Due 5/10 at 3:00PM on Gradescope

Please write your answers in the boxes provided for Part 2.

You are not required to submit the solutions to Part 1.

Part 1 (Practice Problems):

Q1. Problem 4.36 from book

Real inductors have series resistance associated with the wire used to wind the coil. Suppose that we want to store energy in a 10-H inductor. Determine the limit on the series resistance so the energy remaining after one hour is at least 75 percent of the initial energy.

Q2. Problem 4.27 from book

The circuit in Figure P4.27 has been connected for a very long time. Determine the values of Vc and iR.

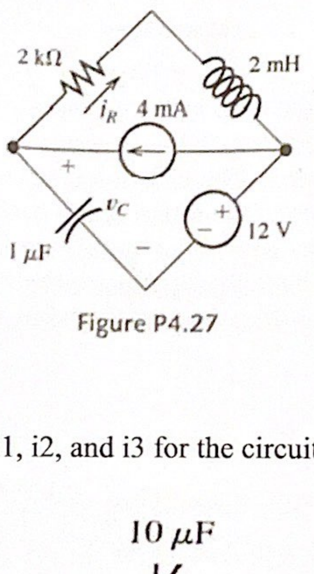


Figure P4.27

Q3. Problem 4.21 from book

Solve for the steady-state values of i1, i2, and i3 for the circuit shown in Figure P4.21.

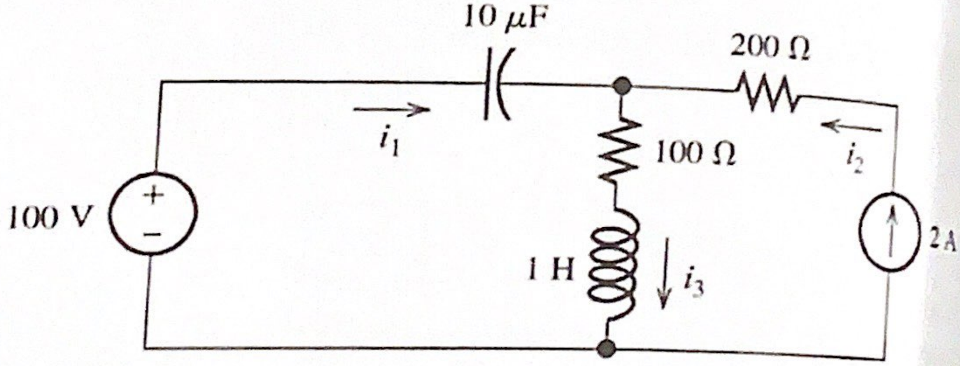


Figure P4.21

Q4. Problem 4.61 from book

A DC source is connected to a series RLC circuit by a switch that closes at t = 0, as shown in Figure P4.61. The initial conditions are i(0+) = 0 and Vc(0+) = 0. Write the differential equation for Vc(t). Solve for Vc(t), if R = 80Ω.

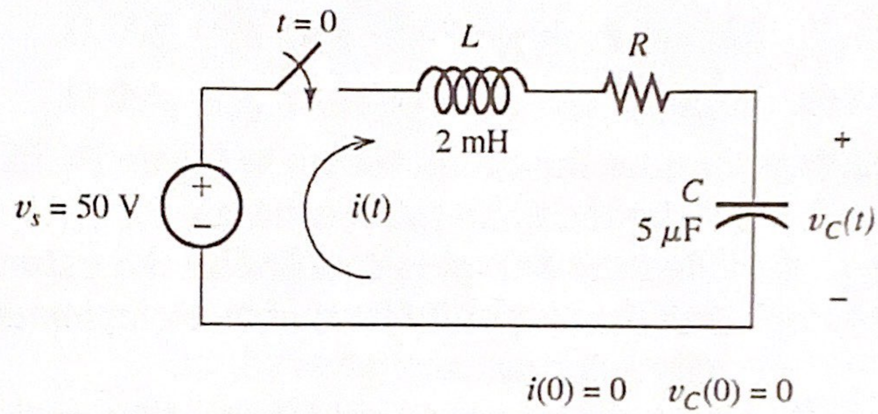


Figure P4.61

Part 2 (Graded)

Q1. Problem 4.42 from book
The switch shown in Figure P4.42 has been closed for a long time prior to t=0, then it opens at t=0 and closes again at t=1 s. Find $i_L(t)$ for all t. (5 points)

