

Part 2:

Q1. Fig 1 below shows 2 equivalent circuits (a) and (b). In circuit (a) the 2 coupled inductors have self-inductances L_1 and L_2 as shown and a mutual inductance M . Find L_A , L_B and L_C in terms of L_1 , L_2 and M .

i_1, i_2, v_1, v_2 must be equal

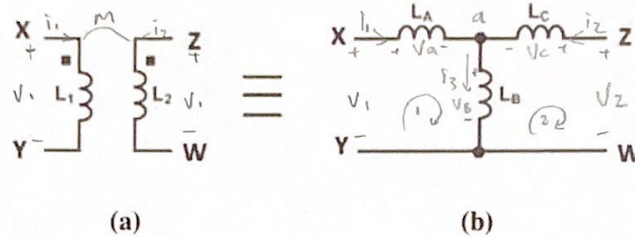


Figure 1

Figure A:

$$v_1 = L_1 \frac{di_1}{dt} + M \frac{di_2}{dt}$$

$$v_2 = L_2 \frac{di_2}{dt} + M \frac{di_1}{dt}$$

$$L_A + L_B = L_1 \quad L_B + L_C = L_2$$

$$L_B = M \quad L_B = M$$

$$L_A = L_1 - L_B \rightarrow L_A = L_1 - M$$

$$L_B = M$$

$$L_C = L_2 - L_B \rightarrow L_C = L_2 - M$$

Figure B:

$$i_1 + i_2 = I_3$$

KVL Loop 1

$$v_1 - v_A - v_B = 0$$

$$v_1 - L_A \frac{di_1}{dt} - L_B \frac{di_2}{dt} = 0$$

$$v_1 - L_A \frac{di_1}{dt} - L_B \frac{d(i_1 + i_2)}{dt} = 0$$

$$v_1 = (L_A + L_B) \frac{di_1}{dt} + L_B \frac{di_2}{dt}$$

KVL Loop 2

$$v_B + v_C - v_2 = 0$$

$$L_B \frac{di_2}{dt} + L_C \frac{di_2}{dt} - v_2 = 0$$

$$L_B \frac{d(i_1 + i_2)}{dt} + L_C \frac{di_2}{dt} - v_2 = 0$$

$$v_2 = (L_B + L_C) \frac{di_2}{dt} + L_B \frac{di_1}{dt}$$

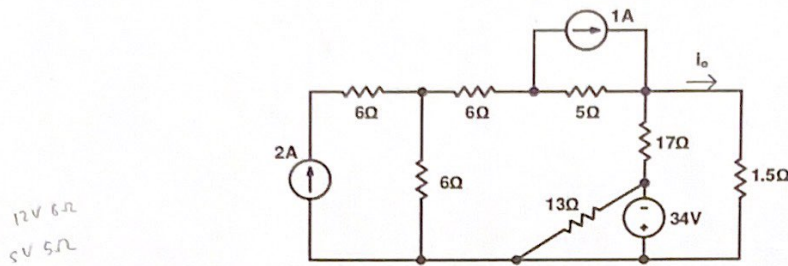
$$L_A = L_1 - M$$

$$L_B = M$$

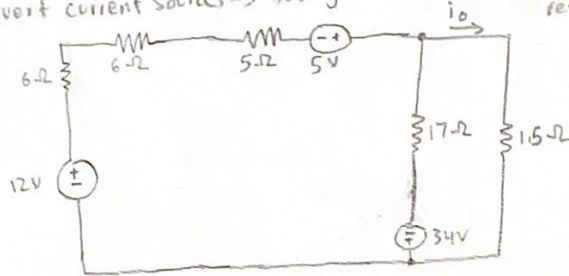
$$L_C = L_2 - M$$

Part 2:

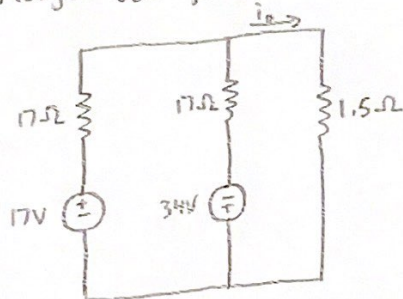
Q2. Use a series of source transformations to find the current i_o in the circuit given in the Fig 2 below.



