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CS 161 HW5

1. a) $P \Rightarrow \neg Q$, $Q \Rightarrow \neg P$

	P	Q	$P \Rightarrow \neg Q$	$Q \Rightarrow \neg P$
w_1	T	T	F	F
w_2	T	F	T	T
w_3	F	T	T	T
w_4	F	F	T	T

- To prove the sentences are equivalent, their models should be the same.

$M(P \Rightarrow \neg Q) = \{w_2, w_3, w_4\}$

$M(Q \Rightarrow \neg P) = \{w_2, w_3, w_4\}$

- Since their models are the same, we know the sentences are equivalent.

b) $P \Leftrightarrow \neg Q$, $((P \wedge \neg Q) \vee (\neg P \wedge Q))$

	P	Q	$P \Leftrightarrow \neg Q$	$((P \wedge \neg Q) \vee (\neg P \wedge Q))$	
w_1	T	T	F	F	F ∨ F
w_2	T	F	T	T	T ∨
w_3	F	T	T	T	F ∨ T
w_4	F	F	F	F	F ∨ F

$M(P \Leftrightarrow \neg Q) = \{w_2, w_3\}$

$M(((P \wedge \neg Q) \vee (\neg P \wedge Q))) = \{w_2, w_3\}$

- Since their models are the same, we know the sentences are equivalent.

2. a) $(Smoke \Rightarrow Fire) \Rightarrow (\neg Smoke \Rightarrow \neg Fire)$

	Smoke	Fire	Statement	
w_1	T	T	T	$(T) \Rightarrow (T)$
w_2	T	F	T	$(F) \Rightarrow (T)$
w_3	F	T	F	$(T) \Rightarrow (F)$
w_4	F	F	T	$(T) \Rightarrow (T)$

- The sentence is not valid, because the statement did not hold true for all worlds (it didn't hold in w_3).
- The sentence is satisfiable because it holds in w_1, w_2, w_3.

2b) (Smoke $\Rightarrow$ Fire) $\Rightarrow$ ((Smoke $\vee$ Heat) $\Rightarrow$ Fire)

	Smoke	Fire	Heat	Statement		
w_1	T	T	T	T		
w_2	T	T	F	T	(T) $\Rightarrow$ (T $\Rightarrow$ T)	T $\Rightarrow$ T
w_3	T	F	T	T	(F) $\Rightarrow$ (T $\Rightarrow$ F)	F $\Rightarrow$ F
w_4	T	F	F	T	(F) $\Rightarrow$ (T $\Rightarrow$ F)	
w_5	F	T	T	T	(T) $\Rightarrow$ (T $\Rightarrow$ T)	
w_6	F	T	F	T	(T) $\Rightarrow$ (F $\Rightarrow$ T)	
w_7	F	F	T	F	(T) $\Rightarrow$ (T $\Rightarrow$ F)	
w_8	F	F	F	T	(T) $\Rightarrow$ (F $\Rightarrow$ F)	

- This sentence is not valid since it doesn't hold for w_7
- This sentence is satisfiable since it holds everywhere except w_7

c) ((Smoke $\wedge$ Heat) $\Rightarrow$ Fire) $\Leftrightarrow$ ((Smoke $\Rightarrow$ Fire) $\vee$ (Heat $\Rightarrow$ Fire))

Smoke	Fire	Heat	Statement	
T	T	T	T	(T) $\Leftrightarrow$ (T)
T	T	F	T	(T) $\Leftrightarrow$ (T)
T	F	T	T	(F) $\Leftrightarrow$ (F)
T	F	F	T	(T) $\Leftrightarrow$ (F $\cup$ T)
F	T	T	T	(T) $\Leftrightarrow$ (T $\vee$
F	T	F	T	(T) $\Leftrightarrow$ (T
F	F	T	T	(T) $\Leftrightarrow$ (T
F	F	F	T	(T) $\Leftrightarrow$ (T

- This sentence is valid since it holds in all worlds
- This sentence is satisfiable since it holds in all worlds

$\vee$ or $\wedge$ and

$(\vee) \wedge (\vee)$

3. a) Set variables to be used in the knowledge base

A = Mythical C = Mammal E = Magical

B = Immortal D = Horned

① $A \Rightarrow B$ ③ $(B \vee C) \Rightarrow D$

② $\neg A \Rightarrow (\neg B \wedge C)$ ④ $D \Rightarrow E$

b) ① $A \Rightarrow B$

$\boxed{\neg A \vee B}$

② $\neg A \Rightarrow (\neg B \wedge C)$

$A \vee (\neg B \wedge C)$

$\boxed{(A \vee \neg B) \wedge (A \vee C)}$

③ $(B \vee C) \Rightarrow D$

$\neg(B \vee C) \vee D$

$(\neg B \vee \neg C) \vee D$

$\boxed{(\neg B \vee D) \wedge (\neg C \vee D)}$

④ $D \Rightarrow E$

$\boxed{\neg D \vee E}$

CNF: $(\neg A \vee B) \wedge (A \vee \neg B) \wedge (A \vee C) \wedge (\neg B \vee D) \wedge (\neg C \vee D) \wedge (\neg D \vee E)$

c) i) Prove the unicorn is mythical $(\alpha = A)$

① $\neg A \vee B$

② $A \vee \neg B$

③ $A \vee C$

④ $\neg B \wedge D$

⑤ $\neg C \wedge D$

⑥ $\neg D \vee E$

$\neg\alpha$ ⑦ $\neg A$

⑧ [2+7] $\neg B$

⑨ [3+7] C

⑩ [5+9] D

⑪ [6+10] E

There are no more rules that can be applied. There are no contradictions, so $\Delta \wedge \neg\alpha$ is satisfiable. $\Delta \nvDash \alpha$, so we can not use our knowledge base to prove that the unicorn is mythical.

