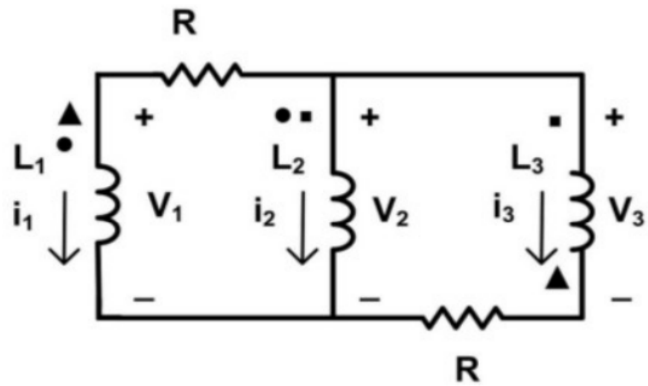
ECE100: Homework 3, Part 1 Solutions

Q1: Use the dot convention to find the voltage on the 3 coils in the following circuit. Assume the mutual inductance between any two inductors in M.

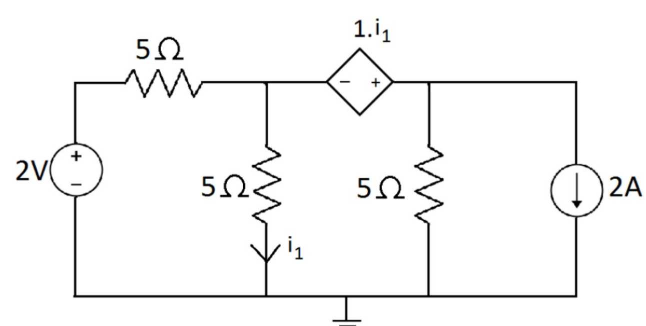


$$V_1 = L_1\frac{di_1}{dt} + M\frac{di_2}{dt} - M\frac{di_3}{dt}$$

$$V_2 = L_2\frac{di_2}{dt} + M\frac{di_1}{dt} + M\frac{di_3}{dt}$$

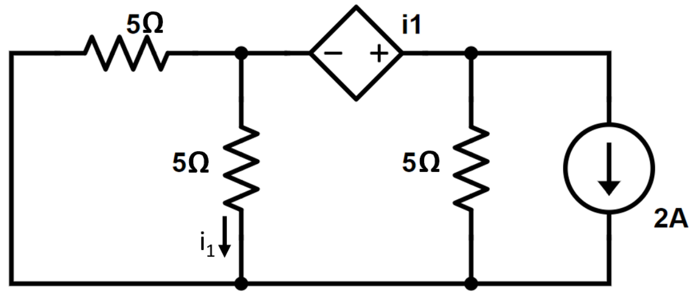
$$V_3 = L_3\frac{di_3}{dt} - M\frac{di_1}{dt} + M\frac{di_2}{dt}$$

Q2: Use the superposition principle to obtain a value for the current i_1 as labeled in the circuit shown below.

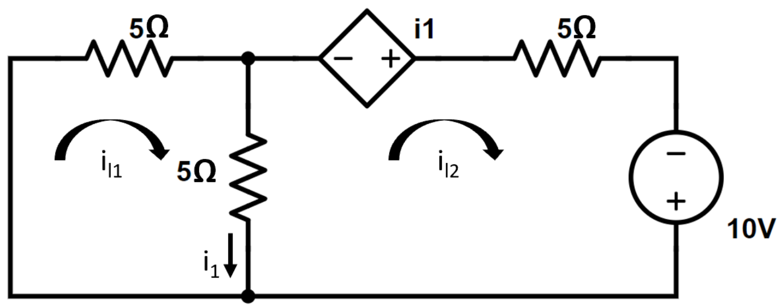


The dependent source cannot be turned off during superposition, so we can write a version for turning off independent voltage source and one for turning off the independent current source.

Turning of the independent voltage source, we replace it with a short as below.



Since we are dealing with a dependent voltage source, this will be easier to deal with if we replace the current source and resistor with a voltage source and resistor. The new voltage source value should be $V_{Th} = IR = 2(5) = 10V$.



Using MCA:

$$0 = 5i_{l1} + 5(i_{l1} - i_{l2}) = 10i_{l1} - 5i_{l2}$$

$$10 + i_1 + 5i_1 = 5i_{l2}$$

Solving the first equation for i_{l1} and plugging in:

$$i_{l1} = \frac{1}{2}i_{l2}$$

$$10 + 6i_1 = 10 + 6i_{l1} - 6i_{l2} = 5i_{l2}$$

$$10 + 6\left(\frac{1}{2}i_{l2}\right) = 11i_{l2}$$

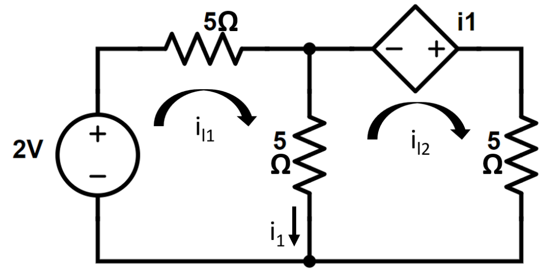
$$10 = 8i_{l2}$$

$$i_{l2} = \frac{5}{4}$$

$$i_{l1} = \frac{5}{8}$$

Now $i_1 = i_{l1} - i_{l2} = -\frac{5}{8}A$ for the response from to the current source.

