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CS 118 - Dis 1B
3 October 2022

CS 118 HW 1

1A) See the derivation below for details.

We see in the final result that the scaling factor is inversely proportional to n because there is the $\frac{1}{2n}$ there. Intuitively, the reason that the scaling factor is inversely proportional to n is because as the frequency of the waves increases, the magnitude of how much they need to be scaled has to decrease in order to continue closer to reaching a perfect straight line for the square function.

$$b_n = \frac{2}{T}\int_0^T f(x)\sin\left(\frac{2\pi n x}{T}\right)dx$$

$$b_n = 2\int_0^1 f(x)\sin(2\pi n x)\,dx$$

$$b_n = 2\left[\int_0^{.5}\frac{\pi}{4}\sin(2\pi n x)x - \int_{0.5}^{1}\frac{\pi}{4}\sin(2\pi n x)dx\right]$$

$$b_n = \frac{\pi}{2}\left[\int_0^{.5}\sin(2\pi n x)dx - \int_{.5}^{1}\sin(2\pi n x)dx\right]$$

$$b_n = \frac{\pi}{2}\left\{\left[-\frac{\cos(2\pi n x)}{2\pi n}\right]_0^{.5} - \left[-\frac{\cos(2\pi n x)}{2\pi n}\right]_{0.5}^{1}\right\}$$

$$b_n = \frac{1}{4}\left\{\left[-\frac{\cos(n\pi)}{n} + \frac{1}{n}\right] - \left[\frac{-1}{4} + \frac{\cos(n\pi)}{n}\right]\right\}$$

$$b_n = \frac{1}{4}\left[\frac{2}{n} - \frac{2\cos(n\pi)}{n}\right]$$

$$b_n = \frac{1}{2n}\left[1 - \cos(n\pi)\right]$$

1B1)

