CHAPTER 1

Exercises

E1.1 Charge = Current $\times$ Time = (2 A) $\times$ (10 s) = 20 C

E1.2 $i(t) = \dfrac{dq(t)}{dt} = \dfrac{d}{dt}(0.01\sin(200t) = 0.01 \times 200\cos(200t) = 2\cos(200t)$ A

E1.3 Because i_2 has a positive value, positive charge moves in the same direction as the reference. Thus, positive charge moves downward in element *C*.

Because i_3 has a negative value, positive charge moves in the opposite direction to the reference. Thus positive charge moves upward in element *E*.

E1.4 Energy = Charge $\times$ Voltage = (2 C) $\times$ (20 V) = 40 J

Because v_{ab} is positive, the positive terminal is *a* and the negative terminal is *b*. Thus the charge moves from the negative terminal to the positive terminal, and energy is removed from the circuit element.

E1.5 i_{ab} enters terminal *a*. Furthermore, v_{ab} is positive at terminal *a*. Thus the current enters the positive reference, and we have the passive reference configuration.

E1.6 (a) $p_a(t) = v_a(t)i_a(t) = 20t^2$

$$w_a = \int_0^{10} p_a(t)dt = \int_0^{10} 20t^2 dt = \left.\frac{20t^3}{3}\right|_0^{10} = \frac{20t^3}{3} = 6667 \text{ J}$$

(b) Notice that the references are opposite to the passive sign convention. Thus we have:

$$p_b(t) = -v_b(t)i_b(t) = 20t - 200$$

$$w_b = \int_0^{10} p_b(t)dt = \int_0^{10} (20t - 200)dt = 10t^2 - 200t\Big|_0^{10} = -1000 \text{ J}$$

E1.7 (a) Sum of currents leaving = Sum of currents entering
$i_a = 1 + 3 = 4$ A

(b) $2 = 1 + 3 + i_b \quad \Rightarrow \quad i_b = -2$ A

(c) $0 = 1 + i_c + 4 + 3 \quad \Rightarrow \quad i_c = -8$ A

E1.8 Elements A and B are in series. Also, elements E, F, and G are in series.

E1.9 Go clockwise around the loop consisting of elements A, B, and C:
$-3 - 5 + v_c = 0 \quad \Rightarrow \quad v_c = 8$ V

Then go clockwise around the loop composed of elements C, D and E:
$-v_c - (-10) + v_e = 0 \quad \Rightarrow \quad v_e = -2$ V

E1.10 Elements E and F are in parallel; elements A and B are in series.

E1.11 The resistance of a wire is given by $R = \dfrac{\rho L}{A}$. Using $A = \pi d^2 / 4$ and substituting values, we have:

$$9.6 = \frac{1.12 \times 10^{-6} \times L}{\pi (1.6 \times 10^{-3})^2 / 4} \quad \Rightarrow \quad L = 17.2 \text{ m}$$

E1.12 $P = V^2/R \quad \Rightarrow \quad R = V^2 / P = 144\ \Omega \quad \Rightarrow \quad I = V/R = 120/144 = 0.833$ A

E1.13 $P = V^2/R \quad \Rightarrow \quad V = \sqrt{PR} = \sqrt{0.25 \times 1000} = 15.8\text{ V}$
$I = V/R = 15.8/1000 = 15.8\text{ mA}$

E1.14 Using KCL at the top node of the circuit, we have $i_1 = i_2$. Then, using KVL going clockwise, we have $-v_1 - v_2 = 0$; but $v_1 = 25$ V, so we have $v_2 = -25$ V. Next we have $i_1 = i_2 = v_2/R = -1$ A. Finally, we have $P_R = v_2 i_2 = (-25) \times (-1) = 25$ W and $P_s = v_1 i_1 = (25) \times (-1) = -25$ W.

E1.15 At the top node we have $i_R = i_s = 2$A. By Ohm's law we have $v_R = Ri_R = 80$ V. By KVL we have $v_s = v_R = 80$ V. Then $p_s = -v_s i_s = -160$ W (the minus sign is due to the fact that the references for v_s and i_s are opposite to the passive sign configuration). Also we have $P_R = v_R i_R = 160$ W.

Problems

P1.1 Broadly, the two objectives of electrical systems are:
1. To gather, store, process, transport, and display information.
2. To distribute, store, and convert energy between various forms.

P1.2 Eight subdivisions of EE are:

1. Communication systems.
2. Computer systems.
3. Control systems.
4. Electromagnetics.
5. Electronics.
6. Photonics.
7. Power systems.
8. Signal Processing.

P1.3 Four important reasons that non-electrical engineering majors need to learn the fundamentals of EE are:

1. To pass the Fundamentals of Engineering Exam.
2. To be able to lead in the design of systems that contain electrical/electronic elements.
3. To be able to operate and maintain systems that contain electrical/electronic functional blocks.
4. To be able to communicate effectively with electrical engineers.

P1.4 Responses to this question are varied.

P1.5 (a) Electrical current is the time rate of flow of net charge through a conductor or circuit element. Its units are amperes, which are equivalent to coulombs per second.
(b) The voltage between two points in a circuit is the amount of energy transferred per unit of charge moving between the points. Voltage has units of volts, which are equivalent to joules per coulomb.
(c) The current through an open switch is zero. The voltage across the switch can be any value depending on the circuit.

(d) The voltage across a closed switch is zero. The current through the switch can be any value depending of the circuit.
(e) Direct current is constant in magnitude and direction with respect to time.
(f) Alternating current varies either in magnitude or direction with time.

P1.6 (a) A conductor is analogous to a frictionless pipe.
(b) A resistance is analogous to a constriction in a pipe or to a pipe with friction.
(c) A battery is analogous to a pump.
(d) A fluid flow is analogous to current flow.

P1.7* The reference direction for i_{ab} points from *a* to *b*. Because i_{ab} has a negative value, the current is equivalent to positive charge moving opposite to the reference direction. Finally, since electrons have negative charge, they are moving in the reference direction (i.e., from *a* to *b*).
For a constant (dc) current, charge equals current times the time interval. Thus, $Q = (3\ A) \times (3\ s) = 9\ C$.

P1.8* $i(t) = \frac{dq(t)}{dt} = \frac{d}{dt}\left(4 + 2t + 5t^2\right) = 2 + 10t\ A$

P1.9* $Q = \int_0^\infty i(t)dt = \int_0^\infty 2e^{-t}dt = -2e^{-t}\ |_0^\infty = 2 \text{ coulombs}$

P1.10* The charge flowing through the battery is

$$Q = (5 \text{ amperes}) \times (24 \times 3600 \text{ seconds}) = 432 \times 10^3 \text{ coulombs}$$

The stored energy is

$$\text{Energy} = QV = (432 \times 10^3) \times (12) = 5.184 \times 10^6 \text{ joules}$$

(a) Equating gravitational potential energy, which is mass times height times the acceleration due to gravity, to the energy stored in the battery and solving for the height, we have

$$h = \frac{\text{Energy}}{mg} = \frac{5.184 \times 10^6}{30 \times 9.8} = 17.6 \text{ km}$$

(b) Equating kinetic energy to stored energy and solving for velocity, we have

$$v = \sqrt{\frac{2 \times \text{Energy}}{m}} = 587.9 \text{ m/s}$$

(c) The energy density of the battery is

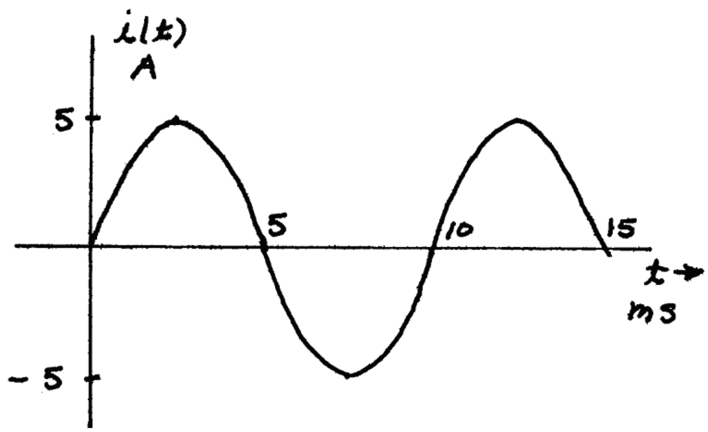
$$\frac{5.184 \times 10^6}{30} = 172.8 \times 10^3 \text{ J/kg}$$

which is about 0.384% of the energy density of gasoline.

P1.11* $Q = \text{current} \times \text{time} = (10 \text{ amperes}) \times (36{,}000 \text{ seconds}) = 3.6 \times 10^5 \text{ coulombs}$

$\text{Energy} = QV = (3.6 \times 10^5) \times (12.6) = 4.536 \times 10^6 \text{ joules}$

P1.12 (a) The sine function completes one cycle for each 2π radian increase in the angle. Because the angle is $200\pi t$, one cycle is completed for each time interval of 0.01 s. The sketch is:



(b) $Q = \int_0^{0.01} i(t)dt = \int_0^{0.01} 5\sin(200\pi t)dt = (5/200\pi)\cos(200\pi t)\Big|_0^{0.01}$

$= 0 \ C$

(b) $Q = \int_0^{0.015} i(t)dt = \int_0^{0.015} 5\sin(200\pi t)dt = (5/200\pi)\cos(200\pi t)\Big|_0^{0.015}$

$= 0.0159 \ C$

P1.13 To cause current to flow, we make contact between the conducting parts of the switch, and we say that the switch is closed. The corresponding fluid analogy is a valve that allows fluid to pass through. This corresponds to an open valve. Thus, an open valve is analogous to a closed switch.

P1.14 $\text{Electrons per second} = \dfrac{5 \text{ coulomb/s}}{1.60 \times 10^{-19} \text{ coulomb/electron}} = 3.125 \times 10^{19}$

P1.15 The positive reference for i is at the tail of the arrow, which is terminal a. As per direction given in fig. P1.15, the current enters at terminal a, but since i = -5A, it actually enters at terminal b. Since it consumes 60W, terminal b is positive with respect to a, correct polarity is v_{ba} and $v = v_{ab} = -v_{ba} = -12\text{V}$. The true polarity is positive at terminal b. Since current enters the device at the positive terminal of voltage, the device consumes energy.

P1.16 $i(t) = \dfrac{dq(t)}{dt} = \dfrac{d}{dt}\left(2t + 3t^2\right) = 2 + 6t \text{ A}$

P1.17 The number of electrons passing through a cross section of the wire per second is

$$N = \frac{15\sqrt{2}}{1.6 \times 10^{-19}} = 1.326 \times 10^{20} \text{ electrons/second}$$

The volume of copper containing this number of electrons is

$$\text{volume} = \frac{1.326 \times 10^{20}}{10^{29}} = 1.326 \times 10^{-9} \text{ m}^3$$

The cross sectional area of the wire is

$$A = \frac{\pi d^2}{4} = 3.301 \times 10^{-6} \text{ m}^2$$

Finally, the average velocity of the electrons is

$$u = \frac{\text{volume}}{A} = 401.7 \ \mu\text{m/s}$$

P1.18 The electron gains $1.6 \times 10^{-19} \times 120 = 19.2 \times 10^{-18}$ joules

P1.19 $Q = \text{current} \times \text{time} = (3 \text{ amperes}) \times (2 \text{ seconds}) = 6 \text{ coulombs}$

The magnitude of the energy transferred is

$\text{Energy} = QV = (6) \times (12) = 72 \text{ joules}$

Notice that i_{ab} is positive. If the current were carried by positive charge, it would be entering terminal *a*. Thus, electrons enter terminal *b*. The energy is taken from the element.

P1.20 If the current is referenced to flow into the positive reference for the voltage, we say that we have the passive reference configuration. Using double subscript notation, if the order of the subscripts are the same for the current and voltage, either *ab* or *ba*, we have a passive reference configuration.

P1.21* (a) $P = -v_a i_a = -60$ W Energy is being supplied by the element.

(b) $P = v_b i_b = 60$ W Energy is being absorbed by the element.

(c) $P = -v_{DE} i_{ED} = -210$ W Energy is being supplied by the element.

P1.22* $Q = w/V = (600 \text{ J})/(12 \text{ V}) = 50\ C$.

To increase the chemical energy stored in the battery, positive charge should move through the battery from the positive terminal to the negative terminal, in other words from *a* to *b*. Electrons move from *b* to *a*.

P1.23 The amount of energy is $W = QV = (4\ C) \times (25 \text{ V}) = 100$ J. Because the reference polarity for v_{ab} is positive at terminal *a* and v_{ab} is positive in value, terminal *a* is actually the positive terminal. Because the charge moves from the positive terminal to the negative terminal, energy is delivered to the device.

P1.24 Notice that the references are opposite to the passive configuration, so we have

$p(t) = -v(t)i(t) = 45e^{-2t}$ W

$$\text{Energy} = \int_0^\infty p(t)dt = (-45/2)e^{-2t} \Big|_0^\infty = 22.5 \text{ joules}$$

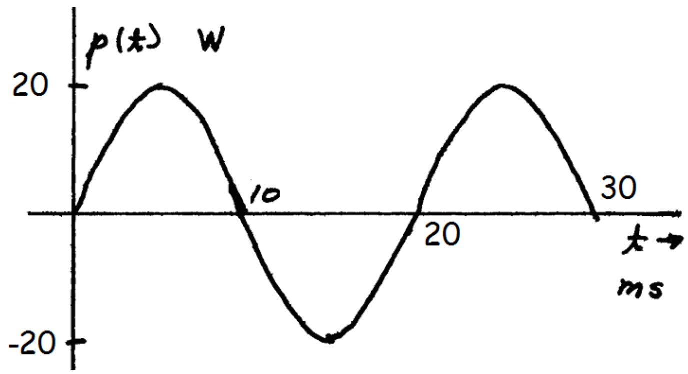
Because the energy is positive, the energy is delivered to the element.

P1.25* $\text{Energy} = \dfrac{\text{Cost}}{\text{Rate}} = \dfrac{\$60}{0.12\ \$/\text{kWh}} = 500\ \text{kWh}$

$$P = \frac{\text{Energy}}{\text{Time}} = \frac{500\ \text{kWh}}{30 \times 24\ \text{h}} = 694.4\ \ \text{W} \qquad I = \frac{P}{V} = \frac{694.4}{120} = 5.787\ \ A$$

$$\text{Reduction} = \frac{60}{694.4} \times 100\% = 8.64\%$$

P1.26 (a) $p(t) = v_{ab} i_{ab} = 20\sin(100\pi t)\ \ \text{W}$



(b) $w = \int_0^{0.010} p(t)dt = \int_0^{0.010} 20\sin(100\pi t)dt = -(20/100\pi)\cos(100\pi t)\Big|_0^{0.010}$

$= 127.3\ \ \text{mJ}$

(c) $w = \int_0^{0.02} p(t)dt = \int_0^{0.02} 20\sin(100\pi t)dt = -(20/100\pi)\cos(100\pi t)\Big|_0^{0.02}$

$= 0\ \ \text{J}$

P1.27 The current supplied to the electronics is $i = p/v = 30/12.6 = 2.381\ A$. The ampere-hour rating of the battery is the operating time to discharge the battery multiplied by the current. Thus, the operating time is $T = 100/2.381 = 42\ \text{h}$. The energy delivered by the battery is $W = pT = 30(42) = 1260\ \text{Wh} = 1.26\ \text{kWh}$. Neglecting the cost of recharging, the cost of energy for 250 discharge cycles is $Cost = 95/(250 \times 1.26) = 0.302\ \$/\text{kWh}$.

***P1.28** (a) P 50 W taken from element A.
(b) P 50 W taken from element A.
(c) P 50 W delivered to element A.

P1.29 (a) P 50 W delivered to element A.
(b) P 50 W delivered to element A.
(c) P 50 W taken from element A.

P1.30 The power that can be delivered by the cell is $p = vi = 0.45$ W. In 10 hours, the energy delivered is $W = pT = 4.5$ Whr $= 0.0045$ kWhr. Thus, the unit cost of the energy is $Cost = (1.95)/(0.0045) = 433.33$ \$/kWhr which is 3611 times the typical cost of energy from electric utilities.

P1.31 A node is a point that joins two or more circuit elements. All points joined by ideal conductors are electrically equivalent. Thus, there are five nodes in the circuit at hand:

1: Joining elements A, B, C, and F
2: Joining elements B and D
3: Joining elements D and G
4: Joining elements E, F, and G
5: Joining elements A, C, and E

P1.32* At the node joining elements A and B, we have $i_a + i_b = 0$. Thus, $i_a = -2$ A. For the node at the top of element C, we have $i_b + i_c = 3$ A. Thus, $i_c = 1$ A. Finally, at the top right-hand corner node, we have $3 + i_e = i_d$. Thus, $i_d = 4$ A. If C is removed then $i_b = -i_a = 3$ A

P1.33 The currents in series-connected elements are equal.

P1.34 The sum of the currents entering a node equals the sum of the currents leaving. It is true because charge cannot collect at a node in an electrical circuit.

P1.35* Elements *B, D* and *G*, together in series, are parallel with element *F*.

P1.36 For a proper fluid analogy to electric circuits, the fluid must be incompressible. Otherwise the fluid flow rate out of an element could be more or less than the inward flow. Similarly the pipes must be inelastic so the flow rate is the same at all points along each pipe.

P1.37* We are given $i_b = 3A$, $i_c = 5A$, $i_g = 6A$, and $i_e = 5A$. Applying KCL, we find

$$i_a = i_b - i_c = -2A \qquad i_g + i_e = i_b + i_d \ \therefore\ i_d = 8A$$

$$i_a + i_d = i_f \therefore\ i_f = 6A \qquad i_f = i_g + i_h \therefore\ i_h = 0$$

Element H is an open circuit

P1.38 We are given $i_a = 2$ A, $i_c = -3$ A, $i_g = 6$ A, and $i_h = 1$ A. Applying KCL, we find

$$i_b = i_c + i_a = -1 \text{ A} \qquad i_e = i_c + i_h = -2 \text{ A}$$

$$i_d = i_f - i_a = 5 \text{ A} \qquad i_f = i_g + i_h = 7 \text{ A}$$

P1.39 (a) Elements A and E are in series, and elements C and D are in series.
(b) Because elements C and D are in series, the currents are equal in magnitude. However, because the reference directions are opposite, the algebraic signs of the current values are opposite. Thus, we have $i_c = -i_d$.
(c) At the node joining elements A, B, and C, we can write the KCL equation $i_b = i_a + i_c = 6 - 2 = 4$ A. Also, we found earlier that $i_d = -i_c = 2$ A.

P1.40 If one travels around a closed path adding the voltages for which one enters the positive reference and subtracting the voltages for which one enters the negative reference, the total is zero. KVL must be true for the law of conservation of energy to hold.

P1.41* Applying KCL and KVL, we have

$$i_c = i_a - i_d = 1 \text{ A} \qquad i_b = -i_a = -2 \text{ A}$$

$$v_b = v_d - v_a = -6 \text{ V} \qquad v_c = v_d = 4 \text{ V}$$

The power for each element is

$$P_A = -v_a i_a = -20 \text{ W} \qquad P_B = v_b i_b = 12 \text{ W}$$

$$P_C = v_c i_c = 4 \text{ W} \qquad P_D = v_d i_d = 4 \text{ W}$$

Thus, $P_A + P_B + P_C + P_D = 0$

P1.42* Summing voltages for the lower left-hand loop, we have $-5 + v_a + 10 = 0$, which yields $v_a = -5$ V. Then for the top-most loop, we have $v_c - 15 - v_a = 0$, which yields $v_c = 10$ V. Finally, writing KCL around the outside loop, we have $-5 + v_c + v_b = 0$, which yields $v_b = -5$ V.

P1.43 We are given $v_a = 10\text{ V}$, $v_b = -3\text{ V}$, $v_f = 12\text{ V}$, and $v_h = 5\text{ V}$. Applying KVL, we find

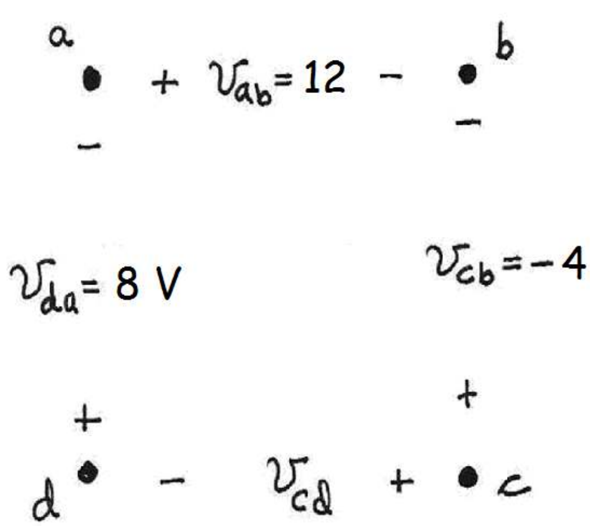
$$v_d = v_a + v_b = 7\text{ V} \qquad v_c = -v_a - v_f - v_h = -27\text{ V}$$
$$v_e = -v_a - v_c + v_d = 24\text{ V} \qquad v_g = v_e - v_h = 19\text{ V}$$

P1.44 (a) Elements *A* and *C* are in parallel.
(b) Because elements *A* and *C* are in parallel, the voltages are equal in magnitude. However because the reference polarities are opposite, the algebraic signs of the voltage values are opposite. Thus, we have $v_A = -v_C$.
(c) Writing a KVL equation while going clockwise around the loop composed of elements *A, E,* and *F*, we obtain $v_A + v_F - v_E = 0$. Solving for v_F and substituting values, we find $v_F = -5\text{ V}$. Also, we have $v_C = -v_A = -4\text{ V}$.

P1.45 (a) In Figure P1.32, elements *C*, *D*, and *E* are in parallel.
(b) In Figure P1.43, no element is in parallel with another element.

P1.46 There are two nodes; one at the center of the diagram and the other at the outer periphery of the circuit. Elements *A*, *B*, *C*, and *D* are in parallel. No elements are in series.

P1.47 The points and the voltages specified in the problem statement are:



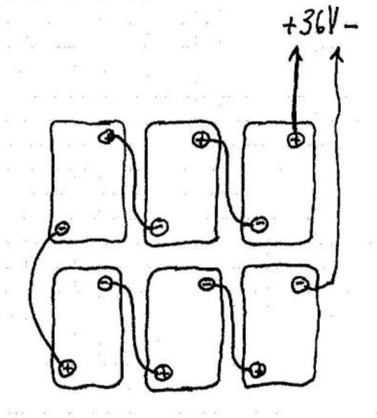
Applying KVL to the loop *abca*, substituting values and solving, we obtain:

$$v_{ab} - v_{cb} - v_{ac} = 0$$
$$12 + 4 - v_{ac} = 0 \qquad v_{ac} = 16\text{ V}$$

Similiarly, applying KVL to the loop *abcda*, substituting values and solving, we obtain:

$$v_{ab} - v_{cb} + v_{cd} + v_{da} = 0$$
$$12 + 4 + v_{cd} + 8 = 0$$
$$v_{cd} = -24\text{ V}$$

P1.48 Six batteries are needed and they need to be connected in series. A typical configuration looking down on the tops of the batteries is shown:



P1.49 Provided that the current reference points into the positive voltage reference, the voltage across a resistance equals the current through the resistance times the resistance. On the other hand, if the current reference points into the negative voltage reference, the voltage equals the negative of the product of the current and the resistance.

P1.50 (a) The voltage between any two points of an ideal conductor is zero regardless of the current flowing.
(b) An ideal voltage source maintains a specified voltage across its terminals.
(c) An ideal current source maintains a specified current through itself.
(d) The voltage across a short circuit is zero regardless of the current flowing. When an ideal conductor is connected between two points, we say that the points are shorted together.
(e) The current through an open circuit is zero regardless of the voltage.

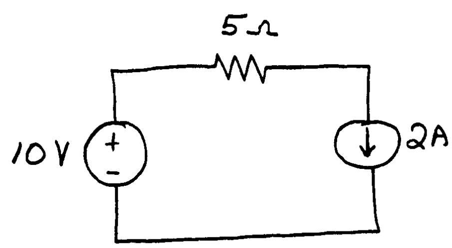
P1.51 Four types of dependent sources and the units for their gain constants are:

1. Voltage-controlled voltage sources. V/V or unitless.
2. Voltage-controlled current sources. A/V or siemens.
3. Current-controlled voltage sources. V/A or ohms.
4. Current-controlled current sources. A/A or unitless.

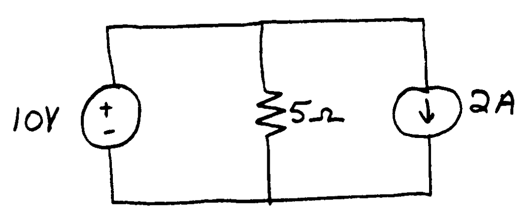
P1.52 (a) The resistance of the copper wire is given by $R_{Cu} = \rho_{Cu}L/A$, and the resistance of the aluminum wire is $R_{Al} = \rho_{Al}L/A$. Taking the ratios of the respective sides of these equations yields $R_{Al}/R_{Cu} = \rho_{Al}/\rho_{Cu}$.
(b) Solving for R_{Al} and substituting values, we have

$$\begin{aligned} R_{Al} &= R_{Cu}\rho_{Al}/\rho_{Cu} \\ &= (1.5) \times (2.73 \times 10^{-8})/(1.72 \times 10^{-8}) \\ &= 2.38 \ \Omega \end{aligned}$$

P1.53*



P1.54



P1.55 Equation 1.10 gives the resistance as

$$R = \frac{\rho L}{A}$$

(a) Thus, if the length of the wire is doubled, the resistance doubles to $20\,\Omega$.

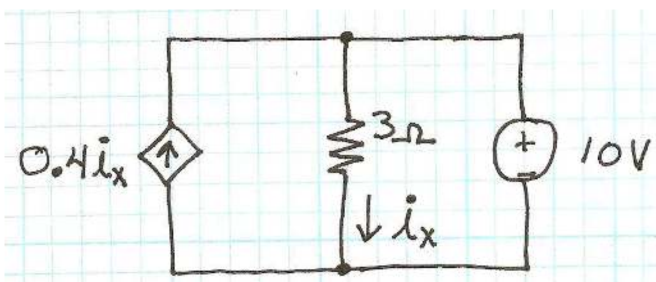
(b) If the diameter of the wire is doubled, the cross sectional area A is increased by a factor of four. Thus, the resistance is decreased by a factor of four to 2.5 Ω.

P1.56 The resistance is proportional to the resistivity and length, and inversely proportional to the square of the wire diameter. In parallel circuits, current division takes place in inverse proportion of resistances. Hence current division will take place in inverse proportion of resistivity of copper and aluminium.

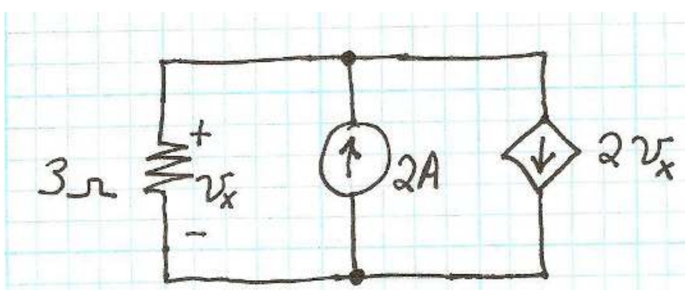
P1.57* $$R = \frac{(V_1)^2}{P_1} = \frac{100^2}{100} = 100\ \Omega$$

$$P_2 = \frac{(V_2)^2}{R} = \frac{90^2}{100} = 81\text{ W for a 19\% reduction in power}$$

P1.58



P1.59



P1.60 The power delivered to the resistor is

$$p(t) = i^2(t)R = 256000\exp(-4t)$$

and the energy delivered is

$$w = \int_{10}^{\infty} p(t)dt = \int_{10}^{\infty} 256000 \exp(-4t)\,dt = 2.7189 \times 10^{-13} \text{ J}$$

P1.61 The power delivered to the resistor is

$$p(t) = v^2(t)/R = 720\sin^2(2\pi t) = 360 - 360\cos(4\pi t)$$

and the energy delivered is

$$w = \int_0^5 p(t)dt = \int_0^5 [360 + 360\cos(4\pi t)]dt = \left[360t - \frac{360}{4\pi}\sin(4\pi t)\right]_0^5 = 1774.513 \text{ J}$$

P1.62* (a) Not contradictory.

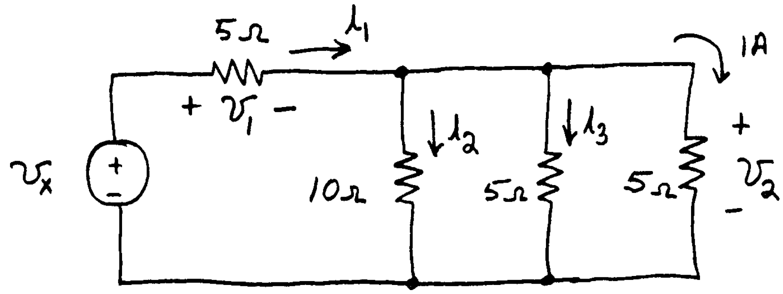
(b) A 2-A current source in series with a 3-A current source is contradictory because the currents in series elements must be equal.

(c) Not contradictory.

(d) A 2-A current source in series with an open circuit is contradictory because the current through a short circuit is zero by definition and currents in series elements must be equal.

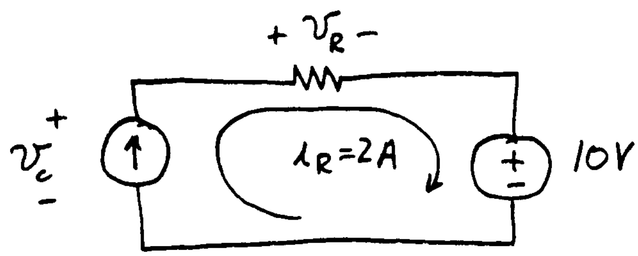
(e) A 5-V voltage source in parallel with a short circuit is contradictory because the voltages across parallel elements must be equal and the voltage across a short circuit is zero by definition.

P1.63*



Applying Ohm's law, we have $v_2 = (5\,\Omega)\times(1\text{ A}) = 5\text{ V}$. However, v_2 is the voltage across all three resistors that are in parallel. Thus, $i_3 = \frac{v_2}{5} = 1\text{ A}$, and $i_2 = \frac{v_2}{10} = 0.5\text{ A}$. Applying KCL, we have $i_1 = i_2 + i_3 + 1 = 2.5\text{ A}$. By Ohm's law: $v_1 = 5i_1 = 12.5\text{ V}$. Finally using KVL, we have $v_x = v_1 + v_2 = 17.5\text{ V}$.

P1.64*



As shown above, the 2 A current circulates clockwise through all three elements in the circuit. Applying KVL, we have

$$v_c = v_R + 10 = 5i_R + 10 = 20 \text{ V}$$

$P_{current-source} = -v_c i_R = -40$ W. Thus, the current source delivers power.

$P_R = (i_R)^2 R = 2^2 \times 5 = 20$ W. The resistor absorbs power.

$P_{voltage-source} = 10 \times i_R = 20$ W. The voltage source absorbs power.

P1.65 (a) The voltage across the voltage source is 10 V independent of the current. The reference direction for v_{ab} is opposite to that of the source. Thus, we have $v_{ab} = -10$ which plots as a vertical line in the i_{ab} versus v_{ab} plane.

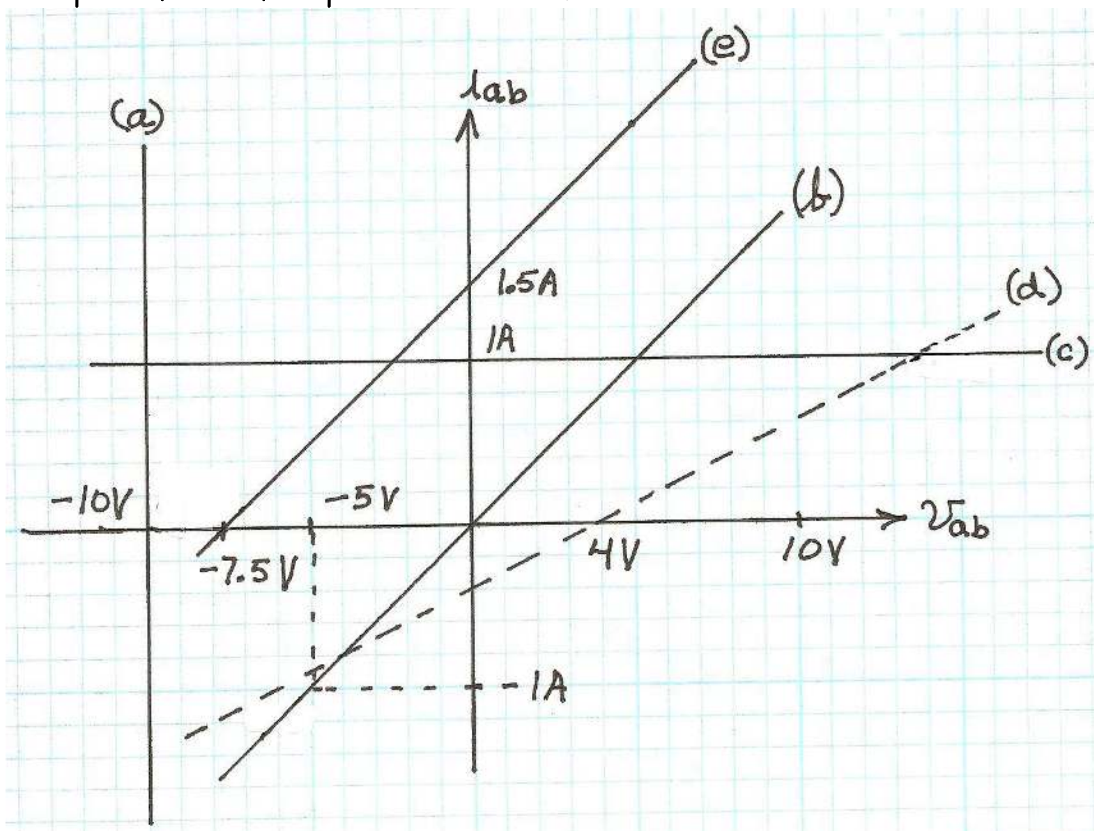
(b) Ohm's law gives $i_{ab} = v_{ab}/5$.

(c) The current source has $i_{ab} = 1$ A independent of v_{ab}, which plots as a horizontal line in the i_{ab} versus v_{ab} plane.

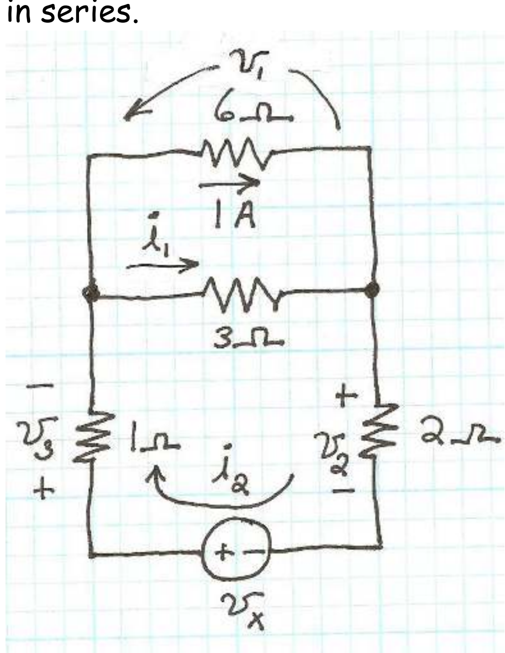
(d) Applying Ohm's law and KVL, we obtain $v_{ab} = 10i_{ab} + 4$ which is equivalent to $i_{ab} = (v_{ab} - 4)/10$

(e) Applying KCL and Ohm's law, we obtain $i_{ab} = (v_{ab}/5) + 1.5$.

The plots for all five parts are shown.



P1.66 (a) The 1-Ω resistance, the 2-Ω resistance, and the voltage source v_x are in series.



(b) The 6-Ω resistance and the 3-Ω resistance are in parallel.

(c) Refer to the sketch of the circuit. Applying Ohm's law to the 6-Ω resistance, we determine that $v_1 = 6$ V. Then, applying Ohm's law to the 3-Ω resistance, we have $i_1 = 2$ A. Next, KCL yields $i_2 = 3$ A. Continuing, we use Ohm's law to find that $v_2 = 6$ V and $v_3 = 3$ V. Finally, applying KVL, we have $v_x = v_3 + v_1 + v_2 = 15$ V.

P1.67 The power for each element is 120 W in magnitude. The voltage source absorbs power and the current source delivers it.

P1.68 This is a parallel circuit, and the voltage across each element is 12 V positive at the bottom end. Thus, the current flowing through the resistor is

$$i_R = -\frac{12\text{ V}}{8\,\Omega} = -1.5\text{ A}$$

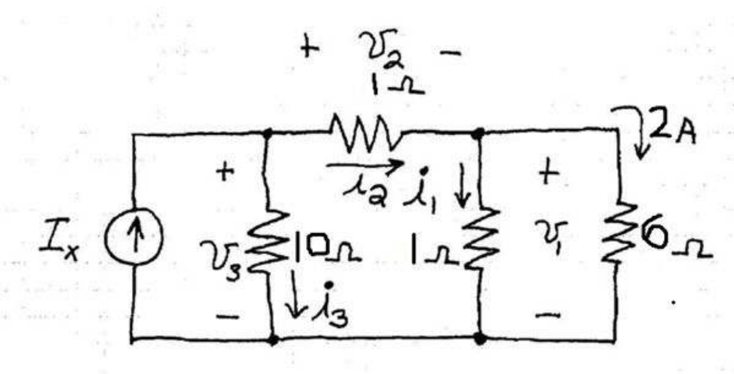
(Notice that current actually flows upward through the resistor.) Applying KCL, we find that the current through the voltage source is 5.5 A flowing downward.

If $i_R = -4$A, then value of voltage source to replace -12V source is

$$i_R = \frac{XV}{8\Omega} = -4A \therefore XV = -4A \times 8\Omega = -32V.$$

A -32V voltage source should replace -12V source

P1.69



Ohm's law for the 6-Ω resistor yields: $v_1 = 12$ V Then, we have $i_1 = v_1/1 = 12$ A. Next, KCL yields $i_2 = i_1 + 2 = 14$ A. Then for the top 2-Ω resistor, we have $v_2 = 14 \times 1 = 14$ V. Using KVL, we have $v_3 = v_2 + v_1 = 26$ V. Next, applying Ohms law, we obtain $i_3 = v_3/10 = 2.6$ A. Finally applying KCL, we have $I_x = i_2 + i_3 = 16.6$ A.

P1.70* (a) Applying KVL, we have $10 = v_x + 5v_x$, which yields $v_x = 10/6 = 1.667$ V

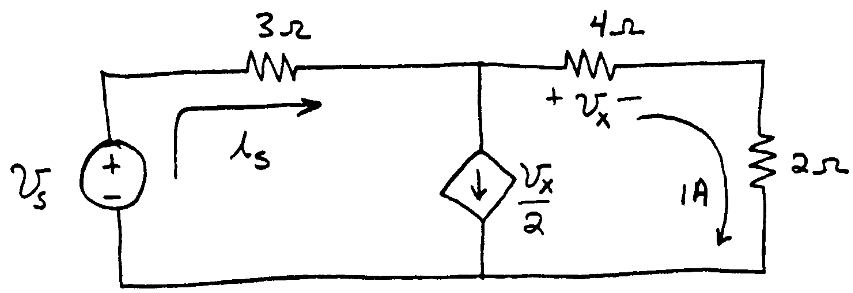
(b) $i_x = v_x/3 = 0.5556$ A

(c) $P_{voltage-source} = -10 i_x = -5.556$ W. (This represents power delivered by the voltage source.)

$$P_R = 3(i_x)^2 = 0.926 \text{ W (absorbed)}$$

$$P_{controlled-source} = 5 v_x i_x = 4.63 \text{ W (absorbed)}$$

P1.71* We have a voltage-controlled current source in this circuit.



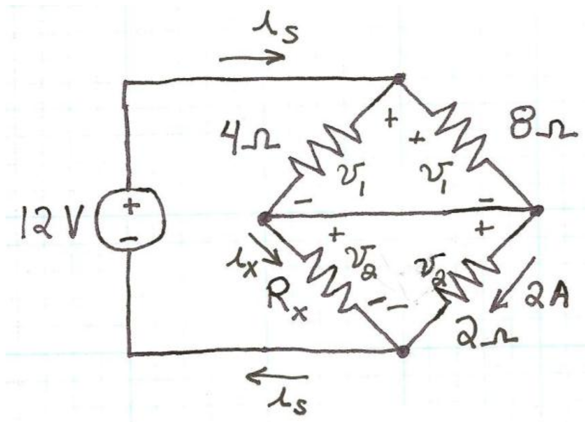
$v_x = (4\,\Omega) \times (1\,A) = 4$ V $\qquad$ $i_s = v_x/2 + 1 = 3$ A

Applying KVL around the outside of the circuit, we have:

$$v_s = 3i_s + 4 + 2 = 15 \text{ V}$$

P1.72 (a) No elements are in series.

(b) R_x and the 2-Ω resistor are in parallel. Also, the 4-Ω resistor and the 8-Ω resistor are in parallel. The voltages across the parallel elements are the same as shown in this figure:

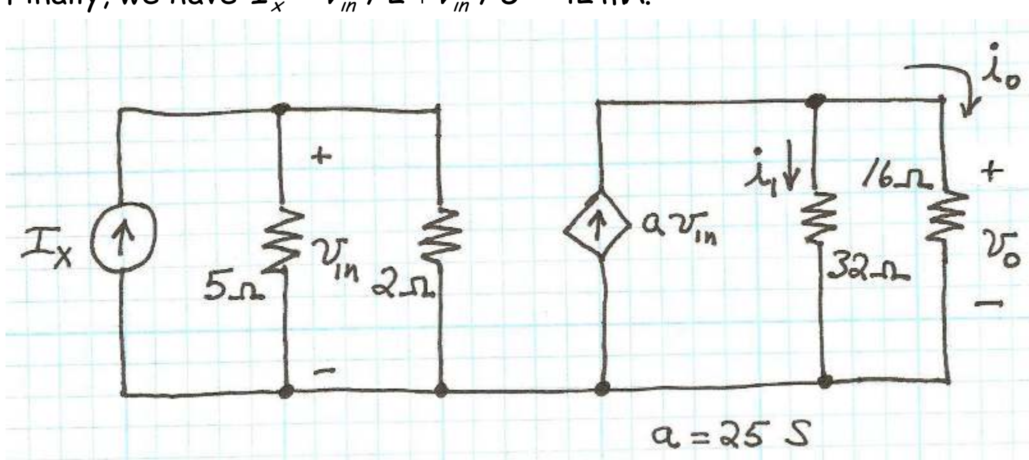


(c) $v_2 = 2 \times 2 = 4$ V
$v_1 = 12 - v_2 = 8$ V

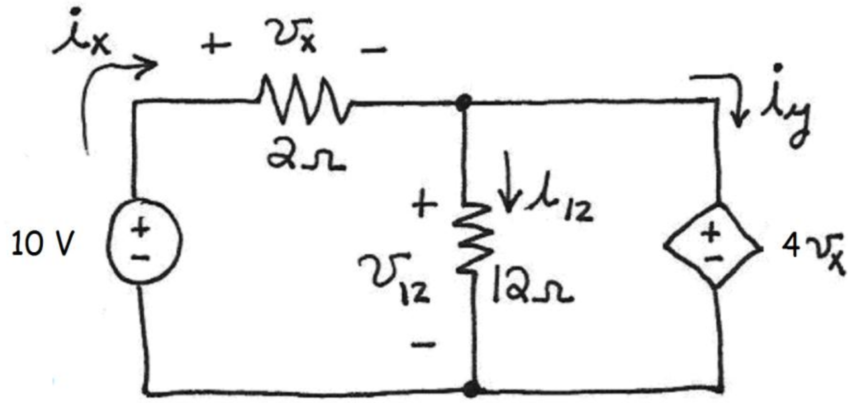
$i_s = \dfrac{v_1}{4} + \dfrac{v_1}{8} = 3\ A$
$i_x = i_s - 2 = 1\ A$
$R_x = v_2 / i_x = 4\ \Omega$

P1.73 $i_2 = v/2$ $i_8 = v/8$ $i_2 + i_8 = v/2 + v/8 = 10$
$v = 16$ V $i_2 = 8\ A$ $i_8 = 2\ A$

P1.74 This is a voltage-controlled current source. First, we have $v_o = \sqrt{P_o 16} = 16$ V. Then, we have $i_1 = v_o / 32 = 0.5\ A$ and $i_o = v_o / 8 = 1\ A$. KCL gives $25 v_{in} = i_1 + i_o = 1.5\ A$. Thus, we have $v_{in} = 1.5/25 = 60$ mV.
Finally, we have $I_x = v_{in}/2 + v_{in}/5 = 42$ mA.

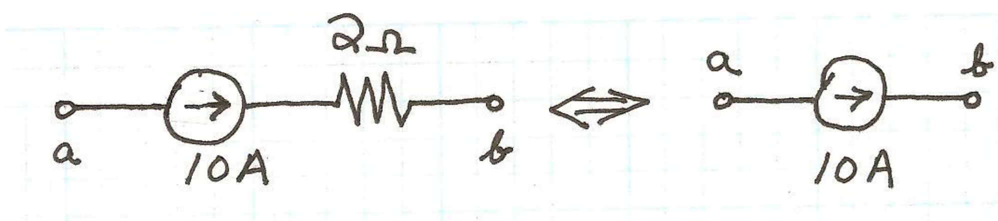


P1.75 This circuit contains a voltage-controlled voltage source.

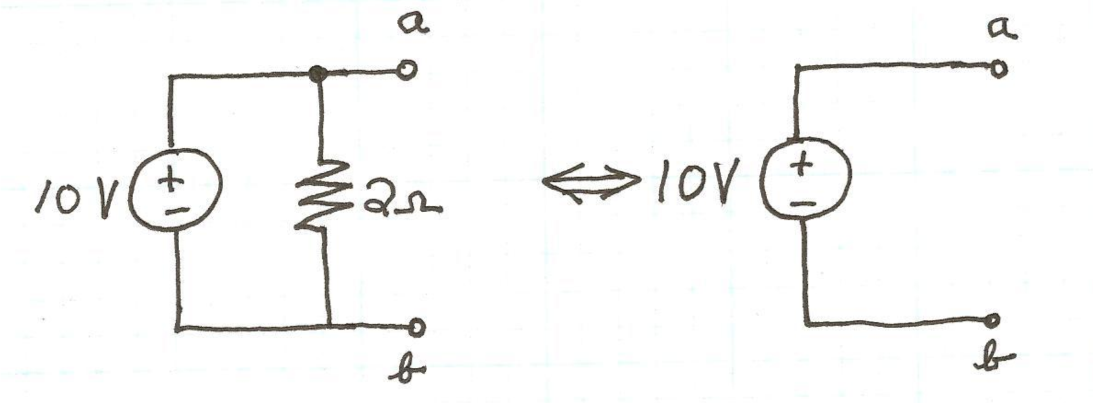


Applying KVL around the periphery of the circuit, we have $-10 + v_x + 4v_x = 0$, which yields $v_x = 2$ V. Then, we have $v_{12} = 4v_x = 8$ V. Using Ohm's law we obtain $i_{12} = v_{12}/12 = 0.667$ A and $i_x = v_x/2 = 1$ A. Then KCL applied to the node at the top of the 12-Ω resistor gives $i_x = i_{12} + i_y$ which yields $i_y = 0.333$ A.

P1.76 Consider the series combination shown below on the left. Because the current for series elements must be the same and the current for the current source is 10 A by definition, the current flowing from *a* to *b* is 10 A. Notice that the current is not affected by the 2-Ω resistance in series. Thus, the series combination is equivalent to a simple current source as far as anything connected to terminals *a* and *b* is concerned. The voltage v_{ab} depends on what else is connected between *a* and *b*.



P1.77 Consider the parallel combination shown below on the left. Because the voltage for parallel elements must be the same, the voltage v_{ab} must be 10 V. Notice that v_{ab} is not affected by the resistance. Thus, the parallel combination is equivalent to a simple voltage source as far as anything connected to terminals *a* and *b* is concerned. The current i_{ab} depends on what else is connected between *a* and *b*.



P1.78 (a) $4 = i_1 + i_2$

(b) $i_1 = v/15$

$i_2 = v/10$

(c) $4 = v/15 + v/10$

$v = 24\ \text{V}$

(d) $P_{currentsource} = -I_s v = -96\ \text{W}$ (Power is supplied by the source.)

$P_1 = v^2/R_1 = 38.4\ \text{W}$ (Power is absorbed by R_1.)

$P_2 = v^2/R_2 = 57.6\ \text{W}$ (Power is absorbed by R_2.)

P1.79 (a) $v = v_1 + v_2$

(b) $v_1 = R_1 I_s = 6\ \text{V}$

$v_2 = R_2 I_s = 8\ \text{V}$

(c) $v = 14\ \text{V}$

(d) $P_{current-source} = -vI_s = -28\ \text{W}$. (Power delivered by the source.)

$P_1 = R_1 I_s^2 = 12\ \text{W}$ (Power absorbed by R_1.)

$P_2 = R_2 I_s^2 = 16\ \text{W}$ (Power absorbed by R_2.)

Notice that power is conserved.

P1.80 The source labeled I_s is an independent current source. The source labeled ai_x is a current-controlled current source. Applying ohm's law to the 20-Ω resistance gives:

$$i_x = 20\text{ V}/20\,\Omega = 1\text{ A}$$

Applying KCL for the node at the top end of the controlled current source:

$$I_s = 0.5i_x + i_x = 1.5i_x = 1.5\text{ A}$$

Then KVL around the outside of the circuit yields

$$v = 15I_s + 10i_x + 20 = 52.5\text{ V}$$

P1.81 (a) $i_3 = i_1 + i_2$

(b) $-v_1 i_3 + v_2 i_3 + v_4 i_1 + v_3 i_2 + v_5 i_2 = 0$

(c) $(-v_1 + v_2 + v_4)i_1 + (-v_1 + v_2 + v_3 + v_5)i_2 = 0$

(d) Setting the coefficients of i_1 and i_2 to zero, we have

$$-v_1 + v_2 + v_4 = 0 \quad \text{and} \quad -v_1 + v_2 + v_3 + v_5 = 0$$

which are the KVL equations written around the left-hand loop and around the outside of the network, respectively.

P1.82 The source labeled 24 V is an independent voltage source. The source labeled ai_x is a current-controlled voltage source.
Applying Ohm's law and KVL, we have $-24 + 5i_x + 3i_x = 0$. Solving, we obtain $i_x = 3\text{ A}$.

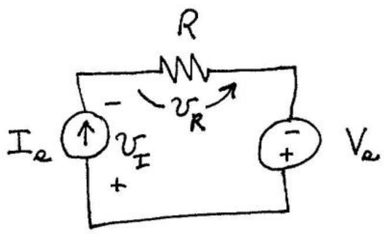
Practice Test

T1.1 (a) 4; (b) 7; (c) 16; (d) 18; (e) 1; (f) 2; (g) 8; (h) 3; (i) 5; (j) 15; (k) 6; (l) 11;

(m) 13; (n) 9; (o) 14.

T1.2 (a) The current I_s = 3 A circulates clockwise through the elements entering the resistance at the negative reference for v_R. Thus, we have $v_R = -I_s R = -6$ V.
(b) Because I_s enters the negative reference for V_s, we have $P_V = -V_s I_s = -30$ W. Because the result is negative, the voltage source is delivering energy.
(c) The circuit has three nodes, one on each of the top corners and one along the bottom of the circuit.

(d) First, we must find the voltage v_I across the current source. We choose the reference shown:



Then, going around the circuit counterclockwise, we have $-v_I + V_s + v_R = 0$, which yields $v_I = V_s + v_R = 10 - 6 = 4$ V. Next, the power for the current source is $P_I = I_s v_I = 12$ W. Because the result is positive, the current source is absorbing energy.

Alternatively, we could compute the power delivered to the resistor as $P_R = I_s^2 R = 18$ W. Then, because we must have a total power of zero for the entire circuit, we have $P_I = -P_V - P_R = 30 - 18 = 12$ W.

T1.3 (a) The currents flowing downward through the resistances are v_{ab}/R_1 and v_{ab}/R_2. Then, the KCL equation for node a (or node b) is

$$I_2 = I_1 + \frac{v_{ab}}{R_1} + \frac{v_{ab}}{R_2}$$

Substituting the values given in the question and solving yields $v_{ab} = -8$ V.
(b) The power for current source I_1 is $P_{I1} = v_{ab} I_1 = -8 \times 3 = -24$ W. Because the result is negative we know that energy is supplied by this current source.
The power for current source I_2 is $P_{I2} = -v_{ab} I_2 = 8 \times 1 = 8$ W. Because the result is positive, we know that energy is absorbed by this current source.
(c) The power absorbed by R_1 is $P_{R1} = v_{ab}^2 / R_1 = (-8)^2/12 = 5.33$ W. The power absorbed by R_2 is $P_{R2} = v_{ab}^2 / R_2 = (-8)^2/6 = 10.67$ W.

T1.4 (a) Applying KVL, we have $-V_s + v_1 + v_2 = 0$. Substituting values given in the problem and solving we find $v_1 = 8$ V.
(b) Then applying Ohm's law, we have $i = v_1/R_1 = 8/4 = 2$ A.
(c) Again applying Ohm's law, we have $R_2 = v_2/i = 4/2 = 2\,\Omega$.

T1.5 Applying KVL, we have $-V_s + v_x = 0$. Thus, $v_x = V_s = 15$ V. Next Ohm's law gives $i_x = v_x / R = 15/10 = 1.5$ A. Finally, KCL yields
$i_{sc} = i_x - av_x = 1.5 - 0.3 \times 15 = -3$ A.

CHAPTER 2

Exercises

E2.1 (a) R_2, R_3, and R_4 are in parallel. Furthermore R_1 is in series with the combination of the other resistors. Thus we have:

$$R_{eq} = R_1 + \frac{1}{1/R_2 + 1/R_3 + 1/R_4} = 3\,\Omega$$

(b) R_3 and R_4 are in parallel. Furthermore, R_2 is in series with the combination of R_3, and R_4. Finally R_1 is in parallel with the combination of the other resistors. Thus we have:

$$R_{eq} = \frac{1}{1/R_1 + 1/[R_2 + 1/(1/R_3 + 1/R_4)]} = 5\,\Omega$$

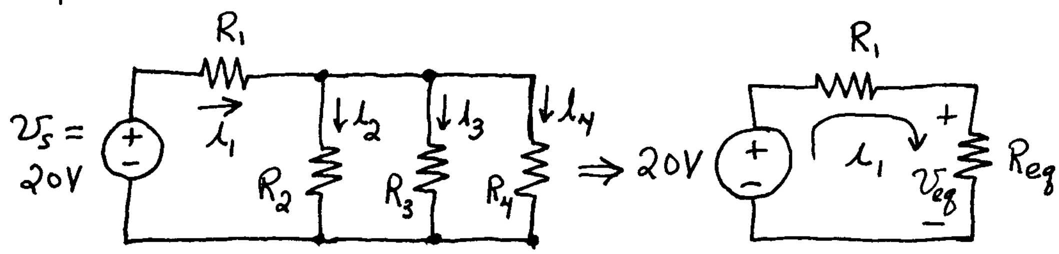
(c) R_1 and R_2 are in parallel. Furthermore, R_3, and R_4 are in parallel. Finally, the two parallel combinations are in series.

$$R_{eq} = \frac{1}{1/R_1 + 1/R_2} + \frac{1}{1/R_3 + 1/R_4} = 52.1\,\Omega$$

(d) R_1 and R_2 are in series. Furthermore, R_3 is in parallel with the series combination of R_1 and R_2.

$$R_{eq} = \frac{1}{1/R_3 + 1/(R_1 + R_2)} = 1.5\ \text{k}\Omega$$

E2.2 (a) First we combine R_2, R_3, and R_4 in parallel. Then R_1 is in series with the parallel combination.

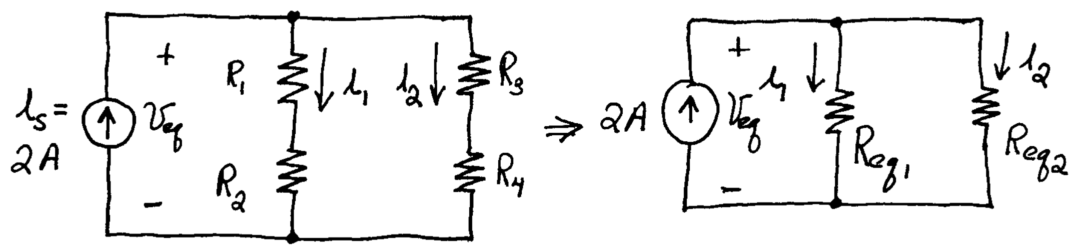


$$R_{eq} = \frac{1}{1/R_2 + 1/R_3 + 1/R_4} = 9.231\,\Omega \qquad i_1 = \frac{20\text{ V}}{R_1 + R_{eq}} = \frac{20}{10 + 9.231} = 1.04\text{ A}$$

$$v_{eq} = R_{eq} i_1 = 9.600\text{ V} \qquad i_2 = v_{eq}/R_2 = 0.480\text{ A} \qquad i_3 = v_{eq}/R_3 = 0.320\text{ A}$$

$$i_4 = v_{eq}/R_4 = 0.240\text{ A}$$

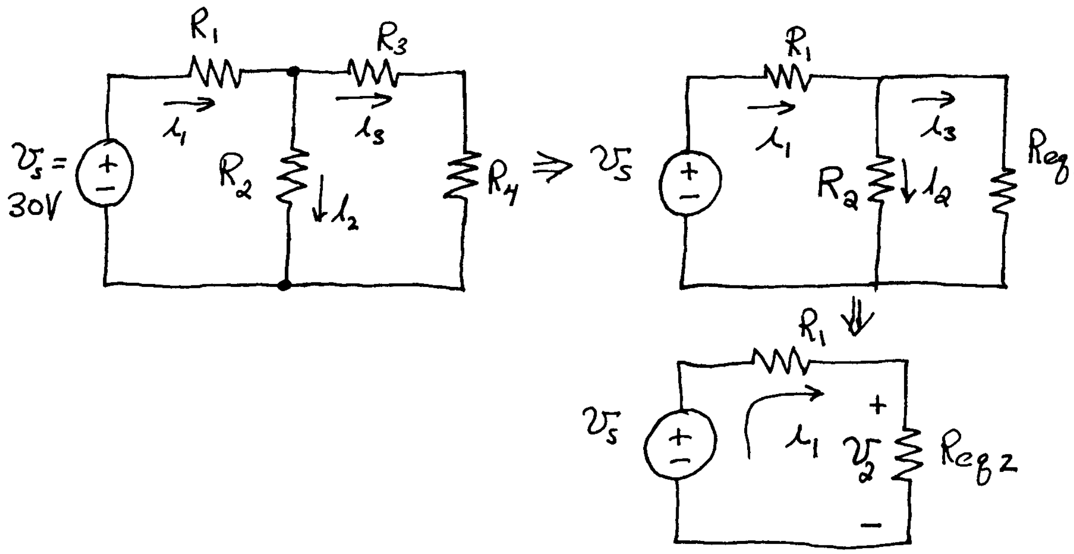
(b) R_1 and R_2 are in series. Furthermore, R_3, and R_4 are in series. Finally, the two series combinations are in parallel.



$R_{eq1} = R_1 + R_2 = 20\ \Omega \quad R_{eq2} = R_3 + R_4 = 20\ \Omega \quad R_{eq} = \dfrac{1}{1/R_{eq1} + 1/R_{eq2}} = 10\ \Omega$

$v_{eq} = 2 \times R_{eq} = 20\ \text{V} \quad i_1 = v_{eq}/R_{eq1} = 1\ A \quad i_2 = v_{eq}/R_{eq2} = 1\ A$

(c) R_3, and R_4 are in series. The combination of R_3 and R_4 is in parallel with R_2. Finally the combination of R_2, R_3, and R_4 is in series with R_1.



$R_{eq1} = R_3 + R_4 = 40\ \Omega \quad R_{eq2} = \dfrac{1}{1/R_{eq1} + 1/R_2} = 20\ \Omega \quad i_1 = \dfrac{v_s}{R_1 + R_{eq2}} = 1\ A$

$v_2 = i_1 R_{eq2} = 20\ \text{V} \quad i_2 = v_2/R_2 = 0.5\ A \quad i_3 = v_2/R_{eq1} = 0.5\ A$

E2.3 (a) $v_1 = v_s \dfrac{R_1}{R_1 + R_2 + R_3 + R_4} = 10\ \text{V}$. $v_2 = v_s \dfrac{R_2}{R_1 + R_2 + R_3 + R_4} = 20\ \text{V}$.

Similarly, we find $v_3 = 30\ \text{V}$ and $v_4 = 60\ \text{V}$.

(b) First combine R_2 and R_3 in parallel: $R_{eq} = 1/(1/R_2 + 1/R_3) = 2.917\ \Omega$.

Then we have $v_1 = v_s \dfrac{R_1}{R_1 + R_{eq} + R_4} = 6.05\text{ V}$. Similarly, we find

$v_2 = v_s \dfrac{R_{eq}}{R_1 + R_{eq} + R_4} = 5.88\text{ V}$ and $v_4 = 8.07\text{ V}$.

E2.4 (a) First combine R_1 and R_2 in series: $R_{eq} = R_1 + R_2 = 30\ \Omega$. Then we have $i_1 = i_s \dfrac{R_3}{R_3 + R_{eq}} = \dfrac{15}{15+30} = 1\ A$ and $i_3 = i_s \dfrac{R_{eq}}{R_3 + R_{eq}} = \dfrac{30}{15+30} = 2\ A$.

(b) The current division principle applies to two resistances in parallel. Therefore, to determine i_1, first combine R_2 and R_3 in parallel: $R_{eq} = 1/(1/R_2 + 1/R_3) = 5\ \Omega$. Then we have $i_1 = i_s \dfrac{R_{eq}}{R_1 + R_{eq}} = \dfrac{5}{10+5} = 1\ A$.

Similarly, $i_2 = 1\ A$ and $i_3 = 1\ A$.

E2.5 Write KVL for the loop consisting of v_1, v_y, and v_2. The result is $-v_1 - v_y + v_2 = 0$ from which we obtain $v_y = v_2 - v_1$. Similarly we obtain $v_z = v_3 - v_1$.

E2.6 Node 1: $\dfrac{v_1 - v_3}{R_1} + \dfrac{v_1 - v_2}{R_2} = i_a$ Node 2: $\dfrac{v_2 - v_1}{R_2} + \dfrac{v_2}{R_3} + \dfrac{v_2 - v_3}{R_4} = 0$

Node 3: $\dfrac{v_3}{R_5} + \dfrac{v_3 - v_2}{R_4} + \dfrac{v_3 - v_1}{R_1} + i_b = 0$

E2.7 Following the step-by-step method in the book, we obtain

$$\begin{bmatrix} \dfrac{1}{R_1} + \dfrac{1}{R_2} & -\dfrac{1}{R_2} & 0 \\ -\dfrac{1}{R_2} & \dfrac{1}{R_2} + \dfrac{1}{R_3} + \dfrac{1}{R_4} & -\dfrac{1}{R_4} \\ 0 & -\dfrac{1}{R_4} & \dfrac{1}{R_4} + \dfrac{1}{R_5} \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} = \begin{bmatrix} -i_s \\ 0 \\ i_s \end{bmatrix}$$

E2.8 Instructions for various calculators vary. The MATLAB solution is given in the book following this exercise.

E2.9 (a) Writing the node equations we obtain:

$$\text{Node 1: } \frac{v_1 - v_3}{20} + \frac{v_1}{5} + \frac{v_1 - v_2}{10} = 0$$

$$\text{Node 2: } \frac{v_2 - v_1}{10} + 10 + \frac{v_2 - v_3}{5} = 0$$

$$\text{Node 3: } \frac{v_3 - v_1}{20} + \frac{v_3}{10} + \frac{v_3 - v_2}{5} = 0$$

(b) Simplifying the equations we obtain:

$$0.35v_1 - 0.10v_2 - 0.05v_3 = 0$$

$$-0.10v_1 + 0.30v_2 - 0.20v_3 = -10$$

$$-0.05v_1 - 0.20v_2 + 0.35v_3 = 0$$

(c) and (d) Solving using Matlab:

```
>>clear
>>G = [0.35 -0.1 -0.05; -0.10 0.30 -0.20; -0.05 -0.20 0.35];
>>I = [0; -10; 0];
>>V = G\I
V =
  -27.2727
  -72.7273
  -45.4545
>>Ix = (V(1) - V(3))/20
Ix =
    0.9091
```

E2.10 Using determinants we can solve for the unknown voltages as follows:

$$v_1 = \frac{\begin{vmatrix} 6 & -0.2 \\ 1 & 0.5 \end{vmatrix}}{\begin{vmatrix} 0.7 & -0.2 \\ -0.2 & 0.5 \end{vmatrix}} = \frac{3 + 0.2}{0.35 - 0.04} = 10.32 \text{ V}$$

$$v_2 = \frac{\begin{vmatrix} 0.7 & 6 \\ -0.2 & 1 \end{vmatrix}}{\begin{vmatrix} 0.7 & -0.2 \\ -0.2 & 0.5 \end{vmatrix}} = \frac{0.7 + 1.2}{0.35 - 0.04} = 6.129 \text{ V}$$

Many other methods exist for solving linear equations.

E2.11 First write KCL equations at nodes 1 and 2:

Node 1: $\frac{v_1 - 10}{2} + \frac{v_1}{5} + \frac{v_1 - v_2}{10} = 0$

Node 2: $\frac{v_2 - 10}{10} + \frac{v_2}{5} + \frac{v_2 - v_1}{10} = 0$

Then, simplify the equations to obtain:

$8v_1 - v_2 = 50$ and $-v_1 + 4v_2 = 10$

Solving manually or with a calculator, we find v_1 = 6.77 V and v_2 = 4.19 V. The MATLAB session using the symbolic approach is:

```
>> clear
[V1,V2] = solve('(V1-10)/2+(V1)/5 +(V1 - V2)/10 = 0' , ...
                 '(V2-10)/10 +V2/5 +(V2-V1)/10 = 0')
V1 =
210/31
V2 =
130/31
```

Next, we solve using the numerical approach.

```
>> clear
G = [8 -1; -1 4];
I = [50; 10];
V = G\I
V =
   6.7742
   4.1935
```

E2.12 The equation for the supernode enclosing the 15-V source is:

$$\frac{v_3 - v_2}{R_3} + \frac{v_3 - v_1}{R_1} = \frac{v_1}{R_2} + \frac{v_2}{R_4}$$

This equation can be readily shown to be equivalent to Equation 2.37 in the book. (Keep in mind that v_3 = -15 V.)

E2.13 Write KVL from the reference to node 1 then through the 10-V source to node 2 then back to the reference node:

$$-v_1 + 10 + v_2 = 0$$

Then write KCL equations. First for a supernode enclosing the 10-V source, we have:

$$\frac{v_1}{R_1} + \frac{v_1 - v_3}{R_2} + \frac{v_2 - v_3}{R_3} = 1$$

Node 3:

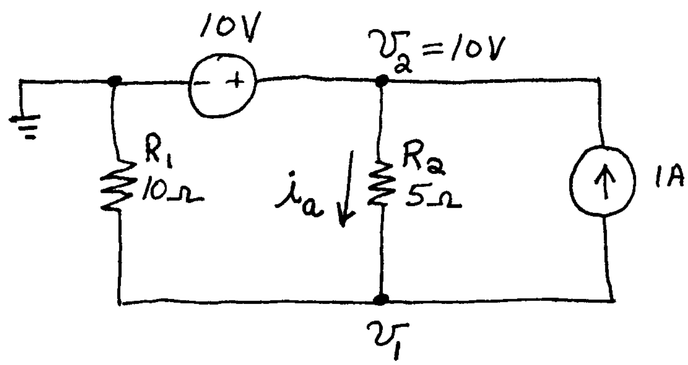
$$\frac{v_3}{R_4} + \frac{v_3 - v_1}{R_2} + \frac{v_3 - v_2}{R_3} = 0$$

Reference node:

$$\frac{v_1}{R_1} + \frac{v_3}{R_4} = 1$$

An independent set consists of the KVL equation and any two of the KCL equations.

E2.14 (a) Select the reference node at the left-hand end of the voltage source as shown at right.



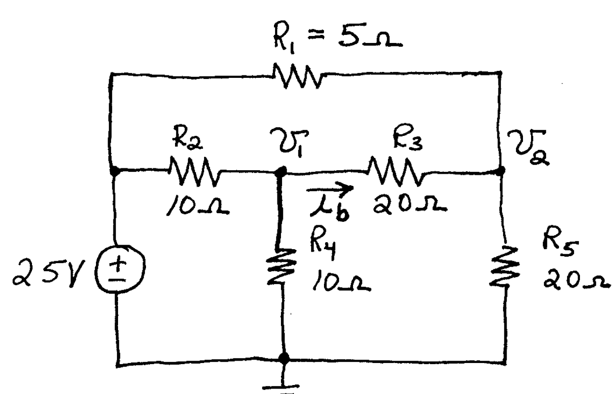
Then write a KCL equation at node 1.

$$\frac{v_1}{R_1} + \frac{v_1 - 10}{R_2} + 1 = 0$$

Substituting values for the resistances and solving, we find v_1 = 3.33 V.

Then we have $i_a = \dfrac{10 - v_1}{R_2} = 1.333$ A.

(b) Select the reference node and assign node voltages as shown.



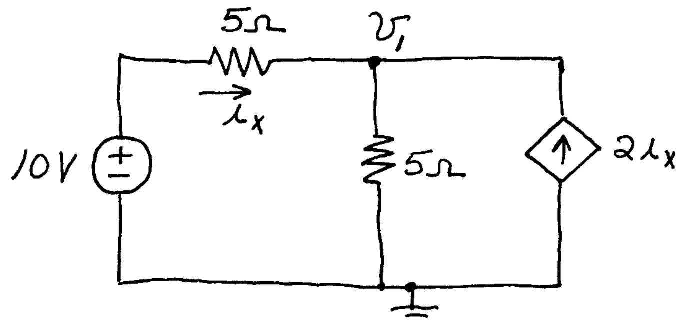
Then write KCL equations at nodes 1 and 2.

$$\frac{v_1 - 25}{R_2} + \frac{v_1}{R_4} + \frac{v_1 - v_2}{R_3} = 0$$

$$\frac{v_2 - 25}{R_1} + \frac{v_2 - v_1}{R_3} + \frac{v_2}{R_5} = 0$$

Substituting values for the resistances and solving, we find v_1 = 13.79 V and v_2 = 18.97 V. Then we have $i_b = \frac{v_1 - v_2}{R_3} = -0.259$ A.

E2.15 (a) Select the reference node and node voltage as shown. Then write a KCL equation at node 1, resulting in

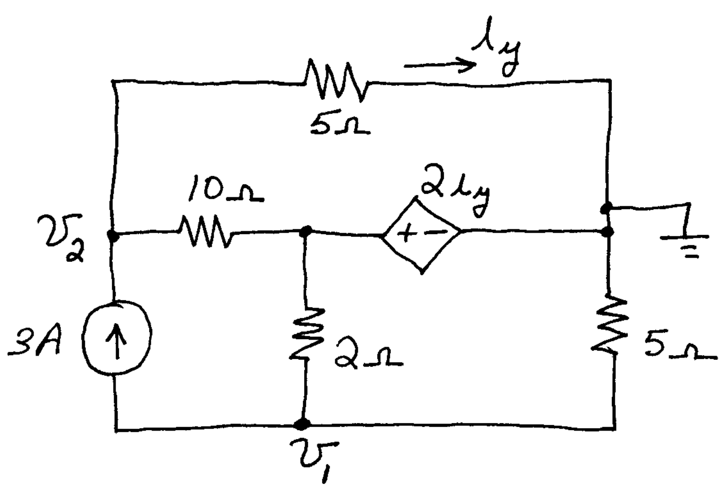


$$\frac{v_1}{5} + \frac{v_1 - 10}{5} - 2i_x = 0$$

Then use $i_x = (10 - v_1)/5$ to substitute and solve. We find v_1 = 7.5 V.

Then we have $i_x = \frac{10 - v_1}{5} = 0.5$ A.

(b) Choose the reference node and node voltages shown:



Then write KCL equations at nodes 1 and 2:

$$\frac{v_1}{5} + \frac{v_1 - 2i_y}{2} + 3 = 0 \qquad \frac{v_2}{5} + \frac{v_2 - 2i_y}{10} = 3$$

Finally use $i_y = v_2 / 5$ to substitute and solve. This yields $v_2 = 11.54$ V and $i_y = 2.31$ A.

E2.16

```
>> clear
>> [V1 V2 V3] = solve('V3/R4 + (V3 - V2)/R3 + (V3 - V1)/R1 = 0', ...
                      'V1/R2 + V3/R4 = Is', ...
                      'V1 = (1/2)*(V3 - V1) + V2' ,'V1','V2','V3');
>> pretty(V1),  pretty(V2),  pretty(V3)

                R2 Is (2 R3 R1 + 3 R4 R1 + 2 R4 R3)
          -----------------------------------------------
          2 R3 R1 + 3 R4 R1 + 3 R1 R2 + 2 R4 R3 + 2 R3 R2

                R2 Is (3 R3 R1 + 3 R4 R1 + 2 R4 R3)
          -----------------------------------------------
          2 R3 R1 + 3 R4 R1 + 3 R1 R2 + 2 R4 R3 + 2 R3 R2

                    Is R2 R4 (3 R1 + 2 R3)
          -----------------------------------------------
          2 R3 R1 + 3 R4 R1 + 3 R1 R2 + 2 R4 R3 + 2 R3 R2
```

E2.17 Refer to Figure 2.33b in the book. (a) Two mesh currents flow through R_2: i_1 flows downward and i_4 flows upward. Thus the current flowing in R_2 referenced upward is i_4 - i_1. (b) Similarly, mesh current i_1 flows to the left through R_4 and mesh current i_2 flows to the right, so the total current referenced to the right is i_2 - i_1. (c) Mesh current i_3 flows downward through R_8 and mesh current i_4 flows upward, so the total current referenced downward is i_3 - i_4. (d) Finally, the total current referenced upward through R_8 is i_4 - i_3.

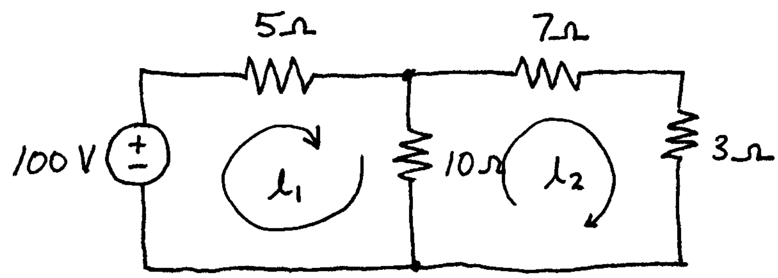
E2.18 Refer to Figure 2.33b in the book. Following each mesh current in turn, we have

$$R_1 i_1 + R_2(i_1 - i_4) + R_4(i_1 - i_2) - v_A = 0$$
$$R_5 i_2 + R_4(i_2 - i_1) + R_6(i_2 - i_3) = 0$$
$$R_7 i_3 + R_6(i_3 - i_2) + R_8(i_3 - i_4) = 0$$
$$R_3 i_4 + R_2(i_4 - i_1) + R_8(i_4 - i_3) = 0$$

In matrix form, these equations become

$$\begin{bmatrix} (R_1+R_2+R_4) & -R_4 & 0 & -R_2 \\ -R_4 & (R_4+R_5+R_6) & -R_6 & 0 \\ 0 & -R_6 & (R_6+R_7+R_8) & -R_8 \\ -R_2 & 0 & -R_8 & (R_2+R_3+R_8) \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ i_3 \\ i_4 \end{bmatrix} = \begin{bmatrix} v_A \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

E2.19 We choose the mesh currents as shown:

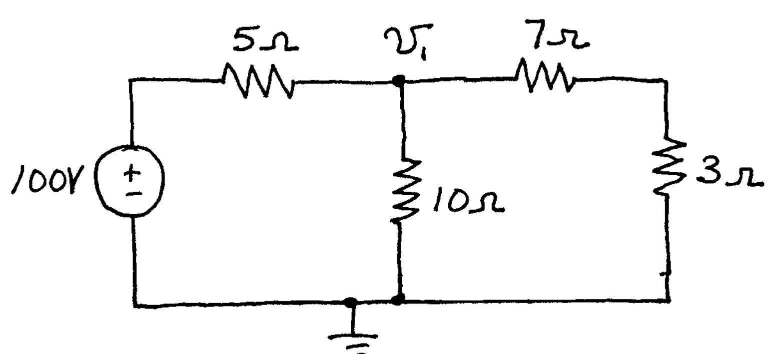


Then, the mesh equations are:

$$5i_1 + 10(i_1 - i_2) = 100 \quad \text{and} \quad 10(i_2 - i_1) + 7i_2 + 3i_2 = 0$$

Simplifying and solving these equations, we find that $i_1 = 10\ A$ and $i_2 = 5\ A$. The net current flowing downward through the 10-Ω resistance is $i_1 - i_2 = 5\ A$.

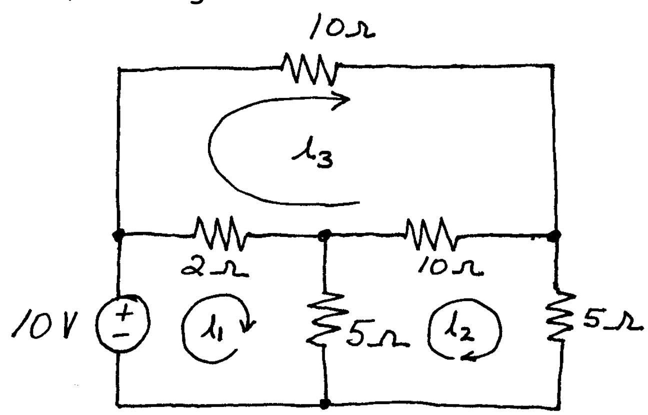
To solve by node voltages, we select the reference node and node voltage shown. (We do not need to assign a node voltage to the connection between the 7-Ω resistance and the 3-Ω resistance because we can treat the series combination as a single 10-Ω resistance.)



The node equation is $(v_1 - 10)/5 + v_1/10 + v_1/10 = 0$. Solving we find that v_1 = 50 V. Thus we again find that the current through the 10-Ω resistance is $i = v_1/10 = 5$ A.

Combining resistances in series and parallel, we find that the resistance "seen" by the voltage source is 10 Ω. Thus the current through the source and 5-Ω resistance is (100 V)/(10 Ω) = 10 A. This current splits equally between the 10-Ω resistance and the series combination of 7 Ω and 3 Ω.

E2.20 First, we assign the mesh currents as shown.



Then we write KVL equations following each mesh current:

$$2(i_1 - i_3) + 5(i_1 - i_2) = 10$$
$$5i_2 + 5(i_2 - i_1) + 10(i_2 - i_3) = 0$$
$$10i_3 + 10(i_3 - i_2) + 2(i_3 - i_1) = 0$$

Simplifying and solving, we find that i_1 = 2.194 A, i_2 = 0.839 A, and i_3 = 0.581 A. Thus the current in the 2-Ω resistance referenced to the right is i_1 - i_3 = 2.194 - 0.581 = 1.613 A.

E2.21 Following the step-by-step process, we obtain

$$\begin{bmatrix} (R_2 + R_3) & -R_3 & -R_2 \\ -R_3 & (R_3 + R_4) & 0 \\ -R_2 & 0 & (R_1 + R_2) \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ i_3 \end{bmatrix} = \begin{bmatrix} v_A \\ -v_B \\ v_B \end{bmatrix}$$

E2.22 Refer to Figure 2.39 in the book. In terms of the mesh currents, the current directed to the right in the 5-A current source is i_1, however by the definition of the current source, the current is 5 A directed to the left. Thus, we conclude that i_1 = -5 A. Then we write a KVL equation following i_2, which results in $10(i_2 - i_1) + 5i_2 = 100$.

E2.23 Refer to Figure 2.40 in the book. First, for the current source, we have

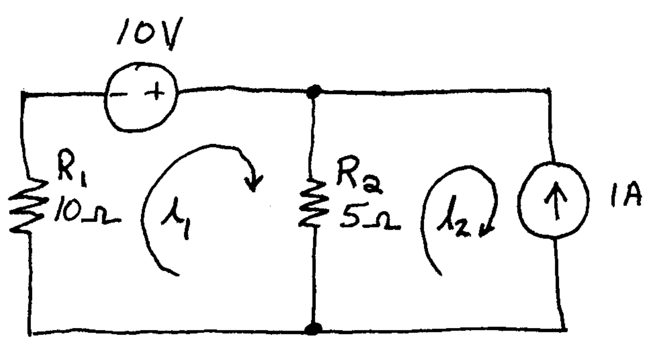
$$i_2 - i_1 = 1$$

Then, we write a KVL equation going around the perimeter of the entire circuit:

$$5i_1 + 10i_2 + 20 - 10 = 0$$

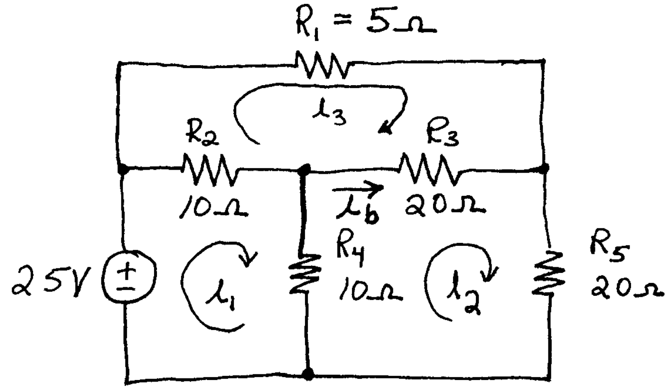
Simplifying and solving these equations we obtain i_1 = -4/3 A and i_2 = -1/3 A.

E2.24 (a) As usual, we select the mesh currents flowing clockwise around the meshes as shown. Then for the current source, we have i_2 = -1 A. This is because we defined the mesh



current i_2 as the current referenced downward through the current source. However, we know that the current through this source is 1 A flowing upward. Next we write a KVL equation around mesh 1: $10i_1 - 10 + 5(i_1 - i_2) = 0$. Solving, we find that i_1 = 1/3 A. Referring to Figure 2.30a in the book we see that the value of the current i_a referenced downward through the 5 Ω resistance is to be found. In terms of the mesh currents, we have $i_a = i_1 - i_2 = 4/3\text{ A}$.

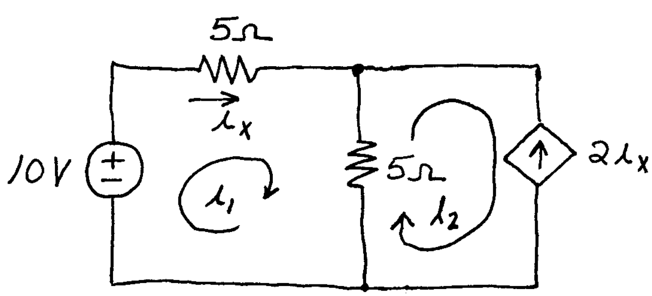
(b) As usual, we select the mesh currents flowing clockwise around the meshes as shown.
Then we write a KVL equation for each mesh.



$$-25+10(i_1-i_3)+10(i_1-i_2)=0$$
$$10(i_2-i_1)+20(i_2-i_3)+20i_2=0$$
$$10(i_3-i_1)+5i_3+20(i_3-i_2)=0$$

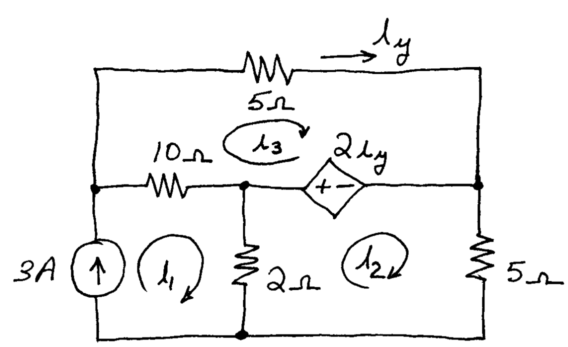
Simplifying and solving, we find i_1 = 2.3276 A, i_2 = 0.9483 A, and i_3 = 1.2069 A. Finally, we have $i_b = i_2 - i_3$ = -0.2586 A.

E2.25 (a) KVL mesh 1:
$$-10+5i_1+5(i_1-i_2)=0$$
For the current source:
$$i_2=-2i_x$$
However, i_x and i_1 are the same current, so we also have $i_1 = i_x$.



Simplifying and solving, we find $i_x = i_1 = 0.5$ A.

(b) First for the current source, we have: $i_1 = 3$ A
Writing KVL around meshes 2 and 3, we have:



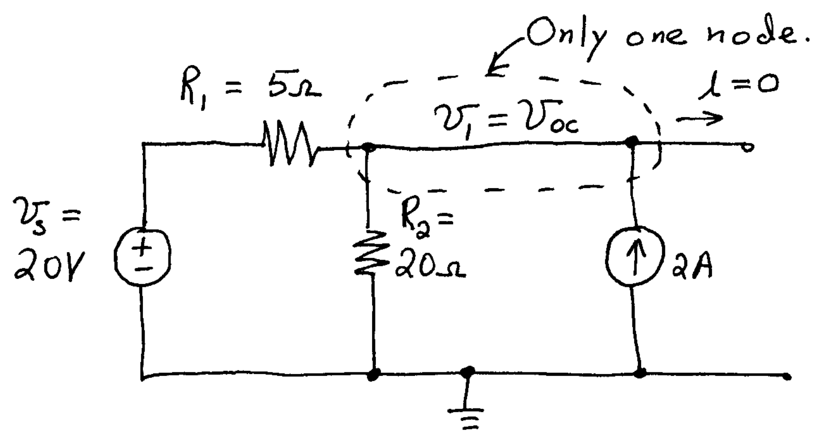
$$2(i_2-i_1)+2i_y+5i_2=0$$
$$10(i_3-i_1)+5i_3-2i_y=0$$

However i_3 and i_y are the same current: $i_y = i_3$. Simplifying and solving, we find that $i_3 = i_y = 2.31$ A.

E2.26 Under open-circuit conditions, 5 A circulates clockwise through the current source and the 10-Ω resistance. The voltage across the 10-Ω resistance is 50 V. No current flows through the 40-Ω resistance so the open circuit voltage is $V_t = 50$ V.

With the output shorted, the 5 A divides between the two resistances in parallel. The short-circuit current is the current through the 40-Ω resistance, which is $i_{sc} = 5\dfrac{10}{10+40} = 1$ A. Then, the Thévenin resistance is $R_t = v_{oc} / i_{sc} = 50\ \Omega$.

E2.27 Choose the reference node at the bottom of the circuit as shown:

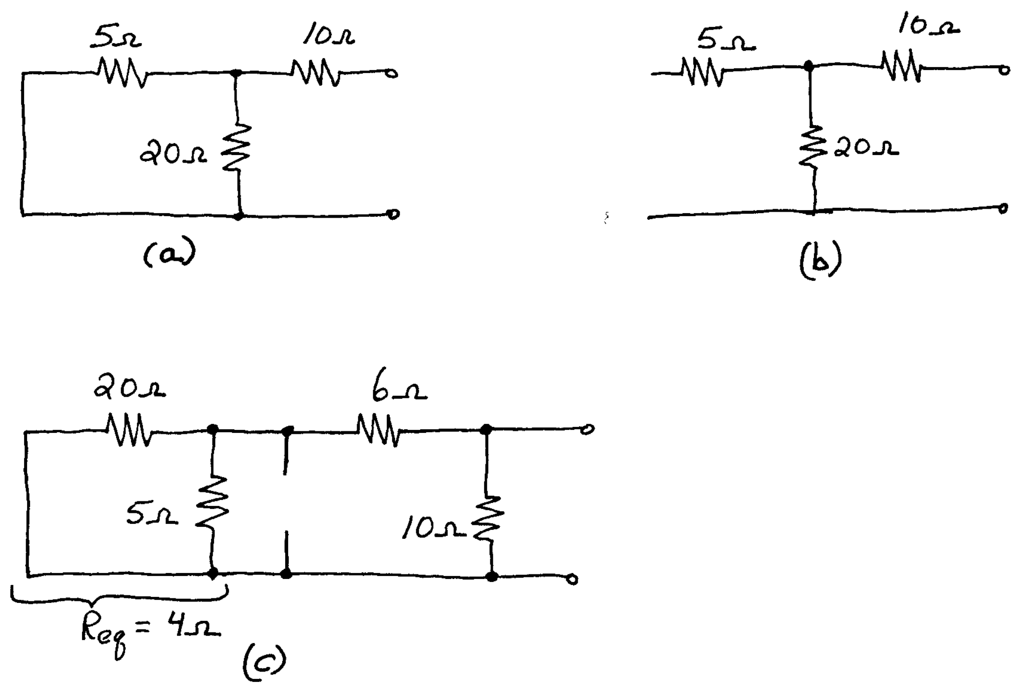


Notice that the node voltage is the open-circuit voltage. Then write a KCL equation:

$$\frac{v_{oc} - 20}{5} + \frac{v_{oc}}{20} = 2$$

Solving we find that v_{oc} = 24 V which agrees with the value found in Example 2.17.

E2.28 To zero the sources, the voltage sources become short circuits and the current sources become open circuits. The resulting circuits are :

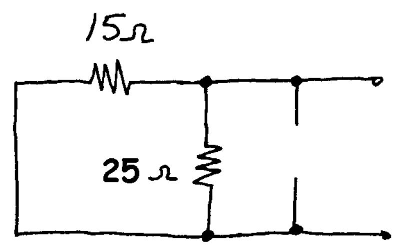


(a) $R_t = 10 + \dfrac{1}{1/5 + 1/20} = 14\ \Omega$ (b) $R_t = 10 + 20 = 30\ \Omega$

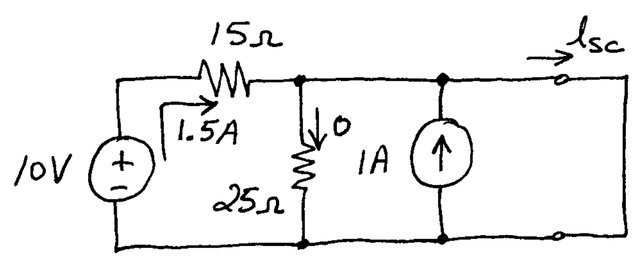
(c) $R_t = \dfrac{1}{\dfrac{1}{10} + \dfrac{1}{6 + \dfrac{1}{(1/5 + 1/20)}}} = 5\ \Omega$

E2.29 (a) Zero sources to determine Thévenin resistance. Thus

$R_t = \dfrac{1}{1/15 + 1/25} = 9.375\ \Omega.$

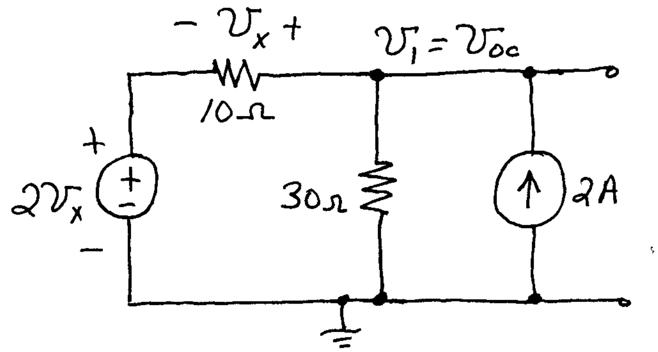


Then find short-circuit current:



$I_n = i_{sc} = 10/15 + 1 = 1.67\ A$

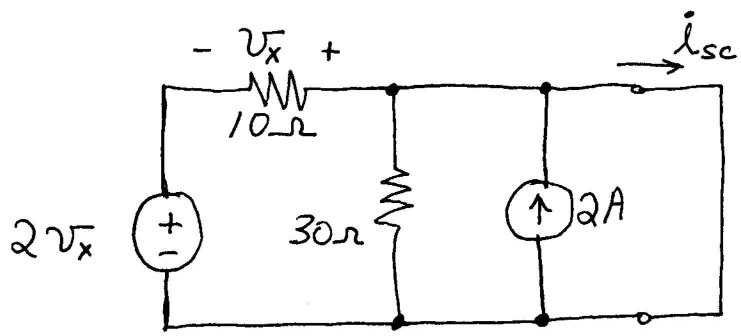
(b) We cannot find the Thévenin resistance by zeroing the sources, because we have a controlled source. Thus, we find the open-circuit voltage and the short-circuit current.



$$\frac{v_{oc} - 2v_x}{10} + \frac{v_{oc}}{30} = 2 \qquad\qquad v_{oc} = 3v_x$$

Solving, we find $V_t = v_{oc} = 30\text{ V}$.

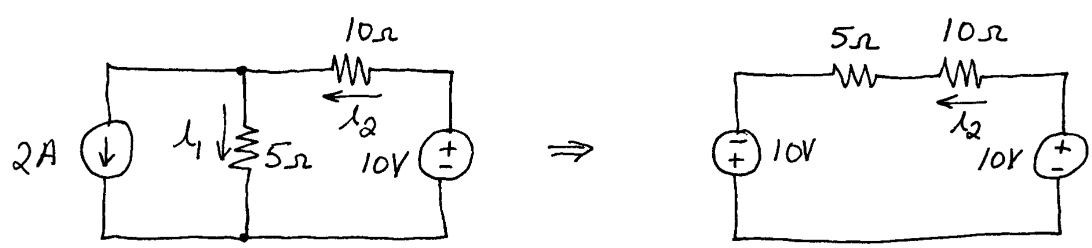
Now, we find the short-circuit current:



$$2v_x + v_x = 0 \qquad \Rightarrow \qquad v_x = 0$$

Therefore $i_{sc} = 2\text{ A}$. Then we have $R_t = v_{oc} / i_{sc} = 15\ \Omega$.

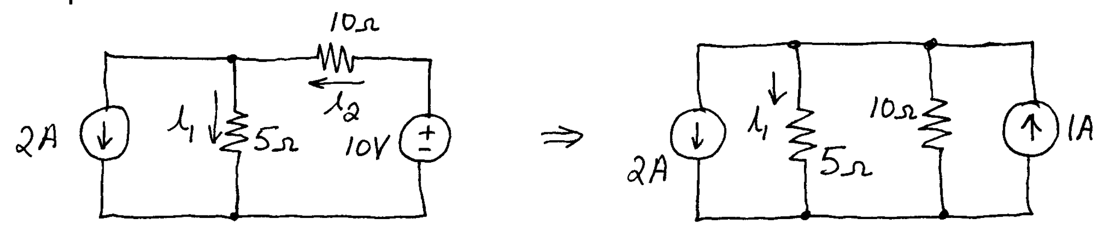
E2.30 First, we transform the 2-A source and the 5-Ω resistance into a voltage source and a series resistance:



Then we have $i_2 = \frac{10+10}{15} = 1.333$ A.

From the original circuit, we have $i_1 = i_2 - 2$, from which we find $i_1 = -0.667$ A.

The other approach is to start from the original circuit and transform the 10-Ω resistance and the 10-V voltage source into a current source and parallel resistance:



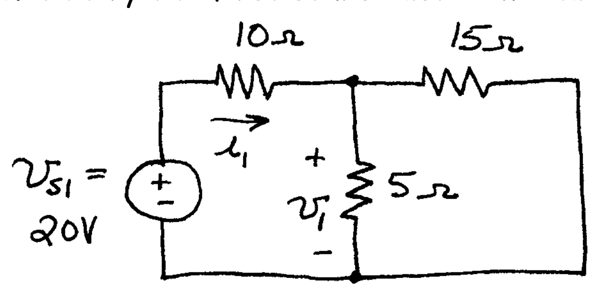
Then we combine the resistances in parallel. $R_{eq} = \frac{1}{1/5 + 1/10} = 3.333\ \Omega$.

The current flowing upward through this resistance is 1 A. Thus the voltage across R_{eq} referenced positive at the bottom is 3.333 V and $i_1 = -3.333/5 = -0.667$ A. Then from the original circuit we have $i_2 = 2 + i_1 = 1.333$ A, as before.

E2.31 Refer to Figure 2.62b. We have $i_1 = 15/15 = 1$ A.

Refer to Figure 2.62c. Using the current division principle, we have $i_2 = -2 \times \frac{5}{5+10} = -0.667$ A. (The minus sign is because of the reference direction of i_2.) Finally, by superposition we have $i_T = i_1 + i_2 = 0.333$ A.

E2.32 With only the first source active we have:

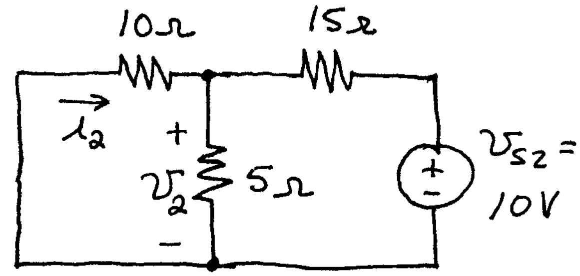


Then we combine resistances in series and parallel:

$$R_{eq} = 10 + \frac{1}{1/5 + 1/15} = 13.75\ \Omega$$

Thus, $i_1 = 20/13.75 = 1.455$ A, and $v_1 = 3.75 i_1 = 5.45$ V.

With only the second source active, we have:



Then we combine resistances in series and parallel:

$$R_{eq2} = 15 + \frac{1}{1/5 + 1/10} = 18.33\ \Omega$$

Thus, $i_s = 10/18.33 = 0.546$ A, and $v_2 = 3.33 i_s = 1.818$ V. Then, we have $i_2 = (-v_2)/10 = -0.1818$ A

Finally we have $v_T = v_1 + v_2 = 5.45 + 1.818 = 7.27$ V and $i_T = i_1 + i_2 = 1.455 - 0.1818 = 1.27$ A.

Problems

P2.1* (a) $R_{eq} = 20\ \Omega$ (b) $R_{eq} = 23\ \Omega$

P2.2* We have $\dfrac{1}{1/20 + 1/12 + R_x} = 10$ which yields $R_x = 8\Omega$

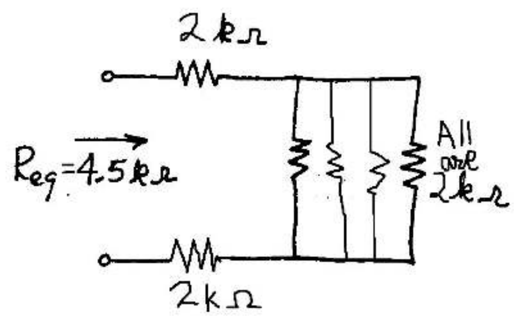
P2.3* The 25-Ω and 50-Ω resistances are in parallel and have an equivalent resistance of $R_{eq1} = 50/3\ \Omega$. Also the 40-Ω and 70-Ω resistances are in parallel with an equivalent resistance of $R_{eq2} = 280/11\ \Omega$. Next we see that R_{eq1} and the 6-Ω resistor are in series and have an equivalent resistance of $R_{eq3} = 6 + R_{eq1} = 68/3\ \Omega$. Finally R_{eq3} and R_{eq2} are in parallel and the overall equivalent resistance is

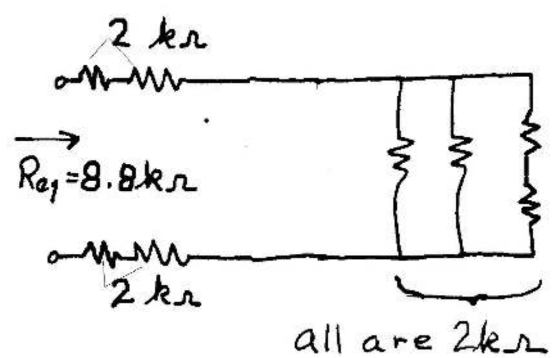
$$R_{ab} = \frac{1}{1/R_{eq3} + 1/R_{eq2}} = 11.99\Omega$$

P2.4* The 12-Ω and 6-Ω resistances are in parallel having an equivalent resistance of 4 Ω. Similarly, the 18-Ω and 9-Ω resistances are in parallel and have an equivalent resistance of 6 Ω. Finally, the two parallel combinations are in series, and we have

$$R_{ab} = 4 + 6 = 10\ \Omega$$

P2.5*



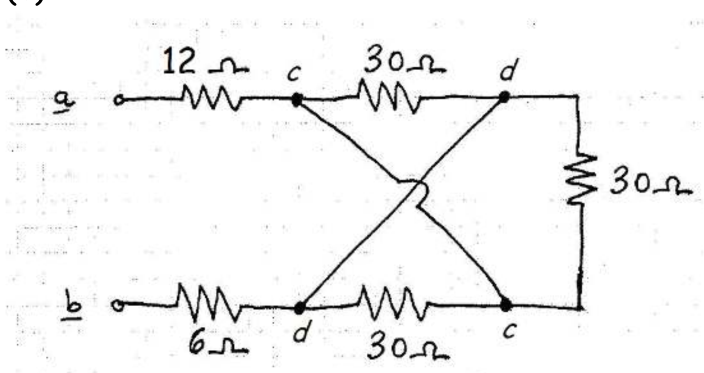


P2.6 (a) $R_{eq} = 8\,\Omega$ (b) $R_{eq} = 12.5\,\Omega$

P2.7 Because the resistances are in parallel, the voltage v across both of them is the same. The voltage across R_1 is $v = 50i$. The voltage across R_2 is $v = 4iR_2 = 180i$. Thus, we have $50i = 4iR_2$, yielding $R_2 = 12.5\Omega$.

P2.8 (a) $R_{eq} = 44\,\Omega$ (b) $R_{eq} = 32\,\Omega$

(c)

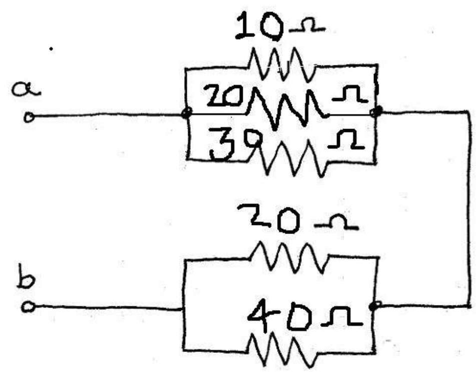


Notice that the points labeled c are the same node and that the points labeled d are another node. Thus, all three of the 30-Ω resistors are in parallel because they are each connected between nodes c and d. The equivalent resistance is 28 Ω.

P2.9 We have $\dfrac{1}{1/70+1/G_x}=20$ which yields $G_x=28$ mho

P2.10 We have $R_{eq}=\dfrac{2R(3R)}{2R+3R}=\dfrac{2R}{5}$. Clearly, for R_{eq} to be an integer, R must be an integer multiple of 5.

P2.11 $R_{ab}=18.78\Omega$



P2.12 In the lowest power mode, the power is $P_{lowest}=\dfrac{120^2}{R_1+R_2}=102.9\ \text{W}$.

For the highest power mode, the two elements should be in parallel with an applied voltage of 240 V. The resulting power is

$$P_{highest}=\frac{240^2}{R_1}+\frac{240^2}{R_2}=1440+576=2016\ \ \text{W}.$$

Some other modes and resulting powers are:

R_1 operated separately from 240 V yielding 1440 W

R_2 operated separately from 240 V yielding 576 W

R_1 in series with R_2 operated from 240 V yielding 411.4 W

R_1 operated separately from 120 V yielding 360 W

P2.13 Combining the resistances shown in Figure P2.13b, we have

$$R_{eq}=8+\frac{1}{1/5+1/R_{eq}}+8=16+\frac{5R_{eq}}{5+R_{eq}}$$

$$\left(R_{eq}\right)^2-16R_{eq}-80=0$$

$$R_{eq}=20\,\Omega$$

($R_{eq}=-4\,\Omega$ is another root, but is not physically reasonable.)

P2.14 $$R_{eq} = \frac{1}{\frac{1}{1000}+\frac{1}{1000}+\frac{1}{1000}+\ldots} = \frac{1}{\frac{n}{1000}} = \frac{1000}{n}$$

P2.15 For operation at the lowest power, we have

$$P = 180 = \frac{120^2}{R_1 + R_2}$$

At the high power setting, we have

$$P = 960 = \frac{120^2}{R_1} + \frac{120^2}{R_2}$$

These equations can be put in the form

$$R_1 + R_2 = 80$$

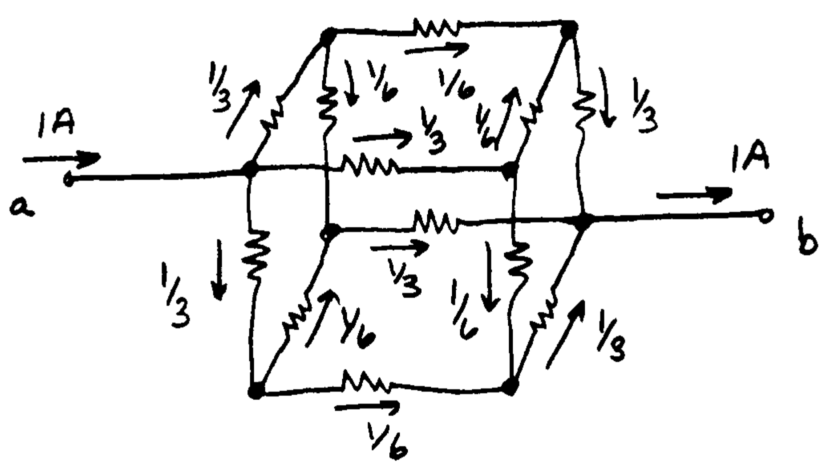
$$\frac{1}{1/R_1 + 1/R_2} = \frac{R_1 R_2}{R_1 + R_2} = 15$$

Solving these equations, we find $R_1 = 20\,\Omega$ and $R_2 = 60\,\Omega$. (A second solution simply has the values of R_1 and R_2 interchanged.)

The intermediate power settings are obtained by operating one of the elements from 120 V resulting in powers of 240 W and 720 W.

P2.16 $R = 20.9\ \Omega$.

P2.17 By symmetry, we find the currents in the resistors as shown below:



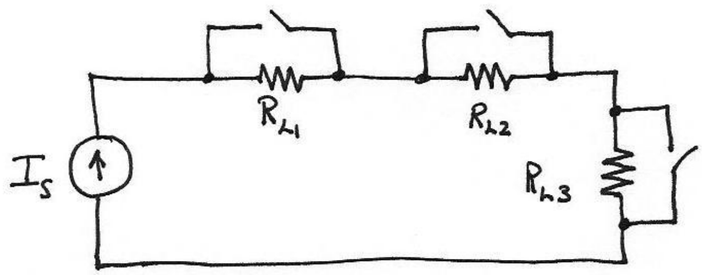
Then, the voltage between terminals a and b is

$$v_{ab} = R_{eq} = 1/3 + 1/6 + 1/3 = 5/6$$

P2.18 (a) For a series combination $G_{eq} = \dfrac{1}{1/G_1 + 1/G_2 + 1/G_3}$

(b) For a parallel combination of conductances $G_{eq} = G_1 + G_2 + G_3$

P2.19 To supply the loads in such a way that turning one load on or off does not affect the other loads, we must connect the loads in series with a switch in parallel with each load:



To turn a load on, we open the corresponding switch, and to turn a load off, we close the switch.

P2.20 The equations for the conductances are

$$G_b + G_c = \frac{1}{R_{as}} = \frac{1}{24} \qquad G_a + G_c = \frac{1}{R_{bs}} = \frac{1}{30} \qquad G_b + G_a = \frac{1}{R_{cs}} = \frac{1}{40}$$

Adding respective sides of the first two equations and subtracting the respective sides of the third equation yields

$2G_c = \dfrac{1}{24} + \dfrac{1}{30} - \dfrac{1}{40} = \dfrac{6}{120}$ from which we obtain $G_c = \dfrac{1}{40}$ S. Then we have $R_c = 40\,\Omega$. Similarly, we find $R_a = 120\,\Omega$ and $R_b = 60\,\Omega$.

P2.21 We have $R_a + R_b = R_{ab} = 30$, $R_b + R_c = R_{bc} = 50$ and $R_a + R_c = R_{ca} = 40$. These equations can be solved to find that $R_a = 10\ \Omega$, $R_b = 20\ \Omega$, and $R_c = 30\ \Omega$. After shorting terminals b and c, the equivalent resistance between terminal a and the shorted terminals is

$$R_{eq} = R_a + \frac{1}{1/R_b + 1/R_c} = 22\ \Omega$$

P2.22 The steps in solving a circuit by network reduction are:

1. Find a series or parallel combination of resistances.
2. Combine them.

3. Repeat until the network is reduced to a single resistance and a single source (if possible).
4. Solve for the currents and voltages in the final circuit. Transfer results back along the chain of equivalent circuits, solving for more currents and voltages along the way.
5. Check to see that KVL and KCL are satisfied in the original network.

The method does not always work because some networks cannot be reduced sufficiently. Then, another method such as node voltages or mesh currents must be used.

P2.23* $i_1 = \frac{15}{R_{eq}} = \frac{15}{16} = 0.9375\ A$

$v_x = 5.625\ \text{V}$

$i_2 = \frac{v_x}{12} = 0.46875\ A$

P2.24* $R_{eq} = \frac{1}{1/5 + 1/15} = 3.75\ \Omega \qquad v_x = 2\ A \times R_{eq} = -7.5\ \text{V}$

$i_1 = v_x / 5 = -1.5\ A \quad i_2 = v_x / 15 = -0.5\ A$

$P_{6A} = 6 \times 7.5 = 45$ W delivering

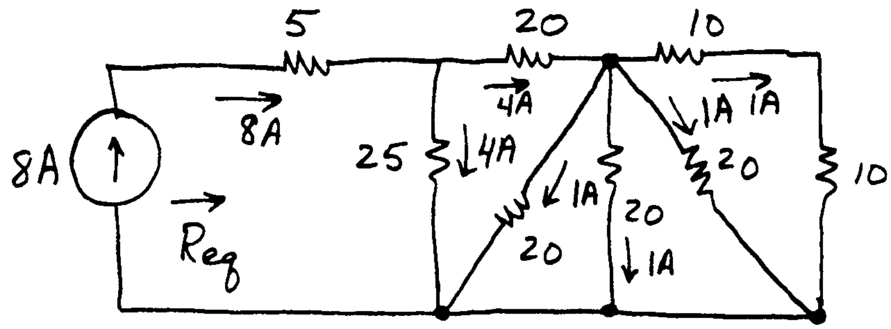
$P_{8A} = 8 \times 7.5 = 60$ W absorbing

$P_{5\Omega} = (-7.5)^2/5 = 11.25$ W absorbing

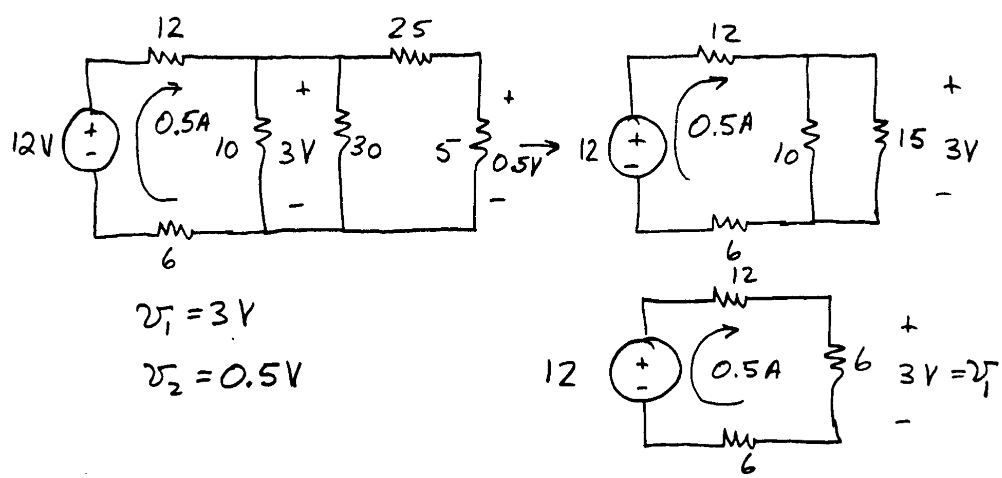
$P_{15\Omega} = (-7.5)^2/15 = 3.75$ W absorbing

P2.25* Combining resistors in series and parallel, we find that the equivalent resistance seen by the current source is $R_{eq} = 17.5\ \Omega$.

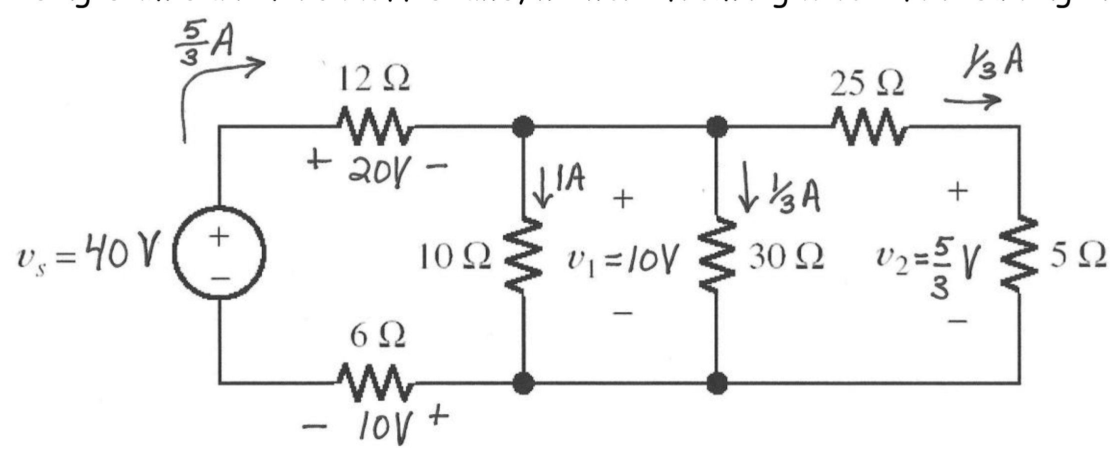
Thus, $v = 8 \times 17.5 = 140$ V. Also, $i = 1$ A.



P2.26* We combine resistances in series and parallel until the circuit becomes an equivalent resistance across the voltage source. Then, we solve the simplified circuit and transfer information back along the chain of equivalents until we have found the desired results.



P2.27 Using Ohm's and Kirchhoff's laws, we work from right to left resulting in



P2.28 The equivalent resistance seen by the current source is

$$R_{eq} = 6 + \frac{1}{1/20 + 1/(25+5)} = 18\ \Omega$$

Then, we have $v_s = 3R_{eq} = 54\ \text{V}$ $\quad v_1 = 3\frac{1}{1/20 + 1/(25+5)} = 36\ \text{V}$

$$i_2 = \frac{v_1}{25+5} = 1.2\ A$$

P2.29 The equivalent resistance seen by the voltage source is

$$R_{eq} = \frac{1}{1/18 + 1/(7+2)} + 4 = 10\ \Omega$$

Then, we have

$$i_1 = \frac{30\ \text{V}}{R_{eq}} = 3\ A \qquad v_2 = i_1 \frac{1}{1/18 + 1/(7+2)} = 18\ \text{V}$$

$$i_2 = \frac{v_2}{18} = 1\ A \qquad i_3 = -\frac{v_2}{9} = -2\ A$$

P2.30 The equivalent resistance seen by the current source is $R_{eq} = 6 + \frac{1}{1/9 + 1/18} + \frac{1}{1/10 + 1/40} = 20\ \Omega$. Then, we have $v = 5R_{eq} = 100\ \text{V}$, $i_2 = 3.333\ A$, and $i_1 = 4\ A$.

P2.31 $i_2 = \frac{12\ \text{V}}{4\ \Omega} = 3\ A \qquad i_1 = i_2 + 2 = 5\ A$

Notice that i_1 is referenced into the negative reference for the voltage source, thus $P_{voltage-source} = -12 i_1 = -60\ \text{W}$. The 2-A of the current source flows from the positive reference for the voltage toward the negative reference, and we have

$P_{current-source} = 2\ A \times 12\ \text{V} = +24\ \text{W}$.

Power is delivered by the voltage source and absorbed by the current source. The resistance absorbs 36 W so power is conserved in the circuit.

P2.32 With the switch open, the current flowing clockwise in the circuit is given by $i = \frac{16}{6 + R_2}$, and we have $v_2 = R_2 i = \frac{16R_2}{6+R_2} = 10.$ Solving, we find $R_2 = 10$ Ω.

With the switch closed, R_2 and R_L are in parallel with an equivalent resistance given by $R_{eq} = \dfrac{1}{1/R_2 + 1/R_L} = \dfrac{1}{1/10 + 1/R_L}$. The current through R_{eq} is given by $i = \dfrac{16}{6 + R_{eq}}$ and we have $v_2 = R_{eq} i = \dfrac{16R_{eq}}{6 + R_{eq}} = 8$.

Solving, we find $R_{eq} = 6\ \Omega$. Then, we can write $R_{eq} = \dfrac{1}{1/10 + 1/R_L} = 6$.

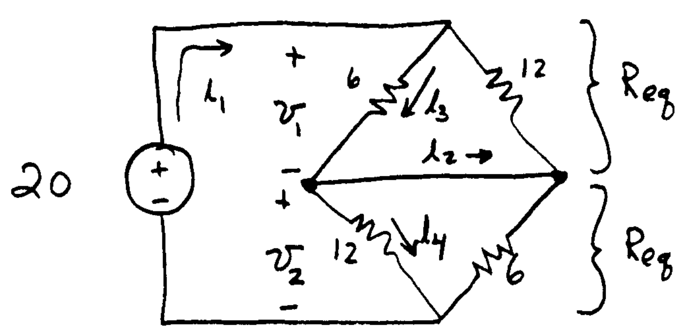
Solving, we find $R_L = 15\ \Omega$.

P2.33 The currents through the 3-Ω resistance and the 4-Ω resistance are zero, because they are series with an open circuit. Thus, we can consider the 8-Ω and the 7-Ω resistances to be in series. The current circulating clockwise in the left-hand loop is given by $i_1 = \dfrac{30}{7+8} = 2\ A$, and we have $v_1 = 14$ V. The current circulating counterclockwise in the right-hand loop is 3 A. By Ohm's law, we have $v_2 = 6\ \text{V}$. Then, using KVL, we have $v_{ab} = v_1 - v_2 = 3\ \text{V}$, $v_{bc} = v_2 = 6\,\text{V}$, and $v_{ca} = -v_1 = -14\,\text{V}$.

P2.34 $i = \dfrac{P}{v} = \dfrac{4.5\ \text{W}}{15\ \text{V}} = 0.3A \qquad R_{eq} = R + \dfrac{1}{1/R + 1/R} + R = 2.5R$

$i = 0.3 = \dfrac{15}{R_{eq}} = \dfrac{15}{2.5R} \qquad\qquad R = 13.33\Omega$

P2.35*



$R_{eq} = \dfrac{1}{1/6 + 1/12} = 4\,\Omega \qquad\qquad i_1 = \dfrac{20\ \text{V}}{2R_{eq}} = 2.5\ A$

$v_1 = v_2 = R_{eq} i_1 = 10\ \text{V} \qquad\qquad i_3 = 10/6 = 1.667\ A$

$i_4 = 10/12 = 0.8333\ A \qquad\qquad i_2 = i_3 - i_4 = 0.8333\ A$

P2.36* $v_1 = \dfrac{R_1}{R_1 + R_2 + R_3} \times v_s = 5\text{ V}$ $\qquad$ $v_2 = \dfrac{R_2}{R_1 + R_2 + R_3} \times v_s = 7\text{ V}$

$$v_3 = \frac{R_3}{R_1 + R_2 + R_3} \times v_s = 13\text{ V}$$

P2.37* Using current division, $i_1 = \dfrac{R_2}{R_1 + R_2} 5 = 2\ A$ $\qquad$ $i_2 = \dfrac{R_1}{R_1 + R_2} 5 = 3A$

P2.38* Combining R_2 and R_3, we have an equivalent resistance $R_{eq} = \dfrac{1}{1/R_2 + 1/R_3} = 10\ \Omega$. Then, using the voltage-division principle, we have $v = \dfrac{R_{eq}}{R_1 + R_{eq}} \times v_s = \dfrac{10}{20 + 10} \times 10 = 3.333\text{ V}$.

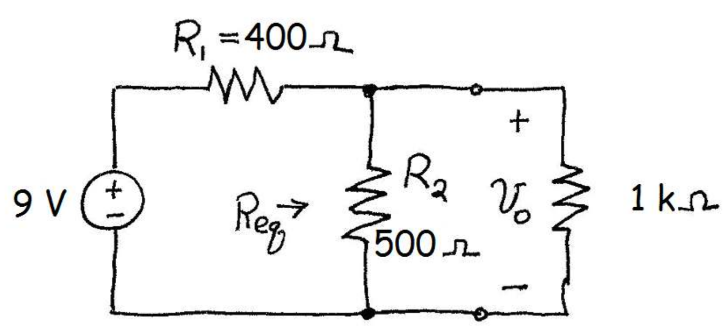
P2.39 $i_2 = \dfrac{R_3}{R_2 + R_3} \times i_s = \dfrac{50}{25 + 50} \times 30 = 20\text{ mA}$

$$i_3 = \frac{R_2}{R_2 + R_3} \times i_s = \frac{25}{25 + 50} \times 30 = 10\text{ mA}$$

P2.40 (a) $R_1 + R_2 = \dfrac{9\text{ V}}{10\text{ mA}} = 900\ \Omega$ $\qquad$ $\dfrac{R_2}{R_1 + R_2} \times 9 = 5$

Solving, we find $R_2 = 500\ \Omega$ and $R_1 = 400\ \Omega$.

(b)



The equivalent resistance for the parallel combination of R_2 and the load is

$$R_{eq} = \frac{1}{1/500 + 1/1000} = 333.3\,\Omega$$

Then, using the voltage division principle, we have

$$v_o = \frac{R_{eq}}{R_1 + R_{eq}} \times 9\text{ V} = 4.091\text{ V}$$

(c) If we choose a larger current in part (a), resulting in smaller values for R_1 and R_2, the loaded voltage in part (b) would be closer to 5 V. However, this would result in shorter battery life.

P2.41 We have $120\dfrac{10}{10 + 15 + R_x} = 45$, which yields $R_x = 1.667\Omega$

P2.42 First, we combine the 60 Ω and 20 Ω resistances in parallel yielding an equivalent resistance of 15 Ω, which is in parallel with R_x. Then, applying the current division principle, we have

$$30\frac{15}{15 + R_x} = 10$$

which yields $R_x = 30\,\Omega$.

P2.43 In a similar fashion to the solution for Problem P2.13, we can write the following expression for the resistance seen by the 16-V source.

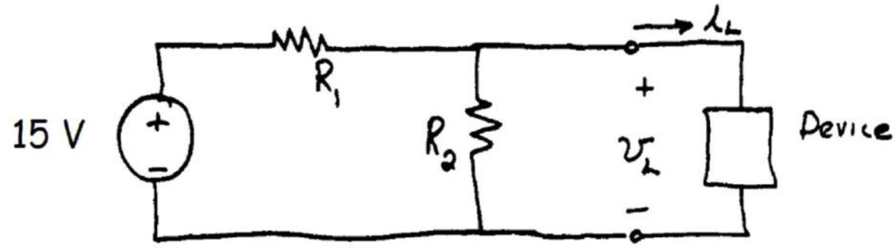
$$R_{eq} = 2 + \frac{1}{1/R_{eq} + 1/4}\ \text{k}\Omega$$

The solutions to this equation are $R_{eq} = 4\,\text{k}\Omega$ and $R_{eq} = -2\,\text{k}\Omega$. However, we reason that the resistance must be positive and discard the negative root. Then, we have $i_1 = \dfrac{16\text{ V}}{R_{eq}} = 4$ mA, $i_2 = i_1\dfrac{R_{eq}}{4 + R_{eq}} = \dfrac{i_1}{2} = 2$ mA, and $i_3 = \dfrac{i_1}{2} = 2$ mA. Similarly, $i_4 = \dfrac{i_3}{2} = \dfrac{i_1}{2^2} = 1$ mA. Clearly, $i_{n+2} = i_n/2$. Thus, $i_{18} = \dfrac{i_1}{2^9} = 7.8125\ \mu$A.

P2.44* $v = 0.1\text{ mA} \times R_w = 50\text{ mV}$

$$R_g = \frac{50\text{ mV}}{2\text{ A} - 0.1\text{ mA}} = 25\text{ m}\Omega$$

P2.45 The circuit diagram is:



With $i_L = 0$ and $v_L = 5\text{ V}$, we must have $\frac{R_2}{R_1 + R_2} \times 15 = 5\text{ V}$. Rearranging, this gives

$$\frac{R_1}{R_2} = 2 \qquad (1)$$

With $i_L = 100\text{ mA}$ and $v_L = 4.7\text{ V}$, we have $15 - R_1(4.7/R_2 + 100\text{ mA}) = 4.7$. Rearranging, this gives

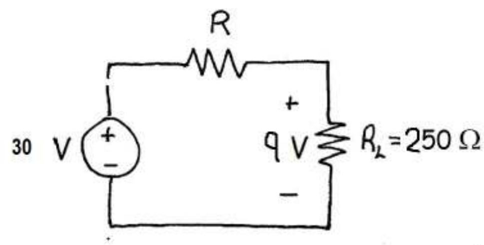
$$4.7\frac{R_1}{R_2} + R_1 \times 0.1 = 10.3. \qquad (2)$$

Using Equation (1) to substitute into Equation (2) and solving, we obtain $R_1 = 9\,\Omega$ and $R_2 = 4.5\,\Omega$.

Maximum power is dissipated in R_1 for $i_L = 100\text{ mA}$, for which the voltage across R_1 is 10.3 V. Thus, $P_{\max R1} = \frac{10.3^2}{9} = 11.8\text{ W}$. Thus, R_1 must be rated for at least 11.8 W of power dissipation.

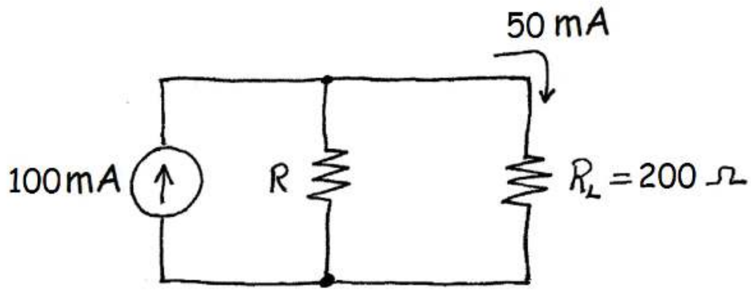
Maximum power is dissipated in R_2 for $i_L = 0$, in which case the voltage across R_2 is 5 V. Thus, $P_{\max R2} = \frac{5^2}{4.5} = 5.56\text{ W}$.

P2.46 We need to place a resistor in series with the load and the voltage source as shown:



Applying the voltage-division principle, we have $30\frac{250}{250+R}=9$. Solving, we find $R=583.33\Omega$.

P2.47 We have $P=500\times10^{-3}=I_L^2R_L=I_L^2\times200$. Solving, we find that the current through the load is $I_L=50$ mA. Thus, we must place a resistor in parallel with the current source and the load.



Then, we have $100\frac{R}{R+R_L}=50$ from which we find $R=200\ \Omega$.

P2.48. 1. Select a reference node and assign variables for the unknown node voltages. If the reference node is chosen at one end of an independent voltage source, one node voltage is known at the start, and fewer need to be computed.
2. Write network equations. First, use KCL to write current equations for nodes and supernodes. Write as many current equations as you can without using all of the nodes. Then if you do not have enough equations because of voltage sources connected between nodes, use KVL to write additional equations.
3. If the circuit contains dependent sources, find expressions for the controlling variables in terms of the node voltages. Substitute into the

network equations, and obtain equations having only the node voltages as unknowns.

4. Put the equations into standard form and solve for the node voltages.

5. Use the values found for the node voltages to calculate any other currents or voltages of interest.

P2.49* At node 1 we have: $\frac{v_1}{20} + \frac{v_1 - v_2}{10} = 1$

At node 2 we have: $\frac{v_2}{5} + \frac{v_2 - v_1}{10} = 2$

In standard form, the equations become

$$0.15v_1 - 0.1v_2 = 1$$
$$-0.1v_1 + 0.3v_2 = 2$$

Solving, we find $v_1 = 14.29$ V and $v_2 = 11.43$ V.

Then we have $i_1 = \frac{v_1 - v_2}{10} = 0.2857$ A.

P2.50 Writing KCL equations, we have

$$\frac{v_1}{10} + \frac{v_1 - v_2}{5} + \frac{v_1 - v_3}{22} = 0$$
$$\frac{v_2 - v_1}{5} + \frac{v_2}{10} = 4$$
$$\frac{v_3}{4} + \frac{v_3 - v_1}{22} + = -4$$

In standard form, we have:

$$0.3455v_1 - 0.2v_2 - 0.04545v_3 = 0$$
$$-0.2v_1 + 0.3v_2 = 4$$
$$-0.04545v_1 + 0.2955v_3 = -4$$

The MATLAB commands needed to solve are:

```
G = [0.3455 -0.2 -0.04545; -0.2 0.3 0; -0.04545 0 0.2955]
I = [0; 4; -4]
V = G\I
```

From this, we find $v_1 = 10.0$ V, $v_2 = 20.0$ V, and $v_3 = -12.0$ V.

If the source is reversed, the algebraic signs are reversed in the **I** matrix and consequently, the node voltages are reversed in sign.

P2.51 Writing KCL equations at nodes 1, 2, and 3, we have

$$\frac{v_1}{R_4} + \frac{v_1 - v_2}{R_2} + \frac{v_1 - v_3}{R_1} = 0$$

$$\frac{v_2 - v_1}{R_2} + \frac{v_2 - v_3}{R_3} = I_s$$

$$\frac{v_3}{R_5} + \frac{v_3 - v_2}{R_3} + \frac{v_3 - v_1}{R_1} = 0$$

In standard form, we have:

$$0.6167v_1 - 0.20v_2 - 0.25v_3 = 0$$

$$-0.20v_1 + 0.325v_2 - 0.125v_3 = 4$$

$$-0.25v_1 - 0.125v_2 + 0.50v_3 = 0$$

Using Matlab, we have

```
G = [0.6167 -0.20 -0.25; -0.20 0.325 -0.125; -0.25 -0.125 0.500];
I = [0; 4; 0];
V = G\I
V =
  13.9016
  26.0398
  13.4608
```

P2.52 Writing KCL equations at nodes 1, 2, and 3, we have

$$\frac{v_1}{R_3} + \frac{v_1 - v_2}{R_4} + I_s = 0$$

$$\frac{v_2 - v_1}{R_4} + \frac{v_2 - v_3}{R_6} + \frac{v_2}{R_5} = 0$$

$$\frac{v_3}{R_1 + R_2} + \frac{v_3 - v_2}{R_6} = I_s$$

In standard form, we have:

$$0.133v_1 - 0.0833v_2 = -4$$

$$-0.0833v_1 + 0.3833v_2 - 0.2v_3 = 0$$

$$-0.2v_2 + 0.236v_3 = 4$$

Solving using Matlab, we have

```
G = [ 0.133 -0.0833 0; -0.0833 0.3833 -0.2; 0 -0.2 0.236]
I = [-4; 0; 4]
V = G\I
```

$v_1 = -26.6476\text{V}$ $\quad v_2 = 5.4726\,\text{V}$ $\quad v_3 = 21.5869\text{V}$

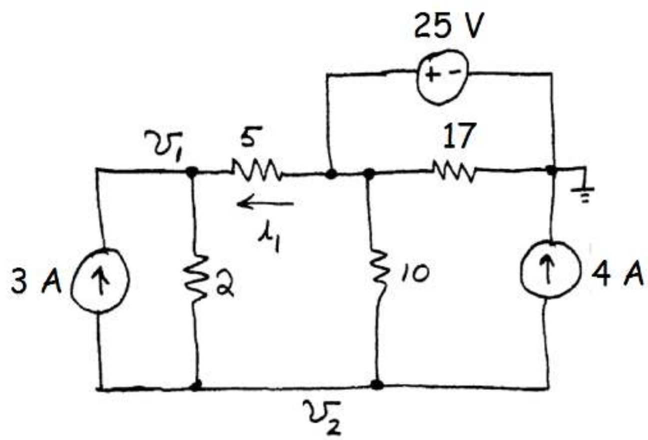
P2.53* Writing a KVL equation, we have $v_1 - v_2 = 10$.

At the reference node, we write a KCL equation: $\frac{v_1}{5} + \frac{v_2}{10} = 1$.

Solving, we find $v_1 = 6.667$ and $v_2 = -3.333$.

Then, writing KCL at node 1, we have $i_s = \frac{v_2 - v_1}{5} - \frac{v_1}{5} = -3.333\text{ A}$.

P2.54 To minimize the number of unknowns, we select the reference node at one end of the voltage source. Then, we define the node voltages and write a KCL equation at each node.



$$\frac{v_1 - 25}{5} + \frac{v_1 - v_2}{2} = 3 \qquad\qquad \frac{v_2 - v_1}{2} + \frac{v_2 - 25}{10} = -7$$

In Matlab, we have

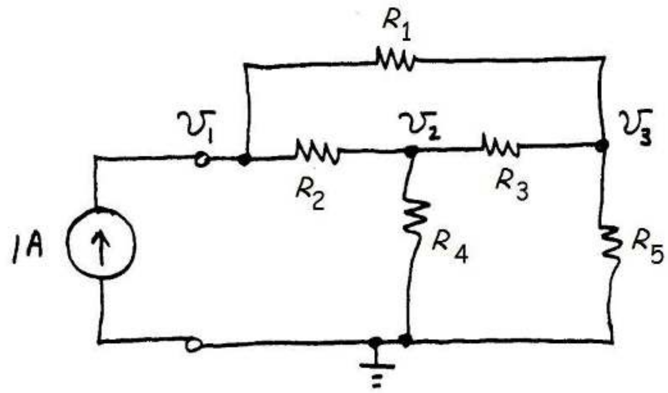
```
G = [0.7 -0.5; -0.5 0.6]
I = [8; -4.5]
V = G\I
I1 = (25 - V(1))/5
```

Then, we have $i_1 = 2\text{ A}$.

The 17-Ω resistance does not appear in the network equations and has no effect on the answer. The voltage at the top end of the 10-Ω resistance is 25 V regardless of the value of the 17-Ω resistance. Thus, any nonzero value could be substituted for the 17-Ω resistance without affecting the answer.

P2.55 We must not use all of the nodes (including those that are inside supernodes) in writing KCL equations. Otherwise, dependent equations result.

P2.56 The circuit with a 1-A source connected is:



$$\frac{V_1 - V_2}{R_2} + \frac{V_1 - V_3}{R_1} = 1 \qquad \frac{V_2}{R_4} + \frac{V_2 - V_1}{R_2} + \frac{V_2 - V_3}{R_3} = 0$$

$$\frac{V_3}{R_5} + \frac{V_3 - V_1}{R_1} + \frac{V_3 - V_2}{R_3} = 0$$

In Matlab, we use the commands

```
[V1,V2,V3] = solve('(V1 - V2)/R2 + (V1 - V3)/R1 = 1' , ...
              ' V2/R4 + (V2 - V1)/R2 + (V2 - V3)/R3 = 0' , ...
              ' V3/R5 + (V3 - V1)/R1 + (V3 - V2)/R3 = 0');
pretty(V1)
```

After some clean up, this produces

$$R_{eq} = \frac{R_4R_5R_2 + R_2R_3R_5 + R_2R_3R_1 + R_5R_2R_1 + R_4R_2R_1 + R_4R_3R_1 + R_4R_1R_5 + R_4R_3R_5}{R_5R_3 + R_5R_1 + R_2R_3 + R_4R_3 + R_4R_1 + R_1R_3 + R_5R_2 + R_4R_2}$$

Then, the command

```
subs(V1, {'R1','R2','R3','R4','R5'}, {15,15,15,10,10})
```

yields $R_{eq} = 12.5\ \Omega$.

P2.57* First, we can write: $i_x = \frac{v_1 - v_2}{5}$.

Then, writing KCL equations at nodes 1 and 2, we have:

$$\frac{v_1}{10} + i_x = 1 \text{ and } \frac{v_2}{20} + 0.5i_x - i_x = 0$$

Substituting for i_x and simplifying, we have

$$0.3v_1 - 0.2v_2 = 1$$
$$-0.1v_1 + 0.15v_2 = 0$$

Solving, we have $v_1 = 6$ and $v_2 = 4$.

Then, we have $i_x = \frac{v_1 - v_2}{5} = 0.4 \text{ A}$.

P2.58* $v_x = v_2 - v_1$

Writing KCL at nodes 1 and 2:

$$\frac{v_1}{5} + \frac{v_1 - 2v_x}{15} + \frac{v_1 - v_2}{10} = 1$$

$$\frac{v_2}{5} + \frac{v_2 - 2v_x}{10} + \frac{v_2 - v_1}{10} = 2$$

Substituting and simplifying, we have

$15v_1 - 7v_2 = 30$ and $v_1 + 2v_2 = 20$.

Solving, we find $v_1 = 5.405$ and $v_2 = 7.297$.

P2.59 First, we can write:

$$i_x = \frac{5i_x - v_2}{10}$$

Simplifying, we find $i_x = -0.2v_2$.

Then write KCL at nodes 1 and 2:

$$\frac{v_1 - 5i_x}{5} = 4 + 2 \qquad \frac{v_2}{10} - i_x = -6$$

Substituting for i_x and simplifying, we have

$$v_1 + v_2 = 30 \quad \text{and} \quad 0.3v_2 = -6$$

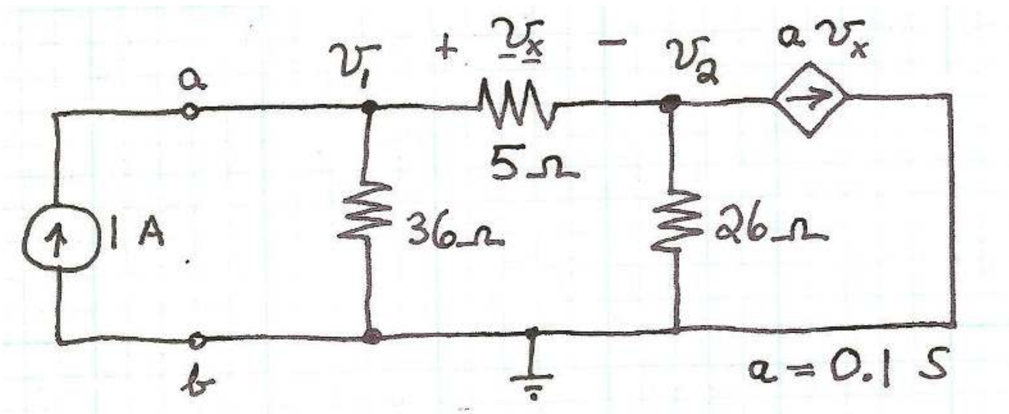
which yield $v_1 = 50 \text{ V}$ and $v_2 = -20 \text{ V}$.

P2.60 First, we can write $i_x = -\frac{v_1}{10}$. Then writing KVL, we have $v_1 - 5i_x - v_2 = 0$. Writing KCL at the reference node, we have $\frac{v_2}{20} + \frac{v_2}{20} = i_x + 8$. Using the first equation to substitute for i_x and simplifying, we have

$$1.5v_1 - v_2 = 0$$
$$v_1 + v_2 = 80$$

Solving, we find $v_1 = 32.0$ V, $v_2 = 48.0$ V, and $i_x = -\frac{v_1}{10} = -3.2$ A. Finally, the power delivered to the 8-Ω resistance is $P = \frac{(v_1 - v_2)^2}{8} = 32.0$ W.

P2.61 The circuit with a 1-A current source connected is:



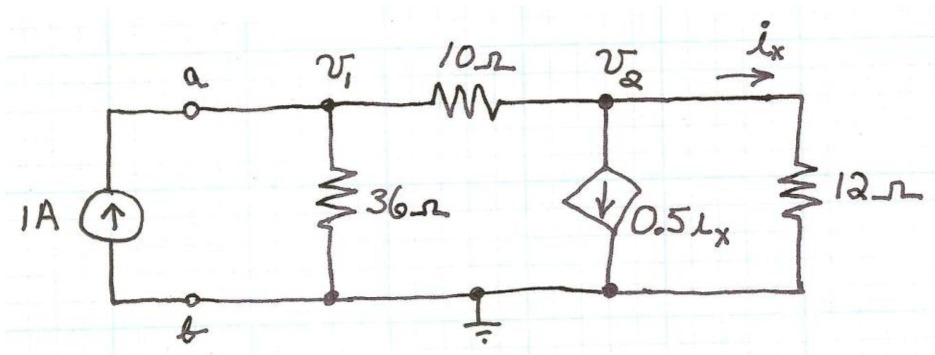
$$v_x = v_1 - v_2$$
$$\frac{v_1}{36} + \frac{v_1 - v_2}{5} = 1$$
$$\frac{v_2 - v_1}{5} + \frac{v_2}{26} + 0.1v_x = 0$$

Using the first equation to substitute for v_x and simplifying, we have

$$0.22778v_1 - 0.2v_2 = 1$$
$$-0.1v_1 + 0.13846v_2 = 0$$

Solving we find $v_1 = 12$. However, the equivalent resistance is equal in value to v_1 so we have R_{eq} = 12 Ω.

P2.62 The circuit with a 1-A current source connected is



$$i_x = \frac{v_2}{12}$$
$$\frac{v_1}{36} + \frac{v_1 - v_2}{10} = 1$$
$$\frac{v_2 - v_1}{10} + \frac{v_2}{12} + 0.5i_x = 0$$

Using the first equation to substitute for i_x and simplifying, we have

$$0.12778v_1 - 0.1v_2 = 1$$
$$-0.1v_1 + 0.225v_2 = 0$$

Solving we find $v_1 = 12$ V. However, the equivalent resistance is equal in value to v_1, so we have R_{eq} = 12 Ω.

P2.63 Elements on the diagonal of **G** equal the sum of the conductances connected to any node, which is 3 S. Element g_{jk} off the diagonal is zero if no resistance is connected between nodes j and k and equal to −1 if there is a resistance connected between the nodes. **G** is the same for all three parts of the problem, only the node to which the current source is attached changes. (We used the MATLAB Array Editor to enter the elements of G.)

```
>> G
G =
     3    -1    -1     0     0     0     0
    -1     3     0    -1     0     0    -1
    -1     0     3    -1    -1     0     0
     0    -1    -1     3     0    -1     0
     0     0    -1     0     3    -1     0
     0     0     0    -1    -1     3    -1
     0    -1     0     0     0    -1     3
>> Ia = [1; 0; 0; 0; 0; 0; 0];
```

```
>> Ib = [0; 1; 0; 0; 0; 0; 0];
>> Ic = [0; 0; 0; 1; 0; 0; 0];
>> Va = G\Ia;
>> Vb = G\Ib;
>> Vc = G\Ic;
>> % Here are the answers:
>> Ra = Va(1)
Ra =
   0.5833
>> Rb = Vb(2)
Rb =
   0.7500
>> Rc = Vc(4)
Rc =
   0.8333
```

By symmetry, shorting nodes with equal node voltages, and series parallel combination, we can obtain $R_a = 7/12\ \Omega$, $R_b = 3/4\ \Omega$, and $R_c = 5/6\ \Omega$.

P2.64 First, we enter the node voltage equations into the solve command and define the unknowns to be V1, V2 and Vout. Then, we use the pretty command to print the answer for Vout:

```
SV = solve('(V1 - Vin)/(2*R1) + (V1 - Vout)/R1 + (V1 - V2)/R1 = 0' , ...
        '(V2 - V1)/R1 + V2/R1 + (V2 - Vout)/R1 = 0', ...
       '(Vout - V1)/R1 + (Vout - V2)/R1 + Vout/R2 = 0' , 'V1','V2','Vout');
pretty(SV.Vout)
```

The result is

```
      4 R2 Vin
   -------------
   13 R1 + 11 R2
```

Thus, we have

$$\frac{V_{out}}{V_{in}} = \frac{4R_2}{13R_1 + 11R_2}$$

P2.65 We write equations in which voltages are in volts, resistances are in kΩ, and currents are in mA.

KCL node 2: $\dfrac{(v_2 - v_1)}{4} + \dfrac{(v_2 - v_3)}{3} + \dfrac{v_2}{2} = 0$

KCL node 3: $\dfrac{(v_3 - v_2)}{3} + \dfrac{(v_3 - v_1)}{1} + \dfrac{(v_3 - v_4)}{2} = 5$

KCL ref node: $\dfrac{v_2}{2} + \dfrac{v_4}{5} = 5$

KVL: $v_1 - v_4 = 20$

Then, using Matlab we have:

```
G = [-1/4 (1/2 + 1/3 + 1/4) -1/3 0; ...
-1 -1/3 (1 + 1/2 + 1/3) -1/2; 0 1/2 0 1/5; 1 0 0 -1];
I = [0; 5; 5; 20];
V = G\I

V =
        20.7317
         9.7073
        16.0000
         0.7317
```

P2.66. 1. If necessary, redraw the network without crossing conductors or elements. Then, define the mesh currents flowing around each of the open areas defined by the network. For consistency, we usually select a clockwise direction for each of the mesh currents, but this is not a requirement.

2. Write network equations, stopping after the number of equations is equal to the number of mesh currents. First, use KVL to write voltage equations for meshes that do not contain current sources. Next, if any current sources are present, write expressions for their currents in terms of the mesh currents. Finally, if a current source is common to two meshes, write a KVL equation for the supermesh.

3. If the circuit contains dependent sources, find expressions for the controlling variables in terms of the mesh currents. Substitute into the network equations, and obtain equations having only the mesh currents as unknowns.

4. Put the equations into standard form. Solve for the mesh currents by use of determinants or other means.

5. Use the values found for the mesh currents to calculate any other currents or voltages of interest.

P2.67* Writing KVL equations around each mesh, we have

$$8i_1 + 24(i_1 - i_2) = 25 \qquad \text{and } 24(i_2 - i_1) + 16i_2 = 15$$

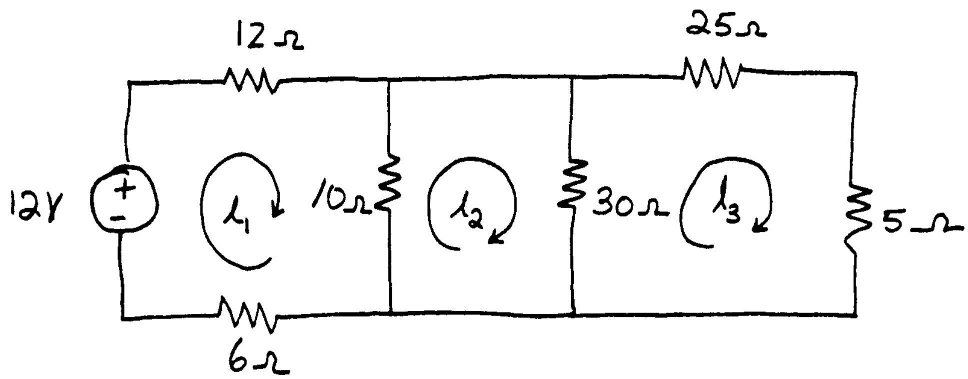
Putting the equations into standard form we have

$$32i_1 - 24i_2 = 25 \qquad \text{and} \qquad -24i_1 + 40i_2 = 15$$

Solving, we obtain $i_1 = 1.932$ A and $i_2 = 1.53$ A.
Then, the power delivered to the 24-Ω resistor is $P = (i_1 - i_2)^2 24 = 3.878$ W.

P2.68* Writing and simplifying the mesh-current equations, we have:

$$28i_1 - 10i_2 = 12$$
$$-10i_1 + 40i_2 - 30i_3 = 0$$
$$-30i_2 + 60i_3 = 0$$

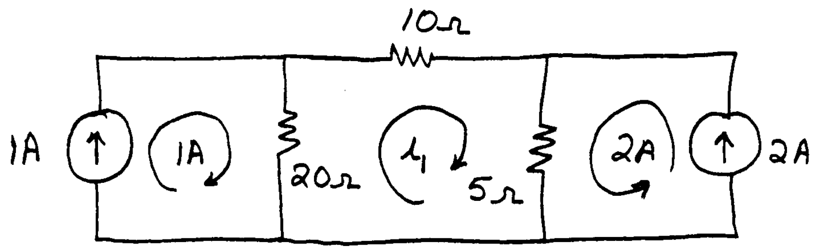


Solving, we obtain

$$i_1 = 0.500 \qquad i_2 = 0.200 \qquad i_3 = 0.100$$

Thus, $v_2 = 5i_3 = 0.500$ V and the power delivered by the source is $P = 12i_1 = 6$ W.

P2.69* Because of the current sources, two of the mesh currents are known.



Writing a KVL equation around the middle loop we have

$$20(i_1 - 1) + 10i_1 + 5(i_1 + 2) = 0$$

Solving, we find $i_1 = 0.2857$ A.

P2.70 Writing KVL equations around each mesh, we have

$$10i_1 + 15(i_1 - i_3) - 75 = 0$$

$$25(i_2 - i_3) + i_2 + 75 = 0$$

$$5i_3 + 25(i_3 - i_2) + 15(i_3 - i_2) = 0$$

Putting the equations into standard from, we have

$$25i_1 - 15i_3 = 75$$

$$26i_2 - 25i_3 = -75$$

$$-15i_1 - 25i_2 + 45i_3 = 0$$

Using Matlab to solve, we have

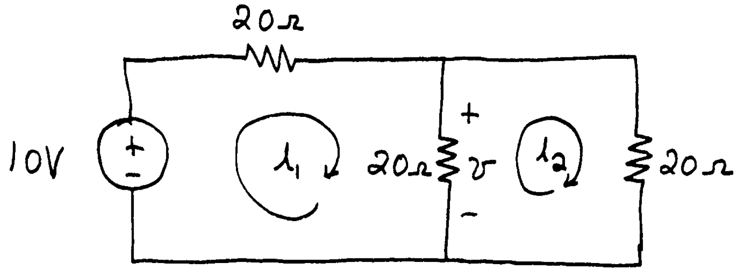
```
R = [25 0 -15; 0 26 -25; -15 -25 45];
V = [75; -75; 0];
 I = R\V
I =

   1.6399
  -5.0643
  -2.2669
```

Then, the power delivered by the source is $P = 75(i_1 - i_2) =$ 502.815 W.

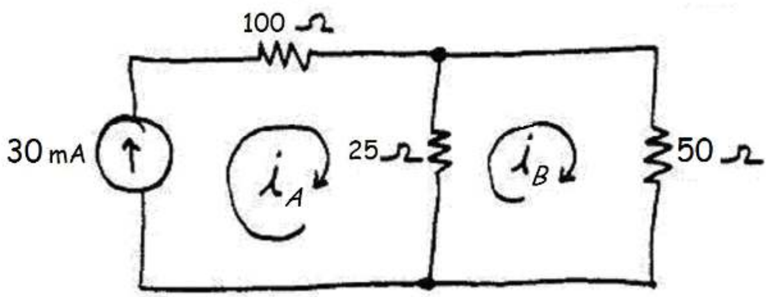
P2.71 Writing and simplifying the mesh equations, we obtain:

$40i_1 - 20i_2 = 10 \qquad -20i_1 + 40i_2 = 0$



Solving, we find $i_1 = 0.3333$ and $i_2 = 0.1667$.
Thus, $v = 20(i_1 - i_2) = 3.333$ V.

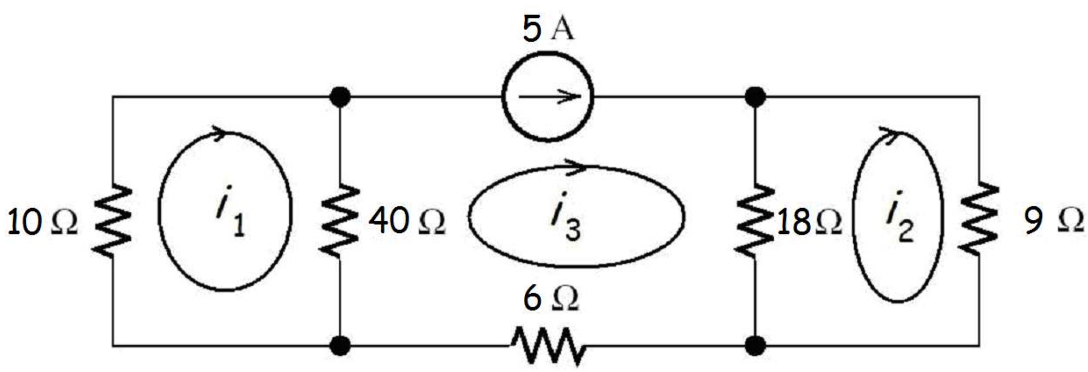
P2.72 The mesh currents and corresponding equations are:



$i_A = 30$ mA $\quad 25(i_B - i_A) + 50i_B = 0$

Solving, we find $i_B = 10$ mA.
However, i_3 shown in Figure P2.39 is the same as i_B, so the answer is i_3 = 10 mA.

P2.73 First, we select the mesh currents and then write three equations.



Mesh 1: $10i_1 + 40(i_1 - i_3) = 0$

Mesh 2: $9i_2 + 18(i_2 - i_3) = 0$

However by inspection, we have $i_3 = 5$. Solving, we obtain $i_1 = 4$ A and $i_2 = 3.333$ A.

P2.74 We assume that i_1 is a mesh current flowing around the left-hand mesh and that i_2 flows around the right-hand mesh. Writing and simplifying the mesh equations yields:

$$14i_1 - 8i_2 = 10$$
$$-8i_1 + 16i_2 = 0$$

Solving, we find $i_1 = 1.000$ and $i_2 = 0.500$.

Finally, the power delivered by the source is $P = 10i_1 = 10$ W.

P2.75 The mesh (KVL) equations are:

$$7i_A + 2i_A + 18(i_A - i_B) = 0$$
$$18(i_B - i_A) + 4i_B = -30$$

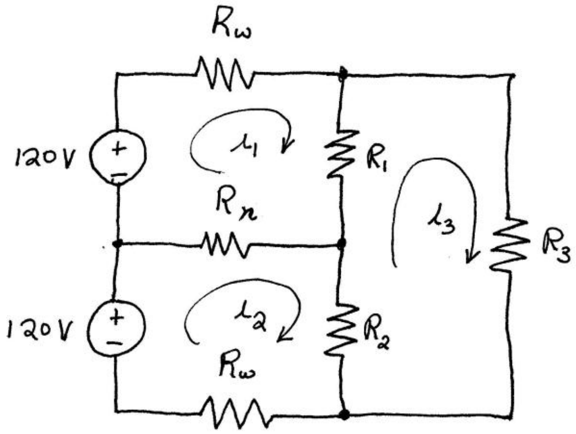
Solving we find $i_A = -2$ A and $i_B = -3$ A. Then, we have $i_1 = -i_B = 3$ A and $i_2 = i_A - i_B = 1.0$ A.

P2.76 By inspection: $i_A = 3$ A.

Mesh B: $25i_B + 5i_B + 20(i_B - i_A) = 0$

Solving, we find $i_B = 1.2$ A. Then, we have $i_1 = i_A - i_B = 1.8$ A and $i_2 = i_B = 1.2$ A.

P2.77 (a) First, we select mesh-current variables as shown.



Then, we can write

$$(R_w + R_n + R_1)i_1 - R_n i_2 - R_1 i_3 = 120$$

$$-R_n i_1 + (R_w + R_n + R_2)i_2 - R_2 i_3 = 120$$
$$-R_1 i_1 - R_2 i_2 + (R_1 + R_2 + R_3)i_3 = 0$$

Alternatively, because the network consists of independent voltage sources and resistances, and all of the mesh currents flow clockwise, we can enter the matrices directly into MATLAB.

```
Rw = 0.1; Rn=0.1; R1 = 20; R2 = 10; R3 = 16;
R = [Rw+Rn+R1 -Rn -R1; -Rn Rw+Rn+R2 -R2; -R1 -R2 R1+R2+R3];
V = [120; 120; 0];
I = R\V;
% Finally, we compute the voltages across the loads.
Vr1 = R1*(I(1) - I(3)), Vr2 = R2*(I(2) - I(3)), Vr3 = R3*I(3)...
% which results in:
Vr1 =
  118.5121
Vr2 =
  116.7862
Vr3 =
  235.2983
```

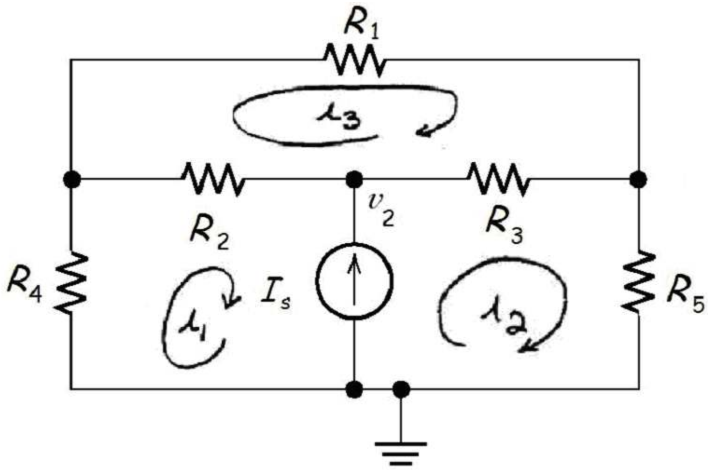
These values are within the normal range for nearly all devices.

(b) Next, we change Rn to a very high value such as 10^9 which for practical calculations is equivalent to an open circuit, and again compute the voltages resulting in:

```
Vr1 =
  156.9910
Vr2 =
   78.4955
Vr3 =
  235.4865
```

The voltage across R_1 is certainly high enough to damage most loads designed to operate at 110 to 120 V.

P2.78



Current source in terms of mesh currents: $-i_1 + i_2 = I_s$

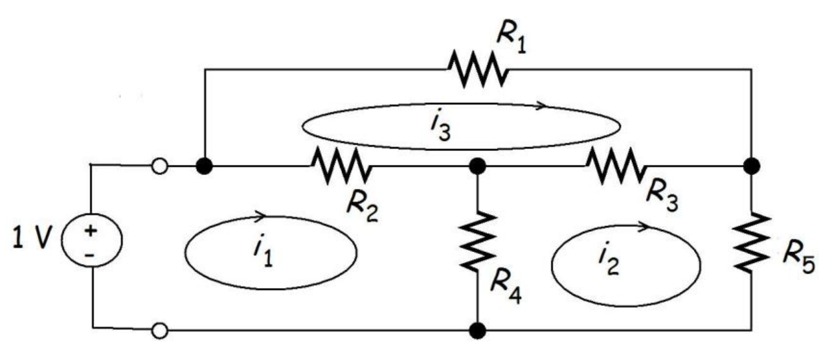
KVL for mesh 3: $\quad -R_2 i_1 - R_3 i_2 + (R_1 + R_2 + R_3) i_3 = 0$

KVL around outside of network: $R_4 i_1 + R_5 i_2 + R_1 i_3 = 0$

Then using MATLAB:

```
R1 = 4; R2 = 5; R3 = 8; R4 = 6; R5 = 8; Is = 4;
R = [-1 1 0; -R2 -R3 (R1+R2+R3); R4 R5 R1];
V = [Is; 0; 0];
I = R\V;
V2 = R5*I(2) + R3*(I(2) - I(3))...
% and the answer is:
V2 =
  26.0414
```

P2.79



$$(R_2 + R_4) i_1 - R_4 i_2 - R_2 i_3 = 1$$

$$-R_4 i_1 + (R_3 + R_4 + R_5) i_2 - R_3 i_3 = 0$$

$$-R_2 i_1 - R_3 i_2 + (R_1 + R_2 + R_3) i_3 = 0$$

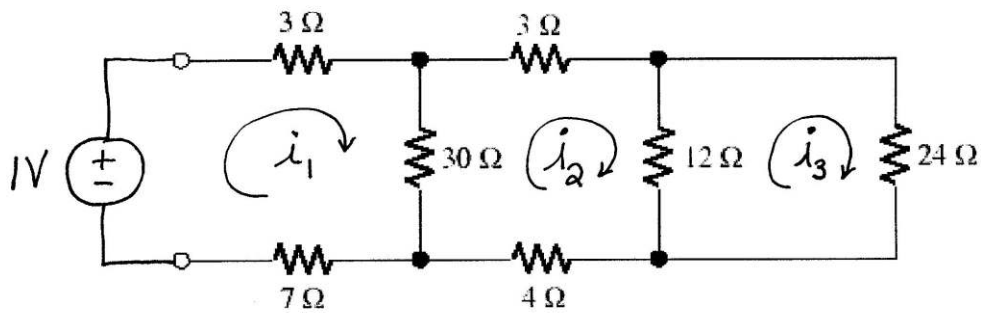
Now using MATLAB:

```
R1 = 15; R2 = 15; R3 = 15; R4 = 10; R5 = 10;
R = [(R2+R4) -R4 -R2; -R4 (R3+R4+R5) -R3; -R2 -R3 (R1+R2+R3)];
V = [1; 0; 0];
I =  R\V;
Req = 1/I(1) % Gives answer in ohms.
Req =
   12.5
```

P2.80 Mesh 1: $3i_1 + 7i_1 + 30(i_1 - i_2) = 1$

Mesh 2: $3i_2 + 12(i_2 - i_3) + 4i_2 + 30(i_2 - i_1) = 0$

Mesh 3: $24i_3 + 12(i_3 - i_2) = 0$



Solving, we find $i_1 = 0.05$ A. Then $R_{eq} = 1/i_1 = 20\ \Omega$.

P2.81 We write equations in which voltages are in volts, resistances are in kΩ, and currents are in mA.

KVL mesh 1: $4i_1 + 1(i_1 - i_2) + 3(i_1 - i_3) = 0$

KVL mesh 2: $1(i_2 - i_1) + 2(i_2 - i_4) = -20$

KVL supermesh: $2i_3 + 3(i_3 - i_1) + 2(i_4 - i_2) + 5i_4 = 0$

Current source: $i_4 - i_3 = 5$

Now, we proceed in Matlab.

```
R = [8 -1 -3 0; -1 3 0 -2; -3 -2 5 7; 0 0 -1 1];
V = [0; -20; 0; 5];
I = R\V % This yields the mesh currents in mA.
I =

  -2.7561
  -7.4878
  -4.8537
   0.1463
```

P2.82. 1. Perform two of these:

a. Determine the open-circuit voltage $V_t = v_{oc}$.

b. Determine the short-circuit current $I_n = i_{sc}$.

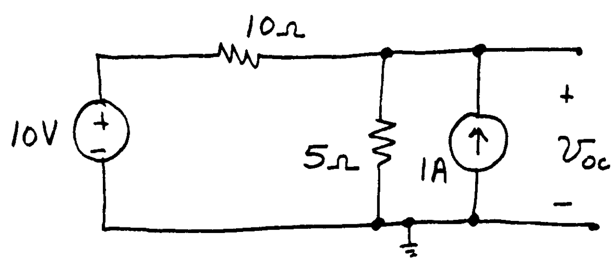
c. Zero the independent sources and find the Thévenin resistance R_t looking back into the terminals. Do not zero dependent sources.

2. Use the equation $V_t = R_t I_n$ to compute the remaining value.

3. The Thévenin equivalent consists of a voltage source V_t in series with R_t.

4. The Norton equivalent consists of a current source I_n in parallel with R_t.

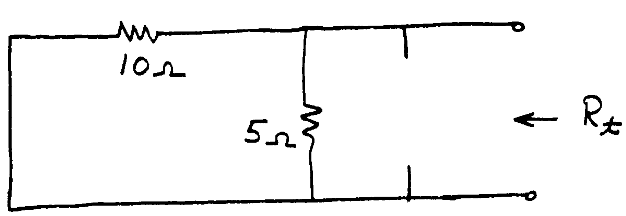
P2.83* First, we write a node voltage equation to solve for the open-circuit voltage:



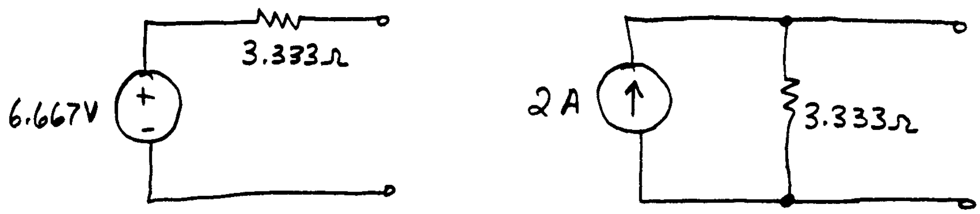
$$\frac{v_{oc} - 10}{10} + \frac{v_{oc}}{5} = 1$$

Solving, we find $v_{oc} = 6.667\text{ V}$.

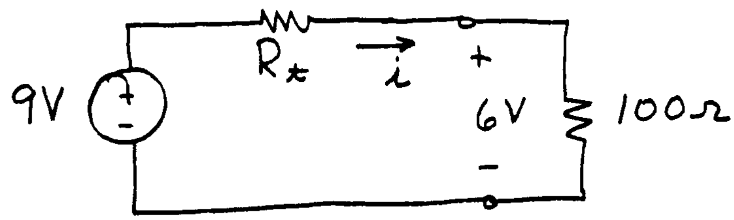
Then zeroing the sources, we have this circuit:



Thus, $R_t = \dfrac{1}{1/10 + 1/5} = 3.333\ \Omega$. The Thévenin and Norton equivalents are:

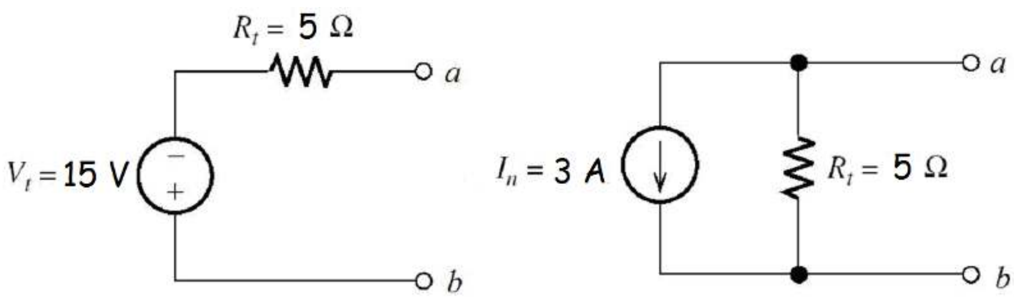


P2.84* The equivalent circuit of the battery with the resistance connected is

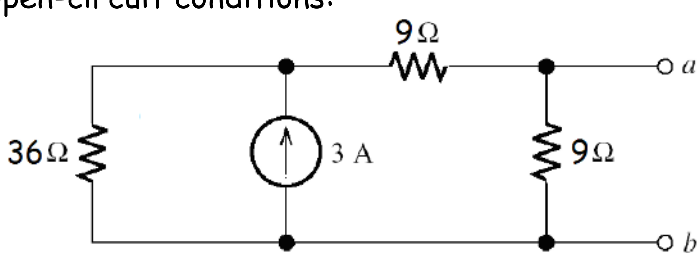


$i = 6/100 = 0.06$ A $\qquad$ $R_t = \dfrac{9-6}{0.06} = 50\ \Omega$

P2.85 The 9-Ω resistor has no effect on the equivalent circuits because the voltage across the 12-V source is independent of the resistor value.

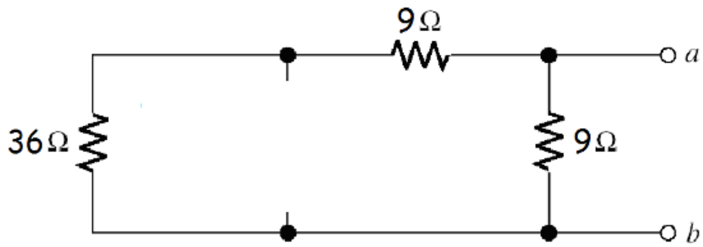


P2.86 With open-circuit conditions:



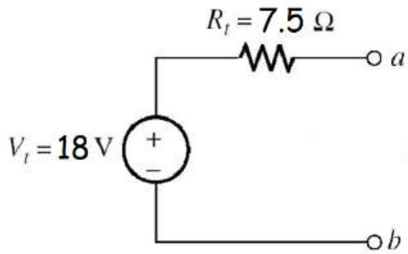
Solving, we find $v_{ab} = 18$ V.