Turning off the independent current source, we replace it with an open as below.



Using MCA:

$$2 = 5i_{l1} + 5(i_{l1} - i_{l2}) = 10i_{l1} - 5i_{l2}$$

$$i_{l1} - i_{l2} = 5i_{l2} + 5(i_{l2} - i_{l1}) = 10i_{l2} - 5i_{l1}$$

Solving the first equation for i_{l2} and plugging in:

$$i_{l2} = 2i_{l1} - \frac{2}{5}$$

$$6i_{l1} = 11i_{l2} = 11\left(2i_{l1} - \frac{2}{5}\right) = 22i_{l1} - \frac{22}{5}$$

$$16i_{l1} = \frac{22}{5}$$

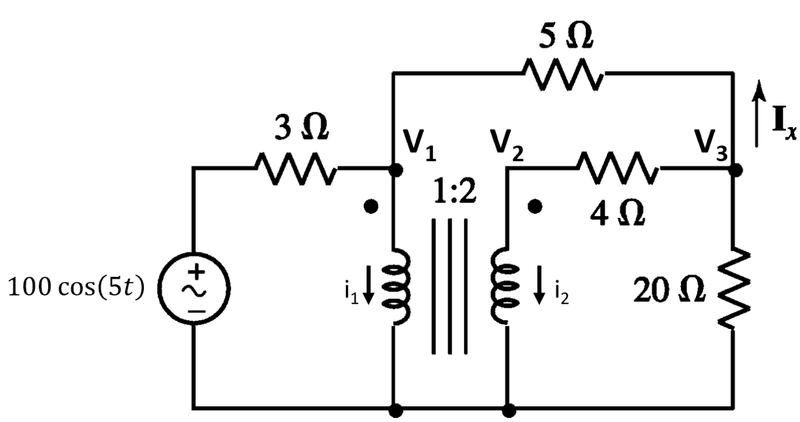
$$i_{l1} = \frac{11}{40}, \qquad i_{l2} = \frac{11}{20} - \frac{2}{5} = \frac{3}{20}$$

So $i_1 = i_{l1} - i_{l2} = \frac{5}{40} = \frac{1}{8}A$ is the response due to the voltage source.

Using the superposition principle, the value of i_1 is equal to the sum of the contributions from the voltage sources, so:

$$i_1 = i_{vsrc} + i_{isrc} = \frac{1}{8} - \frac{5}{8} = -\frac{1}{2}A$$

Q3: Find I_X in the circuit below. The transformer is ideal. The voltage source is $v(t) = 100\cos(5t)$.



$$v_s = 100\cos(5t)$$

Using KCL (implicitly using the source transform on the voltage source and resistor):

$$\frac{v_s}{3} = \frac{v_1}{3} + i_1 + \frac{(v_1 - v_3)}{5}$$

$$0 = i_2 + \frac{v_2 - v_3}{4}$$

$$0 = \frac{v_3 - v_1}{5} + \frac{v_3 - v_2}{4} + \frac{v_3}{20}$$

Additionally, because the transformer is ideal, we know:

$$i_2 = -\frac{1}{2} i_1$$

$$v_2 = 2v_1$$

Substituting these equations into the KCL equations:

$$\frac{v_s}{3} = \frac{v_1}{3} + i_1 + \frac{(v_1 - v_3)}{5} = \frac{8}{15} v_1 + i_1 - \frac{v_3}{5}$$

$$0 = -\frac{1}{2} i_1 + \frac{2v_1 - v_3}{4}$$

$$0 = \frac{v_3 - v_1}{5} + \frac{v_3 - 2v_1}{4} + \frac{v_3}{20}$$

Solving:

$$i_1 = v_1 - \frac{v_3}{2}$$

$$0 = \left(\frac{1}{5} + \frac{1}{4} + \frac{1}{20}\right) v_3 - \left(\frac{1}{5} + \frac{1}{2}\right) v_1 = \frac{1}{2} v_3 - \frac{7}{10} v_1$$

$$v_1 = \frac{5}{7} v_3$$

$$i_1 = \frac{5}{7} v_3 - \frac{v_3}{2} = \frac{3}{14} v_3$$

$$\frac{v_s}{3} = \left(\frac{8}{15}\right)\frac{5}{7} v_3 + \frac{3}{14} v_3 - \frac{v_3}{5} = \frac{83}{210} v_3$$