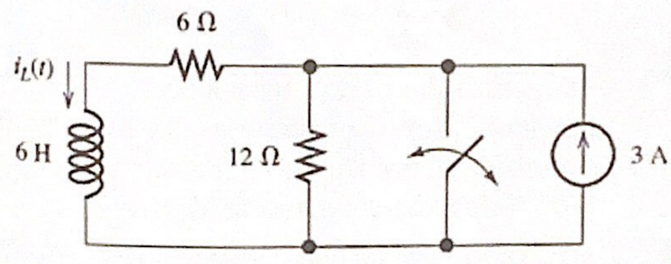
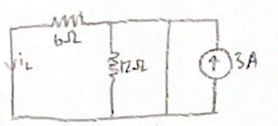


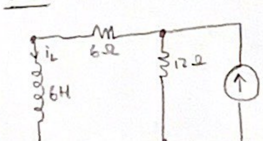
Figure P4.42

$t=0$
steady state, inductor is like a short circuit



We can conclude:
$\boxed{i_L(0^-) = i_L(0^+) = 0}$

$t>0$ [but before 1s]



time constant
$\tau = \frac{L}{R} \quad \tau = \frac{6}{18} = \frac{1}{3}$

current through inductor at $t=\infty$ (steady state)
(current divider)

$i_L = i_2 = I\left[\frac{12}{12+6}\right] = 3\left[\frac{12}{12+6}\right]$

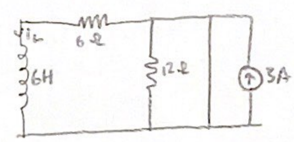
$i_L(\infty) = 2A$

$i_L(t) = i_L(\infty) - [i_L(\infty) - i(0)]e^{-t/\tau}$

$i_L(t) = 2 - [2-0]e^{-t/1/3}$

$\boxed{i_L(t) = 2 - 2e^{-3t}}$

$t>1$



12 Ω resistor is shorted now that the switch is closed

time constant
$\tau = \frac{L}{R} \quad \tau = \frac{6}{6} = 1$

current just flows through the short circuit

$i_L(\infty) = 0\,A$

$i_L(1) = 2 - 2e^{-3(1)}$

$i_L(1) = 1.9\,A$

$i_L(t) = i_L(\infty) - [i_L(\infty) - i_L(1)]e^{-t/\tau}$

$i_L(t) = 0 - [0 - 1.9]e^{-(t-1)}$

$i_L(t) = 1.9e^{-(t-1)}$

$i_L(t)=$	$i_L(0^-) = i_L(0^+) = 0$	A
$0<t\le 1$	$i_L(t) = 2 - 2e^{-3t}$	A
$t \ge 1$	$i_L(t) = 1.9e^{-(t-1)}$	A

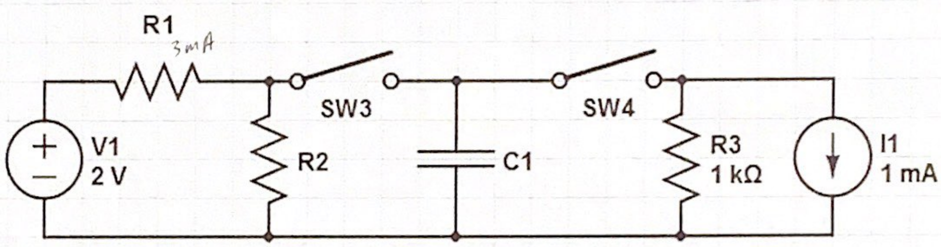
Q2. In the circuit below switch SW3 was closed and SW4 was open prior to t=0. Switch SW3 was opened at t=0 and SW4 was closed at t=0. It was found that the change in capacitor voltage between t=0 & t=∞ was 1.5 V.

a. What is R1/R2 (3 points)

