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$p = 13$

CS 181 HW6

1. a) $\forall n \; \exists p, x, y \; \neg(x * y = p) \wedge \neg(x = 1) \wedge \neg(x = p)$

There exists infinitely many prime numbers "p" such that there is not any x, y where $\neg xy = p$ and $x \neq 1$ and $x \neq p$.

The key is to not allow one of the numbers (x) be 1 or p, as this will violate the constraint of the prime numbers.

b) "1729 is the smallest natural number that can be expressed as a sum of two cubes in two different ways."

$$P(n) \equiv \exists a,b,c,d \; (n = a*a*a + b*b*b) \wedge (n = c*c*c + d*d*d) \wedge \neg(a = c) \wedge \neg(a = d)$$

$\left[a^3 + b^3 \wedge c^3 + d^3 \right.$
$\left. a \neq c, \; a \neq d \right]$
to ensure at least one number is different

$$P(1729) \wedge (\forall n ((n < 1729) \Rightarrow \neg P(n)))$$

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2. FINDPROOF : $\{0,1\}^* \to \{0,1\}$

V = a Turing machine (a verifier) x = a string

$$FINDPROOF(V,x) = \begin{cases} 1 & \text{i.f.f there exists } w \text{ such that } V(x,w)=1 \\ 0 & \text{else} \end{cases}$$

X is a statement. w is a proof.

$V(x,w) = 1$ if the statement/proof are valid.

$FINDPROOF(V,x) = 1$ if a proof exists for the statement.

- Intuitively, this is uncomputable due to Godel's First Incompleteness Theorem. There will always be true statements with no proof.

Reduction using NOTEMPTY

- For any TM M:

$$NOTEMPTY(M) = \begin{cases} 1 & \text{if there exists } x \text{ such that } M(x)=1 \\ 0 & \text{else} \end{cases}$$

- Reduce NOTEMPTY to FINDPROOF

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def N(z,w):
    Return EVAL(M,w)        # Run M on input w
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$R(M) = (N, 0)$

$NOTEMPTY(M) = FINDPROOF(R(M))$

Case 1 There exists a w such that $M(w) = 1$

- $N(0,w)$ returns 1
- $FINDPROOF(R(M)) = FINDPROOF(N,0) = 1$
- $NOTEMPTY(M) = 1$ since $M(w)=1$

Case 2 There does not exist a w such that $M(w)=1$

- $N(0,w)$ does not halt (let's say it returns 0)
- $FINDPROOF(R(M)) = FINDPROOF(N,0) = 0$
- $NOTEMPTY(M) = 0$ since $M(w) = 0$ (did not halt)

Done