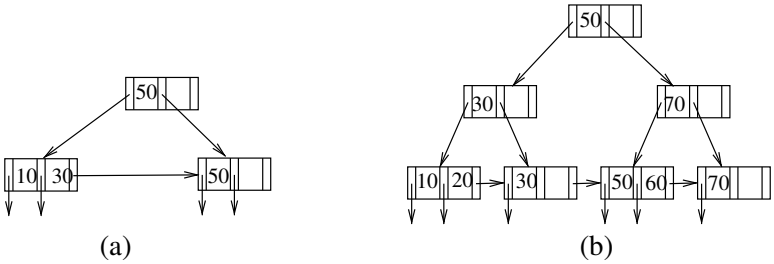
CS143: Homework #6 (Index)

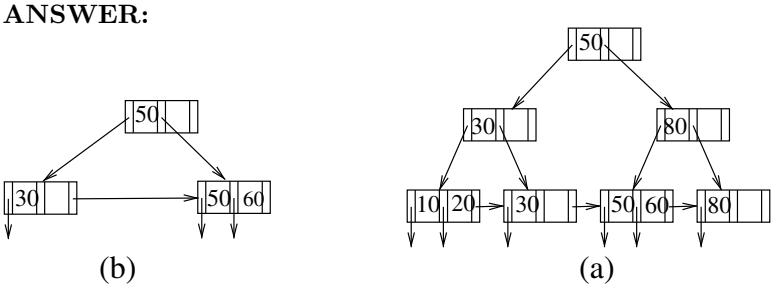
1. Consider the following two B+trees for this problem.



   (a) Show the final B+tree structure after we insert 60, 20, and 80 into Figure (a) in the given order.

   (b) Show the final B+tree structure after we delete 20, 10, and 70 from Figure (b) in the given order.

   ANSWER:



2. Consider a B+tree that indexes 300 records. Assume that $n = 5$ for this B+tree (i.e., each node has at most 5 pointers), what is the minimum and maximum height (depth) of the tree? (A tree with only the root node has a height of 1.)

   ANSWER:

   Minimum 4. (maximum 4 record pointers per node at leaf. $\lceil 300/4 \rceil = 75$ leaf nodes are needed when full. maximum branching factor 5 at non-leaf nodes. $\lceil 75/5 \rceil = 15$ nodes are needed at level 2. $\lceil 15/5 \rceil = 3$ nodes are needed at level 3. One more level of root node that points to these three nodes.)

   Maximum 5. (minimum 2 record pointers per node at leaf. $\lfloor 300/2 \rfloor = 150$ leaf nodes. minimum branching factor 3 at non-leaf nodes. $\lfloor 150/3 \rfloor = 50$ nodes at level 2. $\lfloor 50/3 \rfloor = 16$ nodes at level 3. $\lfloor 16/3 \rfloor = 5$ nodes at level 4. $\lfloor 5/3 \rfloor = 1$ nodes at level 5. Since there is only one node at level 5, this is the root node.)

3. Consider the following key values:

```
106, 115, 916, 0, 96, 126, 16, 15, 31
```

These keys are to be inserted in the above order into an (initially empty) extendible hash table. The hash function $h(n)$ for key n is $h(n) = n \bmod 256$; that is, the hash value is the remainder when the key value is divided by 256 (2^8). Thus, the hash value is an 8-bit value. Each block can hold 3 data items. Draw the extendible hash table after all data items are inserted. Show the keys themselves in the buckets, not the hash value. The bucket numbers are drawn from the bits at the high order end of the hash value. Be sure to indicate i for the directory, the number of hash value bits used. Also indicate i for each bucket, the number of hash function bits that are used for that bucket.

ANSWER:

```
Data: 106, 115, 916, 0, 96, 126, 16, 15, 31

Keys:

H(106) = 106 = 01101010
H(115) = 115 = 01110011
H(916) = 148 = 10010100
H(0)   =   0 = 00000000
H(96)  =  96 = 01100000
H(126) = 126 = 01111110
H(16)  =  16 = 00010000
H(15)  =  15 = 00001111
H(31)  =  31 = 00011111

The cells with no pointers point to empty buckets
```