Homework 6. Due December 4th, 9:59AM.

CS181: Fall 2021

Guidelines:

- Upload your assignments to Gradescope by 9:59 PM.
- Follow the instructions mentioned on the course webpage for uploading to Gradescope very carefully (including starting each problem on a new page and matching the pages with the assignments); this makes it easy and smooth for everyone. As the guidelines are simple enough, bad uploads will not be graded.
- You may use results proved in class without proofs as long as you state them clearly.
- Most importantly, make sure you adhere to the policies for academic honesty set out on the course webpage. The policies will be enforced strictly. Homework is a stepping stone for exams; keep in mind that reasonable partial credit will be awarded and trying the problems will help you a lot for the exams.
- Note that we have a **modified grading scheme for this assignment**: A sincere attempt will get you 100% of the credit and a reasonable attempt will get you 50% for each problem. Nevertheless, please attempt the problems honestly and write down the solutions the best way you can - this is really the most helpful way to flex your neurons in preparation for the exam.
- All problem numbers correspond to our text 'Introduction to Theory of Computation' by Boaz Barak. So, exercise a.b refers to Chapter a, exercise b.

1. Give a quantified integer statement to express the following: [2 points]

   (a) "There are an infinite number of primes". [Hint: First design a logical statement (that only uses arithmetic operations and other quantified integer variables) with one free variable p, $Prime(p)$ that is true only when p is a prime. You can then combine this expression with an idea similar to examples we saw in class.]

   (b) "1729 is the smallest natural number that can be expressed as sum of two cubes in two different ways".

2. Exercise 11.2, part (a). [2 points]

   [Hint: Try to find a reduction from any of the problems in class. For example, you can try to reduce from NOTEMPTY. Can you find a reduction R such that for any Turing machine M, there exists an input w such that $M(w) = 1$ if and only if $Findproof(R(M)) = 1$?]