Practice Midterm 2 solution

Q1:

(a)Circuit time constant is R*C = 100 MΩ*1 µF=100s.

(b)At t=0, energy store in capacitor is 1mJ

$$\frac{1}{2}CV^2 = E$$
$$\frac{1}{2}1\mu V^2 = 1m$$

Hence, V = 44.72V at t=0.

The voltage across the capacitor is $v = 44.72e^{-\frac{t}{100}}$.

At t = 20s, $v = 44.72e^{-\frac{20}{100}} = 36.61V$.

The current $i = \frac{36.61}{100M} = 3.66 * 10^{-7}A$.

Q2:

(a)At t=0, the capacitor is charged to $V_c = 1 * 4.7k = 4700V$. After the switch is opened, the current through inductor $i_L(0) = 0A$. Since the voltage across the resistor $V_R(0) = i_L(0) * R = 0$, $V_L(0) = V_C(0) - V_R(0) = 4700V$. After t=0, V_L will be lower than $4700V$ because current is greater than 0 and there will be voltage across the resistor. Hence, the peak voltage magnitude is 4700V across the inductor.

(b)Write down the equation,

$$0 = V_R + V_L + V_C$$
$$0 = i_L(t)R + L\frac{di_L(t)}{dt} + V_C(t)$$
$$0 = i_L(t)R + L\frac{di_L(t)}{dt} + \frac{1}{C}\int_{-\infty}^{t} i_L(t)$$

Take derivative,

$$0 = \frac{R}{L}\frac{di_L(t)}{dt} + \frac{d^2 i_L(t)}{dt^2} + \frac{1}{LC}i_L(t)$$

$\alpha = \frac{R}{2L} = \frac{4700}{2*500*10^{-3}} = 4700$ and $w_0 = \frac{1}{\sqrt{LC}} = 447.2$, this is overdamped.

Write the differential equation, $S^2 + 9400S + 200000 = 0$

$$S_1 = -21.32, S_2 = -9378.68$$

$$i_L(t) = K_1 e^{-21.32t} + K_2 e^{-9378.68t}$$

At t = 0, $i_L(0) = 0 = K_1 + K_2$

$$V_L(0) = -4700 = L\frac{di_L(0)}{dt} = -21.32K_1 - 9378.68K_2$$

$$K_1 = -0.502 and\ K_2 = 0.502$$

Hence, $i_L(t) = -0.502e^{-21.32t} + 0.502e^{-9378.68t}$

Q3:

Both D_1 and D_2 are on. $V_x = \frac{1K}{2.2K+1K} * 2V = 0.606V$.