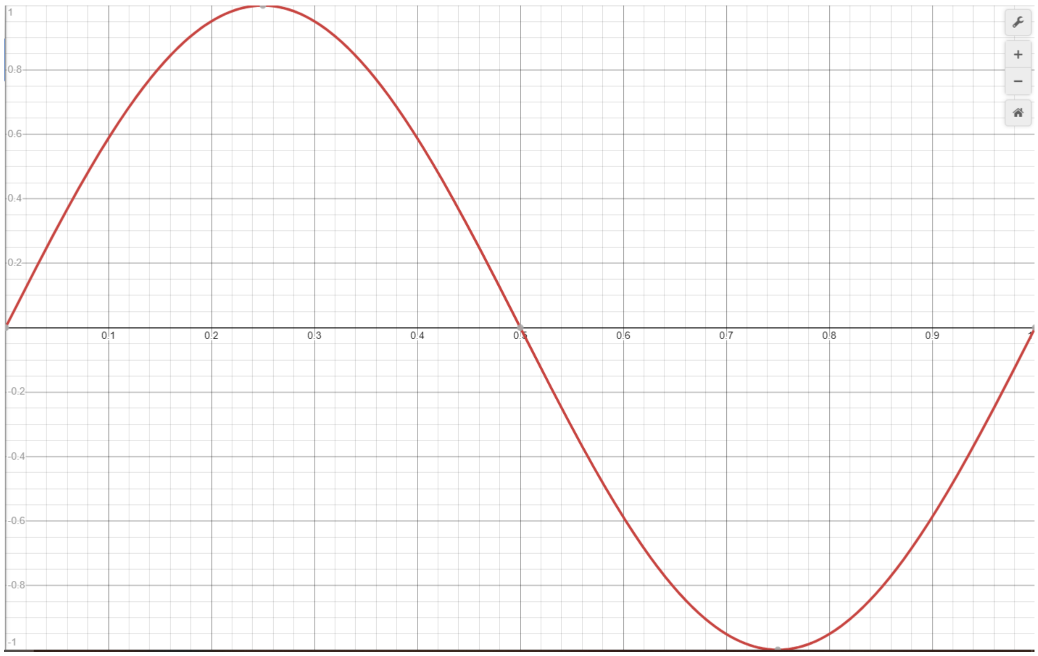

Figure 2: $f(t) = sin(2\pi t)$

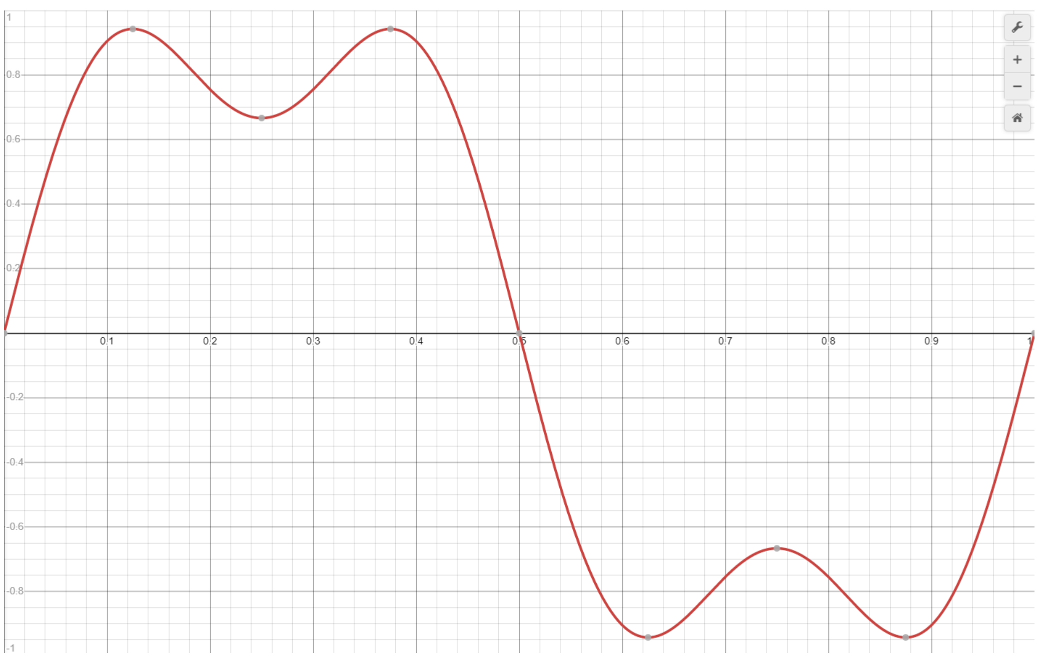

Figure 3: $f(t) = sin(2\pi t) + \frac{1}{3}sin(6\pi t)$

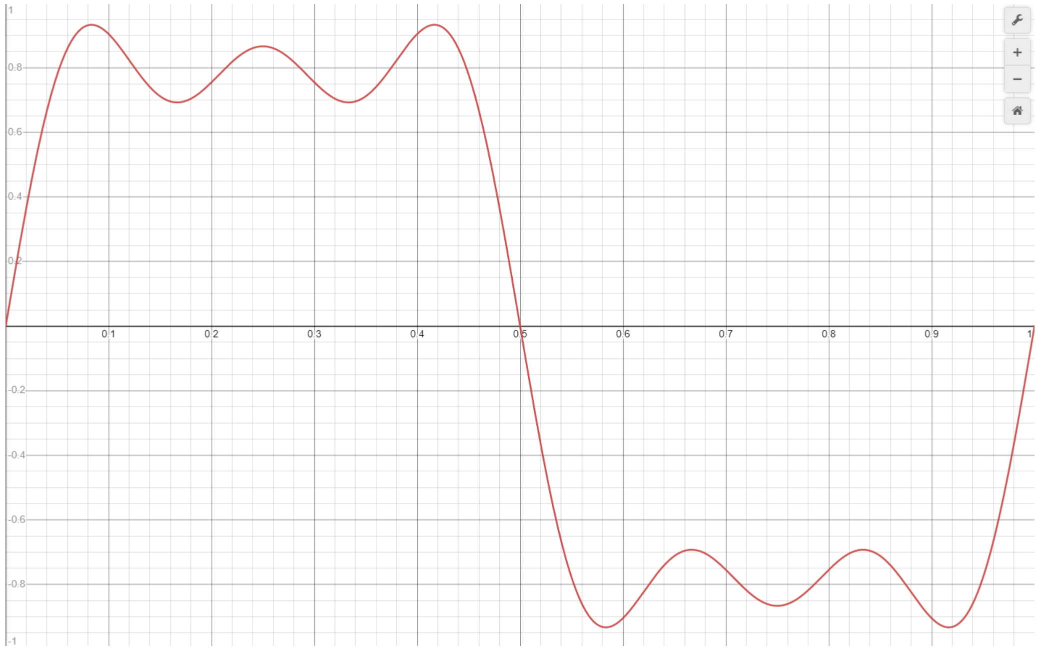

Figure 4: $f(t) = sin(2\pi t) + \frac{1}{3}sin(6\pi t) + \frac{1}{5}sin(10\pi t)$

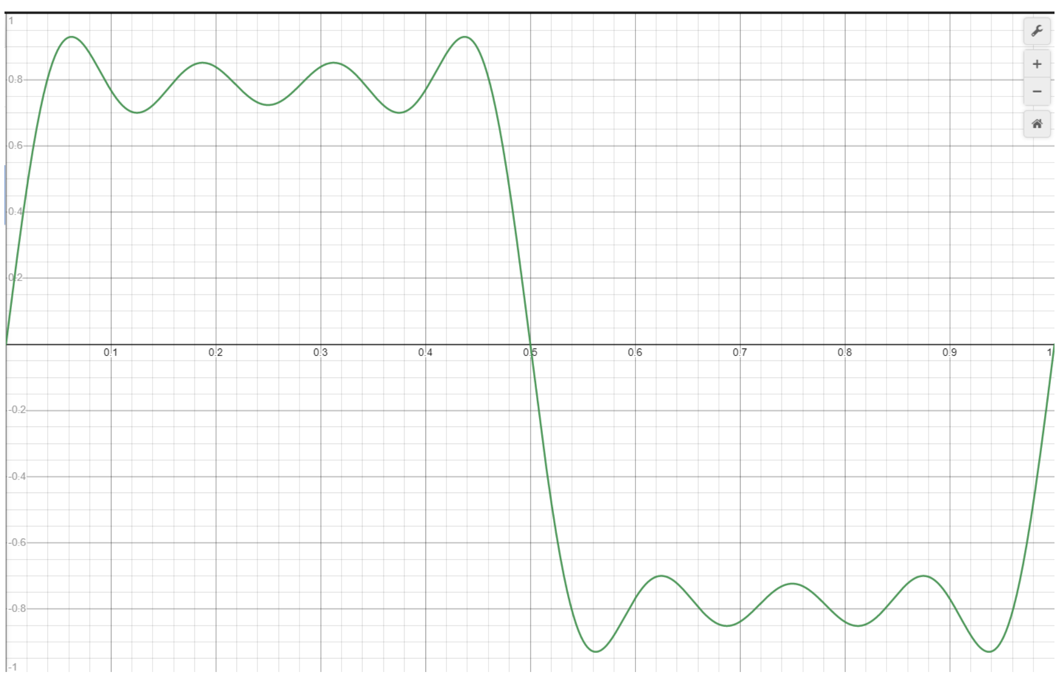

Figure 5: $f(t) = sin(2\pi t) + \frac{1}{3}sin(6\pi t) + \frac{1}{5}sin(10\pi t) + \frac{1}{7}sin(14\pi t)$

Figure 6: $f(t) = sin(2\pi t) + \frac{1}{3}sin(6\pi t) + \frac{1}{5}sin(10\pi t) + \frac{1}{7}sin(14\pi t) + \frac{1}{9}sin(18\pi t)$

1B2) The bandwidth (in Hertz) for the approximations are 1, 3, 5, 7, 9 respectively for the five approximations shown above.

