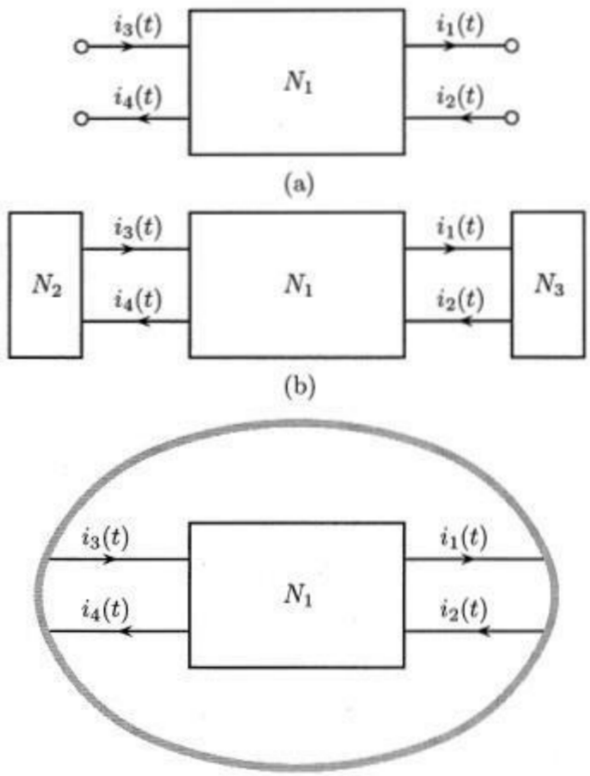
ECE100: Homework 1, Part 2 Solutions

Q1: Consider the network N1 shown in the figures below. Answer the questions following the figures. (5 points)



(i) When network **N1** is connected to the two sub-networks **N2** and **N3** as shown in Figure (b), what is the relation between currents $i_1(t)$ and $i_2(t)$?

Since the current in to a subcircuit must be equal to the current out of the subcircuit:

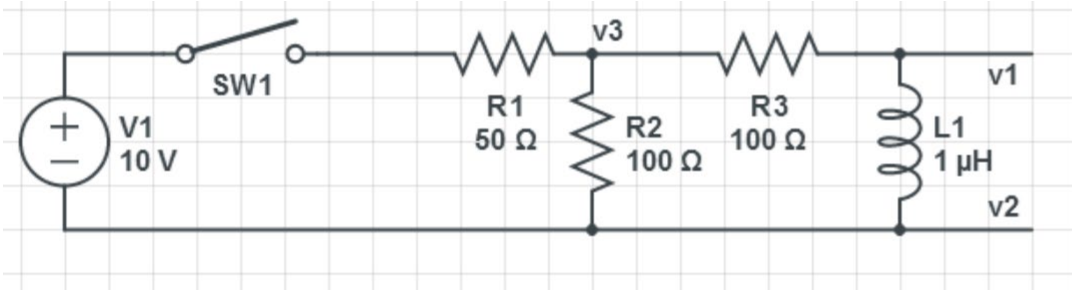
$$i_1(t) = i_2(t)$$

(ii) Does the result that you derived in (i) apply to $i_1(t)$ and $i_2(t)$ when **N1** is embedded in a larger (but unknown) network as shown in Figure (c)? Explain your answer.

The same relation does not apply in this case, because we do not know that the wires carrying i_1 and i_2 are the only paths in and out of a subcircuit. In this case, we can write the relationship:

$$i_1(t) + i_4(t) = i_2(t) + i_3(t)$$

Q2: At time $t = 0$ switch SW1 was closed. Assuming current through the inductor L1 is i_2.



(a) Just after the switch is closed, what is the value of $v_3 - v_2$? (2 points)

Assuming the switch has been open for a long time, there is no current flowing in the circuit before time $t = 0$. Since inductors resist changes in current, the current through the inductor at time $t = 0^+$ will still be zero. This means that all current in the circuit will flow along the path provided by R_1 and R_2. The current will then be given by:

$$i_{R_1} = \frac{10V}{150\Omega} = 0.0667A$$

The voltage across R_2 is then:

$$v_3 - v_2 = i_{R_1} R_2 = 0.0667 * 100 = 6.67V$$

Since R_2 has twice the resistance of R_1, we expect twice the voltage drop if the resistors share the same current, so this makes sense.

(b) At $t = t_0$, $v_1 - v_2 = +4V$. Find the value of $i_2(t_0)$ and $\frac{di_2(t_0)}{dt}$. (2 points)

First, we can find the derivative:

$$v_1 - v_2 = L\frac{di_2(t_0)}{dt}$$

$$\frac{di_2(t_0)}{dt} = \frac{4}{10^{-6}} = 4{,}000{,}000A/s$$

Since the voltage across the inductor is given and we prefer not to solve an integral for the current, we can find the values of components around it to solve this problem.

Using KVL, we get two equations:

$$10 = i_{R_1} * 50 + i_{R_2} * 100$$

$$i_{R_2} * 100 = (i_{R_1} - i_{R_2})100 + 4 = 100i_{R_1} - 100i_{R_2} + 4$$

Rearranging the second equation:

$$200i_{R_2} = 100i_{R_1} + 4$$

$$100i_{R_2} = 50i_{R_1} + 2$$

Plugging in to the first equation:

$$10 = 50i_{R_1} + 50i_{R_1} + 2$$

$$i_{R_1} = \frac{8}{100} = 0.08A$$

$$i_{R_2} = 0.5i_{R_1} + 0.02 = 0.06A$$

Since the sum of the currents through R_2 and R_3 must equal the current through R_1 by KCL:

$$i_{R_3} = i_2(t_0) = i_{R_1} - i_{R_2} = 0.02A$$

(c) At time $t = t_1$, $v_3 - v_2 = +5V$. Calculate $i_2(t_1)$ and $\frac{di_2(t_1)}{dt}$. (1 point)

This problem can be solved similarly to problem (b) above, but we can also notice that the voltage across R_2 is equal to the voltage across R_1 by KVL since both are 5V. Since R_1 has half the resistance of R_2, we will see twice as much current through R_1 as R_2. This means the same current must flow through the inductor branch as flows through R_2. Since R_3 has the same resistance as R_2. If these resistors share the same value of current, they must also share the same voltage of 5V. By KVL, the voltage across the inductor must be 0.

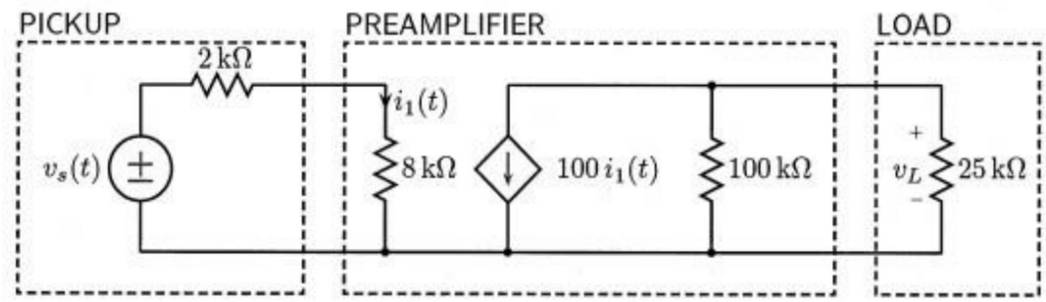
$$v_1 - v_2 = 0 = L\frac{di_2(t_1)}{dt}$$

$$\frac{di_2(t_1)}{dt} = 0$$

The current through the inductor must be the same as the current through R_3 since they are in series.

$$i_2(t_1) = \frac{5}{100} = 0.05A$$

Q3: Shown below is a circuit model of a "single transistor preamplifier" that is used to amplify the output of a low amplitude magnetic pickup ($v_s(t)$) and drive a 25kOhm load. Express $v_L(t)$, the voltage measured across the load, as a function of the magnetic pick up (i.e. $v_s(t)$). (10 points)



The right portion of the circuit depends on the left portion of the circuit only because of the dependent voltage source and there is no relationship the other way. This allows us to easily solve for the current $i_1(t)$ by taking the equivalent resistance in this portion of the circuit.

$$i_1(t) = \frac{v_s(t)}{2k\Omega + 8k\Omega} = \frac{v_s(t)}{10{,}000\Omega}$$

The voltage across the load is shared across both resistors, so we can combine these in parallel to find the voltage.

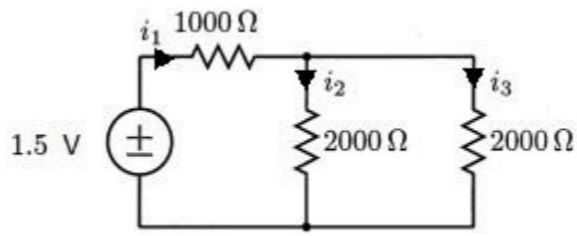
$$R_{eq} = \frac{100k\Omega * 25k\Omega}{100k\Omega + 25k\Omega} = 20k\Omega$$

$$v_L(t) = -100i_1(t)R_{eq} = -2{,}000{,}000i_1(t)$$

Plugging in:

$$v_L(t) = -2{,}000{,}000\frac{v_s(t)}{10{,}000} = -200v_s(t)$$

Q4: Consider the circuit below and answer the questions that follow. (10 points)



(a) This circuit contains three elements and thus there are six element variables, all of which are constant since the voltage source is constant. The currents are labelled and the voltages across the terminals of the three resistors are implied by the default sign convention. Write a set of six linear equations in the variables $v_1, v_2, v_3, i_1, i_2,$ and i_3 that specify the complete solution. These should take the form of three element relations, one KCL equation, and two KVL equations.

Element relations:

$$v_1 = 1000i_1$$

$$v_2 = 2000i_2$$

$$v_3 = 2000i_3$$

KVL (The equations for the left and right loops give all information about the circuit.):

$$1.5 = v_1 + v_2$$

$$v_2 = v_3$$

KCL (Only one node provides an interesting equation if we consider the bottom node to be ground.):

$$i_1 = i_2 + i_3$$

(b) Solve the above set of equations to determine the values of the element variables.

$$v_2 = v_3$$

$$2000i_2 = 2000i_3$$

$$i_2 = i_3$$

$$i_1 = i_2 + i_3 = 2i_2$$

$$1.5 = v_1 + v_2 = 1000i_1 + 2000i_2$$

$$1.5 = 1000 * 2i_2 + 2000i_2 = 4000i_2$$

$$i_2 = i_3 = \frac{1.5}{4000} = 375\mu A$$

$$i_1 = 2i_2 = 750\mu A$$

$$v_1 = 1000i_1 = 750mV$$

$$v_2 = v_3 = 2000i_2 = 750mV$$

(c) Evaluate the power absorbed by all of the elements and sources. Show that the total power absorbed in the resistors is equal to the total power supplied by the source.

Power is given by $P = IV$. Since the direction of current through the voltage source is opposite the direction for positive current given by the passive sign convention, we get negative current through the voltage source.

$$P_{source} = -0.75mA * 1.5V = -1.125mW$$

We expect the power dissipated in the resistors to add up to this power.

$$P_1 = 0.75mA * 0.75V = 0.5625mW$$

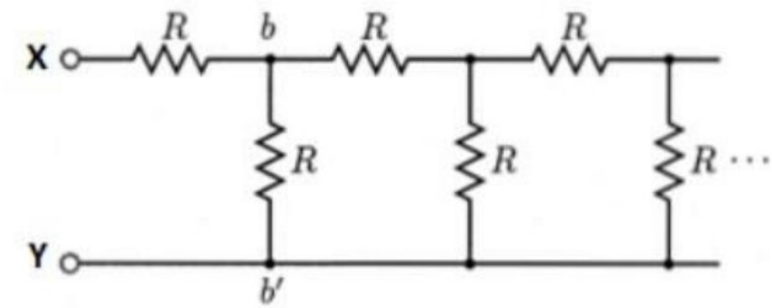
$$P_2 = P_3 = 0.375mA * 0.75V = 0.28125mW$$

$$P_1 + P_2 + P_3 + P_{source} = 0$$

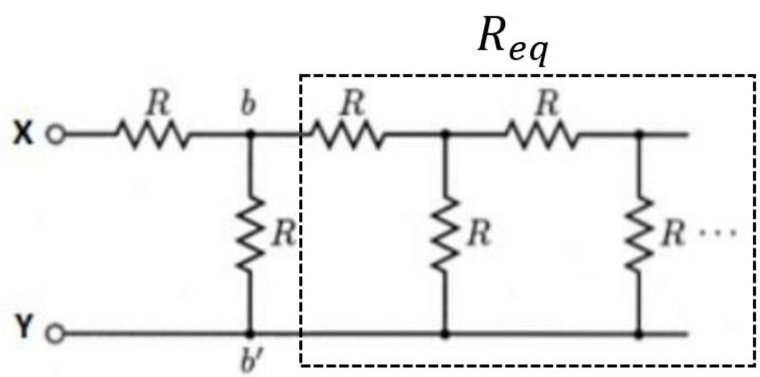
$$0.5625mW + 0.28125mW + 0.28125mW - 1.125mW = 0$$

Note that positive values for power indicates power is consumed while negative values indicate power is provided.

Q5: Find the equivalent resistance between **X** and **Y** of the infinite ladder network shown below. (5 points)



The key to this problem is observing that we can replace part of this circuit with R_{eq}.



The problem then boils down to a simple resistor network and solving a quadratic equation. (The || operator refers to adding components in parallel.)

$$R_{eq} = R + R||R_{eq}$$

$$R_{eq} = R + \frac{RR_{eq}}{R + R_{eq}}$$

$$R_{eq}^2 + RR_{eq} = R^2 + 2RR_{eq}$$

$$R_{eq}^2 - RR_{eq} - R^2 = 0$$

$$R_{eq} = \frac{R \pm \sqrt{R^2 + 4R^2}}{2}$$

We throw out the negative solution since resistance must be a positive value.

$$R_{eq} = \frac{\left(1 + \sqrt{5}\right)}{2} R$$