ii) Prove that the unicorn is magical ($\alpha = E$)

① $\neg A \vee B$

② $A \vee \neg B$

③ $A \vee C$

④ $\neg B \vee D$

⑤ $\neg C \vee D$

⑥ $\neg D \vee E$

$\neg\alpha$ ⑦ $\neg E$

⑧ [6+7] $\neg D$

⑨ [5+8] $\neg C$

⑩ [4+8] $\neg B$

⑪ [1+10] $\neg A$ } contradiction

⑫ [3+9] A

There is a contradiction [$A, \neg A$] so we know that $\Delta \wedge \neg\alpha$ is unsatisfiable. This means that $\Delta \models \alpha$, and we can use our knowledge base to prove that the unicorn is magical.

iii) Prove that the unicorn is horned ($\alpha = D$)

① $\neg A \vee B$

② $A \vee \neg B$

③ $A \vee C$

④ $\neg B \vee D$

⑤ $\neg C \vee D$

⑥ $\neg D \vee E$

$\neg\alpha$ ⑦ $\neg D$

⑧ [5+7] $\neg C$

⑨ [4+7] $\neg B$

⑩ [3+8] A } contradiction

⑪ [1+9] $\neg A$

There is a contradiction [$A, \neg A$] so we know that $\Delta \wedge \neg\alpha$ is unsatisfiable. This means that $\Delta \models \alpha$, and we can use our knowledge base to prove that the unicorn is horned.

(Would it be a unicorn if it wasn't horned??)

4. a) Figure 1

Decomposable - the sub-circuits feeding into the and-gates do not share any of the same variables

Not Deterministic - the OR at the very top of the circuit does not have at most 1 true input.

AND #1: $[(\neg A \wedge B) \vee (A \wedge \neg B)] \wedge [C \vee (\neg C \wedge \neg D)]$

AND #2: $[A \vee (\neg A \wedge \neg B)] \wedge [(C \wedge \neg D) \vee (\neg C \wedge D)]$

If we set $\{A = T, B = F, C = T, D = F\}$ then both AND #1 and AND #2 are T, so there are <u>two</u> true inputs to the OR.

Not Smooth - the second OR at depth 3 has variables $\{C\}$ and $\{C, D\}$, so we conclude that the values are not shared. The third OR at depth 3 has variables $\{A\}$ and $\{A, B\}$, so we conclude that the values are not shared.

b) Figure 2

Decomposable - the sub-circuits feeding into the AND-gates do not share any of the same variables

Not deterministic - the first and third OR at depth 3 do not have at most one true input.

OR #1 : $(\neg A \wedge B) \vee (\neg A \vee B)$

$\hookrightarrow A = F,\ B = T \Rightarrow OR\ \#1 = (T) \vee (T)$

OR #3: $(\neg A \wedge B) \vee (\neg A \vee B)$

$\hookrightarrow$ same as OR #1

These both have 2 true inputs to the OR gate.

Smooth - the sub-circuits feeding into the AND gates share the same variables

5. $w(A) = 0.1$ $w(\neg A) = 0.9$

$w(B) = 0.3$ $w(\neg B) = 0.7$

$w(C) = 0.5$ $w(\neg C) = 0.5$

$w(D) = 0.7$ $w(\neg D) = 0.3$

a) $(\neg A \wedge B) \vee (\neg B \wedge A)$

	A	B	Statement	
w_1	T	T	F	$F \vee F$
w_2	T	F	T	$F \vee T$
w_3	F	T	T	$T \vee$
w_4	F	F	F	$F \vee F$

$WMC = w_2 \, w_3$

$WMC = [w(A)w(\neg B)] + [w(\neg A)w(B)]$

$WMC = [0.1 \times 0.7] + [0.9 \times 0.3]$

$\boxed{WMC = 0.34}$

b) AND : product OR : Sum

- AND #1 : $w(\neg A)w(B) = 0.9 \times 0.3 = 0.27$
- AND #2 : $w(\neg B)w(A) = 0.7 \times 0.1 = 0.07$
- OR : $0.27 + 0.07 = 0.34$

The count of the root yields the same result that we found when using the WMC. Both methods arrived at the value 0.34.

c) $\{[(\neg A \wedge B) \vee (\neg B \wedge A)] \wedge [(C \wedge D) \vee (\neg D \wedge \neg C)]\} \vee$
$\{[(\neg A \wedge \neg B) \vee (B \wedge A)] \wedge [(C \wedge \neg D) \vee (D \wedge \neg C]\}$

$\Downarrow$

$\{[(0.9 \times 0.3) + (0.7 \times 0.1)] \times [(0.5 \times 0.7) + (0.3 \times 0.5)]\} +$
$\{[(0.9 \times 0.7) + (0.3 \times 0.1)] \times [(0.5 \times 0.3) + (0.7 \times 0.5)]\}$

$\Downarrow$

$\boxed{WMC = 0.5}$