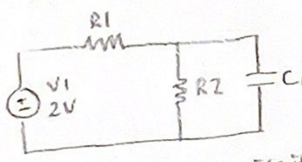
b. Now suppose SW3 was also closed after the circuit reached steady state. It was found that current through R1 is 3 mA just after closing SW3. Find R1 & R2 (2 points)

t>0 SW3 open

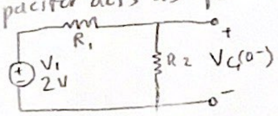
t>0 SW4 closed



t<0



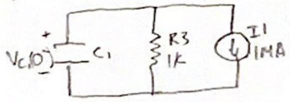
Capacitor acts as open circuit



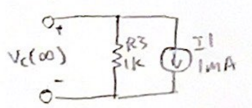
Voltage Divider Method

$V_C(0^-) = 2\frac{R_2}{R_1+R_2} = V_C(0)$

t>0



t=∞ => steady state
capacitor is open circuit



$V_C(\infty) = (1000)(-0.001) = -1$

Given

$V_C(0) - V_C(\infty) = 1.5$

$2\frac{R_2}{R_1+R_2} - (-1) = 1.5$

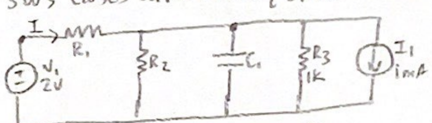
$2\frac{R_2}{R_1+R_2} = 0.5$

$2\frac{1}{\frac{R_1}{R_2}+1} = 0.5$

$1 + \frac{R_1}{R_2} = \frac{2}{0.5}$

$\boxed{\frac{R_1}{R_2} = 3}$

SW3 closes after steady state



$V_{C_1}(\infty) = -1$ [from part A]

$\frac{2V - (-1V)}{R_1} = I$

$\frac{3}{R_1} = \frac{3}{1000}$

$R_1 = 1000\ \Omega$

$R_2 = 333.33\ \Omega$

a. $R1/R2$ =	3
b. $R1$ =	1000 Ω
b. $R2$ =	333.33 Ω

Q3. Problem 4.63 from book

A DC source is connected to a series RLC circuit by a switch that closes at t = 0, as shown in Figure P4.61. The initial conditions are $i(0+) = 0$ and $v_C(0+) = 0$. Write the differential equation for $v_C(t)$ if R = 20Ω. (5 points)

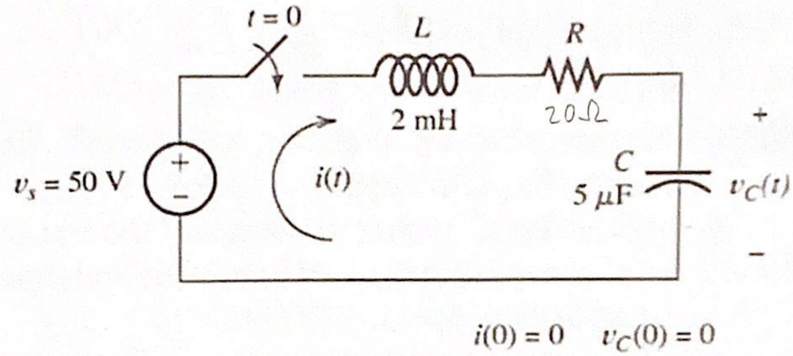


Figure P4.61

KVL: $V_s - iR - L\frac{di}{dt} - V_c(t) = 0$

$V_c(t) = V_s - iR - L\frac{di}{dt}$

$V_c(t) = 50 - 20i(t) - 0.002\frac{di(t)}{dt}$

$\alpha = \frac{R}{2L} = \frac{20}{2(0.002)} = 5000$

$W_o = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{0.002 \times (5\times10^{-6})}} = 10000$

$\alpha < W_o$, underdamped

natural frequency:

$W_n = \sqrt{W_o^2 - \alpha^2}$

$W_n = \sqrt{10000^2 - 5000^2} = 8660$

Complementary solution

$V_{cc}(t) = K_1 e^{-\alpha t}\cos(W_n t) + K_2 e^{-\alpha t}\sin(W_n t)$

Complete solution

$V_c(t) = 50 + K_1 e^{-\alpha t}\cos(W_n t) + K_2 e^{-\alpha t}\sin(W_n t)$

Initial Condition

$i(0) = 0$ $\quad$ $V_c(0) = 0$

$\Downarrow$

$C\frac{dV_c(t)}{dt} = 0$ @ $t = 0$

$$\begin{cases} V_c(0) = 0 = 50 + K_1 \\ \frac{dV_c(t)}{dt_0} = 0 = -\alpha K_1 + W_n K_2 \end{cases}$$

