I completed this assignment entirely on my own.

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CS M51A
DIS 1D

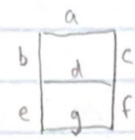
Assignment 2

1\.

Truth Table

Key
1 : ON
0 : OFF

#	w	x	y	z	a	b	c	d	e	f	g
0	0	0	0	0	1	1	1	0	1	1	1
1	0	0	0	1	0	0	1	0	0	1	0
2	0	0	1	0	1	0	1	1	1	0	1
3	0	0	1	1	1	0	1	1	0	1	1
4	0	1	0	0	0	1	1	1	0	1	0
5	0	1	0	1	1	1	0	1	0	1	1
6	0	1	1	0	1	1	0	1	1	1	1
7	0	1	1	1	1	0	1	0	0	1	0
8	1	0	0	0	1	1	1	1	1	1	1
9	1	0	0	1	1	1	1	1	0	1	0



Karnaugh Maps

ⓐ

Sum of Products (SoP)

	y'	y'	y	y	
w'	1	0	1	1	x'
w'	0	1	1	1	x
w	X	X	X	X	x
w	1	1	X	X	x'
	z'	z	z	z'	

Product of Sums (PoS)

	y'	y'	y	y	
w'	1	0	1	1	x'
w'	0	1	1	1	x
w	X	X	X	X	x
w	1	1	X	X	x'
	z'	z	z	z'	

SoP: $a = (w) + (y) + (x \cdot z) + (x' \cdot z')$
PoS: $a = (x' + y + z) \cdot (w + x + y + z')$

ⓑ

(SoP)

	y'	y'	y	y	
w'	1	0	0	0	x'
w'	1	1	0	1	x
w	X	X	X	X	x
w	1	1	X	X	x'
	z'	z	z	z'	

(PoS)

	y'	y'	y	y	
w'	1	0	0	0	x'
w'	1	1	0	1	x
w	X	X	X	X	x
w	1	1	X	X	x'
	z'	z	z	z'	

SoP: $b = (w) + (x \cdot y') + (y' \cdot z') + (x \cdot z')$

PoS: $b = (x + y') \cdot (y' + z') \cdot (w + x + z')$

(c)

(S∘P)

	y'	y'	y	y	
w'	1	1	1	1	x'
w'	1	0	1	0	x
w	X	X	X	X	x
w	1	1	X	X	x'
	z'	z	z	z'	

(P∘S)

	y'	y'	y	y	
w'	1	1	1	1	x'
w'	1	0	1	0	x
w	X	X	X	X	x
w	1	1	X	X	x'
	z'	z	z	z'	

S∘P: $c = (x') + (y \cdot z) + (y' \cdot z')$

P∘S: $c = (x' + y + z') \cdot (x' + y' + z)$

(d)

(S∘P)

	y'	y'	y	y	
w'	0	0	1	1	x'
w'	1	1	0	1	x
w	X	X	X	X	x
w	1	1	X	X	x'
	z'	z	z	z'	

(P∘S)

	y'	y'	y	y	
w'	0	0	1	1	x'
w'	1	1	0	1	x
w	X	X	X	X	x
w	1	1	X	X	x'
	z'	z	z	z'	

S∘P: $d = (w) + (x \cdot y') + (x' \cdot y) + (y \cdot z')$

P∘S: $d = (w + x + y) \cdot (x' + y' + z')$

(e)

(S∘P)

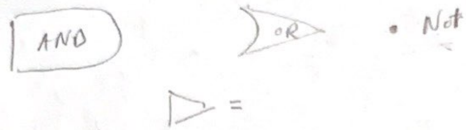
	y'	y'	y	y	
w'	1	0	0	1	x'
w'	0	0	0	1	x
w	X	X	X	X	x
w	1	0	X	X	x'
	z'	z	z	z'	

(P∘S)

	y'	y'	y	y	
w'	1	0	0	1	x'
w'	0	0	0	1	x
w	X	X	X	X	x
w	1	0	X	X	x'
	z'	z	z	z'	

S∘P: $e = (x' \cdot z') + (y \cdot z')$

P∘S: $e = (z') \cdot (x' + y)$



(f)

(SoP)

	y'	y'	y	y	
w'	1	1	1	0	x'
w'	1	1	1	1	x
w	X	X	X	X	x
w	1	1	X	X	x'
	z'	z	z	z'	

	y'	y'	y	y	
w'	1	1	1	0	x'
w'	1	1	1	1	x
w	X	X	X	X	x
w	1	1	X	X	y'
	z'	z	z	z'	

SoP: $f = x + y' + z$

PoS: $f = x + y' + z$

(g)

(SoP)

	y'	y'	y	y	
w'	1	0	1	1	x'
w'	0	1	0	1	x
w	X	X	X	X	x
w	1	0	X	X	x'
	z'	z	z	z'	

(PoS)

	y'	y'	y	y	
w'	1	0	1	1	x'
w'	0	1	0	1	x
w	X	X	X	X	x
w	1	0	X	X	x'
	z'	z	z	z'	

SoP: $g = (x' \cdot y) + (x' \cdot z') + (y \cdot z') + (x \cdot y' \cdot z)$

PoS: $g = (x + y + z') \cdot (x' + y + z) \cdot (x' + y' + z')$

2. $f1 = A \cdot B'$ (A AND NOT B)

NOT : $1 \cdot A' \to A'$ ✓	NOT : $1 \; f1 \; A \to A'$ ✓
AND : $(1 \cdot B') \to B'$	AND : $1 \; f1 \; B = B'$
$(A \cdot (B')') = A \cdot B$ ✓	$A \; f1 \; B' = A \cdot B$
OR : $(1 \cdot B') = B'$	OR: $1 \; f1 \; B = B'$
$(A \cdot (B')') = A \cdot B$	$A \; f1 \; (1 \; f1 \; B) = A \cdot B$
$(1 \cdot (A \cdot B)') = (A \cdot B)'$	$1 \; f1 \; (A \cdot B) = (A \cdot B)'$
$(A \cdot B)' \to A + B$ ✓	$(A \cdot B)' \to A + B$ ✓

* For the proof of "OR", the derivation reached a "NAND". Since NAND itself is universal, that means "OR" can be achieved, making f1 universal.

$f2 = A' \cdot B$ (NOT A AND B)

NOT : $A' \cdot 1 \to A''$ ✓	NOT : $A \; f2 \; 1 \to A'$ ✓
AND : $(A' \cdot 1) = A'$	AND : $A \; f2 \; 1 = A'$
$((A')' \cdot B) = A \cdot B$ ✓	$(A \; f2 \; 1) \; f2 \; B = A \cdot B$ ✓
OR : $(A' \cdot 1) = A'$	$A \; f2 \; 1 = A'$
$((A')' \cdot B = A \cdot B$	$(A \; f2 \; 1) \; f2 \; B = A \cdot B$
$((A \cdot B)' \cdot 1) = (A \cdot B)'$	$[(A \; f2 \; 1) \; f2 \; B] \; f2 \; 1 = (A \cdot B)'$
$(A \cdot B)' \to A + B$ ✓	$(A \cdot B)' \to A \cdot B$ ✓

* For the proof of "OR", the derivation reached a "NAND. Since NAND itself is universal, that means "OR" can be achieved, making f2 universal. f2 is also logically equivalent to f1.

$f3 = A + B'$ (A OR NOT B)

NOT : $0 + A' \rightarrow A'$ ✓	NOT : $0\ f3\ A \rightarrow A'$ ✓
AND : $0 + A' = A'$	AND : $0\ f3\ A = A'$
$A' + B' = (A \cdot B)'$	$(0\ f3\ A)\ f3\ B = (A \cdot B)'$
$0 + (A \cdot B)'' = A \cdot B$ ✓	$0\ f3\ [(0\ f3\ A)\ f3\ B] = A \cdot B$ ✓
OR : $0 + A' = A'$	OR : $0\ f3\ A = A'$
$A' + B' = (A \cdot B)'$ ✓	$(0\ f3\ A)\ f3\ B = (A \cdot B)'$ ✓

* For the proof of "OR", the derivation reached a "NAND". Because NAND is universal, that means OR can be achieved and f3 is universal.

$f4 = A' + B$ (NOT A OR B)

NOT : $A' + 0 \rightarrow A'$ ✓	NOT : $A\ f4\ 0 \rightarrow A'$ ✓
AND : $A' + 0 = A'$	AND : $A\ f4\ 0 = A'$
$B' + A' = (B \cdot A)'$	$B\ f4\ (A\ f4\ 0) = (B \cdot A)'$
$(B \cdot A)'' + 0 = B \cdot A$ ✓	$[B\ f4\ (A\ f4\ 0)]\ f4\ 0 = B \cdot A$ ✓
OR : $A' + 0 = A'$	OR : $A\ f4\ 0 = A'$
$B' + A' = (A \cdot B)'$ ✓	$B\ f4\ (A\ f4\ 0) = (A \cdot B)'$ ✓

* For the proof of "OR", the derivation reached a "NAND". Because NAND is universal, that means OR can be achieved and f4 is universal.
f4 is logically equivalent to f3.

Explanation for how NAND is universal:

$X' = (X \cdot X)' = X \text{ NAND } X$ ✓

$X + Y = ((X+Y)')' = (X' \cdot Y')' = ((X \cdot X)' \cdot (Y \cdot Y)')'$
$= (X \text{ NAND } X) \text{ NAND } (Y \text{ NAND } Y)$ ✓

$X \cdot Y = ((X \cdot Y)')' = ((X \cdot Y)' \cdot (X \cdot Y)')'$
$= (X \text{ NAND } Y) \text{ NAND } (X \text{ NAND } Y)$ ✓