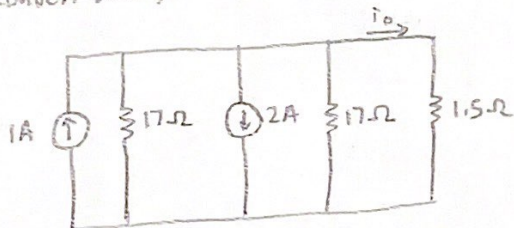
① Convert current sources \rightarrow voltage sources, remove 13Ω resistor Figure 2



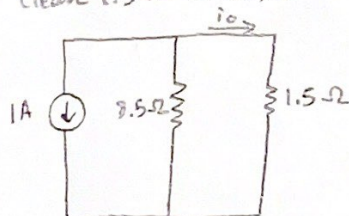
② Merge Voltage Sources + Resistors



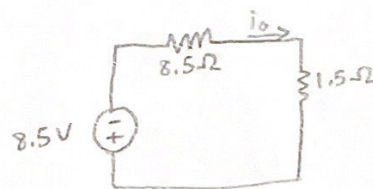
③ Convert Voltage Sources \rightarrow current sources



④ Merge current sources + resistors (leave 1.5Ω alone, need it to calculate i_o)



⑤ Convert current source \rightarrow voltage source



Note: i_o should be negative due to sign convention

$$8.5\Omega + 1.5\Omega = 10\Omega \quad (\text{resistors in series})$$

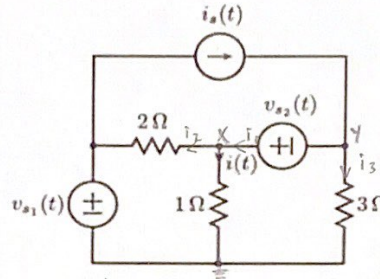
$$i_o = \frac{-8.5V}{10\Omega}$$

$$i_o = -0.85A$$

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Part 2:

Q3. Refer circuit below. Find $i(t)$ (current flowing through the 1 ohm resistor) in terms of $i_s(t)$, $v_{s1}(t)$, $v_{s2}(t)$.



$$i(t) = \frac{v_x}{1}$$

KCL @ Node X

$$i_1 = i_2 + i(t)$$

$$i_1 = \frac{v_x - v_{s1}(t)}{2} + \frac{v_x}{1}$$

KCL @ Node Y

$$i_s(t) = i_1 + i_3$$

$$i_s(t) = i_1 + \frac{v_y}{3}$$

$$i_s(t) = i_1 + \frac{v_x - v_{s2}(t)}{3}$$

$$i_s(t) = \left[\frac{v_x - v_{s1}(t)}{2} + \frac{v_x}{1} \right] + \left[\frac{v_x - v_{s2}(t)}{3} \right]$$

$$i_s(t) = \frac{11v_x - 3v_{s1}(t) - 2v_{s2}(t)}{3}$$

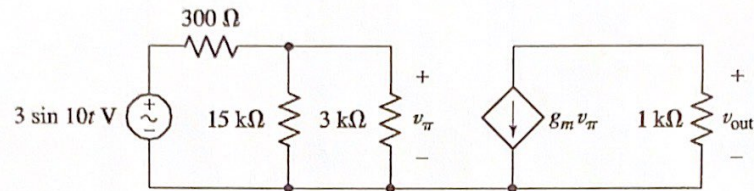
$$v_x = \frac{6i_s(t) + 3v_{s1}(t) + 2v_{s2}(t)}{11}$$

$$i(t) = \frac{6i_s(t) + 3v_{s1}(t) + 2v_{s2}(t)}{11}$$

$$i(t) = \frac{6i_s(t) + 3v_{s1}(t) + 2v_{s2}(t)}{11}$$

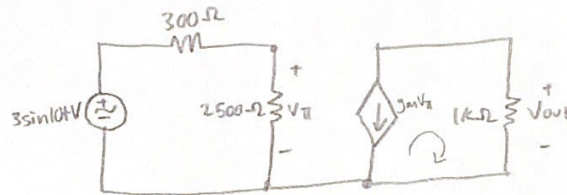
Part 2:

Q4. The circuit below is a commonly used equivalent circuit used to model the ac behavior of a bipolar junction transistor amplifier circuit. If $g_m = 38 \text{ mS}$ compute v_{out} .



$$15 \text{ k}\Omega \parallel 3 \text{ k}\Omega$$

$$\frac{(15000)(3000)}{15000 + 3000} = 2500 \Omega$$



Voltage Divider Method

$$V_x = V_{total} \cdot \frac{R_x}{R_{total}}$$

$$V_{\pi} = 3 \sin 10t \cdot \frac{2500}{2500 + 300}$$

$$V_{\pi} = 3 \sin 10t (0.8929)$$

$$V_{\pi} = 2.6787 \sin 10t \text{ V}$$

$$V = IR$$

$$V_{out} = (-g_m V_{\pi}) (1000)$$

$$V_{out} = [(-38 \times 10^{-3}) \times 2.6787 \sin 10t] (1000)$$

$$V_{out} = -101.8 \sin 10t \text{ V}$$

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