Discussion 6 Notes

Tuesday, May 17, 2022 1:01 PM

1st Order Circuits

$$\frac{dx(t)}{dt} + \frac{1}{\tau}x(t) = f(t)$$

Time constant ↗ ↑ forcing function

$$x(t) = x_p(t) + x_c(t)$$

Complementary Solution

$$x_c(t) = Ke^{-t/\tau}$$

Particular Solution

$f(t) = \text{Constant} \rightarrow x_p(t) = K_p$

$f(t) = \text{sinusoid}(\omega t) \rightarrow x_p(t) = A\cos(\omega t) + B\sin(\omega t)$

General Form

$$x(t) = x_0 + (x_0 - x_\infty)e^{-t/\tau}$$

2nd Order

$$\frac{d^2x(t)}{dt^2} + 2\alpha\frac{dx(t)}{dt} + \omega_0^2 x(t) = f(t)$$

$$x(t) = x_p(t) + x_c(t)$$

Complementary Solution — Result for $f(t) = 0$

Solving symbolic form above

$$x_c(t) \longrightarrow Ke^{st}$$

Characteristic Equation

$$s^2 + 2\alpha s + \omega_0^2 = 0$$

$$s = \frac{-2\alpha \pm \sqrt{4\alpha^2 - 4\omega_0^2}}{2} = -\alpha \pm \sqrt{\alpha^2 - \omega_0^2}$$

$s_1 = -\alpha + \sqrt{\alpha^2 - \omega_0^2}$, $s_2 = -\alpha - \sqrt{\alpha^2 - \omega_0^2}$

Damping Ratio

Overdamped $(\zeta > 1)$

Damping Ratio $\zeta = \frac{\alpha}{\omega_0}$

$s_1 = -\alpha + \sqrt{\alpha^2 - \omega_0^2}$, $s_2 = \ldots$

$\omega_0 < \alpha \rightarrow$ Overdamped $(\zeta > 1)$

$\omega_0 = \alpha \rightarrow$ Critically Damped $(\zeta = 1)$

$\omega_0 > \alpha \rightarrow$ Underdamped $(\zeta < 1)$

Overdamped

$$x_c(t) = K_1 e^{s_1 t} + K_2 e^{s_2 t}$$

Critically Damped

$s_1 = s_2$

$$x_c(t) = K_1 e^{s_1 t} + K_2 t e^{s_1 t}$$

Underdamped

$s_1 = -\alpha + j\omega_n$, $s_2 = -\alpha - j\omega_n$

$\omega_n = \sqrt{\omega_0^2 - \alpha^2}$

$$x_c(t) = K_1 e^{-\alpha t}\cos(\omega_n t) + K_2 e^{-\alpha t}\sin(\omega_n t)$$

Particular Solution

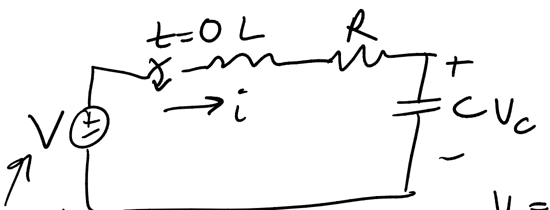
$\boxed{f(t) = \text{Constant}}$

↳ L and C $\rightarrow$ Steady State Equivalents

L $\rightarrow$ short

C $\rightarrow$ Open

↳ Find Value in question



Const

$$V = L\frac{di}{dt} + Ri + \frac{1}{C}\int i\,dt$$

Solve for i

$$0 = L\frac{d^2 i}{dt^2} + R\frac{di}{dt} + \frac{1}{C}i$$

$$0 = \frac{d^2 i}{dt^2} + \frac{R}{L}\frac{di}{dt} + \frac{1}{LC}i$$

Already homogeneous
No particular Solution

$V_c = \frac{1}{C}\int i\,dt$

$i = C\frac{dV_c}{dt}$

$$V = CL\frac{d^2 V_c}{dt^2} + RC\frac{dV_c}{dt} + V_c$$

$$\frac{V}{LC} = \frac{d^2 V_c}{dt^2} + \frac{R}{L}\frac{dV_c}{dt} + \frac{1}{LC}V_c$$

$\frac{R}{L} = 2\alpha$ $\qquad \omega_0^2 = \frac{1}{LC}$

$\alpha = \frac{R}{2L}$ $\qquad \omega_0 = \frac{1}{\sqrt{LC}}$

$$x_p(t) = V$$

i = complementary

$V_c(t) = V +$ complementary