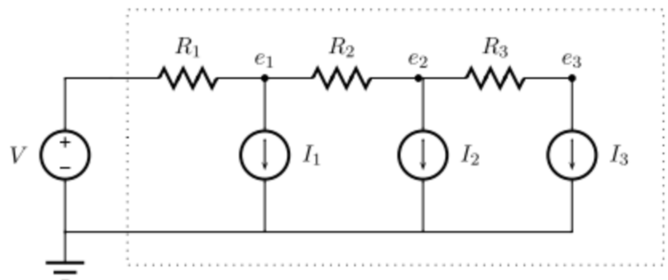
ECE100 Midterm Solutions Spring 2022

Q1: The dotted line below shows a model for power distribution network of a typical processor. The voltage source models the external supply that powers the processor, the resistors model the power distribution wiring internal to the processor, and the current sources model the loads presented by the individual parts of the processor. The source values V, I_1, I_2, and I_3 are all positive, as are the three internal node voltages e_1, e_2, and e_3. Further, depending upon whether the corresponding part of the processor is in use or not, I_1, I_2, and I_3 can each take on only the value of either I or zero.



Q1.1: Using the node method, develop a set of simultaneous equations for the power distribution network that can be solved for the three unknown node voltages e1, e2 and e3. Express these equations in the form:

$$G\begin{bmatrix} e1 \\ e2 \\ e3 \end{bmatrix} = S$$

Solution:

$$S = \begin{bmatrix} \frac{V}{R_1} - I_1 \\ -I_2 \\ -I_3 \end{bmatrix}$$

$$G = \begin{bmatrix} \frac{1}{R_1} + \frac{1}{R_2} & -\frac{1}{R_2} & 0 \\ -\frac{1}{R_2} & \frac{1}{R_2} + \frac{1}{R_3} & -\frac{1}{R_3} \\ 0 & -\frac{1}{R_3} & \frac{1}{R_3} \end{bmatrix}$$

Q1.2: Given the possible values for I1, I2 and I3, what is the maximum power that the voltage source must be able to supply to the processor? Express your answer in terms of V, I, R1, R2 and R3.

Solution: The maximum power will be provided by the source when all the current sources are on at the same time.

$$P_{Max} = VI_{Max} = V(3I) = 3VI$$

Q2: You were taught that superposition works for RLC circuits. So to find out the loop current in the adjacent circuit, you remove V1 calculate current and then remove V2 and calculate current and then add them up to find total current as follows:

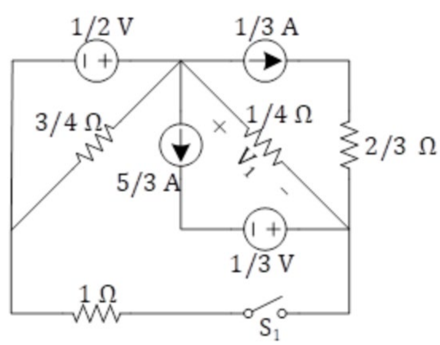
$$I_1 = \frac{V_2 + V_3}{R} \; ; I_2 = \frac{V_1 + V_3}{R}$$

$$I = I_1 + I_2 \text{ (by superposition)} = \frac{V_1+V_2+2V_3}{R}$$

But for this simple circuit, you know I should be (V1+V2+V3)/R. Now you are confused and start cursing your professor who told you that RLC circuit are linear and superposition works. What went wrong here?

Solution: For superposition, each independent source can only be on for one of the cases. Instead of turning off one source at a time, we should turn off all source *except* one in each case and break the circuit into 3 cases, one for each source. Alternatively, if we want to use 2 cases, we could leave sources 1 and 2 on with 3 turned off as one case, then leave 3 on with 1 and 2 off for the second case. As described in the problem, we double-count source 3.

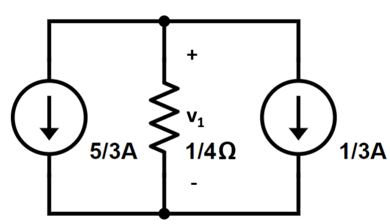
Q3: Use source transformation to solve the following question, what is the voltage V_1 when switch S_1 is open?



