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CS 161 HW7

1. $Pr(\alpha_1, \ldots, \alpha_n | \beta) = Pr(\alpha_1 | \alpha_2 \ldots \alpha_n, \beta) Pr(\alpha_2 | \alpha_3, \ldots, \alpha_n, \beta) \ldots Pr(\alpha_n | \beta)$

Base Case

- $n = 1$ $\quad Pr(\alpha_1 | \beta) = Pr(\alpha_1 | \beta)$
- $n = 2$ $\quad Pr(\alpha_1, \alpha_2 | \beta) = Pr(\alpha_1 | \alpha_2, \beta) Pr(\alpha_2 | \beta)$ [Bayes Conditioning Rule]

Inductive Step

- Assume $Pr(\alpha_1, \ldots, \alpha_n | \beta) = Pr((\alpha_1, \ldots, \alpha_{n-1}), \alpha_n | \beta)$ is true for all values of n between 1 and $n-1$
- This means that $Pr(\alpha_1, \ldots, \alpha_n | \beta) = Pr(\alpha_1, \ldots, \alpha_{n-1} | \alpha_n, \beta) Pr(\alpha_n | \beta)$ [BCR]
- Now apply our assumption

$Pr(\alpha_1, \ldots \alpha_{n-1} | \alpha_n, \beta) = Pr(\alpha_1 | \alpha_2 \ldots \alpha_n, \beta) Pr(\alpha_2 | \alpha_3, \ldots, \alpha_n, \beta) \ldots Pr(\alpha_{n-1} | \alpha_n, \beta)$

$Pr(\alpha_1, \ldots \alpha_n | \beta) = Pr(\alpha_1 | \alpha_2, \ldots, \alpha_n, \beta) Pr(\alpha_2 | \alpha_3, \ldots, \alpha_n, \beta) \ldots Pr(\alpha_n | \beta)$ ✓

We have proved the identity.

2. Oil = O Gas = G (Positive) Test = T

$P(O) = 0.5$ $P(\neg O) = 0.5$

$P(G) = 0.2$ $P(\neg G) = 0.8$

$P(\neg O \land \neg G) = 0.3$ $P(O \lor G) = 0.7$

$P(T \mid O) = 0.9$ $P(\neg T \mid O) = 0.1$

$P(T \mid G) = 0.3$ $P(\neg T \mid G) = 0.7$

$P(T \mid \neg O \land \neg G) = 0.1$ $P(\neg T \mid \neg O \land \neg G) = 0.9$

Want to Find: $P(O \mid T)$

Use Bayes Rule

$$P(O \mid T) = \frac{P(T \mid O)\,P(O)}{P(T)} \Longrightarrow P(O \mid T) = \frac{(0.9)(0.5)}{P(T)}$$

must solve for $P(T)$

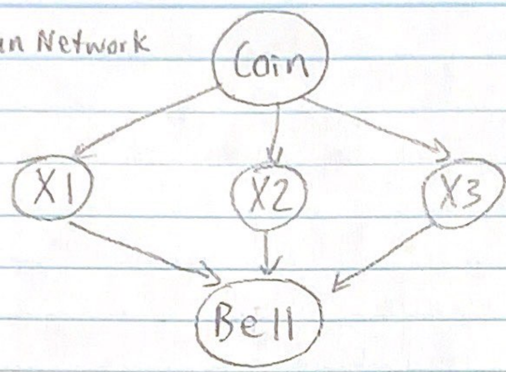
- perform case analysis for $P(T)$

$$P(T) = P(T \mid O)P(O) + P(T \mid G)P(G) + P(T \mid \neg O \land \neg G)P(\neg O \land \neg G)$$
$$= (0.9 \times 0.5) + (0.3 \times 0.2) + (0.1 \times 0.3)$$
$$= 0.45 + 0.06 + 0.03$$
$$= 0.54$$

$$P(O \mid T) = \frac{(0.9)(0.5)}{0.54} = \frac{0.45}{0.54}$$

$$\boxed{P(O \mid T) = 0.8\overline{333} \approx 83.3\%}$$

3. Bayesian Network



Coin : {a, b, c}

X_1 : {H, T} X_2 : {H, T} X_3 : {H, T}

Bell : {T, F} (T - Bell rings "on", F - Bell doesn't ring)

Coin	θ_{coin}
a	1/3
b	1/3
c	1/3

Coin	X_1	$\theta_{X_1 \mid coin}$
a	H	0.2
a	T	0.8
b	H	0.4
b	T	0.6
c	H	0.8
c	T	0.2

Coin	X_2	$\theta_{X_2 \mid coin}$
a	H	0.2
a	T	0.8
b	H	0.4
b	T	0.6
c	H	0.8
c	T	0.2

Coin	X_3	$\theta_{X_3 \mid coin}$
a	H	0.2
a	T	0.8
b	H	0.4
b	T	0.6
c	H	0.8
c	T	0.2