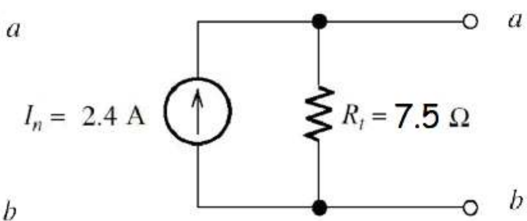
With the source zeroed:



$$R_t = \frac{1}{1/9 + 1/(9+36)} = 7.5\,\Omega$$

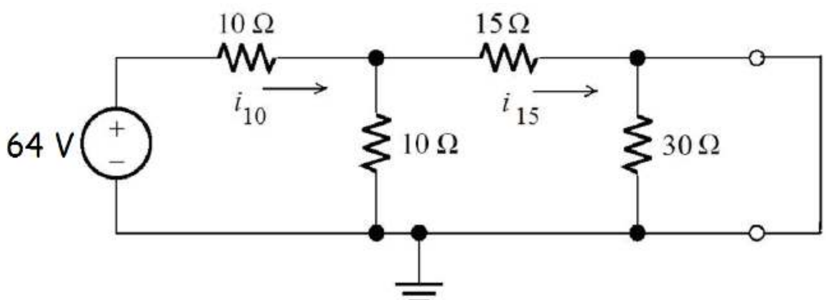
The equivalent circuits are:





Notice the source polarity relative to terminals *a* and *b*.

P2.87 First, we combine the 30-Ω resistances that are in parallel replacing them with a 15-Ω resistance. Then, we solve the network with a short circuit:



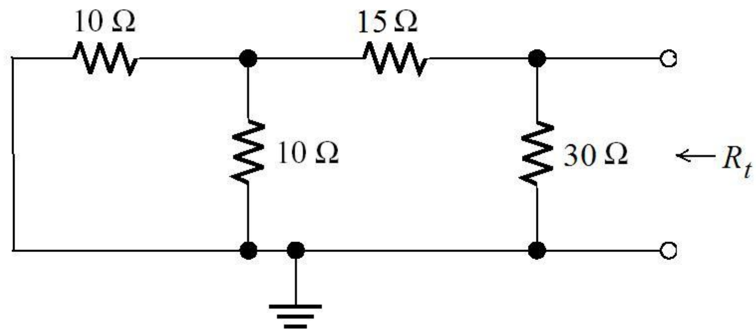
$$R_{eq} = 10 + \frac{1}{1/10 + 1/15} = 16\,\Omega$$

$$i_{10} = 64/R_{eq} = 4\ A$$

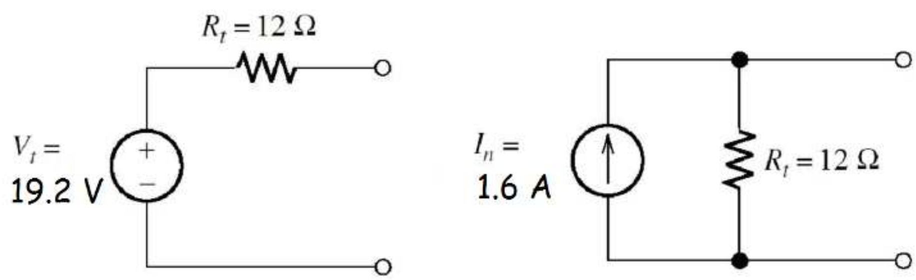
$$i_{15} = i_{10}\frac{10}{10+15} = 1.6\ \ A$$

$$i_{sc} = i_{15} = 1.6\ \ A$$

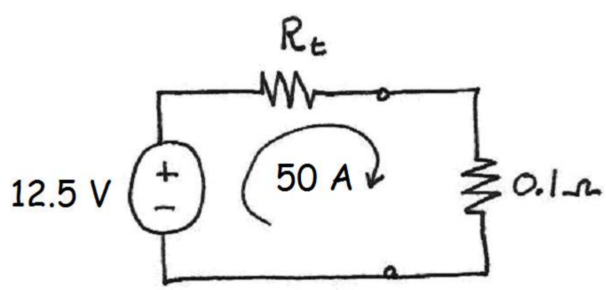
Zeroing the source, we have:



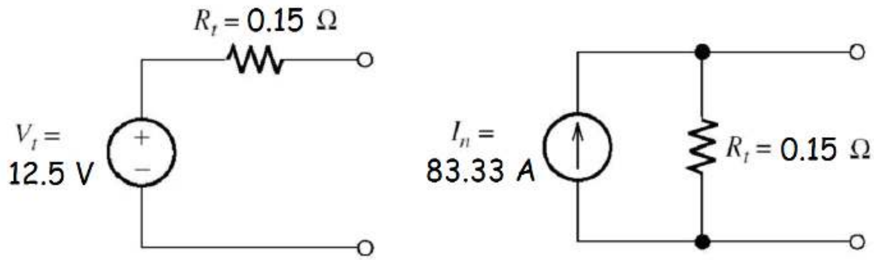
Combining resistances in series and parallel we find $R_t = 12\ \Omega$.
Then the Thévenin voltage is $v_t = i_{sc}R_t = 19.2\ \text{V}$. The Thévenin and Norton equivalents are:



P2.88 The Thévenin voltage is equal to the open-circuit voltage which is 12.5 V. The equivalent circuit with the 0.1-Ω load connected is:

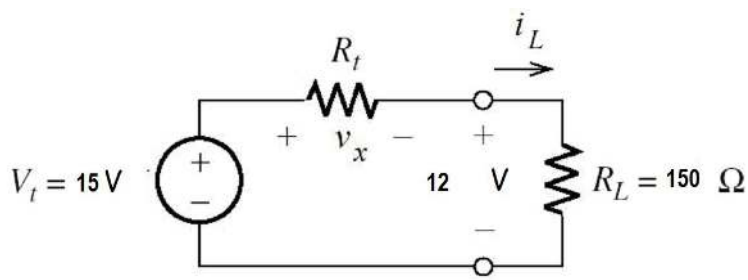


We have $12.5/(R_t + 0.1) = 500$ from which we find $R_t = 0.15\ \Omega$. The Thévenin and Norton equivalent circuits are:



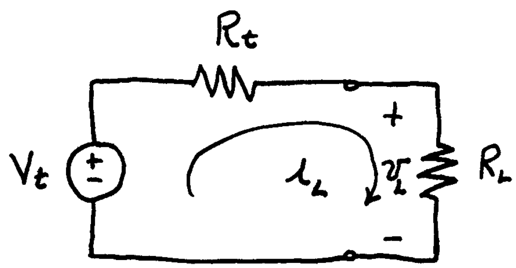
The short-circuit current is 83.33 A. Because no energy is converted from chemical form to heat in a battery under open-circuit conditions, the Thévenin equivalent seems more realistic from an energy conversion standpoint.

P2.89 The Thévenin voltage is equal to the open-circuit voltage, which is 15 V. The circuit with the load attached is:



We have $i_L = \frac{12}{150} = 80\text{ mA}$ and $v_x = V_t - 12 = 3$ V Thus, the Thévenin resistance is $R_t = \frac{3\text{ V}}{80\text{ mA}} = 37.5\ \Omega$

P2.90 The equivalent circuit with a load attached is:



For a load of 1 kΩ, we have $i_L = 8/1000 = 8\text{ mA}$, and we can write $v_L = V_t - R_t i_L$. Substituting values this becomes

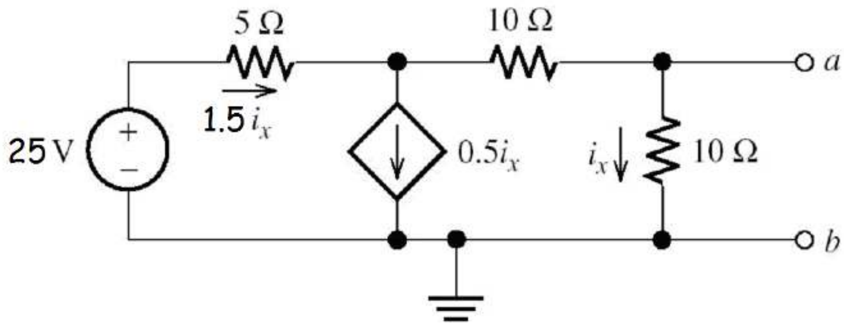
$$8 = V_t - 0.008R_t \qquad (1)$$

Similarly, for the 2-kΩ load we obtain

$$10 = V_t - 0.005R_t \quad (2)$$

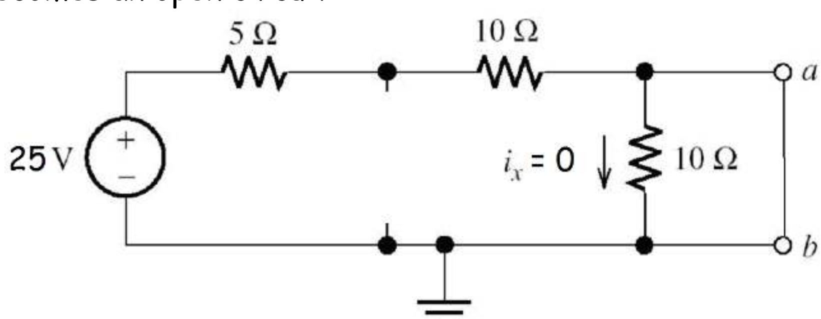
Solving Equations (1) and (2), we find $V_t = 13.33$ V and $R_t = 666.7\ \Omega$.

P2.91 Open-circuit conditions:

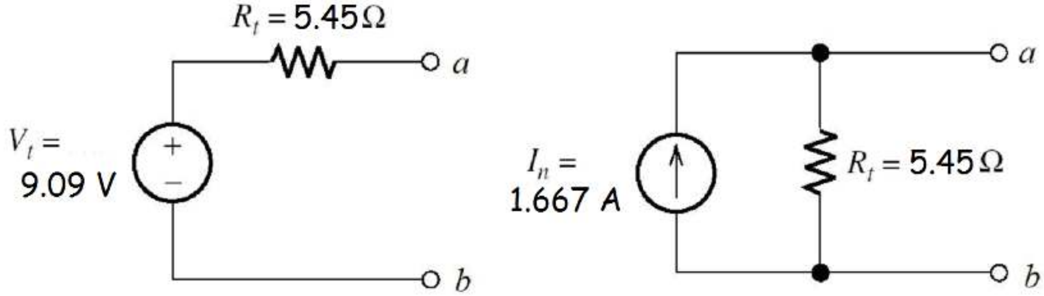


Using KVL, we have $25 = 5(1.5i_x) + 10i_x + 10i_x$ Solving, we find $i_x = 0.90909$ A and then we have $V_t = v_{oc} = 10i_x = 9.0909$ V.

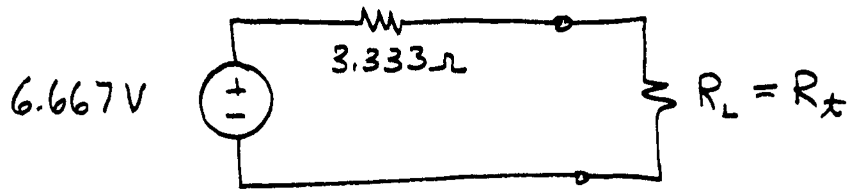
Under short-circuit conditions, we have $i_x = 0$ and the controlled source becomes an open circuit:



$i_{sc} = \dfrac{25}{15} = 1.667$ A. Then, we have $R_t = v_{oc}/i_{sc} = 5.45\ \Omega$. Thus, the equivalents are:



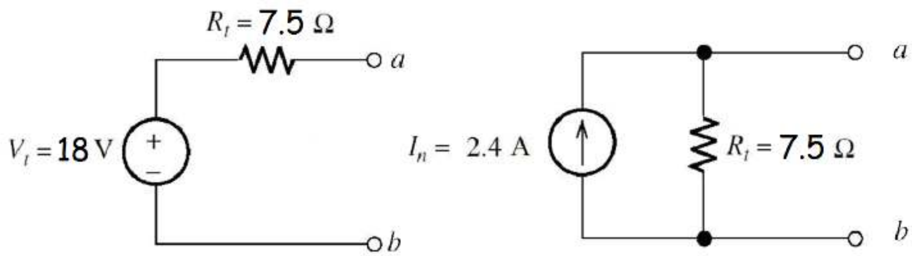
P2.92 As is Problem P2.83, we find the Thévenin equivalent:



Then maximum power is obtained for a load resistance equal to the Thévenin resistance.

$$P_{max} = \frac{(V_t/2)^2}{R_t} = 3.333 \text{ W}$$

P2.93 As in Problem P2.86, we find the Thévenin equivalent:



Then, maximum power is obtained for a load resistance equal to the Thévenin resistance.

$$P_{max} = \frac{(V_t/2)^2}{R_t} = 10.8 \text{ W}$$

P2.94 For maximum power conditions, we have $R_L = R_t$. The power taken from the voltage source is

$$P_s = \frac{(V_t)^2}{R_t + R_L} = \frac{(V_t)^2}{2R_t}$$

Then, half of V_t appears across the load and the power delivered to the load is

$$P_L = \frac{(0.5V_t)^2}{R_t}$$

Thus, the percentage of the power taken from the source that is delivered to the load is

$$\eta = \frac{P_L}{P_s} \times 100\% = 50\%$$

On the other hand, for $R_L = 9R_t$, we have

$$P_s = \frac{(V_t)^2}{R_t + R_L} = \frac{(V_t)^2}{10R_t}$$

$$P_L = \frac{(0.9V_t)^2}{9R_t}$$

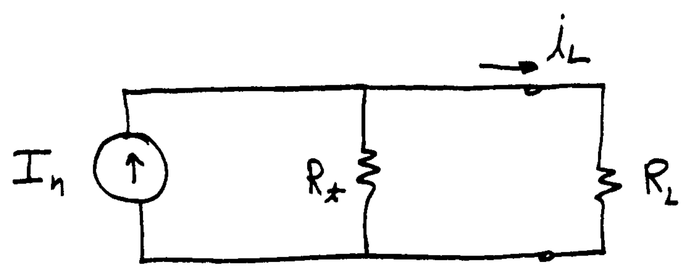
$$\eta = \frac{P_L}{P_s} \times 100\% = 90\%$$

Design for maximum power transfer is relatively inefficient. Thus, systems in which power efficiency is important are almost never designed for maximum power transfer.

P2.95* To maximize the power to R_L, we must maximize the voltage across it. Thus, we need to have $R_t = 0$. The maximum power is

$$P_{max} = \frac{20^2}{5} = 80 \text{ W}$$

P2.96 The circuit is



By the current division principle:

$$i_L = I_n \frac{R_t}{R_L + R_t}$$

The power delivered to the load is

$$P_L = (i_L)^2 R_L = (I_n)^2 \frac{(R_t)^2 R_L}{(R_L + R_t)^2}$$

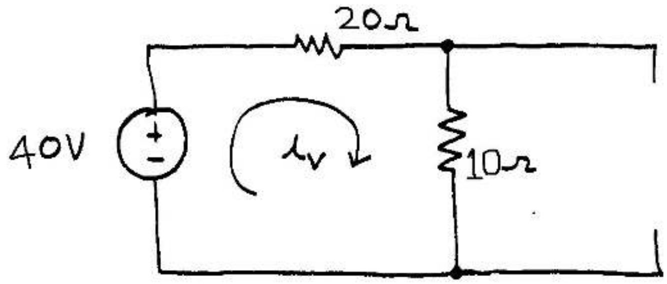
Taking the derivative and setting it equal to zero, we have

$$\frac{dP_L}{dR_L} = 0 = (I_n)^2 \frac{(R_t)^2 (R_t + R_L)^2 - 2(R_t)^2 R_L (R_t + R_L)}{(R_t + R_L)^4}$$

which yields $R_L = R_t$.

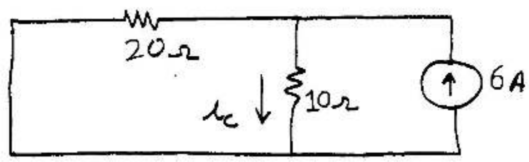
The maximum power is $P_{L\max} = (I_n)^2 R_t/4$.

P2.97* First, we zero the current source and find the current due to the voltage source.



$i_v = 40/30 = 1.33\ \Omega$

Then, we zero the voltage source and use the current-division principle to find the current due to the current source.

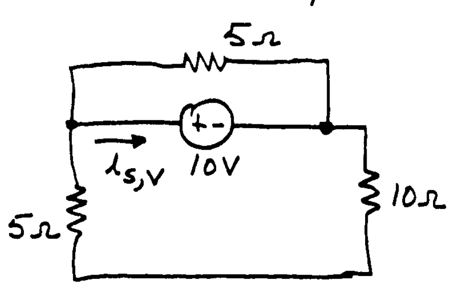


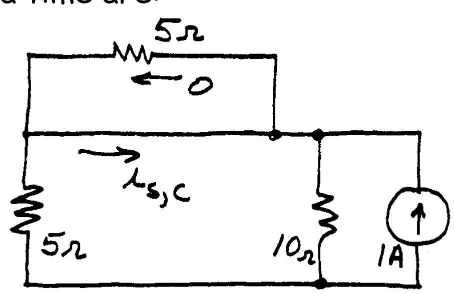
$$i_c = 6\frac{20}{10+20} = 4\ A$$

Finally, the total current is the sum of the contributions from each source.

$$i = i_v + i_c = 5.33\ A$$

P2.98* The circuits with only one source active at a time are:





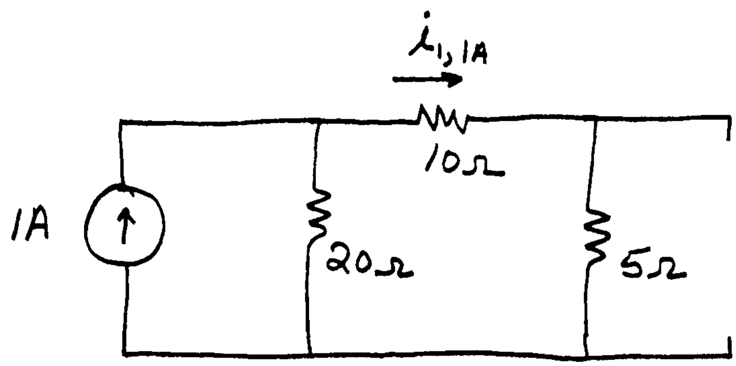
$$R_{eq} = \frac{1}{1/5 + 1/15} = 3.75\ \Omega$$

$$i_{s,c} = -1\frac{10}{10+5} = -0.667\ A$$

$$i_{s,v} = -\frac{10\text{ V}}{R_{eq}} = -2.667\text{ A}$$

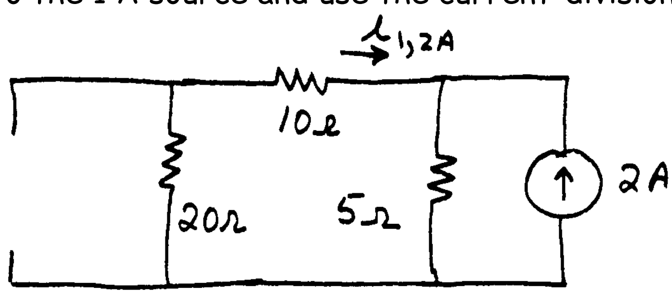
Then the total current due to both sources is $i_s = i_{s,v} + i_{s,c} = -3.333\text{ A}$.

P2.99 Zero the 2-A source and use the current-division principle:



$$i_{1,1A} = 1\frac{20}{15+20} = 0.5714\text{ A}$$

Then zero the 1 A source and use the current-division principle:

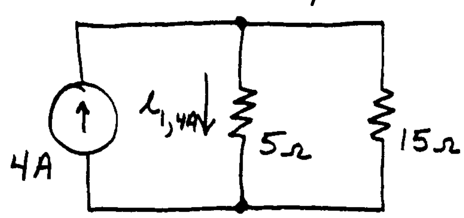


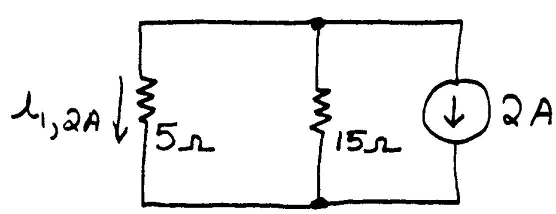
$$i_{1,2A} = -2\frac{5}{5+30} = -0.2857\text{ A}$$

Finally,

$$i_1 = i_{1,1A} + i_{1,2A} = 0.2857\text{ A}$$

P2.100 The circuits with only one source active at a time are:



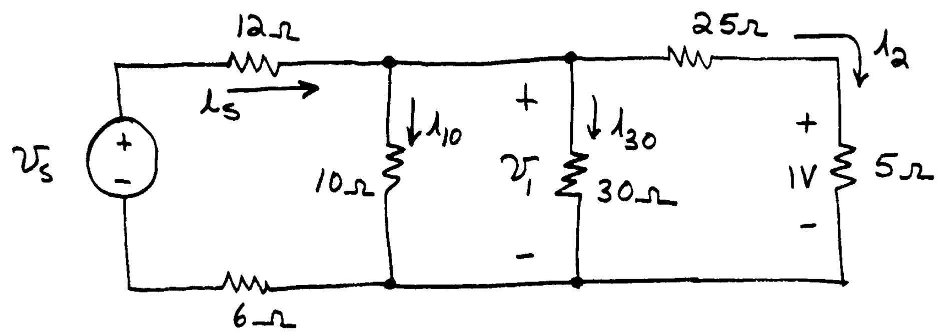


$$i_{1.4A} = 4 \times \frac{15}{15+5} = 3\ A \qquad i_{1,2A} = -2 \times \frac{15}{15+5} = -1.5$$

Finally, we add the components to find the current with both sources active.

$$i_1 = i_{1,4A} + i_{1,2A} = 1.5\ A$$

P2.101 The circuit, assuming that $v_2 = 1\ \text{V}$ is:

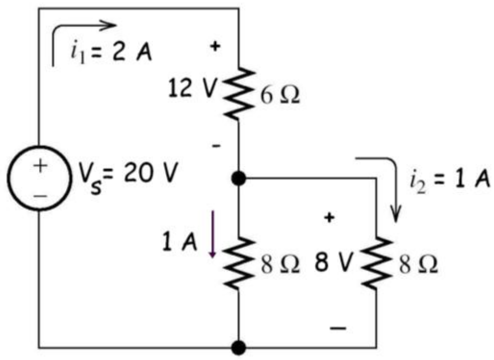


$$i_2 = (v_2/5) = 0.2\ A$$
$$v_1 = 30 i_2 = 6\ \text{V}$$
$$i_{10} = v_1/10 = 0.6\ A$$
$$i_{30} = v_1/30 = 0.2\ A$$
$$i_s = i_2 + i_{10} + i_{30} = 1\ A$$
$$v_s = 12 i_s + v_1 + 6 i_s = 24\ \text{V}$$

We have established that for $v_s = 24\ \text{V}$, we have $v_2 = 1\ \text{V}$. Thus, for $v_s = 12\ \text{V}$, we have:

$$v_2 = 1 \times \frac{12}{24} = 0.5\ \text{V}$$

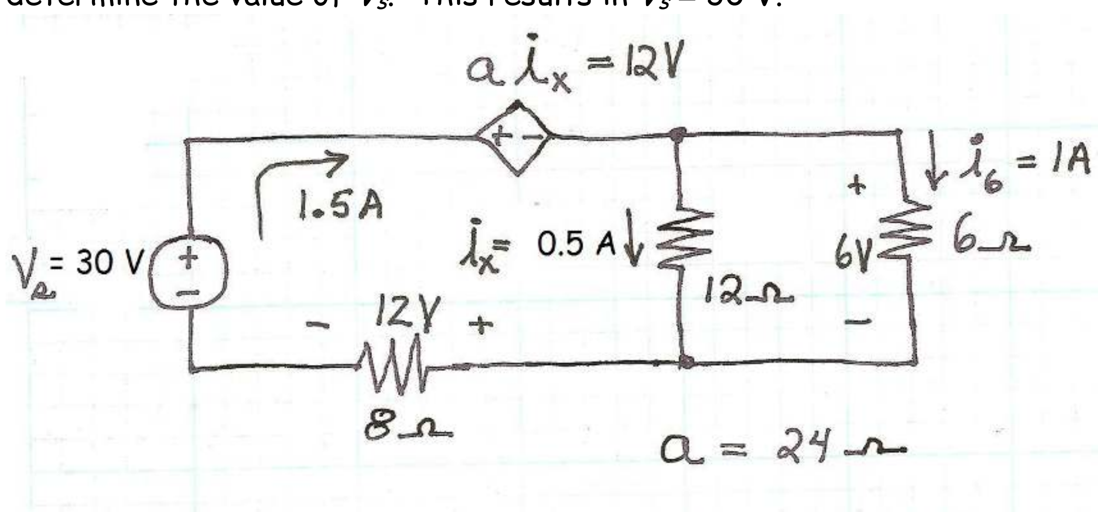
P2.102 We start by assuming $i_2 = 1\ A$ and work back through the circuit to determine the value of v_s. The results are shown on the circuit diagram.



However, the circuit actually has v_s = 10 V, so the actual value of i_2 is

$$\frac{10}{20} \times (1\ A) = 0.5\ A.$$

P2.103 We start by assuming $i_6 = 1$ A and work back through the circuit to determine the value of V_s. This results in $V_s = 30$ V.



However, the circuit actually has V_s = 10 V, so the actual value of i_6 is

$$\frac{10}{30} \times (1\ A) = 0.3333\ A.$$

P2.104 (a) With only the 2-A source activated, we have

$$i_2 = 2 \text{ and } v_2 = 2(i_2)^3 = 16 \text{ V}$$

(b) With only the 1-A source activated, we have

$$i_1 = -1 \text{ A and } v_1 = 2(i_1)^3 = -2 \text{ V}$$

(c) With both sources activated, we have

$i = 1\text{ A}$ and $v = 2(i)^3 = 2\text{ V}$

Notice that $i \neq i_1 + i_2$. Superposition does not apply because device A has a nonlinear relationship between v and i.

P2.105 From Equation 2.91, we have

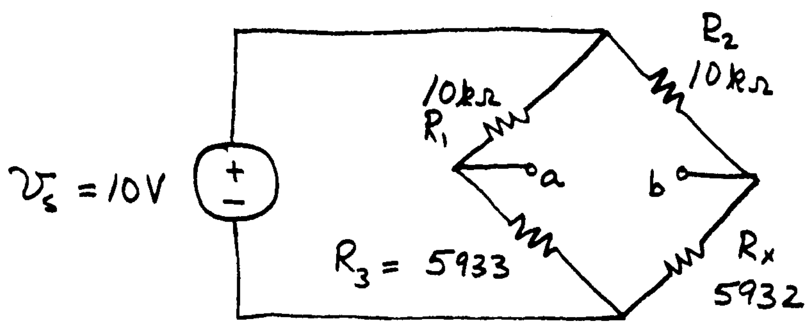
(a) $$R_x = \frac{R_2}{R_1}R_3 = \frac{1\text{ k}\Omega}{1\text{ k}\Omega} \times 3419 = 3419\ \Omega$$

(b) $$R_x = \frac{R_2}{R_1}R_3 = \frac{100\text{ k}\Omega}{10\text{ k}\Omega} \times 3419 = 34.19\text{ k}\Omega$$

P2.106* (a) Rearranging Equation 2.91, we have

$$R_3 = \frac{R_1}{R_2}R_x = \frac{10^4}{10^4} \times 5932 = 5932\ \Omega$$

(b) The circuit is:



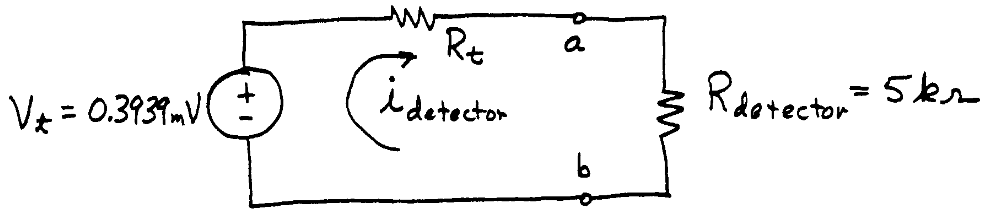
The Thévenin resistance is

$$R_t = \frac{1}{1/R_3 + 1/R_1} + \frac{1}{1/R_2 + 1/R_x} = 7447\ \Omega$$

The Thévenin voltage is

$$v_t = v_s\frac{R_3}{R_1 + R_3} - v_s\frac{R_x}{R_x + R_2}$$
$$= 0.3939\text{ mV}$$

Thus, the equivalent circuit is:



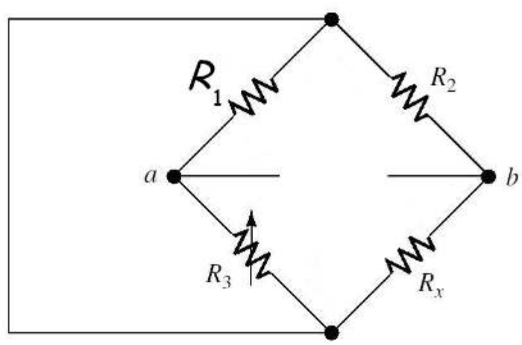
$$i_{\text{detector}} = \frac{V_t}{R_t + R_{\text{detector}}} = 31.65 \times 10^{-9}\ A$$

Thus, the detector must be sensitive to very small currents if the bridge is to be accurately balanced.

P2.107 If R_1 and R_3 are too small, large currents are drawn from the source. If the source were a battery, it would need to be replaced frequently. Large power dissipation could occur, leading to heating of the components and inaccuracy due to changes in resistance values with temperature.

If R_1 and R_3 are too large, we would have very small detector current when the bridge is not balanced, and it would be difficult to balance the bridge accurately.

P2.108 With the source replaced by a short circuit and the detector removed, the Wheatstone bridge circuit becomes



The Thévenin resistance seen looking back into the detector terminals is

$$R_t = \frac{1}{1/R_3 + 1/R_1} + \frac{1}{1/R_2 + 1/R_x}$$

The Thévenin voltage is zero when the bridge is balanced.

P2.109 Using the voltage-division principle, the voltage at node *a* is

$$v_a = V_s \frac{R_0 + \Delta R}{R_0 + \Delta R + R_0 - \Delta R} = V_s \frac{R_0 + \Delta R}{2R_0}$$

Similarly at node *b*, we have

$$v_b = V_s \frac{R_0 - \Delta R}{2R_0}$$

Then, the output voltage is

$$v_o = v_{ab} = v_a - v_b = V_s \frac{\Delta R}{R_0}$$

Finally using Equation 2.92 to substitute for ΔR, we have

$$v_o = V_s G \frac{\Delta L}{L}$$

P2.110 Before strain is applied, the resistance is

$$R_0 = \frac{\rho L}{A}$$

After strain is applied, the length becomes $L + \Delta L = L(1 + \Delta L / L)$, and the cross sectional area becomes $A /((1 + \Delta L / L)$ so the volume is constant. Thus, the resistance becomes

$$R_0 + \Delta R = R_0(1 + \frac{\Delta R}{R_0}) = \frac{\rho L(1 + \Delta L / L)}{A/(1 + \Delta L / L)} = R_0(1 + \Delta L / L)^2$$

$$R_0(1 + \frac{\Delta R}{R_0}) = R_0(1 + 2\Delta L / L + (\Delta L / L)^2)$$

However, we have $\Delta L / L << 1$ so we can neglect the $(\Delta L / L)^2$ term to a good approximation. This results in

$$R_0(1 + \frac{\Delta R}{R_0}) \cong R_0(1 + 2\Delta L / L)$$

$$G = \frac{\Delta R / R_0}{\Delta L / L} \cong 2$$

P2.111 In this case, the bridge would be balanced for any value of ΔR and the output voltage v_o would be zero regardless of the strain.

Practice Test

T2.1 (a) 6, (b) 10, (c) 2, (d) 7, (e) 10 or 13 (perhaps 13 is the better answer), (f) 1 or 4 (perhaps 4 is the better answer), (g) 11, (h) 3, (i) 8, (j) 15, (k) 17, (l) 14.

T2.2 The equivalent resistance seen by the voltage source is:

$$R_{eq} = R_1 + \frac{1}{1/R_2 + 1/R_3 + 1/R_4} = 16\,\Omega$$

$$i_s = \frac{v_s}{R_{eq}} = 6\ A$$

Then, using the current division principle, we have

$$i_4 = \frac{G_4}{G_2 + G_3 + G_4} i_s = \frac{1/60}{1/48 + 1/16 + 1/60} 6 = 1\ \ A$$

T2.3 Writing KCL equations at each node gives

$$\frac{v_1}{4} + \frac{v_1 - v_2}{5} + \frac{v_1 - v_3}{2} = 0$$

$$\frac{v_2 - v_1}{5} + \frac{v_2}{10} = 2$$

$$\frac{v_3}{1} + \frac{v_3 - v_1}{2} = -2$$

In standard form, we have:

$$0.95v_1 - 0.20v_2 - 0.50v_3 = 0$$

$$-0.20v_1 + 0.30v_2 = 2$$

$$-0.50v_1 + 1.50v_3 = -2$$

In matrix form, we have

$$\mathbf{GV} = \mathbf{I}$$

$$\begin{bmatrix} 0.95 & -0.20 & -0.50 \\ -0.20 & 0.30 & 0 \\ -0.50 & 0 & 1.50 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 2 \\ -2 \end{bmatrix}$$

The MATLAB commands needed to obtain the column vector of the node voltages are

```
G = [0.95 -0.20 -0.50; -0.20 0.30 0; -0.50 0 1.50]
I = [0; 2; -2]
V = G\I      % As an alternative we could use V = inv(G)*I
```

Actually, because the circuit contains only resistances and independent current sources, we could have used the short-cut method to obtain the **G** and **I** matrices.

T2.4 We can write the following equations:

KVL mesh 1: $R_1 i_1 - V_s + R_3(i_1 - i_3) + R_2(i_1 - i_2) = 0$

KVL for the supermesh obtained by combining meshes 2 and 3:

$$R_4 i_2 + R_2(i_2 - i_1) + R_3(i_3 - i_1) + R_5 i_3 = 0$$

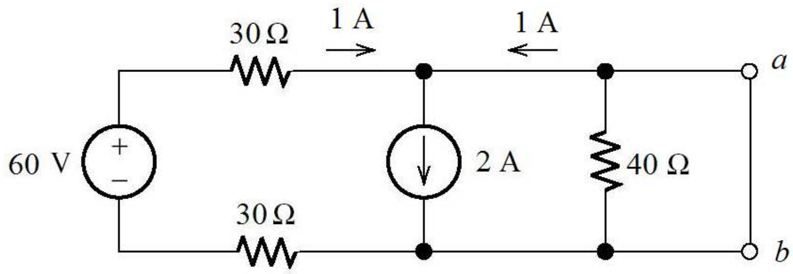
KVL around the periphery of the circuit:

$$R_1 i_1 - V_s + R_4 i_2 + R_5 i_3 = 0$$

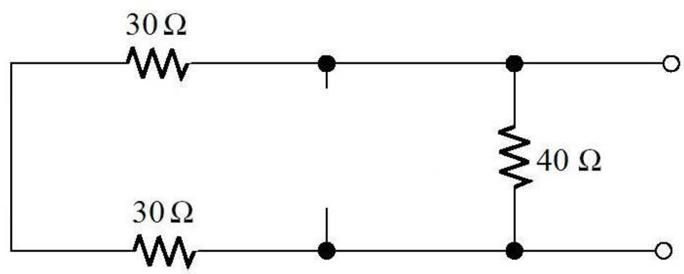
Current source: $i_2 - i_3 = I_s$

A set of equations for solving the network must include the current source equation plus two of the mesh equations. The three mesh equations are dependent and will not provide a solution by themselves.

T2.5 Under short-circuit conditions, the circuit becomes



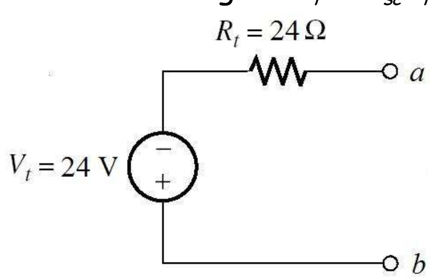
Thus, the short-circuit current is 1 A flowing out of *b* and into *a*. Zeroing the sources, we have

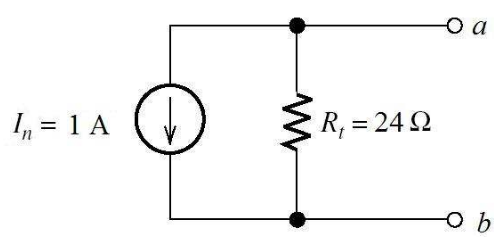


Thus, the Thévenin resistance is

$$R_t = \frac{1}{1/40 + 1/(30+30)} = 24\ \Omega$$

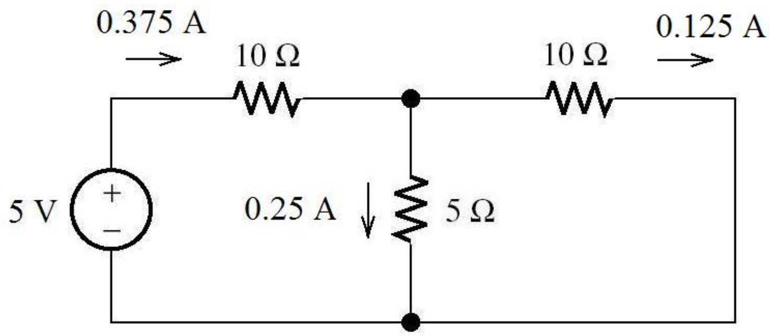
and the Thévenin voltage is $V_t = I_{sc} R_t = 24\ \text{V}$. The equivalent circuits are:

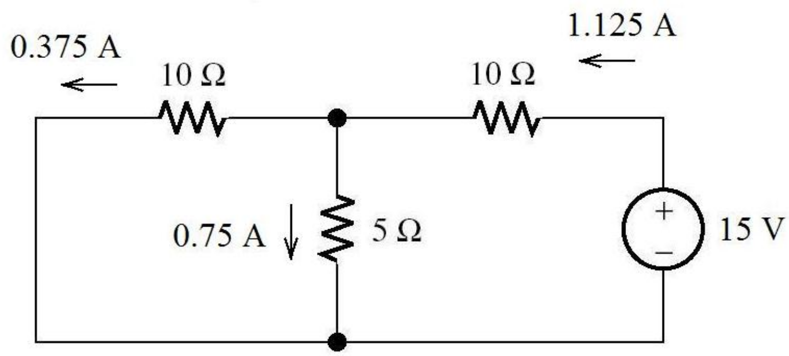




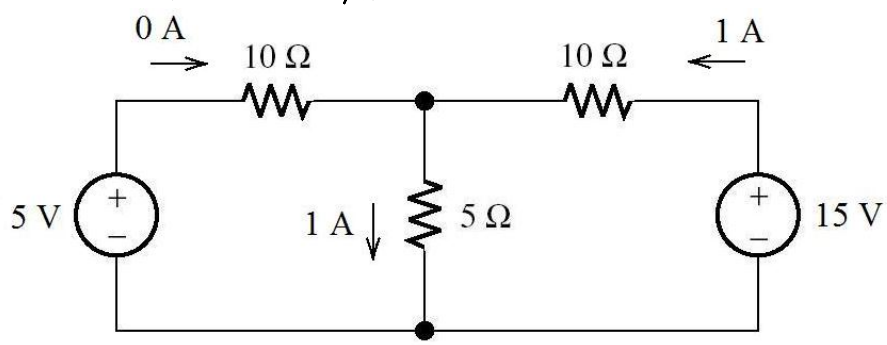
Because the short-circuit current flows out of terminal *b*, we have oriented the voltage polarity positive toward *b* and pointed the current source reference toward *b*.

T2.6 With one source active at a time, we have





Then, with both sources active, we have



We see that the 5-V source produces 25% of the total current through the 5-Ω resistance. However, the power produced by the 5-V source with both sources active is zero. Thus, the 5-V source produces 0% of the power delivered to the 5-Ω resistance. Strange, but true! Because power is a nonlinear function of current (i.e., $P = Ri^2$), the superposition principle does not apply to power.

CHAPTER 3

Exercises

E3.1 $v(t) = q(t)/C = 10^{-6}\sin(10^5 t)/(2\times 10^{-6}) = 0.5\sin(10^5 t)$ V

$$i(t) = C\frac{dv}{dt} = (2\times 10^{-6})(0.5\times 10^5)\cos(10^5 t) = 0.1\cos(10^5 t)\ A$$

E3.2 Because the capacitor voltage is zero at $t = 0$, the charge on the capacitor is zero at $t = 0$.

$$\begin{aligned} q(t) &= \int_0^t i(x)dx + 0 \\ &= \int_0^t 10^{-3}dx = 10^{-3}t \ \text{ for } 0 \le t \le 2\text{ ms} \\ &= \int_0^{2E-3} 10^{-3}dx + \int_{2E-3}^t -10^{-3}dx = 4\times 10^{-6} - 10^{-3}t \ \text{ for } 2\text{ ms} \le t \le 4\text{ ms} \end{aligned}$$

$$\begin{aligned} v(t) &= q(t)/C \\ &= 10^4 t \text{ for } 0 \le t \le 2\text{ ms} \\ &= 40 - 10^4 t \ \text{ for } 2\text{ ms} \le t \le 4\text{ ms} \end{aligned}$$

$$\begin{aligned} p(t) &= i(t)v(t) \\ &= 10t \text{ for } 0 \le t \le 2\text{ ms} \\ &= -40\times 10^{-3} + 10t \ \text{ for } 2\text{ ms} \le t \le 4\text{ ms} \end{aligned}$$

$$\begin{aligned} w(t) &= Cv^2(t)/2 \\ &= 5t^2 \quad \text{for } 0 \le t \le 2\text{ ms} \\ &= 0.5\times 10^{-7}(40 - 10^4 t)^2 \text{ for } 2\text{ ms} \le t \le 4\text{ ms} \end{aligned}$$

in which the units of charge, electrical potential, power, and energy are coulombs, volts, watts and joules, respectively. Plots of these quantities are shown in Figure 3.8 in the book.

E3.3 Refer to Figure 3.10 in the book. Applying KVL, we have

$$v = v_1 + v_2 + v_3$$

Then using Equation 3.8 to substitute for the voltages we have

$$v(t) = \frac{1}{C_1}\int_0^t i(t)dt + v_1(0) + \frac{1}{C_2}\int_0^t i(t)dt + v_2(0) + \frac{1}{C_3}\int_0^t i(t)dt + v_3(0)$$

This can be written as

$$v(t) = \left(\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}\right)\int_0^t i(t)dt + v_1(0) + v_2(0) + v_3(0) \qquad (1)$$

Now if we define

$$\frac{1}{C_{eq}} = \left(\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}\right) \quad \text{and} \quad v(0) = v_1(0) + v_2(0) + v_3(0)$$

we can write Equation (1) as

$$v(t) = \frac{1}{C_{eq}}\int_0^t i(t)dt + v(0)$$

Thus the three capacitances in series have an equivalent capacitance given by Equation 3.25 in the book.

E3.4 (a) For series capacitances:

$$C_{eq} = \frac{1}{1/C_1 + 1/C_2} = \frac{1}{1/2 + 1/1} = 2/3\ \mu\text{F}$$

(b) For parallel capacitances:

$$C_{eq} = C_1 + C_2 = 1 + 2 = 3\ \mu\text{F}$$

E3.5 From Table 3.1 we find that the relative dielectric constant of polyester is 3.4. We solve Equation 3.26 for the area of each sheet:

$$A = \frac{Cd}{\varepsilon} = \frac{Cd}{\varepsilon_r\varepsilon_0} = \frac{10^{-6} \times 15 \times 10^{-6}}{3.4 \times 8.85 \times 10^{-12}} = 0.4985\ \text{m}^2$$

Then the length of the strip is

$$L = A/W = 0.4985/(2 \times 10^{-2}) = 24.93\ \text{m}$$

E3.6 $$v(t) = L\frac{di(t)}{dt} = (10 \times 10^{-3})\frac{d}{dt}\left[0.1\cos\left(10^4 t\right)\right] = -10\sin\left(10^4 t\right)\ \text{V}$$

$$w(t) = \tfrac{1}{2}Li^2(t) = 5 \times 10^{-3} \times \left[0.1\cos\left(10^4 t\right)\right]^2 = 50 \times 10^{-6}\cos^2\left(10^4 t\right)\ \text{J}$$

E3.7

$$i(t) = \frac{1}{L}\int_0^t v(x)dx + i(0) = \frac{1}{150 \times 10^{-6}}\int_0^t v(x)dx$$

$$= 6667\int_0^t 7.5 \times 10^6\, x dx = 25 \times 10^9 t^2 \text{ V for } 0 \le t \le 2\ \mu s$$

$$= 6667\int_0^{2E-6} 7.5 \times 10^6\, x dx = 0.1\text{ V for } 2\mu s \le t \le 4\ \mu s$$

$$= 6667\left(\int_0^{2E-6} 7.5 \times 10^6\, x dx + \int_{4E-6}^{t}(-15)dx\right) = 0.5 - 10^5 t\ \text{ V for } 4\mu s \le t \le 5\ \mu s$$

A plot of $i(t)$ versus t is shown in Figure 3.19b in the book.

E3.8 Refer to Figure 3.20a in the book. Using KVL we can write:

$$v(t) = v_1(t) + v_2(t) + v_3(t)$$

Using Equation 3.28 to substitute, this becomes

$$v(t) = L_1\frac{di(t)}{dt} + L_2\frac{di(t)}{dt} + L_3\frac{di(t)}{dt} \qquad (1)$$

Then if we define $L_{eq} = L_1 + L_2 + L_3$, Equation (1) becomes:

$$v(t) = L_{eq}\frac{di(t)}{dt}$$

which shows that the series combination of the three inductances has the same terminal equation as the equivalent inductance.

E3.9 Refer to Figure 3.20b in the book. Using KCL we can write:

$$i(t) = i_1(t) + i_2(t) + i_3(t)$$

Using Equation 3.32 to substitute, this becomes

$$i(t) = \frac{1}{L_1}\int_0^t v(t)dt + i_1(0) + \frac{1}{L_2}\int_0^t v(t)dt + i_2(0) + \frac{1}{L_3}\int_0^t v(t)dt + i_3(0)$$

This can be written as

$$v(t) = \left(\frac{1}{L_1} + \frac{1}{L_2} + \frac{1}{L_3}\right)\int_0^t v(t)dt + i_1(0) + i_2(0) + i_3(0) \qquad (1)$$

Now if we define

$$\frac{1}{L_{eq}} = \left(\frac{1}{L_1} + \frac{1}{L_2} + \frac{1}{L_3}\right) \quad \text{and} \quad i(0) = i_1(0) + i_2(0) + i_3(0)$$

we can write Equation (1) as

$$i(t) = \frac{1}{L_{eq}} \int_0^t v(t)dt + i(0)$$

Thus, the three inductances in parallel have the equivalent inductance shown in Figure 3.20b in the book.

E3.10 Refer to Figure 3.21 in the book.

(a) The 2-H and 3-H inductances are in series and are equivalent to a 5-H inductance, which in turn is in parallel with the other 5-H inductance. This combination has an equivalent inductance of 1/(1/5 + 1/5) = 2.5 H. Finally the 1-H inductance is in series with the combination of the other inductances so the equivalent inductance is 1 + 2.5 = 3.5 H.

(b) The 2-H and 3-H inductances are in series and have an equivalent inductance of 5 H. This equivalent inductance is in parallel with both the 5-H and 4-H inductances. The equivalent inductance of the parallel combination is 1/(1/5 + 1/4 + 1/5) = 1.538 H. This combination is in series with the 1-H and 6-H inductances so the overall equivalent inductance is 1.538 + 1 + 6 = 8.538 H.

E3.11 The MATLAB commands including some explanatory comments are:

```
% We avoid using i alone as a symbol for current because
% we reserve i for the square root of -1 in MATLAB. Thus, we
% will use iC for the capacitor current.
syms t iC qC vC % Define t, iC, qC and vC as symbolic objects.
iC = 0.5*sin((1e4)*t);
ezplot(iC, [0 3*pi*1e-4])
qC=int(iC,t,0,t); % qC equals the integral of iC.
figure % Plot the charge in a new window.
ezplot(qC, [0 3*pi*1e-4])
vC = 1e7*qC;
figure % Plot the voltage in a new window.
ezplot(vC, [0 3*pi*1e-4])
```

The plots are very similar to those of Figure 3.5 in the book. An m-file (named Exercise_3_11) containing these commands can be found in the MATLAB folder on the OrCAD disk.

Problems

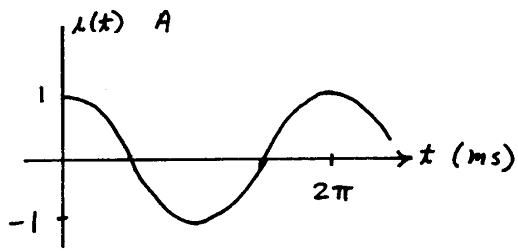
P3.1 Capacitors consist of two conductors separated by an insulating material. Frequently, the conductors are sheets of metal that are separated by a thin layer of the insulating material.

P3.2 Because we have $i = C dv / dt$ for a capacitance, the current is zero if the voltage is constant. Thus, we say that capacitances act as open circuits for constant (dc) voltages.

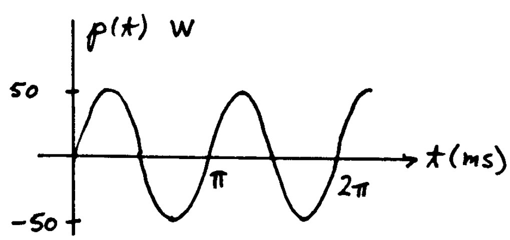
P3.3 A dielectric material acts as an insulator and allows no current flow. It allows storage of energy by creating an electrostatic field. Examples include air, Mylar and mica. Permittivity is the property that determines the capacitance offered by the material for given physical dimensions.

P3.4 Charge (usually in the form of electrons) flows in and accumulates on one plate. Meanwhile, an equal amount of charge flows out of the other plate. Thus, current seems to flow through a capacitor.

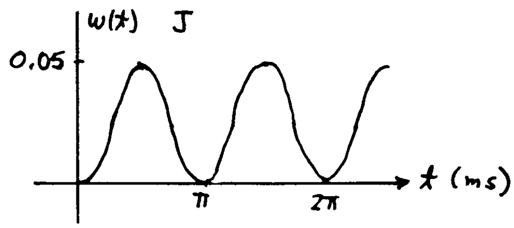
P3.5* $i(t) = C\dfrac{dv}{dt} = 10^{-5}\dfrac{d}{dt}(100\sin 1000t) = \cos(1000t)$



$p(t) = v(t)i(t) = 100\cos(1000t)\sin(1000t) = 50\sin(2000t)$



$w(t) = \dfrac{1}{2}C[v(t)]^2 = 0.05\sin^2(1000t)$



P3.6* $i = C\dfrac{dv}{dt}$

$$\frac{dv}{dt} = \frac{i}{C} = \frac{100 \times 10^{-6}}{10000 \times 10^{-9}} = 10 \text{ V/s} \qquad \Delta t = \frac{\Delta v}{dv/dt} = \frac{10}{10} = 1 \text{ s}$$

P3.7* $v(t) = \dfrac{1}{C}\displaystyle\int_0^t i(t)dt + v(0) \qquad v(t) = 2 \times 10^4 \int_0^t 3 \times 10^{-3} dt - 20$

$v(t) = 60t - 20 \text{ V}$

$p(t) = i(t)v(t) = 3 \times 10^{-3}(60t - 20) \text{ W}$

Evaluating at $t = 0$, we have $p(0) = -60$ mW. Because the power has a negative value, the capacitor is delivering energy.
At $t = 1$ s, we have $p(1) = 120$ mW. Because the power is positive, we know that the capacitor is absorbing energy.

P3.8* $W = \text{power} \times \text{time} = 5 \text{ hp} \times 746 \text{ W/hp} \times 3600 \text{ s}$
$= 13.4 \times 10^6$ J

$$V = \sqrt{\frac{2W}{C}} = \sqrt{\frac{2 \times 13.4 \times 10^6}{0.01}} = 51.8 \text{ kV}$$

It turns out that a 0.01-F capacitor rated for this voltage would be much too large and massive for powering an automobile. Besides, to have reasonable performance, an automobile would need much more than 5 hp for an hour.

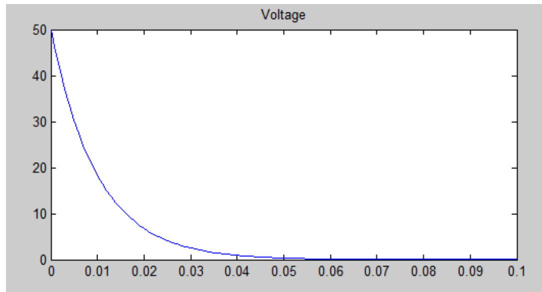
P3.9 The net charge on each plate is $Q = CV = (5 \times 10^{-6}) \times 100 = 500\ \mu C$. One plate has a net positive charge and the other has a net negative charge so the net charge for both plates is zero.

P3.10 $v(t) = \frac{1}{C}\int_0^t i(t)dt + v(0) = \frac{1}{C}\int_0^t I_m \cos(\omega t)dt = \frac{I_m}{\omega C}[\sin(\omega t) - \sin(0)]$

$$= \frac{I_m}{\omega C}\sin(\omega t)$$

Clearly for $\omega \to \infty$, the voltage becomes zero, so the capacitance becomes the equivalent of a short circuit.

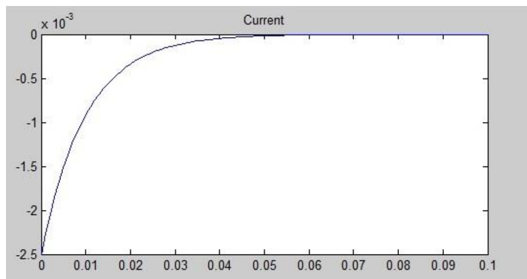
P3.11



$$i(t) = C\frac{dv}{dt}$$

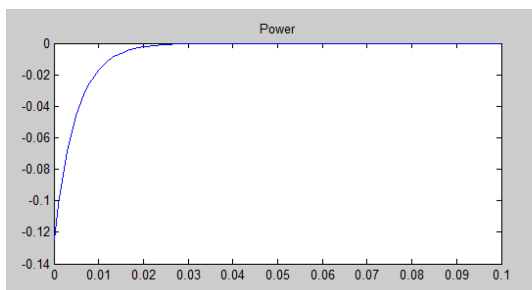
$$= 5 \times 10^{-6}\frac{d}{dt}\left(50e^{-100t}\right)$$

$$= -0.025e^{-100t}\text{ A}$$



$$p(t) = v(t)i(t)$$

$$= -1.25e^{-200t}\text{ W}$$



$$w(t) = \frac{1}{2}C[v(t)]^2$$

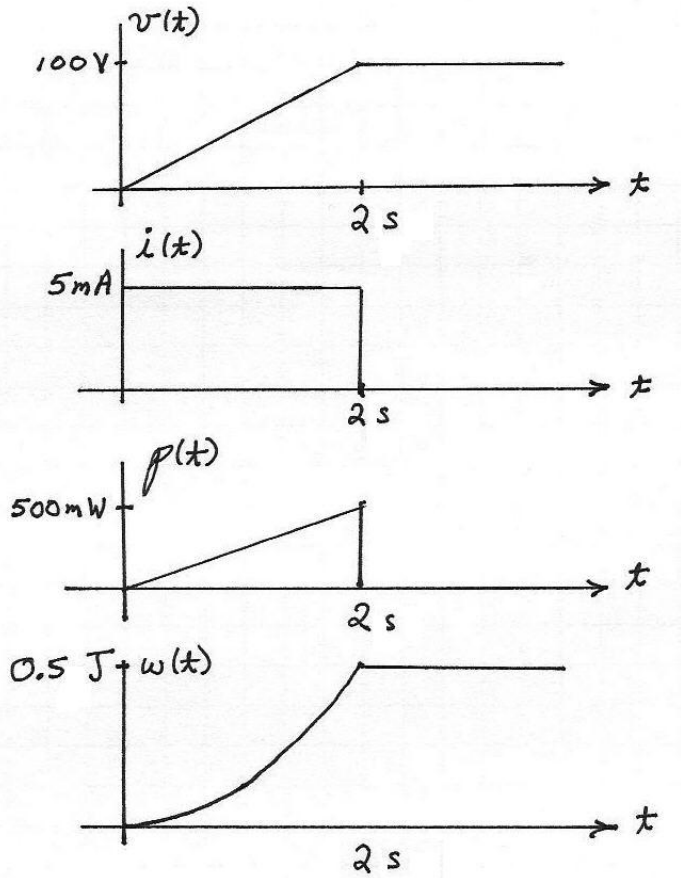
$$= 6.25 \times 10^{-3} \times e^{-200t}\text{ J}$$

P3.12 $Q = Cv = 5 \times 10^{-6} \times 200 = 1\text{ mC}$

$$W = \frac{1}{2}Cv^2 = \frac{1}{2} \times 5 \times 10^{-6} \times (200)^2 = 0.1\text{ J}$$

$$P = \frac{\Delta W}{\Delta t} = \frac{0.1}{10^{-6}} = 100\text{ kW}$$

P3.13

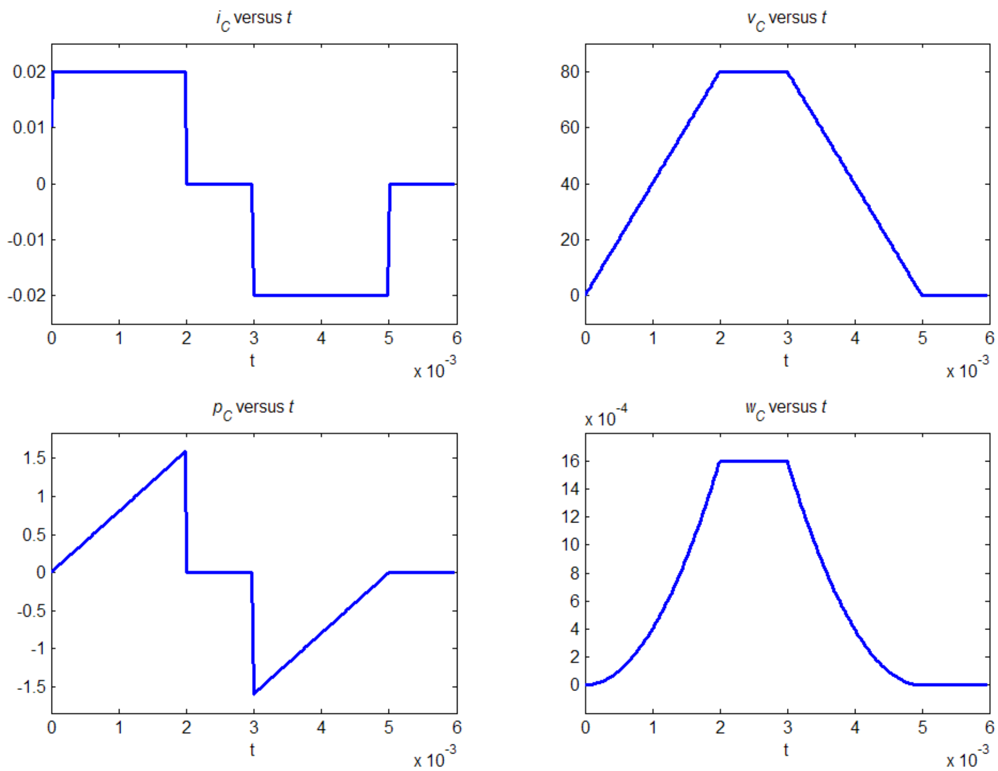


P3.14 $v(t) = \frac{1}{C}\int_0^t i(t)dt + v(0)$

$$v(t) = 2 \times 10^6 \int_0^t i(t)dt \qquad p(t) = v(t)i(t)$$

$$w(t) = \frac{1}{2}Cv^2(t) = 0.25 \times 10^{-6} \times v^2(t)$$

The sketches should be similar to the following plots. The units for the quantities in these plots are A, V, W, J and s.



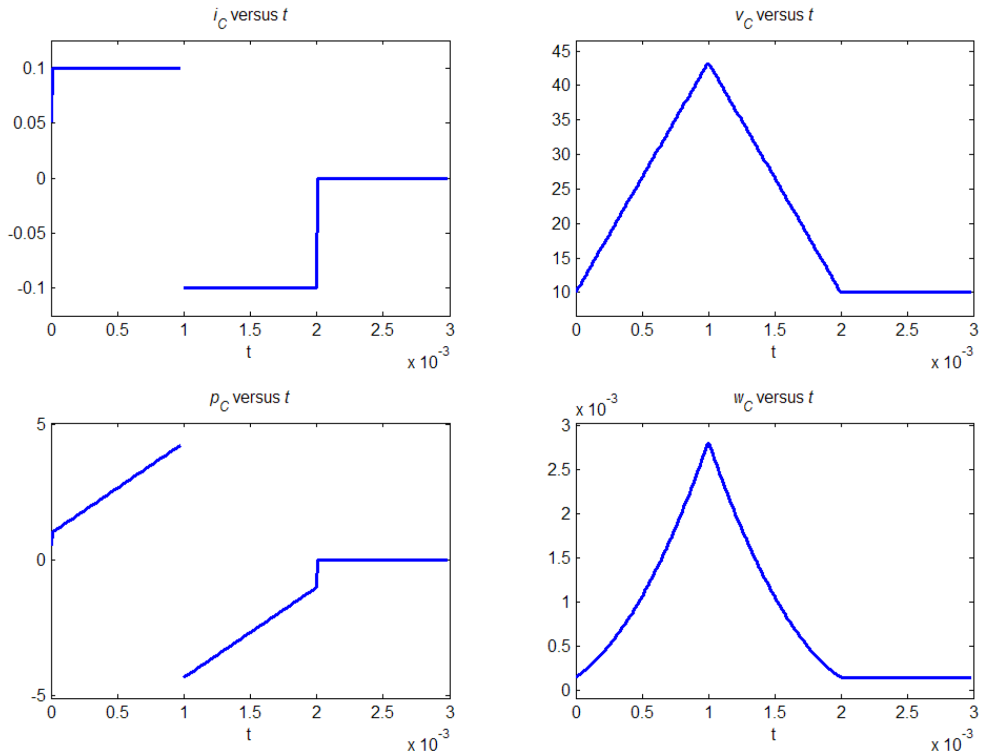
See End of document for solution P3.15

P3.16 $$v(t) = \frac{1}{C}\int_0^t i(t)dt + v(0) = 0.333 \times 10^6 \int_0^t i(t)dt + 10$$

$$p(t) = v(t)i(t)$$

$$w(t) = \frac{1}{2}Cv^2(t) = 1.5 \times 10^{-6} \times v^2(t)$$

The sketches should be similar to the following plots. The units for the quantities in these plots are A, V, W, J and s.



P3.17 We can write

$$w = \frac{1}{2}Cv^2 = 5 \times 10^{-6} v^2 = 200$$

Solving, we find $v = 6325$ V. Then, because the stored energy is decreasing, the power is negative. Thus, we have

$$i = \frac{p}{v} = \frac{-500}{6325} = -79.06 \text{ mA}$$

The minus sign shows that the current actually flows opposite to the passive convention. Thus, current flows out of the positive terminal of the capacitor.

P3.18 A capacitance initially charged to 10 V has

$$v(t) = \frac{1}{C}\int_0^t i(t)dt + 10$$

However, if the capacitance is infinite, this becomes $v(t) = 10$ V which describes a 10-V voltage source. Thus, a very large capacitance initially charged to 10 V is an approximate 10-V voltage source.

P3.19 $$i(t) = \frac{dq(t)}{dt} = \frac{d}{dt}[C(t)v(t)] = \frac{d}{dt}\left[200 \times 10^{-6} + 100 \times 10^{-6} \sin(200t)\right]$$
$$i(t) = 20\cos(200t) \text{ mA}$$

P3.20 We can write $v(t_0) = V_f = \frac{1}{C}\int_{t_0}^{t_0+\Delta t} i(t)dt$. The integral represents the area of the current pulse, which has units of ampere seconds or coulombs and must equal $V_f C$. The pulse area represents the net charge transferred by the current pulse. Because a constant-amplitude pulse has the largest area for a given peak amplitude, we can say that the peak amplitude of the current pulse must be at least as large as $V_f C / \Delta t$.

We conclude that the area of the pulse remains constant and that the peak amplitude approaches infinity as Δt approaches zero. In the limit, this type of pulse is called an impulse.

P3.21 By definition, the voltage across a short circuit must be zero. Since we have $v = Ri$ for a resistor, zero resistance corresponds to a short circuit. For an initially uncharged capacitance, we have

$$v(t) = \frac{1}{C}\int_0^t i(t)dt$$

For the voltage to be zero for all values of current and time, the capacitance must be infinite. Thus, an infinite initially uncharged capacitance is equivalent to a short circuit.

For an open circuit, the current must be zero. This requires infinite resistance. However for a capacitance, we have

$$i(t) = C\frac{dv(t)}{dt}$$

Thus, a capacitance of zero is equivalent to an open circuit.

P3.22 $$i(t) = C\frac{dv(t)}{dt}$$
$$= 20 \times 10^{-6} \frac{d}{dt}\left[3\cos\left(10^{5t}\right) + 2\sin\left(10^{5t}\right)\right]$$

$$= -6\sin(10^5 t) + 4\cos(10^5 t)$$

$$p(t) = v(t)i(t)$$
$$= [3\cos(10^5 t) + 2\sin(10^5 t)][-6\sin(10^{5t}) + 4\cos(10^5 t)]$$

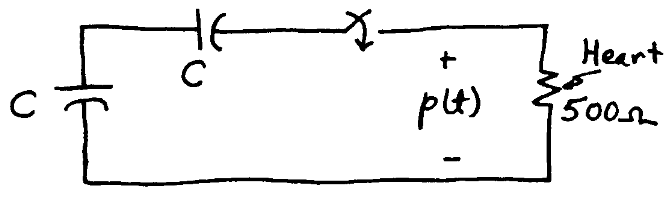
Evaluating at $t = 0$, we have $p(0) = 12$ W. Because $p(0)$ is positive, we know that the capacitor is absorbing energy at $t = 0$.
Evaluating at $t_2 = \pi/2 \times 10^{-5}$, we have $p(t_2) = -12$ W. Because $p(t_2)$ is negative, we know that the capacitor is supplying energy at $t = t_2$.

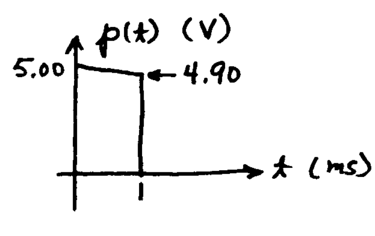
P3.23 Capacitances in parallel are combined by adding their values. Thus, capacitances in parallel are combined as resistances in series are. Capacitances in series are combined by taking the reciprocal of the sum of the reciprocals of the individual capacitances. Thus, capacitances in series are combined as resistances in parallel are.

P3.24* (a) $C_{eq} = 1 + \dfrac{1}{1/2 + 1/2} = 2\ \mu\text{F}$

(b) The two 4- F capacitances are in series and have an equivalent capacitance of $\dfrac{1}{1/4 + 1/4} = 2\ \mu\text{F}$. This combination is a parallel with the 2- F capacitance, giving an equivalent of 4 F. Then the 12 F is in series, giving a capacitance of $\dfrac{1}{1/12 + 1/4} = 3\ \mu\text{F}$. Finally, the 5 F is in parallel, giving an equivalent capacitance of $C_{eq} = 3 + 5 = 8\ \mu\text{F}$.

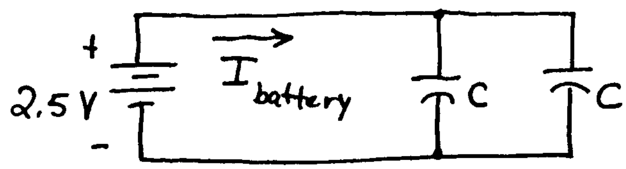
P3.25* As shown below, the two capacitors are placed in series with the heart to produce the output pulse.





While the capacitors are connected, the average voltage supplied to the heart is 4.95 V. Thus, the average current is $I_{pulse} = 4.95/500 = 9.9\ \text{mA}$.

The charge removed from each capacitor during the pulse is $\Delta Q = 9.9\ \text{mA} \times 1\ \text{ms} = 9.9\ \mu C$. This results in a 0.1 V change in voltage, so we have $C_{eq} = \frac{C}{2} = \frac{\Delta Q}{\Delta v} = \frac{9.9 \times 10^{-6}}{0.1} = 99\ \mu\text{F}$. Thus, each capacitor has a capacitance of $C = 198\ \mu\text{F}$. Then as shown below, the capacitors are placed in parallel with the 2.5-V battery to recharge them.



The battery must supply 9.9   C to each battery. Thus, the average current supplied by the battery is $I_{battery} = \frac{2 \times 9.9\ \mu C}{1\text{s}} = 19.8\ \mu A$. The ampere-hour rating of the battery is

$$19.8 \times 10^{-6} \times 5 \times 365 \times 24 = 0.867 \text{ Ampere hours}.$$

P3.26 (a) $C_{eq} = 3 + \frac{1}{1/2 + 1/1} + \frac{1}{1/2 + 1/(1+1)} = 4.667\ \mu\text{F}$

(b) $C_{eq} = \frac{1}{1/(2+1) + 1/(4+2)} = 2\ \mu\text{F}$

P3.27 We obtain the maximum capacitance of 6 μ F by connecting a 2- μ F capacitor in parallel with a 4- μ F . We obtain the minimum capacitance of 1.33 μ F by connecting the 2- μ F capacitor in series with the 4- μ F.

P3.28 $C_{eq} = \frac{1}{1/C_1 + 1/C_2} = 6\ \mu\text{F}$

The charges stored on each capacitor and on the equivalent capacitance are equal because the current through each is the same.

$$Q = C_{eq} \times 50\ \text{V} = 300\ \mu C$$

$$v_1 = \frac{Q}{C_1} = 20\ \text{V}$$

$$v_2 = \frac{Q}{C_2} = 30\ \text{V}$$

As a check, we verify that $v_1 + v_2 = 50\ \text{V}$.

P3.29 The equivalent capacitance is $C_{eq} = 100\ \ \mu\text{F}$ and its initial voltage is 150 V.

The energies are $w_1 = \frac{1}{2} C V_1^2 = \frac{1}{2} 200 \times 10^{-6} \times 50^2 = 0.25$ J and $w_2 = 1.0$ J.
Thus, the total energy stored in the two capacitors is $w_{\text{total}} = 1.25$ J.
On the other hand, the energy stored in the equivalent capacitance is
$w_{eq} = \frac{1}{2} C_{eq} V_{eq}^2 = \frac{1}{2} 100 \times 10^{-6} \times 150^2 = 1.125$ J.
The reason for the discrepancy is that all of the energy stored in the original capacitances cannot be accessed as long as they are connected in series. Net charge is trapped on the plates that are connected together.

See end of document for P3.30 & P3.31

P3.32 $C = \dfrac{\varepsilon_r \varepsilon_0 A}{d} = \dfrac{\varepsilon_r \varepsilon_0 WL}{d}$

(a) Thus if W and L are both doubled, the capacitance is increased by a factor of four resulting in $C = 400$ pF.
(b) If d is halved, the capacitance is doubled resulting in $C = 200$ pF.
(c) The relative dielectric constant of air is approximately unity. Thus, replacing air with oil increases ε_r by a factor of 35 increasing the capacitance to 3500 pF.

P3.33* The charge Q remains constant because the terminals of the capacitor are open-circuited.

$$Q = C_1 V_1 = 1000 \times 10^{-12} \times 1000 = 1\ \mu C$$

$$W_1 = (1/2) C_1 (V_1)^2 = 500\ \mu J$$

After the distance between the plates is doubled, the capacitance becomes $C_2 = 500$ pF.

The voltage increases to $V_2 = \frac{Q}{C_2} = \frac{10^{-6}}{500 \times 10^{-12}} = 2000 \text{ V}$ and the stored energy is $W_2 = (1/2)C_2(V_2)^2 = 1000\ \mu\text{J}$. The force needed to pull the plates apart supplies the additional energy.

P3.34 Using $C = \frac{\varepsilon_r \varepsilon_0 A}{d} = \frac{\varepsilon_r \varepsilon_0 WL}{d}$ and $V_{max} = Kd$ to substitute into $W_{max} = \frac{1}{2}CV_{max}^2$, we have $W_{max} = \frac{1}{2}\frac{\varepsilon_r \varepsilon_0 WL}{d}K^2 d^2 = \frac{1}{2}\varepsilon_r \varepsilon_0 K^2 WLd$.
However, the volume of the dielectric is Vol = WLd, so we have

$$W_{max} = \frac{1}{2}\varepsilon_r \varepsilon_0 K^2 (\text{Vol})$$

Thus, we conclude that the maximum energy stored is independent of W, L, and d if the volume is constant and if both W and L are much larger than d. To achieve large energy storage per unit volume, we should look for a dielectric having a large value for the product $\varepsilon_r K^2$. The dielectric should have high relative dielectric constant and high breakdown strength.

P3.35 The capacitance of the microphone is

$$C = \frac{\varepsilon_0 A}{d} = \frac{8.85 \times 10^{-12} \times 10 \times 10^{-4}}{100[1 + 0.003\cos(1000t)]10^{-6}}$$
$$\cong 88.5 \times 10^{-12}[1 - 0.003\cos(1000t)]$$

The current flowing through the microphone is

$$i(t) = \frac{dq(t)}{dt} = \frac{d[Cv]}{dt} \cong 53.1 \times 10^{-9}\sin(1000t) \quad A$$

P3.36 Referring to Figure P3.36 in the book, we see that the transducer consists of two capacitors in parallel: one above the surface of the liquid and one below. Furthermore, the capacitance of each portion is proportional to its length and the relative dielectric constant of the material between the plates. Thus for the portion above the liquid, the capacitance in pF is

$$C_{above} = 200\frac{100 - x}{100} \quad \text{pF}$$

in which x is the height of the liquid in cm. For the portion of the plates below the surface of the liquid:

$$C_{below} = 200(15)\frac{x}{100}$$

Then the total capacitance is:

$$C = C_{above} + C_{below}$$
$$C = 200 + 28x \quad \text{pF}$$

P3.37 With the tank full, we have

$$w_{full} = \tfrac{1}{2} C_{full} V_{full}^2 = \tfrac{1}{2} \times 2500 \times 10^{-12} \times 1000^2 = 1.25 \text{ mJ}$$
$$Q = C_{full} V_{full} = 2500 \times 10^{-12} \times 1000 = 2.5 \ \mu C$$

The charge cannot change when the tank is drained, so we have

$$V_{empty} = \frac{Q}{C_{empty}} = \frac{2.5 \times 10^{-6}}{100 \times 10^{-12}} = 25000 \text{ V}$$
$$w_{empty} = \tfrac{1}{2} C_{empty} V_{empty}^2 = \tfrac{1}{2} \times 100 \times 10^{-12} \times 25000^2 = 31.25 \text{ mJ}$$

The added energy is supplied from the gravitational potential energy of the insulating fluid. When there is liquid between the plates, the charge separation of the dielectric partly cancels the electrical forces of the charges on the plates. When the dielectric fluid drains, this cancellation effect is lost, which is why the voltage increases. The charge on the plates creates a small force of attraction on the fluid, and it is the force of gravity acting against this force of attraction as the fluid drains that accounts for the added energy.

P3.38

$$i_c(t) = C\frac{dv_c(t)}{dt} = 10^{-7}\frac{d}{dt}[10\cos(100t)] = -10^{-4}\sin(100t)$$
$$v_r(t) = Ri_c(t) = -10^{-3}\sin(100t)$$
$$v(t) = v_c(t) + v_r(t)$$
$$v(t) = 10\cos(100t) - 10^{-3}\sin(100t)$$

Thus, $v(t) = v_c(t)$ to within 1% accuracy, and the resistance can be neglected.

Repeating for $v_c(t) = 0.1\cos(10^7 t)$, we find

$$i_c(t) = -0.1\sin(10^7 t)$$
$$v_r(t) = -\sin(10^7)$$
$$v(t) = v_c(t) + v_r(t) = 0.1\cos(10^7 t) - \sin(10^7 t)$$

Thus, in this case, the voltage across the parasitic resistance is larger in magnitude than the voltage across the capacitance.

P3.39 The required volume is

$$\text{Vol} = Ad = \frac{2W_{max}}{\varepsilon_0 \varepsilon_r K^2} = \frac{2 \times 132 \times 10^6}{8.85 \times 10^{-12} (32 \times 10^5)^2} = 2.9 \times 10^6 \text{ m}^3$$

Clearly an air dielectric capacitor is not a practical energy storage device for an electric car!

The thickness of the dielectric is

$$d_{min} = \frac{V_{max}}{K} = \frac{10^3}{32 \times 10^5} = 0.3125 \text{ mm}$$

P3.40 Before the switch closes, the energies are

$$W_1 = (1/2)C_1(V_1)^2 = 50 \text{ mJ}$$
$$W_2 = (1/2)C_2(V_2)^2 = 50 \text{ mJ}$$

Thus, the total stored energy is 100 mJ. The charge on the top plate of C_1 is $Q_1 = C_1 V_1 = +1000\ \mu C$. The charge on the top plate of C_2 is $Q_2 = C_2 V_2 = -1000\ \mu C$. Thus, the total charge on the top plates is zero. When the switch closes, the charges cancel, the voltage becomes zero, and the stored energy becomes zero.

Where did the energy go? Usually, the resistance of the wires absorbs it. If the superconductors are used so that the resistance is zero, the energy can be accounted for by considering the inductance of the circuit. (It is not possible to have a real circuit that is precisely modeled by Figure P3.40; there is always resistance and inductance associated with the wires that connect the capacitances.)

P3.41 A fluid-flow analogy for an inductor consists of an incompressible fluid flowing through a frictionless pipe of constant diameter. The pressure differential between the ends of the pipe is analogous to the voltage across the inductor and the flow rate of the liquid is proportional to the current. (If the pipe had friction, the electrical analog would have series resistance. If the ends of the pipe had different diameters, a pressure differential would exist for constant flow rate, whereas an inductance has zero voltage for constant flow rate.)

P3.42 Inductors consist of coils of wire wound on coil forms such as toroids.

P3.43* See end of document for solution

P3.44* $v_L = L\dfrac{di}{dt} = 0.5\dfrac{2}{0.4} = 2.5$ V with such polarity as to aid the initial current.

P3.45*

$$i_L(t) = \frac{1}{L}\int_0^t v_L(t)dt + i_L(0) = 20\times10^3\int_0^t 10dt - 0.1 = 2\times10^5 t - 0.1\text{ A}$$

Solving for the time that the current reaches $+100$ mA, we have

$$i_L(t_x) = 0.1 = 2\times10^5 t_0 - 0.1$$
$$t_x = 1\ \mu s$$

P3.46 $i_L(t_1) = \frac{1}{L}\int_0^{t_1} v_L(t)dt + i_L(0)$

$i_L(1) = \frac{1}{5}\int_0^1 20dt + (-0.2)A = 3.98\ A$ $\qquad$ $i_L(6) = \frac{1}{5}\int_0^6 20dt + (-0.2)\ A = 23.98\ A$

$p(1) = v_L(1)i_L(1) = 79.6\ \text{W}$ $\qquad$ $p(6) = v_L(6)i_L(6) = 479.6\ \text{W}$

$w(1) = \frac{1}{2}Li_L^2(1) = 39.601\ \text{J}$ $\qquad$ $w(6) = \frac{1}{2}Li_L^2(6) = 1437.601\ \text{J}$

P3.47 Because the energy stored in an inductor is $w = \frac{1}{2}Li^2$, the energy stored in the inductor decreases when the current magnitude decreases. Therefore, energy is flowing out of the inductor.
Because we have $v_L(t) = L\frac{di_L(t)}{dt}$, the voltage is zero when the current is constant. Thus the power, which is the product of current and voltage, is zero if the current is constant.

P3.48 Because we have $v_L(t) = L\frac{di_L(t)}{dt}$, the voltage is zero when the current is constant. Thus, we say that inductors act as short circuits for steady dc currents.

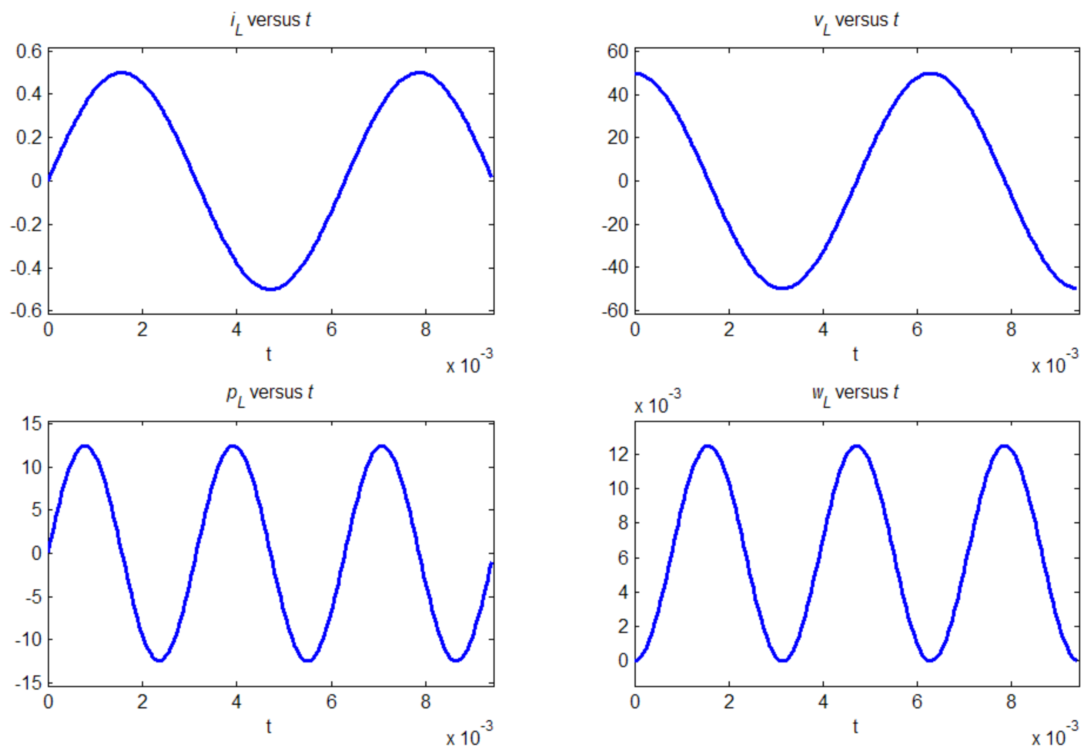
P3.49 $L = 0.1\,\text{H} \qquad i_L(t) = 0.5\sin(1000t)\,A$

$$v_L(t) = L\frac{di_L(t)}{dt} = 50\cos(1000t)\,\text{V}$$

$$p(t) = v_L(t)i_L(t) = 25\cos(1000t)\sin(1000t) = 12.5\sin(2000t)\,\text{W}$$

$$w(t) = \frac{1}{2}L[i_L(t)]^2 = 0.0125\sin^2(1000t)\,\text{J}$$

The sketches should be similar to the following plots. The units for the quantities in these plots are A, V, W, J and s.



P3.50 $L = 0.3\,\mathrm{H}$

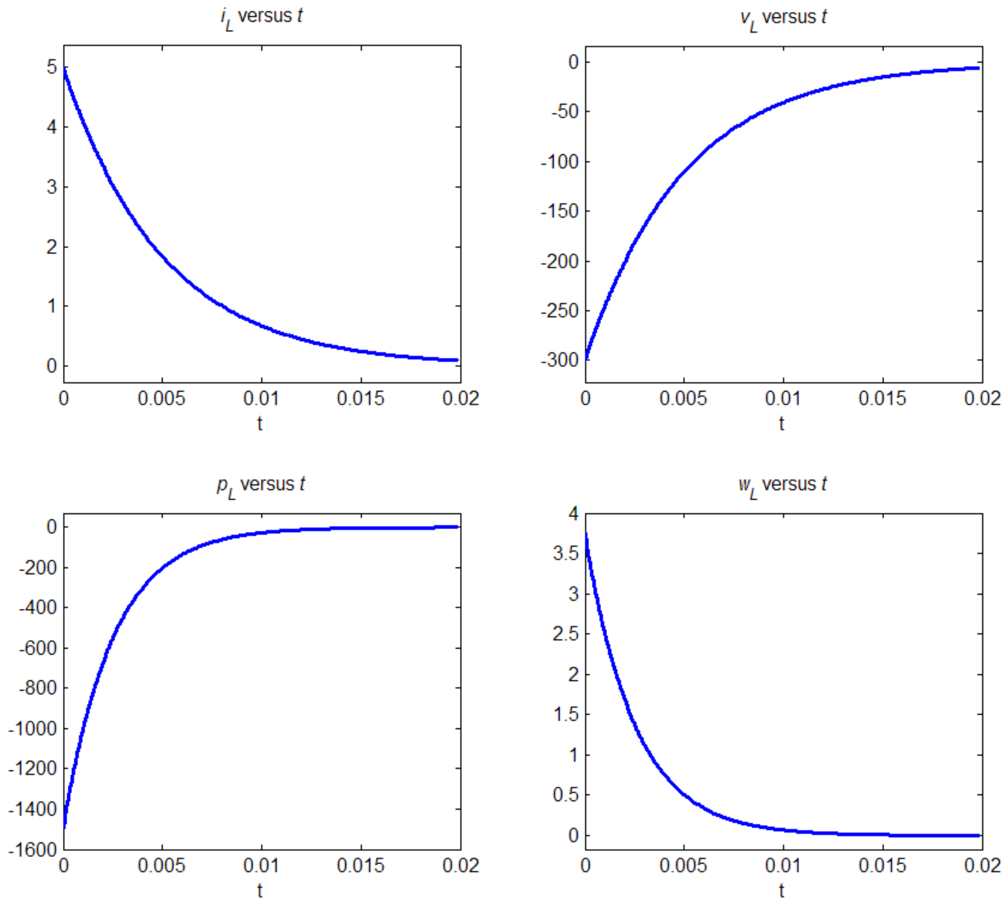
$$i_L(t) = 5e^{-200t}$$

$$v_L(t) = L\frac{di_L(t)}{dt} = -300e^{-200t}\ \mathrm{V}$$

$$p(t) = v_L(t)i_L(t) = \left(-300e^{-200t}\right)\left(5e^{-200t}\right) = -1500e^{-400t}\ \mathrm{W}$$

$$w(t) = \frac{1}{2}L\left[i_L(t)\right]^2 = 3.75e^{-400t}\ \mathrm{J}$$

The sketches should be similar to the following plots. The units for the quantities in these plots are A, V, W, J and s.

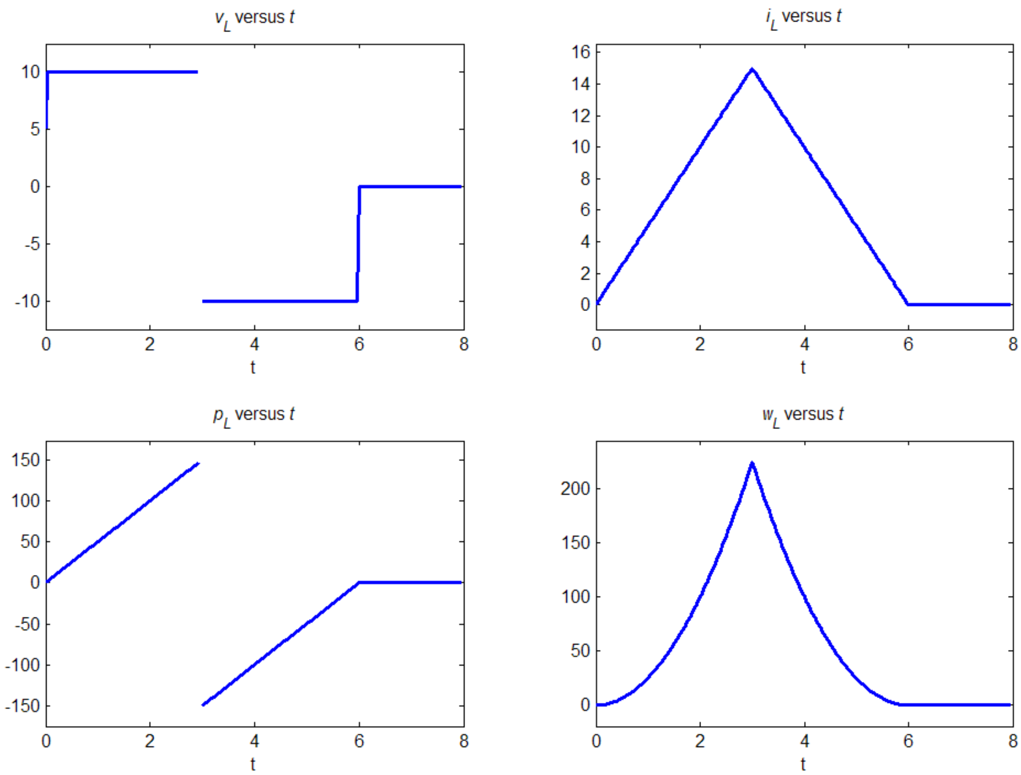


P3.51 $L = 2\,\text{H}$

$$i_L(t) = \frac{1}{L}\int_0^t v_L(t)dt + i_L(0) = \frac{1}{2}\int_0^t v_L(t)dt$$

$$w(t) = \frac{1}{2}L[i_L(t)]^2 = [i_L(t)]^2$$

The sketches should be similar to the following plots. The units for the quantities in these plots are A, V, W, J and s



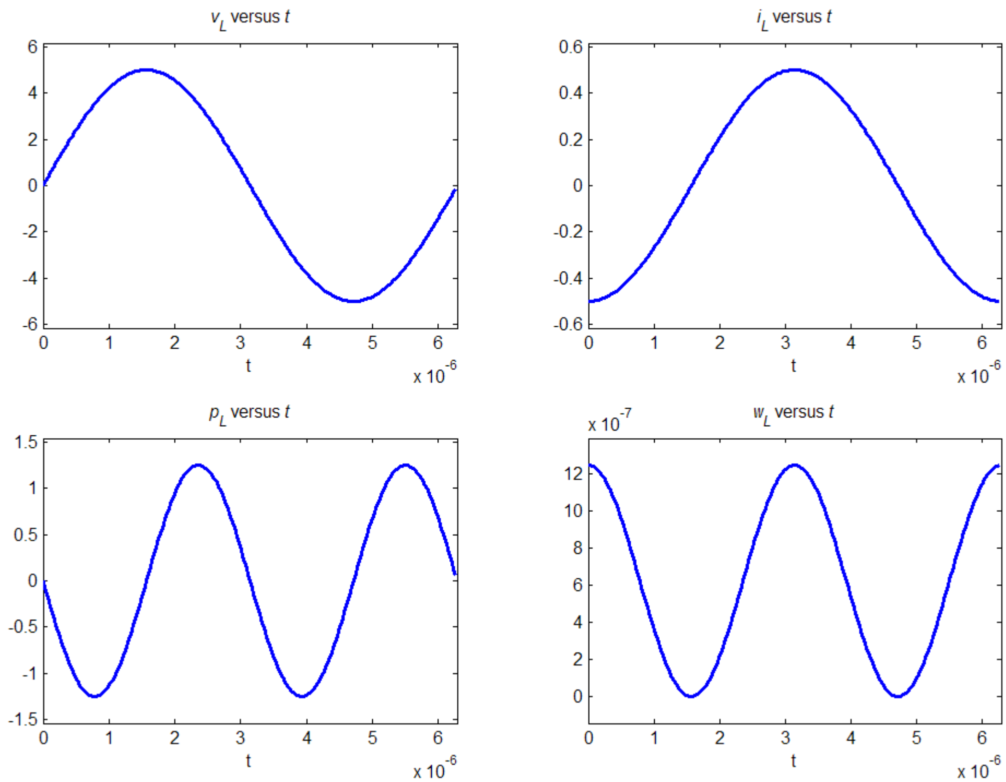
P3.52 $L = 10\ \mu\text{H} \qquad v_L(t) = 5\sin(10^6 t)$

$$i_L(t) = \frac{1}{L}\int_0^t v_L(t)dt + i_L(0) = 10^5\int_0^t 5\sin(10^6 t)dt - 0.5 = -0.5\cos(10^6 t)\,A$$

$$p(t) = v_L(t)i_L(t) = -2.5\sin(10^6 t)\cos(10^6 t) = -1.25\sin(2\times 10^6 t)$$

$$w(t) = \frac{1}{2}L[i_L(t)]^2 = 1.25\cos^2(10^6 t)\,\mu J$$

The sketches should be similar to the following plots. The units for the quantities in these plots are A, V, W, J and s



P3.53 For an inductor with an initial current of 10 A, we have

$i(t) = \frac{1}{L}\int_{t_0}^{t} v(t)dt + 10$. However for an infinite inductance, this becomes $i(t) = +10$ which is the specification for a 10-A current source. Thus, a very large inductance with an initial current of 10 A is an approximation to a 10-A current source.

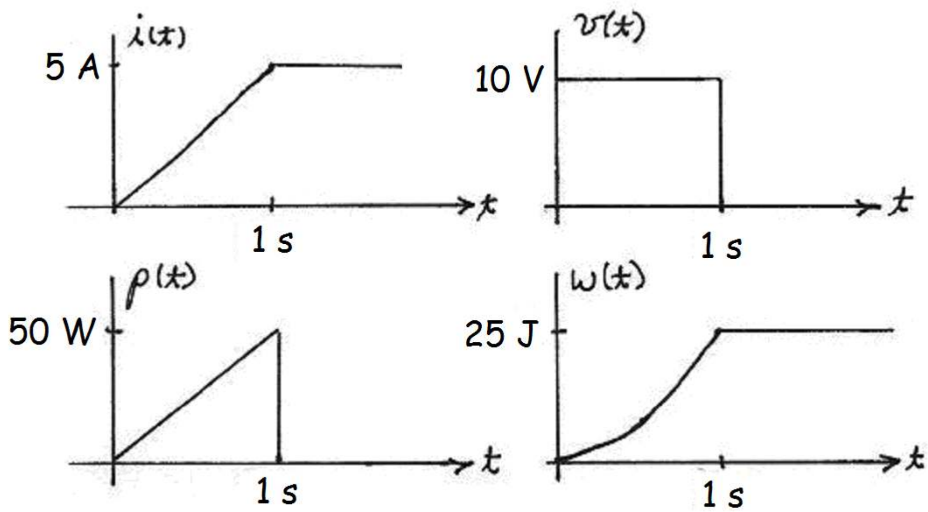
P3.54 $w(4) = \frac{1}{2}Li_L^2(4) = 100\text{ J} \Rightarrow |i_L(4)| = 10\text{ A}$

Since a reference is not specified, we can choose $i_L(4) = +10$ A. Also, because the stored energy is increasing, the power for the inductor carries a plus sign. Thus $p(4) = 200 = v_L(4)i_L(4)$ and we have $v_L(4) = 20$ V. Finally, because the current and voltage have the same algebraic signs, the current flows into the positive polarity.

P3.55 Because the current through an open circuit is zero by definition, and we have $i(t) = \frac{1}{L}\int_{t_0}^{t} v(t)dt$ (assuming zero initial current), infinite inductance corresponds to an open circuit.

Because the voltage across a short circuit is zero by definition, and $v_L = L\frac{di_L(t)}{dt}$, we see that $L = 0$ corresponds to a short circuit.

P3.56



P3.57 $i(t) = \frac{1}{L}\int_0^t v(t)dt + i(0) = \frac{1}{L}\int_0^t V_m \cos(\omega t)dt = \frac{V_m}{\omega L}\left[\sin(\omega t) - \sin(0)\right]$

$$= \frac{V_m}{\omega L}\sin(\omega t)$$

Clearly for $\omega \to \infty$, the current becomes zero, so the inductance becomes the equivalent of an open circuit.

P3.58 We can write $i_L(t_0 + \Delta t) = I_f = \frac{1}{L}\int_{t_0}^{t_0+\Delta t} v_L(t)dt$. The integral represents the area of the voltage pulse, which has units of volt seconds and must equal $I_f L$. Because a constant-amplitude pulse has the largest area for a given peak amplitude, we can say that the peak amplitude of the voltage pulse must be at least as large as $I_f L / \Delta t$.

We conclude that the area of the pulse remains constant and that the peak amplitude approaches infinity as Δt approaches zero. In the limit, this type of pulse is called an impulse.

P3.59 Inductances are combined in the same way as resistances. Inductances in series are added. Inductances in parallel are combined by taking the reciprocal of the sum of the reciprocals of the several inductances.

P3.60* (a) $L_{eq} = 1 + \dfrac{1}{1/6 + 1/(1+2)} = 3\,\text{H}$

(b) 9 H in parallel with 18 H is equivalent to 6H. Also, 20 H in parallel with 5 H is equivalent to 4 H. Finally, we have $L_{eq} = \dfrac{1}{1/15 + 1/(6+4)} = 6\,\text{H}$

P3.61 (a) The 2 H inductors and 0.5 H inductor have no effect because they are in parallel with a short circuit. Thus, $L_{eq} = 1\,\text{H}$.

(b) The two 2-H inductances in parallel are equivalent to 1 H. Also, the 1 H in parallel with the 3 H inductance is equivalent to 0.75 H. Thus,

$$L_{eq} = 1 + \frac{1}{1/(1+1) + 1/(2+0.75)} = 2.158\,\text{H}.$$

P3.62 Let the inductor be of L henry. When in series

$$4L = 16\ \text{H} \quad \therefore L = 4\ \text{H}$$

By connecting all four inductors in parallel, we obtain the minimum inductance:

$$L_{\min} = \frac{1}{1/4 + 1/4 + 1/4 + 1/4} = 1\ \text{H}$$

P3.63* $i(t) = \dfrac{1}{L_{eq}}\displaystyle\int_0^t v(t)dt = \dfrac{L_1 + L_2}{L_1 L_2}\int_0^t v(t)dt$

$i_2(t) = \dfrac{1}{L_2}\displaystyle\int_0^t v(t)dt$

Thus, we can write $i_2(t) = \dfrac{L_1}{L_1 + L_2} i(t) = \dfrac{2}{5} i(t)$.

Similarly, we have $i_1(t) = \dfrac{L_2}{L_1 + L_2} i(t) = \dfrac{3}{5} i(t)$.

This is similar to the current-division principle for resistances. Keep in mind that these formulas assume that the initial currents are zero.

P3.64 Ordinarily, negative inductance is not practical. Thus, adding inductance in series always increases the equivalent inductance. However, placing inductance in parallel results in smaller inductance. Thus, we need to consider a parallel inductance such that

$$\frac{1}{1/L + 1/4} = 3$$

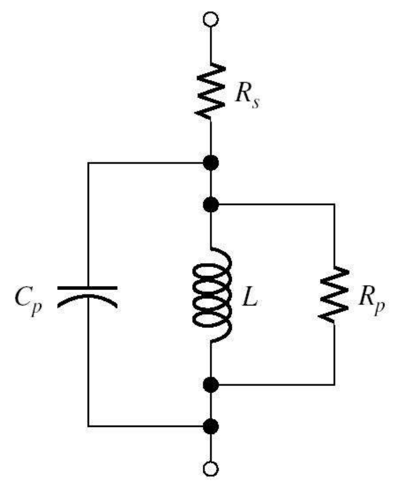
Solving, we find that $L = 12$ H.

P3.65 Given that $(L+4) \,||\, 10 = 5$ H

$$\therefore \frac{(L+4)10}{L+4+10} = 5$$

Solving, we find that $L = 6$ H.

P3.66



P3.67 (a) $v_R(t) = Ri(t) = 0.1\cos(10^5)$

$$v_L(t) = L\frac{di(t)}{dt} = -100\sin(10^5)$$

$$v(t) = v_R(t) + v_L(t)$$

$$= 0.1\cos(10^5 t) - 100\sin(10^5 t)$$

In this case, to obtain 1% accuracy, the resistance can be neglected.

(b) For $i(t) = 0.1\cos(10t)$, we have

$$v_R(t) = 0.1\cos(10t)$$
$$v_L(t) = -0.01\sin(10t)$$
$$v(t) = 0.1\cos(10t) - 0.01\sin(10t)$$

Thus, in this case, the parasitic resistance cannot be neglected.

P3.68 Because $v_L = L\dfrac{di_L(t)}{dt} = 0$ for currents that are constant in time, we conclude that the inductance behaves as a short circuit for dc currents. The capacitance acts as an open circuit for constant voltages. Thus, the circuit simplifies to the single resistance R_s, which is given by $R_s = \dfrac{400 \text{ mV}}{100 \text{ mA}} = 4\,\Omega.$

P3.69
$$v(t) = L\frac{di_L(t)}{dt} = 10\times10^{-3}\times10^4[\cos(10^4t)] = 100\cos(10^4t)\ \text{V}$$
$$i_C(t) = C\frac{dv(t)}{dt} = -\sin(10^4t)\ \text{A}$$
$$i(t) = i_C(t) + i_L(t) = 0$$
$$w_C(t) = \tfrac{1}{2}Cv^2(t) = 5\cos^2(10^4t) = 2.5 + 2.5\cos(2\times10^4t)\ \text{mJ}$$
$$w_L(t) = \tfrac{1}{2}Li_L^2(t) = 5\sin^2(10^4t) = 2.5 - 2.5\cos(2\times10^4t)\ \text{mJ}$$
$$w(t) = w_C(t) + w_L(t) = 5\ \text{mJ}$$

The values in this circuit have been carefully selected so the source current is zero. Because the source current is zero and there are no resistances, there is no source or sink for energy in the circuit. Thus, we would expect the total energy to be constant, as the equations show. The total energy surges back and forth between the capacitance and the inductance. In a real circuit, the parasitic resistances would eventually absorb the energy. The circuit is analogous to a swinging pendulum or a ringing bell.

P3.70
$$i(t) = C\frac{dv_C(t)}{dt} = 250\times10^{-6}\times40\times1000[-\sin(1000t)] = -10\sin(1000t)\ A$$
$$v_L(t) = L\frac{di(t)}{dt} = -40\cos(1000t)\ \text{V}$$
$$v(t) = v_C(t) + v_L(t) = 0$$

$w_C(t) = \frac{1}{2} C v_C^2(t) = 200\cos^2(1000t) = 100 + 100\cos(2000t)$ mJ

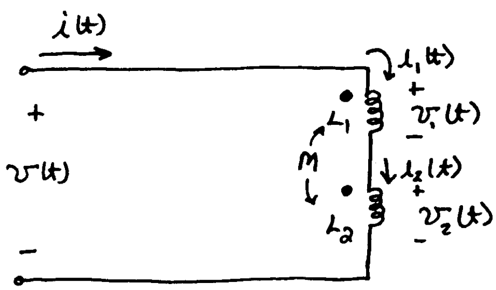
$w_L(t) = \frac{1}{2} L i^2(t) = 200\sin^2(1000t) = 100 - 100\cos(2000t)$ mJ

$w(t) = w_C(t) + w_L(t) = 200$ mJ

The values in this circuit have been carefully selected so the source voltage is zero. Because the source voltage is zero and there are no resistances, there is no source or sink for energy in the circuit. Thus, we would expect the total energy to be constant, as the equations show. The total energy surges back and forth between the capacitance and the inductance. In a real circuit, the parasitic resistances would eventually absorb the energy. The circuit is analogous to a swinging pendulum or a ringing bell.

P3.71 When a time-varying current flows in a coil, a time-varying magnetic field is produced. If some of this field links a second coil, voltage is induced in it. Thus, time-varying current in one coil results in a contribution to the voltage across a second coil.

P3.72* (a)



As in Figure 3.23a, we can write

$$v_1(t) = L_1 \frac{di_1(t)}{dt} + M\frac{di_2(t)}{dt}$$

$$v_2(t) = M\frac{di_1(t)}{dt} + L_2\frac{di_2(t)}{dt}$$

However, for the circuit at hand, we have $i(t) = i_1(t) = i_2(t)$.

Thus,

$$v_1(t) = (L_1 + M)\frac{di(t)}{dt}$$

$$v_2(t) = (L_2 + M)\frac{di(t)}{dt}$$

Also, we have $v(t) = v_1(t) + v_2(t)$.

Substituting, we obtain $v(t) = (L_1 + 2M + L_2)\dfrac{di(t)}{dt}$.

Thus, we can write $v(t) = L_{eq}\dfrac{di(t)}{dt}$, in which

$L_{eq} = L_1 + 2M + L_2$.

(b) Similarly, for the dot at the bottom end of L_2, we have

$$L_{eq} = L_1 - 2M + L_2$$

P3.73 (a) Refer to Figures 3.23 and P3.73. For the dots as shown in Figure P3.73, we have

$$v_1(t) = L_1\frac{di_1(t)}{dt} + M\frac{di_2(t)}{dt} = 40\cos(20t) - 15\sin(30t)\ \text{V}$$

$$v_2(t) = M\frac{di_1(t)}{dt} + L_2\frac{di_2(t)}{dt} = 20\cos(20t) - 45\sin(30t)\ \text{V}$$

(b) With the dot moved to the bottom of L_2, we have

$$v_1(t) = L_1\frac{di_1(t)}{dt} - M\frac{di_2(t)}{dt} = 40\cos(20t) + 15\sin(30t)\ \text{V}$$

$$v_2(t) = -M\frac{di_1(t)}{dt} + L_2\frac{di_2(t)}{dt} = -20\cos(20t) - 45\sin(30t)$$

P3.74 In general, we have

$$v_1(t) = L_1\frac{di_1}{dt} \pm M\frac{di_2}{dt}$$

$$v_2(t) = \pm M\frac{di_1}{dt} + L_2\frac{di_2}{dt}$$

Substituting the given information, we have

$$v_1(t) = -4000\sin(1000t)$$

and

$$2000\sin(1000t) = \mp 2000M\sin(1000t)$$

We deduce that $M = 1$ H. Furthermore, because the lower of the two algebraic signs applies, we know that the currents are referenced into unlike (i.e., one dotted and one undotted) terminals. Similarly the positive reference for the voltages is at unlike terminals.

P3.75 With a short circuit across the terminals of the second coil, we have

$$v_1(t) = L_1 \frac{di_1(t)}{dt} + M \frac{di_2(t)}{dt}$$

$$v_2(t) = 0 = M \frac{di_1(t)}{dt} + L_2 \frac{di_2(t)}{dt}$$

Solving the second equation for $di_2(t)/dt$ and substituting into the first equation, we have

$$v_1(t) = L_1 \frac{di_1(t)}{dt} - \frac{M^2}{L_2} \frac{di_1(t)}{dt} = \frac{L_1L_2 - M^2}{L_2} \frac{di_1(t)}{dt}$$

from which we can see that

$$L_{eq} = \frac{L_1L_2 - M^2}{L_2}$$

P3.76 Because of the parallel connection, we have $v_1(t) = v_2(t) = v(t)$ and the equations for the mutually coupled inductors become

$$v(t) = L_1 \frac{di_1(t)}{dt} - M \frac{di_2(t)}{dt}$$

$$v(t) = -M \frac{di_1(t)}{dt} + L_2 \frac{di_2(t)}{dt}$$

Using determinants to solve for the derivatives, we have

$$\frac{di_1(t)}{dt} = \frac{\begin{vmatrix} v(t) & -M \\ v(t) & L_2 \end{vmatrix}}{\begin{vmatrix} L_1 & -M \\ -M & L_2 \end{vmatrix}} = \frac{L_2 + M}{L_1L_2 - M^2} v(t) \qquad \frac{di_2(t)}{dt} = \frac{\begin{vmatrix} L_1 & v(t) \\ -M & v(t) \end{vmatrix}}{\begin{vmatrix} L_1 & -M \\ -M & L_2 \end{vmatrix}} = \frac{L_1 + M}{L_1L_2 - M^2} v(t)$$

Then, we have

$$i(t) = i_1(t) + i_2(t)$$

$$\frac{di(t)}{dt} = \frac{di_1(t)}{dt} + \frac{di_1(t)}{dt} = \frac{L_1 + L_2 + 2M}{L_1L_2 - M^2} v(t)$$

from which we conclude that

$$L_{eq} = \frac{L_1L_2 + M^2}{L_1 + L_2 - 2M}$$

P3.77 $v_L = L\dfrac{di_L}{dt} = 0.2\dfrac{d}{dt}\left[\exp(-2t)\sin(4\pi t)\right]$

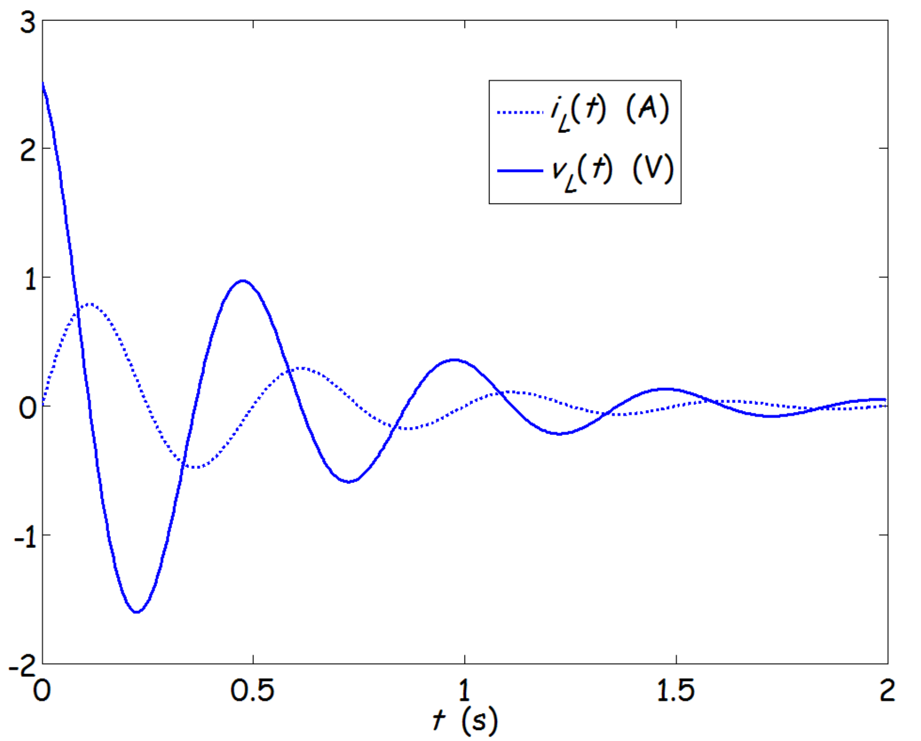
$v_L = 0.8\pi \exp(-2t)\cos(4\pi t) - 0.4\exp(-2t)\sin(4\pi t)$

A sequence of MATLAB commands that produces the desired plots is

```
syms iL vL t
```

```
iL = exp(-2*t)*sin(4*pi*t);
vL = 0.2*diff(iL,t);
ezplot(iL, [0 2])
hold on
ezplot(vL, [0 2])
```

The result is:



P3.78 Using either tables or integration by parts, we have

$$i_L(t) = \frac{1}{L}\int_0^t v_L(t)dt + i_L(0) = \frac{1}{L}\int_0^t t\exp(-t)dt = \frac{1}{L}\left[-\exp(-t)(t+1)\right]_0^t$$

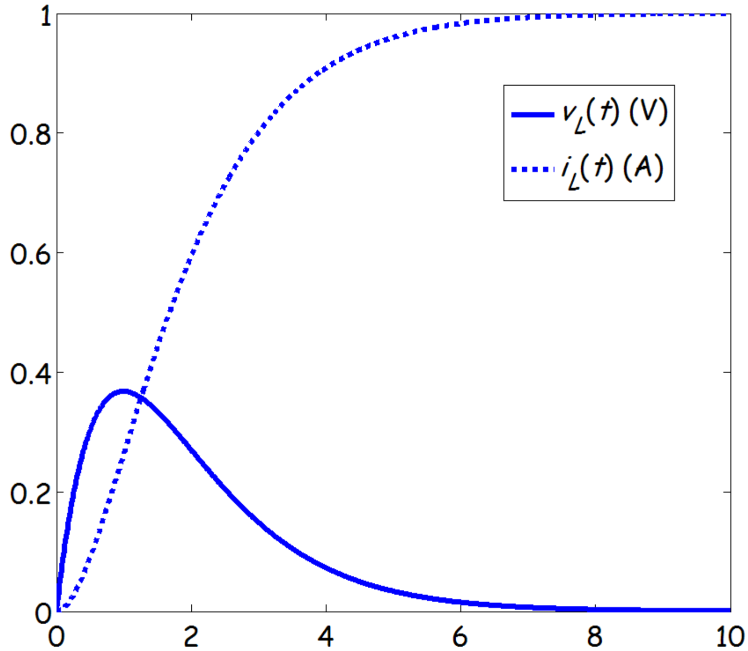
$$i_L(t) = 1 - t\exp(-t) - \exp(-t)$$

A sequence of MATLAB commands to verify our result for $i_L(t)$ and obtain the desired plots is

```
syms iL vL t
vL = t*exp(-t);
iL = int(vL,t,0,t)
ezplot(vL, [0 10])
```

```
hold on
ezplot(iL, [0 10])
```

The result is:



Practice Test

T3.1

$$v_{ab}(t) = \frac{1}{C}\int_0^t i_{ab}(t)dt + v_C(0) = 10^5 \int_0^t 0.3\exp(-2000t)dt$$

$$v_{ab}(t) = -15\exp(-2000t)\Big|_0^t$$

$$v_{ab}(t) = 15 - 15\exp(-2000t) \text{ V}$$

$$w_C(\infty) = \frac{1}{2}Cv_C^2(\infty) = \frac{1}{2}10^{-5}(15)^2 = 1.125 \text{ mJ}$$

T3.2 The 6-μF and 3-μF capacitances are in series and have an equivalent capacitance of

$$C_{eq1} = \frac{1}{1/6 + 1/3} = 2\ \mu\text{F}$$

C_{eq1} is in parallel with the 4-μF capacitance, and the combination has an equivalent capacitance of

$$C_{eq2} = C_{eq1} + 4 = 6 \ \mu\text{F}$$

C_{eq2} is in series with the 12-μF and the combination, has an equivalent capacitance of

$$C_{eq3} = \frac{1}{1/12 + 1/6} = 4 \ \mu\text{F}$$

Finally, C_{eq3} is in parallel with the 1-μF capacitance, and the equivalent capacitance is

$$C_{eq} = C_{eq3} + 1 = 5 \ \mu\text{F}$$

T3.3 $$C = \frac{\varepsilon_r \varepsilon_0 A}{d} = \frac{80 \times 8.85 \times 10^{-12} \times 2 \times 10^{-2} \times 3 \times 10^{-2}}{0.1 \times 10^{-3}} = 4248 \text{ pF}$$

T3.4 $$v_{ab}(t) = L\frac{di_{ab}}{dt} = 2 \times 10^{-3} \times 0.3 \times 2000 \cos(2000t) = 1.2 \cos(2000t) \ \text{V}$$

The maximum value of sin(2000t) is unity. Thus the peak current is 0.3 A, and the peak energy stored is

$$w_{peak} = \frac{1}{2} L i_{peak}^2 = \frac{1}{2} \times 2 \times 10^{-3} (0.3)^2 = 90 \ \mu\text{J}$$

T3.5 The 2-H and 4-H inductances are in parallel and the combination has an equivalent inductance of

$$L_{eq1} = \frac{1}{1/2 + 1/4} = 1.333 \ \text{H}$$

Also, the 3-H and 5-H inductances are in parallel, and the combination has an equivalent inductance of

$$L_{eq2} = \frac{1}{1/3 + 1/5} = 1.875 \ \text{H}$$

Finally, L_{eq1} and L_{eq2} are in series. The equivalent inductance between terminals a and b is

$$L_{eq} = L_{eq1} + L_{eq2} = 3.208 \ \text{H}$$

T3.6 For these mutually coupled inductances, we have

$$v_1(t) = L_1 \frac{di_1(t)}{dt} - M \frac{di_2(t)}{dt}$$
$$v_2(t) = -M \frac{di_1(t)}{dt} + L_2 \frac{di_2(t)}{dt}$$

in which the currents are referenced into the positive polarities. Thus the currents are

$i_1(t) = 2\cos(500t)$ and $i_2(t) = -2\exp(-400t)$

Substituting the inductance values and the current expressions we have

$$v_1(t) = -40 \times 10^{-3} \times 1000 \sin(500t) - 20 \times 10^{-3} \times 800 \exp(-400t)$$
$$v_1(t) = -40 \sin(500t) - 16 \exp(-400t) \text{ V}$$

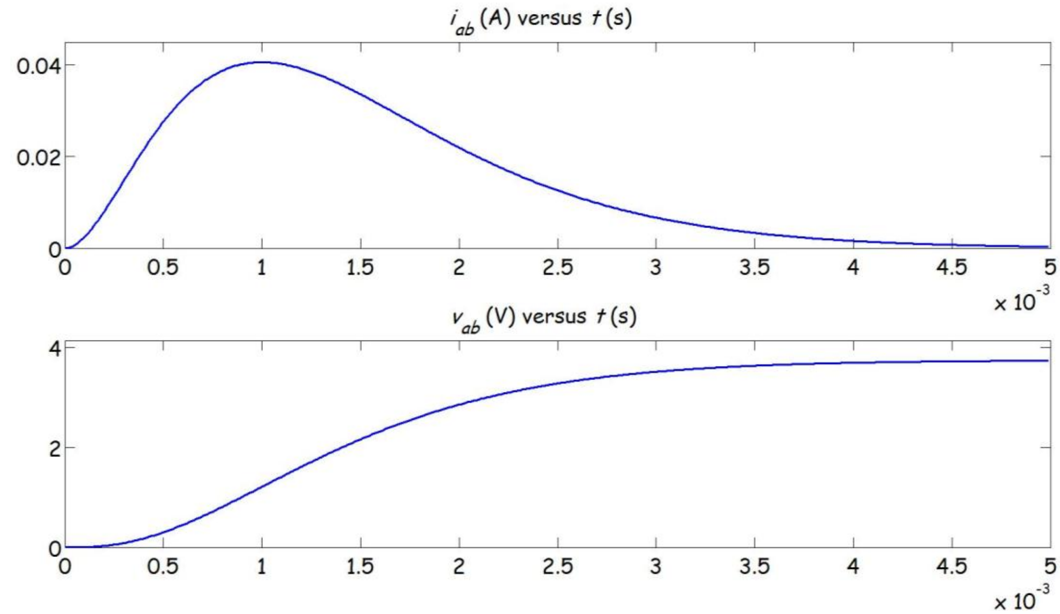
$$v_2(t) = 20 \times 10^{-3} \times 1000 \sin(500t) - 30 \times 10^{-3} \times 800 \exp(-400t)$$
$$v_2(t) = 20 \sin(500t) - 24 \exp(-400t) \text{ V}$$

T3.7 One set of commands is

```
syms vab iab t
iab = 3*(10^5)*(t^2)*exp(-2000*t);
vab = (1/20e-6)*int(iab,t,0,t)
subplot(2,1,1)
ezplot(iab, [0 5e-3]), title('\iti_a_b\rm (A) versus \itt\rm (s)')
subplot(2,1,2)
ezplot(vab, [0 5e-3]), title('\itv_a_b\rm (V) versus \itt\rm (s)')
```

The results are

$$v_{ab} = \frac{15}{4} - \frac{15}{4}\exp(-2000t) - 7500\exp(-2000t) - 7.5 \times 10^6 t^2 \exp(-2000t)$$



i_{ab} (A) versus t (s)
0.04
0.02
0
0
0.5
1
1.5
2
2.5
3
3.5
4
4.5
5
$\times 10^{-3}$
v_{ab} (V) versus t (s)
4
2
0
0
0.5
1
1.5
2
2.5
3
3.5
4
4.5
5
$\times 10^{-3}$

P3.15 Because the switch is closed prior to t = 0, the initial voltage is zero, and we have

$v(t) = \frac{1}{C}\int_0^t i(t)dt + v(0);\ \therefore v(25ms) = 10^5 \int_0^{25x10^{-3}} 5x10^{-3}\,dt + 0 = 500\,x25\,x10^{-3}\ = 12.5\,V$

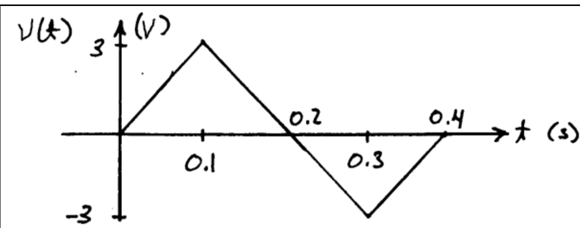
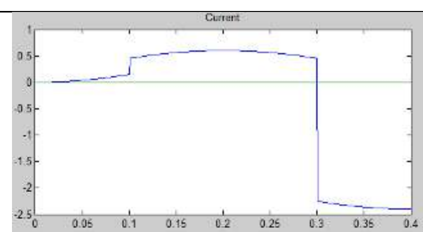
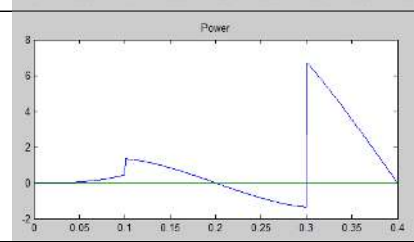
p =vi = 12.5 x 5 x 10^{-3} W = 62.5 mW

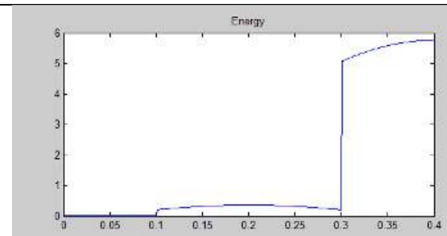
w(t) = $\frac{1}{2}\ Cv(t)^2;\ \therefore w(25\,ms) = \frac{1}{2}\ 10x10^{-6}\,x\,(12.5)^2 = 781.25\,\mu J$

P3.30 Q = CV, hence the charge deposited on both the capacitors will be in proportion to their capacitances as applied voltage v = 50 V is common to both of them. Hence, Q1 = C1V = 750 μC and Q2 = C2V = 500 μC.

*P3.31 $C = \frac{\varepsilon A}{d} = \frac{\varepsilon_0 \varepsilon_r A}{d} = \frac{100\ x\ 10^{-12}\ x\ 100\ x\ 10^{-3}\ x\ 0.3}{0.04\ x\ 10^{-3}} = 7500\ pF$

*P3.43

L = 2 H	
$i_L(t) = 1/L\int_0^t v(t)dt$	
P(t) = v(t) i(t)	

w(t) = $\frac{1}{2}\ L\ [i_L(t)]^2$	

CHAPTER 4

Exercises

E4.1 The voltage across the circuit is given by Equation 4.8:

$$v_C(t) = V_i \exp(-t / RC)$$

in which V_i is the initial voltage. At the time $t_{1\%}$ for which the voltage reaches 1% of the initial value, we have

$$0.01 = \exp(-t_{1\%} / RC)$$

Taking the natural logarithm of both sides of the equation, we obtain

$$\ln(0.01) = -4.605 = -t_{1\%} / RC$$

Solving and substituting values, we find $t_{1\%}$ = 4.605RC = 23.03 ms.

E4.2 The exponential transient shown in Figure 4.4 is given by

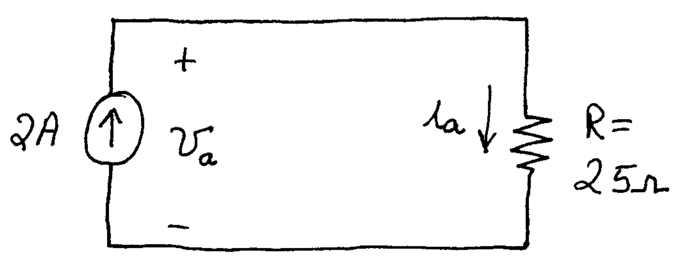
$$v_C(t) = V_s - V_s \exp(-t / \tau)$$

Taking the derivative with respect to time, we have

$$\frac{dv_C(t)}{dt} = \frac{V_s}{\tau}\exp(-t / \tau)$$

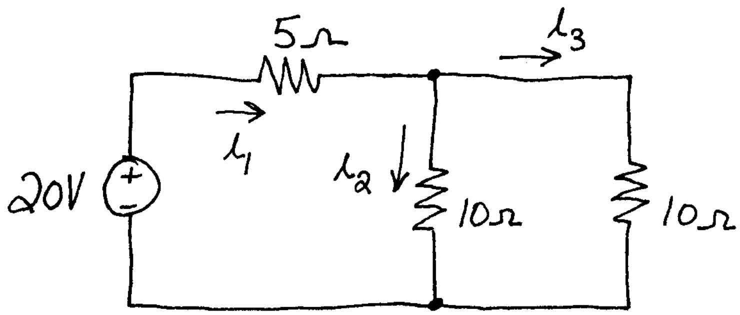
Evaluating at t = 0, we find that the initial slope is V_s / τ. Because this matches the slope of the straight line shown in Figure 4.4, we have shown that a line tangent to the exponential transient at the origin reaches the final value in one time constant.

E4.3 (a) In dc steady state, the capacitances act as open circuits and the inductances act as short circuits. Thus the steady-state (i.e., t approaching infinity) equivalent circuit is:



From this circuit, we see that $i_a = 2$ A. Then, Ohm's law gives the voltage as $v_a = Ri_a = 50$ V.

(b) The dc steady-state equivalent circuit is:



Here the two 10-Ω resistances are in parallel with an equivalent resistance of 1/(1/10 + 1/10) = 5 Ω. This equivalent resistance is in series with the 5-Ω resistance. Thus the equivalent resistance seen by the source is 10 Ω, and $i_1 = 20/10 = 2\text{ A}$. Using the current division principle, this current splits equally between the two 10-Ω resistances, so we have $i_2 = i_3 = 1\text{ A}$.

E4.4 (a) $\tau = L/R_2 = 0.1/100 = 1\text{ ms}$

(b) Just before the switch opens, the circuit is in dc steady state with an inductor current of $V_s/R_1 = 1.5\text{ A}$. This current continues to flow in the inductor immediately after the switch opens so we have $i(0+) = 1.5\text{ A}$. This current must flow (upward) through R_2 so the initial value of the voltage is $v(0+) = -R_2 i(0+) = -150\text{ V}$.

(c) We see that the initial magnitude of $v(t)$ is ten times larger than the source voltage.

(d) The voltage is given by

$$v(t) = -\frac{V_s L}{R_1 \tau}\exp(-t/\tau) = -150\exp(-1000t)$$

Let us denote the time at which the voltage reaches half of its initial magnitude as t_H. Then we have

$$0.5 = \exp(-1000t_H)$$

Solving and substituting values we obtain

$$t_H = -10^{-3}\ln(0.5) = 10^{-3}\ln(2) = 0.6931\text{ ms}$$

E4.5 First we write a KCL equation for $t \geq 0$.

$$\frac{v(t)}{R} + \frac{1}{L}\int_0^t v(x)dx + 0 = 2$$

Taking the derivative of each term of this equation with respect to time and multiplying each term by R, we obtain:

$$\frac{dv(t)}{dt} + \frac{R}{L}v(t) = 0$$

The solution to this equation is of the form:

$$v(t) = K\exp(-t/\tau)$$

in which $\tau = L/R = 0.2\text{ s}$ is the time constant and K is a constant that must be chosen to fit the initial conditions in the circuit. Since the initial (t = 0+) inductor current is specified to be zero, the initial current in the resistor must be 2 A and the initial voltage is 20 V:

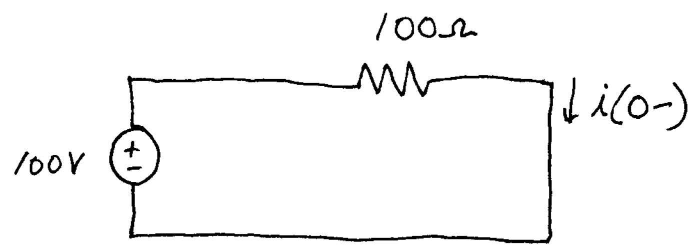
$$v(0+) = 20 = K$$

Thus, we have

$$v(t) = 20\exp(-t/\tau) \qquad i_R = v/R = 2\exp(-t/\tau)$$

$$i_L(t) = \frac{1}{L}\int_0^t v(x)dx = \frac{1}{2}\left[-20\tau\exp(-x/\tau)\right]_0^t = 2 - 2\exp(-t/\tau)$$

E4.6 Prior to t = 0, the circuit is in DC steady state and the equivalent circuit is



Thus we have i(0-) = 1 A. However the current through the inductor cannot change instantaneously so we also have i(0+) = 1 A. With the switch open, we can write the KVL equation:

$$\frac{di(t)}{dt} + 200i(t) = 100$$

The solution to this equation is of the form

$$i(t) = K_1 + K_2\exp(-t/\tau)$$

in which the time constant is $\tau = 1/200 = 5\text{ ms}$. In steady state with the switch open, we have $i(\infty) = K_1 = 100/200 = 0.5\text{ A}$. Then using the initial current, we have $i(0+) = 1 = K_1 + K_2$, from which we determine that $K_2 = 0.5$. Thus we have

$$\begin{aligned} i(t) &= 1.0 \text{ A for } t < 0 \\ &= 0.5 + 0.5\exp(-t/\tau) \text{ for t} > 0. \end{aligned}$$

$$\begin{aligned} v(t) &= L\frac{di(t)}{dt} \\ &= 0 \text{ V for } t < 0 \\ &= -100\exp(-t/\tau) \text{ for t} > 0. \end{aligned}$$

E4.7 As in Example 4.4, the KVL equation is

$$Ri(t) + \frac{1}{C}\int_0^t i(x)dx + v_C(0+) - 2\cos(200t) = 0$$

Taking the derivative and multiplying by C, we obtain

$$RC\frac{di(t)}{dt} + i(t) + 400C\sin(200t) = 0$$

Substituting values and rearranging the equation becomes

$$5\times10^{-3}\frac{di(t)}{dt} + i(t) = -400\times10^{-6}\sin(200t)$$

The particular solution is of the form

$$i_p(t) = A\cos(200t) + B\sin(200t)$$

Substituting this into the differential equation and rearranging terms results in

$$\begin{aligned} &5\times10^{-3}\left[-200A\sin(200t) + 200B\cos(200t)\right] + A\cos(200t) + B\sin(200t) \\ &= -400\times10^{-6}\sin(200t) \end{aligned}$$

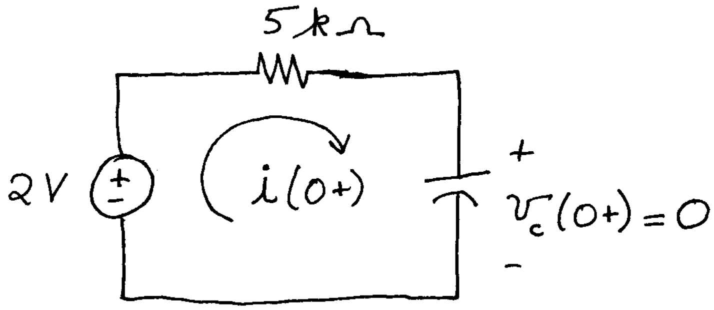
Equating the coefficients of the cos and sin terms gives the following equations:

$$-A + B = -400\times10^{-6} \quad \text{and } B + A = 0$$

from which we determine that $A = 200\times10^{-6}$ and $B = -200\times10^{-6}$. Furthermore, the complementary solution is $i_C(t) = K\exp(-t/\tau)$, and the complete solution is of the form

$$i(t) = 200\cos(200t) - 200\sin(200t) + K\exp(-t/\tau) \;\; \mu A$$

At t = 0+, the equivalent circuit is



from which we determine that $i(0+) = 2/5000 = 400\ \mu A$. Then evaluating our solution at t = 0+, we have $i(0+) = 400 = 200 + K$, from which we determine that $K = 200$ μA. Thus the complete solution is

$$i(t) = 200\cos(200t) - 200\sin(200t) + 200\exp(-t/\tau)\ \ \mu A$$

E4.8 The KVL equation is

$$Ri(t) + \frac{1}{C}\int_0^t i(x)dx + v_C(0+) - 10\exp(-t) = 0$$

Taking the derivative and multiplying by C, we obtain

$$RC\frac{di(t)}{dt} + i(t) + 10C\exp(-t) = 0$$

Substituting values and rearranging, the equation becomes

$$2\frac{di(t)}{dt} + i(t) = -20\times 10^{-6}\exp(-t)$$

The particular solution is of the form

$$i_p(t) = A\exp(-t)$$

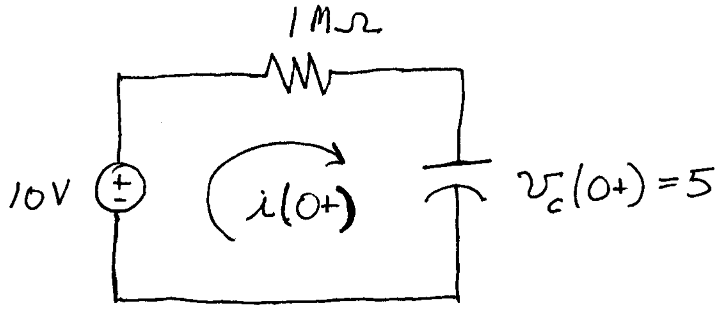
Substituting this into the differential equation and rearranging terms results in

$$-2A\exp(-t) + A\exp(-t) = -20\times 10^{-6}\exp(-t)$$

Equating the coefficients gives $A = 20\times 10^{-6}$. Furthermore, the complementary solution is $i_C(t) = K\exp(-t/2)$, and the complete solution is of the form

$$i(t) = 20\exp(-t) + K\exp(-t/2)\ \ \mu A$$

At t = 0+, the equivalent circuit is



from which we determine that $i(0+) = 5/10^6 = 5\ \mu A$. Then evaluating our solution at t = 0+, we have $i(0+) = 5 = 20 + K$, from which we determine that $K = -15\ \mu A$. Thus the complete solution is

$$i(t) = 20\exp(-t) - 15\exp(-t/2)\ \ \mu A$$

E4.9 (a) $\omega_0 = \dfrac{1}{\sqrt{LC}} = \dfrac{1}{\sqrt{10^{-3} \times 10^{-7}}} = 10^5 \quad \alpha = \dfrac{1}{2RC} = 2 \times 10^5 \quad \zeta = \dfrac{\alpha}{\omega_0} = 2$

(b) At t = 0+, the KCL equation for the circuit is

$$0.1 = \frac{v(0+)}{R} + i_L(0+) + Cv'(0+) \qquad (1)$$

However, $v(0+) = v(0-) = 0$, because the voltage across the capacitor cannot change instantaneously. Furthermore, $i_L(0+) = i_L(0-) = 0$, because the current through the inductance cannot change value instantaneously. Solving Equation (1) for $v'(0+)$ and substituting values, we find that $v'(0+) = 10^6$ V/s.

(c) To find the particular solution or forced response, we can solve the circuit in steady-state conditions. For a dc source, we treat the capacitance as an open and the inductance as a short. Because the inductance acts as a short $v_p(t) = 0$.

(d) Because the circuit is overdamped ($\zeta > 1$), the homogeneous solution is the sum of two exponentials. The roots of the characteristic solution are given by Equations 4.72 and 4.73:

$$s_1 = -\alpha - \sqrt{\alpha^2 - \omega_0^2} = -373.2 \times 10^3$$

$$s_2 = -\alpha + \sqrt{\alpha^2 - \omega_0^2} = -26.79 \times 10^3$$

Adding the particular solution to the homogeneous solution gives the general solution:

$$v(t) = K_1 \exp(s_1 t) + K_2 \exp(s_2 t)$$

Now using the initial conditions, we have

$$v(0+) = 0 = K_1 + K_2 \qquad v'(0+) = 10^6 = K_1 s_1 + K_2 s_2$$

Solving we find $K_1 = -2.887$ and $K_2 = 2.887$. Thus the solution is:

$$v(t) = 2.887[\exp(s_2 t) - \exp(s_1 t)]$$

E4.10 (a) $\omega_0 = \dfrac{1}{\sqrt{LC}} = \dfrac{1}{\sqrt{10^{-3} \times 10^{-7}}} = 10^5 \quad \alpha = \dfrac{1}{2RC} = 10^5 \quad \zeta = \dfrac{\alpha}{\omega_0} = 1$

(b) The solution for this part is the same as that for Exercise 4.9b in which we found that $v'(0+) = 10^6$ V/s.

(c) The solution for this part is the same as that for Exercise 4.9c in which we found $v_p(t) = 0$.

(d) The roots of the characteristic solution are given by Equations 4.72 and 4.73:

$$s_1 = -\alpha - \sqrt{\alpha^2 - \omega_0^2} = -10^5 \qquad s_2 = -\alpha + \sqrt{\alpha^2 - \omega_0^2} = -10^5$$

Because the circuit is critically damped $(\zeta = 1)$, the roots are equal and the homogeneous solution is of the form:

$$v(t) = K_1 \exp(s_1 t) + K_2 t \exp(s_1 t)$$

Adding the particular solution to the homogeneous solution gives the general solution:

$$v(t) = K_1 \exp(s_1 t) + K_2 t \exp(s_1 t)$$

Now using the initial conditions we have

$$v(0+) = 0 = K_1 \qquad v'(0+) = 10^6 = K_1 s_1 + K_2$$

Solving we find $K_2 = 10^6$ Thus the solution is:

$$v(t) = 10^6 t \exp(-10^5 t)$$

E4.11 (a) $\omega_0 = \dfrac{1}{\sqrt{LC}} = \dfrac{1}{\sqrt{10^{-3} \times 10^{-7}}} = 10^5 \quad \alpha = \dfrac{1}{2RC} = 2 \times 10^4 \quad \zeta = \dfrac{\alpha}{\omega_0} = 0.2$

(b) The solution for this part is the same as that for Exercise 4.9b in which we found that $v'(0+) = 10^6$ V/s.

(c) The solution for this part is the same as that for Exercise 4.9c in which we found $v_p(t) = 0$.

(d) Because we have $(\zeta < 1)$, this is the underdamped case and we have

$$\omega_n = \sqrt{\omega_0^2 - \alpha^2} = 97.98 \times 10^3$$

Adding the particular solution to the homogeneous solution gives the general solution:

$$v(t) = K_1 \exp(-\alpha t)\cos(\omega_n t) + K_2 \exp(-\alpha t)\sin(\omega_n t)$$

Now using the initial conditions we have

$$v(0+) = 0 = K_1 \qquad v'(0+) = 10^6 = -\alpha K_1 + \omega_n K_2$$

Solving we find $K_2 = 10.21$ Thus the solution is:

$$v(t) = 10.21\exp(-2\times 10^4 t)\sin(97.98\times 10^3 t)\ \text{V}$$

E4.12 The commands are:

```
syms ix t R C vCinitial w
ix = dsolve('(R*C)*Dix + ix = (w*C)*2*cos(w*t)', 'ix(0)=-vCinitial/R');
ians =subs(ix,[R C vCinitial w],[5000 1e-6 1 200]);
pretty(vpa(ians, 4))
ezplot(ians,[0 80e-3])
```

An m-file named Exercise_4_12 containing these commands can be found in the MATLAB folder.

E4.13 The commands are:

```
syms vc t
% Case I R = 300:
vc = dsolve('(1e-8)*D2vc + (1e-6)*300*Dvc+ vc =10', ...
            'vc(0) = 0','Dvc(0)=0');
vpa(vc,4)
ezplot(vc, [0 1e-3])
hold on % Turn hold on so all plots are on the same axes
% Case II R = 200:
vc = dsolve('(1e-8)*D2vc + (1e-6)*200*Dvc+ vc =10',...
            'vc(0) = 0','Dvc(0)=0');
vpa(vc,4)
ezplot(vc, [0 1e-3])
% Case III R = 100:
vc = dsolve('(1e-8)*D2vc + (1e-6)*100*Dvc+ vc =10',...
            'vc(0) = 0','Dvc(0)=0');
vpa(vc,4)
```

ezplot(vc, [0 1e-3])

An m-file named Exercise_4_13 containing these commands can be found in the MATLAB folder.

Problems

P4.1 The time constant τ is the interval required for the voltage to fall to exp(-1) $\cong$ 0.368 times its initial value. The time constant is given by $\tau =$ RC. The higher the time constant, the slower the rate of discharge of the capacitor.

P4.2 We have $v(t) = V_i \exp(-t/\tau)$ and $w(t) = \frac{1}{2}Cv^2(t)$ in which V_i is the initial voltage. The initial stored energy is $W_i = \frac{1}{2}CV_i^2$. After five time constants, we have $v(t) = V_i exp(-5) \approx 0.00674\ V_i$ so that 0.674 percent of the initial voltage remains.

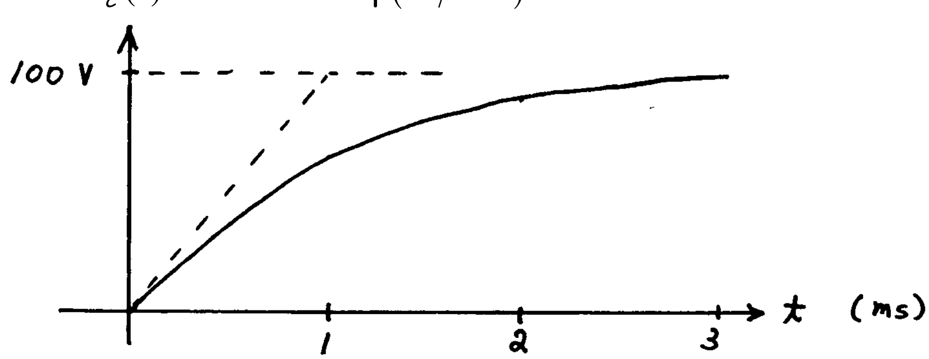
The energy is $w(5\tau) = \frac{1}{2}Cv^2(5\tau) = \frac{1}{2}\ Cv^2exp(-10) = w_i\, exp(-10) \approx 0.00005 w_i$ so only 0.005 ($\approx$0) percent of the initial energy remains.

P4.3 The solution is of the form given in Equation 4.19:

$$v_C(t) = V_s - V_s \exp(-t/RC)$$
$$RC = 10^5 \times 0.01 \times 10^{-6} = 1\ \text{ms}$$

Thus, we have

$$v_C(t) = 100 - 100\exp(-t/10^{-3})$$



P4.4* The solution is of the form of Equation 4.17:

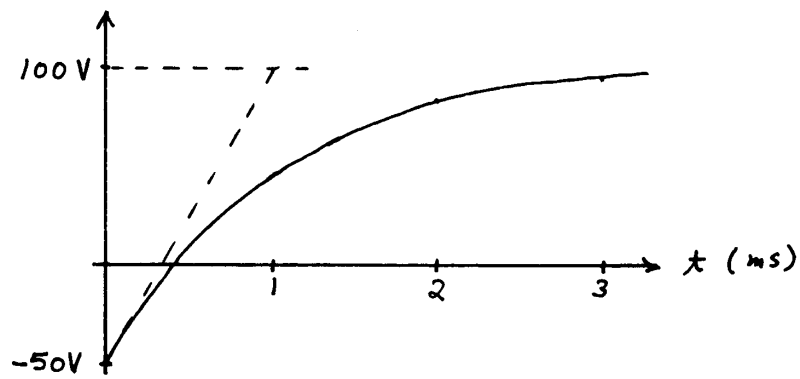
$$v_C(t) = V_s + K_2 \exp(-t/RC) = 100 + K_2 \exp(-t/10^{-3})$$

in which K_2 is a constant to be determined. At $t = 0+$, we have

$$v_C(0+) = -50 = 100 + K_2$$

Solving, we find that $K_2 = -150$ and the solution is

$$v_C(t) = 100 - 150 \exp(-t/10^{-3})$$



P4.5* The voltage across the capacitor is given by Equation 4.8.

$$v(t) = V_i \exp(-t / RC)$$

in which $V_i = 100$ V is the initial voltage, C = 100 μF is the capacitance, and R is the leakage resistance.

The energy stored in the capacitance is

$$w = \frac{1}{2} C v^2(t) = 0.5 \times 10^{-4} \times 100^2 \exp(-2t / RC)$$

Since we require the energy to be 90% of the initial value after one minute, we can write

$$0.9 \times 0.5 \times 10^{-4} \times 100^2 \leq 0.5 \times 10^{-4} \times 100^2 \exp(-120 / RC)$$

Solving we determine that RC must be greater than 1139 s. Thus, the leakage resistance must be greater than 11.39 MΩ.

P4.6 The voltage across the capacitor is given by Equation 4.8.

$$v(t) = V_i \exp(-t / RC)$$

in which V_i is the initial voltage. Substituting values, we have

$$v(10^{-3}) = 10 = V_i \exp(-0.001 / 0.0006) = V_i \exp(-1.667)$$

$$V_i = 52.94 \text{ V}$$

P4.7 (a)

$$RC = 20 \text{ ms}$$
$$v_C(t) = 50 \text{ for } t < 0$$
$$= 50\exp(-t/0.02) = 50\exp(-50t) \text{ for } t > 0$$
$$v_R(t) = 0 \text{ for } t < 0$$
$$= 50\exp(-t/0.02) = 50\exp(-50t) \text{ for } t > 0$$

(b) $$p_R(t) = \frac{[v_R(t)]^2}{R} = 2500\exp(-100t)\ \mu\text{W}$$

(c)
$$W = \int_0^\infty p_R(t)dt$$
$$= \int_0^\infty 2500\exp(-100t)dt$$
$$= -25\exp(-100t)\Big|_0^\infty$$
$$= 25\ \mu\text{J}$$

(d) The initial energy stored in the capacitance is

$$W = \frac{1}{2}C[v_C(0)]^2$$
$$= \frac{1}{2}\times 0.02\times 10^{-6}\times 50^2$$
$$= 25\ \mu\text{J}$$

P4.8 Equation 4.8 gives the expression for the voltage across a capacitance discharging through a resistance:

$$v_C(t) = V_i\exp(-t/RC)$$

After one-half-life, we have $v_C(t_{half}) = \frac{V_i}{2}$ and $\frac{V_i}{2} = V_i\exp(-t_{half}/RC)$.

Dividing by V_i and taking the natural logarithm of both sides, we have

$$-\ln(2) = -t_{half}/RC$$

Solving, we obtain

$$t_{half} = RC\ln(2) = 0.6931RC = 0.6931\tau$$

P4.9 Prior to $t = 0$, we have $v(t) = 0$ because the switch is closed. After $t = 0$, we can write the following KCL equation at the top node of the circuit:

$$\frac{v(t)}{R} + C\frac{dv(t)}{dt} = 1\text{ mA}$$

Multiplying both sides by R and substituting values, we have

$$0.01\frac{dv(t)}{dt} + v(t) = 10 \qquad (1)$$

The solution is of the form

$$v(t) = K_1 + K_2\exp(-t/RC) = K_1 + K_2\exp(-100t) \qquad (2)$$

Substituting Equation (2) into Equation (1), we eventually obtain

$$K_1 = 10$$

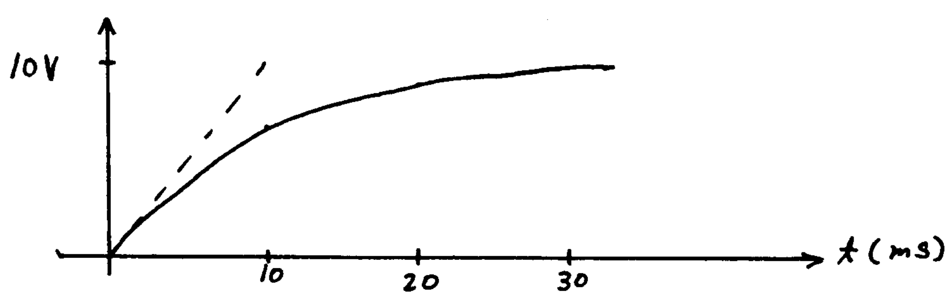
The voltage across the capacitance cannot change instantaneously, so we have

$$v(0+) = v(0-) = 0$$

$$v(0+) = 0 = K_1 + K_2$$

Thus, $K_2 = -K_1 = -10$, and the solution is

$$v(t) = 10 - 10\exp(-100t) \text{ for } t > 0$$



P4.10* See end of document for solution.

P4.11* This is a case of a capacitance discharging through a resistance. The voltage is given by Equation 4.8:

$$v_C(t) = V_i \exp(-t/RC)$$

At $t = 0$, we have $v_C(0) = 50 = V_i$. At $t = 30$, we have $v_C(30) = 25$. Thus, we can write $25 = 50\exp(-30/RC)$. Dividing by 50 and taking the natural logarithm of both sides, we obtain $-\ln(2) = -30/RC$. Rearranging, we have $R = \dfrac{30}{C\ln(2)} = 4.328\ \text{M}\Omega$.

P4.12 We have $P = P_i(0.97)^t = P_i\exp(-t/\tau)$ in which t is time in years, P_i is the initial purchasing power and τ is the time constant in years. Solving we have

$$(0.97)^t = \exp(-t/\tau)$$
$$t\ln(0.97) = -t/\tau$$
$$\tau = -1/\ln(0.97) = 32.83\ \text{years}$$

P4.13 During the charging interval, the time constant is $\tau_1 = R_1 C = 10\ \text{s}$, and the voltage across the capacitor is given by

$$v_C(t) = 1000[1 - \exp(-t/\tau_1)] \quad 0 \le t \le 25$$

At the end of the charging interval (t = 25 s), this yields

$$v_C(25) = 1000[1 - \exp(-2.5)] = 917.9\ \text{V}$$

The time constant during the discharge interval is $\tau_2 = R_2 C = 20\,\text{s}$. Working in terms of the time variable $t' = t - 25$, the voltage during the discharge interval is

$$v_C(t') = 917.9\exp(-t'/\tau_2) \quad 0 \le t'$$

At t = 50, or equivalently, $t' = 50 - 25 = 25$, this yields

$$917.9\exp(-1.25) = 263.0\ \text{V}$$

P4.14 The initial current is $V_i / R = 20000/100 = 200\ \text{A}$. No wonder one jumps!!! The time constant is $\tau = RC = 0.01\ \mu s$.

P4.15 The voltage across the resistance and capacitance is

$$v_C(t) = V_i \exp(-t/RC)$$

The initial charge stored on the capacitance is

$$Q_i = CV_i$$

The current through the resistance is

$$i_R(t) = \frac{V_i}{R}\exp(-t/RC)$$

The total charge passing through the resistance is

$$Q = \int_0^\infty i_R(t)dt$$

$$= \int_0^\infty \frac{V_i}{R}\exp(-t/RC)dt$$

$$= \frac{V_i}{R}\left[-RC\exp(-t/RC)\right]_0^\infty$$

$$= CV_1$$

P4.16* $v(t) = V_1\exp[-(t-t_1)/RC]$ for $t \geq t_1$

P4.17 See end of document for solution

P4.18 (a) The voltages across the capacitors cannot change instantaneously. Thus, $v_1(0+) = v_1(0-) = 100$ V and $v_2(0+) = v_2(0-) = 0$. Then, we can write

$$i(0+) = \frac{v_1(0+) - v_2(0+)}{R} = \frac{100 - 0}{100 \times 10^3} = 1\text{ mA}$$

(b) Applying KVL, we have

$$-v_1(t) + Ri(t) + v_2(t) = 0$$

$$\frac{1}{C_1}\int_0^t i(t)dt - 100 + Ri(t) + \frac{1}{C_2}\int_0^t i(t)dt + 0 = 0$$

Taking a derivative with respect to time and rearranging, we obtain

$$\frac{di(t)}{dt} + \frac{1}{R}\left(\frac{1}{C_1} + \frac{1}{C_2}\right)i(t) = 0 \qquad (1)$$

(c) The time constant is $\tau = R\dfrac{C_1C_2}{C_1 + C_2} = 50\text{ ms}$.

(d) The solution to Equation (1) is of the form

$$i(t) = K_1 \exp(-t/\tau)$$

However, $i(0+) = 1\text{ mA}$, so we have $K_1 = 1\text{ mA}$ and $i(t) = \exp(-20t)\text{ mA}$.

(e) The final value of $v_2(t)$ is

$$v_2(\infty) = \frac{1}{C_2}\int_0^\infty i(t)dt + v_2(0+)$$

$$= 10^6\int_0^t 10^{-3}\exp(-t/0.05)dt + 0$$

$$= 10^3(-0.05)\exp(-t/0.05)\Big|_0^\infty$$

$$= 50\text{ V}$$

Thus, the initial charge on C_1 is eventually divided equally between C_1 and C_2.

P4.19 For a dc steady-state analysis:

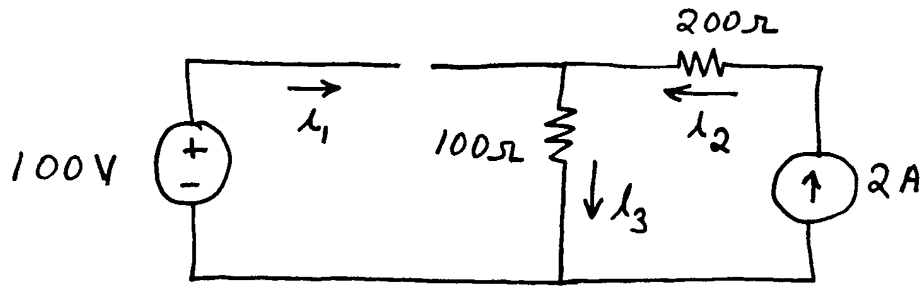
1. Replace capacitances with open circuits.
2. Replace inductances with short circuits.
3. Solve the resulting circuit, which consists of dc sources and resistances.

P4.20 In dc steady state conditions, the voltages across the capacitors are constant. Therefore, the currents through the capacitances, which are given by $i_C = C\,{dv}/{dt}$, are zero. Open circuits also have zero current.

Similarly, the currents through the inductances are constant. Therefore, the voltages across inductances, which are given by $v = L\,{di}/{dt}$, are zero. Short circuits also have zero voltage.

Therefore, we replace capacitances with open circuits and inductances with short circuits in a dc steady-state analysis

P4.21* In steady state, the equivalent circuit is:



Thus, we have

$$i_1 = 0$$
$$i_3 = i_2 = 2\ A$$

P4.22* After the switch opens and the circuit reaches steady state, the 10-mA current flows through the 1-kΩ resistance, and the voltage is 10 V. The initial voltage is $v_C(0+) = 0$. The time constant of the circuit is $RC = 10\ \text{ms}$. As a function of time, we have

$$v_C(t) = 10 - 10\exp(-t/RC) = 10 - 10\exp(-100t)$$

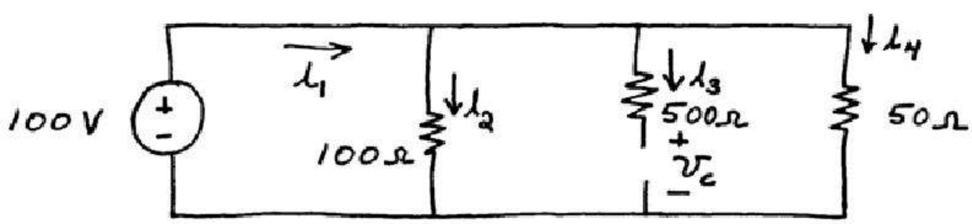
Let t_{99} denote the time at which the voltage reaches 99% is its final value. Then, we have

$$v_C(t_{99}) = 9.9 = 10 - 10\exp(-100t_{99})$$

Solving, we find

$$t_{99} = 46.05\ \text{ms}$$

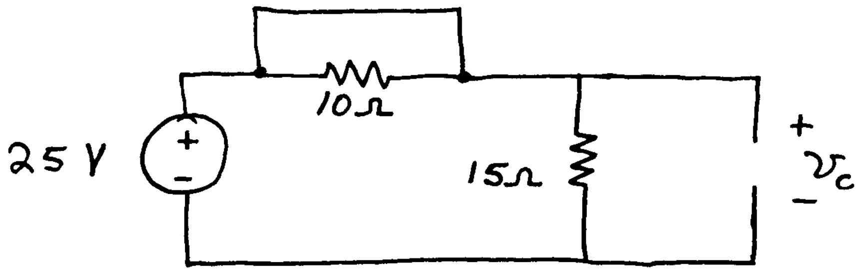
P4.23 In steady state with a dc source, the inductance acts as a short circuit and the capacitance acts as an open circuit. The equivalent circuit is:



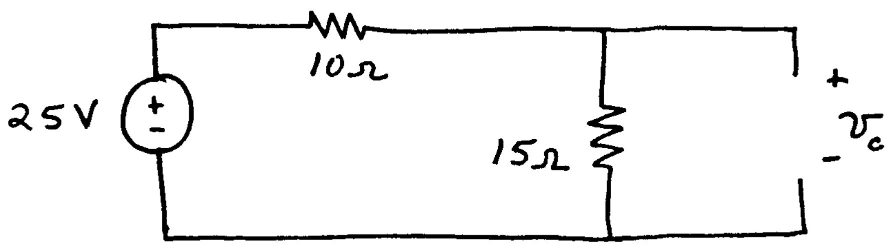
$$i_4 = (100\text{V})/(50\Omega) = 100\ A$$
$$i_3 = 0$$
$$i_2 = (100\text{V})/(100\Omega) = 1A$$
$$i_1 = i_2 + i_3 + i_4 = 3A$$
$$v_C = 100\ \text{V}$$

P4.24 $i_L = 5\text{ mA}$ $v_x = 10\text{ V}$ $v_C = -15\text{ V}$

P4.25 Prior to $t = 0$, the steady-state equivalent circuit is:



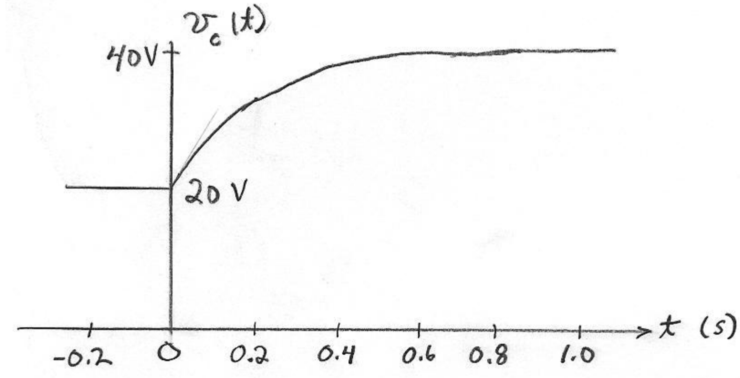
and we see that $v_c = 25\text{ V}$.
A long time after $t = 0$, the steady-state equivalent circuit is:



and we have $v_c = 25\dfrac{15}{15+10} = 15\text{ V}$.

P4.26 With the circuit in steady state prior to the capacitor behaves as an open circuit, the two 2-kΩ resistors are in parallel, and $v_C(0-) = (20\text{ mA}) \times (1\text{ k}\Omega) = 20\text{ V}$. Because there cannot be infinite current in this circuit, we have $v_C(0+) = 20\text{ V}$. After the switch opens and the circuit reaches steady state, we have $v_C(\infty) = (20\text{ mA}) \times (2\text{ k}\Omega) = 40\text{ V}$. For the Thévenin resistance seen by the capacitor is R_t = 2 kΩ and the time constant is $\tau = R_t C = 0.2$ s. The general form of the solution for is $= +$ $-$ v_C However, we know that $v_C(0+) = 20 = K_1 + K_2$ and $v_C(\infty) = 40 = K_1$. Solving we find that $K_2 = -20$. Thus, we have

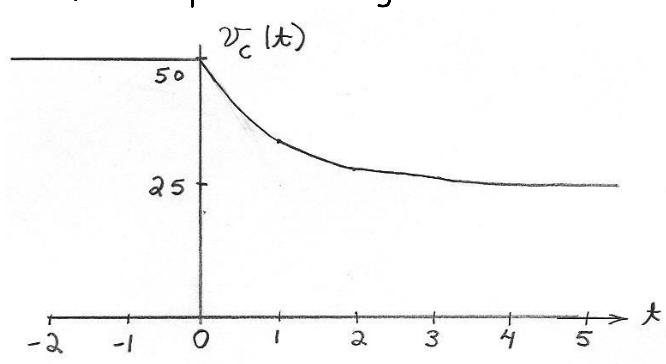
$$v_C(t) = 20 \quad t \le 0$$
$$= 40 - 20\exp(-t/\tau) \quad t \ge 0$$



P4.27 With the circuit in steady state prior to the capacitor behaves as an open circuit; the current is zero, and $v_C(0-) = 50$ V. Because there cannot be infinite currents in this circuit, we have $v_C(0+) = 50$ V. After the switch closes and the circuit reaches steady state, the source divides equally between the two 1-MΩ resistances, and we have $v_C(\infty) = 25$ V. For the Thévenin resistance seen by the capacitor is R_t = 500 kΩ and the time constant is $\tau = R_t C = 1$ s. The general form of the solution for is $v_C(t) = K_1 + K_2 \exp(-t/\tau)$. However, we know that $v_C(0+) = 50 = K_1 + K_2$ and $v_C(\infty) = 25 = K_1$. Solving we find that $K_2 = 25$. Thus we have

$$v_C(t) = 50 \quad t \le 0$$
$$= 25 + 25\exp(-t/\tau) \quad t \ge 0$$

A sketch of the capacitor voltage is:

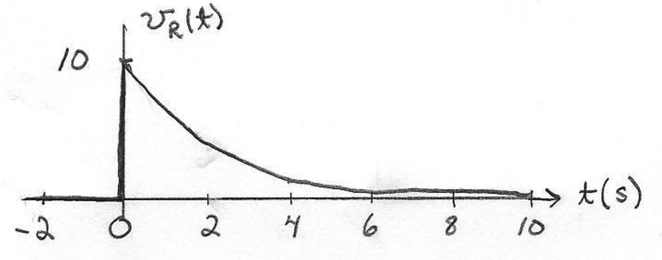


P4.28 $i_R = 2\text{ mA} \qquad v_C = 35\text{ V}$

P4.29* With the switch in position *A* and the circuit in steady state prior to the capacitor behaves as an open circuit, $v_R(0-) = 0$, and

$v_C(0-) = 30\dfrac{10\text{ k}\Omega}{10\text{ k}\Omega + 20\ \Omega\text{k}} = 10\text{ V}$. Because there cannot be infinite current in this circuit, we have $v_C(0+) = v_R(0+) = 10\text{ V}$. After switching and the circuit reaches steady state, we have $v_R(\infty) = 0\text{ V}$. For the resistance across the capacitor is R = 200 kΩ and the time constant is $\tau = RC = 2$ s. The general form of the solution for is $v_R(t) = K_1 + K_2\exp(-t/\tau)$. However, we know that $v_R(0+) = 10 = K_1 + K_2$ and $v_R(\infty) = 0 = K_1$. Solving we find that Thus, we have

$$v_R(t) = 0 \quad t < 0$$
$$= 10\exp(-t/\tau) \quad t \geq 0$$

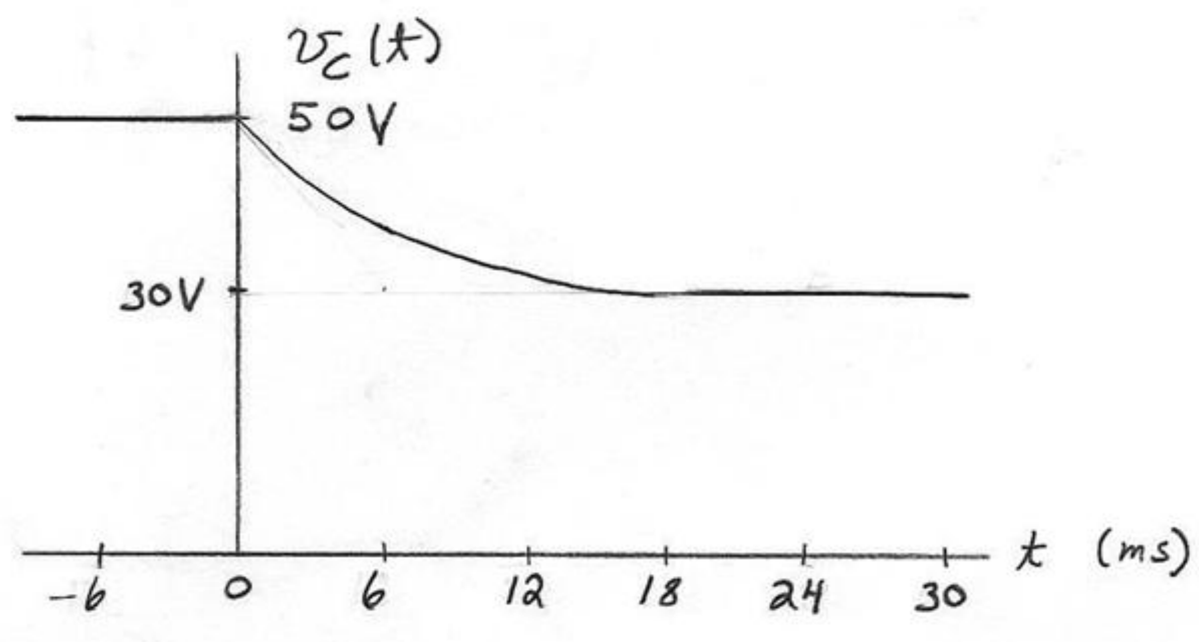


P4.30 With the switch closed and the circuit in steady state prior to t = 0, the capacitor behaves as an open circuit, and $v_C(0-) = 50$ V. Because there cannot be infinite current in this circuit when the switch opens, we have $v_C(0+) = 50$ V. After switching and the circuit reaches steady state, we $v_C(\infty) = 50\frac{15k\Omega}{15k\Omega+10k\Omega} = 30\text{V}$.

For t ≥ 0, the Thévenin resistance seen by the capacitor is $R_t = \frac{1}{1/10+1/15}$ =6 kΩ and the time constant is $\tau = R_t C = 6$ ms. The general form of the solution for t ≥ 0 is $v_C(t) = K_1 + K_2 \exp(-t/\tau)$. However, we know that $v_C(0+) = 50 = K_1 + K_2$ and $v_C(\infty) = 30 = K_1$. Solving, we find that $K_2 = 20$ .Thus, we have

$v_C(t) = 50 \quad t \le 0$

$v_C(t) = 30 + 20\exp(-t/\tau) \quad t \ge 0$



P4.31 The time constant τ is the interval required for the current to fall to $\exp(-1) \cong 0.368$ times its initial value. The time constant is given by $\tau = L/R$. Thus, to attain a long time constant, we need a large value for L and a small value for R.

P4.32 The general form of the solution is $x(t) = A + B\exp(-t/\tau)$. A is the steady-state solution for $t >> 0$. To determine the value of A, we replace the inductor with a short or the capacitance with an open and solve the resulting circuit. Next, we determine the value of the desired current or voltage immediately after $t = 0$, denoted by $x(0+)$. Then, we solve $x(0+) = A + B$ for the value of B. Finally, we determine the Thévenin resistance R_t from the perspective of the energy storage element (i.e., the resistance seen looking back into the circuit with the energy storage element removed) and compute the time constant: $\tau = R_t C$ for a capacitance or $\tau = L/R_t$ for an inductance.

P4.33* In steady state with the switch closed, we have $i(t) = 0$ for $t < 0$ because the closed switch shorts the source.

