Ethan Wong
11/2/21
305314001

CS 181 HW3

1. For any NFA N, there exists an equivalent NFA N' with only one accept state. This can be accomplished by having the 0,1 output from the start state loop back to itself. There is also an outgoing ε edge from the start state to the rest of the NFA. The rest of the NFA includes whatever you are looking for and additional states with 0,1 output until the final state. This NFA essentially "guesses" until it finds the desired result and fails if it is unable to "guess" the result.

2. RegEx (r) has length m:
The base cases are that 0 is a regex, 1 is a regex, and ε is a regex. It takes at most two states to represent these. The compound cases then include concatenation, union, and Kleene*. These operations will at most double the number of states. Thus, the regex is O(2m), which simplifies to O(m).

3. L = {x : x has an equal number of 10's and 01's}
$\left\{\begin{array}{l} w = 1,\ w = 0,\ w = \varepsilon \rightarrow 0 \text{ instances of both} \\ w \text{ starts with } 0 \text{ and ends with } 0 \\ w \text{ starts with } 1 \text{ and ends with } 1 \end{array}\right.$

$(1|0|\varepsilon|0(0|1)^*0|1(0|1)^*1)$

00 000 $\underline{010}$ $\underline{0110}$ 0101010 $\underline{011110}$
$\underline{010001010}$

regular - computable by DFA

4. a) $L = \{x : x = 0^m 1^n \quad m > n\}$

This is basically another way of expressing the MAJ problem.

- Suppose that L is regular
  - ↳ There exists a P such that Pumping Lemma holds
  - ↳ Suppose $x = abc$ such that properties of PL hold
    - → $b \cdot c$ has only 1's
    - → $a \cdot b \cdot b \cdot c$ has more 1's than 0's
    - → $a \cdot b \cdot b \cdot c$ is not in L because there are more 1's than 0's in the string, leading to a contradiction.

Therefore L is not regular

b) $F : \{0,1\}^* \to \{0,1\}$

$$F(x) = \begin{cases} 1 & \text{iff } x = 1^{3^i} \text{ for some } i > 0 \\ 0 & \text{else} \end{cases}$$

1

111

111111111.

- Basically, F returns 1 if x is a string with "power of three" 1's
- Suppose that F is regular
  - ↳ There exists a P such that Pumping Lemma holds
  - ↳ Suppose $x = abc$ such that properties of PL holds
    - → $a^{\frac{p}{2}} \cdot b^{\frac{p}{2}} \cdot c^{p}$ where all $a = 1$, all $b = 1$, all $c = 1$ and $F(abc) = 1$
    - → $a \cdot b \cdot b \cdot c$ as input would not return 1 since we are only doubling the length of the middle portion
    - → i.e input = $\underbrace{111}_{a}\underbrace{111}_{b}\underbrace{111}_{c}$, returns 1

$a \cdot b \cdot b \cdot c$ = $\underbrace{111}_{a}\underbrace{111111}_{b}\underbrace{111}_{c}$, returns 0

Therefore F is not regular

c) $ADD = \{0^m 1 0^n 1 0^{m+n} : m,n \geq 1\}$

- some 0's, a 1, same 0's, a 1, lots of 0's
- Suppose that ADD is regular

↳ There exists P such that Pumping Lemma holds

↳ Suppose $x = a \cdot b \cdot c$ such that properties of L hold

Ex : $\underbrace{0^{\frac{P}{2}} 1 0^{\frac{P}{2}-1}}_{ab \leq P} \underbrace{1 0^{P-1}}_{c}$

- b includes a 1 in it
- abc is in the language
- $x = a \cdot b \cdot b \cdot c$ is not in the language because there will be 3 1's in x, and this fails the specification of the language.
- Therefore ADD is not regular