$$v_3 = \frac{70}{83} v_s$$

$$v_1 = \frac{50}{83} v_s$$

$$i_1 = \frac{15}{83} v_s$$

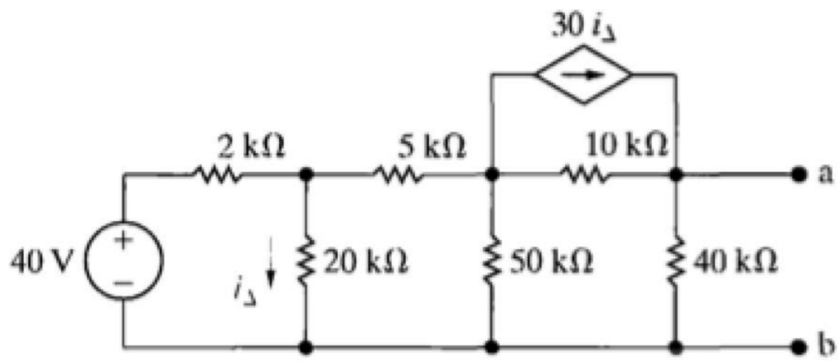
$$i_2 = -\frac{15}{166}v_s$$

$$v_2 = \frac{100}{83}v_s$$

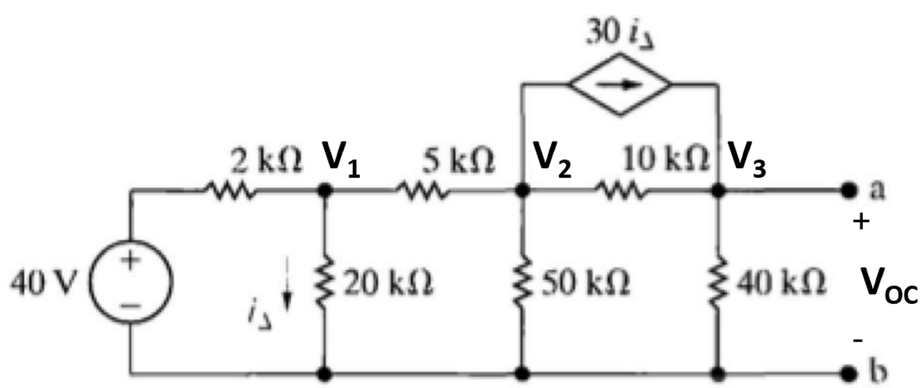
It follows:

$$I_x = \frac{v_3 - v_1}{5} = \frac{4}{83}v_s = \frac{400}{83}\cos(5t)$$

Q4: Find the Norton equivalent with respect to the terminals a, b for the circuit shown below.



To find the Norton equivalent we solve for the open circuit voltage and short circuit current.



Using KCL:

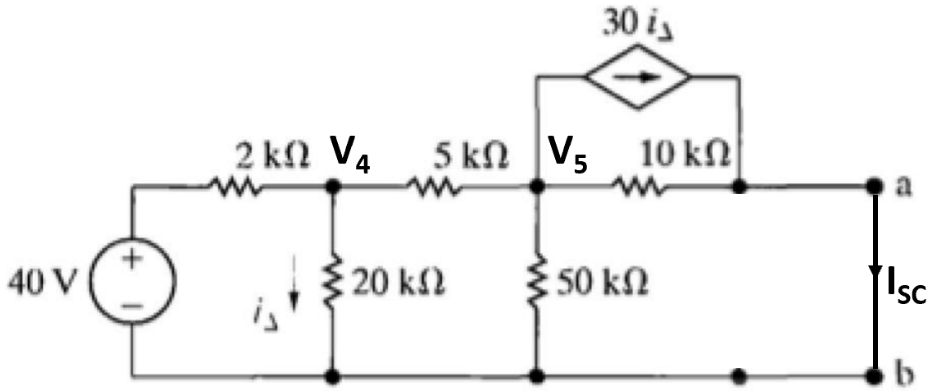
$$\frac{40}{2k} = \left(\frac{1}{2k} + \frac{1}{20k} + \frac{1}{5k}\right)V_1 - \frac{1}{5k}V_2$$

$$0 = -\frac{1}{5k}V_1 + \left(\frac{1}{5k} + \frac{1}{50k} + \frac{1}{10k}\right)V_2 - \frac{1}{10k}V_3 + \frac{30}{20k}V_1$$

$$0 = -\frac{1}{10k}V_2 + \left(\frac{1}{10k} + \frac{1}{40k}\right)V_3 - \frac{30}{20k}V_1$$

Solving:

$$V_1 = 24, V_2 = -10, V_3 = 280 = V_{OC}$$



Using KCL:

$$\frac{40}{2k} = \left(\frac{1}{2k} + \frac{1}{20k} + \frac{1}{5k}\right) V_4 - \frac{1}{5k} V_5$$

$$0 = -\frac{1}{5k} V_4 + \left(\frac{1}{5k} + \frac{1}{50k} + \frac{1}{10k}\right) V_5 + \frac{30}{20k} V_4 = 0$$

Solving:

$$V_4 = 12.8, V_5 = -52$$

$$I_{SC} = 30 i_\Delta + \frac{V_5}{10k} = 0.014$$

$$R_N = \frac{V_{OC}}{I_{SC}} = 20k\Omega$$

$$I_N = 0.014A$$