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CEE 110 HW #4

1. 1 in 88 people infected (0.0114)

a) 200 people in sample

Poisson Distribution

$\lambda = np$ $\quad \lambda = 200 \cdot 0.0114$ $\quad \lambda = 2.27$

$E(x) = \lambda$ $\quad E(x) = 2.27$

$V(x) = \lambda$ $\quad V(x) = 2.27$ $\quad$ standard deviation $= \sqrt{2.27} = 1.51$

$\boxed{E(x) = 2.27 \quad \text{standard deviation} = 1.51}$

b) x = at least two people

P(at least two people) = 1 − P(0) − P(1)

$P(x=0) = e^{-2.27} \times \frac{2.27^0}{0!} = 0.1033$

$P(x=1) = e^{-2.27} \times \frac{2.27^1}{1!} = 0.2345$

$1 - 0.1033 - 0.2345 = 0.6622$

$\boxed{\text{The approximate probability that at least 2 people have been infected is } 0.6622}$

c) P(fewer than 5 infections) $\quad n = 352$

$\lambda = np$ $\quad \lambda = 352 \cdot 0.0114$ $\quad \lambda = 4.013$

$P(x < 5) = P(0) + P(1) + P(2) + P(3) + P(4)$

$= e^{-4.013}\left[\frac{4.013^0}{0!} + \frac{4.013^1}{1!} + \frac{4.013^2}{2!} + \frac{4.013^3}{3!} + \frac{4.013^4}{4!}\right]$

$= 0.62$

$\boxed{\text{The approximate probability that fewer than 5 people have been infected is } 0.62}$

2. 1000 computer servers 0.003 probability of failure

a) Poisson distribution

$\lambda = 1000 \cdot 0.003 = 3$

$P(X=2) = e^{-3} \cdot \frac{3^2}{2!} = 0.224$

$\boxed{\text{The probability that exactly two servers fail is } 0.224}$

b) $1 - P(X=1000) - P(X=999) - P(X=998)$

- $P(X=1000) = e^{-3} \cdot \frac{3^{1000}}{1000!} = 0$
- $P(X=999) = e^{-3} \cdot \frac{3^{999}}{999!} = 0$
- $P(X=998) = e^{-3} \cdot \frac{3^{998}}{998!} = 0$

The probability of any of these events happening is so miniscule that they are practically 0.

$1 - 0 - 0 - 0 = 1$

The approximate probability that fewer than 998 servers fail is 1.

c) mean $= E(x) = \lambda = 3$

standard deviation $= \sqrt{P(x)} = \sqrt{\lambda} = \sqrt{3} = 1.732$

The mean for the number of servers that fail is 3.
The standard deviation for the servers that fail is 1.732

3. 10 numbers / m^3

a) Assuming $\lambda = 10$ numbers/m^3

$$P(X \geq 8) = 1 - P(X \leq 7)$$
$$= 1 - \sum_{n=0}^{7} e^{-10} \cdot \frac{10^n}{n!}$$
$$= 1 - 0.2202$$
$$= 0.7798$$

The probability that 1 m^3 of discharge contains at least 8 bacteria is 0.7798.

b) $E(x) = \frac{10}{m^3} \cdot 1.5m^3 = 15$

standard deviation $= \sqrt{15} = 3.873$

The mean number of bacteria in $1.5m^3$ of discharge is 15 and the standard deviation is 3.873

c) mean value : 15

standard deviation : 3.873

$$P(X > 18.873) \approx P(X > 19)$$
$$= 1 - P(X \leq 18)$$
$$= 1 - \sum_{n=0}^{18} e^{-15} \cdot \frac{15^n}{n!}$$
$$= 1 - 0.8195$$
$$= 0.1805$$

The probability that the number of organisms in $1.5m^3$ of discharge exceeds its mean value by more than one standard deviation is 0.1805

4. Continuous random variable X positive constant c

$$f(x) = \begin{cases} ce^{-4x} & x \geq 0 \\ 0 & \text{else} \end{cases}$$

a) $\int_0^{\infty} f(x)\,dx = 1$

$\int_0^{\infty} ce^{-4x}\,dx = 1$

$\frac{1}{4}C = 1 \qquad \boxed{C = 4}$

b) $F(x) = \int_{-\infty}^{x} f(x)\,dx$

$= \int_0^{x} 4e^{-4x}\,dx$

$= \left[-e^{u}\right]_0^{-4x}$

$= -e^{-4x} + 1$

$$\boxed{F(x) = \begin{cases} 1 - e^{-4x} & x \geq 0 \\ 0 & \text{else} \end{cases}}$$

