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Due Tuesday (4/12) @3PM

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ECE 100 HW1

1\. i) KCL states that the sum of all currents entering a node is the same as the sum of all currents leaving a node. Since $i_1(t)$ is the only current entering N_3 and $i_2(t)$ is the only current leaving N_3, we know that $i_1(t) = i_2(t)$

KCL: $i_1(t) - i_2(t) = 0$ KCL: $\Sigma i(t) = 0$

$i_1(t) = i_2(t)$ ✓

ii) KCL can also be applied in this situation.

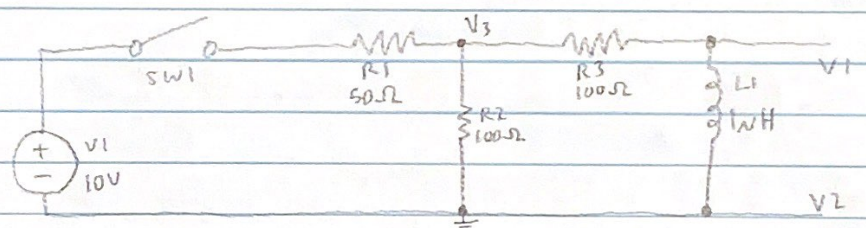
KCL: $\Sigma i(t) = 0$

$$i_1(t) - i_2(t) - i_3(t) + i_4(t) = 0$$

$$i_1(t) + i_4(t) = i_2(t) + i_3(t)$$

The result derived from (i) is not necessarily applicable in this situation. The KCL equation now includes $i_3(t)$ and $i_4(t)$ so we can't say for certain that $i_1(t) = i_2(t)$ anymore.

2\.



$V = IR$

$10 = I\,50$ $I = 0.2$

a) Right when the switch is closed, the inductor (L1) still acts as an open circuit because it doesn't change instantly. Thus $V_3 - V_2$ is essentially the same as the voltage across R2. To find voltage across R2, you can kind of ignore right side of the circuit (because it's an open circuit).

- R1 is in series with R2 → $50\Omega + 100\Omega = 150\Omega$

$V = IR$

$10V = I(150\Omega)$

$I = 0.0667$

Voltage across R2: $V = IR$ $\boxed{V_3 - V_2 = 6.67\,V}$

$$V = 0.0667 \times 100\Omega = 6.67V$$

2

b) $t = t_0 \quad V_1 - V_2 = 4V$

$V_2 = 0V$, it's connected to ground $\quad V_1 = 4V$

KCL @ V_3

$$\underbrace{\frac{10 - V_3}{50}}_{\text{entering } V_3} = \underbrace{\frac{V_3 - 0}{100} + \frac{V_3 - V_1}{100}}_{\text{Leaving } V_3}$$

$$\frac{10 - V_3}{50} = \frac{V_3}{100} + \frac{V_3 - 4}{100}$$

$$-4V_3 = -24$$

$$V_3 = 6V$$

Next find current through R3

$$\frac{V_3 - V_1}{100} = \frac{6 - 4}{100} = 0.02A$$

KCL

Current through R3 flows into L1 (current entering V_1 = current leaving V_1)

$$i_2(t_0) = 0.02A$$

$$V = L\frac{di_2}{dt}$$

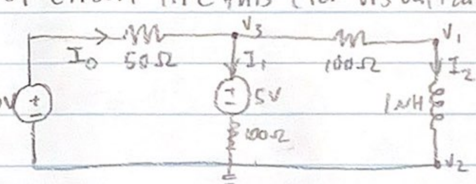
$$4 = 10^{-6}\frac{di_2}{dt} \rightarrow \frac{di_2}{dt} = 4000000 \text{ A/s}$$

$$\boxed{\begin{aligned} i_2(t_0) &= 0.02A \\ \frac{di_2(t_0)}{dt} &= 4000000 \text{ A/s} \end{aligned}}$$

c) $t = t_1 \quad V_3 - V_2 = 5V$

$V_2 = 0V$, it's connected to ground $\quad V_3 = 5V \quad$ (voltage across R2 is 5V)

Think of circuit like this (for my visualization purposes):



$$I_0 = \frac{10 - 5}{50} = 0.1A$$

$$I_1 = \frac{R_3 - 0}{100} = \frac{5}{100} = 0.05A$$

$$I_2 = I_0 - I_1 \rightarrow I_2 = 0.05A \qquad i_2(t_1) = 0.05A$$

Use KCL at V_3

$$\frac{10 - V_3}{50} = \frac{V_3 - 0}{100} + \frac{V_3 - V_1}{100} \qquad V = L\frac{di}{dt}$$

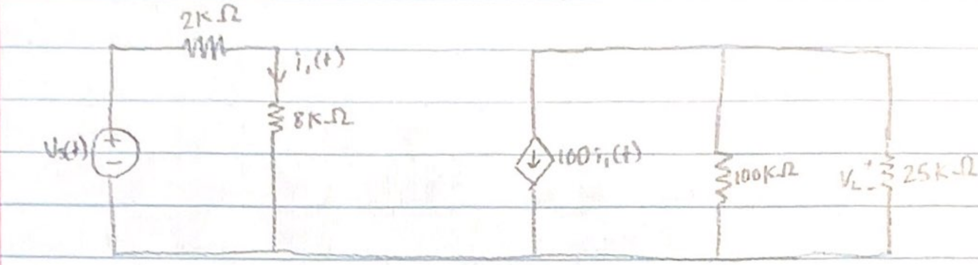
$$\frac{10 - 5}{50} = \frac{5}{100} + \frac{5 - V_1}{100} \qquad 0 = 10^{-6}\frac{di}{dt}$$

$$V_1 = 0 \qquad 0 = \frac{di}{dt}$$

$$V_1 - V_2 = 0 \qquad 0 = \frac{di_2(t_1)}{dt}$$

$$\boxed{\begin{aligned} i_2(t_1) &= 0.05A \\ \frac{di_2(t_1)}{dt} &= 0 \text{ A/s} \end{aligned}}$$

