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I completed this assignment entirely on my own, except for discussions with Jiamin Xu

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Assignment 7

2 4-bit inputs

1 start input \rightarrow normally 0, initiates multiplication with pulse to 1

1 clock input

1 8-bit output

Demonstration of Binary Multiplication (with variables)

2bit:

$$\begin{array}{r}
 \begin{array}{cc} & W1 & W0 \\ & \times & X1 & X0 \\ \hline & 0 & W1X1 & W0X0 \\ + & W1X1 & W0X1 & 0 \\ \hline P3 & P2 & P1 & P0 \end{array}
 \end{array}$$

$$P0 = W0X0$$

$$P1 = W1X0 + W0X1$$

$$P2 = W1X1 + C_0 \text{ from } P1$$

$$P3 = C_0 \text{ from } P2$$

4bit:

$$\begin{array}{r}
 \begin{array}{cccccccc} & & & & W3 & W2 & W1 & W0 \\ & & & & \times & X3 & X2 & X1 & X0 \\ \hline 0 & 0 & 0 & 0 & W3X0 & W2X0 & W1X0 & W0X0 & (1) \\ 0 & 0 & 0 & W3X1 & W2X1 & W1X1 & W0X1 & 0 & (2) \\ 0 & 0 & W3X2 & W2X2 & W1X2 & W0X2 & 0 & 0 & (3) \\ + \text{extra} & & W3X3 & W2X3 & W1X3 & W0X3 & 0 & 0 & 0 & (4) \\ \hline P7 & P6 & P5 & P4 & P3 & P2 & P1 & P0 \end{array}
 \end{array}$$

Method of Multiplication

I have shown above two examples of binary multiplication. The logic for the 4-bit multiplication is very similar to that of the 2-bit multiplication. The basic multiplication of each bit is performed with an AND gate because the truth table for AND matches the truth table for multiplication. The addition will be performed by a full-adder.

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Explanation of Overall Circuit Logic

The circuit does not do anything until a user presses the start button. ~~While the start button is not~~ clicked, the circuit will clear itself to ensure its registers are empty. Once the start button is pressed, the circuit will clear itself to make sure it starts fresh and a clock pulse is sent to the 4-bit Parallel load Right Shift Registers. W ($w_3 w_2 w_1 w_0$) gets placed in the top register and X ($x_3 x_2 x_1 x_0$) gets placed in the bottom register. After this is done, the circuit will be in S_1 .

The input no longer matters for the rest of the states. It only mattered for Init. When changing from S_1 to S_2 , the output includes "m", which will tell the circuit to shift the rightmost bit from X and every bit of W into the Multipliers. Since our select inputs S_0 and S_1 are both zero in this stage, the vector MUX receives the T_0 input from Multiplier X_0 .

The clock pulse allows the output to be added with whatever was previously stored in the 8-bit parallel-load register that is connected to the adder (00000000). The second 8-bit parallel register holds the value as well. This will be useful when it is time to output. After this is done, the circuit will be in S_2 .

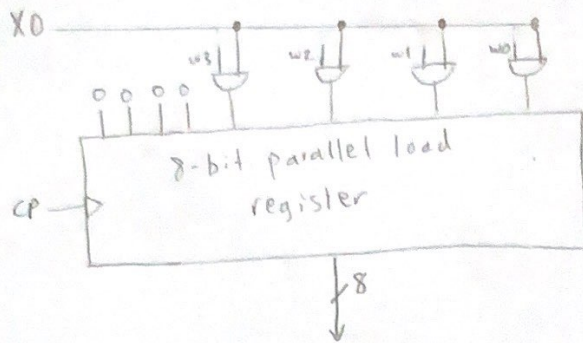
When changing from S_2 to S_3 , the process is almost identical. The only difference is that the S_1 select input is now 1, so the vector MUX selects the T_1 input from Multiplier X_1 . When changing from S_3 to S_4 , the S_0 select input is 1, so the vector Mux selects the T_2 input from Multiplier X_2 .