c) $P(2 < X < 5) = F(5) - F(2)$

$[1 - e^{-20}] - [1 - e^{-8}]$

$\boxed{P(2 < X < 5) = e^{-8} - e^{-20} \approx 0.00033}$

d) $E(x) = \int_{-\infty}^{\infty} x \cdot f(x)\,dx$

$E(x) = \int_{-\infty}^{\infty} 4xe^{-4x}\,dx$

$E(x) = \int_0^{\infty} 4xe^{-4x}\,dx$

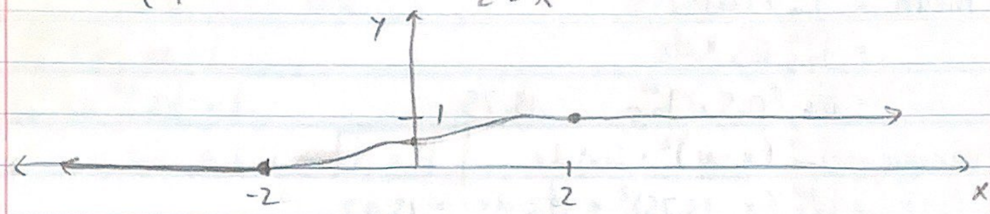
$E(x) = \left[-\frac{1}{4}e^{-4x}(4x+1)\right]_0^{\infty}$

$E(x) = \left[-\frac{1}{4}e^{-\infty}(4\infty + 1)\right] - \left[-\frac{1}{4}e^{0}(4 \cdot 0 + 1)\right]$

$\boxed{E(x) = \frac{1}{4}}$

5.

$$F(x) = \begin{cases} 0 & x < -2 \\ \frac{1}{2} + \frac{3}{32}\left(4x - \frac{x^3}{3}\right) & -2 \le x < 2 \\ 1 & 2 \le x \end{cases}$$



a) $P(X < 0)$ $\qquad P(x > a) = 1 - F(a)$

$P(-\infty \le X \le 0) = F(0) - F(-\infty)$

$= \frac{1}{2} - 0$

$= \frac{1}{2}$

$\boxed{P(X < 0) = \frac{1}{2}}$

b) Want to find : $F(x) = 0.5$, what is x?

Based on the answer from part a

$P(X < 0) = \frac{1}{2}$

Therefore $x = 0$

$\boxed{x = 0}$

c) The cdf was given

$\frac{d}{dx}$ cdf = pdf

$\frac{d}{dx}\left(\frac{1}{2} + \frac{3}{32}\left(4x - \frac{x^3}{3}\right)\right)$

$\frac{d}{dx}\left(\frac{1}{2} + \frac{12}{32}x - \frac{x^3}{32}\right)$

$-\frac{3x^2}{32} + \frac{3}{8}$

$$\boxed{\text{pdf} = f(x) = \begin{cases} -\frac{3x^2}{32} + \frac{3}{8} & -2 \le x \le 2 \\ 0 & \text{otherwise} \end{cases}}$$

6. Uniform distribution $(7.5, 20)$

$f(x) = \int_{7.5}^{20} \frac{1}{12.5}\,dx$

a) mean $= \int_{-\infty}^{\infty} x f(x)\,dx$

$= \int_{7.5}^{20} \frac{1}{12.5} x\,dx$

$= \frac{1}{12.5}\left[0.5x^2\right]_{7.5}^{20} = 13.75$

variance $= \int_{-\infty}^{\infty} (x-\mu)^2 \cdot f(x)\,dx$

$= \int_{7.5}^{20} (x-13.75)^2 \cdot \frac{1}{12.5}\,dx = 13.02$

The mean is 13.75 and the variance is 13.02 for the waiting time.

b) $f(x) = \int_{7.5}^{20} \frac{1}{12.5}\,dx$

$F(x) = \int_{7.5}^{x} \frac{1}{12.5}\,dx$

$= \left[\frac{1}{12.5}x\right]_{0.75}^{x} = \frac{x}{12.5} - 0.6$

$\boxed{F(x) = \frac{x}{12.5} - 0.6}$

c) $P(10 \le x \le 15) = F(15) - F(10)$

$F(15) = 1.14$

$F(10) = 0.74$

$1.14 - 0.74 = 0.4$

$\boxed{P(10 \le x \le 15) = 0.4}$

d) $\mu = 13.75$ std. deviation $= \sqrt{13.02} = 3.61$

$\mu \pm 3.61 \rightarrow [10.14, 17.36]$ minutes

$P(10.14 \le x \le 17.36) = F(17.36) - F(10.14)$

$F(17.36) = 1.3288$

$F(10.14) = 0.7512$ $1.3288 - 0.7512 = 0.5776$

The probability that the observed waiting time is within 1 standard deviation of the mean value is 0.5776