3. Express $V_L(t)$ as a function of $V_s(t)$

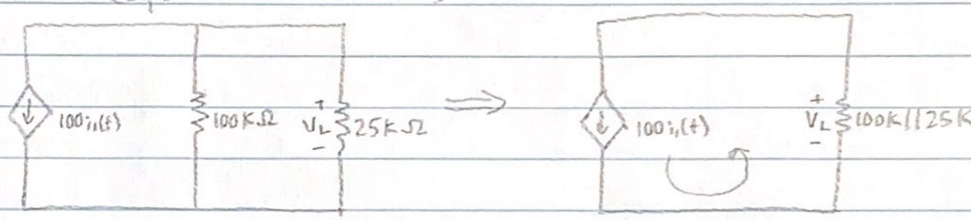


Ohm's Law: $V = iR$

$V_s = i_1(10K\Omega)$

$i_1 = \frac{V_s}{10000}$ A

Right Side (Equivalent Circuits)



$100K \| 25K = \frac{100K \times 25K}{100K + 25K} = 20K$

$V_L = iR$

$V_L = -100 i_1(t) \times 20K\Omega$

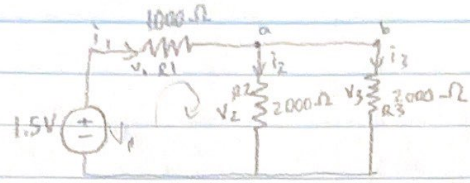
$V_L = -100 \frac{V_s}{10000} \times 20K\Omega$

$V_L = -\frac{V_s}{100} \times 20000$

$\boxed{V_L = -200 V_s}$

$V = iR$

4.



a) Ohm's Law

$V_1 = 1000 i_1$

$V_2 = 2000 i_2$

$V_3 = 2000 i_3$

KVL

$1.5 = 1000 i_1 + 2000 i_2$ $(1.5 = V_1 + V_2)$

$V_2 = V_3$ ($i_2 = i_3$ bc node a and node b are same)

KCL: $i_1 = i_2 + i_3$

entering node a   leaving node a (node a and node b are equivalent)

b) $V_2 = V_3$

$2000 i_2 = 2000 i_3 \rightarrow i_2 = i_3$ ✓

$i_1 = i_2 + i_3$

$i_1 = 2 i_2$ ✓

$1.5 = 1000 i_1 + 2000 i_2$

$1.5 = 2000 i_2 + 2000 i_2$

$1.5 = 4000 i_2$

$i_2 = 0.000375A = 0.375mA$

$\boxed{\begin{array}{l} i_1 = 0.75mA \\ i_2 = 0.375mA \\ i_3 = 0.375mA \\ V_1 = 0.75V \\ V_2 = 0.75V \\ V_3 = 0.75V \end{array}}$

c)

$P = vi$

Element	Power Absorbed
V_1	$1.5 \times (-i_1) = 1.5 \times (-0.75mA) = -1.125mW$
R_1	$V_1 \times (i_1) = 0.75V \times 0.75mA = 0.5625mW$
R_2	$V_2 \times (i_2) = 0.75V \times 0.375mA = 0.28125mW$
R_3	$V_3 \times (i_3) = 0.75V \times 0.375mA = 0.28125mW$

Total Power Absorbed by Source = −(Power absorbed from V_1)

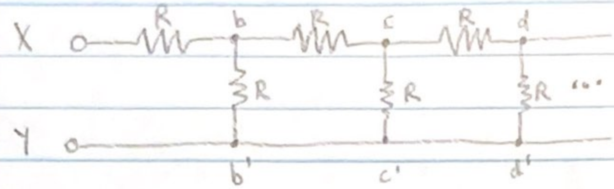
= 1.125 mW

Total Power Absorbed by Resistors = 0.5625mW + 0.28125mW + 0.28125mW

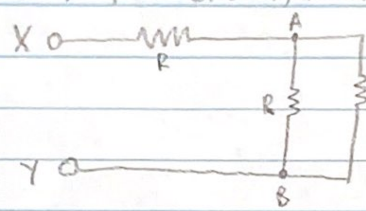
= 1.125mW

This shows us that the total power absorbed in the resistors is equal to the total power supplied by the source. This means that overall, the circuit absorbed 0 power. (0W)

5.



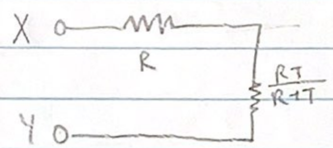
To simplify the circuit, think of it like this:



(T represents equivalent resistance between nodes X and Y)

Now observe that $R \parallel T$

$R \parallel T = \frac{RT}{R+T}$



Now the resistors are in series

$T(R+T)$ $\qquad T = R + \frac{RT}{R+T}$

$TR + T^2$ $\qquad RT + T^2 = R^2 + 2RT$

$T^2 = R^2 + RT$

$0 = R^2 - T^2 + RT$

Solve for T

$T = -\frac{-R - \sqrt{5}R}{2} \Rightarrow T = \frac{R + \sqrt{5}R}{2} \Rightarrow T = \left(\frac{1+\sqrt{5}}{2}\right)R$

$T = 1.618R$

$R_{eq} = 1.618R$

$\boxed{\text{The equivalent resistance between X and Y is } 1.618R.}$