$\Downarrow$ simplify

$$\begin{cases} 50 + K_1 = 0 \\ -5000K_1 + 8660K_2 = 0 \end{cases}$$

$K_1 = -50 \quad K_2 = -28.86$

Plug back into complete solution

$V_c(t) = 50 - 50e^{-\alpha t}\cos(W_n t) - 28.86e^{-\alpha t}\sin(W_n t)$

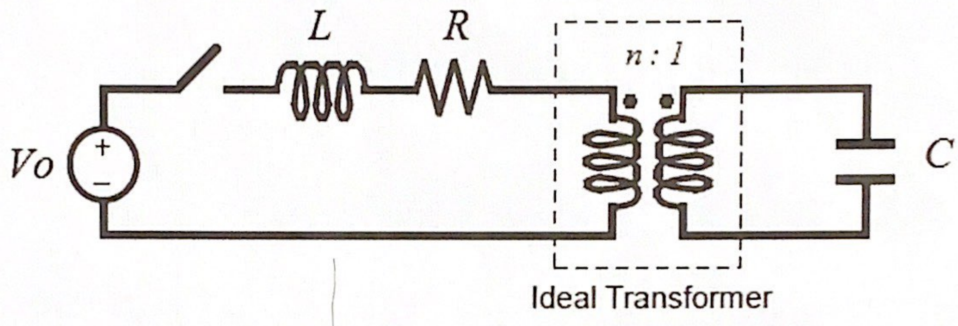
Plug in values

$\boxed{V_c(t) = 50 - 50e^{-5000t}\cos(8660t) - 28.86e^{-5000t}\sin(8660t)}$

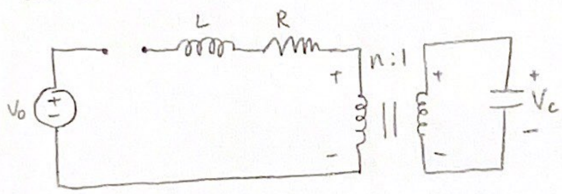
$\boxed{v_C(t) = \begin{matrix} 50 - 50e^{-5000t}\cos(8660t) \\ -28.86e^{-5000t}\sin(8660t) \end{matrix}}$

Q4. For the circuit shown in the figure below, the switch is open for a long time and the capacitor is fully discharged. The switch closes at time t = 0s. Find an expression for the current in the inductor after the switch closes. Draw a plot for the inductor current and the energy stored in the inductor. (5 points)

Given: $Vo = 2V$, $L = 1\mu H$, $R = 1\Omega$, $C = 0.04\mu F$ and $n = 2$.



$t=0^-$

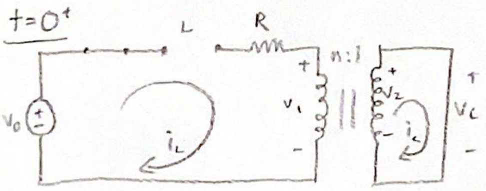


No current to inductor or capacitor due to the open circuit (switch still open)

$i_L(0^-) = 0A \quad i_L(0^+) = 0A$

$V_c(0^-) = 0V \quad V_c(0^+) = 0V$

$t=0^+$



Right when switch closes, inductor acts as open circuit and capacitor acts as short

Use KVL on left loop

$V_o = L\frac{di_L}{dt} + i_L(0^+)R + V_1(0^+)$

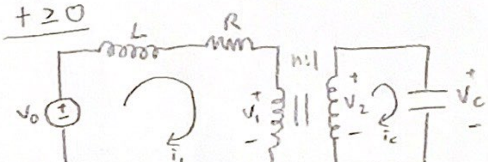
$[V_c(0^+) = 0, \text{ so } V_1(0^+) = nV_c(0^+) = 0]$

$\hookrightarrow V_1(0^+) = 0,\ i_L(0^-) = 0$

$V_o = L\frac{di_L}{dt}$

$\frac{di_L}{dt}\Big|_{t=0} = \frac{V_o}{L} = \frac{2}{L}$

$t \geq 0$



$[V_2 = V_c]$

Solve for transformer.

$V_1 = nV_c,\ i_c = n i_L$

Use KVL on left loop.

$V_o = L\frac{di_L}{dt} + Ri_L + V_1$

$V_o = L\frac{di_L}{dt} + Ri_L + nV_c$

$V_o = L\frac{di_L}{dt} + Ri_L + n\frac{1}{C}\int_{-\infty}^{t} i_c\,dt$

$V_o = L\frac{di_L}{dt} + Ri_L + n\frac{1}{C}\int_{-\infty}^{t} n i_L\,dt$

$V_o = L\frac{di_L}{dt} + Ri_L + \frac{n^2}{C}\int_{-\infty}^{t} i_L\,dt$

Differentiate, with respect to t

$0 = L\frac{d^2i_L}{dt^2} + R\frac{di_L}{dt} + \frac{n^2}{C}i_L$

Divide by L

$0 = \frac{d^2i_L}{dt^2} + \frac{R}{L}\frac{di_L}{dt} + \frac{n^2}{LC}i_L$

Work continued on last page!