1B3) With the $\frac{4}{\pi}$ factor:

Approximation 1: Furthest Sample = 1.2732; 27.32% error
Approximation 2: Furthest Sample = 1.2004; 20.04% error
Approximation 3: Furthest Sample = 0.8821; 11.79% error
Approximation 4: Furthest Sample = 0.8917; 10.83% error
Approximation 5: Furthest Sample = 0.8959; 10.41% error

1B4) Approximation 1: t = 0.18
Approximation 2: t = 0.086
Approximation 3: t = 0.057
Approximation 4: t = 0.043
Approximation 5: t = 0.034

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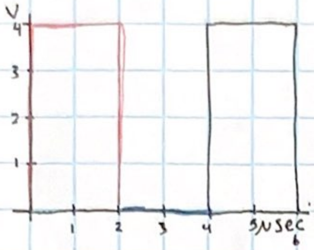
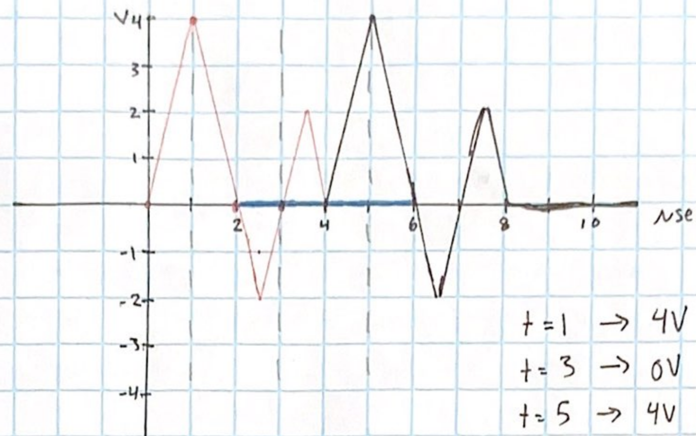
2A)



Sender
(101)

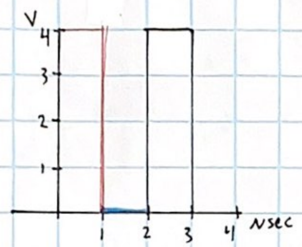


$t=1 \to 4V$
$t=3 \to 0V$
$t=5 \to 4V$

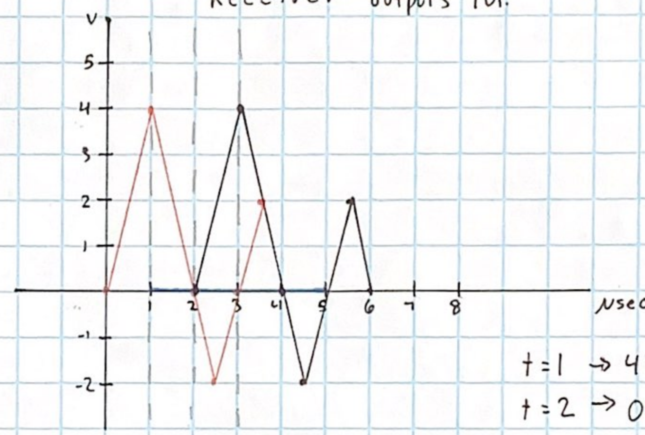
$\begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$

Receiver outputs 101.

B)



Sender
(101)

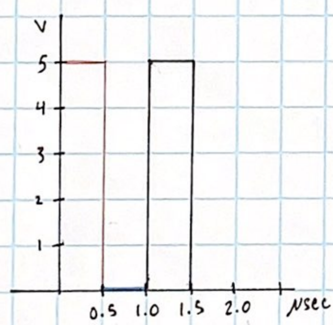


Receiver outputs 101.