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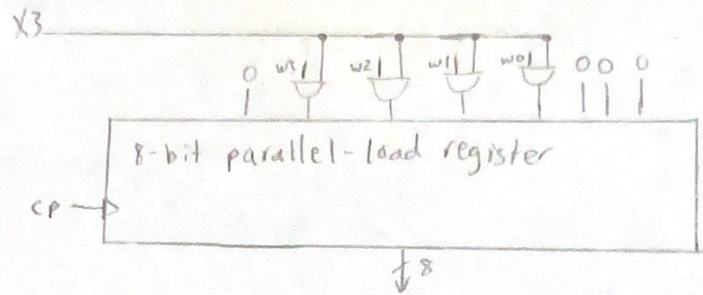
When changing from $S4$ back to initial, both the select inputs are 1. This means the vector MUX receives the $T3$ input from Multiplier $\times 3$.

After all of this is done, the second 8-bit Parallel load register will hold the final product as all the multiplication will have been completed. The multiplication will have been completed with shifts, ANDs, and an adder. We can then use the output clock to clock the product from the 8-bit parallel load register. The process takes 4 total clock pulses. The result will remain available in the register until the next multiplication (when the start button is pressed again).

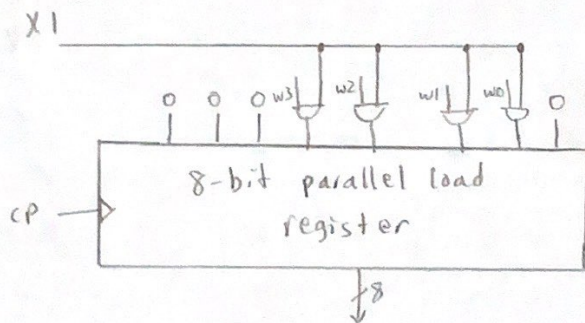
Multiplier X0



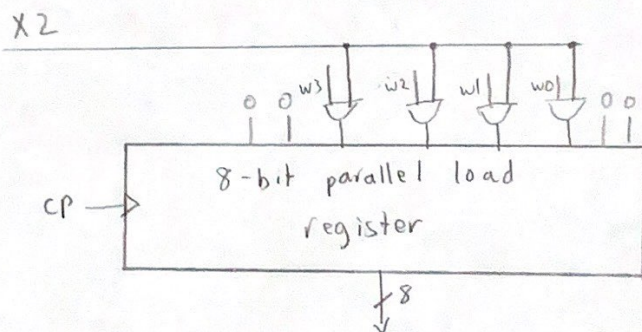
Multiplier X3



Multiplier X1



Multiplier X2

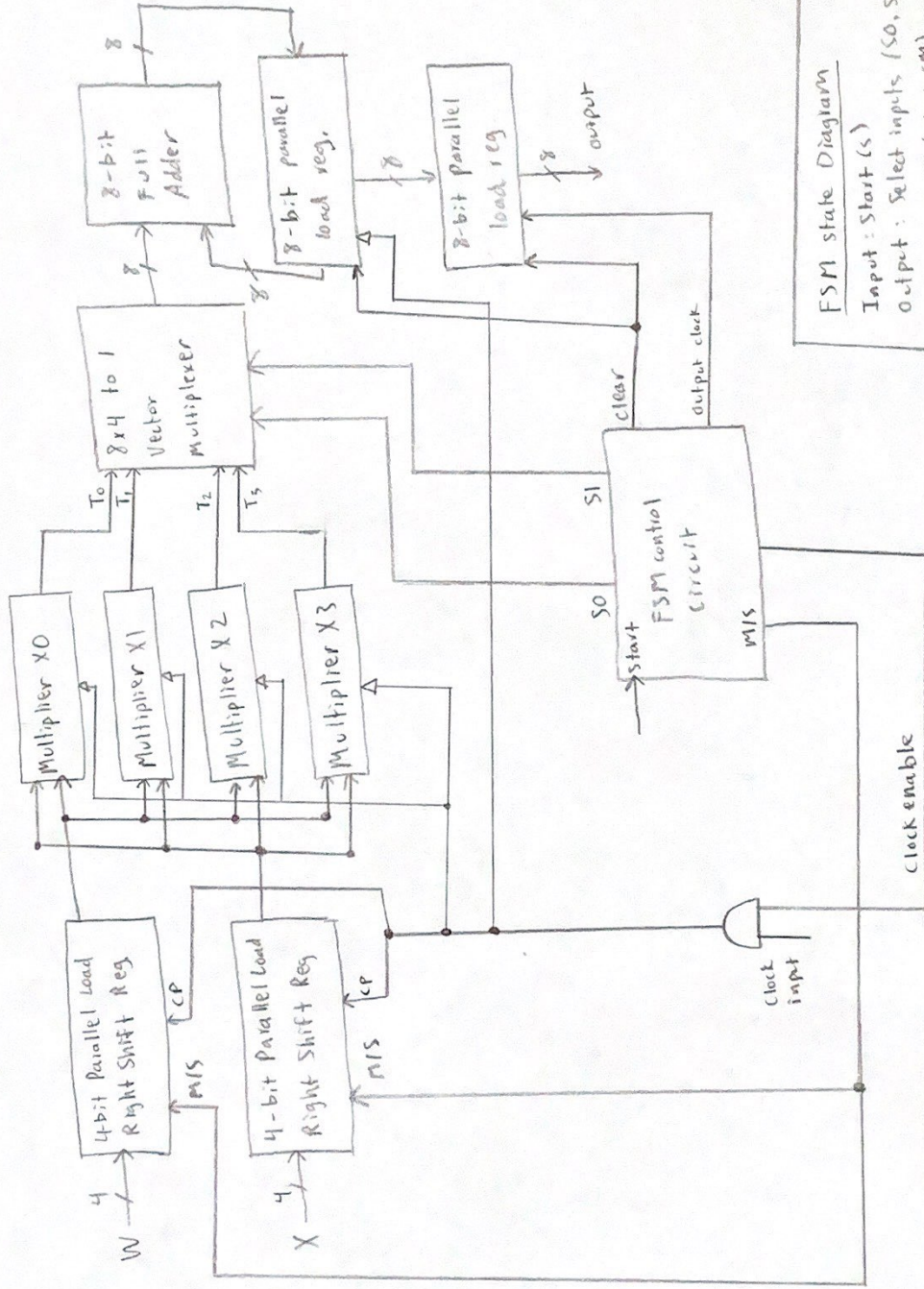


The presence of the zeros are so that each result is shifted to the right place to ensure multiplication works properly. See page 1 for example of multiplication; it demonstrates why the zeroes are needed.

FULL Circuit Diagram

Logic for Vector MUX

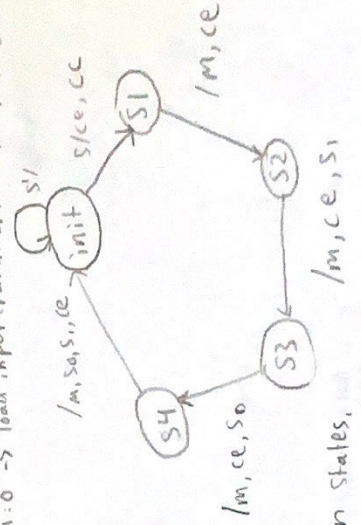
S0	S1	T
0	0	T0
0	1	T1
1	0	T2
1	1	T3



FSM state Diagram

Input: Start (s)