In steady state with the switch open, the inductance acts as a short circuit and the current becomes $i(\infty) = 1\,A$. The current is of the form

$$i(t) = K_1 + K_2 \exp(-Rt/L) \text{ for } t \geq 0$$

in which $R = 20\,\Omega$, because that is the Thévenin resistance seen looking back from the terminals of the inductance with the switch open. Also, we have

$$i(0+) = i(0-) = 0 = K_1 + K_2$$

$$i(\infty) = 1 = K_1$$

Thus, $K_2 = -1$ and the current (in amperes) is given by

$$\begin{aligned} i(t) &= 0 && \text{for } t < 0 \\ &= 1 - \exp(-20t) && \text{for } t \geq 0 \end{aligned}$$

P4.34 The general form of the solution is

$$i_L(t) = K_1 + K_2 \exp(-Rt/L)$$

At $t = 0+$, we have

$$i_L(0+) = i_L((0-) = 0 = K_1 + K_2$$

At , the inductance behaves as a short circuit, and we have

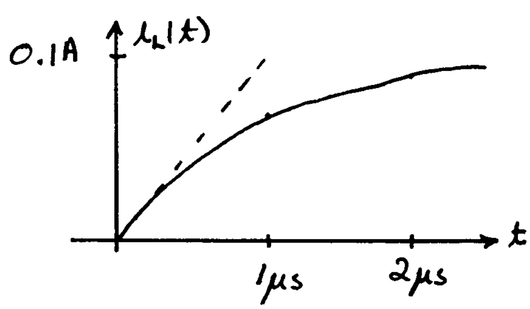
$$i_L(\infty) = 0.1 = K_1$$

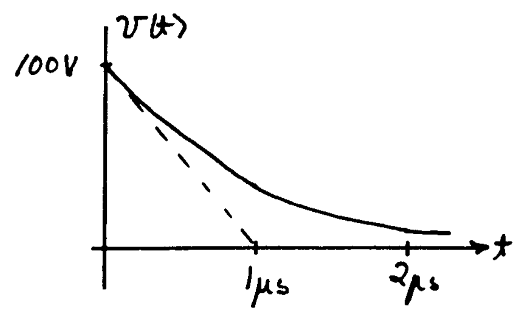
Thus, the solution for the current is

$$\begin{aligned} i_L(t) &= 0 \quad \text{for } t < 0 \\ &= 0.1 - 0.1\exp(-10^6 t) \quad \text{for } t > 0 \end{aligned}$$

The voltage is

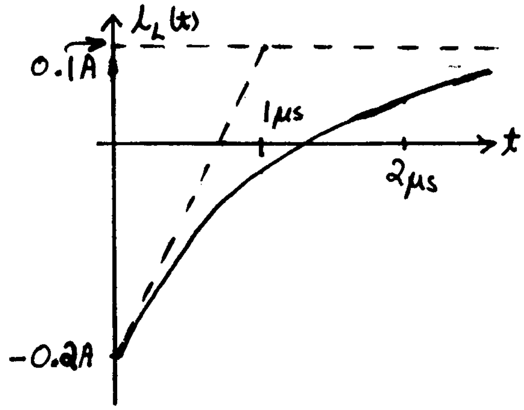
$$\begin{aligned} v(t) &= L\frac{di(t)}{dt} \\ &= 0 \text{ for } t < 0 \\ &= 100\exp(-10^6 t) \text{ for } t > 0 \end{aligned}$$

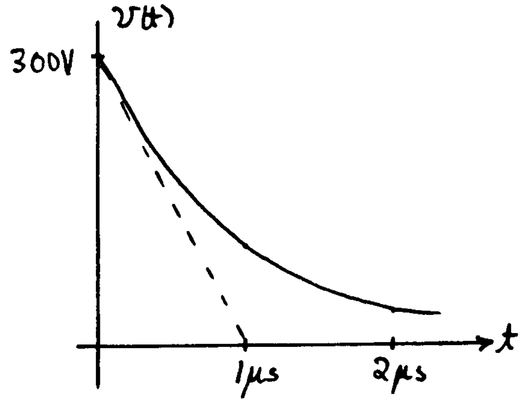




P4.35* The solution is similar to that for Problem P4.34.

$$\begin{aligned} i_L(t) &= 0.1 - 0.3\exp(-10^6 t) \text{ for } t > 0 \\ v(t) &= 300\exp(-10^6 t) \text{ for } t > 0 \end{aligned}$$





P4.36 The expression for the current $i_L(t)$ and volatge $v_L(t)$ is given by
$i_L(t)$: 0.5 * 0.5exp(*200t) and $v_L = 100\exp(-200t)$

The general solution is of the form

$i(t) = K_1 + K_2 \exp(-Rt/L)$

Comparing it with the given expression of current,

$K_1 = 0.5$, $K_2 = -0.5$ and $R/L = 200$

At $t = 0+$, we have

$i(0+) = 0 = K_1 + K_2$

and at $t = \infty$, we have

$i(\infty) = V/R = K_1 = 0.5$

$$v_L(t) = L\frac{di}{dt} = L \times 100\exp(-200t) = 100\exp(-200t)$$

$L = 1$

$R = 200L = 200 \times 1 = 200$
$V = 0.5R = 0.5 \times 200 = 100\text{V}$

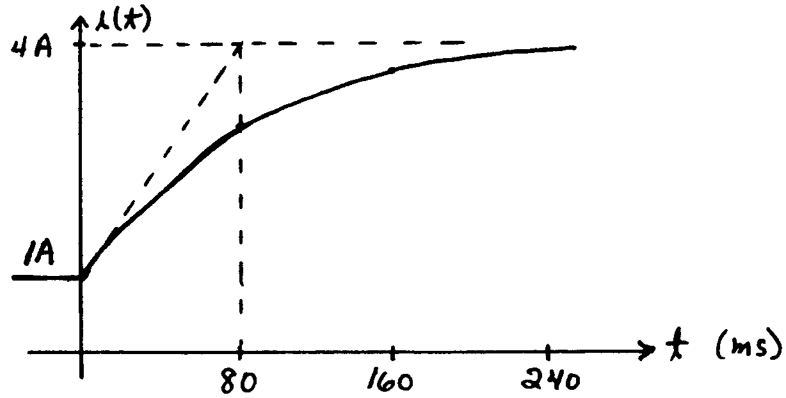
P4.37 In steady state, the inductor acts as a short circuit. With the switch open, the steady-state current is $(100\text{ V})/(100\ \Omega) = 1\ A$. With the switch closed, the current eventually approaches $i(\infty) = (100\text{ V})/(25\ \Omega) = 4\ A$. For $t > 0$, the current has the form

where $R = 25\ \Omega$, because that is the resistance with the switch closed. Now, we have

$$i(0+) = i(0-) = 1 = K_1 + K_2$$
$$i(\infty) = 4 = K_1$$

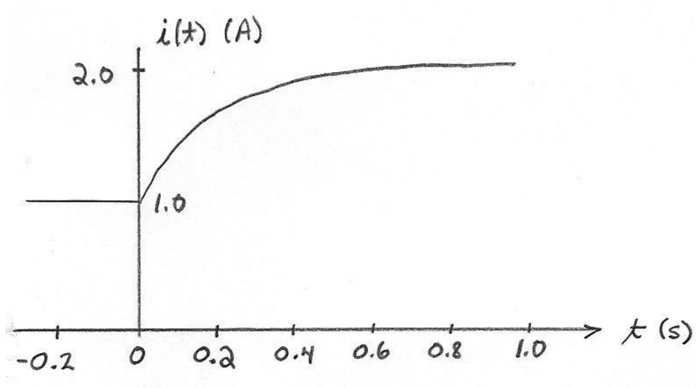
Thus, we have $K_2 = -3$. The current is

$$i(t) = 1 \qquad t < 0 \text{ (switch open)}$$
$$= 4 - 3\exp(-12.5t) \quad t \geq 0 \text{ (switch closed)}$$



P4.38 Before the switch closes, 1 A of current circulates through the source and the two 10-Ω resistors. Immediately after the switch closes, the inductor current remains 0 A, because infinite voltage is not possible in this circuit. (Because the inductor current is zero we can consider the inductor to be an open circuit at $t = 0+$.) Therefore, the current through the resistors is unchanged, and $i(0+) = 1$ A. In steady state, the inductor acts as a short circuit, and we have $i(\infty) = 2$ A. The Thévenin resistance seen by the inductor is 5 Ω because the two 10-Ω resistors are in parallel when we zero the source and look back into the circuit from the inductor terminals. Thus, the time constant is $\tau = L / R = 200$ ms. The general form of the solution is $i(t) = K_1 + K_2 \exp(-t / \tau)$. Using the initial and final values, we have $i(0+) = 1 = K_1 + K_2$ and $i(\infty) = 2 = K_1$ which yields $K_2 = -1$. Thus, the current is

$$i(t) = 1 \quad t \leq 0$$
$$= 2 - \exp(-t / \tau) \quad t \geq 0$$



P4.39 Prior to the current source is shorted, so we have

$$i_L(t) = 0 \text{ for } t < 0$$

After the switch opens at the current $i_L(t)$ increases from zero, headed for 1 A. The inductance sees a Thévenin resistance of 4 Ω, and the time constant is $\tau = L/4 = 1$ s, so we have

$$i_L(t) = 1 - \exp(-t) \text{ for } 0 < t < 1$$

At $t = 1$, the current reaches 0.632 A. Then, the switch closes, the source is shorted, and the current $i_L(t)$ decays toward zero. Because the inductance sees a resistance of 2 Ω, the time constant is $\tau = L/2 = 2$ s.

$$i_L(t) = 0.632\exp[-(t-1)/2] \text{ for } 1 < t$$

P4.40 (a) $i(t) = I_i \exp(-Rt/L)$ for $t \geq 0$

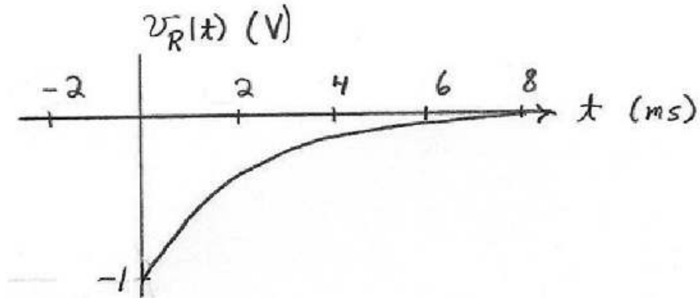
(b) $p_R(t) = Ri^2(t) = R(I_i)^2 \exp(-2Rt/L)$ for $t \geq 0$

(c)
$$
\begin{aligned}
W &= \int_0^{\infty} p_R(t)dt \\
&= \int_0^{\infty} R(I_i)^2 \exp(-2Rt/L)dt \\
&= R(I_i)^2 \left[-\frac{L}{2R} \exp(-2Rt/L) \right]_{t=0}^{t=\infty} \\
&= \frac{1}{2} L(I_i)^2
\end{aligned}
$$

which is precisely the expression for the energy stored in the inductance at $t = 0$.

P4.41 With the circuit in steady state before the switch opens, the inductor acts as a short circuit, the current through the inductor is $i_L(t) = 0.1\,A$, and $v_R(t) = 0$. Immediately after the switch opens, the inductor current remains 0.1 A because infinite voltage is not possible in this circuit. Then the return path for the inductor current is through the 10 kΩ resistor so $v_R(0+) = -1$V. After the switch opens, the current and voltage decay exponentially with a time constant $\tau = L/R = 2$ ms. Thus, we have

$$v_R(t) = 0 \quad t < 0$$
$$= -\exp(-t/\tau) \qquad t > 0$$



P4.42 In steady state, with the switch closed, the current is $i(t) = 2$ A for $t < 0$. The resistance of a voltmeter is very high -- ideally infinite. Thus, there is no path for the current in the inductance when the switch opens, and di/dt is very large in magnitude at $t = 0$. Consequently, the voltage induced in the inductance is very large in magnitude and an arc occurs across the switch. With an ideal meter and switch, the voltage would be infinite. The voltmeter could be damaged in this circuit.

P4.43* The current in a circuit consisting of an inductance L and series resistance R is given by $i(t) = I_i \exp(-Rt/L)$ in which I_i is the initial current. The initial energy stored in the inductance is $w_i = (1/2)LI_i^2$ and the energy stored as a function of time is

$$w(t) = (1/2)LI_i^2 \exp(-2Rt/L)$$

Thus, we require

$$0.75 \times (1/2)LI_i^2 \le (1/2)LI_i^2 \exp[-2R(3600)/10]$$

Solving, we determine that we require $R \le 399.6\ \mu\Omega$. In practice, the only practical way to attain such a small resistance for a 10-H inductance is to use superconductors.

P4.44 1. Write the circuit equation and, if it includes an integral, reduce the equation to a differential equation by differentiating.
2. Form the particular solution. Often this can be accomplished by adding terms like those found in the forcing function and its derivatives, including an unknown coefficient in each term. Next, solve for the unknown coefficients by substituting the trial solution into the differential equation and requiring the two sides of the equation to be identical.

3. Form the complete solution by adding the complementary solution $x_c(t) = K\exp(-t/\tau)$ to the particular solution.

4. Use initial conditions to determine the value of K.

P4.45* Applying KVL, we obtain the differential equation:

$$L\frac{di(t)}{dt} + Ri(t) = 5\exp(-t) \text{ for } t > 0 \qquad (1)$$

We try a particular solution of the form:

$$i_p(t) = A\exp(-t) \qquad (2)$$

in which A is a constant to be determined. Substituting Equation (2) into Equation (1), we have

$$-LA\exp(-t) + RA\exp(-t) = 5\exp(-t)$$

which yields

$$A = \frac{5}{R-L} = -1$$

The complementary solution is of the form

$$i_c(t) = K_1\exp(-Rt/L)$$

The complete solution is

$$i(t) = i_p(t) + i_c(t) = -\exp(-t) + K_1\exp(-Rt/L)$$

Before the switch closes, the current must be zero. Furthermore, the current cannot change instantaneously, so we have $i(0+) = 0$. Therefore, we have $i(0+) = 0 = -1 + K_1$ which yields $K_1 = 1$. Finally, the current is given by $i(t) = -\exp(-t) + \exp(-Rt/L)$ for $t \geq 0$.

P4.46 The differential equation is obtained by applying KVL for the node at the top end of the capacitance:

$$\frac{v_C(t) - v(t)}{R} + C\frac{dv_C(t)}{dt} = 0$$

Rearranging this equation and substituting $v(t) = t$, we have

$$RC\frac{dv_C(t)}{dt} + v_C(t) = t \text{ for } t > 0 \qquad (1)$$

We try a particular solution of the form

$$v_{Cp}(t) = A + Bt \qquad (2)$$

in which A and B are constants to be determined. Substituting Equation (2) into Equation (1), we have

$$RCB + A + Bt = t$$

Solving, we have

$B = 1$

$A = -RC$

Thus, the particular solution is

$$v_{Cp}(t) = -RC + t$$

The complementary solution (to the homogeneous equation) is of the form

$$v_{Cc}(t) = K_1 \exp(-t/RC)$$

Thus, the complete solution is

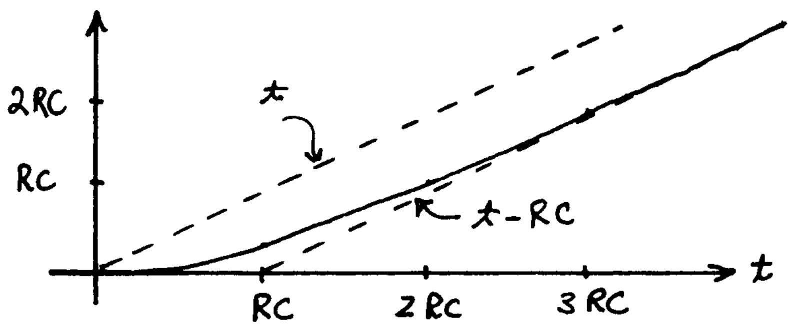
$$v_C(t) = v_{Cp}(t) + v_{Cc}(t) = -RC + t + K_1 \exp(-t/RC)$$

However, the solution must meet the given initial condition:

$$v_C(0) = 0 = -RC + K_1$$

Thus, $K_1 = RC$ and we have

$$v_C(t) = v_{Cp}(t) + v_{Cc}(t) = -RC + t + RC \exp(-t/RC)$$



P4.47* Write a current equation at the top node:

$$2\exp(-3t) = \frac{v_C(t)}{R} + C\frac{dv_C(t)}{dt}$$

Substitute the particular solution suggested in the hint:

$$2\exp(-3t) \equiv \frac{A}{R}\exp(-3t) - 3AC\exp(-3t)$$

Solving for A and substituting values of the circuit parameters, we find $A = -10^6$. The time constant is $\tau = RC = 1$ s, and the general form of the solution is

$$v_C(t) = K_1 \exp(-t) + v_p(t) = K_1 \exp(-t) - 10^6 \exp(-3t)$$

However because of the closed switch, we have $v_C(0+) = 0$. Substituting this into the general solution we find $K_1 = 10^6$. Thus

$$v_C(t) = 10^6 \exp(-t) - 10^6 \exp(-3t) \quad t > 0$$

P4.48* Write a current equation at the top node:

$$5\cos(10t) = \frac{v(t)}{R} + \frac{1}{L}\int_0^t v(t)dt + i_L(0)$$

Differentiate each term with respect to time to obtain a differential equation:

$$-50\sin(10t) = \frac{1}{R}\frac{dv(t)}{dt} + \frac{v(t)}{L}$$

Substitute the particular solution suggested in the hint:

$$-50\sin(10t) \equiv \frac{1}{R}[-10A\sin(10t) + 10B\cos(10t)] + \frac{1}{L}[A\cos(10t) + B\sin(10t)]$$

Equating coefficients of sine and cosine terms, we have

-50=-10A/R+B/L
0=10B/R+A/L

Solving for A and B and substituting values of the circuit parameters, we find A= 500/11 and B $= -50/11$. The time constant is $\tau = L/R = 1\text{s}$, and the general form of the solution is

$$v(t) = K_1\exp(-t/\tau) + v_p(t) = K_1\exp(-t/\tau) + \tfrac{500}{11}\cos(10t) - \tfrac{50}{11}\sin(10t)$$

However, because the current in the inductor is zero at t = 0+, the 5 A supplied by the source must flow through the 10-Ω resistor, and we have $v(0+) = 50$. Substituting this into the general solution we find $K_1 = \frac{50}{11}$

Thus

$$v(t) = \tfrac{50}{11}\exp(-t/\tau) + \tfrac{500}{11}\cos(10t) - \tfrac{50}{11}\sin(10t)$$

P4.49 Using KVL, we obtain the differential equation

$$L\frac{di(t)}{dt} + Ri(t) = v(t)$$

$$\frac{di(t)}{dt} + 300i(t) = 10\sin(300t)$$

We try a particular solution of the form

$$i_p(t) = A\cos(300t) + B\sin(300t)$$

in which A and B are constants to be determined. Substituting the proposed solution into the differential equation yields:

$$-300A\sin(300t) + 300B\cos(300t) + 300A\cos(300t) + 300B\sin(300t) = 10\sin(300t)$$

Equating coefficients of cosines yields

$$300B + 300A = 0$$

Equating coefficients of sines yields

$$-300A + 300B = 10$$

From these equations, we find that $B = 1/60$ and $A = -1/60$. The complementary solution is

$$i_c(t) = K_1 \exp(-Rt/L) = K_1 \exp(-300t)$$

and the complete solution is

$$i(t) = i_c(t) = i_p(t)$$
$$= K_1 \exp(-300t) - (1/60)\cos(300t) + (1/60)\sin(300t)$$

Finally, we use the given initial condition

$$i(0) = 0 = K_1 - 1/60$$

to determine that $K_1 = 1/60$. Thus, the solution for the current in amperes is

$$i(t) = (1/60)\exp(-300t) - (1/60)\cos(300t) + (1/60)\sin(300t)$$

P4.50 Applying KVL to the circuit, we have

$$L\frac{di(t)}{dt} + Ri(t) = v(t)$$

$$2\frac{di(t)}{dt} + 10i(t) = 10t$$

We try a particular solution of the form $i_p(t) = A + Bt$ in which A and B are constants to be determined. Substituting the proposed solution into the differential equation yields $2B + 10A + 10Bt = 10t$. From this, we find that $B = 1$ and $A = -0.2$. The complementary solution is

$$i_c(t) = K_1 \exp(-Rt/L) = K_1 \exp(-5t)$$

and the complete solution is

$$i(t) = i_c(t) + i_p(t) = K_1 \exp(-5t) - 0.2 + t$$

Finally, we use the given initial condition:

$$i(0) = 0 = K_1 - 0.2$$

to determine that $K_1 = 0.2$. Thus, the solution is

$$i(t) = 0.2\exp(-5t) - 0.2 + t$$

P4.51. Applying KVL, we obtain the differential equation:

$$2\frac{di(t)}{dt} + i(t) = 5\exp(-t)\sin(t) \quad \text{for } t > 0 \qquad (1)$$

Because the derivative of the forcing function is

$$-5\exp(-t)\sin(t) + 5\exp(-t)\cos(t)$$

we try a particular solution of the form:

$$i_p(t) = A\exp(-t)\sin(t) + B\exp(-t)\cos(t) \qquad (2)$$

in which A and B are constants to be determined. Substituting Equation (2) into Equation (1), we have

$$-2A\exp(-t)\sin(t) - 2B\exp(-t)\cos(t) + 2A\exp(-t)\cos(t) - 2B\exp(-t)\sin(t)$$
$$+ A\exp(-t)\sin(t) + B\exp(-t)\cos(t) \equiv 5\exp(-t)\sin(t)$$

which yields

$$-A - 2B = 5 \qquad \text{and} \qquad 2A - B = 0$$

Solving, we find $A = -1$ and $B = -2$

The complementary solution is of the form

$$i_c(t) = K_1 \exp(-Rt/L) = K_1 \exp(-t/2)$$

The complete solution is

$$i(t) = i_p(t) + i_c(t) = -\exp(-t)\sin(t) - 2\exp(-t)\cos(t) + K_1\exp(-t/2)$$

Before the switch closes, the current must be zero. Furthermore, the current cannot change instantaneously, so we have $i(0+) = 0$. Therefore, we have $i(0+) = 0 = -2 + K_1$ which yields $K_1 = 2$. Thus, the current is given by

$$i(t) = -\exp(-t)\sin(t) - 2\exp(-t)\cos(t) + 2\exp(-t/2) \quad \text{for } t \geq 0$$

P4.52 Usually, the particular solution includes terms with the same functional forms as the terms found in the forcing function and its derivatives. In this case, there are four different types of terms in the forcing function and its derivatives, namely $t\sin(t)$, $t\cos(t)$, $\sin(t)$, and $\cos(t)$.

Thus, we are led to try a particular solution of the form

$$v_p(t) = At\sin(t) + Bt\cos(t) + C\sin(t) + D\cos(t)$$

Substituting into the differential equation, we have

$$2\left[A\sin(t) + At\cos(t) + B\cos(t) - Bt\sin(t) + C\cos(t) - D\sin(t)\right] +$$
$$At\sin(t) + Bt\cos(t) + C\sin(t) + D\cos(t) \equiv 5t\sin(t)$$

We require the two sides of the equation to be identical. Equating coefficients of like terms, we have

$$2A - 2D + C = 0$$
$$2B + 2C + D = 0$$
$$2A + B = 0$$

$A - 2B = 5$

Solving these equations, we obtain $A = 1$, $B = -2$, $C = 6/5$, and $D = 8/5$. Thus, the particular solution is

$$v_p(t) = t\sin(t) - 2t\cos(t) + \frac{6}{5}\sin(t) + \frac{8}{5}\cos(t)$$

P4.53 The particular solution includes terms with the same functional forms as the terms found in the forcing function and its derivatives. In this case, there are three different types of terms in the forcing function and its derivatives, namely $t^2\exp(-t)$, $t\exp(-t)$, and $\exp(-t)$.

Thus, we are led to try a particular solution of the form

$$v_p(t) = At^2\exp(-t) + Bt\exp(-t) + C\exp(-t)$$

Substituting into the differential equation, we have

$$2At\exp(-t) - At^2\exp(-t) + B\exp(-t) - Bt\exp(-t) - C\exp(-t)$$
$$+ 3At^2\exp(-t) + 3Bt\exp(-t) + 3C\exp(-t) \equiv t^2\exp(-t)$$

We require the two sides of the equation to be identical. Equating coefficients of like terms, we have

$$2A = 1$$
$$2A + 2B = 0$$
$$B + 2C = 0$$

Solving these equations, we obtain $A = 1/2$, $B = -1/2$, and $C = 1/4$. Thus, the particular solution is

$$v_p(t) = (1/2)t^2\exp(-t) - (1/2)t\exp(-t) + (1/4)\exp(-t)$$

P4.54 (a) $\dfrac{di(t)}{dt} + 2i = 3\exp(-2t)$

(b) $\tau = L/R = 0.5$ s $\qquad i_c(t) = A\exp(-2t)$

(c) A particular solution of the form $i_p(t) = K\exp(-2t)$ does not work because the left-hand side of the differential equation is identically zero for this choice.

(d) Substituting $i_p(t) = Kt\exp(-2t)$ into the differential equation produces

$$-2Kt\exp(-2t) + K\exp(-2t) + 2Kt\exp(-2t) \equiv 3\exp(-2t)$$

from which we have $K = 3$.

(e) Adding the particular solution and the complementary solution we have

$$i(t) = A\exp(-2t) + 3t\exp(-2t)$$

However, the current must be zero in the inductor prior to $t = 0$ because of the open switch, and the current cannot change instantaneously in this circuit, so we have $i(0+) = 0$. This yields $A = 0$. Thus, the solution is

$$i(t) = 3t\exp(-2t)$$

P4.55 (a) $2\times 10^{-6}\dfrac{dv(t)}{dt} + \dfrac{v(t)}{50\times 10^3} = 5\times 10^{-6}\exp(-10t)$

(b) $\tau = RC = 0.1$ s $\qquad v_c(t) = A\exp(-10t)$

(c) A particular solution of the form $v_p(t) = K\exp(-10t)$ does not work because the left-hand side of the differential equation is identically zero for this choice.

(d) Substituting $v_p(t) = Kt\exp(-10t)$ into the differential equation produces

$$2\times 10^{-6}[-10Kt\exp(-10t) + K\exp(-10t)] + \frac{Kt\exp(-10t)}{50\times 10^3}$$
$$= 5\times 10^{-6}\exp(-10t)$$

from which we have $K = 2.5$.

(e) Adding the particular solution and the complementary solution we have

$$v(t) = A\exp(-10t) + 2.5t\exp(-10t)$$

However, the voltage across the capacitance must be zero prior to $t = 0$ because of the closed switch, and the voltage cannot change instantaneously in this circuit, so we have $v(0+) = 0$. This yields $A = 0$. Thus, the solution is

$$v(t) = 2.5t\exp(-10t)\ \text{V}$$

P4.56 First, we write the differential equation for the system and put it in the form

$$\frac{d^2x(t)}{dt^2} + 2\alpha\frac{dx(t)}{dt} + \omega_0^2(t) = f(t)$$

Then compute the damping ratio $\zeta = \alpha / \omega_0$.

If we have $\zeta < 1$, the system is underdamped, and the complementary solution is of the form

$$x_c(t) = K_1\exp(-\alpha t)\cos(\omega_n t) + K_2\exp(-\alpha t)\sin(\omega_n t)$$

in which $\omega_n = \sqrt{\omega_0^2 - \alpha^2}$.

If we have $\zeta = 1$, the system is critically damped, and the complementary solution is of the form

$$x_c(t) = K_1 \exp(s_1 t) + K_2 t \exp(s_1 t)$$

in which s_1 is the root of the characteristic equation $s^2 + 2\alpha s + \omega_0^2 = 0$.

If we have $\zeta > 1$, the system is overdamped, and the complementary solution is of the form

$$x_c(t) = K_1 \exp(s_1 t) + K_2 \exp(s_2 t)$$

in which s_1 and s_2 are the roots of the characteristic equation $s^2 + 2\alpha s + \omega_0^2 = 0$.

P4.57 The mechanical analog of the series RLC circuit is shown in Figure 4.20(b). The displacement x of the mass is analogous to electrical charge, the velocity $\frac{dx}{dt}$ is analogous to current, and force is analogous to voltage. The mass plays the role of the inductance, the spring plays the role of the capacitance, and the damper plays the role of the resistance. The equation of motion for the mechanical system can be put into the form of Equation 4.63

P4.58 We look at a circuit diagram and combine all of the inductors that are in series or parallel. Then, we combine all of the capacitances that are in series or parallel. Next, we count the number of energy storage elements (inductances and capacitances) in the reduced circuit. If there is only one energy-storage element, we have a first-order circuit. If there are two, we have a second-order circuit, and so forth.

P4.59 The unit step function is defined by

$$\begin{aligned} u(t) &= 0 \quad \text{for } t < 0 \\ &= 1 \quad \text{for } t \geq 0 \end{aligned}$$

The unit step function is illustrated in Figure 4.27 in the book. . DC unity voltage is a unit step function as it is 0 for t<0 and becomes 1 for t>0

P4.60 The sketch should resemble the response shown in Figure 4.29 for ζ =0.1. Second-order circuits that are severely underdamped$\zeta(<<1)$ have step responses that display considerable overshoot and ringing.

P4.61* Applying KVL to the circuit, we obtain

$$L\frac{di(t)}{dt} + Ri(t) + v_C(t) = v_s = 50 \qquad (1)$$

For the capacitance, we have

$$i(t) = C\frac{dv_C(t)}{dt} \tag{2}$$

Using Equation (2) to substitute into Equation (1) and rearranging, we have

$$\frac{d^2v_C(t)}{dt^2} + (R/L)\frac{dv_C(t)}{dt} + (1/LC)v_C(t) = 50/LC \tag{3}$$

$$\frac{d^2v_C(t)}{dt^2} + 4\times 10^4\frac{dv_C(t)}{dt} + 10^8 v_C(t) = 50\times 10^8$$

We try a particular solution of the form $v_{Cp}(t) = A$, resulting in $A = 50$. Thus, $v_{Cp}(t) = 50$. (An alternative method to find the particular solution is to solve the circuit in dc steady state. Since the capacitance acts as an open circuit, the steady-state voltage across it is 50 V.) Comparing Equation (3) with Equation 4.67 in the text, we find

$$\alpha = \frac{R}{2L} = 2\times 10^4$$

$$\omega_0 = \frac{1}{\sqrt{LC}} = 10^4$$

Since we have $\alpha > \omega_0$, this is the overdamped case. The roots of the characteristic equation are found from Equations 4.72 and 4.73 in the text.

$$s_1 = -\alpha + \sqrt{\alpha^2 - \omega_0^2} = -0.2679\times 10^4$$

$$s_2 = -\alpha - \sqrt{\alpha^2 - \omega_0^2} = -3.732\times 10^4$$

The complementary solution is

$$v_{Cc}(t) = K_1\exp(s_1 t) + K_2\exp(s_2 t)$$

and the complete solution is

$$v_C(t) = 50 + K_1\exp(s_1 t) + K_2\exp(s_2 t)$$

The initial conditions are

$$v_C(0) = 0 \quad \text{and} \quad i(0) = 0 = C\frac{dv_C(t)}{dt}\bigg|_{t=0}$$

Thus, we have

$$v_C(0) = 0 = 50 + K_1 + K_2$$

$$\frac{dv_C(t)}{dt}\Big|_{t=0} = 0 = s_1K_1 + s_2K_2$$

Solving, we find $K_1 = -53.87$ and $K_2 = 3.867$. Finally, the solution is

$$v_C(t) = 50 - 53.87\exp(s_1t) + 3.867\exp(s_2t)$$

P4.62* As in the solution to P4.61, we have

$$v_{Cp}(t) = 50$$

$$\alpha = \frac{R}{2L} = 10^4$$

$$\omega_0 = \frac{1}{\sqrt{LC}} = 10^4$$

Since we have $\alpha = \omega_0$,this is the critically damped case. The roots of the characteristic equation are equal:

$$s_1 = -\alpha + \sqrt{\alpha^2 - \omega_0^2} = -10^4$$

$$s_2 = -\alpha - \sqrt{\alpha^2 - \omega_0^2} = -10^4$$

The complementary solution is given in Equation 4.75 in the text:

$$v_{Cc}(t) = K_1\exp(s_1t) + K_2t\exp(s_1t)$$

and the complete solution is

$$v_C(t) = 50 + K_1\exp(s_1t) + K_2t\exp(s_1t)$$

The initial conditions are

$$v_C(0) = 0 \quad \text{and} \quad i(0) = 0 = C\frac{dv_C(t)}{dt}\Big|_{t=0}$$

Thus, we have

$$v_C(0) = 0 = 50 + K_1$$

$$\frac{dv_C(t)}{dt}\Big|_{t=0} = 0 = s_1K_1 + K_2$$

Solving, we find $K_1 = -50$ and $K_2 = -50 \times 10^4$. Finally, the solution is

$$v_C(t) = 50 - 50\exp(s_1t) - (50 \times 10^4)t\exp(s_1t)$$

P4.63* As in the solution to P4.61, we have

$$v_{Cp}(t) = 50$$

$$\alpha = \frac{R}{2L} = 0.5 \times 10^4$$

$$\omega_0 = \frac{1}{\sqrt{LC}} = 10^4$$

Since we have $\alpha < \omega_0$, this is the under-damped case. The natural frequency is given by Equation 4.76 in the text:

$$\omega_n = \sqrt{\omega_0^2 - \alpha^2} = 8.660 \times 10^3$$

The complementary solution is given in Equation 4.77 in the text:

$$v_{Cc}(t) = K_1 \exp(-\alpha t)\cos(\omega_n t) + K_2 \exp(-\alpha t)\sin(\omega_n t)$$

and the complete solution is

$$v_C(t) = 50 + K_1 \exp(-\alpha t)\cos(\omega_n t) + K_2 \exp(-\alpha t)\sin(\omega_n t)$$

The initial conditions are

$$v_C(0) = 0 \quad \text{and} \quad i(0) = 0 = C\frac{dv_C(t)}{dt}\bigg|_{t=0}$$

Thus, we have

$$v_C(0) = 0 = 50 + K_1$$

$$\frac{dv_C(t)}{dt}\bigg|_{t=0} = 0 = -\alpha K_1 + \omega_n K_2$$

Solving, we find $K_1 = -50$ and $K_2 = -28.86$. Finally, the solution is

$$v_C(t) = 50 - 50\exp(-\alpha t)\cos(\omega_n t) - (28.86)\exp(-\alpha t)\sin(\omega_n t)$$

P4.64 (a) Using Equation 4.103 from the text, the damping coefficient is

$$\alpha = \frac{1}{2RC} = 20 \times 10^6$$

Equation 4.104 gives the undamped resonant frequency

$$\omega_0 = \frac{1}{\sqrt{LC}} = 10 \times 10^6$$

Equation 4.71 gives the damping ratio

$$\zeta = \alpha/\omega_0 = 2$$

Thus, we have an overdamped circuit.

(b) Writing a current equation at $t = 0+$, we have

$$\frac{v(0+)}{R} + i_L(0+) + Cv'(0+) = 1$$

Substituting $v(0+) = 0$ and $i_L(0+) = 0$, yields

$$v'(0+) = 1/C = 10^9$$

(c) Under steady-state conditions, the inductance acts as a short circuit. Therefore, the particular solution for $v(t)$ is:

$$v_p(t) = 0$$

(d) The roots of the characteristic equation are found from Equations 4.72 and 4.73 in the text.

$$s_1 = -\alpha + \sqrt{\alpha^2 - \omega_0^2} = -2.679 \times 10^6$$

$$s_2 = -\alpha - \sqrt{\alpha^2 - \omega_0^2} = -37.32 \times 10^6$$

The complementary solution is

$$v_c(t) = K_1 \exp(s_1 t) + K_2 \exp(s_2 t)$$

and (since the particular solution is zero) the complete solution is

$$v(t) = K_1 \exp(s_1 t) + K_2 (s_2 t)$$

The initial conditions are

$$v(0+) = 0 \quad \text{and} \quad v'(0+) = 10^9$$

Thus, we have

$$v(0+) = 0 = K_1 + K_2$$

$$v'(0+) = 10^9 = s_1 K_1 + s_2 K_2$$

Solving, we find $K_1 = 28.87$ and $K_2 = -28.87$. Finally, the solution is

$$v(t) = 28.87 \exp(s_1 t) - 28.87 \exp(s_2 t)$$

P4.65 (a) Using Equation 4.103 from the text, the damping coefficient is

$$\alpha = \frac{1}{2RC} = 10 \times 10^6$$

Equation 4.104 gives the undamped resonant frequency:

$$\omega_0 = \frac{1}{\sqrt{LC}} = 10 \times 10^6$$

Equation 4.71 gives the damping ratio:

$$\zeta = \alpha / \omega_0 = 1$$

Thus, we have a critically damped circuit.

(b) Writing a current equation at $t = 0+$, we have

$$\frac{v(0+)}{R} + i_L(0+) + Cv'(0+) = 1$$

Substituting $v(0+) = 0$ and $i_L(0+) = 0$, yields

$$v'(0+) = 1/C = 10^9$$

(c) Under steady-state conditions, the inductance acts as a short circuit. Therefore, the particular solution for $v(t)$ is:

$$v_p(t) = 0$$

(d) The roots of the characteristic equation are found from Equations 4.72 and 4.73 in the text.

$$s_1 = -\alpha + \sqrt{\alpha^2 - \omega_0^2} = -10 \times 10^6$$

$$s_2 = -\alpha - \sqrt{\alpha^2 - \omega_0^2} = -10 \times 10^6$$

The complementary solution is given in Equation 4.75 in the text:

$$v_C(t) = K_1 \exp(s_1 t) + K_2 t \exp(s_1 t)$$

and the complete solution is

$$v(t) = v_C(t) + v_p(t) = K_1 \exp(s_1 t) + K_2 t \exp(s_1 t)$$

The initial conditions are

$$v(0+) = 0 \quad \text{and} \quad v(0+) = 10^9$$

Thus, we have

$$v(0+) = 0 = K_1$$

$$v'(0+) = 10^9 = s_1 K_1 + K_2$$

Solving, we find $K_1 = 0$ and $K_2 = 10^9$. Finally, the solution is

$$v_C(t) = 10^9 t \exp(-10^7 t)$$

P4.66 (a) Using Equation 4.103 from the text, the damping coefficient is

$$\alpha = \frac{1}{2RC} = 10^6$$

Equation 4.104 gives the undamped resonant frequency:

$$\omega_0 = \frac{1}{\sqrt{LC}} = 10 \times 10^6$$

Equation 4.71 gives the damping ratio:

$$\zeta = \alpha/\omega_0 = 0.1$$

Thus, we have an underdamped circuit.

(b) Writing a current equation at $t = 0+$, we have

$$\frac{v(0+)}{R} + i_L(0+) + Cv'(0+) = 1$$

Substituting $v(0+) = 0$ and $i_L(0+) = 0$, yields

$$v'(0+) = 1/C = 10^9$$

(c) Under steady-state conditions, the inductance acts as a short circuit. Therefore, the particular solution for $v(t)$ is:

$$v_p(t) = 0$$

(d) The natural frequency is given by Equation 4.76 in the text:

$$\omega_n = \sqrt{\omega_0^2 - \alpha^2} = 9.950 \times 10^6$$

The complementary solution is given in Equation 4.77 in the text:

$$v_C(t) = K_1 \exp(-\alpha t)\cos(\omega_n t) + K_2 \exp(-\alpha t)\sin(\omega_n t)$$

and the complete solution is

$$v(t) = K_1 \exp(-\alpha t)\cos(\omega_n t) + K_2 \exp(-\alpha t)\sin(\omega_n t)$$

The initial conditions are

$$v(0+) = 0 \quad \text{and} \quad v'(0+) = 10^9$$

Thus, we have

$$v(0+) = 0 = K_1$$

$$v'(0+) = 10^9 = -\alpha K_1 + \omega_n K_2$$

Solving, we find $K_1 = 0$ and $K_2 = 100.5$. Finally, the solution is

$$v(t) = 100.5 \exp(-\alpha t)\sin(\omega_n t)$$

P4.67 Write a KVL equation for the circuit:

$$10\cos(100t) = L\frac{di(t)}{dt} + Ri(t) + \frac{1}{C}\int_0^t i(t)dt + v_C(0)$$

Differentiate each term with respect to time to obtain a differential equation:

$$-1000\sin(10t) = L\frac{d^2 i(t)}{dt^2} + R\frac{di(t)}{dt} + \frac{i(t)}{C}$$

Substitute the particular solution suggested in the hint to obtain:

$$-10^3\sin(10t) \equiv L[-10^4 A\cos(100t) - 10^4 B\sin(100t)]$$
$$+ R[-100A\sin(100t) + 100B\cos(100t)] + \frac{1}{C}[A\cos(100t) + B\sin(100t)]$$

Equating coefficients of sine and cosine terms, we have

$$-10^3 = -10^4 BL - 100AR + \frac{B}{C}$$

$$0 = -10^4 AL + 100BR + \frac{A}{C}$$

Solving for A and B and substituting values of the circuit parameters, we find $A = 0.2$ and $B = 0$. Thus, the particular solution is

$$i_p(t) = 0.2\cos(100t)$$

Using Equations 4.60 and 4.61 from the text, we have

$$\alpha = \frac{R}{2L} = 25$$

$$\omega_0 = \frac{1}{\sqrt{LC}} = 100$$

Because we have $\alpha < \omega_0$, this is the under-damped case. The natural frequency is given by Equation 4.76 in the text:

$$\omega_n = \sqrt{\omega_0^2 - \alpha^2} = 96.82$$

The complementary solution is given in Equation 4.77 in the text:

$$i_c(t) = K_1 \exp(-\alpha t)\cos(\omega_n t) + K_2 \exp(-\alpha t)\sin(\omega_n t)$$

and the complete solution is

$$i(t) = 0.2\cos(100t) + K_1 \exp(-\alpha t)\cos(\omega_n t) + K_2 \exp(-\alpha t)\sin(\omega_n t)$$

However, because the current is zero at t = 0+, the voltage across the inductor must be 10 V which implies that $di(0+)/dt = 10$. Thus, we can write

$$i(0+) = 0 = 0.2 + K_1$$

$$\frac{di(0+)}{dt} = 10 = -\alpha K_1 + \omega_n K_2$$

Solving we find $K_1 = -0.2$ and $K_2 = 0.05164$. Finally, the solution is:

$$i(t) = 0.2\cos(100t) - 0.2\exp(-\alpha t)\cos(\omega_n t) + 0.05164\exp(-\alpha t)\sin(\omega_n t)$$

P4.68 As in the solution to P4.67, we have

$$-10^3 = -10^4 BL - 100AR + \frac{B}{C}$$

$$0 = -10^4 AL + 100BR + \frac{A}{C}$$

Solving for A and B and substituting values of the circuit parameters, we find $A = 0.05$ and $B = 0$. Thus the particular solution is

$$i_p(t) = 0.05\cos(100t)$$

Using Equations 4.60 and 4.61 from the text, we have

$$\alpha = \frac{R}{2L} = 100$$

$$\omega_0 = \frac{1}{\sqrt{LC}} = 100$$

Since we have $\alpha = \omega_0$, this is the critically damped case. The roots of the characteristic equation are equal:

$$s_1 = -\alpha + \sqrt{\alpha^2 - \omega_0^2} = -100$$

$$s_2 = -\alpha - \sqrt{\alpha^2 - \omega_0^2} = -100$$

The complementary solution is given in Equation 4.75 in the text:

$$i_c(t) = K_1 \exp(s_1 t) + K_2 t \exp(s_1 t)$$

and the complete solution is

$$i(t) = 0.05\cos(100t) + K_1 \exp(s_1 t) + K_2 t \exp(s_1 t)$$

As in the solution to P4.51, The initial conditions are

$$i(0+) = 0 = 0.05 + K_1$$

$$\frac{di(0+)}{dt} = 10 = s_1 K_1 + K_2$$

Solving we find $K_1 = -0.05$ and $K_2 = 5$. Finally, the solution is

$$i(t) = 0.05\cos(100t) - 0.05\exp(-100t) + 5t\exp(-100t)$$

P4.69 As in the solution to P4.67, we have

$$-10^3 = -10^4 BL - 100AR + \frac{B}{C}$$

$$0 = -10^4 AL + 100BR + \frac{A}{C}$$

Solving for A and B and substituting values of the circuit parameters, we find $A = 0.025$ and $B = 0$. Thus, the particular solution is

$$i_p(t) = 0.025\cos(100t)$$

Using Equations 4.60 and 4.61 from the text, we have

$$\alpha = \frac{R}{2L} = 200$$

$$\omega_0 = \frac{1}{\sqrt{LC}} = 100$$

Since we have $\alpha > \omega_0$, this is the overdamped case. The roots of the characteristic equation are found from Equations 4.72 and 4.73 in the text.

$$s_1 = -\alpha + \sqrt{\alpha^2 - \omega_0^2} = -26.79$$

$$s_2 = -\alpha - \sqrt{\alpha^2 - \omega_0^2} = -373.2$$

The complementary solution is

$$i_c(t) = K_1 \exp(s_1 t) + K_2 \exp(s_2 t)$$

and the complete solution is

$$i(t) = 0.025\cos(100t) + K_1 \exp(s_1 t) + K_2 \exp(s_2 t)$$

As in the solution to P4.51, The initial conditions are

$$i(0+) = 0 = 0.025 + K_1 + K_2$$

$$\frac{di(0+)}{dt} = 10 = s_1 K_1 + s_2 K_2$$

Solving, we find $K_1 = 0.00193$ and $K_2 = -0.02693$. Finally, the solution is

$$i(t) = 0.025\cos(100t) + 0.00193\exp(s_1 t) - 0.02693\exp(s_2 t)$$

P4.70 (a) Applying KCL, we have:

$$\frac{1}{L}\int_0^t v(t)dt + C\frac{dv}{dt} = 2\sin(10^4 t)$$

Taking the derivative, multiplying by *L,* and rearranging, we have

$$LC\frac{d^2v}{dt^2} + v(t) = 2L10^4\cos(10^4 t)$$

(b) This is a parallel *RLC* circuit having $R = \infty$. Using Equation 4.103 from the text, the damping coefficient is

$$\alpha = \frac{1}{2RC} = 0$$

Equation 4.104 gives the undamped resonant frequency:

$$\omega_0 = \frac{1}{LC} = 10^4$$

Equation 4.76 in the text gives the natural frequency:

$$\omega_n = \sqrt{\omega_0^2 - \alpha^2} = 10^4$$

The complementary solution given in Equation 4.77 becomes:

$$v_C(t) = K_1\exp(-\alpha t)\cos(\omega_n t) + K_2\exp(-\alpha t)\sin(\omega_n t)$$
$$= K_1\cos(10^4 t) + K_2\sin(10^4 t)$$

(c)The usual form for the particular solution doesn't work because it has the same form as the complementary solution. In other words, if we substitute a trial particular solution of the form $v_p(t) = A\cos(10^4 t) + B\sin(10^4 t)$, the left-hand side of the differential equation becomes zero and cannot match the forcing function.

(d) When we substitute $v_p(t) = At\cos(10^4 t) + Bt\sin(10^4 t)$ into the differential equation, we eventually obtain

$$-2A10^{-4}\sin(10^4 t)+2B10^{-4}\cos(10^4 t)\equiv 200\cos(10^4 t)$$

from which we obtain $B=10^6$ and $A=0$. Thus, the particular solution is

$$v_p(t)=10^6 t\sin\left(10^4 t\right)$$

(e) The complete solution is

$$v(t)=v_p(t)+v_C(t)=10^6 t\sin\left(10^4 t\right)+K_1\cos\left(10^4 t\right)+K_2\sin\left(10^4 t\right)$$

However, we have $v(0)=0$, which yields $K_1=0$. Also, by KCL, the current through the capacitance is zero at $t=0+$, so we have

$$\frac{dv(0+)}{dt}=0=10^4 K_2$$

which yields $K_2=0$, so the complete solution is

$$v(t)=v_p(t)+v_C(t)=10^6 t\sin\left(10^4 t\right)$$

4.71 Note: We use $v_C(t)=v(t)$ in this solution. Prior to $t=0$, we have $v_C(t)=0$ because the switch is closed. After $t=0$, we can write the following KCL equation at the top node of the circuit:

$$\frac{v_C(t)}{R}+C\frac{dv_C(t)}{dt}=1\text{ mA}$$

Multiplying both sides by R and substituting values, we have

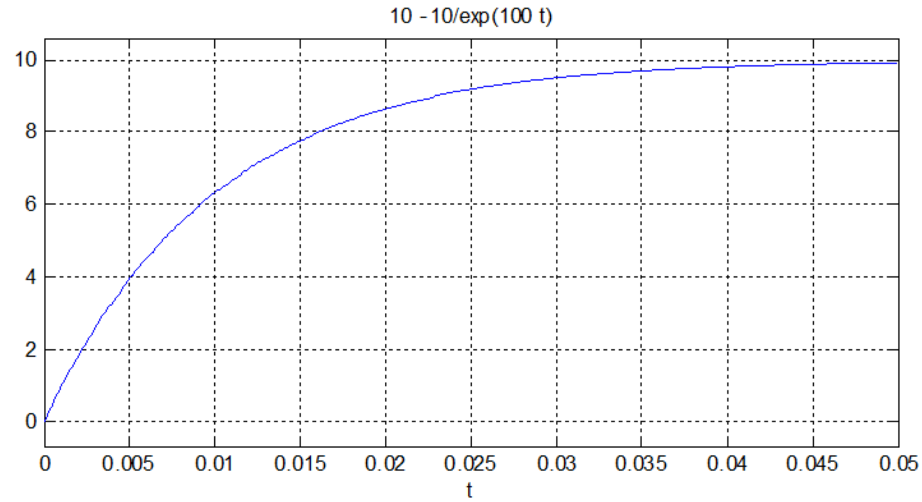
$$0.01\frac{dv_C(t)}{dt}+v_C(t)=10$$

The voltage across the capacitance cannot change instantaneously, so we have

$v_C(0+)=v_C(0-)=0$

```
syms vC t
vC = dsolve('DvC*(0.01) + vC = 10', 'vC(0) = 0')
ezplot(vC,[0 50e-3])
```

The results are: $v_C(t)=10-10\exp(-100t)$ for $t>0$ and this plot:



P4.72 The differential equation is obtained by applying KVL for the node at the top end of the capacitance:

$$\frac{v_C(t) - v(t)}{R} + C\frac{dv_C(t)}{dt} = 0$$

Rearranging this equation and substituting $v(t) = t$, we have

$$RC\frac{dv_C(t)}{dt} + v_C(t) = t \text{ for } t > 0$$

Because the source is zero prior to zero time, $v_C(0) = 0$.

The MATLAB commands are

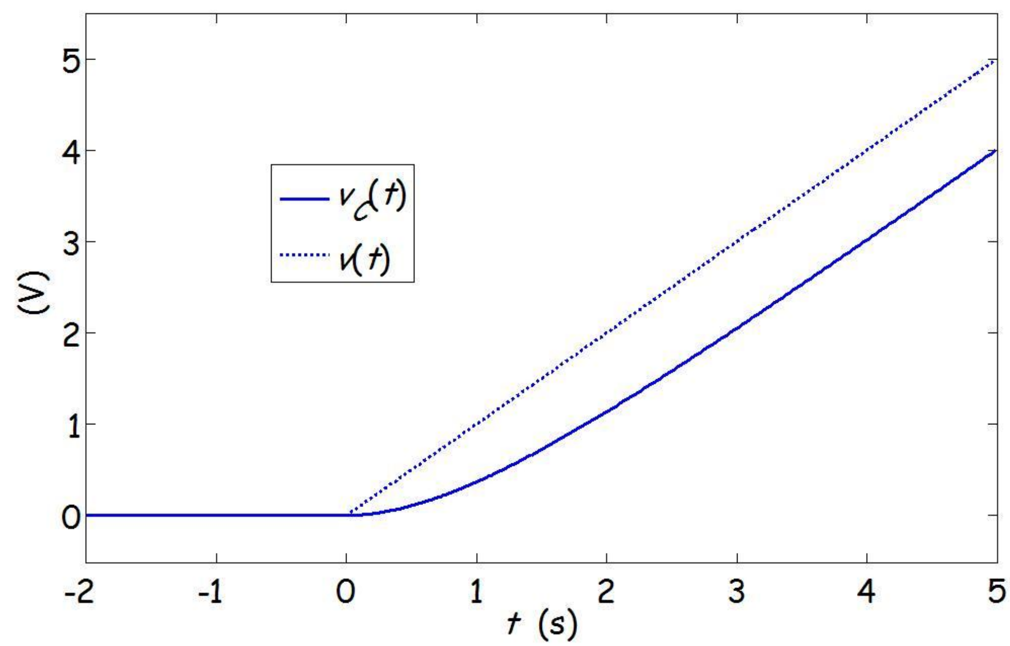
```
syms v vC t R C
v = t
vC = dsolve('R*C*DvC + vC = t', 'vC(0) = 0')
vC = subs(vC, [R C], [1e6 1e-6])
ezplot(vC, [0 5])
hold on
ezplot(v, [0 5])
```

The resulting expression is

```
vC = t + 1/exp(t) - 1
```

In more standard notation, we have:

$$v_C(t) = t - 1 + \exp(t)$$



P4.73 Using KVL, we obtain the differential equation

$$L\frac{di_s(t)}{dt} + Ri_s(t) = v(t)$$

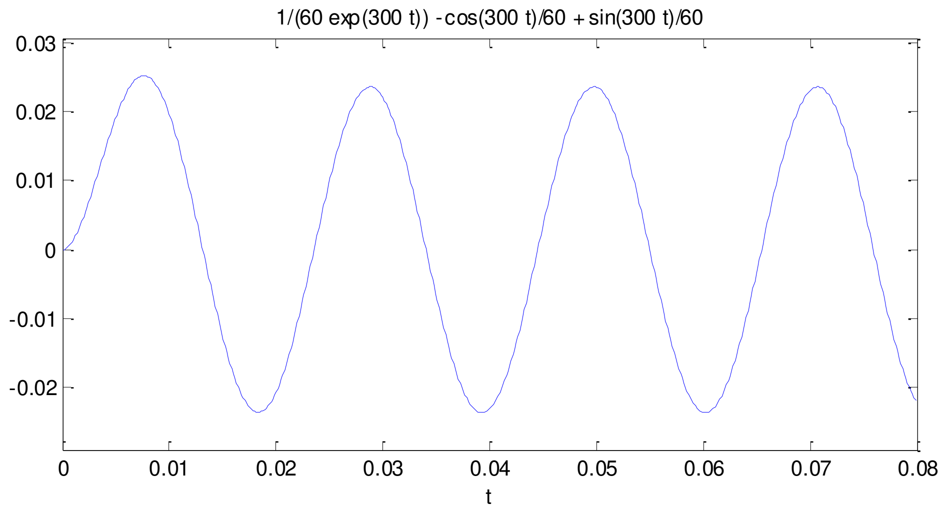
$$\frac{di_s(t)}{dt} + 300i_s(t) = 10\sin(300t)$$

The MATLAB commands are:

```
syms Is t
Is = dsolve('DIs + 300*Is =10*sin(300*t)', 'Is(0) = 0')
ezplot(Is, [0 80e-3])
```

The result is

```
Is = 1/(60*exp(300*t)) - cos(300*t)/60 + sin(300*t)/60
```



P4.74 (a) Writing a KCL equation at the top node after $t = 0$, we have

$$C\frac{dv(t)}{dt} + \frac{v(t)}{R} + \frac{1}{L}\int_0^t v(t)dt + i_L(0+) = 1$$

Taking the derivative to eliminate the integral and substituting component values, we have

$$10^{-9}\frac{d^2v(t)}{dt^2} + 0.04\frac{dv(t)}{dt} + 10^5 v(t) = 0$$

(b) Because the switch has been open for a long time, the inductor acts as a short circuit, and we have $i_L(0-) = 1$ A. Because the current in the inductance cannot change instantaneously in this circuit, we have $i_L(0+) = 1$ A. Writing a current equation at $t = 0+$, we have

$$\frac{v(0+)}{R} + i_L(0+) + Cv'(0+) = 1$$

Substituting $R = 25\ \Omega$, $v(0+) = 50$ V and $i_L(0+) = 1$ A, yields

$$v'(0+) = -2\times 10^9 \text{ V/s}$$

(c) The MATLAB commands are:

```
syms v t
S=dsolve('(1e-9)*D2v + 0.04*Dv + (1e5)*v = 0', 'v(0) = 50, Dv(0)= -2e9');
simple(vpa(S,6))
```

The result is:

```
ans =  53.8675/exp(3.73205e7*t) - 3.86751/exp(2679499.0*t)
```

P4.75 (a) Writing a KCL equation at the top node after $t = 0$, we have

$$C\frac{dv(t)}{dt} + \frac{1}{L}\int_0^t v(t)dt + i_L(0+) = 2\sin(10^4 t)$$

Taking the derivative to eliminate the integral, and substituting component values, we have

$$10^{-6}\frac{d^2v(t)}{dt^2}+100v(t)=2\times10^4\cos(10^4 t)$$

(b) Because the switch is closed prior to $t=0,$ we know that $v_C(0-)=0.$ The voltage across the capacitor cannot change instantaneously, so we have $v_C(0+)=0.$ At $t=0,$ the KCL equation becomes

$$C\frac{dv(0+)}{dt}=0$$

so we have $v'(0+)=0.$

(c) The MATLAB commands are:

```
syms v t
v=dsolve('(1e-6)*D2v + 100*v = (2e4)*cos((1e4)*t)', 'v(0) = 0, Dv(0)= 0')
```

The result is

v = 25*cos(30000*t) - 25*cos(10000*t) + sin(10000*t)*(1000000*t + 50*sin(20000*t))

which can be simplified to

$v(t)=10^6 t\sin(10^4 t).$

P4.76 The KVL equations around meshes 1 and 2 are

$$\frac{d[i_1(t)-i_2(t)]}{dt}+2i_1(t)=12$$

$$\frac{d[i_2(t)-i_1(t)]}{dt}+4i_2(t)+2\frac{di_2(t)}{dt}=0$$

Because the inductor currents are zero before $t=0,$ and the circuit does not force the currents to change, we have $i_1(0+)=0$ and $i_2(0+)=0.$

The MATLAB commands are:

```
syms I1 I2 t
S = dsolve('DI1 - DI2 +2*I1 = 12','3*DI2 - DI1 + 4*I2 = 0',...
        'I1(0) = 0','I2(0) = 0');
I1 = S.I1
I2 = S.I2
```

The results are:

I1 = -4*exp(-4*t) - 2*exp(-t) + 6 and I2 = -2*exp(-4*t) + 2*exp(-t)

P4.10* The initial energy is $w_i = 1/2\ Cv_i^2 = (1/2) * 100 * 10^{-6} * 1000^2 = 50J$
At $t=t_2$, half of the energy remains, and we have $25=(1/2)\ Cv(t_2)^2$, which yields $v(t_2)=707.1V$. The voltage across the capacitance is given by
$v_c(t) = V_i \exp(-t/RC) = 1000 \exp(-t/10)$ for $t>0$.
Substituting, we have $707.1 = 1000 \exp(-t_2/10)$. Solving, we obtain
$\mathrm{Ln}(0.7071) = -t_2/10$
$t_2 = 3.466$ seconds.

P4.17 The final voltage for each 1 s interval is the initial voltage for the succeeding interval.
We have $\tau=RC=2$ s. For $0 \le t \le 1$, we have $v(t) = 10-10\exp(-t/2)$ which yields $v(1)=3.93$ V
For $1 \le t \le 2$, we have $v(t) = 3.93\exp[-(t-1)/2)]$ which yields $v(2)= 2.38V$
For $2 \le t \le 3$, we have $v(t) = 10-(10-2.38)\exp[-(t-2)/2)]$ which yields $v(3)=4.62$ V.
Finally, for $3 \le t \le 4$, we have $v(t) = 4.62\exp[-(t-3)/2]$ which yields $v(4)=2.80$ V.

Practice Test

T4.1 (a) Prior to the switch opening, the circuit is operating in DC steady state, so the inductor acts as a short circuit, and the capacitor acts as an open circuit.

$$i_1(0-) = 10/1000 = 10 \text{ mA} \qquad i_2(0-) = 10/2000 = 5 \text{ mA}$$

$$i_3(0-) = 0 \qquad i_L(0-) = i_1(0-) + i_2(0-) + i_3(0-) = 15 \text{ mA}$$

$$v_C(0-) = 10 \text{ V}$$

(b) Because infinite voltage or infinite current are not possible in this circuit, the current in the inductor and the voltage across the capacitor cannot change instantaneously. Thus, we have $i_L(0+) = i_L(0-) = 15$ mA and $v_C(0+) = v_C(0-) = 10$ V. Also, we have $i_1(0+) = i_L(0+) = 15$ mA, $i_2(0+) = v_C(0+)/5000 = 2$ mA, and $i_3(0+) = -i_2(0+) = -2$ mA.

(c) The current is of the form $i_L(t) = A + B\exp(-t/\tau)$. Because the inductor acts as a short circuit in steady state, we have

$$i_L(\infty) = A = 10/1000 = 10 \text{ mA}$$

At $t = 0+$, we have $i_L(0+) = A + B = 15$ mA, so we find $B = 5$ mA.
The time constant is $\tau = L/R = 2\times10^{-3}/1000 = 2\times10^{-6}$ s.
Thus, we have $i_L(t) = 10 + 5\exp(-5\times10^{5}t)$ mA.

(d) This is a case of an initially charged capacitance discharging through a resistance. The time constant is $\tau = RC = 5000\times10^{-6} = 5\times10^{-3}$ s. Thus we have $v_C(t) = V_i\exp(-t/\tau) = 10\exp(-200t)$ V.

T4.2 (a) $2\dfrac{di(t)}{dt} + i(t) = 5\exp(-3t)$

(b) The time constant is $\tau = L/R = 2$ s and the complementary solution is of the form $i_c(t) = A\exp(-0.5t)$.

(c) The particular solution is of the form $i_p(t) = K\exp(-3t)$. Substituting into the differential equation produces

$$-6K\exp(-3t) + K\exp(-3t) \equiv 5\exp(-3t)$$

from which we have $K = -1$.

(d) Adding the particular solution and the complementary solution, we have

$$i(t) = A\exp(-0.5t) - \exp(-3t)$$

However, the current must be zero in the inductor prior to $t = 0$ because of the open switch, and the current cannot change instantaneously in this circuit, so we have $i(0+) = 0$. This yields $A = 1$. Thus, the solution is

$$i(t) = \exp(-0.5t) - \exp(-3t) \text{ A}$$

T4.3 (a) Applying KVL to the circuit, we obtain

$$L\frac{di(t)}{dt} + Ri(t) + v_C(t) = 15 \qquad (1)$$

For the capacitance, we have

$$i(t) = C\frac{dv_C(t)}{dt} \qquad (2)$$

Using Equation (2) to substitute into Equation (1) and rearranging, we have

$$\frac{d^2v_C(t)}{dt^2} + (R/L)\frac{dv_C(t)}{dt} + (1/LC)v_C(t) = 15/LC \qquad (3)$$

$$\frac{d^2v_C(t)}{dt^2} + 2000\frac{dv_C(t)}{dt} + 25\times10^6 v_C(t) = 375\times10^6$$

(b) We try a particular solution of the form $v_{Cp}(t) = A$, resulting in $A = 15$. Thus, $v_{Cp}(t) = 15$. (An alternative method to find the particular solution is to solve the circuit in dc steady state. Since the capacitance acts as an open circuit, the steady-state voltage across it is 15 V.)

(c) We have

$$\omega_0 = \frac{1}{\sqrt{LC}} = 5000 \text{ and } \alpha = \frac{R}{2L} = 1000$$

Since we have $\alpha < \omega_0$, this is the underdamped case. The natural frequency is given by:

$$\omega_n = \sqrt{\omega_0^2 - \alpha^2} = 4899$$

The complementary solution is given by:

$$v_{Cc}(t) = K_1\exp(-1000t)\cos(4899t) + K_2\exp(-1000t)\sin(4899t)$$

(d) The complete solution is

$$v_C(t) = 15 + K_1\exp(-\alpha t)\cos(\omega_n t) + K_2\exp(-\alpha t)\sin(\omega_n t)$$

The initial conditions are

$$v_C(0) = 0 \quad \text{and} \quad i(0) = 0 = C\frac{dv_C(t)}{dt}\Big|_{t=0}$$

Thus, we have

$$v_C(0) = 0 = 15 + K_1$$

$$\frac{dv_C(t)}{dt}\Big|_{t=0} = 0 = -\alpha K_1 + \omega_n K_2$$

Solving, we find $K_1 = -15$ and $K_2 = -3.062$. Finally, the solution is

$$v_C(t) = 15 - 15\exp(-1000t)\cos(4899t) - (3.062)\exp(-1000t)\sin(4899t) \text{ V}$$

T4.4 One set of commands is

```
syms vC t
S = dsolve('D2vC + 2000*DvC + (25e6)*vC = 375e6',...
           'vC(0) = 0, DvC(0) = 0');
simple(vpa(S,4))
```

These commands are stored in the m-file named T_4_4 .

CHAPTER 5

Exercises

E5.1 (a) We are given $v(t) = 150\cos(200\pi t - 30^\circ)$. The angular frequency is the coefficient of t so we have $\omega = 200\pi$ radian/s . Then

$$f = \omega / 2\pi = 100 \text{ Hz} \qquad T = 1/f = 10 \text{ ms}$$

$$V_{rms} = V_m / \sqrt{2} = 150/\sqrt{2} = 106.1 \text{ V}$$

Furthermore, $v(t)$ attains a positive peak when the argument of the cosine function is zero. Thus keeping in mind that ωt has units of radians, the positive peak occurs when

$$\omega t_{\max} = 30 \times \frac{\pi}{180} \quad \Rightarrow \quad t_{\max} = 0.8333 \text{ ms}$$

(b) $P_{avg} = V_{rms}^2 / R = 225$ W

(c) A plot of $v(t)$ is shown in Figure 5.4 in the book.

E5.2 We use the trigonometric identity $\sin(z) = \cos(z - 90^\circ)$. Thus

$$100\sin(300\pi t + 60^\circ) = 100\cos(300\pi t - 30^\circ)$$

E5.3 $\omega = 2\pi f \cong 377$ radian/s $\qquad T = 1/f \cong 16.67$ ms $\qquad V_m = V_{rms}\sqrt{2} \cong 155.6$ V

The period corresponds to 360° therefore 5 ms corresponds to a phase angle of $(5/16.67) \times 360^\circ = 108^\circ$. Thus the voltage is

$$v(t) = 155.6\cos(377t - 108^\circ)$$

E5.4 (a)

$$\mathbf{V}_1 = 10\angle 0^\circ + 10\angle -90^\circ = 10 - j10 \cong 14.14\angle -45^\circ$$

$$10\cos(\omega t) + 10\sin(\omega t) = 14.14\cos(\omega t - 45^\circ)$$

(b)

$$\mathbf{I}_1 = 10\angle 30^\circ + 5\angle -60^\circ \cong 8.660 + j5 + 2.5 - j4.330$$

$$\cong 11.16 + j0.670 \cong 11.18\angle 3.44^\circ$$

$$10\cos(\omega t + 30^\circ) + 5\sin(\omega t + 30^\circ) = 11.18\cos(\omega t + 3.44^\circ)$$

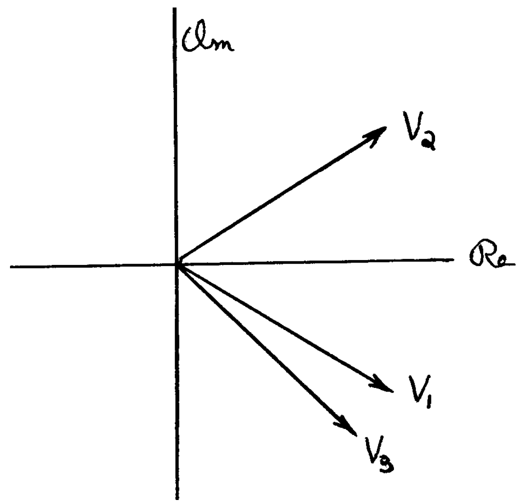
(c)

$$\mathbf{I}_2 = 20\angle 0^\circ + 15\angle -60^\circ \cong 20 + j0 + 7.5 - j12.99$$

$$\cong 27.5 - j12.99 \cong 30.41\angle -25.28^\circ$$

$$20\sin(\omega t + 90^\circ) + 15\cos(\omega t - 60^\circ) = 30.41\cos(\omega t - 25.28^\circ)$$

E5.5 The phasors are $\mathbf{V}_1 = 10\angle -30^\circ$ $\mathbf{V}_2 = 10\angle +30^\circ$ and $\mathbf{V}_3 = 10\angle -45^\circ$



v_1 lags v_2 by 60° (or we could say v_2 leads v_1 by 60°)

v_1 leads v_3 by 15° (or we could say v_3 lags v_1 by 15°)

v_2 leads v_3 by 75° (or we could say v_3 lags v_2 by 75°)

E5.6 (a) $Z_L = j\omega L = j50 = 50\angle 90^\circ$ $\mathbf{V}_L = 100\angle 0^\circ$

$\mathbf{I}_L = \mathbf{V}_L / Z_L = 100 / j50 = 2\angle -90^\circ$

(b) The phasor diagram is shown in Figure 5.11a in the book.

E5.7 (a) $Z_C = 1/ j\omega C = -j50 = 50\angle -90^\circ$ $\mathbf{V}_C = 100\angle 0^\circ$

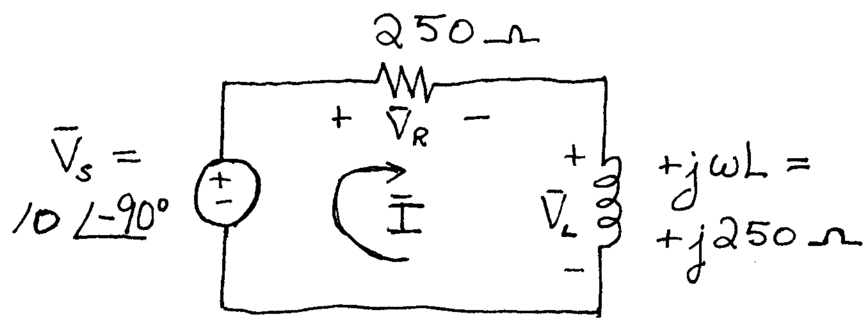
$\mathbf{I}_C = \mathbf{V}_C / Z_C = 100 /(-j50) = 2\angle 90^\circ$

(b) The phasor diagram is shown in Figure 5.11b in the book.

E5.8 (a) $Z_R = R = 50 = 50\angle 0^\circ$ $\mathbf{V}_R = 100\angle 0^\circ$ $\mathbf{I}_R = \mathbf{V}_R / R = 100 /(50) = 2\angle 0^\circ$

(b) The phasor diagram is shown in Figure 5.11c in the book.

E5.9 (a) The transformed network is:



$$\mathbf{I} = \frac{\mathbf{V}_s}{Z} = \frac{10\angle -90^\circ}{250 + j250} = 28.28\angle -135^\circ \text{ mA}$$

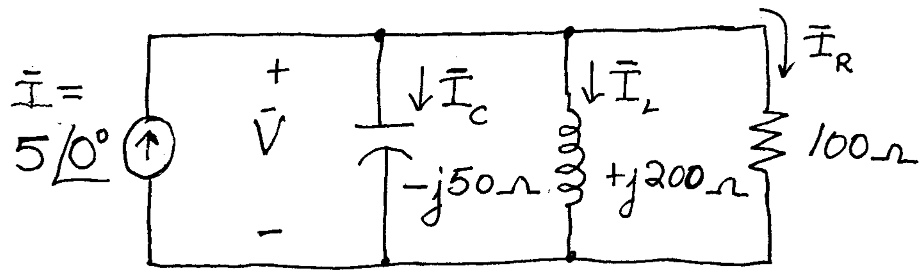
$$i(t) = 28.28\cos(500t - 135^\circ) \text{ mA}$$

$\mathbf{V}_R = R\mathbf{I} = 7.07\angle -135^\circ \qquad \mathbf{V}_L = j\omega L\mathbf{I} = 7.07\angle -45^\circ$

(b) The phasor diagram is shown in Figure 5.17b in the book.

(c) $i(t)$ lags $v_s(t)$ by 45°.

E5.10 The transformed network is:



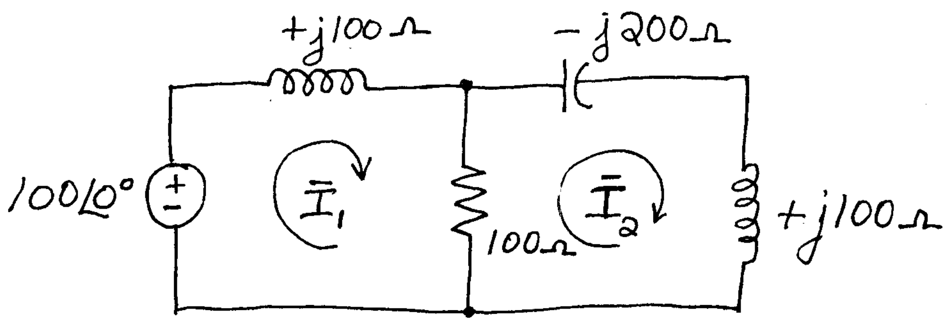
$$Z = \frac{1}{1/100 + 1/(-j50) + 1/(+j200)} = 55.47\angle -56.31^\circ \ \ \Omega$$

$\mathbf{V} = Z\mathbf{I} = 277.4\angle -56.31^\circ \ \text{V} \qquad \mathbf{I}_C = \mathbf{V}/(-j50) = 5.547\angle 33.69^\circ \ \ A$

$\mathbf{I}_L = \mathbf{V}/(j200) = 1.387\angle -146.31^\circ \ \ A$

$\mathbf{I}_R = \mathbf{V}/(100) = 2.774\angle -56.31^\circ \ \ A$

E5.11 The transformed network is:



We write KVL equations for each of the meshes:

$$j100\mathbf{I}_1 + 100(\mathbf{I}_1 - \mathbf{I}_2) = 100$$
$$-j200\mathbf{I}_2 + j100\mathbf{I}_2 + 100(\mathbf{I}_2 - \mathbf{I}_1) = 0$$

Simplifying, we have

$$(100 + j100)\mathbf{I}_1 - 100\mathbf{I}_2 = 100$$
$$-100\mathbf{I}_1 + (100 - j100)\mathbf{I}_2 = 0$$

Solving we find $\mathbf{I}_1 = 1.414\angle -45^\circ$ A and $\mathbf{I}_2 = 1\angle 0^\circ$ A. Thus we have $i_1(t) = 1.414\cos(1000t - 45^\circ)$ A and $i_2(t) = \cos(1000t)$.

E5.12 (a) For a power factor of 100%, we have $\cos(\theta) = 1$, which implies that the current and voltage are in phase and $\theta = 0$. Thus, $Q = P\tan(\theta) = 0$. Also $I_{rms} = P/[V_{rms}\cos(\theta)] = 5000/[500\cos(0)] = 10$ A. Thus we have $I_m = I_{rms}\sqrt{2} = 14.14$ and $\mathbf{I} = 14.14\angle 40^\circ$.

(b) For a power factor of 20% lagging, we have $\cos(\theta) = 0.2$, which implies that the current lags the voltage by $\theta = \cos^{-1}(0.2) = 78.46^\circ$. Thus, $Q = P\tan(\theta) = 24.49$ kVAR. Also, we have $I_{rms} = P/[V_{rms}\cos(\theta)] = 50.0$ A. Thus we have $I_m = I_{rms}\sqrt{2} = 70.71$ A and $\mathbf{I} = 70.71\angle -38.46^\circ$.

(c) The current ratings would need to be five times higher for the load of part (b) than for that of part (a). Wiring costs would be lower for the load of part (a).

E5.13 The first load is a 10 μF capacitor for which we have

$$Z_C = 1/(j\omega C) = 265.3\angle -90^\circ\ \Omega \qquad \theta_C = -90^\circ \qquad I_{Crms} = V_{rms}/|Z_C| = 3.770\ \text{A}$$

$$P_C = V_{rms}I_{Crms}\cos(\theta_C) = 0 \qquad Q_C = V_{rms}I_{Crms}\sin(\theta_C) = -3.770\ \text{kVAR}$$

The second load absorbs an apparent power of $V_{rms}I_{rms} = 10$ kVA with a power factor of 80% lagging from which we have $\theta_2 = \cos^{-1}(0.8) = 36.87^\circ$. Notice that we select a positive angle for θ_2 because the load has a lagging power factor. Thus we have $P_2 = V_{rms}I_{2rms}\cos(\theta_2) = 8.0$ kW and $Q_2 = V_{rms}I_{2rms}\sin(\theta) = 6$ kVAR.

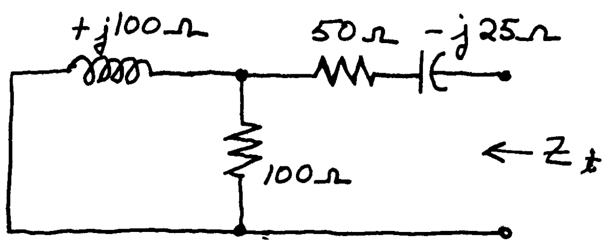
Now for the source we have:

$$P_s = P_C + P_2 = 8\ \text{kW} \quad Q_s = Q_C + Q_2 = 2.23\ \text{kVAR}$$

$$V_{rms}I_{srms} = \sqrt{P_s^2 + Q_s^2} = 8.305\ \text{kVA} \quad I_{srms} = V_{rms}I_{srms}/V_{rms} = 8.305\ \text{A}$$

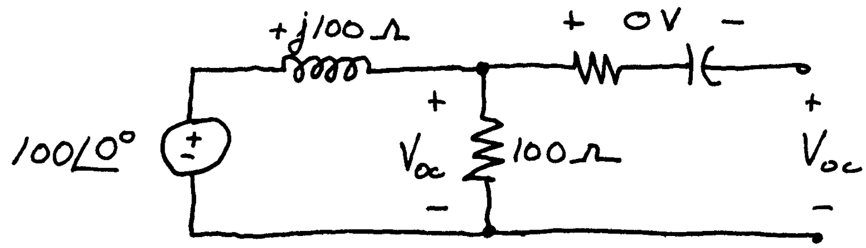
$$\text{power factor} = P_s/(V_{rms}I_{srms}) \times 100\% = 96.33\%$$

E5.14 First, we zero the source and combine impedances in series and parallel to determine the Thévenin impedance.



$$Z_t = 50 - j25 + \frac{1}{1/100 + 1/j100} = 50 - j25 + 50 + j50$$
$$= 100 + \mathrm{j}25 = 103.1\angle 14.04^\circ$$

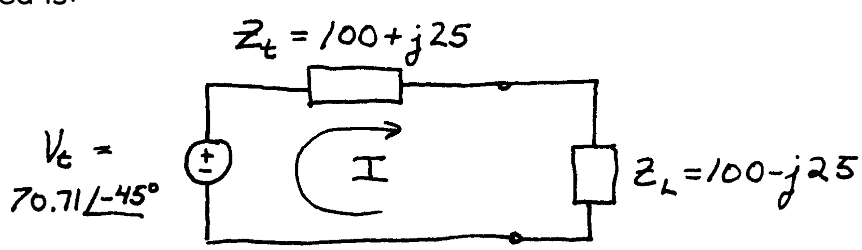
Then we analyze the circuit to determine the open-circuit voltage.



$$\mathbf{V}_t = \mathbf{V}_{oc} = 100 \times \frac{100}{100 + j100} = 70.71\angle -45^\circ$$

$$\mathbf{I}_n = V_t / Z_t = 0.6858\angle -59.04^\circ$$

E5.15 (a) For a complex load, maximum power is transferred for $Z_L = Z_t^* = 100 - j25 = R_L + jX_L$. The Thévenin equivalent with the load attached is:



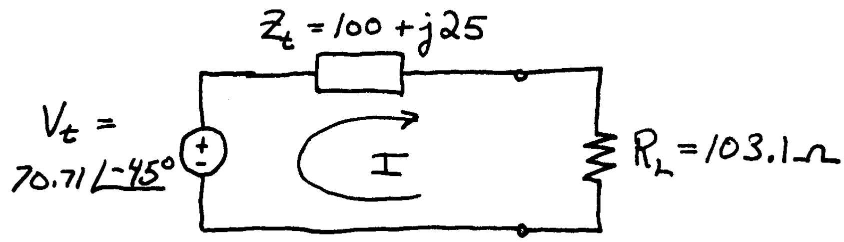
The current is given by

$$\mathbf{I} = \frac{70.71\angle -45^\circ}{100 + j25 + 100 - j25} = 0.3536\angle -45^\circ$$

The load power is

$$P_L = R_L I_{rms}^2 = 100(0.3536/\sqrt{2})^2 = 6.25\ \mathrm{W}$$

(b) For a purely resistive load, maximum power is transferred for $R_L = |Z_t| = \sqrt{100^2 + 25^2} = 103.1\,\Omega$. The Thévenin equivalent with the load attached is:



The current is given by

$$\mathbf{I} = \frac{70.71\angle -45^\circ}{103.1 + 100 - j25} = 0.3456\angle -37.98^\circ$$

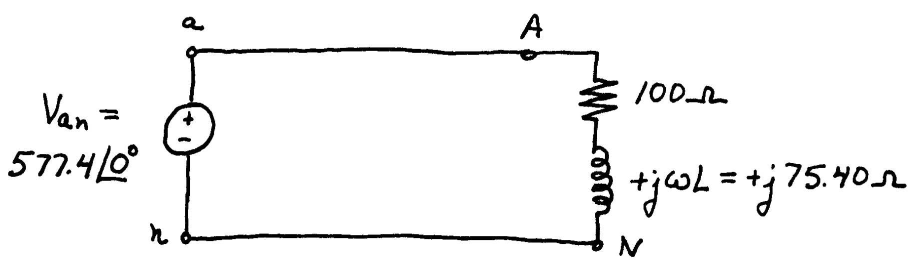
The load power is

$$P_L = R_L I_{rms}^2 = 103.1(0.3456/\sqrt{2})^2 = 6.157 \text{ W}$$

E5.16 The line-to-neutral voltage is $1000/\sqrt{3} = 577.4$ V. No phase angle was specified in the problem statement, so we will assume that the phase of $\mathbf{V}_{an}$ is zero. Then we have

$$\mathbf{V}_{an} = 577.4\angle 0^\circ \qquad \mathbf{V}_{bn} = 577.4\angle -120^\circ \qquad \mathbf{V}_{cn} = 577.4\angle 120^\circ$$

The circuit for the a phase is shown below. (We can consider a neutral connection to exist in a balanced Y-Y connection even if one is not physically present.)



The a-phase line current is

$$\mathbf{I}_{aA} = \frac{\mathbf{V}_{an}}{Z_L} = \frac{577.4\angle 0^\circ}{100 + j75.40} = 4.610\angle -37.02^\circ$$

The currents for phases b and c are the same except for phase.

$$\mathbf{I}_{bB} = 4.610\angle -157.02^\circ \qquad \mathbf{I}_{cC} = 4.610\angle 82.98^\circ$$

$$P = 3\frac{V_Y I_L}{2}\cos(\theta) = 3\frac{577.4 \times 4.610}{2}\cos(37.02^\circ) = 3.188 \text{ kW}$$

$$Q = 3\frac{V_Y I_L}{2}\sin(\theta) = 3\frac{577.4 \times 4.610}{2}\sin(37.02^\circ) = 2.404 \text{ kVAR}$$

E5.17 The *a*-phase line-to-neutral voltage is

$$\mathbf{V}_{an} = 1000/\sqrt{3}\angle 0^\circ = 577.4\angle 0^\circ$$

The phase impedance of the equivalent Y is $Z_Y = Z_\Delta / 3 = 50/3 = 16.67\ \Omega$.

Thus the line current is

$$\mathbf{I}_{aA} = \frac{\mathbf{V}_{an}}{Z_Y} = \frac{577.4\angle 0^\circ}{16.67} = 34.63\angle 0^\circ \text{ A}$$

Similarly, $\mathbf{I}_{bB} = 34.63\angle -120^\circ$ A and $\mathbf{I}_{cC} = 34.63\angle 120^\circ$ A.

Finally, the power is

$$P = 3(I_{aA}/\sqrt{2})^2 R_Y = 30.00 \text{ kW}$$

E5.18 Writing KCL equations at nodes 1 and 2 we obtain

$$\frac{\mathbf{V}_1}{100 + j30} + \frac{\mathbf{V}_1 - \mathbf{V}_2}{50 - j80} = 1\angle 60^\circ$$

$$\frac{\mathbf{V}_2}{j50} + \frac{\mathbf{V}_2 - \mathbf{V}_1}{50 - j80} = 2\angle 30^\circ$$

In matrix form, these become

$$\begin{bmatrix} \left(\dfrac{1}{100 + j30} + \dfrac{1}{50 - j80}\right) & -\dfrac{1}{50 - j80} \\ -\dfrac{1}{50 - j80} & \left(\dfrac{1}{j50} + \dfrac{1}{50 - j80}\right) \end{bmatrix} \begin{bmatrix} \mathbf{V}_1 \\ \mathbf{V}_2 \end{bmatrix} = \begin{bmatrix} 1\angle 60^\circ \\ 2\angle 30^\circ \end{bmatrix}$$

The MATLAB commands are

```
Y = [(1/(100+j*30)+1/(50-j*80))  (-1/(50-j*80));...
     (-1/(50-j*80))  (1/(j*50)+1/(50-j*80))];
I = [pin(1,60); pin(2,30)];
V = inv(Y)*I;
pout(V(1))
pout(V(2))
```

The results are

$$\mathbf{V}_1 = 79.98\angle 106.21^\circ \quad \text{and} \quad \mathbf{V}_2 = 124.13\angle 116.30^\circ$$

Problems

P5.1 The units of angular frequency ω are radians per second. The units of frequency f are hertz, which are equivalent to inverse seconds. The relationship between them is $\omega = 2\pi f$.

P5.2 An angle in radians is defined to be arc length divided by radius. Thus, radian measure of angle is length divided by length. In terms of physical units, radians are unitless. Thus, the physical units of ω are s^{-1}.

P5.3 (a) Increasing the peak amplitude V_m? 5. Stretches the sinusoidal curve vertically.
(b) Increasing the frequency f? 2. Compresses the sinusoidal curve horizontally.
(c) Increasing θ? 4. Translates the sinusoidal curve to the left.
(d) Decreasing the angular frequency ω? 1. Stretches the sinusoidal curve horizontally.
(e) Decreasing the period? 2. Compresses the sinusoidal curve horizontally.

P5.4* $v(t) = 10\sin(1000\pi t + 30^\circ) = 10\cos(1000\pi - 60^\circ)$

$\omega = 1000\pi$ rad/s $\qquad f = 500$ Hz $\qquad T = 1/f = 2$ ms

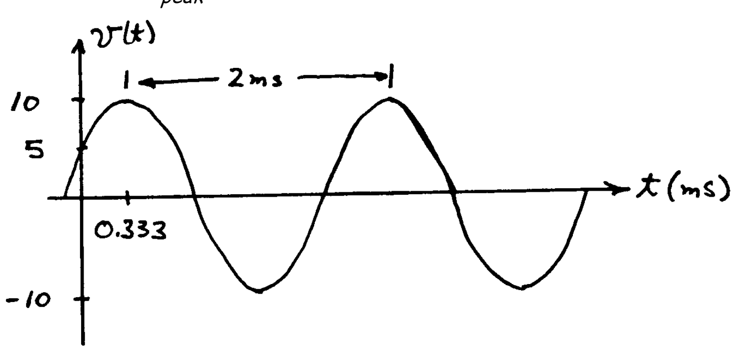
phase angle $= \theta = -60^\circ = -\pi/3$ radians

$V_{rms} = V_m/\sqrt{2} = 10/\sqrt{2} = 7.071$ V $\qquad P = (V_{rms})^2/R = 1$ W

The first positive peak occurs for

$$1000\pi t_{peak} - \pi/3 = 0$$

$$t_{peak} = 0.3333 \text{ ms}$$



P5.5 $v(t) = 50\sin(500\pi t + 150^\circ) = 50\cos(500\pi t + 60^\circ)$

$\omega = 500\pi$ rad/s

$f = 250$ Hz

phase angle $= \theta = 60^\circ = \pi/3$ radians

$T = 1/f = 4$ ms

$V_{rms} = V_m/\sqrt{2} = 50/\sqrt{2} = 35.36$ V

$P = (V_{rms})^2/R = 25$ W

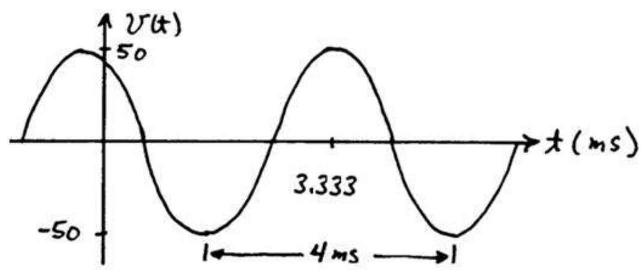
Positive peak occurs for

$$500\pi t_1 + \pi/3 = 0$$

$$t_1 = -0.6666 \text{ ms}$$

The first positive peak <u>after</u> $t = 0$ is at

$$t_{peak} = t_1 + T = 3.333 \text{ ms}$$



P5.6* Sinusoidal voltages can be expressed in the form $v(t) = V_m\cos(\omega t + \theta)$. The peak voltage is $V_m = \sqrt{2}V_{rms} = \sqrt{2}\times 20 = 28.28$ V. The frequency is $f = 1/T =$ 10 kHz and the angular frequency is $\omega = 2\pi f = 2\pi 10^4$ radians/s. The phase corresponding to a time interval of $\Delta t =$ 20 μs is $\theta = (\Delta t/T)\times 360^\circ = 72^\circ$. Thus, we have $v(t) = 28.28\cos(2\pi 10^4 t - 72^\circ)$ V.

P5.7 $I_m = I_{rms}\sqrt{2} = 20\sqrt{2} = 28.28$ A $\qquad f = \dfrac{1}{T} = 1000$ Hz

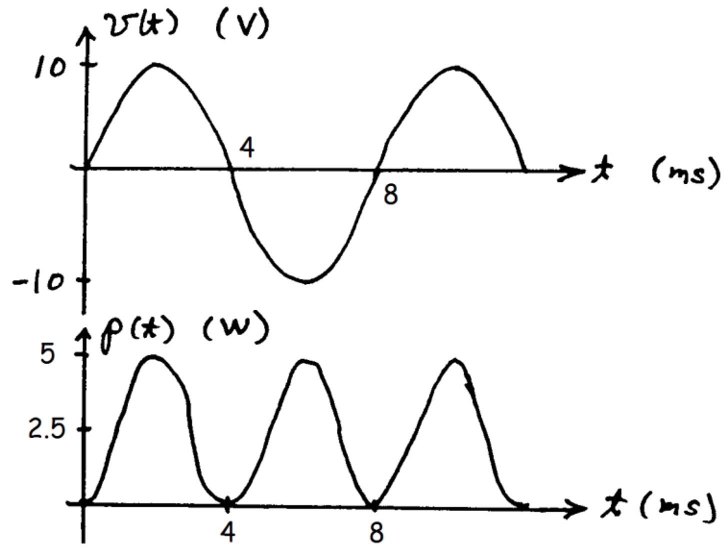
$\omega = 2\pi f = 2000\pi$ rad/s $\qquad \theta = -360\dfrac{t_{\max}}{T} = -108^\circ$

$i(t) = 28.28\cos(2000\pi t - 108^\circ)$

P5.8 $v(t) = 10\sin(250\pi t)\,\text{V}$

$$p(t) = v^2(t)/R = 5\sin^2(250\pi t) = 2.5[1 - \sin(500\pi t)] \quad \text{W}$$

$$P_{avg} = (V_{rms})^2/R = 2.5\ \text{W}$$

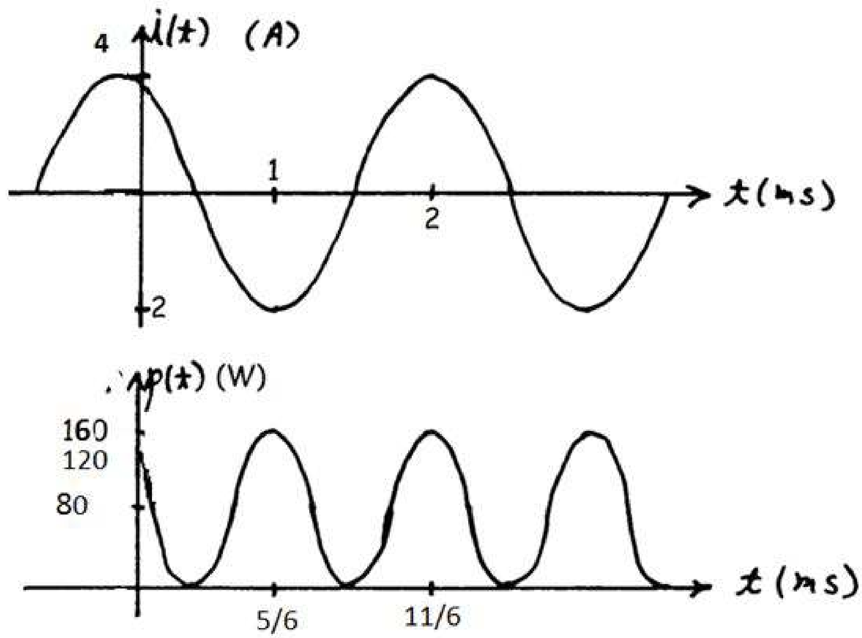


P5.9 The angular frequency is $\omega = 2\pi f = 1000\pi$ radians/s. We have $T = 1/f = 2$ ms. The sinusoid reaches a positive peak one quarter of a cycle after crossing zero with a positive slope. Thus, the first positive peak occurs at $t = 0.6$ ms. The phase corresponding to a time interval of $\Delta t = 0.6$ ms is $\theta = (\Delta t / T) \times 360^\circ = 108^\circ$. Thus, we have $v(t) = 15\cos(1000\pi t - 108^\circ)$ V.

P5.10 $i(t) = 4\cos(1000\pi t + 30^\circ) = 4\cos\left(1000\pi t + \frac{\pi}{6}\right)$

$$p(t) = Ri^2(t) = 10 \times 16\cos^2\left(1000\pi t + \frac{\pi}{6}\right) = 80\left[1 + \cos\left(2000\pi t + \frac{\pi}{6}\right)\right]\text{W}$$

$$P_{avg} = R(I_{rms})^2 = 10 \times (4/\sqrt{2})^2 = 80\ \text{W}$$



P5.11 The rms values of <u>all</u> periodic waveforms are not equal to their peak values divided by the square root of two. However, they are for sinusoids, which are important special cases.

P5.12* $$I_{rms} = \sqrt{\frac{1}{T}\int_0^T i^2(t)dt} = \sqrt{\frac{1}{4}\left(\int_0^2 25dt + \int_2^4 4dt\right)} = 3.808\ A$$

P5.13 $$V_{rms} = \sqrt{\frac{i}{T}\int_0^T v^2(t)dt} = \sqrt{\frac{1}{2}\int_0^1 15^2\,dt} = 10.61\,\mathrm{V}$$

P5.14 $$V_{rms} = \sqrt{\frac{1}{T}\int_0^T v^2(t)dt} = \sqrt{\int_0^1 [10\exp(-t)]^2\,dt} = \sqrt{\int_0^1 [100\exp(-2t)]dt}$$

$$= \sqrt{[-50\exp(-2t)]_{t=0}^{t=1}} = \sqrt{50[1-\exp(-2)]} = 6.575\ \mathrm{V}$$

$A^2 + B^2\cos^2(2\pi t) + 4C^2\sin^2(2\pi t) + 2AB\cos(2\pi t) +$
$4AC\sin(2\pi t)$

$\left.\right]^2 dt$

P5.15 $$V_{rms} = \sqrt{\frac{1}{T}\int_0^T v^2(t)dt} = \sqrt{\int_0^1 \left[A + B\cos(2\pi t) + 2C\sin(2\pi t)\right]^2 dt}$$

$$V_{rms} = \sqrt{\int_0^1 \left[\right.}$$

$\left.\right]$

$$= \sqrt{\left(\frac{A^2}{2}t + 0 + \frac{4B^2}{2}t\right)_{t=0}^{t=1}}$$

$$= \sqrt{\left(\frac{A^2 + 4B^2}{2}\right)}$$

The MATLAB Symbolic Toolbox commands are

```
syms Vrms t A B
Vrms = sqrt((int((A*cos(2*pi*t)+2*B*sin(2*pi*t))^2,t,0,1)))
```

and the result is

```
Vrms =
(A^2/2 + 2*B^2)^(1/2)
```

P5.16 The limits on the integral don't matter as long as they cover one period

$$V_{rms} = \sqrt{\frac{1}{T}\int_{-T/2}^{T/2} v^2(t)dt} = \sqrt{20\int_{-0.025}^{0.025}\left[4\cos(20\pi t)\right]^2 dt} = \sqrt{20\int_{-0.025}^{0.025}\left[8 + 8\cos(40\pi t)\right]dt}$$

$$V_{rms} = \sqrt{\left(160t + \frac{160}{40\pi}\sin(40\pi t)\right)_{t=-0.025}^{t=0.025}} = \sqrt{8} = 2\sqrt{2}\ \text{V}$$

P5.17 $$V_{rms} = \sqrt{\frac{1}{T}\int_0^T v^2(t)dt} = \sqrt{\frac{1}{2}\int_0^2 (5t)^2 dt} = \sqrt{\frac{1}{2}\left(25\frac{t^3}{3}\right)_{t=0}^{t=2}} = 5.774\ \text{V}$$

P5.18

$$V_{rms} = \sqrt{\frac{1}{T}\int_0^T v^2(t)\,dt}$$

$$V_{rms} = \sqrt{\frac{1}{0.1}\int_0^{0.1}\left[5 + 4\sqrt{2}sin(2\pi t + 60^\circ)\right]^2 dt}$$

$$= \sqrt{\frac{1}{0.1}\int_0^{0.1}\left[25 + 32sin^2(2\pi t + 60^\circ) + 40\sqrt{2}Sin(2\pi t + 60^\circ)\right]}\,dt$$

$$= \sqrt{\frac{1}{0.1}\left[\int_0^{0.1} 25\,dt + \int_0^{0.1} 40\sqrt{2}Sin(2\pi t + 60^\circ)dt + \int_0^{0.1} 16\,dt - \int_0^{0.1} cos2(2\pi t + 60^\circ)dt\right]}$$

$$= \sqrt{[25 + 0 + 16 - 0]}$$

$$= \sqrt{41}$$

P5.19 To add sinusoids we:

1. Convert each of the sinusoids to be added into a phasor, which is a complex number whose magnitude equals the peak amplitude of the sinusoid and whose phase is the phase angle of the sinusoid written as a cosine function.
2. Add the phasors and convert the sum to polar form using complex arithimetic.
3. Convert the resulting phasor to a sinusoid.

All of the sinusoids to be added must have the same frequency.

P5.20 (a) Examine the phasor diagram and consider it to rotate counterclockwise. If phasor A points in a given direction before phasor B by an angle θ, we say that A leads B by the angle θ or that B lags A by the angle θ.

(b) Examine plots of the sinusoidal waveforms versus time. If A reaches a point (such as a positive peak or a zero crossing with positive slope) by an interval Δt before B reaches the corresponding point, we say that A leads B by the angle $\theta = (\Delta t / T) \times 360^\circ$ or that B lags A by the angle θ.

P5.21* We are given the expression

$$5\cos(\omega t + 75^\circ) - 3\cos(\omega t - 75^\circ) + 4\sin(\omega t)$$

Converting to phasors we obtain

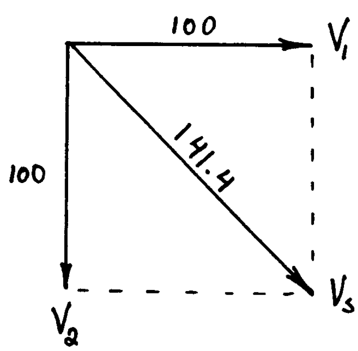
$$5\angle 75^\circ - 3\angle -75^\circ + 4\angle -90^\circ =$$
$$1.2941 + j4.8296 - (0.7765 - j2.8978) - j4 =$$
$$0.5176 + j3.7274 = 3.763\angle 82.09^\circ$$

Thus, we have

$$5\cos(\omega t + 75^\circ) - 3\cos(\omega t - 75^\circ) + 4\sin(\omega t) =$$
$$3.763\cos(\omega t + 82.09^\circ)$$

P5.22*

$$v_1(t) = 100\cos(\omega t)$$
$$v_2(t) = 100\sin(\omega t) = 100\cos(\omega t - 90^\circ)$$
$$\mathbf{V}_1 = 100\angle 0^\circ = 100$$
$$\mathbf{V}_2 = 100\angle -90^\circ = -j100$$
$$\mathbf{V}_s = \mathbf{V}_1 + \mathbf{V}_2 = 100 - j100 = 141.4\angle -45^\circ$$
$$v_s(t) = 141.4\cos(\omega t - 45^\circ)$$



$\mathbf{V}_2$ lags $\mathbf{V}_1$ by 90°
$\mathbf{V}_s$ lags $\mathbf{V}_1$ by 45°
$\mathbf{V}_s$ leads $\mathbf{V}_2$ by 45°

P5.23*

$$\omega = 2\pi f = 400\pi$$

$$v_1(t) = 10\cos(400\pi t + 30^\circ)$$
$$v_2(t) = 5\cos(400\pi t + 150^\circ)$$
$$v_3(t) = 10\cos(400\pi t + 90^\circ)$$

$v_1(t)$ lags $v_2(t)$ by 120°
$v_1(t)$ lags $v_3(t)$ by 60°
$v_2(t)$ leads $v_3(t)$ by 60°

P5.24 The magnitudes of the phasors for the two voltages are $12\sqrt{2}$ and $7\sqrt{2}$ V. The phase angles are not known. If the phase angles are the same, the phasor sum would have its maximum magnitude which is $19\sqrt{2}$. On the other hand, if the phase angles differ by 180°, the phasor sum would have its minimum magnitude, which is $5\sqrt{2}$. Thus, the maximum rms value of the sum is 19 V and the minimum is 5 V.

P5.25 $V_m = 15 \text{ V} \qquad T = 20 \text{ ms}$

$$f = \frac{1}{T} = 50 \text{ Hz} \qquad \omega = 2\pi f = 100\pi \text{ rad/s}$$

$$\theta = -360^\circ \frac{t_{max}}{T} = 72^\circ$$

$$v(t) = 15\cos(100\pi t + 72^\circ) \text{ V}$$

$$\mathbf{V} = 15\angle 72^\circ \text{ V}$$

$$V_{rms} = \frac{15}{\sqrt{2}} = 10.61 \text{ V}$$

P5.26 $T = 1/f = 1/1000 = 1 \text{ ms}$

i_1 lags i_2 by the angle $\Delta\theta = (\Delta t/T) \times 360^\circ = (0.25/1) \times 360^\circ = 90^\circ$.

Therefore, the angle for i_2 is $\theta_2 = \theta_1 + \Delta\theta = 60^\circ$.

P5.27

$$v_1(t) = 90\cos(\omega t - 15^\circ)$$

$$v_2(t) = 50\sin(\omega t - 60^\circ) = 50\cos(\omega t - 150^\circ)$$

$$\mathbf{V}_1 = 90\angle -15^\circ = 86.93 - j23.29$$

$$\mathbf{V}_2 = 50\angle -150^\circ = -43.30 - j25.00$$

$$\mathbf{V}_s = \mathbf{V}_1 - \mathbf{V}_2 = 130.23 + j1.7063 = 130.25\angle 0.75^\circ$$

$$v_s(t) = 130.25\cos(\omega t + 0.75^\circ)$$

$\mathbf{V}_2$ lags $\mathbf{V}_1$ by 135°

$\mathbf{V}_s$ leads $\mathbf{V}_1$ by 15.75°

$\mathbf{V}_s$ leads $\mathbf{V}_2$ by 150.75°

P5.28 We are given the expression

$$5\sin(\omega t) + 15\cos(\omega t - 30^\circ) + 20\cos(\omega t - 120^\circ) = 5\sin(\omega t - 90^\circ)$$
$$+ 15\cos(\omega t - 30^\circ) + 20\cos(\omega t - 120^\circ)$$

Converting to phasors we obtain

$$-j5 + 12.9904 - j7.5000 - 10 - j17.32 = 2.9904 - j29.84$$

$=29.97\angle -84.27^\circ$

Thus, we have

$$5\sin(\omega t) + 15\cos(\omega t - 30^\circ) + 20\cos(\omega t - 120^\circ) =$$

$$29.97\cos(\omega t \quad 84.27\ )^\circ$$

P5.29 $v_1(t) = 15\cos(\omega t + 30^\circ)$ V $\quad I_{1m} = \sqrt{2} \times I_{1rms} = 7.071$ A

$\mathbf{V}_1 = 15\angle 30^\circ$ V $\quad \mathbf{I}_1 = 7.071\angle -10^\circ$ A

$i_1(t) = 7.071\cos(\omega t - 10^\circ)$ A

P5.30 For a pure resistance, current and voltage are in phase.
For a pure inductance, current lags voltage by 90°.
For a pure capacitance, current leads voltage by 90°.

P5.31 For an inductance, we have $\mathbf{V}_L = j\omega L\mathbf{I}_L$.

For a capacitance, we have $\mathbf{V}_C = \dfrac{\mathbf{I}_C}{j\omega C}$.

For a resistance, we have $\mathbf{V}_R = R\mathbf{I}_R$.

P5.32*

$$v_L(t) = 10\cos(2000\pi t)$$

$$\omega = 2000\pi$$

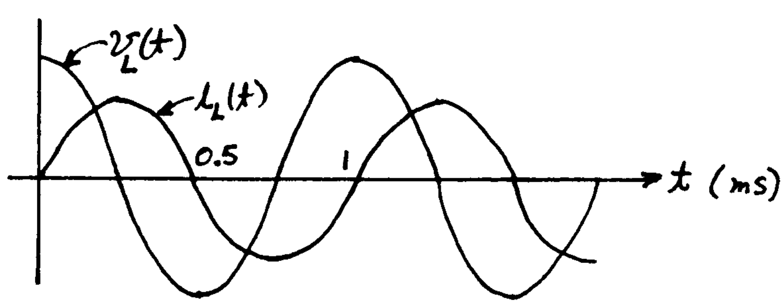
$$Z_L = j\omega L = j200\pi = 200\pi\angle 90^\circ$$

$$\mathbf{V}_L = 10\angle 0^\circ$$

$$\mathbf{I}_L = \mathbf{V}_L / Z_L = (1/20\pi)\angle -90^\circ$$

$$i_L(t) = (1/20\pi)\cos(2000\pi t - 90^\circ) = (1/20\pi)\sin(2000\pi t)$$

$i_L(t)$ lags $v_L(t)$ by 90°



P5.33*

$$v_C(t) = 10\cos(2000\pi t)$$

$$\omega = 2000\pi$$

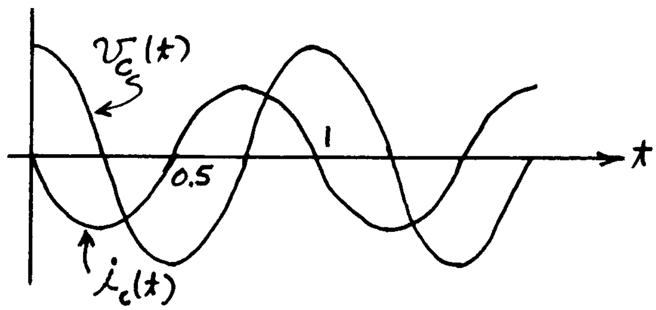
$$Z_C = \frac{-j}{\omega C} = -j15.92 = 15.92\angle -90^\circ\ \Omega$$

$$\mathbf{V}_C = 10\angle 0^\circ$$

$$\mathbf{I}_C = \mathbf{V}_C / Z_C = 0.6283\angle 90^\circ$$

$$i_C(t) = 0.6283\cos(2000\pi t + 90^\circ) = -0.6283\sin(2000\pi t)$$

$i_C(t)$ leads $v_C(t)$ by 90°



P5.34 (a) Notice that the current is a sine rather than a cosine.

$$\mathbf{V} = 100\angle 30^\circ \quad \mathbf{I} = 2.5\angle -60^\circ \quad Z = \frac{\mathbf{V}}{\mathbf{I}} = 40\angle 90^\circ = j40$$

Because Z is pure imaginary and positive, the element is an inductance.

$$\omega = 200 \qquad L = \frac{|Z|}{\omega} = 200 \text{ mH}$$

(b) Notice that the voltage is a sine rather than a cosine.

$$\mathbf{V} = 100\angle -60^\circ \quad \mathbf{I} = 4\angle 30^\circ \quad Z = \frac{\mathbf{V}}{\mathbf{I}} = 25\angle -90^\circ = -j25$$

Because Z is pure imaginary and negative, the element is a capacitance.

$$\omega = 200 \qquad C = \frac{1}{|Z|\omega} = 200\ \mu\text{F}$$

(c) $\mathbf{V} = 100\angle 30^\circ \quad \mathbf{I} = 5\angle 30^\circ \quad Z = \frac{\mathbf{V}}{\mathbf{I}} = 20\angle 0^\circ = 20 + j0$

Because Z is pure real, the element is a resistance of 20 Ω.

5.35 (a) From the plot, we see that $T = 4$ ms, so we have $f = 1/T = 250$ Hz and $\omega = 500\pi$. Also, we see that the current lags the voltage by 1 ms or

90°, so we have an inductance. . Finally, $\omega L = V_m / I_m = 5000\ \Omega$, from which we find that $L = 3.183$ H.

(b) From the plot, we see that $T = 16$ ms, so we have $f = 1/T = 62.5$ Hz and $\omega = 125\pi$. Also, we see that the current leads the voltage by 4 ms or 90°, so we have a capacitance. Finally, $1/\omega C = V_m / I_m = 2500\ \Omega$, from which we find that $C = 1.019\ \mu$F.

P5.36 (a) $Z = \dfrac{\mathbf{V}}{\mathbf{I}} = 20\angle -90^\circ = -j20$

Because Z is pure imaginary and negative, the element is a capacitance.

$$\omega = 1000 \qquad C = \frac{1}{|Z|\omega} = 50\ \mu\text{F}$$

(b) $Z = \dfrac{\mathbf{V}}{\mathbf{I}} = 10\angle 90^\circ = j10$

Because Z is pure imaginary and positive, the element is an inductance.

$$\omega = 1000 \qquad L = \frac{|Z|}{\omega} = 10\text{ mH}$$

(c) $Z = \dfrac{\mathbf{V}}{\mathbf{I}} = 20\angle 0^\circ = 20$

Because Z is pure real, the element is a resistance of 20 Ω.

P5.37 The steps in analysis of steady-state ac circuits are:

1. Replace sources with their phasors.
2. Replace inductances and capacitances with their complex impedances.
3. Use series/parallel, node voltages, or mesh currents to solve for the quantities of interest.

All of the sources must have the same frequency.

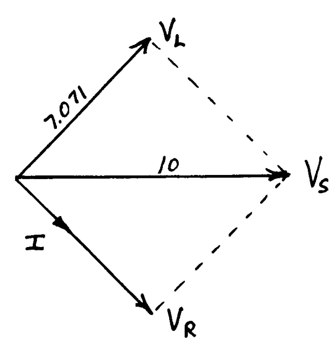
P5.38*

$$\mathbf{I} = \frac{\mathbf{V}_s}{R + j\omega L} = \frac{10\angle 0^\circ}{100 + j100} = 70.71\angle -45^\circ \text{ mA}$$

$$\mathbf{V}_R = R\mathbf{I} = 7.071\angle -45^\circ \text{ V}$$

$$\mathbf{V}_L = j\omega L\mathbf{I} = 7.071\angle 45^\circ \text{ V}$$

$\mathbf{I}$ lags $\mathbf{V}_s$ by 45°

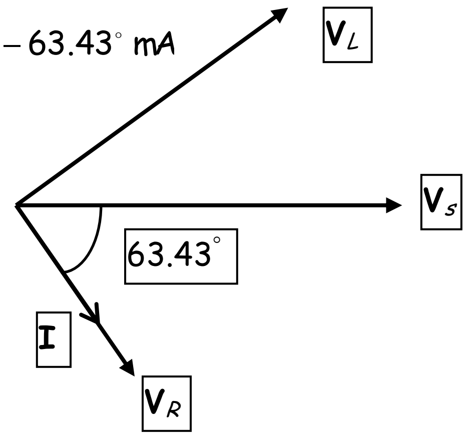


P5.39 $\mathbf{I} = \dfrac{\mathbf{V}_s}{R + j\omega L} = \dfrac{10\angle 0^\circ}{100 + j200} = 44.72\angle -63.43^\circ \text{ mA}$

$\mathbf{V}_R = R\mathbf{I} = 4.472\angle -63.43^\circ \text{ V}$

$\mathbf{V}_L = j\omega L\mathbf{I} = 8.944\angle 26.57^\circ \text{ V}$

$\mathbf{I}$ lags $\mathbf{V}_s$ by 63.43°



P5.40*

$\mathbf{I} = \dfrac{\mathbf{V}_s}{R - j/\omega C}$

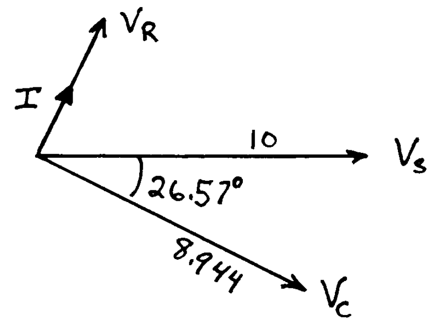
$= \dfrac{10\angle 0^\circ}{1000 - j2000}$

$= 4.472\angle 63.43^\circ \text{ mA}$

$\mathbf{V}_R = R\mathbf{I} = 4.472\angle 63.43^\circ \text{ V}$

$\mathbf{V}_C = (-j/\omega C)\mathbf{I} = 8.944\angle -26.57^\circ \text{V}$

$\mathbf{I}$ leads $\mathbf{V}_s$ by 63.43°

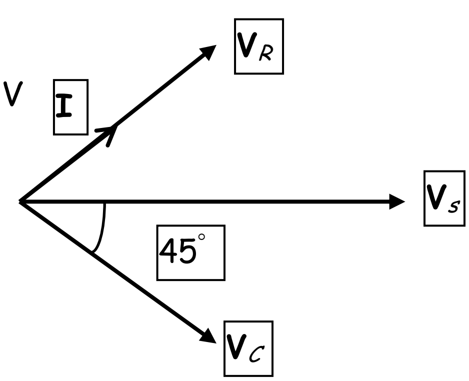


P5.41 $\mathbf{I} = \dfrac{\mathbf{V}_s}{R - j/\omega C} = \dfrac{10\angle 0^\circ}{1000 - j1000} = 5 + j5 = 7.071\angle 45^\circ \text{ mA}$

$\mathbf{V}_R = R\mathbf{I} = 7.071\angle 45^\circ \text{ V}$

$\mathbf{V}_C = (-j/\omega C)\mathbf{I} = 7.071\angle -45^\circ \text{ V}$

$\mathbf{I}$ leads $\mathbf{V}_s$ by 45°



P5.42* $Z = j\omega L + R - j\dfrac{1}{\omega C}$

$$\omega = 500: \quad Z = j50 + 50 - j200$$
$$= 50 - j150\ \Omega = 158.1\angle -71.57^\circ$$

$$\omega = 1000: \quad Z = j100 + 50 - j100$$
$$= 50\ \Omega = 50\angle 0^\circ$$

$$\omega = 2000: \quad Z = j200 + 50 - j50$$
$$= 50 + j150\ \Omega = 158.1\angle 71.57^\circ$$

P5.43 $Z = \dfrac{1}{1/Z_L + 1/Z_c + 1/R} = \dfrac{1}{1/j\omega L + j\omega C + 1/R}$

$$\omega = 500: \quad Z = \frac{1}{1/(j50) + j0.005 + 0.01}$$
$$= 30.7692 + j46.1538 = 55.47\angle 56.31^\circ\ \Omega$$

$$\omega = 1000: \quad Z = 100 + j0 = 100\angle 0^\circ\ \Omega$$

$$\omega = 2000: \quad Z = 30.7692 - j46.1538 = 55.47\angle -56.31^\circ\ \Omega$$

P5.44 $Z = \dfrac{1}{1/(R + j\omega L) + j\omega C}$

$$\omega = 500: \quad Z = \frac{1}{1/(5 + j5) + j0.05} = 8 + j4 = 8.944\angle 26.57^\circ\ \Omega$$

$$\omega = 1000: \quad Z = 20 - j10 = 22.36\angle -26.57^\circ\ \Omega$$

$$\omega = 2000: \quad Z = 0.5 - j6.5 = 6.519\angle -85.60^\circ\ \Omega$$

P5.45*

$$\mathbf{I}_s = 10\angle 0^\circ \text{ mA}$$

$$\mathbf{V} = \mathbf{I}_s \frac{1}{1/R + 1/j\omega L + j\omega C}$$

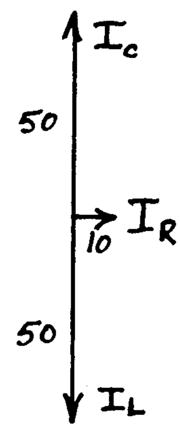
$$= 10^{-2} \frac{1}{1/1000 - j0.005 + j0.005}$$

$$= 10\angle 0^\circ \text{ V}$$

$$\mathbf{I}_R = \mathbf{V}/R = 10\angle 0^\circ \text{ mA}$$

$$\mathbf{I}_L = \mathbf{V}/j\omega L = 50\angle -90^\circ \text{ mA}$$

$$\mathbf{I}_C = \mathbf{V}(j\omega C) = 50\angle 90^\circ \text{ mA}$$



The peak value of $i_L(t)$ is five times larger than the source current! This is possible because current in the capacitance balances the current in the inductance (i.e., $\mathbf{I}_L + \mathbf{I}_C = 0$).

5.46

$$\mathbf{I}_s = 0.5\angle{-90^\circ}$$

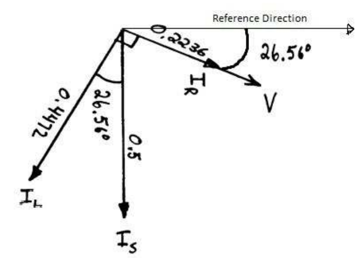
$$\mathbf{V} = \mathbf{I}_s \frac{1}{1/200 + 1/j100}$$

$$= 44.72\angle{-26.56^\circ}$$

$$\mathbf{I}_R = \mathbf{V}/R = 0.2236\angle{-26.56^\circ}$$

$$\mathbf{I}_L = \mathbf{V}/j\omega L = 0.4472\angle -116.56^\circ$$

$\mathbf{V}$ leads $\mathbf{I}_s$ by 63.44°



P5.47

$$\mathbf{V}_s = 10\angle -45^\circ \text{ V}$$

$$\mathbf{I} = \frac{\mathbf{V}_s}{j\omega L + R - j/\omega C} = \frac{10\angle -45^\circ}{j500 + 10 - j500} = 1\angle -45^\circ \text{ A}$$

$$\mathbf{V}_L = j500 \times \mathbf{I} = 500\angle 45^\circ \text{ V}$$

$$\mathbf{V}_R = 100\mathbf{I} = 10\angle -45^\circ \text{ V}$$

$$\mathbf{V}_C = -j500\mathbf{I} = 500\angle -135^\circ \text{ V}$$

The peak value of $v_L(t)$ is 50 times larger than the source voltage! This is possible because the impedance of the capacitor cancels the impedance of the inductance.

P5.48*

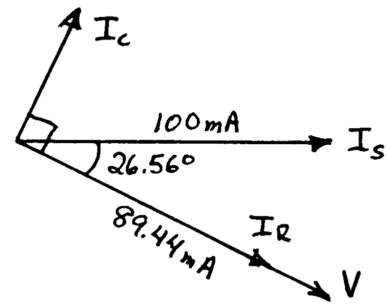
$$\mathbf{I}_s = 100\angle 0^\circ \text{ mA}$$

$$\mathbf{V} = \mathbf{I}_s \frac{1}{1/100 + 1/(-j200)}$$

$$= 8.944\angle -26.56^\circ \text{ V}$$

$$\mathbf{I}_R = \mathbf{V}/R = 89.44\angle -26.56^\circ \text{ mA}$$

$$\mathbf{I}_C = \mathbf{V}(j\omega C) = 44.72\angle 63.44^\circ \text{ mA}$$



$\mathbf{V}$ lags $\mathbf{I}_s$ by 26.56°

P5.49

$$\mathbf{V}_1 = 100\angle -90^\circ = -j100$$

$$\mathbf{V}_2 = 100\angle 90^\circ = j100$$

$$\mathbf{I} = \frac{\mathbf{V}_1 - \mathbf{V}_2}{R + j\omega L} = \frac{-j100 - j100}{100 + j100}$$

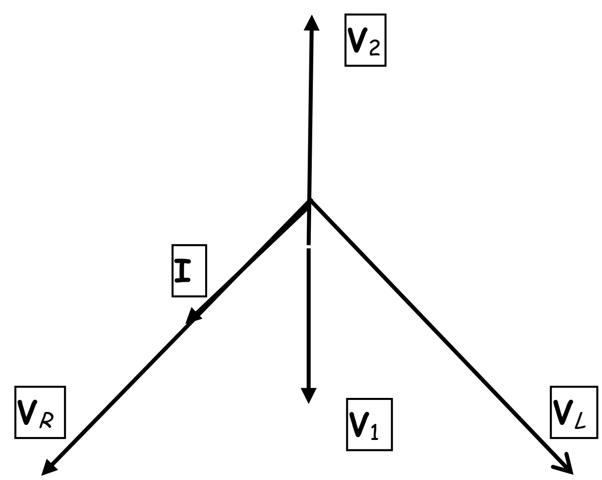
$$= 1.414\angle -135^\circ \text{ A}$$

$$\mathbf{V}_R = 100\mathbf{I} = 141.4\angle -135^\circ \text{ V}$$

$$\mathbf{V}_L = j100\mathbf{I} = 141.4\angle -45^\circ \text{ V}$$

$\mathbf{I}$ lags $\mathbf{V}_1$ by 45°

$\mathbf{I}$ lags $\mathbf{V}_L$ by 90°

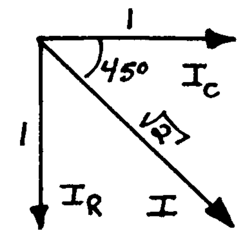


P5.50 The KCL equation is $\dfrac{\mathbf{V}_1 - j20}{5} + \dfrac{\mathbf{V}_1}{5 + j5} + 3 = 0$. Solving, we find

$$\mathbf{V}_1 = -13 + j9 = 15.81\angle 145.30^\circ \text{ V}.$$

P5.51

$$Z_{total} = j\omega L + \frac{1}{1/R + j\omega C}$$
$$= j100 + \frac{1}{0.01 + j0.01}$$
$$= j100 + 50 - j50$$
$$= 50 + j50$$
$$= 70.71\angle 45^\circ$$



$$\mathbf{I} = \frac{100\angle 0^\circ}{Z_{total}} = 1.414\angle -45^\circ$$

$$\mathbf{I}_R = \mathbf{I}\frac{Z_C}{R + Z_C} = (1.414\angle -45^\circ)\times\frac{-j100}{100 - j100}$$
$$= 1\angle -90^\circ$$

$$\mathbf{I}_C = \mathbf{I}\frac{R}{R + Z_C} = (1.414\angle -45^\circ)\times\frac{100}{100 - j100}$$
$$= 1\angle 0^\circ$$

P5.52 (a) The impedance is given by $Z = j\omega(0.02) - j/(\omega 50\times 10^{-6})$ which is infinite for zero frequency and for infinite frequency. Thus, the combination is equivalent to an open circuit for zero frequency and for infinite frequency.
Setting the impedance equal to zero, we solve to find $\omega = 1000$. Thus, the combination is equivalent to a short circuit for $f = 1000/2\pi = 159.2$ Hz.

(b) The impedance is given by $Z = \dfrac{1}{j\omega(50\times 10^{-6}) - j/(0.02\omega)}$ which is infinite when the denominator is zero. This occurs for $\omega = 1000$. Thus, the parallel combination is equivalent to an open circuit for $f = 1000/2\pi = 159.2$ Hz.
The impedance is zero for zero frequency and for infinite frequency. Thus, the parallel combination is equivalent to a short circuit for zero frequency and for infinite frequency.

P5.53 The KCL equation is $\dfrac{\mathbf{V}_1 - j20}{10} + \dfrac{\mathbf{V}_1}{j10} + \dfrac{\mathbf{V}_1 - (17.32 + j10)}{10} = 0$. Solving, we find $\mathbf{V}_1 = 17.32 + j37.32 = 41.14\angle 65.10^\circ$ V.

P5.54 (a) The impedance is given by $Z = 50 + j\omega(0.02)$ which is infinite for infinite frequency. Thus, the series combination is equivalent to an open circuit for infinite frequency.

There is no real value of frequency for which the combination is equivalent to a short circuit. (We consider ω to be a real value.)

(b) The impedance is given by $Z = \dfrac{1}{0.02 - j/(0.02\omega)}$ which is infinite when the denominator is zero. This does not occur for any real value of frequency. Thus, the parallel combination is nor equivalent to an open circuit for any value of frequency.

The impedance is zero for zero frequency. Thus, the parallel combination is equivalent to a short circuit for zero frequency.

P5.55 The **real power** is the average power P consumed by a general load which is given by

$$P = V_{rms} I_{rms} \cos(\theta)$$

The peak instantaneous power associated with the energy storage elements contained in a general load is called **reactive power** and is given by

$$Q = V_{rms} I_{rms} \sin(\theta)$$

where θ is the power angle given by Equation 5.62. V_{rms} is the effective (or rms) voltage across the load, and I_{rms} is the effective current through the load.

The **apparent power** is defined as the product of the effective voltage and the effective current, or

$$\text{Apparent Power} = V_{rms} I_{rms}$$

The units for real power are watts (W). For reactive power, the units are volt-amperes reactive (VAR). For apparent power, the units are volt-amperes (VA).

P5.56 Complex power delivered to a circuit element is equal to one half of the phasor voltage across the element times the complex conjugate of the phasor current, provided that the current reference direction points into the positive reference for the voltage. Average power P equals the real part of the complex power and reactive power Q equals the imaginary part.

P5.57 Power factor is the cosine of the power angle. It is often expressed as a percentage. $\text{PF} = \cos(\theta) \times 100\%$. The power angle is the phase of the voltage minus the phase of the current provided that the current reference direction points into the positive reference for the voltage.

P5.58 A load with a leading power factor is capacitive and has negative reactive power. A load with a lagging power factor is inductive and has positive reactive power.

P5.59 (a) For a pure resistance, the power is positive and the reactive power is zero.

(b) For a pure inductance, the power is zero and the reactive power is positive.

(c) For a pure capacitance, the power is zero, and the reactive power is negative.

P5.60 Usually, power factor correction refers to adding capacitances in parallel with an inductive load to reduce the reactive power flowing from the power plant through the distribution system to the load. To minimize power system losses, we need 100% power factor for the combined load. Ultimately, an economic analysis is needed to balance the costs of power factor correction against the costs of losses and additional distribution capacity needed because of reactive power.

P5.61 See Figure 5.23 in the book.

P5.62 Real power represents a net flow, over time, of energy from the source to the load. This energy must be supplied to the system from hydro, fossil-fuel, nuclear, or other energy sources.
Reactive power represents energy that flows back and forth from the source to the load. Aside from losses in the transmission system (lines and transformers), no net energy must be supplied to the system in steady state to create the reactive power. Reactive power is important mainly because of the increased system losses associated with it.

P5.63* $\mathbf{I} = \dfrac{1000\sqrt{2}\angle 0^\circ}{100} + \dfrac{1000\sqrt{2}\angle 0^\circ}{-j265.3} = 14.14 + j5.331 = 15.11\angle 20.66^\circ$

$P = V_{rms} I_{rms} \cos\theta = 10 \text{ kW}$

$Q = V_{rms} I_{rms} \sin\theta = -3.770 \text{ kVAR}$

$\text{Apparent power} = V_{rms} I_{rms} = 10.68 \text{ kVA}$

$\text{Power factor} = \cos(20.66^\circ) = 0.9357 = 93.57\% \text{ leading}$

P5.64 $\mathbf{I} = \dfrac{1000\sqrt{2}\angle 90^\circ}{100} + \dfrac{1000\sqrt{2}\angle 90^\circ}{j377} = j14.14 + 3.751$

$= 14.63\angle 75.14^\circ$

$\text{Power angle } \theta = 90 - 75.14 = 14.86^\circ$

$P = V_{rms} I_{rms} \cos\theta = 10 \text{ kW}$

$Q = V_{rms} I_{rms} \sin\theta = 2.653 \text{ kVAR}$

$\text{Apparent power} = V_{rms} I_{rms} = 10.345 \text{ kVA}$

$\text{Power factor} = \cos(14.86^\circ) = 0.9666 = 96.66\% \text{ lagging}$

P5.65* This is a capacitive load because the reactance is negative.

$P = I_{rms}^2 R = (15)^2 100 = 22.5 \text{ kW}$

$Q = I_{rms}^2 X = (15)^2(-50) = -11.25 \text{ kVAR}$

$\theta = \tan^{-1}\left(\frac{Q}{P}\right) = \tan^{-1}(-0.5) = 26.57^\circ$

power factor $= \cos(\theta) = 89.44\%$

apparent power $= \sqrt{P^2 + Q^2} = 25.16 \text{ KVA}$

P5.66 We have

$\mathbf{S} = \frac{1}{2}\mathbf{VI^*} = \frac{1}{2}(1500\sqrt{2}\angle 60^\circ)(15\sqrt{2}\angle -75^\circ) = 22.5\angle -15^\circ \text{ kVA}$

$= 21.73 - j5.823 \text{ kVA}$

$\theta = \theta_v - \theta_i = 60^\circ - 75^\circ = -15^\circ$

power factor $= \cos(\theta) = 96.59\%$ leading

$P = V_{rms} I_{rms} \cos(\theta) = 21.73 \text{ kW}$

$Q = V_{rms} I_{rms} \sin(\theta) = -5.823 \text{ kVAR}$

apparent power $= V_{rms} I_{rms} = 1500 \times 15 = 22.50 \text{ KVA}$

$$Z = \frac{\mathbf{V}}{\mathbf{I}} = \frac{1500\sqrt{2}\angle 60^\circ}{15\sqrt{2}\angle 75^\circ} = 100\angle -15^\circ\ \Omega$$

P5.67 $\mathbf{I} = \dfrac{240\sqrt{2}\angle 60^\circ - 280\sqrt{2}\angle 30^\circ}{5 + j8} = 14.84\sqrt{2}\angle 93.02^\circ$

$I_{rms} = 14.84 \text{ A}$

Delivered by Source *A*:

$P_A = 240 I_{rms} \cos(60 - 93.02) = 2.987 \text{ kW}$

$Q_A = 240 I_{rms} \sin(60 - 93.02) = -1.941 \text{ kVAR}$

Absorbed by Source *B*:

$P_B = 280 I_{rms} \cos(30 - 93.02) = 1.885 \text{ kW}$

$Q_A = 280 I_{rms} \sin(30 - 93.02) = -3.704 \text{ kVAR}$

Absorbed by resistor:

$P_R = I_{rms}^2 R = 1.101 \text{ kW}$

Absorbed by inductor:

$Q_L = I_{rms}^2 X = 1.762 \text{ kVAR}$

P5.68 This is a capacitive load because the reactance is negative.

$Z = 40 - j30 = 50\angle -36.87^\circ \quad I = \dfrac{\mathbf{V}}{Z} = \dfrac{1500\sqrt{2}\angle 30^\circ}{50\angle -36.87^\circ} = 30\sqrt{2}\angle 66.87^\circ$

$\mathbf{S} = \frac{1}{2}\mathbf{VI}^* = 45\angle -36.87^\circ = 36 - j27 \text{ kVA}$

$P = I_{rms}^2 R = (30)^2 40 = 36 \text{ kW}$

$Q = I_{rms}^2 X = (30)^2(-30) = -27 \text{ kVAR}$

$\theta = -36.87^\circ$

power factor $= \cos(\theta) = 80\%$

apparent power $= V_{rms} I_{rms} = 1500 \times 30 = 45$ KVA

P5.69 $\mathbf{S} = \frac{1}{2}\mathbf{VI}^* = \frac{1}{2}(10^4\sqrt{2}\angle 75^\circ) \times (25\sqrt{2}\angle 30^\circ)^* = 176.8 + j176.8 \text{ kVA}$

$V_{rms} = 10^4$ V $\qquad I_{rms} = 25$ A $\qquad \theta = \theta_v - \theta_i = 75^\circ - 30^\circ = 45^\circ$

power factor $= \cos\theta \times 100\% = 70.71\%$ lagging

$P = V_{rms} I_{rms} \cos(\theta) = 176.8$ kW $\quad Q = V_{rms} I_{rms} \sin(\theta) = 176.8$ kVAR

Apparent Power $= V_{rms} I_{rms} = 250$ KVA

This is an inductive load.

P5.70 (a) For a pure capacitance, real power is zero and reactive power is negative.

(b) For a resistance in series with an inductance, real power is positive and reactive power is positive.

(c) For a resistance in series with a capacitance, real power is positive and reactive power is negative.

(d) For a pure resistance, real power is positive and reactive power is zero.

P5.71 If the inductive reactance is greater than the capacitive reactance, the total impedance is inductive, real power is zero, and reactive power is positive.

If the inductive reactance equals the capacitive reactance, the total impedance is zero, and the current is infinite. Real power and reactive power are indeterminate. This case is rarely, if ever, of practical importance.

If the inductive reactance is less than the capacitive reactance, the total impedance is capacitive, real power is zero, and reactive power is negative.

P5.72 If the inductive reactance is greater than the capacitive reactance, the total impedance is capacitive, real power is zero, and reactive power is negative.

If the inductive reactance equals the capacitive reactance, the total impedance is infinite, and the current is zero. Real power and reactive power are both zero.

If the inductive reactance is less than the capacitive reactance, the total impedance is inductive, real power is zero, and reactive power is positive.

P5.73 Apparent power $= V_{rms} I_{rms} \quad \Rightarrow \quad 2500 = 240 I_{rms} \quad \Rightarrow \quad I_{rms} = 10.417 \text{ A}$

$$|Z| = \frac{V_{rms}}{I_{rms}} = 23.04\ \Omega$$

$\cos(\theta) = \dfrac{P}{V_{rms} I_{rms}} = \dfrac{1500}{2500} \quad \Rightarrow \quad \theta = 53.13^\circ$ (We know that θ is positive because the impedance is inductive.)

$$Z = 23.04\angle 53.13^\circ = 13.82 + j18.43 = R + j\omega L$$

$$R = 13.82\ \Omega \qquad L = \frac{18.43}{(2\pi 60)} = 48.89 \text{ mH}$$

P5.74 $\mathbf{V}_A = (15 - j15)20\sqrt{2}\angle 10^\circ + 240\sqrt{2}\angle -20^\circ = 659.0\sqrt{2}\angle -29.59^\circ$

$V_{Arms} = 659.0 \text{ V rms}$

Delivered by Source *A*:

$$P_A = 659.0(20)\cos(-29.59 - 10) = 10.16 \text{ kW}$$

$$Q_A = 659.0(20)\sin(-29.59 - 10) = -8.40 \text{ kVAR}$$

Absorbed by Source *B*:

$$P_B = 240(20)\cos(-20 - 10) = 4.160 \text{ kW}$$

$$Q_B = 240(20)\sin(-20 - 10) = -2.400 \text{ kVAR}$$

Absorbed by resistor:

$$P_R = I_{rms}^2 R = 6.00 \text{ kW}$$

Absorbed by capacitor:

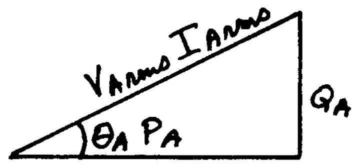
$$Q_L = I_{rms}^2 X = -6.00 \text{ kVAR}$$

P5.75* Load A:

$P_A = 10\text{ kW}$

$\theta_A = \cos^{-1}(0.9) = 25.84^\circ$

$Q_A = P_A \tan\theta_A = 4.843\text{ kVAR}$



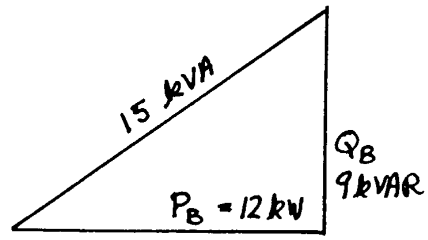
Load B:

$V_{rms} I_{Brms} = 15\text{ kVA}$

$\theta_B = \cos^{-1}(0.8) = 36.87^\circ$

$Q_B = V_{rms} I_{Brms} \sin(\theta_B) = 9\text{ kVAR}$

$P_B = V_{rms} I_{Brms} \cos(\theta_B) = 12\text{ kW}$



Source:

$P_s = P_A + P_B = 22\text{ kW}$

$Q_s = Q_A + Q_B = 13.84\text{ kVAR}$

$$\text{Apparent power} = \sqrt{(P_s)^2 + (Q_s)^2} = 26\text{ kVA}$$

$$\text{Power factor} = \frac{P_s}{\text{Apparent power}} = 0.8462 = 84.62\%\text{ lagging}$$

P5.76 Load A:

$P_A = 50\text{ kW}$

$\theta_A = \cos^{-1}(0.6) = 53.13^\circ$

$Q_A = P_A \tan\theta_A = 66.67\text{ kVAR}$

Load B:

The phase angle is negative for a leading power factor.

$P_B = 75\text{ kW}$

$\theta_B = \cos^{-1}(0.8) = -36.86^\circ$

$Q_B = P_B \tan\theta_B = -56.25\text{ kVAR}$

Source:

$P_s = P_A + P_B = 125\text{ kW}$

$Q_s = Q_A + Q_B = 10.42\text{ kVAR}$

$$\text{Apparent power} = \sqrt{(P_s)^2 + (Q_s)^2} = 125.43\text{ kVA}$$

$$\text{Power factor} = \frac{P_s}{\text{Apparent power}} = 0.9965 = 99.65\%\text{ lagging}$$

P5.77 $P = \frac{(V_{rms})^2}{R} = \frac{1500^2}{25} = 90\text{ kW}$

$$Q = Q_L + Q_C = \frac{(V_{rms})^2}{X_L} + \frac{(V_{rms})^2}{X_C} = \frac{1500^2}{100} + \frac{1500^2}{(-250)}$$

$$Q = 13.5\text{ kVAR}$$

$$\text{Apparent power} = \sqrt{P^2 + Q^2} = 91.01\text{ kVA}$$

$$\text{Power factor} = \frac{P}{\text{Apparent power}} = 98.89\%\text{ lagging}$$

P5.78 $\mathbf{I} = \frac{1500\sqrt{2}\angle 0^\circ}{R + j\omega L - j/\omega C} = \frac{1500\sqrt{2}\angle 0^\circ}{25 + j100 - j250} = 9.864\sqrt{2}\angle 80.54^\circ$

$$\mathbf{S} = \tfrac{1}{2}\mathbf{VI}^* = 2432 - j14595$$

$$P = \text{Re}(\mathbf{S}) = 2432\text{ W}$$

$$Q = \text{Im}(\mathbf{S}) = -14595\text{ VAR}$$

$$\text{Apparent power} = |\mathbf{S}| = 14796\text{ VA}$$

$$\text{Power factor} = \cos(\theta) = \cos(-80.54^\circ) = 16.43\%\text{ leading}$$

P5.79* (a)

$$\cos\theta = 0.25$$

$$\theta = 75.52^\circ$$

$$P = V_{rms} I_{rms} \cos(\theta)$$

$$I_{rms} = \frac{P}{V_{rms}\cos(\theta)} = \frac{100\text{ kW}}{1\text{ kV}(0.25)} = 400\text{ A}$$

$$\mathbf{I} = 400\sqrt{2}\angle -75.52^\circ$$

(b)

$$Q_{load} = V_{rms} I_{rms} \sin\theta = 387.3\text{ kVAR}$$

$$Q_{\text{total}} = 0 = Q_{load} + Q_C$$

$$Q_C = -387.3 \times 10^3 = \frac{(V_{rms})^2}{X_C}$$

$$X_C = -2.582 = -\frac{1}{\omega C}$$

$$C = 1027\,\mu\text{F}$$

The capacitor must be rated for at least 387.3 kVAR. With the capacitor in place, we have:

$$P = 100\text{ kW} = V_{rms}I_{rms}$$
$$I_{rms} = 100\text{ A}$$
$$\mathbf{I} = 100\angle 0^\circ$$

(c) The line current is smaller by a factor of 4 with the capacitor in place, reducing I^2R losses in the line by a factor of 16.

P5.80 The ac steady state Thévenin equivalent circuit for a two-terminal circuit consists of a phasor voltage source $\mathbf{V}_t$ in series with a complex impedance Z_t.

The ac steady state Norton equivalent circuit for a two-terminal circuit consists of a phasor current source $\mathbf{I}_n$ in parallel with a complex impedance Z_t.

$\mathbf{V}_t$ is the open-circuit voltage of the original circuit. $\mathbf{I}_n$ is the short circuit current of the original network. The impedance can be found by zeroing the independent sources and finding the impedance looking into the terminals of the original network. Also, we have $\mathbf{V}_t = Z_t\mathbf{I}_t$.

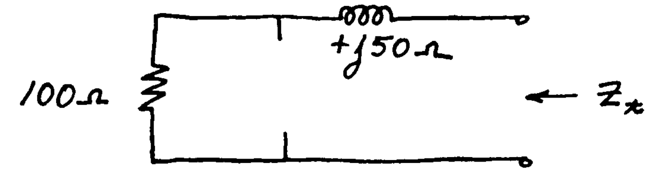
P5.81 The load voltage is given by the voltage divider principle.

$$\mathbf{V}_L = \mathbf{V}_t \frac{Z_L}{Z_t + Z_L}$$

The load voltage $\mathbf{V}_L$ is larger than the Thévenin voltage in magnitude if the magnitude of $Z_t + Z_L$ is less than the magnitude of Z_L which can happen when the imaginary parts of Z_t and Z_L have opposite signs. This does not happen in purely resistive circuits.

P5.82 To attain maximum power, the load must equal (a) the complex conjugate of the Thévenin impedance if the load can have any complex value; (b) the magnitude of the Thévenin impedance if the load must be a pure resistance.

P5.83* (a) Zeroing the current source, we have:



Thus, the Thévenin impedance is

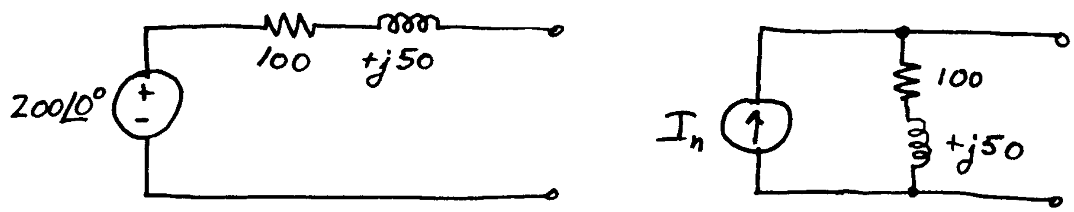
$$Z_t = 100 + j50 = 111.8\angle 26.57^\circ\ \Omega$$

Under open circuit conditions, there is zero voltage across the inductance, the current flows through the resistance, and the Thévenin voltage is

$$\mathbf{V}_t = \mathbf{V}_{oc} = 200\angle 0^\circ$$

$$\mathbf{I}_n = \mathbf{V}_t / Z_t = 1.789\angle -26.57^\circ$$

Thus, the Thévenin and Norton equivalent circuits are:



(b) For maximum power transfer, the load impedance is

$$Z_{load} = 100 - j50$$

$$\mathbf{I}_{load} = \frac{\mathbf{V}_t}{Z_t + Z_{load}} = \frac{200}{100 + j50 + 100 - j50} = 1$$

$$P_{load} = R_{load}\left(I_{rms-load}\right)^2 = 100(1/\sqrt{2})^2 = 50\text{ W}$$

(c) In the case for which the load must be pure resistance, the load for maximum power transfer is

$$Z_{load} = |Z_t| = 111.8$$

$$I_{load} = \frac{\mathbf{V}_t}{Z_t + Z_{load}} = \frac{200}{100 + j50 + 111.8} = 0.9190\angle -13.28^\circ$$

$$P_{load} = R_{load}\left(I_{rms-load}\right)^2 = 47.21\text{ W}$$

P5.84 At the lower left-hand node under open-circuit conditions, KCL yields $\mathbf{I}_x + 0.5\mathbf{I}_x = 0$ from which we have $\mathbf{I}_x = 0$. Then, the voltages across the 6-Ω resistance and the $-j10$-Ω capacitance are zero, and KVL yields

$$\mathbf{V}_t = \mathbf{V}_{ba-oc} = 15\angle 0^\circ\text{ V}$$

With short circuit conditions, we have

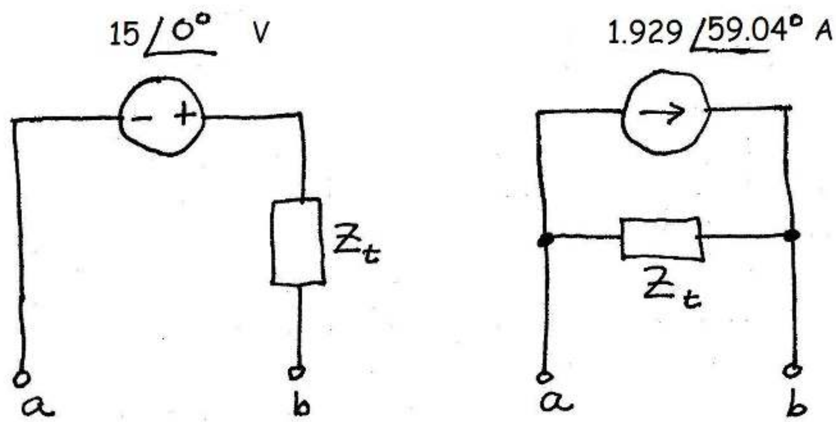
$$\mathbf{I}_x = \frac{15}{6 - j10} = 1.286\angle 59.04^\circ$$

$$\mathbf{I}_n = \mathbf{I}_{ba-sc} = 0.5\mathbf{I}_x + \mathbf{I}_x = 1.929\angle 59.04^\circ$$

The Thévenin impedance is given by

$$Z_t = \frac{\mathbf{V}_t}{\mathbf{I}_n} = \frac{15}{1.929\angle 59.04^\circ} = 7.774\angle -59.04^\circ$$

Finally, the equivalent circuits are:



P5.85 Under open-circuit conditions, we have

$$\mathbf{V}_t = \mathbf{V}_{ab-oc} = 10.00\angle 90^\circ \text{ V}$$

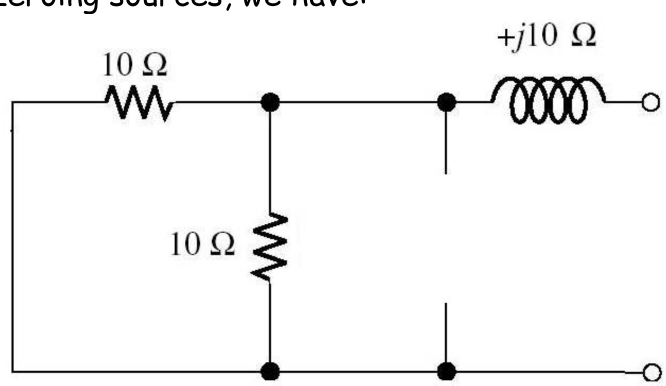
With the source zeroed, we look back into the terminals and see

$$Z_t = 4 - j3 \ \ \Omega$$

Next, the Norton current is

$$\mathbf{I}_n = \frac{\mathbf{V}_t}{Z_t} = 2\angle 126.87^\circ \text{ A}$$

P5.86 Zeroing sources, we have:



Thus, the Thévenin impedance is

$$Z_t = \frac{1}{1/10 + 1/10} + j10 = 5 + j10 = 11.18\angle 63.43^\circ$$

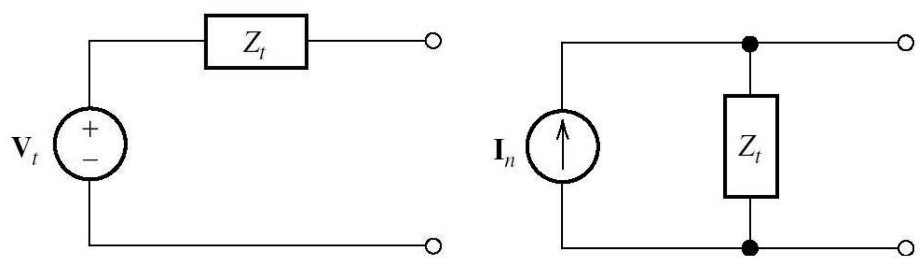
Writing a current equation for the node at the upper end of the current source under open circuit conditions, we have

$$\frac{\mathbf{V}_{oc} - 50}{10} + \frac{\mathbf{V}_{oc}}{10} = 5$$

$$\mathbf{V}_t = \mathbf{V}_{oc} = 50.0\angle 0^\circ$$

$$\mathbf{I}_n = \mathbf{V}_t / Z_t = 4.472\angle -63.43^\circ$$

The Thévenin and Norton equivalent circuits are:



For the maximum power transfer, the load impedance is

$$Z_{load} = 5 - j10$$

$$\mathbf{I}_{load} = \frac{\mathbf{V}_t}{Z_t + Z_{load}} = \frac{50}{5 - j10 + 5 + j10} = 5\,A$$

$$P_{load} = R_{load}\left(I_{rms-load}\right)^2 = 62.5\text{ W}$$

In the case for which the load must be pure resistance, the load for maximum power transfer is

$$Z_{load} = |Z_t| = 11.18\ \Omega$$

$$I_{load} = \frac{\mathbf{V}_t}{Z_t + Z_{load}} = \frac{50}{5 + j10 + 11.18} = 2.629\angle -31.72^\circ$$

$$P_{load} = R_{load}\left(I_{rms-load}\right)^2 = 38.62\text{ W}$$

P5.87* For maximum power transfer, the impedance of the load should be the complex conjugate of the Thévenin impedance:

$$Z_{load} = 10 - j5$$

$$Y_{load} = 1/Z_{load} = 0.08 + j0.04$$

$$Y_{load} = 1/R_{load} + j\omega C_{load} = 0.08 + j0.04$$

Setting real parts equal:

$$1/R_{load} = 0.08 \qquad R_{load} = 12.5\ \Omega$$

Setting imaginary parts equal:

$$\omega C_{load} = 0.04 \qquad C_{load} = 106.1\ \mu\text{F}$$

P5.88 For maximum power transfer, the impedance of the load should be the complex conjugate of the Thévenin impedance:

$$Z_{load} = 10 - j5 = R_{load} - j/(\omega C_{load})$$

Setting real parts equal:

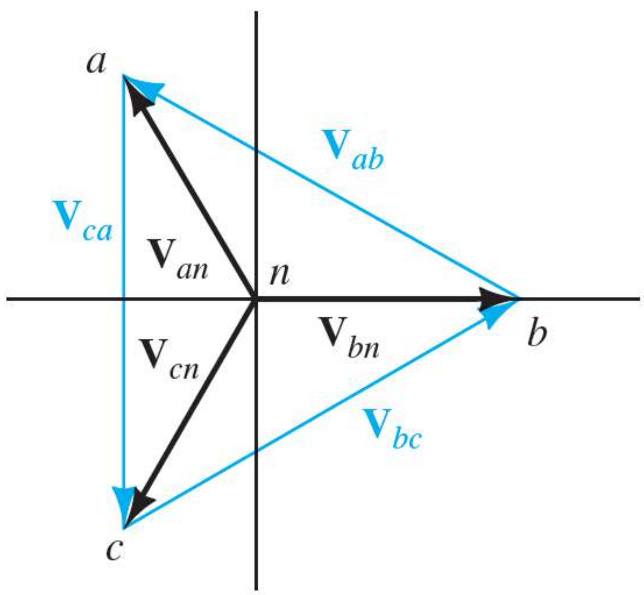
$$R_{load} = 10\ \Omega$$

Setting imaginary parts equal:

$$-5 = -1/(\omega C_{load}) \qquad C_{load} = 1/(5\omega) = 530.5\ \mu\text{F}$$

P5.89 We are given: $v_{an}(t) = 200\cos(\omega t + 120^\circ)$ and a positive-sequence source. As a phasor $\mathbf{V}_{an} = 200\angle 120^\circ$. For counterclockwise rotation, the sequence of phasors is abc.

The phasor diagram is:



From the phasor diagram, we can determine that

$$\mathbf{V}_{bn} = 200\angle 0^\circ$$

$$\mathbf{V}_{cn} = 200\angle -120^\circ$$

$$\mathbf{V}_{ab} = \mathbf{V}_{an} - \mathbf{V}_{bn} = 200\sqrt{3}\angle 150^\circ$$

$$\mathbf{V}_{bc} = \mathbf{V}_{bn} - \mathbf{V}_{cn} = 200\sqrt{3}\angle +30^\circ$$

$$\mathbf{V}_{ca} = \mathbf{V}_{cn} - \mathbf{V}_{an} = 200\sqrt{3}\angle -90^\circ$$

$$v_{bn}(t) = 200\cos(\omega t)$$

$$v_{cn}(t) = 200\cos(\omega t - 120^\circ)$$
$$v_{ab}(t) = 200\sqrt{3}\cos(\omega t + 150^\circ)$$
$$v_{bc}(t) = 200\sqrt{3}\cos(\omega t + 30^\circ)$$
$$v_{ca}(t) = 200\sqrt{3}\cos(\omega t - 90^\circ)$$

P5.90 We are given $v_{an}(t) = 150\cos(400\pi t)$ V

(a) By inspection, $\omega = 2\pi f = 400\pi$ and we have $f = 200$ Hz.

(b)
$$v_{bn}(t) = 150\cos(400\pi t - 120^\circ) \text{ V}$$
$$v_{cn}(t) = 150\cos(400\pi t + 120^\circ) \text{ V}$$

(c)
$$v_{bn}(t) = 150\cos(400\pi t + 120^\circ) \text{ V}$$
$$v_{cn}(t) = 150\cos(400\pi t - 120^\circ) \text{ V}$$

P5.91*

$$Z_Y = \frac{1}{1/R + j\omega C}$$
$$= \frac{1}{1/50 + j377 \times 10^{-4}}$$
$$= 10.98 - j20.70$$
$$= 23.43\angle -62.05^\circ$$

$$Z_\Delta = 3Z_Y$$
$$= 70.29\angle -62.05^\circ \ \Omega$$

P5.92*

$$V_L = \sqrt{3} \times V_Y = \sqrt{3} \times 440 = 762.1 \text{ V rms}$$
$$I_L = \frac{V_Y}{R} = \frac{440}{30} = 14.67 \text{ A rms}$$
$$P = 3V_Y I_L \cos(\theta) = 3 \times 440 \times 14.67 \times \cos(0)$$
$$= 19.36 \text{ kW}$$

P5.93 Total power flow in a balanced system is constant with time. For a single phase system the power flow pulsates. Reduced vibration in generators and motors is a potential advantage for the three-phase system. In addition, less wire is needed for the same power flow in a balanced three-phase system.

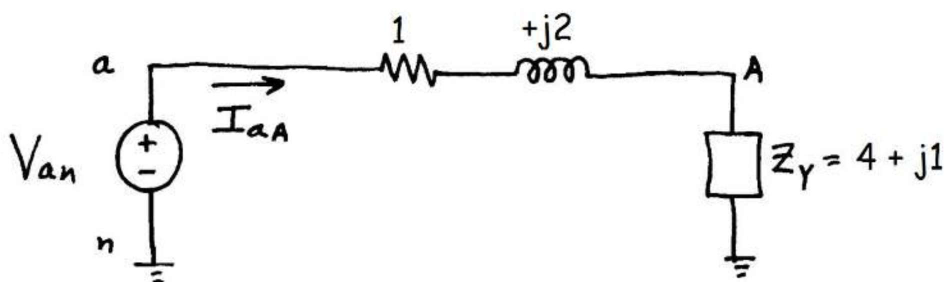
P5.94 This is a positive sequence source. The phasor diagram is shown in Figure 5.41 in the book. Thus, we have

$$\mathbf{V}_{an} = \frac{440\sqrt{2}}{\sqrt{3}}\angle 0^\circ \quad \text{V}$$

The impedance of an equivalent wye-connected load is

$$Z_y = \frac{Z_\Delta}{3} = 4 + j1 \quad \Omega$$

The equivalent circuit for the *a*-phase of an equivalent wye-wye circuit is:



Thus, the line current is

$$\mathbf{I}_{aA} = \frac{\mathbf{V}_{an}}{1 + j2 + Z_y}$$

$$= 61.61\angle -30.96^\circ$$

$$\mathbf{V}_{An} = \mathbf{V}_{an} - \mathbf{I}_{aA}(1 + j2)$$

$$= 254.03\angle -16.93^\circ$$

$$\mathbf{V}_{AB} = 440\angle 13.07^\circ$$

$$\mathbf{I}_{AB} = \frac{\mathbf{V}_{AB}}{Z_\Delta}$$

$$= 35.57\angle -0.96^\circ$$

$$P_{load} = 3(I_{ABrms})^2 \times 12 = 22.77 \text{ kW}$$

$$P_{line} = 3(I_{aArms})^2 \times 1 = 5.69 \text{ kW}$$

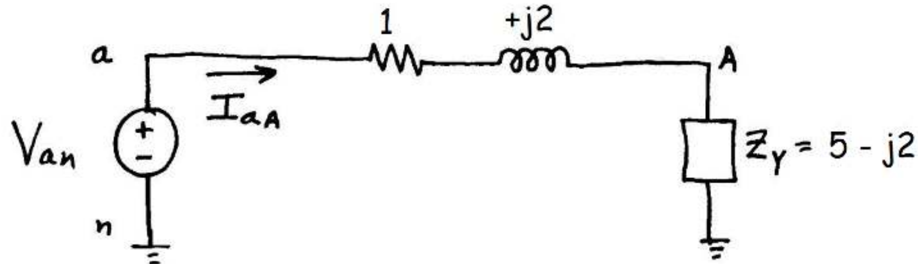
P5.95* This is a positive sequence source. The phasor diagram is shown in Figure 5.41 in the book. Thus, we have:

$$\mathbf{V}_{an} = \frac{440\sqrt{2}}{\sqrt{3}} \angle 0^\circ$$

The impedance of a equivalent wye-connected load is

$$Z_y = \frac{Z_\Delta}{3} = 5 - j2\,\Omega$$

The equivalent circuit for the a-phase of an equivalent wye-wye circuit is:



Thus, the line current is:

$$\mathbf{I}_{aA} = \frac{\mathbf{V}_{an}}{1 + j2 + Z_y}$$

$$= 59.87\angle 0^\circ$$

$$\mathbf{V}_{An} = \mathbf{V}_{an} - \mathbf{I}_{aA}(1 + j2)$$

$$= 322.44\angle -21.80^\circ$$

$$\mathbf{V}_{AB} = 558\angle 8.20^\circ$$

$$\mathbf{I}_{AB} = \frac{\mathbf{V}_{AB}}{Z_\Delta}$$

$$= 34.56\angle 30^\circ$$

$$P_{load} = 3(I_{ABrms})^2 \times 12 = 26.89\text{ kW}$$

$$P_{line} = 3(I_{aArms})^2 \times 1 = 5.38\text{ kW}$$

P5.96 The line-to-line voltage is $277\sqrt{3} = 480$ V rms.
The impedance of each arm of the delta is

$$Z_\Delta = \frac{1}{1/15 + 1/(j30)} = 13.42\angle 26.57^\circ$$

The equivalent wye has impedances of

$$Z_y = \frac{Z_\Delta}{3} = 4.472\angle 26.57^\circ$$

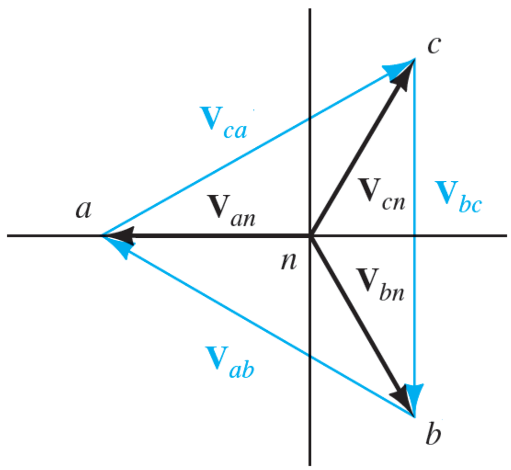
Then working with one phase of the wye-wye, we have

$$I_{line} = \frac{277}{|Z_y|} = 61.94 \quad \text{A rms}$$

$$\text{power factor} = \cos(26.57^\circ) \times 100\% = 89.44\%$$

$$P = 3(277)(61.94)(0.8944) = 46.04 \ \text{kW}$$

P5.97 The phasor diagram is:



$$\mathbf{V}_{ab} = \mathbf{V}_{an} - \mathbf{V}_{bn} = \sqrt{3} \times V_y \angle 150^\circ$$

$$\mathbf{V}_{bc} = \mathbf{V}_{\mathbf{bn}} - \mathbf{V}_{\mathbf{cn}} = \sqrt{3} \times V_y \angle -90^\circ$$

$$\mathbf{V}_{ca} = \mathbf{V}_{cn} - \mathbf{V}_{an} = \sqrt{3} \times V_y \angle 30^\circ$$

P5.98

$$V_y = \frac{V_L}{\sqrt{3}} = \frac{208}{\sqrt{3}} = 120 \text{ V rms}$$

$$\mathbf{V}_{an} = 120\sqrt{2}\angle 0^\circ$$

$$\mathbf{V}_{bn} = 120\sqrt{2}\angle -120^\circ$$

$$\mathbf{V}_{cn} = 120\sqrt{2}\angle 120^\circ$$

$$\mathbf{V}_{ab} = 208\sqrt{2}\angle 30^\circ$$

$$\mathbf{V}_{bc} = 208\sqrt{2}\angle -90^\circ$$

$$\mathbf{V}_{ca} = 208\sqrt{2}\angle 150^\circ$$

$$\mathbf{I}_a = \frac{\mathbf{V}_{an}}{30 + j40} = 2.4\sqrt{2}\angle -53.13^\circ$$

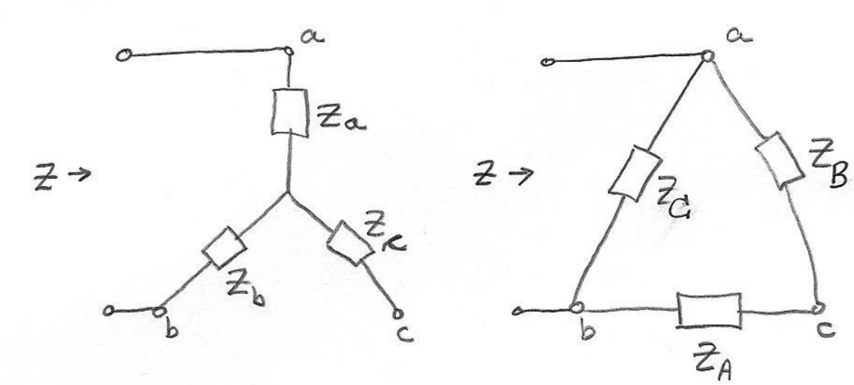
$$\mathbf{I}_b = 2.4\sqrt{2}\angle -173.13^\circ$$

$$\mathbf{I}_c = 2.4\sqrt{2}\angle 66.86^\circ$$

$$P = 3V_{Yrms} I_{Lrms} \cos(\theta) = 3 \times 120 \times 2.4 \times \cos(53.13)$$
$$= 518.4 \text{ W}$$

$$Q = 3V_{Yrms} I_{Lrsm} \sin(\theta) = 3 \times 120 \times 2.4 \times \sin(53.13^\circ)$$
$$= 691.2 \text{ VAR}$$

P5.99 As suggested in the hint given in the book, the impedances of the circuits between terminals a and b with c open must be identical.



Equating the impedances, we obtain:

$$Z_a + Z_b = \frac{1}{1/Z_C + 1/(Z_A + Z_B)} = \frac{Z_A Z_C + Z_B Z_C}{Z_A + Z_B + Z_C} \qquad (1)$$

Similarly for the other pairs of terminals, we obtain

$$Z_a + Z_c = \frac{1}{1/Z_B + 1/(Z_A + Z_C)} = \frac{Z_A Z_B + Z_B Z_C}{Z_A + Z_B + Z_C} \qquad (2)$$

$$Z_b + Z_c = \frac{1}{1/Z_A + 1/(Z_C + Z_B)} = \frac{Z_A Z_C + Z_A Z_B}{Z_A + Z_B + Z_C} \qquad (3)$$

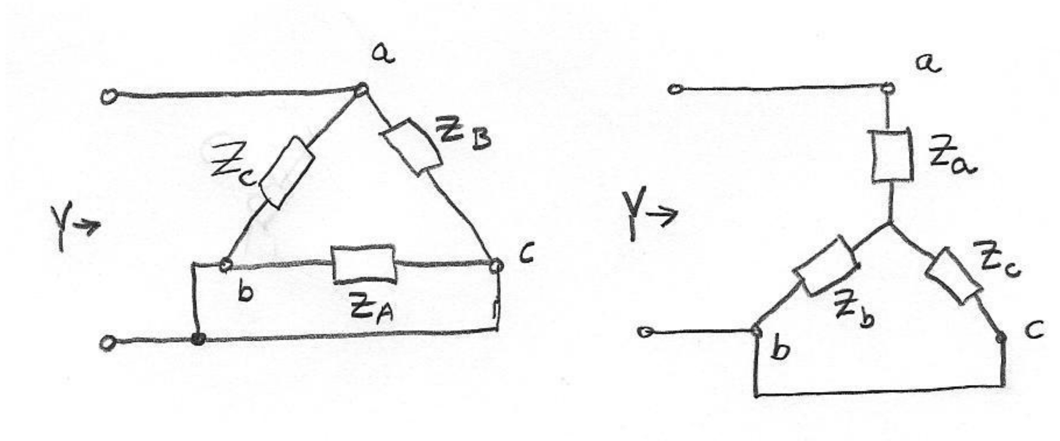
Then adding the respective sides of Equations 1 and 2, subtracting the corresponding sides of Equation 3, and dividing both sides of the result by 2, we have:

$$Z_a = \frac{Z_B Z_C}{Z_A + Z_B + Z_C}$$

Similarly we obtain:

$$Z_b = \frac{Z_A Z_C}{Z_A + Z_B + Z_C} \quad \text{and} \quad Z_c = \frac{Z_A Z_B}{Z_A + Z_B + Z_C}$$

P5.100 As suggested in the hint, consider the circuits shown below. The admittances of the circuits between terminals must be identical.