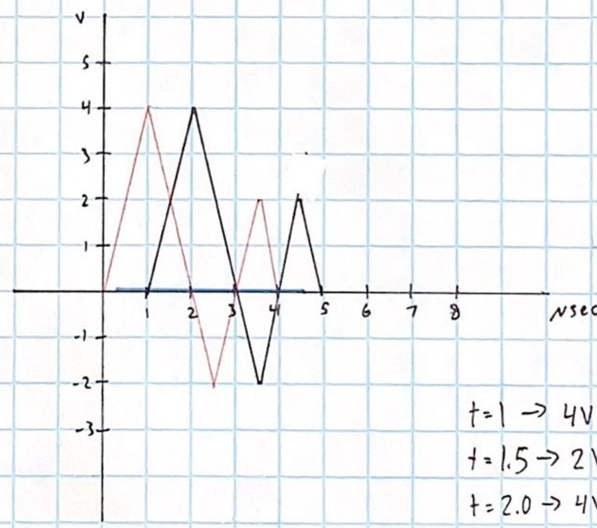
$t=1 \to 4V$
$t=2 \to 0V$
$t=3 \to 4V$

$\begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$

C)



Sender
(101)

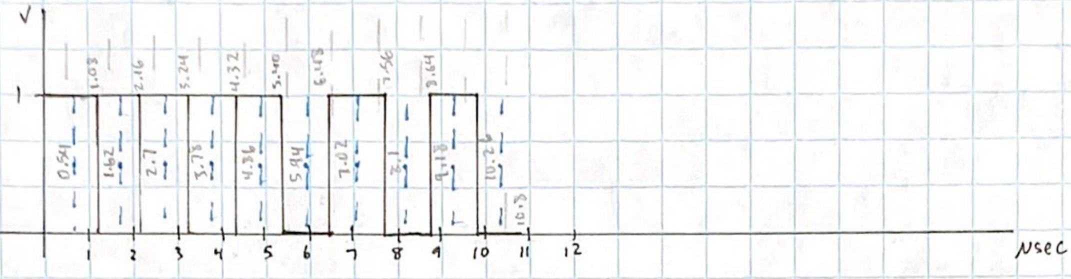


$t=1 \to 4V$
$t=1.5 \to 2V+2V=4V$
$t=2.0 \to 4V$

Receiver outputs 111.
At $t=1.5$ there is interference between the signals, and the receiver mistakenly interprets it as 1.

3A) $1\mu sec$ 8% slower $\left\{ \begin{array}{ccc} 0 & 1.08 & 2.16 \\ 1.08 & 2.16 & 3.24 \end{array} \cdots \right\}$ $\begin{array}{c} 3.24 \\ 4.32 \end{array}$ 5.40 6.48 7.56 8.64 9.72 10.80

1 1 1 1 1 0 1 0 1 0



3B) The 10^{th} bit is sent from 9.72 μsec - 10.80 μsec. Without clock recovery code, the sampling should occur at 9.5 μsec.

$$\frac{9.72 + 10.80}{2} = 10.26 \qquad 10.26 - 9.5 = 0.76$$

The sampling is off by 0.76 μsec by the 10^{th} bit.

3C) Sampling Normally Occurs At: 0.5 1.5 2.5 3.5 4.5 5.5 6.5 7.5 8.5 9.5

Sampling Should Occur At: 0.54 1.62 2.7 3.78 4.86 5.94 7.02 8.1 9.18 10.26

See the graph above in 3A for the modified sampling instances. They are represented by blue dotted lines.

3D) A sharp spike of noise at time 0.3 μsec won't affect sampling times because the sampling will not have started yet. Sampling starts at 0.5 μsec.

3E) ~~A sharp noise spike at time 2.7 μsec will force the lag to be adjusted. This will alter the sampling times - they will be altered and get corrected at the 5.40 μsec mark where there is a transition from 1 to 0.~~

A sharp noise spike at 2.7 μsec would coincide with one of the sampling times. Since the spike is at -1V, this would cause the bit to be interpreted as a 0 instead of 1. Sampling time is unaffected.