HW4 part1 answers

Q1.

The energy store in an inductor is $E = \frac{1}{2}LI^2 = \frac{1}{2}10I^2 = 5I^2$.

After 1hr, the inductor still has 75% energy, E = 0.75*5$I^2$=3.75$I^2$, and the resistance consumes 1.25$I^2$.

Since $I(t) = Ie^{\frac{-t}{£}}, P_R(t) = I^2(t)R = I^2 e^{\frac{-2t}{£}} R$.

$$E_R = \int_0^{60*60} I^2 e^{\frac{-2t}{£}} R\, dt = I^2 e^{\frac{-2t}{£}} R \frac{£}{-2} |_0^{3600}$$

$$= I^2 R \frac{£}{-2}\left[e^{\frac{-2*3600}{£}} - e^{\frac{-2*0}{£}}\right] \leq 1.25I^2$$

$$-R£\left[e^{\frac{-7200}{£}} - 1\right] \leq 2.5, \text{ and } £=\frac{L}{R} = \frac{10}{R}$$

$$-10[e^{-720R} - 1] \leq 2.5$$

$$e^{-720R} \leq 0.75$$

$$-720R \leq \ln(0.75)$$

$$R \leq 0.4m\Omega$$

Q2.

Since the circuit is connected for a long time, the capacitor is opened and iR = 4mA. The inductor is a short circuit, so Vc = 12 + 4mA * 2KΩ = 20 V.

Q3.

For the steady-state, capacitor is opened. So i1 = 0A, i2 = i3 = 2A.

Q4.

First, we write down the KVL of this circuit.

$$V_x = L\frac{di(t)}{dt} + i(t)R + v_c(t), \text{ and } i(t) = C\frac{dV_C(t)}{dt}$$

$$\text{So, } V_x = LC\frac{d^2V_C(t)}{dt^2} + RC\frac{dV_C(t)}{dt} + v_c(t)$$

$$50 = 2 * 10^{-3} * 5 * 10^{-6}\frac{d^2V_c(t)}{dt^2} + 80 * 5 * 10^{-6}\frac{dV_c(t)}{dt} + v_c(t)$$

$$50 * 10^8 = \frac{d^2V_c(t)}{dt^2} + 40000\frac{dV_c(t)}{dt} + 10^8 v_c(t)$$

$$\text{Since } \alpha = \frac{R}{2L} = \frac{80}{2*2*10^{-3}} = 20000 \text{ and } w_0 = \frac{1}{\sqrt{LC}} = 10000$$

$$\frac{\alpha}{w_0} = 2, \text{ this is the overdamped case.}$$

Write the differential equation,

$$s^2 + 40000S + 10^8 = 0$$

$$S = -37320.5\ or - 2679.5$$

$$\text{So } V_c(t) = K_1e^{-37320.5t} + K_2e^{-2679.5t} + 50$$

$$\text{When t = 0, } 0 = K_1 + K_2 + 50$$

$$\text{We know } i(t) = C\frac{dV_C(t)}{dt}$$

$$= 5 * 10^{-6}(-37320.5 * K_1e^{-37320.5t} - 2679.5 * K_2e^{-37320.5t})$$

$$\text{So } i(0) = 0 = 5 * 10^{-6}(-37320.5 * K_1 - 2679.5 * K_2)$$

We have $K_1 = 3.8675$ and $K_2 = -53.8675$

$$V_c(t) = 3.8675e^{-37320.5t} - 53.8675e^{-2679.5t} + 50$$