First, we will solve for the admittances of the delta in terms of the impedances of the wye. Then we will invert the results to obtain relationships between the impedances.

$$Y_C + Y_B = \frac{1}{Z_a + \dfrac{1}{1/Z_b + 1/Z_c}} = \frac{Z_b + Z_c}{Z_a Z_b + Z_b Z_c + Z_a Z_c} \quad (1)$$

Similarly working with the other terminials, we obtain

$$Y_A + Y_B = \frac{Z_a + Z_b}{Z_a Z_b + Z_b Z_c + Z_a Z_c} \quad (2)$$

$$Y_A + Y_C = \frac{Z_a + Z_c}{Z_a Z_b + Z_b Z_c + Z_a Z_c} \quad (3)$$

Then adding the respective sides of Equations 2 and 3, subtracting the corresponding sides of Equation 1, and dividing both sides of the result by 2, we have:

$$Y_A = \frac{Z_a}{Z_a Z_b + Z_b Z_c + Z_a Z_c}$$

Inverting both sides of this result yields:

$$Z_A = \frac{Z_a Z_b + Z_b Z_c + Z_a Z_c}{Z_a}$$

Similarly, we obtain:

$$Z_B = \frac{Z_a Z_b + Z_b Z_c + Z_a Z_c}{Z_b} \quad \text{and} \quad Z_C = \frac{Z_a Z_b + Z_b Z_c + Z_a Z_c}{Z_c}$$

P5.101* First we write the KVL equation:

$$\mathbf{V}_1 - \mathbf{V}_2 = 10\angle 0^\circ$$

Then, we enclose nodes 1 and 2 in a closed surface to form a supernode and write a KCL equation:

$$\frac{\mathbf{V}_1}{10} + \frac{\mathbf{V}_1}{j20} + \frac{\mathbf{V}_2}{15} + \frac{\mathbf{V}_2}{-j5} = 0$$

The MATLAB commands are:

```
echo on
Y=[1 -1;(0.1+1/(j*20)) (1/15+(1/(-j*5)))]
I=[10;0]
V=inv(Y)*I
pout(V(1))
pout(V(2))
```

The answers are:

$$\mathbf{V}_1 = 9.402\angle 29.58^\circ$$

$$\mathbf{V}_2 = 4.986\angle 111.45^\circ$$

P5.102 Writing KVL equations around the meshes, we obtain

$$10\mathbf{I}_1 + j20(\mathbf{I}_1 - \mathbf{I}_2) = 0$$

$$j20(\mathbf{I}_2 - \mathbf{I}_1) + 15(\mathbf{I}_2 - \mathbf{I}_3) = -10$$

$$-j5\mathbf{I}_3 + 15(\mathbf{I}_3 - \mathbf{I}_2) = 0$$

The MATLAB commands are:

```
echo on
Z=[(10+j*20) -j*20 0; -j*20 (15+j*20) -15;0 -15 (15-j*5)]
V=[0;-10;0]
I=inv(Z)*V
pout(I)
```

Solving, we obtain:

$$\mathbf{I}_1 = 0.9402\angle -150.4^\circ$$

$$\mathbf{I}_2 = 1.051\angle -177.0^\circ$$

$$\mathbf{I}_3 = 0.9972\angle -158.6^\circ$$

P5.103* Writing KVL equations around the meshes, we obtain

$$10\mathbf{I}_1 + j10(\mathbf{I}_1 - \mathbf{I}_2) = j20$$

$$-j5\mathbf{I}_2 + j10(\mathbf{I}_2 - \mathbf{I}_1) = -10$$

The MATLAB commands are:

```
j=i
```

```
Z=[(10+j*10) -j*10;-j*10  j*5]
V=[j*20;-10]
I=inv(Z)*V
pout(I)
```

Solving, we obtain:

$$\mathbf{I}_1 = 2.000\angle 180^\circ$$

$$\mathbf{I}_2 = 4.472\angle 153.43^\circ$$

P5.104 The current through the current source is

$$\mathbf{I}_1 - \mathbf{I}_2 = 3$$

Writing KVL around the perimeter of the circuit, we have

$$5\mathbf{I}_1 + (5 + j5)\mathbf{I}_2 = j20$$

The MATLAB commands are:

```
j=i
Z=[1 -1; 5 (5+j*5)]
V=[3;j*20]
I=inv(Z)*V
pout(I)
```

Solving, we obtain:

$$\mathbf{I}_1 = 3.406\angle 40.24^\circ \quad \text{and} \quad \mathbf{I}_2 = 2.236\angle 100.30^\circ$$

P5.105 Writing KCL equations at nodes 1 and 2 we obtain

$$\frac{\mathbf{V}_1}{20} + \frac{\mathbf{V}_1 - \mathbf{V}_2}{5 + j8} = 1\angle 0^\circ$$

$$\frac{\mathbf{V}_2}{-j20} + \frac{\mathbf{V}_2 - \mathbf{V}_1}{5 + j8} = 1\angle 30^\circ$$

```
Y=[(1/20+1/(5+j*8)) -1/(5+j*8);...
 -1/(5+j*8) (1/(-j*20)+1/(5+j*8))]
I=[1;pin(1,30)]
V=inv(Y)*I
pout(V)
```

Solving, we obtain

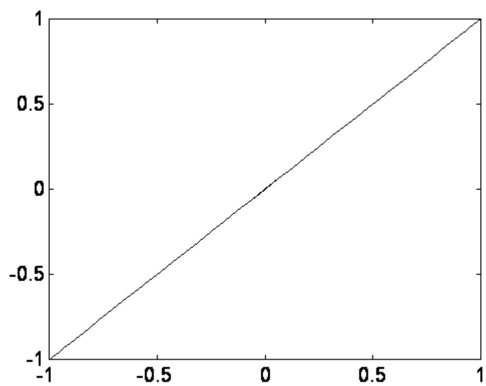
$$\mathbf{V}_1 = 23.75\angle -37.26^\circ$$

$$\mathbf{V}_2 = 30.55\angle -37.06^\circ$$

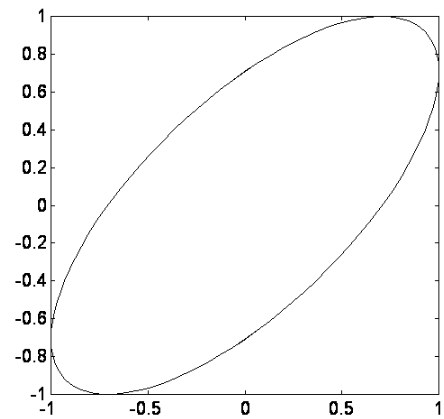
P5.106 A sequence of MATLAB instructions to accomplish the desired plot for part (b) is

```
Wx = 2*pi;
Wy = 2*pi;
Theta = 45*pi/180;
t = 0:0.01:20;
x = cos(Wx*t);
y = cos(Wy*t + Theta);
plot(x,y)
```

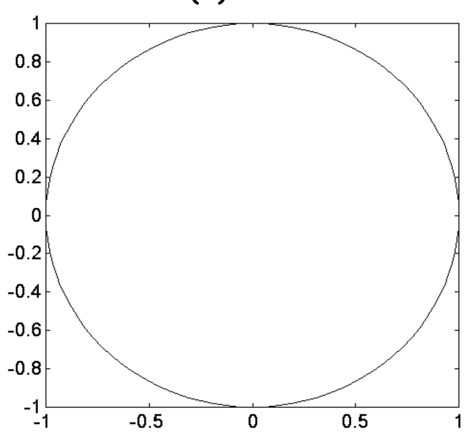
By changing the parameters, we can obtain the plots for parts a, c, and d. The resulting plots are



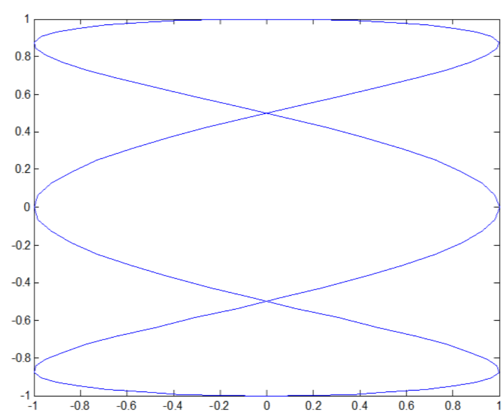
(a)



(b)



(c)



(d)

P5.107 $V_{rms} = \sqrt{\frac{1}{T}\int_0^T v^2(t)dt} = \sqrt{\int_0^1 [10\exp(-5t)\sin(20\pi t)]^2 dt}$

The MATLAB Symbolic Toolbox commands are:

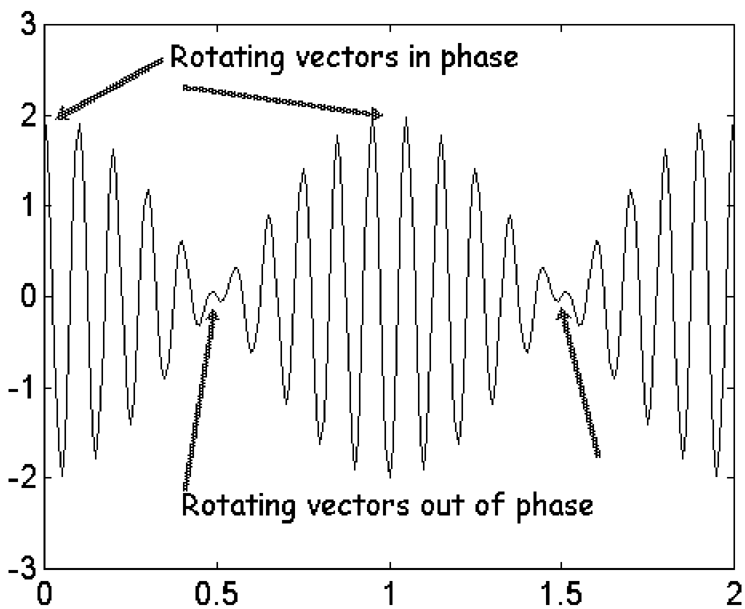
```
syms Vrms t
Vrms = sqrt((int((10*exp(-5*t)*sin(20*pi*t))^2,t,0,1)))
vpa(Vrms,4)
```

The result is $V_{rms} = 2.229$ V.

P5.108 A sequence of MATLAB commands to generate the desired plot is:

```
t = 0:0.01:2;
v = cos(19*pi*t) + cos(21*pi*t);
plot(v,t)
```

The resulting plot is

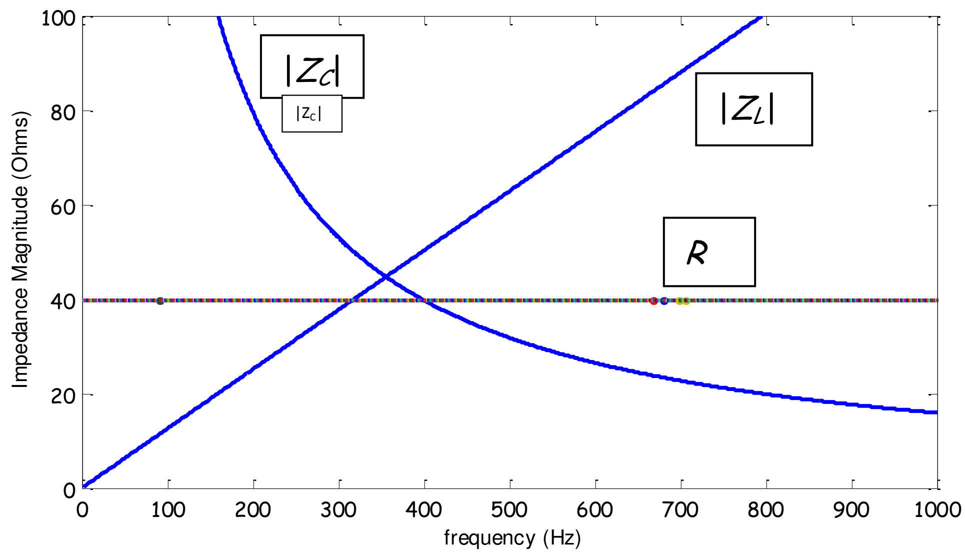


Notice that the first term $\cos(19\pi t)$ has a frequency of 9.5 Hz while the second term $\cos(21\pi t)$ has a frequency of 10.5 Hz. At $t = 0$, the rotating vectors are in phase and add constructively. At $t = 0.5$, the vector for the second term has rotated one half turn more than the vector for the first term and the vectors cancel. At $t = 1$, the first vector has rotated 9.5 turns and the second vector has rotated 10.5 turns so they both point in the same direction and add constructively.

P5.109 A Matlab m-file that produces the desired plots is:

```
f=0:1:1000;
ZL=2*pi*f*0.02;
ZC=1./(2*pi*f*10e-6);
R=40
plot(f,ZL)
axis([0 1000 0 100])
hold
plot(f,ZC)
plot(f,R)
```

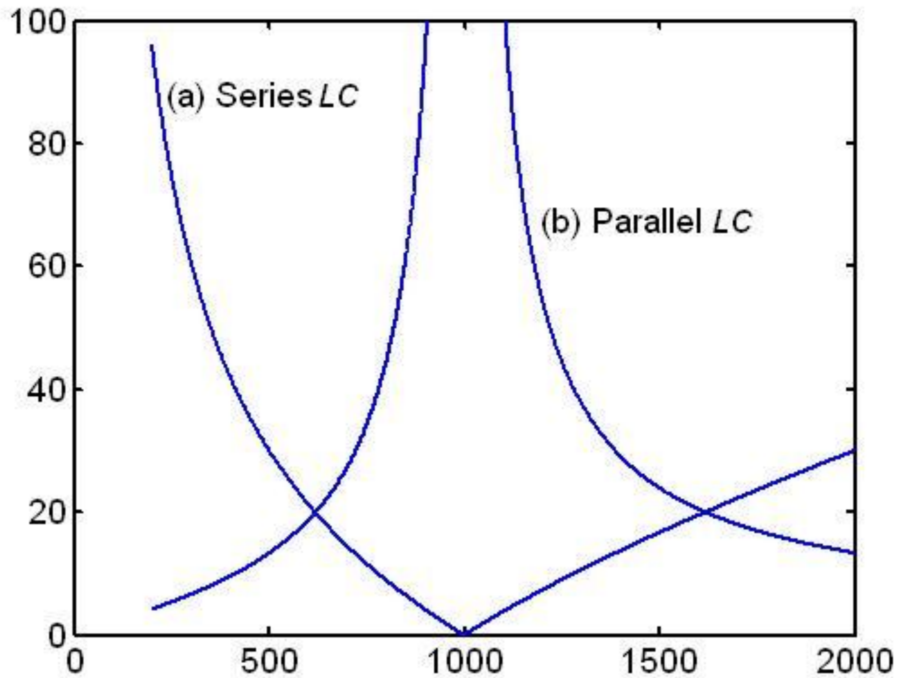
The resulting plot is:



P5.110 A Matlab program that produces the desired plots is:

```
w=0:1:2000;
L=20e-3;
C=50e-6;
Zmaga=abs(w*L-1./(w*C));
Zmagb=abs(1./((1./(i*w*L))+i*w*C));
plot(w,Zmaga)
hold on
plot(w,Zmagb)
axis([0 2000 0 100])
hold off
```

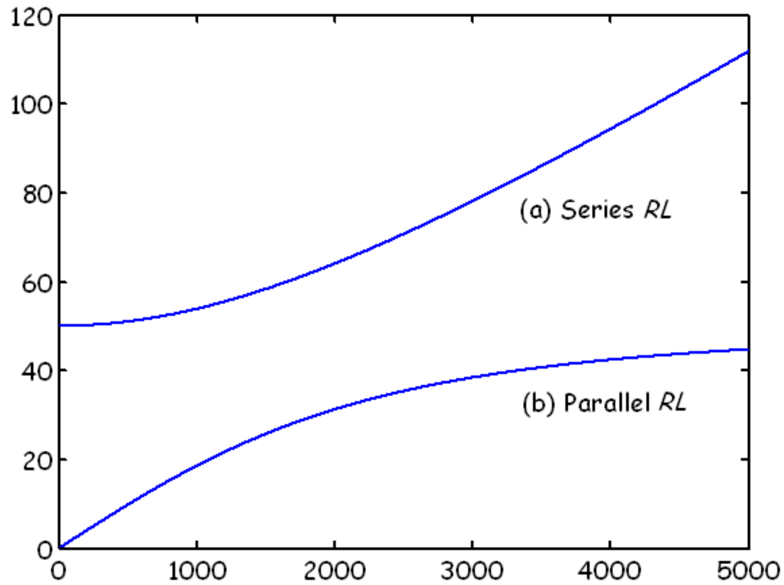
The result is:



P5.111 A Matlab program that produces the desired plots is:

```
w=0:1:5000;
L=20e-3;
R=50;
Zmaga=abs(R+i*w*L);
Zmagb=abs(1./((1./(i*w*L))+1/R));
plot(w,Zmaga)
hold on
plot(w,Zmagb)
hold off
```

The result is:



Practice Test

T5.1 $I_{rms} = \sqrt{\frac{1}{T}\int_0^T i^2(t)dt} = \sqrt{\frac{1}{3}\int_0^2 (3t)^2 dt} = \sqrt{t^3\Big|_0^2} = \sqrt{8} = 2.828\ A$

$P = I_{rms}^2 R = 8(50) = 400\ \ \text{W}$

T5.2 $\mathbf{V} = 5\angle -45° + 5\angle -30° = 3.5355 - j3.5355 + 4.3301 - j2.5000$

$\mathbf{V} = 7.8657 - j6.0355 = 9.9144\angle -37.50°$

$v(t) = 9.914\cos(\omega t - 37.50°)$

T5.3 (a) $V_{1rms} = \frac{15}{\sqrt{2}} = 10.61\text{ V}$

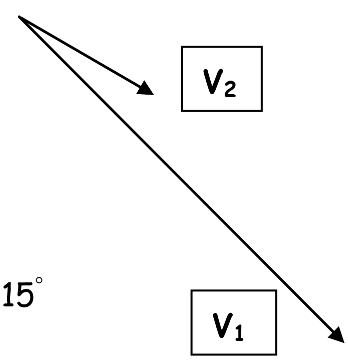
(b) $f = 200\ \ \text{Hz}$

(c) $\omega = 400\pi$ radians/s

(d) $T = 1/f = 5\ \ \text{ms}$

(e) $\mathbf{V}_1 = 15\angle -45°$ and $\mathbf{V}_2 = 5\angle -30°$

$\mathbf{V}_1$ lags $\mathbf{V}_2$ by 15° or $\mathbf{V}_2$ leads $\mathbf{V}_1$ by 15°



T5.4 $\mathbf{I} = \dfrac{\mathbf{V}_s}{R + j\omega L - j/\omega C} = \dfrac{10\angle 0^\circ}{10 + j15 - j5} = \dfrac{10\angle 0^\circ}{14.14\angle 45^\circ} = 0.7071\angle -45^\circ \ A$

$\mathbf{V}_R = 10\mathbf{I} = 7.071\angle -45^\circ \text{ V} \qquad \mathbf{V}_L = j15\mathbf{I} = 10.606\angle 45^\circ \ \text{V}$

$\mathbf{V}_C = -j5\mathbf{I} = 5.303\angle -135^\circ \ \text{V}$

T5.5 $\mathbf{S} = \frac{1}{2}\mathbf{VI}^* = \frac{1}{2}(440\angle 30^\circ)(25\angle 10^\circ) = 5500\angle 40^\circ = 4213 + j3535 \ \text{VA}$

$P = \text{Re}(\mathbf{S}) = 4213 \text{ W}$

$Q = \text{Im}(\mathbf{S}) = 3535 \text{ VAR}$

Apparent power $= |\mathbf{S}| = 5500$ VA

Power factor $= \cos(\theta_v - \theta_I) = \cos(40^\circ) = 76.6\%$ lagging

T5.6 We convert the delta to a wye and connect the neutral points with an ideal conductor.

$Z_Y = Z_\Delta / 3 = 2 + j8/3$

$Z_{total} = Z_{line} + Z_Y = 0.3 + j0.4 + 2 + j2.667 = 2.3 + j3.067$

$Z_{total} = 3.833\angle 53.13^\circ$

$\mathbf{I}_{aA} = \dfrac{\mathbf{V}_{an}}{Z_{total}} = \dfrac{208\angle 30^\circ}{3.833\angle 53.13^\circ} = 54.26\angle -23.13^\circ \ A$

T5.7 The mesh equations are:

$j10\mathbf{I}_1 + 15(\mathbf{I}_1 - \mathbf{I}_2) = 10\angle 45^\circ$

$-j5\mathbf{I}_2 + 15(\mathbf{I}_2 - \mathbf{I}_1) = -15$

In matrix form these become

$$\begin{bmatrix} (15 + j10) & -15 \\ -15 & (15 - j5) \end{bmatrix} \begin{bmatrix} \mathbf{I}_1 \\ \mathbf{I}_2 \end{bmatrix} = \begin{bmatrix} 10\angle 45^\circ \\ -15 \end{bmatrix}$$

The commands are:

```
Z = [(15+j*10)  -15; -15  (15-j*5)]
V = [pin(10,45); -15]
I = inv(Z)*V
pout(I(1))
pout(I(2))
```

CHAPTER 6

Exercises

E6.1 (a) The frequency of $v_{in}(t) = 2\cos(2\pi \cdot 2000t)$ is 2000 Hz. For this frequency $H(f) = 2\angle 60^\circ$. Thus, $\mathbf{V}_{out} = H(f)\mathbf{V}_{in} = 2\angle 60^\circ \times 2\angle 0^\circ = 4\angle 60^\circ$ and we have $v_{out}(t) = 4\cos(2\pi \cdot 2000t + 60^\circ)$.

(b) The frequency of $v_{in}(t) = \cos(2\pi \cdot 3000t - 20^\circ)$ is 3000 Hz. For this frequency $H(f) = 0$. Thus, $\mathbf{V}_{out} = H(f)\mathbf{V}_{in} = 0 \times 2\angle 0^\circ = 0$ and we have $v_{out}(t) = 0$.

E6.2 The input signal $v(t) = 2\cos(2\pi \cdot 500t + 20^\circ) + 3\cos(2\pi \cdot 1500t)$ has two components with frequencies of 500 Hz and 1500 Hz. For the 500-Hz component we have:

$$\mathbf{V}_{out,1} = H(500)\mathbf{V}_{in} = 3.5\angle 15^\circ \times 2\angle 20^\circ = 7\angle 35^\circ$$

$$v_{out,1}(t) = 7\cos(2\pi \cdot 500t + 35^\circ)$$

For the 1500-Hz component:

$$\mathbf{V}_{out,2} = H(1500)\mathbf{V}_{in} = 2.5\angle 45^\circ \times 3\angle 0^\circ = 7.5\angle 45^\circ$$

$$v_{out,2}(t) = 7.5\cos(2\pi \cdot 1500t + 45^\circ)$$

Thus the output for both components is

$$v_{out}(t) = 7\cos(2\pi \cdot 500t + 35^\circ) + 7.5\cos(2\pi \cdot 1500t + 45^\circ)$$

E6.3 The input signal $v(t) = 1 + 2\cos(2\pi \cdot 1000t) + 3\cos(2\pi \cdot 3000t)$ has three components with frequencies of 0, 1000 Hz and 3000 Hz.
For the dc component, we have

$$v_{out,1}(t) = H(0) \times v_{in,1}(t) = 4 \times 1 = 4$$

For the 1000-Hz component, we have:

$$\mathbf{V}_{out,2} = H(1000)\mathbf{V}_{in,2} = 3\angle 30^\circ \times 2\angle 0^\circ = 6\angle 30^\circ$$

$$v_{out,1}(t) = 6\cos(2\pi \cdot 1000t + 30^\circ)$$

For the 3000-Hz component:

$$\mathbf{V}_{out,3} = H(3000)\mathbf{V}_{in,3} = 0 \times 3\angle 0^\circ = 0$$

$$v_{out,3}(t) = 0$$

Thus, the output for all three components is

$$v_{out}(t) = 4 + 6\cos(2\pi \cdot 1000t + 30^\circ)$$

E6.4 Using the voltage-division principle, we have:

$$\mathbf{V}_{\text{out}} = \mathbf{V}_{\text{in}} \times \frac{R}{R + j2\pi fL}$$

Then the transfer function is:

$$H(f) = \frac{\mathbf{V}_{\text{out}}}{\mathbf{V}_{\text{in}}} = \frac{R}{R + j2\pi fL} = \frac{1}{1 + j2\pi fL / R} = \frac{1}{1 + jf / f_B}$$

E6.5 From Equation 6.9, we have $f_B = 1/(2\pi RC) = 200\text{ Hz}$, and from Equation 6.9, we have $H(f) = \dfrac{\mathbf{V}_{\text{out}}}{\mathbf{V}_{\text{in}}} = \dfrac{1}{1 + jf / f_B}$.

For the first component of the input, the frequency is 20 Hz,
$H(f) = 0.995\angle -5.71^\circ$, $\mathbf{V}_{\text{in}} = 10\angle 0^\circ$, and $\mathbf{V}_{\text{out}} = H(f)\mathbf{V}_{\text{in}} = 9.95\angle -5.71^\circ$
Thus the first component of the output is

$$v_{\text{out},1}(t) = 9.95\cos(40\pi t - 5.71^\circ)$$

For the second component of the input, the frequency is 500 Hz,
$H(f) = 0.371\angle -68.2^\circ$, $\mathbf{V}_{\text{in}} = 5\angle 0^\circ$, and $\mathbf{V}_{\text{out}} = H(f)\mathbf{V}_{\text{in}} = 1.86\angle -68.2^\circ$
Thus the second component of the output is

$$v_{\text{out},2}(t) = 1.86\cos(40\pi t - 68.2^\circ)$$

For the third component of the input, the frequency is 10 kHz,
$H(f) = 0.020\angle -88.9^\circ$, $\mathbf{V}_{\text{in}} = 5\angle 0^\circ$, and $\mathbf{V}_{\text{out}} = H(f)\mathbf{V}_{\text{in}} = 0.100\angle -88.9^\circ$
Thus the third component of the output is

$$v_{\text{out},2}(t) = 0.100\cos(2\pi \times 10^4 t - 88.9^\circ)$$

Finally, the output with for all three components is:

$$v_{\text{out}}(t) = 9.95\cos(40\pi t - 5.71^\circ) + 1.86\cos(40\pi t - 68.2^\circ)$$
$$+ 0.100\cos(2\pi \times 10^4 t - 88.9^\circ)$$

E6.6 $\left|H(f)\right|_{\text{dB}} = 20\log\left|H(f)\right| = 20\log(50) = 33.98\text{ dB}$

E6.7 (a) $\left|H(f)\right|_{\text{dB}} = 20\log\left|H(f)\right| = 15\text{ dB}$

$$\log\left|H(f)\right| = 15/20 = 0.75$$

$$H(f) = 10^{0.75} = 5.623$$

(b) $|H(f)|_{\text{dB}} = 20\log|H(f)| = 30\text{ dB}$

$\log|H(f)| = 30/20 = 1.5$

$H(f) = 10^{1.5} = 31.62$

E6.8 (a) $1000 \times 2^2 = 4000\text{ Hz}$ is two octaves higher than 1000 Hz.

(b) $1000/2^3 = 125$ Hz is three octaves lower than 1000 Hz.

(c) $1000 \times 10^2 = 100$ kHz is two decades higher than 1000 Hz.

(d) $1000/10 = 100$ Hz is one decade lower than 1000 Hz.

E6.9 (a) To find the frequency halfway between two frequencies on a logarithmic scale, we take the logarithm of each frequency, average the logarithms, and then take the antilogarithm. Thus

$$f = 10^{[\log(100)+\log(1000)]/2} = 10^{2.5} = 316.2\text{ Hz}$$

is half way between 100 Hz and 1000 Hz on a logarithmic scale.

(b) To find the frequency halfway between two frequencies on a linear scale, we simply average the two frequencies. Thus (100 + 1000)/2 = 550 Hz is halfway between 100 and 1000 Hz on a linear scale.

E6.10 To determine the number of decades between two frequencies we take the difference between the common (base-ten) logarithms of the two frequencies. Thus 20 Hz and 15 kHz are $\log(15\times 10^3) - \log(20) = 2.875$ decades apart.

Similarly, to determine the number of octaves between two frequencies we take the difference between the base-two logarithms of the two frequencies. One formula for the base-two logarithm of z is

$$\log_2(z) = \frac{\log(z)}{\log(2)} \cong 3.322\log(z)$$

Thus the number of octaves between 20 Hz and 15 kHz is

$$\frac{\log(15\times 10^3)}{\log(2)} - \frac{\log(20)}{\log(2)} = 9.551$$

E6.11 The transfer function for the circuit shown in Figure 6.17 in the book is

$$H(f) = \frac{\mathbf{V}_{\text{out}}}{\mathbf{V}_{\text{in}}} = \frac{1/(j2\pi fC)}{R + 1/(j2\pi fC)} = \frac{1}{1 + j2\pi RCf} = \frac{1}{1 + jf/f_B}$$

in which $f_B = 1/(2\pi RC) = 1000$ Hz. Thus the magnitude plot is approximated by 0 dB below 1000 Hz and by a straight line sloping downward at 20 dB/decade above 1000 Hz. This is shown in Figure 6.18a in the book.

The phase plot is approximated by 0° below 100 Hz, by -90° above 10 kHz and by a line sloping downward between 0° at 100 Hz and -90° at 10 kHz. This is shown in Figure 6.18b in the book.

E6.12 Using the voltage division principle, the transfer function for the circuit shown in Figure 6.19 in the book is

$$H(f) = \frac{\mathbf{V}_{\text{out}}}{\mathbf{V}_{\text{in}}} = \frac{R}{R + 1/(j2\pi fC)} = \frac{j2\pi RC}{1 + j2\pi RCf} = \frac{j(f/f_B)}{1 + j(f/f_B)}$$

in which $f_B = 1/(2\pi RC)$.

E6.13 Using the voltage division principle, the transfer function for the circuit shown in Figure 6.22 in the book is

$$H(f) = \frac{\mathbf{V}_{\text{out}}}{\mathbf{V}_{\text{in}}} = \frac{j2\pi fL}{R + j2\pi fL} = \frac{j2\pi fL/R}{1 + j2\pi fL/R} = \frac{j(f/f_B)}{1 + j(f/f_B)}$$

in which $f_B = R/(2\pi L)$.

E6.14 A first-order filter has a transfer characteristic that decreases by 20 dB/decade below the break frequency. To attain an attenuation of 50 dB the signal frequency must be 50/20 = 2.5 decades below the break frequency. 2.5 decades corresponds to a frequency ratio of $10^{2.5} = 316.2$. Thus to attenuate a 1000 Hz signal by 50 dB the high-pass filter must have a break frequency of 316.2 kHz. Solving Equation 6.22 for capacitance and substituting values, we have

$$C = \frac{1}{2\pi f_B R} = \frac{1}{2\pi \times 1000 \times 316.2 \times 10^3} = 503.3\,\text{pF}$$

E6.15 $C = \dfrac{1}{\omega_0^2 L} = \dfrac{1}{(2\pi f_0)^2 L} = \dfrac{1}{(2\pi 10^6)^2 10 \times 10^{-6}} = 2533\ \text{pF}$

$R = \omega_0 L / Q_s = 1.257\ \Omega$

$B = f_0 / Q_s = 20\ \text{kHz}$

$f_L \cong f_0 - B/2 = 990\ \text{kHz}$

$f_H \cong f_0 + B/2 = 1010\ \text{kHz}$

E6.16 At resonance we have

$$\mathbf{V}_R = \mathbf{V}_s = 1\angle 0^\circ$$

$$\mathbf{V}_L = j\omega_0 L\mathbf{I} = j\omega_0 L\mathbf{V}_s / R = jQ_s\mathbf{V}_s = 50\angle 90^\circ \text{ V}$$

$$\mathbf{V}_C = (1/j\omega_0 C)\mathbf{I} = (1/j\omega_0 C)\mathbf{V}_s / R = -jQ_s\mathbf{V}_s = 50\angle -90^\circ \text{ V}$$

E6.17 $$L = \frac{1}{\omega_0^2 C} = \frac{1}{(2\pi f_0)^2 C} = \frac{1}{(2\pi \times 5 \times 10^6)^2 470 \times 10^{-12}} = 2.156\ \mu\text{H}$$

$$Q_s = f_0 / B = (5 \times 10^6)/(200 \times 10^3) = 25$$

$$R = \frac{1}{\omega_0 C Q_s} = \frac{1}{2\pi \times 5 \times 10^6 \times 470 \times 10^{-12} \times 25} = 2.709\ \Omega$$

E6.18 $$f_0 = \frac{1}{2\pi\sqrt{LC}} = 711.8 \text{ kHz} \qquad Q_p = \frac{R}{\omega_0 L} = 22.36 \qquad B = f_0 / Q_p = 31.83 \text{ kHz}$$

E6.19 $$Q_p = f_0 / B = 50 \qquad L = \frac{R}{\omega_0 Q_p} = 0.3183\ \mu\text{H} \qquad C = \frac{Q_p}{\omega_0 R} = 795.8 \text{ pF}$$

E6.20 A second order lowpass filter with f_0 = 5 kHz is needed. The circuit configuration is shown in Figure 6.34a in the book. The normalized transfer function is shown in Figure 6.34c. Usually we would want a filter without peaking and would design for Q = 1. Given that L = 5 mH, the other component values are

$$R = \frac{2\pi f_0 L}{Q} = 157.1\ \Omega \qquad C = \frac{1}{(2\pi f_0)^2 L} = 0.2026\ \mu\text{F}$$

The circuit is shown in Figure 6.39 in the book.

E6.21 We need a bandpass filter with $f_L = 45$ kHz and $f_H = 55$ kHz. Thus we have

$$f_0 \cong \frac{f_L + f_H}{2} = 50 \text{ kHz} \qquad B = f_H - f_L = 10 \text{ kHz} \qquad Q = f_0 / B = 5$$

$$R = \frac{2\pi f_0 L}{Q} = 62.83\ \Omega \qquad C = \frac{1}{(2\pi f_0)^2 L} = 10.13 \text{ nF}$$

The circuit is shown in Figure 6.40 in the book.

E6.22 The files Example_6_8 and Example_6_9 can be found in the MATLAB folder on the OrCAD disk. The results should be similar to Figures 6.42 and 6.44.

E6.23 (a) Rearranging Equation 6.56, we have

$$\frac{\tau}{T} = \frac{a}{1-a} = \frac{0.9}{1-0.9} = 0.9$$

Thus we have $\tau = 9T$.

(b) From Figure 6.49 in the book we see that the step response of the digital filter reaches 0.632 at approximately $n = 9$. Thus the speed of response of the *RC* filter and the corresponding digital filter are comparable.

E6.24 Writing a current equation at the node joining the resistance and capacitance, we have

$$\frac{y(t)}{R} + C\frac{d[y(t)-x(t)]}{dt} = 0$$

Multiplying both sides by R and using the fact that the time constant is $\tau = RC$, we have

$$y(t) + \tau\frac{dy(t)}{dt} - \tau\frac{dx(t)}{dt} = 0$$

Next we approximate the derivatives as

$$\frac{dx(t)}{dt} \cong \frac{\Delta x}{\Delta t} = \frac{x(n)-x(n-1)}{T} \quad \text{and} \quad \frac{dy(t)}{dt} \cong \frac{\Delta y}{\Delta t} = \frac{y(n)-y(n-1)}{T}$$

which yields

$$y(n) + \tau\frac{y(n)-y(n-1)}{T} - \tau\frac{x(n)-x(n-1)}{T} = 0$$

Solving for $y(n)$, we obtain

$$y(n) = a_1 y(n-1) + b_0 x(n) + b_1 x(n-1)$$

in which

$$a_1 = b_0 = -b_1 = \frac{\tau/T}{1+\tau/T}$$

E6.25 (a) Solving Equation 6.58 for d and substituting values, we obtain

$$d = \frac{f_s}{2f_{notch}} = \frac{10^4}{2\times 500} = 10$$

(b) Repeating for f_{notch} = 300 Hz, we have

$$d = \frac{f_s}{2f_{notch}} = \frac{10^4}{2 \times 300} = 16.67$$

However, d is required to be an integer value so we cannot obtain a notch filter for 300 Hz exactly for this sampling frequency. (Possibly other more complex filters could provide the desired performance.)

Problems

P6.1 The fundamental concept of Fourier theory is that all signals are sums of sinewaves of various amplitudes, frequencies, and phases.

P6.2 The transfer function shows how a filter affects the amplitude and phase of input components as a function of frequency. It is defined as the output phasor divided by the input phasor as a function of frequency.

To determine the complex value of the transfer function for a given frequency, we apply a sinusoidal input of that frequency, wait for the output to achieve steady state, and then measure the amplitude and phase of both the input and the output using voltmeters, oscilloscopes or other instruments. Then, the value of the transfer function is computed as the ratio of the output phasor divided by the input phasor. We change the frequency and repeat to determine the transfer function for other frequencies.

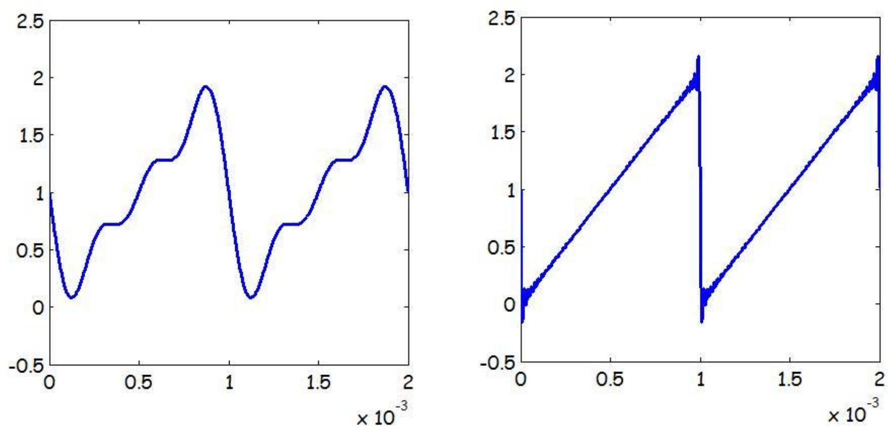
P6.3 Filters (and other systems described by linear time-invariant differential equations) separate the input into sinusoidal components with various frequencies, modify the amplitudes and phases of the components using the transfer function, and then add the modified components to create the output.

P6.4 A MATLAB program to create the plots is:

```
t = 0:2e-6:2e-3;
vst = ones(size(t));
for n=1:3
vst = vst - (2/(n*pi))*sin(2000*n*pi*t);
end
subplot(2,2,1)
plot(t,vst)
axis([0 2e-3 -0.5 2.5])
```

```
for n=4:50
vst = vst - (2/(n*pi))*sin(2000*n*pi*t);
end
subplot(2,2,2)
plot(t,vst)
axis([0 2e-3 -0.5 2.5])
```

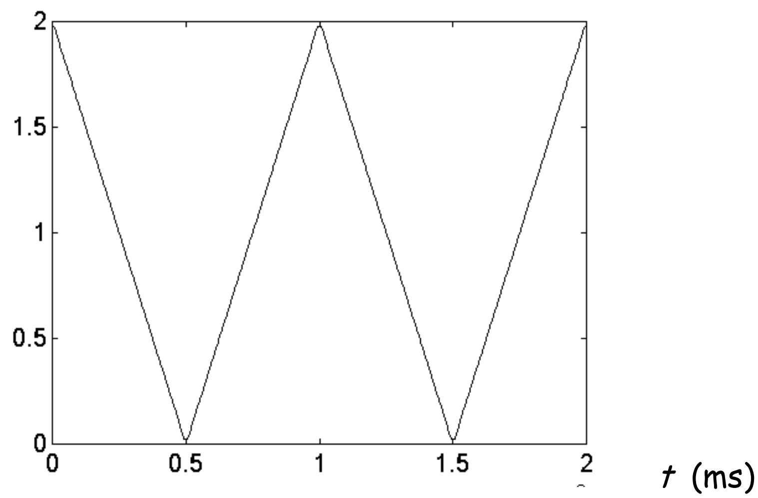
The resulting plots are:



P6.5 A MATLAB program to create the plot is:

```
t = 0:2e-6:2e-3;
vt = ones(size(t));
for n=1:2:19
vt = vt +(8/(n*pi)^2)*cos(2000*n*pi*t);
end
plot(t,vt)
```

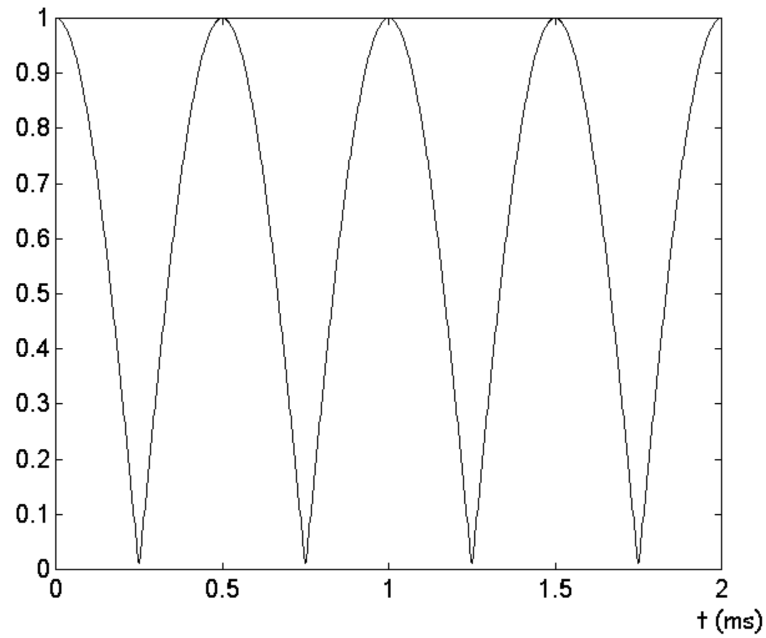
The resulting plot is:



P6.6 A MATLAB program to create the plot is:

```
t = 0:2e-6:2e-3;
vfw = (2/pi)*ones(size(t));
for n=2:2:60
vfw = vfw + (4/(pi*(n-1)*(n+1)))*(-1)^(1+ n/2)*cos(2000*n*pi*t);
end
plot(t,vfw)
```

The resulting plot is:

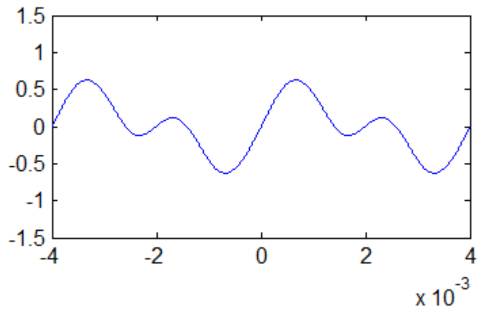


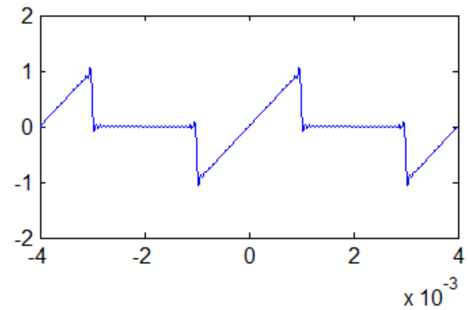
P6.7 A MATLAB program to create the plots is:

```
clear
t = -4e-3:2e-6:4e-3;
vst = zeros(size(t));
for n = 1:3
coef = (sin(n*pi/2))/(n*pi/2)^2  -  cos(n*pi/2)/(n*pi/2);
vst = vst + coef* sin(500*n*pi*t);
end
subplot(2,2,1)
plot(t,vst)
axis([-4e-3 4e-3 -1.5 1.5])
for n = 4:50
coef = (sin(n*pi/2))/(n*pi/2)^2  -  cos(n*pi/2)/(n*pi/2);
vst = vst + coef* sin(500*n*pi*t);
```

```
end
subplot(2,2,2)
axis([-4e-3 4e-3 -1.5 1.5])
plot(t,vst)
```

The resulting plots are:





P6.8* The given input signal is

$$v_{in}(t) = 5 + 2\cos(2\pi 2500t + 30^\circ) + 2\cos(2\pi 7500t)$$

This signal has a component $v_{in1}(t) = 5$ with $f = 0$, a second component $v_{in2}(t) = 2\cos(2\pi 2500t + 30^\circ)$ with $f = 2500$, and a third component $v_{in3}(t) = 2\cos(2\pi 7500t)$ with $f = 7500$. From Figure P6.8, we find the transfer function values for these frequencies:

$$H(0) = 2, \quad H(2500) = 1.75\angle -45^\circ, \text{ and } H(7500) = 1.25\angle -135^\circ$$

The dc output is $v_{out1} = H(0)v_{in1} = 10$

The phasors for the sinusoidal input components are

$$\mathbf{V}_{in2} = 2\angle 30^\circ \quad \text{and} \quad \mathbf{V}_{in3} = 2\angle 0^\circ$$

Multiplying the input phasors by the transfer function values , results in:

$$\mathbf{V}_{out2} = \mathbf{V}_{in2} \times H(2500) = 3.5\angle -15^\circ \qquad \mathbf{V}_{out3} = \mathbf{V}_{in3} \times H(7500) = 2.5\angle -135^\circ$$

The corresponding output components are:

$$v_{out2}(t) = 3.5\cos(2\pi 2500t - 15^\circ)$$

$$v_{out3}(t) = 2.5\cos(2\pi 7500t - 135^\circ)$$

Thus, the output signal is

$$v_{out}(t) = 10 + 3.5\cos(2\pi 2500t - 15^\circ) + 2.5\cos(2\pi 7500t - 135^\circ)$$

P6.9 The solution is similar to that for Problem 6.8. The answer is:

$$v_{out}(t) = 8 + 7.5\cos(2\pi 5000t - 120^\circ) \text{ V}$$

P6.10 The solution is similar to that for Problem 6.8. The answer is:

$$v_{out}(t) = 12 + 3.4\cos(6000\pi t - 54^\circ) + 5.6\cos(12000\pi t - 108^\circ) \text{ V}$$

P6.11* The phasors for the input and output are:

$$\mathbf{V}_{in} = 2\angle -25^\circ \quad \text{and} \quad \mathbf{V}_{out} = 1\angle 20^\circ$$

The transfer function for $f = 5000$ Hz is

$$H(5000) = \mathbf{V}_{out}/\mathbf{V}_{in} = 0.5\angle 45^\circ$$

P6.12 From Figure P6.12, we see that the period of the signals is 20 ms. Therefore, the frequency is 50 Hz. Because the input reaches a positive peak at t = 8 ms, it has a phase angle of

$$\theta_{\text{in}} = -(t_d / T) \times 360^\circ = -(8/20) \times 360^\circ = -144^\circ$$

The output reaches its peak at 4 ms, and its phase angle is

$$\theta_{\text{out}} = -(t_d / T) \times 360^\circ = -(4/20) \times 360^\circ = -72^\circ$$

The transfer function is

$$H(100) = \frac{\mathbf{V}_{\text{out}}}{\mathbf{V}_{\text{in}}} = \frac{1\angle -72^\circ}{2\angle -144^\circ} = 0.5\angle 72^\circ$$

P6.13* The input has a peak value of 5 V and reaches a positive peak at 1 ms. Since the period is 4 ms, the frequency is 250 Hz and 1 ms corresponds to a phase angle of $\theta_{in} = -(t_d / T) \times 360^\circ = -(1/4) \times 360^\circ = -90^\circ$. Thus, the phasor for the input is $\mathbf{V}_{\text{in}} = 5\angle -90^\circ$. Similarly, the phasor for the output is $\mathbf{V}_{\text{out}} = 15\angle -135^\circ$. Then, the transfer function for a frequency of 250 Hz is

$$H(250) = \frac{\mathbf{V}_{\text{out}}}{\mathbf{V}_{\text{in}}} = 3\angle -45^\circ$$

P6.14* The triangular waveform is given in Problem P6.5 as

$$v_t(t) = 1 + \frac{8}{\pi^2}\cos(2000\pi t) + \frac{8}{(3\pi)^2}\cos(6000\pi t) + \cdots$$
$$+ \frac{8}{(n\pi)^2}\cos(2000n\pi t) + \cdots$$

in which n assumes only odd integer values. This waveform has components with frequencies of 0 (dc), 1000 Hz, 3000 Hz, etc. The transfer function shown in Figure P6.14 is zero for all components except the dc term for which we have $H(0) = 2$. Thus, the dc term is multiplied by 2 and all of the other terms are rejected. Thus, $v_o(t) = 2$.

P6.15* Given

$$v_{in}(t) = V_{\max}\cos(2\pi ft)$$

$$v_{out}(t) = \int_0^t V_{\max}\cos(2\pi ft)dt = \frac{V_{\max}}{2\pi f}\sin(2\pi ft)$$

The phasors are

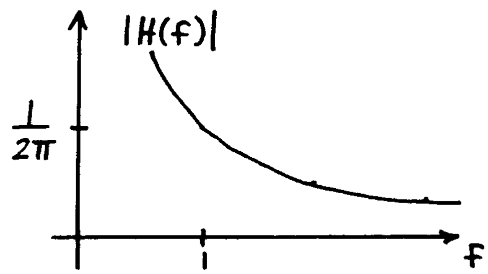
$$\mathbf{V}_{in} = V_{\max}\angle 0^\circ$$

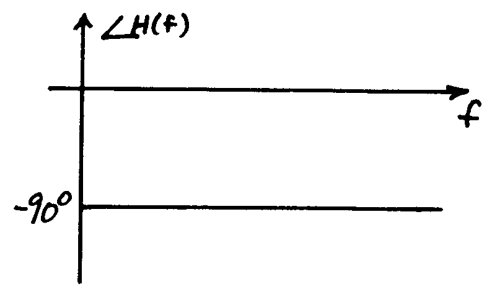
$$\mathbf{V}_{out} = \frac{V_{\max}}{2\pi f}\angle -90^\circ$$

The transfer function is

$$H(f) = \frac{\mathbf{V}_{out}}{\mathbf{V}_{in}} = \frac{1}{2\pi f}\angle -90^\circ = \frac{-j}{2\pi f}$$

Plots of the magnitude and phase of this transfer function are:





P6.16 Given $v_{in}(t) = V_{\max}\cos(2\pi ft + 60^\circ)$

$$v_{out}(t) = \frac{d[v_{in}(t)]}{dt} = -2\pi f V_{\max}\sin(2\pi ft + 60^\circ)$$

$$= 2\pi f V_{\max}\cos(2\pi ft + 60^\circ + 90^\circ) = 2\pi f V_{\max}\cos(2\pi ft + 150^\circ)$$

The phasors are

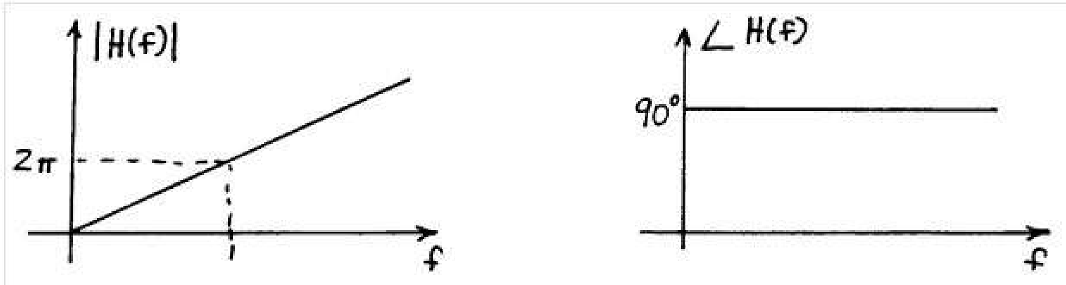
$$\mathbf{V}_{in} = V_{\max}\angle 60^\circ$$

$$\mathbf{V}_{out} = 2\pi f V_{\max}\angle 150^\circ$$

The transfer function is

$$H(f) = \frac{\mathbf{V}_{out}}{\mathbf{V}_{in}} = 2\pi f\angle(150^\circ - 60^\circ) = 2\pi f\angle 90^\circ = j2\pi f$$

Plots of the magnitude and phase of this transfer function are:



P6.17 The input signal given in Problem P6.4 is:

$$v_{st}(t) = 1 - \frac{2}{\pi}\sin(2000\pi t) - \frac{2}{2\pi}\sin(4000\pi t) - \frac{2}{3\pi}\sin(6000\pi t)$$
$$- \cdots - \frac{2}{n\pi}\sin(2000 n\pi t) - \cdots$$

The frequencies of the various components are $f = 0$, 1000, 2000, 3000, and so forth. The transfer function $H(f)$ is zero for all components except for the one with a frequency of 3000 Hz. For the 3000-Hz component, we have:

$$\begin{aligned}\mathbf{V}_{out} &= \mathbf{V}_{in} \times H(3000) \\ &= [(2/3\pi)\angle 90^\circ] \times 5\angle 0^\circ \\ &= (10/3\pi)\angle 90^\circ\end{aligned}$$

Thus, the output is:

$$\begin{aligned}v_{out}(t) &= (10/3\pi)\cos(6000\pi t + 90^\circ) \\ &= -(10/3\pi)\sin(6000\pi t)\end{aligned}$$

P6.18 $v_{in}(t) = V_{max}\cos(2\pi f t)$

$$v_{out}(t) = 1000\int_{t-1e-3}^{t} V_{max}\cos(2\pi f t)dt$$

$$v_{out}(t) = \frac{1000 V_{max}}{2\pi f}[\sin(2\pi f t) - \sin(2\pi f t - 2\pi f 10^{-3})]$$

The phasors are

$$\mathbf{V}_{in} = V_{max}\angle 0^\circ$$

$$\mathbf{V}_{out} = \frac{1000 V_{max}}{2\pi f}[1\angle -90^\circ - 1\angle(-90^\circ - 2\pi f/1000)]$$

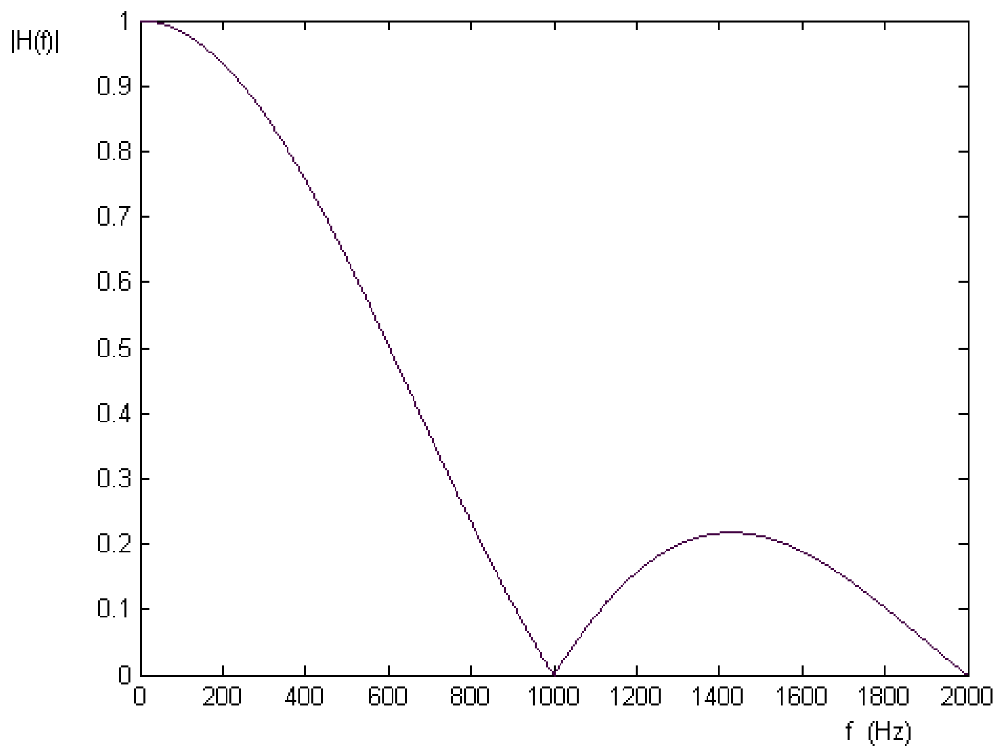
The transfer function is

$$H(f) = \frac{\mathbf{V}_{out}}{\mathbf{V}_{in}} = \frac{-j1000[1-1\angle(-2\pi f/1000)]}{2\pi f} = \frac{1000[1-\exp(-j2\pi f/1000)]}{j2\pi f}$$

in which the angle is expressed in radians. A MATLAB program to plot the magnitude of this transfer function is

```
f = 0:1:2000;
Hmag = abs((1000)*((1 - exp(-j*2*pi*f./1000))./(j*2*pi*f)));
plot(f,Hmag)
```

The resulting plot of the magnitude of the transfer function is:



For integer multiples of 1000 Hz, an integer multiple of cycles appear in the integration interval (1 ms). Because the integral of an integer number of cycles of a sine wave is zero, the transfer function magnitude is zero for those frequencies.

P6.19 The input signal is

$$v_{in}(t) = 2 + 3\cos(1000\pi t) + 3\sin(2000\pi t) + \cos(3000\pi t)$$

which has components with frequencies of 0, 500, 1000, and 1500 Hz. We can determine the transfer function at these frequencies by dividing the corresponding output phasor by the input phasor.

The output is

$$v_{out}(t) = 3 + 2\cos(1000\pi t + 30^\circ) + 4\sin(3000\pi t)$$

Thus, we have

$$H(0) = 3/2 = 1.5 \qquad H(500) = \frac{2\angle 30^\circ}{3\angle 0^\circ} = 0.6667\angle 30^\circ$$

$$H(1000) = \frac{0}{3\angle -90^\circ} = 0 \qquad H(1500) = \frac{4\angle -90^\circ}{1\angle 0^\circ} = 4\angle -90^\circ$$

P6.20 $v_{in}(t) = V_{max}\cos(2\pi f t)$

$$v_{out}(t) = V_{max}\cos(2\pi f t) + V_{max}\cos[2\pi f(t - 2\times 10^{-3})]$$

$$v_{out}(t) = V_{max}\cos(2\pi f t) + V_{max}\cos(2\pi f t - 4\pi f \times 10^{-3})$$

The phasors are

$$\mathbf{V}_{in} = V_{max}\angle 0^\circ$$

$$\mathbf{V}_{out} = V_{max}[1 + 1\angle -2\pi f / 500]$$

The transfer function is

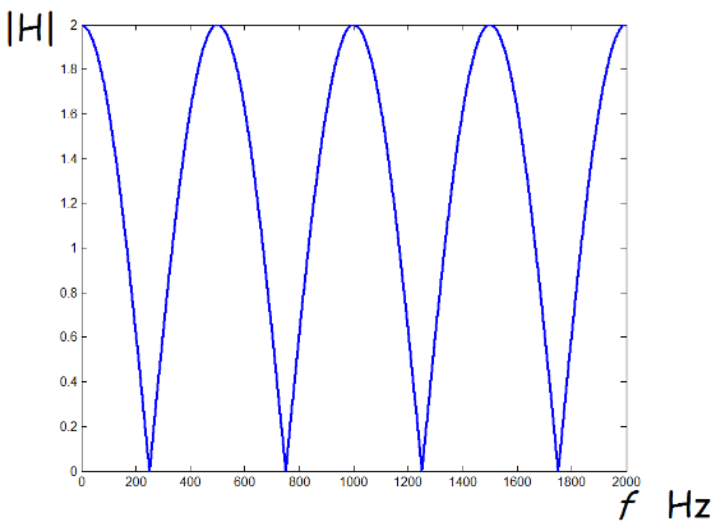
$$H(f) = \frac{\mathbf{V}_{out}}{\mathbf{V}_{in}} = 1 + 1\angle -2\pi f / 500 = 1 + \exp(-j2\pi f / 500)$$

in which the angle is expressed in radians.

A MATLAB program to plot the magnitude of this transfer function is

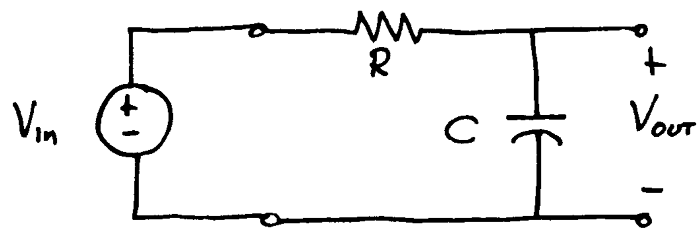
```
f = 0:1:2000;
Hmag = abs(1 + exp((-i*2*pi/500)*f));
plot(f,Hmag)
```

A plot of the magnitude of the transfer function is:



For a 250 Hz sinewave, a delay of 2 ms corresponds to a 180 degree phase shift, so the phasors for $v_{in}(t)$ and for $v_{in}(t - 2\times 10^{-3})$ oppose and cancel. For a 500 Hz sinewave, a delay of 2 ms corresponds to a 360 degree phase shift, so the phasors for $v_{in}(t)$ and for $v_{in}(t - 2 \times 10^{-3})$ add to twice the amplitude.

P6.21 The circuit diagram is:

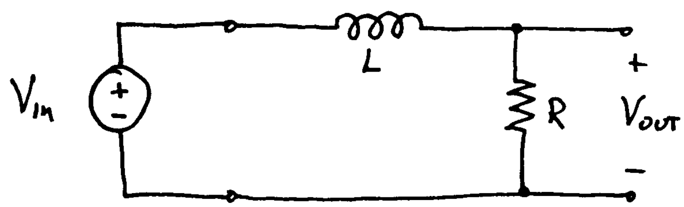


The half-power frequency is:

$$f_B = \frac{1}{2\pi RC}$$

Plots of the magnitude and phase of the transfer function are shown in Figure 6.8 in the text.

P6.22 The circuit diagram of a first-order *RL* lowpass filter is:



The half-power frequency is:

$$f_B = \frac{R}{2\pi L}$$

Plots of the magnitude and phase of the transfer function are identical to those shown in Figure 6.8 in the text.

P6.23 The time constant is given by $\tau = RC$ and the half-power frequency is $f_B = \dfrac{1}{2\pi RC}$. Thus, we have $f_B = \dfrac{1}{2\pi\tau}$.

P6.24 Rearranging Equation 6.8 in the text yields:

$$C = \frac{1}{2\pi f_B R} = \frac{1}{2\pi(2000)15000} = 5.305 \text{ nF}$$

P6.25* The phase of the transfer function is given by Equation 6.11 in the text:

$$\angle H(f) = -\arctan(f/f_B)$$

Thus, $f = -f_B \tan[\angle H(f)]$.

For $\angle H(f) = -1^\circ$, we have $f = 0.01746 f_B$.

For $\angle H(f) = -10^\circ$, we have $f = 0.1763 f_B$.

For $\angle H(f) = -89^\circ$, we have $f = 57.29 f_B$.

P6.26* The half-power frequency of the filter is

$$f_B = \frac{1}{2\pi RC} = 500 \text{ Hz}$$

The transfer function is given by Equation 6.9 in the text:

$$H(f) = \frac{1}{1 + j(f/f_B)}$$

The given input signal is

$$v_{in}(t) = 5\cos(500\pi t) + 5\cos(1000\pi t) + 5\cos(2000\pi t)$$

which has components with frequencies of 250, 500, and 1000 Hz.

Evaluating the transfer function for these frequencies yields:

$$H(250) = \frac{1}{1 + j(250/500)} = 0.8944\angle -26.57^\circ$$

$$H(500) = 0.7071\angle -45^\circ$$

$$H(1000) = 0.4472\angle -63.43^\circ$$

Applying the appropriate value of the transfer function to each component of the input signal yields the output:

$$v_{out}(t) = 4.472\cos(500\pi t - 26.57^\circ) + 3.535\cos(1000\pi t - 45^\circ)$$
$$+ 2.236\cos(2000\pi t - 63.43^\circ)$$

P6.27 The transfer function is given by Equation 6.9 in the text:

$$H(f) = \frac{1}{1 + j(f/f_B)}$$

The given input signal is

$$v_{\text{in}}(t) = 1 + 2\sin(800\pi t + 120^\circ) + 3\cos(20 \times 10^3 \pi t)$$

$$= 1 + 2\cos(800\pi t + 120^\circ - 90^\circ) + 3\cos(20 \times 10^3 \pi t)$$

$$= 1 + 2\cos(800\pi t + 30^\circ) + 3\cos(20 \times 10^3 \pi t)$$

which has components with frequencies of 0, 400, and 10,000 Hz.

Evaluating the transfer function for these frequencies yields:

$$H(0) = \frac{1}{1 + j(0/400)} = 1$$

$$H(400) = \frac{1}{1 + j(400/400)} = 0.7071\angle -45^\circ$$

$$H(10{,}000) = \frac{1}{1 + j(10000/400)} = 0.0400\angle -87.71^\circ$$

Applying the appropriate value of the transfer function to each component of the input signal yields the output:

$$v_{out}(t) = 1 + 1.4142\cos(800\pi t - 15^\circ) + 0.1200\cos(20\times 10^3 \pi t - 87.71^\circ)$$

P6.28 The period of a 5-kHz sinusoid is 200 μs. The phase shift corresponding to a delay of 20 μs is $\theta = -36^\circ$. (We give the phase shift as negative because the output lags the input.) Equation 6.11 gives the phase shift of the first-order filter. Thus, we can write:

$$\angle H(f) = -\arctan\left(\frac{5000}{f_B}\right) = -36^\circ$$

$$f_B = 6882 \text{ Hz}$$

P6.29 To achieve a reduction of the 20-kHz component by a factor of 100, we must have

$$\left|H(20\times 10^3)\right| = \frac{1}{\sqrt{1 + ((20\times 10^3)/f_B)^2}} = \frac{1}{100}$$

Solving, we find

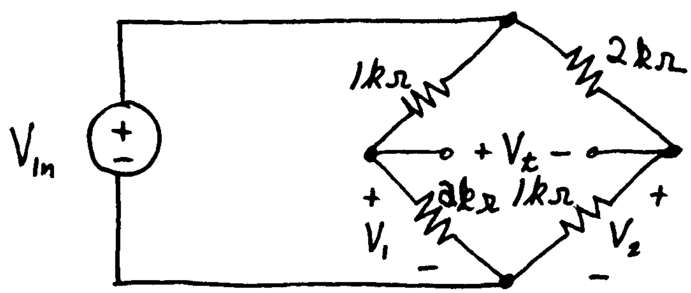
$$f_B = 200.0 \text{ Hz}$$

At 2 kHz, the resulting value of the transfer function is

$$H(f) = \frac{1}{1 + j(f/f_B)} = \frac{1}{1 + j(2000/200)} = 0.0995\angle -84.29^\circ$$

Thus, the 2-kHz component is changed in amplitude by a factor of 0.0995.

P6.30* The circuit seen by the capacitance is:



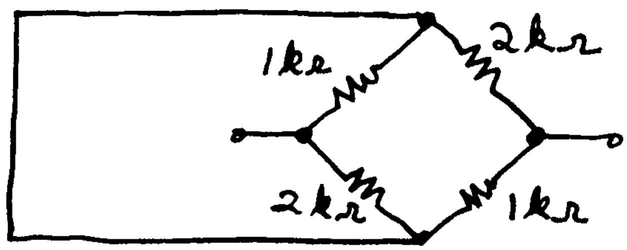
The open-circuit or Thévenin voltage is

$$\mathbf{V}_t = \mathbf{V}_1 - \mathbf{V}_2$$
$$= \mathbf{V}_{in}\frac{2000}{2000+1000} - \mathbf{V}_{in}\frac{1000}{1000+2000}$$

Thus, we obtain:

$$\mathbf{V}_t = \frac{1}{3}\mathbf{V}_{in} \qquad (1)$$

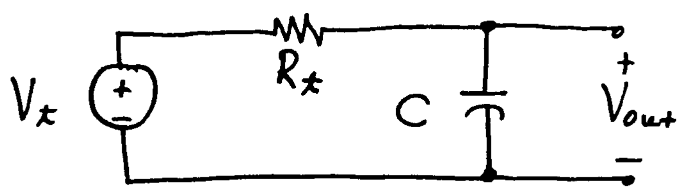
Zeroing the source, we have



The Thévenin resistance is

$$R_t = \frac{1}{1/1000 + 1/2000} + \frac{1}{1/2000 + 1/1000}$$
$$= 1333\ \Omega$$

Thus, the equivalent circuit is:



As in the text, this circuit has the transfer function:

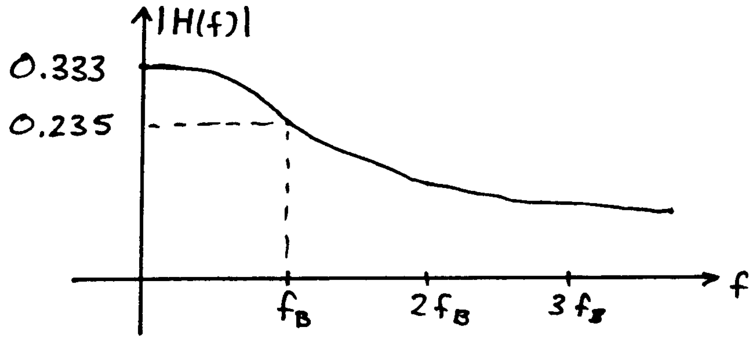
$$\frac{\mathbf{V}_{out}}{\mathbf{V}_t} = \frac{1}{1 + j(f/f_B)} \qquad (2)$$

where $f_B = \dfrac{1}{2\pi R_t C} = \dfrac{1}{2\pi 1333 \times 10^{-5}} = 11.94\ \text{Hz}$

Using Equation (1) to substitute for $\mathbf{V}_t$ in Equation (2) and rearranging, we have

$$\frac{\mathbf{V}_{out}}{\mathbf{V}_{in}} = \frac{1/3}{1 + j(f/f_B)}$$

A sketch of the transfer-function magnitude is:



P6.31 The output voltages given have a frequency of 10 kHz. Assume that the input voltage has the same frequency.

Dividing the output phasor by the input phasor, we obtain the transfer function:

$$H(10^4) = \frac{\mathbf{V}_{out}}{\mathbf{V}_{in}} = \frac{0.5\angle -84.26^\circ}{|\mathrm{V}_{in}|\angle\theta} = \frac{0.5\angle(-84.26^\circ - \theta)}{|\mathrm{V}_{in}|}$$

Using Equation 6.10 from the book, we have

$$\left|H(10^{4)}\right| = \frac{1}{\sqrt{1+(f/f_B)^2}} = \frac{1}{\sqrt{1+(10000/1005)^2}} = \frac{0.5}{|\mathrm{V}_{in}|}$$

Solving, we obtain $|\mathrm{V}_{in}| = 5$ Then using Equation 6.11, we have

$$\angle H(f) = -\arctan\left(\frac{10000}{1005}\right) = -84.26^\circ = -84.26^\circ - \theta$$

$$\theta = 84.26^\circ$$

P6.32 For the 10-kHz signal, the transfer function magnitude is

$$\left|H(f)\right| = \frac{|\mathbf{V}_{out}|}{|\mathbf{V}_{in}|} = \frac{0.2\times\sqrt{2}}{5\times\sqrt{2}} = 0.04 = \frac{1}{\sqrt{1+(f/f_b)^2}} = \frac{1}{\sqrt{1+(10\times 10^3/f_b)^2}}$$

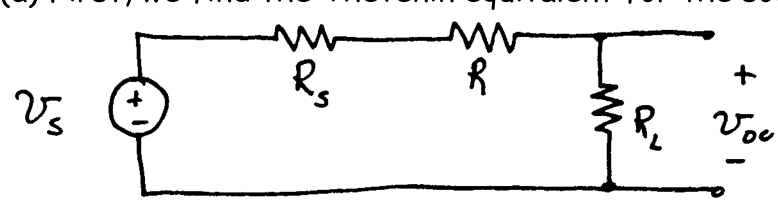
Solving, we find $f_b = 400.3$ Hz.

Then, for the 50-kHz signal, we have

$$\left|H(f)\right| = \frac{|\mathbf{V}_{out}|}{|\mathbf{V}_{in}|} = \frac{V_{orms}\times\sqrt{2}}{5\times\sqrt{2}} = \frac{1}{\sqrt{1+[50\times 10^3/(400.3)]^2}}$$

which yields $V_{orms} = 40$ mV.

P6.33 (a) First, we find the Thévenin equivalent for the source and resistances.



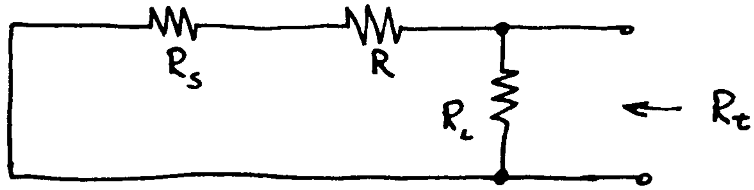
The open-circuit voltage is given by

$$v_t(t) = v_{oc}(t) = v_s(t)\frac{R_L}{R_s + R + R_L}$$

In terms of phasors, this becomes:

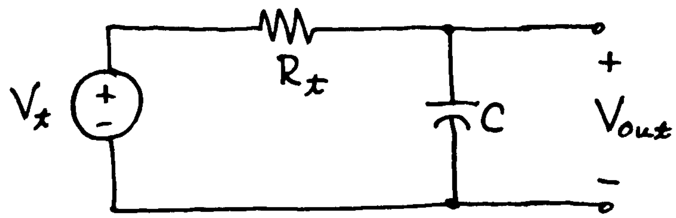
$$\mathbf{V}_t = \mathbf{V}_s \frac{R_L}{R_s + R + R_L} \qquad (1)$$

Zeroing the source, we find the Thévenin resistance:



$$R_t = \frac{1}{1/R_L + 1/(R + R_s)}$$

Thus, the original circuit has the equivalent:



The transfer function for this circuit is:

$$\frac{\mathbf{V}_{out}}{\mathbf{V}_t} = \frac{1}{1 + j(f/f_B)} \qquad (2)$$

where, $f_B = \dfrac{1}{2\pi R_t C}$

Using Equation (1) to substitute for $\mathbf{V}_t$ in Equation (2) and rearranging, we have:

$$H(f) = \frac{\mathbf{V}_{out}}{\mathbf{V}_s} = \frac{R_L}{R_s + R + R_L} \times \frac{1}{1 + j(f/f_B)} \qquad (3)$$

(b) Evaluating for the circuit components given, we have:

$$R_t = 980\ \Omega$$

$$f_B = 812.0\ \text{Hz}$$

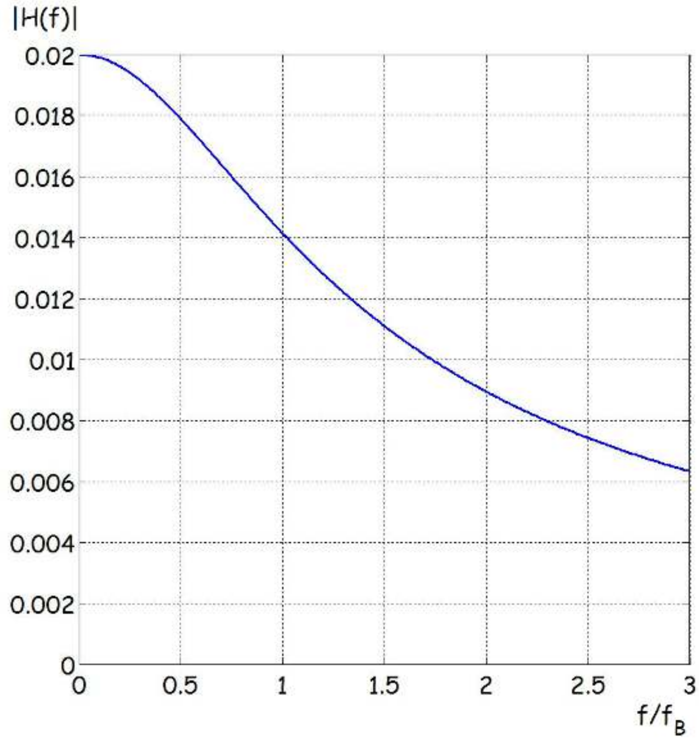
$$H(f) = \frac{0.02}{1 + j(f/f_B)}$$

A MATLAB program to plot the transfer-function magnitude is:

```
foverfb=0:0.01:3;
```

```
Hmag=abs(0.02./(1 + i*foverfb));
plot(foverfb,Hmag)
axis([0 3 0 0.02])
```

The resulting plot is:



P6.34 (a) Applying the voltage-division principle, we have:

$$H(f) = \frac{\mathbf{V}_{out}}{\mathbf{V}_{in}} = \frac{R_2}{R_1 + R_2 + j2\pi fL} = \frac{R_2/(R_1+R_2)}{1 + j2\pi fL/(R_1+R_2)} = \frac{R_2/(R_1+R_2)}{1 + j(f/f_B)}$$

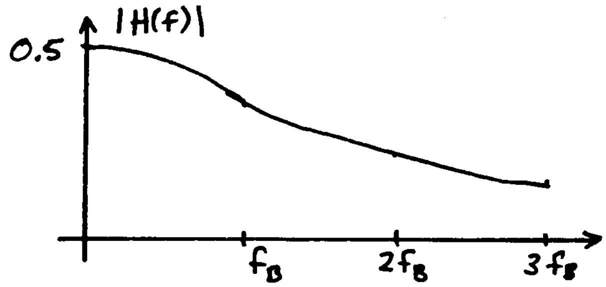
where $f_B = (R_1 + R_2)/(2\pi L)$

(b) Evaluating for the component values given, we have:

$$f_B = 1.061 \text{ MHz}$$

$$H(f) = \frac{0.5}{1 + j(f/f_B)}$$

A sketch of the transfer function magnitude is:



P6.35 With an assumed velocity $v = V_m \cos(2\pi ft)$, the force is

$$f = m\frac{dv}{dt} + kv = -m2\pi fV_m \sin(2\pi ft) + kV_m \cos(2\pi ft)$$

The phasors for the velocity and the force are

$$\mathbf{V} = V_m\angle 0^\circ \quad \text{and} \quad \mathbf{F} = jm2\pi fV_m + kV_m$$

Then the transfer function is

$$H(f) = \frac{\mathbf{V}}{\mathbf{F}} = \frac{V_m}{jm2\pi fV_m + kV_m} = \frac{1/k}{1 + jm2\pi f/k} = \frac{1/k}{1 + jf/f_B}$$

where $f_B = \dfrac{k}{2\pi m}$ is the half-power frequency.

P6.36 A Bode plot is a plot of the decibel magnitude of a network function versus frequency, using a logarithmic scale for frequency. Because it can clearly illustrate very large .and very small magnitudes for a wide range of frequencies on one plot, the Bode plot is particularly useful for displaying transfer functions

P6.37 The passband of a simple *RC* lowpass filter is the range of frequencies of the components that are passed to the output with relatively little change in amplitude. This is (approximately) the range from DC to the break frequency.

P6.38 A logarithmic frequency scale is one for which equal distances correspond to multiplying frequency by the same factor. A linear frequency scale is one for which equal distances correspond to adding the same amount to the starting frequency.

P6.39* (a) $\log(f_a) = \dfrac{\log(100) + \log(3000)}{2}$

or equivalently: $f_a = \sqrt{100 \times 3000} = 547.7\ \text{Hz}$

(b) $f_b = \dfrac{100 + 3000}{2} = 1550 \text{ Hz}$

P6.40 (a) 250 Hz is one octave lower than 500 Hz.
(b) 2000 Hz is two octaves higher than 500 Hz.
(c) 5 Hz is two decades lower than 500 Hz.
(d) 5 kHz is one decade higher than 500 Hz.

P6.41 A notch filter rejects components with frequencies in a narrow band while passing components with frequencies higher or lower than those in the rejection band. One application is to eliminate 60-Hz power line interference from audio signals.

P6.42* (a) We have:

$$20\log|H(f)| = -40$$
$$\log|H(f)| = -2$$
$$|H(f)| = 10^{-2} \quad = 0.01$$

(b) Similarly,

$$20\log|H(f)| = 40$$
$$\log|H(f)| = 2$$
$$|H(f)| = 10^{2} \quad = 100$$

P6.43 To convert to decibels, we take 20 times the common logarithm of the transfer function. Thus, we have:

20log(0.5) = -6.021 dB
20log(2) = +6.021 dB
$20\log(1/\sqrt{2}) = -3.010 \text{ dB}$
$20\log(\sqrt{2}) = 3.010 \text{ dB}$

P6.44 If the output terminals of one filter are connected to the input terminals of a second filter, we say that the filters are cascaded.

P6.45 On a linear scale, the frequencies are 5, 14, 23, 32, 41, 50 Hz. (We add (50 – 5)/5 = 9 Hz to each value to obtain the next value.) On a logarithmic scale, the frequencies are 5, 7.924, 19.91, 12.56, 31.55, 50 Hz. (We multiply each value by $\sqrt[5]{50/5} = 1.5849$ to obtain the next value.)

P6.46 (a) We have $10^{N_d} \times 25 = 10000$.

Taking the common logarithm of both sides, we have:

$$N_d + 1.398 = 4$$

$$N_d = 2.602 \text{ decades}$$

(b) Similarly, in octaves, $2^{N_{oct}} \times 25 = 10000$.

Taking the common logarithm of both sides, we have:

$$N_{oct} \log(2) + 1.398 = 4$$

$$N_{oct} = 8.644 \text{ octaves}$$

P6.47 We have $H(f) = H_1(f)H_2(f)$ and $|H(f)|_{\text{dB}} = |H_1(f)|_{\text{dB}} + |H_2(f)|_{\text{dB}}$. For these formulas to be valid, $H_1(f)$ must be the transfer function of the first filter *with the second attached.*

P6.48* (a) The overall transfer function is the product of the transfer functions of the filters in cascade:

$$H(f) = H_1(f) \times H_2(f) = \frac{1}{[1 + j(f/f_B)]^2}$$

(b)

$$|H(f)| = \frac{1}{1 + (f/f_B)^2}$$

$$|H(f_{3dB})| = \frac{1}{\sqrt{2}} = \frac{1}{1 + (f_{3dB}/f_B)^2}$$

$$(f_{3dB}/f_B)^2 = \sqrt{2} - 1$$

$$f_{3dB} = f_B\sqrt{\sqrt{2} - 1} = 0.6436 f_B$$

P6.49 see end of document for solution

P6.50 A Bode magnitude plot is a plot of the decibel magnitude of a network function versus frequency using a logarithmic scale for frequency.

A phase Bode plot is a plot of the phase of a network function versus frequency using a logarithmic scale for frequency.

P6.51 The slope of the high-frequency asymptote is -20 dB/decade. The slope of the low-frequency asymptote is zero. The asymptotes meet at the half-power frequency f_B.

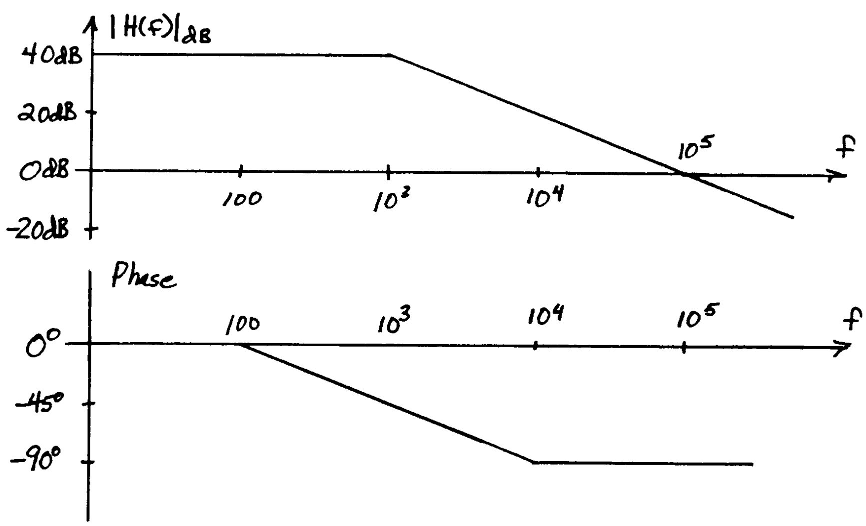
P6.52 Because the transfer functions in decibels add for cascaded systems, the slope declines at 80 dB/decade.

P6.53* $$H(f) = \frac{100}{1 + j(f/1000)} = \frac{100}{\sqrt{1 + (f/1000)^2}}$$

$$|H(f)|_{dB} = 20\log(100) - 20\log\sqrt{1 + (f/1000)^2}$$
$$= 40 - 20\log\sqrt{1 + (f/1000)^2}$$

This is similar to the transfer function treated in Section 6.4 in the text except for the additional 40 dB constant. The half-power frequency is $f_B = 1000$.

The asymptotic Bode plots are:



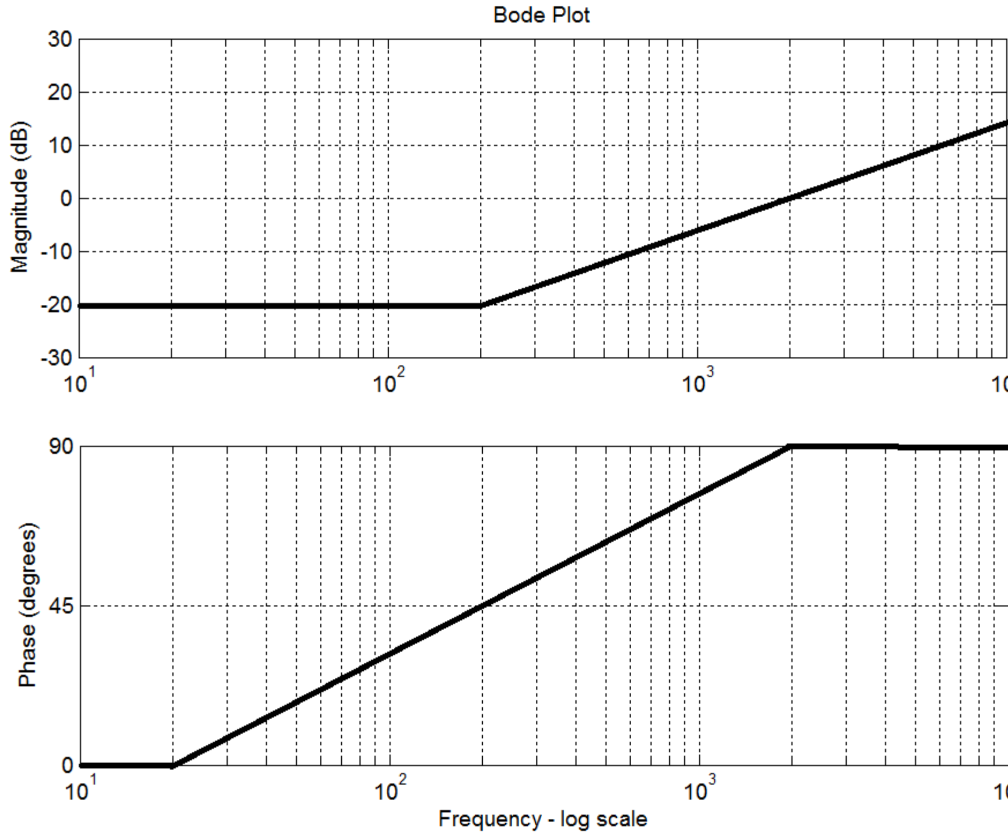
P6.54 $H(f) = 0.1[1 + j(f/200)]$ $\quad |H(f)| = 0.1\sqrt{1 + (f/200)^2}$

$$|H(f)|_{dB} = 20\log(0.1) + 20\log\sqrt{1 + (f/200)^2} = -20 + 20\log\sqrt{1 + (f/200)^2}$$

The break frequency is $f_B = 200$.

The phase is $+\arctan(f/f_B)$.

The asymptotic Bode Plots are:

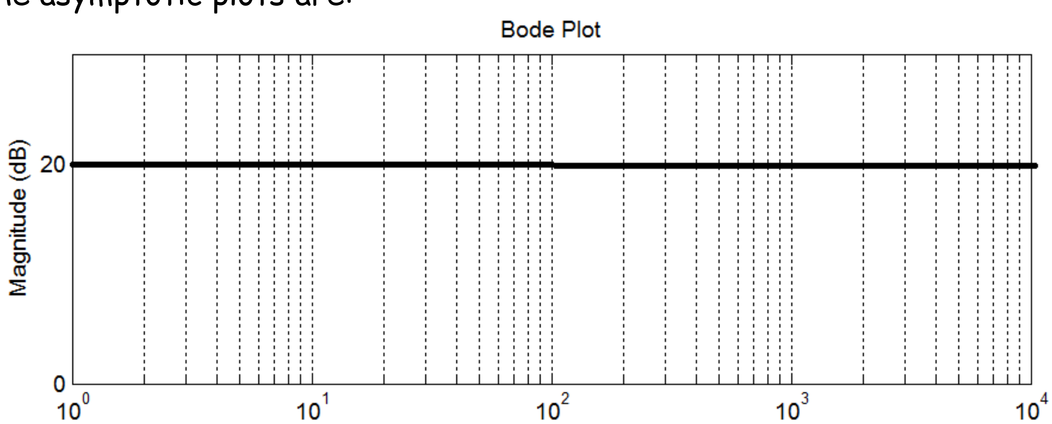


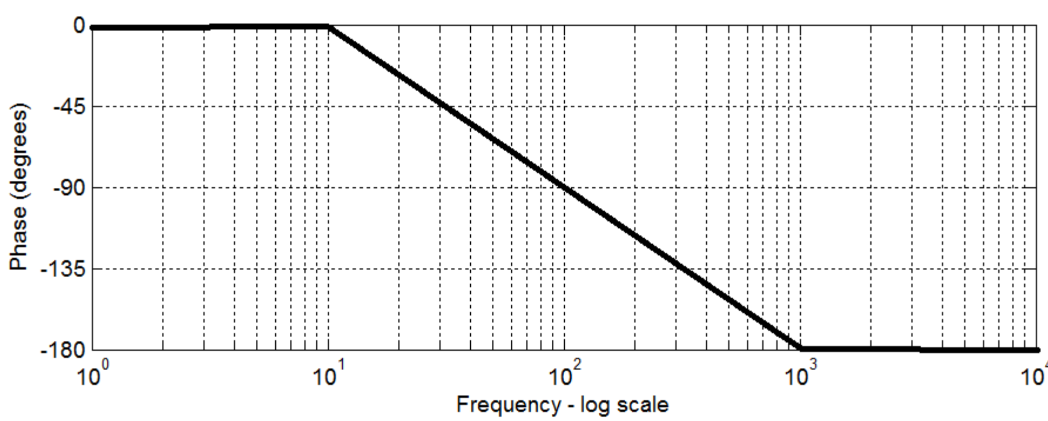
P6.55 $H(f) = 10\dfrac{1 - j(f/100)}{1 + j(f/100)} \qquad |H(f)| = 10\dfrac{\sqrt{1+(f/100)^2}}{\sqrt{1+(f/100)^2}} = 10$

Thus, $|H(f)|_{dB} = 20\log(10) = 20\ \text{dB}$

The phase is $-2 \times \arctan(f/100)$.

The asymptotic plots are:





P6.56 This is a first-order lowpass RC filter. The break frequency is:

$$f_B = \frac{1}{2\pi RC} = 5.895\text{ MHz}$$

The Bode plots are like Figures 6.15 and 6.16 in the text.

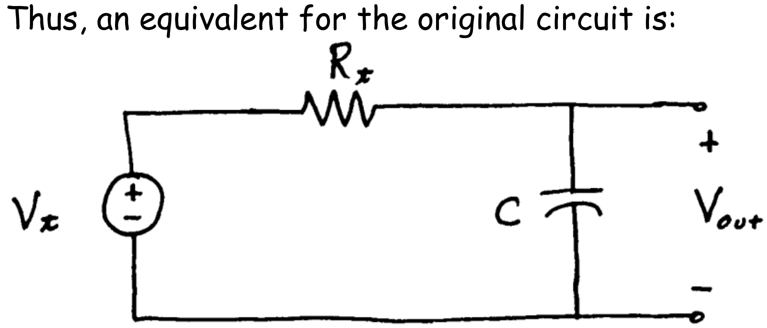
P6.57 First, we find the Thévenin equivalent for the source and the resistances. The Thévenin resistance is

$$R_t = \frac{1}{1/R_1 + 1/R_2} = 1000\ \Omega$$

and the Thévenin voltage is

$$\mathbf{V}_t = \frac{R_2}{R_1 + R_2}\mathbf{V}_{in} = 0.5\mathbf{V}_{in}$$

Thus, an equivalent for the original circuit is:



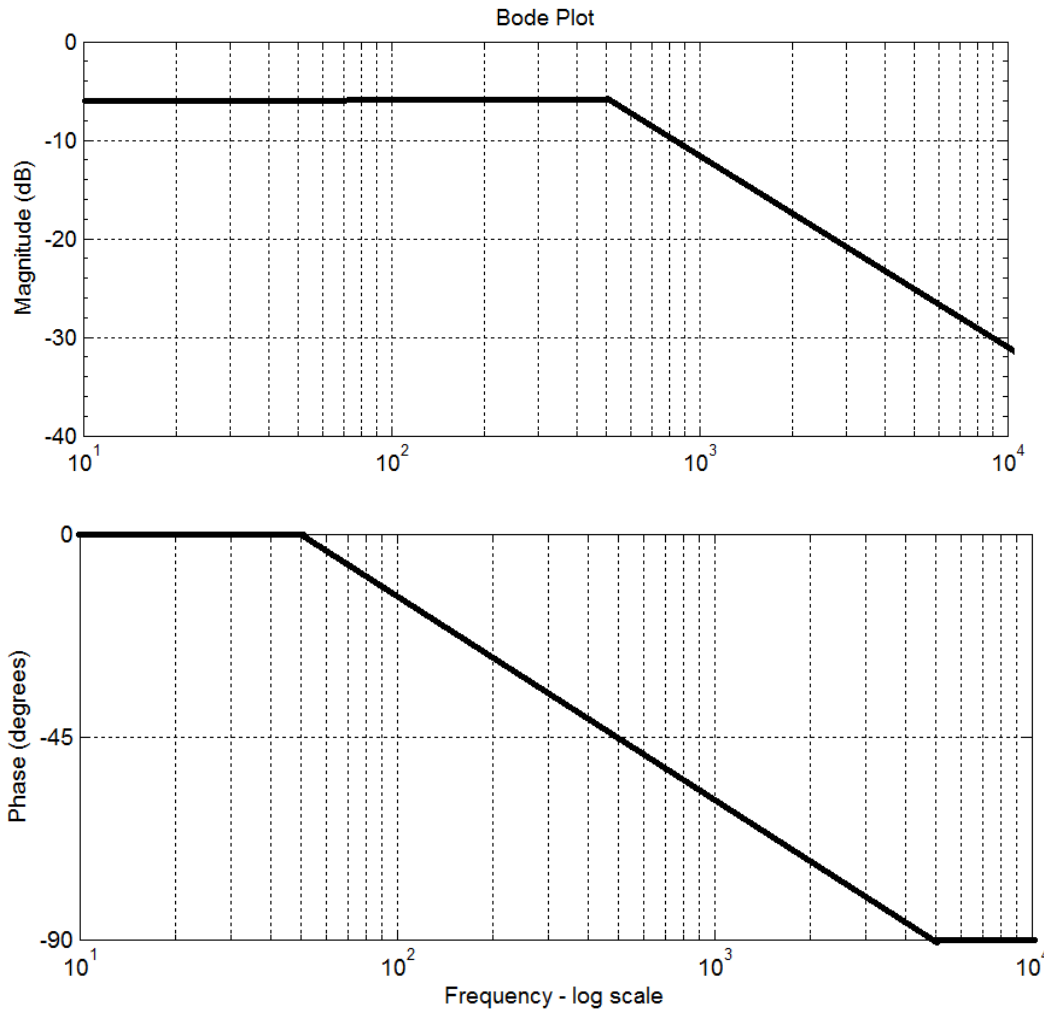
This is a lowpass filter having a transfer function given by Equation 6.8 (with changes in notation):

$$\frac{\mathbf{V}_{out}}{\mathbf{V}_t} = \frac{1}{1 + j(f/f_B)}$$

where $f_B = 1/(2\pi R_f C) = 500$ Hz.

Using the fact that $\mathbf{V}_f = 0.1\mathbf{V}_{in}$, we have $H(f) = \dfrac{\mathbf{V}_{out}}{\mathbf{V}_{in}} = \dfrac{0.5}{1 + j(f/f_B)}$

The plots are:



P6.58 (a) Solving for the input voltage, we have

$$v_{in}(t) = 0.1v_{out}(t) + 40\pi\int_0^t v_{out}(t)dt$$

$$= 0.1A\cos(2\pi ft) + 40\pi\int_0^t A\cos(2\pi ft)dt$$

$$= 0.1A\cos(2\pi ft) + \frac{40\pi}{2\pi f}A\sin(2\pi ft)\Big|_0^t$$

$$= 0.1A\cos(2\pi ft) + \frac{20}{f}A\sin(2\pi ft)$$

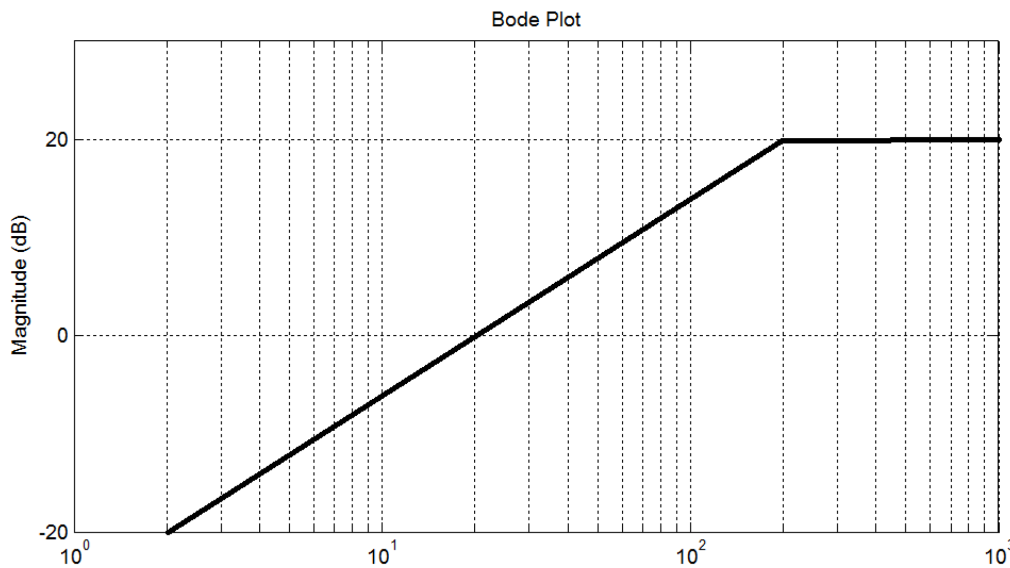
(b) Then the transfer function for the system is

$$H(f) = \frac{\mathbf{V}_{out}}{\mathbf{V}_{in}} = \frac{A}{0.1A - jA(20/f)} = \frac{10}{1 - j(200/f)}$$

(c) For $f >> 200$, the asymptote is constant at 20 dB. For $f << 200$, the transfer function becomes

$$20\log|H(f)| = 20 - 20\log\sqrt{1 + (200/f)^2} \cong 20 - 20\log(200/f)$$

which describes an asymptote sloping downward at 20 dB/decade as f becomes smaller. The asymptotic plot for the magnitude is



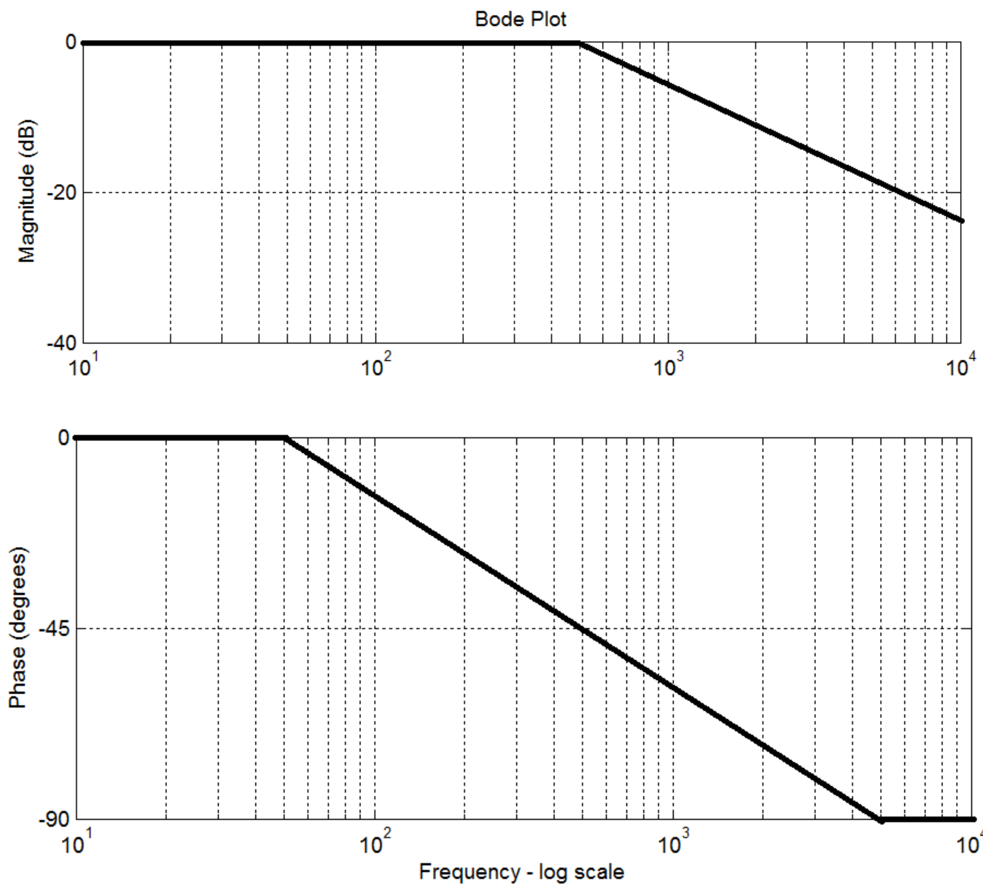
P6.59 Applying the voltage-division principle, we find the transfer function:

$$H(f) = \frac{\mathbf{V}_{out}}{\mathbf{V}_{in}} = \frac{R}{R + j2\pi fL} = \frac{1}{1 + j(f/f_B)}$$

where $f_B = R/2\pi L$. Thus, this is a first-order lowpass filter.

The break frequency is $f_B = R/2\pi L = 500\text{ Hz}$.

The plots are

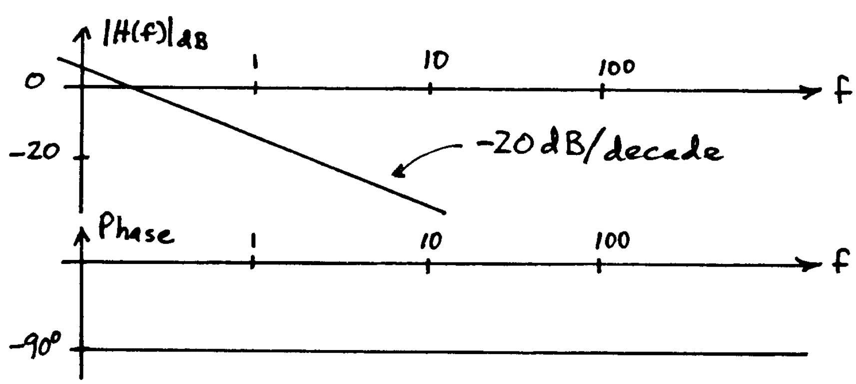


P6.60*

$$H(f) = \frac{1}{j2\pi f}$$

$$|H(f)|_{dB} = -20\log(2\pi f) = -15.96 - 20\log(f)$$

The slope of the magnitude is -20 dB per decade.
The phase is -90° at all frequencies.
The Bode plots are:



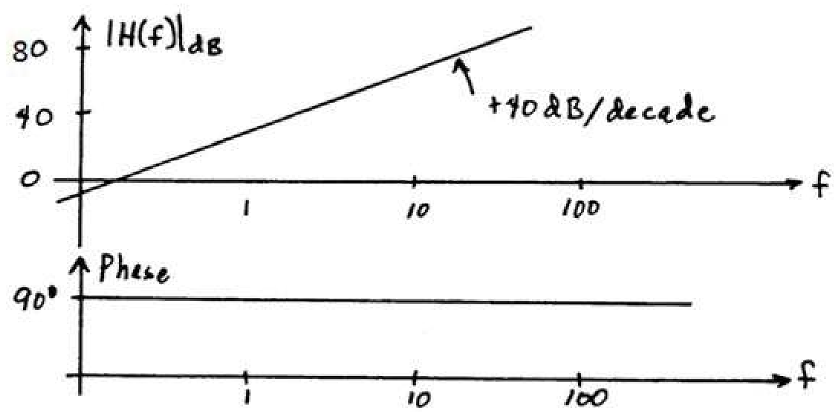
P6.61

$$H(f) = j4\pi f$$

$$|H(f)|_{dB} = 20\log(4\pi f) = 21.98 + 20\log(f)$$

The slope of the magnitude is 40 dB per decade.
The phase is 90° at all frequencies.

The Bode plots are:



P6.62 The slope of the high-frequency asymptote is zero. The slope of the low-frequency asymptote is 20 dB/decade. The asymptotes meet at the half-power frequency .

P6.63 The first-order highpass *RC* filter is shown in Figure 6.19 in the book. The half-power frequency is given by $f_B = 1/(2\pi RC)$.

P6.64* Applying the voltage-division principle, we have:

$$H(f) = \frac{\mathbf{V}_{out}}{\mathbf{V}_{in}}$$

$$= \frac{R_2}{R_1 + R_2 + 1/j2\pi fC}$$

$$= \frac{R_2/(R_1 + R_2)}{1 + 1/j2\pi fC(R_1 + R_2)}$$

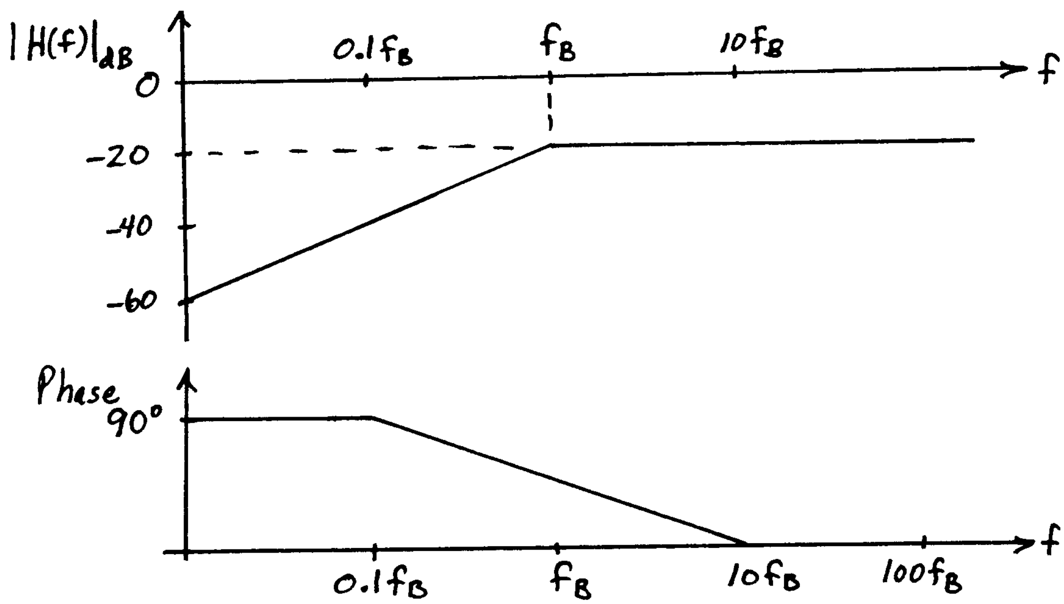
$$= \frac{R_2}{R_1 + R_2} \frac{j2\pi fC(R_1 + R_2)}{1 + j2\pi fC(R_1 + R_2)}$$

$$= \frac{R_2}{R_1 + R_2} \frac{j(f/f_B)}{1 + j(f/f_B)}$$

$$= 0.1 \frac{j(f/f_B)}{1 + j(f/f_B)}$$

where $f_B = 1/2\pi C(R_1 + R_2) = 15.92\text{ Hz}$.

The asymptotic Bode plots are:



P6.65* This is the first-order high-pass filter analyzed in Section 6.5 in the text. The transfer function is

$$H(f) = \frac{\mathbf{V}_{\text{out}}}{\mathbf{V}_{in}} = \frac{j(f/f_B)}{1 + j(f/f_B)}$$

where $f_B = \dfrac{1}{2\pi RC} = 1000\text{ Hz}$.

The input signal is given by

$$v_{in}(t) = 5 + 5\cos(2000\pi t)$$

This signal has components at $f = 0\text{ Hz}$ and $f = 1000\text{ Hz}$. The transfer-function values at these frequencies are:

$$H(0) = \frac{j0}{1 + j0} = 0$$

$$H(1000) = \frac{j1}{1 + j1} = 0.7071\angle 45^\circ$$

Applying these transfer-function values to the respective components yields:

$$v_{out}(t) = 3.536\cos(2000\pi t + 45^\circ)$$

P6.66 This is the first-order high-pass filter analyzed in Section 6.5 in the text. The transfer function is

$$H(f) = \frac{\mathbf{V}_{\text{out}}}{\mathbf{V}_{in}} = \frac{j(f/f_B)}{1 + j(f/f_B)}$$

where $f_B = \dfrac{1}{2\pi RC} = 1000\text{ Hz}$.

The input signal is given by

$v_{\text{in}}(t) = 10\cos(400\pi t + 30^\circ) + 20\cos(4000\pi t) + 5\sin(4 \times 10^4 \pi t)$

This signal has components with frequencies of 200 Hz, 2 kHz and 20 kHz. The transfer-function values at these frequencies are:

$$H(200) = \frac{j0.2}{1 + j0.2} = 0.1961\angle 78.69^\circ$$

$$H(2000) = \frac{j2}{1 + j2} = 0.8944\angle 26.57^\circ$$

$$H(20000) = \frac{j20}{1 + j20} = 0.9988\angle 2.86^\circ$$

Applying these transfer-function values to the respective components yields:

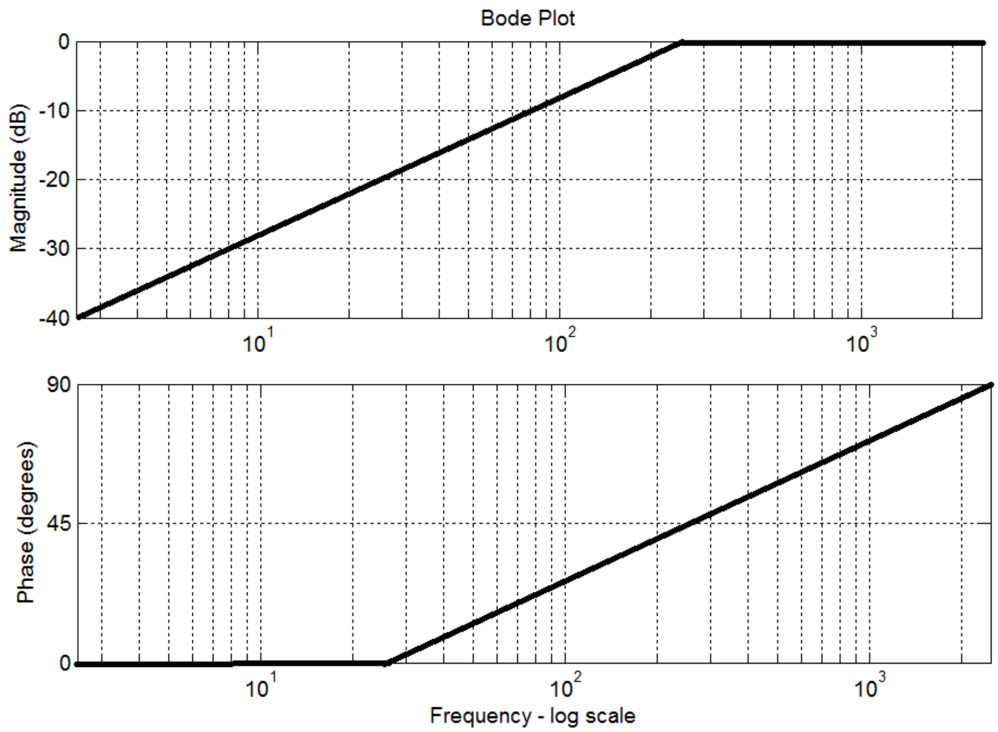
$$v_{out}(t) = 1.961\cos(400\pi t + 78.69^\circ) + 17.889\cos(4000\pi t + 26.57^\circ)$$
$$+ 4.994\cos(4 \times 10^4 \pi t - 87.14^\circ)$$

P6.67 This is a first-order high-pass filter analyzed in Section 6.5 in the text. The transfer function is

$$H(f) = \frac{\mathbf{V}_{out}}{\mathbf{V}_{in}} = \frac{j(f/f_B)}{1 + j(f/f_B)}$$

where $f_B = \dfrac{1}{2\pi RC} = 250\text{ Hz}$.

The Bode plots are



P6.68 Applying the voltage-division principle, we have:

$$H(f) = \frac{\mathbf{V}_{out}}{\mathbf{V}_{in}} = \frac{j2\pi fL}{R + j2\pi fL} = \frac{j2\pi fL/R}{1 + j2\pi fL/R}$$

$$= \frac{j(f/f_B)}{1 + j(f/f_B)}$$

in which $f_B = R/2\pi L = 2\ \text{MHz}$. The Bode plots are the same as Figure 6.21 in the text.

P6.69 To attenuate the 60-Hz component by 80 dB, the break frequency must be four decades higher than 60 Hz because the roll-off slope is 20 dB/decade. Thus, the break frequency must be $f_B = 600\ \text{kHz}$.

The 600-Hz component is attenuated by 60 dB.

Since $f_B = \dfrac{1}{2\pi RC}$, we have $C = \dfrac{1}{2\pi R f_B} = \dfrac{1}{2\pi(100)600000} = 2653\ \text{pF}$

P6.70 The resonant frequency f_o is defined to be the frequency at which the impedance is purely resistive (i.e., the total reactance is zero). In other words, impedance is minimum in a series *RLC* circuit. Therefore, the current being the ratio of voltage and impedance will be maximum at resonance.

The quality factor Q_s is defined to be the ratio of the reactance of the inductance at the resonant frequency to the resistance.

$$Q_s = \frac{2\pi f_o L}{R}$$

P6.71*

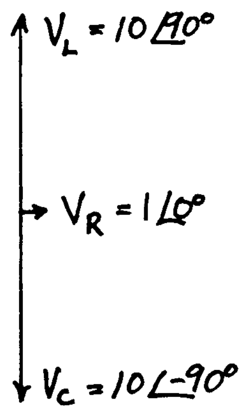
$$f_0 = \frac{1}{2\pi\sqrt{LC}} = 1.125 \text{ MHz}$$

$$Q_s = \frac{2\pi f_0 L}{R} = 10$$

$$B = \frac{f_0}{Q_s} = 112.5 \text{ kHz}$$

$$f_H \cong f_0 + \frac{B}{2} = 1.181 \text{ MHz}$$

$$f_L \cong f_0 - \frac{B}{2} = 1.069 \text{ MHz}$$



At the resonant frequency: $\mathbf{V}_R = 1\angle 0^\circ$ $\quad \mathbf{V}_L = 10\angle 90^\circ$ $\quad \mathbf{V}_C = 10\angle -90^\circ$

P6.72

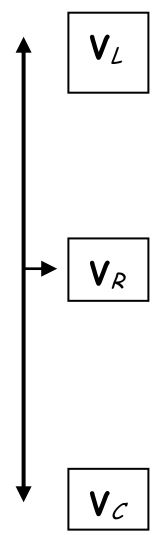
$$f_0 = \frac{1}{2\pi\sqrt{LC}} = 562.7 \text{ kHz}$$

$$Q_s = \frac{2\pi f_0 L}{R} = 20$$

$$B = \frac{f_0}{Q_s} = 28.13 \text{ kHz}$$

$$f_H \cong f_0 + \frac{B}{2} = 576.8 \text{ kHz}$$

$$f_L \cong f_0 - \frac{B}{2} = 548.6 \text{ kHz}$$



At the resonant frequency:

$\mathbf{V}_R = 1\angle 0^\circ$

$\mathbf{V}_L = 20\angle 90^\circ$ $\quad \mathbf{V}_C = 20\angle -90^\circ$

P6.73

$$Q_s = \frac{f_0}{B} = \frac{300 \text{ kHz}}{15 \text{ kHz}} = 20$$

$$L = \frac{RQ_s}{2\pi f_0}$$

$$C = \frac{1}{Q_s R(2\pi f_0)} = 663.1 \times 10^{-12}$$

Solving, we obtain $L = 424.4\mu\text{H}$, $R = 40\Omega$

P6.74 The impedance of the circuit is given by

$$Z(j\omega) = \frac{1}{j\omega C} + \frac{1}{1/R - j/\omega L} = \frac{-j}{\omega C} + \frac{1/R + j/\omega L}{1/R^2 + 1/\omega^2 L^2}$$

At the resonant frequency, the imaginary part equals zero.

$$\frac{-1}{\omega_0 C} + \frac{1/\omega_0 L}{1/R^2 + 1/\omega_0^2 L^2} = 0$$

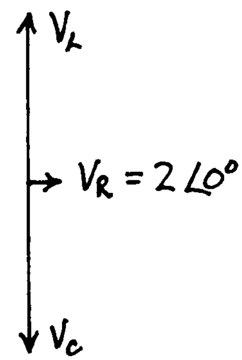
which eventually yields

$$\omega_0 = \frac{1}{\sqrt{LC - L^2/R^2}} \qquad \text{or} \qquad f_0 = \frac{1}{2\pi\sqrt{LC - L^2/R^2}}$$

P6.75* Assuming zero phase for $\mathbf{V}_R$, the phasor diagram at the resonant frequency is shown.

Thus, $\mathbf{V}_L = 20\angle 90^\circ$ and $\mathbf{V}_C = 20\angle -90^\circ$

and



$$\mathbf{I} = \frac{\mathbf{V}_R}{R} = 40\angle 0^\circ \text{ mA}$$

$$2\pi f_0 L \angle 90^\circ = \frac{\mathbf{V}_L}{\mathbf{I}} = 500\angle 90^\circ$$

$$L = 79.57\ \mu\text{H}$$

$$\frac{1}{2\pi f_0 C}\angle -90^\circ = \frac{\mathbf{V}_C}{\mathbf{I}} = 500\angle -90^\circ$$

$$C = 318.3\text{ pF}$$

P6.76

$$|Z_s|_{\min} = R = 5\,\Omega \qquad Q_s = \frac{f_0}{B} = \frac{12\text{ MHz}}{400\text{ kHz}} = 30$$

$$L = \frac{RQ_s}{2\pi f_0} = 19.64\ \mu\text{H} \qquad C = \frac{1}{Q_s R(2\pi f_0)} = 88.42\text{ pF}$$

P6.77 A bandpass filter is a filter that passes components in a band of frequencies, rejecting components with higher and lower frequencies. Bandwidth is the span of frequencies for which the transfer function magnitude is higher than its maximum value divided by the square root of two.

P6.78 As in the series resonant circuit, the resonant frequency f_o is the frequency for which the impedance is purely resistive.

Referring Equation (6.38), $\left|Z_p\right| = \dfrac{1}{\sqrt{\left[\left(\frac{1}{R}\right)^2 + \left(2\pi f C - \frac{1}{2\pi f L}\right)^2\right]}}$

At resonance, second term in denominator becomes zero (i.e. denominator becomes minimum). Therefore, impedance becomes maximum.

For the parallel circuit, we define the quality factor Q_p as the ratio of the resistance to the reactance of the inductance at resonance, given by

$$Q_p = \frac{R}{2\pi f_o L}$$

Notice that this is the reciprocal of the expression for the quality factor Q_s of the series resonant circuit.

P6.79*

$$f_0 = \frac{1}{2\pi\sqrt{LC}} = 1.592 \text{ MHz}$$

$$Q_p = \frac{R}{2\pi f_0 L} = 10.00 \qquad B = \frac{f_0}{Q_p} = 159.2 \text{ kHz}$$

P6.80

$$\left|Z_p\right|_{\max} = R = 5 \text{ k}\Omega$$

$$Q_p = \frac{f_0}{B} = 100$$

$$L = \frac{R}{2\pi f_0 Q_p} = 0.3988\ \mu\text{H} \quad C = \frac{Q_p}{2\pi f_0 R} = 159.2 \text{ pF}$$

P6.81

$$Q_p = \frac{f_0}{B} \qquad L = \frac{R}{2\pi f_0 Q_p} = 64 \times 10^{-9}$$

$$C = \frac{Q_p}{2\pi f_0 R} = 40 \times 10^{-12}$$

Solving, we obtain $f_0 = 99.47$ MHz, $B = 4$MHz

P6.82

$$Q_p = \frac{f_0}{B} = 16$$

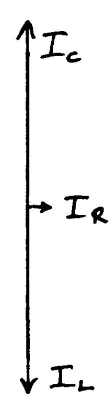
$$C = \frac{Q_p}{2\pi f_0 R} = 159.2 \text{ pF}$$

$$L = \frac{R}{2\pi f_0 Q_p} = 2.487\ \mu\text{H}$$

$$\mathbf{I} = \mathbf{I}_R = 1\angle 0^\circ \text{ mA}$$

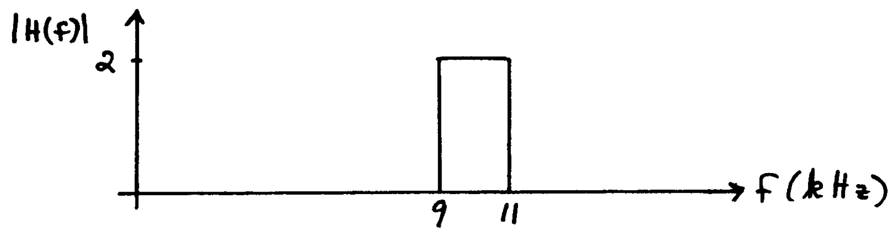
$$\mathbf{I}_L = \frac{\mathbf{V}}{j2\pi f_0 L} = \frac{R\mathbf{I}}{j2\pi f_0 L} = 16\angle -90^\circ \text{ mA}$$

$$\mathbf{I}_C = \frac{\mathbf{V}}{1/(j2\pi f_o C)} = \frac{R\mathbf{I}}{1/(j2\pi f_0 C)} = 16\angle +90^\circ \text{ mA}$$

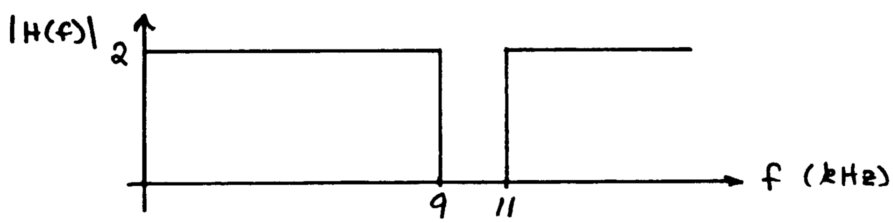


P6.83 Four types of ideal filters are lowpass, highpass, bandpass, and band reject (or notch) filters. Their transfer functions are shown in Figure 6.32 in the book.

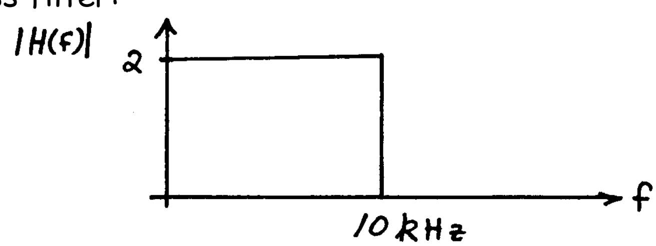
P6.84* Bandpass filter:



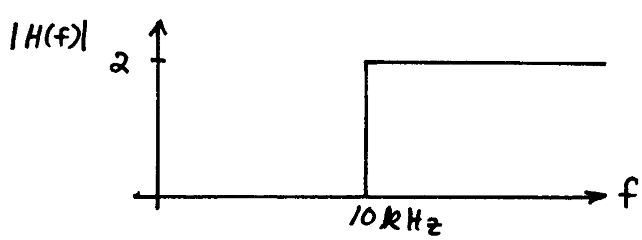
Band-reject filter:



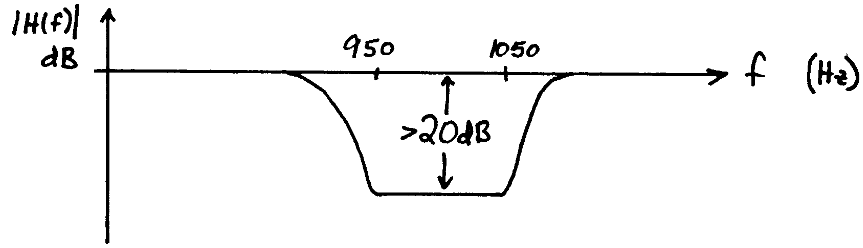
P6.85 Lowpass filter:



Highpass filter:



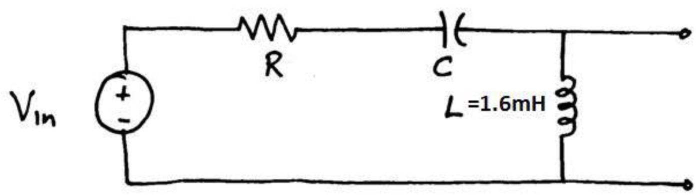
P6.86 A band-reject filter is needed with cutoff frequencies of approximately 950 and 1050 Hz. The magnitude plot of a suitable filter is:



P6.87 A lowpass filter having a cutoff frequency of 100 Hz will pass the heart signals and reject most of the noise.

P6.88 An AM radio signal having a carrier frequency of 980 kHz has components ranging in frequency from 970 kHz to 990 kHz. A bandpass filter is needed to pass this signal and reject the signals from other AM radio transmitters. The cutoff frequencies should be 970 and 990 kHz.

P6.89* The circuit diagram of a second-order highpass filter is:

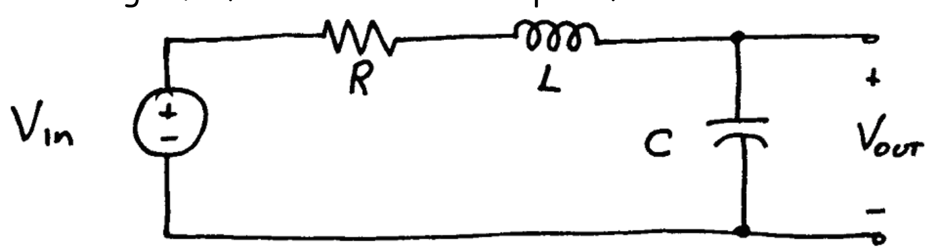


$$L = \frac{RQ_s}{2\pi f_0} = \frac{1R}{2\pi 100 \times 10^3} = 1.6 \times 10^{-3}$$

$$C = \frac{1}{Q_s R(2\pi f_0)} = \frac{1}{1R(2\pi 100 \times 10^3)}$$

Solving, we obtain $R = 1005\Omega$, $C = 1583.6$ pF

P6.90 The circuit diagram of a second-order lowpass filter is:



$$L = \frac{RQ_s}{2\pi f_0} = 0.1326\ \mu\text{H}$$

$$C = \frac{1}{Q_s R(2\pi f_0)} = 212.2\ \text{pF}$$

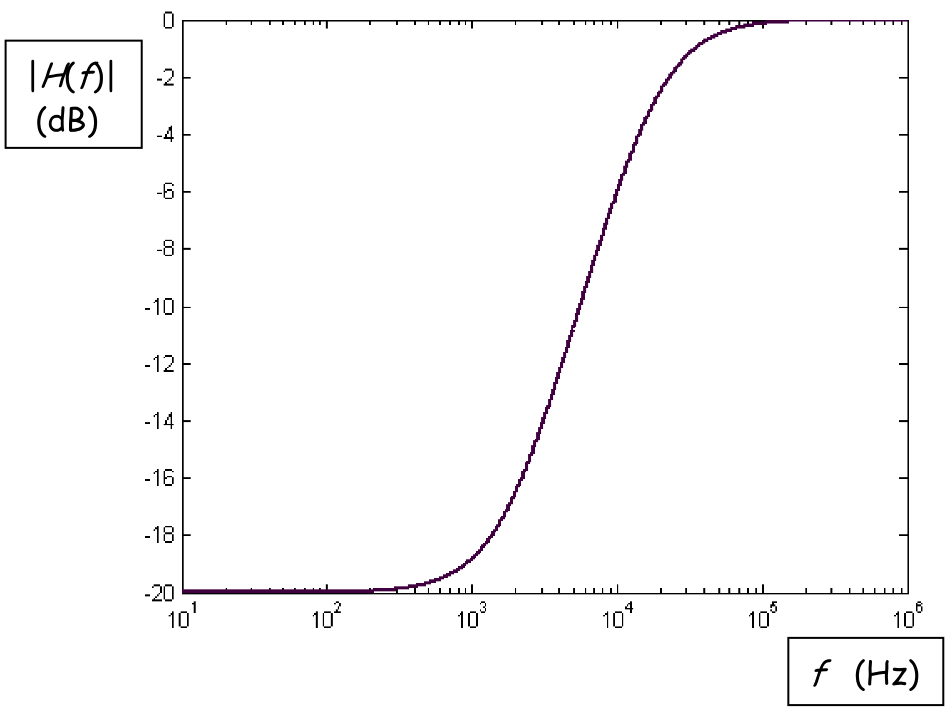
P6.91 (a) Applying the voltage-division principle, we have

$$H(f) = \frac{R_2}{R_2 + \dfrac{1}{1/R_1 + j2\pi fC}}$$

(b) A MATLAB program to produce the desired plot is

```
R1 = 9000;
R2 = 1000;
C = 1e-8;
f = logspace(1,6,600);
w = 2*pi*f;
H = R2./(R2+1./(j*w*C + 1/(R1)));
semilogx(f,20*log10(abs(H)))
```

The resulting plot is



(c) At very low frequencies, with the capacitance considered to be an open circuit, we have a two-resistance voltage divider and

$$H(f) = \frac{R_2}{R_2 + R_1} = 0.1$$

which is equivalent to -20 dB as shown in the plot.

(d) At very high frequencies with the capacitance considered as a short circuit, we have $\mathbf{V}_{out} = \mathbf{V}_{in}$ so $H(f) = 1$ which is equivalent to 0 dB as shown in the plot.

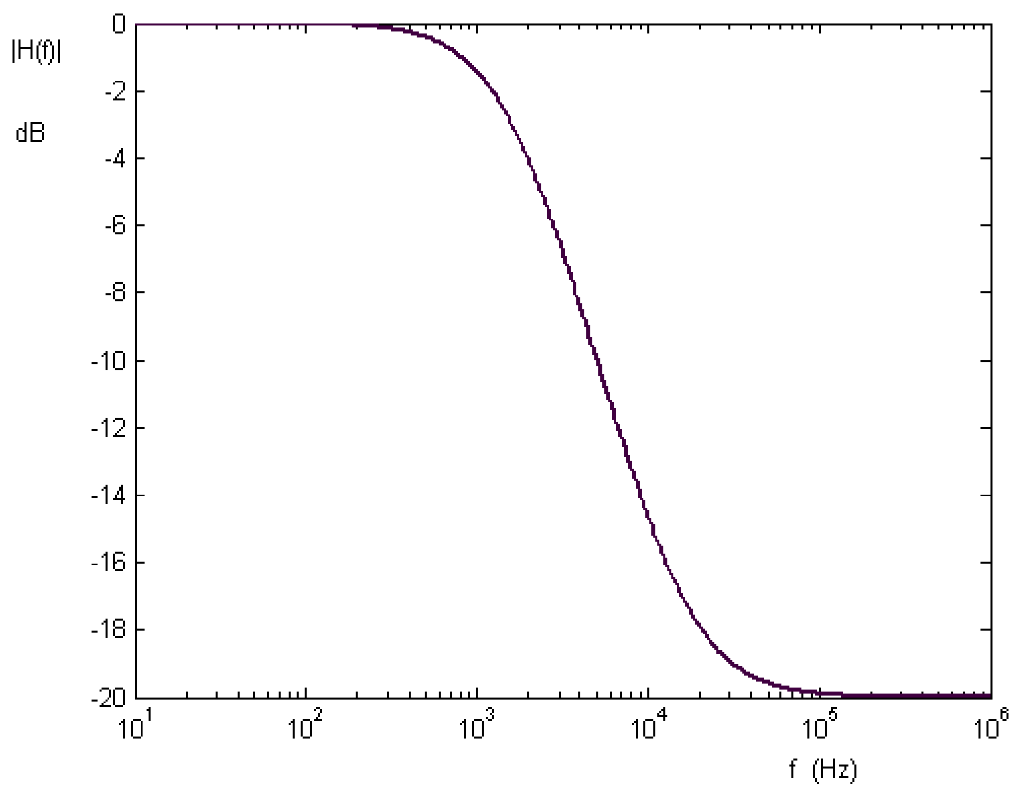
P6.92 (a) Applying the voltage-division principle, we have

$$H(f) = \frac{R_2 + 1/(j2\pi fC)}{R_1 + R_2 + 1/(j2\pi fC)}$$

(b) A MATLAB program to produce the desired plot is

```
R1 = 9000;
R2 = 1000;
C = 1e-8;
f = logspace(1,6,600);
w = 2*pi*f;
H = (R2+1./(j*w*C))./(R1+R2+1./(j*w*C ));
semilogx(f,20*log10(abs(H)))
```

The resulting plot is



(c) At very low frequencies, with the capacitance considered to be an open circuit, no current flows in the circuit, and we have $\mathbf{V}_{out} = \mathbf{V}_{in}$ so $H(f) = 1$ which is equivalent to 0 dB as shown in the plot.

(d) At very high frequencies with the capacitance considered as a short circuit, we have a two-resistance voltage divider and

$$H(f) = \frac{R_2}{R_2 + R_1} = 0.1$$

which is equivalent to -20 dB as shown in the plot.

P6.93 (a and b) R_1 and C_1 form the highpass circuit for which we want a break frequency of 100 Hz. Thus, we have $C_1 = 1/2\pi R_1(100) = 1.592\,\mu\text{F}$.
R_2 and C_2 form the lowpass circuit for which we want a break frequency of 10 kHz. Thus, we have $C_2 = 1/2\pi R_2(10^4) = 159.2\text{ pF}$.

(c) First, we write this expression for the impedance in series with C_1:

$$Z = \frac{1}{\dfrac{1}{R_1} + \dfrac{1}{R_2 + 1/j2\pi f C_2}}$$

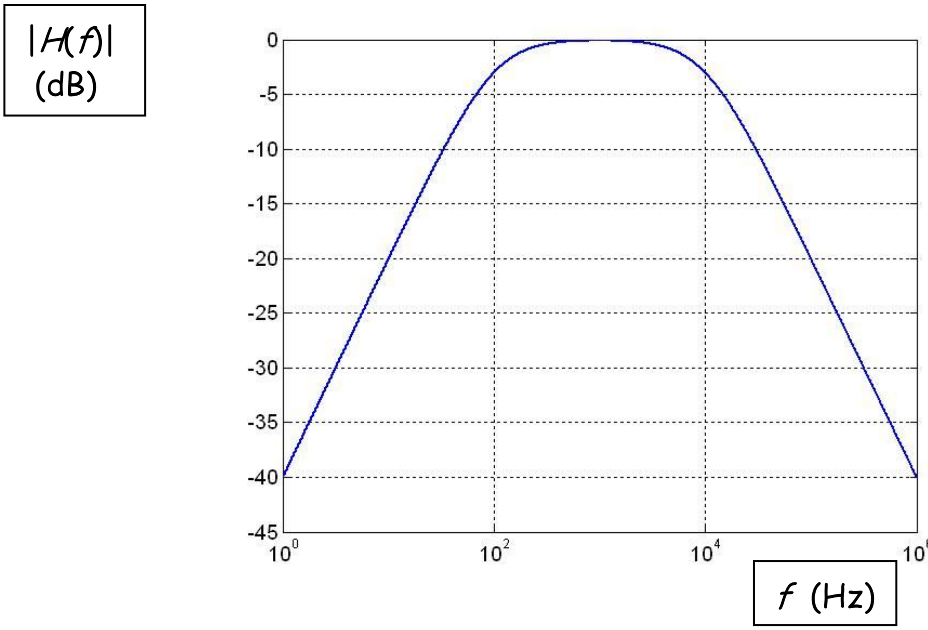
Then, the transfer function is the product of two voltage division ratios:

$$H(f) = \frac{Z}{Z + 1/j2\pi f C_1} \times \frac{1/j2\pi f C_2}{R_2 + 1/j2\pi f C_2} = \frac{Z}{Z + 1/j2\pi f C_1} \times \frac{1}{1 + j2\pi f C_2 R_2}$$

A MATLAB program to produce the desired Bode plot is:

```
R1=1e3;
R2=100e3;
C1=1.592E-6;
C2=159.2E-12;
f=logspace(0,6,700);
Z=1./(1/R1 +1./(R2 +1./(i*2*pi*f*C2)));
H=Z./(Z+1./(i*2*pi*f*C1))*1./(1+i*2*pi*f*C2*R2);
semilogx(f,20*log10(abs(H)))
```

The resulting Bode plot is:



P6.94 (a and b) R_1 and C_1 form the lowpass circuit for which we want a break frequency of 10 kHz. Thus, we have $R_1 = 1/2\pi C_1(10^4) = 159.2\ \Omega$.

R_2 and C_2 form the highpass circuit for which we want a break frequency of 100 Hz. Thus, we have $R_2 = 1/2\pi C_2(100) = 1.592\ \text{M}\Omega$.

(c) First, we write this expression for the impedance in series with R_1:

$$Z = \frac{1}{\dfrac{1}{1/j\omega C_1} + \dfrac{1}{R_2 + 1/j2\pi f C_2}}$$

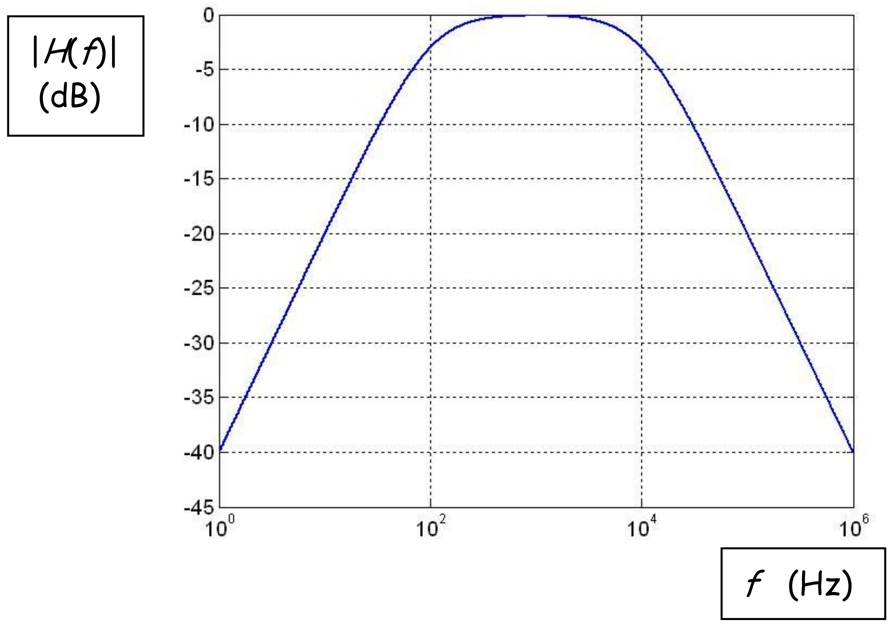
Then, the transfer function is the product of two voltage-division ratios:

$$H(f) = \frac{Z}{Z+R_1} \times \frac{R_2}{R_2 + 1/j2\pi f C_2} = \frac{Z}{Z+R_1} \times \frac{j2\pi f C_2 R_2}{1 + j2\pi f C_2 R_2}$$

A MATLAB program to produce the desired Bode plot is:

```
C1=1e-7;
C2=1e-9;
R1=159.2;
R2=1.592e6;
f=logspace(0,6,600);
Z=1./(i*2*pi*f*C1 +1./(R2 +1./(i*2*pi*f*C2)));
H=(Z./(Z+R1)).*( i*2*pi*f*C2*R2./(1+i*2*pi*f*C2*R2));
semilogx(f,20*log10(abs(H)))
```

The resulting Bode plot is:



P6.95 (a) The impedance of the circuit is given by

$$Z(j\omega) = \frac{1}{1/R - j/(\omega L)} - j\frac{1}{\omega C} = \frac{1/R + j/(\omega L)}{1/R^2 + 1/(\omega^2 L^2)} - j\frac{1}{\omega C}$$

At the resonant frequency, we set the imaginary part equal to zero.

$$\frac{1/(\omega_0 L)}{1/R^2 + 1/(\omega_0^2 L^2)} - \frac{1}{\omega_0 C} = 0$$

from which we obtain

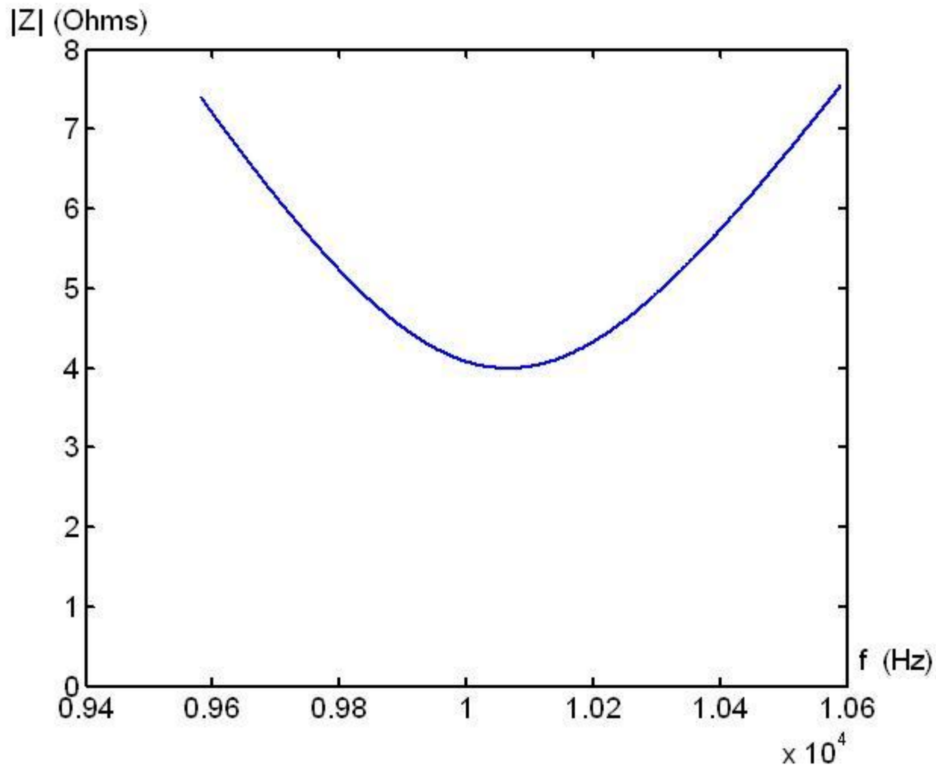
$$\omega_0 = \sqrt{\frac{1}{LC - L^2/R^2}} \quad \text{or } f_0 = \frac{1}{2\pi}\sqrt{\frac{1}{LC - L^2/R^2}}$$

(b) $f_0 = 10.09 \text{ kHz}$

(c) A MATLAB program to plot the impedance magnitude is

```
R = 1000;
L = 1e-3;
C = 0.25e-6;
f0 = (1/(2*pi))*sqrt(1./(L*C - L^2/R^2));
f = 0.95*f0:1:1.05*f0;
Z = (1./(1/R - i./(2*pi*f*L)) - i./(2*pi*f*C));
plot(f,abs(Z))
```

The resulting plot is:



P6.96 (a) The impedance of the circuit is given by

$$Z(j\omega) = \frac{1}{j\omega C} j\omega L + \frac{1}{1/R + 1/(j\omega L)}$$

A MATLAB program to plot the impedance magnitude is

```
R = 1000;
L = 1e-3;
C = 0.25e-6;
f = 9000:1:11000;
Z = (1./(i*2*pi*f*C) + 1./(1/R +1./( i*2*pi*f*L)) );
plot(f,abs(Z))
hold
```

(b) Looking through the values for abs(Z) computed by MATLAB, we find that the minimum impedance is 3.992 Ω which occurs at a frequency of 10,066 Hz. The bandwidth is 637 Hz.

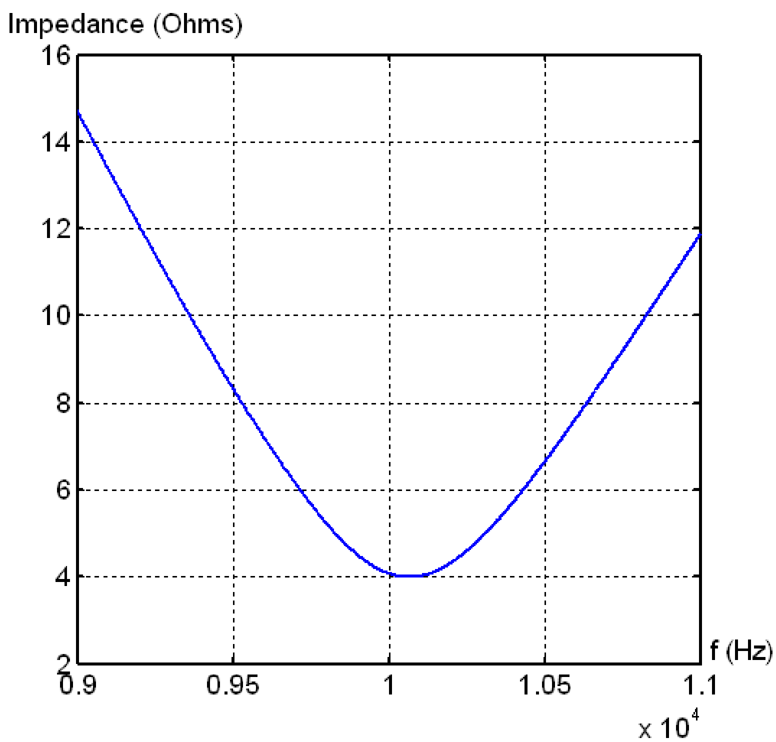
(c) For a series circuit with these parameters, the resistance equals the minimum impedance $R_s = 3.992\ \Omega$, the resonant frequency is the frequency of minimum impedance $f_0 = 10{,}066$ Hz, and the quality factor is the resonant frequency divided by the bandwidth $Q = f_0 / B =$ 10066/637 $=$ 15.80. Then, we can compute the inductance and capacitance

$$L_s = \frac{RQ}{2\pi f_0} = 0.9974 \text{ mH} \qquad C_s = \frac{1}{QR(2\pi f_0)} = 0.2506 \ \mu\text{F}$$

(d) After executing the program of part (a) if we use the following commands, we get the plot for the series circuit.

```
Ls = 0.9974e-3;
Cs = 0.2506e-6;
Rs = 3.992;
Zs = Rs + i*2*pi*f*Ls + 1./(i*2*pi*f*Cs);
plot(f,abs(Zs))
```

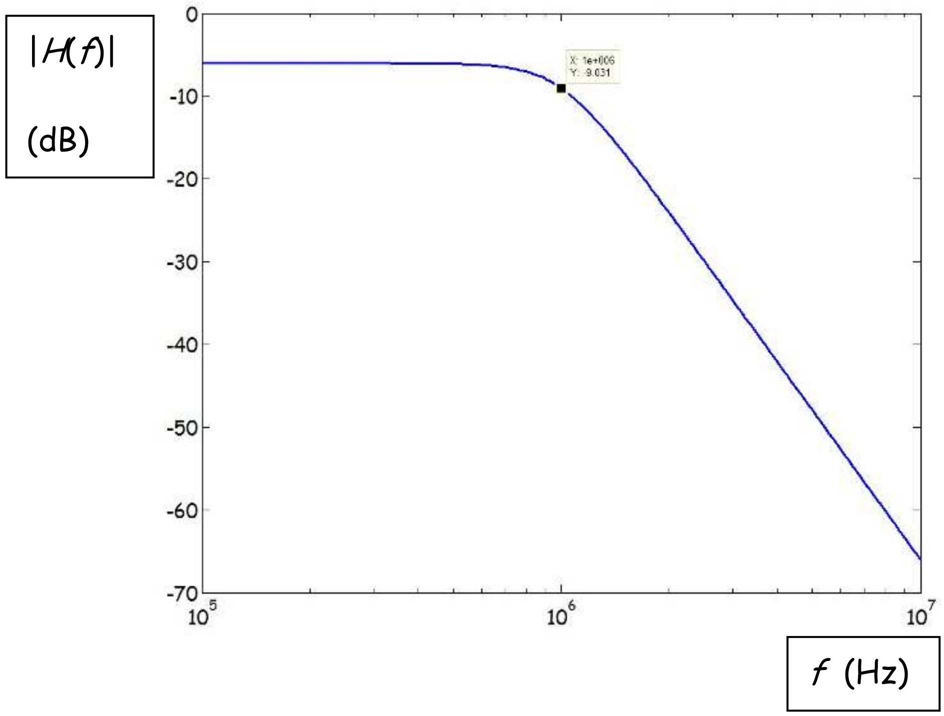
The impedances have virtually the same plot:



P6.97 The MATLAB commands are:

```
% Construct the symbolic objects that appear in the circuit:
syms V1 V2
syms w Rs RL C1 C2 L real
% Notice that V1 and V2 are complex quantities
% while w, Rs, etc. are real.
% Solve the node voltage equations for V1, and V2:
T = solve('(V1-1)/Rs + i*w*C1*V1 + (V1-V2)/(i*w*L) = 0',...
        '(V2-V1)/(i*w*L) + i*w*C2*V2 + V2/RL = 0',...
        'V1','V2');
% Enter the component values:
C1 = 3.1831e-9; C2 = 3.1831e-9;
L = 15.915e-6; Rs = 50; RL = 50;
% Substitute the component values into the solution for V2
% and define result as the transfer function H:
H = subs(T.V2);
% Set up a row matrix of logarithmically equally spaced
% frequencies at 100 points per decade from 10^5 to 10^7 Hz:
f = logspace(5,7,201);
wn = 2*pi*f;
% Substitute the frequency values into the transfer function
% and convert to numeric form by using the double command:
H = double(subs(H,w,wn));
% Convert the transfer function magnitude to dB and plot:
HmagdB = 20*log10(abs(H));
semilogx(f,HmagdB)
```

The resulting plot is:



From the plot, we determine that the half-power frequency is 1 MHz.

To check the plotted transfer function at very low frequencies, we replace the inductances by shorts and the capacitors with opens. Then the circuit becomes a simple voltage divider and the transfer function is

$$H(0) = \frac{R_L}{R_s + R_L} = 0.5$$

which is equivalent to -6 dB, agreeing very well with the value plotted at 100 kHz.

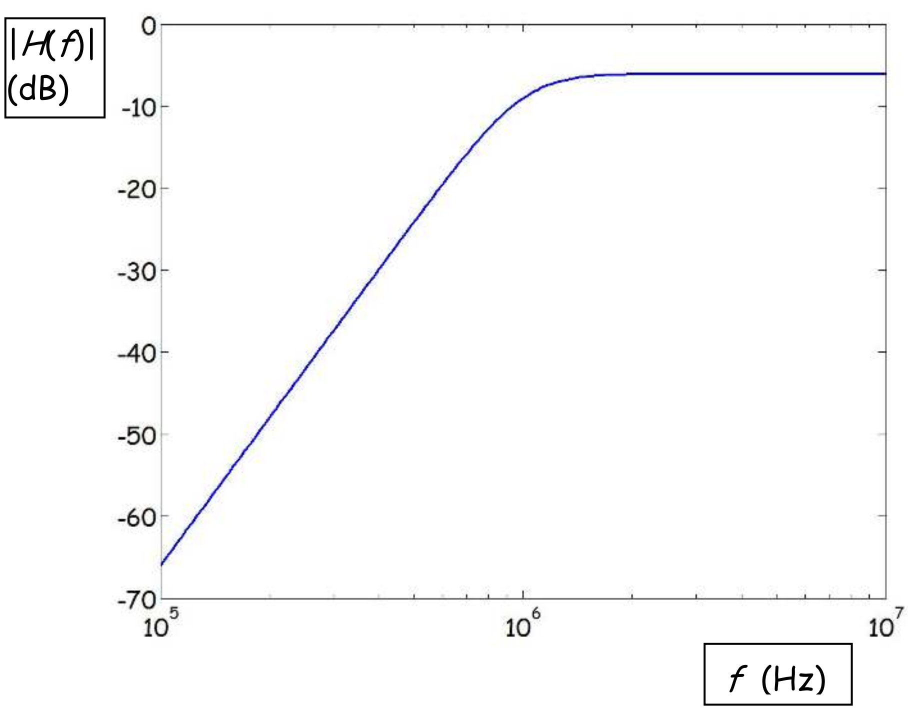
At very high frequencies, the capacitors become shorts and the inductors become opens. Then, the output voltage tends toward zero, and the transfer function tends toward $-\infty$ dB. This agrees with the trend of the plot at high frequencies.

P6.98 The MATLAB commands are:

```
% Construct the symbolic objects that appear in the circuit:
syms V1 V2
syms w Rs RL L1 L2 C real
% Notice that V1 and V2 are complex quantities
% while w, Rs, etc. are real.
% Solve the node voltage equations for V1, and V2:
T = solve('(V1-1)/Rs + V1/(i*w*L1) + (V1-V2)*i*w*C = 0',...
```

```
        '(V2-V1)*i*w*C + V2/(i*w*L2) + V2/RL = 0',...
        'V1','V2');
% Enter the component values:
L1 = 7.9577e-6; L2 = 7.9577e-6;
C=1.5915e-9; Rs = 50; RL = 50;
% Substitute the component values into the solution for V2
% and define result as the transfer function H:
H = subs(T.V2);
% Set up a row matrix of logarithmically equally spaced
% frequencies at 100 points per decade from 10^5 to 10^7 Hz:
f = logspace(5,7,201);
wn = 2*pi*f;
% Substitute the frequency values into the transfer function
% and convert to numeric form by using the double command:
H = double(subs(H,w,wn));
% Convert the transfer function magnitude to dB and plot:
HmagdB = 20*log10(abs(H));
semilogx(f,HmagdB)
```

The resulting plot is:



To check the plotted transfer function at very high frequencies, we replace the capacitance by a short and the inductances with opens. Then the circuit becomes a simple voltage divider and the transfer function is

$$H(0) = \frac{R_L}{R_s + R_L} = 0.5$$

which is equivalent to –6 dB, agreeing very well with the value plotted at 10 MHz.

At very low frequencies, the inductors become shorts and the capacitance become an open. Then, the output voltage tends toward zero, and the transfer function tends toward $-\infty$ dB. This agrees with the trend of the plot at low frequencies.

P6.99 (a) The admittance of the circuit is given by

$$Y(j\omega) = \frac{1}{R + j\omega L} + j\omega C = \frac{R - j\omega L}{R^2 + \omega^2 L^2} + j\omega C$$

At the resonant frequency, we set the imaginary part equal to zero.

$$\frac{-\omega_0 L}{R^2 + \omega_0^2 L^2} + \omega_0 C = 0$$

from which we obtain

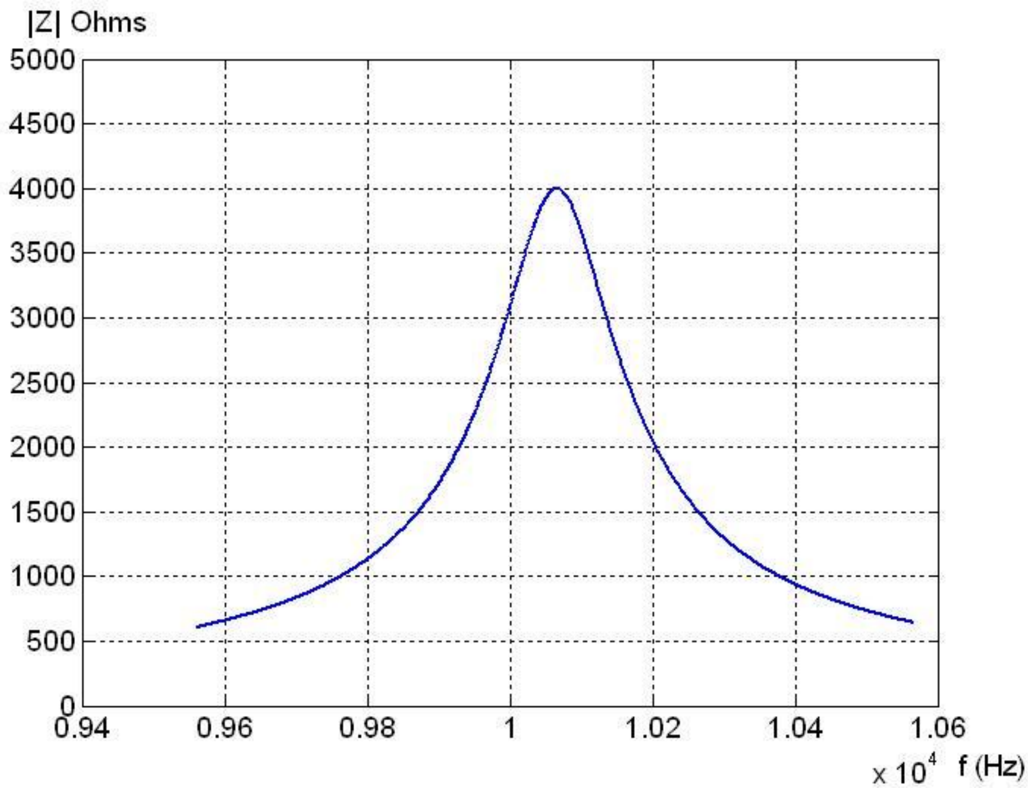
$$\omega_0 = \sqrt{\frac{L - R^2 C}{L^2 C}} \quad \text{or} \quad f_0 = \frac{1}{2\pi}\sqrt{\frac{L - R^2 C}{L^2 C}}$$

(b) $f_0 = 10.06\ \text{kHz}$

(c) A MATLAB program to plot the impedance magnitude is

```
R = 1;
L = 1e-3;
C = 0.25e-6;
f0 = (1/(2*pi))*sqrt((L - C*R^2)/(C*L^2));
f = 0.95*f0:1:1.05*f0;
Y = 1./(R + i*2*pi*f*L) + i*2*pi*f*C;
Z = 1./Y;
plot(f,abs(Z))
```

The resulting plot is:



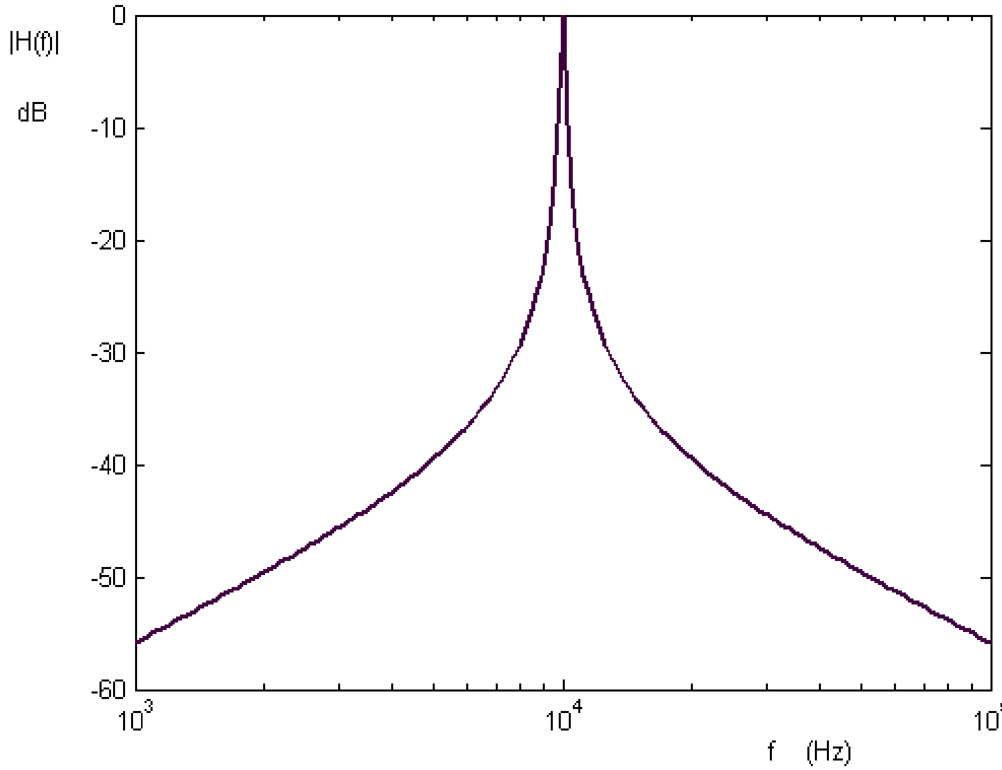
P6.100 (a) Applying the voltage-division principle, we have

$$H(f) = \frac{R}{R + j2\pi fL - j/(2\pi fC)}$$

(b) A MATLAB program to produce the desired plot is

```
R = 10;
L = 0.01;
C = 2.533e-8;
f = logspace(3,5,2000);
w = 2*pi*f;
H = R./(R+j*w*L + 1./(j*w*C ));
semilogx(f,20*log10(abs(H)))
```

The resulting plot is



(c) At very low frequencies, with the capacitance considered to be an open circuit, no current flows and $H(f)$ becomes very small in magnitude as shown in the plot.
(d) At very high frequencies with the inductance considered as an open circuit, no current flows and $H(f)$ becomes very small in magnitude as shown in the plot.

P6.101 (a) Applying the voltage-division principle, we have

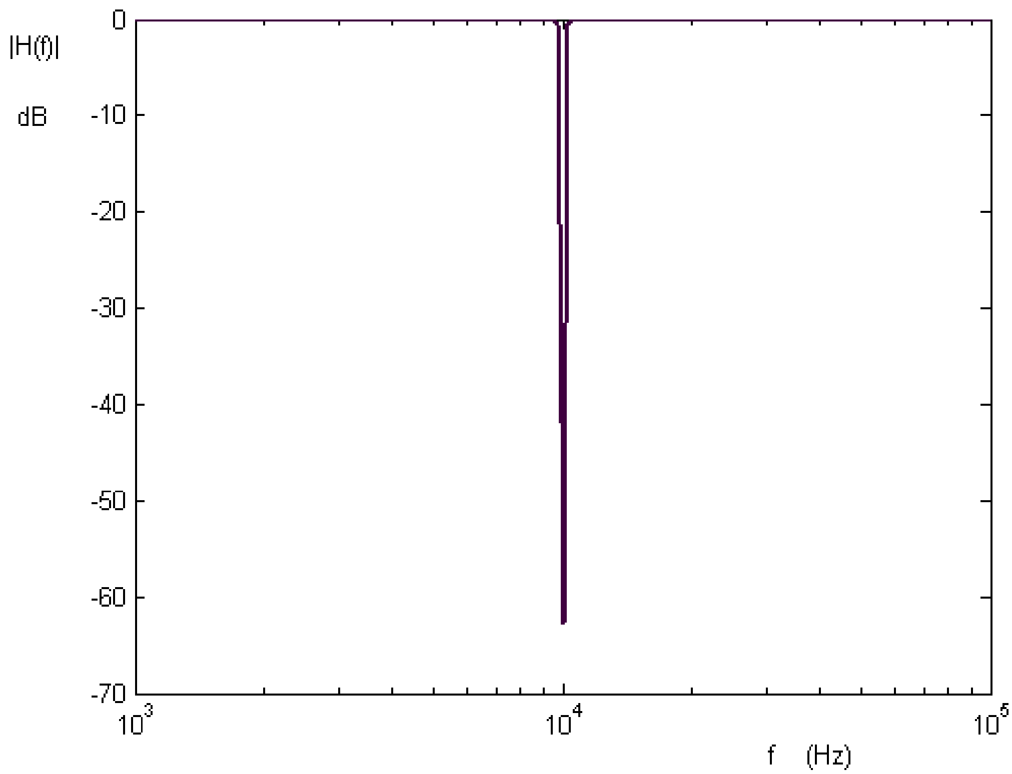
$$H(f) = \frac{j2\pi fL - j/(2\pi fC)}{R + j2\pi fL - j/(2\pi fC)}$$

(b) A MATLAB program to produce the desired plot is

```
R = 10
L = 0.01
C = 2.533e-8
logf = 3:0.01:5;
f = 10.^logf;
w = 2*pi*f;
H = (j*w*L + 1./(j*w*C )).(R+j*w*L + 1./(j*w*C ));
```

```
semilogx(f,20*log10(abs(H)))
```

The resulting plot is



(c) At very low frequencies, with the capacitance considered to be an open circuit, no current flows and $H(f) \cong 1$ which is equivalent to 0 dB as shown in the plot.

(d) At very high frequencies with the inductance considered as an open circuit, no current flows and $H(f) \cong 1$ which is equivalent to 0 dB as shown in the plot.