$\boxed{i_L(t) = 0.2e^{-\sigma t}\cdot \sin\omega_d t}$

$\sigma = 5\times10^5\ \frac{rad}{s}$

$\omega_d = 10\times10^6\ \frac{rad}{s}$

[Box was too small to write all of it in]

Use Candidate Solution

$i_L(t) = Ke^{st}$

Characteristic equations

$$s^2 + \frac{R}{L}s + \frac{n^2}{LC} = 0$$

$$s = \frac{-R}{2L} \pm \frac{1}{2}\sqrt{\left(\frac{R}{L}\right)^2 - \frac{4n^2}{LC}}$$

Plug in values for R, L, C, n

$$s = \frac{-1}{2\times10^{-6}} \pm \frac{1}{2}\sqrt{\left(\frac{1}{10^{-6}}\right)^2 - \frac{16}{(10^{-6}\times 0.04\times10^{-6})}}$$

$$s = -5\times10^5 \pm \frac{1}{2}\sqrt{10^{12} - 4\times10^{14}}$$

$$s \approx -5\times10^5 \pm j10\times10^6$$

$$s = -\sigma \pm j\omega_d$$

General Solution

$$i_L(t) = e^{-\sigma t}(k_1\cos\omega_d t + k_2\sin\omega_d t)$$

$$\frac{di_L}{dt} = e^{-\sigma t}(-k_1\omega_d\sin\omega_d t + k_2\omega_d\cos\omega_d t) - \sigma e^{-\sigma t}(k_1\cos\omega_d t + k_2\sin\omega_d t)$$

Initial conditions

$$i_L(0^+) = 0 \quad \frac{di_L}{dt_{0^+}} = \frac{2}{L}$$

Solve

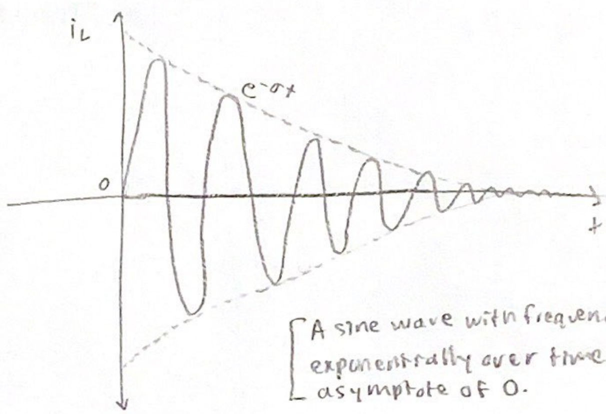
$K_1 = 0$

$$K_2 = \frac{2}{\omega_d L} = 0.2$$

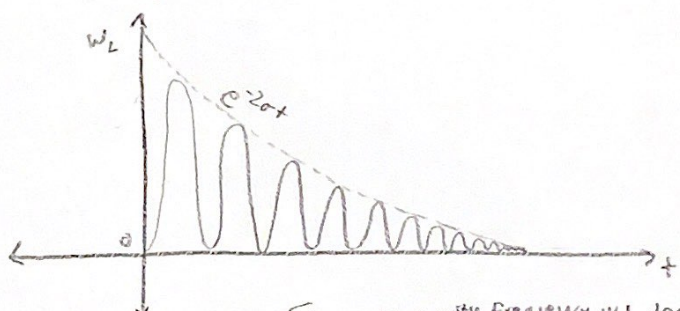
$$i_L(t) = 0.2e^{-\sigma t}\sin\omega_d t$$

$\sigma = 5\times10^5 \ \frac{rad}{s}$

$\omega_d = 10\times10^6 \ \frac{rad}{s}$



[A sine wave with frequency ω_d, decays exponentially over time. Approaches asymptote of 0.]



[A sine wave with frequency ω_d, decays exponentially overtime. Approaches asymptote of 0.]

$$W_L(t) = 0.02Le^{-2\sigma t}\sin^2\omega_d t$$

$$W_L(t) = 0.02Le^{-2\sigma t}\left(\frac{1-\cos 2\omega_d t}{2}\right)$$