
25	0.256	0.684	1.316	1.708	2.060	2.485	2.787	3.450	3.725
26	0.256	0.684	1.315	1.706	2.056	2.479	2.779	3.435	3.707
27	0.256	0.684	1.314	1.703	2.052	2.473	2.771	3.421	3.690
28	0.256	0.683	1.313	1.701	2.048	2.467	2.763	3.408	3.674
29	0.256	0.683	1.311	1.699	2.045	2.462	2.756	3.396	3.659
30	0.256	0.683	1.310	1.697	2.042	2.457	2.750	3.385	3.646
35	0.255	0.682	1.306	1.690	2.030	2.438	2.724	3.340	3.591
40	0.255	0.681	1.303	1.684	2.021	2.423	2.704	3.307	3.551
50	0.255	0.679	1.299	1.676	2.009	2.403	2.678	3.261	3.496
60	0.254	0.679	1.296	1.671	2.000	2.390	2.660	3.232	3.460
120	0.254	0.677	1.289	1.658	1.980	2.358	2.617	3.160	3.373
inf.	0.253	0.674	1.282	1.645	1.960	2.326	2.576	3.090	3.291

Hypothesis Testing: Normal with Unknown STD

- H_0: $\mu = \mu_0$
- Our test statistic is: $t = \frac{\bar{x} - \mu_0}{s/\sqrt{n}}$

Alternative Hypothesis	Rejection region for level α test
H_a: $\mu > \mu_0$	$t \geq t_{\alpha, n-1}$
H_a: $\mu < \mu_0$	$t \leq - t_{\alpha, n-1}$
H_a: $\mu \neq \mu_0$	$t \geq t_{\alpha/2, n-1}$ or $t \leq - t_{\alpha/2, n-1}$