P6.102 (a) Writing a current equation at the node joining the inductance and resistance, we have

$$\frac{1}{L}\int_0^t [y(t) - x(t)]dt + i_L(0) + \frac{y(t)}{R} = 0$$

Taking the derivative with respect to time we have

$$\frac{1}{L}y(t) - \frac{1}{L}x(t) + \frac{1}{R}\frac{dy(t)}{dt} = 0$$

Multiplying each term by L and using the fact that the time constant is $\tau = L/R$, we obtain

$$y(t) - x(t) + \tau \frac{dy(t)}{dt} = 0$$

Then we approximate the derivative and write the following approximation to the differential equation.

$$y(n) - x(n) + \tau \frac{y(n) - y(n-1)}{T} = 0$$

Solving for $y(n)$, we obtain the equation for the digital filter:

$$y(n) = \frac{\tau / T}{1 + \tau / T} y(n-1) + \frac{1}{1 + \tau / T} x(n)$$

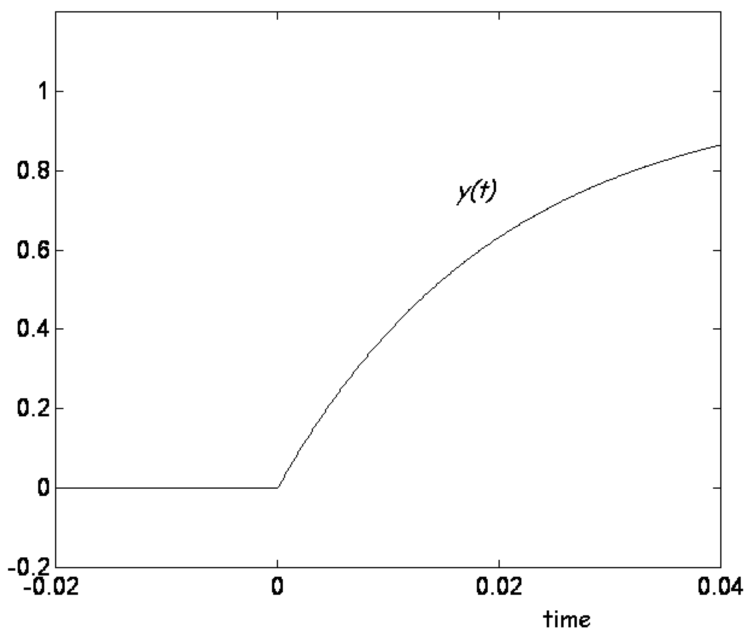
(b) For the values given the time constant is $\tau = L/R = 20$ ms. The step input is

$$\begin{aligned} x(t) &= 0 \quad \text{for } t < 0 \\ &= 1 \quad \text{for } t \geq 0 \end{aligned}$$

Using the methods of Chapter 4, we have

$$\begin{aligned} y(t) &= 0 \quad \text{for } t < 0 \\ &= 1 - \exp(t/\tau) \quad \text{for } t \geq 0 \end{aligned}$$

A plot of $y(t)$ versus t is:



(c) The sampling interval is T = 1/500 = 2 ms, and we have τ/T = 10. Thus the defining equation for the digital filter is:

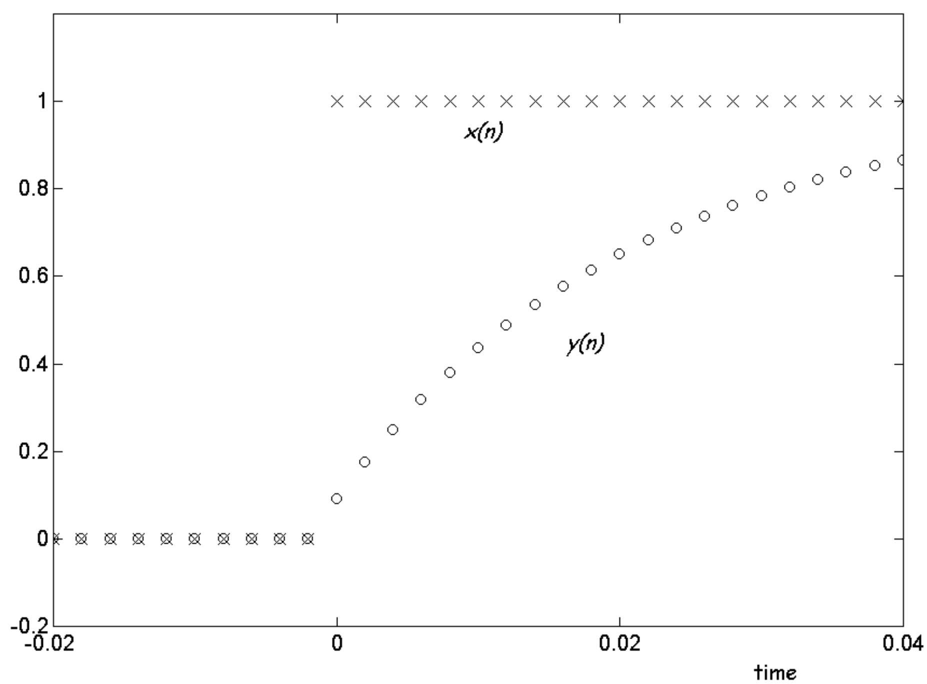
$$y(n) = \frac{10}{11} y(n-1) + \frac{1}{11} x(n)$$

The step input to the digital filter is defined as

$$x(n) = 0 \quad \text{for } n < 0$$
$$= 1 \text{ for } n \geq 0$$

A list of MATLAB commands to compute and plot x and y is:

```
t = -20e-3:2e-3:60e-3;
x = ones(size(t));
for n = 1:10
x(n) = 0;
end
y =zeros(size(t))
for n = 2:31
y(n) = (10/11)*y(n-1) +(1/11)*x(n);
end
plot(t,y,'wo')
hold
plot(t,x, 'wx')
axis([-20e-3 40e-3 -0.2 1.2])
```



P6.103 (a) Writing a current equation at the node joining the inductance and resistance, we have

$$\frac{1}{L}\int_0^t y(t)dt + i_L(0) + \frac{y(t) - x(t)}{R} = 0$$

Taking the derivative with respect to time we have

$$\frac{1}{L}y(t)+\frac{1}{R}\frac{dy(t)}{dt}-\frac{1}{R}\frac{dx(t)}{dt}=0$$

Multiplying each term by L and using the fact that the time constant is τ = L/R, we obtain

$$y(t)+\tau\frac{dy(t)}{dt}-\tau\frac{dx(t)}{dt}=0$$

Then we approximate the derivatives and write the following approximation to the differential equation.

$$y(n)+\tau\frac{y(n)-y(n-1)}{T}-\tau\frac{x(n)-x(n-1)}{T}=0$$

Solving for $y(n)$, we obtain the equation for the digital filter:

$$y(n)=\frac{\tau/T}{1+\tau/T}y(n-1)+\frac{\tau/T}{1+\tau/T}[x(n)-x(n-1)]$$

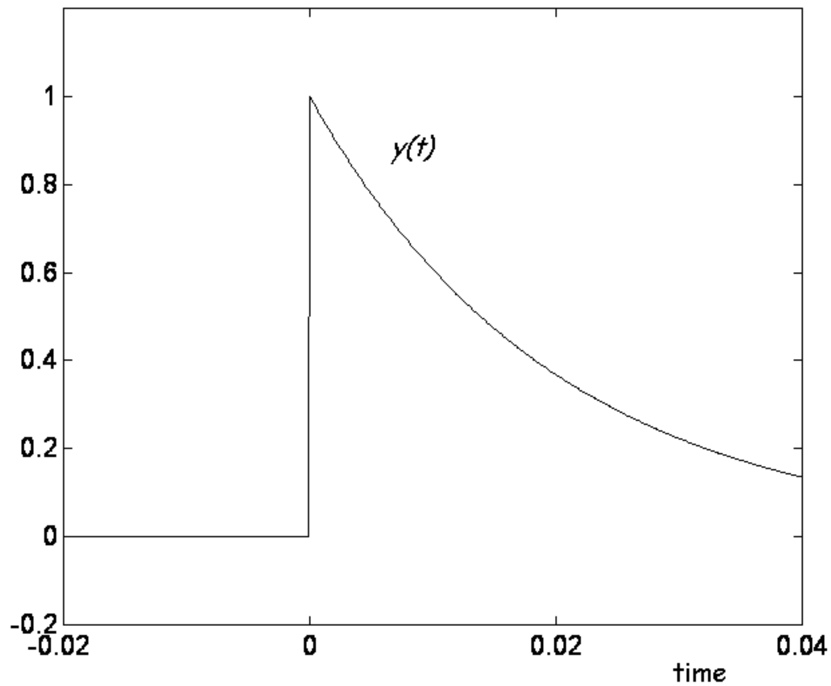
(b) For the values given the time constant is $\tau = L/R = 20$ ms. The step input is

$$\begin{aligned} x(t) &= 0 \quad \text{for } t < 0 \\ &= 1 \quad \text{for } t \geq 0 \end{aligned}$$

Using the methods of Chapter 4, we have

$$\begin{aligned} y(t) &= 0 \quad \text{for } t < 0 \\ &= \exp(t/\tau) \quad \text{for } t \geq 0 \end{aligned}$$

A plot of $y(t)$ versus t is:



(c) The sampling interval is T = 1/500 = 2 ms, and we have $\tau/T = 10$. Thus the defining equation for the digital filter is:

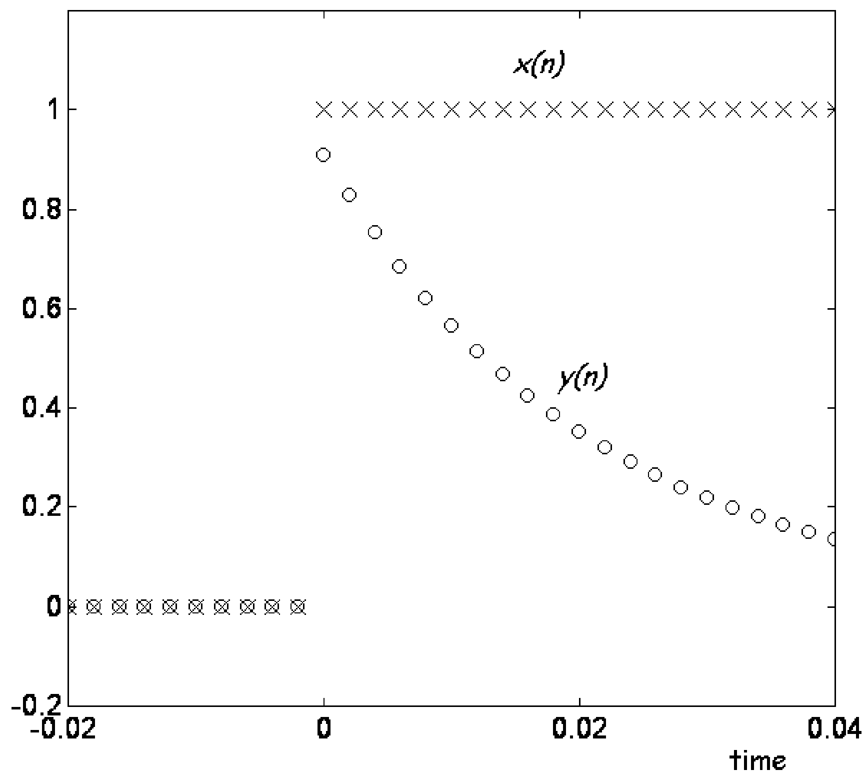
$$y(n) = \frac{10}{11} y(n-1) + \frac{10}{11}[x(n) - x(n-1)]$$

The step input to the digital filter is defined as

$$x(n) = 0 \quad \text{for } n < 0$$
$$= 1 \text{ for } n \geq 0$$

A list of MATLAB commands to compute and plot x and y is:

```
t = -20e-3:2e-3:60e-3;
x = ones(size(t));
for n = 1:10
x(n) = 0;
end
y =zeros(size(t))
for n = 2:31
y(n) = (10/11)*y(n-1) +(10/11)*(x(n) - x(n - 1));
end
plot(t, y, 'wo')
hold
plot(t, x, 'wx')
axis([-20e-3 40e-3 -0.2 1.2])
```



P6.104* (a) Refer to Figure P6.104 in the book. This is a series *RLC* circuit. Equations 6.30 and 6.31 state

$$Q_s = \frac{\omega_0 L}{R} \quad \text{and} \quad \omega_0^2 = \frac{1}{LC}$$

Substituting $R = 1$ and solving, we have

$$L = \frac{Q_s}{\omega_0} \quad \text{and} \quad C = \frac{1}{\omega_0 Q_s}$$

(b) Using the fact that $i(t) = y(t)$, and writing a KVL equation for the circuit, we have

$$x(t) = L\frac{dy(t)}{dt} + \frac{1}{C}\int_0^t y(t)dt + v_C(0) + y(t)$$

in which $v_C(0)$ is the initial capacitor voltage. Taking the derivative of each term with respect to time and using the results of part (a) to substitute for L and C, we obtain

$$\frac{dx(t)}{dt} = \frac{Q_s}{\omega_0}\frac{d^2y(t)}{dt^2} + \omega_0 Q_s y(t) + \frac{dy(t)}{dt}$$

Next approximate the derivatives as

$$\frac{dx(t)}{dt} \cong \frac{x(n)-x(n-1)}{T} \qquad \frac{dy(t)}{dt} \cong \frac{y(n)-y(n-1)}{T}$$

$$\frac{d^2y(t)}{dt^2} \cong \frac{\dfrac{y(n)-y(n-1)}{T} - \dfrac{y(n-1)-y(n-2)}{T}}{T}$$

$$= \frac{y(n)-2y(n-1)+y(n-2)}{T^2}$$

This yields the discrete equation

$$\frac{x(n)-x(n-1)}{T} = \frac{Q_s}{\omega_0}\frac{y(n)-2y(n-1)+y(n-2)}{T^2}$$
$$+\omega_0 Q_s y(n) + \frac{y(n)-y(n-1)}{T}$$

Solving for $y(n)$ yields the equation for the equivalent digital filter.

$$y(n) = \frac{\omega_0 T + 2Q_s}{Q_s + \omega_0^2 T^2 Q_s + \omega_0 T} y(n-1) - \frac{Q_s}{Q_s + \omega_0^2 T^2 Q_s + \omega_0 T} y(n-2)$$

$$+ \frac{\omega_0 T}{Q_s + \omega_0^2 T^2 Q_s + \omega_0 T}[x(n) - x(n-1)]$$

(Note: In case you try to simulate this filter, the coefficients must be calculated to a very high degree of accuracy to attain behavior comparable to that of the *RLC* circuit.)

Practice Test

T6.1 All real-world signals (which are usually time-varying currents or voltages) are sums of sinewaves of various frequencies, amplitudes, and phases. The transfer function of a filter is a function of frequency that shows how the amplitudes and phases of the input components are altered to produce the output components.

T6.2 Applying the voltage-division principle, we have:

$$H(f) = \frac{\mathbf{V}_{out}}{\mathbf{V}_{in}} = \frac{j2\pi fL}{R + j2\pi fL} = \frac{j2\pi fL/R}{1 + j2\pi fL/R}$$

$$= \frac{j(f/f_B)}{1 + j(f/f_B)}$$

in which $f_B = R/2\pi L = 1000$ Hz. The input signal has components with frequencies of 0 (dc), 500 Hz, and 1000 Hz. The transfer function values for these frequencies are: $H(0) = 0$, $H(500) = 0.4472\angle 63.43°$, and $H(1000) = 0.7071\angle 45°$. Applying the transfer function values to each of the input components, we have $H(0) \times 3 = 0$, $H(500) \times 4\angle 0° = 1.789\angle 63.43°$, and $H(1000) \times 5\angle -30° = 3.535\angle 15°$. Thus, the output is

$$v_{out}(t) = 1.789\cos(1000\pi t - 63.43°) + 3.535\cos(2000\pi t + 15°)$$

T6.3 (a) The slope of the low-frequency asymptote is +20 dB/decade.
(b) The slope of the high-frequency asymptote is zero.
(c) The coordinates at which the asymptotes meet are 20log(50) = 34 dB and 200 Hz.
(d) This is a first-order highpass filter.
(e) The break frequency is 200 Hz.

T6.4 (a) $f_0 = \dfrac{1}{2\pi\sqrt{LC}} = 1125\text{ Hz}$

(b) $Q_s = \dfrac{2\pi f_0 L}{R} = 28.28$

(c) $B = \dfrac{f_0}{Q_s} = 39.79\text{ Hz}$

(d) At resonance, the impedance equals the resistance, which is $5\,\Omega$.

(e) At dc, the capacitance becomes an open circuit so the impedance is infinite.

(f) At infinite frequency the inductance becomes an open circuit, so the impedance is infinite.

T6.5 (a) $f_0 = \dfrac{1}{2\pi\sqrt{LC}} = 159.2\text{ kHz}$

(b) $Q_p = \dfrac{R}{2\pi f_0 L} = 10.00$

(c) $B = \dfrac{f_0}{Q_p} = 15.92\text{ kHz}$

(d) At resonance, the impedance equals the resistance which is $10\text{ k}\Omega$.

(e) At dc, the inductance becomes a short circuit, so the impedance is zero.

(f) At infinite frequency the capacitance becomes a short circuit, so the impedance is zero.

T6.6 (a) This is a first-order circuit because there is a single energy-storage element (L or C). At very low frequencies, the capacitance approaches an open circuit, the current is zero, $\mathbf{V}_{out} = \mathbf{V}_{in}$ and $|H| = 1$. At very high frequencies, the capacitance approaches a short circuit, $\mathbf{V}_{out} = 0$, and $|H| = 0$. Thus, we have a first-order lowpass filter.

(b) This is a second-order circuit because there are two energy-storage elements (L or C). At very low frequencies, the capacitance approaches an open circuit, the inductance approaches a short circuit, the current is zero, $\mathbf{V}_{out} = \mathbf{V}_{in}$ and $|H| = 1$. At very high frequencies, the inductance approaches an open circuit, the capacitance approaches a short circuit, $\mathbf{V}_{out} = 0$, and $|H| = 0$. Thus we have a second-order lowpass filter.

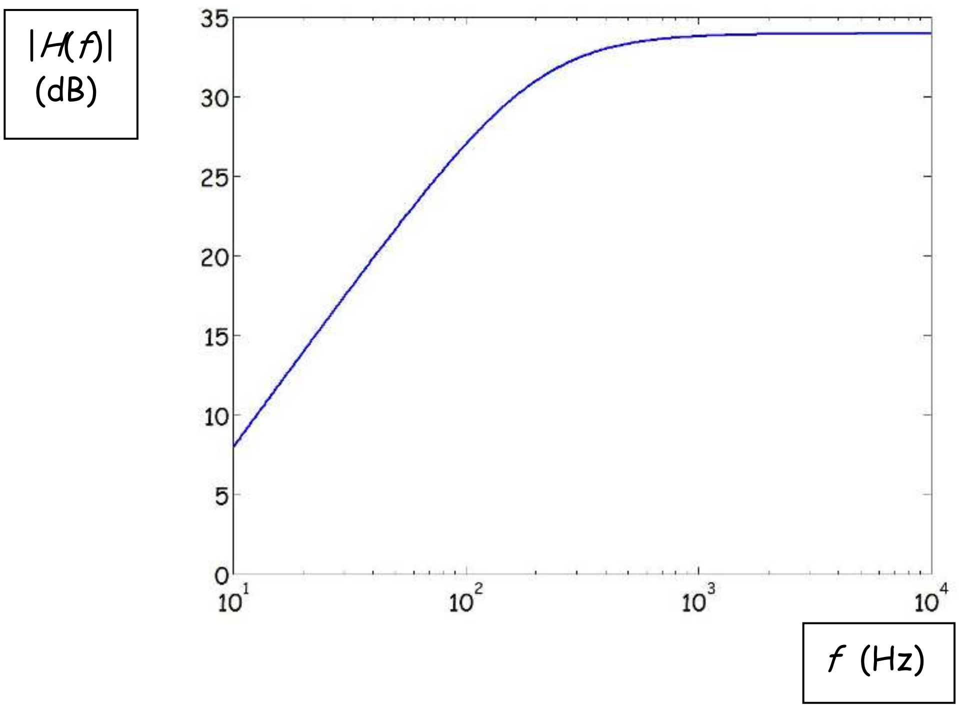
(c) This is a second-order circuit because there are two energy-storage elements (L or C). At very low frequencies, the inductance approaches a short circuit, $\mathbf{V}_{out} = \mathbf{V}_{in}$ and $|H| = 1$. At very high frequencies, the capacitance approaches a short circuit, $\mathbf{V}_{out} = \mathbf{V}_{in}$ and $|H| = 1$. At the resonant frequency, the LC combination becomes an open circuit, the current is zero, $\mathbf{V}_{out} = 0$, and $|H| = 0$. Thus, we have a second-order band-reject (or notch) filter.

(d) This is a first-order circuit because there is a single energy-storage element (L or C). At very low frequencies, the inductance approaches a short circuit, $\mathbf{V}_{out} = 0$, and $|H| = 0$. At very high frequencies the inductance approaches an open circuit, the current is zero, $\mathbf{V}_{out} = \mathbf{V}_{in}$ and $|H| = 1$. Thus we have a first-order highpass filter.

T6.7 One set of commands is:

```
f = logspace(1,4,400);
H = 50*i*(f/200)./(1+i*f/200);
semilogx(f,20*log10(abs(H)))
```

Other sets of commands are also correct. You can use MATLAB to see if your commands give a plot equivalent to:



P6.49 For filters in cascade, the transfer functions in decibels are added.

$$|H(f_1)|_{db} = |H_1(f_1)|_{db} + |H_2(f_1)|_{db}$$

$$20log|H(f_1)| = 40$$

$$log|H(f_1)| = 2$$

$$|H(f_1)| = 10^2 = 100$$

CHAPTER 7

Exercises

E7.1 (a) For the whole part, we have:

	Quotient	Remainders
23/2	11	1
11/2	5	1
5/2	2	1
2/2	1	0
1/2	0	1

Reading the remainders in reverse order, we obtain:

$23_{10} = 10111_2$

For the fractional part, we have

$2 \times 0.75 = 1 + 0.5$

$2 \times 0.50 = 1 + 0$

Thus, we have

$0.75_{10} = 0.110000_2$

Finally, the answer is $23.75_{10} = 10111.11_2$

(b) For the whole part, we have:

	Quotient	Remainders
17/2	8	1
8/2	4	0
4/2	2	0
2/2	1	0
1/2	0	1

Reading the remainders in reverse order, we obtain:

$17_{10} = 10001_2$

For the fractional part, we have

$2 \times 0.25 = 0 + 0.5$

$2 \times 0.50 = 1 + 0$

Thus, we have

$0.25_{10} = 0.010000_2$

Finally, the answer is $17.25_{10} = 10001.01_2$

(c) For the whole part we have:

	Quotient	Remainders
4/2	2	0
2/2	1	0
1/2	0	1

Reading the remainders in reverse order we obtain:

$4_{10} = 100_2$

For the fractional part, we have

$2 \times 0.30 = 0 + 0.6$

$2 \times 0.60 = 1 + 0.2$

$2 \times 0.20 = 0 + 0.4$

$2 \times 0.40 = 0 + 0.8$

$2 \times 0.80 = 1 + 0.6$

$2 \times 0.60 = 1 + 0.2$

Thus we have

$0.30_{10} = 0.010011_2$

Finally, the answer is $4.3_{10} = 100.010011_2$

E7.2 (a) $1101.111_2 = 1\times2^3 + 1\times2^2 + 0\times2^1 + 1\times2^0 + 1\times2^{-1} + 1\times2^{-2} + 1\times2^{-3} = 13.875_{10}$

(b) $100.001_2 = 1\times2^2 + 0\times2^1 + 0\times2^0 + 0\times2^{-1} + 0\times2^{-2} + 1\times2^{-3} = 4.125_{10}$

E7.3 (a) Using the procedure of Exercise 7.1, we have

$97_{10} = 1100001_2$

Then adding two leading zeros and forming groups of three bits we have

$001\ 100\ 001_2 = 141_8$

Adding a leading zero and forming groups of four bits we obtain

$0110\ 0001 = 61_{16}$

(b) Similarly

$229_{10} = 11100101_2 = 345_8 = E5_{16}$

E7.4 (a) $72_8 = 111\ 010 = 111010_2$

(b) $FA6_{16} = 1111\ 1010\ 0110 = 111110100110_2$

E7.5 $197_{10} = 0001\ 1001\ 0111 = 000110010111_{BCD}$

E7.6 To represent a distance of 20 inches with a resolution of 0.01 inches, we need 20/0.01 = 2000 code words. The number of code words in a Gray code is 2^L in which L is the length of the code words. Thus we need L = 11, which produces 2048 code words.

E7.7 (a) First we convert to binary

$22_{10} = 16 + 4 + 2 = 10110_2$

Because an eight-bit representation is called for, we append three leading zeros. Thus +22 becomes

00010110

in two's complement notation.

(b) First we convert +30 to binary form

$30_{10} = 16 + 8 + 4 + 2 = 11110_2$

Attaching leading zeros to make an eight-bit result we have

$30_{10} = 00011110_2$

Then we take the ones complement and add 1 to find the two's complement:

one's complement:	11100001
add 1	+1
	11100010

Thus the eight-bit two's complement representation for -30_{10} is 11100010.

E7.8 First we convert 19_{10} and -4_{10} to eight-bit two's complement form then we add the results.

19	00010011
-4	111111100
15	00001111

Notice that we neglect the carry out of the left-most bit.

E7.9 See Tables 7.3 and 7.4 in the book.

E7.10 See Table 7.5 in the book.

E7.11 (a) To apply De Morgan's laws to the expression

$$AB + \bar{B}C$$

first we replace each logic variable by its inverse

$$\bar{A}\,\bar{B} + B\bar{C}$$

then we replace AND operations by OR operations and vice versa

$$(\bar{A} + \bar{B})(B + \bar{C})$$

finally we invert the entire expression so we have

$$D = AB + \bar{B}C = \overline{(\bar{A} + \bar{B})(B + \bar{C})}$$

(b) Following the steps of part (a) we have

$$\overline{[F(G + \bar{H}) + F\bar{G}]}$$

$$\overline{[\bar{F}(\bar{G} + H) + \bar{F}G]}$$

$$\overline{[(\bar{F} + \bar{G}H)(\bar{F} + G)]}$$

$$E = \overline{[F(G + \bar{H}) + F\bar{G}]} = [(\bar{F} + \bar{G}H)(\bar{F} + G)]$$

E7.12 For the AND gate we use De Morgan's laws to write

$$AB = \overline{(\bar{A} + \bar{B})}$$

See Figure 7.21 in the book for the logic diagrams.

E7.13 The truth table for the exclusive-OR operation is

A	B	$A \oplus B$
0	0	0
0	1	1
1	0	1
1	1	0

Focusing on the rows in which the result is 1, we can write the SOP expression

$$A \oplus B = \bar{A}B + A\bar{B}$$

The corresponding logic diagram is shown in Figure 7.25a in the book.

Focusing on the rows in which the result is 0, we can write the POS expression

$$A \oplus B = (A + B)(\bar{A} + \bar{B})$$

The corresponding logic diagram is shown in Figure 7.25b in the book.

E7.14 The truth table is shown in Table 7.7 in the book. Focusing on the rows in which the result is 1, we can write the SOP expression

$$A = \sum m(3, 6, 7, 8, 9, 12)$$
$$= \bar{F}\,\bar{D}GR + \bar{F}\,DG\bar{R} + \bar{F}DGR + F\bar{D}\,\bar{G}\,\bar{R} + F\bar{D}\,\bar{G}R + FD\bar{G}\,\bar{R}$$

Focusing on the rows in which the result is 0, we can write the POS expression

$$A = \prod M(0, 1, 2, 4, 5, 10, 11, 13, 14, 15)$$
$$= (F + D + G + R)(F + D + G + \bar{R})(F + D + \bar{G} + R)\cdots(\bar{F} + \bar{D} + \bar{G} + \bar{R})$$

E7.15 The Truth table is shown in Table 7.8.

E7.16 (a) $\bar{A}\,\bar{B}C\bar{D}$

(b) $\bar{A}\,\bar{B}\,\bar{C}\,D$

E7.17 See Figure 7.34 in the book.

E7.18 See Figure 7.35 in the book.

E7.19 Because S is high at $t = 0$, Q is high and remains so until R becomes high at $t = 3$. Q remains low until S becomes high at $t = 7$. Then Q remains high until R becomes high at $t = 11$.

E7.20 See Table 7.9.

E7.21 See Figure 7.49 in the book.

Problems

P7.1* 1. When noise is added to a digital signal, the logic levels can still be exactly determined, provided that the noise is not too large in amplitude. Usually, noise cannot be completely removed from an analog signal.

2. Component values in digital circuits do not need to be as precise as in analog circuits.

3. Very complex digital logic circuits (containing millions of components) can be economically produced. Analog circuits often call for large

capacitances and/or precise component values that are impossible to manufacture as large-scale integrated circuits.

P7.2 The noise margins for a logic circuit are defined as

$$NM_L = V_{IL} - V_{OL} \text{ and } NM_H = V_{OH} - V_{IH}$$

in which V_{IL} is the highest input voltage accepted as logic 0, V_{IH} is the lowest input voltage accepted as logic 1, V_{OL} is the highest logic-0 output voltage, and V_{OH} is the lowest logic-1 output voltage.

Large values for the noise margins are important so noise added to or subtracted from logic signals in the interconnections does not change the operation of the logic circuit.

P7.3 A bit is a binary symbol that can assume values of 0 or 1. A byte is a word consisting of eight bits. A nibble is a four-bit word.

P7.4 In positive logic, the logic value 1 is represented by a higher voltage range than the voltage range for logic 0. The reverse is true for negative logic.

P7.5 The noise margins for the logic circuits in use are

$$NM_L = V_{IL} - V_{OL} = 2.2 - 0.5 = 1.7 \text{ V}$$

$$NM_H = V_{OH} - V_{IH} = 4.7 - 3.7 = 1.0 \text{ V}$$

Noises in the range from -0.3 V to +0.6 V do not exceed the noise margins so the system is reliable.

Reference to Figure 7.3 in the book, shows that as long as the noise amplitude is in the range from $-NM_H$ to $+NM_L$ the logic levels will be interpreted properly at the receiving end. Thus, as long as the noise amplitudes are in the range from -1.0 V to +1.7 V, this system will be reliable.

P7.6 Parallel transmission is faster and is used for short distance data transfer, such as the internal data transfer in a computer, but requires more wires than serial transmission

Serial transmission is used mainly for long-distance digital communication systems, as it requires only one wire. However, it is slower than parallel transmission.

P7.7 Seven-bit words are needed to express the decimal integers 0 through 100 in binary form (because $2^7 = 128$).

Ten-bit words are needed to express the decimal integers 0 through 1000 in binary form (because $2^{10} = 1024$).

Twenty-bit words are needed to express the decimal integers 0 through 10^6 in binary form (because 2^{20} = 1048576).

P7.8
(a)* 5.625
(b)* 21.375
(c) 14.125
(d) 13.125
(e) 9.3125
(f) 15.75

P7.9
(a)* 1101.11 + 101.111 = 10011.101
(b) 1011.01 + 1010.11 = 10110.00
(c) 10101.101 + 1101.001 = 100010.110
(d) 1011000.1000 + 10001001.1001 = 11100010.0001

P7.10
(a)* $9.75_{10} = 1001.11_2 = 1001.01110101_{BCD}$

(b) $6.5_{10} = 0110.1_2 = 0110.0101_{BCD}$
(c) $11.75_{10} = 1011.11_2 = 00010001.01110101_{BCD}$
(d) $63.03125_{10} = 111111.00001_2 = 01100011.00000011000100100101_{BCD}$
(e) $67.375_{10} = 1000011.011_2 = 01100111.001101110101_{BCD}$

P7.11
(a)* $77.7_8 = 111111.111_2 = 3F.E_{16} = 63.875_{10}$
(b) $36.5_8 = 011110.101_2 = 1E.A_{16} = 30.625_{10}$
(c) $123.4_8 = 001010011.100_2 = 53.8_{16} = 83.5_{10}$
(d) $57.5_8 = 101111.101_2 = 2F.A_{16} = 47.625_{10}$

P7.12
(a)* $10010011.0101_{BCD} + 00110111.0001_{BCD} = 93.5_{10} + 37.1_{10} = 130.6_{10} = 000100110000.0110_{BCD}$

(b) $01010100.1000_{BCD} + 01001001.1001_{BCD} = 54.8_{10} + 49.9_{10} = 104.7_{10} = 000100000100.0111_{BCD}$

(c) $0111\ 1001\ 0110.0011_{BCD} + 0011\ 0101.1001_{BCD} = 796.3_{10} + 35.9_{10} = 832.2_{10} = 100000110010.0010_{BCD}$

P7.13
(a) Counting in decimal, 378 follows 377.
(b) Counting in octal, 400 follows 377.

(c) Counting in hexadecimal, 378 follows 377.

P7.14 (a)* $313.0625_{10} = 100111001.0001_2 = 471.04_8 = 139.1_{16}$

(b) $253.25_{10} = 11111101.01_2 = 375.20_8 = FD.4_{16}$

(c) $349.75_{10} = 101011101.11_2 = 535.6_8 = 15D.C_{16}$

(d) $835.25_{10} = 1101000011.01_2 = 1503.2_8 = 343.4_{16}$

(e) $212.5_{10} = 11010100.1_2 = 324.4_8 = D4.8_{16}$

P7.15 (a)* 75 = 01001011 (eight-bit signed two's complement)

(b)* -87 = 10101001

(c) 19 = 00010011

(d) -19 = 11101101

(e) -95 = 10100001

(f) 125 = 01111101

P7.16 (a)* $777.7_8 = 111111111.111_2 = 1FF.E_{16} = 511.875_{10}$

(b) $123.5_8 = 1010011.101_2 = 53.A_{16} = 83.625_{10}$

(c) $24.4_8 = 10100.100_2 = 14.8_{16} = 20.5_{10}$

(d) $644.2_8 = 110100100.0100_2 = 1A4.4_{16} = 420.25_{10}$

P7.17 (a) A 4-bit binary number can represent the decimal integers 0 through $2^4 - 1 = 15$.

(b) A 4-digit octal number can represent the decimal integers 0 through $8^4 - 1 = 4095$.

(c) A 4-digit hexadecimal number can represent the decimal integers 0 through $16^4 - 1 = 65535$.

P7.18* Gray code is a coding scheme in which each code word differs from its next in only one bit

An example comparison between 4-bit binary and gray code is shown below –

Binary Code - 0000,0001,0010,0011,0100,0101,0110,0111,1000,1001,
1010,1011,1100,1101,1110,1111

Gray Code - 0000,0001,0011,0010,0110,0111,0101,0100,1100,1101,
1111,1110,1010,1011,1001,1000

Gray codes are used to encode the angular position of rotating shafts. In such a case, the successive positions of the shaft differ in code by one bit only, allowing less erroneous results to be obtained. Change in position of the shaft is quickly reflected in the output, of which only one bit would need to change.

On the other hand, if binary code had been used for the same, a change from position 3 (0011) to position 4 (0100) would have entailed a 3-bit change. If the sensors for the three bits are not aligned, the code changes from 0011 to 0001 to 0000 and finally to 0100. The intermediate values are all incorrect

P7.19

(a) * $11101000 \Rightarrow \overbrace{00010111}^{\text{One's Complement}} \Rightarrow \overbrace{00011000}^{\text{Two's Complement}}$

(b) $00000000 \Rightarrow 11111111 \Rightarrow 00000000$

(c) $01010101 \Rightarrow 10101010 \Rightarrow 10101011$

(d) $01111100 \Rightarrow 10000111 \Rightarrow 10000100$

(e) $11000000 \Rightarrow 00111111 \Rightarrow 01000000$

P7.20

(a)* $EB4.5_{16} = 111010110100.0101_2 = 3764.3125_{10}$

(b)* $657.7_8 = 110101111.111_2 = 431.875_{10}$

(c) $DC.9_{16} = 11011100.1001_2 = 220.5625_{10}$

(d) $235.43_8 = 010011101.100011_2 = 157.546875_{10}$

(e) $BA.C_{16} = 10111010.1100_2 = 186.75_{10}$

P7.21

(a)*

$$\begin{array}{r} 43 \\ \underline{-45} \\ -2 \end{array} \qquad \begin{array}{r} 00101011 \\ \underline{-00101101} \end{array} \qquad \begin{array}{r} 00101011 \\ \underline{+11010011} \\ 11111110 \end{array}$$

(b)

$$\begin{array}{r} 27 \\ \underline{+15} \\ 42 \end{array} \qquad \begin{array}{r} 00011011 \\ \underline{00001111} \\ 00101010 \end{array}$$

(c)

$$\begin{array}{r} 34 \\ \underline{-45} \\ -11 \end{array} \qquad \begin{array}{r} 00100010 \\ \underline{-00101101} \end{array} \qquad \begin{array}{r} 00100010 \\ \underline{+11010011} \\ 11110101 \end{array}$$

(d)

```
 25      00011001      00011001
-39     -00100111     +11011001
---     ---------     ---------
-14                    11110010
```

(e)

```
 59      00111011      00111011
-34     -00100010     +11011110
---     ---------     ---------
 25                 (1)00011011
```

P7.22 Overflow and underflow are not possible if the two numbers to be added have opposite signs. If the two numbers to be added have the same sign and the result has the opposite sign, underflow or overflow has occurred.

P7.23 (a) $0.1_{10} = 0.000110011\,\overline{0011}_2$

(b) $0.6_{10} = 0.100110011\,\overline{0011}_2$

(c) Most calculators give $(0.1 \times 1024 - 102) \times 10 - 4 = 0$ exactly.

(d) Using MATLAB, I have

```
>> (0.1*1024 - 102)*10 - 4
ans = 5.6843e-014
```

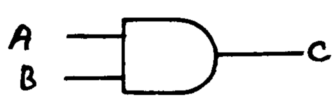
which is not zero because of the error associated with representing 0.1_{10} in binary with a finite number of bits.

P7.24 A truth table lists all of the combinations of the variables in a logic expression as well as the value of the expression.

P7.25 If the variables in a logic expression are replaced by their inverses, the AND operation is replaced by OR, the OR operation is replaced by AND, and the entire expression is inverted, the resulting logic expression yields the same values as before the changes. In equation form, we have:

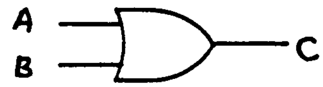
$$ABC = \overline{\overline{A} + \overline{B} + \overline{C}} \qquad\qquad (A + B + C) = \overline{\overline{A}\,\overline{B}\,\overline{C}}$$

P7.26 AND gate:



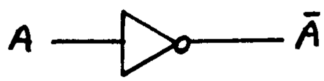
AB	C
00	0
01	0
10	0
11	1

OR gate:



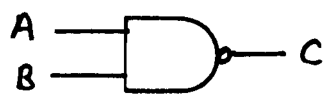
AB	C
00	0
01	1
10	1
11	1

Inverter:



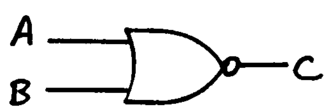
A	$\overline{A}$
0	1
1	0

NAND gate:



AB	C
00	1
01	1
10	1
11	0

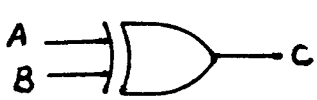
NOR gate:



AB	C
00	1
01	0
10	0
11	0

Exclusive OR gate:

AB	C
00	0
01	1
10	1
11	0



P7.27 (a)* $E = AB + A\overline{B}C + \overline{C}D$

A	B	C	D	E
0	0	0	0	0
0	0	0	1	0
0	0	1	0	1
0	0	1	1	0
0	1	0	0	0
0	1	0	1	0
0	1	1	0	1
0	1	1	1	0
1	0	0	0	1
1	0	0	1	1
1	0	1	0	1
1	0	1	1	1
1	1	0	0	0
1	1	0	1	0
1	1	1	0	1
1	1	1	1	1

(b) $D = A\overline{B}C + \overline{AC}$

A	B	C	D
0	0	0	1
0	0	1	0
0	1	0	1
0	1	1	0
1	0	0	0
1	0	1	1
1	1	0	0
1	1	1	0

(c) $Z = \overline{W}Y + \overline{(\overline{X} + Y + W)}$

W	X	Y	Z
0	0	0	0
0	0	1	1
0	1	0	1
0	1	1	1
1	0	0	0
1	0	1	0

1	1	0	0
1	1	1	0

(d) $D = \overline{A}\,\overline{C}B + \overline{B}$

A	B	C	D
0	0	0	1
0	0	1	1
0	1	0	1
0	1	1	0
1	0	0	1
1	0	1	1
1	1	0	0
1	1	1	0

(e) $D = \overline{\left(\overline{C} + AB\right)}$

A	B	C	D
0	0	0	1
0	0	1	0
0	1	0	0
0	1	1	0
1	0	0	0
1	0	1	0
1	1	0	0
1	1	1	0

P7.28 One method to prove the validity of a Boolean identity is to list the truth table and show that both sides of the identity give the same result for each combination of logic variables.

P7.29* $AB + \overline{B}C = \overline{(\overline{A} + \overline{B})(B + \overline{C})}$

A	B	C	AB	$\overline{B}C$	$AB + \overline{B}C$	$\overline{A} + \overline{B}$	$B + \overline{C}$	$(\overline{A} + \overline{B})(B + \overline{C})$	$\overline{(\overline{A} + \overline{B})(B + \overline{C})}$
0	0	0	0	0	0	1	1	1	0
0	0	1	0	1	1	1	0	0	1
0	1	0	0	0	0	1	1	1	0
0	1	1	0	0	0	1	1	1	0
1	0	0	0	0	0	1	1	1	0
1	0	1	0	1	1	1	0	0	1
1	1	0	1	0	1	0	1	0	1
1	1	1	1	0	1	0	1	0	1

P7.30

A	B	C	$\overline{A}+\overline{B}+\overline{C}$	$\overline{\left(\overline{A}+\overline{B}+\overline{C}\right)}$	ABC
0	0	0	1	0	0
0	0	1	1	0	0
0	1	0	1	0	0
0	1	1	1	0	0
1	0	0	1	0	0
1	0	1	1	0	0
1	1	0	1	0	0
1	1	1	0	1	1

Thus, we have $ABC = \overline{\overline{A}+\overline{B}+\overline{C}}$

A	B	C	$\overline{A}\,\overline{B}\,\overline{C}$	$\overline{\left(\overline{A}\,\overline{B}\,\overline{C}\right)}$	$A+B+C$
0	0	0	1	0	0
0	0	1	0	1	1
0	1	0	0	1	1
0	1	1	0	1	1
1	0	0	0	1	1
1	0	1	0	1	1
1	1	0	0	1	1
1	1	1	0	1	1

Thus, we have $A+B+C = \overline{\overline{A}\,\overline{B}\,\overline{C}}$

P7.31

A	B	$A+B$	$\overline{A}+AB$	$(A+B)(\overline{A}+AB)$	$(A+B)(\overline{A}+AB)$	$\overline{B}$
0	0	0	1	0	1	1
0	1	1	1	1	0	0
1	0	1	0	0	1	1
1	1	1	1	1	0	0

Notice that the column for $\overline{B}$ matches that for $(A+B)(\overline{A}+AB)$.

P7.32

A	B	$A+\overline{A}B$	$A+B$
0	0	0	0
0	1	1	1
1	0	1	1
1	1	1	1

P7.33

A	B	C	$(ABC + AB\overline{C} + A\overline{B}\,\overline{C} + A\overline{B}C)$
0	0	0	0
0	0	1	0
0	1	0	0
0	1	1	0
1	0	0	1
1	0	1	1
1	1	0	1
1	1	1	1

P7.34 (a) $F = (A + B)\overline{C}$

A	B	C	F
0	0	0	0
0	0	1	0
0	1	0	1
0	1	1	0
1	0	0	1
1	0	1	0
1	1	0	1
1	1	1	0

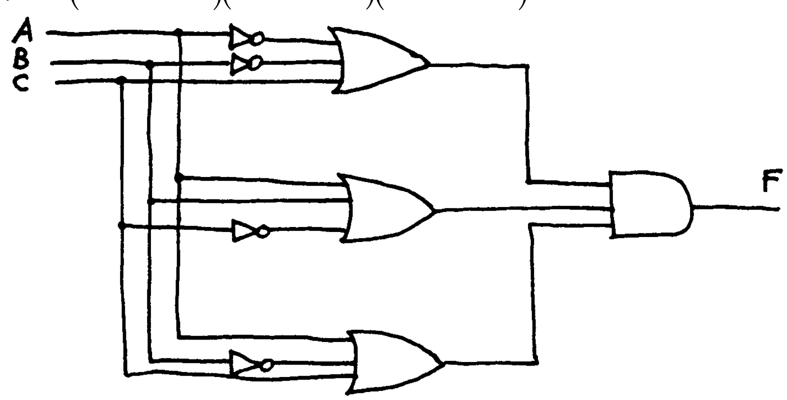
(b) $F = A + B + \overline{(BC)}$

A	B	C	F
0	0	0	1
0	0	1	1
0	1	0	1
0	1	1	1
1	0	0	1
1	0	1	1
1	1	0	1
1	1	1	1

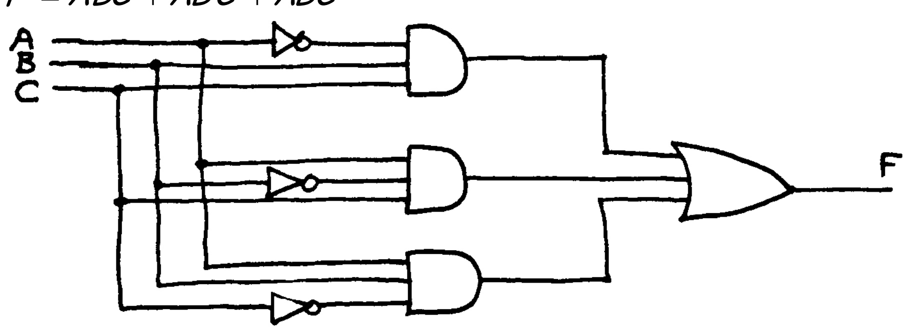
(c) $F = AB + \overline{(BC)} + D$

A	B	C	D	F
0	0	0	0	1
0	0	0	1	1
0	0	1	0	0
0	0	1	1	1
0	1	0	0	0
0	1	0	1	1
0	1	1	0	0
0	1	1	1	1
1	0	0	0	1
1	0	0	1	1
1	0	1	0	0
1	0	1	1	1
1	1	0	0	1
1	1	0	1	1
1	1	1	0	1
1	1	1	1	1

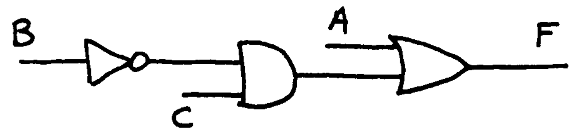
P7.35 (a)* $F = (\overline{A} + \overline{B} + C)(A + B + \overline{C})(A + \overline{B} + C)$



(b) $F = A\overline{B}C + AB\overline{C} + \overline{A}BC$



(c) $F = A + \overline{B}C$



P7.36 (a)* $F = (A + B + C)(A + \overline{B} + C)(\overline{A} + B + \overline{C}) = \overline{\overline{A}\,\overline{B}\,\overline{C} + \overline{A}B\overline{C} + A\overline{B}C}$

(b)* $F = ABC + A\overline{B}C + \overline{A}B\overline{C} = \overline{(\overline{A} + \overline{B} + \overline{C})(\overline{A} + B + \overline{C})(A + \overline{B} + C)}$

(c) $F = AB + (\overline{C} + A)\overline{D} = \overline{(\overline{A} + \overline{B})(C\overline{A} + D)}$

(d) $F = A(\overline{B} + C) + D = \overline{(\overline{A} + B\overline{C})\overline{D}}$

(e) $F = A\overline{B}C + A(B + C) = \overline{(\overline{A} + B + \overline{C})(\overline{A} + \overline{B}\,\overline{C})}$

P7.37 (a) Applying De Morgan's laws to the output, we have

$$C = \overline{A} + \overline{B} = \overline{AB}$$

Thus, the gate shown is equivalent to a NAND gate.

(b) Applying De Morgan's laws to the output, we have

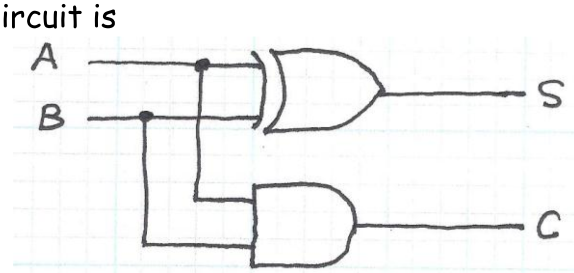
$$F = \overline{D}\,\overline{E} = \overline{D + E}$$

Thus, the gate shown is equivalent to a NOR gate.

P7.38 The truth table is

A	B	S	C
0	0	0	0
0	1	1	0
1	0	1	0
1	1	0	1

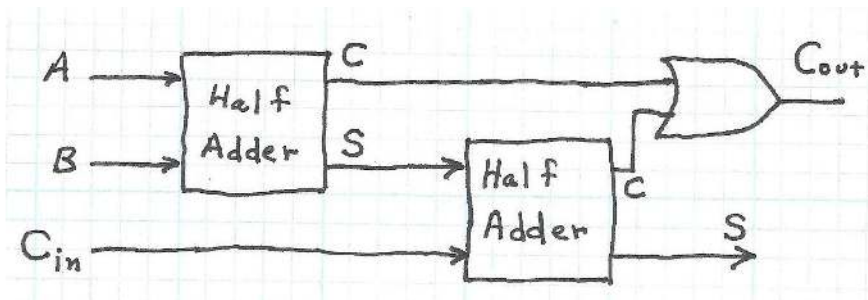
The circuit is



P7.39 The truth table is

A	B	C_{in}	S	C_{out}
0	0	0	0	0
0	0	1	1	0
0	1	0	1	0
0	1	1	0	1
1	0	0	1	0
1	0	1	0	1
1	1	0	0	1
1	1	1	1	1

The circuit is

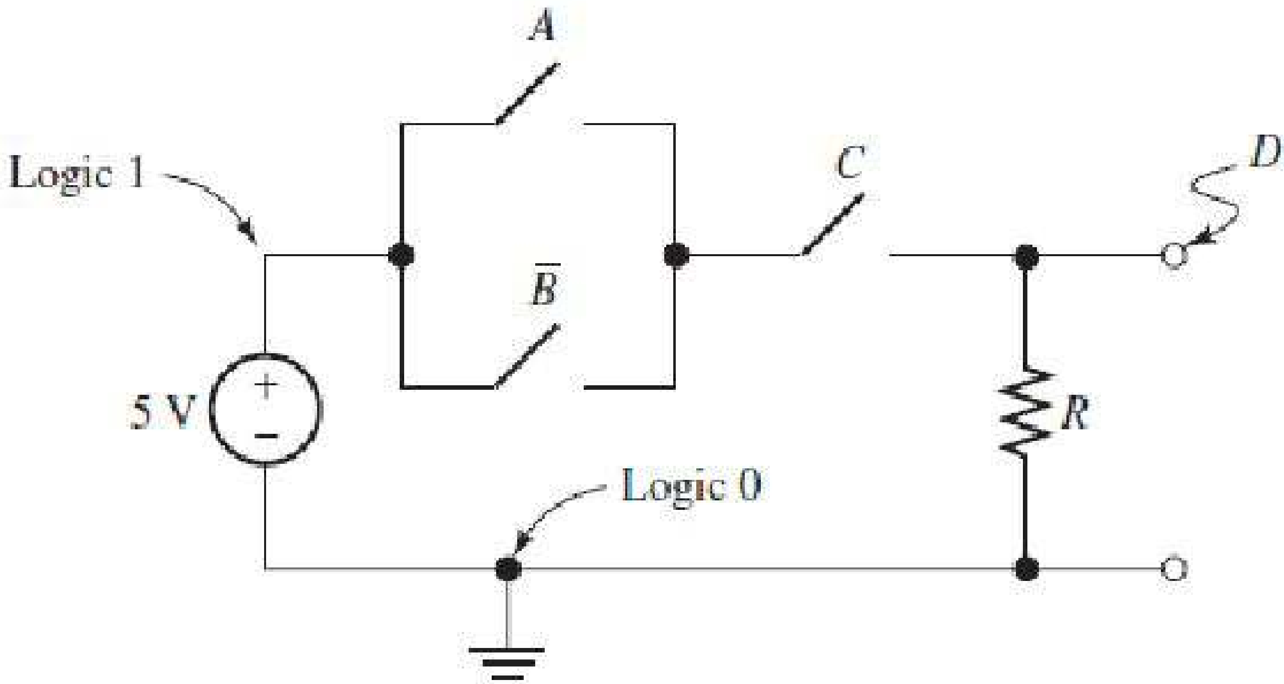


P7.40 NAND gates are said to be sufficient for combinatorial logic because any Boolean expression can be implemented solely with NAND gates. Similarly, NOR gates are sufficient.

P7.41 $$D = (\bar{A} + B)\bar{C}$$

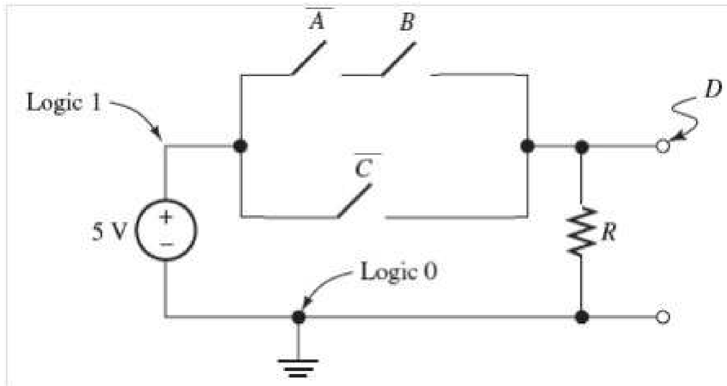
The truth table is:

A	B	C	$D = (\bar{A} + B)\bar{C}$
0	0	0	0
0	0	1	1
0	1	0	0
0	1	1	1
1	0	0	0
1	0	1	0
1	1	0	0
1	1	1	1



A
Logic 1
C
D
B̄
5 V
+
−
R
Logic 0

P7.42



P7.43 In this circuit, the output is high only if switch A is open (A low) and if either of the other two switches is open. Thus, we can write

$$D = \overline{A}(\overline{C} + B)$$

The truth table is:

A	B	C	D
0	0	0	1
0	0	1	0
0	1	0	1
0	1	1	1
1	0	0	0
1	0	1	0
1	1	0	0
1	1	1	0

P7.44 In synthesizing a logic expression as a sum of products, we focus on the lines of the truth table for which the result is 1. A logical product of logic variables and their inverses is written that yields 1 for each of these lines. Then, the products are summed.

In synthesizing a logic expression as a product of sums, we focus on the lines of the truth table for which the result is 0. A logical sum of logic variables and their inverses is written that yields 0 for each of these lines. Then, the sums are combined in an AND gate.

P7.45* $F = \overline{A}\,\overline{B}\,\overline{C} + \overline{A}B\overline{C} + A\overline{B}C + ABC$
$= \sum m(0,2,5,7)$

$F = \left(A + B + \overline{C}\right)\left(A + \overline{B} + \overline{C}\right)\left(\overline{A} + B + C\right)\left(\overline{A} + \overline{B} + C\right)$
$= \prod M(1,3,4,6)$

P7.46 $G = \overline{A}\,\overline{B}\,\overline{C} + \overline{A}\,\overline{B}C + \overline{A}BC$
$= \sum m(0,1,3)$
$G = (A + \overline{B} + C)(\overline{A} + B + C)(\overline{A} + B + \overline{C})(\overline{A} + \overline{B} + C)(\overline{A} + \overline{B} + \overline{C})$
$= \prod M(2,4,5,6,7)$

P7.47 $H = \overline{A}\,\overline{B}\,\overline{C} + \overline{A}\,\overline{B}C + \overline{A}B\overline{C} + AB\overline{C} + ABC$
$= \sum m(0,1,2,6,7)$
$H = (A + \overline{B} + \overline{C})(\overline{A} + B + C)(\overline{A} + B + \overline{C})$
$= \prod M(3,4,5)$

P7.48 $I = \overline{A}B\overline{C} + \overline{A}BC + AB\overline{C} + ABC$
$= \sum m(2,3,6,7)$
$I = (A + B + C)(A + B + \overline{C})(\overline{A} + B + C)(\overline{A} + B + \overline{C})$
$= \prod M(0,1,4,5)$

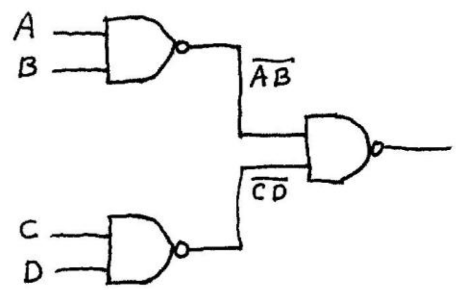
P7.49 $J = \overline{A}\,\overline{B}C + \overline{A}BC + AB\overline{C} + ABC$
$= \sum m(1,3,6,7)$
$J = (A + B + C)(A + \overline{B} + C)(\overline{A} + B + C)(\overline{A} + B + \overline{C})$
$= \prod M(0,2,4,5)$

P7.50 $K = \overline{A}B\overline{C} + \overline{A}BC + A\overline{B}\,\overline{C} + A\overline{B}C$
$= \sum m(2,3,4,5)$
$K = (A + B + C)(A + B + \overline{C})(\overline{A} + \overline{B} + C)(\overline{A} + \overline{B} + \overline{C})$
$= \prod M(0,1,6,7)$

P7.51 Applying De Morgan's Laws to the output of the circuit, we have

$$AB + CD = \overline{\overline{AB}\;\overline{CD}}$$

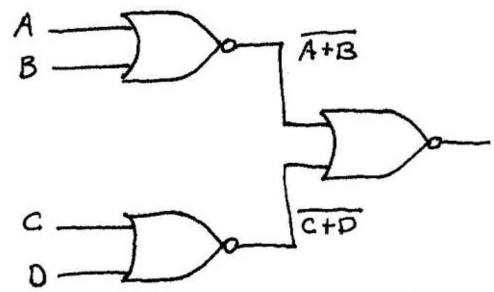
Thus, the circuit implemented with NAND gates is



P7.52 Applying De Morgan's Laws to the output of the circuit, we have

$$(A+B)(C+D) = \overline{\overline{(A+B)} + \overline{(C+D)}}$$

Thus, the circuit implemented with NOR gates is



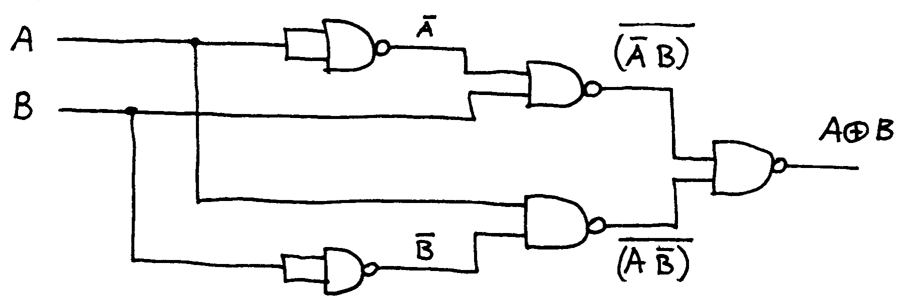
P7.53* The truth table is:

A	B	$A \oplus B$
0	0	0
0	1	1
1	0	1
1	1	0

Thus, we can write the product of sums expression and apply De Morgan's Laws to obtain:

$$A \oplus B = A\overline{B} + \overline{A}B = \overline{\overline{(A\overline{B})}\ \overline{(\overline{A}B)}}$$

The circuit is:



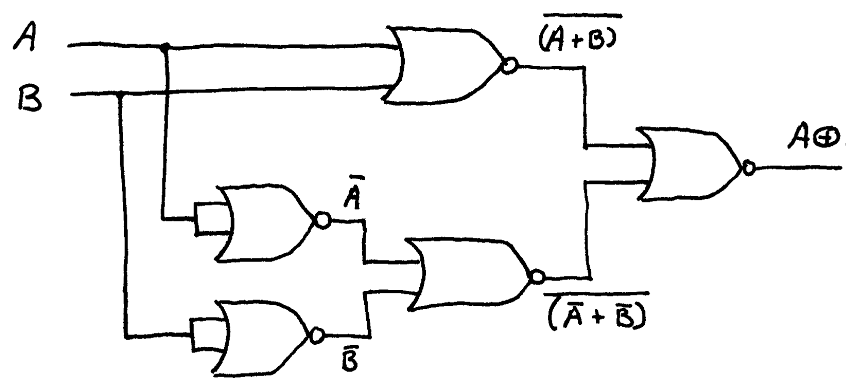
P7.54 The truth table is:

A	B	$A \oplus B$
0	0	0
0	1	1
1	0	1
1	1	0

Thus, we can write the product of sums expression and apply De Morgan's Laws to obtain:

$$A \oplus B = (A+B)(\bar{A}+\bar{B}) = \overline{\overline{(A+B)}+\overline{(\bar{A}+\bar{B})}}$$

The circuit is:



P7.55 An example of a decoder is a circuit that uses a BCD input to produce the logic signals needed to drive the elements of a seven-segment display.

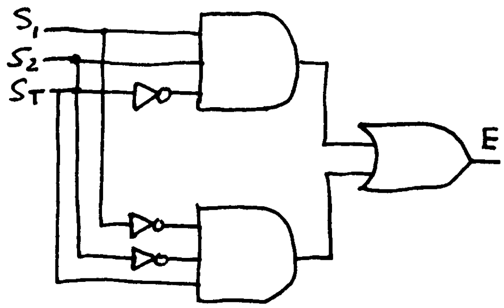
Another example is the three-to-eight-line decoder that has a three-bit input and eight output lines. The 3-bit input word selects one of the output lines and that output becomes high.

P7.56* (a)

S_1	S_2	S_T	E
0	0	0	0
0	0	1	1
0	1	0	0
0	1	1	0
1	0	0	0
1	0	1	0
1	1	0	1
1	1	1	0

(b) $E = \sum m(1,6) = \overline{S_1}\,\overline{S_2} S_T + S_1 S_2 \overline{S_T}$

(c) Circuit diagram:



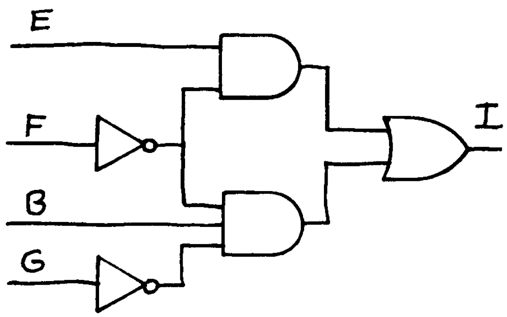
P7.57 (a) The truth table is:

B	E	F	G	I
0	0	0	0	0
0	0	0	1	0
0	0	1	0	0
0	0	1	1	0
0	1	0	0	1
0	1	0	1	1
0	1	1	0	0
0	1	1	1	0
1	0	0	0	1
1	0	0	1	0
1	0	1	0	0
1	0	1	1	0
1	1	0	0	1
1	1	0	1	1
1	1	1	0	0
1	1	1	1	0

(b) $I = \sum m(4,5,8,12,13)$

(c) $I = \prod M(0,1,2,3,6,7,9,10,11,14,15)$

(d) $I = E\overline{F} + B\overline{F}\,\overline{G}$

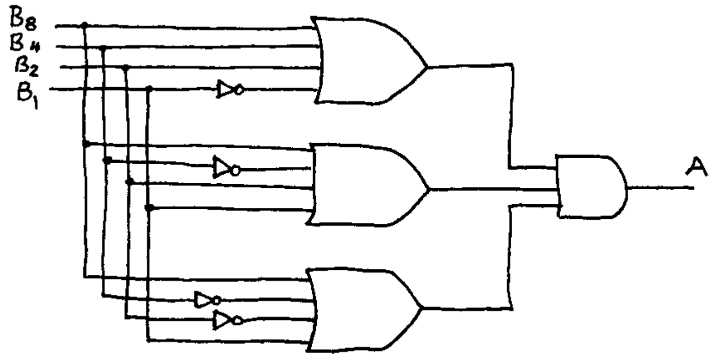


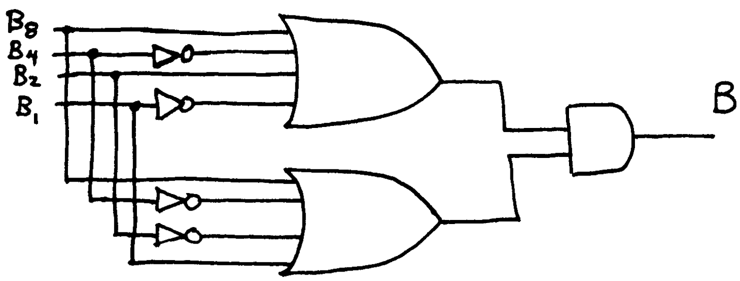
P7.58 The truth table (x = don't cares) is:

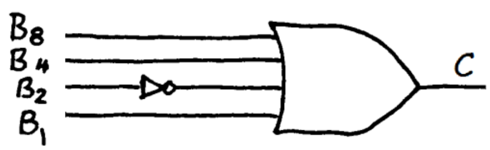
B_8	B_4	B_2	B_1	A	B	C	D
0	0	0	0	1	1	1	1
0	0	0	1	0	1	1	0
0	0	1	0	1	1	0	1
0	0	1	1	1	1	1	1
0	1	0	0	0	1	1	0
0	1	0	1	1	0	1	1
0	1	1	0	0	0	1	1
0	1	1	1	1	1	1	0
1	0	0	0	1	1	1	1
1	0	0	1	1	1	1	0
1	0	1	0	x	x	x	x
1	0	1	1	x	x	x	x
1	1	0	0	x	x	x	x
1	1	1	0	x	x	x	x
1	1	1	1	x	x	x	x

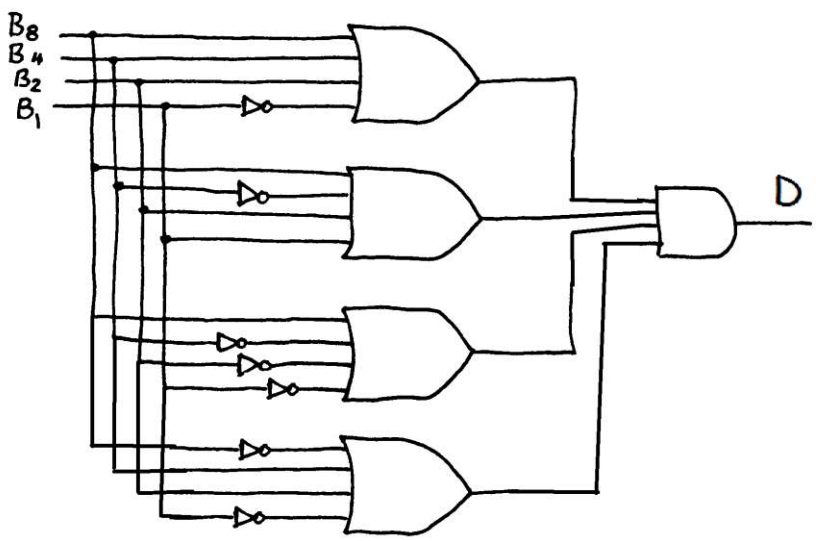
$$A = \prod M(1,4,6)$$
$$B = \prod M(5,6)$$
$$C = M(2) = B_8 + B_4 + \overline{B_2} + B_1$$
$$D = \prod M(1,4,7,9)$$

The circuits are:


B_8
B_4
B_2
B_1
A


B_8
B_4
B_2
B_1
B


B_8
B_4
B_2
B_1
C


B_8
B_4
B_2
B_1
D

P7.59

a.

AB (down) CD (across)	00	01	11	10
00			1	
01		1		1
11	1			1
10				1

Two singlets and three pairs in the K-map.

b.

Combining, we get $F = \bar{A}B\bar{C}D + \bar{A}\bar{B}CD + AB\bar{D} + BC\bar{D} + AC\bar{D}$

c.

Combining the POS terms (0s in the K-map),

AB (down) CD (across)	00	01	11	10
00	0	0		0
01	0		0	
11		0	0	
10	0	0	0	

Five pairs and a quad in the K-map.

Combining, we get $F = (A + B + \bar{C})(A + B + D)(A + C + D)(\bar{B} + \bar{C} + \bar{D})(\bar{A} + B + C)(\bar{A} + \bar{D})$

P7.60

a.

A (down) BC (across)	00	01	11	10

0	1		1	1
1				

Two pairs in the K-map

b.

Combining, we get $D = \bar{A}B + AC$

c.

Combining the POS terms (0s in the K-map),

A (down) BC (across)	00	01	11	10
0		0		
1	0	0	0	0

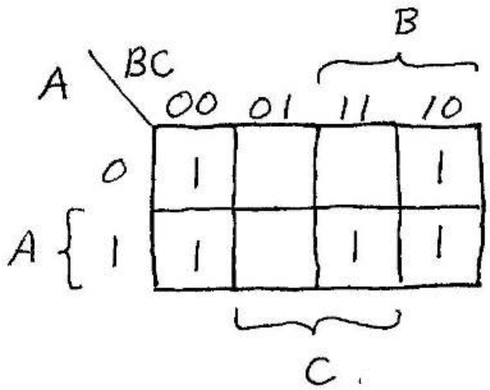
One quad and one pair in the K-map.

Combining, we get $D = \bar{A}(B + \bar{C})$

Also, we can get three pairs in the K-map for another solution.

$$D = (\bar{A} + B)(\bar{A} + \bar{B})(B + \bar{C})$$

P7.61 (a) The Karnaugh map is:



(b) $D = \bar{C} + AB$

(c) Inverting the map, and writing the minimum SOP expression yields

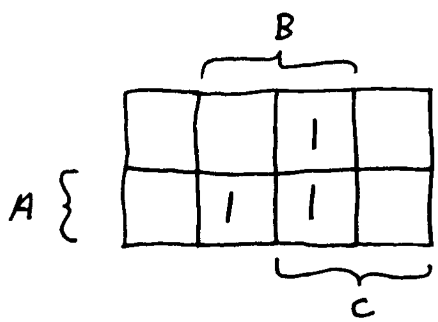
$$\bar{D} = \bar{A}C + \bar{B}C$$

Then, applying De Morgan's laws gives the POS expression:

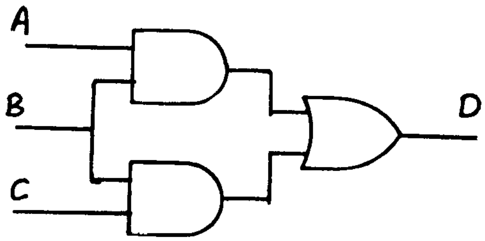
$$D = (A + \bar{C})(B + \bar{C})$$

See end of document for solution P7.62.

P7.63 (a) The Karnaugh map is:

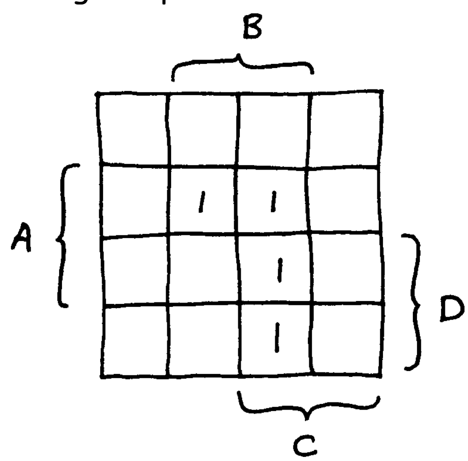


(b) $D = AB + BC$



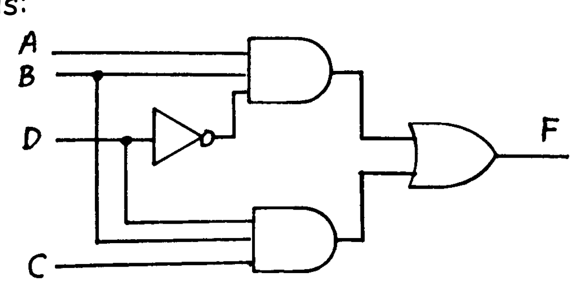
(c) Inverting the map, and writing the minimum SOP expression yields $\overline{D} = \overline{B} + \overline{A}\,\overline{C}$. Then, applying De Morgan's laws gives $D = B(A + C) = AB + BC$ which is the same as the expression found in part (b) so the implementation is the same.

P7.64 (a) The Karnaugh map is:



(b) $F = AB\overline{D} + BCD$

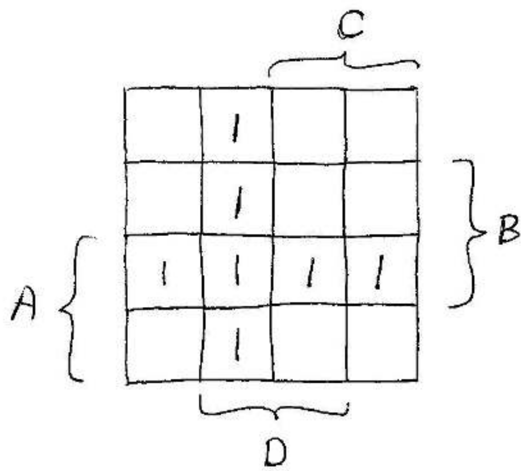
(c) The circuit is:



(d) Inverting the map, and writing the minimum SOP expression yields $\overline{F} = \overline{A}\,\overline{D} + \overline{B} + \overline{C}D$. Then applying DeMorgan's laws gives

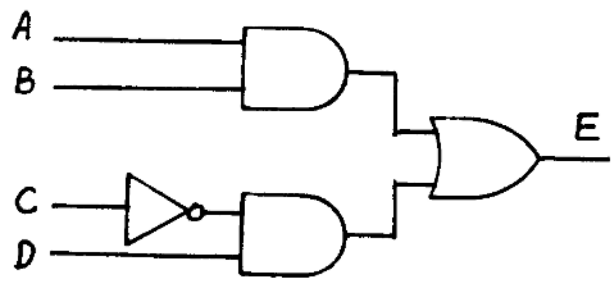
$$F = (A + D)B(C + \overline{D})$$

P7.65* (a) The Karnaugh map is:



(b) $E = AB + \overline{C}D$

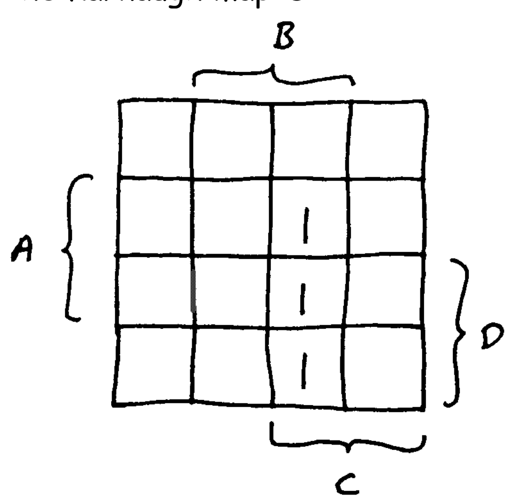
(c) The circuit is:



(d) Inverting the map, and writing the minimum SOP expression yields $\overline{E} = \overline{A}\,C + \overline{A}\,\overline{D} + \overline{B}\,C + \overline{B}\,\overline{D}$. Then, applying DeMorgan's laws gives

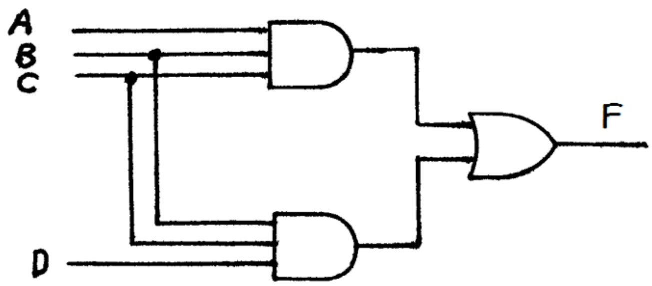
$$E = (A + \overline{C})(A + D)(B + \overline{C})(B + D)$$

P7.66 (a) The Karnaugh map is:



(b) $F = ABC + BCD$

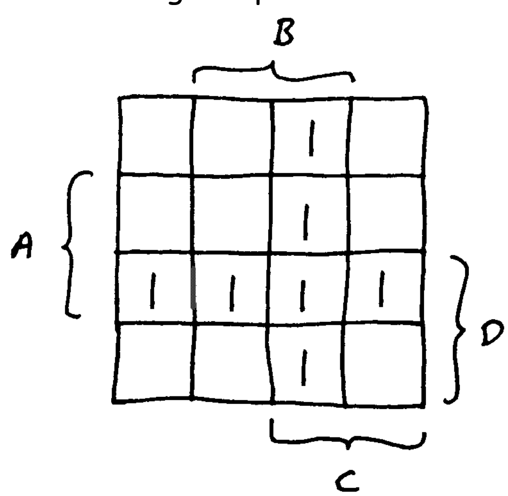
(c) The circuit is:



(d) Inverting the map, and writing the minimum SOP expression yields $\overline{F} = \overline{A}\,\overline{D} + \overline{B} + \overline{C}$. Then, applying De Morgan's laws gives

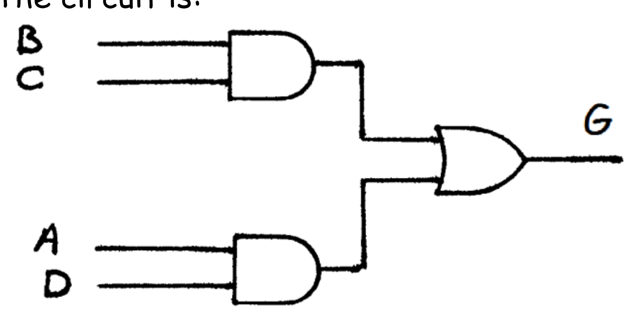
$$F = (A + D)BC$$

P7.67 (a) The Karnaugh map is:



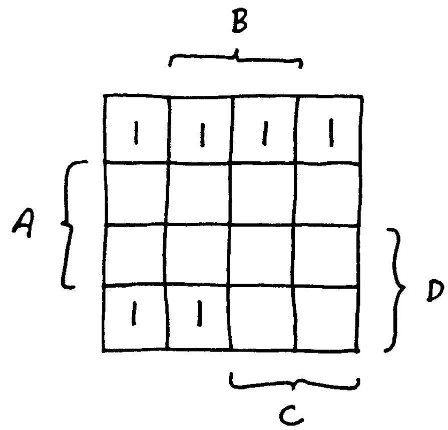
(b) $G = BC + AD$

(c) The circuit is:



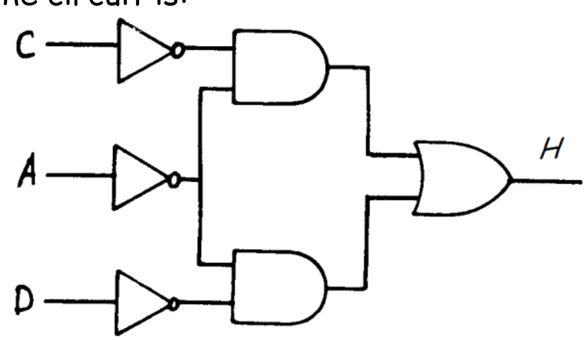
(d) Inverting the map, and writing the minimum SOP expression yields $\overline{G} = \overline{A}\,\overline{B} + \overline{A}\,\overline{C} + \overline{B}\,\overline{D} + \overline{C}\,\overline{D}$. Then, applying DeMorgan's laws gives $G = (A + B)(A + C)(B + D)(C + D)$.

P7.68 (a) The Karnaugh map is:



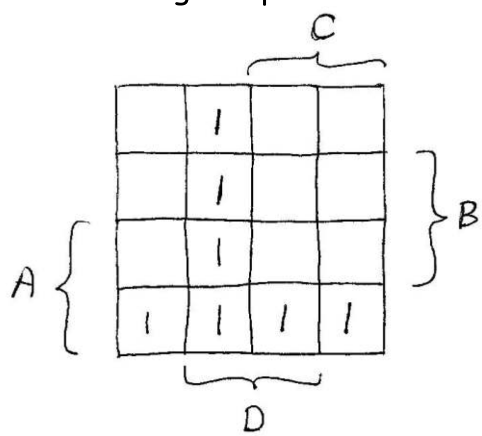
(b) $H = \overline{A}\,\overline{C} + \overline{A}\,\overline{D}$

(c) The circuit is:



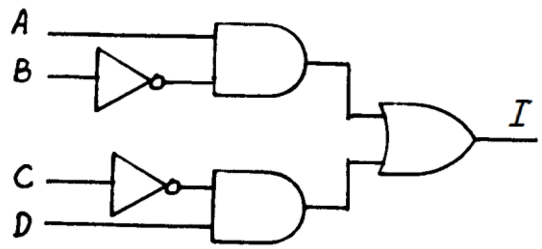
(d) Inverting the map, and writing the minimum SOP expression yields $\overline{H} = A + CD$. Then, applying De Morgan's laws gives $H = \overline{A}(\overline{C} + \overline{D})$.

P7.69 (a) The Karnaugh map is:



(b) $I = A\overline{B} + \overline{C}D$

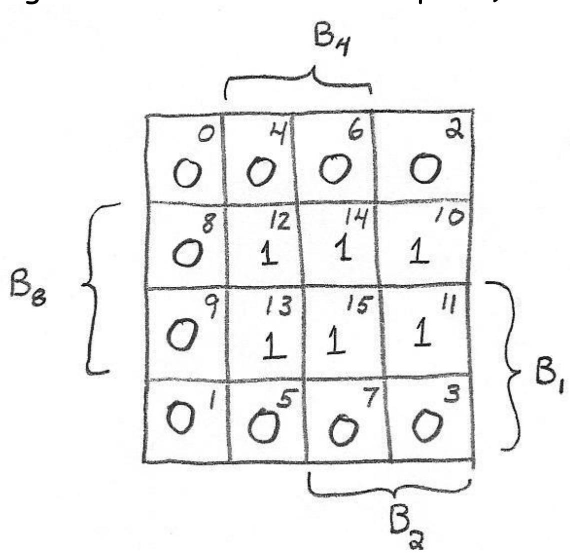
(c) The circuit is:



(d) Inverting the map, and writing the minimum SOP expression yields $\overline{I} = \overline{A}\,\overline{C} + \overline{A}\,\overline{D} + BC + B\overline{D}$. Then, applying De Morgan's laws gives

$$I = (A + \overline{C})(A + D)(\overline{B} + \overline{C})(\overline{B} + D)$$

P7.70 The Karnaugh map (with the decimal equivalent of each word in the upper right hand corner of each square) is:

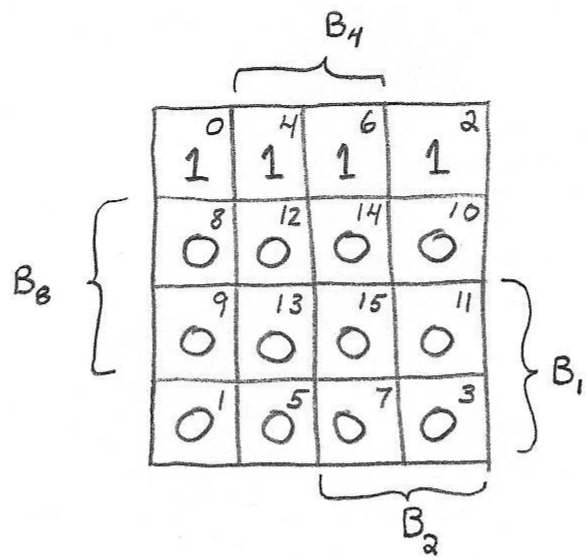


By inspection, the minimal SOP expression is:

$$X = B_8 B_4 + B_8 B_2$$

Inverting the map, and writing the minimum SOP expression yields $\overline{X} = \overline{B}_8 + \overline{B}_2 \overline{B}_4$. Then, applying DeMorgan's laws gives $X = B_8(B_2 + B_4)$.

P7.71 The Karnaugh map (with the decimal equivalent of each word in the upper right hand corner of each square) is:



By inspection the minimal SOP expression is:

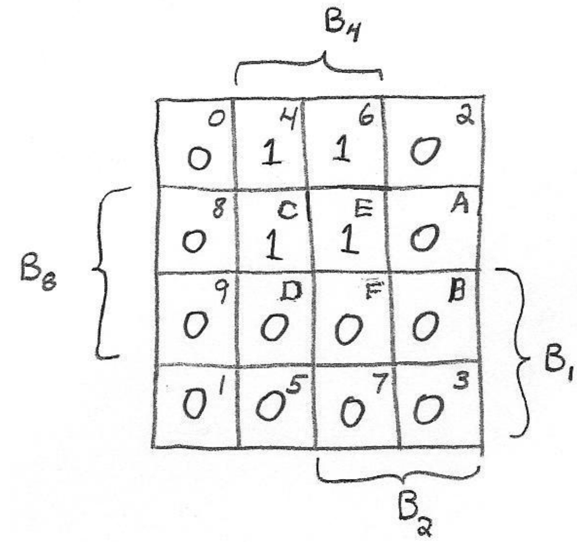
$$X = \overline{B}_8 \, \overline{B}_1$$

P7.72 If A denotes the supply from the utility and B denotes the supply from the DG set, the logic circuit should have the following combinational logic

$$F = \overline{A}B + A\overline{B}$$

This 'F is the supply that is provided to the residents. It can be seen that at any time, only either of the two can be ON.

P7.73 The Karnaugh map (with the hexadecimal equivalent of each word in the upper right hand corner of each square) is:

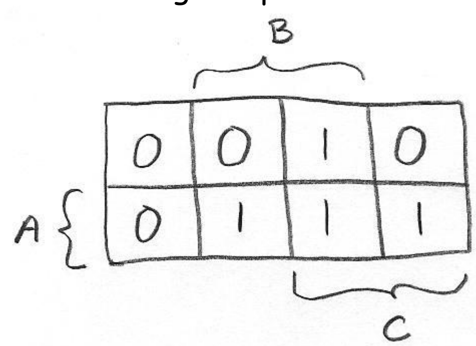


By inspection the minimal SOP expression is:

$$X = \overline{B}_1 B_4$$

Inverting the map, and writing the minimum SOP expression yields $\overline{X} = B_1 + \overline{B}_4$. Then, applying DeMorgan's laws gives $X = \overline{B}_1 B_4$.

P7.74 The Karnaugh map for a three-member council is:

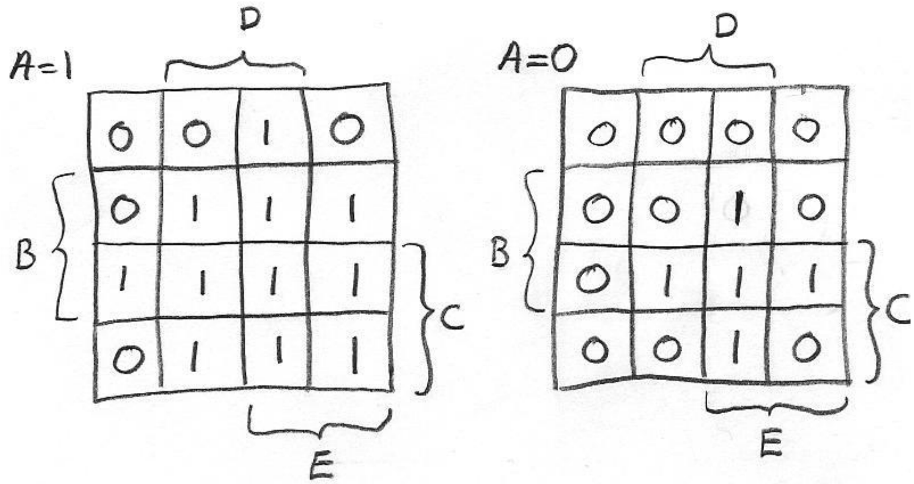


By inspection, we see that three two-cubes are needed and the minimal SOP expression is

$$X = AB + AC + BC$$

Clearly, the minimal SOP checks to see if at least one group of two members has voted yes.

For a five-member council, the Karnaugh map consists of two four-variable maps, one for $A = 1$ and one for $A = 0$.

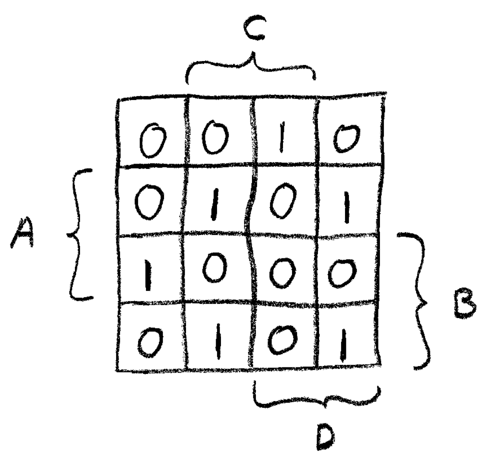


Notice that six four-cubes are needed to cover the A = 1 part of the map. Think of the A = 0 map as being (over or under) the A = 1 map. Four two cubes are needed to cover the A = 0 part of the map. However each of these cubes can be combined with corresponding cubes in the A = 1 part of the map to form four 4-cubes. The minimum SOP expression is:

$$X = ABC + ABD + ABE + ADE + ACD + ACE + BCD + BCE + BDE + CDE$$

Here, the minimal SOP expression checks to see if at least one group of three members has voted yes.

P7.75 The Karnaugh map is



By inspection, we see that six one cubes are needed. Thus the minimal SOP expression is:

$$X = AB\overline{C}\,\overline{D} + A\overline{B}C\overline{D} + A\overline{B}\,\overline{C}D + \overline{A}BC\overline{D} + \overline{A}B\overline{C}D + \overline{A}\,\overline{B}CD$$

P7.76 (a) The truth table for the circuit of Figure P7.76 is

A	*B*	*C*	*D*	*P*
0	0	0	0	0
0	0	0	1	1
0	0	1	0	1
0	0	1	1	0
0	1	0	0	1
0	1	0	1	0
0	1	1	0	0
0	1	1	1	1
1	0	0	0	1
1	0	0	1	0
1	0	1	0	0
1	0	1	1	1
1	1	0	0	0
1	1	0	1	1
1	1	1	0	1
1	1	1	1	0

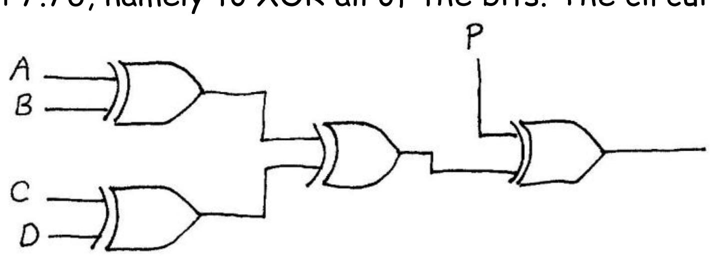
(b) The Karnaugh map for *P* is:

AB \ CD	00	01	11	10
00		1		1
01	1		1	
11		1		1
10	1		1	

From the map, we see that no simplification is possible and the minimum SOP is actually the sum of minterms:

$$P = \sum m(1, 2, 4, 7, 8, 11, 13, 14)$$

(c) The parity check can be performed by the same method as used in Figure P7.76, namely to XOR all of the bits. The circuit is:



P7.77 By inspection, we see that

$$X = A$$

The Karnaugh maps for Y and Z are:

Y

A \ BC	00	01	11	10
0			1	1
1	1	1		

Z

A \ BC	00	01	11	10
0		1		1
1		1		1

$$Y = \overline{A}B + A\overline{B} \qquad Z = B\overline{C} + \overline{B}C$$

P7.78 By inspection, we see that

$$A = X$$

The Karnaugh maps for B and C are:

B

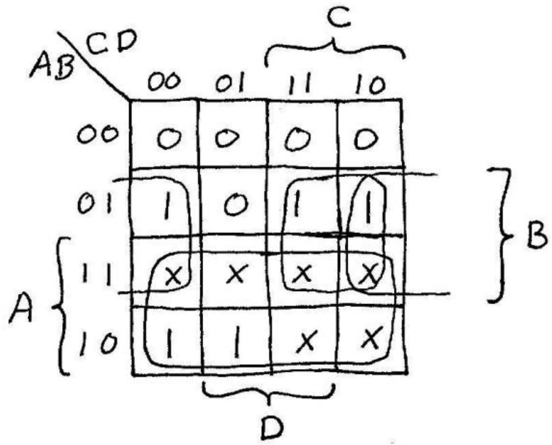
X \ YZ	00	01	10	11
0	0	0	1	1
1	1	1	0	0

C

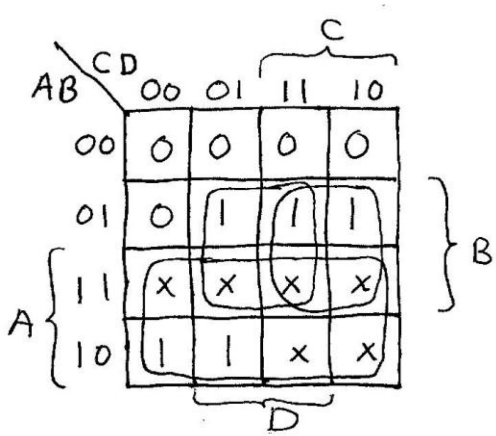
X \ YZ	00	01	11	10
0	0	1	0	1
1	1	0	1	0

$$B = \overline{X}Y + X\overline{Y} \qquad C = \overline{X}\,\overline{Y}Z + \overline{X}Y\overline{Z} + X\overline{Y}\,\overline{Z} + XYZ$$

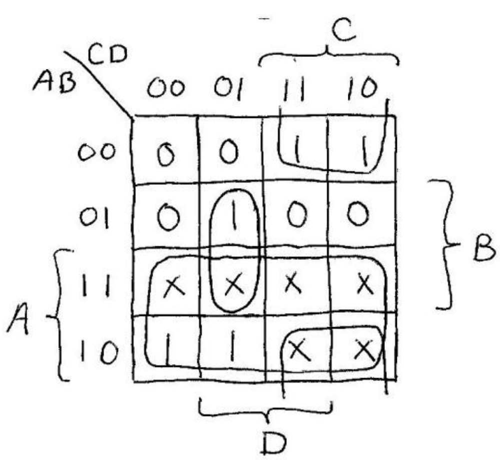
P7.79* (a) $F = A + BC + B\overline{D}$



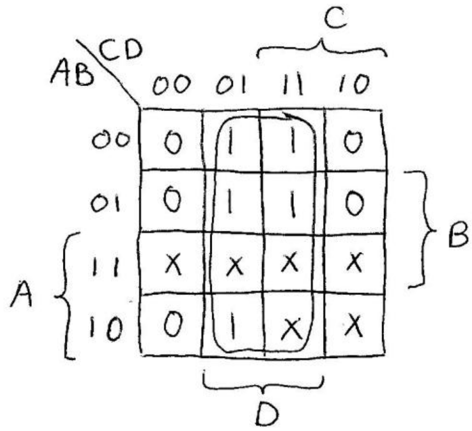
(b) $G = A + BD + BC$



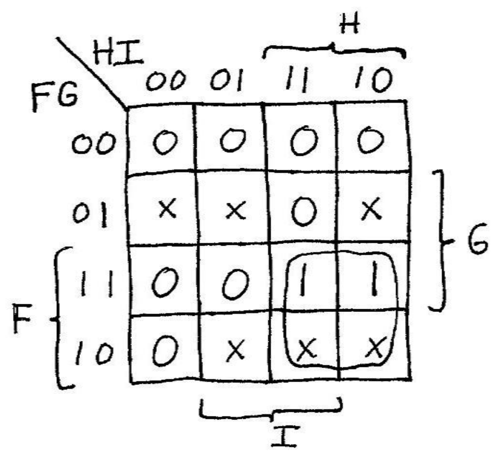
(c) $H = A + \bar{B}C + B\bar{C}D$



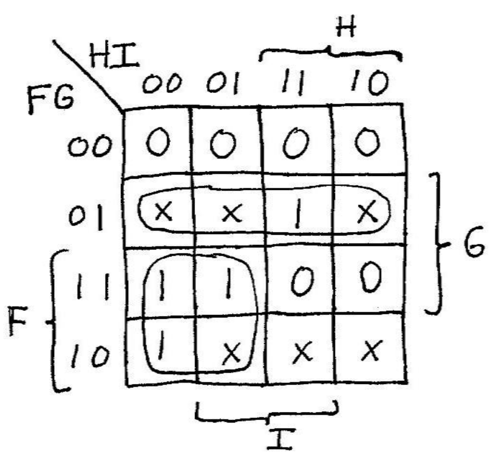
(d) $I = D$



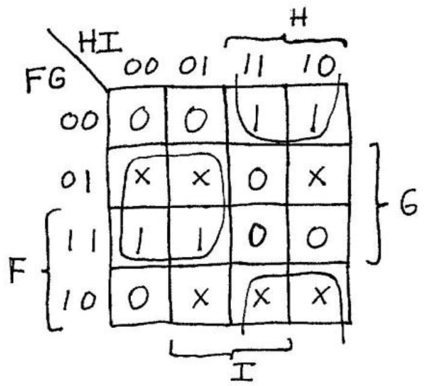
P7.80 (a) $A = FH$



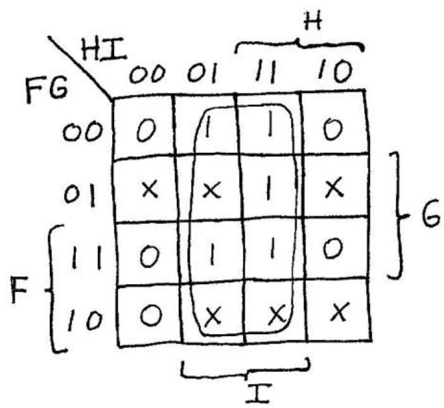
(b) $B = \bar{F}G + F\bar{H}$



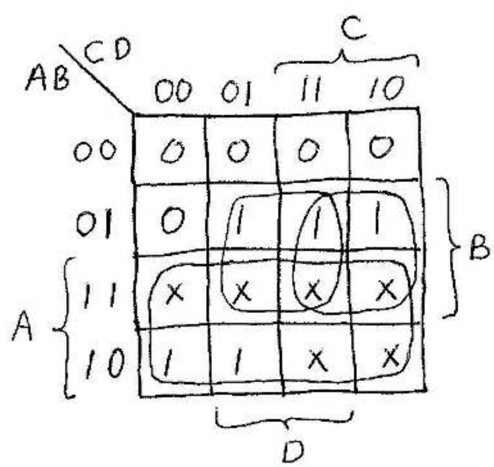
(c) $C = G\overline{H} + \overline{G}H$



(d) $D = I$



P7.81 (a) $W = A + BC + BD$



(b) $X = \overline{B}C + \overline{B}D + B\overline{C}\,\overline{D}$

AB \ CD	00	01	11	10
00	0	1	1	1
01	1	0	0	0
11	X	X	X	X
10	0	1	X	X

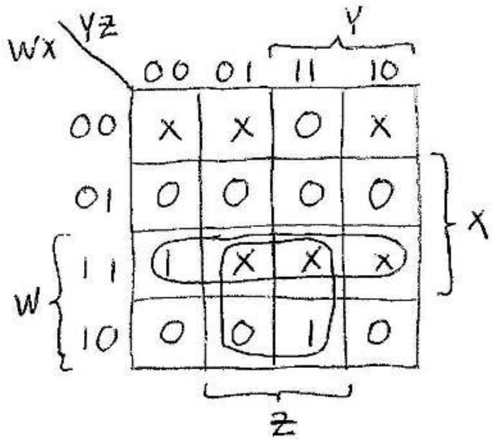
(c) $Y = CD + \overline{C}\,\overline{D}$

AB \ CD	00	01	11	10
00	1	0	1	0
01	1	0	1	0
11	X	X	X	X
10	1	0	X	X

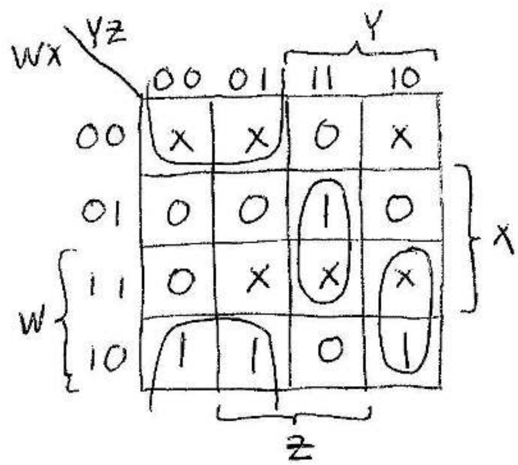
(d) $Z = \overline{D}$

AB \ CD	00	01	11	10
00	1	0	0	1
01	1	0	0	1
11	X	X	X	X
10	1	0	X	X

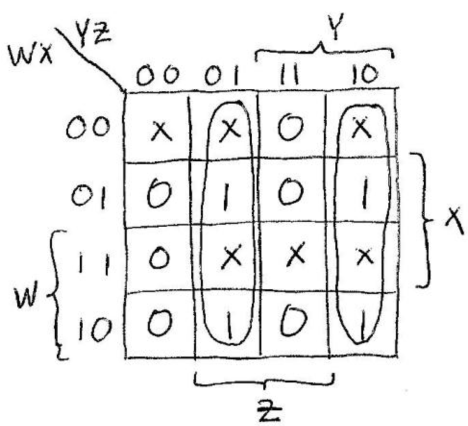
P7.82 (a) $A = WX + WZ$



(b) $B = \overline{X}\,\overline{Y} + XYZ + WY\overline{Z}$



(c) $C = \overline{Y}Z + Y\overline{Z}$

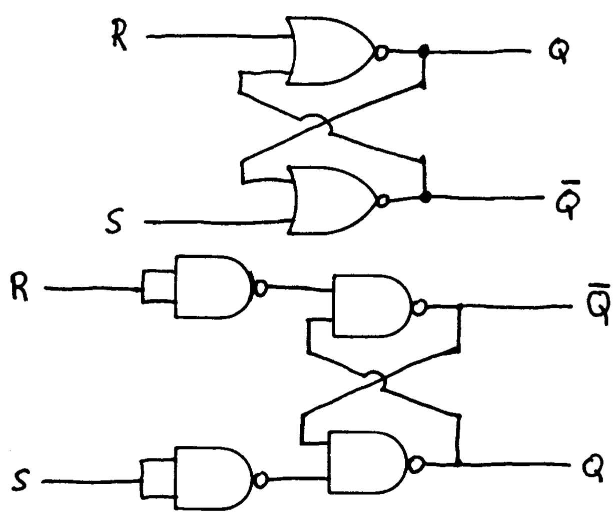


(d) $D = \overline{Z}$

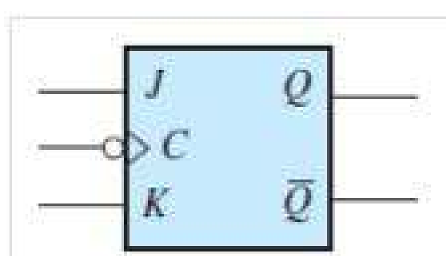
$WX \backslash YZ$	00	01	11	10
00	x	x	0	x
01	1	0	0	1
11	1	x	x	x
10	1	0	0	1

P7.83 See Figure 7.39 in the text.

P7.84



P7.85 The circuit symbol for the JK flip-flop is as follows –



The truth table for the JK flip-flop is as below –

C	J	K	Q_n	Comment
0	×	×	Q_{n-1}	Memory
1	×	×	Q_{n-1}	Memory
↓	0	0	Q_{n-1}	Memory
↓	0	1	0	Reset
↓	1	0	1	Set
↓	1	1	$\overline{Q}_{n-1}$	Toggle

P7.86 Asynchronous inputs are recognized independently of the clock signal. Synchronous inputs are recognized only if the clock is high.

P7.87 In edge triggering, the input values present immediately prior to a transition of the clock signal are recognized. Input values (and changes in input values) at other times are ignored.

P7.88 See Figure 7.47 in the text.

P7.89* The successive states are:

Q_0	Q_1	Q_2
1	0	0
0	1	0
1	0	1
1	1	0
1	1	1
0	1	1
0	0	1

(repeats)

Thus, the register returns to the initial state after seven shifts.

P7.90 (a) With an OR gate, we have:

Q_0	Q_1	Q_2
1	0	0
0	1	0
1	0	1
1	1	0
1	1	1
1	1	1

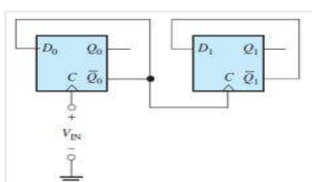
After the register reaches the 111 state, it remains in that state and never returns to the starting state.

(b) With an AND gate, we have:

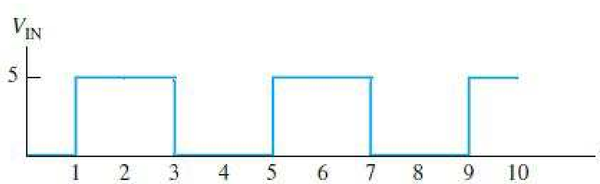
Q_0	Q_1	Q_2
1	0	0
0	1	0
0	0	1
0	0	0
0	0	0

After the register reaches the 000 state, it remains in that state and never returns to the starting state.

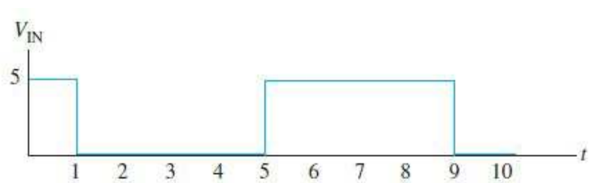
P7.91



The variation of the output Q_0 will be as shown below –

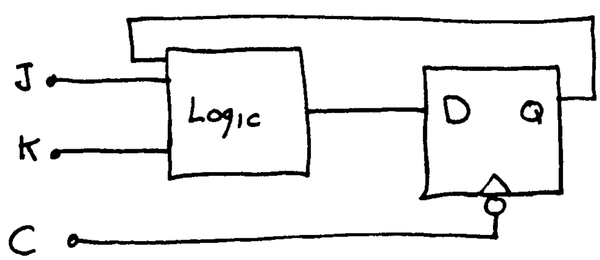


The variation of the output Q_1 will be as shown below –



P7.92 See end of document for solution

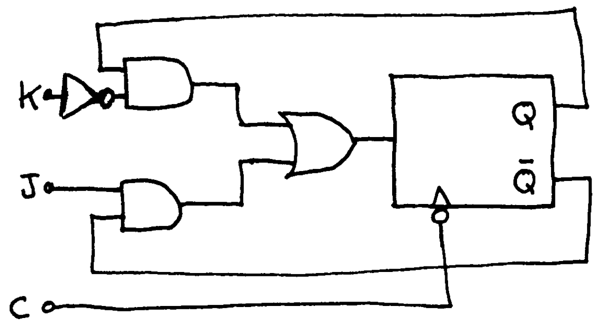
P7.93*



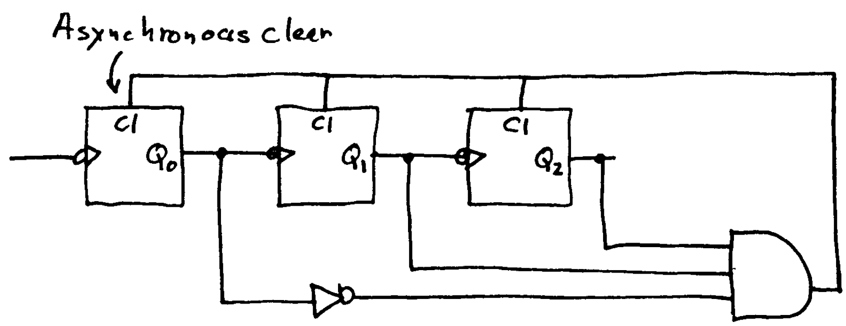
	Q	J	K	D
memory	0	0	0	0
reset	0	0	1	0
set	0	1	0	1
toggle	0	1	1	1
memory	1	0	0	1
reset	1	0	1	0
set	1	1	0	1
toggle	1	1	1	0

$$D = \overline{Q}J\overline{K} + \overline{Q}JK + Q\overline{J}\,\overline{K} + QJ\overline{K}$$

$$= \overline{Q}J\left(\overline{K}+K\right)+Q\overline{K}\left(\overline{J}+J\right)$$
$$= \overline{Q}J+Q\overline{K}$$

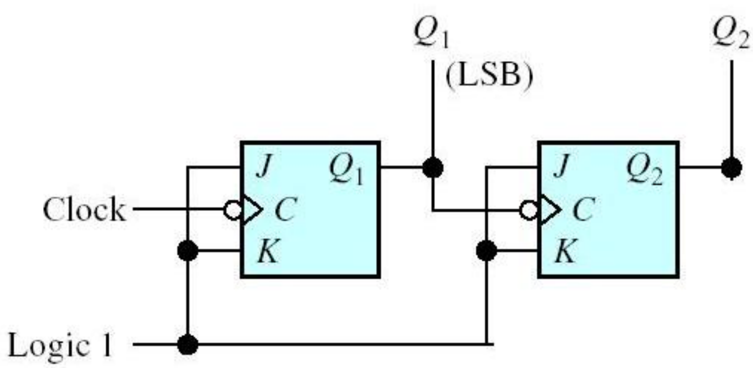


P7.94



P7.95 (a) There are four diodes and to make one revolution in two seconds each diode must be on for 0.5 s. Thus, the frequency of the clock is 2 Hz.

(b) The modulo-4 counter is:



(c) The truth table is

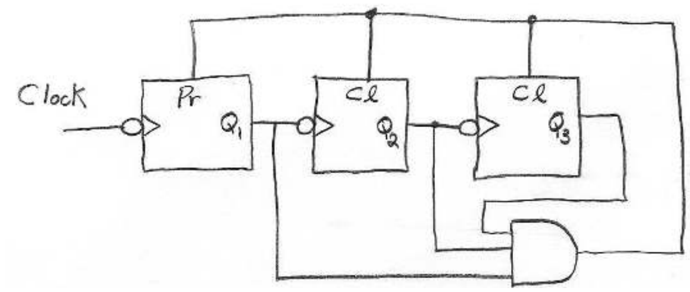
S	Q_2	Q_1	D_1	D_2	D_3	D_4
0	0	0	1	0	0	0
0	0	1	0	1	0	0
0	1	0	0	0	1	0
0	1	1	0	0	0	1

1	0	0	1	0	0	0
1	0	1	0	0	0	1
1	1	0	0	0	1	0
1	1	1	0	1	0	0

The minimal SOP expressions are:

$$D_1 = \overline{Q}_1\overline{Q}_2 \quad D_2 = Q_1\overline{Q}_2\overline{S} + Q_1Q_2S \quad D_3 = \overline{Q}_1Q_2 \quad D_4 = Q_1Q_2\overline{S} + Q_1\overline{Q}_2S$$

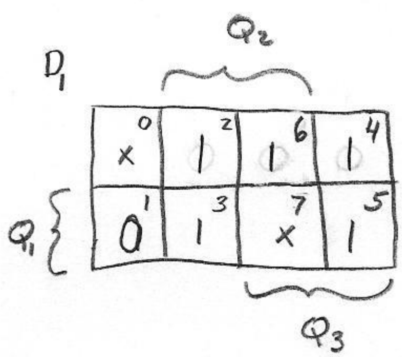
P7.96 (a) The logic diagram for the counter is:

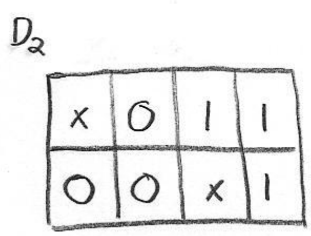


(b) The truth table for the encoder is:

Q_3	Q_2	Q_1	D_1	D_2	D_3	D_4	D_5	D_6	D_7
0	0	0	x	x	x	x	x	x	x
0	0	1	0	0	0	1	0	0	0
0	1	0	1	0	0	0	0	0	1
0	1	1	1	0	0	1	0	0	1
1	0	0	1	1	0	0	0	1	1
1	0	1	1	1	0	1	0	1	1
1	1	0	1	1	1	0	1	1	1
1	1	1	x	x	x	x	x	x	x

The Karnaugh maps are:





D_3

x	0	1	0
0	0	x	0

D_4

x	0	0	0
1	1	x	1

D_5

x	0	1	0
0	0	x	0

D_6

x	0	1	1
0	0	x	1

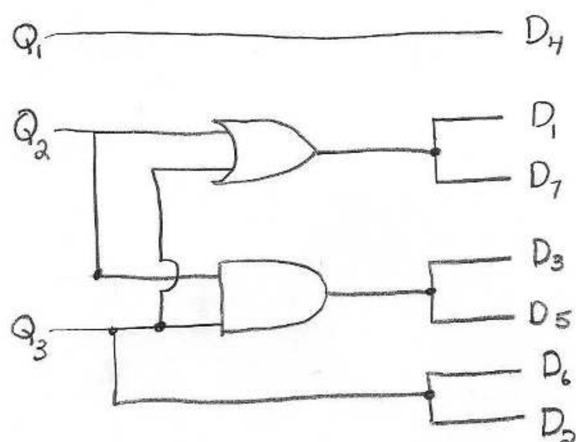
D_7

x	1	1	1
0	1	x	1

From the maps we can write:

$D_1 = D_7 = Q_2 + Q_3 \quad D_2 = D_6 = Q_3 \quad D_4 = Q_1$ and $D_3 = D_5 = Q_2Q_3$

The logic diagram is:



Practice Test

T7.1 (a) 12, (b) 19 (18 is incorrect because it omits the first step, inverting the variables), (c) 20, (d) 23, (e) 21, (f) 24, (g) 16, (h) 25, (i) 7, (j) 10, (k) 8, (l) 1 (the binary codes for hexadecimal symbols *A* through *F* do not occur in BCD).

T7.2 (a) For the whole part, we have:

	Quotient	Remainders
353/2	176	1
176/2	88	0
88/2	44	0
44/2	22	0
22/2	11	0
11/2	5	1
5/2	2	1
2/2	1	0
1/2	0	1

Reading the remainders in reverse order, we obtain:

$353_{10} = 101100001_2$

For the fractional part, we have

$2 \times 0.875 = 1 + 0.75$

$2 \times 0.75 = 1 + 0.5$

$2 \times 0.5 = 1 + 0$

Thus, we have

$0.875_{10} = 0.111_2$

Finally, combining the whole and fractional parts, we have

$353.875_{10} = 101100001.111_2$

(b) For the octal version, we form groups of three bits, working outward from the decimal point, and then write the octal symbol for each group.

$101\ 100\ 001.111_2 = 541.7_8$

(c) For the hexadecimal version, we form groups of four bits, working outward from the decimal point, and then write the hexadecimal symbol for each group.

$0001\ 0110\ 0001.1110_2 = 161.E_{16}$

(d) To obtain binary coded decimal, we simply write the binary equivalent for each decimal digit.

$353.875_{10} = 0011\ 0101\ 0011.1000\ 0111\ 0101_{BCD}$

T7.3 (a) Because the left-most bit is zero, this is a positive number. We simply convert from binary to decimal:

$01100001_2 = 1 \times 2^6 + 1 \times 2^5 + 1 \times 2^0 = 64 + 32 + 1 = +97_{10}$

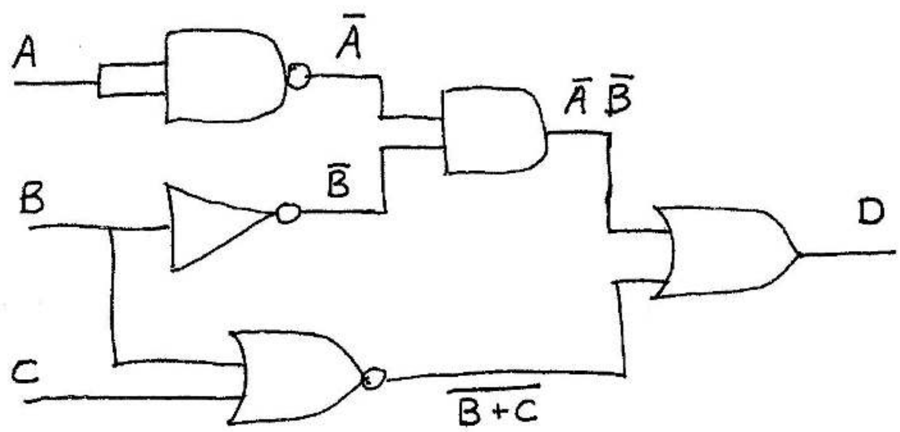
(b) Because the left-most bit is one, this is a negative number. We form the two's complement, which is 01000110. Then, we convert from binary to decimal:

$$01000110_2 = 1 \times 2^6 + 1 \times 2^2 + 1 \times 2^1 = 64 + 4 + 2 = +70_{10}$$

Thus, the decimal equivalent for eight-bit signed two's complement

integer 10111010 is −70.

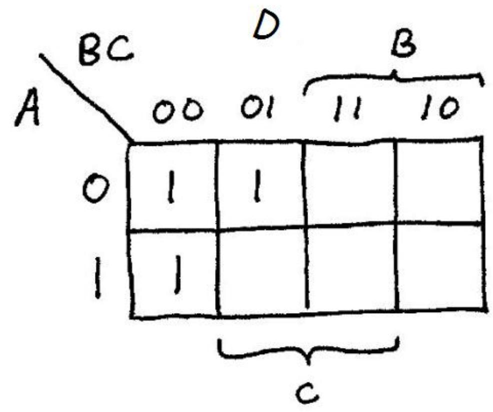
T7.4. (a) The logic expression is $D = \overline{A}\,\overline{B} + \overline{(B + C)}$.



(b) The truth table is:

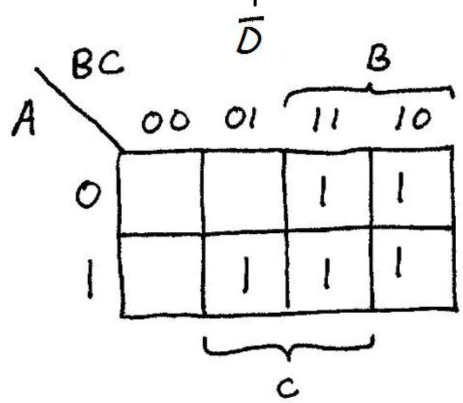
A	B	C	D
0	0	0	1
0	0	1	1
0	1	0	0
0	1	1	0
1	0	0	1
1	0	1	0
1	1	0	0
1	1	1	0

The Karnaugh map is:



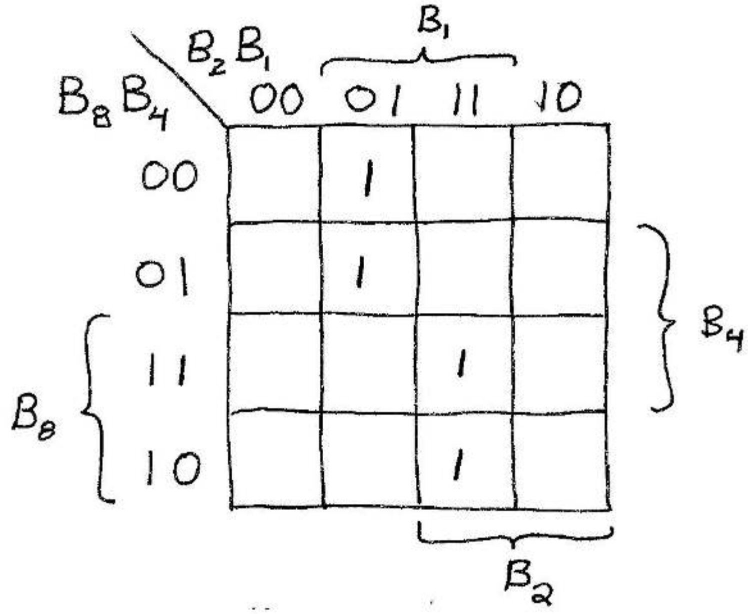
(c) The map can be covered by two 2-cubes and the minimum SOP expression is $D = \overline{A}\,\overline{B} + \overline{B}\,\overline{C}$.

(d) First, we invert the map to find:



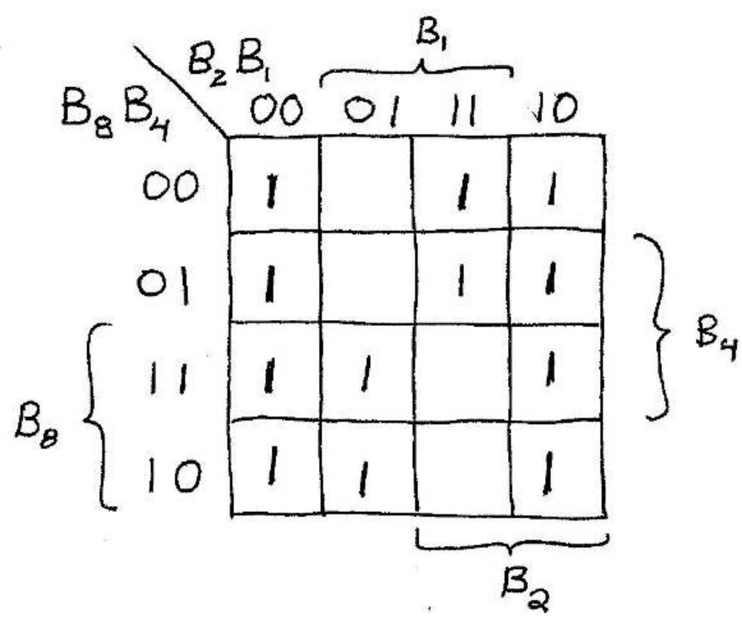
This map can be covered by one 4-cube and one 2-cube and the minimum SOP expression is $\overline{D} = B + AC$. Applying DeMorgan's laws to this yields the minimum POS expression $D = \overline{B}(\overline{A} + \overline{C})$.

T7.5 (a) The completed Karnaugh map is:



(b) The map can be covered by two 2-cubes and the minimum SOP expression is $G = B_1\overline{B_2}\,\overline{B_8} + B_1B_2B_8$.

(c) First, we invert the map to find:



This map can be covered by one 8-cube and two 4-cubes and the minimum SOP expression is $\overline{G} = \overline{B_1} + B_2\overline{B_8} + \overline{B_2}B_8$. Applying De Morgan's laws to this yields the minimum POS expression $G = B_1(\overline{B_2} + B_8)(B_2 + \overline{B_8})$.

T7.6 Clearly, the next value for Q_0 is the NAND combination of the current values of Q_1 and Q_2. The next value for Q_2 is the present value for Q_1. Similarly, the next value for Q_1 is the present value for Q_0. Thus, the successive states (Q_0 Q_1 Q_2) of the shift register are:

100 (initial state)
110
111
011
001
100
111

The state of the register returns to its initial state after 5 shifts.

P7.62

a. The K-map is as below.

A (down) BC (across)	00	01	11	10
0	1			
1	1		1	1

b.

Combining the SOP terms (1s in the K-map),

Two pairs in the K-map.

Combining, we get $D = \bar{B}\bar{C} + AB$

c.

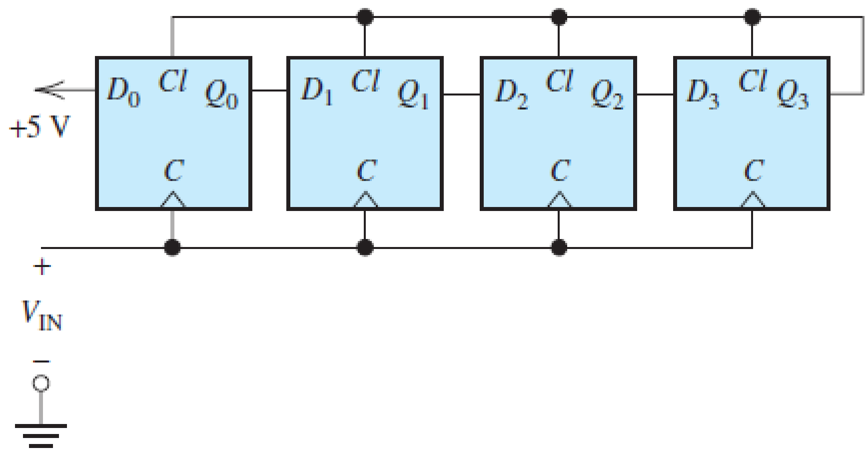
Combining the POS terms (0s in the K-map),

A (down) BC (across)	00	01	11	10
0		0	0	0
1		0		

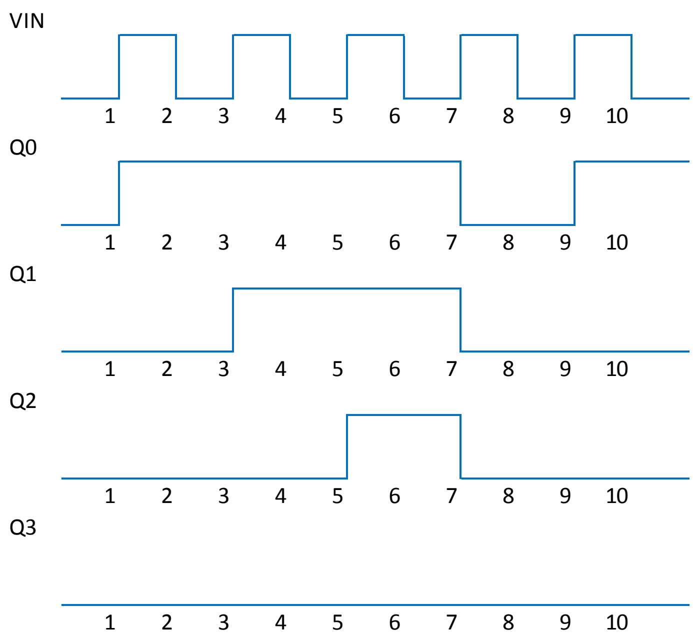
Three pairs in the K-map.

Combining, we get $D = (\bar{A} + \bar{C})(\bar{A} + \bar{B})(B + \bar{C})$

P7.92



The variations in outputs Q_0, Q_1, Q_2 and Q_3 will be as follows –



CHAPTER 8

Exercises

E8.1 The number of bits in the memory addresses is the same as the address bus width, which is 20. Thus, the number of unique addresses is 2^{20} = 1,048,576 = 1024 × 1024 = 1024K.

E8.2 (8 bits/byte) × (64 Kbytes) = 8 × 64 × 1024 = 524,288 bits

E8.3 Starting from the initial situation shown in Figure 8.7a in the book, execution of the command PSHB results in:

	0907:
	0908:
	0909:
SP →	090A: A2

Then, execution of the command PSHA results in:

	0907:
	0908:
SP →	0909: 34
	090A: A2

Then, the PULX command reads two bytes from the stack and we have:

	0907:	X: 34A2
	0908:	
	0909: 34	
	090A: A2	
SP →	090B:	

E8.4 Starting from the initial situation shown in Figure 8.7a in the book, execution of the command PSHX results in:

0907:
0908:

SP → 0909: 00
090A: 00

Then, the command PSHA results in:

0907:
SP → 0908: 34
0909: 00
090A: 00

Next the PULX command reads two bytes from the stack, and we have:

	0907:	X: 3400
	0908: 34	
	0909: 00	
SP →	090A: 00	

E8.5 The results are given in the book.

E8.6 (a) LDAA $0202

This instruction uses extended addressing. The effective address is 0202. In Figure 8.11 we see that this location contains 1A. Thus, the content of the A register after this instruction is 1A. The content of X is not changed by this instruction.

(b) LDAA #$43

This instruction uses immediate addressing. The effective address is the one immediately following the op code. This location contains the hexadecimal digits 43. Thus the content of the A register after this instruction is 43. The content of X is not changed by this instruction.

(c) LDAA $05,X

This instruction uses constant offset indexed addressing. The effective address is the content of the X register plus the offset which is 05. Thus the effective address is 0205. In Figure 8.10 we see that this location contains FF. Thus, the content of the A register after this instruction is FF. The content of X is not changed by this instruction.

(d) LDAA $06
This instruction uses direct addressing. The effective address is 0006. In Figure 8.10 we see that this location contains 13. Thus, the content of the A register after this instruction is 13. The content of X is not changed by this instruction.

(e) LDAA $07,X-
This instruction uses auto post-decremented indexed addressing. The effective address is the content of the X register which is 0200. In Figure 8.10 we see that this location contains 10. Thus the content of the A register after this instruction is 10. Finally, the content of X is decremented by 07. Thus, X contains 01F9 after the instruction is completed.

(f) LDAA $05,+X
This instruction uses auto pre-incremented indexed addressing. The effective address is the content of the X register plus 05 which is 0205. In Figure 8.10 we see that this location contains FF. Thus the content of the A register after this instruction is FF. Register X contains 0205 after the instruction is completed.

E8.7 (a) Referring to Table 8.1 in the book, we see that CLRA is the clear accumulator A instruction with a single byte op code 87. Furthermore execution of this command sets the Z bit of the condition code register. The BEQ $15 command occupies two memory locations with the op code 27 in the first byte and the offset of 15 in the second byte. Thus starting in location 0200, the instructions appear in memory as:

    0200: 87
    0201: 27
    0202: 15

(b) When the instructions are executed, the CLRA command sets the Z bit. Then if the Z-bit was clear the next instruction would be the one starting in location 0203 following the BEQ $15 command. However since the Z bit is set the next instruction is located at 0203 + 15 = 0218.

E8.8 One answer is given in the book. Of course, other correct answers exist.

E8.9 One answer is given in the book. Of course, other correct answers exist.

Problems

P8.1 The functional parts of a computer are the central processing unit (consisting of the control unit and the arithmetic/logic unit), input/output, and memory.

P8.2 Tristate buffers act as switches that can be open or closed depending on a control signal. They allow data to be placed without conflict on a common bus by the CPU, by memory, or by I/O devices.

P8.3 Some examples of input/output devices are keyboards, display devices, and printers. An important category of input devices in control applications are sensors that convert temperatures, pressures, displacements, flow rates, and other physical values to digital form that can be read by the computer. Actuators are output devices that allow the computer to affect the system being controlled. Examples of actuators are valves, motors, and switches.

P8.4 I/O mapped I/O (also known as port mapped I/O) uses a separate, dedicated address space and is accessed via a dedicated set of microprocessor instructions.

P8.5 The collection of signal lines is called the computer's internal bus. There are three such bus lines -

1. Address Bus
2. Data Bus
3. Control Bus

P8.6 An embedded computer is part of another product, such as an automobile, printer, microwave oven, or dishwasher, that is not called a computer.

P8.7* The width of the address bus defines the memory addressing capability of a computer. For example, if the bus is *k* bits wide, the computer can address 2^k bits of memory.

P8.8 A microprocessor is a CPU contained on a single integrated-circuit chip. A microcomputer, such as a PC or a laptop, combines a microprocessor with memory and I/O chips. A microcontroller (MCU) combines all of the functions CPU, memory, buses, and I/O on a single chip and is optimized for embedded control applications.

P8.9 In computers with Harvard architecture, there are separate memories for data and instructions. If the same memory contains both data and instructions, we have von Neumann architecture.

P8.10 As the term is currently used, RAM is read and write memory (originally random access memory) that is based on semiconductor technology and is volatile (data are lost when the power is turned off). Two types of RAM are static and dynamic. Because it is volatile, RAM is generally not useful for storing programs in embedded computers.

P8.11* ROM is read-only memory. Some types are:

1. Mask-programmable ROM in which the data is written when the memory is manufactured.

2. Programmable read-only memory (PROM) in which data is written by special circuits that blow tiny fuses or not depending on whether the data bits are zeros or ones.

3. Erasable PROMs (EPROMs) that can be erased by exposure to ultraviolet light (through a window in the chip package) and rewritten using special circuits.

4. Electrically erasable PROMs (EEPROMs) that can be erased by applying proper voltages to the chip.

All types of ROM are nonvolatile and are used for program storage in embedded computers.

P8.12 Some types of mass storage devices are hard disks, flash memory, and CD-ROM or DVD-ROM disks.

P8.13 For storage of large amounts of data (many megabytes) the least expensive type of memory is mass storage.

P8.14 An address bus having a width of 32 bits can address $2^{32} = 4.29 \times 10^9$ memory locations.

P8.15* In the ignition control system for automobiles, we need to use ROM for the programs and fixed data, because ROM is nonvolatile. Some RAM would be needed for temporary data such as air temperature and throttle setting. Presumably many units would be needed for mass production and mask programmable ROM would be least expensive.

P8.16 EEPROM is preferable whenever we need to change the data frequently as in system development.

P8.17 RAM is volatile. ROM and mass storage are nonvolatile.

P8.18 Elements of MCU-based control systems include analog and digital actuators, sensors, operator input devices, and displays, as well as the microcontroller.

P8.19 A sensor produces an analog or digital signal that is related to a physical variable. Examples are a temperature sensor that produces an output voltage proportional to temperature, a limit switch that produces a logic signal which is high when a robot arm reaches its extreme position, a pressure sensor that produces a voltage proportional to pressure, or a digital flow meter that produces a digital word whose value is proportional to volumetric flow rate.

P8.20 Actuators produce physical outputs in response to digital or analog electrical signals. Examples of digital actuators are switches or valves that are either on or off depending on the logic value of their control signals. Motors of various types produce torque, rotational speed, or rotational position in response to electrical signals. Electrical heaters are used to adjust the temperature of reactants in chemical processes.

P8.21* A digital sensor produces a logic signal or a digital word as its output. An analog sensor produces an analog output signal that varies continuously with the variable being measured.

P8.22 Washing machines, clothes dryers, microwave ovens, refrigerators, ovens, clocks, radios, garage-door openers, heating-system controls, bread machines, and so forth.

P8.23 Mechanical engineering:

Automotive applications such as ignition, anti-skid brakes, and adaptive suspension.

Automated manufacturing applications.

Materials testing.

Chemical engineering:

Process control.

Instruments for pH, temperature, flow rate, and pressure.

Civil engineering:

Traffic control.

Surveying instrumentation.

Building ventilation, heating, and lighting.

Aeronautical engineering:

Navigation.

Flight control.

Instrumentation such as wind speed, ground speed, engine performance, and collision avoidance.

P8.24 An A/D is an analog-to-digital converter that converts samples of an analog signal into a sequence of digital words. They are needed to convert signals from analog sensors into digital form so they can be processed by a digital computer.

P8.25* A D/A is a digital-to-analog converter that converts a sequence of digital words into an analog signal. They are needed when an analog actuator must be controlled by a digital computer.

P8.26 Polling is the process of periodically checking digital input signals to a microprocessor to determine when actions are needed.

An interrupt is caused by a high logic value applied to the interrupt pin. Then, the MCU stops its current activity and starts a program called an interrupt handler.

The main advantage of interrupts compared to polling is that the MCU does not waste time checking input signals.

P8.27 The A, B, and D accumulators are registers that hold one of the arguments and the results of all arithmetic and logical operations. The register can be accessed either as two 8-bit registers (A and B) or as a single 16-bit register (D). A is the upper (most significant) 8 bits of D, and B is the lower (least significant) 8 bits of D.

P8.28 The program counter (PC) is a 16-bit register that contains the address of the first byte of the next instruction to be fetched (read) from memory by the control unit.

The condition-code register (C) is an 8-bit register in which each bit depends on a condition of the processor or on the result of a previous operation. The details of the condition-code register are shown in Figure 8.6 in the text.

P8.29* A stack is a sequence of locations in memory used to store information such as the contents of the program counter and other registers when a subroutine is executed or when an interrupt occurs. The SP is an internal register that points to the top of the stack.

The SP points to the top of the stackwhereas the PC points to the next instruction to be executed in the program sequence. They work together during function calls, when the current PC is storedon the stack loactions SP and SP+1 so that the PC may point to the subroutine'sfirst insrtuction.

P8.30 For microprocessors using the pipeline structure, the FIFO structure memory is better due to its faster performance and reduced instruction cycle time.

P8.31* Initially, we have:

A: 07	0048: 00
B: A9	0049: 00
SP: 004F	004A: 00

X: 34BF	004B: 00
	004C: 00
	004D: 00
	004E: 00
	004F: 00

After the command PSHA, we have:

A: 07	0048: 00
B: A9	0049: 00
SP: 004E	004A: 00
X: 34BF	004B: 00
	004C: 00
	004D: 00
	004E: 07
	004F: 00

After the command PSHB, we have:

A: 07	0048: 00
B: A9	0049: 00
SP: 004D	004A: 00
X: 34BF	004B: 00
	004C: 00
	004D: A9
	004E: 07
	004F: 00

After the command PSHX, we have:

A: A9	0048: 00
B: 07	0049: 00
SP: 004B	004A: 00
X: 34BF	004B: 34
	004C: BF
	004D: A9
	004E: 07
	004F: 00

After the command PULB, we have:

A: 07	0048: 00
B: 34	0049: 00
SP: 004C	004A: 00
X: 34BF	004B: 34
	004C: BF
	004D: A9
	004E: 07
	004F: 00

After the command PULA, we have:

A: BF	0048: 00
B: 34	0049: 00
SP: 004D	004A: 00
X: 34BF	004B: 34
	004C: BF
	004D: A9
	004E: 07
	004F: 00

P8.32 Initially, we have:

A: A7	0048: 00
B: 69	0049: 00
SP: 004E	004A: 00
Y: B804	004B: 00
	004C: 00
	004D: 00
	004E: 00
	004F: 00

After the command PSHY, we have:

A: A7	0048: 00
B: 69	0049: 00
SP: 004C	004A: 00
Y: B804	004B: 00
	004C: B8
	004D: 04
	004E: 00
	004F: 00

After the command PSHB, we have:

A: A7	0048: 00
B: 69	0049: 00
SP: 004B	004A: 00
Y: B804	004B: 69
	004C: B8
	004D: 04
	004E: 00
	004F: 00

After the command PULY, we have:

A: A7	0048: 00
B: 69	0049: 00
SP: 004D	004A: 00
Y: 69B8	004B: 69
	004C: B8
	004D: 04
	004E: 00
	004F: 00

After the command PSHA, we have

A: A7	0048: 00
B: 69	0049: 00
SP: 004C	004A: 00
Y: 69B8	004B: 69
	004C: A7
	004D: 04
	004E: 00
	004F: 00

P8.33* A sequence of instructions that results in swapping the high and low bytes of the X register is:

```
PSHA    ;save the original content of A
PSHB    ;save the original content of B
PSHX    ;put content of X on the stack
PULA    ;pull upper byte of X into A
PULB    ;pull lower byte of X into B
PSHA    ;push upper byte of X onto stack
PSHB    ;push lower byte of X onto stack
```

```
        PULX        ;pull X with upper and lower bytes interchanged
        PULB        ;restore original content of B
        PULA        ;restore original content of A
```

Comments (following the semicolon on each line) explain the effect of each command.

P8.34 For each part of this problem, the initial contents of the registers are A:01 and X:2000. The contents of various memory locations are shown in Figure P8.34 in the book.

*(a) LDAA	$2002	Extended addressing	A:20
(b) LDAA	#$43	Immediate addressing	A:43
*(c) LDAA	$04	Direct addressing	A:9A
(d) LDAA	6,X	Indexed addressing	A:FF
*(e) INCA		Inherent addressing	A:02
(f) CLRA		Inherent addressing	A:00
*(g) LDAA	$2007	Extended addressing	A:F3
(h) INX		Inherent addressing	A:01 (unchanged)

P8.35

Part	Name of Addressing	(D) = (A):(B)	(X)	(Y)
(a) ADDB $1002,Y	Constant-offset IDX	$0023	$1FFF	$1000
(b) LDAA B,X	Accumulator-offset IDX	$2003	$1FFF	$1000
(c) LDAB 7,+X	Auto pre-incremented IDX	$00FF	$2006	$1000
(d) LDX [$1004,Y]	Indexed indirect	$0003	$37AF	$1000
(e) LDAA [D,X]	Indexed indirect	$F303	$1FFF	$1000

P8.36 *(a) The A register contains FF. Because the most significant bit is 1, the TSTA command results in the N bit of the condition code register being set (i.e., N = 1). Thus the BMI $07 instruction results in a branch. Instead of executing the instruction following the BMI command in location 2003, the next instruction to be executed is 2003 plus the offset, which is 7. Thus the command starting at location 2003 + 7 = 200A is the next to be executed after the BMI $07 command.

(b) The A register initially contains FF. The INCA instruction causes the content of the A register to become zero. Because the most significant bit is 0, the N bit of the condition code register is clear (i.e., N = 0). Thus the BMI $11 instruction does not result in a branch, and the next instruction to be executed is in location 2003.

(c) The A register contains FF. Because the most significant bit is 1, the TSTA command results in the N bit of the condition code register being set (i.e., N = 1). Thus the BMI -$0A instruction results in a branch. Instead of executing the instruction following the BMI command in location 2003, the next instruction to be executed is 2003 plus the offset, which is -0A. Thus, the command starting at location 2003 - A = 1FF9 is the next to be executed after the BMI -$0A command.

P8.37

	Mnemonics	Machine codes
*(a)	INCB	42
*(b)	ADDD $4A	D3 4A
(c)	BLO $05	25 05
(d)	NOP	A7
(e)	ADDD #$E0	D3 E0

P8.38

Instruction sequence	Content of A after execution	Z	N
LDAA #$05	05	0	0
ADDA #$E5	EB	0	1
CLRA	00	1	0

P8.39* (a) $(97)' = (1001\ 0111)' = (0110\ 1000) = 68$
(b) $A + B = F0 + 0F = FF$

P8.40* The MUL instruction multiplies the content of A by that of B with the result in D. The contents of A and B are treated as unsigned integers. The most significant byte of the product resides in A, and the least significant byte resides in B.

First let us carry out the multiplication in decimal form. We have A7 = 167_{10} and 20 = 32_{10}. Then $167_{10} \times 32_{10} = 5344_{10}$. Converting the result to a 4-digit hexadecimal number we have A7 $\times$ 20 = 14E0. Thus the content of the A and B registers are (A) =14 and (B) = E0.

P8.41* The program listing is:

```
; ANSWER FOR PROBLEM 8.41
        ORG     0200        ;Directive to begin in location 0200
        LDAB    #$0B        ;0B is hex equivalent of decimal 11
        MUL                 ;compute product
        STD     $FF00       ;store result
        STOP
        END
```

P8.42 The program listing is:

```
;AN ANSWER FOR PROBLEM P8.42
        ORG     $0400
        CLAA
        STAA    $0800
        INCA
        STAA    $0801
        INCA
        STAA    $0802
        INCA
        STAA    $0803
        STOP
        END
```

P8.43* This subroutine first clears register B which will hold the quotient after the program has been executed. Then 3 is repeatedly subtracted from the content of A, and the content of B is incremented until the value in A is negative.

```
;DIVIDE BY 3
;
DIV3     CLRB
LOOP     SUBA    #$03    ;subtract 3 from content of A.
         BMI     END     ;quit if result is negative
         INCB
         JMP     LOOP
END      ADDA    #$03    ;restore remainder to register A
         RTS
```

P8.44

Memory Location	Content of Location	Label	Mnemonic	Operand	Description
			ORG	$800	Directive to begin program in location $800
800	86	START	LDAA	$07	Load A using immediate addressing
801	07				Continuation
802	C6		LDAB	$06	Load B using immediate addressing
803	06				Continuation
804	9B		ADDB	$03	Add memory to B
805	03				Continuation
806	18		STOP		Halt Processor
807	3E				Continuation

The contents of any other locations are indeterminate.

P8.45 The program is:

```
; Answer for Problem P8.45
;
MUL3      STAA     $0A        ;save original content of A
LOOP      ADDA     #$FD       ;add -3  to content of A
           BMI     OUT        ;exit loop if result is negative
          JMP      LOOP
;
;   On exit from the loop, A contains in two's complement form:
;         -3 if A initially contained an integer multiple of 3
;         -2 if A initially contained 1 plus an integer multiple of 3
;         -1  if A initially contained 2 plus an integer multiple of 3
;
OUT       INCA
;
; Branch to ADD1 if A initially contained 2 plus an integer multiple of 3:
;
          BEQ      ADD1
          INCA
;
; Branch to SUB1 if A initially contained 1 plus an integer multiple of 3:
;
          BEQ      SUB1
;
; Restore original content of A and return if A initially contained an
;  integer multiple of 3:
;
          LDAA     $0A
JMP       RETURN
ADD1      LDAA     $0A
          INCA
          JMP      RETURN
SUB1      LDAA     $0A
          ADDA     #$FF       ;add -1 to original content of A
RETURN    RTS
```

P8.46

```
STAB $001A
LDAA #$10
MUL
STAA $001B
LDAA #$10
MUL
STAA $001C
LDAB $001B
LDAA #$0A
MUL
ADDD $001C
```

Practice Test

T8.1 **a.** 11, **b.** 17, **c.** 21, **d.** 24, **e.** 27, **f.** 13, **g.** 26, **h.** 9, **i.** 20, **j.** 12, **k.** 15, **l.** 16, **m.** 8, **n.** 29, **o.** 23, **p.** 30.

T8.2 **a.** direct, 61; **b.** indexed, F3; **c.** inherent, FF; **d.** inherent, 01; **e.** immediate, 05; **f.** immediate, A1.

T8.3 Initially, we have:

A: A6	1034: 00
B: 32	1035: 00
SP: 1039	1036: 00
X: 1958	1037: 00
	1038: 00
	1039: 00
	103A: 00
	103B: 00
	103C: 00

After the command PSHX, we have:

A: A6	1034: 00
B: 32	1035: 00
SP: 1037	1036: 00
X: 1958	1037: 19
	1038: 58
	1039: 00
	103A: 00
	103B: 00
	103C: 00

After the command PSHB, we have:

A: A6	1034: 00
B: 32	1035: 00
SP: 1036	1036: 32
X: 1958	1037: 19
	1038: 58
	1039: 00
	103A: 00
	103B: 00
	103C: 00

After the command PULA, we have:

A: 32	1034: 00
B: 32	1035: 00
SP: 1037	1036: 32
X: 1958	1037: 19
	1038: 58
	1039: 00
	103A: 00
	103B: 00
	103C: 00

After the command PSHX, we have:

A: 32	1034: 00
B: 32	1035: 19
SP: 1035	1036: 58
X: 1958	1037: 19
	1038: 58
	1039: 00
	103A: 00
	103B: 00
	103C: 00

CHAPTER 9

Exercises

E9.1 The equivalent circuit for the sensor and the input resistance of the amplifier is shown i Figure 9.2 in the book. Thus the input voltage is

We want the input voltage with an internal sensor resistance of 10 kΩ to be at least 0.995 times the input voltage with an internal sensor resistance of 5 kΩ. Thus with resistances in kΩ, we have

$$v_{sensor}\frac{R_{in}}{10+R_{in}} \geq 0.995 v_{sensor}\frac{R_{in}}{5+R_{in}}$$

Solving, we determine that R_{in} is required to be greater than 990 kΩ.

E9.2 (a) A very precise instrument can be very inaccurate because precision implies that the measurements are repeatable, however they could have large bias errors.

(b) A very accurate instrument cannot be very imprecise. If repeated measurements vary a great deal under apparently identical conditions, some of the measurements must have large errors and therefore are inaccurate.

E9.3 $v_d = v_1 - v_2 = 5.7 - 5.5 = 0.2\text{ V} \qquad v_{cm} = \frac{1}{2}(v_1 + v_2) = \frac{1}{2}(5.5 + 5.7) = 5.6\text{ V}$

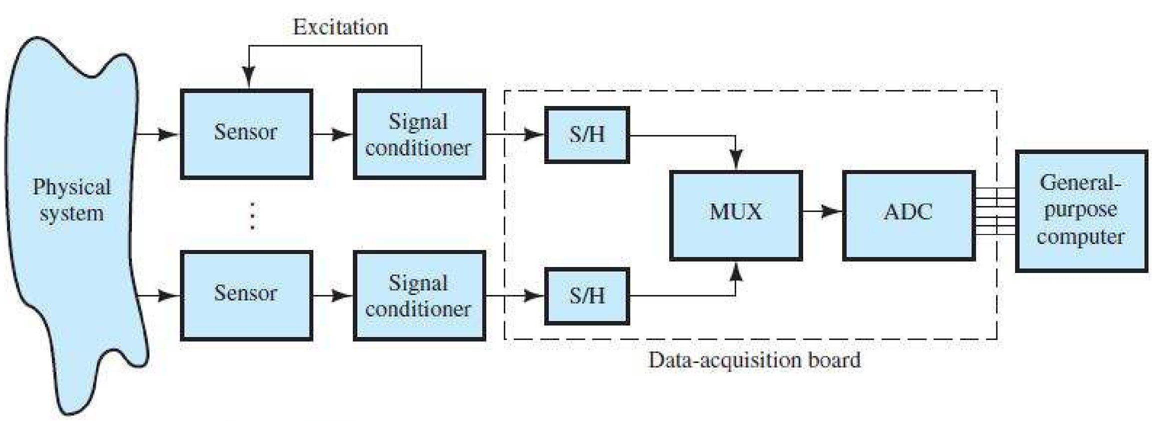
E9.4 The range of input voltages is from -5 V to +5 V or 10 V in all. We have $N = 2^k = 2^8 = 256$ zones. Thus the width of each zone is $\Delta = \frac{10}{N} = 39.1$ mV. The quantization noise is approximately $N_{qrms} \cong \frac{\Delta}{2\sqrt{3}} = 11.3$ mV.

E9.5 Look at Figure 9.14 in the book. In this case, we have f_s = 30 kHz and f = 25 kHz. Thus, the alias frequency is f_{alias} = f_s - f = 5 kHz.

E9.6 The file containing the vi is named Figure 9.17.vi and can be found on the CD that accompanies this book.

Problems

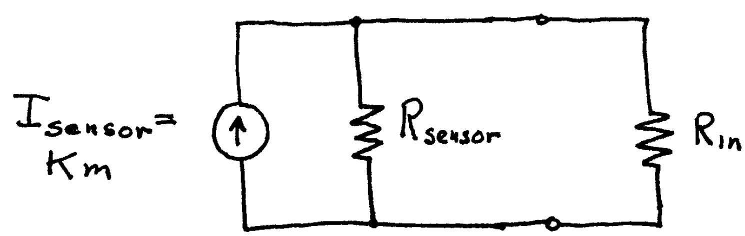
P9.1 The figure shows a computer-based system for instrumentation of a physical system such as an automobile or chemical process. Physical phenomena such as temperatures, angular speeds, etc., produce changes in the voltages, currents, resistances, capacitances, or inductances of the sensors. If the sensor output is not already a voltage, signal conditioners provide an excitation source that transforms the changes in electrical parameters to voltages. Furthermore, the signal conditioner amplifies and filters these voltages. The conditioned signals are input to a data-acquisition (DAQ) board. On the DAQ board, each of the conditioned signals is sent to a sample-and-hold circuit (S/H) that periodically samples the signal and holds the value steady while the multiplexer (MUX) connects it to the analog- to- digital converter (A/D or ADC) that converts the values to digital words. The words are read by the computer, which then processes the data further before storing and displaying the results



Computer-based DAQ system.

P9.2* The equivalent circuit of a sensor is shown in Figure 9.2 in the book. Loading effects are caused by the voltage drop across R_{sensor} that occurs when the input resistance of the amplifier draws current from the sensor. Then the input voltage to the amplifier (and therefore overall sensitivity) depends on the resistances as well as the internal voltage of the sensor. To avoid loading effects, we need to have R_{in} much greater than R_{sensor}.

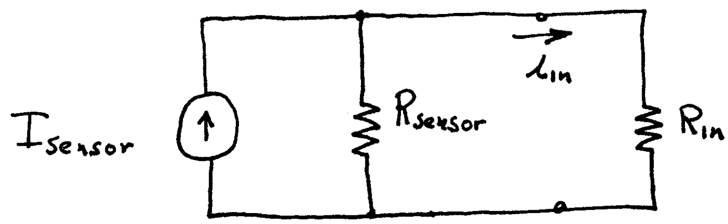
P9.3 The equivalent circuit for a sensor for which the short-circuit current is proportional to the measurand is



In this case we want the short-circuit current to flow into the amplifier (which is often a current-to-voltage converter). Thus we need to have R_{in} much smaller than R_{sensor}.

P9.4* Sensors used for Temperature measurements are: Thermistors, Thermocouples, Diode Thermometers and RTD's, for Pressure measurement: Capacitive sensors, Bourdon tube and LVDT combine; for Velocity

P9.5* We need a current-to-voltage converter that converts the short-circuit current into a voltage that can be applied to an analog-to-digital converter. The equivalent circuit is:



The input current to the converter is

$$i_{in} = I_{sensor} \frac{R_{sensor}}{R_{sensor} + R_{in}}$$

Furthermore we want the overall sensitivity to change by no more than 1% as R_{sensor} varies from 10 kΩ to 1 MΩ. Thus, with resistances in kΩ, we require

$$\frac{10}{10 + R_{in}} \geq 0.99 \frac{1000}{1000 + R_{in}}$$

Solving we find that we need $R_{in} \leq 102\ \Omega$.

P9.6 Bias errors are the same each time a measurement is repeated under apparently identical conditions. Random errors vary between repeated measurements.

P9.7 Bias errors include offset, scale error, nonlinearity, and hysteresis, which are illustrated in Figure 9.5 in the book.

P9.8* Because the accuracy is stated as ±0.2% of full scale which is 200 miles, the maximum error is ±0.4 miles. Thus if the measured value is 140 miles, the true value lies in the range from 139.6 miles to 140.4 miles.

P9.9 Accuracy is the maximum expected difference in magnitude between measured and true values (often expressed as a percentage of the full-scale value).

Precision is the ability of the instrument to repeat the measurement of a constant measurand.

Resolution is the smallest possible increment discernible between measured values.

P9.10* (a) Instrument *B* is the most precise because the repeated measurements vary the least. Instrument *A* is the least precise.

(b) Instrument *C* is the most accurate because the maximum error of its measurements is least. Instrument *A* is the least accurate because it has the larges maximum error.

(c) Instrument *C* has the best resolution, and instrument *A* has the worst resolution.

P9.11 Signal conditioners are an important and integral part of electronic instrumentation systems. Many quantities to be measured are non-electric and have to be converted to suit the requirements of electronic systems. The functions of signal conditioners include amplification, conversion of currents to voltages, supply of (ac or dc) excitations to the sensors so changes in resistance, inductance, or capacitance are converted to changes in voltage, and filtering to eliminate noise or other unwanted signal components.

P9.12 Differntial amplifier can attend to both inverting and non-inverting amplifiers. Also it can eliminate common mode signals with a proper design with high CMRR.

P9.13 If the input voltages are equal, the differential input voltage is zero. Thus the output of an ideal differential amplifier is zero.

P9.14*

$$v_d = v_1 - v_2 = 0.004 \text{ V}$$

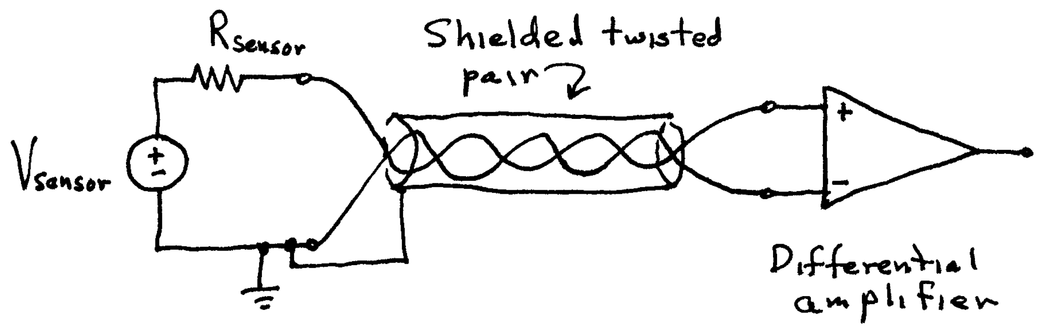
$$v_{cm} = \tfrac{1}{2}(v_1 + v_2) = 5\cos(\omega t) \text{ V}$$

$$v_o = A_d v_d = 4 \text{ V}$$

P9.15 Refer to Figure 9.7 in the book. We can write $v_1 = v_{cm} + \frac{1}{2}v_d$ and $v_2 = v_{cm} - \frac{1}{2}v_d$. The common mode signal is a 2-V rms 60-Hz sine wave which is given as $v_{cm} = 2\sqrt{2}\cos(2\pi 60t)$ in which we have assumed that the phase angle is zero. The differential signal is $v_d = 0.006$ V. Thus we have

$$v_1 = v_{cm} + \tfrac{1}{2} v_d = 0.003 + 2\sqrt{2}\cos(2\pi t)\ \text{V}$$
$$v_2 = v_{cm} - \tfrac{1}{2} v_d = -0.003 + 2\sqrt{2}\cos(2\pi t)\ \text{V}$$

P9.16* To avoid ground loops, we must not have grounds at both ends of the 5-m cable. Because the sensor is grounded, we need to use a differential amplifier. To reduce interference from magnetic fields, we should use a twisted pair or coaxial cable. To reduce interference from electric fields we should choose a shielded cable and connect the shield to ground at the sensor. A schematic diagram of the sensor, cable and amplifier is:



P9.17 An Ungrounded sensor is also known as floating sensor. Floating sensors are used when the sensor and signal conditioning units are located apart and various grounding points are involved which may not be at the same potential due to small circulating currents in ground wires.

P9.18* 60-Hz interference can be caused by magnetic fields linked with the sensor circuit. We could try a coaxial or twisted pair cable and/or move the sensor cable away from sources of 60-Hz magnetic fields such as transformers.

Another possibility is that the interference could be caused by a ground loop which we should eliminate.
Also electric field coupling is a possibility, in which case we should use a shielded cable with the shield grounded.

P9.19 In principle, analog-to-digital conversion consists of first sampling (i.e., measuring) the signal amplitude at periodically spaced points in time, and second representing the approximate amplitude of each sample by a digital word.

P9.20 Aliasing is a condition in which a component of a sampled signal has a frequency different from the frequency of the corresponding component before sampling. It occurs anytime a signal component has a frequency greater than half of the sampling frequency.

P9.21 Quantization noise occurs because finite length digital words can represent signal amplitudes only approximately. If we reconstruct a signal using the output of an analog-to-digital converter, the reconstruction is slightly different from the original signal. The difference is quantization noise.

P9.22* The minimum sampling rate is twice the highest frequency. Thus in theory the sampling rate should be at least 60 kHz. In practice it is usually better to use at least three times the highest frequency. To achieve a resolution of 0.1%, we need an analog-to-digital converter with at least 1000 steps. Because the number of steps is an integer power of two we should use a 10-bit converter.

P9.23* (a) The converter has 2^8 steps, and the width of each step is $\Delta = 10/2^8 = 0.03906$ V = 39 mV

(b) The rms quantization noise is $N_{qrms} = \Delta/(2\sqrt{3}) = 0.01127 = 11.275\ m\text{V}$ This noise voltage would deliver

$$P_q = (11.275 \times 10^{-3})^2/R = (127 \times 10^{-6})/R \text{ watts}$$

to a resistor of value R.

(c) The power delivered by a 3-V peak sinewave is

$$P_s = (3/\sqrt{2})^2/R = 4.5/R \text{ watts}$$

(d) The signal-to-noise ratio is

$$\text{SNR}_{\text{dB}} = 10\log\left(\frac{P_s}{P_q}\right) = 45.489 \text{ dB}$$

P9.24* We need $N = Range / \Delta = 10/0.01 = 1000$ zones. Because $N = 2^k$ we require a $k = 10$ bit ADC.

We need $N = Range / \Delta = 10/0.02 = 500$ zones. Because $N = 2^k$ we require a $k = 9$ bit ADC

P9.25* Refer to Figure 9.14 in the book. For $f = 10$ kHz, the sampling frequency must be greater than 20 kHz to avoid aliasing. For each sampling frequency the apparent frequency of the samples is

(a) 1 kHz (aliasing has occurred)

(b) 2 kHz (aliasing has occurred)

(c) 10 kHz (no aliasing)

P9.26 (a) We have

$$x(n) = A\cos(120\pi n T_s + \phi) = A\cos(\pi n / 3 + \phi)$$

$$x(n+3) = A\cos(\pi n / 3 + \pi + \phi)$$

Thus $x(n+3)$ is phase shifted by π radians with respect to $x(n)$, and we have $y(n) = \frac{1}{2}\left[x(n) + x(n+3)\right] = 0$.

(b) In this case, we have $y(n) = \frac{1}{2}\left[x(n) + x(n+3)\right] = V_{signal}$.

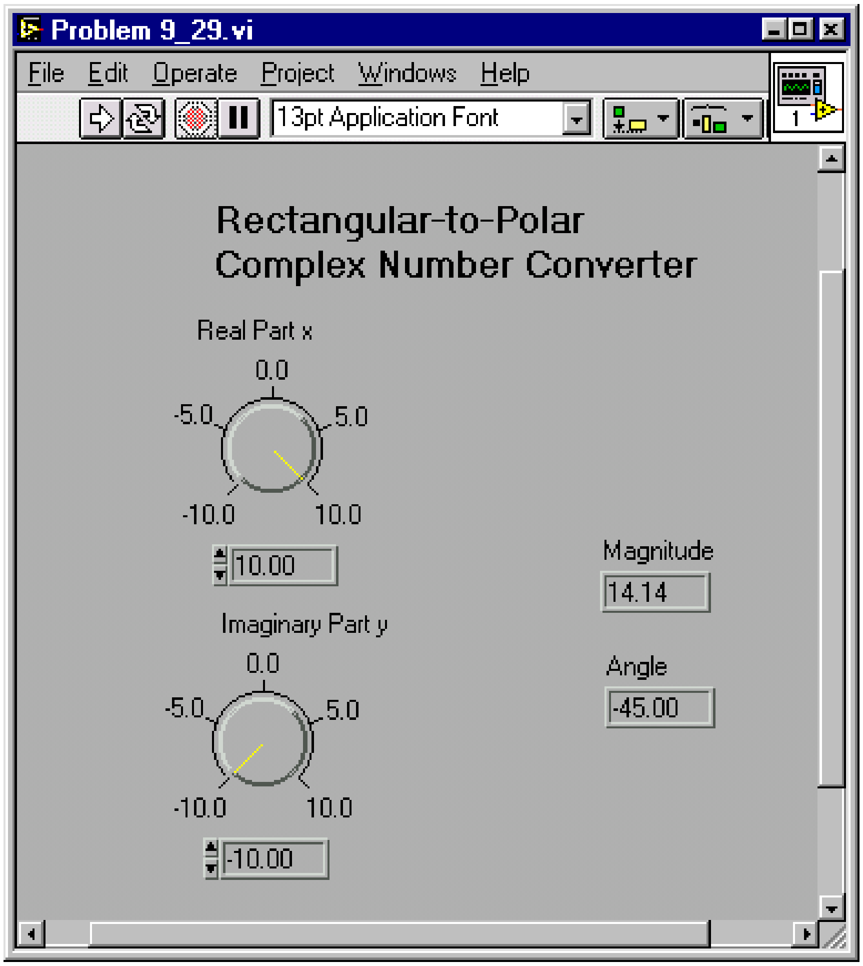
(c) This filter would be very good for removing 60-Hz interference due to electromagnetic field coupling or ground loops from a dc or low-frequency signal.

P9.27 LabVIEW stands for *Lab*oratory *V*irtual *I*nstrument *E*ngineering *W*orkbench.

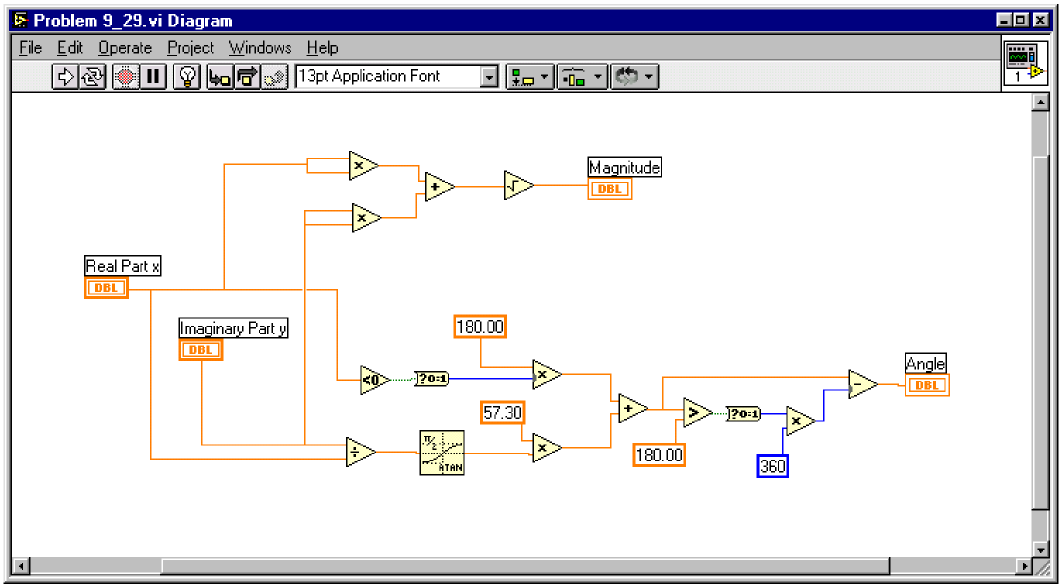
VI is a LabVIEW program which is based on a graphical tool,which uses an object-oriented graphical programming language i.e. programs are created by positioning and interconnecting icons on the computer screen which appear as front panel and block diagram to the programmer.

P9.28 LabVIEW uses data flow concepts. This means that the calculations associated with a specific block are not carried out until all of the input data are available to that block. Data flows in from sensors through various processing blocks to the display or, in the case of a controller, out through the DAQ board to actuators in the physical system.

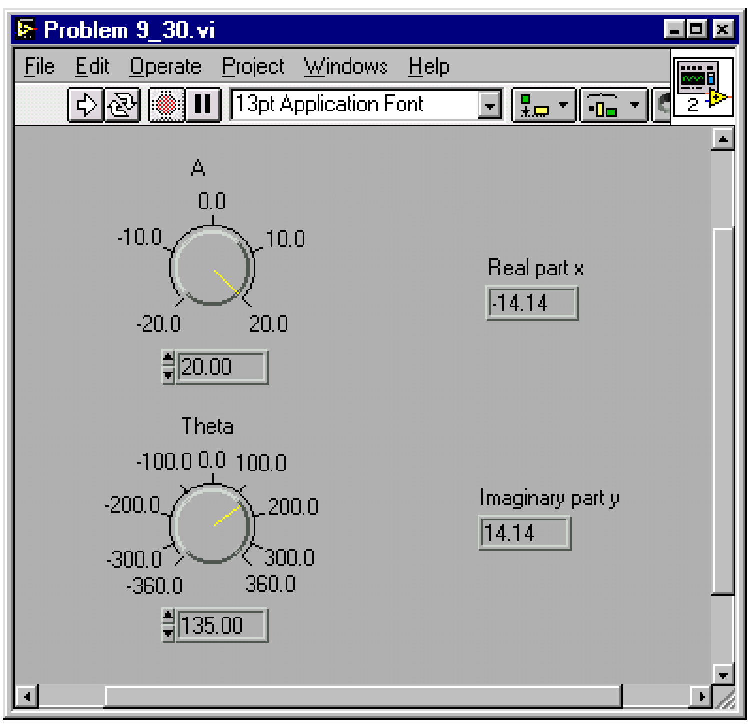
P9.29* The front panel is:



The magnitude of the complex number is computed as $A = \sqrt{x^2 + y^2}$. To find the angle, first we compute $\arctan(y/x)$ and convert the result to degrees by multiplying by 57.30. Then if x is negative, we add 180° which gives the correct angle. However we want the angle to fall between -180° and +180°. Thus if the angle is greater than 180°, we subtract 360°. The block diagram is:



P9.30 The front panel is:

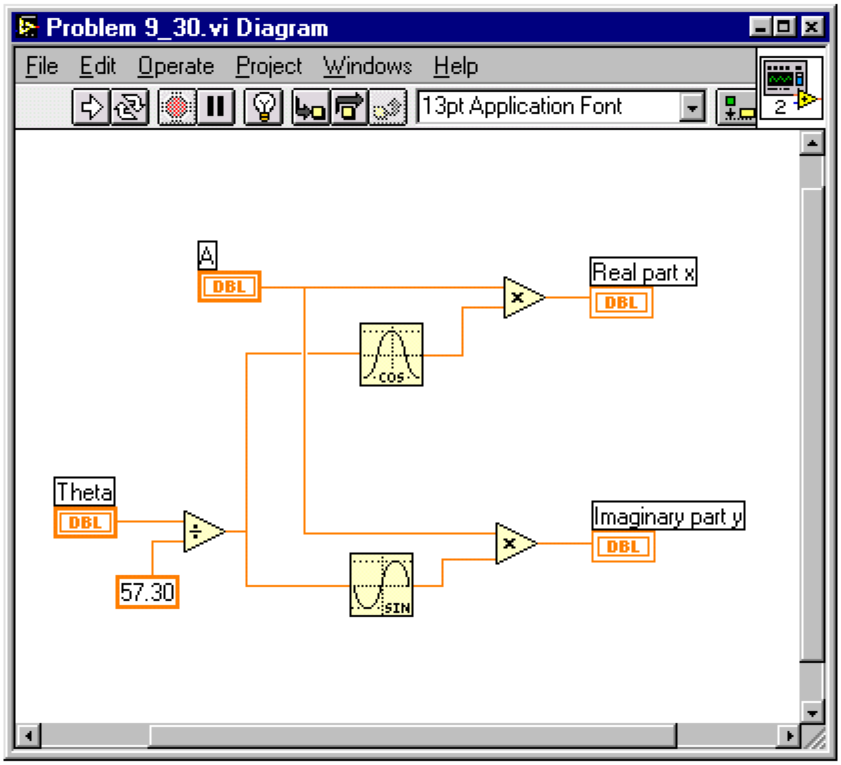


We compute the real and imaginary parts using the equations

$$x = A\cos(\theta)$$

$$y = A\sin(\theta)$$

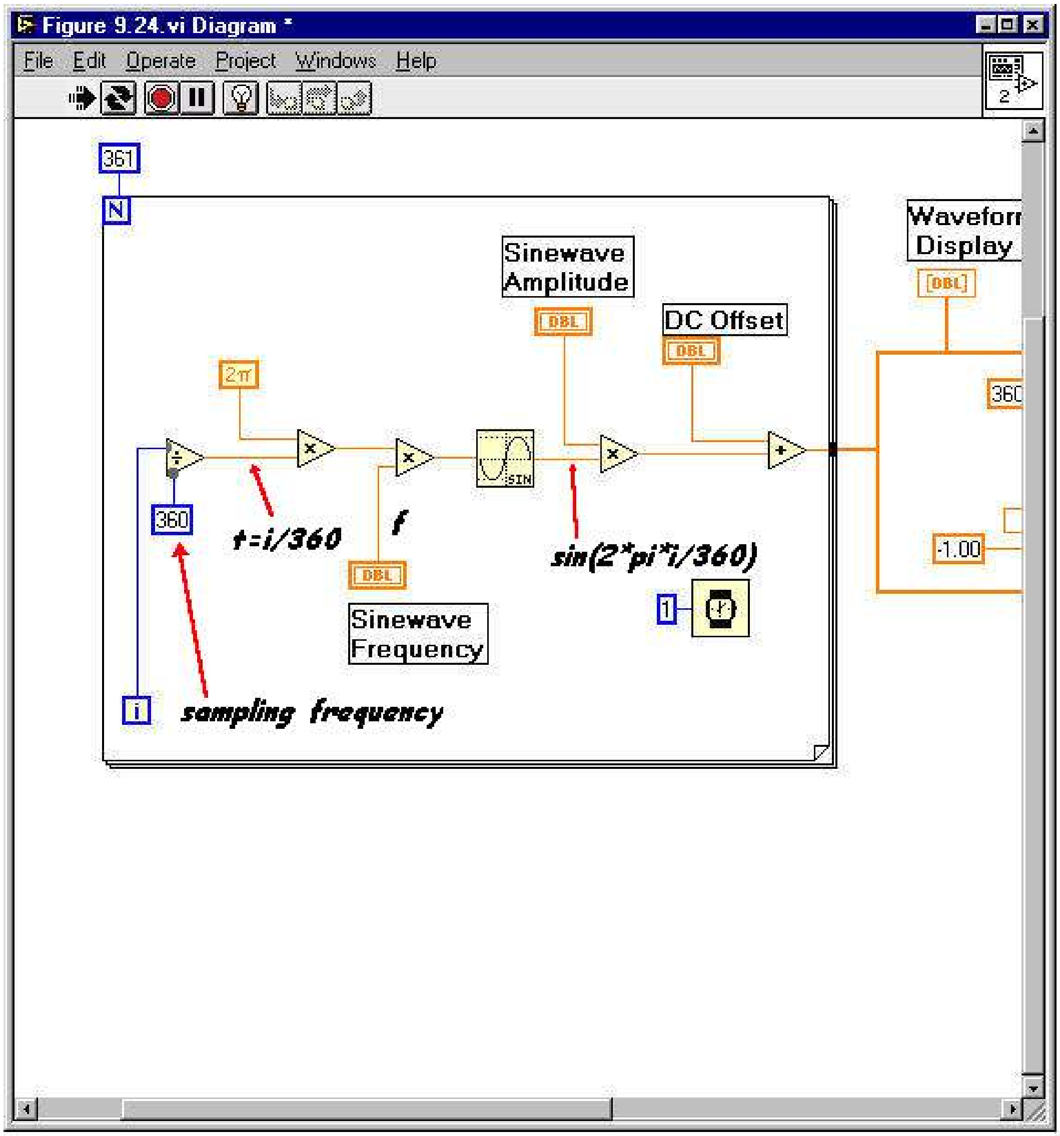
However the sin and cos functions in G, assume that the angles are in radians. Therefore we must convert the angles from degrees to radians. The block diagram is:



P9.31* We must keep in mind that G-programs deal with sampled signals even though they may appear to be continuous in time on the displays. In the virtual instrument Figure 9.24.vi, the sampling frequency for the sinewave is 360 samples per second. Thus we can expect aliasing if the frequency is greater than 180 Hz. Referring to Figure 9.14 in the book, we see that for each frequency f, the alias frequency is

f	f_{alias}
355	5
356	4
357	3
358	2
359	1
360	0
361	1
362	2
363	3
364	4

Thus as the frequency f increases, the apparent frequency decreases as we observe on the front panel. The partial block diagram and the quantities at various points are shown below:



P9.32 Many excellent answers exist.

Practice Test

T9.1. The four main elements are sensors, a DAQ board, software, and a general-purpose computer.

T9.2. The four types of systematic (bias) errors are offset, scale error, nonlinearity, and hysteresis.

T9.3. Bias errors are the same for measurements repeated under identical conditions, while random errors are different for each measurement.

T9.4. Ground loops occur when the sensor and the input of the amplifier are connected to ground by separate connections. The effect is to add noise (often with frequencies equal to that of the power line and its harmonics) to the desired signal.

T9.5. If we are using a sensor that has one end grounded, we should choose an amplifier with a differential input to avoid a ground loop.

T9.6. Coaxial cable or shielded twisted pair cable.

T9.7. If we need to sense the open-circuit voltage, the input impedance of the amplifier should be very large compared to the internal impedance of the sensor.

T9.8. The sampling rate should be more than twice the highest frequency of the components in the signal. Otherwise, higher frequency components can appear as lower frequency components known as aliases.

CHAPTER 10

Exercises

E10.1 Solving Equation 10.1 for the saturation current and substituting values, we have

$$\mathcal{I}_s = \frac{i_D}{\exp(v_D / nV_T) - 1}$$
$$= \frac{10^{-4}}{\exp(0.600/0.026) - 1}$$
$$= 9.502 \times 10^{-15} \text{ A}$$

Then for $v_D = 0.650$ V, we have

$$i_D = \mathcal{I}_s\left[\exp(v_D / nV_T) - 1\right] = 9.502 \times 10^{-15} \times \left[\exp(0.650/0.026) - 1\right]$$
$$= 0.6841 \text{ mA}$$

Similarly for $v_D = 0.800$ V, $i_D = 4.681$ mA.

E10.2 The approximate form of the Shockley Equation is $i_D = \mathcal{I}_s \exp(v_D / nV_T)$. Taking the ratio of currents for two different voltages, we have

$$\frac{i_{D1}}{i_{D2}} = \frac{\exp(v_{D1} / nV_T)}{\exp(v_{D2} / nV_T)} = \exp\left[(v_{D1} - v_{D2}) / nV_T\right]$$

Solving for the difference in the voltages, we have:

$$\Delta v_D = nV_T \ln(i_{D1} / i_{D2})$$

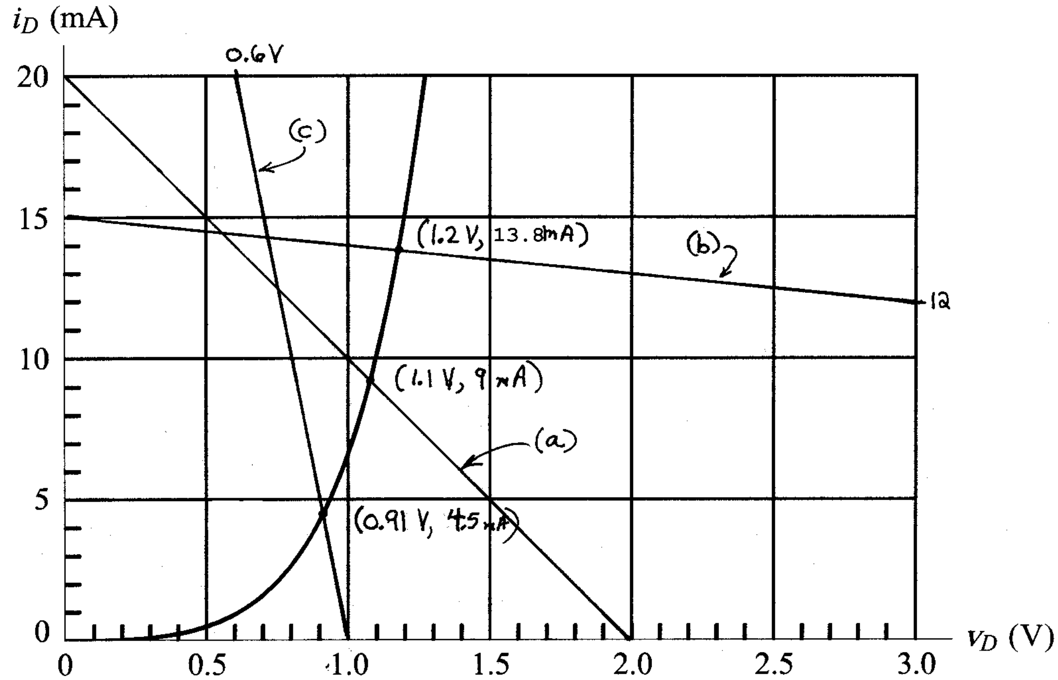
Thus to double the diode current we must increase the voltage by $\Delta v_D = 0.026\ln(2) = 18.02$ mV and to increase the current by an order of magnitude we need $\Delta v_D = 0.026\ln(10) = 59.87$ mV

E10.3 The load line equation is $V_{SS} = Ri_D + v_D$. The load-line plots are shown on the next page. From the plots we find the following operating points:

(a) $V_{DQ} = 1.1$ V $\quad \mathcal{I}_{DQ} = 9$ mA

(b) $V_{DQ} = 1.2$ V $\quad \mathcal{I}_{DQ} = 13.8$ mA

(c) $V_{DQ} = 0.91$ V $\quad \mathcal{I}_{DQ} = 4.5$ mA

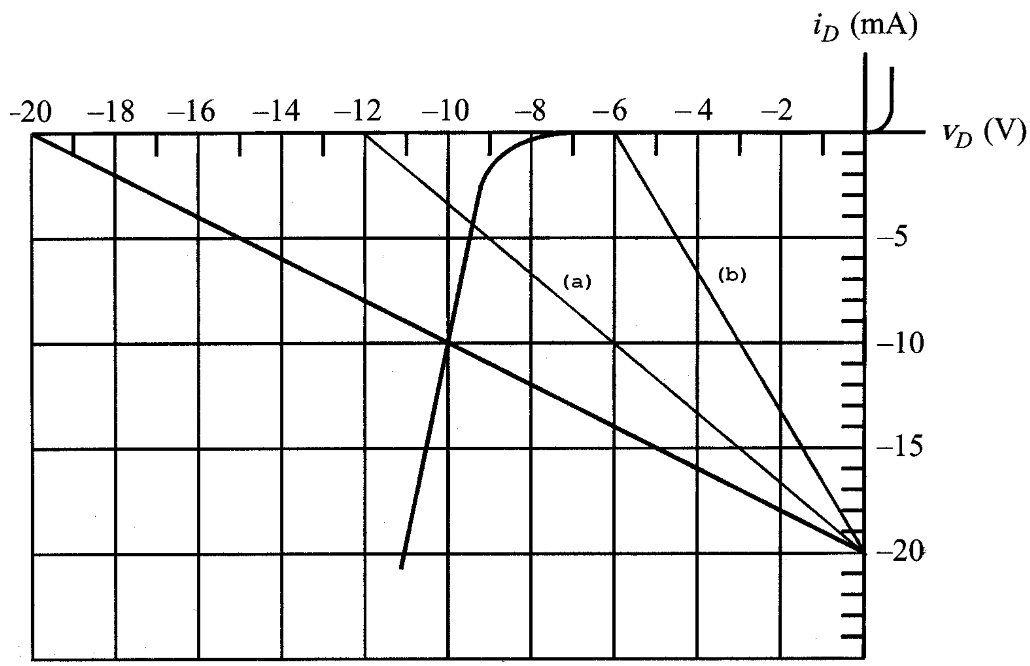


E10.4 Following the methods of Example 10.4 in the book, we determine that:

(a) For $R_L = 1200\ \Omega$, $R_T = 600\ \Omega$, and $V_T = 12$ V.

(b) For $R_L = 400\ \Omega$, $R_T = 300\ \Omega$, and $V_T = 6$ V.

The corresponding load lines are:

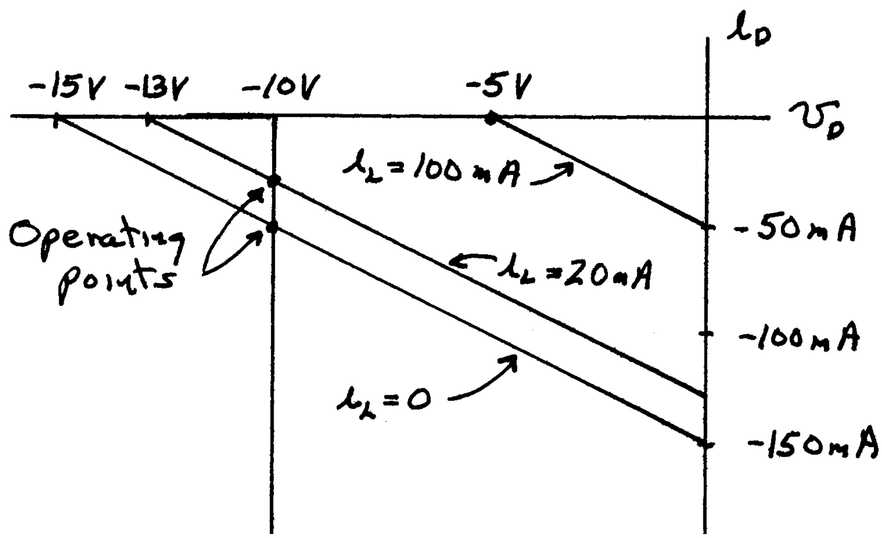


At the intersections of the load lines with the diode characteristic we find (a) $v_L = -v_D \cong 9.4$ V; (b) $v_L = -v_D \cong 6.0$ V.

E10.5 Writing a KVL equation for the loop consisting of the source, the resistor, and the load, we obtain:

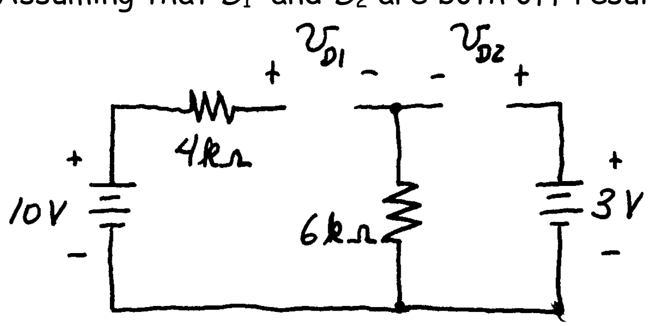
$$15 = 100(i_L - i_D) - v_D$$

The corresponding load lines for the three specified values of i_L are shown:



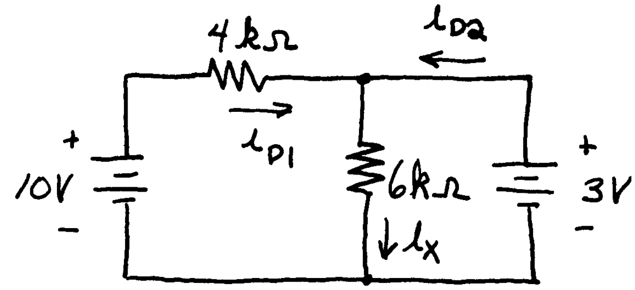
At the intersections of the load lines with the diode characteristic, we find (a) $v_o = -v_D = 10$ V; (b) $v_o = -v_D = 10$ V; (c) $v_o = -v_D = 5$ V. Notice that the regulator is effective only for values of load current up to 50 mA.

E10.6 Assuming that D_1 and D_2 are both off results in this equivalent circuit:



Because the diodes are assumed off, no current flows in any part of the circuit, and the voltages across the resistors are zero. Writing a KVL equation around the left-hand loop we obtain $v_{D1} = 10$ V, which is not consistent with the assumption that D_1 is off.

E10.7 Assuming that D_1 and D_2 are both on results in this equivalent circuit:

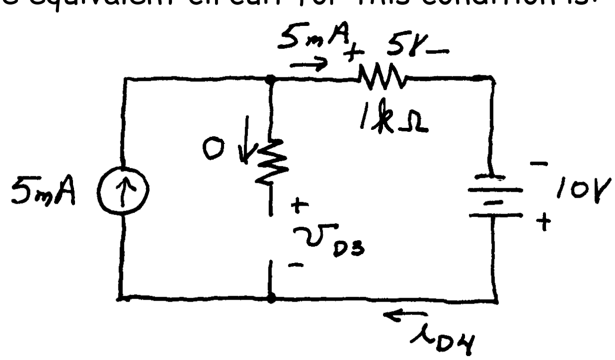


Writing a KVL equation around the outside loop, we find that the voltage across the 4-kΩ resistor is 7 V and then we use Ohm's law to find that i_{D1} equals 1.75 mA. The voltage across the 6-kΩ resistance is 3 V so i_x is 0.5 mA. Then we have $i_{D2} = i_x - i_{D1} = -1.25$ mA, which is not consistent with the assumption that D_2 is on.

E10.8 (a) If we assume that D_1 is off, no current flows, the voltage across the resistor is zero, and the voltage across the diode is 2 V, which is not consistent with the assumption. If we assume that the diode is on, 2 V appears across the resistor, and a current of 0.5 mA circulates clockwise which is consistent with the assumption that the diode is on. Thus the diode is on.

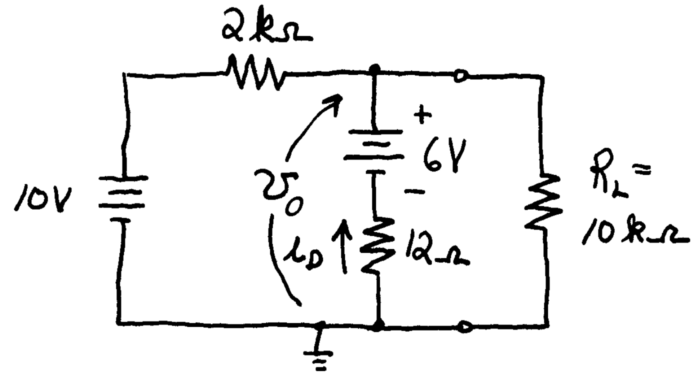
(b) If we assume that D_2 is on, a current of 1.5 mA circulates counterclockwise in the circuit, which is not consistent with the assumption. On the other hand, if we assume that D_2 is off we find that $v_{D2} = -3$ where as usual we have referenced v_{D2} positive at the anode. This is consistent with the assumption, so D_2 is off.

(c) It turns out that the correct assumption is that D_3 is off and D_4 is on. The equivalent circuit for this condition is:



For this circuit we find that $i_{D4} = 5$ mA and $v_{D3} = -5$ V. These results are consistent with the assumptions.

E10.9 (a) With R_L = 10 kΩ, it turns out that the diode is operating on line segment *C* of Figure 10.19 in the book. Then the equivalent circuit is:

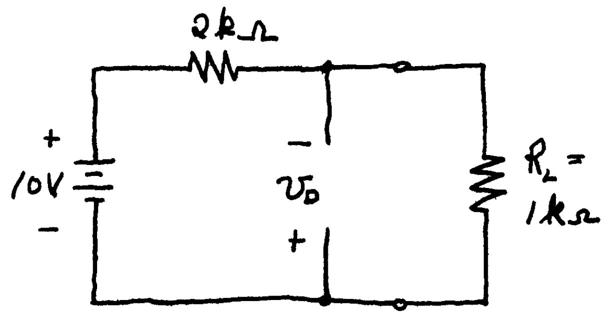


We can solve this circuit by using the node-voltage technique, treating v_o as the node voltage-variable. Notice that $v_o = -v_D$. Writing a KCL equation, we obtain

$$\frac{v_o - 10}{2000} + \frac{v_o - 6}{12} + \frac{v_o}{10000} = 0$$

Solving, we find $v_D = -v_o = -6.017$ V. Furthermore, we find that $i_D = -1.39$ mA. Since we have $v_D \le -6$ V and $i_D \le 0$, the diode is in fact operating on line segment *C*.

(b) With R_L = 1 kΩ, it turns out that the diode is operating on line segment *B* of Figure 10.19 in the book, for which the diode equivalent is an open circuit. Then the equivalent circuit is:



Using the voltage division principle, we determine that $v_D = -3.333$ V. Because we have $-6 \le v_D \le 0$, the result is consistent with the assumption that the diode operates on segment *B*.

E10.10 The piecewise linear model consists of a voltage source and resistance in series for each segment. Refer to Figure 10.18 in the book and notice that the *x*-axis intercept of the line segment is the value of the voltage source, and the reciprocal of the slope is the resistance. Now look at Figure 10.22a and notice that the intercept for segment *A* is zero and the reciprocal of the slope is (2 V)/(5 mA) = 400 Ω. Thus as shown in Figure 10.22b, the equivalent circuit for segment *A* consists of a 400-Ω resistance.

Similarly for segment *B*, the *x*-axis intercept is +1.5 V and the reciprocal slope is (0.5 mA)/(5 V) = 10 kΩ.

For segment *C*, the intercept is -5.5 V and the reciprocal slope is 800 Ω. Notice that the polarity of the voltage source is reversed in the equivalent circuit because the intercept is negative.

E10.11 Refer to Figure 10.25 in the book.

(a) The peak current occurs when the sine wave source attains its peak amplitude, then the voltage across the resistor is $V_m - V_B = 20 - 14 = 6$ V and the peak current is 0.6 *A*.

(b) Refer to Figure 10.25 in the book. The diode changes state at the instants for which $V_m \sin(\omega t) = V_B$. Thus we need the roots of $20\sin(\omega t) = 14$. These turn out to be $\omega t_1 = 0.7754$ radians and $\omega t_2 = \pi - 0.7754$ radians.

The interval that the diode is on is $t_2 - t_1 = \dfrac{1.591}{\omega} = \dfrac{1.591T}{2\pi} = 0.2532T$.

Thus the diode is on for 25.32% of the period.

E10.12 As suggested in the Exercise statement, we design for a peak load voltage of 15.2 V. Then allowing for a forward drop of 0.7 V we require $V_m = 15.9$ V. Then we use Equation 10.10 to determine the capacitance required. $C = (I_L T)/V_r = (0.1/60)/0.4 = 4167\ \mu$F.

E10.13 For the circuit of Figure 10.28, we need to allow for two diode drops. Thus the peak input voltage required is $V_m = 15 + V_r/2 + 2 \times 0.7 = 16.6$ V.

Because this is a full-wave rectifier, the capacitance is given by Equation 10.12. $C = (I_L T)/(2V_r) = (0.1/60)/0.8 = 2083\ \mu$F.

E10.14 Refer to Figure 10.31 in the book.

(a) For this circuit all of the diodes are off if $-1.8 < v_o < 10$. With the diodes off, no current flows and $v_o = v_{\text{in}}$. When v_{in} exceeds 10 V, D_1 turns on and D_2 is in reverse breakdown. Then $v_o = 9.4 + 0.6 = 10$ V. When v_{in} becomes less than -1.8 V diodes D_3, D_4, and D_5 turn on and $v_o = -3 \times 0.6 = -1.8$ V. The transfer characteristic is shown in Figure 10.31c.

(b)) For this circuit both diodes are off if $-5 < v_o < 5$. With the diodes off, no current flows and $v_o = v_{\text{in}}$.

When v_{in} exceeds 5 V, D_6 turns on and D_7 is in reverse breakdown. Then a current given by $i = \dfrac{v_{in} - 5}{2000}$ (i is referenced clockwise) flows in the circuit, and the output voltage is $v_o = 5 + 1000i = 0.5v_{\text{in}} + 2.5$ V

When v_{in} is less than -5 V, D_7 turns on and D_6 is in reverse breakdown. Then a current given by $i = \dfrac{v_{\text{in}} + 5}{2000}$ (still referenced clockwise) flows in the circuit, and the output voltage is $v_o = -5 + 1000i = 0.5v_{\text{in}} - 2.5$ V

E10.15 Answers are shown in Figure 10.32c and d. Other correct answers exist.

E10.16 Refer to Figure 10.34a in the book.

(a) If $v_{\text{in}}(t) = 0$, we have only a dc source in the circuit. In steady state, the capacitor acts as an open circuit. Then we see that D_2 is forward conducting and D_1 is in reverse breakdown. Allowing 0.6 V for the forward diode voltage the output voltage is -5 V.

(b) If the output voltage begins to fall below -5 V, the diodes conduct large amounts of current and change the voltage v_C across the capacitor. Once the capacitor voltage is changed so that the output cannot fall below -5 V, the capacitor voltage remains constant. Thus the output voltage is $v_o = v_{\text{in}} - v_C = 2\sin(\omega t) - 3$ V.

(c) If the 15-V source is replaced by a short circuit, the diodes do not conduct, $v_C = 0$, and $v_o = v_{\text{in}}$.

E10.17 One answer is shown in Figure 10.35. Other correct answers exist.

E10.18 One design is shown in Figure 10.36. Other correct answers are possible.

E10.19 Equation 10.22 gives the dynamic resistance of a semiconductor diode as $r_d = nV_T / I_{DQ}$.

I_{DQ} (mA)	r_d (Ω)
0.1	26,000
1.0	2600
10	26

E10.20 For the Q-point analysis, refer to Figure 10.42 in the book. Allowing for a forward diode drop of 0.6 V, the diode current is

$$I_{DQ} = \frac{V_C - 0.6}{R_C}$$

The dynamic resistance of the diode is

$$r_d = \frac{nV_T}{I_{DQ}}$$

the resistance R_p is given by Equation 10.23 which is

$$R_p = \frac{1}{1/R_C + 1/R_L + 1/r_d}$$

and the voltage gain of the circuit is given by Equation 10.24.

$$A_v = \frac{R_p}{R + R_p}$$

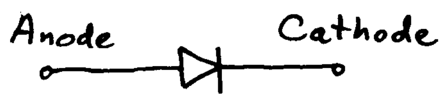
Evaluating we have

V_C (V)	1.6	10.6
I_{DQ} (mA)	0.5	5.0
r_d (Ω)	52	5.2
R_p (Ω)	49.43	5.173
A_v	0.3308	0.04919

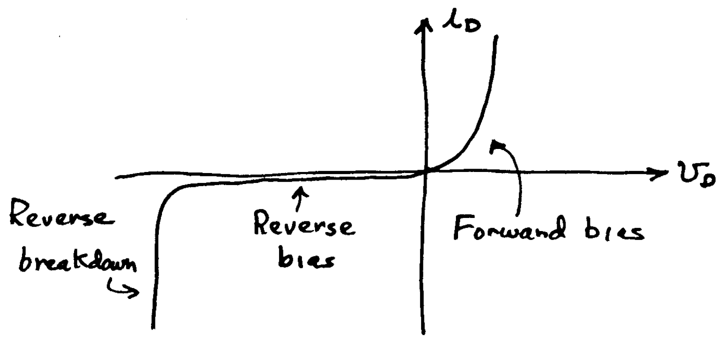
Problems

P10.1 A one-way valve that allows fluid to flow in one direction but not in the other is an analogy for a diode.

P10.2



P10.3



P10.4 The Shockley equation gives the diode current i_D in terms of the applied voltage v_D:

$$i_D = I_s\left[\exp\left(\frac{v_D}{nV_T}\right) - 1\right]$$

where I_s is the **saturation current**, and n is the **emission coefficient** which takes values between 1 and 2. The voltage V_T is the **thermal voltage** given by

$$V_T = \frac{kT}{q}$$

where T is the temperature of the junction in kelvins, $k = 1.38 \times 10^{-23}$ joules/kelvin is Boltzmann's constant, and $q = 1.60 \times 10^{-19}$ coulombs is the magnitude of the electrical charge of an electron.

P10.5 $V_T = kT/q = (1.38 \times 10^{-23}T)/(1.60 \times 10^{-19})$

Temperature °C	Absolute temperature	V_T (mV)
20	293	25.3
30	303	26.2

P10.6 For $v_D = 0.6$ V, we have

$$i_D = 0.5 \times 10^{-3} = I_s[\exp(v_D / nV_T) - 1]$$
$$\cong I_s \exp(v_D / nV_T)$$

Thus, we determine that:

$$I_s = \frac{i_D}{\exp(v_D / nV_T)} = 2.2651 \times 10^{-14} \text{ A}$$

Then, for $v_D = 0.7$ V, we have

$$i_D = I_s[\exp(v_D / nV_T) - 1] = 0.328 \text{ mA}$$

Similarly, for $v_D = 0.800$ V, we find $i_D = 1.7886$ mA.

P10.7* The approximate form of the Shockley Equation is $i_D = I_s \exp(v_D / nV_T)$.
Taking the ratio of currents for two different voltages, we have

$$\frac{i_{D1}}{i_{D2}} = \frac{\exp(v_{D1} / nV_T)}{\exp(v_{D2} / nV_T)} = \exp\left[(v_{D1} - v_{D2}) / nV_T\right]$$

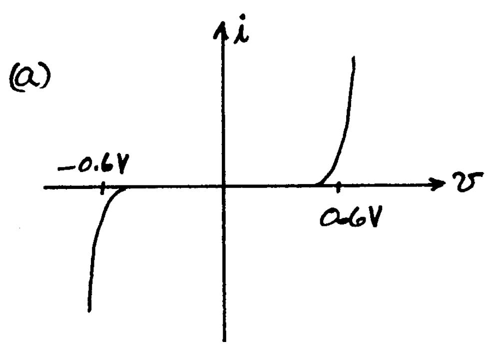
Solving for n we obtain:

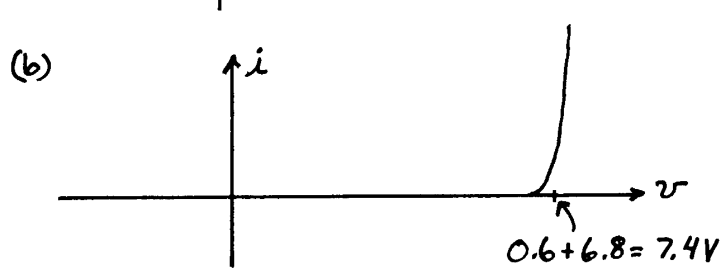
$$n = \frac{v_{D1} - v_{D2}}{V_T \ln(i_{D1} / i_{D2})} = \frac{0.650 - 0.700}{0.025 \ln(1.2/12)} = 1.336$$

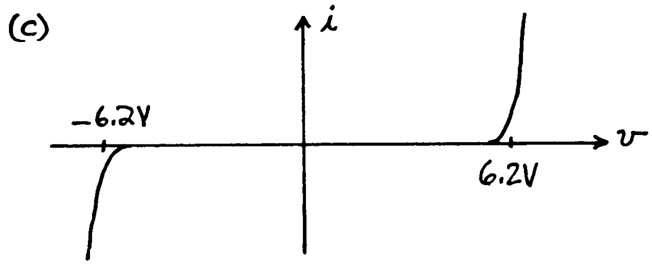
Then, we have

$$I_s = \frac{i_{D1}}{\exp(v_{D1} / nV_T)} = 1.201 \times 10^{-16} \text{ A}$$

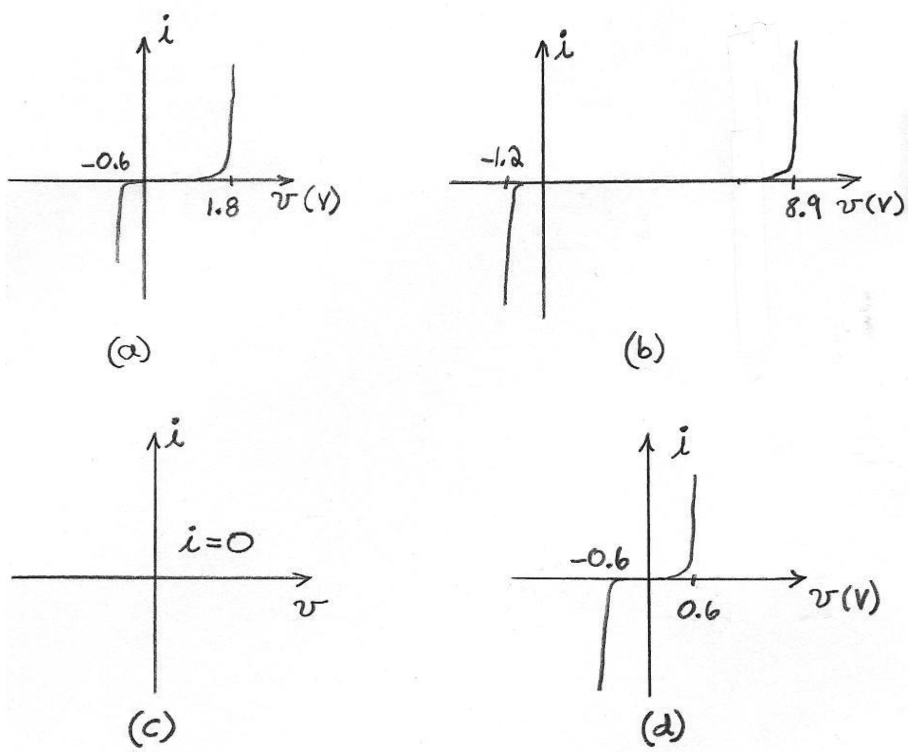
P10.8*







P10.9



P10.10 At $T = 175$ K, we have:

$$v_D = 0.65 - 0.002(150 - 25)$$
$$= 0.40\text{ V}$$

P10.11 $V_T = kT/q = (1.38 \times 10^{-23} \times 373)/(1.60 \times 10^{-19}) = 32.17$ mV

$$I_s = \frac{i_{D1}}{[\exp(v_{D1}/nV_T) - 1]} = 421.9 \times 0^{-9}\ A$$

$$i_{D2} = I_s[\exp(v_{D2}/nV_T) - 1] = 4.733\text{ mA}$$

P10.12 Using the approximate form of the Shockley Equation, we have

$$10^{-3} = I_s \exp(0.600/nV_T) \qquad (1)$$

$$10^{-2} = I_s \exp(0.700/nV_T) \qquad (2)$$

Dividing the respective sides of Equation (2) by those of Equation (1), we have

$$10 = \frac{I_s \exp(0.700/nV_T)}{I_s \exp(0.600/nV_T)} = \exp(-0.100/nV_T)$$

$$\ln(10) = 0.100/nV_T$$

$$n = 0.100/[V_T \ln(10)] = 1.670$$

P10.13 For part (a), Equation 10.3 gives the diode voltage in terms of the current as

$$v_D = nV_T \ln[(i_D / I_s) + 1]$$

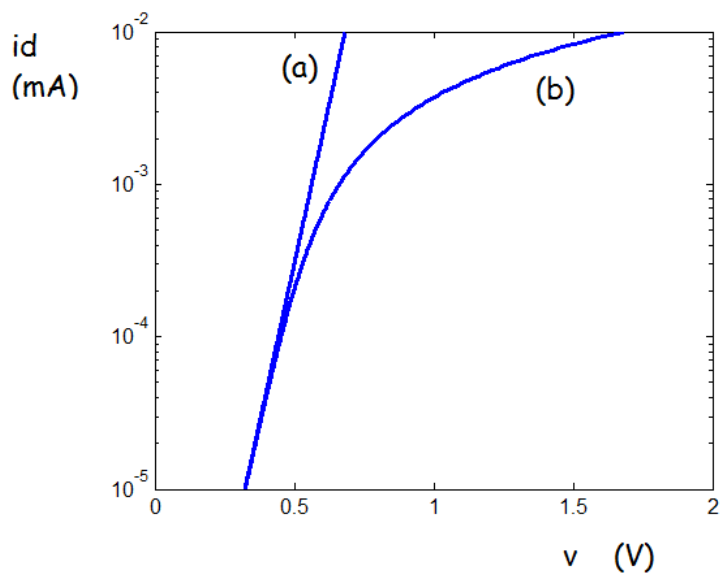
For part (b) with a 100-Ω resistance in series, the terminal voltage is

$$v = v_D + 100 i_D$$

A MATLAB program to obtain the desired plots is:

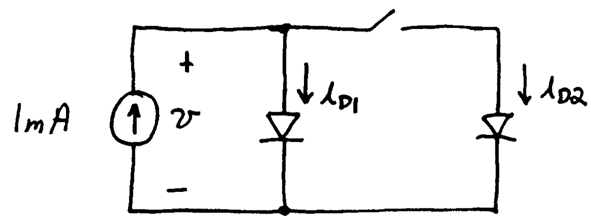
```
log10_of_id = -5:0.01:-2;
id = 10.^log10_of_id;
vd = 2*0.026*log((id/20e-9) + 1);
v = vd + 100*id;
semilogy(v,id)
hold
semilogy(vd,id)
```

(Note in MATLAB log is the natural logarithm.) The resulting plots are shown:



The plot for part (a) is a straight line. The series resistance is relatively insignificant for currents less than 0.1 mA and is certainly significant for currents greater than 1 mA.

P10.14*



With the switch open, we have:

$$i_{D1} = 10^{-3} = I_s\left[\exp(v/nV_T) - 1\right]$$
$$\cong I_s \exp(v/nV_T)$$

Thus, we determine that:

$$I_s \cong \frac{10^{-3}}{\exp(v/V_T)} = \frac{10^{-3}}{\exp(0.7/0.005)} = 8.3153 \times 10^{-10}\ A$$

With the switch closed, by symmetry, we have:

$$i_{D1} = i_{D2} = 0.5\ \text{mA}$$
$$0.5 \times 10^{-3} = I_s \exp(v/V_T)$$
$$v = nV_T \ln \frac{0.5 \times 10^{-3}}{I_s} = 665\ \text{mV}$$

Repeating the calculations with n =1, we obtain:

$$I_s = 6.914 \times 10^{-16}\ A$$
$$v = 682.67\ \text{mV}$$

P10.15* (a) By symmetry, the current divides equally and we have

$$I_A = I_B = 100\ \text{mA}$$

(b) We have

$$i_D = I_s\left[\exp(v/nV_T) - 1\right]$$
$$\cong I_s \exp(v/nV_T)$$

Solving for I_s, we obtain

$$I_s = \frac{i_D}{\exp(v/nV_T)}$$

For diode A, the temperature is $T_A = 300$ K, and we have

$$V_{TA} = \frac{kT_A}{q} = \frac{1.38 \times 10^{-23} \times 300}{1.6 \times 10^{-19}} = 25.88 \text{ mV}$$

$$I_{sA} = \frac{0.100}{\exp(0.700/0.02588)} = 1.792 \times 10^{-13} \text{ A}$$

For diode B, we have $T = 305$ K, and

$$I_{sB} = 2I_{sA} = 3.583 \times 10^{-13} \text{ A}$$

$$V_{TB} = 26.31 \text{ mV}$$

Applying Kirchhoff's current law, we have

$$0.2 = I_A + I_B$$

$$0.2 = (1.792 \times 10^{-13})\exp(v/0.02588) + (3.583 \times 10^{-13})\exp(v/0.02631)$$

Solving for v by trial and err, we obtain $v \cong 697.1$ mV, $I_A = 87$ mA, and $I_B = 113$ mA.

P10.16* The load-line equation is

$$V_s = R_s i_x + v_x$$

Substituting values, this becomes

$$3 = i_x + v_x$$

Next, we plot the nonlinear device characteristic equation

$$i_x = [\exp(v_x) - 1]/10$$

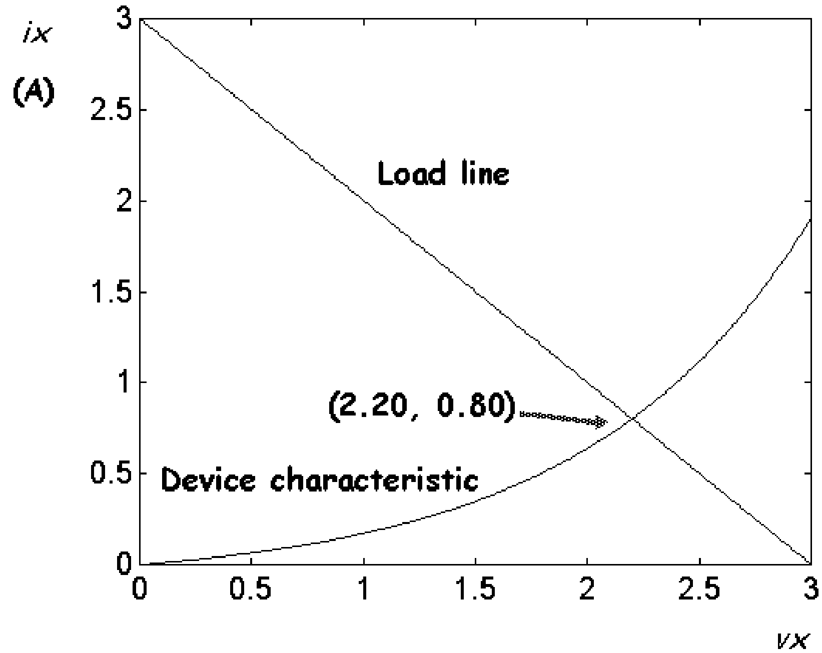
and the load line on the same set of axes.

The MATLAB commands are:

```
clear
%plot load line
vx=0:0.01:3;
ix= 3-vx;
plot(vx,ix)
hold
%plot nonlinear device characteristic
ix=(exp(vx)-1)/10;
plot(vx,ix)
grid minor
ylabel('ix (A)')
```

```
xlabel('vx (V)')
```

Finally, the solution is at the intersection of the load line and the characteristic as shown:



P10.17 The load-line equation is

$$V_s = R_s i_x + v_x$$

Substituting values this becomes

$$4 = i_x + v_x$$

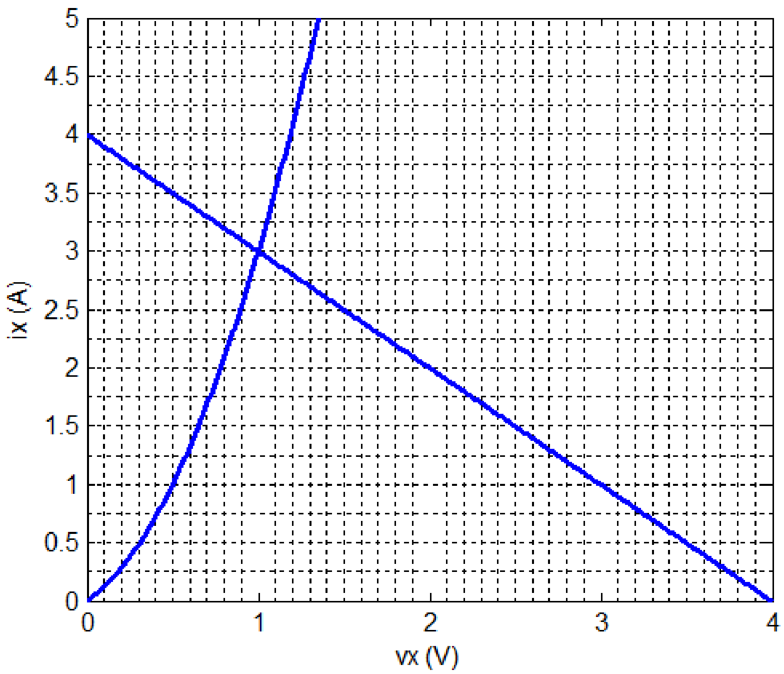
Next we plot the nonlinear device characteristic equation

$$i_x = v_x + 2v_x^2$$

and the load line on the same set of axes. The MATLAB commands are:

```
clear
%plot load line
vx=0:0.01:4;
ix= 4-vx;
plot(vx,ix)
hold
%plot nonlinear device characteristic
ix=vx+2*(vx.^2);
plot(vx,ix)
grid minor
axis([0 4 0 5])
ylabel('ix (A)')
xlabel('vx (V)')
```

Finally, the solution is at the intersection of the load line and the characteristic as shown:



P10.18 The load-line equation is

$$V_s = R_s i_x + v_x$$

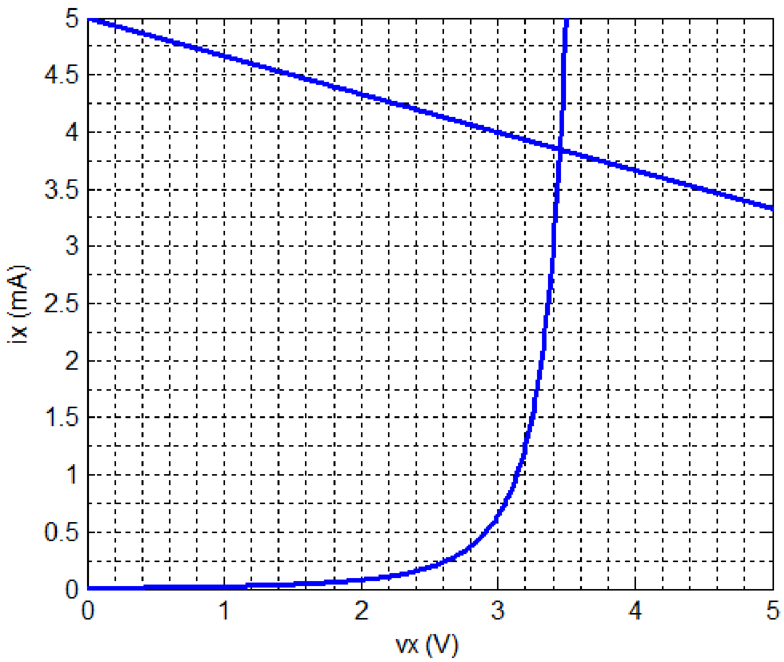
Substituting values, this becomes

$$15 = 3i_x + v_x$$

in which i_x is in milliamperes and v_x is in volts. Next, we plot the nonlinear device characteristic equation and the load line on the same set of axes using the commands:

```
clear
%plot load line
vx=0:0.01:15;
ix= 5-vx/3;
plot(vx,ix)
hold
%plot nonlinear device characteristic
ix=0.01./(1-vx/4).^3;
plot(vx,ix)
axis([0 5 0 5])
grid minor
ylabel('ix (mA)')
xlabel('vx (V)')
```

Finally, the solution is at the intersection of the load line and the characteristic as shown:



P10.19 The load line equation is

$$V_s = R_s i_x + v_x$$

Substituting values' this becomes

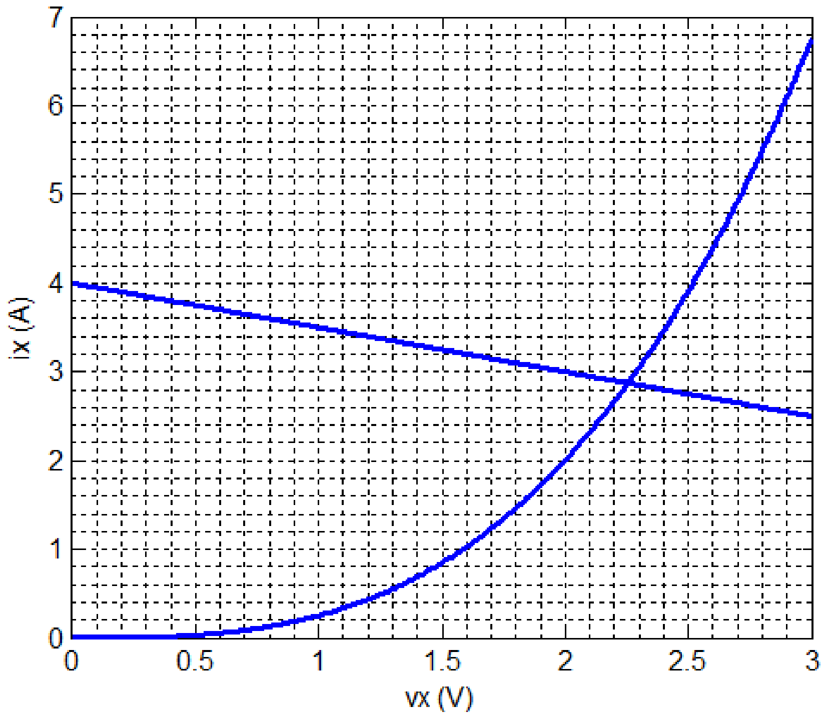
$$8 = 2i_x + v_x$$

Next we plot the nonlinear device characteristic equation $i_x = v_x^3 / 4$ and the load line on the same set of axes.

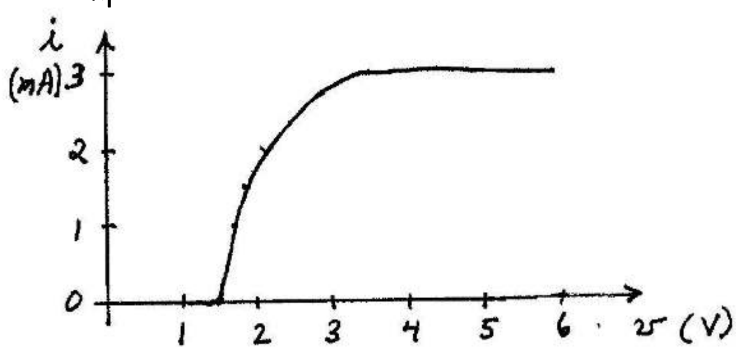
The MATLAB commands are:

```
clear
%plot load line
vx=0:0.01:3;
ix= 4-vx/2;
plot(vx,ix)
hold
%plot nonlinear device characteristic
ix=(vx.^3)./4;
plot(vx,ix)
grid minor
ylabel('ix (A)')
xlabel('vx (V)')
```

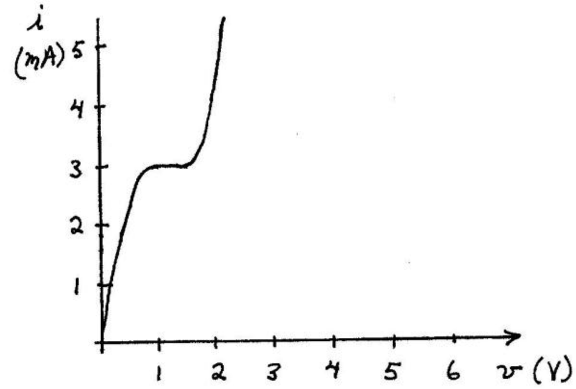
Finally, the solution is at the intersection of the load line and the characteristic as shown:



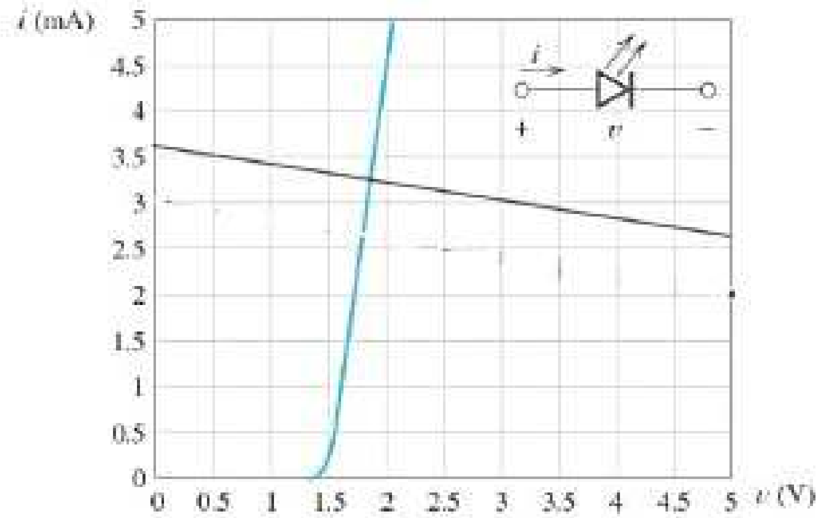
P10.20 (a) In a series circuit, the total voltage is the sum of the voltages across the individual devices. Thus, we add the characteristics horizontally. The overall volt-ampere characteristic is



(b) In a parallel circuit, the overall current is the sum of the currents through the individual devices. Thus, we add the characteristics vertically. The overall volt-ampere characteristic is

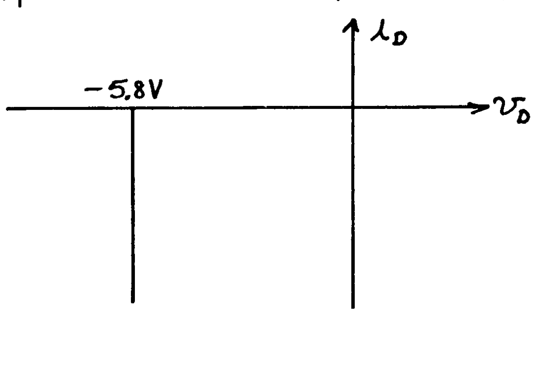


P10.21 The load-line construction is:

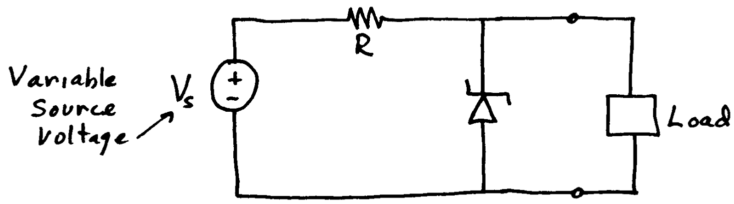


At the intersection of the characteristic and the load line, we have $v = 1.87$ V and $i = 3.31$ mA.

P10.22 A Zener diode is a diode intended for operation in the reverse breakdown region. It is typically used to provide a source of constant voltage. The volt-ampere characteristic of an ideal 5.8-V Zener diode is:



P10.23* The circuit diagram of a simple voltage regulator is:



P10.24 Refer to Figure 10.14 in the book. As the load resistance becomes smaller, the reverse current through the Zener diode becomes smaller in magnitude. The smallest load resistance for which the load voltage remains at 10 V corresponds to zero diode current. Then, the load current is equal to the current through the 100-Ω resistor given by $i_L = (15-10)/100 = 50$ mA. Thus, the smallest load resistance is $R_L = v_o / i_L = 200\ \Omega$.

P10.25 We need to choose R_s so the minimum reverse current through the Zener diode is zero. Minimum current through the Zener occurs with minimum V_s and maximum i_L. Also, we can write:

$$i_Z = \frac{V_s - v_L}{R_s} - i_L$$

Substituting values, we have

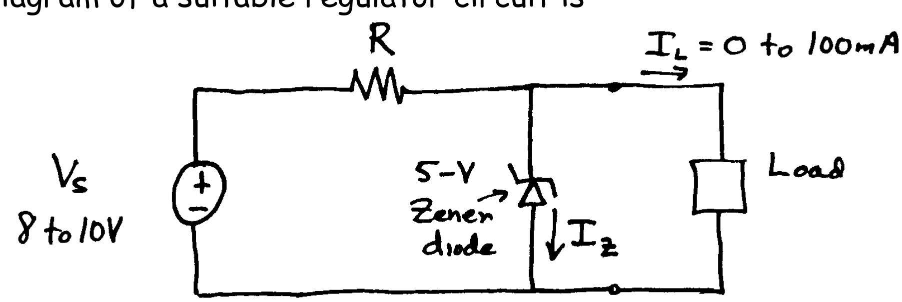
$$i_Z = 0 = \frac{8-5}{R_s} - 0.15$$

Solving for the resistance we find $R_s = 20\ \Omega$.

Maximum power dissipation in the resistance occurs for maximum V_s.

$$P_{\max} = \frac{(V_{s\max} - v_L)^2}{R_s} = \frac{(12-5)^2}{R_s} = 2.45\text{ W}$$

P10.26 The diagram of a suitable regulator circuit is



We must be careful to choose the value of R small enough so I_z remains positive for all values of source voltage and load current. (Keep in mind that the Zener diode cannot supply power.) From the circuit, we can write

$$I_z = \frac{V_s - V_L}{R} - I_L$$

Minimum I_z occurs for $I_L = 100$ mA and $V_s = 8$ V. Solving for the maximum value allowed for R, we have

$$R_{\max} = \frac{V_s - V_L}{I_z + I_L} = \frac{8-5}{0+0.1} = 30\ \Omega$$

Thus, we must choose the value of R to be less than 30 Ω. We need to allow some margin for component tolerances and some design margin. However, we do not want to choose R too small because the current and power dissipation in the diode becomes larger as R becomes smaller. Thus, a value of about 24 Ω would be suitable. (This is a standard value.) With this value of R, we have

$$I_{R\max} = \frac{V_{s\max} - 5}{R} = 208 \text{ mA}$$

$$I_{Z\max} = I_{R\max} = 208 \text{ mA}$$

$$P_{R\max} = \left(I_{R\max}\right)^2 R = 1.04 \text{ W}$$

$$P_{Z\max} = 5 I_{Z\max} = 1.04 \text{ W}$$

P10.27 Refer to the solution to Problem P10.26. In the present case, we have $R_{\max} = 10\ \Omega$, and we could choose $R = 8.2\ \Omega$ because this is a standard value and we need to provide some design margin. With this value, we have

$$I_{R\max} = I_{Z\max} = 610 \text{ mA}$$

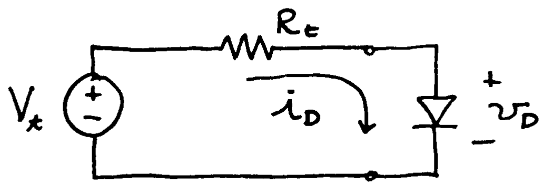
$$P_{R\max} = P_{Z\max} = 3.05 \text{ W}$$

P10.28 Refer to the solution to Problem P10.26. In the present case, we have $R_{\max} = 3.0\ \Omega$, and we would choose $R = 2.4\ \Omega$ because this is a standard value and we need to provide some design margin. With this value, we have

$$I_{R\max} = I_{Z\max} = 2.08 \text{ A}$$

$$P_{R\max} = P_{Z\max} = 10.4 \text{ W}$$

P10.29 First, find the Thévenin equivalent for the circuit as seen looking back from the terminals of the nonlinear device:

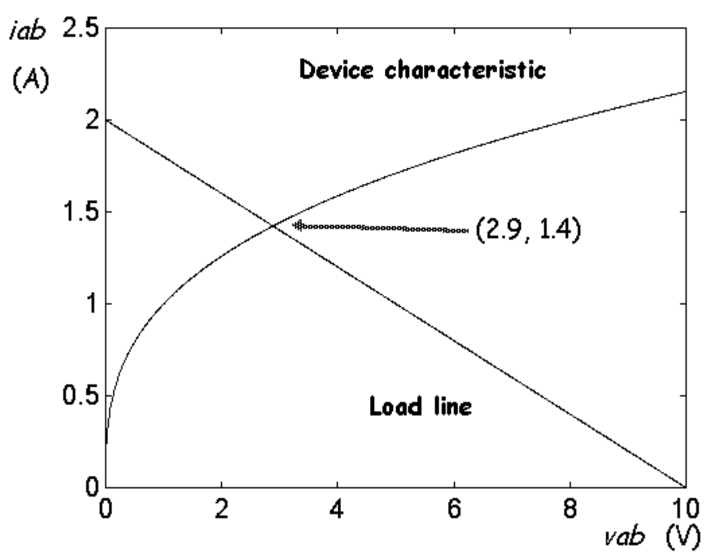


Then, write the KVL equation for the equivalent circuit:

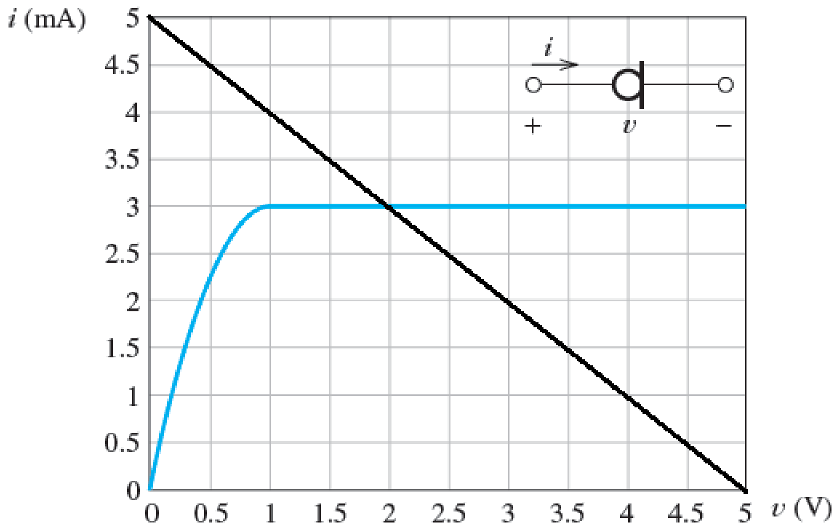
$$V_t = R_t i_D + v_D$$

Next, plot this equation on the device characteristics and find the value of v_D and i_D at the intersection of the load line and the device characteristic. After the device current and voltage have been found, the original circuit can be solved by standard methods.

P10.30* The Thévenin resistance is $R_t = V_{oc} / I_{sc} = 10/2 = 5\ \Omega$. Also the Thévenin voltage is $V_t = V_{oc} = 10$ V. The load line equation is $10 = 5i_{ab} + v_{ab}$. We plot the load line and nonlinear device characteristic and find the solution at the intersection as shown:

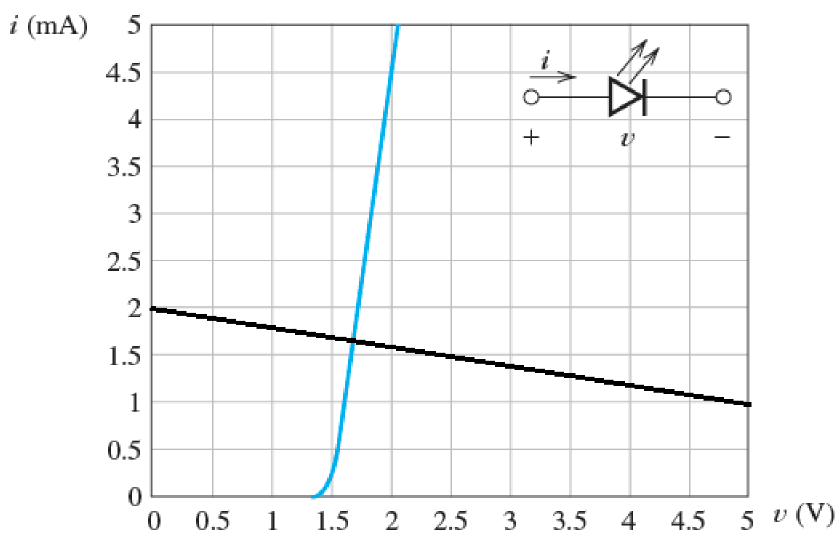


P10.31 If we remove the diode, the Thévenin equivalent for the remaining circuit consists of a 5-V source in series with a 1-kΩ resistance. The load line is



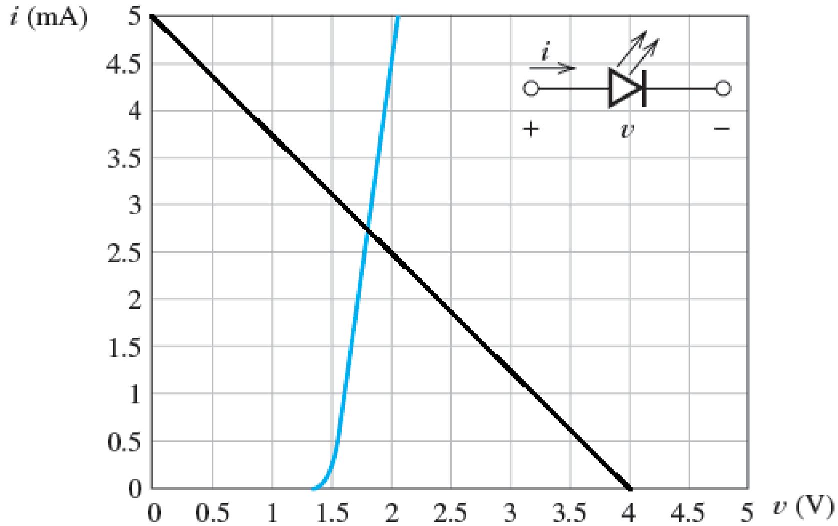
At the intersection of the characteristic and the load line, we have the device current $i_1 = 3.0$ mA. Then, applying KCL to the original circuit, we have $i_2 = 10 - i_1 = 7.0$ mA.

P10.32 If we remove the diode, the Thévenin equivalent for the remaining circuit consists of a 10-V source in series with a 5-kΩ resistance. The load line is



At the intersection of the characteristic and the load line, we have $v \cong 1.65$ V and $i \cong 1.67$ mA.

P10.33 If we remove the diode, the Thévenin equivalent for the remaining circuit consists of a 4-V source in series with an 800-Ω resistance. The load line is



At the intersection of the characteristic and the load line, we have $v \cong 1.80$ V and $i \cong 2.75$ mA.

P10.34 An ideal diode acts as a short circuit as long as current flows in the forward direction. It acts as an open circuit provided that there is reverse voltage across it. The volt-ampere characteristic is shown in Figure 10.15 in the text. After solving a circuit with ideal diodes, we must check to see that forward current flows in diodes assumed to be on, and we must check to see that reverse voltage appears across all diodes assumed to be off.

P10.35 The equivalent circuit for two ideal diodes in series pointing in opposite directions is an open circuit because current cannot flow in the reverse direction for either diode.

The equivalent circuit for two ideal diodes in parallel pointing in opposite directions is a short circuit because one of the diodes is forward conducting for either direction of current flow.

P10.36* (a) D_1 is on and D_2 is off. $V = 10$ volts and $I = 0$.

(b) D_1 is on and D_2 is off. $V = 6$ volts and $I = 6$ mA.

(c) Both D_1 and D_2 are on. $V = 30$ volts and $I = 33.6$ mA.

P10.37 (a) The diode is on, $V = 0$ and $I = \dfrac{10}{5000} = 2\text{ mA}$.

(b) The diode is off, $I = 0$ and $V = 5\text{ V}$.

(c) The diode is on, $V = 0$ and $I = \dfrac{6}{3000} = 2\text{ mA}$.

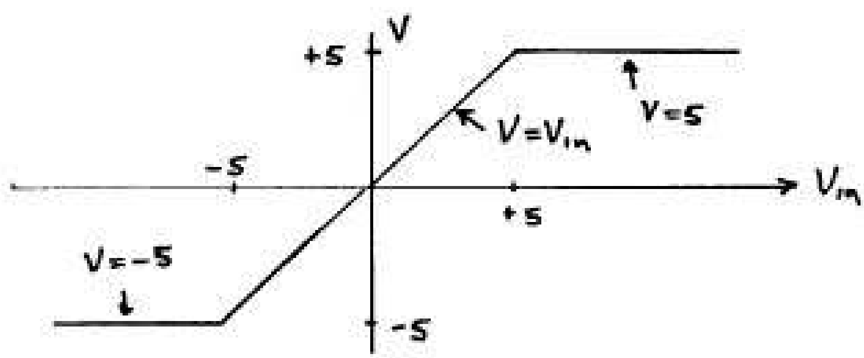
(d) The diode is on, $I = 3\text{ mA}$ and $V = 6\text{ V}$.

P10.38 (a) D_1 is on, D_2 is on, and D_3 is off. $V = 7.5$ volts and $I = 0$.

(b)

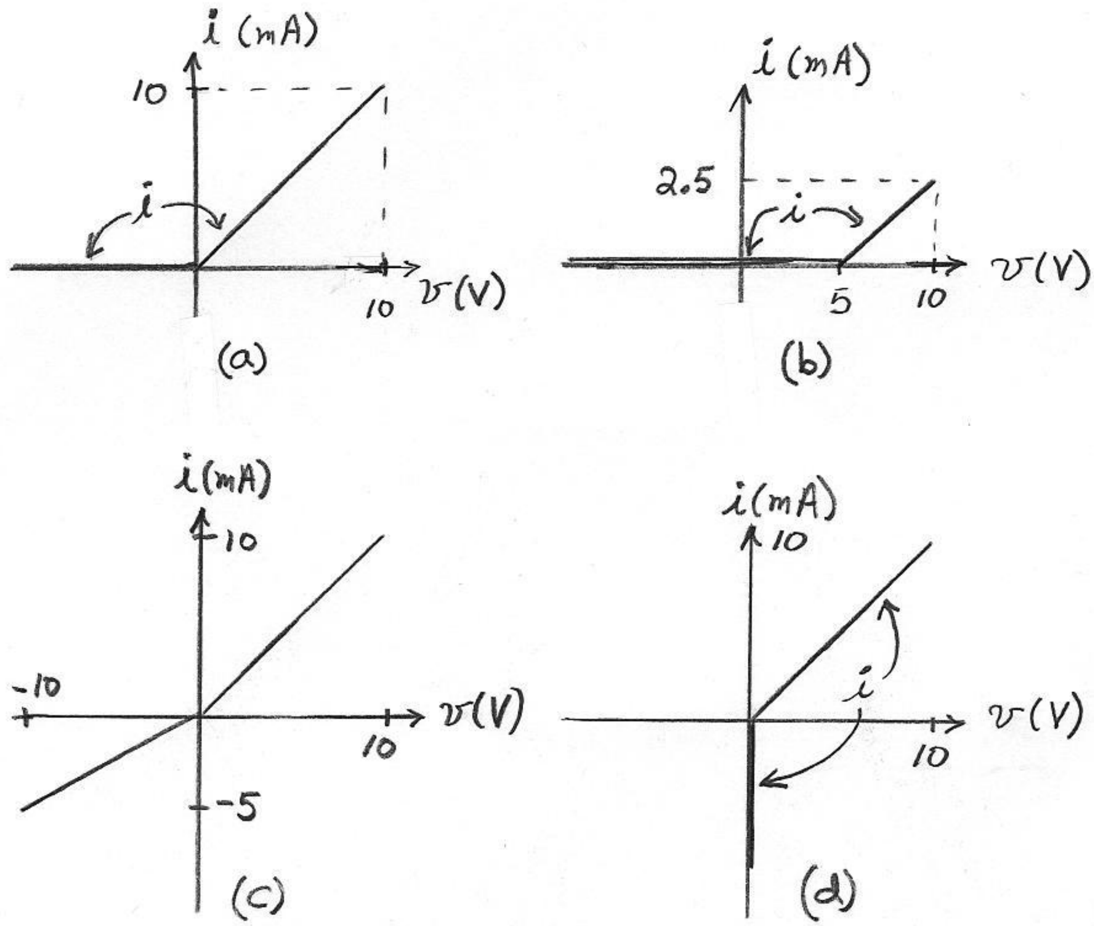
V_{in}	D_1	D_2	D_3	D_4	V	I
0	on	on	on	on	0	0
5	on	on	on	on	2 V	2 mA
10	off	on	on	off	5 V	5 mA
20	off	on	on	off	5 V	5 mA

The plot of V versus V_{in} is:



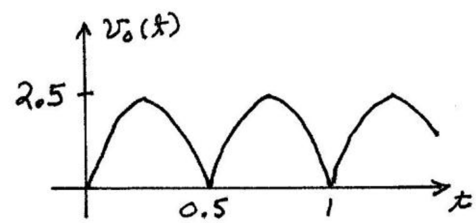
P10.39 (a) The output is high if either or both of the inputs are high. If both inputs are low the output is low. This is an OR gate.
(b) The output is high only if both inputs are high. This is an AND gate.

P10.40



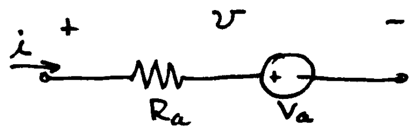
P10.41 When the sinusoidal source is positive, D_2 is on and D_1 is off. Then, we have $v_o(t) = 2.5\sin(2\pi t)$.

When the source is negative, D_1 is on and D_2 is off. Then, we have $v_o(t) = -2.5\sin(2\pi t)$.

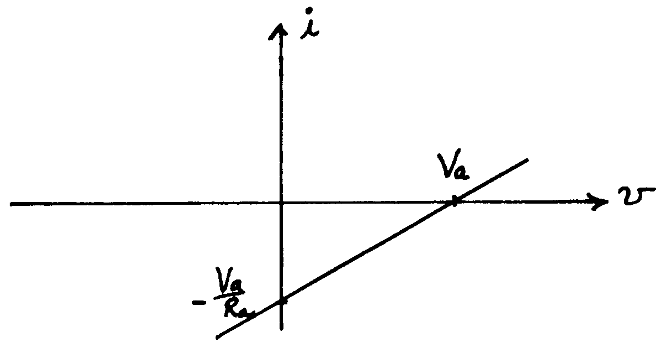


P10.42 If a nonlinear two-terminal device is modeled by the piecewise-linear approach, the equivalent circuit of the device for each linear segment consists of a voltage source in series with a resistance.

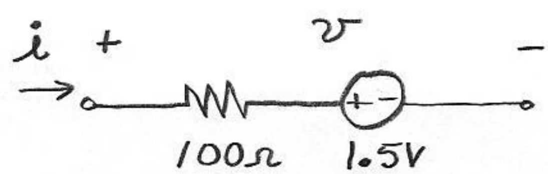
P10.43



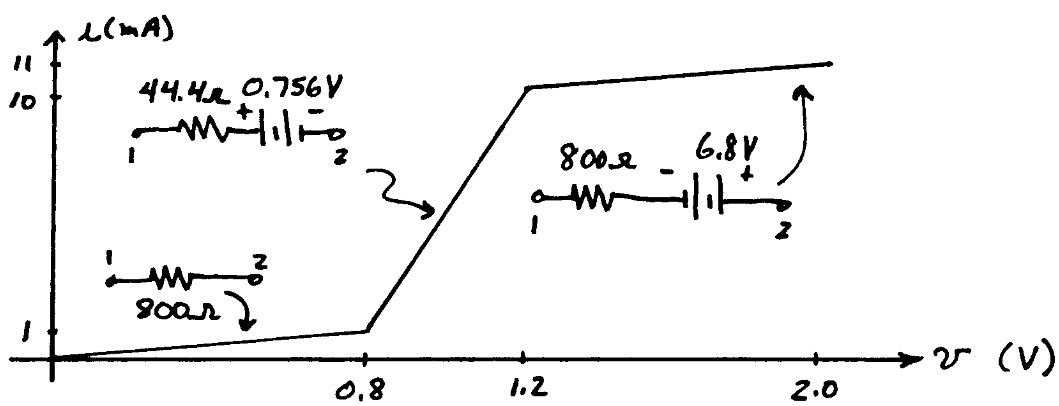
$$v = R_a i + V_a$$



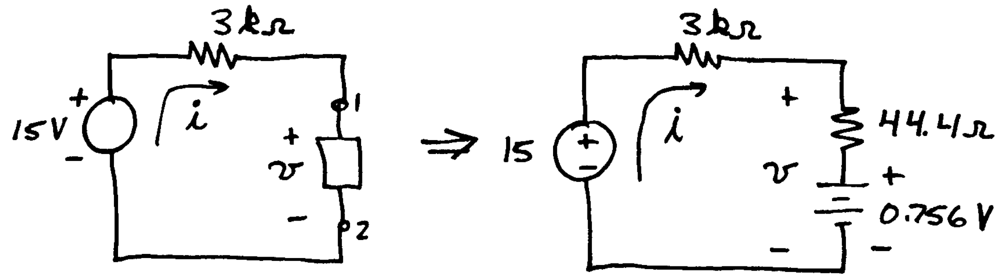
P10.44 We know that the line passes through the points (2 V, 5 mA) and (3 V, 15 mA). Thus, the slope of the line is $-1/R$ = (-10 mA)/(1 V), and we have R = 100 Ω. Furthermore, the intercept on the voltage axis is at v = 1.5 V. Thus, the equivalent circuit is



10.45* The equivalent circuits for each segment are shown below:

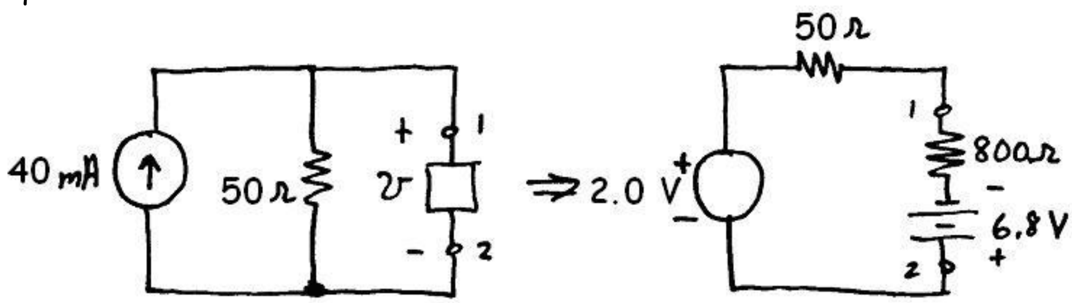


For the circuit of Figure P10.45a, we can determine by trial and error (or by a load-line analysis) that the device operates on the middle line segment. Thus, the equivalent circuit is:



$$i = \frac{15 - 0.756}{3000 + 44.4} = 4.68 \text{ mA}$$
$$v = 0.756 + 44.4i = 0.964 \text{ V}$$

For the circuit of Figure P10.45b, we can determine by trail and error that the device operates on the upper right-hand line segment. Thus, the equivalent circuit is:



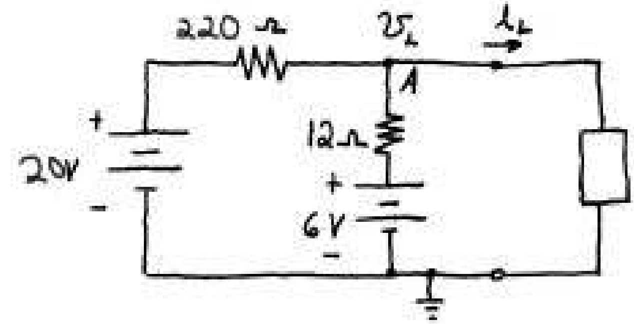
$$i = \frac{2 + 6.8}{50 + 800} = 10.4 \text{ mA}$$
$$v = 800i - 6.8 = 1.48 \text{ V}$$

P10.46 In the forward bias region ($i_D > 0$), the equivalent circuit is a short circuit. Thus in the equivalent circuit, the voltage source is zero and the resistance is zero.

In the reverse bias region ($0 > v_D > -10$ V) the equivalent circuit is an open circuit. Thus in the equivalent circuit, the voltage is indeterminate and the resistance is infinite.

In the reverse breakdown region ($0 > i_D$), the equivalent circuit consists of a 10-V voltage source in series with zero resistance.

P10.47* For small values of i_L, the Zener diode is operating on line segment C of Figure 10.19, and the equivalent circuit is



Writing a KCL equation at node A, we obtain:

$$\frac{v_L - 20}{100} + \frac{v_L - 6}{12} + i_L = 0$$

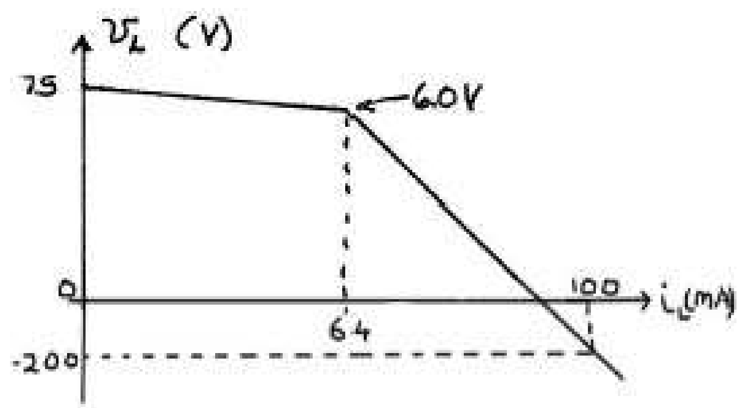
Solving we obtain

$$v_L = 7.5i_L - 10.71$$

This equation is valid for $v_L \geq 6$ V. When $0 \leq v_L \leq 6$ V, the Zener diode operates on line segment B, for which the Zener is modeled as an open circuit and we have

$$v_L = 20 - 220 i_L$$

Plotting these equations results in



P10.48 (a) Assuming that the diode is an open circuit, we can compute the node voltages using the voltage-division principle.

$$v_1 = 16\frac{300}{100 + 300} = 12\text{ V} \quad v_2 = 16\frac{200}{200 + 200} = 8\text{ V}$$

Then, the voltage across the diode is $v_D = v_1 - v_2 = 4$ V. Because v_D is greater than $V_f = 0.7$ V, the diode is in fact operating as an closed circuit.

(b) Assuming that the diode operates as a voltage source, we can use KVL to write:

$$v_1 - v_2 = 0.7$$

Placing a closed surface around the diode to form a super node and writing a KCL equation gives

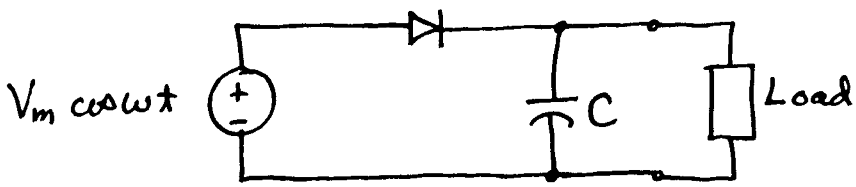
$$\frac{v_1 - 16}{100} + \frac{v_1}{300} + \frac{v_2 - 16}{100} + \frac{v_2}{200} = 0$$

Solving these equations, we find $v_1 = 11.665$ V and $v_2 = 9.965$ V. Then, writing a KCL equation at node 1 gives the diode current.

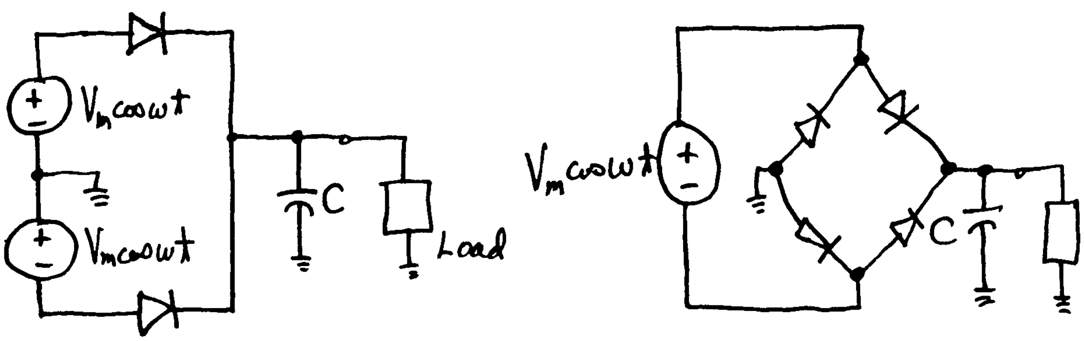
$$i_D = \frac{16 - v_1}{100} - \frac{v_1}{300} = 4.47\ \text{mA}$$

Because the diode current is positive, the diode operation is consistent with the model.

P10.49 Half-wave rectifier with a capacitance to smooth the output voltage:



Full-wave circuits:



P10.50 The peak value of the ac source is $V_m = 25\sqrt{2} = 35.36$ V. Thus the PIV is 35.36 V and the peak current is 35.36/100 = 353.6 mA.

P10.51 The dc output voltage is equal to the peak value of the ac source, which is $v_L = 25\sqrt{2} = 35.36$ V. The Load current is $i_L = v_L / R_L = 35.36$ mA. The charge that passes through the load must also pass through the diode. The charge is $Q = i_L T_L = 0.03536/60 = 0.589$mC. The peak inverse voltage is 70.72 V. The charge passes through the diode in a very short interval, thus the peak diode current is much larger than the load current.

P10.52 The diode is on for $V_B \le V_m \sin(\omega t)$. Substituting values and solving, we find that during the first cycle after $t = 0$ the diode is on for

$$\arcsin(12/24) = \pi/6 \le \omega t \le \pi - \arcsin(12/24) = 5\pi/6$$

The current is given by

$$i(t) = \frac{24\sin(\omega t) - 12}{0.5} = 48\sin(\omega t) - 24 \text{ A}$$

The charge passing through the circuit during the first cycle is

$$Q_1 = \int_{\pi/6\omega}^{5\pi/6\omega} [48\sin(\omega t) - 24]dt = \left[-\frac{48}{\omega}\cos(\omega t) - 24t\right]_{\pi/6\omega}^{5\pi/6\omega} = \frac{48\sqrt{3} - 16\pi}{\omega}$$

The average current is the charge passing through the circuit in 1 second (or 60 cycles). Also, we have $\omega = 120\pi$ Thus

$$I_{avg} = 60\frac{48\sqrt{3} - 16\pi}{\omega} = 60\frac{48\sqrt{3} - 16\pi}{120\pi} = \frac{24\sqrt{3}}{\pi} - 8 = 5.232 \text{ A}$$

Then, the time required to fully charge the battery is

$$T = \frac{100}{5.232} = 19.1 \text{ hours}$$

P10.53 (a) The integral of $V_m \sin(\omega t)$ over one cycle is zero, so the dc voltmeter reads zero.

(b) $$V_{avg} = \frac{1}{T}\left[\int_0^{T/2} V_m \sin(\omega t)dt + \int_{T/2}^{T} 0dt\right] = \frac{1}{T}\left[-\frac{V_m}{\omega}\cos(\omega t)\right]_{t=0}^{t=T/2} = \frac{V_m}{\pi}$$

(c) $$V_{avg} = \frac{1}{T}\left[\int_0^{T/2} V_m \sin(\omega t)dt + \int_{T/2}^{T} -V_m \sin(\omega t)dt\right] = \frac{2V_m}{\pi}$$

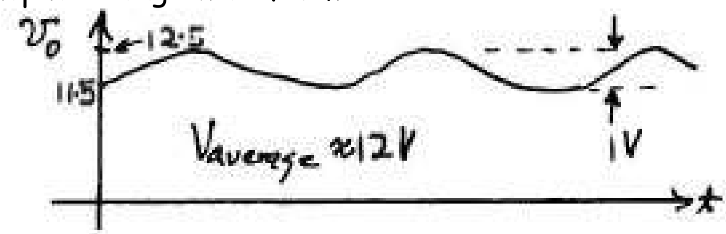
P10.54* For a half-wave rectifier, the capacitance required is given by Equation 10.10 in the text.

$$C = \frac{I_L T}{V_r} = \frac{0.25(1/60)}{0.2} = 20833 \ \mu\text{F}$$

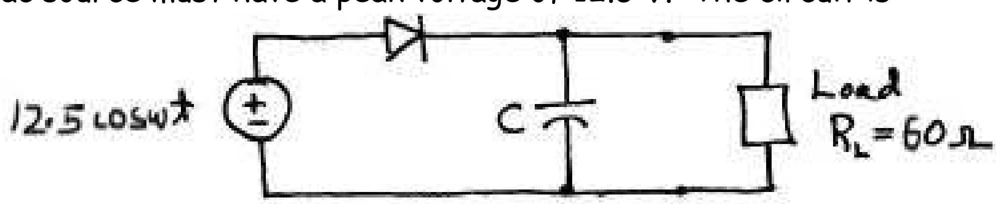
For a full-wave rectifier, the capacitance is given by Equation 10.12 in the text:

$$C = \frac{I_L T}{2V_r} = \frac{0.25(1/60)}{2(0.2)} = 10416\ \mu\text{F}$$

P10.55* The output voltage waveform is:



The peak voltage is approximately 12.5 V. Assuming an ideal diode, the ac source must have a peak voltage of 12.5 V. The circuit is:



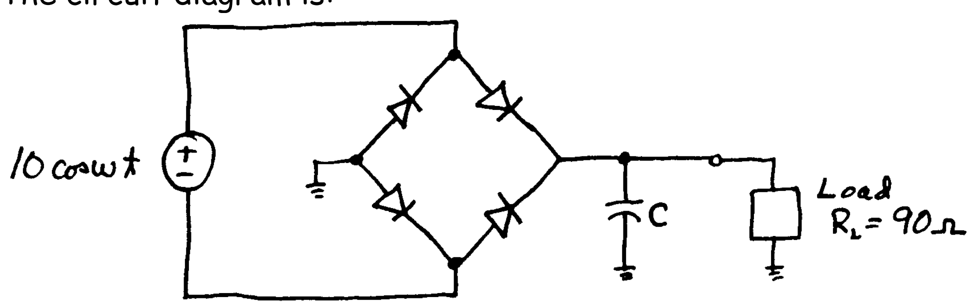
The capacitance required is given by Equation 10.10 in the text.

$$C = \frac{I_L T}{V_r} = \frac{(0.2)/(12\times 60)}{2} = 277.78\ \mu\text{F}$$

P10.56 As in Problem P10.55, the peak voltage must be 10 V. For a full-wave rectifier, the capacitance is given by Equation 10.12 in the text:

$$C = \frac{I_L T}{2V_r} = \frac{0.1(1/60)}{2(2)} = 417\ \mu\text{F}$$

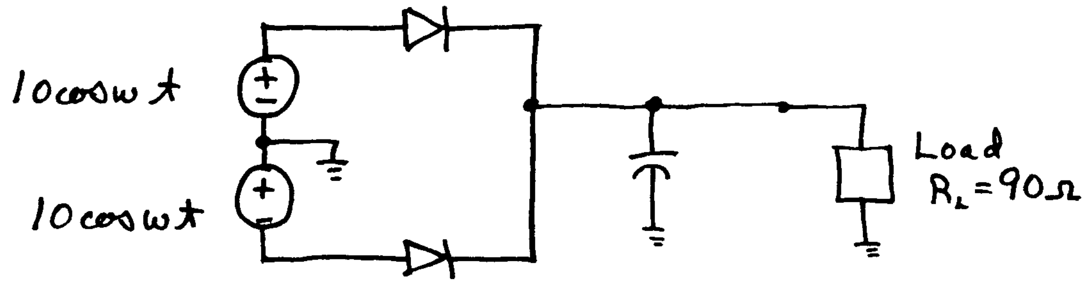
The circuit diagram is:



P10.57 As in Problem P10.55, the peak voltage must be 10 V. For a full-wave rectifier, the capacitance is given by Equation 10.12 in the text:

$$C = \frac{I_L T}{2V_r} = \frac{0.1(1/60)}{2(2)} = 417\ \mu\text{F}$$

The circuit diagram is:



P10.58 If we allow for a forward diode drop of 0.8 V, the peak ac voltage must be 10.8 V. Otherwise, the circuit is the same as in the solution to Problem P10.55.

P10.59 (a) The current pulse starts and ends at the times for which

$$v_s(t) = V_B$$

$$110\sin(200\pi t) = 12$$

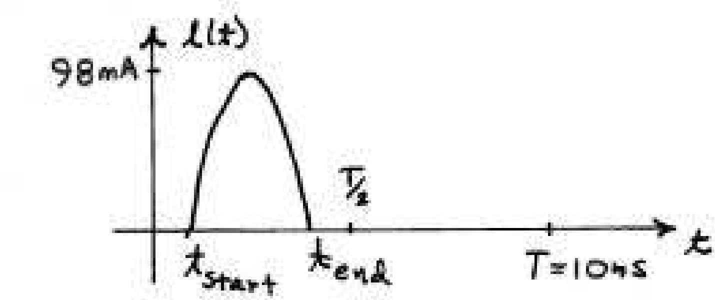
Solving we find that

$$t_{\text{start}} = \frac{\sin^{-1}(0.11)}{200\pi} = 0.174 \text{ ms} \text{ and } t_{\text{end}} = \frac{T}{2} - t_{\text{start}} = 31.242 \text{ ms}$$

Between these two times the current is

$$i(t) = \frac{110\sin(200\pi t) - 12}{1000} = 0.11\sin(200\pi t) - 0.12$$

A sketch of the current to scale versus time is



(b) The charge flowing through the battery in one period is

$$Q = \int_{t_{start}}^{t_{end}} i(t)dt = \int_{t_{start}}^{t_{end}} \frac{110\sin(200\pi t) - 12}{1000} dt$$

$$= \left[\frac{-\cos(200\pi t)}{1818.18\pi} - \frac{12t}{1000} \right]_{t_{start}}^{t_{end}}$$

$$Q = 42.85 \times 10^{-6}\ C$$

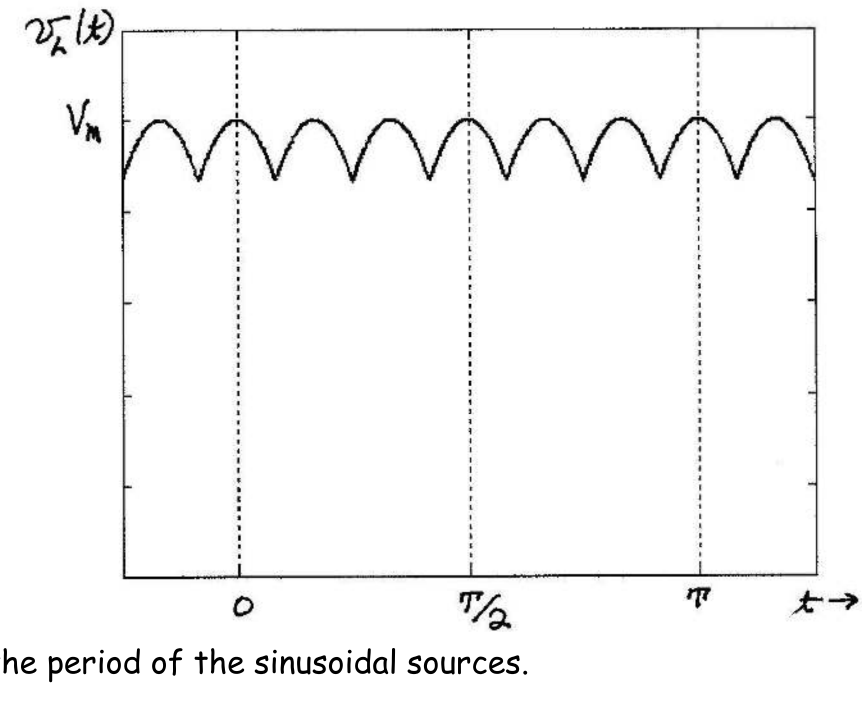
Finally, the average current is the charge divided by the period.

$$I_{avg} = \frac{Q}{T} = \frac{42.85 \times 10^{-6}}{10 \times 10^{-3}} = 4.285\ \text{mA}$$

P10.60 (a) With ideal diodes and a large smoothing capacitance, the load voltage equals the peak source voltage which is V_m = 12 V. Then the PIV is 2 V_m = 24 V.

(b) Here again with ideal diodes and a large smoothing capacitance, the load voltage equals the peak source voltage which is V_m = 12 V. However, the PIV is only V_m = 12 V.

P10.61 (a) The circuit operates as three full-wave rectifiers with a common load and shared diodes. Thus, the load voltage at any instant is equal to the largest source-voltage magnitude. The plot of the load voltage is



T is the period of the sinusoidal sources.

(b) The minimum voltage occurs at $t = T/12$ and is given by $V_{\min} = V_m \cos \omega T/12 = V_m \cos(\pi/6) = 0.866V_m$. Thus the peak-to-peak ripple is 0.134 V_m.

The average load voltage is given by

$$V_{\text{avg}} = \frac{1}{T}\int_0^T v_L(t)dt$$

However, since $v_L(t)$ has 12 intervals with the same area, we can write:

$$V_{\text{avg}} = \frac{1}{T/12}\int_0^{T/12} v_L(t)dt = \frac{12}{T}\int_0^{T/12} V_m \cos(\omega t)dt$$

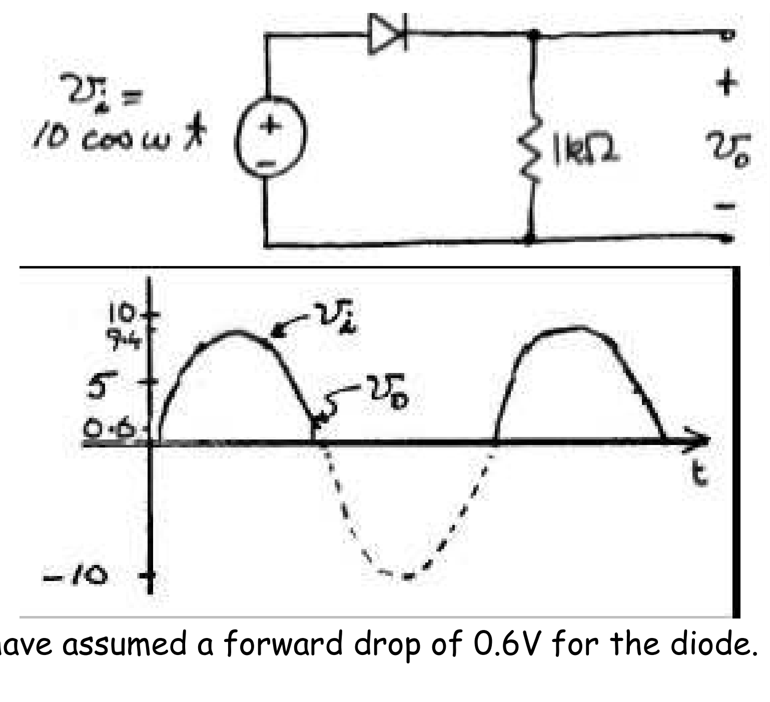
$$V_{\text{avg}} = V_m \frac{6\sin(\pi/6)}{\pi} \cong 0.955V_m$$

(c) To produce an average charging current of 30 A, we require

$$V_{avg} = 12 + 0.1 \times 30 = 15 \text{ V}$$

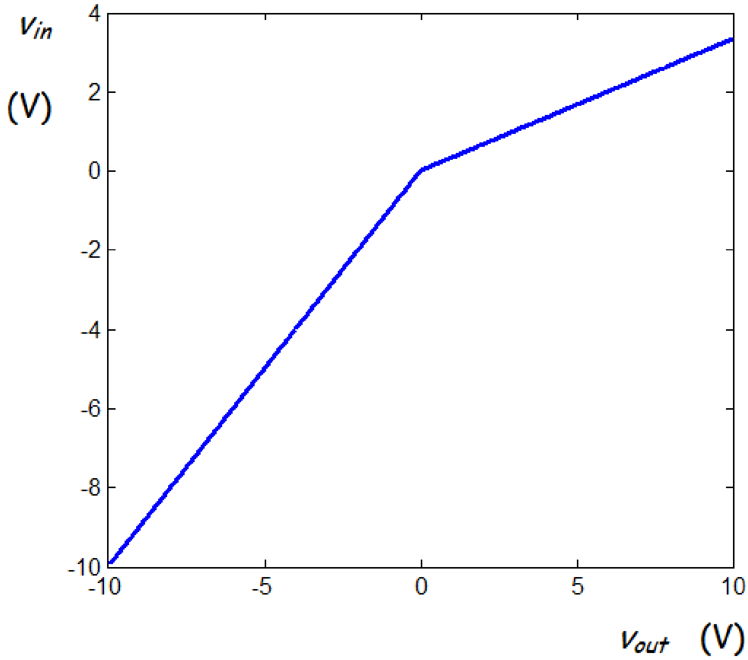
(d) In practice, we would need to allow for forward drops of the diodes, drops across the slip rings, and resistances of the stator windings and wiring.

P10.62 A clipper circuit removes or clips part of the input waveform. An example circuit with waveforms is:

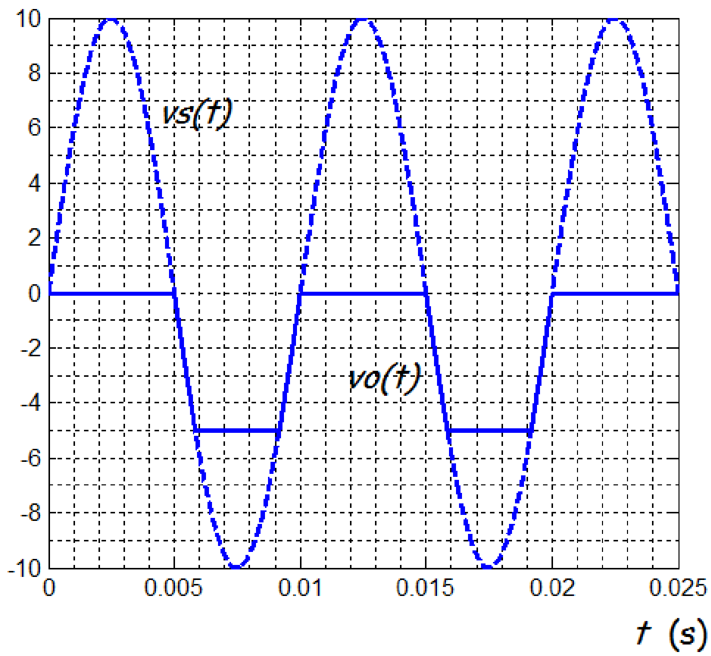


We have assumed a forward drop of 0.6V for the diode.

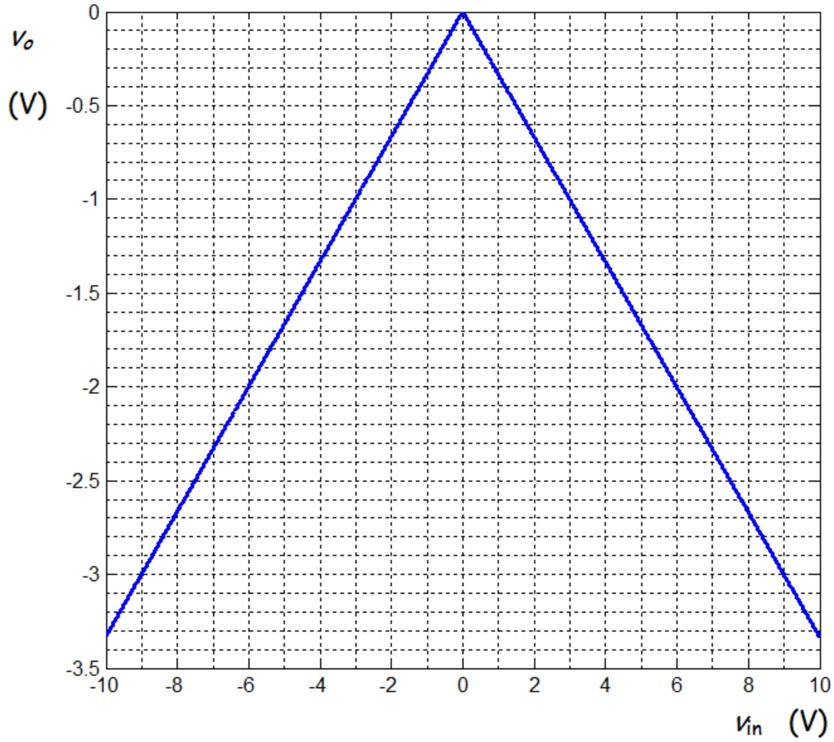
P10.63



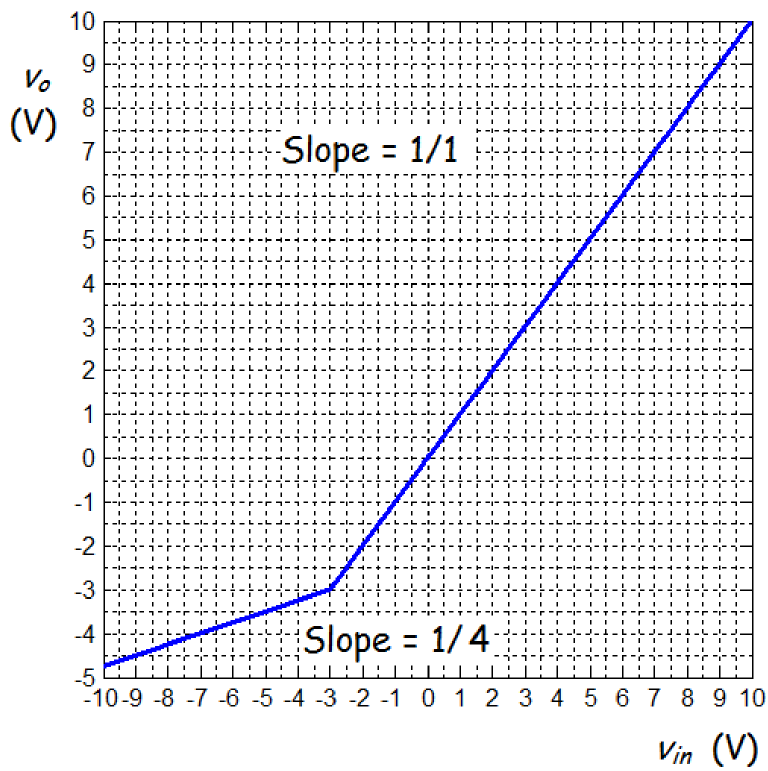
P10.64 Refer to Figure P10.64 in the book. When the source voltage is positive, diode D_3 is on and the output $v_o(t)$ is zero. For source voltages between 0 and 5 V, none of the diodes conducts and $v_o(t) = v_s(t)$. Finally, when the source voltage falls below -5 V, D_1 is on and D_2 is in the breakdown region so the output voltage is 5 V. The waveforms are:



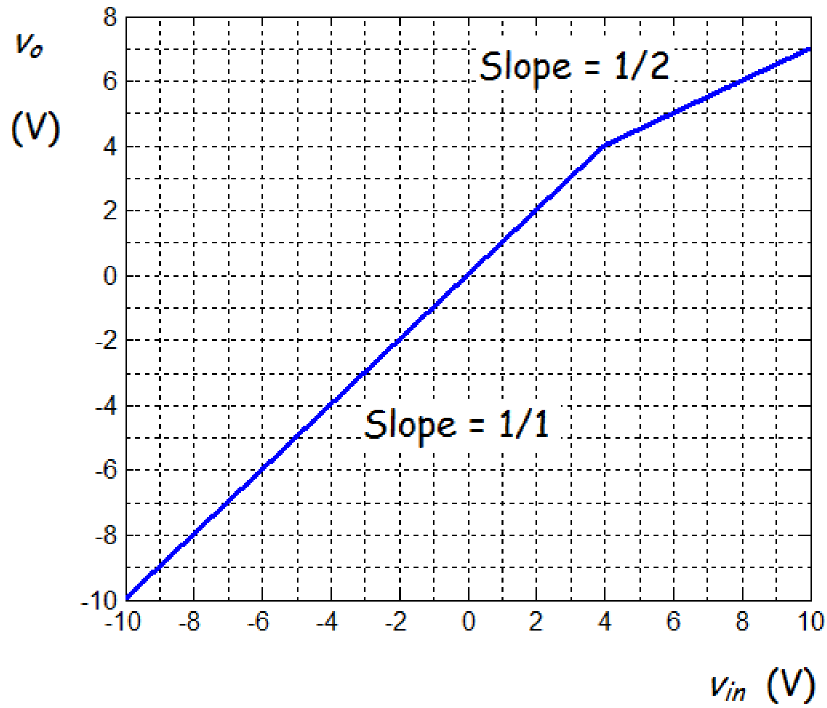
P10.65


v_o
(V)
0
-0.5
-1
-1.5
-2
-2.5
-3
-3.5
-10 -8 -6 -4 -2 0 2 4 6 8 10
v_{in} (V)

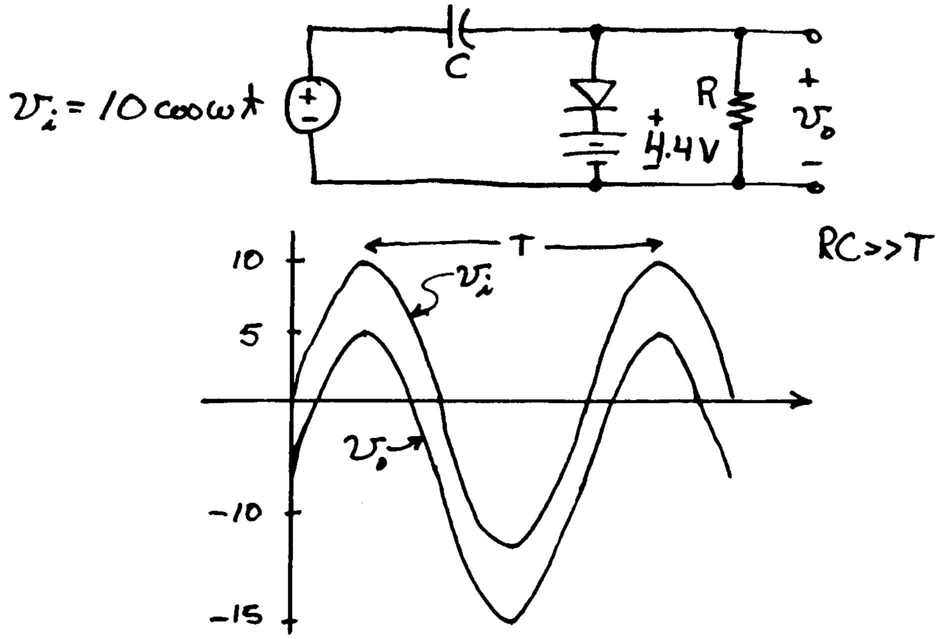
P10.66


v_o
(V)
10 9 8 7 6 5 4 3 2 1 0 -1 -2 -3 -4 -5
Slope = 1/1
Slope = 1/4
-10 -9 -8 -7 -6 -5 -4 -3 -2 -1 0 1 2 3 4 5 6 7 8 9 10
v_{in} (V)

P10.67

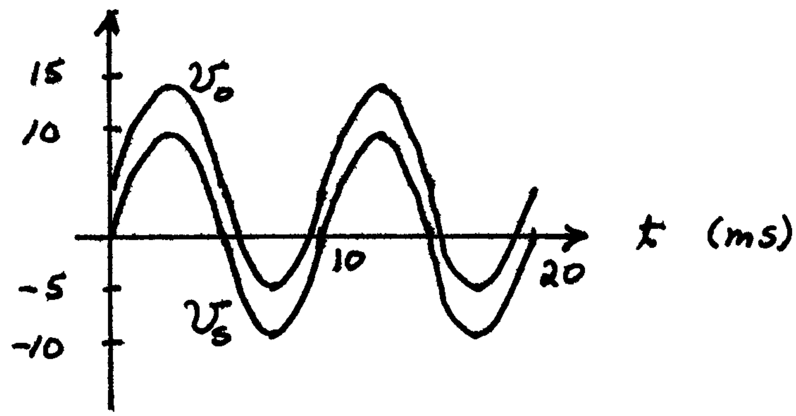


P10.68 A clamp circuit adds or subtracts a dc component to the input waveform such that either the positive peak or the negative peak is forced to assume a predetermined value. An example circuit that clamps the positive peak to +5 V is shown below:

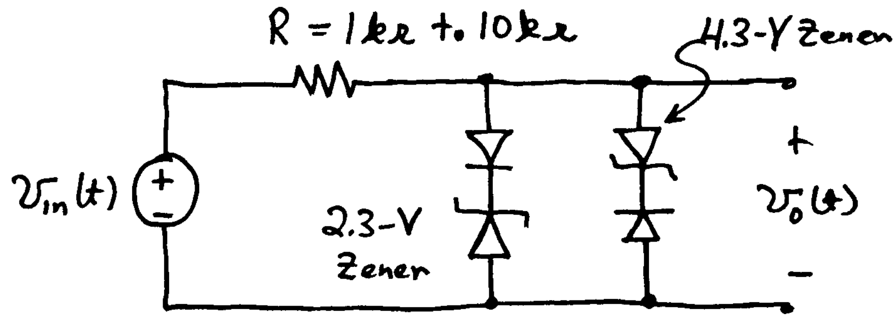


We have allowed a forward drop of 0.6 V for the diode.

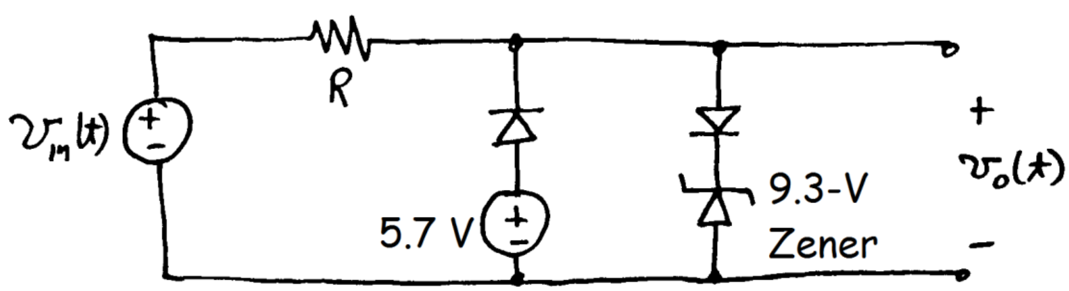
P10.69* Refer to the circuit shown in Figure P10.69 in the book. If the output voltage attempts to become less than -5 V, the Zener diode breaks down and current flows, charging the capacitance. Thus, the negative peak is clamped to -5 V. The input and output waveforms are:



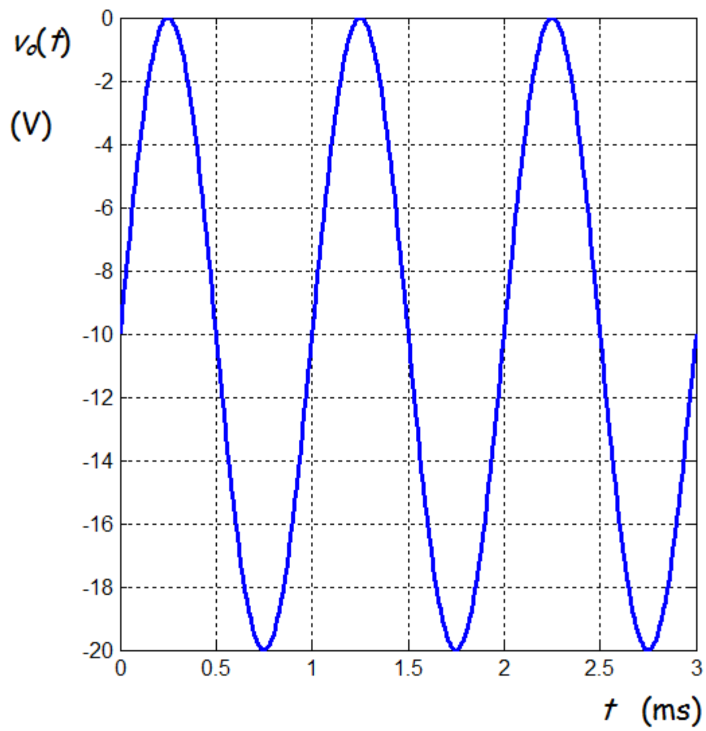
P10.70* A suitable circuit is:



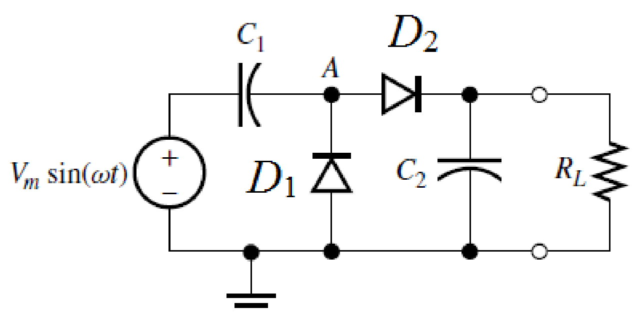
P10.71 A suitable circuit is:



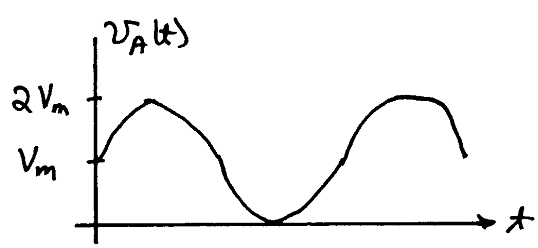
P10.72 This is a clamp circuit that clamps the positive peaks to zero.



P10.73

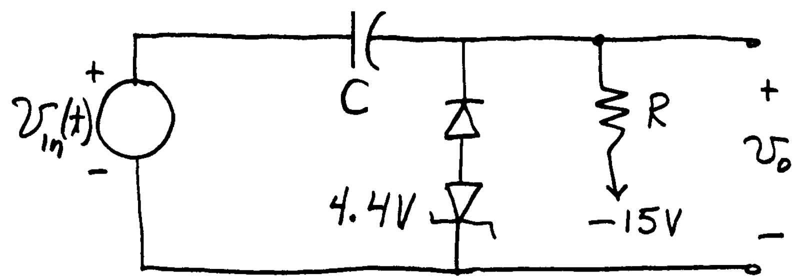


The capacitor C_1 and diode D_1 act as a clamp circuit that clamps the negative peak of $v_A(t)$ to zero. Thus, the waveform at point A is:



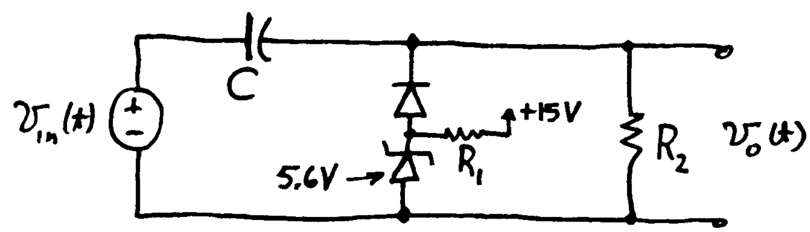
Diode D_2 and capacitor C_2 act as a half-wave peak rectifier. Thus, the voltage across R_L is the peak value of $v_A(t)$. Thus, $v_L(t) \cong 2V_m$. This is called a voltage-doubler circuit because the load voltage is twice the peak value of the ac input. The peak inverse voltage is $2V_m$ for both diodes.

P10.74* A suitable circuit is:



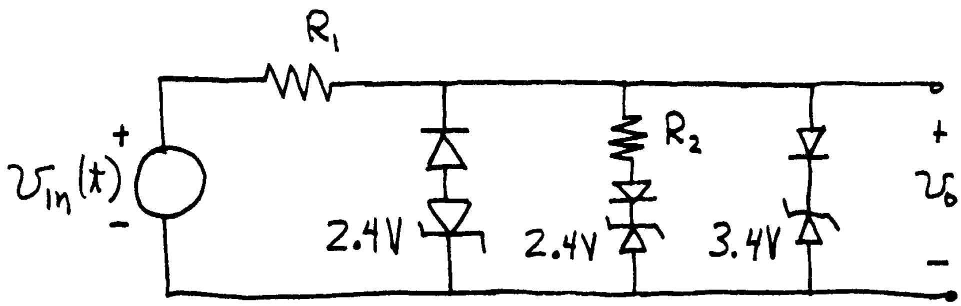
We must choose the time constant $RC \gg T$, where T is the period of the input waveform.

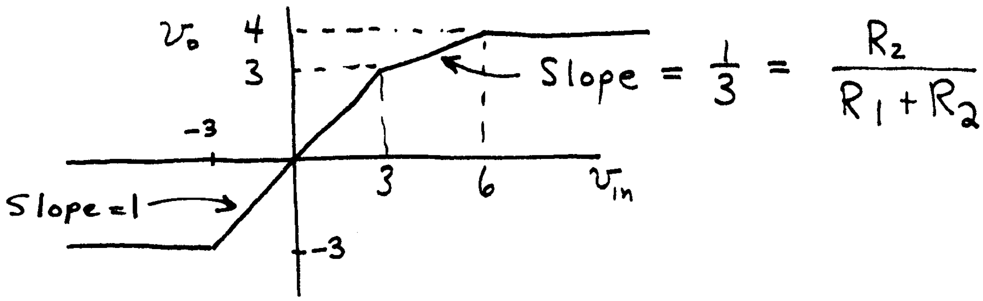
P10.75 A suitable circuit is:



We must choose R_1 to ensure that the 5.6-V Zener is in the breakdown region at all times and choose the time constant $R_2C \gg T$, where T is the period of the input waveform.

P10.76 (a) A suitable circuit is:



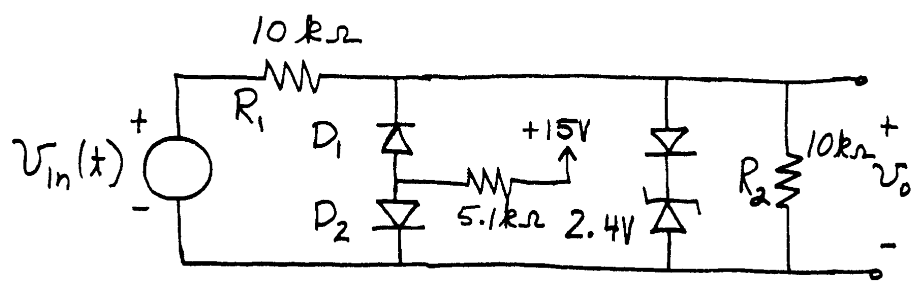


We choose the resistors R_1 and R_2 to achieve the desired slope.

$$\text{Slope } = \frac{1}{3} = \frac{R_2}{R_1 + R_2}$$

Thus, choose $R_1 = 2R_2$. For example, $R_1 = 2\ \text{k}\Omega$ and $R_2 = 1\ \text{k}\Omega$.

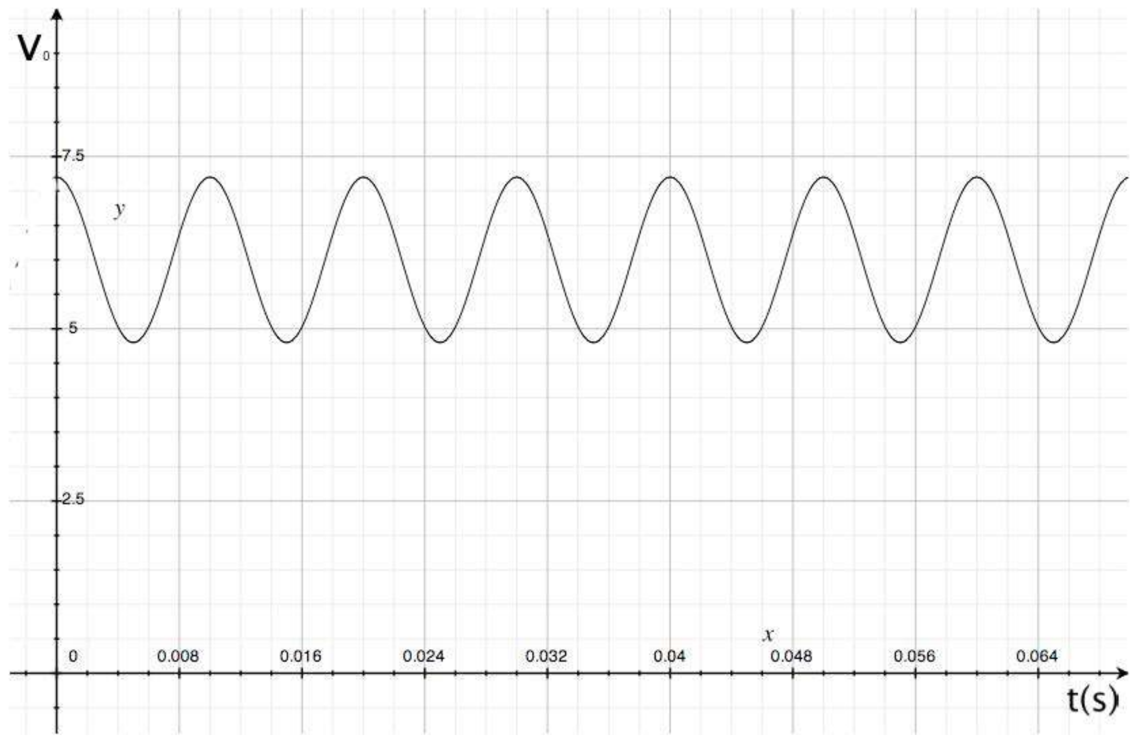
(b) A suitable circuit is:



Other resistor values will work, but we must make sure that D_2 remains forward biased for all values of v_{in}, including $v_{in} = -10$ V. To achieve the desired slope (i.e., the slope is 0.5) for the transfer characteristic, we must have $R_1 = R_2$.

P10.77 I_{DQ} represents the dc component of the diode current with no signal applied to the circuit, and $i_d(t)$ represents the changes from the Q-point current when the signal is applied. Furthermore, $i_D(t)$ is the total diode current. Thus, we have

$$i_D(t) = I_{DQ} + i_d(t) = 6 + 1.2\cos(200\pi t)\text{mA}$$

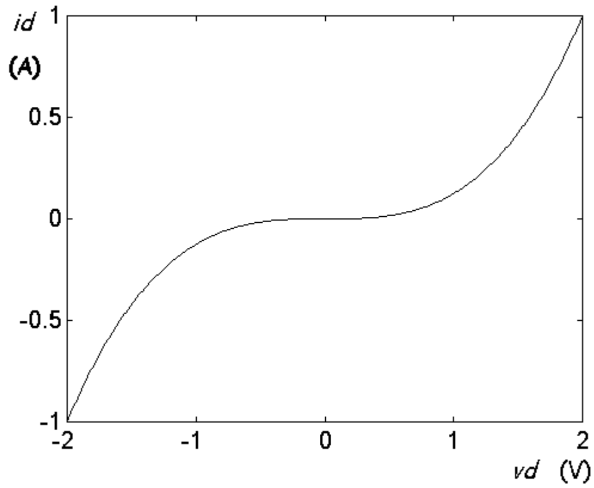


P10.78 The small signal equivalent circuit of a diode is a resistance known as the dynamic resistance. The dynamic resistance is the reciprocal of the slope of the i_D versus v_D characteristic at the operating point.

P10.79 Dc sources voltage sources are replaced by short circuits in a small-signal equivalent circuit. By definition the voltage across a dc voltage source is constant. Thus, even if there is ac current flowing through the dc source the ac voltage across it is zero as is the case for a short circuit.

P10.80 We should replace dc current sources by open circuits in a small-signal equivalent circuit. The current through a dc current source is constant. Thus, the ac current must be zero even if we apply an ac voltage. Zero current for a non-zero applied voltage implies that we have an open circuit.

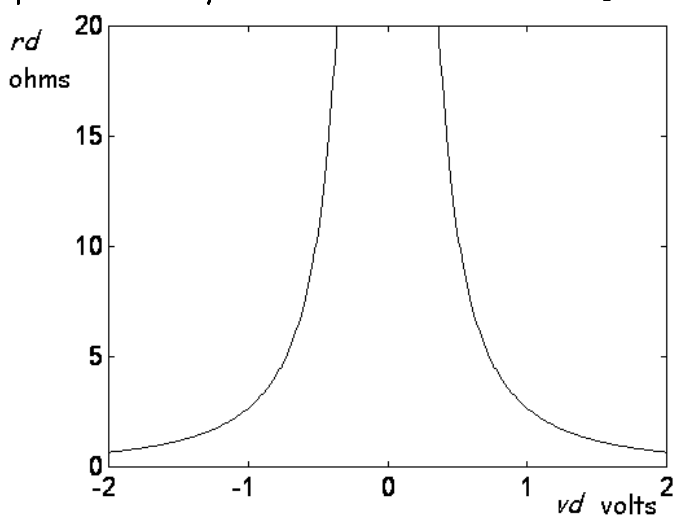
P10.81* A plot of the device characteristic is:



Clearly this device is not a diode because it conducts current in both directions. The dynamic resistance is given by:

$$r_D = \left(\frac{di_D}{dv_D}\right)^{-1} = \frac{8}{3v_D^2}$$

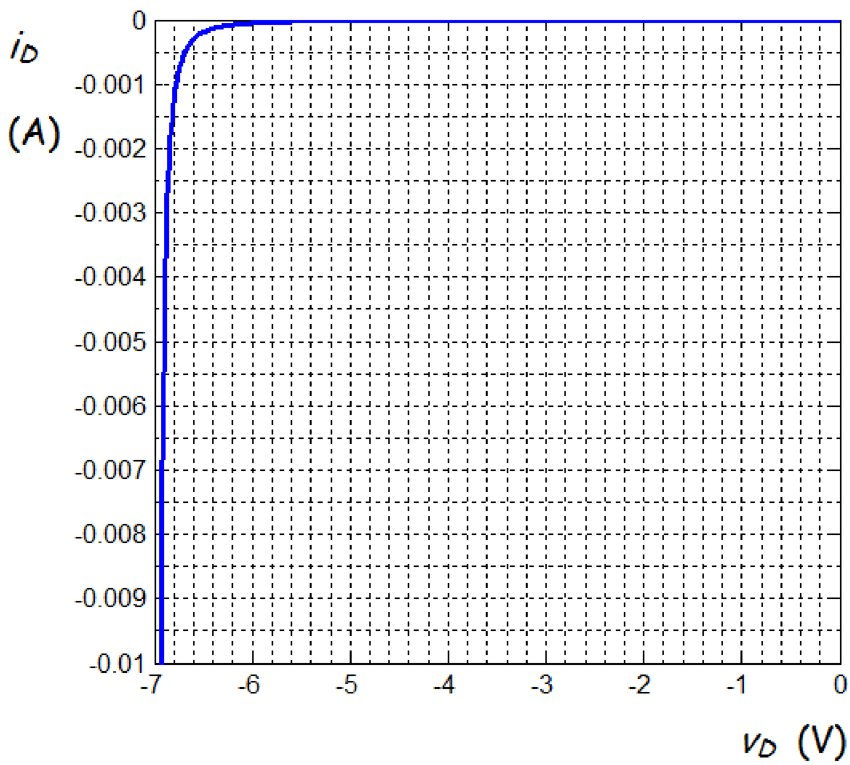
A plot of the dynamic resistance versus v_D is:



P10.82 We are given

$$i_D = \frac{-10^{-6}}{(1 + v_D/7)^2} \quad \text{for} -7\text{ V} < v_D < 0$$

A plot of this is:



The dynamic resistance is:

$$r_D = (dv_D / di_D)^{-1} = 3.5 \times 10^6 (1 + v_D / 7)^3$$

To find the dynamic resistance at a given Q-point, we evaluate this expression for $v_D = V_{DQ}$.

For $I_{DQ} = -0.5$ mA, we have $V_{DQ} = -6.68695$ V and $r_D = 22.7\ \Omega$.

For $I_{DQ} = -10$ mA, we have $V_{DQ} = -6.930$ V and $r_D = 3.5\ \Omega$.

P10.83 We are given $v_D(t) = 5 + \cos(\omega t)$ V and $i_D(t) = 10 + 0.2\cos(\omega t)$ mA. The dynamic resistance is the ratio of the ac voltage amplitude to the ac current amplitude.

$$r_D = \frac{v_d}{i_d} = \frac{0.03}{0.2 \times 10^{-3}} = 150\ \Omega$$

The Q-point results if we set the ac signals to zero. Thus, we have

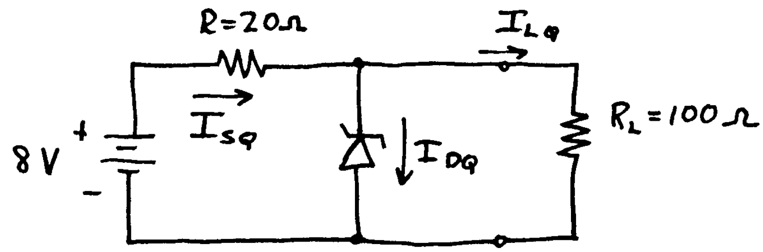
$$V_{DQ} = 5 \text{ V} \quad \text{and} \quad I_{DQ} = 10 \text{mA}$$

P10.84 Dynamic resistance is given by

$$r_D = \left(\frac{di_D}{dv_D}\right)^{-1} = \frac{dv_D}{di_D}$$

Because voltage is constant for changes in current, the dynamic resistance is zero for an ideal Zener diode in the breakdown region.

P10.85* To find the Q-point, we ignore the ac ripple voltage and the circuit becomes:



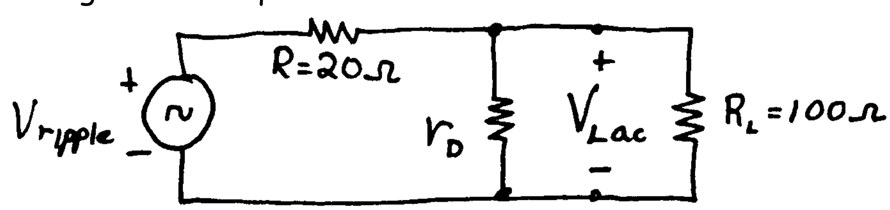
Thus, we have:

$$I_{sQ} = \frac{8-5}{20} = 150 \text{ mA}$$

$$I_{LQ} = 5/100 = 50 \text{ mA}$$

$$I_{DQ} = I_{sQ} - I_{LQ} = 100 \text{ mA}$$

The small-signal or ac equivalent circuit is:



where r_D is the dynamic resistance of the Zener diode. Using the voltage-division principle, the ripple voltage across the load is

$$v_{Lac} = V_{ripple} \times \frac{R_p}{R + R_p}$$

where $R_p = \dfrac{1}{1/R_L + 1/r_D}$ is the parallel combination of the load resistance and the dynamic resistance of the diode. Substituting values, we find

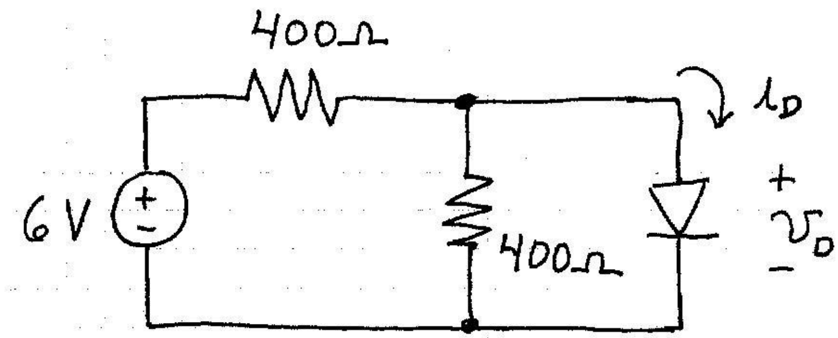
$$V_{Lac} = 10 \times 10^{-3} = 1 \times \frac{R_p}{20 + R_p}$$

Solving, we find $R_p = 0.202\ \Omega$. Then, we have:

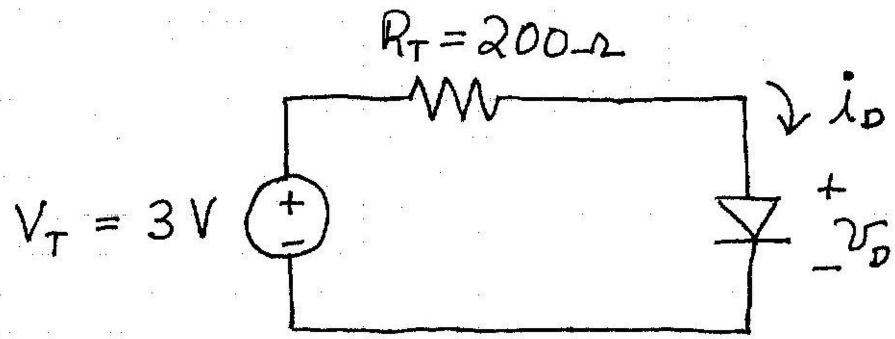
$R_p = 0.202 = \dfrac{1}{1/100 + 1/r_D}$ which yields $r_D = 0.202\ \Omega$.

Practice Test

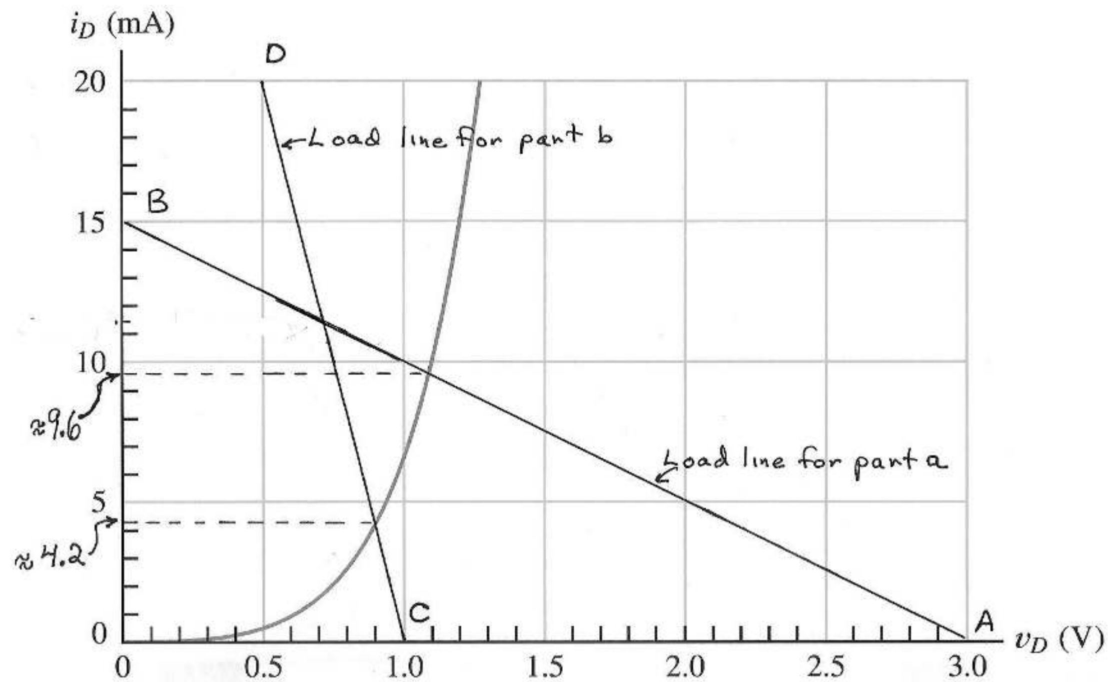
T10.1 (a) First, we redraw the circuit, grouping the linear elements to the left of the diode.



Then, we determine the Thévenin equivalent for the circuit looking back from the diode terminals.



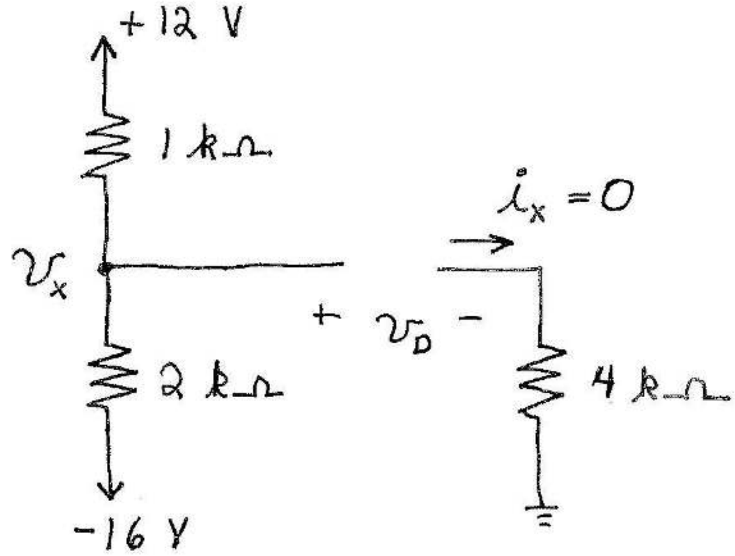
Next, we write the KVL equation for the network, which yields $V_T = R_T i_D + v_D$. Substituting the values for the Thévenin voltage and resistance, we have the load-line equation, $3 = 200 i_D + v_D$. For $i_D = 0$, we have $v_D = 3$ V which are the coordinates for Point *A* on the load line, as shown below. For $v_D = 0$, the load-line equation gives $i_D = 15$ mA which are the coordinates for Point *B* on the load line. Using these two points to plot the load line on Figure 10.8, we have



The intersection of the load line and the diode characteristic gives the current at the operating point as $i_D \cong 9.6$ mA.

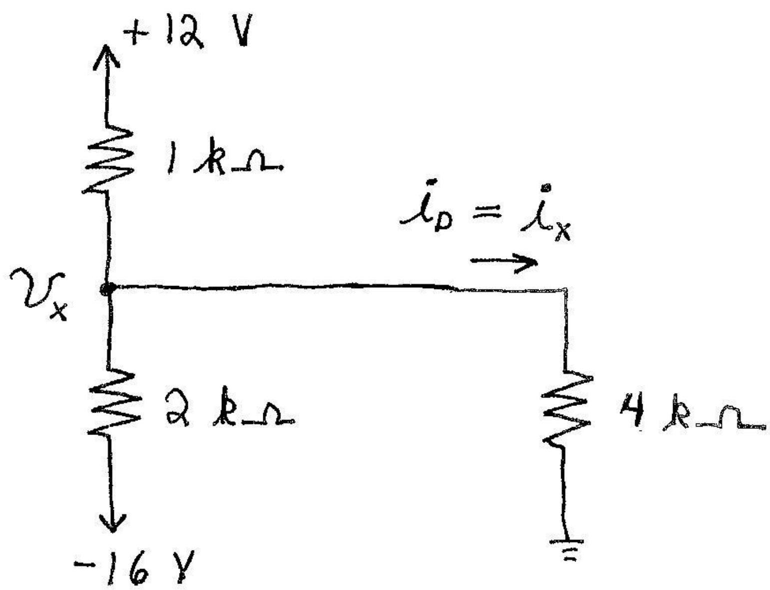
(b) First, we write the KCL equation at the top node of the network, which yields $i_D + v_D / 25 = 40$ mA. For $i_D = 0$, we have $v_D = 1$ V which are the coordinates for Point *C* on the load line shown above. For $v_D = 0$, the load-line equation gives $i_D = 40$ mA which plots off the vertical scale. Therefore, we substitute $i_D = 20$ mA, and the KCL equation then yields $v_D = 0.5$ V. These values are shown as point D. Using Points C and D we plot the load line on Figure 10.8 as shown above. The intersection of the load line and the diode characteristic gives the current at the operating point as $i_D \cong 4.2$ mA.

T10.2 If we assume that the diode is off (i.e., an open circuit), the circuit becomes



Writing a KCL equation with resistances in kΩ, currents in mA, and voltages in V, we have $\frac{v_x - 12}{1} + \frac{v_x - (-16)}{2} = 0$. Solving, we find that $v_x = 2.667$ V. However, the voltage across the diode is $v_D = v_x$, which must be negative for the diode to be off. Therefore, the diode must be on.

With the diode assumed to be on (i.e. a short circuit) the circuit becomes

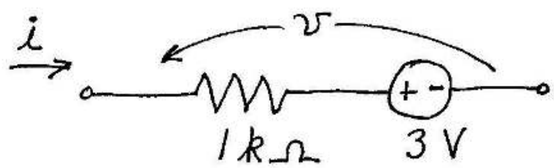


Writing a KCL equation with resistances in kΩ, currents in mA and voltages in V, we have $\frac{v_x - 12}{1} + \frac{v_x - (-16)}{2} + \frac{v_x}{4} = 0$. Solving, we find that

$v_x = 2.286$ V. Then, the current through the diode is

$i_D = i_x = \dfrac{v_x}{4} = 0.571$ mA. Of course, a positive value for i_D is consistent with the assumption that the diode is on.

T10.3 We know that the line passes through the points (5 V, 2 mA) and (10 V, 7 mA). The slope of the line is $-1/R = -\Delta i/\Delta v = (-5 \text{ mA})/(5 \text{ V})$, and we have $R = 1$ kΩ. Furthermore, the intercept on the voltage axis is at $v = 3$ V. Thus, the equivalent circuit for the device is



T10.4 The circuit diagram is:

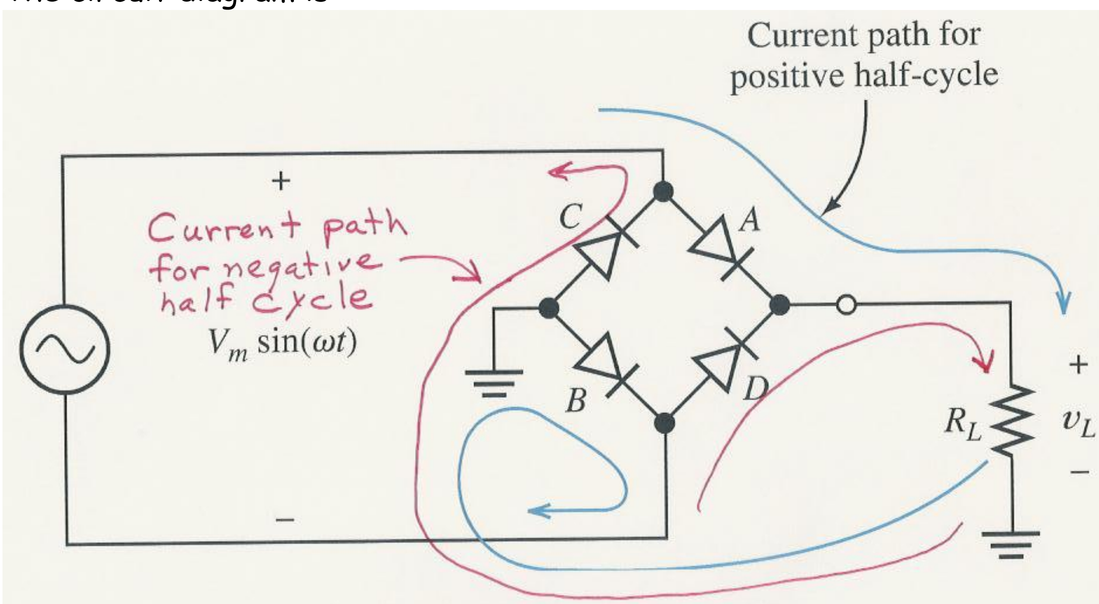
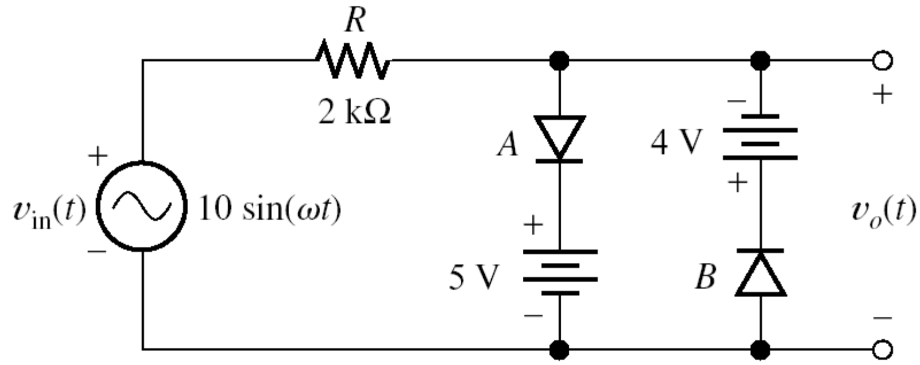


Figure 10.28 Diode-bridge full-wave rectifier.

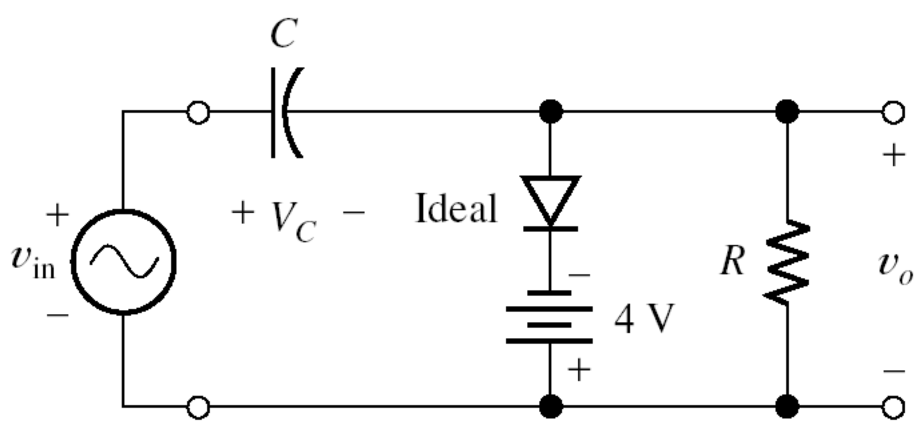
Your diagram may be correct even if it is laid out differently. Check to see that you have four diodes and that current flows from the source through a diode in the forward direction then through the load and finally through a second diode in the forward direction back to the opposite end of the source. On the opposite half cycle, the path should be through the other two diodes and through the load in the same direction as before. Notice in the diagram that current flows downward through the load on both half cycles.

T10.5 An acceptable circuit diagram is:



Your diagram may be somewhat different in appearance. For example, the 4-V source and diode *B* can be interchanged as long as the source polarity and direction of the diode don't change; similarly for the 5-V source and diode *A*. The parallel branches can be interchanged in position. The problem does not give enough information to properly select the value of the resistance, however, any value from about 1 kΩ to 1 MΩ is acceptable.

T10.6 An acceptable circuit diagram is:



The time constant *RC* should be much longer than the period of the source voltage. Thus, we should select component values so that $RC >> 0.1$ s.

T10.7 We have

$$V_T = \frac{kT}{q} = \frac{1.38 \times 10^{-23} \times 300}{1.60 \times 10^{-19}} = 25.88 \text{ mV}$$

$$r_d = \frac{nV_T}{I_{DQ}} = \frac{2 \times 25.88 \times 10^{-3}}{5 \times 10^{-3}} = 10.35\ \Omega$$

The small-signal equivalent circuit for the diode is a 10.35 Ω resistance.

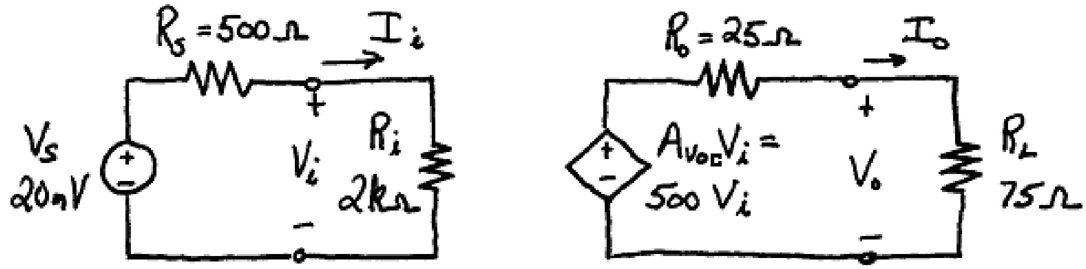
CHAPTER 11

Exercises

E11.1 (a) A noninverting amplifier has positive gain. Thus
$$v_o(t) = A_v v_i(t) = 50 v_i(t) = 5.0\sin(2000\pi t)$$

(b) An inverting amplifier has negative gain. Thus
$$v_o(t) = A_v v_i(t) = -50 v_i(t) = -5.0\sin(2000\pi t)$$

E11.2

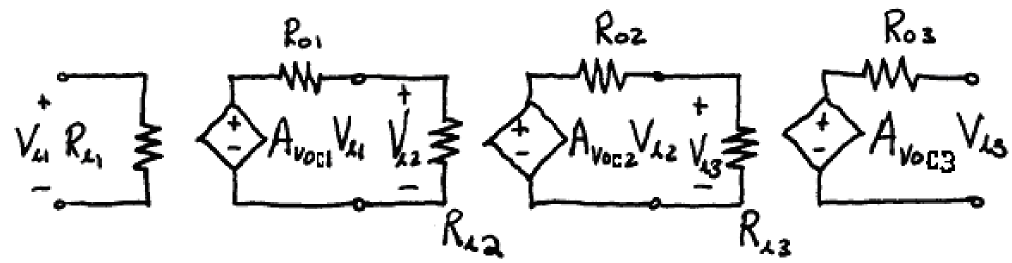


$$A_v = \frac{V_o}{V_i} = A_{voc}\frac{R_L}{R_o + R_L} = 500\frac{75}{25+75} = 375$$
$$A_{vs} = \frac{V_o}{V_s} = \frac{R_i}{R_s + R_i}A_{voc}\frac{R_L}{R_o + R_L} = \frac{2000}{500+2000}500\frac{75}{25+75} = 300$$
$$A_i = \frac{I_o}{I_i} = A_v\frac{R_i}{R_L} = 375 \times \frac{2000}{75} = 10^4$$
$$G = A_v A_i = 3.75 \times 10^6$$

E11.3 Recall that to maximize the power delivered to a load from a source with fixed internal resistance, we make the load resistance equal to the internal (or Thévenin) resistance. Thus we make $R_L = R_o = 25\ \Omega$. Repeating the calculations of Exercise 11.2 with the new value of R_L, we have

$$A_v = \frac{V_o}{V_i} = A_{voc}\frac{R_L}{R_o + R_L} = 500\frac{25}{25+25} = 250$$
$$A_i = \frac{I_o}{I_i} = A_v\frac{R_i}{R_L} = 250 \times \frac{2000}{25} = 2\times 10^4$$
$$G = A_v A_i = 5 \times 10^6$$

E11.4



By inspection, $R_i = R_{i1} = 1000\ \Omega$ and $R_o = R_{o3} = 30\ \Omega$.

$$A_{oc} = \frac{V_{o3}}{V_{i1}} = A_{oc1}\frac{R_{i2}}{R_{o1}+R_{i2}}A_{oc2}\frac{R_{i3}}{R_{o2}+R_{i3}}A_{oc3}$$

$$A_{oc} = \frac{V_{o3}}{V_{i1}} = 10\frac{2000}{100+2000}20\frac{3000}{200+3000}30 = 5357$$

E11.5 Switching the order of the amplifiers of Exercise 11.4 to 3-2-1, we have

$R_i = R_{i3} = 3000\ \Omega$ and $R_o = R_{o1} = 100\ \Omega$

$$A_{oc} = \frac{V_{o1}}{V_{i3}} = A_{oc3}\frac{R_{i2}}{R_{o3}+R_{i2}}A_{oc2}\frac{R_{i1}}{R_{o2}+R_{i1}}A_{oc1}$$

$$A_{oc} = \frac{V_{o1}}{V_{i3}} = 30\frac{2000}{300+2000}20\frac{1000}{200+1000}10 = 4348$$

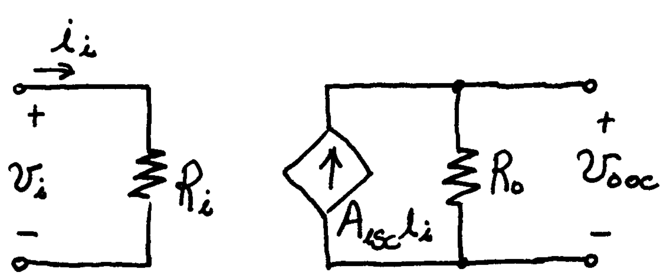
E11.6

$$P_s = (15\text{ V})\times(1.5\ A) = 22.5\text{ W}$$

$$P_d = P_s + P_i - P_o = 22.5 + 0.5 - 2.5 = 20.5\text{ W}$$

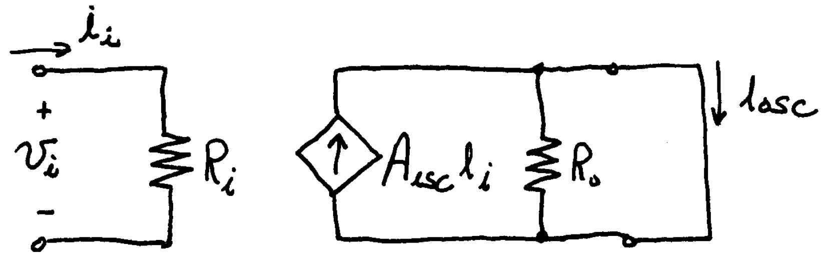
$$\eta = \frac{P_o}{P_s}\times 100\% = 11.11\%$$

E11.7 The input resistance and output resistance are the same for all of the amplifier models. Only the circuit configuration and the gain parameter are different. Thus we have $R_i = 1\text{ k}\Omega$ and $R_o = 20\ \Omega$ and we need to find the open-circuit voltage gain. The current amplifier with an open-circuit load is:



$$A_{voc} = \frac{v_{ooc}}{v_i} = \frac{A_{isc} i_i R_o}{R_i i_i} = \frac{A_{isc} R_o}{R_i} = \frac{200 \times 20}{1000} = 4$$

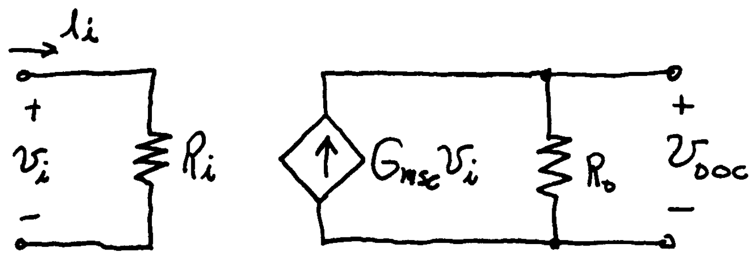
E11.8 For a transconductance-amplifier model, we need to find the short-circuit transconductance gain. The current-amplifier model with a short-circuit load is:



$$G_{msc} = \frac{i_{osc}}{v_i} = \frac{A_{isc} i_i}{R_i i_i} = \frac{A_{isc}}{R_i} = \frac{100}{500} = 0.2 \text{ S}$$

The impedances are the same for all of the amplifier models, so we have $R_i = 500\ \Omega$ and $R_o = 50\ \Omega$.

E11.9 For a transresistance-amplifier model, we need to find the open-circuit transresistance gain. The transconductance-amplifier model with an open-circuit load is:



$$R_{moc} = \frac{v_{ooc}}{i_i} = \frac{G_{msc} v_i R_o}{v_i / R_i} = G_{msc} R_o R_i = 0.05 \times 10 \times 10^6 = 500\ \text{k}\Omega$$

The impedances are the same for all of the amplifier models, so we have $R_i = 1\ \text{M}\Omega$ and $R_o = 10\ \Omega$.

E11.10 The amplifier has $R_i = 1\ \text{k}\Omega$ and $R_o = 1\ \text{k}\Omega$.

(a) We have $R_s < 10\ \Omega$ which is much less than R_i, and we also have $R_L > 100\ \text{k}\Omega$ which is much larger than R_o. Therefore for this source and load, the amplifier is approximately an ideal voltage amplifier.

(b) We have $R_s > 100\ \text{k}\Omega$ which is much greater than R_i, and we also have $R_L < 10\ \Omega$ which is much smaller than R_o. Therefore for this source and load, the amplifier is approximately an ideal current amplifier.

(c) We have $R_s < 10\ \Omega$ which is much less than R_i, and we also have $R_L < 10\ \Omega$ which is much smaller than R_o. Therefore for this source and load, the amplifier is approximately an ideal transconductance amplifier.

(d) We have $R_s > 100\ \text{k}\Omega$ which is much larger than R_i, and we also have $R_L > 100\ \text{k}\Omega$ which is much larger than R_o. Therefore for this source and load, the amplifier is approximately an ideal transresistance amplifier.

(e) Because we have $R_s \cong R_i$, the amplifier does not approximate any type of ideal amplifier.

E11.11 We want the amplifier to respond to the short-circuit current of the source. Therefore, we need to have $R_i << R_s$. Because the amplifier should deliver a voltage to the load that is independent of the load resistance, the output resistance R_o should be very small compared to the smallest load resistance. These facts (R_s very small and R_o very small) indicate that we need a nearly ideal transresistance amplifier.

E11.12 The gain magnitude should be constant for all components of the input signal, and the phase should by proportional to the frequency of each component. The input signal has components with frequencies of 500 Hz, 1000 Hz and 1500 Hz, respectively. The gain is $5\angle 30^\circ$ at a frequency of 1000 Hz. Therefore the gain should be $5\angle 15^\circ$ at 500 Hz, and $5\angle 45^\circ$ at 1500 Hz.

E11.13 We have

$$v_{\text{in}}(t) = V_m \cos(\omega t)$$

$$v_o(t) = 10v_{\text{in}}(t - 0.01) = 10V_m \cos[\omega(t - 0.01)] = 10V_m \cos(\omega t - 0.01\omega)$$

The corresponding phasors are $\mathbf{V}_{\text{in}} = V_m\angle 0$ and $\mathbf{V}_\text{o} = 10V_m\angle -0.01\omega$. Thus the complex gain is

$$A_v = \frac{\mathbf{V}_o}{\mathbf{V}_{\text{in}}} = \frac{10V_m\angle -0.01\omega}{V_m\angle 0} = 10\angle -0.01\omega$$

E11.14 $B \cong \frac{0.35}{t_r} = \frac{0.35}{66.7 \times 10^{-9}} = 5.247 \text{ MHz}$

E11.15 Equation 11.13 states

$$\text{Percentage tilt} \cong 200\pi f_L T$$

Solving for f_L and substituting values, we obtain

$$f_L \cong \frac{\text{percentage tilt}}{200\pi T} = \frac{1}{200\pi \times 100 \times 10^{-6}} = 15.92 \text{ Hz}$$

as the upper limit for the lower half-power frequency.

E11.16 (a) $v_o(t) = 100v_i(t) + v_i^2(t)$

$$= 100\cos(\omega t) + \cos^2(\omega t)$$
$$= 100\cos(\omega t) + 0.5 + 0.5\cos(2\omega t)$$

The desired term has an amplitude of V_1 = 100 and a second-harmonic distortion term with an amplitude of V_2 = 0.5. There are no higher order distortion terms so we have $D_2 = V_2 / V_1 = 0.005$ or 0.5%.

$$D = \sqrt{D_2^2 + D_3^2 + D_4^2 \ldots} = D_2 = 0.5\%$$

(b) $v_o(t) = 100v_i(t) + v_i^2(t)$

$$= 500\cos(\omega t) + 25\cos^2(\omega t)$$
$$= 500\cos(\omega t) + 12.5 + 12.5\cos(2\omega t)$$

The desired term has an amplitude of V_1 = 500 and a second-harmonic distortion term with an amplitude of V_2 = 12.5. There are no higher order distortion terms so we have $D_2 = V_2 / V_1 = 0.025$ or 2.5%.

$$D = \sqrt{D_2^2 + D_3^2 + D_4^2 \ldots} = D_2 = 2.5\%$$

E11.17 With the input terminals tied together and a 1-V signal applied, the differential signal is zero and the common-mode signal is 1 V. The common-mode gain is $A_{cm} = V_o / V_{icm} = 0.1/1 = 0.1$, which is equivalent to -20 dB. Then we have $CMRR = 20\log(|A_d|/|A_{cm}|) = 20\log(500{,}000) = 114.0 \text{ dB}$.

E11.18 (a) $v_{id} = v_{i1} - v_{i2} = 1\text{ V}$ $\qquad v_{icm} = (v_{i1} + v_{i2})/2 = 0\text{ V}$

$$v_o = A_1 v_{i1} - A_2 v_{i2} = (A_1 + A_2)/2$$
$$= A_d v_{id} + A_{cm} v_{icm} = A_d$$

Thus $A_d = (A_1 + A_2)/2$.

(b) $v_{id} = v_{i1} - v_{i2} = 0\text{ V} \qquad v_{icm} = (v_{i1} + v_{i2})/2 = 1\text{ V}$

$$v_o = A_1 v_{i1} - A_2 v_{i2} = (A_1 - A_2)$$
$$= A_d v_{id} + A_{cm} v_{icm} = A_{cm}$$

Thus $A_{cm} = A_1 - A_2$.

(c) $A_d = (A_1 + A_2)/2 = (100 + 101)/2 = 100.5$

$$A_{cm} = A_1 - A_2 = 100 - 101 = -1$$

$$CMRR = 20\log\left(\frac{|A_d|}{|A_{cm}|}\right) = 20\log\left(\frac{|A_1 + A_2|}{2|A_1 - A_2|}\right)$$

$$CMRR = 20\log\left(\frac{|A_1 + A_2|}{2|A_1 - A_2|}\right) = 20\log\left(\frac{|100 + 101|}{2|100 - 101|}\right) = 40.0\text{ dB.}$$

E11.19 Except for numerical values this Exercise is the same as Example 11.13 in the book. With equal resistances at the input terminals, the bias currents make no contribution to the output voltage. The extreme contributions to the output due to the offset voltage are

$$A_d V_{Voff} = A_d V_{off} \frac{R_{in}}{R_{in} + R_{s1} + R_{s2}}$$
$$= 500 \times (\pm 10 \times 10^{-3}) \frac{100 \times 10^3}{(100 + 50 + 50)10^3} = \pm 2.5\text{ V}$$

The extreme contributions to the output voltage due to the offset current are

$$A_d V_{Ioff} = A_d \frac{I_{off}}{2} \frac{R_{in}(R_{s1} + R_{s2})}{R_{in} + R_{s1} + R_{s2}}$$
$$= 500 \times \frac{\pm 100 \times 10^{-9}}{2} \frac{100 \times 10^3 (50 + 50) \times 10^3}{(100 + 50 + 50)10^3} = \pm 1.25\text{ V}$$

Thus, the extreme output voltages due to all sources are ± 3.75 V.

E11.20 This Exercise is similar to Example 11.13 in the book with R_{s1} = 50 kΩ and R_{s2} = 0. With unequal resistances at the input terminals, the bias currents make a contribution to the output voltage given by

$$V_{oBias} = A_d I_B \frac{R_{s1} R_{in}}{R_{s1} + R_{in}}$$
$$= 500 \times 400 \times 10^{-9} \frac{50 \times 10^3 \times 100 \times 10^3}{50 \times 10^3 + 100 \times 10^3} = +6.667\text{ V}$$

The extreme contributions to the output due to the offset voltage are

$$A_d V_{Voff} = A_d V_{\text{off}} \frac{R_{\text{in}}}{R_{\text{in}} + R_{s1} + R_{s2}}$$

$$= 500 \times (\pm 10 \times 10^{-3}) \frac{100 \times 10^3}{(100 + 50 + 0)10^3} = \pm 3.333 \text{ V}$$

The extreme contributions to the output voltage due to the offset current are

$$A_d V_{I\text{off}} = A_d \frac{I_{\text{off}}}{2} \frac{R_{\text{in}}(R_{s1} + R_{s2})}{R_{\text{in}} + R_{s1} + R_{s2}}$$

$$= 500 \times \frac{\pm 100 \times 10^{-9}}{2} \frac{100 \times 10^3 (50 + 0) \times 10^3}{(100 + 50 + 0)10^3} = \pm 0.8333 \text{ V}$$

Thus, the extreme output voltages due to all sources are a minimum of 2.5 V and a maximum of 10.83 V.

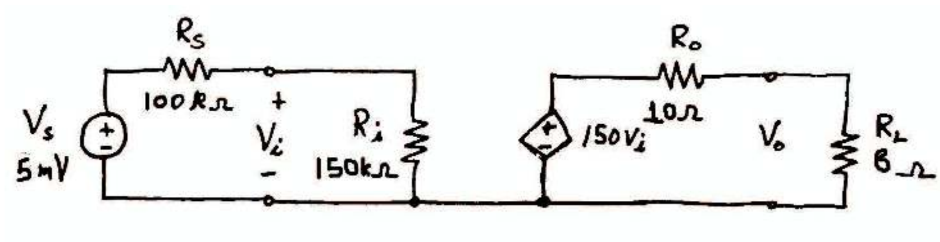
Problems

P11.1 Selection of Inverting and Non-inverting amplifier will be based upon application. For example, in case of monaural audio, the inversion in signal has no effect on the output, whereas in case of stereo both the speakers/channels require the same type of input, either inverting or non-inverting. In case of video signals, inversion results in negative image with black and white interchange.

P11.2 See Figure 11.3 in the book.

P11.3 Because of loading effects, the voltage gains (A_v or A_{vs}) realized are less than the internal gain A_{voc} of the amplifier. Loading effects can occur either at the input or output of an amplifier. The output voltage decreases when a load is connected because the current drawn by the load causes a voltage drop across the output impedance of the amplifier. The voltage at the source terminals decreases when the amplifier is connected because the current drawn by the amplifier results in a voltage drop across the internal (Thévenin) resistance of the source.

P11.4* The equivalent circuit is:



$$A_v = \frac{V_o}{V_i} = A_{voc}\frac{R_L}{R_o + R_L} = 150\frac{8}{10+8} = 66.66$$

$$A_{vs} = \frac{V_o}{V_s} = A_{voc}\frac{R_i}{R_i + R_s}\frac{R_L}{R_o + R_L}$$

$$= 150\frac{150\times10^3}{100\times10^3 + 150\times10^3}\;\frac{8}{10+8}$$

$$= 40$$

$$A_i = A_v\frac{R_i}{R_L} = 66.66\frac{150\times10^3}{8} = 1250\times10^3$$

$$G = A_v A_i = 83.325\times10^6$$

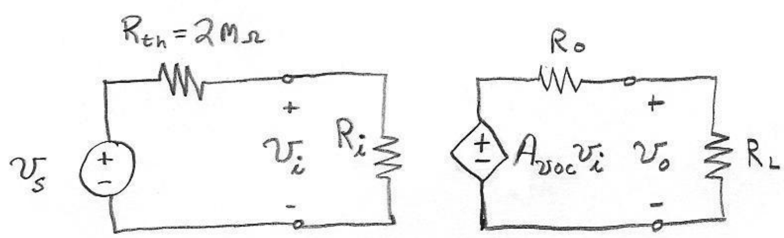
P11.5* $A_i = \dfrac{G}{A_v} = \dfrac{8000}{75} = 106.66$

$$R_i = \frac{A_i}{A_v}R_L = \frac{106.66}{75}150 = 213.33$$

P11.6 $A_v = \dfrac{v_o}{v_i} = \dfrac{R_L i_L}{R_i i_i} = \dfrac{R_L}{R_i}\times A_i = \dfrac{100}{10^6}\times 500 = 0.05$

$G = A_v A_i = 25$

P11.7 The equivalent circuit is:



$$v_i(t) = \frac{R_i}{R_i + R_{th}} v_s = \frac{10^6}{10^6 + 2\times 10^6}[3\times 10^{-3}\cos(200\pi t)] = 10^{-3}\cos(200\pi t) \quad \text{V}$$

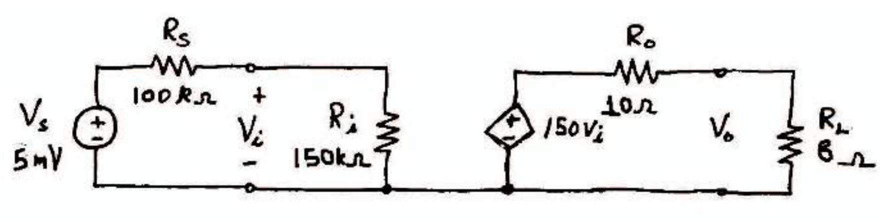
$$v_o(t) = A_{voc} v_i \frac{R_L}{R_o + R_L} = -10^4[10^{-3}\cos(200\pi t)]\frac{1000}{1000 + 1000} = -5\cos(200\pi t)$$

$$P_i = \frac{V_{i-rms}^2}{R_i} = \frac{(10^{-3}/\sqrt{2})^2}{10^6} = 0.5\times 10^{-12} \quad \text{W}$$

$$P_o = \frac{V_{o-rms}^2}{R_L} = \frac{(5/\sqrt{2})^2}{10^3} = 12.5\times 10^{-3} \quad \text{W}$$

$$G = \frac{P_o}{P_i} = 25\times 10^9$$

P11.8 The equivalent circuit using the amplifier is:



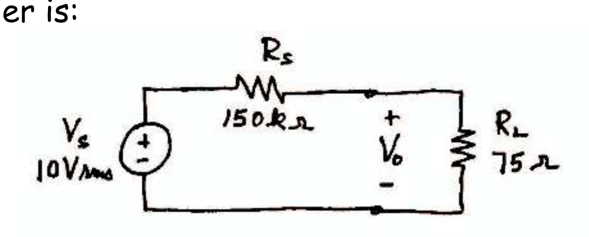
We have

$$A_{vs} = \frac{V_o}{V_s} = A_{oc}\frac{R_i}{R_i + R_s}\frac{R_L}{R_o + R_L} = 1\frac{8\times 10^6}{8\times 10^6 + 150\times 10^3}\frac{75}{150 + 75} = 0.327$$

$$V_o = A_{vs}V_s = 0.327\times 10 = 3.27 \text{ V rms}$$

$$P_o = (V_o)^2/R_L = 142.73 \text{ mW}$$

The equivalent for the load connected directly to the source without the amplifier is:



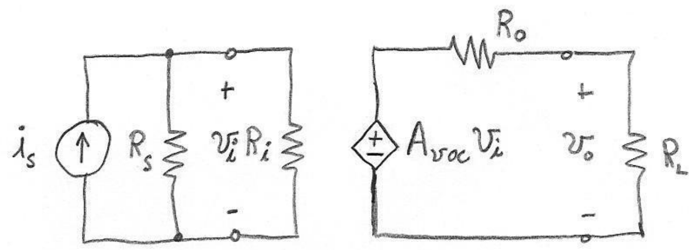
In this case, we have:

$$V_o = V_s\frac{R_L}{R_L + R_s} = 10\frac{75}{75 + 150\times 10^3} = 4.99 \text{ mV rms}$$

$$P_o = 333 \times 10^{-9} \text{ W}$$

Thus, the output voltage and output power is much higher when the amplifier is used, even though the open-circuit voltage gain of the amplifier is unity, because the amplifier alleviates source loading.

P11.9 The equivalent circuit is:



$$v_i(t) = \frac{1}{1/R_i + 1/R_s} i_s = 3000[2\times10^{-3}\cos(200\pi t)] = 6\cos(200\pi t) \quad \text{V}$$

$$v_o(t) = A_{voc} v_i \frac{R_L}{R_o + R_L} = -10[6\cos(200\pi t)]\frac{1000}{1000+1000} = -30\cos(200\pi t) \quad \text{V}$$

$$P_i = \frac{V_{i-rms}^2}{R_i} = \frac{(6/\sqrt{2})^2}{12000} = 1.5\times10^{-3} \text{ W}$$

$$P_o = \frac{V_{o-rms}^2}{R_L} = \frac{(30/\sqrt{2})^2}{10^3} = 450\times10^{-3} \text{ W} \qquad G = \frac{P_o}{P_i} = 300$$

P11.10* Before the 2-kΩ resistance is placed across the input terminals, the output voltage is given by

$$V_o = 2 = A_{oc} I_s R_i \frac{R_L}{R_o + R_L} \qquad (1)$$

After the resistance is placed in parallel with the input terminals, the output voltage is given by

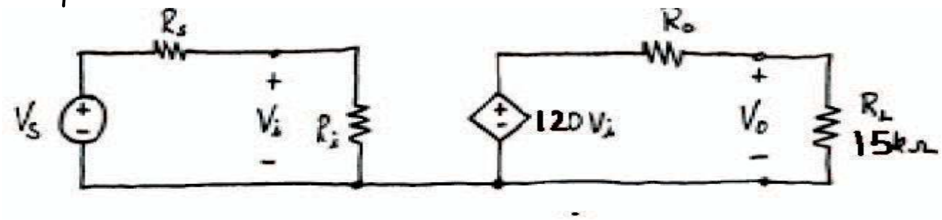
$$V_o' = 1.5 = A_{oc} I_s \frac{1}{1/2000 + 1/R_i} \frac{R_L}{R_o + R_L} \qquad (2)$$

Dividing the respective sides of Equation 2 by those of Equation 1, we have

$$\frac{V_o'}{V_o} = \frac{1.5}{2} = \frac{1}{R_i/2000 + 1}$$

Solving we obtain $R_i = 666.7\,\Omega$.

P11.11 The equivalent circuit is:



$$V_o = 120V_i \frac{R_L}{R_o + R_L}$$

$$A_v = \frac{V_o}{V_i} = 75 = \frac{120V_i \dfrac{R_L}{R_o + R_L}}{V_i} = 120\frac{15\ \text{k}\Omega}{R_o + 15\ \text{k}\Omega}$$

Solving, we find

$$R_o = 9\ \text{k}\Omega$$

P11.12 We have

$$v_o = A_{voc} v_i \frac{R_L}{R_L + R_o}$$

In avoiding changes in v_o with changes in R_L, the important parameter of the amplifier is its output resistance.

We need to choose R_o so that

$$0.99\frac{R_{L\max}}{R_{L\max} + R_o} \le \frac{R_{L\min}}{R_{L\min} + R_o}$$

$$0.99\frac{10^4}{10^4 + R_o} \le \frac{5000}{5000 + R_o}$$

Simplifying, we find that we need $R_o \le 102.0\,\Omega$.

P11.13 Because Equation 11. 3 states

$$A_i = A_v \frac{R_i}{R_L}$$

we conclude that if the current and voltage gains are equal, then the input and load resistances are equal.

P11.14

$$A_v = \frac{V_o}{V_i} = A_{oc}\frac{R_L}{R_o + R_L} = 1000\frac{8}{2+8} = 800$$

$$A_{vs} = \frac{V_o}{V_s} = A_{oc}\frac{R_i}{R_i + R_s}\frac{R_L}{R_o + R_L}$$

$$= 1000 \frac{20 \times 10^3}{20 \times 10^3 + 10 \times 10^3} \frac{8}{2+8}$$
$$= 533.3$$
$$A_i = A_v \frac{R_i}{R_L} = 800 \frac{20 \times 10^3}{8} = 2 \times 10^6$$
$$G = A_v A_i = 1.6 \times 10^9$$

P11.15* With the switch closed, we have:

$$V_o = 150 \text{ mV} = A_v \frac{R_L}{R_o + R_L} V_s \qquad (1)$$

With the switch open, we have:

$$V_o = 80 \text{ mV} = A_v \frac{R_{in}}{R_{in} + 3 \times 10^6} \frac{R_L}{R_o + R_L} V_s \qquad (2)$$

Dividing the respective sides of Equation (2) by those of Equation (1), we obtain:

$$0.533 = \frac{R_{in}}{R_{in} + 3 \times 10^6}$$

Solving, we get $R_{in} = 3.424$ MΩ

P11.16 Because we have $G = A_v A_i$, we can have $G = 10$ for $A_v = 0.1$ provided that $A_i = 100$. Then because $A_i = A_v R_i / R_L$, we have $R_i / R_L = 100$.

P11.17 We have

$$v_o = v_s \frac{R_i}{R_i + R_s} A_{voc} \frac{R_L}{R_L + R_o}$$

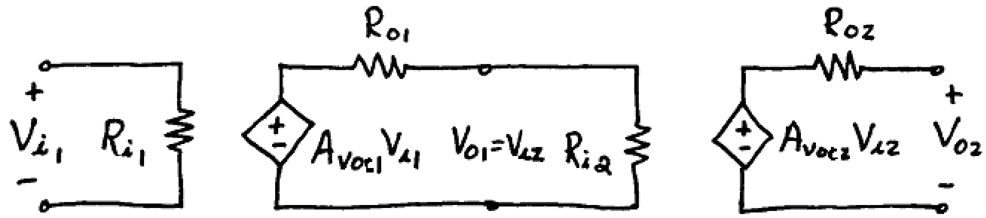
In avoiding changes in v_o with changes in R_s, the important parameter of the amplifier is its input resistance.

We need to choose R_i so that

$$0.98 \frac{R_i}{R_i + R_{s\min}} \leq \frac{R_i}{R_i + R_{s\max}}$$
$$0.98 \frac{R_i}{R_i + 0} \leq \frac{R_i}{R_i + 10^4}$$

Simplifying, we find that we need $R_i \geq 490$ kΩ.

P11.18 The equivalent circuit for the cascaded amplifiers is:



We can write:

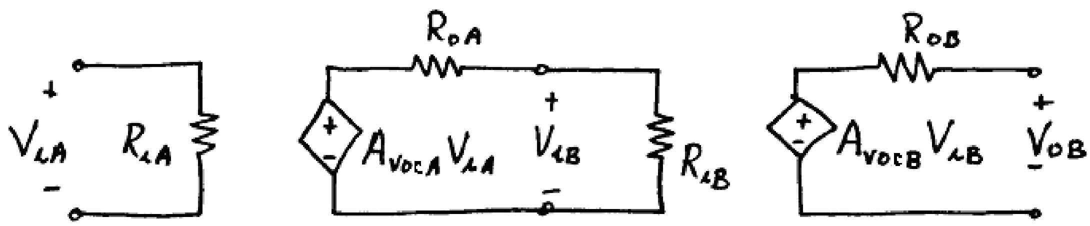
$$V_{i2} = A_{voc1}V_{i1} \times \frac{R_{i2}}{R_{i2} + R_{o1}}$$

$$V_{o2} = A_{voc2}V_{i2} = A_{voc2}A_{voc1}V_{i1}\frac{R_{i2}}{R_{i2} + R_{o1}}$$

Thus, the open-circuit voltage gain is:

$$A_{voc} = \frac{V_{o2}}{V_{i1}} = A_{voc2}A_{voc1}\frac{R_{i2}}{R_{i2} + R_{o1}}$$

P11.19 For the amplifiers in the order *A-B*, the equivalent circuit is:



Thus, we have:

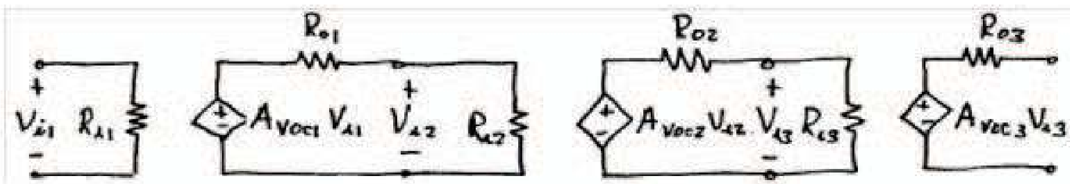
$$R_i = R_{iA} = 3\ \text{k}\Omega$$
$$R_o = R_{oB} = 2\ \text{k}\Omega$$
$$A_{voc} = A_{vocA}A_{vocB}\frac{R_{iB}}{R_{iB} + R_{oA}}$$
$$= 100(500)\frac{10^6}{10^6 + 400}$$
$$= 49.98 \times 10^3$$

For the amplifiers cascaded in the order *B-A*, we have:

$$R_i = R_{iB} = 1\text{M}\Omega$$

$$R_o = R_{oA} = 400\ \Omega$$
$$A_{oc} = A_{ocA}A_{ocB}\frac{R_{iA}}{R_{iA}+R_{oB}}$$
$$= 100\left(500\frac{3000}{3000+2000}\right)$$
$$= 30\times 10^3$$

P11.20* The equivalent circuit for the cascade is:



We have:

$$R_i = R_{i1} = 2\ \text{k}\Omega$$
$$R_o = R_{o3} = 3\ \text{k}\Omega$$
$$A_{oc} = A_{oc1}A_{oc2}A_{oc3}\frac{R_{i2}}{R_{i2}+R_{o1}}\frac{R_{i3}}{R_{i3}+R_{o2}}$$
$$= 50(100)(150)\frac{2000}{2000+1000}\frac{2000}{2000+2000}$$
$$= 2.5\times 10^5$$

P11.21 Reversing the order of the amplifiers of Problem P11.20, we have

$$R_i = R_{i3} = 6\ \text{k}\Omega$$
$$R_o = R_{o1} = 1\ \text{k}\Omega$$
$$A_{oc} = A_{oc1}A_{oc2}A_{oc3}\frac{R_{i2}}{R_{i2}+R_{o3}}\frac{R_{i1}}{R_{i1}+R_{o2}}$$
$$= 100(200)(300)\frac{4000}{4000+3000}\frac{2000}{2000+2000}$$
$$= 1.714\times 10^6$$

P11.22* The voltage gain of an n-stage cascade is given by

$$A_v = A_{oc}^n\left(\frac{R_i}{R_o+R_i}\right)^{n-1}\left(\frac{R_L}{R_o+R_L}\right) = 10^n\left(\tfrac{1}{2}\right)^{n-1}\left(\tfrac{1}{3}\right)$$

in which we have assumed that $n \geq 2$. Evaluating for various values of n we have:

n	A_v
2	16.67
3	83.33
4	416.7
5	2083

Thus, five amplifiers must be cascaded to attain a voltage gain in excess of 1000.

P11.23 The input resistance of the cascade is that of the first stage which is $R_i = 2\ \text{k}\Omega$. The open-circuit voltage gain is

$$A_v = A_{voc}^3\left(\frac{R_i}{R_o + R_i}\right)^2 = 25^3\left(\frac{2}{3+2}\right)^2 = 2500$$

The output resistance of the cascade is the output resistance of the last stage which is $R_o = 3\ \text{k}\Omega$.

P11.24 The power efficiency of an amplifier is the percentage of the power from the dc power supply that is converted to output signal power. We can write:

$$\eta = \frac{P_o}{P_s} \times 100\%$$

where P_o is the output signal power and P_s is the power taken from the power supply.

The remainder of the supplied power is converted to heat and is called dissipated power P_d. High internal power dissipation is undesirable and as such, circuits are designed to have low power dissipation.

P11.25* The 20-V source delivers power P_1: $(20) \times (2) = 40\text{W}$

The 25-V source delivers power P_2 : $(25) \times (4) = 100\text{W}$

The 10-V source absorbs power P_3 : $(10) \times (2) = -20\text{W}$

Thus, the net power supplied to the amplifier is:

$$P_s = P_1 + P_2 + P_3 = 120 \text{ W}$$

P11.26 $P_i = (V_i)^2 / R_i = 10^{-7}$ W

$$P_o = (V_o)^2 / R_L = 12.5 \text{ W}$$
$$P_s = V_s I_s = 30 \text{ W}$$
$$P_d = P_s + P_i - P_o = 17.5 \text{ W}$$
$$\eta = \frac{P_o}{P_s} \times 100\% = 41.67\%$$

P11.27 $P_i = I_i^2 R_i = (10^{-6})^2 \times 10^5 = 0.1\ \mu\text{W}$

$$P_o = (V_o)^2 / R_L = 10 \text{ W}$$
$$P_s = V_s I_s = 18 \text{ W}$$
$$P_d = P_s + P_i - P_o = 8 \text{ W}$$
$$\eta = \frac{P_o}{P_s} \times 100\% = 55.56\%$$

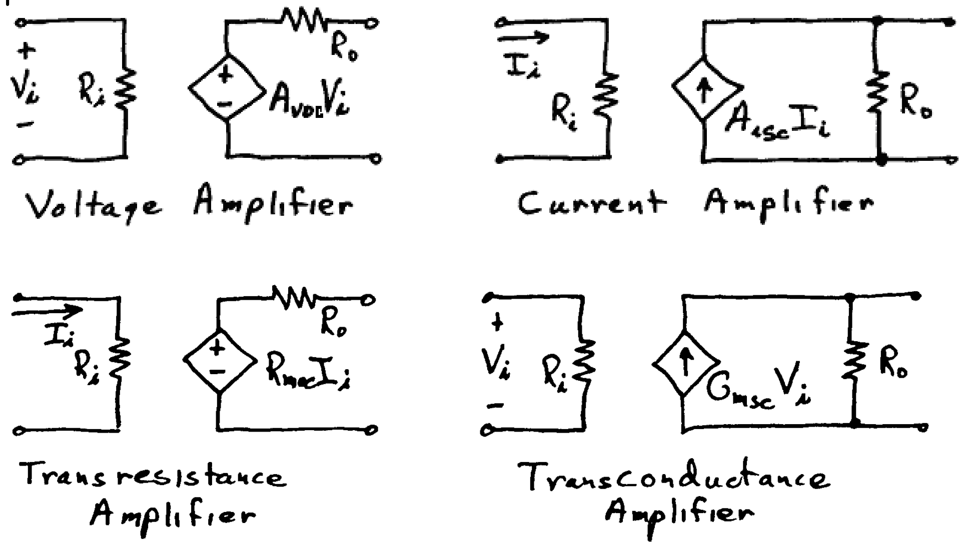
P11.28 $P_o = \dfrac{V_{o-rms}^2}{R_L} = \dfrac{(24)^2}{8} = 72 \text{ W}$

$$P_s = V_s I_s = 200 \text{ W}$$
$$\eta = \frac{P_o}{P_s} \times 100\% = 36\%$$
$$P_d = P_s + P_i - P_o = 200 + 0 - 72 = 128 \text{ W}$$

P11.29 $P_i = P_{i1} = \dfrac{V_{i1-rms}^2}{R_{i1}} = \dfrac{(2 \times 10^{-3})^2}{10^6} = 4 \text{ pW}$

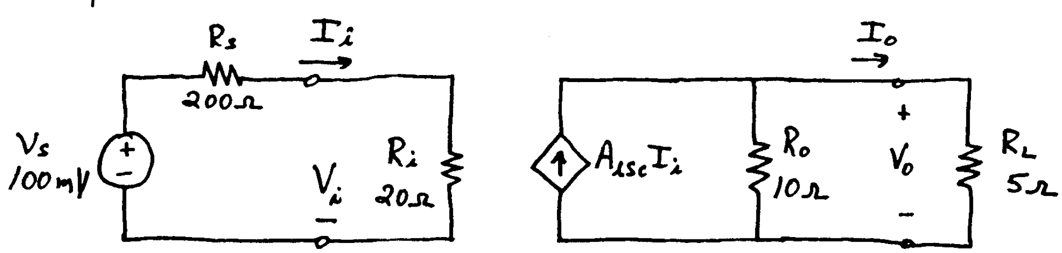
$$P_o = P_{o2} = \frac{V_{o2-rms}^2}{R_L} = \frac{(12)^2}{8} = 18 \text{ W}$$
$$P_{\text{supply}} = 2 + 22 = 24 \text{ W}$$
$$G = \frac{P_o}{P_i} = 4.5 \times 10^{12}$$
$$P_{\text{dissipated}} = P_{\text{supply}} - P_o = 24 - 18 = 6 \text{ W}$$
$$\eta = \frac{P_o}{P_{\text{supply}}} \times 100\% = 75\%$$

P11.30 The voltage gain A_{voc} is measured under open-circuit conditions.
The current gain A_{isc} is measured under short-circuit conditions.
The transresistance gain R_{moc} is measured under open-circuit conditions.
The transconductance gain G_{msc} is measured under short-circuit conditions.
The amplifier models are:



P11.31 (a) The transresistance amplifier model contains a current-controlled voltage source.
(b) The current amplifier model contains a current-controlled current source.
(c) The transconductance model contains a voltage-controlled current source.

P11.32* The equivalent circuit is:



We have:

$$I_i = V_s/(R_s + R_i) = 454.5\mu A \text{ rms}$$
$$V_i = I_i R_i = 9.090 \text{ mV rms}$$
$$I_o = A_{isc} I_i \frac{R_o}{R_o + R_L} = 0.909 \text{ A rms}$$

$$V_o = R_L I_o = 4.545 \text{ V rms}$$
$$A_i = I_o / I_i = 2000$$
$$A_v = V_o / V_i = 500$$
$$G = A_i A_v = 10^6$$
$$P_i = (V_i)^2 / R_i = 4.131\ \mu\text{W} \qquad P_o = (V_o)^2 / R_L = 4.131 \text{ W}$$
$$P_s = V_s I_s = 12 \times 2 = 24 \text{ W}$$
$$P_d = P_s + P_i - P_o = 19.87 \text{ W}$$
$$\eta = \frac{P_o}{P_s} \times 100\% = 17.2\%$$

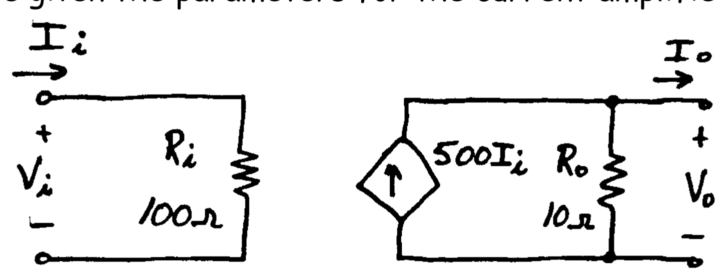
P11.33* $A_{voc} = \dfrac{V_o}{V_i} = \dfrac{R_{moc}(V_i / R_i)}{V_i} = \dfrac{R_{moc}}{R_i} = 100 \text{ V/V}$

$$G_{msc} = \frac{I_o}{V_i} = \frac{[R_{moc}(V_i / R_i)] / R_o}{V_i} = \frac{R_{moc}}{R_i R_o} = 0.05 \text{ S}$$
$$A_{isc} = \frac{I_o}{I_i} = \frac{R_{moc} I_i / R_o}{I_i} = \frac{R_{moc}}{R_o} = 7.5 \text{ A/A}$$

P11.34 $A_{voc} = \dfrac{V_o}{V_i} = \dfrac{G_{msc} V_i R_o}{V_i} = G_{msc} R_o = 50 \text{ V/V}$

$$R_{moc} = \frac{V_o}{I_i} = \frac{G_{msc} V_i R_o}{V_i / R_i} = G_{msc} R_i R_o = 500 \text{ k}\Omega$$
$$A_{isc} = \frac{I_o}{I_i} = \frac{G_{msc} V_i}{V_i / R_i} = G_{msc} R_i = 5000 \text{ A/A}$$

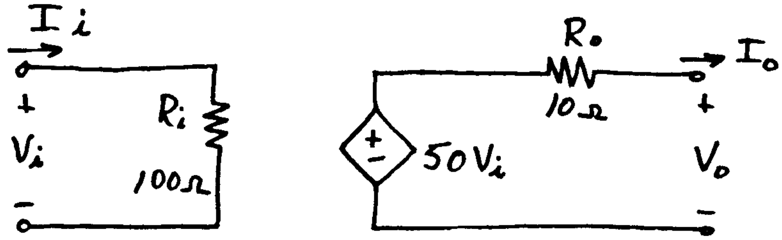
P11.35 We are given the parameters for the current amplifier model:



(a) The open circuit voltage gain is:

$$A_{voc} = \frac{V_{ooc}}{V_i} = \frac{500 I_i R_o}{R_i I_i} = 50$$

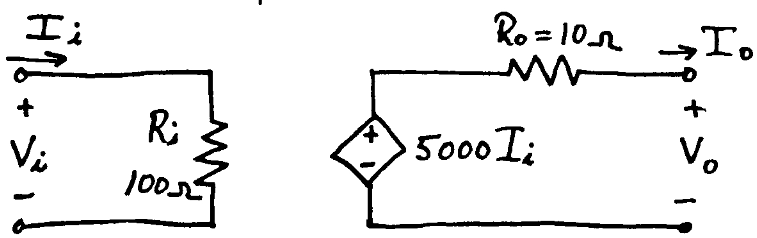
The voltage-amplifier model is:



(b) The transresistance gain is:

$$R_{moc} = \frac{V_{ooc}}{I_i} = \frac{500 I_i R_o}{I_i} = 5000\ \Omega$$

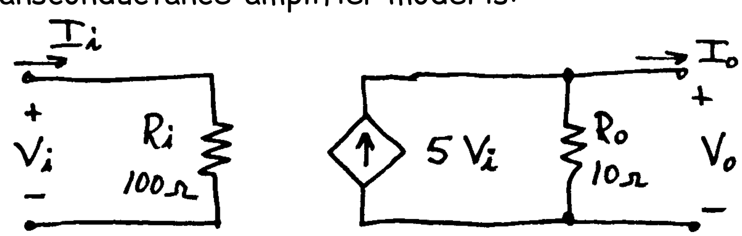
The transresistance-amplifier model is:



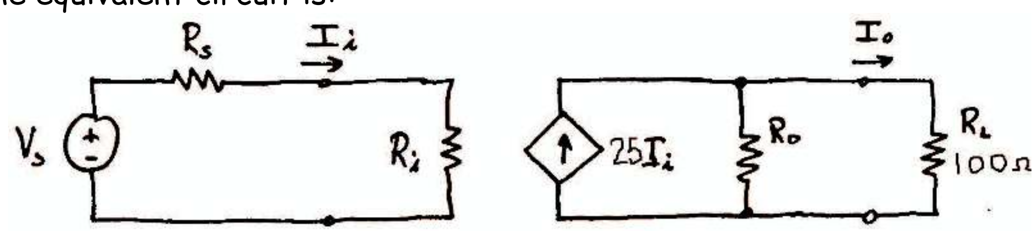
(c) The transconductance gain is:

$$G_{msc} = \frac{I_{osc}}{V_i} = \frac{500 I_i}{R_i I_i} = 5\ \mathrm{S}$$

The transconductance-amplifier model is:



P11.36 The equivalent circuit is:



We can write:

$$I_o = 25 I_i \frac{R_o}{R_o + R_L}$$

$$A_i = \frac{I_o}{I_i} = 12 = 25 \frac{R_o}{R_o + 50}$$

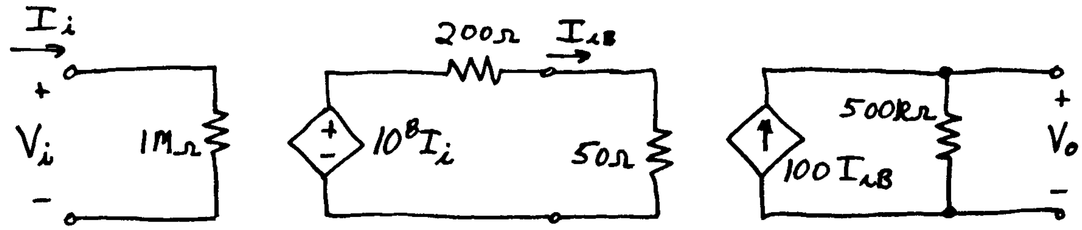
Solving, we find that $R_o = 92.307\Omega$

P11.37 $A_{voc} = \dfrac{V_o}{V_i} = \dfrac{A_{isc}(V_i / R_i)R_o}{V_i} = \dfrac{A_{isc}R_o}{R_i} = 30 \text{ V/V}$

$$R_{moc} = \frac{V_o}{I_i} = \frac{A_{isc} I_i R_o}{I_i} = A_{isc} R_o = 60 \text{ k}\Omega$$

$$G_{msc} = \frac{I_o}{V_i} = \frac{A_{isc} I_i}{I_i R_i} = \frac{A_{isc}}{R_i} = 0.1 \text{ S}$$

P11.38* The equivalent circuit is:

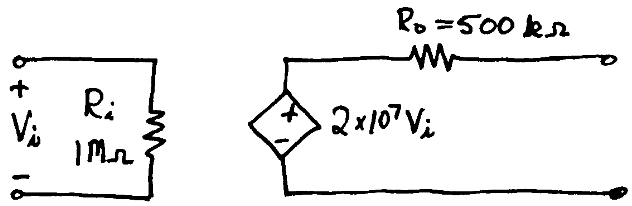


$$A_{voc} = \frac{V_{ooc}}{V_i} = \frac{500 \times 10^3 \times 100 \times \dfrac{10^8 I_i}{200 + 50}}{10^6 I_i} = 2 \times 10^7$$

$$R_i = R_{iA} = 1 \text{ M}\Omega$$

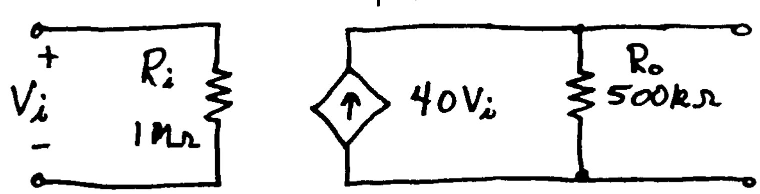
$$R_o = R_{oB} = 500 \text{ k}\Omega$$

Thus, the voltage-amplifier model is:

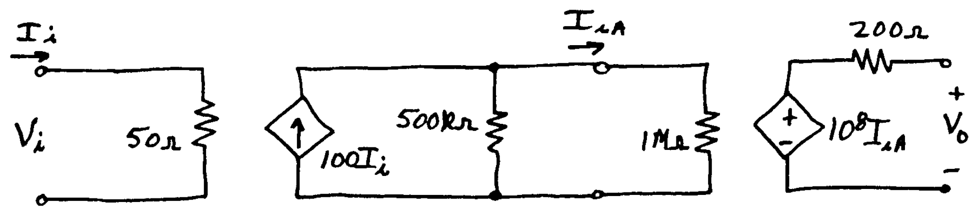


Then we can write $G_{msc} = \dfrac{I_{osc}}{V_i} = \dfrac{(A_{voc}V_i)/R_o}{V_i} = 40 \text{ S}$

Thus, the transconductance-amplifier model is:



P11.39 The equivalent circuit is:

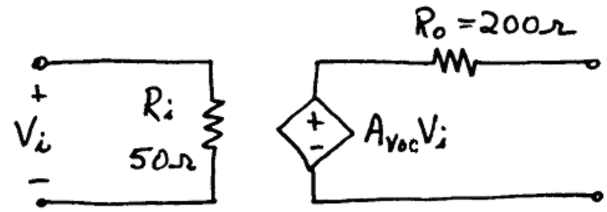


$$A_{voc} = \frac{V_{ooc}}{V_i} = \frac{10^8 \times \dfrac{500 \times 10^3}{10^6 + 500 \times 10^3} \times 100 I_i}{50 I_i} = 6.667 \times 10^7$$

$$R_i = R_{iB} = 50\ \Omega$$

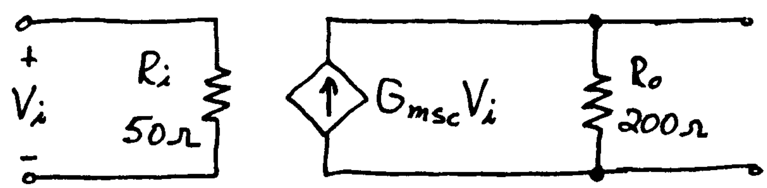
$$R_o = R_{oA} = 200\ \Omega$$

Thus, the voltage-amplifier model is:

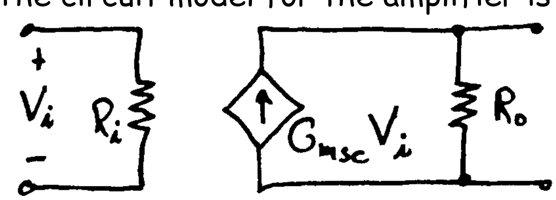


Then we can write $G_{msc} = \dfrac{I_{osc}}{V_i} = \dfrac{(A_{voc}V_i)/R_o}{V_i} = 333 \times 10^3$ S

Thus, the transconductance-amplifier model is:



P11.40* The circuit model for the amplifier is

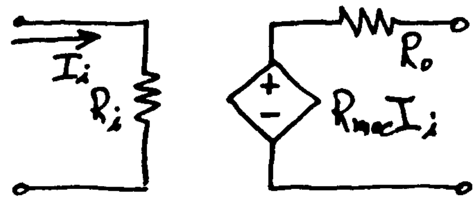


$$A_{voc} = \frac{V_{ooc}}{V_i} = \frac{G_{msc}V_iR_o}{V_i} = G_{msc}R_o = (0.5\text{ S})(200\ \Omega) = 100$$

$$A_{isc} = \frac{I_{osc}}{I_i} = \frac{G_{msc}V_i}{V_i / R_i} = G_{msc}R_i = (0.5\text{ S})(1000\ \Omega) = 500$$

$$R_{moc} = \frac{V_{ooc}}{I_i} = \frac{G_{msc}V_iR_o}{V_i / R_i} = G_{msc}R_oR_i = (0.5\text{ S})(200\ \Omega)(1000\ \Omega) = 100\text{ k}\Omega$$

P11.41* The circuit model for the amplifier is:



$$A_{oc} = \frac{V_{ooc}}{V_i} = \frac{R_{moc}I_i}{R_iI_i} = \frac{R_{moc}}{R_i} = \frac{200\ \text{k}\Omega}{10\ \text{k}\Omega} = 20$$

$$A_{isc} = \frac{I_{osc}}{I_i} = \frac{R_{moc}I_i / R_o}{I_i} = \frac{R_{moc}}{R_o} = \frac{200\ \text{k}\Omega}{2\ \text{k}\Omega} = 100$$

$$G_{msc} = \frac{I_{osc}}{V_i} = \frac{R_{moc}I_i / R_o}{R_iI_i} = \frac{R_{moc}}{R_iR_o} = \frac{200\ \text{k}\Omega}{(10\ \text{k}\Omega)(2\ \text{k}\Omega)} = 0.01\ \text{S}$$

P11.42

$$R_i = \frac{V_i}{I_i} = \frac{I_{osc} / G_{msc}}{I_{osc} / A_{isc}} = \frac{A_{isc}}{G_{msc}} = \frac{50}{0.2} = 250\ \Omega$$

$$R_o = \frac{A_{voc}V_i}{I_{osc}} = \frac{A_{voc}V_i}{G_{msc}V_i} = \frac{A_{voc}}{G_{msc}} = 500\ \Omega$$

$$R_{moc} = \frac{V_{ooc}}{I_i} = \frac{G_{msc}V_iR_o}{V_i / R_i} = G_{msc}R_oR_i = (0.2\ \text{S})(500\ \Omega)(250\ \Omega) = 25\ \text{k}\Omega$$

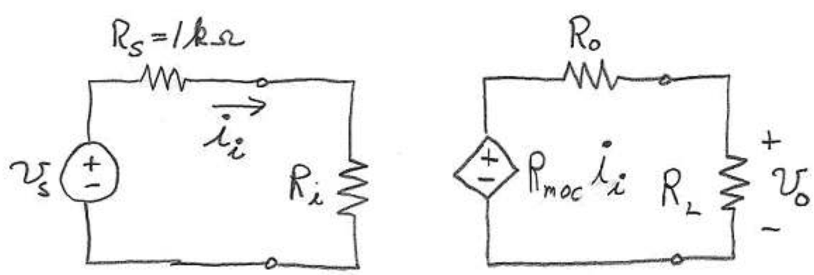
P11.43

$$R_i = \frac{V_i}{I_i} = \frac{I_{osc} / G_{msc}}{I_{osc} / A_{isc}} = \frac{A_{isc}}{G_{msc}} = \frac{50}{0.5} = 100\ \Omega$$

$$R_o = \frac{V_{ooc}}{G_{msc}V_i} = \frac{R_{moc}I_i}{G_{msc}R_iI_i} = \frac{R_{moc}}{G_{msc}R_i} = \frac{200}{(0.5)(100)} = 4\ \Omega$$

$$A_{voc} = \frac{V_{ooc}}{V_i} = \frac{R_{moc}I_i}{R_iI_i} = \frac{R_{moc}}{R_i} = \frac{200}{(100)} = 2\ \text{V/V}$$

P11.44 The equivalent circuit is:



$$i_i = \frac{v_s}{R_s + R_i} = \frac{2\times 10^{-3}\cos(200\pi t)}{1000 + 2000} = 0.6667\times 10^{-6}\cos(200\pi t)\ A$$

$$v_o = R_{moc} i_i \frac{R_L}{R_o + R_L} = -4.444\cos(200\pi t)\text{ V}$$

$$P_i = R_i I_{i-rms}^2 = 2000\left(\frac{0.6667\times 10^{-6}}{\sqrt{2}}\right)^2 = 444.4 \text{ pW}$$

$$P_o = \frac{V_{o-rms}^2}{R_L} = \frac{(4.444/\sqrt{2})^2}{1000} = 9.876 \text{ mW}$$

$$G = \frac{P_o}{P_i} = 22.22\times 10^6$$

P11.45 An amplifier with a very high input resistance is needed if we want the amplifier output to be proportional to the open-circuit voltage of the source and to be independent of the Thévenin impedance of the source. A good example is in an electrocardiograph in which we want the amplifier output to show the voltages produced by the heart independent of the impedance of the skin-electrode interface.

P11.46 An amplifier with a very low input impedance is needed if we want the amplifier output to be proportional to the short-circuit current of the source. A good example is an electronic ammeter.

P11.47 To supply a constant voltage to a variable number of parallel loads, we need an amplifier with a very low output impedance so the drop across the output impedance is negligible.

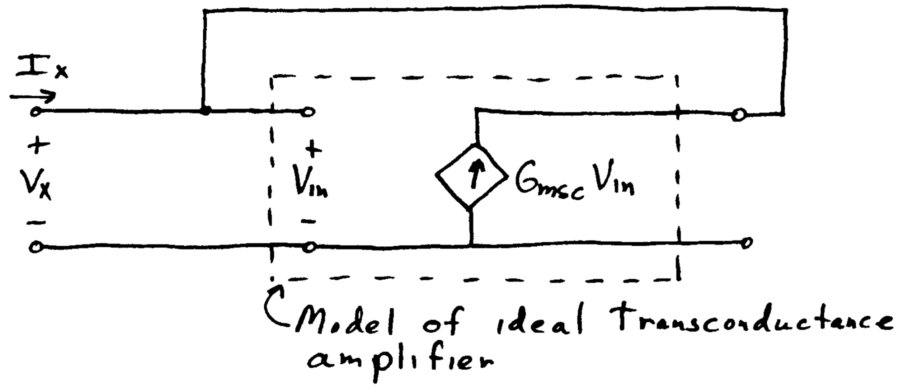
In a series connection, we would want the current (which is the same for all of the loads) to be constant. This calls for an amplifier with a very large output impedance.

P11.48 (a) To force a current that is proportional to the input signal through the load, we need an amplifier with a very large output impedance.
(b) To force a voltage that is proportional to the input signal to appear across the load, we need an amplifier with a very small output impedance.

P11.49 If a transmission line is connected to the input of an amplifier and if we need to avoid reflections on the line, the input resistance of the amplifier should equal the characteristic impedance of the line.

P11.50 The characteristics of various ideal amplifiers are summarized in Table 11.1 in the text.

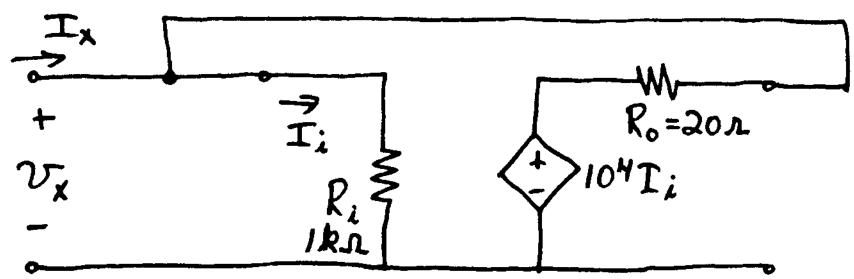
P11.51 The equivalent circuit is:



$$R_x = \frac{V_x}{I_x} = \frac{V_{in}}{-G_{msc}V_{in}} = -\frac{1}{G_{msc}} = -10\ \Omega$$

Thus if G_{msc} is positive, the circuit behaves as a negative resistance.

P11.52* The equivalent circuit is:



We can write:

$$I_i = \frac{V_x}{R_i} \qquad (1)$$

$$I_x = I_i + \frac{V_x - R_{moc}I_i}{R_o} \qquad (2)$$

Using Equation (1) to substitute for I_i in Equation (2) and solving, we have:

$$R_x = \frac{V_x}{I_x} = \frac{1}{1/R_i + 1/R_o - R_{moc}/(R_iR_o)} = -2.23\ \Omega$$

P11.53 We have $R_i << R_s$ and $R_o << R_L$. Thus, we have an approximately ideal transresistance amplifier. As in Example 11.7, we have:

$$R_{moc} = A_{vo} R_i = 10\ \Omega$$

P11.54 We have $R_i >> R_s$ and $R_o >> R_L$. Thus, we have an approximately ideal transconductance amplifier.

$$G_{msc} = \frac{A_{vo}}{R_o} = \frac{100}{10^6} = 10^{-4}\ \text{S}$$

P11.55* To sense the open-circuit voltage of a sensor, we need an amplifier with very high input resistance resistance and very low output resistance. Thus, we need a nearly ideal non-inverting voltage amplifier with a gain of 10 so that 0 - 500 mV range of sensor can be converted to 0 – 5 V range of the ADC.

P11.56* The input resistance is that of the ideal transresistance amplifier which is zero. The output resistance of the cascade is the output resistance of the ideal transconductance amplifier which is infinite. An amplifier having zero input resistance and infinite output resistance is an ideal current amplifier. Also, we have $A_{isc} = R_{moc} G_{msc}$.

P11.57 To sense the short-circuit current of a sensor, we need an amplifier with very low input resistance (compared to the Thévenin resistance of the sensor). To avoid loading effects by the potenitally variable load resistance, we need an amplifier with very low output resistance (compared to the smallest load resistance). Thus, we need a nearly ideal transresistance amplifier.

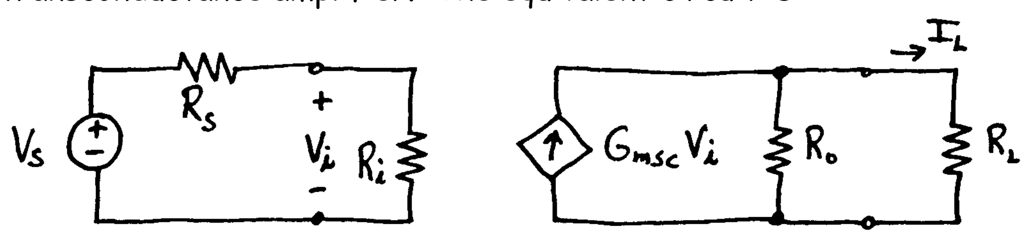
P11.58 To sense the short-circuit current of a sensor, we need an amplifier with very low input resistance (compared to the Thévenin resistance of the sensor). For the load current to be independent of the variable load resistance, we need an amplifier with very high output resistance (compared to the largest load resistance). Thus, we need a nearly ideal current amplifier.

P11.59 The input resistance is that of the voltage amplifier which is infinite. The output resistance of the cascade is the output resistance of the transconductance amplifier which is infinite. An amplifier having infinite input resistance and infinite output resistance is an ideal transconductance amplifier. Also, we have $G_{msc\text{-cascade}} = A_{voc}G_{msc}$.

P11.60 The input resistance is that of the transconductance amplifier which is infinite. The output resistance of the cascade is the output resistance of the transresistance amplifier which is zero. An amplifier having infinite input resistance and zero output resistance is an ideal voltage amplifier. Also, we have $A_{voc\text{-cascade}} = R_{moc}G_{msc}$.

With the order reversed, the input resistance is that of the transresistance amplifier which is zero. The output resistance of the cascade is the output resistance of the transconductance amplifier which is infinite. An amplifier having zero input resistance and infinite output resistance is an ideal current amplifier. Also, we have $A_{isc\text{-cascade}} = R_{moc}G_{msc}$.

P11.61* To sense the source voltage with minimal loading effects, we need $R_i >> R_s$. To force a current through the load independent of its resistance, we need $R_o >> R_L$. Thus, we need a nearly ideal transconductance amplifier. The equivalent circuit is:



We have $I_L = \dfrac{R_i}{R_i + R_s}\dfrac{R_o}{R_o + R_L}G_{msc}V_s$.

For the two given values of R_s, we require:

$$0.99\frac{R_i}{R_i + 1000} = \frac{R_i}{R_i + 2000}$$

Solving, we have $R_i = 98\ \text{k}\Omega$.

For the two given values of R_L, we require:

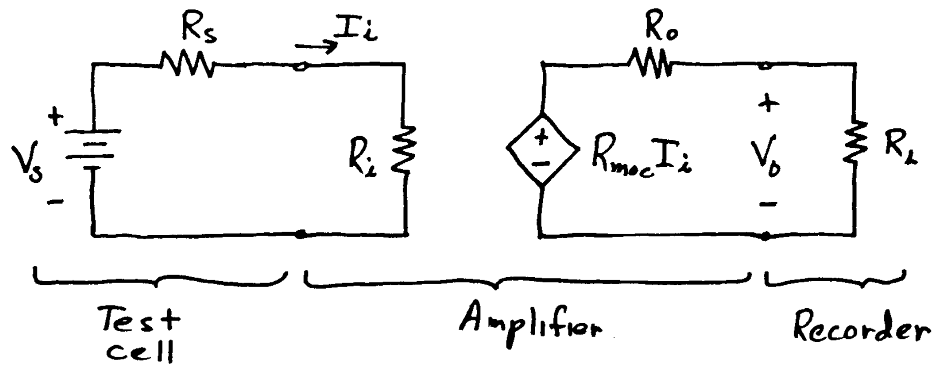
$$0.99\frac{R_o}{R_o + 100} = \frac{R_o}{R_o + 300}$$

Solving, we find: $R_o = 19.7\ \text{k}\Omega$

P11.62 We need $R_i << 10\ \Omega$ and $R_o << 10\ \text{k}\Omega$. Because we want the chart calibration to be 1 mA/cm and the chart pen deflects 1 cm per volt applied, the required transresistance gain is

$$R_{moc} = \frac{1\ \text{V}}{1\ \text{mA}} = 1000\ \Omega$$

Thus, a nearly ideal transresistance amplifier is needed. The equivalent circuit is



To achieve approximately ±3% accuracy, we will allow ±1% each for variations in the transresistance gain, in chart sensitivity, and in load resistance. From the equivalent circuit, we have

$$V_o = \frac{V_s}{R_s + R_i} \times R_{moc} \times \frac{R_L}{R_o + R_L}$$

Because are allowing a 1% change in V_o as R_L varies from $10\ \text{k}\Omega$ to an open circuit, we have:

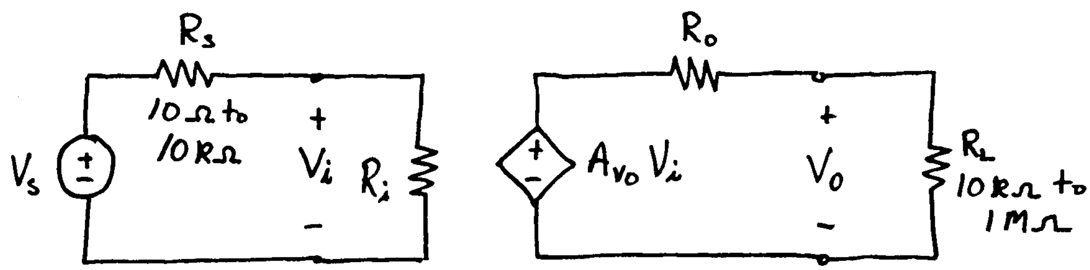
$$\frac{10\ \text{k}\Omega}{R_o + 10\ \text{k}\Omega} = 0.99$$

Solving, we find $R_o \cong 100\ \Omega$

Thus, we specify an amplifier having:

$$R_{moc} = 1000\ \Omega \pm 1\%$$
$$R_i < 10\ \Omega$$
$$R_o < 100\ \Omega$$

P11.63 We need an amplifier with high input resistance, low output resistance, and a voltage gain of 10. Thus, a nearly ideal voltage amplifier is required. Let us allow $\pm 1\%$ variation in the output voltage due to changes in R_s, in amplifier gain, and in load resistance. The equivalent circuit is:



We have:

$$V_o = V_s \frac{R_i}{R_i + R_s} A_{vo} \frac{R_L}{R_L + R_o}$$

We require:

$$0.99 \frac{R_i}{R_i + 10} = \frac{R_i}{R_i + 10^4} \text{ which yields } R_i = 989 \text{ k}\Omega$$

$$0.99 \frac{10^6}{10^6 + R_o} = \frac{10^4}{10^4 + R_o} \text{ which yields } R_o = 102\ \Omega$$

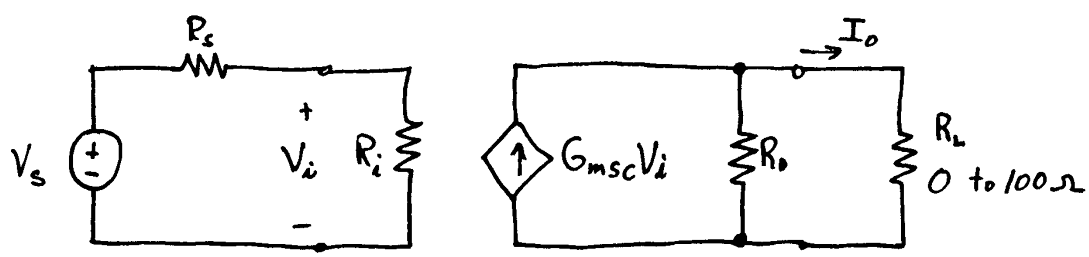
Thus, we specify an amplifier having:

$A_{vo} = 1000 \pm 1\%$

$R_i \geq 989 \text{ k}\Omega$

$R_o \leq 102\ \Omega$

P11.64 We need an amplifier with high input resistance, high output resistance, and a gain of $G_m = (1\text{ mA})/(0.1\text{ V}) = 10^{-2}$ S. Thus, a nearly ideal transconductance amplifier is needed. The sensitivity of the recorder varies by $\pm 1\%$; thus, we budget a total of $\pm 2\%$ for variations in amplifier gain, source resistance, and load resistance. We will allow 0.667% for each of these. The equivalent circuit is:



$$I_o = V_s \frac{R_i}{R_i + R_s} G_{msc} \frac{R_o}{R_L + R_o}$$

We require:

$$0.9933 \frac{R_i}{R_i + 10} = \frac{R_i}{R_i + 10^4} \text{ which yields } R_i = 1.49 \text{ M}\Omega$$

$$0.9933\frac{R_o}{R_o+0}=\frac{R_o}{R_o+100} \text{ which yields } R_o=14.9\text{ k}\Omega$$

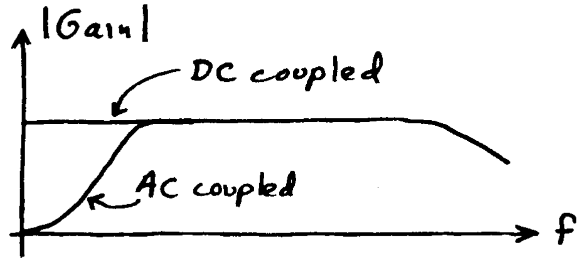
Thus, we specify an amplifier having:

$$G_{msc}=10\pm 2/3\%$$

$$R_i\geq 1.49\text{ M}\Omega$$

$$R_o\geq 14.9\text{ k}\Omega$$

P11.65



P11.66 A wideband amplifier has constant gain over a wide range of frequency. (i.e., The ratio of the upper half-power frequency to the lower half-power frequency f_H/f_L is much greater than unity.) A narrowband amplifier has constant gain over a narrow range of frequency (i.e., f_H/f_L is nearly unity) and falls to zero outside that range.

P11.67* We are given

$$v_{\text{in}}(t)=0.1\cos(2000\pi t)+0.2\cos(4000\pi t+30^\circ)$$

and

$$v_o(t)=10\cos(2000\pi t-20^\circ)+15\cos(4000\pi t+20^\circ)$$

The phasors for the 1000-Hz components are $\mathbf{V}_{\text{in}}=0.1\angle 0^\circ$ and $\mathbf{V}_o=10\angle-20^\circ$. Thus the complex gain for the 1000-Hz component is

$$A_v=\frac{\mathbf{V}_o}{\mathbf{V}_{\text{in}}}=\frac{10\angle-20^\circ}{0.1\angle 0^\circ}=100\angle-20^\circ$$

Similarly, the complex gain for the 2000-Hz component is

$$A_v=\frac{\mathbf{V}_o}{\mathbf{V}_{\text{in}}}=\frac{15\angle 20^\circ}{0.2\angle 30^\circ}=75\angle-10^\circ$$

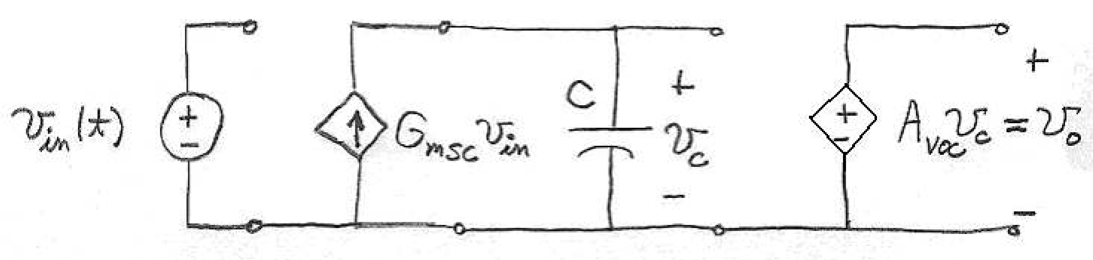
P11.68* The signal to be amplified is the short-circuit current of an electrochemical cell (or battery). This signal is dc and therefore a dc-coupled amplifier is needed. (The dc gain of an ac-coupled amplifier is zero and the signal of interest would not be amplified. Thus an ac-coupled amplifier would not be appropriate.)

P11.69 We need a midband voltage gain of (10 V)/(10 mV) = 1000 for the audio signal. Thus if a dc coupled amplifier were to be used the dc component of the output would be 2000 V. This is impractical, potentially dangerous, and would destroy most loudspeakers. Thus an ac-coupled amplifier is needed. Appropriate values for the half-power frequencies are f_L = 20 Hz or less and f_H = 10 kHz or more.

P11.70* The frequency response of a voltage amplifier is shown in fig. 11.24, the half-power points are also mentioned.

Usually, we specify the approximate useful frequency range of an amplifier by giving the frequencies for which the voltage (or current) gain magnitude is $1/\sqrt{2}$ times the midband gain magnitude. These are known as the half-power frequencies because the output power level is half the value for the midband region if a constant amplitude variable-frequency input test signal is used. Expressing the factor $1/\sqrt{2}$ in decibels, we have $20 \log(1/\sqrt{2}) = -3.01$ dB. Thus, at the half-power frequencies, the voltage (or current) gain is approximately 3 dB lower than the midband gain.The bandwidth B of an amplifier is the distance between the half-power frequencies.

P11.71 The equivalent circuit is:



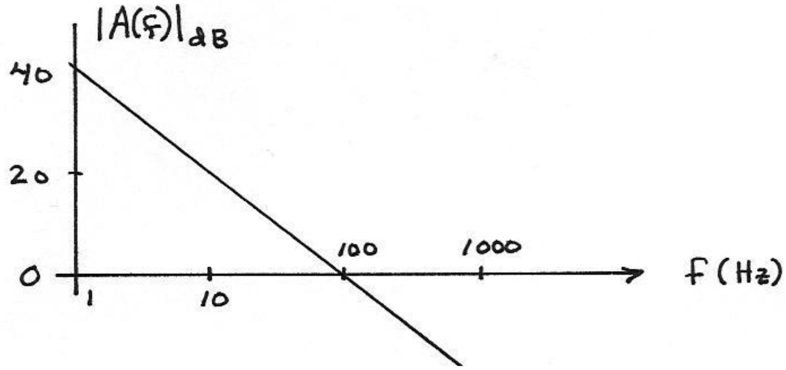
(a) $$v_o(t) = A_{oc} v_c(t) = A_{oc} \frac{1}{C} \int_0^t G_{msc} v_{in}(t) dt = \frac{A_{oc} G_{msc}}{C} \int_0^t v_{in}(t) dt$$

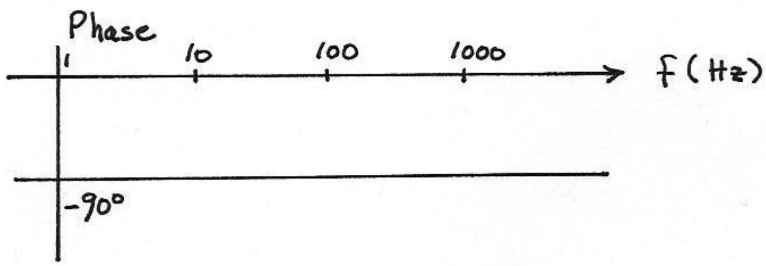
(b) $$v_o(t) = \frac{A_{oc}G_{msc}}{C}\int_0^t V_m \cos(2\pi f t)dt = \frac{V_m A_{oc} G_{msc}}{2\pi f C}\left[\sin(2\pi f t)\right]_0^t$$

$$v_o(t) = \frac{V_m A_{oc} G_{msc}}{2\pi f C}\sin(2\pi f t)$$

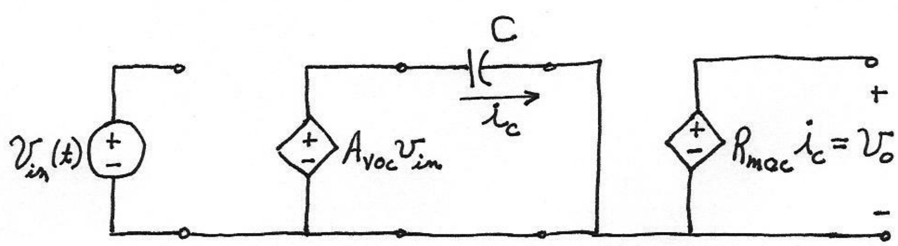
$$A(f) = \frac{\mathbf{V}_o}{\mathbf{V}_{in}} = \frac{-j\dfrac{V_m A_{oc} G_{msc}}{2\pi f C}}{V_m} = \frac{A_{oc}G_{msc}}{j2\pi f C} = \frac{A_{oc}G_{msc}}{2\pi f C}\angle -90^\circ$$

(c) For the values given, we have $A(f) = \dfrac{100}{f}\angle -90^\circ$. Then, the magnitude of the gain in dB is $|A(f)|_{dB} = 40 - 20\log(f)$. The Bode Plots of magnitude and phase are:





P11.72 The equivalent circuit is:

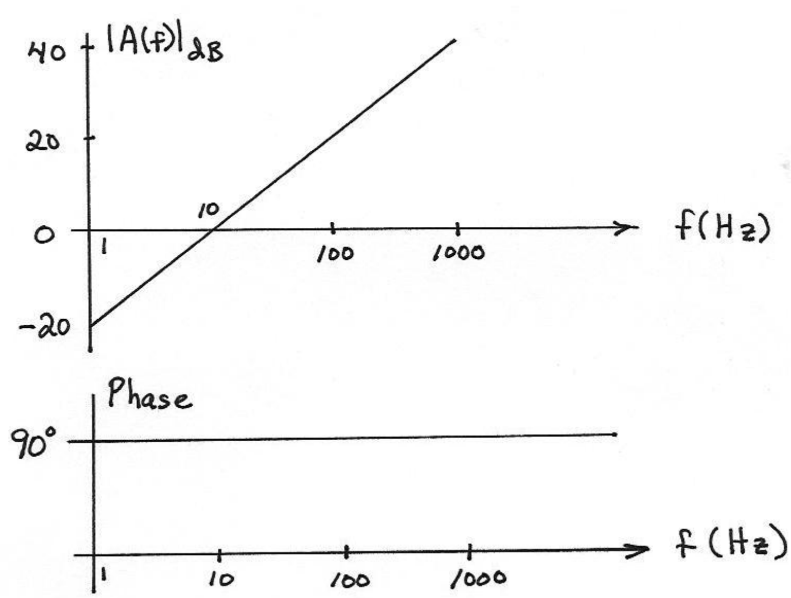


(a) $$v_o(t) = R_{moc}i_c(t) = R_{moc}C\frac{d(A_{oc}v_{in})}{dt} = A_{oc}R_{moc}C\frac{dv_{in}}{dt}$$

(b) $v_o(t) = A_{oc}R_{moc}C\dfrac{d}{dt}[V_m\cos(2\pi f t)] = -2\pi f A_{oc}R_{moc}CV_m\sin(2\pi f t)$

$$A(f) = \frac{\mathbf{V}_o}{\mathbf{V}_{in}} = \frac{j2\pi f A_{oc}R_{mo}\,CV_m}{V_m} = 2\pi f A_{oc}R_{moc}C\angle 90^\circ$$

(c) For the values given, we have $A(f) = 0.1f\angle 90^\circ$. Then, the magnitude of the gain in dB is $|A(f)|_{dB} = -20 + 20\log(f)$. The Bode Plots of magnitude and phase are:



P11.73 Amplitude and phase distortion are sometimes called linear distortion because they occur even though the amplifier is linear (i.e., obeys superposition). If the gain of an amplifier has a different magnitude for the various frequency components of the input signal, a form of distortion known as amplitude distortion occurs whereas If phase is not proportional to frequency, the waveform shape is changed in passing through the amplifier and phase distortion occurs. To avoid linear waveform distortion, an amplifier should have constant gain magnitude and a phase response that is linear versus frequency for the range of frequencies contained in the input signal.

P11.74 The input signal is given as

$$v_i(t) = 0.01\cos(2000\pi t) + 0.02\cos(4000\pi t)$$

which has components with frequencies of 1000 Hz and 2000 Hz respectively.

The gain of the amplifier, as a function of frequency, is given by

$$A = \frac{100}{1 + j(f/1000)}$$

Evaluating for $f = 1000$ and for $f = 2000$, we have

$$A(1000) = \frac{100}{1 + j(1000/1000)} = 70.71\angle -45^\circ$$
$$A(2000) = 44.72\angle -63.43^\circ$$

Thus, the output is:

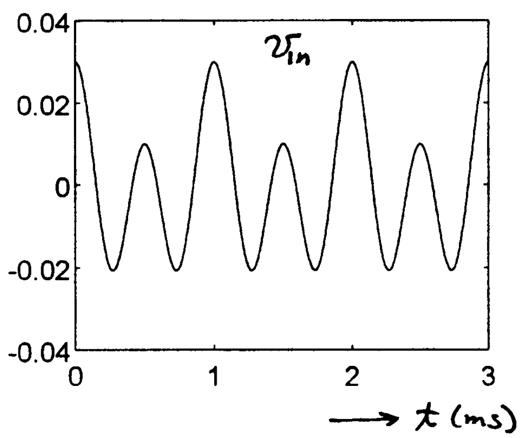
$$v_o(t) = 0.7071\cos(2000\pi t - 45^\circ) + 0.8944\cos(4000\pi t - 63.43^\circ)$$

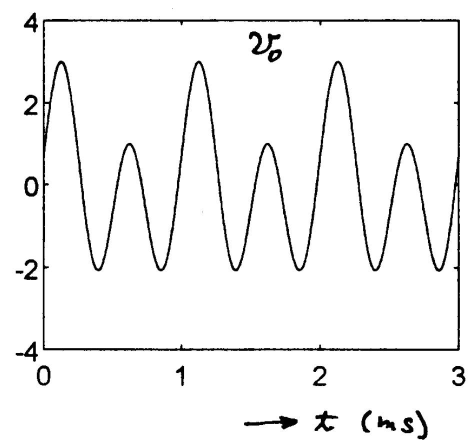
P11.75* $v_{in}(t) = 0.01\cos(2000\pi t) + 0.02\cos(4000\pi t)$

Thus, $v_{in}(t)$ contains a 1000 Hz component and a 2000 Hz component. The gain magnitude must be the same for both components. The phase shift must be proportional to frequency. Therefore, the gain at 2000 Hz must be $100\angle -90^\circ$. The output signal is

$$v_o(t) = 1\cos(2000\pi t - 45^\circ) + 2\cos(4000\pi t - 90^\circ)$$

The plots are:





P11.76 (a) The amplifier is linear because

$$[v_{inA}(t) + Kv_{inA}(t - t_d)] + [v_{inB}(t) + Kv_{inB}(t - t_d)] =$$
$$[v_{inA}(t) + v_{inB}(t)] + K[v_{inA}(t - t_d) + v_{inB}(t - t_d)]$$

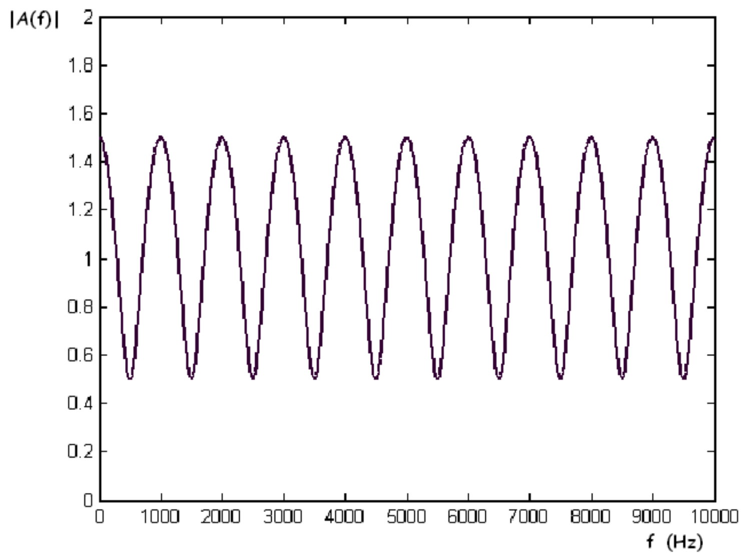
(b) For $v_{in}(t) = V_m\cos(2\pi ft)$, we have

$$v_o(t) = v_{in}(t) + Kv_{in}(t - t_d) =$$
$$V_m\cos(2\pi ft) + KV_m\cos(2\pi ft - 2\pi ft_d)$$
$$A(f) = \frac{\mathbf{V}_o}{\mathbf{V}_{in}} = \frac{V_m + KV_m\angle -2\pi ft_d}{V_m} = 1 + K\exp(-j2\pi ft_d)$$

(c) In MATLAB, a plot of the magnitude can be obtained with the commands:

```
f=0:10:10000;
A=1 + 0.5*exp(-i*2*pi*f*0.001);
plot(f,abs(A))
```

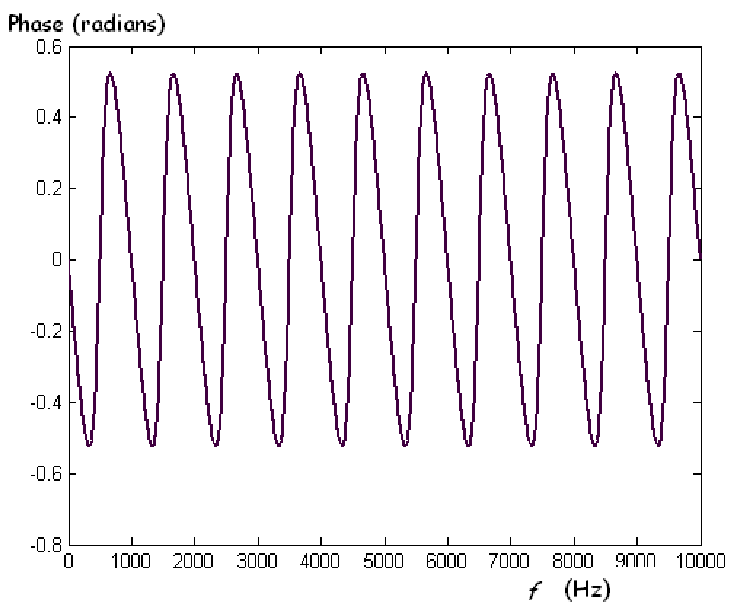
The resulting magnitude plot is:



Then a plot of the phase can be obtained with the command:

```
plot(f, angle(A))
```

The resulting plot is



(d) This amplifer produces amplitude distortion because $|A(f)|$ is not constant with f. The amplifier produces phase distortion because the phase is not proportional to frequency.

P11.77 (a) The amplifier is linear because

$$[v_{inA}(t) + K\frac{dv_{inA}(t)}{dt}] + [v_{inB}(t) + K\frac{dv_{inB}(t)}{dt}] =$$

$$[v_{inA}(t) + v_{inB}(t)] + K\frac{d}{dt}[v_{inA}(t) + v_{inB}(t)]$$

(b) For $v_{in}(t) = V_m\cos(2\pi ft)$, we have

$$v_o(t) = v_{in}(t) + K\frac{dv_{in}(t)}{dt} =$$

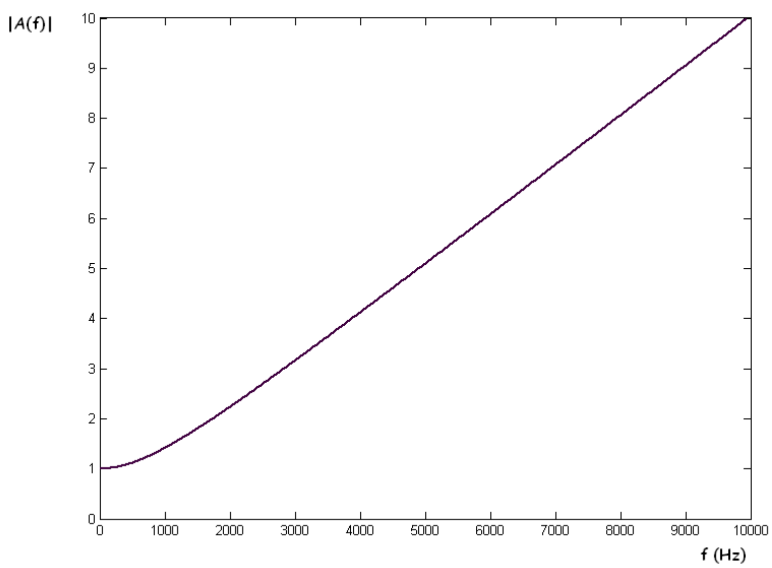
$$V_m\cos(2\pi ft) - KV_m 2\pi f\sin(2\pi ft)$$

$$A(f) = \frac{\mathbf{V}_o}{\mathbf{V}_{in}} = \frac{V_m + KV_m 2\pi f\angle\pi/2}{V_m} = 1 + jK2\pi f$$

(c) In MATLAB, a plot of the magnitude can be obtained with the commands:

```
f=0:10:10000;
A=1 + i*f/1000;
plot(f,abs(A))
```

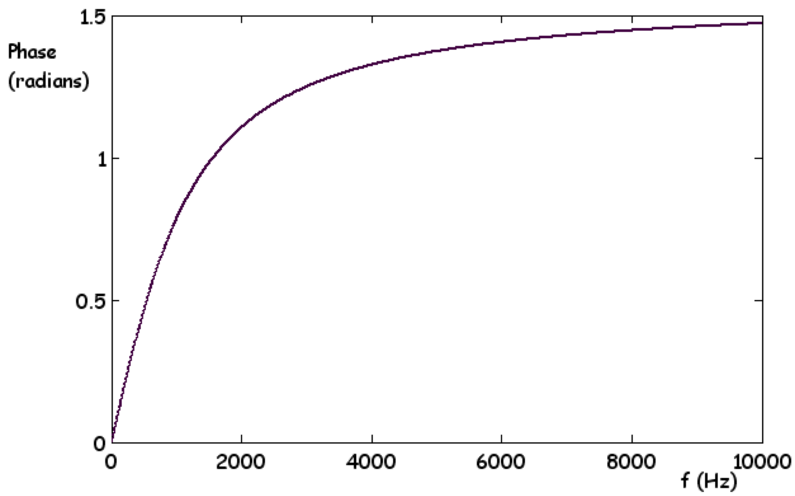
The resulting magnitude plot is:



Then a plot of the phase can be obtained with the command:

plot(f, angle(A))

The resulting plot is



(d) This amplifer produces amplitude distortion because $|A(f)|$ is not constant with frequency. The amplifier produces phase distortion because the phase is not proportional to frequency.

P11.78 (a) The amplifier is linear because

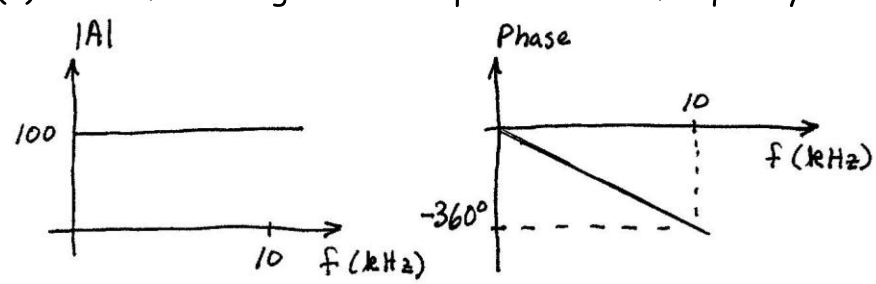
$$Kv_{\text{inA}}(t-t_d)+Kv_{\text{inB}}(t-t_d)=K[v_{\text{inA}}(t-t_d)+v_{\text{inB}}(t-t_d)]$$

(b) For $v_{\text{in}}(t)=V_m\cos(2\pi ft)$, we have

$v_o(t)=Kv_{\text{in}}(t-t_d)=KV_m\cos(2\pi ft-2\pi ft_d)$

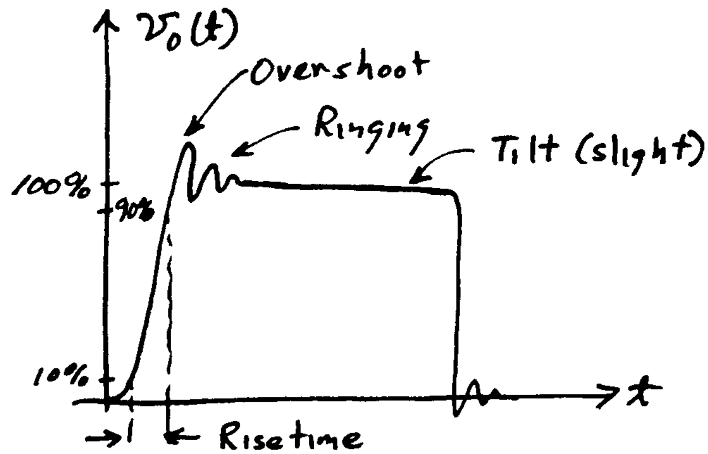
$$A(f)=\frac{\mathbf{V}_o}{\mathbf{V}_{\text{in}}}=\frac{KV_m\angle-2\pi ft_d}{V_m}=K\angle-2\pi ft_d$$

(c) Plots of the magnitude and phase versus frequency are:



(d) The gain magnitude is constant and the phase is proportional to frequency so this amplifier produces no amplitude or phase distortion.

P11.79 The sketch is



The relationship between rise time and bandwidth is:

$$t_r = \frac{0.35}{B}$$

Percentage tilt, the lower half-power frequency, and the pulse duration are related by:

$$\text{percentage tilt} \approx 200\pi f_L T$$

P11.80 (a) Applying the voltage-division principle, we have

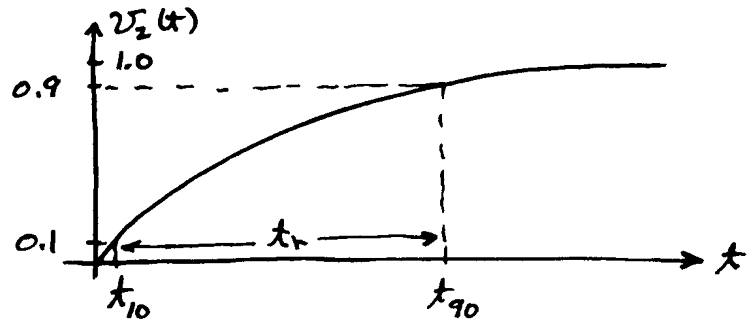
$$A = \frac{\mathbf{V}_2}{\mathbf{V}_1} = \frac{1/(j\omega C)}{R + 1/(j\omega C)} = \frac{1}{1 + j\omega RC}$$

Defining $B = 1/2\pi RC$, we have $A = \dfrac{1}{1 + j(f/B)}$

where B is the half-power bandwidth. The gain magnitude approaches zero as frequency becomes large.

(b) The transient response is:

$$v_2(t) = [1 - \exp(-t/RC)]u(t)$$



We have $0.1 = 1 - \exp(-t_{10}/RC)$ and $0.9 = 1 - \exp(-t_{90}/RC)$

Solving, we obtain:

$$t_{10} = -RC\ln(0.9)$$
$$t_{90} = -RC\ln(0.1)$$
$$t_r = t_{90} - t_{10} = RC\ln(9)$$

(c) Combining the results of parts (a) and (b), we obtain

$$t_r = \frac{\ln(9)}{2\pi B} \cong \frac{0.35}{B}$$

This is the basis for the rule-of-thumb given in Equation 11.11.

P11.81 (a) Applying the voltage-division principle, we have

$$A = \frac{\mathbf{V}_2}{\mathbf{V}_1} = \frac{R}{R + 1/(j\omega C)} = \frac{1}{1 + 1/j\omega RC}$$

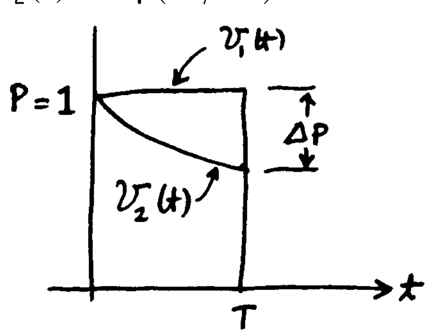
Defining, $f_L = 1/2\pi RC$, we have

$$A = \frac{1}{1 - j(f_L/f)}$$

(b) $f_L = 1/2\pi RC$ is the half-power frequency. The gain magnitude approaches unity as frequency becomes large. At dc, the gain is zero.

(c) Solving for transient response, we obtain:

$$v_2(t) = \exp(-t/RC) \text{ for } 0 < t < T$$



Thus, we have:

$$\Delta P = 1 - \exp(-T/RC) \qquad (1)$$

$$\text{Percentage tilt} = \frac{\Delta P}{P} \times 100\% = [1 - \exp(-T/RC)] \times 100\%$$

However,

$$\exp(-T/RC) = 1 - \left(\frac{T}{RC}\right) - \left(\frac{T}{RC}\right)^2 - \left(\frac{T}{RC}\right)^3 - \cdots$$

For $T << RC$

$$\exp(-T/RC) = 1 - \left(\frac{T}{RC}\right)$$

Using this to substitute in Equation (1), we have

$$\text{Percentage tilt} = \frac{T}{RC} \times 100\%$$

(d) Combining the results of parts (b) and (c), we obtain

$$\text{Percentage tilt} \cong 2\pi f_L T \times 100\%$$

This result is precise only for a first-order circuit for which $RC >> T$ but it provides a useful rule-of-thumb for other high pass filters provided that the percentage tilt is small (i.e., less than 10%).

P11.82*

$$B \cong f_H = 15 \text{ kHz}$$

$$t_r \cong \frac{0.35}{B} = 23.3\ \mu\text{s}$$

$$\text{Percentage tilt} \cong 2\pi f_L T \times 100\%$$

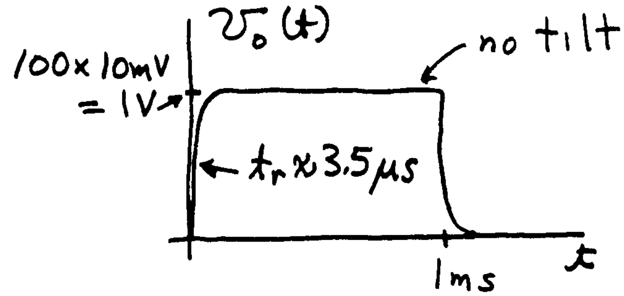
$$\cong 2\pi(15)(2 \times 10^{-3}) \times 100\%$$

$$\cong 18.8\%$$

P11.83* (a) Because the amplifier is dc-coupled (i.e., $f_L = 0$), the tilt is zero. The upper half-power frequency is $f_H = 100$ kHz, and we estimate the rise time as

$$t_r = \frac{0.35}{f_H} = 3.5\ \mu\text{s}$$

Because the frequency response shows no peaking, we do not expect overshoot or ringing. The gain magnitude is 100 in the passband, so we expect the output pulse to be 100 times larger in amplitude than the input. Thus, the output pulse is approximately like this:

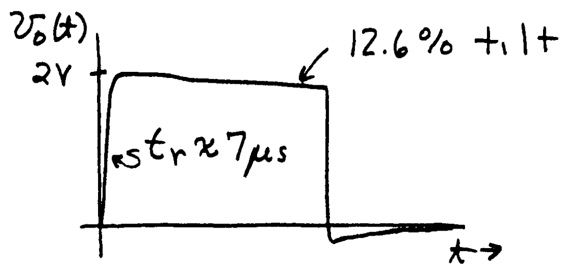


(b) We have $f_L = 20$ Hz and $f_H = 50$ kHz. Thus,

$$\text{Percentage tilt} \cong 200\pi f_L T = 200\pi(20)10^{-3} = 12.6\%$$

$$t_r = \frac{0.35}{f_H} = \frac{0.35}{50 \times 10^3} = 7\ \mu s$$

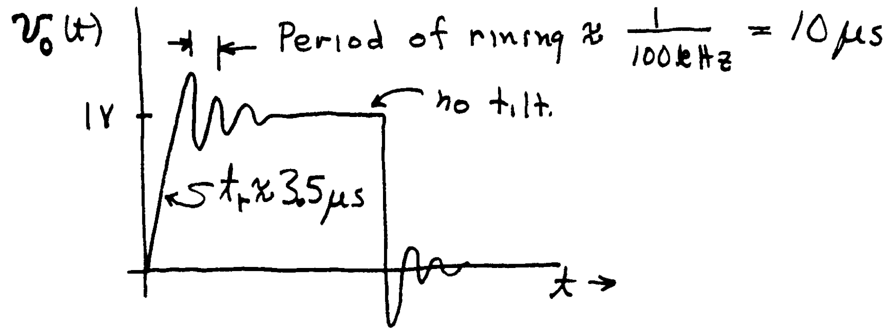
Because the frequency response shows no peaking, we do not expect overshoot or ringing. The gain magnitude is 200 in the passband, so we expect the output pulse to be 200 times larger in amplitude than the input. Thus, the output pulse is about like this:



(c) Because the amplifier is dc-coupled (i.e., $f_L = 0$), the tilt is zero. The upper half-power frequency is $f_H \cong 100$ kHz, and we estimate the rise time as

$$t_r \cong \frac{0.35}{f_H} = 3.5\ \mu s$$

Because the frequency response displays peaking, we expect overshoot and ringing. Furthermore, the period of the ringing will be approximately $1/(100\text{ kHz}) = 10\ \mu s$. The gain magnitude is 100 in the passband, so we expect the output pulse to be 100 times larger in amplitude than the input. Thus, the output pulse will be about like this:

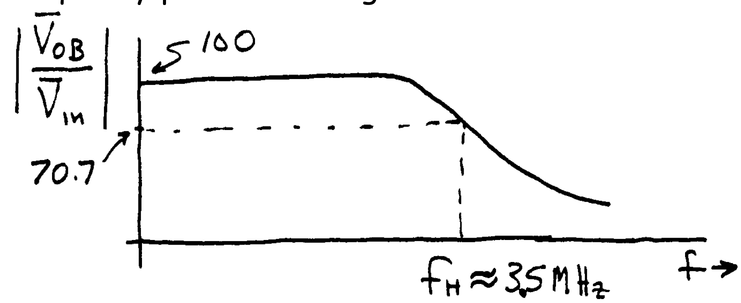


P11.84 (a) Part (a) of Figure P11.84 shows the input pulse. Thus there is no part (a) of the solution.

(b) Since $tilt = 0$, we expect $f_L = 0$.

$$f_H \cong \frac{0.35}{t_r} = \frac{0.35}{0.1 \times 10^{-6}} = 3.5 \text{ MHz}$$

Because the output pulse is 100 times larger in amplitude than the input pulse, the midband gain of the amplifier is 100. Because the pulse shows little overshoot and ringing, we expect the amplifier gain to roll off smoothly versus frequency. Thus, we expect a gain versus frequency plot something like this:

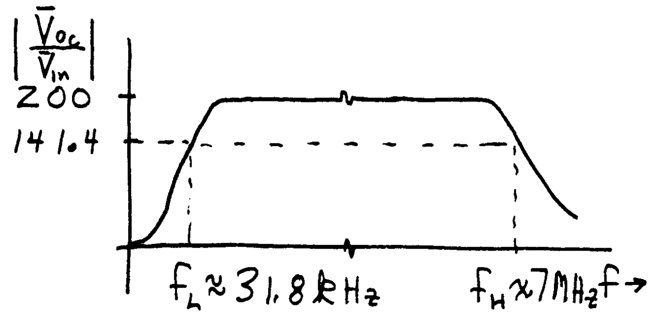


(c) Percentage tilt $= \frac{\Delta P}{P} = \frac{2 - 1.6}{2} = 20\%$

$$f_L \cong \frac{\text{percentage tilt}}{200\pi T} = \frac{20}{200\pi 10^{-6}} = 31.8 \text{ kHz}$$

$$f_H \cong \frac{0.35}{t_r} = \frac{0.35}{0.05 \times 10^{-6}} = 7 \text{ MHz}$$

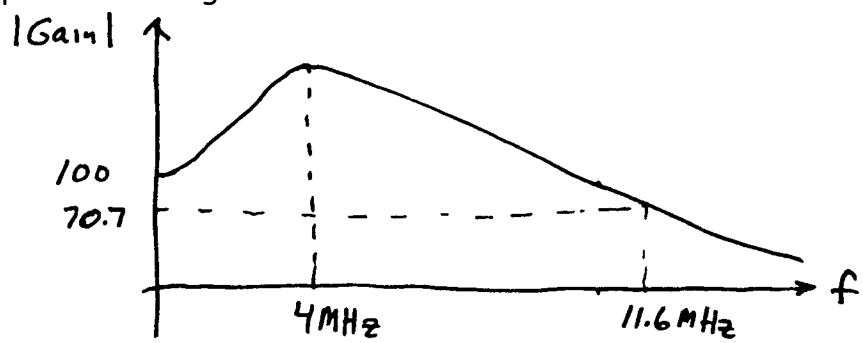
Because the output pulse is 200 times larger in amplitude than the input pulse, the midband gain of the amplifier is 200. Because the pulse shows little overshoot and ringing, we expect the amplifier gain to roll off smoothly versus frequency. Thus, we expect a gain versus frequency plot something like this:



(d) Since tilt$\cong 0$, we expect $f_L = 0$.

$$f_H \cong \frac{0.35}{t_r} = \frac{0.35}{0.03\times 10^{-6}} = 11.7 \text{ MHz}$$

Because the output pulse in 100 times larger in amplitude than the input pulse, the midband gain of the amplifier is 100. Because the pulse shows overshoot and ringing with a period of about 0.25 μs, we expect the amplifier gain to display a gain peak at about $f = 1/(0.25\ \mu s) = 4$ MHz. Thus, we expect a gain versus frequency plot something like this:



P11.85 When a sinewave input signal is passed through an amplifier having a nonlinear transfer characteristic, distortion consisting of harmonics is produced. Harmonics are components having frequencies that are integer multiples of the input frequency.

P11.86* We are given

$$v_{in}(t) = 0.1\cos(2000\pi t)$$

and

$$v_o(t) = 10\cos(2000\pi t) + 0.2\cos(4000\pi t) + 0.1\cos(6000\pi t)$$

in which $10\cos(2000\pi t)$ is the desired term with an amplitude V_1 = 10 V. The second term $0.2\cos(4000\pi t)$ is second harmonic distortion with an amplitude V_2 = 0.2 V. Finally the term $0.1\cos(6000\pi t)$ is third harmonic distortion with amplitude V_3 = 0.1 V. There is no fourth or higher order harmonic distortion. Then using Equations 11.17, 11.18, and 11.19 we have

$$D_2 = \frac{V_2}{V_1} = \frac{0.2}{10} = 0.02$$
$$D_3 = \frac{V_3}{V_1} = \frac{0.1}{10} = 0.01$$
$$D_4 = \frac{V_4}{V_1} = \frac{0}{10} = 0$$
$$D = \sqrt{D_2^2 + D_3^2 + D_4^4 + \cdots} = \sqrt{(0.02)^2 + (0.01)^2} = 0.02236 = 2.236\%$$

P11.87 Substituting the input into the equation for the transfer characteristic, we obtain:

$$v_o(t) = 20\cos(200\pi t) + 2.4\cos^2(200\pi t) + 3.2\cos^3(200\pi t)$$

Applying the trigonometric identities suggested in the problem statement, we obtain

$$v_o(t) = 1.2 + 22.4\cos(200\pi t) + 1.2\cos(400\pi t) + 0.8\cos^3(600\pi t)$$

Thus, the amplitude of the desired output term is V_1 = 22.4, the amplitude of the second harmonic is V_2 = 1.2 V, and the amplitude of the third harmonic is V_3 = 0.8 V. The amplitudes of higher order terms are zero. Then using Equations 11.17, 11.18, and 11.19, we have

$$D_2 = \frac{V_2}{V_1} = \frac{1.2}{22.4} = 0.05357$$
$$D_3 = \frac{V_3}{V_1} = \frac{0.8}{22.4} = 0.03571$$
$$D_4 = \frac{V_4}{V_1} = \frac{0}{10} = 0$$
$$D = \sqrt{D_2^2 + D_3^2 + D_4^4 + \cdots} = \sqrt{(0.05357)^2 + (0.03571)^2}$$
$$= 0.06438$$
$$= 6.438\%$$

P11.88 Substituting the input into the equation for the output we obtain

$$v_o(t) = \cos(\omega_1 t) + \cos(\omega_1 t) + 0.1[\cos(\omega_1 t) + \cos(\omega_1 t)]^2$$

$$v_o(t) = \cos(\omega_1 t) + \cos(\omega_2 t) + 0.1[\cos^2(\omega_1 t) + 2\cos(\omega_1 t)\cos(\omega_2 t) + \cos^2(\omega_2 t)]$$

Next, applying the identities suggested in the problem and simplifying, we have

$$v_o(t) = \cos(\omega_1 t) + \cos(\omega_2 t) + 0.1 + 0.05\cos(2\omega_1 t) + 0.05\cos(2\omega_2 t)$$
$$+ 0.1\cos[(\omega_1 - \omega_2)t] + 0.1\cos[(\omega_1 + \omega_2)t]$$

The frequencies and amplitudes of the components are:

Frequency (Hz)	Amplitude (V)
f_1	1
f_2	1
0 (dc)	0.1
$2f_1$	0.05
$2f_2$	0.05
$f_1 - f_2$	0.1
$f_1 + f_2$	0.1

P11.89 A differential amplifier has two input terminals, one is known as the inverting input and the other is known as the noninverting input. If the input signals applied to the input terminals of a differential amplifier are denoted as v_{i1} and v_{i2}, the common-mode signal is $v_{icm} = (v_{i1} + v_{i2})/2$ and the differential signal is $v_{id} = v_{i1} - v_{i2}$. The output signal is $v_o = A_{cm}v_{icm} + A_d v_{id}$ where A_d is called the differential gain and is called the common-mode gain. Ideally, the common-mode gain is zero so the amplifier responds to only the differential signal.

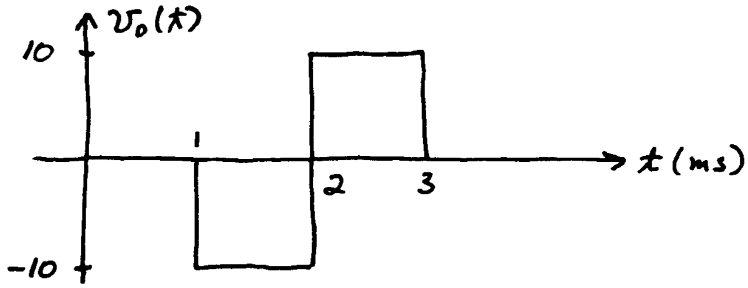
P11.90 The common-mode rejection ratio (in decibels) is defined as the ratio of the magnitude of the differential gain to the magnitude of the common-mode gain. Often, CMRR is expressed in decibels as

$$CMRR = 20\log\frac{|A_d|}{|A_{cm}|}$$

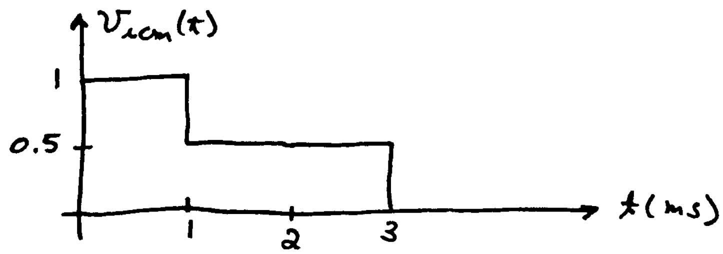
The CMRR of an amplifier is generally a function of frequency, becoming lower as frequency is raised. High CMRR means very low common mode gain, which in turn means common mode signals like noise are rather atteneuated and their effect is reduced.

P11.91 Electrocardiography provides a classic example of differential and common-mode signals. The differential signal between a pair of electrodes connected to a patient is produced by the patient's heart and has a peak amplitude on the order of 1 mV. The common-mode voltages are capacitively coupled power-line voltages on the order of 10 V peak that appear at both electrodes with respect to power-system ground.

P11.92 $v_o = A_d v_{id} = 10(v_{i1} - v_{i2})$



$v_{icm} = (v_{i1} + v_{i2})/2$



P11.93* With the input terminals tied together, the differential signal is zero and the common-mode signal is:

$$v_{icm} = (v_{i1} + v_{i2})/2 = 5 \text{ mV rms}$$

The common-mode gain is:

$$A_{cm} = v_o / v_{icm} = 20/5 = 6$$

The common-mode rejection ratio is:

$$CMRR = 20\log\frac{|A_d|}{|A_{cm}|} = 20\log\frac{1000}{6} = 44.436 \text{ dB}$$

P11.94 We want

$$60 = 20\log\left(\frac{|V_{od}|}{|V_{ocm}|}\right) = 20\log\left(\frac{|A_d V_{id}|}{|A_{cm} V_{icm}|}\right)$$

$$= 20\log\left[\frac{|A_d|(20 \text{ mV})}{|A_{cm}|(5 \text{ V})}\right]$$

$$= 20\log\left(\frac{|A_d|}{|A_{cm}|}\right) + 20\log\left(\frac{20 \text{ mV}}{5 \text{ V}}\right)$$

$$= 20\log\left(\frac{|A_d|}{|A_{cm}|}\right) - 48.0$$

Thus,

$$CMRR = 20\log\left(\frac{|A_d|}{|A_{cm}|}\right) = 108 \text{ dB}$$

P11.95 If we apply $v_{i1} = 0.5$ and $v_{i2} = -0.5$, we have a pure differential signal with $v_{id} = 1.0$. The resulting output is $v_o = 750 v_{i1} - 450 v_{i2} = 1000.5$. Thus, the differential gain is $A_{vd} = 600$

If we apply $v_{i1} = 0.5\text{V}$ and $v_{i2} = 0.5\text{V}$, we have a pure common-mode signal with $v_{icm} = 0.5$. The resulting output is $v_o = 750\ v_{i1(t)} - 450 v_{i2(t)} = 150$. Thus, the common-mode gain is $A_{vcm} = 150$

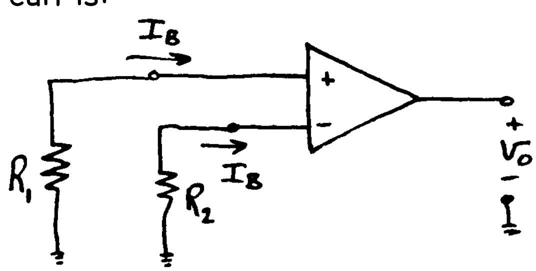
Then, we have

$$CMRR = 20\log\left(\frac{|A_d|}{|A_{cm}|}\right) = 12.04 \text{ dB}$$

P11.96 See Figure 11.40 in the text. The effect of these sources is to add a dc component to the output.

P11.97 See Figure 11.43 in the text.

P11.98* The circuit is:



The bias currents are equal. However, because the resistances may not be equal, a differential input voltage is possible. In the extreme case, one resistor could have a value of 1050 Ω and the other could have a value of 950 Ω. Then the differential input voltage is:

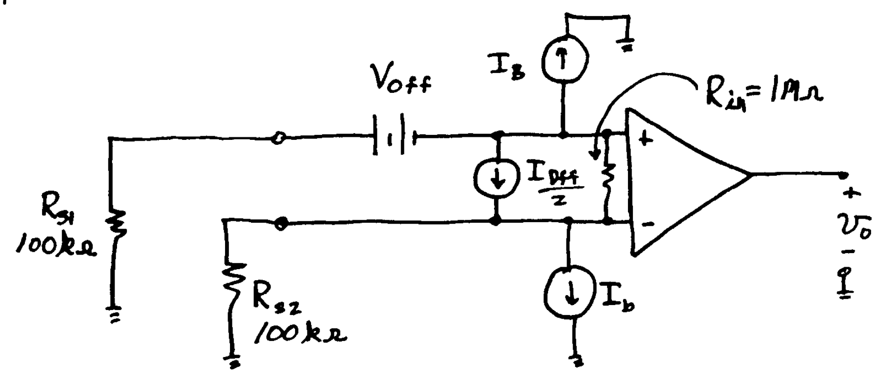
$$\begin{aligned} v_{id} &= R_1 I_{bias} - R_2 I_{bias} \\ &= (R_1 - R_2) I_{bias} \\ &= \pm 100\ \Omega \times 100 \text{ nA} \\ &= \pm 10\ \mu\text{V} \end{aligned}$$

Thus, the extreme values of the output voltage are:

$$v_o = A_d v_{id} = \pm 5 \text{ mV}$$

If the resistors are exactly equal, then the output voltage is zero.

P11.99* The equivalent circuit is:



The solution is similar to that for Example 11.13 in the text.
The bias currents produce a common-mode input voltage of $v_{icm} = I_B R_{s1} = I_s R_{s2} = 10 \text{ mV}$. However, the common-mode gain is assumed to be zero so the common-mode input signal does not contribute to the output voltage.

The differential input voltage due to the offset current is:

$$V_{Ioff} = \frac{I_{off}}{2} \frac{R_{in}(R_{s1} + R_{s2})}{R_{in} + R_{s1} + R_{s2}} = 1.667 \text{ mV}$$

The differential input voltage due to the offset voltage is:

$$V_{Voff} = V_{off} \frac{R_{in}}{R_{in} + R_{s1} + R_{s2}} = 1.667 \text{ mV}$$

If these two voltages have the same polarity, the total differential input voltage is $1.667 + 1.667 = 3.333 \text{ mV}$ in magnitude. Then the output voltage is:

$$v_o = A_d v_{id} = 3.333 \text{ V}$$

Thus the output voltage can range from -3.333 to +3.333 V.

P11.100 The common-mode rejection ratio is:

$$CMRR = 20 \log\left(\frac{|A_d|}{|A_{cm}|}\right) = 60 \text{ dB}$$

Using the fact that $|A_d| = 1000$ and solving, we find that $|A_{cm}| = 1$. As in Problem P11.99, the common-mode input voltage is 10 mV. Thus, the contribution to the output voltage is $|A_{cm}| \times v_{icm} = 10 \text{ mV}$. The contributions due the offset current and offset voltage are the same as in Problem P11.99. Thus, the extreme output voltage is:

$$v_o = 0.01 + 3.333 = 3.343 \text{ V}$$

Thus, the contribution due to the common-mode gain is negligible.

P11.101 In Figure P11.101(a), we have $v_o = -A_v V_{\text{off}} = 150$ mV, from which we obtain $V_{\text{off}} = -1.5$ mV.

For Figure P11.101(b), we can write

$$v_o = A_v(-V_{\text{off}} - RI_B - RI_{\text{off}}/2) = 0.05$$

For Figure P11.101(c), we can write

$$v_o = A_v(-V_{\text{off}} + RI_B - RI_{\text{off}}/2) = 0.300$$

Substituting $R = 100 \text{ k}\Omega$, $A_v = 100$, and $V_{\text{off}} = -1.5$ mV into the previous two equations and simplifying we obtain

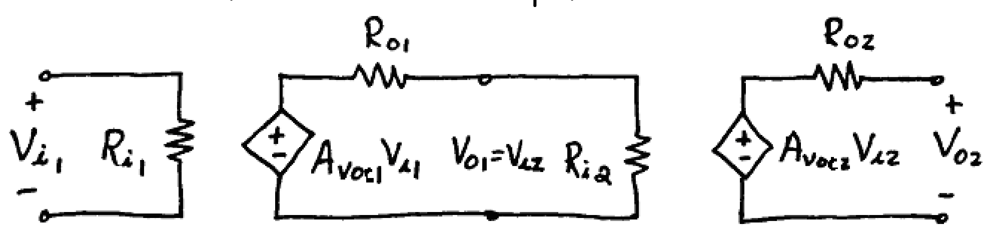
$$-I_B - I_{\text{off}}/2 = -10 \text{ nA}$$

$$I_B - I_{\text{off}}/2 = 15 \text{ nA}$$

Solving, we find $I_{\text{off}} = -5 \text{ nA}$ and $I_B = 12.5 \text{ nA}$.

Practice Test

T11.1 The equivalent circuit for the cascaded amplifiers is:



We can write:

$$V_{i2} = A_{voc1} V_{i1} \times \frac{R_{i2}}{R_{i2} + R_{o1}}$$

$$V_{o2} = A_{voc2} V_{i2} = A_{voc2} A_{voc1} V_{i1} \frac{R_{i2}}{R_{i2} + R_{o1}}$$

Thus, the open-circuit voltage gain is:

$$A_{voc} = \frac{V_{o2}}{V_{i1}} = A_{voc2}A_{voc1}\frac{R_{i2}}{R_{i2} + R_{o1}} = 50 \times 50\frac{60}{60 + 40} = 1500$$

The input resistance of the cascade is that of the first stage which is $R_i = 60\ \Omega$. The output resistance of the cascade is the output resistance of the last stage which is $R_o = 40\ \Omega$.

T11.2 Your answer should be similar to Table 11.1.

Table 11.1. Characteristics of Ideal Amplifiers

Amplifier Type	Input Impedance	Output Impedance	Gain Parameter
Voltage	∞	0	A_{voc}
Current	0	∞	A_{isc}
Transconductance	∞	∞	G_{msc}
Transresistance	0	0	R_{moc}

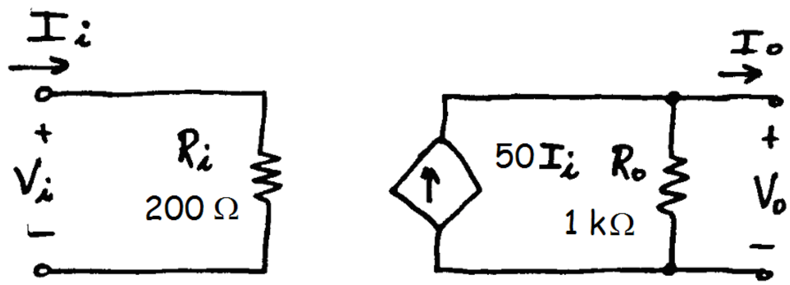
T11.3 **a.** The amplifier should sense the open-circuit source voltage, thus the input impedance should be infinite. The load current should be independent of the variable load, so the output impedance should be infinite. Thus, we need an ideal transconductance amplifier.

b. The amplifier should respond to the short-circuit source current, thus the input impedance should be zero. The load current should be independent of the variable load impedance so the output impedance should be infinite. Therefore, we need an ideal current amplifier.

c. The amplifier should sense the open-circuit source voltage, thus the input impedance should be infinite. The load voltage should be independent of the variable load, so the output impedance should be zero. Thus, we need an ideal voltage amplifier.

d. The amplifier should respond to the short-circuit source current, thus the input impedance should be zero. The load voltage should be independent of the variable load impedance, so the output impedance should be zero. Therefore, we need an ideal transresistance amplifier.

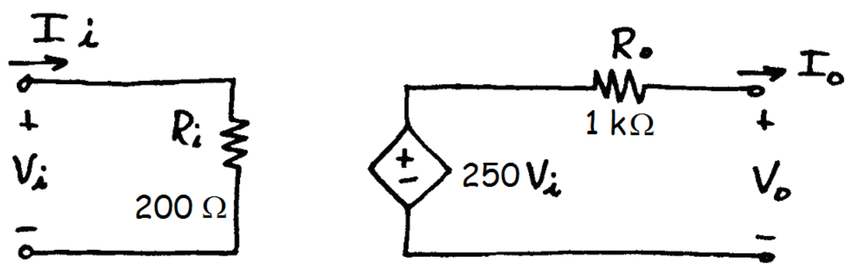
T11.4 We are given the parameters for the current-amplifier model, which is:



The open-circuit voltage gain is:

$$A_{voc} = \frac{V_{ooc}}{V_i} = \frac{50I_iR_o}{R_iI_i} = 250$$

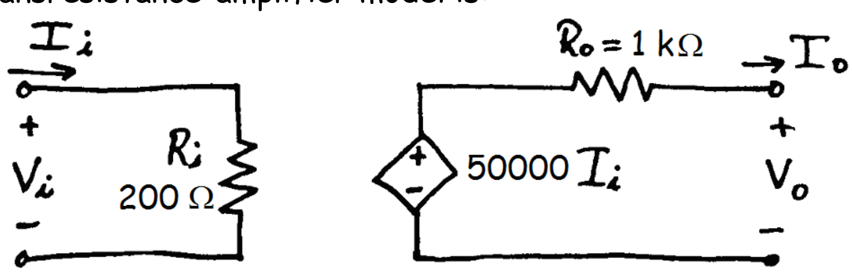
A_{voc} is unitless. (Sometimes we give the units as V/V.) The voltage-amplifier model is:



The transresistance gain is:

$$R_{moc} = \frac{V_{ooc}}{I_i} = \frac{50I_iR_o}{I_i} = 50\ \text{k}\Omega$$

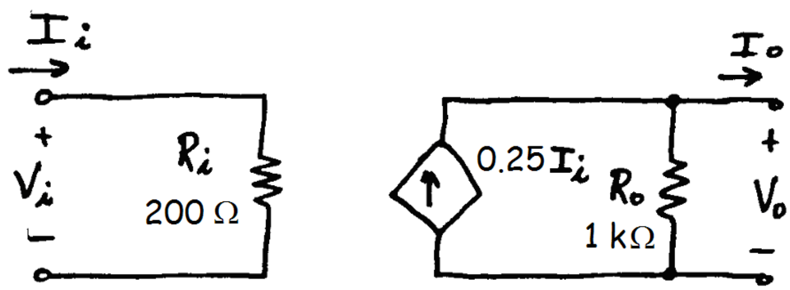
The transresistance-amplifier model is:



The transconductance gain is:

$$G_{msc} = \frac{I_{osc}}{V_i} = \frac{50I_i}{R_iI_i} = 0.25\ \text{S}$$

The transconductance-amplifier model is:



T11.5 $P_i = I_i^2 R_i = (10^{-3})^2 \times 2 \times 10^3 = 2\text{ mW}$

$P_o = (V_o)^2 / R_L = (12)^2 / 8 = 18\text{ W}$ $\qquad P_s = V_s I_s = 15 \times 2 = 30\text{ W}$

$P_d = P_s + P_i - P_o \cong 12\text{ W}$ $\qquad \eta = \dfrac{P_o}{P_s} \times 100\% = 60\%$

T11.6 To avoid linear waveform distortion, the gain magnitude should be constant and the phase response should be a linear function of frequency over the frequency range from 1 to 10 kHz. Because the gain is 100 and the peak input amplitude is 100 mV, the peak output amplitude should be 10 V. The amplifier must not display clipping or unacceptable nonlinear distortion for output amplitudes of this value.

T11.7 The principal effect of offset current, bias current, and offset voltage of an amplifier is to add a dc component to the signal being amplified.

T11.8 Harmonic distortion can occur when a pure sinewave test signal is applied to the input of an amplifier. The distortion appears in the output as components whose frequencies are integer multiples of the input frequency. Harmonic distortion is caused by a nonlinear relationship between the input voltage and output voltage.

T11.9 Common mode rejection ratio (CMRR) is the ratio of the differential gain to the common mode gain of a differential amplifier. Ideally, the common mode gain is zero, and the amplifier produces an output only for the differential signal. CMRR is important when we have a differential signal of interest in the presence of a large common-mode signal not of interest. For example, in recording an electrocardiogram, two electrodes are connected to the patient; the differential signal is the heart signal of interest to the cardiologist; and the common mode signal is due to the 60-Hz power line.

CHAPTER 12

Exercises

E12.1 (a) $v_{GS} = 1$ V and $v_{DS} = 5$ V: Because we have $v_{GS} < V_{to}$, the FET is in cutoff.

(b) $v_{GS} = 3$ V and $v_{DS} = 0.5$ V: Because $v_{GS} > V_{to}$ and $v_{GD} = v_{GS} - v_{DS} = 2.5 > V_{to}$, the FET is in the triode region.

(c) $v_{GS} = 3$ V and $v_{DS} = 6$ V: Because $v_{GS} > V_{to}$ and $v_{GD} = v_{GS} - v_{DS} = -3$ V $< V_{to}$, the FET is in the saturation region.

(d) $v_{GS} = 5$ V and $v_{DS} = 6$ V: Because $v_{GS} > V_{to}$ and $v_{GD} = v_{GS} - v_{DS} = 1$ V which is less than V_{to}, the FET is in the saturation region.

E12.2 First we notice that for $v_{GS} = 0 \text{ or } 1\text{ V}$, the transistor is in cutoff, and the drain current is zero. Next we compute the drain current in the saturation region for each value of v_{GS}:

$$K = \tfrac{1}{2}KP(W/L) = \tfrac{1}{2}(50 \times 10^{-6})(80/2) = 1 \text{ mA/V}^2$$

$$i_D = K(v_{GS} - V_{to})^2$$

The boundary between the triode and saturation regions occurs at

$$v_{DS} = v_{GS} - V_{to}$$

v_{GS} (V)	i_D (mA)	v_{DS} at boundary
2	1	1
3	4	2
4	9	3

In saturation, i_D is constant, and in the triode region the characteristics are parabolas passing through the origin. The apex of the parabolas are on the boundary between the triode and saturation regions. The plots are shown in Figure 12.7 in the book.

E12.3 First we notice that for $v_{GS} = 0 \text{ or } -1\text{ V}$, the transistor is in cutoff, and the drain current is zero. Next we compute the drain current in the saturation region for each value of v_{GS}:

$$K = \frac{1}{2}KP(W/L) = \frac{1}{2}(25\times10^{-6})(200/2) = 1.25\text{ mA/V}^2$$

The boundary between the triode and saturation regions occurs at $v_{DS} = v_{GS} - V_{to}$.

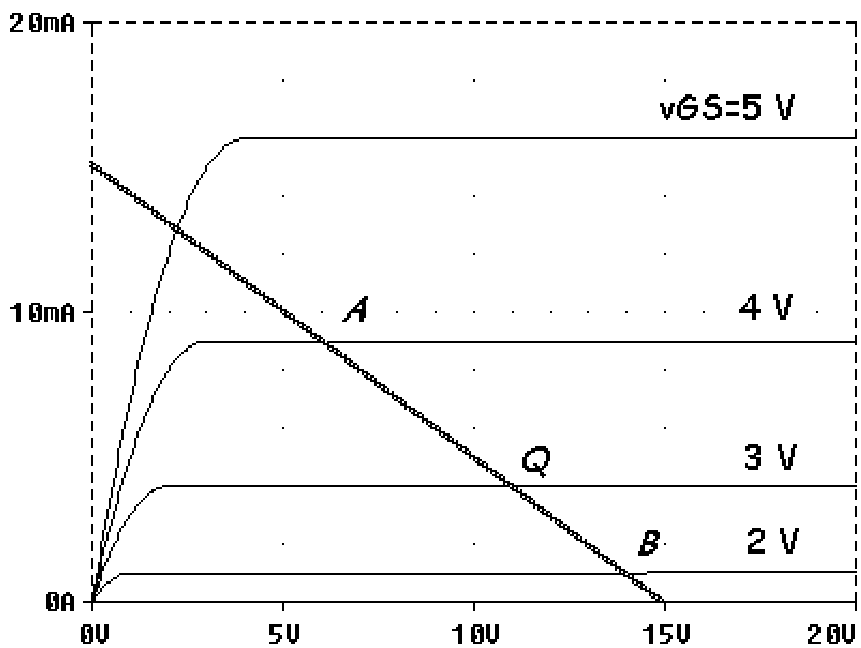
v_{GS} (V)	i_D (mA)	v_{DS} at boundary
−2	1.25	−1
−3	5	−2
−4	11.25	−3

In saturation, i_D is constant, and in the triode region the characteristics are parabolas passing through the origin. The apex of the parabolas are on the boundary between the triode and saturation regions. The plots are shown in Figure 12.9 in the book.

E12.4 We have

$$v_{GS}(t) = v_{in}(t) + V_{GG} = \sin(2000\pi t) + 3$$

Thus we have $V_{GS\max} = 4$ V, $V_{GSQ} = 3$ V, and $V_{GS\min} = 2$ V. The characteristics and the load line are:



For $v_{in} = +1$ we have $v_{GS} = 4$ and the instantaneous operating point is A. Similarly for $v_{in} = -1$ we have $v_{GS} = 2$ V and the instantaneous operating point is at B. We find $V_{DSQ} \cong 11$ V, $V_{DS\min} \cong 6$ V, $V_{DS\max} \cong 14$ V.

E12.5 First, we compute

$$V_G = V_{DD}\frac{R_2}{R_1 + R_2} = 7\text{ V}$$

and $K = \frac{1}{2}KP(W/L) = \frac{1}{2}(50 \times 10^{-6})(200/10) = 0.5\text{ mA/V}^2$

As in Example 12.2, we need to solve:

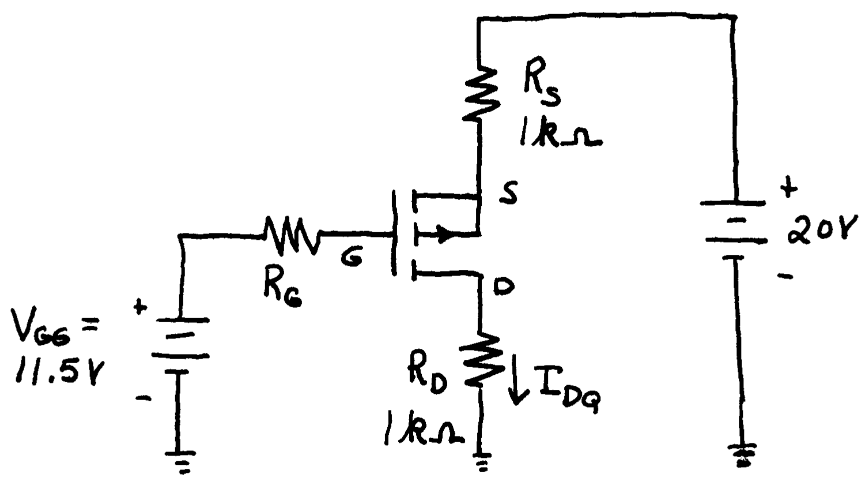
$$V_{GSQ}^2 + \left(\frac{1}{R_S K} - 2V_{to}\right)V_{GSQ} + (V_{to})^2 - \frac{V_G}{R_S K} = 0$$

Substituting values, we have

$$V_{GSQ}^2 - V_{GSQ} - 6 = 0$$

The roots are $V_{GSQ} = -2$ V and 3 V. The correct root is $V_{GSQ} = 3$ V which yields $I_{DQ} = K(V_{GSQ} - V_{to})^2 = 2$ mA. Finally, we have $V_{DSQ} = V_{DD} - R_S I_{DQ} =$ 16 V.

E12.6 First, we replace the gate bias circuit with its equivalent circuit:



Then we can write the following equations:

$$K = \frac{1}{2}KP(W/L) = \frac{1}{2}(25 \times 10^{-6})(400/10) = 0.5\text{ mA/V}^2$$

$$V_{GG} = 11.5 = V_{GSQ} - R_s I_{DQ} + 20 \qquad (1)$$

(2)

Using Equation (2) to substitute into Equation (1), substituting values, and rearranging, we have $V_{GSQ}^2 - 16 = 0$. The roots of this equation are $V_{GSQ} = \pm 4$ V. However $V_{GSQ} = -4$ V is the correct root for a PMOS transistor. Thus we have

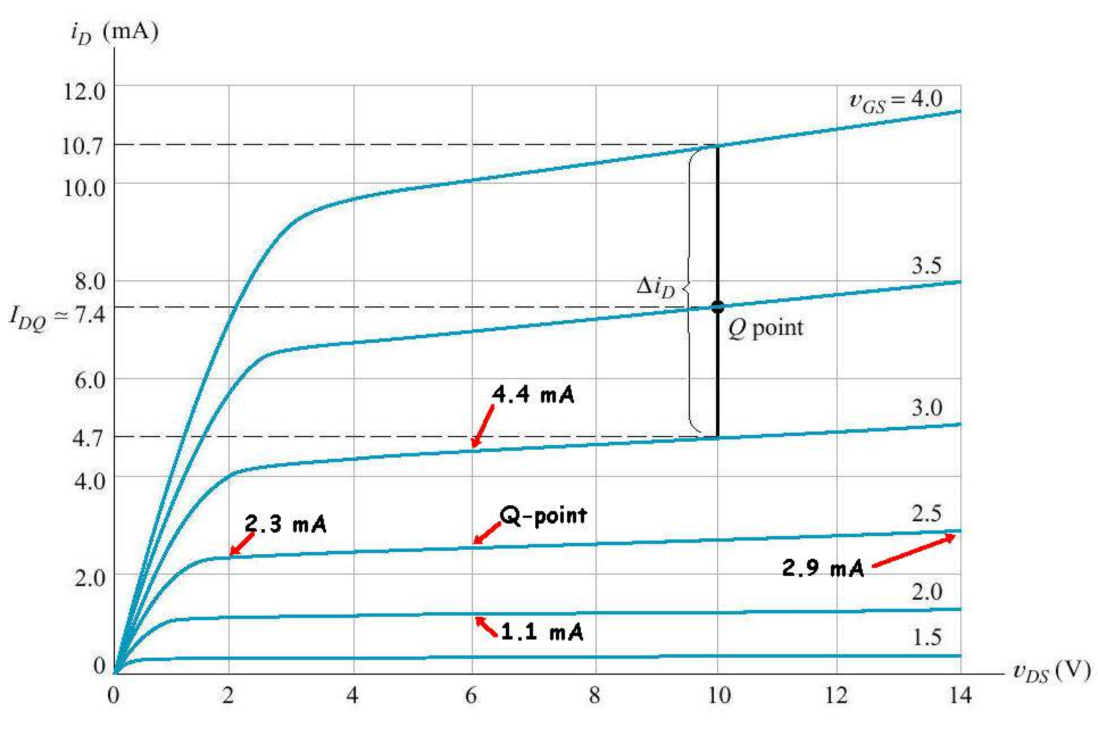
$$I_{DQ} = 4.5 \text{ mA}$$

and

$$V_{DSQ} = R_s I_{DQ} + R_D I_{DQ} - 20 = -11 \text{ V}.$$

E12.7 From Figure 12.21 at an operating point defined by $V_{GSQ} = 2.5$ V and $V_{DSQ} = 6$ V, we estimate

$$g_m = \frac{\Delta i_D}{\Delta v_{GS}} = \frac{(4.4 - 1.1)\text{mA}}{1\text{V}} = 3.3\,\text{mS}$$



$$1/r_d = \frac{\Delta i_D}{\Delta v_{GS}} \cong \frac{(2.9 - 2.3)\text{mA}}{(14 - 2)\text{V}} = 0.05 \times 10^{-3}$$

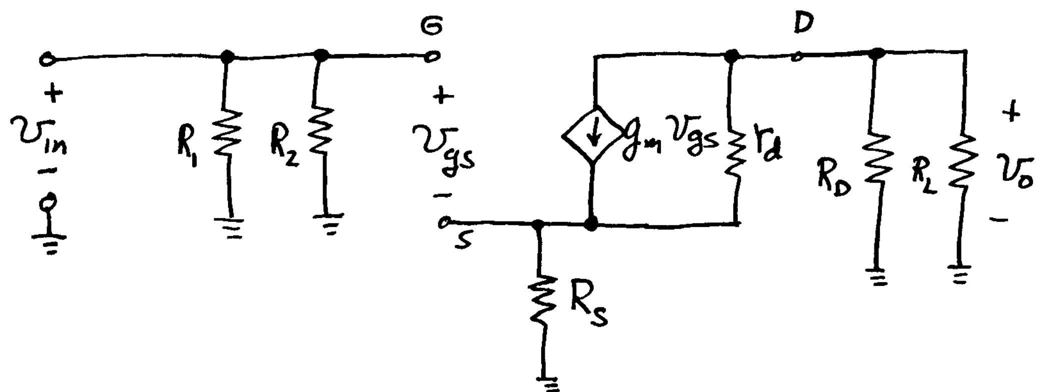
Taking the reciprocal, we find $r_d = 20$ kΩ.

E12.8 $$g_m = \left.\frac{\partial i_D}{\partial v_{GS}}\right|_{Q-point} = \left.\frac{\partial}{\partial v_{GS}} K(v_{GS} - V_{to})^2\right|_{Q-point} = 2K(V_{GSQ} - V_{to})$$

E12.9 $$R_L' = \frac{1}{1/r_d + 1/R_D + 1/R_L} = R_D = 4.7\ \text{k}\Omega$$

$$A_{oc} = -g_m R_L' = -(1.77\ \text{mS}) \times (4.7\ \text{k}\Omega) = -8.32$$

E12.10 For simplicity we treat r_d as an open circuit and let $R_L' = R_D || R_L$.



$$v_{in} = v_{gs} + R_s g_m v_{gs}$$

$$v_o = -R_L' g_m v_{gs}$$

$$A_v = \frac{v_o}{v_{in}} = \frac{-R_L' g_m}{1 + R_L' g_m}$$

E12.11 $R_L' = R_D \| R_L = 3.197\ \text{k}\Omega$

$$A_v = \frac{v_o}{v_{in}} = \frac{-R_L' g_m}{1 + R_L' g_m} = \frac{-(3.197\ \text{k}\Omega)(1.77\ \text{mS})}{1 + (2.7\ \text{k}\Omega)(1.77\ \text{mS})} = -0.979$$

E12.12 The equivalent circuit is shown in Figure 12.28 in the book from which we can write

$$v_{\text{in}} = 0 \qquad v_{gs} = -v_x \qquad i_x = \frac{v_x}{R_S} + \frac{v_x}{r_d} - g_m v_{gs} = \frac{v_x}{R_S} + \frac{v_x}{r_d} + g_m v_x$$

Solving, we have

$$R_o = \frac{v_x}{i_x} = \frac{1}{g_m + \frac{1}{R_S} + \frac{1}{r_d}}$$

E12.13 Refer to the small-signal equivalent circuit shown in Figure 12.30 in the book. Let $R'_L = R_D \| R_L$.

$$v_{\text{in}} = -v_{gs}$$

$$v_o = -R'_L g_m v_{gs}$$

$$A_v = v_o / v_{\text{in}} = R'_L g_m$$

$$i_{\text{in}} = v_{\text{in}} / R_s - g_m v_{gs} = v_{\text{in}} / R_s + g_m v_{\text{in}}$$

$$R_{\text{in}} = \frac{v_{\text{in}}}{i_{\text{in}}} = \frac{1}{g_m + 1/R_s}$$

If we set $v(t) = 0$, then we have $v_{gs} = 0$. Removing the load and looking back into the amplifier, we see the resistance R_D. Thus we have $R_o = R_D$.

E12.14 See Figure 12.34 in the book.

E12.15 See Figure 12.35 in the book.

Problems

P12.1 Cutoff: $i_D = 0$ for $v_{GS} \le V_{to}$

Triode: $i_D = K[2(v_{GS} - V_{to})v_{DS} - v_{DS}^2]$
for $v_{DS} \le v_{GS} - V_{to}$ (or $v_{GD} \ge V_{to}$) and $v_{GS} \ge V_{to}$

Saturation: $i_D = K(v_{GS} - V_{to})^2$
for $v_{GS} \ge V_{to}$ and $v_{DS} \ge v_{GS} - V_{to}$ (or $v_{GD} \le V_{to}$)

P12.2 See Figures 12.1 and 12.2 in the book.

P12.3* $K = \frac{1}{2} KP(W/L) = 0.25 \text{ mA/V}^2$

(a) Saturation because we have $v_{GS} \geq V_{to}$ and $v_{DS} \geq v_{GS} - V_{to}$.

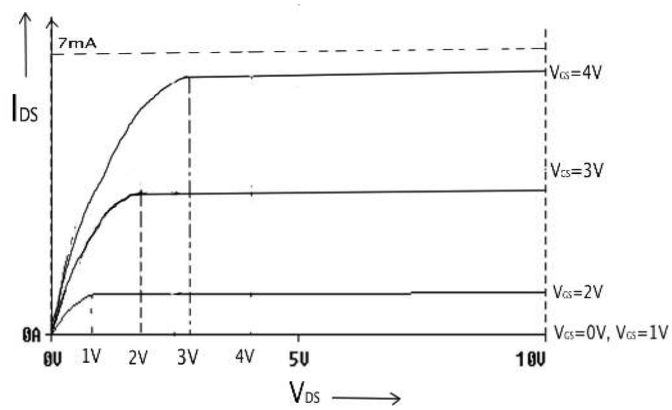
$i_D = K(v_{GS} - V_{to})^2 = 2.25$ mA

(b) Triode because we have $v_{DS} < v_{GS} - V_{to}$ and $v_{GS} \geq V_{to}$.

$i_D = K[2(v_{GS} - V_{to})v_{DS} - v_{DS}^2] = 2$ mA

(c) Cutoff because we have $v_{GS} \leq V_{to}$. $i_D = 0$.

P12.4*



P12.5 We have $i_D = 0$ for $v_{GS} \leq 0.5$ V. Therefore, we have $V_{to} = 0.5$ V. In the saturation region with $v_{GS} = 2$ V, we have $i_D = 4.5$ mA. Then, we have

$$K = \frac{i_D}{(v_{GS} - V_{to})^2} = \frac{4.5}{(2 - 0.5)^2} = 2 \text{ mA/V}^2$$

and $W/L = 2K/KP = 80$.

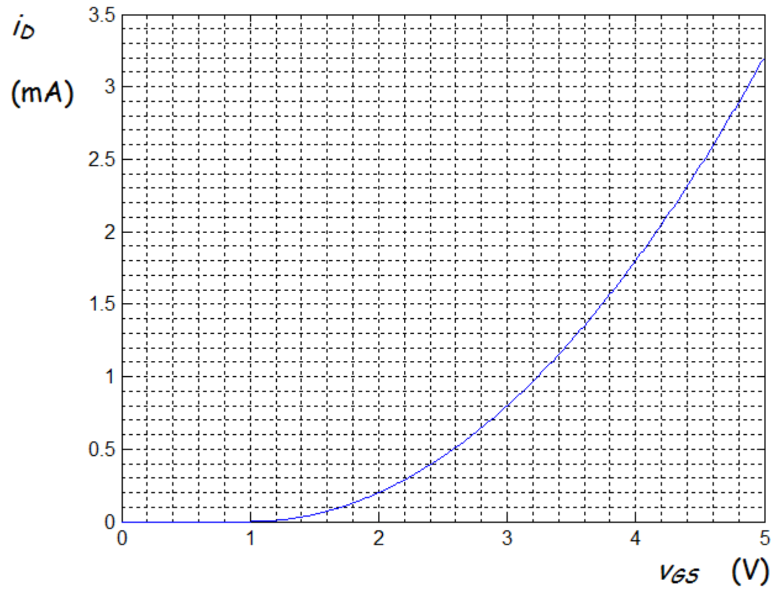
P12.7 The device is in saturation for $v_{DS} \geq v_{GS} - V_{to} = 2$ V. The device is in the triode region for $v_{DS} \leq 2$ V. In the saturation region, we have

$$i_D = K(v_{GS} - V_{to})^2 = 0.2(v_{GS} - 1)^2 \text{ for } v_{GS} \geq 1$$

In the cutoff region, we have

$$i_D = 0 \text{ for } v_{GS} \leq 1$$

The plot of i_D versus v_{GS} in the saturation region is:



P12.8 With the gate connected to the drain, we have $v_{DS} = v_{GS}$ so $v_{DS} \geq v_{GS} - V_{to}$. Then, if v_{GS} is greater than the threshold voltage, the device is operating in the saturation region. If v_{GS} is less than the threshold voltage, the device is operating in the cutoff region.

P12.9 (a) This NMOS transistor is operating in saturation because we have $v_{GS} \geq V_{to}$ and $v_{DS} \geq v_{GS} - V_{to}$. Thus, $I_a = K(v_{GS} - V_{to})^2 = 1.8$ mA.

(b) This PMOS transistor is operating in saturation because we have $v_{GS} \leq V_{to}$ and $v_{DS} = -4 \leq v_{GS} - V_{to} = -3 - (-1) = -2$. Thus, $I_b = K(v_{GS} - V_{to})^2 = 0.8$ mA.

(c) This PMOS transistor is operating in the triode region because we have $v_{GS} \leq V_{to}$ and $v_{DS} = -1 \geq v_{GS} - V_{to} = -5 - (-1) = -4$. Thus,
$I_c = K[2(v_{GS} - V_{to})v_{DS} - v_{DS}^2] = 1.4$ mA.

(d) This NMOS transistor is operating in the triode region because we have $v_{GS} \geq V_{to}$ and $v_{DS} = 1 \leq v_{GS} - V_{to} = 3 - 1 = 2$. Thus,
$I_d = K[2(v_{GS} - V_{to})v_{DS} - v_{DS}^2] = 0.6$ mA.

P12.12* In the saturation region, we have $i_D = K(v_{GS} - V_{to})^2$. Substituting values, we obtain two equations:

$$0.2\text{ mA} = K(2 - V_{to})^2$$

$$1.8\text{ mA} = K(3 - V_{to})^2$$

Dividing each side of the second equation by the respective side of the first, we obtain

$$9 = \frac{(3 - V_{to})^2}{(2 - V_{to})^2}$$

Solving we determine that $V_{to} = 1.5$ V. (We disregard the other root, which is $V_{to} = 2.25$ V, because the transistor would then be in cutoff with $v_{DS} = 2$ V and would not give a current of 0.2 mA as required.) Then, using either of the two equations, we find $K = 0.8\text{ mA/V}^2$.

.

P12.14 With $v_{GS} = v_{DS} = 5$ V, the transistor operates in the saturation region for which we have $i_D = K(v_{GS} - V_{to})^2$. Solving for K and substituting values we obtain $K = 312.5\ \mu\text{A/V}^2$. However we have $K = (W/L)(KP/2)$. Solving for W/L and substituting values we obtain $W/L = 12.5$. Thus for $L = 2\ \mu$m, we need $W = 25\ \mu$m.

P12.15 To obtain the least drain current choose minimum W and maximum L (i.e., $W_1 = 0.25\ \mu$m and $L_1 = 2\ \mu$m). To obtain the greatest drain current choose maximum W and minimum L (i.e., $W_2 = 2\ \mu$m and $L_2 = 0.25\ \mu$m). The ratio $_{GS}$=1V or V_{GS}=–3

P12.16 For a device operating in the triode region, we have

$$i_D = K[2(v_{GS} - V_{to})v_{DS} - v_{DS}^2]$$

Assuming that $v_{DS} << v_{GS} - V_{to}$, we can neglect the v_{DS}^2 term, and this becomes

$$i_D \cong K2(v_{GS} - V_{to})v_{DS}$$

Then, the resistance between drain and source is given by

$$r_d = v_{DS} / i_D = \frac{1}{K2(v_{GS} - V_{to})}$$

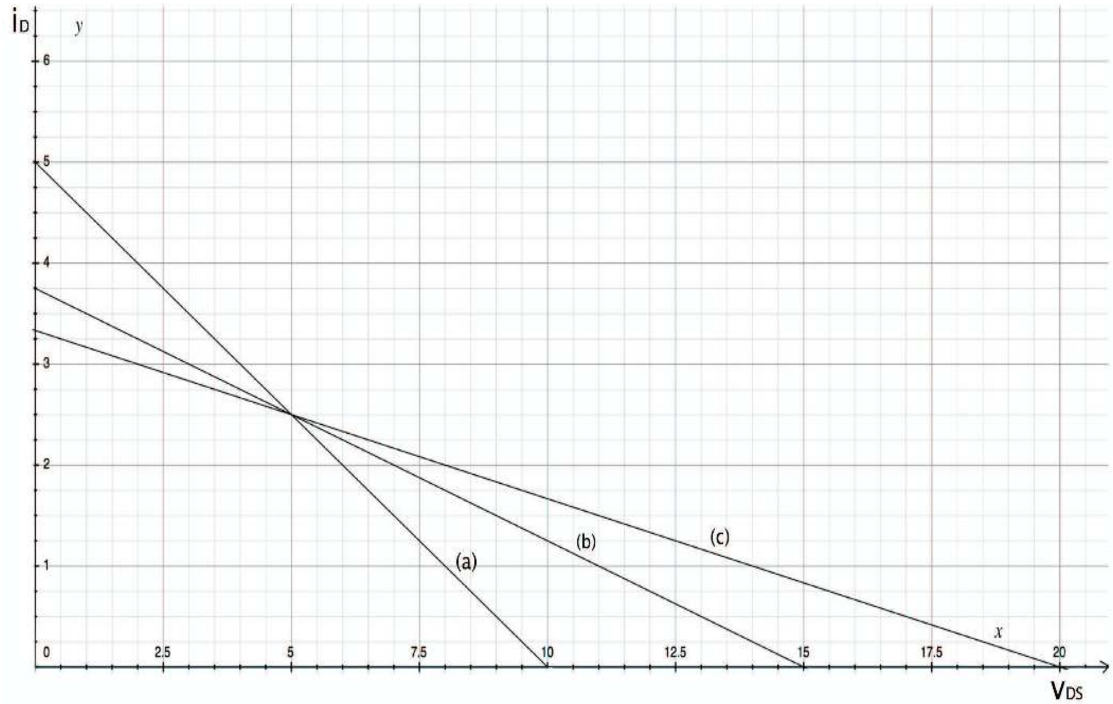
With the device in cutoff (i.e., $v_{GS} \le V_{to}$), the drain current is zero and r_d is infinite. Evaluating, we have:

v_{GS} (V)	r_d (kΩ)
1	∞
2	2.5
3	1.25
4	0.833

P12.18 Because $v_{GD} = 4 - 5 = -1$ V is less than V_{to}, the transistor is operating in saturation. Thus, we have $i_D = K(v_{GS} - V_{to})^2$. Substituting values gives $0.5 = 0.5(v_{GS} - 1)^2$ which yields two roots: $v_{GS} = 2$ V and $v_{GS} = 0$ V. However, the second root is extraneous, so we have $v_{GS} = 2 = 4 - 0.0005R$ which yields $R = 4000\ \Omega$.

P12.19 Distortion occurs in FET amplifiers because of curvature and nonuniform spacing of the characteristic curves.

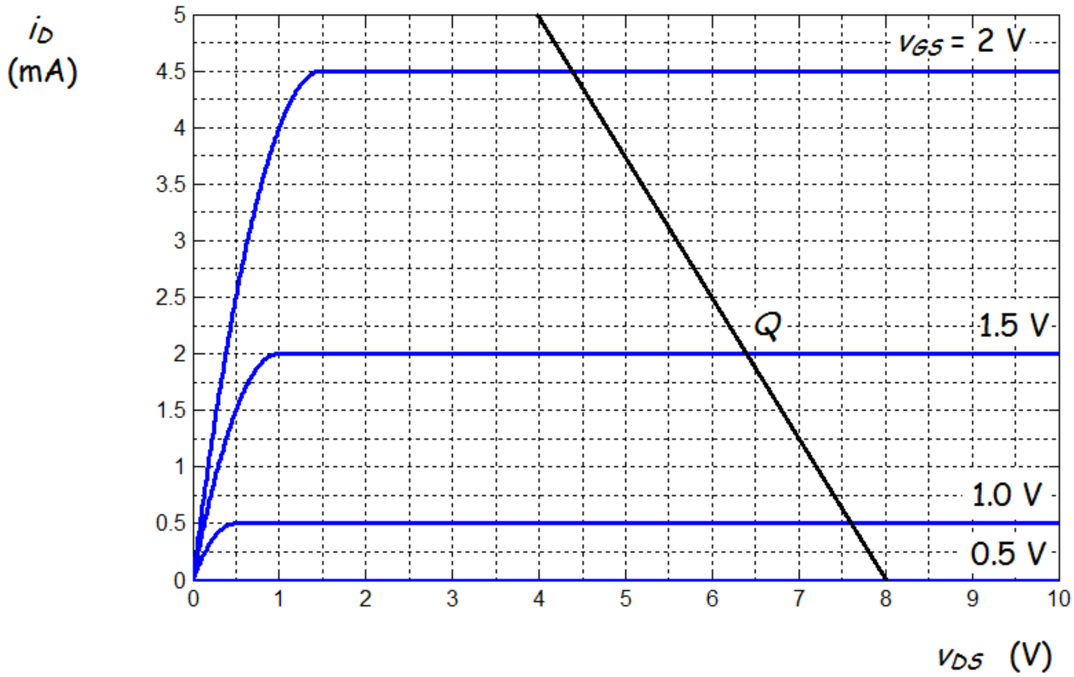
P12.20* The load-line equation is $V_{DD} = R_D i_D + v_{DS}$, and the plots are:



Notice that the load line rotates around the point (V_{DD}, 0) as the resistance changes.

P12.23 (a) The 1625 kΩ and 375 kΩ resistors act as a voltage divider that establishes a dc voltage $V_{GSQ} = 1.5$ V. Then, if the capacitor is treated as a short for the ac signal, we have $v_{GS}(t) = 1.5 + 0.5\sin(2000\pi t)$. Thus, v_{GS} swings between extremes of 1 and 2 V .

(b) and (c)

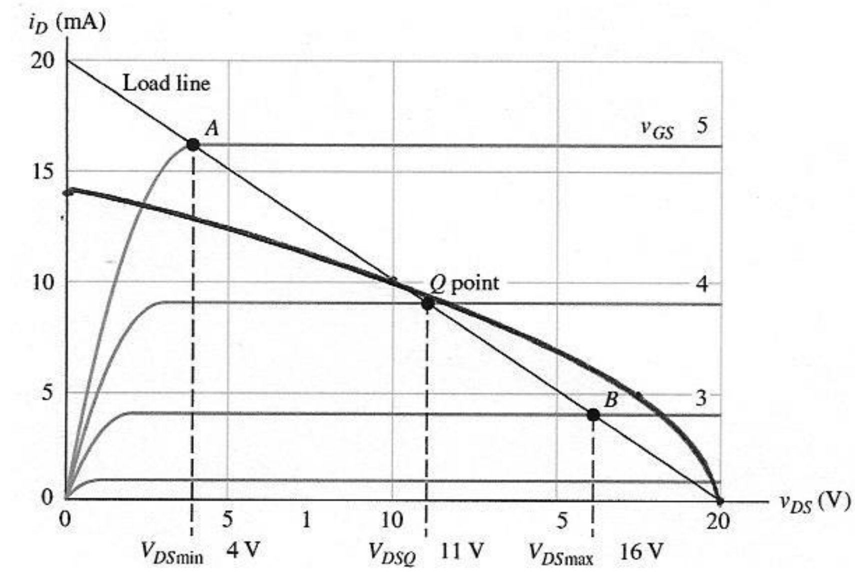


From the load line, we find $V_{DSQ} = 6.4$ V, $V_{DS\max} = 7.6$ V, and $V_{DS\min} = 4.4$ V.

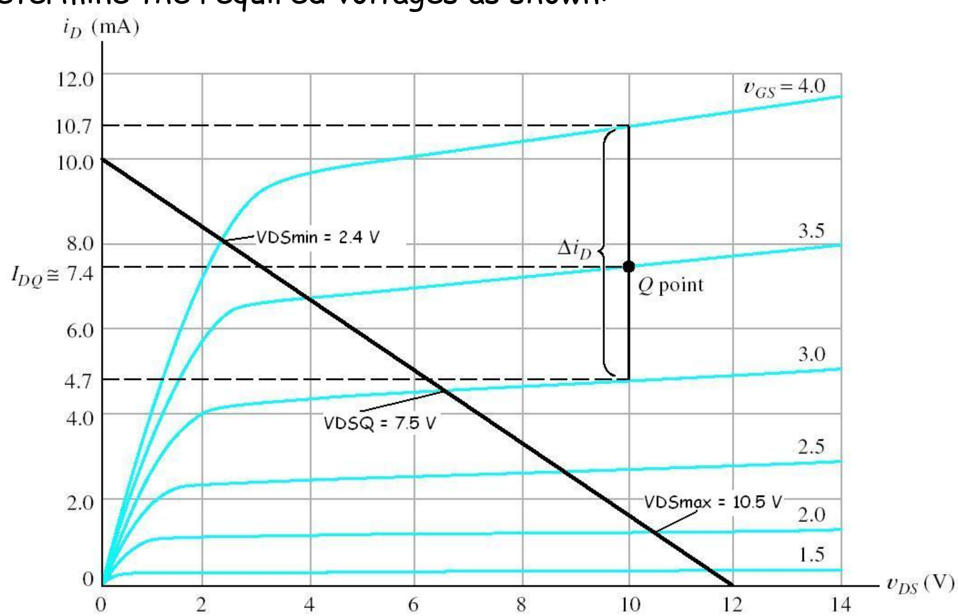
P12.24 For $v_{in} = +0.5$ V we have $v_{GS} = 2$ V. For the FET to remain in saturation, we must have $V_{DS\min} \geq 1.5$ V at which point the drain current is 4.5 mA. Thus, the maximum value of R_D is $R_{D\max} = (8 - 1.5)/4.5\text{ mA} = 1.444\text{ k}\Omega$.

P12.25 The KVL equation around the loop consisting of the V_{DD} source, the nonlinear element, and the drain/source is $V_{DD} = 20 = 0.1 i_D^2 + v_{DS}$

The load line in this case is a parabola with its apex at $i_D = 0$, $v_{DS} = 20$ V as shown:



P12.26 The Thévenin equivalent for the drain circuit contains a 12-V source in series with a 1.2-kΩ resistance. Then, we can construct the load line and determine the required voltages as shown:



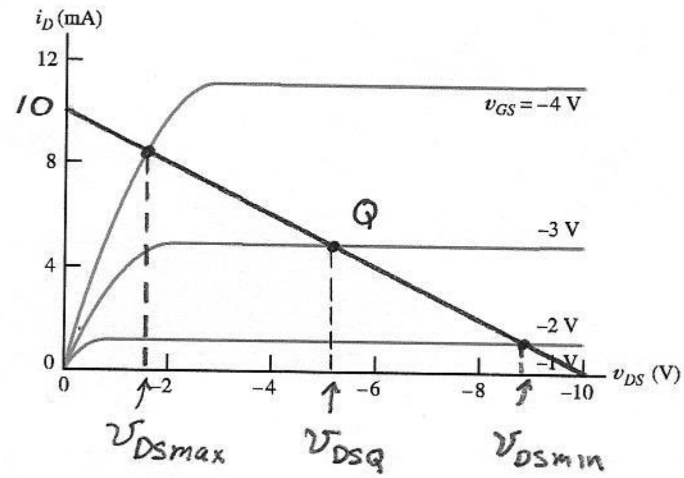
P12.27 Using KVL, we have

$$v_{GS}(t) = \sin(2000\pi t) + 7 - 10 = -3 + \sin(2000\pi t)$$

Thus, the maximum value of v_{GS} is -2 V, the Q-point value is -3 V, and the minimum value is -4 V. Applying KVL again, we obtain the load-line equation:

$$-10 = v_{DS} - i_D$$

in which the current is in mA. The load line is:



From the load line, we determine that the maximum value of v_{DS} is approximately -1.6 V, the Q-point is -5.5 V, and the minimum value is -8.8 V. The corresponding output voltages are 8.4 V, 4.5 V, and 1.2 V.

P12.28 We are given

$$v_{DS}(t) = V_{DC} + V_{1m}\sin(2000\pi t) + V_{2m}\cos(4000\pi t)$$

Evaluating at $t = 0.25$ ms and observing that the plot gives $v_{DS} = 4$ V at that instant we have

$$4 = V_{DC} + V_{1m} - V_{2m}$$

Similarly at $t = 0$ we have

$$11 = V_{DC} + V_{2m}$$

and at $t = 0.75$ ms we have

$$16 = V_{DC} - V_{1m} - V_{2m}$$

Solving the previous three equations we have $V_{DC} = 10.5$ V, $V_{2m} = 0.5$ V and $V_{1m} = -6$ V. Thus the percentage second–harmonic distortion is $|V_{2m}/V_{1m}| \times 100\% = 8.33\%$. Thus, this amplifier has a very large amount of distortion compared to that of a high quality amplifier.

P12.29 In an amplifier circuit, we need to bias the MOSFET so the ac signal to be amplified can cause changes in the currents and voltages resulting in an amplified signal. If the signal peak amplitude was smaller than 1 V, the transistor had $V_{to} = 1$ V, and we biased the transistor at $V_{GSQ} = 0$, the transistor would remain in cutoff, $i_D(t)$ would be zero for all t, and the signal would not be amplified.

P12.30* For this circuit, we can write

$$V_{GSQ} = 15 - I_{DQ}R_S$$

Assuming operation in saturation, we have

$$I_{DQ} = K(V_{GSQ} - V_{to})^2$$

using the first equation to substitute into the second equation, we have

$$I_{DQ} = K(15 - I_{DQ}R_S - V_{to})^2 = 0.4(14.2 - 2I_{DQ})^2$$

where we have assumed that I_{DQ} is in mA. Rearranging and substituting values, we have

$$I_{DQ}^2 - 14.825 I_{DQ} + 50.41 = 0$$

Which gives $I_{DQ} = 5.289$ mA and 9.542 mA. $I_{DQ} = 5.289$ mA is selected. Then we have $V_{DSQ} = 30 - R_D I_{DQ} - R_S I_{DQ} = 3.585$ V. $V_{GSQ} = 4.434$ V
<u>As such Eqns. 12.2 and 12.6 give the same value of I_{DQ} and the MOSFET operates in the boundary region between triode operation and saturation</u>

P12.31* We can write $V_{DD} = V_{DSQ} + R_S I_{DQ}$. Substituting values and solving, we obtain $R_S = 3\ \mathrm{k\Omega}$. Next we have $K = \frac{1}{2}KP(W/L) = 2\ \mathrm{mA/V^2}$. Assuming that the NMOS operates in saturation, we have

Substituting values and solving, we find $V_{GSQ} = 0$ V and $V_{GSQ} = 2$ V. The correct root is $V_{GSQ} = 2$ V. (As a check we see that the device does operate in saturation because we have greater than) Then we have $V_G = V_{GSQ} + R_S I_{DQ} = 8$ V. However we also have

Substituting values and solving, we obtain $R_2 = 2\ \mathrm{M\Omega}$.

*P12.32	We can write $V_{DQ} = 20 = 2\,I_{DQ} + 8 + 2$ in which I_{DQ} is in mA. Solving, we obtain I_{DQ} = 5 mA. Then, we find Rs = 2/I_{DQ} = 400 Ω. Next, we have $= \frac{1}{2} KP\left(\frac{W}{L}\right) = 0.6\; mA/V^2$. Assuming that the NMOS operates in saturation, we have $I_{DQ} = K\left(V_{GSQ} - V_{to}\right)^2$ Substituting values and solving we find V_{GSQ} = 3.7865 V and V_{GSQ} = -1.9865 V. The correct root is V_{GSQ} = 3.7865 V. (As a check, we see that the device does operate in saturation because we have V_{DSQ} = 8 V, which is greater than $V_{GSQ} - V_{to}$.) Then, we have VG = V_{GSQ} + 2 = 5.7865 V. However we also have: $V_G = V_{DD} \frac{R_2}{R_1+R_2}$ Substituting values and solving, we obtain R1 = 2.456 MΩ

P12.33 We have $V_G = V_{GSQ} = 5R_2/(R_1 + R_2) = 2.5$ V. Then we have $I_{DQ} = K(V_{GSQ} - V_{to})^2 = 1.28$ mA. $V_{DSQ} = V_{DD} - R_D I_{DQ} = -0.12$ V. For the MOSFET to operate in saturation R_D cannot exceed 2.65 kΩ

P12.34* We have $V_{GSQ} = V_{DSQ} = V_{DD} - R_D I_{DQ}$. Then substituting $I_{DQ} = K(V_{GSQ} - V_{to})^2$, we have

$$V_{GSQ} = V_{DD} - R_D K(V_{GSQ} - V_{to})^2$$

Substituting values and rearranging, we have

$$V_{GSQ}^2 + 2V_{GSQ} - 39 = 0$$

Solving we determine that $V_{GSQ} = 5.325$ V and then we have $I_{DQ} = K(V_{GSQ} - V_{to})^2 = 4.675$ mA.

P12.35 First, we use Equation 12.11 to compute

$$V_G = V_{DD}\frac{R_2}{R_1 + R_2} = 4\text{ V}$$

As in Example 12.2, we need to solve:

$$V_{GSQ}^2 + \left(\frac{1}{R_S K} - 2V_{to}\right)V_{GSQ} + (V_{to})^2 - \frac{V_G}{R_S K} = 0$$

Substituting values, we have

$$V_{GSQ}^2 - V_{GSQ} - 3 = 0$$

The roots are $V_{GSQ} = -1.3028$ V and 2.3028 V. The correct root is $V_{GSQ} = 2.3028$ V which yields $I_{DQ} = K(V_{GSQ} - V_{to})^2 = 0.8486$ mA. Finally, we have $V_{DSQ} = V_{DD} - R_D I_{DQ} - R_S I_{DQ} = 8.6055$ V.

P12.36 Assuming that the MOSFET is in saturation, we have

$$V_{GSQ} = 3.75 - I_{DQ}$$

$$I_{DQ} = K(V_{GSQ} - V_{to})^2$$

where we have assumed that I_{DQ} and K are in mA and mA/V^2 respectively. Using the second equation to substitute in the first, substituting values and rearranging, we have

$$V_{GSQ}^2 - 1.75\,V_{GSQ} + 0.0625 = 0$$

Solving for V_{GSQ}, we find $V_{GSQ} = 1.7135$V or 0.0365V.

Since in saturation, $V_{GSQ} > V_{to}$, therefore $V_{GSQ} = 1.7135$V.

The corresponding $I_{DQ} = 2.0365$ mA

P12.37 Applying voltage equation in the source-drain circuit,

$V_{DD} - I_{DQ}R_D - V_{DSQ} - I_{SQ}R_S = 0$

$R_S = \{15 - (2\text{mA})/(3\text{k}\Omega) - 8\}/\{2\text{mA}\} = 0.5\ \text{k}\Omega$

For calculation of R_2, we apply voltage equation in the gate-source circuit as follows,

$V_{GG} = V_{GSQ} + (I_{DQ} \times R_S) \qquad (1)$

$V_{GSQ} = [(I_{DQ}/\text{K})^{0.5} + V_{to}]$

$\Rightarrow \qquad V_{GSQ} = 4.4641$ V

Substituting this value in Equation (1), we get

$$V_{GG} = 4.4641 + (2\text{mA} \times 0.5\ \text{k}\Omega) = 5.4641\ \text{V}$$

V_{GG} can be calculated by voltage divider rule as follows:

$$V_{GG} = R_1/(R_1 + R_2)$$

Substituting the value of V_{GG} and R_1, we obtain $R_2 = 1.1460$ MΩ

P12.38 Because v_{GD1} is zero, the first transistor operates in saturation. We have $K_1 = \frac{1}{2}KP(W_1/L_1) = 50\ \mu\text{A/V}^2$. Then, we have

$$i_{D1} = K_1(V_{GSQ} - V_{to})^2$$

Substituting values and solving, we find $V_{GSQ} = -1.0$ V and $V_{GSQ} = 3$ V.

The correct root is $V_{GSQ} = 3$ V.

Then, the resistance is $R = \dfrac{5 - V_{GSQ}}{i_{D1}} = \dfrac{2.0}{0.2} = 10.0\ \text{k}\Omega$

The second transistor has $K_2 = \frac{1}{2}KP(W_2 / L_2) = 200\ \mu A/V^2$ and
$i_{D2} = K_2(V_{GSQ} - V_{to})^2 = 0.8\ \text{mA}$.
Provided that V_x is larger than 2 V, the second transistor operates in saturation, i_{D2} is constant, and the transistor is equivalent to an ideal 0.8-mA current source.

P12.39 $$g_m = \left.\frac{\partial i_D}{\partial v_{GS}}\right|_{Q\text{-point}} \qquad 1/r_d = \left.\frac{\partial i_D}{\partial v_{DS}}\right|_{Q\text{-point}}$$

P12.40 For constant drain current in the saturation region, we have $r_d = \infty$.

P12.41 See Figure 12.20 in the book.

P12.42 From Figure P12.42 at an operating point defined by $V_{GSQ} = 2.5$ V and $V_{DSQ} = 6$ V, we have

$$g_m = \frac{\Delta i_D}{\Delta v_{GS}} = \frac{(6.4 - 1.5)\,\text{mA}}{1\,\text{V}} = 4.9\,\text{mS}$$

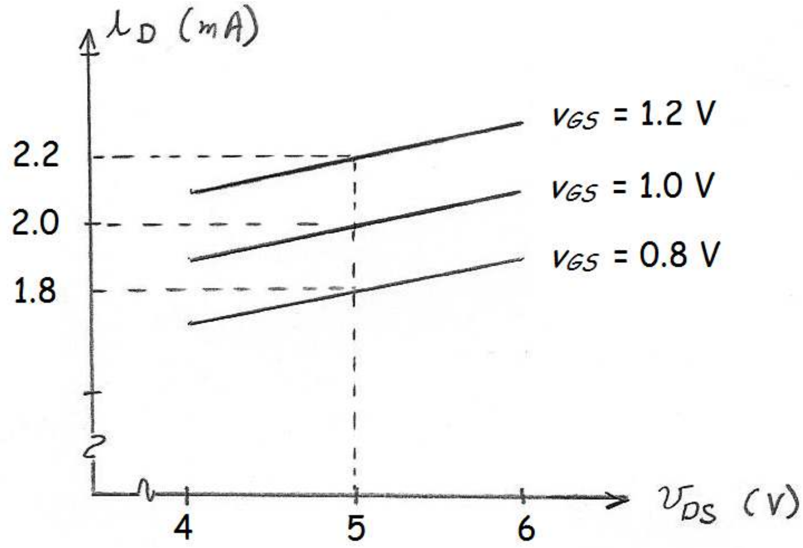
$$1/r_d = \frac{\Delta i_D}{\Delta v_{DS}} \cong \frac{(4.0 - 3.1)\,\text{mA}}{(8 - 4)\,\text{V}} = 0.225 \times 10^{-3}$$

Taking the reciprocal, we find $r_d = 4.44$ kΩ.

P12.43 We will sketch the characteristics for v_{GS} ranging a few tenths of a volt on either side of the Q point. g_m determines the spacing between the characteristic curves. For $g_m = 1$ mS, the curves move upward by 0.2 mA for each 0.2 V increase in v_{GS}.

Also, we will sketch the characteristics for v_{DS} ranging a few volts on either side of the Q point. r_d determines the slope of the characteristic curves. For $r_d = 10$ kΩ, the curves slope upward by 0.1 mA for each 1 V increase in v_{DS}.

The sketch of the curves is:



P12.44 For $V_{DSQ} = 0$, the vertical spacing of the drain characteristics is zero. Therefore, $g_m = 0$ at this operating point. Then, the small-signal equivalent circuit consists only of r_d. FETs are used as electronically controllable resistances at this operating point.

P12.45* In the triode region, we have

$$i_D = K[2(v_{GS} - V_{to})v_{DS} - v_{DS}^2]$$

$$g_m = \left.\frac{\partial i_D}{\partial v_{GS}}\right|_{Q\text{-point}} = 2Kv_{DS}\big|_{Q\text{-point}} = 2KV_{DSQ}$$

P12.46* In the triode region, we have

$$i_D = K[2(v_{GS} - V_{to})v_{DS} - v_{DS}^2]$$

$$1/r_d = \left.\frac{\partial i_D}{\partial v_{DS}}\right|_{Q\text{-point}} = 2K(v_{GS} - V_{to} - v_{DS})\big|_{Q\text{-point}}$$

$$r_d = \frac{1}{2K(V_{GSQ} - V_{to} - V_{DSQ})}$$

P12.47

$$g_m = \left.\frac{\partial i_D}{\partial v_{GS}}\right|_{Q\text{-point}} = 8v_{GS}\big|_{Q\text{-point}} = 16\text{ mS}$$

$$1/r_d = \left.\frac{\partial i_D}{\partial v_{DS}}\right|_{Q\text{-point}} = 0.2\big|_{Q\text{-point}} = 0.2\text{ mS}$$

$$r_d = 5\text{ k}\Omega$$

P12.48

$$g_m = \left.\frac{\partial i_D}{\partial v_{GS}}\right|_{Q\text{-point}} = \exp(v_{GS})\big|_{Q\text{-point}} = \exp(1) = 2.718\text{ mS}$$

$$1/r_d = \left.\frac{\partial i_D}{\partial v_{DS}}\right|_{Q\text{-point}} = 0.04v_{DS}\big|_{Q\text{-point}} = 0.4\text{ mS}$$

$$r_d = 2.5\text{ k}\Omega$$

P12.49 The parameter r_D can be calculated as

$$r_d = \left.\frac{\Delta v_{DS}}{\Delta i_D}\right|_{v_{GS}=V_{GSQ}} = \frac{1\text{ V}}{0.08\text{ mA}} = 12.5\text{ k}\Omega$$

The Q-point is $V_{DSQ} = 5\text{V}$, $I_{DQ} = 5\text{mA}$, $V_{GSQ} = 2\text{V}$

P12.50 This transistor is operating with constant v_{DS}. Thus, we can determine g_m by dividing the peak ac drain current by the peak ac gate-to-source voltage.

$$g_m = \left.\frac{\Delta i_D}{\Delta v_G}\right|_{v_{DS}=V_{DSQ}} = \frac{0.2\text{ mA}}{0.2\text{ V}} = 1.0\text{ mS}$$

The Q-point is $V_{DSQ} = 5\text{ V}$, $V_{GSQ} = 2\text{ V}$, and $I_{DQ} = 1\text{ mA}$.

P12.51 See Figure 12.22 in the book.

P12.52 Coupling capacitors act as open circuits for dc and as approximate short circuits for the ac signals to be amplified. Coupling capacitors are used in discrete circuits to isolate the various stages for dc so the bias points of the various stages can be determined independently while connecting the ac signal. The output coupling capacitor prevents dc current from flowing through the load and causes the ac signal to appear across the load. Furthermore, the input coupling capacitor connects the ac signal and prevents the dc component of the source from affecting the bias point of the input stage.

Coupling capacitors are replaced by short circuits in midband small-signal equivalent circuits. They cause the gain of an amplifier to decline as the signal frequency becomes small.

P12.53* (a)

$$V_G = V_{DD}\frac{R_2}{R_1+R_2} = 20\frac{0.3}{1.7+0.3} = 3\text{ V}$$

$$V_{GSQ} = V_G = 3\text{ V}$$

$$K = \tfrac{1}{2}KP(W/L) = 2.5\text{ mA/V}^2$$

$$I_{DQ} = K(V_{GSQ}-V_{to})^2 = 10\text{ mA}$$

$$V_{DSQ} = V_{DD} - R_D I_{DSQ} = 10\text{ V}$$

$$g_m = 2\sqrt{KI_{DQ}} = 0.01\text{ S}$$

(b)

$$R_L' = \frac{1}{1/R_D + 1/R_L} = 500\ \Omega$$

$$A_v = -g_m R_L' = -5$$

$$R_{in} = \frac{1}{1/R_1 + 1/R_2} = 255\text{ k}\Omega$$

$$R_o = R_D = 1\text{ k}\Omega$$

P12.54 (a)

$$V_G = V_{DD}\frac{R_2}{R_1+R_2} = 20\frac{0.3}{1.7+0.3} = 3\text{ V}$$

$$V_{GSQ} = V_G = 3\text{ V}$$

$$K = \tfrac{1}{2}KP(W/L) = 0.75\text{ mA/V}^2$$

$$I_{DQ} = K(V_{GSQ}-V_{to})^2 = 0.75\text{ mA}$$

$$V_{DSQ} = V_{DD} - R_D I_{DSQ} = 19.25\text{ V}$$

$$g_m = 2\sqrt{KI_{DQ}} = 0.0015\text{ S}$$

(b)

$$R_L' = \frac{1}{1/R_D + 1/R_L} = 500\ \Omega$$

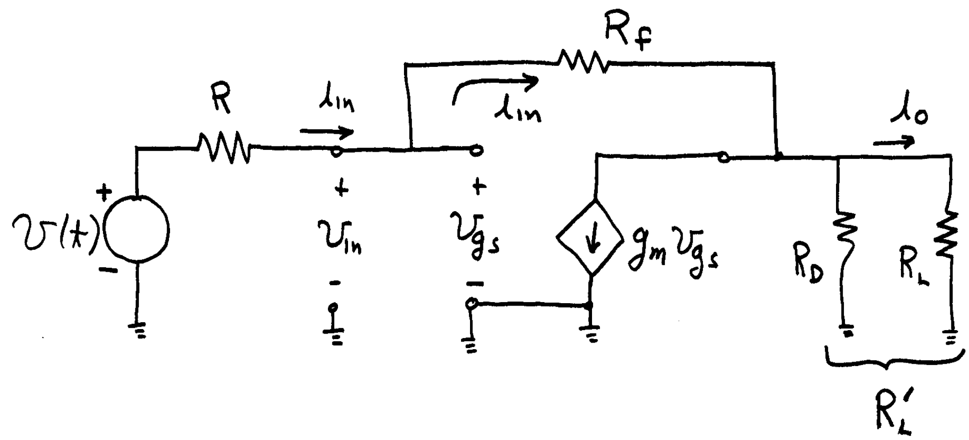
$$A_v = -g_m R_L' = -0.75$$

$$R_{in} = \frac{1}{1/R_1 + 1/R_2} = 255\text{ k}\Omega$$

$$R_o = R_D = 1\text{ k}\Omega$$

Notice that the gain of the circuit can change a great deal as the parameters of the FET change.

P12.55 (a)

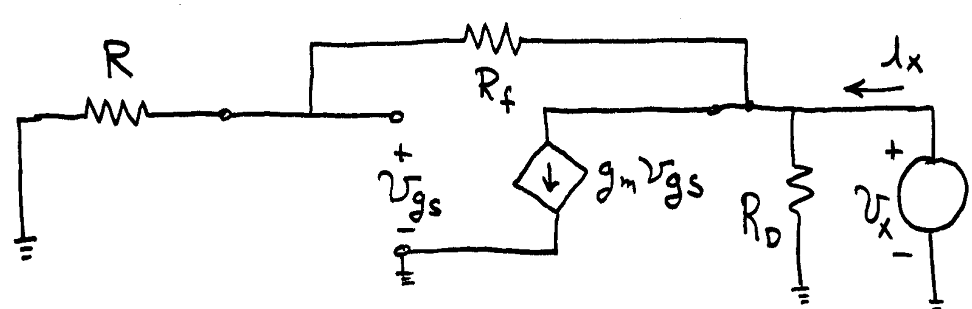


(b) $v_o = R_L'(i_{in} - g_m v_{in}) \qquad i_{in} = (v_{in} - v_o)/R_f$

$$A_v = \frac{v_o}{v_{in}} = \frac{R_L' - g_m R_L' R_f}{R_L' + R_f}$$

$$R_{in} = \frac{v_{in}}{i_{in}} = \frac{R_f}{1 - A_v}$$

The circuit used to determine output impedance is:

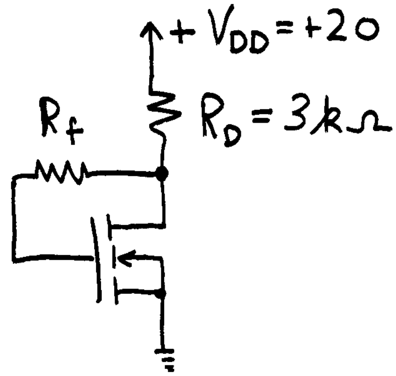


We define $R_D' = R_D \| (R + R_f)$. Then we can write

$$v_{gs} = v_x \frac{R}{R + R_f} \quad \text{and} \quad i_x = \frac{v_x}{R_D'} + g_m v_{gs}$$

$$R_o = \frac{v_x}{i_x} = \frac{1}{\dfrac{1}{R_D'} + \dfrac{g_m R}{R_f + R}}$$

(c) The dc circuit is:



$V_{GSQ} = V_{DSQ}$ $\quad I_{DQ} = K(V_{DSQ} - V_{to})^2$ $\quad I_{DQ} = (V_{DD} - V_{DSQ})/R_D$

Using the above equations, we obtain

$$3V_{DSQ}^2 - 29V_{DSQ} + 55 = 0$$

$V_{DSQ} = 7.08$ V and $I_{DQ} = 4.31$ mA

$$g_m = \left.\frac{\partial i_D}{\partial v_{GS}}\right|_{Q\text{-point}} = 2K\left(V_{GSQ} - V_{to}\right) = 4.16 \times 10^{-3} \text{ S}$$

(d) $R_L' = R_D \| R_L = 2.31\ \text{k}\Omega$

$A_v = -9.37$

$R_{in} = 9.64\ \text{k}\Omega$

$R_o = 414\ \Omega$

(e) $v_o(t) = v(t) \times \dfrac{R_{in}}{R + R_{in}} \times A_v = -0.164\sin(2000\pi t)$

(f) This is an inverting amplifier that has very low input impedance compared to many other FET amplifiers.

P12.56* Referring to the circuit shown in Figure P12.56, we have

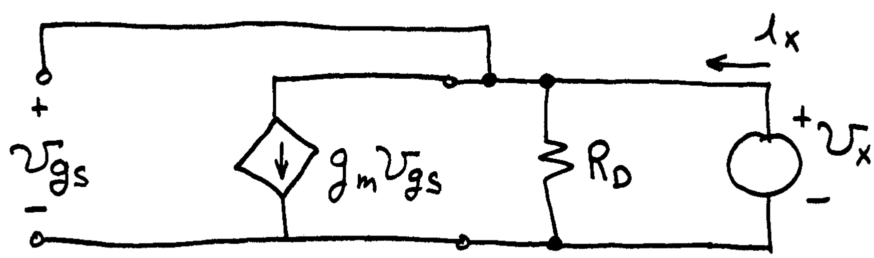
$V_{GSQ} = V_{DSQ}$ $\quad I_{DQ} = K(V_{GSQ} - V_{to})^2$ $\quad I_{DQ} = (V_{DD} - V_{DSQ})/R_D$

From the previous three equations we obtain:

$$1.1V_{DSQ}^2 - 5.6V_{DSQ} - 10.1 = 0$$

$V_{DSQ} = 6.50$ V and $I_{DQ} = 6.135$ mA

$$g_m = \left.\frac{\partial i_D}{\partial v_{GS}}\right|_{Q-\text{point}} = 2K\left(V_{GSQ} - V_{to}\right) = 3.5 \text{ mS}$$



$$R_o = \frac{v_x}{i_x} = \frac{v_x}{v_x/R_D + g_m v_x} = \frac{1}{1/R_D + g_m} = 253\ \Omega$$

P12.57 If we need a voltage-gain magnitude greater than unity, we choose a common-source amplifier. To attain lowest output impedance usually a source follower is better.

P12.58 See Figure 12.26 in the book.