X_1	X_2	X_3	Bell	$\theta_{Bell \mid x_1, x_2, x_3}$
H	H	H	T	1
H	H	H	F	0
H	H	T	T	0
H	H	T	F	1
H	T	H	T	0
H	T	H	F	1
H	T	T	T	0
H	T	T	F	1
T	H	H	T	0
T	H	H	F	1
T	H	T	T	0
T	H	T	F	1
T	T	H	T	0
T	T	H	F	1
T	T	T	T	1
T	T	T	F	0

4. a)
- I(A, ∅, BE)
- I(B, ∅, AC)
- I(C, A, BDE)
- I(D, AB, CE)
- I(E, B, ACDFG)
- I(F, CD, ABE)
- I(G, F, ABCDEH)
- I(H, FE, ABCDG)

b) ① d_separated(A, F, E)

A → D ← B [open]

D ← B → E [open]

Path is open. $\boxed{\text{FALSE}}$

② d_separated(G, B, E)

Path 1 - [G, F, D, B, E]

G ← F ← D [open]

F ← D ← B [open]

D ← B → E [closed - B is known] Blocked

Path 2 - [G, F, H, E]

G ← F → H [open]

F → H ← E [closed - H is descendant of B, which is known] Blocked

Path 3 - [G, F, C, A, D, B, E]

This path eventually reaches D ← B → E, which we determined was blocked in path 1. Thus path 3 is Blocked.

Since all possible paths are blocked, we conclude $\boxed{\text{True}}$

③ dsep(AB, CDE, GH)

C, D, and E are all sequential values. Every possible path from AB to GH passes through these sequential values. C, D, and E are all known, meaning these three sequential values are blocked. Thus all paths from AB to GH are also blocked. $\boxed{\text{True}}$

c) $Pr(a, b, c, d, e, f, g, h)$

$\boxed{Pr(a)Pr(b)Pr(c|a)Pr(d|a,b)Pr(e|b)Pr(f|c,d)Pr(h|e,f)Pr(g|f)}$

d) ① $Pr(A=1, B=1)$

↳ $Pr(A=1) \wedge Pr(B=1)$ Note: A and B are independent

$Pr(A=1) \times Pr(B=1)$ [see Markovian Assumptions]

$0.2 \times 0.7 = 0.14$

$\boxed{Pr(A=1, B=1) = 0.14}$

② $Pr(E=0 | A=0)$

- we can't directly find values for these like in part 1
- Need to use Bayes Conditioning Rules

$$Pr(E=0 | A=0) = \frac{Pr(E=0, A=0)}{Pr(A=0)}$$

- A and E are also independent [see Markovian assumptions]

$Pr(E=0, A=0) \rightarrow Pr(E=0) \times Pr(A=0)$

- $\frac{Pr(E=0)Pr(A=0)}{Pr(A=0)} = Pr(E=0)$

- Now solve for $Pr(E=0)$ using Case Analysis (since it isn't explicitly given)

$Pr(E=0) = [Pr(E=0 | B=0) Pr(B=0)] + [Pr(E=0 | B=1) Pr(B=1)]$

$= [0.1 \times 0.3] + [0.9 \times 0.7]$

$= 0.03 + 0.63$

$= 0.66$

$\boxed{Pr(E=0 | A=0) = 0.66}$

5.

	A	B	Pr(A,B)	A ⇒ B
w_0	T	T	0.3	T
w_1	T	F	0.2	F
w_2	F	T	0.1	T
w_3	F	F	0.4	T

$\alpha : A \Rightarrow B$

a) $\boxed{M(\alpha) = \{w_0, w_2, w_3\}}$

b) $Pr(\alpha) = 0.3 + 0.1 + 0.4 = 0.8$

$\boxed{Pr(\alpha) = 0.8}$

c) $Pr(A, B \mid \alpha)$

- "Given that α is True" → anything where α = False becomes 0
- Normalize the rest by dividing by $Pr(\alpha)$

	A	B	α	$Pr(A, B \mid \alpha)$	
w_0	T	T	T	0.375	$0.3/0.8$
w_1	T	F	F	0	
w_2	F	T	T	0.125	$0.1/0.8$
w_3	F	F	T	0.5	$0.4/0.8$

↳ Probabilities add up to 1. ✓

d) $Pr(A \Rightarrow \neg B \mid \alpha)$

	A	B	$A \Rightarrow \neg B$	$Pr(A, B \mid \alpha)$
w_0	T	T	F	0.375
w_1	T	F	T	0
w_2	F	T	T	0.125
w_3	F	F	T	0.5

→ probability of A and B having these values given that α is true

$M(A \Rightarrow \neg B) = \{w_1, w_2, w_3\}$

$Pr(A \Rightarrow \neg B \mid \alpha) = 0 + 0.125 + 0.5$

$\boxed{Pr(A \Rightarrow \neg B \mid \alpha) = 0.625}$