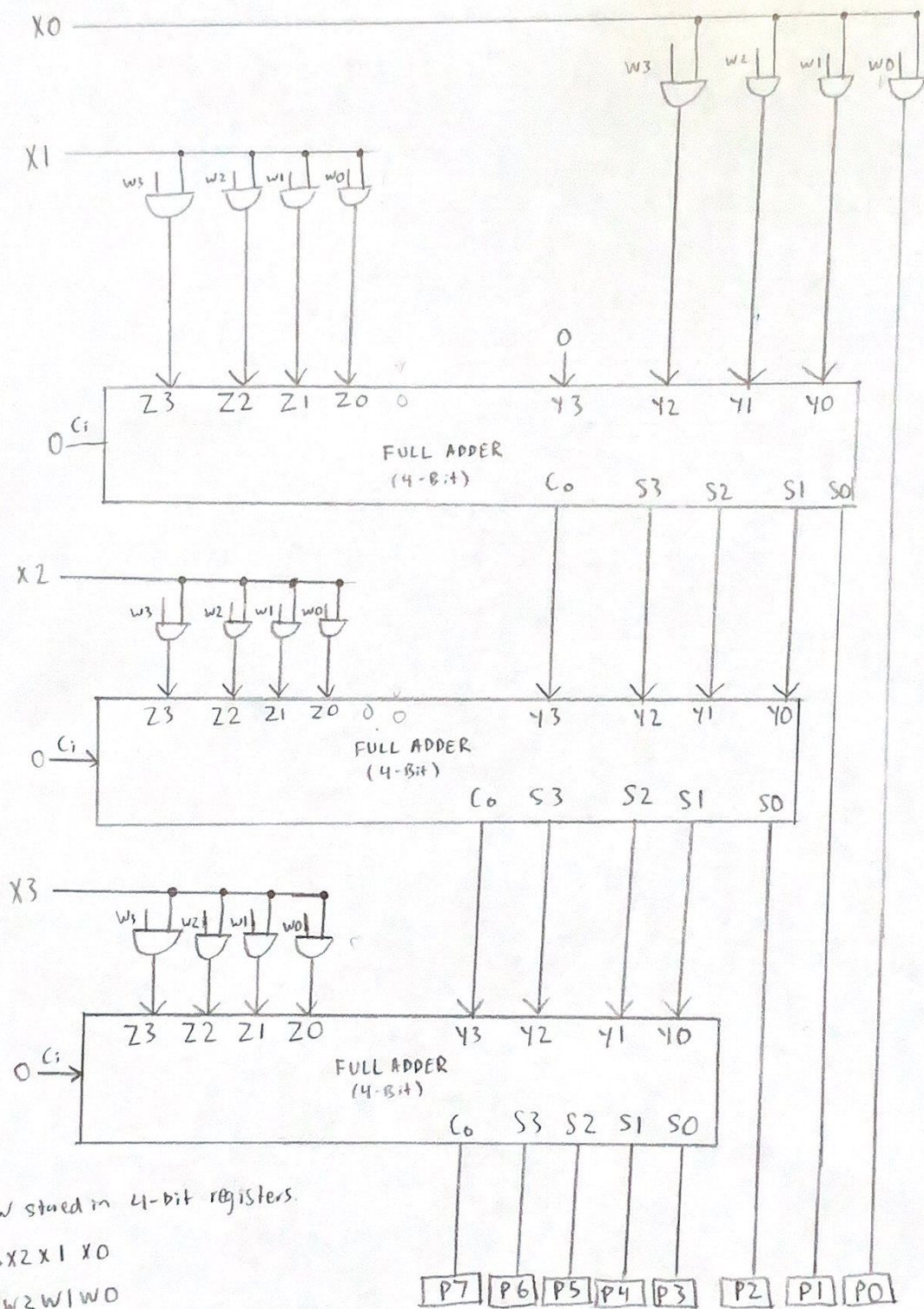
Output: Select inputs ($S0, S1$), output clock (0), clear (cc)
mode/select (m), clock enable (ce)
m: 0 → load input (parallel) m: 1 → Shift!



Blank input → any input
Blank output → no output
S1: 1st shift
S2: 2nd shift
S3: 3rd shift
S4: 4th shift
State reduction is impossible because there is never the same outputs shared between states.

Circuit Diagram for 4-bit multiplier

→ This is extra, but I thought this could be another way to implement 4-bit multiplication. I feel like this is easier and simpler to understand. ②



Both X, W stored in 4-bit registers.

$$X = X_3 X_2 X_1 X_0$$

$$W = W_3 W_2 W_1 W_0$$

For the sake of organization and clarity I did not connect every single X_3, X_2, X_1, X_0 back to X and every single w_3, w_2, w_1, w_0 back to W . It would've

made the diagram very cluttered and I believe my diagram effectively gets the point across.