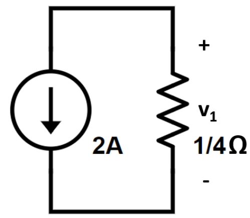
Solution: With switch S_1 open, we can recognize a few things:

1. The 1Ω resistor is only connected on one end, so its current must be 0.
2. The 1/2V source and the 3/4Ω resistor are only connected to the part of the circuit with V_1 at a single point, so it will not affect V_1.
3. The two circuit branches containing current sources can be simplified to just contain current sources since the voltage across the entire branch will always be whatever is necessary to maintain the current. This allows us to drop the 2/3Ω resistor and the 1/3V source.

Our simplified circuit now looks like this:



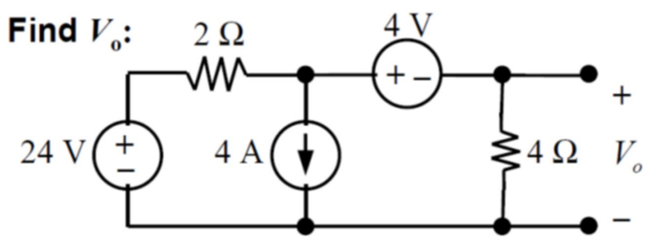
Since the current sources are now in parallel, we can add them:



Using Ohm's Law and the passive sign convention:

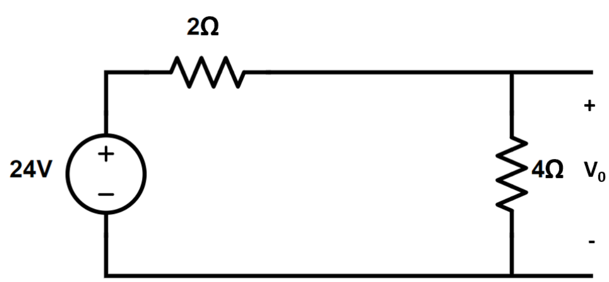
$$V_1 = -2\left(\frac{1}{4}\right) = -\frac{1}{2}V$$

Q4:



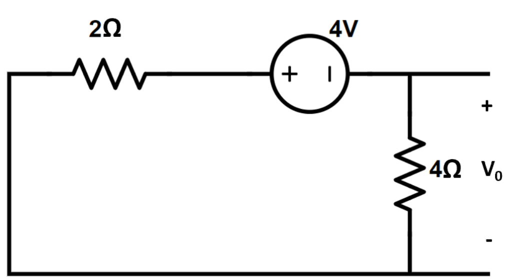
Q4.1: Use superposition to solve for V_0.

Case 1 (24V source only):



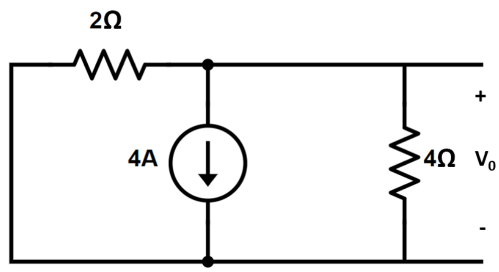
We can use the voltage divider equation to get $V_{0,1} = 24\left(\frac{4}{4+2}\right) = 16V$

Case 2 (4V source only):



Another voltage divider $V_{0,2} = -4\left(\frac{4}{2+4}\right) = -\frac{8}{3}V$

Case 3 (4A source only):



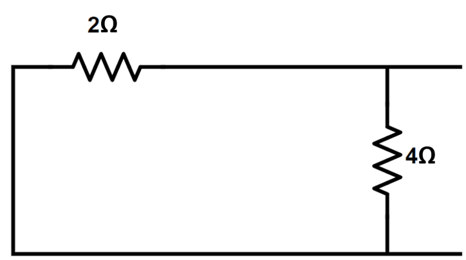
In this case, we can find the voltage over the parallel combination of the resistors.

$$V_{0,3} = -4\left(\frac{2(4)}{2+4}\right) = -\frac{16}{3}V$$

Combining $V_0 = V_{0,1} + V_{0,2} + V_{0,3} = 8V$

Q4.2: Find Thevenin Equivalent across V_0 terminals.

In the previous part we found the Thevenin voltage, so now we need to find the Thevenin resistance. If we turn all the sources off, we just have a parallel combination of the two resistors.



$$R_{Th} = \frac{2(4)}{2+4} = \frac{4}{3}\Omega$$

$$V_{Th} = 8V$$

The Thevenin Equivalent circuit is shown below: