Homework 4, Part2 Solutions

Q1:

$i_L(0) = 0A$, $i_L(\infty) = 3 * \frac{12}{12+6} = 2A$, $£ = \frac{L}{R} = \frac{6}{12+6} = \frac{1}{3}s$,

So $i_L(t) = i_L(\infty) + \left(i_L(0) - i_L(\infty)\right)e^{-\frac{t}{£}}$

$= 2 - 2e^{-3t}$A

When t=1s, the switch closes, $i_L(1) = 2 - 2e^{-3} = 1.9A$, $i_L(\infty) = 0A$

and $£ = \frac{L}{R} = \frac{6}{6} = 1s$

$i_L(t) = i_L(\infty) + \left(i_L(0) - i_L(\infty)\right)e^{-\frac{t}{£}}$

$= 1.9e^{-(t-1)}$A

Q2:

a. at t=0, $V_{C1} = \frac{R_2}{R_1+R_2} * 2$. At t = ∞, $V_{C1} = -1mA * 1K = -1V$.

So $\frac{R_2}{R_1+R_2} * 2 - (-1) = 1.5V, \frac{R_1}{R_2} = 3$

b. After SW3 is closed, $V_{C1} = -1V$ and $i_{R_1} = \frac{2-(-1)}{R_1} = 3mA$, so $R_1 = 1K\Omega$

$R_2 = \frac{R_1}{3} = 333.3\Omega$.

Q3:

Similar to part1 Q4, $V_x = LC\frac{d^2V_c(t)}{dt^2} + RC\frac{dV_c(t)}{dt} + v_c(t)$

$\alpha = \frac{R}{2L} = \frac{20}{2*2*10^{-3}} = 5000$ and $w_0 = \frac{1}{\sqrt{LC}} = 10000$

$\frac{\alpha}{w_0} = 0.5$, this is the underdamped case.

$w_n = \sqrt{{w_0}^2 - \alpha^2} = 8660.25,$

Write the solution (complementary + particular),

$$V_c(t) = K_1 e^{-\alpha t}\cos(w_n t) + K_2 e^{-\alpha t} sin(w_n t) + V_p(t)$$

$$V_c(t) = K_1 e^{-5000t}\cos(8660.25t) + K_2 e^{-5000t} sin(8660.25t) + 50$$

Solve K_1 and K_2,

$V_c(0) = 0 = K_1 + 50, K_1 = -50.$

Since $i(0) = 0 = C\frac{dV_c(0)}{dt}$,

$$\frac{dV_c(0)}{dt} = -5000K_1 + 8660.25 * K_2 = 0, K_2 = -28.26$$

$$V_c(t) = -50e^{-5000t}\cos(8660.25t) - 28.26e^{-5000t} sin(8660.25t) + 50$$

(You can get full points if you write the differential equation)

Q4.

at t=0, $i_L(0) = 0A$

After the switch is closed, the voltage of the left transformer is $nv_c(t)$

$$V_0(t) = L\frac{di_L(t)}{dt} + Ri_L(t) + 2v_c(t)$$

$$V_0(t) = L\frac{di_L(t)}{dt} + Ri_L(t) + \frac{2}{C}\int_{-\infty}^{t} i_c(t)dt$$

Since $i_c(t) = 2i_L(t)$,

$$V_0(t) = L\frac{di_L(t)}{dt} + Ri_L(t) + \frac{4}{C}\int_{-\infty}^{t} i_L(t)dt$$

Take derivative,

$$0 = \frac{d^2 i_L(t)}{dt^2} + \frac{R}{L}\frac{di_L(t)}{dt} + \frac{4}{LC}i_L(t)$$

$\alpha = \frac{R}{2L} = \frac{1}{2*1*10^{-6}} = 500000$ and $w_0 = \frac{1}{\sqrt{\frac{LC}{4}}} = 1 * 10^7$

$\frac{\alpha}{w_0} = 0.5$, this is the underdamped case.

$w_n = \sqrt{{w_0}^2 - \alpha^2} = 9.975 * 10^6$ (or $1 * 10^7$)

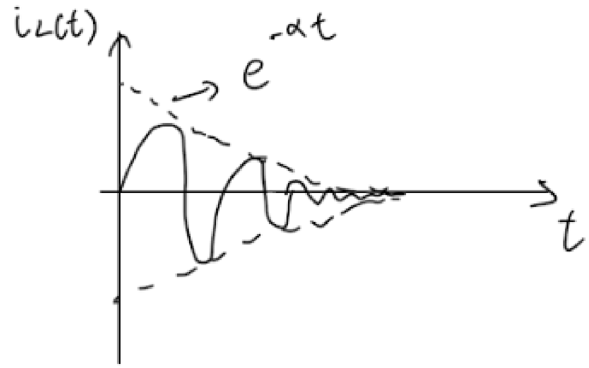
Write the solution (complementary + particular),

$$i_L(t) = K_1 e^{-500000t}\cos(1 * 10^7 t) + K_2 e^{-500000t} sin(9.975 * 10^6 t) + 0$$

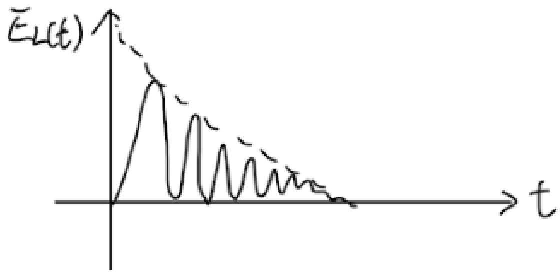
The initial condition $i_L(0) = K_1 = 0, i_L(t) = K_2 e^{-500000t} sin(9.975 * 10^6 t)$

$$V_L(0) = 2 = L\frac{di_L(0)}{dt} = 9.975 * 10^6 * K_2 * 1 * 10^{-6}$$

$K_2 = 0.2, i_L(t) = 0.2e^{-500000t} sin(9.975 * 10^6 t)$



iL(t)
e^{-αt}
t



EL(t)
t