P12.59* We have

$$K = \left(\frac{W}{L}\right)\frac{KP}{2} = 400\ \mu A/V^2$$

Assuming operation in saturation, we have

$$I_{DQ} = K(V_{GSQ} - V_{to})^2$$

Solving for V_{GSQ} and evaluating we have

$$V_{GSQ} = V_{to} + \sqrt{I_{DQ}/K} = 3.236 \text{ V}$$

$$V_G = V_{DD}\frac{R_2}{R_1 + R_2} = 10 \text{ V}$$

$$V_G = V_{GSQ} + R_S I_{DQ}$$

Solving for R_S and substituting values we have

$$R_S = (V_G - V_{GSQ})/I_{DQ} = 3.382\ \text{k}\Omega$$

We have $g_m = 2\sqrt{KI_{DQ}} = 1.789$ mS

$$R_L' = \frac{1}{1/R_L + 1/R_S + 1/r_d} = 1.257\ \text{k}\Omega$$

$$A_v = \frac{v_o}{v_{in}} = \frac{g_m R_L'}{1 + g_m R_L'} = 0.6922$$

$$R_{in} = \frac{v_{in}}{i_{in}} = R_G = R_1 \| R_2 = 666.7\ \text{k}\Omega$$

$$R_o = \frac{1}{g_m + \frac{1}{R_s} + \frac{1}{r_d}} = 386.9\ \Omega$$

P12.60 (a) Applying voltage equations in the gate-source circuit, we obtain,

$$0 - V_{GSQ} - (I_D)(4\text{k}\Omega) = -15$$

Also we know that in saturation,

$$I_D = 0.5K_P(W/L)[V_{GSQ} - V_{to}]^2$$

Substituting in the previous equations, we obtain $V_{GS} = 3.1114$V and $I_D = 2.9721$ mA

Using the equation for g_m,

$$g_m = 2\sqrt{(KI_D)} = 2.8152\ \text{mS}$$

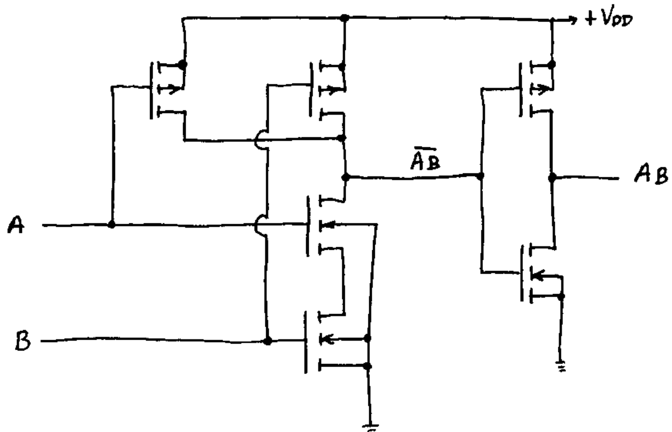
(b) $R_L' = (R_D \| R_L) = 1.7647\ \text{k}\Omega$

$A_V = R_L' g_m = 4.968$

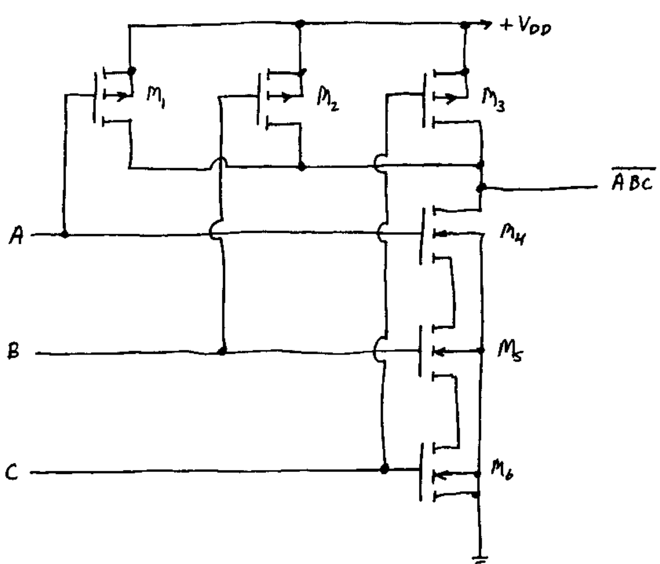
$$R_{in} = \frac{1}{g_m + 1/R_S} = 0.3262\ \text{k}\Omega$$

P12.61 See Figure 12.31 in the text.

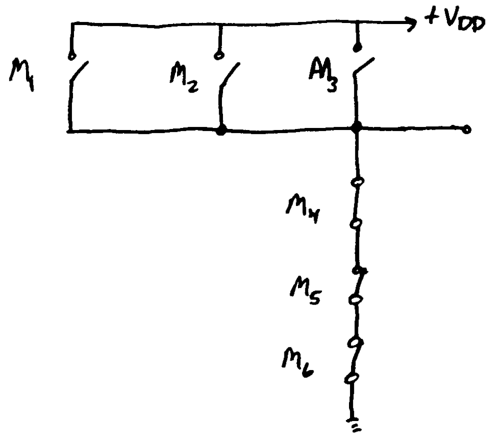
P12.62



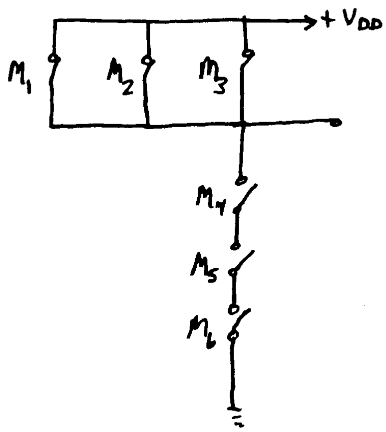
P12.63 (a)



(b) All inputs high:



(c) All inputs low:



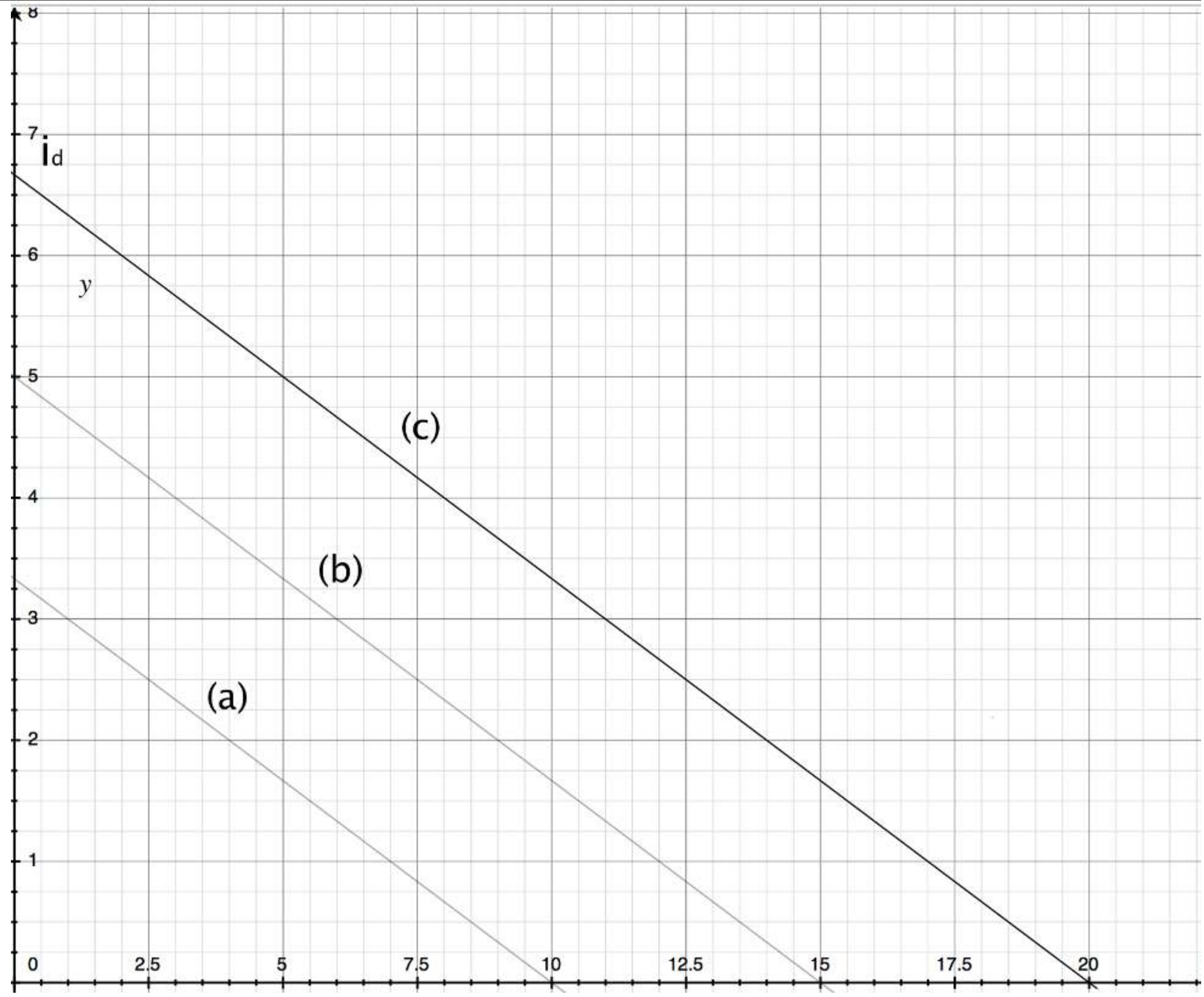
P12.64 (a) The charge flowing from the source to charge the load capacitance each time the output switches from low to high is $Q = V_{DD}C_L$. The energy E delivered by the source is the voltage times charge. $E = V_{DD}^2 C_L$. Half of this energy is dissipated as heat in R_P and half is stored in the capacitance. Then, when the output switches low, the energy stored in the capacitance is dissipated as heat in R_n. Thus, the average power taken from the source and dissipated as heat is $P_{Gate} = fE = fV_{DD}^2 C_L$.

(b) Clearly, when V_{DD} is cut in half, the power is cut by a factor of 4. Thus, lower power supply voltages are desirable.

(c) $P = 10^9 \times (0.01f)V_{DD}^2 C_L = 20\ \text{W}$.

P12.6	(a) Cutoff because we have $v_{GS} \le V_{to}$. (b)Saturation because we have $v_{GS} \ge V_{t0}$ and $v_{DS} \ge v_{GS} - V_{to}$. (c) Triode because we have $v_{GS} \ge V_{to}$ and $v_{DS} \le v_{GS} - V_{to}$. (d) Saturation because we have $v_{GS} \ge V_{t0}$ and $v_{DS} \ge v_{GS} - V_{to}$.
P12.10	From the given values of K_P, W and L, K can be calculated as: $K = \frac{W}{2L} K_P = 0.25 mA/V^2$ (a) If R_1=4kΩ and R_2=1kΩ, V_{GS}= 0.2x5=1V. Also it is given that V_{DS}=5V and V_{to}=1V. Since $V_{GS} \ge V_{to}$ and $V_{DS} \ge (V_{GS} - V_{to})$, the transistor is in the saturation region. Thus $i_D = K(V_{GS} - V_{to})^2 = 0$ mA (b) If R_1=6kΩ and R_2=4kΩ, V_{GS}= 0.4x5=2V. Also it is given that V_{DS}=5V and V_{to}=1V. Since $V_{GS} \ge V_{to}$ and $V_{DS} \ge (V_{GS} - V_{to})$, the transistor is in the saturation region. Thus $i_D = K(V_{GS} - V_{to})^2 = 0.25$ mA
P12.11	From the given values of K_P, W and L, K can be calculated as: $K = \frac{W}{2L} K_P = 0.25 mA/V^2$ (a) If R_1=1kΩ and R_2=9kΩ, V_{GS}= -0.1x5= -0.5V. Also it is given that V_{DS} = -5V and V_{to} = -0.8V. Since $V_{GS} \le V_{to}$ and $V_{DS} \le (V_{GS} - V_{to})$, the transistor is in the saturation region. Thus $i_D = K(V_{GS} - V_{to})^2 = 0.225$ mA (b) If R_1= 3kΩ and R_2= 2kΩ, V_{GS}= -0.6x5= -3V. Also it is given that V_{DS}= -5V and V_{to}= -0.8V. Since $V_{GS} \le V_{to}$ and $V_{DS} \le (V_{GS} - V_{to})$, the transistor is in the saturation region. Thus $i_D = K(V_{GS} - V_{to})^2 = 1.21$ mA

*P12.13	V_{t0} = -1V, K = 0.2mA/V^2. In the saturation region for the p-channel MOSFET, $$i_D = K(V_{GS}-V_{to})^2$$ $$\Rightarrow\ 0.8 = 0.2(V_{GS} - (-1))^2$$ $$\Rightarrow\ V_{GS} = 1V \text{ or } -3V$$ But since in the saturation region $V_{GS} \le V_{t0}$, V_{GS} = -3V
P12.17	For the n-channel MOSFET, v_{t0} = 1V and K = 0.2mA/V^2 • for v_{in} = 0V, v_{GS} = 0V, $v_{GS} \le v_{to}$, thus the MOSFET operates in the cutoff region thus $i_D = 0$ • for v_{in} = 5V, v_{GS} = 5V, $v_{GS} \ge v_{to}$ and $v_{DS} \ge (v_{GS} - v_{t0})$, thus the MOSFET operates in the saturation regi thus $$i_D = K(v_{GS} - v_{t0})^2$$ $$\Rightarrow\ i_D = 3.2mA$$ For the p-channel MOSFET, v_{t0} = -1V and K = 0.2mA/V^2 • for v_{in} = 0V, v_{GS} = -5V, $v_{GS} \le v_{to}$ and $v_{DS} \le (v_{GS} - v_{t0})$, thus the MOSFET operates in the saturation region thus $$i_D = K(v_{GS} - v_{t0})^2$$ $$\Rightarrow\ i_D = 3.2mA$$ • for v_{in} = 5V, v_{GS} = 0V, $v_{GS} \ge v_{to}$, thus the MOSFET operates in the cutoff region thus $$i_D = o$$

P12.21	
P12.22	For V_{GG} = 1V saturation is only when $v_{in} + V_{GG} \geq v_{t0}$ i.e. when $v_{in} \geq 0$. From the Figure 12.11, $V_{DS\ min}$ = 19V, $V_{DSQ} = V_{DS\ max}$ = 20V For V_{GG} = -2V, the MOSFET is always at cutoff, thus $v_{DS\ max} = v_{DS\ min} = v_{DSQ}$

Practice Test

T12.1 Drain characteristics are plots of i_D versus v_{DS} for various values of v_{GS}.

First, we notice that for $v_{GS} = 0.5\text{ V}$, the transistor is in cutoff, and the drain current is zero, because v_{GS} is less than the threshold voltage V_{to}. Thus, the drain characteristic for $v_{GS} = 0.5\text{ V}$ lies on the horizontal axis.

Next, we compute the drain current in the saturation region for $v_{GS} = 4\text{ V}$.

$$K = \tfrac{1}{2}KP(W/L) = \tfrac{1}{2}(80\times10^{-6})(100/4) = 1\text{ mA/V}^2$$

$$i_D = K(v_{GS} - V_{to})^2 = K(4-1)^2 = 9\text{ mA} \quad \text{for } v_{DS} > v_{GS} - V_{to} = 3\text{ V}$$

Thus, the characteristic is constant at 9 mA in the saturation region.

The transistor is in the triode region for $v_{DS} < v_{GS} - V_{to} = 3\text{ V}$, and the drain current (in mA) is given by

$$i_D = K[2(v_{GS} - V_{to})v_{DS} - v_{DS}^2] = 6v_{DS} - v_{DS}^2$$

with v_{DS} in volts. This plots as a parabola that passes through the origin and reaches its apex at $i_D = 9\text{ mA}$ and $v_{DS} = 3\text{ V}$.

The drain characteristic for $v_{GS} = 4\text{ V}$ is identical to that of Figure 12.11 in the book.

T12.2 We have $v_{GS}(t) = v_{in}(t) + V_{GG} = \sin(2000\pi t) + 3\text{ V}$. Thus, we have $V_{GS\max} = 4$ V, $V_{GSQ} = 3$ V, and $V_{GS\min} = 2$ V. Writing KVL around the drain circuit, we have

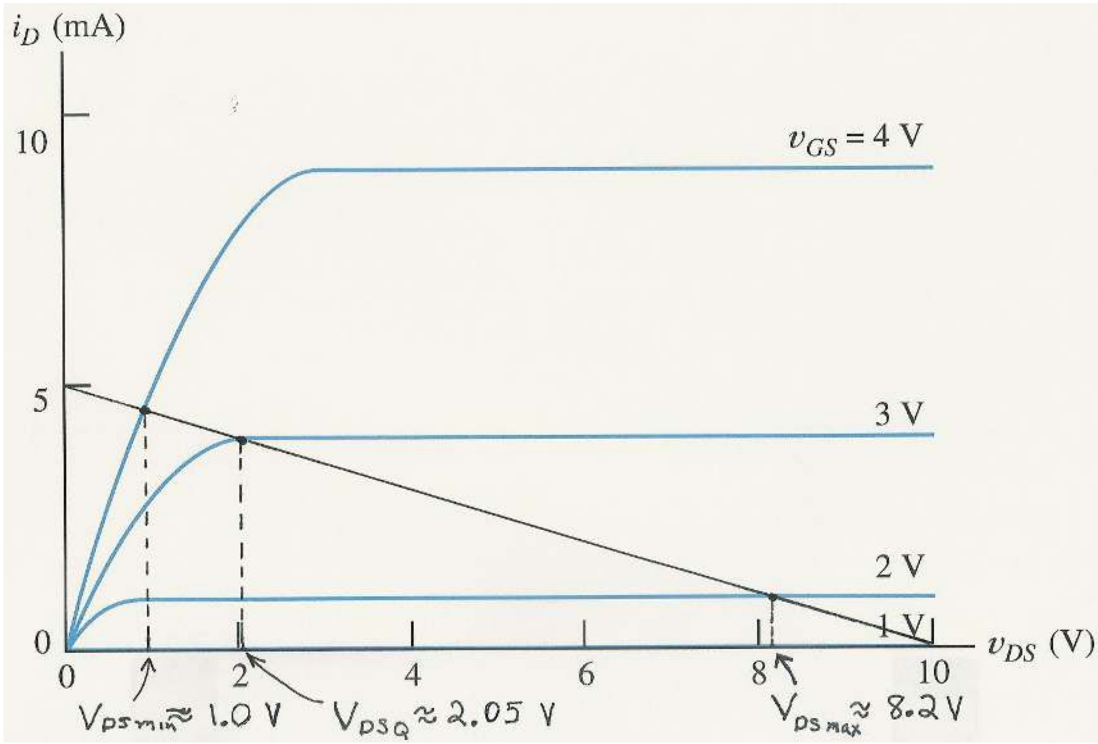
$$V_{DD} = R_D i_D + v_{DS}$$

With voltages in volts, currents in mA, and resistances in kΩ, this becomes

$$10 = 2i_D + v_{DS}$$

which is the equation for the load line.

The characteristics and the load line are:



The results of the load-line analysis are $V_{DS\min} \cong 1.0$ V, $V_{DSQ} \cong 2.05$ V, and $V_{DS\max} \cong 8.2$ V.

T12.3 Because the gate current is zero, we can apply the voltage division principle to determine the voltage at the gate with respect to ground.

$$V_G = \frac{10\text{ k}\Omega}{(10+30)\text{ k}\Omega} \times 12 = 3\text{ V}$$

For the transistor, we have

$$K = \tfrac{1}{2} KP(W/L) = \tfrac{1}{2}(80 \times 10^{-6})(100/4) = 1\text{ mA/V}^2$$

Because the drain voltage is 12 V, which is higher than the gate voltage, we conclude that the transistor is operating in the saturation region. Thus, we have

$$I_{DQ} = K(V_{GSQ} - V_{to})^2$$

$$I_{DQ} = (V_{GSQ} - 1)^2 = 0.5\text{ mA}$$

Solving, we have $V_{GSQ} = 1.707$ V or $V_{GSQ} = 0.293$ V. However, V_{GSQ} must be larger than V_{to} for current to flow, so the second root is extraneous. Then, the voltage across R_s is $V_S = V_G - V_{GSQ} = 1.293$ V. The current through R_S is I_{DQ}. Thus, the required value is $R_S = 1.293/0.5 = 2.586$ kΩ.

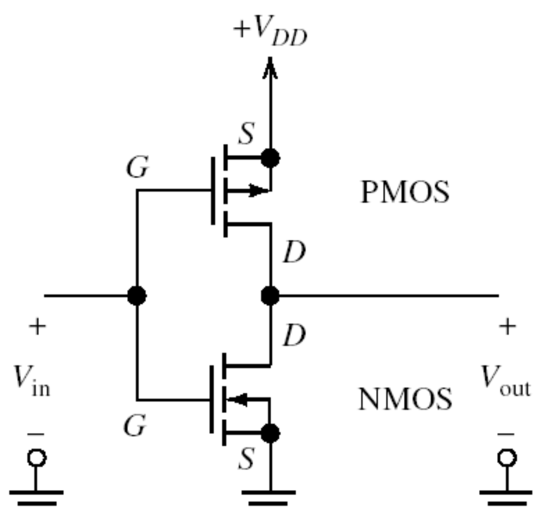
T12.4 This transistor is operating with constant v_{DS}. Thus, we can determine g_m by dividing the peak ac drain current by the peak ac gate-to-source voltage.

$$g_m = \left.\frac{\Delta i_D}{\Delta v_{GS}}\right|_{v_{DS}=V_{DSQ}} = \frac{0.05\ \text{mA}}{0.02\ \text{V}} = 2.5\,\text{mS}$$

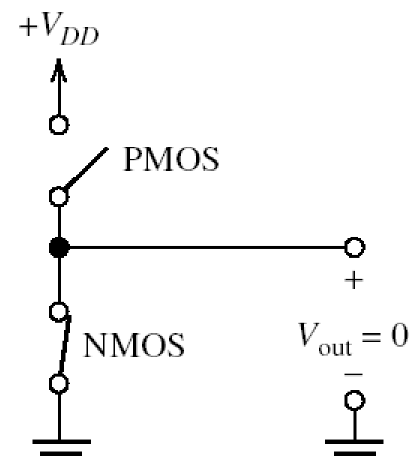
The Q-point is $V_{DSQ} = 5\,\text{V}$, $V_{GSQ} = 2\,\text{V}$, and $I_{DQ} = 0.5\,\text{mA}$.

T12.5 (a) A dc voltage source is replaced with a short circuit in the small-signal equivalent. (b) A coupling capacitor becomes a short circuit. (c) A dc current source is replaced with an open circuit, because even if an ac voltage appears across it, the current through it is constant (i.e., zero ac current flows through a dc current source).

T12.6 See Figure 12.31(b) and (c) in the text.



(b) Circuit diagram



(c) Equivalent circuit with V_{in} high

CHAPTER 13

Exercises

$$i_E = I_{ES}\left[\exp\left(\frac{v_{BE}}{V_T}\right) - 1\right]$$

E13.1 The emitter current is given by the Shockley equation:

$$i_E \cong I_{ES}\exp\left(\frac{v_{BE}}{V_T}\right)$$

For operation with , and we can write

Solving for , we have

$$v_{BE} \cong V_T \ln\left(\frac{i_E}{I_{ES}}\right) = 26\ln\left(\frac{10^{-2}}{10^{-14}}\right) = 718.4\text{ mV}$$

$$v_{BC} = v_{BE} - v_{CE} = 0.7184 - 5 = -4.2816\text{ V}$$

$$\alpha = \frac{\beta}{\beta+1} = \frac{50}{51} = 0.9804$$

$$i_C = \alpha i_E = 9.804\text{ mA}$$

$$i_B = \frac{i_C}{\beta} = 196.1\ \mu\text{A}$$

E13.2 $\beta = \dfrac{\alpha}{1-\alpha}$

	β
0.9	9
0.99	99
0.999	999

E13.3 $i_B = i_E - i_C = 0.5\text{ mA}$ $\alpha = i_C / i_E = 0.95$ $\beta = i_C / i_B = 19$

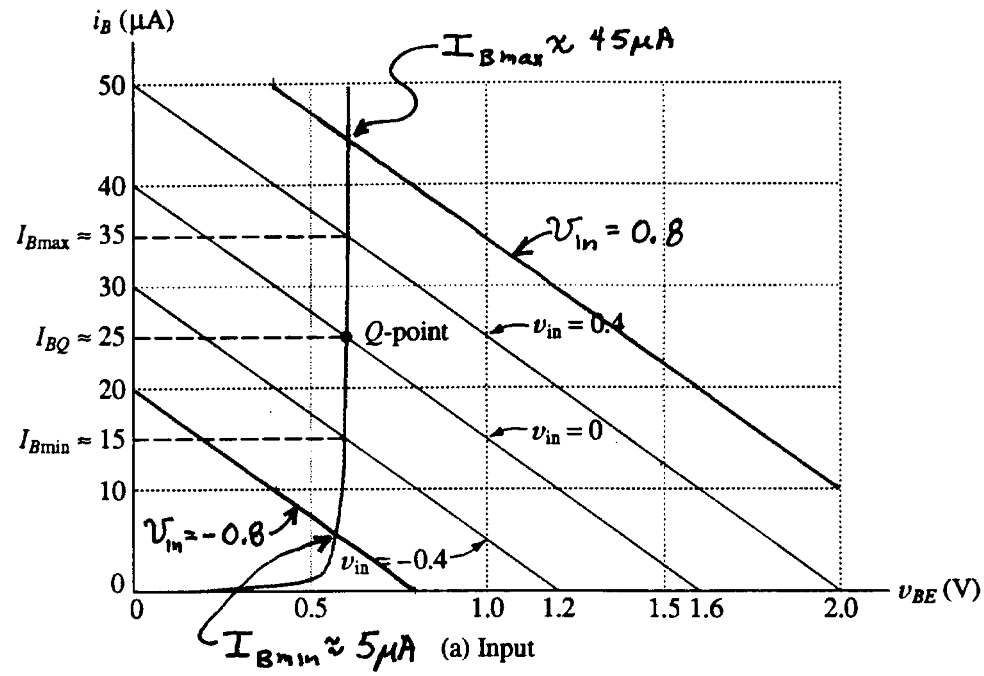
E13.4 The base current is given by Equation 13.8:

$$i_B = (1-\alpha)I_{ES}\left[\exp\left(\frac{v_{BE}}{V_T}\right) - 1\right] = 1.961\times 10^{-16}\left[\exp\left(\frac{v_{BE}}{0.026}\right) - 1\right]$$

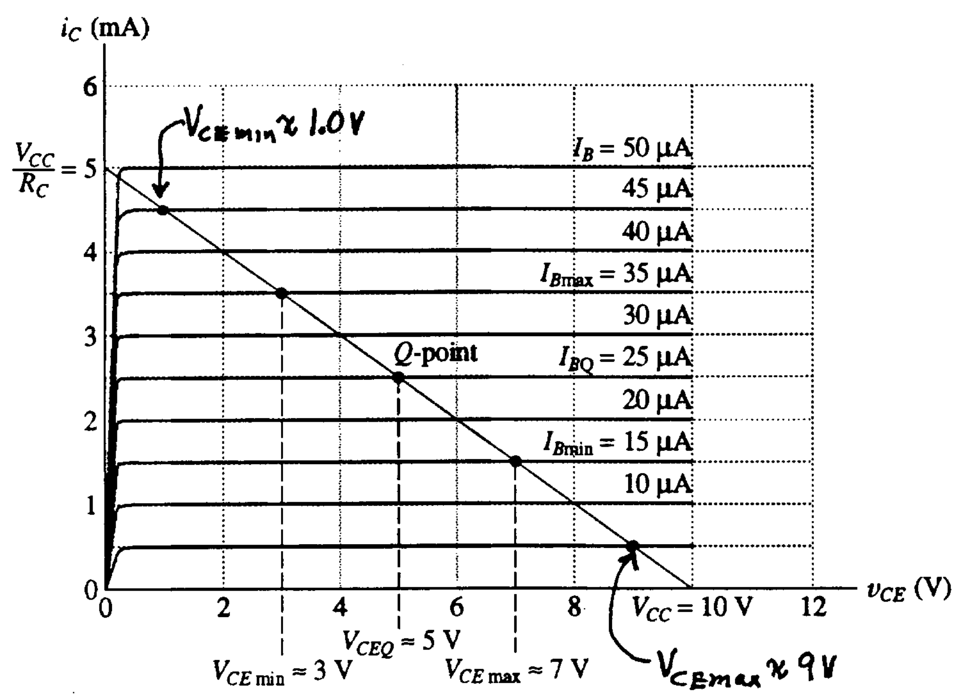
which can be plotted to obtain the input characteristic shown in Figure 13.6a. For the output characteristic, we have $_C$ $_B$ provided that

$v_{CE} \geq$ approximately 0.2 V. For $v_{CE} \leq 0.2$ V, i_C falls rapidly to zero at $v_{CE} = 0$. The output characteristics are shown in Figure 13.6b.

E13.5 The load lines for $v_{in} = 0.8$ V and - 0.8 V are shown:



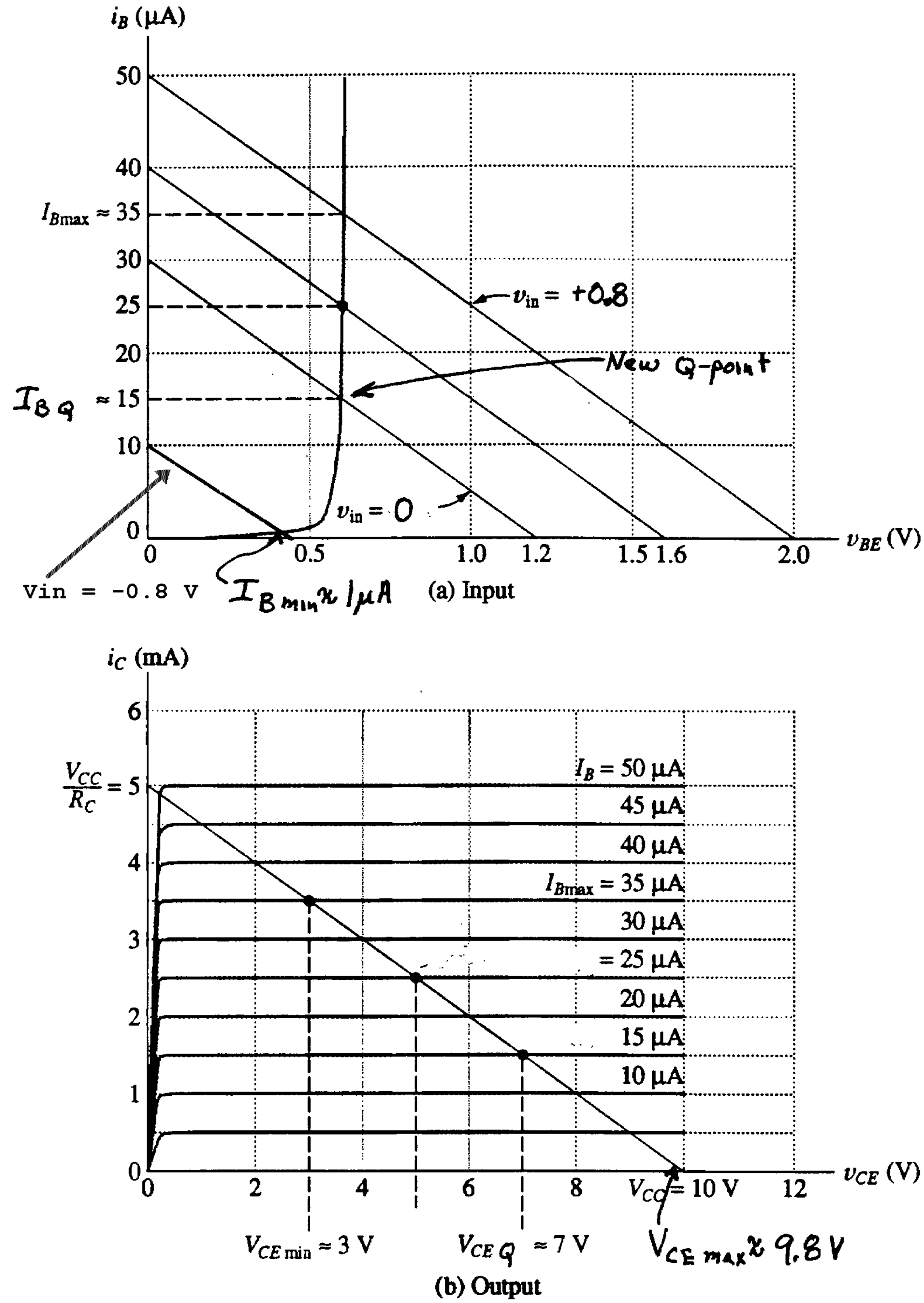
(a) Input



(b) Output

As shown on the output load line, we find $V_{CE\max} \cong 9$ V, $V_{CEQ} \cong 5$ V, and $V_{CE\min} \cong 1.0$ V.

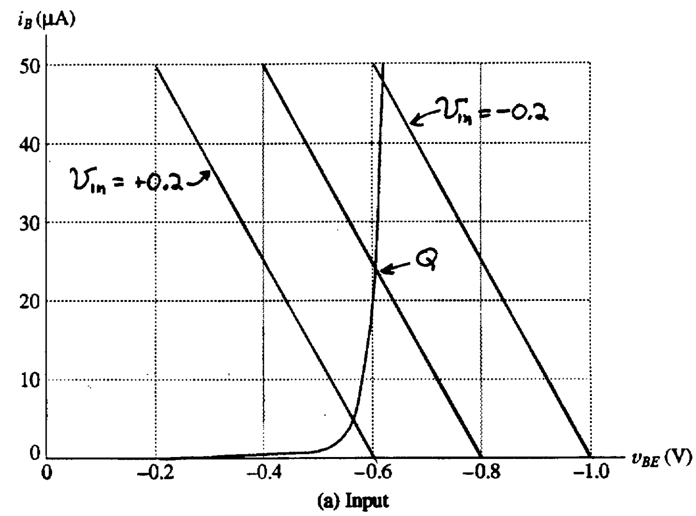
E13.6 The load lines for the new values are shown:



As shown on the output load line, we have
$V_{CE\max} \cong 9.8\,\text{V}, V_{CEQ} \cong 7\,\text{V}$, and $V_{CE\min} \cong 3.0\,\text{V}$.

E13.7 Refer to the characteristics shown in Figure 13.7 in the book. Select a point in the active region of the output characteristics. For example, we could choose the point defined by $v_{CE} = -6$ V and $i_C = 2.5$ mA at which we find $i_B = 50\ \mu A$. Then we have $\beta = i_C / i_B = 50$. (For many transistors the value found for β depends slightly on the point selected.)

E13.8 (a) Writing a KVL equation around the input loop we have the equation for the input load lines: $0.8 - v_{\text{in}}(t) - 8000 i_B + v_{BE} = 0$ The load lines are shown:

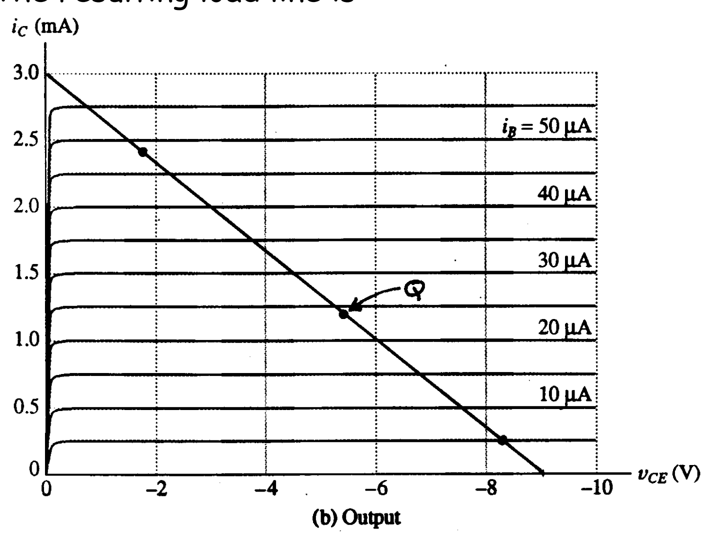


(a) Input

Then we write a KCL equation for the output circuit:

$$9 + 3000 i_C = v_{CE}$$

The resulting load line is:



(b) Output

From these load lines we find

$$I_{B\max} \cong 48\ \mu A,\ I_{BQ} \cong 24\ \mu A,\ I_{B\min} \cong 5\ \mu A,$$
$$V_{CE\max} \cong -1.8\text{ V}, V_{CEQ} \cong -5.3\text{ V}, V_{CE\min} \cong -8.3\text{ V}$$

(b) Inspecting the load lines, we see that the maximum of v_{in} corresponds to $I_{B\min}$ which in turn corresponds to $V_{CE\min}$. Because the maximum of v_{in} corresponds to minimum V_{CE}, the amplifier is inverting. This may be a little confusing because V_{CE} takes on negative values, so the minimum value has the largest magnitude.

E13.9 (a) Cutoff because we have $V_{BE} < 0.5$ V and $V_{BC} = V_{BE} - V_{CE} = -4.5$ V which is less than 0.5 V.

(b) Saturation because we have $I_C < \beta I_B$.

(c) Active because we have $I_B > 0$ and $V_{CE} > 0.2$ V.

E13.10 (a) In this case ($\beta = 50$) the BJT operates in the active region. Thus the equivalent circuit is shown in Figure 13.18d. We have

$$I_B = \frac{V_{CC} - 0.7}{R_B} = 71.5\ \mu A \qquad I_C = \beta I_B = 3.575\text{ mA}$$

$$V_{CE} = V_{CC} - R_C I_C = 11.43\text{ V}$$

Because we have $V_{CE} > 0.2$, we are justified in assuming that the transistor operates in the active region.

(b) In this case ($\beta = 250$), the BJT operates in the saturation region. Thus the equivalent circuit is shown in Figure 13.18c. We have

$$V_{CE} = 0.2\text{ V} \quad I_B = \frac{V_{CC} - 0.7}{R_B} = 71.5\ \mu A \quad I_C = \frac{V_{CC} - 0.2}{R_C} = 14.8\text{ mA}$$

Because we have $\beta I_B > I_C$, we are justified in assuming that the transistor operates in the saturation region.

E13.11 For the operating point to be in the middle of the load line, we want $V_{CE} = V_{CC}/2 = 10$ V and $I_C = \dfrac{V_{CC} - V_{CE}}{R_C} = 2$ mA. Then we have

(a) $I_B = I_C / \beta = 20\ \mu A \quad R_B = \dfrac{V_{CC} - 0.7}{I_B} = 965\text{ k}\Omega$

(b) $I_B = I_C / \beta = 6.667\ \mu A \quad R_B = \dfrac{V_{CC} - 0.7}{I_B} = 2.985\text{ M}\Omega$

E13.12 Notice that a *pnp* BJT appears in this circuit.

(a) For $\beta = 50$, it turns out that the BJT operates in the active region.

$$I_B = \frac{20 - 0.7}{R_B} = 19.3\ \mu A \qquad I_C = \beta I_B = 0.965\ \text{mA}$$

$$V_{CE} = R_C I_C - 20 = -10.35\ \text{V}$$

(b) For $\beta = 250$, it turns out that the BJT operates in the saturation region.

$$V_{CE} = -0.2\ \text{V} \quad I_B = \frac{20 - 0.7}{R_B} = 19.3\ \mu A \quad I_C = \frac{20 - 0.2}{R_C} = 1.98\ \text{mA}$$

Because we have $\beta I_B > I_C$, we are assured that the transistor operates in the active region.

E13.13

$$I_B = \frac{V_B - V_{BE}}{R_B + (\beta + 1)R_E}$$

$$V_{CE} = V_{CC} - R_C I_C - R_E(I_C + I_B)$$

β	(μA)	I_C (mA)	V_{CE} (V)
100	32.01	3.201	8.566
300	12.86	3.858	7.271

For the larger values of R_1 and R_2 used in this Exercise, the ratio of the collector currents for the two values of β is 1.205, whereas for the smaller values of R_1 and R_2 used in Example 13.7, the ratio of the collector currents for the two values of β is 1.0213. In general in the four-resistor bias network smaller values for R_1 and R_2 lead to more nearly constant collector currents with changes in β.

$I_C = \beta I_B$

E13.14

$$R_B = \frac{1}{1/R_1 + 1/R_2} = 3.333\ \text{k}\Omega$$

$$I_{BQ} = \frac{V_B - V_{BE}}{R_B + (\beta + 1)R_E} = 14.13\ \mu A \qquad I_{CQ} = \beta I_{BQ} = 4.239\ \text{mA}$$

$$r_\pi = \frac{\beta V_T}{I_{CQ}} = \frac{300(26\ \text{mV})}{4.238\ \text{mA}} = 1840\ \Omega$$

$$R_L' = \frac{1}{1/R_L + 1/R_C} = 666.7\ \Omega \qquad A_v = -\frac{\beta R_L'}{r_\pi} = -108.7$$

$$A_{voc} = \frac{R_L \beta}{r_\pi} = -163.0 \qquad Z_{in} = \frac{1}{1/R_1 + 1/R_2 + 1/r_\pi} = 1186\ \Omega$$

$$A_i = A_v \frac{Z_{in}}{R_L} = -64.43 \qquad G = A_v A_i = 7004$$

$$Z_o = R_C = 1\ \mathrm{k\Omega}$$

$$v_o = A_v v_{\mathrm{in}} = A_v v_s \frac{Z_{\mathrm{in}}}{Z_{\mathrm{in}} + R_s} = -76.46 \sin(\omega t)$$

E13.15 First, we determine the bias point:

$$R_B = \frac{1}{1/R_1 + 1/R_2} = 50.00\ \mathrm{k\Omega} \qquad V_B = V_{CC} \frac{R_2}{R_1 + R_2} = 10\ \mathrm{V}$$

$$I_{BQ} = \frac{V_B - V_{BE}}{R_B + (\beta + 1) R_E} = 14.26\ \mu A \qquad I_{CQ} = \beta I_{BQ} = 4.279\ \mathrm{mA}$$

Now we can compute and the ac performance.

$$r_\pi = \frac{\beta V_T}{I_{CQ}} = \frac{300(26\ \mathrm{mV})}{4.279\ \mathrm{mA}} = 1823\ \Omega \qquad R_L' = \frac{1}{1/R_L + 1/R_E} = 666.7\ \Omega$$

$$A_v = \frac{R_L'(\beta + 1)}{r_\pi + (\beta + 1) R_L'} = 0.9910 \qquad A_{voc} = \frac{R_E(\beta + 1)}{r_\pi + (\beta + 1) R_E} = 0.9970$$

$$Z_{in} = \frac{1}{1/R_B + 1/[r_\pi + (\beta + 1) R_L']} = 40.10\ \mathrm{k\Omega} \qquad A_i = A_v \frac{Z_{in}}{R_L} = 39.74$$

$$G = A_v A_i = 39.38 \qquad R_s' = \frac{1}{1/R_B + 1/R_s} = 8.333\ \mathrm{k\Omega}$$

$$Z_o = \frac{1}{\dfrac{(\beta + 1)}{R_s' + r_\pi} + \dfrac{1}{R_E}} = 33.18\ \Omega$$

Problems

P13.1 To forward bias a *pn* junction, the *p*-side of the junction should be connected to the positive voltage. In normal operation of a BJT, the emitter-base junction is forward biased and the collector-base junction is reverse biased.

P13.2 The emitter current is given by:

$$i_E = I_{ES}\left[\exp\left(\frac{v_{BE}}{V_T}\right) - 1\right]$$

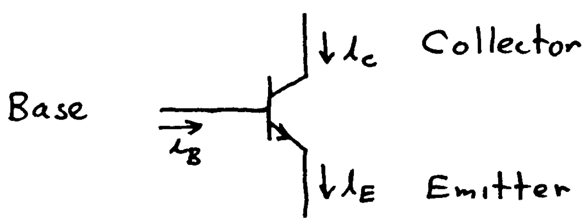
P13.3 For a BJT, the parameters are defined as:

$$\alpha = \frac{i_C}{i_E}$$

$$\beta = \frac{i_C}{i_B} = \frac{\alpha}{1-\alpha}$$

It is assumed that the base-emitter junction is forward biased and that the collector-base junction is reverse biased.

P13.4



P13.5 The sketch should resemble Figure 13.1a in the book. In normal operation, current flows into the base, into the collector, and out of the emitter.

P13.6*

$$i_E = i_C + i_B$$
$$= 9 + 50 = 59 \text{ mA}$$
$$\alpha = \frac{i_C}{i_E} = \frac{9}{59} = 0.084$$
$$\beta = \frac{i_C}{i_B} = \frac{9}{50} = 0.18$$

P13.7* The emitter current is given by the Shockley equation:

$$i_E = I_{ES}\left[\exp\left(\frac{v_{BE}}{V_T}\right)-1\right]$$

In the conduction mode, $v_{BE} >> v_T$

Thus the equation can be approximated to

$$i_E \cong I_{ES}\exp\left(\frac{v_{BE}}{V_T}\right)$$

Taking log on both sides, we obtain

$$v_{BE} = v_T \ln\left(\frac{I_E}{I_{ES}}\right)$$

$$= 26\ln(15\times 10^{-3}/10^{-12})\text{ mV}$$

$$= 0.6092\text{ V}$$

Now $V_{CE} = 10$ V

Thus, $V_{BC} = V_{BE} - V_{CE} = 0.6092 - 10 = -9.3908$ V

$I_B = 0.5$ mA

$I_E = 15$ mA

$I_C = I_E - I_B = 14.5$ mA

$\alpha = I_C/I_E = 14.5/15 = 0.9667$

$\beta = I_C/I_B = 14.5/0.5 = 29$

P13.8

$$i_B = i_E - i_C = 0.5\text{ mA}$$

$$\alpha = \frac{i_C}{i_E} = \frac{10}{10.5} = 0.9524$$

$$\beta = \frac{i_C}{i_B} = \frac{10}{0.5} = 20$$

P13.9 From Equation 13.9, we have

$$\beta = \frac{\alpha}{1-\alpha}$$

$\alpha = 0.986$ (given)

Thus $\beta = 70.8386$

P13.10

$$i_E = I_{ES}\left[\exp\left(\frac{v_{BE}}{V_T}\right)-1\right]$$

$$i_C = \beta i_B = 200\times(10\ \mu A) = 2\text{ mA}$$

$$i_E = i_C + i_B = 2.01\text{ mA}$$

P13.11 From the Shockley equation,

$I_{E2} = I_{E1}\exp((V_{BE2} - V_{BE1})/V_T)$

Thus,
$V_{BE2} = V_{BE1} + V_T \ln(I_{E2}/I_{E1})$

For $I_{E2} = 1.5$ mA and the given values of the rest
$V_{BE2} = 0.7 + 0.026\ln(1.5/15)$
$= 0.6401$ V

For $I_{E2} = 0.015$ mA and the given values of the rest ,
$V_{BE2} = 0.7 + 0.026\ln(0.015/15)$
$= 0.5204$ V

P13.12 Solving the Shockley equation, we have

$$I_{ES} = \frac{i_E}{\left[\exp\left(\frac{v_{BE}}{V_T}\right) - 1\right]}$$

Also, we have $V_T = kT / q$

Substituting values for 300 K, we obtain

$$V_T = 25.875 \text{ mV}$$

$$I_{ES} = 8.500 \times 10^{-13} \text{ A}$$

At 310 K, we have

$$V_T = 26.74 \text{ mV}$$

$$I_{ES} = 3.794 \times 10^{-12} \text{ A}$$

For the 10 K increase in temperature, we find that I_{ES} has increased by a factor of 4.464.

P13.13 From the circuit, we can write:

$$i_E = \frac{8 - 0.8}{15\text{k}\Omega} = 0.48 \text{ mA} \qquad i_B = \frac{8 - 0.7}{800\text{k}\Omega} = 9.125\ \mu A$$

$$i_C = i_E - i_B = 0.471 \text{ mA}$$

Then, we have $\beta = \frac{i_C}{i_B} = 51.6027$

P13.14* Since $V_{BE1} = V_{BE2}$, adding the Shockley equation for both the transistors we obtain

$$i_{Eeq} = i_{E1} + i_{E2} = \left(I_{ES1} + I_{ES2}\right)\left[\exp\left(\frac{v_{BE}}{V_T}\right) - 1\right]$$

Thus, we have:

$$I_{ESeq} = I_{ES1} + I_{ES2} = 2 \times 10^{-13} \text{ A}$$

$$\beta_{eq} = \frac{i_{C1} + i_{C2}}{i_{B1} + i_{B2}} = \frac{\beta_1 i_{B1} + \beta_2 i_{B2}}{i_{B1} + i_{B2}}$$

$$= \beta_1 + \beta_2$$

$$= 150 + 50 = 200$$

P13.15 Writing a current equation at the collector of Q_1, we have:

$$i_{C1} + i_{B1} + i_{B2} = 1\text{ mA}$$

However, $i_{B2} = i_{B1}$ and $i_{C1} = \beta i_{B1}$, so we have

$$\beta i_{B1} + i_{B1} + i_{B1} = 1\text{ mA}$$

$$i_{B1} = 9.804\ \mu A$$

$$i_{B2} = 9.804\ \mu A$$

$$i_{C1} = i_{C2} = \beta i_{B1} = 0.9804\text{ mA}$$

$$i_{E1} = i_{E2} = i_{B1} + i_{C1} = 0.9902\text{ mA}$$

As in the solution to Problem P13.7, we have:

$$v_{BE1} = v_{BE2} \cong V_T \ln\left(\frac{i_E}{I_{ES}}\right) = 0.026 \ln\left(\frac{0.9902 \times 10^{-3}}{10^{-14}}\right) = 0.6583\text{ V}$$

P13.16 We have $i_{Ceq} = i_{C1} + i_{C2} = \beta_1 i_{B1} + \beta_2 i_{E1} = \beta_1 i_{B1} + \beta_2(\beta_1 + 1) i_{B1}$. Then we can write

$$\beta_{eq} = \frac{i_{Ceq}}{i_{Beq}} = \frac{i_{C1} + i_{C2}}{i_{B1}} = \frac{\beta_1 i_{B1} + \beta_2(\beta_1 + 1) i_{B1}}{i_{B1}} = \beta_1 + \beta_2(\beta_1 + 1)$$

P13.17 The Shockley equation states:

$$i_E = I_{ES}\left[\exp\left(\frac{v_{BE}}{V_T}\right) - 1\right]$$

Thus, i_E is proportional to I_{ES}, and we can write:

$$i_{E2} = 10 i_{E1}$$

Because α and β are the same for both transistors, we also have:

$$i_{B2} = 10 i_{B1} \text{ and } i_{C2} = 10 i_{C1}$$

Writing a current equation at the collector of Q_1, we have

$$i_{C1} + i_{B1} + i_{B2} = 1\text{ mA}$$

$$\beta i_{B1} + i_{B1} + 10 i_{B1} = 1 \text{ mA}$$

$$i_{B1} = 9.009\ \mu A$$

$$i_{B2} = 10 i_{B1} = 90.09\ \mu A$$

$$i_{C1} = \beta i_{B1} = 0.9009 \text{ mA}$$

$$i_{C2} = \beta i_{B2} = 9.009 \text{ mA}$$

$$i_{E1} = i_{B1} + i_{C1} = 0.9099 \text{ mA}$$

$$i_{E2} = 10 i_{E1} = 9.099 \text{ mA}$$

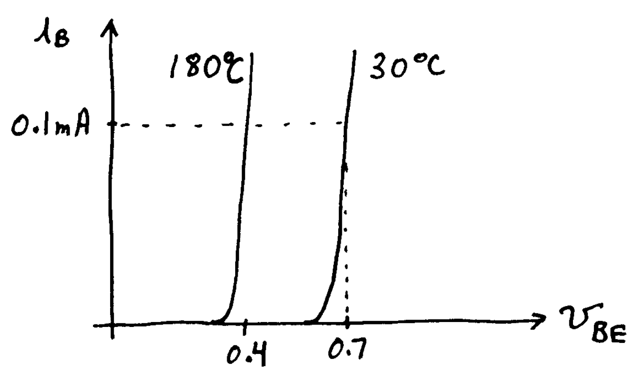
As in the solution to Problem 13.11, we have

$$v_{BE2} = v_{BE1} \cong V_T \ln\left(\frac{i_{E1}}{I_{ES1}}\right) = 0.026 \ln\left(\frac{0.9099 \times 10^{-3}}{10^{-14}}\right) = 0.6561 \text{ V}$$

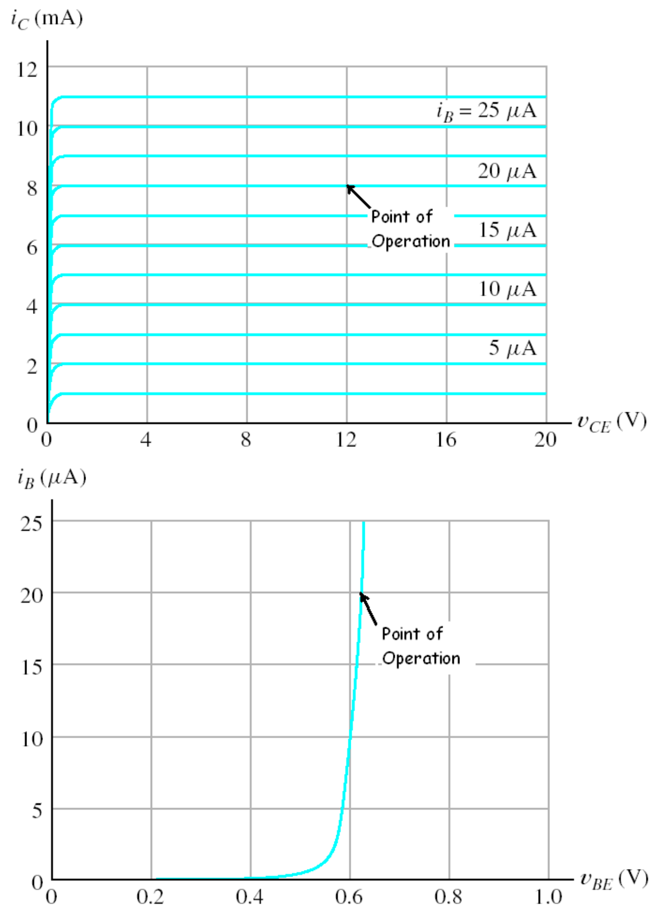
P13.18* We select a point on the output characteristics in the active region and compute $\beta = i_C / i_B$. For example on the curve for $i_B = 20\ \mu A$, we have $i_C = 8 \text{ mA}$ in the active region. Thus, $\beta = (8 \text{ mA})/(20\ \ \mu A) = 400$. Then, we have $\alpha = \beta/(\beta + 1) = 0.9975$.

P13.19* At $180^\circ C$ and $i_B = 0.1 \text{ mA}$, the base-to-emitter voltage is approximately:

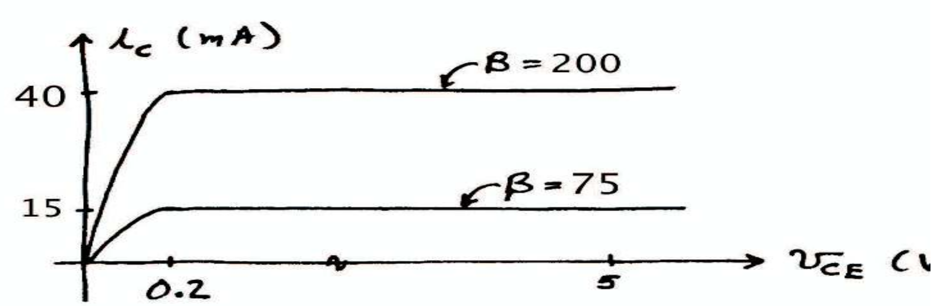
$$v_{BE} = 0.7 - 0.002(180 - 30) = 0.4 \text{ V}$$



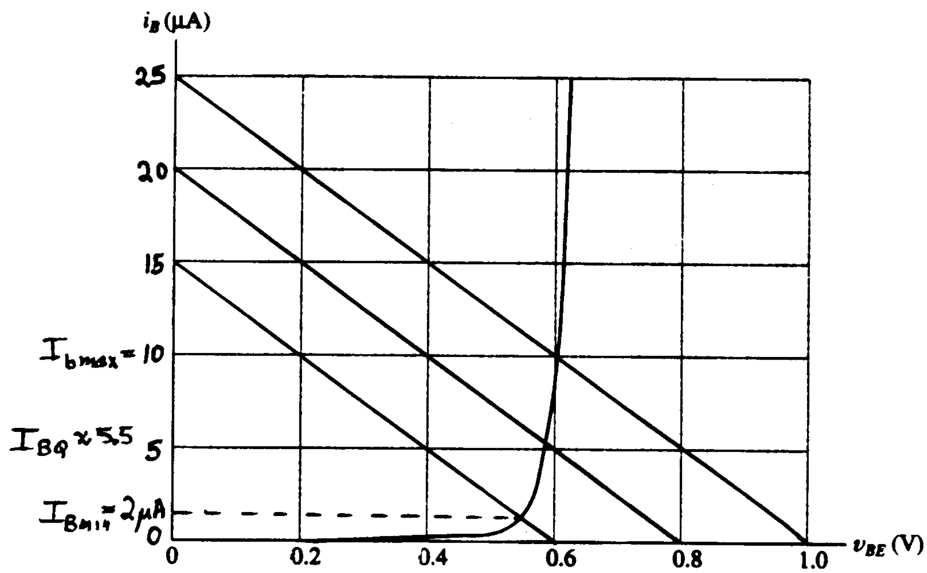
P13.20

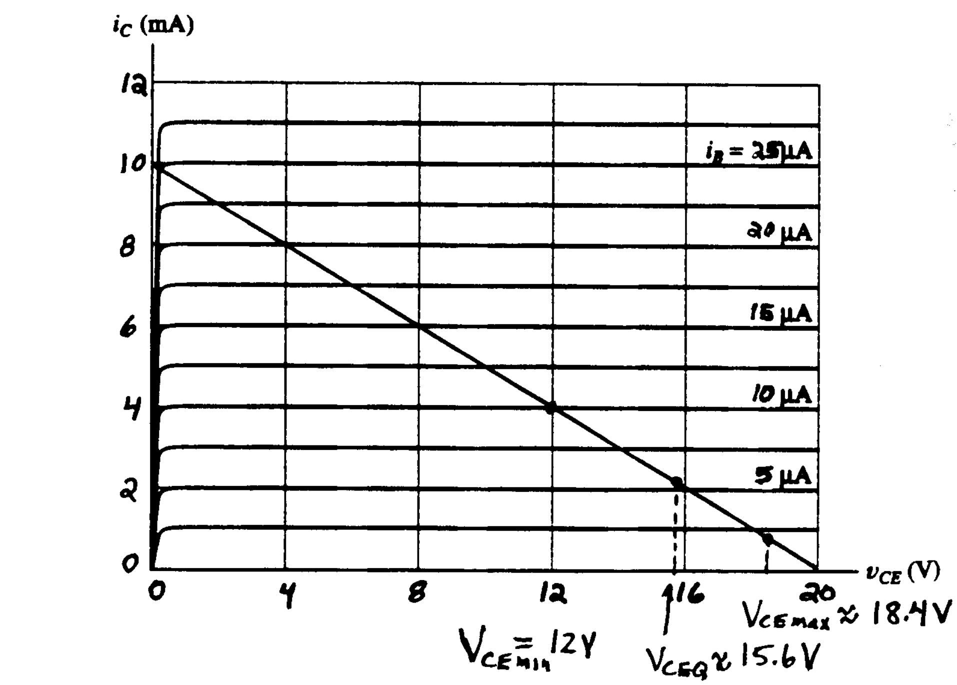


P13.21 In the active region (which is for v_{CE} <0.2 V), we have $i_C < \beta i_B$.
The sketches are:



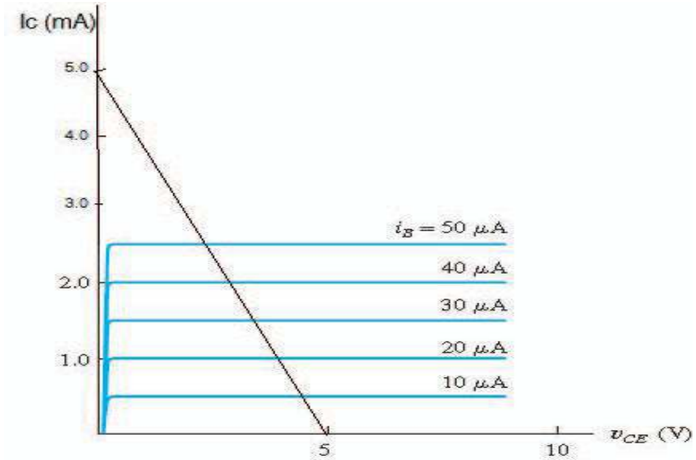
P13.22* Following the approach of Example 13.2, we construct the load lines shown. We estimate that $V_{CE\max} = 18.4\text{ V}, V_{CEQ} = 15.6\text{ V}$, and $V_{CE\min} = 12\text{ V}$. Thus, the voltage gain magnitude is $|A_v| = (18.4 - 12)/0.4 = 16$





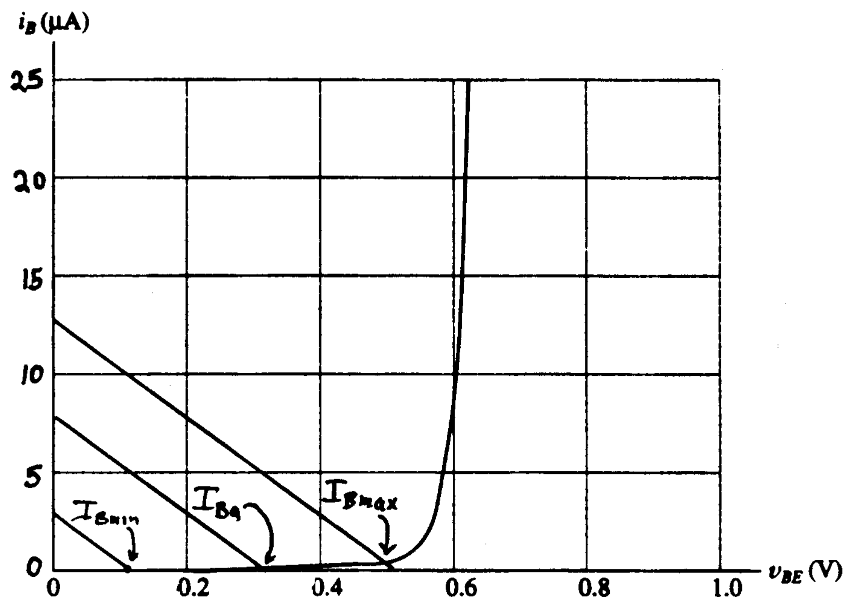
P13.23 Distortion occurs in BJT amplifiers because the input characteristic of the BJT is curved (rather than straight) and because the output characteristic curves are not uniformly spaced.

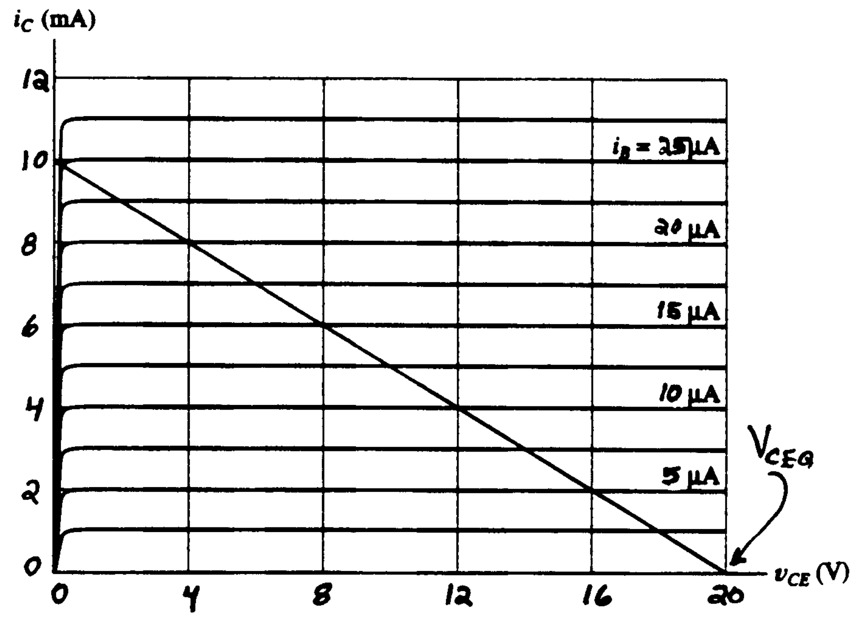
P13.24 The Load line equation is given by $I_C = (5 - V_{CE})1000$ A. Thus it can be drawn as:



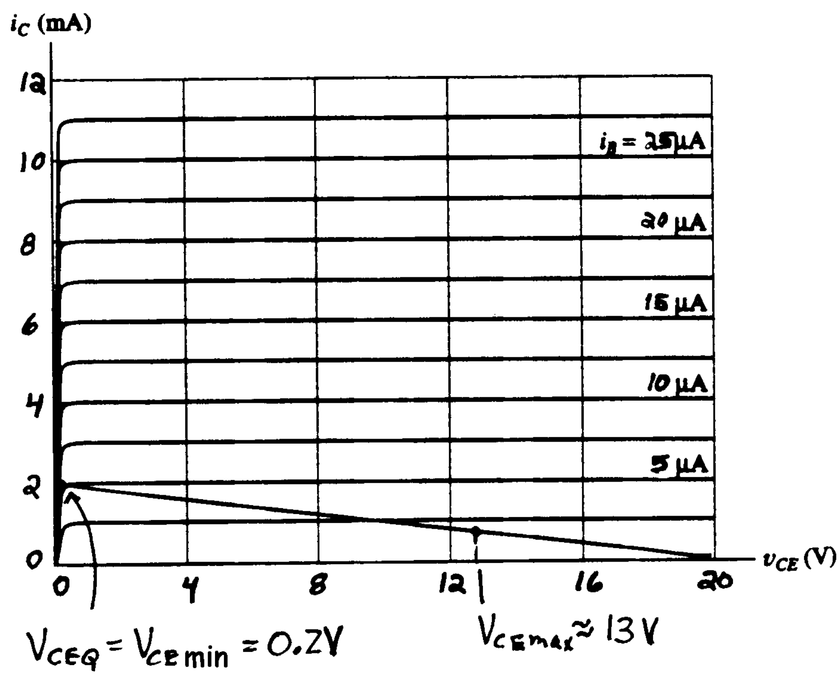
P13.25 Following the approach of Example 13.2, we construct the load lines shown. From the input load line, we have $I_{B\min} \cong I_{BQ} \cong I_{B\max} \cong 0$. Thus, we have $V_{CE\max} \cong V_{CE\min} \cong 20$. Thus, the voltage gain magnitude is:

$$A_v = (20 - 20)/0.4 = 0$$

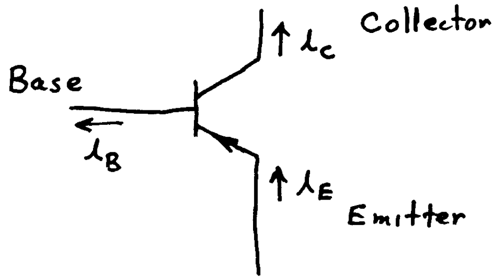




P13.26 The input load line is the same as in Problem P13.22. Thus, we again have $I_{B\min} = 2\ \mu A$, $I_{BQ} = 5.5\ \mu A$ and $I_{B\max} = 10\ \mu A$. The output load line is shown below. Because $V_{CEQ} = V_{CE\min}$, we know that the output waveform is severely distorted. It is not appropriate to compute voltage gain for such a severely distorted output waveform.

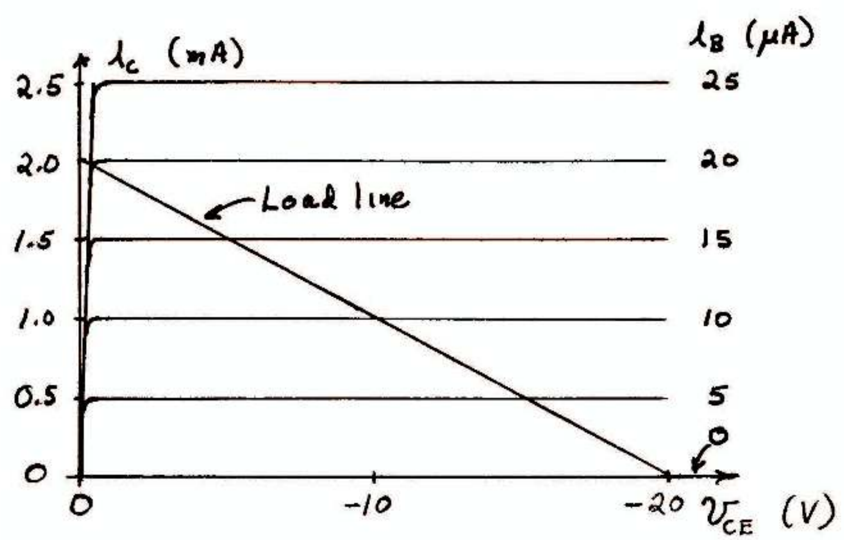


P13.27



P13.28* Since $\alpha = 0.989$, $\beta = 89.91$

(a) and (b) The characteristics and the load line are:



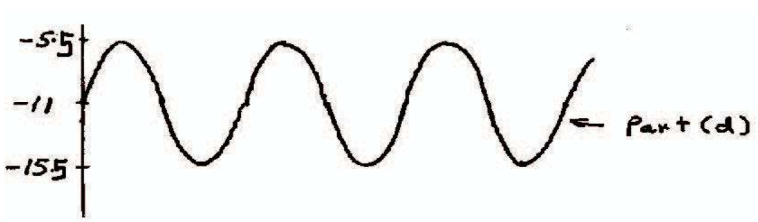
(c) $i_{Bmax} = 15 + 5 = 20\ \mu A$ $\qquad$ $i_{Bmin} = 15 - 5 = 10\ \mu A$

$\beta = 89.91$

Thus $i_{Cmax} = 89.91 \times 20 = 1.7982$ mA and $i_{Cmin} = 89.91 \times 10 = 0.8991$ mA

$i_{CQ} = i_B \times \beta = 15 \times 89.91 = 1.3486$ mA

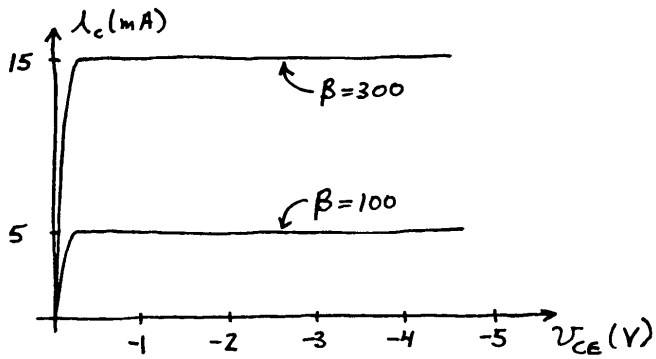
(d)



(e) $i_{Cmax} = 89.91 \times 30 = 2.6971$ mA and $i_{Cmin} = 89.91 \times 20 = 1.7982$ mA

$i_{CQ} = i_B \times \beta$
$= 25 \times 89.91$
$= 2.2478$ mA

P13.29*



P13.30 We can write

$$\beta_{eq} = \frac{i_{Ceq}}{i_{Beq}} = \frac{i_{E2}}{i_{B1}} = \frac{(\beta_2+1)i_{C1}}{i_{B1}} = \frac{(\beta_2+1)\beta_1 i_{B1}}{i_{B1}} = \beta_1(\beta_2+1)$$

P13.31 $I_E = I_C/\alpha = 0.98/0.95 = 1.0316$ mA

$$\beta = \frac{\alpha}{1-\alpha} = 0.95/0.05 = 19$$

P13.32 Since $V_T = kT/q$, V_{T1} (at 150°C) = 36.4514 mV

V_{T2} (at 30°C) = 26.1105 mV

From the Shockley equation,
$I_E = I_{ES}\exp[(V_{BE}/V_T) - 1] \approx I_{ES}\exp(V_{BE}/V_T)$

Since I_{ES} is constant w.r.t temperature,

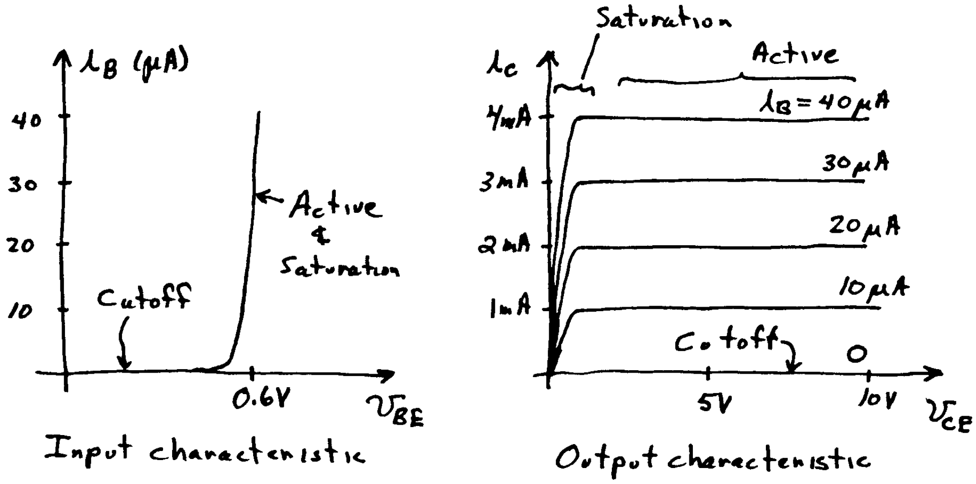
$$\frac{I_{E2}}{I_{E1}} = \frac{\exp(V_{BE2}/V_{T2})}{\exp(V_{BE1}/V_{T1})} = \exp\left[\frac{V_{BE2}}{V_{T2}} - \frac{V_{BE1}}{V_{T1}}\right]$$

Taking log on both sides,

$$V_{BE2} = V_{T2}\left[\frac{V_{BE1}}{V_{T1}} + \ln\frac{I_{E2}}{I_{E1}}\right]$$

$V_{BE2} = 26.1105((0.5/36.4514) + \ln(1.5/1.5)) = 0.3582$ mV

P13.33



P13.34 See Figure 13.16 in the text.

P13.35* In the active region, the base-collector junction is reverse biased and the base-emitter junction is forward biased.

In the saturation region, both junctions are forward biased.

In the cutoff region, both junctions are reverse biased. (Actually, cutoff applies for slight forward bias of the base-emitter junction as well, provided that the base current is negligible.)

P13.36 (a) $I_B = I_E - I_C = 0.5$ mA
The transistor is in saturation because we have $I_C < \beta I_B$
Thus, we have $I_C = 1$ mA, $I_E = 1.5$ mA, and $V_{CE} = 0.2$ V.

(b) The transistor is in the active region because we have $V_{CE} > 0.2$ and $I_B > 0$. Thus, we have
$I_C = \beta I_B = 10$ mA, $I_E = 10.1$ mA, and $V_{CE} = 5$ V.

(c) This *pnp* transistor is in the active region because we have $V_{CE} < -0.2$ and $I_B > 0$. Thus, we have
$I_C = \beta I_B = 2$ mA, $I_E = 2.02$ mA, and $V_{CE} = -4.6$ V.

(d) This pnp transistor is in cutoff because both junctions are reverse biased. Thus, we have $I_C = I_B = I_E = 0$, and $V_{CE} = -3$ V.

P13.37 For the Darlington pair, we have $V_{BEeq} = V_{BE1} + V_{BE2} = 1.4$ V

For the Sziklai pair, we have $V_{BEeq} = V_{BE1} = 0.7$V.

P13.38 (a) Active Region

(b) Saturation region (because I_C is less than βI_B).

(c) Cutoff region ($I_B > 0$ and $V_{CE} < 0.2$V)

P13.39 (a) Active region.

(b) Cutoff region.

(c) Cutoff region (because $I_B \cong 0$ for $V_{BE} = 0.4$ V at room temperature).

(d) Saturation region (because I_C is less than βI_B).

P13.40 See Figure 13.16 in the text.

P13.41* 1. Assume operation in saturation, cutoff, or active region.

2. Use the corresponding equivalent circuit to solve for currents and voltages.

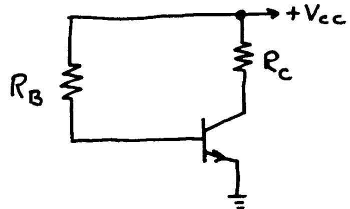
3. Check to see if the results are consistent with the assumption made in step 1. If so, the circuit is solved. If not, repeat with a different assumption.

P13.42* The results are given in the table:

Circuit	β	Region of operation	I_C (mA)	V_{CE} (volts)
(a)	100	active	1.93	10.9
(a)	300	saturation	4.21	0.2
(b)	100	active	1.47	5.00
(b)	300	saturation	2.18	0.2
(c)	100	cutoff	0	15
(c)	300	cutoff	0	15
(d)	100	active	6.5	8.5
(d)	300	saturation	14.8	0.2

P13.43 See Figure 13.22 in the text.

P13.44 The fixed base bias circuit is:



It is unsuitable for mass production because the value of β varies by typically a factor of 3:1 between BJTs of the same type. Consequently, the bias point varies too much from one circuit to another.

P13.45* The BJT operates in the active region. We can write the voltage equation:

$$5 = R_B I_B + 0.7 + R_E I_E$$

However, we can substitute using the relations:

$$I_E = \frac{\beta+1}{\beta} I_C \text{ and } I_B = \frac{I_C}{\beta}$$

Thus, we have:

$$4.3 = R_B \frac{I_C}{\beta} + R_E \frac{\beta+1}{\beta} I_C$$

For $\beta = 100$, we require $I_C = 4$ mA. Furthermore, for $\beta = 300$, we require $I_C = 5$ mA. Thus, we have two equations:

$$4.3 = R_B\left(0.04 \times 10^{-3}\right) + R_E\left(4.04 \times 10^{-3}\right)$$

$$4.3 = R_B\left(0.01667 \times 10^{-3}\right) + R_E\left(5.017 \times 10^{-3}\right)$$

Solving, we find:

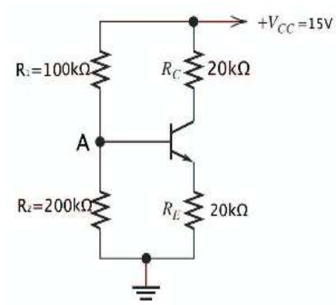
$$R_B = 31.5\ \text{k}\Omega \text{ and } R_E = 753\ \Omega$$

P13.46 The results are given in the table:

Circuit	β	Region of operation	I (mA)	V (volts)
(a)	100	active	2.38	5.25
(a)	300	saturation	4.45	9.80
(b)	100	cutoff	0	10
(b)	300	cutoff	0	10
(c)	100	active	4.26	-10.74
(c)	300	active	4.29	-10.71

(d)	100	Q_1 active Q_2 active	9.53	9.53
(d)	300	Q_1 active Q_2 saturated	14.8	14.8

P13.47



According to the voltage divider rule (assuming I_B to be small),

$$V_A = (200\text{k}\Omega/100\text{k}\Omega + 200\text{k}\Omega) \times 15 = 10\text{V}$$

Applying voltage equation in the A-B-E-GND loop We can write

$$V_A + V_{BE} - (20\text{k}\Omega)I_E = 0$$

Since β=150, α=0.9934, also it is given that V_{BE}=0.7V, Thus

$$10 - 0.7 - [(20\text{k}\Omega)(I_{CQ}/0.9934)] = 0$$

$$\Rightarrow \quad I_{CQ} = 0.4619 \text{ mA}$$

Now V_{CEQ} can be calculated as

$$V_{CEQ} = [V_{CC} - (20\text{k}\Omega)(I_{CQ})] - [(I_{CQ}/\alpha)(20\text{k}\Omega)$$

$$= [15 - (20\text{k}\Omega)(0.4619\text{mA})] - [(0.4619\text{mA}/0.9934)(20\text{k}\Omega)]$$

$$= -3.5374\text{V}$$

Thus the transistor is in the reverse active region.

P13.48* From Equations 13.20 through 13.23, we have:

$$I_C = \beta I_B = \beta \frac{V_B - V_{BE}}{R_B + (\beta + 1)R_E}$$

$$R_B = \frac{1}{1/R_1 + 1/R_2}$$

$$V_B = V_{CC} \frac{R_2}{R_1 + R_2}$$

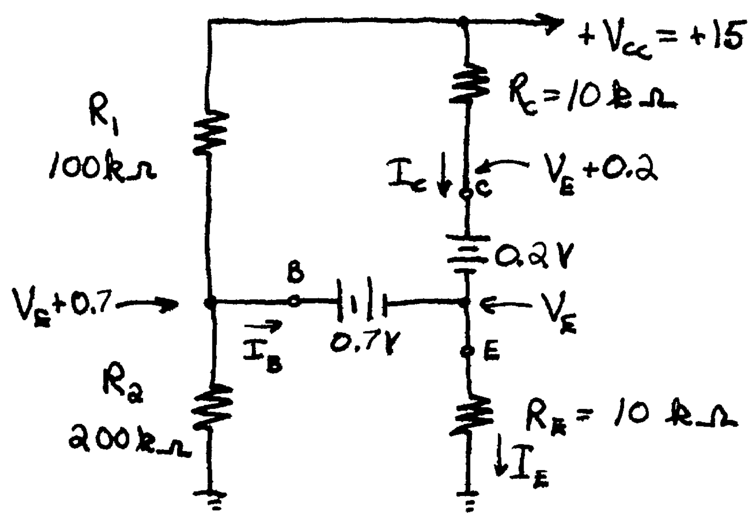
Maximum I_C occurs for $\beta = \beta_{max} = 200$, $R_E = R_{E\min} = 0.95 \times 4.7 \text{ k}\Omega = 4465\ \Omega$, $R_1 = R_{1\min} = 0.95 \times 100 \text{ k}\Omega = 95 \text{ k}\Omega$, and $R_2 = R_{2\max} = 1.05 \times 47 \text{ k}\Omega = 49.35 \text{ k}\Omega$. With these values, we calculate:

$R_B = 32.48 \text{ k}\Omega \qquad V_B = 5.128 \text{ V} \qquad I_{C\max} = 0.952 \text{ mA}$

Minimum I_C occurs for $\beta = \beta_{\min} = 50$, $R_E = R_{E\max} = 1.05 \times 4.7\ \text{k}\Omega =$ $4.935\ \text{k}\Omega$, $R_1 = R_{1\max} = 1.05 \times 100\ \text{k}\Omega = 105\ \text{k}\Omega$, and $R_2 = R_{2\min} =$ $0.95 \times 47\ \text{k}\Omega = 44.65\ \text{k}\Omega$. With these values, we calculate:

$$R_B = 31.33\ \text{k}\Omega \qquad V_B = 4.475\ \text{V} \qquad I_{C\min} = 0.6667\ \text{mA}$$

P13.49 It turns out that for the values given the BJT operates in the saturation region and the equivalent circuit is:



Enclosing the transistor in a closed surface (supernode) and writing a KCL equation, we have

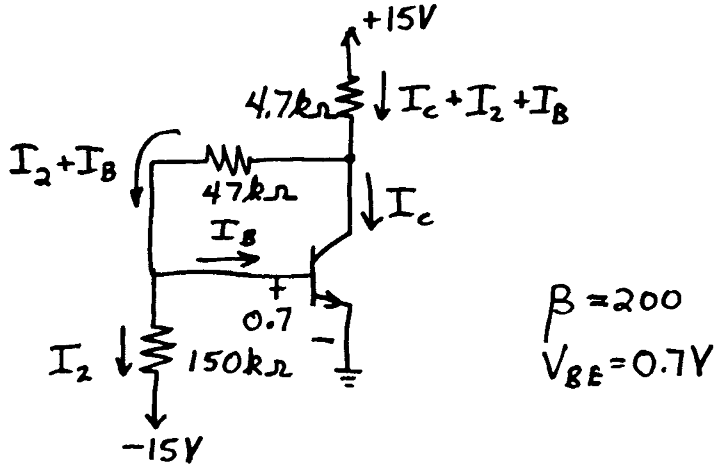
$$\frac{V_E}{R_E} + \frac{V_E + 0.2 - 15}{R_C} + \frac{V_E + 0.7 - 15}{R_1} + \frac{V_E + 0.7}{R_2} = 0$$

Substituting the resistance values and solving we obtain $V_E = 7.533$ V.

Then we have $I_C = \dfrac{15 - (V_E + 0.2)}{R_C} = 0.7267$ mA and

$I_B = I_E - I_C = 0.0266$ mA. As a check we note that we have $\beta I_B > I_C$ as required for operation in the saturation region.

P13.50



$$I_2 = \frac{15 + 0.7}{150\ \text{k}\Omega} = 0.104\ \text{mA}$$

$$15 = 4700(I_C + I_2 + I_B) + 47 \times 10^3 (I_2 + I_B) + 0.7$$

Substituting $I_C = \beta I_B$ and solving for I_B, we obtain:

$$I_B = \frac{15 - 0.7 - (51700)I_2}{4700(\beta + 1) + 51700} = 8.96\ \mu A$$

$$I_C = \beta I_B = 1.79\ \text{mA}$$

$$V_{CE} = 15 - 4700(I_C + I_2 + I_B) = 6.06\ \text{V}$$

P13.51 Current across the 100 kΩ resistance = (0.7 − (−15))/100kΩ = 0.1570 mA

Since β = 300, I_B = 0.5 mA/300 = 1.6667 μA. Thus current across resistance R_1 is [{15 − 7} − {−15 + (0.1570 mA)(100 kΩ)}] = 7.3 V Thus,

R_1 = 7.3/0.1587 mA = 45.9987 kΩ

Also current across R_C is 0.1587 mA + 0.5 mA = 0.6587 mA and potential difference is 8 V, thus

R_C = 8/0.6587 mA = 12.1451 kΩ

P13.52 $I_{B1} = \dfrac{15 - V_{BE1}}{1.43 \times 10^6} = 10\ \mu A \quad I_{C1} = \beta I_{B1} = 1\ \text{mA}$

$$15 = 10^4(I_{C1} + I_{B2}) + V_{BE2} + 10^3 I_{E2} = 10^4(10^{-3} + I_{B2}) + 0.7 + 10^3(101 I_{B2})$$

$$I_{B2} = 38.74\ \mu A \quad I_{C2} = \beta I_{B2} = 3.874\ \text{mA} \quad I_{E2} = (\beta + 1) I_{B2} = 3.913\ \text{mA}$$

$$V_{C1} = 10^3 I_{E2} + V_{B2} = 4.613\ \text{V}$$

$$V_{C2} = 15 - 10^3 I_{C2} - 10^3 I_{E2} = 7.213\ \text{V}$$

P13.53 $r_\pi = \dfrac{\beta V_T}{I_{CQ}}$

P13.54 See Figure 13.26 in the text.

P13.55* We use the same approach as in Section 13.7. We can write

$$i_B(t) = 10^{-5} v_{BE}^2(t)$$

$$I_{BQ} + i_b(t) = 10^{-5}[V_{BEQ} + v_{be}(t)]^2$$

$$I_{BQ} + i_b(t) = 10^{-5} V_{BEQ}^2 + 2 \times 10^{-5} V_{BEQ} v_{be}(t) + 10^{-5} v_{be}^2(t) \qquad (1)$$

However for the Q-point, we have $I_{BQ} = 10^{-5} V_{BEQ}^2$. Therefore, the first term on each side of Equation (1) can be canceled. Furthermore, the last term on the right hand side of Equation (1) is negligible for small signals [i.e., $v_{be}(t)$ much less than V_{BEQ} at all times]. Thus, Equation (1) becomes

$$i_b(t) \cong 2 \times 10^{-5} V_{BEQ} v_{be}(t)$$

We have defined

$$r_\pi = \frac{v_{be}(t)}{i_b(t)}$$

and we have

$$r_\pi = \frac{1}{2 \times 10^{-5} V_{BEQ}} = \frac{5 \times 10^4}{\sqrt{10^5 I_{BQ}}} = \frac{5 \times 10^4}{\sqrt{10^5 I_{CQ} / 100}} = \frac{1581}{\sqrt{I_{CQ}}}$$

For $I_{CQ} = 1$ mA, we obtain $r_\pi = 50$ kΩ.

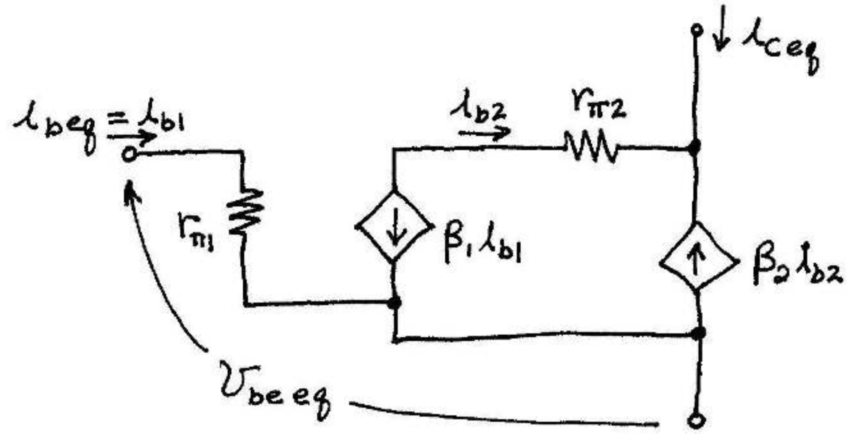
P13.56 $r_\pi = \dfrac{\beta V_T}{I_{CQ}}$

Now $V_T = 26$ mV and $\beta = 200$. Thus,

$$r_\pi = \frac{5.2}{I_{CQ}}$$

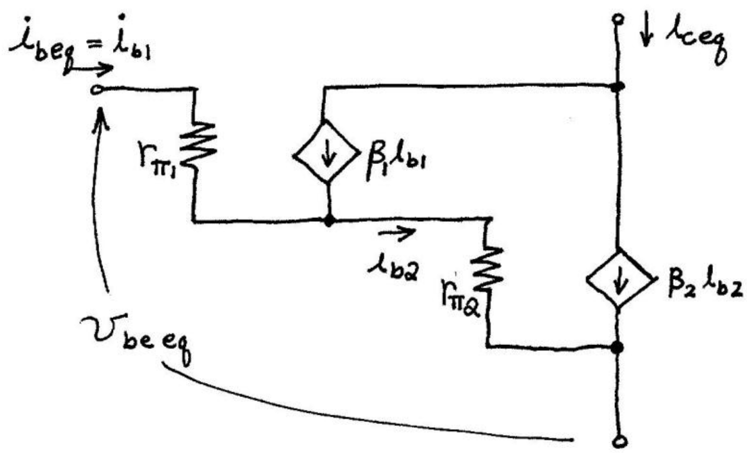
I_{CQ}	r_π
3 mA	1.73 kΩ
0.3 mA	17.3 kΩ
3 μA	1.73 MΩ

P13.57 The equivalent circuit is:



$$r_{\pi eq} = \frac{v_{beeq}}{i_{beq}} = \frac{r_{\pi 1} i_{b1}}{i_{b1}} = r_{\pi 1}$$

P13.58 The equivalent circuit is:



$$r_{\pi eq} = \frac{v_{beeq}}{i_{beq}} = \frac{r_{\pi 1} i_{b1} + r_{\pi 2}(i_{b1} + \beta_1 i_{b1})}{i_{b1}} = r_{\pi 1} + r_{\pi 2}(1 + \beta_1)$$

P13.59 See Figure 13.27a in the text.

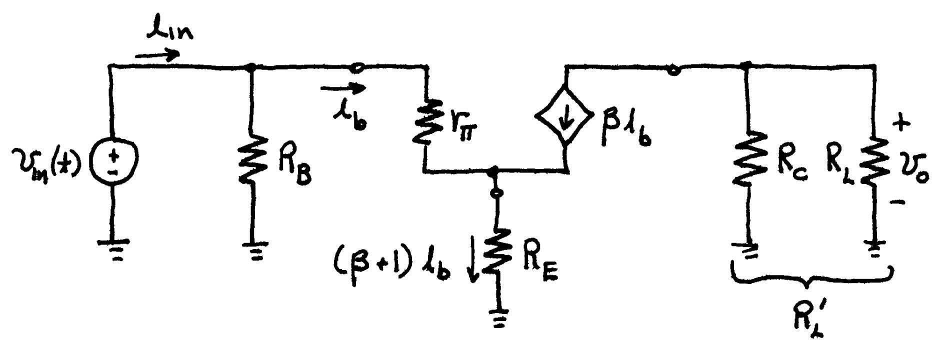
P13.60 Coupling capacitors are used to provide a path for ac signals while maintaining an open circuit for dc. In connecting the signal source or load, coupling capacitors prevent the source or load from affecting the bias point of the amplifier. When it is desired to amplify dc signals, coupling capacitors should not be used.

P13.61 The common-emitter amplifier is inverting. Both the voltage gain and current gain magnitudes are potentially greater than unity.

P13.62* The solution is similar to that of Problem P13.64. The results are:

	High impedance amplifier (Problem 13.62)	Low impedance amplifier (Problem 13.64)
I_{CQ}	0.0393 mA	3.93 mA
	66.2 kΩ	662 Ω
A_v	-75.5	-75.5
A_{voc}	-151	-151
Z_{in}	54.8 kΩ	548 Ω
A_i	-41.4	-41.4
G	3124	3124
Z_o	100 kΩ	1 kΩ

P13.63 (a) The small-signal equivalent circuit is:



in which we have defined $R_L' = \dfrac{1}{1/R_C + 1/R_L}$

(b) From the equivalent circuit we can write:

$$v_{in} = r_\pi i_b + (\beta+1)R_E i_b \qquad (1)$$

$$v_o = -\beta R_L' i_b$$

Then we have

$$A_v = \frac{v_o}{v_{in}} = \frac{\beta R_L' i_b}{r_\pi i_b + (\beta+1)R_E i_b} = \frac{-\beta R_L'}{r_\pi + (\beta+1)R_E}$$

(c) From the equivalent circuit we can write:

$$i_{in} = \frac{v_{in}}{R_B} + i_b$$

Then solving Equation (1) for i_b and substituting, we have

$$i_{in} = \frac{v_{in}}{R_B} + \frac{v_{in}}{r_\pi + (\beta + 1)R_E}$$

$$R_{in} = \frac{v_{in}}{i_{in}} = \frac{1}{1/R_B + 1/\left[r_\pi + (\beta + 1)R_E\right]}$$

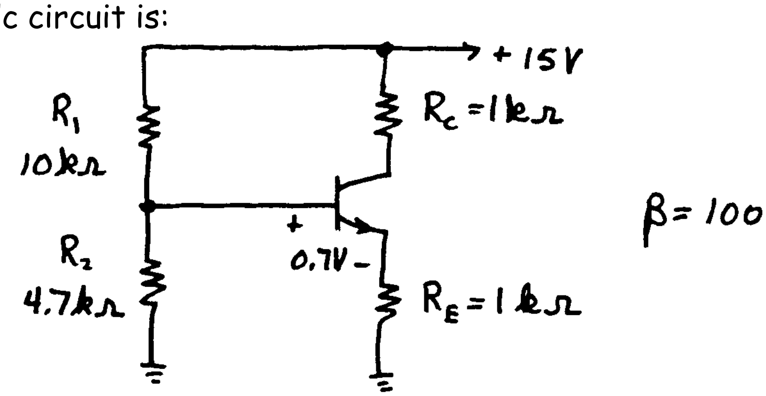
(d) Because the coupling capacitors are open circuits for dc, we can ignore the signal source and the load in the dc analysis. To attain I_{CQ} = 1 mA, we must have I_{BQ} = (1 mA)/β = 50 μA. Writing a voltage equation we have

$$V_{CC} = R_B I_{BQ} + V_{BEQ} + (\beta + 1)I_{BQ}R_E$$

Substituting values and solving, we obtain R_B = 375.9 kΩ.

(e) First, we have $r_\pi = \beta V_T / I_{CQ} = (100 \times 0.026)/0.005 = 520\ \Omega$. Then using the equations from parts (a) and (b) we determine that A_v = -9.416 and R_{in} = 10.33 kΩ.

P13.64 The dc circuit is:



The bias point calculations are:

$$R_B = \frac{1}{1/R_1 + 1/R_2} = 3.197\ \text{k}\Omega \qquad V_B = V_{CC}\frac{R_2}{R_1 + R_2} = 4.796\ \text{V}$$

$$I_{BQ} = \frac{V_B - V_{BE}}{R_B + (\beta + 1)R_E} = 39.3\ \mu\text{A} \qquad I_{CQ} = \beta I_{BQ} = 3.93\ \text{mA}$$

$$r_\pi = \frac{\beta V_T}{I_{CQ}} = \frac{100(26\text{ mV})}{3.93\text{ mA}} = 662\ \Omega$$

Then, we compute the amplifier performance:

$$R'_L = \frac{1}{1/R_L + 1/R_C} = 500\ \Omega \qquad A_v = -\frac{\beta R'_L}{r_\pi} = -75.5$$

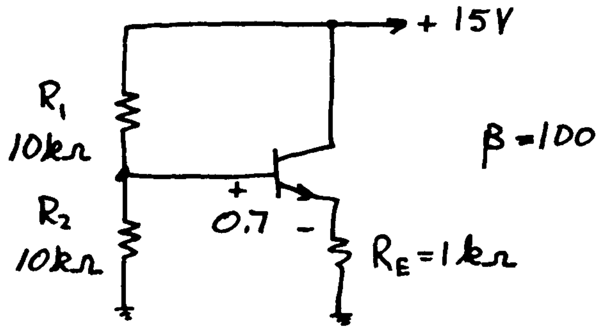
$$A_{voc} = \frac{R_C \beta}{r_\pi} = -151.0 \qquad Z_{in} = \frac{1}{1/R_1 + 1/R_2 + 1/r_\pi} = 548\ \Omega$$

$$A_i = A_v \frac{Z_{in}}{R_L} = -41.4 \qquad G = A_v A_i = 3124$$

$$Z_o = R_C = 1\text{ k}\Omega$$

P13.65 See Figure 13.30a in the text.

P13.66* The dc circuit is:



The bias point calculations are:

$$R_B = \frac{1}{1/R_1 + 1/R_2} = 5\text{ k}\Omega$$

$$V_B = V_{CC}\frac{R_2}{R_1 + R_2} = 7.5\text{ V}$$

$$I_{BQ} = \frac{V_B - V_{BE}}{R_B + (\beta+1)R_E} = 64.1\ \mu A$$

$$I_{CQ} = \beta I_{BQ} = 6.41\text{ mA}$$

Now we can compute and the ac performance.

$$r_\pi = \frac{\beta V_T}{I_{CQ}} = \frac{100(26\text{ mV})}{6.41\text{ mA}} = 405\ \Omega$$

$$R'_L = \frac{1}{1/R_L + 1/R_E} = 333\ \Omega$$

$$A_v = \frac{R_L'(\beta+1)}{r_\pi + (\beta+1)R_L'} = 0.98$$

$$A_{voc} = \frac{R_E(\beta+1)}{r_\pi + (\beta+1)R_E} = 0.996$$

$$Z_{in} = \frac{1}{1/R_B + 1/\left[r_\pi + (\beta+1)R_L'\right]} = 4.36\ \text{k}\Omega$$

$$A_i = A_v \frac{Z_{in}}{R_L} = 8.61 \qquad G = A_v A_i = 8.51$$

$$Z_o = \frac{1}{\dfrac{(\beta+1)}{R_s' + r_\pi} + \dfrac{1}{R_E}} \text{ in which } R_s' = \frac{1}{1/R_B + 1/R_s} = 833\ \Omega$$

$$Z_o = 12.1\ \Omega$$

P13.67 The voltage gain of an emitter follower is positive and less than unity in magnitude. The current gain and power gains are potentially greater than unity.

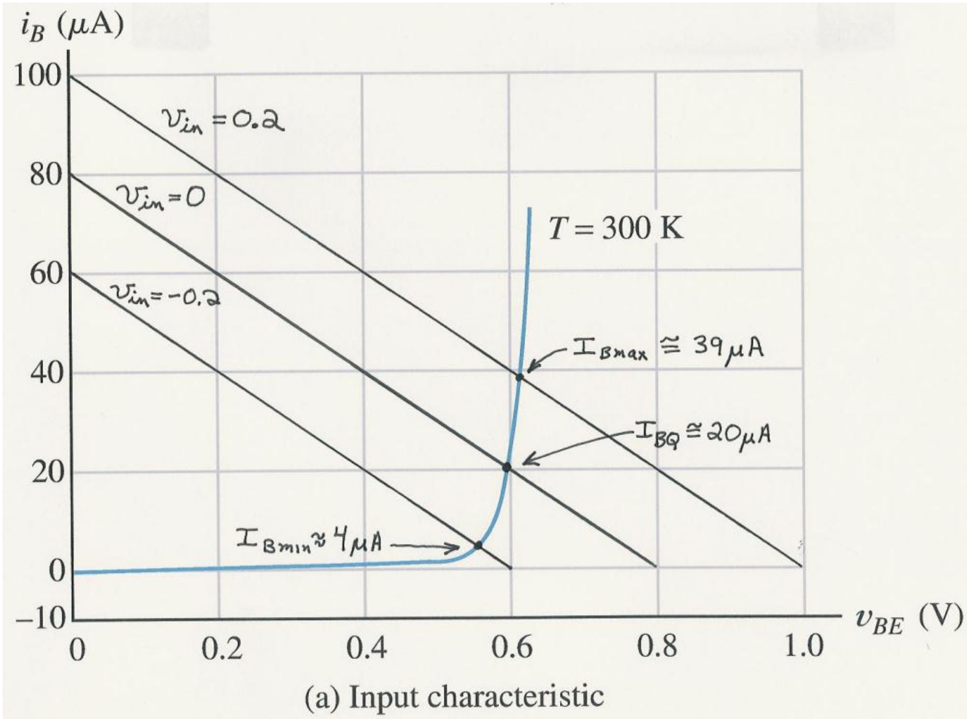
P13.68 The solution is similar to that of Problem P13.66. The results are:

	High impedance amplifier (Problem 13.68)	Low impedance amplifier (Problem 13.66)
I_{CQ}	64.1 μA	6.41 mA
	40.5 kΩ	405 Ω
A_v	0.988	0.988
A_{voc}	0.996	0.996
Z_{in}	436 kΩ	4.36 kΩ
A_i	8.61	8.61
G	8.51	8.51
Z_o	1210 Ω	12.1 Ω

Practice Test

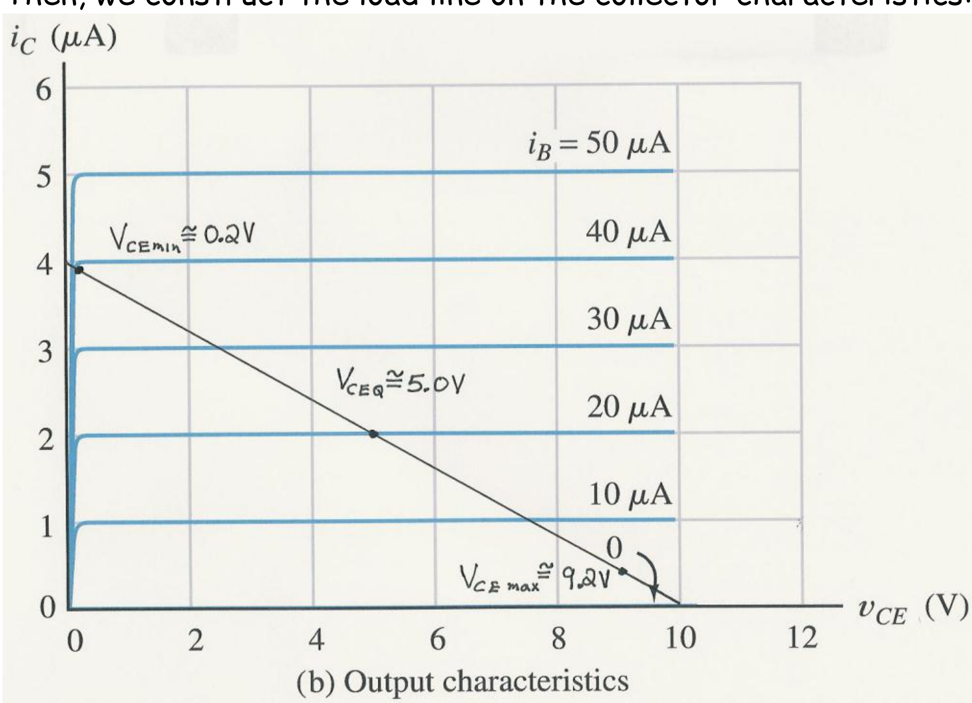
T13.1 **a.** 3, **b.** 2, **c.** 5, **d.** 7 and 1 (either order), **e.** 10, **f.** 7, **g.** 1, **h.** 7, **i.** 15, **j.** 12, **k.** 19.

T13.2 First, we construct the load lines on the input characteristics for $v_{in} = 0$, -0.2 V, and $+0.2$ V:



(a) Input characteristic

At the intersections of the characteristic with the load lines, we find the minimum, *Q*-point, and maximum values of the base current as shown. Then, we construct the load line on the collector characteristics:



(b) Output characteristics

Interpolating between collector characteristics when necessary, we find $V_{CE\min} \cong 0.2\text{ V}$, $V_{CEQ} \cong 5.0\text{ V}$, and $V_{CE\max} \cong 9.2\text{ V}$.

T13.3 $\alpha = \dfrac{I_{CQ}}{I_{EQ}} = \dfrac{1.0}{1.04} = 0.9615 \quad I_{BQ} = I_{EQ} - I_{CQ} = 0.04\text{ mA}$

$$\beta = \frac{I_{CQ}}{I_{BQ}} = \frac{\alpha}{1-\alpha} = 25 \qquad r_\pi = \frac{\beta V_T}{I_{CQ}} = \frac{25 \times 0.026}{0.001} = 650\ \Omega$$

The small-signal equivalent circuit is shown in Figure 13.26.

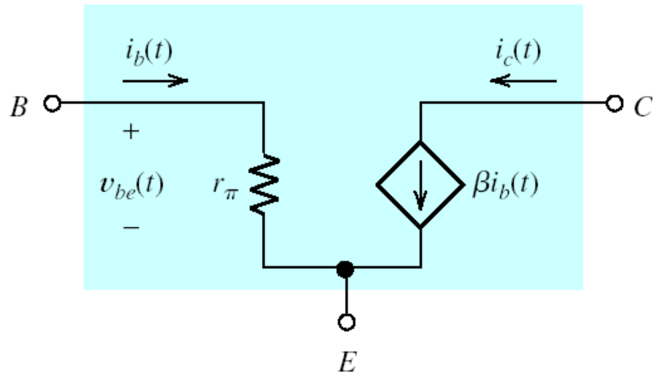
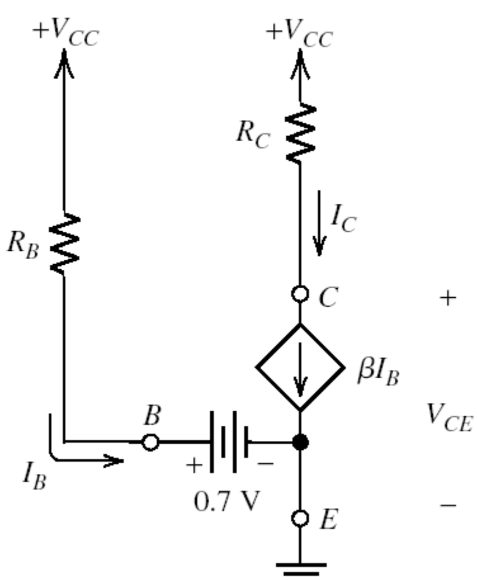


Figure 13.26 Small-signal equivalent circuit for the BJT.

T13.4 (a) It turns out that, in this case ($\beta = 50$), the BJT operates in the active region. The equivalent circuit is:



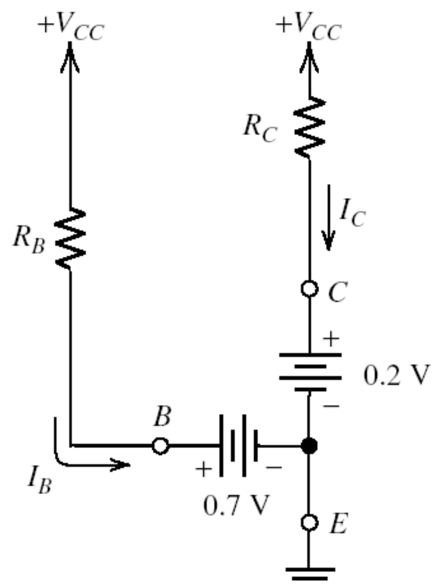
in which we have $V_{CC} = 9$ V, $R_C = 4.7$ kΩ, and $R_B = 470$ kΩ.
We have

$$I_B = \frac{V_{CC} - 0.7}{R_B} = 17.66\ \mu A \qquad I_C = \beta I_B = 0.8830\ \text{mA}$$

$$V_{CE} = V_{CC} - R_C I_C = 4.850\ \text{V}$$

Because we have $V_{CE} > 0.2$, we are justified in assuming that the transistor operates in the active region.

(b) In this case ($\beta = 250$), the BJT operates in the saturation region. The equivalent circuit is:

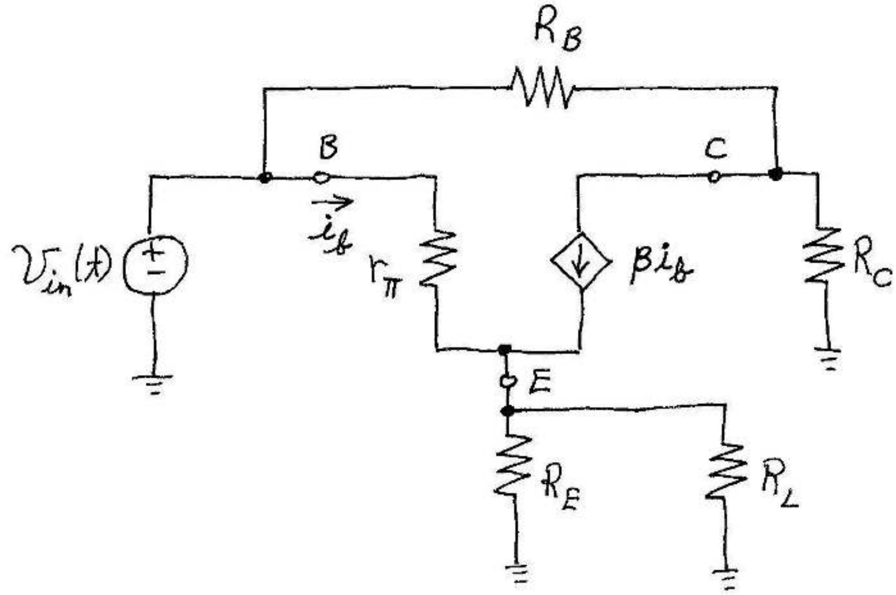


We have

$$V_{CE} = 0.2\,\text{V} \quad I_B = \frac{V_{CC} - 0.7}{R_B} = 17.66\ \mu A \quad I_C = \frac{V_{CC} - 0.2}{R_C} = 1.872\ \text{mA}$$

Because we have $\beta I_B > I_C$, we are justified in assuming that the transistor operates in the saturation region.

T13.5 We need to replace V_{CC} by a short circuit to ground, the coupling capacitances with short circuits, and the BJT with its equivalent circuit. The result is:



T13.6 This problem is similar to parts of Example 13.8.

$$r_\pi = \frac{\beta V_T}{I_{CQ}} = \frac{120(26 \text{ mV})}{4 \text{ mA}} = 780\ \Omega$$

$$R_L' = \frac{1}{1/R_L + 1/R_C} = 1.579\ \text{k}\Omega$$

$$A_v = -\frac{\beta R_L'}{r_\pi} = -243.0$$

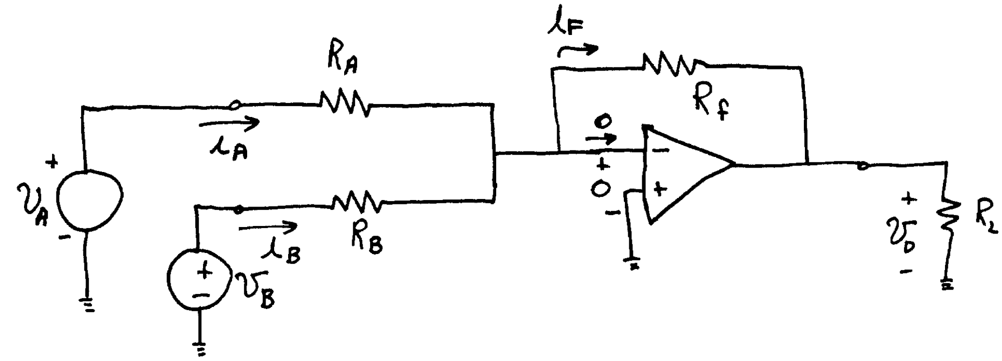
$$R_B = \frac{1}{1/R_1 + 1/R_2} = 31.97\ \text{k}\Omega$$

$$Z_{in} = \frac{1}{1/R_B + 1/r_\pi} = 761.4\ \Omega$$

CHAPTER 14

Exercises

E14.1



(a) $i_A = \dfrac{v_A}{R_A}$ $\quad i_B = \dfrac{v_B}{R_B}$ $\quad i_F = i_A + i_B = \dfrac{v_A}{R_A} + \dfrac{v_B}{R_B}$

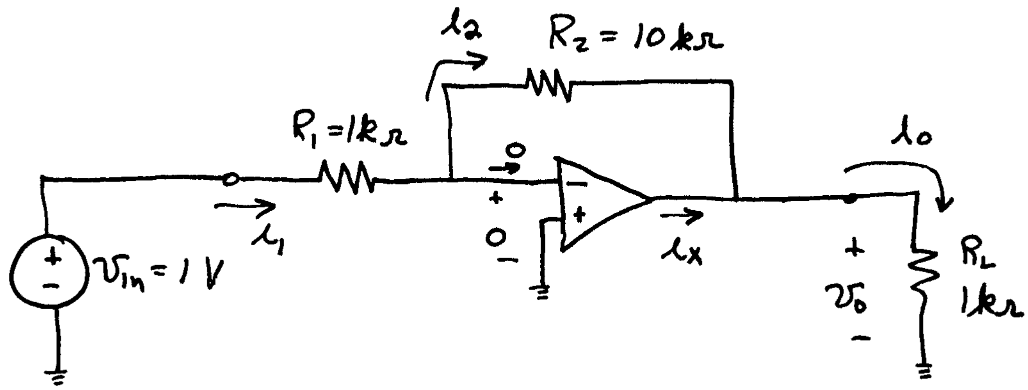
$$v_o = -R_F i_F = -R_F\left(\frac{v_A}{R_A} + \frac{v_B}{R_B}\right)$$

(b) For the v_A source, $R_{\text{in}A} = \dfrac{v_A}{i_A} = R_A$.

(c) Similarly $R_{\text{in}B} = R_B$.

(d) In part (a) we found that the output voltage is independent of the load resistance. Therefore, the output resistance is zero.

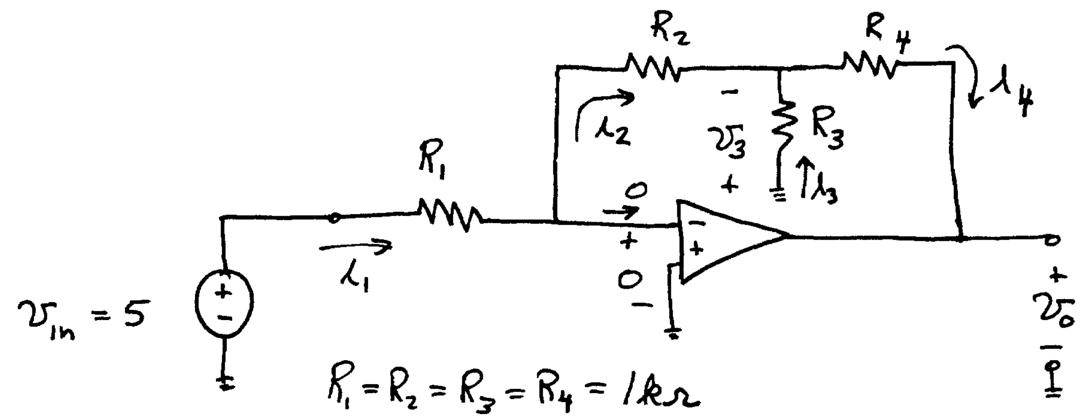
E14.2 (a)



$i_1 = \dfrac{v_{in}}{R_1} = 1\,\text{mA}$ $\quad i_2 = i_1 = 1\,\text{mA}$ $\quad v_o = -R_2 i_2 = -10\,\text{V}$

$$i_o = \frac{v_o}{R_L} = -10 \text{ mA} \qquad i_x = i_o - i_2 = -11 \text{ mA}$$

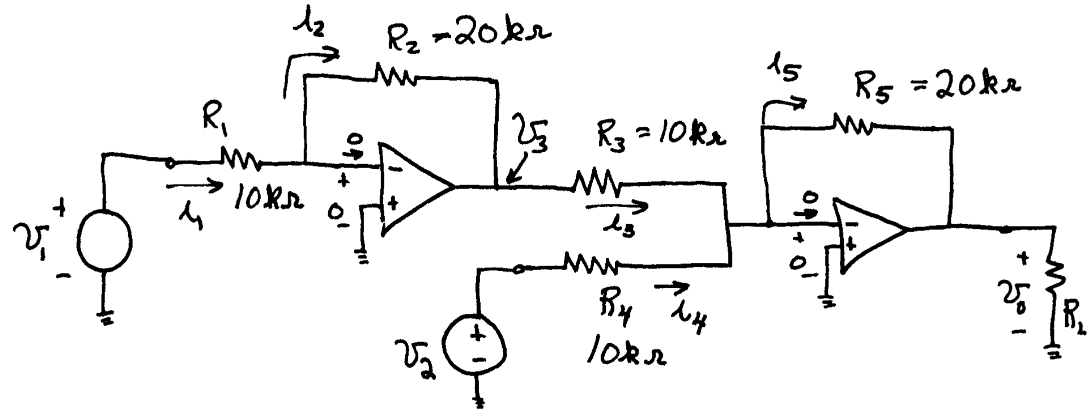
(b)



$$i_1 = \frac{v_{in}}{R_1} = 5 \text{ mA} \qquad i_2 = i_1 = 5 \text{ mA} \qquad v_3 = R_2 i_2 = 5 \text{ V}$$

$$i_3 = \frac{v_3}{R_3} = 5 \text{ mA} \qquad i_4 = i_2 + i_3 = 10 \text{ mA} \qquad v_o = -R_4 i_4 - R_2 i_2 = -15 \text{ V}$$

E14.3



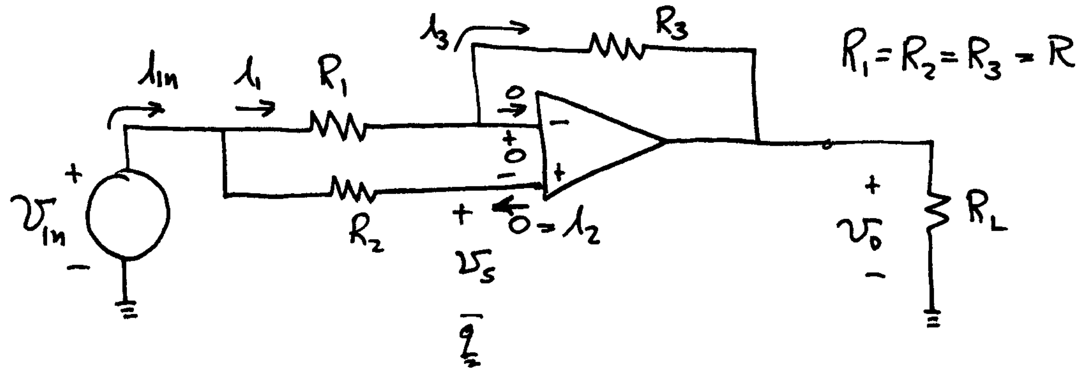
Direct application of circuit laws gives $i_1 = \frac{v_1}{R_1}$, $i_2 = i_1$, and $v_3 = -R_2 i_2$.

From the previous three equations, we obtain $v_3 = -\frac{R_2}{R_1} v_1 = -2v_1$. Then

applying circuit laws gives $i_3 = \frac{v_3}{R_3}$, $i_4 = \frac{v_2}{R_4}$, $i_5 = i_3 + i_4$, and $v_o = -R_5 i_5$.

These equations yield $v_o = -\frac{R_5}{R_3}v_3 - \frac{R_5}{R_4}v_2$. Then substituting values and using the fact that $v_3 = -2v_1$, we find $v_o = 4v_1 - 2v_2$.

E14.4 (a)

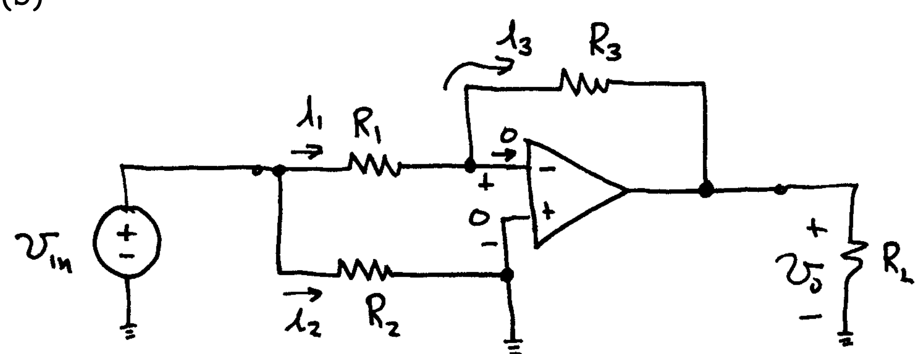


$v_s = v_{in} + R_2 i_2 = v_{in}$ (Because of the summing-point restraint, $i_2 = 0$.)

$i_1 = \frac{v_{in} - v_s}{R_1} = 0$ (Because $v_s = v_{in}$.) $\qquad i_{in} = i_1 - i_2 = 0$

$i_3 = i_1 = 0 \qquad v_o = R_3 i_3 + v_s = v_{in}$ Thus, $A_v = \frac{v_o}{v_{in}} = +1$ and $R_{in} = \frac{v_{in}}{i_{in}} = \infty$.

(b)

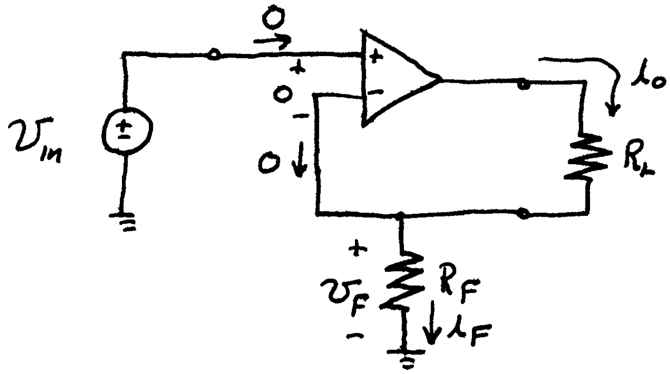


(Note: We assume that $R_1 = R_2 = R_3$.)

$i_1 = \frac{v_{in}}{R_1} = \frac{v_{in}}{R} \qquad i_2 = \frac{v_{in}}{R_2} = \frac{v_{in}}{R} \qquad i_{in} = i_1 + i_2 = \frac{2v_{in}}{R} \qquad R_{in} = \frac{R}{2}$

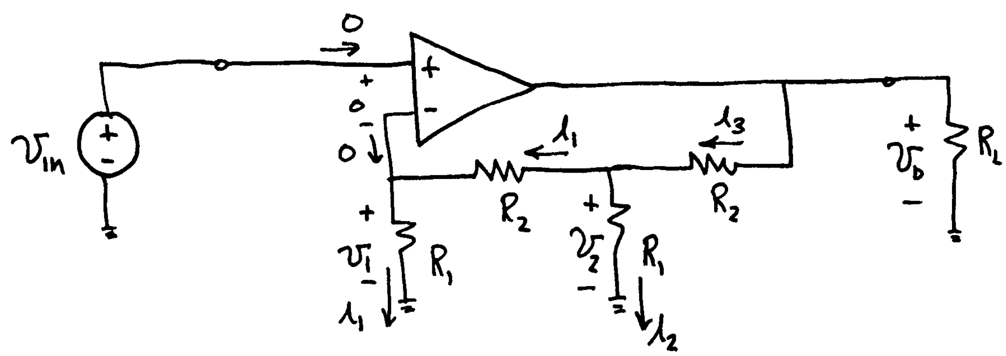
$i_3 = i_1 = \frac{v_{in}}{R_1} \qquad v_o = -R_3 i_3 = -\frac{R_3}{R_1}v_{in} = -v_{in} \qquad A_v = \frac{v_o}{v_{in}} = -1$

E14.5



From the circuit, we can write $v_F = v_{\text{in}}$, $i_F = \dfrac{v_F}{R_F}$, and $i_o = i_F$. From these equations, we find that $i_o = \dfrac{v_{\text{in}}}{R_F}$. Then because i_o is independent of R_L, we conclude that the output impedance of the amplifier is infinite. Also R_{in} is infinite because i_{in} is zero.

E14.6 (a)



$v_1 = v_{\text{in}} \quad i_1 = \dfrac{v_1}{R_1} \quad v_2 = R_2 i_1 + R_1 i_1 \quad i_2 = \dfrac{v_2}{R_1} \quad i_3 = i_1 + i_2 \quad v_o = R_2 i_3 + v_2$

Using the above equations we eventually find that

$$A_v = \frac{v_o}{v_{\text{in}}} = 1 + 3\frac{R_2}{R_1} + \left(\frac{R_2}{R_1}\right)^2$$

(b) Substituting the values given, we find $A_v = 131$.

(c) Because $i_{\text{in}} = 0$, the input resistance is infinite.

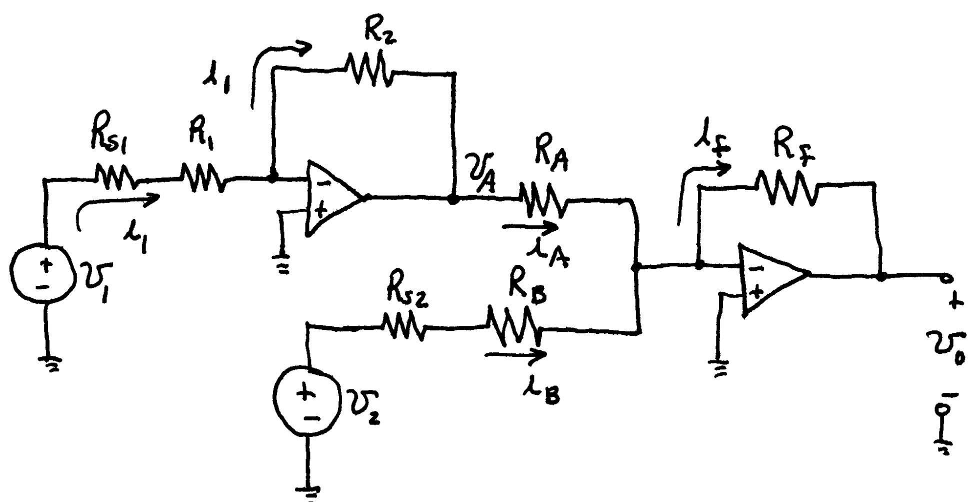
(d) Because $v_o = A_v v_{in}$ is independent of R_L, the output resistance is zero.

E14.7 We have $A_{vs} = -\dfrac{R_2}{R_s + R_1}$ from which we conclude that

$$A_{vs\max} = -\frac{R_{2\max}}{R_{s\min} + R_{1\min}} = -\frac{499 \times 1.01}{0 + 49.9 \times 0.99} = -10.20$$

$$A_{vs\min} = -\frac{R_{2\min}}{R_{s\max} + R_{1\max}} = -\frac{499 \times 0.99}{0.500 + 49.9 \times 1.01} = -9.706$$

E14.8



Applying basic circuit principles, we obtain:

$$i_1 = \frac{v_1}{R_1 + R_{s1}} \qquad v_A = -R_2 i_1 \qquad i_A = \frac{v_A}{R_A}$$

$$i_B = \frac{v_2}{R_B + R_{s2}} \qquad i_f = i_A + i_B \qquad v_o = -R_f i_f$$

From these equations, we eventually find

$$v_o = \frac{R_2}{R_{s1} + R_1}\frac{R_f}{R_A}v_1 - \frac{R_f}{R_{s2} + R_B}v_2$$

E14.9 Many correct answers exist. A good solution is the circuit of Figure 14.11 in the book with $R_2 \cong 19R_1$. We could use standard 1%-tolerance resistors with nominal values of $R_1 = 1\text{ k}\Omega$ and $R_2 = 19.1\text{ k}\Omega$.

E14.10 Many correct answers exist. A good solution is the circuit of Figure 14.18 in the book with $R_1 \geq 20R_s$ and $R_2 \cong 25(R_1 + R_s)$. We could use standard 1%-tolerance resistors with nominal values of $R_1 = 20\ \text{k}\Omega$ and $R_2 = 515\ \ \text{k}\Omega$.

E14.11 Many correct selections of component values can be found that meet the desired specifications. One possibility is the circuit of Figure 14.19 with:

$R_1 =$ a 453-kΩ fixed resistor in series with a 100-kΩ trimmer (nominal design value is 500 kΩ)
R_B is the same as R_1
$R_2 = 499\ \ \text{k}\Omega$
$R_A = 1.5\ \ \text{M}\Omega$
$R_f = 1.5\ \ \text{M}\Omega$

After constructing the circuit we could adjust the trimmers to achieve the desired gains.

E14.12 $f_{BCL} = \dfrac{f_t}{A_{0CL}} = \dfrac{A_{0OL} f_{BOL}}{A_{0CL}} = \dfrac{10^5 \times 40}{100} = 40\ \text{kHz}$ The corresponding Bode plot is shown in Figure 14.22 in the book.

E14.13 (a) $f_{FP} = \dfrac{SR}{2\pi V_{om}} = \dfrac{5 \times 10^6}{2\pi(4)} = 198.9\ \text{kHz}$

(b) The input frequency is less than f_{FP} and the current limit of the op amp is not exceeded, so the maximum output amplitude is 4 V.

(c) With a load of 100 Ω the current limit is reached when the output amplitude is 10 mA $\times$ 100 Ω = 1 V. Thus the maximum output amplitude without clipping is 1 V.

(d) In deriving the full-power bandwidth we obtained the equation:

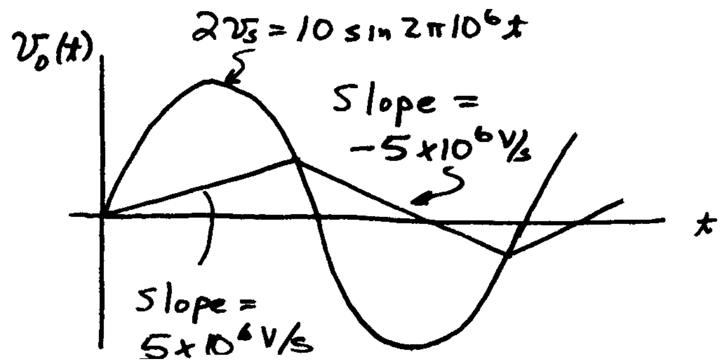
$$2\pi f V_{om} = SR$$

Solving for V_{om} and substituting values, we have

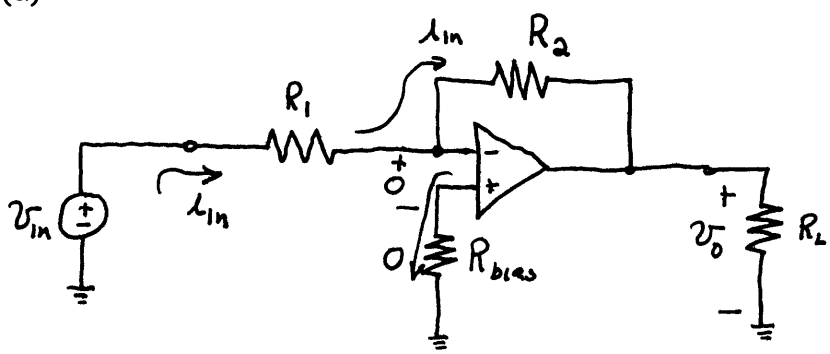
$$V_{om} = \frac{SR}{2\pi f} = \frac{5 \times 10^6}{2\pi 10^6} = 0.7958\ \ \text{V}$$

With this peak voltage and R_L = 1 kΩ, the current limit is not exceeded.

(e) Because the output, assuming an ideal op amp, has a rate of change exceeding the slew-rate limit, the op amp cannot follow the ideal output, which is $v_o(t) = 10\sin(2\pi 10^6 t)$. Instead, the output changes at the slew-rate limit and the output waveform eventually becomes a triangular waveform with a peak-to-peak amplitude of SR × ($T/2$) = 2.5 V.

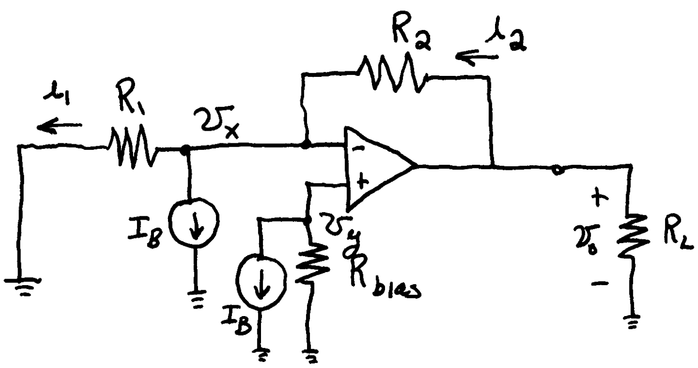


E14.14 (a)



Applying basic circuit laws, we have $i_{in} = \frac{v_{in}}{R_1}$ and $v_o = -R_2 i_{in}$. These equations yield $A_v = \frac{v_o}{v_{in}} = -\frac{R_2}{R_1}$.

(b)



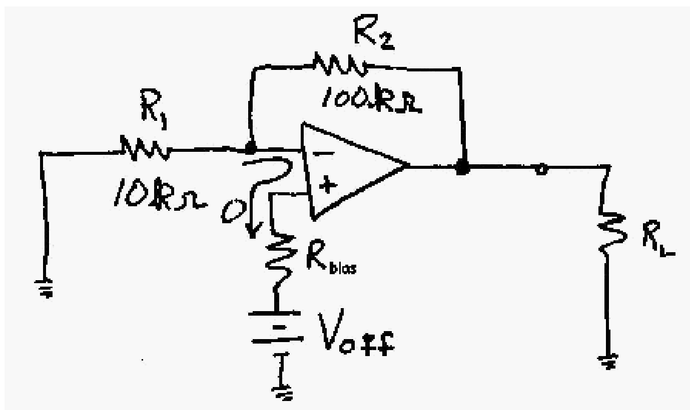
Applying basic circuit principles, algebra, and the summing-point restraint, we have

$$v_x = v_y = -R_{bias} I_B \qquad i_1 = \frac{v_x}{R_1} = -\frac{R_{bias}}{R_1} I_B = -\frac{R_2}{R_1 + R_2} I_B$$

$$i_2 = I_B + i_1 = \left(1 - \frac{R_2}{R_1 + R_2}\right) I_B = \frac{R_1}{R_1 + R_2} I_B$$

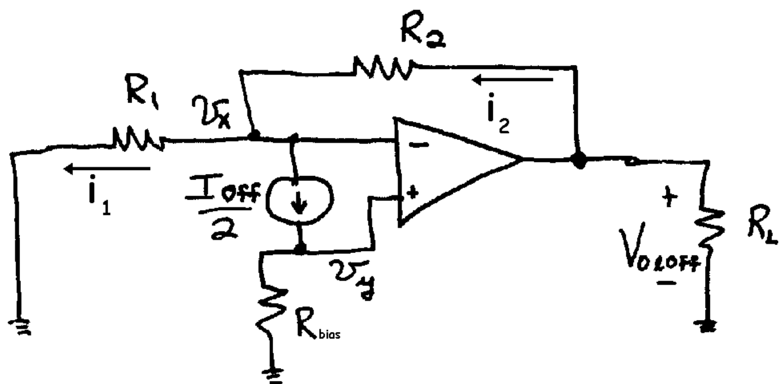
$$v_o = R_2 i_2 + v_x = R_2 \frac{R_1}{R_1 + R_2} I_B - R_{bias} I_B = 0$$

(c)



The drop across R_{bias} is zero because the current through it is zero. For the source V_{off} the circuit acts as a noninverting amplifier with a gain $A_v = 1 + \frac{R_2}{R_1} = 11$. Therefore, the extreme output voltages are given by $v_o = A_v V_{\text{off}} = \pm 33$ mV.

(d)



Applying basic circuit principles, algebra, and the summing-point restraint, we have

$$v_x = v_y = R_{bias}\frac{I_{\text{off}}}{2} \qquad i_1 = \frac{v_x}{R_1} = \frac{R_{bias}}{R_1}\frac{I_{\text{off}}}{2} = \frac{R_2}{R_1 + R_2}\frac{I_{\text{off}}}{2}$$

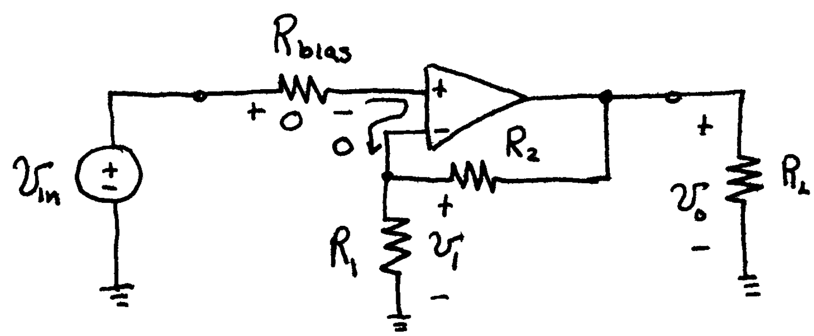
$$i_2 = \frac{I_{\text{off}}}{2} + i_1 = \left(1 + \frac{R_2}{R_1 + R_2}\right)\frac{I_{\text{off}}}{2} = \frac{R_1 + 2R_2}{R_1 + R_2}\frac{I_{\text{off}}}{2}$$

$$v_o = R_2 i_2 + v_x = R_2\frac{R_1 + 2R_2}{R_1 + R_2}\frac{I_{\text{off}}}{2} + R_{bias}\frac{I_{\text{off}}}{2} = R_2 I_{off}$$

Thus the extreme values of v_o caused by I_{off} are $V_{o,\text{Ioff}} = \pm 4$ mV.

(e) The cumulative effect of the offset voltage and offset current is that V_o ranges from -37 to +37 mV.

E14.15 (a)



Because of the summing-point constraint, no current flows through R_{bias} so the voltage across it is zero. Because the currents through R_1 and R_2 are the same, we use the voltage division principle to write

$$v_1 = v_o\frac{R_1}{R_1 + R_2}$$

Then using KVL we have

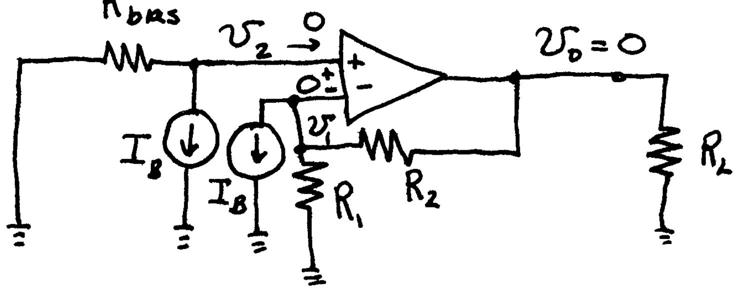
$$v_{\text{in}} = 0 + v_1$$

These equations yield

$$A_v = \frac{v_o}{v_{\text{in}}} = 1 + \frac{R_2}{R_1}$$

Assuming an ideal op amp, the resistor R_{bias} does not affect the gain since the voltage across it it zero.

(b) The circuit with the signal set to zero and including the bias current sources is shown.

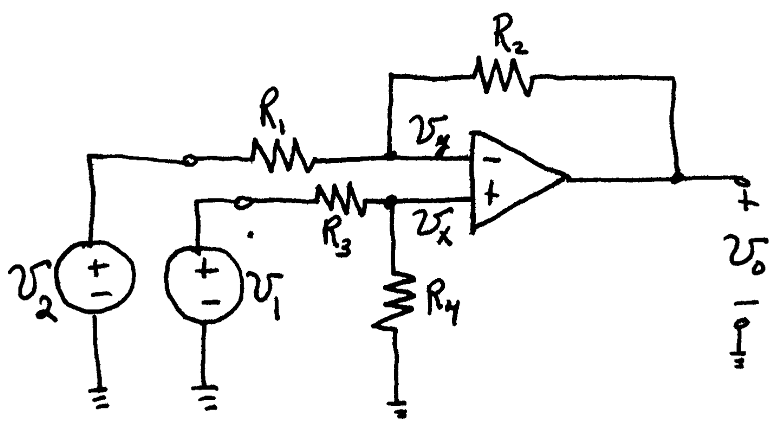


We want the output voltage to equal zero. Using Ohm's law, we can write $v_2 = -R_{\text{bias}} I_B$. Then writing a current equation at the inverting input, we have $I_B + \frac{v_1}{R_1} + \frac{v_1}{R_2} = 0$. Finally, because of the summing-point restraint, we have $v_2 = v_1$. These equations eventually yield

$$R_{\text{bias}} = \frac{1}{1/R_1 + 1/R_2}$$

as the condition for zero output due to the bias current sources.

E14.16



Because no current flows into the op-amp input terminals, we can use the voltage division principle to write

$$v_x = v_1 \frac{R_4}{R_3 + R_4}$$

Because of the summing-point restraint, we have

$$v_x = v_y = v_1 \frac{R_4}{R_3 + R_4}$$

Writing a KCL equation at the inverting input, we obtain

$$\frac{v_y - v_2}{R_1} + \frac{v_y - v_o}{R_2} = 0$$

Substituting for v_y and solving for the output voltage, we obtain

$$v_o = v_1 \frac{R_4}{R_3 + R_4} \frac{R_1 + R_2}{R_1} - v_2 \frac{R_2}{R_1}$$

If we have $R_4 / R_3 = R_2 / R_1$, the equation for the output voltage reduces to

$$v_o = \frac{R_2}{R_1}(v_1 - v_2)$$

E14.17 (a) $v_o(t) = -\frac{1}{RC}\int_0^t v_{in}(t)dt = -1000\int_0^t v_{in}(t)dt$

$$= -1000\int_0^t 5dt = -5000t \quad \text{for } 0 \le t \le 1 \text{ ms}$$

$$= -1000\left(\int_0^{1\text{ ms}} 5dt + \int_{1\text{ ms}}^{t} -5dt\right) = -10 + 5000t \quad \text{for } 1 \text{ ms} \le t \le 3 \text{ ms}$$

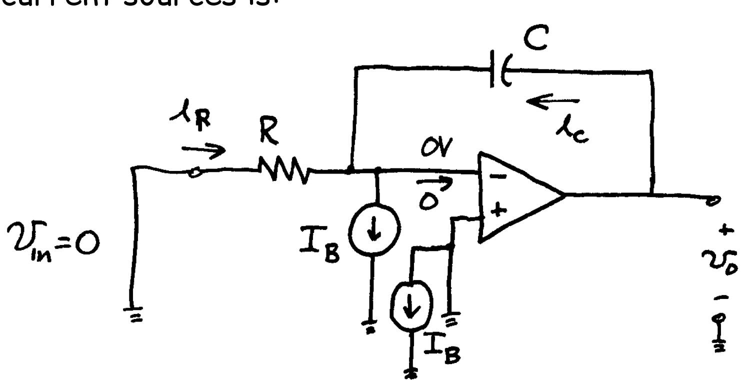
and so forth. A plot of $v_o(t)$ versus t is shown in Figure 14.37 in the book.

(b) A peak-to-peak amplitude of 2 V implies a peak amplitude of 1 V. The first (negative) peak amplitude occurs at $t = 1$ ms. Thus we can write

$$-1 = -\frac{1}{RC}\int_0^{1\text{ ms}} v_{in}dt = -\frac{1}{10^4 C}\int_0^{1\text{ ms}} 5dt = -\frac{1}{10^4 C} \times 5 \times 10^{-3}$$

which yields $C = 0.5\ \mu$F.

E14.18 The circuit with the input source set to zero and including the bias current sources is:



Because the voltage across R is zero, we have $i_C = I_B$, and we can write

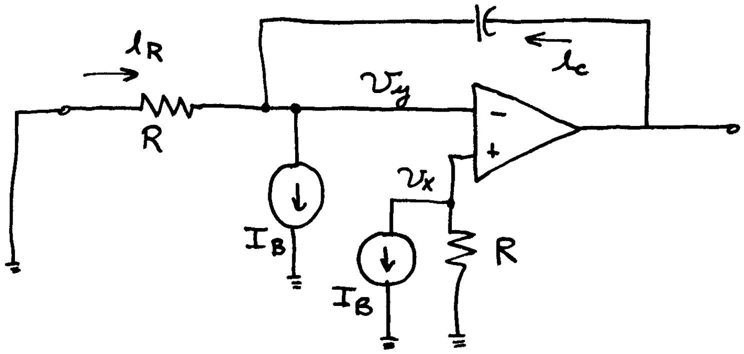
$$v_o = \frac{1}{C}\int_0^t i_C dt = \frac{1}{C}\int_0^t I_B dt = \frac{100 \times 10^{-9} t}{C}$$

(a) For C = 0.01 μF we have $v_o(t) = 10t$ V.

(b) For C = 1 μF we have $v_o(t) = 0.1t$ V.

Notice that larger capacitances lead to smaller output voltages.

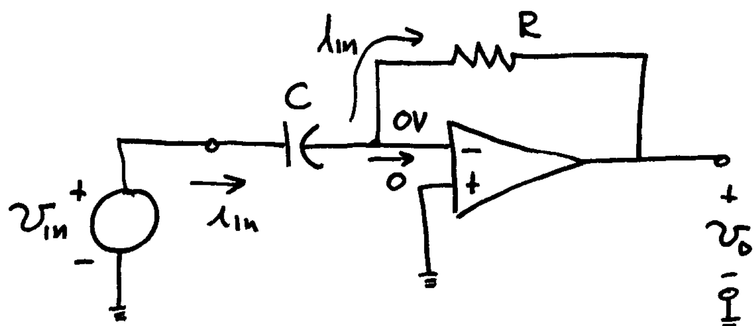
E14.19



$v_y = v_x = -I_B R_B \quad i_R = -v_y / R_B = I_B \quad i_C = i_R + I_B = 0$

Because $i_C = 0$, we have $v_C = 0$, and $v_o = v_y = -I_B R = 1$ mV.

E14.20



$$i_{\text{in}} = C\frac{dv_{\text{in}}}{dt} \qquad v_o(t) = -Ri_{\text{in}} = -RC\frac{dv_{\text{in}}}{dt}$$

E14.21 The transfer function in decibels is

$$|H(f)|_{dB} = 20\log\left[\frac{H_0}{\sqrt{1+(f/f_B)^{2n}}}\right]$$

For $f >> f_B$, we have

$$\left|H(f)\right|_{dB} \cong 20\log\left[\frac{H_0}{\sqrt{(f/f_B)^{2n}}}\right] = 20\log\left|H_0\right| + 20n\log(f_B) - 20n\log(f)$$

This expression shows that the gain magnitude is reduced by $20n$ decibels for each decade increase in f.

E14.22 Three stages each like that of Figure 14.40 must be cascaded. From Table 14.1, we find that the gains of the stages should be 1.068, 1.586, and 2.483. Many combinations of component values will satisfy the requirements of the problem. A good choice for the capacitance value is 0.01 μF, for which we need $R = 1/(2\pi C f_B) = 3.183\ \text{k}\Omega$. Also $R_f = 10\ \text{k}\Omega$ is a good choice.

Problems

P14.1 The most probable functions of the five op amp terminals are the inverting input, the noninverting input, the output, and two power-supply terminals (or one power-supply and one ground terminal).

P14.2 An ideal operational amplifier has the following characteristics:

1. Infinite input impedance.
2. Infinite gain for the differential input signal.
3. Zero gain for the common-mode input signal.
4. Zero output impedance.
5. Infinite bandwidth.

P14.3 The open-loop gain is the voltage gain of the op amp for the differential input voltage with no feedback applied. Closed loop gain is the gain of circuit containing an op amp with feedback.

P14.4 The differential voltage is:

$$v_{id} = v_1 - v_2$$

and the common-mode voltage is:

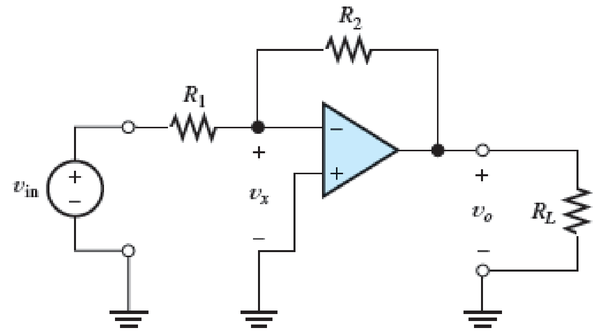
$$v_{icm} = \tfrac{1}{2}(v_1 + v_2)$$

P14.5* $v_{id} = v_1 - v_2 = \cos(2000\pi t)$ $\qquad$ $v_{icm} = \tfrac{1}{2}(v_1 + v_2) = 20\cos(120\pi t)$

P14.6* The steps in analysis of an amplifier containing an ideal op amp are:

1. Verify that negative feedback is present.
2. Assume that the differential input voltage and the input currents are zero.
3. Apply circuit analysis principles including Kirchhoff's and Ohm's laws to write circuit equations. Then, solve for the quantities of interest.

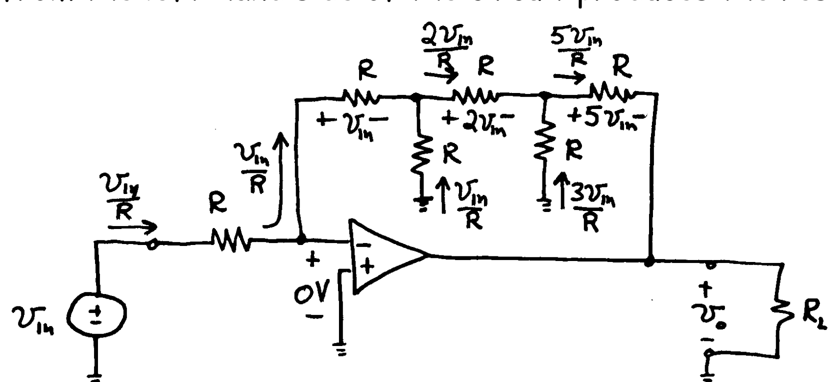
P14.7 The inverting amplifier configuration is shown in Figure 14.4 in the text.



The voltage gain is given by $A_v = -R_2/R_1$, the input impedance is equal to R_1, and the output impedance is zero.

P14.8 According to the summing-point constraint, the output voltage of an op amp assumes the value required to produce zero differential input voltage and zero current into the op-amp input terminals. This principle applies when negative feedback is present but not when positive feedback is present.

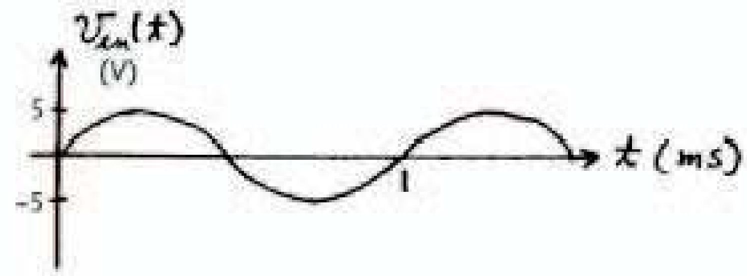
P14.9* The circuit has negative feedback so we can employ the summing-point constraint. Successive application of Ohm's and Kirchhoff's laws starting from the left-hand side of the circuit produces the results shown:

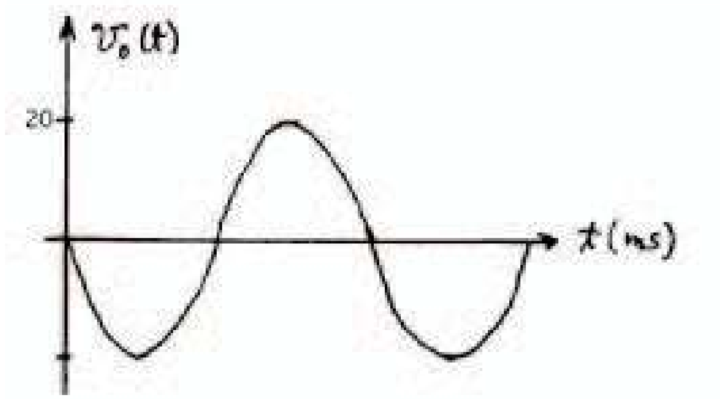


From these results we can use KVL to determine that $v_o = -8v_{\text{in}}$ from which we have $A_v = -8$.

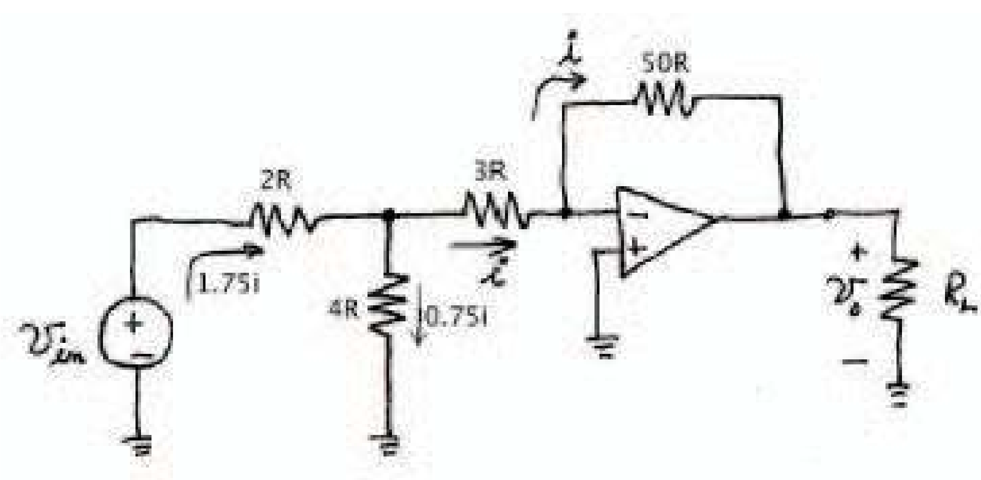
P14.10 This is an inverting amplifier having a voltage gain given by Av=-R2/R1=-4 / Thus, we have $v_o(t) = -4 \times [5\cos(2000\pi t)]$

Sketches of $v_{\text{in}}(t)$ and $v_o(t)$ are





P14.11 Because of the summing-point constraint, the voltages across the resistors of values 4R and 3R are equal. Thus, the current in the resistor of value 4R is 0.75 times that of the current in the 3R resistor.



Applying KCL, we find the other currents as shown. Then, applying KVL, we have $v_{in} = 2R(1.75i) + 3R(0.75i)$ and $v_o = -50Ri$. Solving, we find

$$A_v = \frac{v_o}{v_{in}} = 7.7.$$

P14.12 Using the summing-point constraint, we have

$$i_D = I_s \exp(v_{\text{in}} / nV_T) \text{ and } v_o = -Ri_D$$

Thus, we have

$$v_o = -RI_s \exp(v_{\text{in}} / nV_T)$$

P14.13 Using the summing-point constraint, we have

$$i_D = \frac{v_{\text{in}}}{R} = I_s \exp(v_D / nV_T) \text{ and } v_o = -v_D$$

Solving, we have

$$v_o = -nV_T \ln\left(\frac{v_{\text{in}}}{RI_s}\right)$$

P14.14 This is an inverting amplifier with a voltage gain of -2. Thus, we have $i_x = 2$ mA, $v_L = -4$ V, $i_L = -4$ mA, and $i_o = -6$ mA. In the circuit as shown, there appears to be 6 mA flowing into the closed surface and no current flowing out. However, a real op amp also has power supply connections, and if the currents in these connections are taken into account, KCL is satisfied.

P14.15 Using the summing-point constraint, we have

$$i_D = \frac{v_{\text{in}}}{R} = Kv_D^3 \text{ and } v_o = -v_D$$

Solving, we have

$$v_o = -\sqrt[3]{\frac{v_{\text{in}}}{KR}}$$

P14.16 This circuit has positive feedback and the output can be either +5 V or -5 V. Writing a current equation at the inverting input terminal of the op amp, we have

$$\frac{v_x - 3}{2000} + \frac{v_x - v_o}{2000} = 0$$

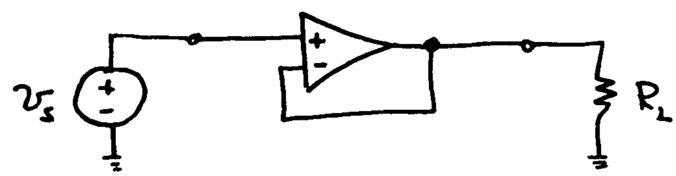
Solving we find

$$v_x = 1.5 + 0.5v_o$$

For $v_o = 5$ V, we have $v_x = 4$V. On the other hand for $v_o = -5$ V, we have $v_x = -1$V. Notice that for v_x positive the output remains stuck at its positive extreme and for v_x negative the output remains stuck at its negative extreme.

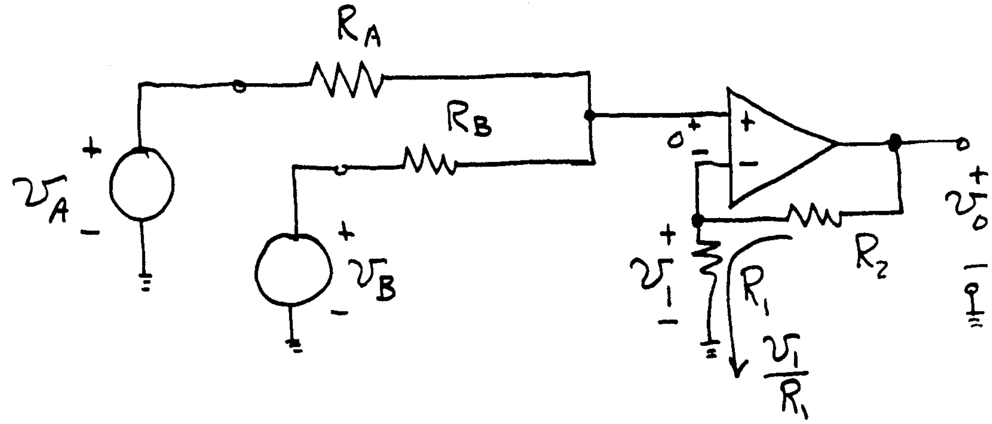
P14.17* If the source has non-zero series impedance, loading (reduction in voltage) will occur when the load is connected directly to the source. On the other hand, the input impedance of the voltage follower is very high (ideally infinite) and loading does not occur. If the source impedance is very high compared to the load impedance, the voltage follower will deliver a much larger voltage to the load than direct connection.

P14.18* The circuit diagram of the voltage follower is:



Assuming an ideal op amp, the voltage gain is unity, the input impedance is infinite, and the output impedance is zero.

P14.19* The circuit diagram is:



Writing a current equation at the noninverting input, we have

$$\frac{v_1 - v_A}{R_A} + \frac{v_1 - v_B}{R_B} = 0 \qquad (1)$$

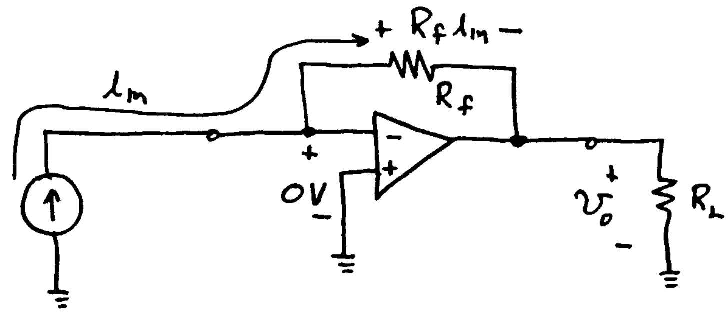
Using the voltage-division principle we can write:

$$v_1 = \frac{R_1}{R_1 + R_2} v_o \qquad (2)$$

Using Equation (2) to substitute for v_1 in Equation (1) and rearranging, we obtain:

$$v_o = \left(\frac{R_1 + R_2}{R_1} \right) \frac{v_A R_B + v_B R_A}{R_A + R_B}$$

P14.20*



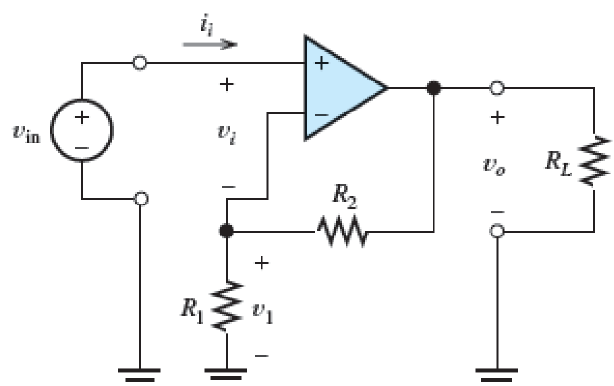
(a) $v_o = -R_f i_{in}$

(b) Since v_o is independent of R_L, the output behaves as a perfect voltage source, and the output impedance is zero.

(c) The input voltage is zero because of the summing-point constraint, and the input impedance is zero.

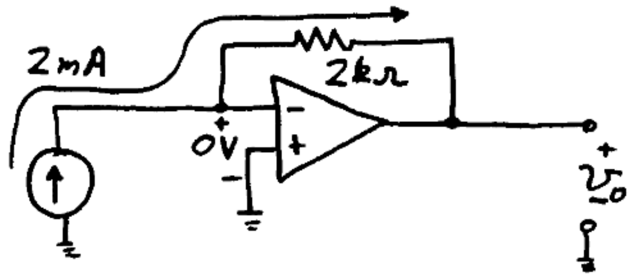
(d) This is an ideal transresistance amplifier.

P14.21 The noninverting amplifier configuration is shown in Figure 14.11 in the text.

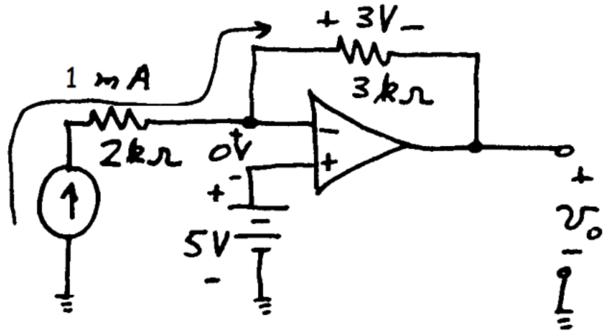


Assuming an ideal op amp, the voltage gain is given by $A_v = 1 + R_2/R_1$, the input impedance is infinite, and the output impedance is zero.

P14.22 (a) $v_o = -(2\ \text{k}\Omega) \times 2\ \text{mA} = -4\ \text{V}$

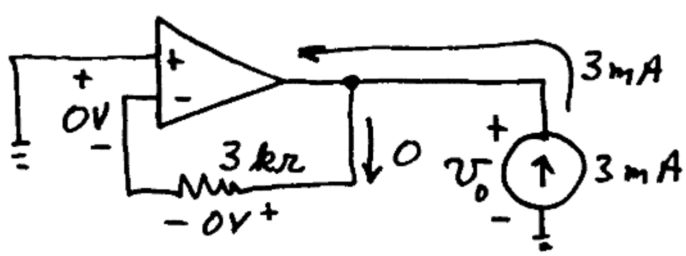


(b) $v_o = -3 + 0 + 5 = 2\ \text{V}$

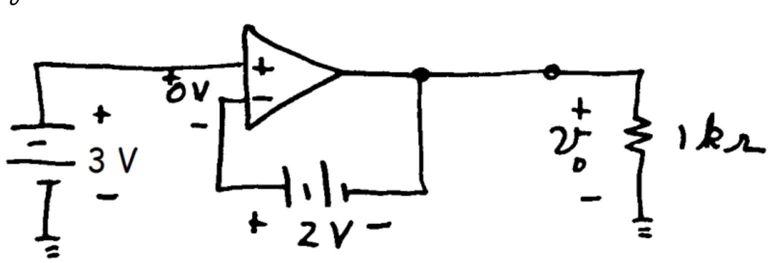


(c) A current of 1 mA flows downward through the 4-kΩ and from right to left through the 3-kΩ resistor. Thus, $v_o = 3 + 4 = 7\ \text{V}$.

(d) $v_o = 0$



(e) $v_o = 3 - 2 = 1\ \text{V}$



P14.23 Analysis of the circuit using the summing-point constraint yields

$$v_o = -\frac{R_2}{10^4} v_{in} + 2\left(1 + \frac{R_2}{10^4}\right)$$

Substituting the expression given for v_{in} yields

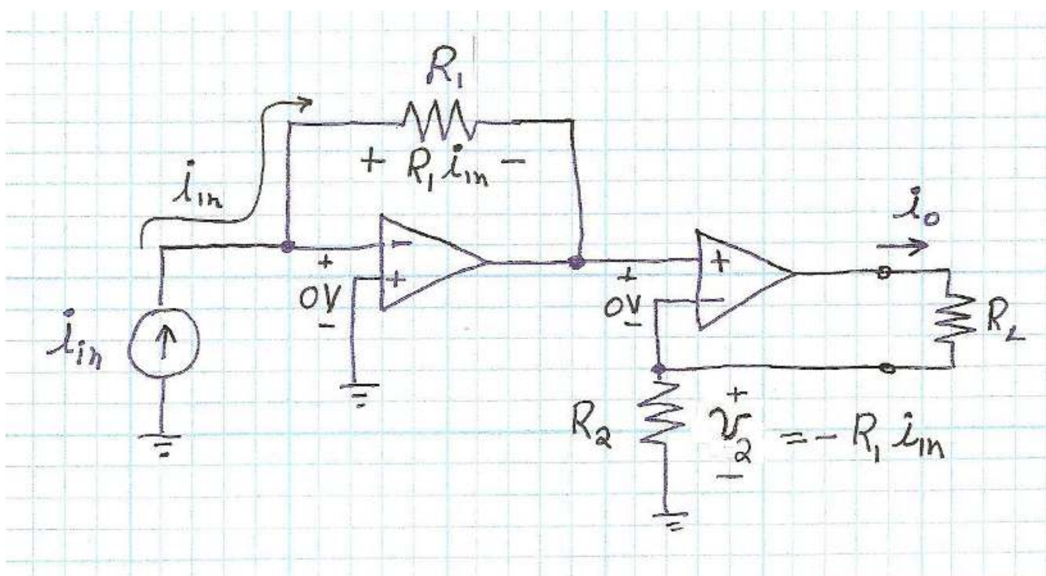
$$v_o = -3\frac{R_2}{10^4} - 3\frac{R_2}{10^4}\cos(2000\pi t) + 2\left(1 + \frac{R_2}{10^4}\right)$$

Then setting the dc component to zero, we have

$$0 = -3\frac{R_2}{10^4} + 2\left(1 + \frac{R_2}{10^4}\right)$$

which yields R_2 = 20 kΩ and then we have $v_o = -6\cos(2000\pi t)$

P14.24 (a) Working from left to right, we find the currents and voltages as shown:



Then, we have

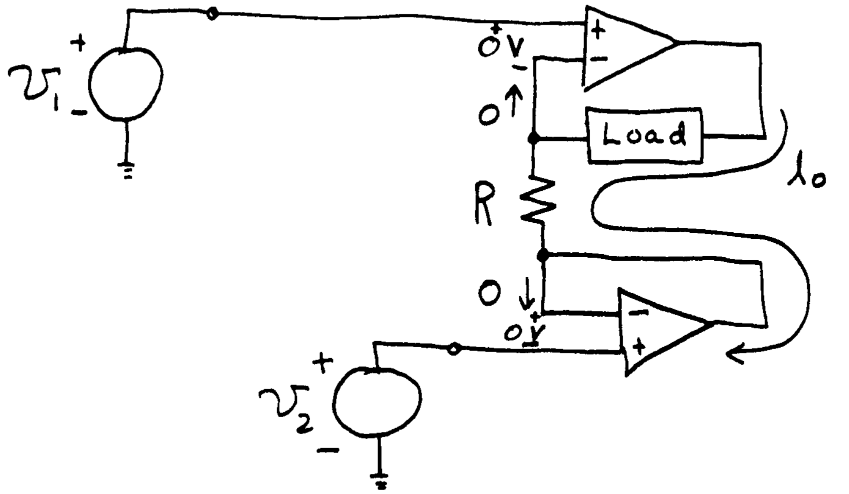
$$i_o = \frac{v_2}{R_2} = -\frac{R_1}{R_2} i_{in}.$$

(b) Since i_o is independent of R_L, the output behaves as a perfect current source, and the output impedance is infinite.

(c) The input voltage is zero because of the summing-point constraint, and the input impedance is zero.

(d) This is an ideal current amplifier.

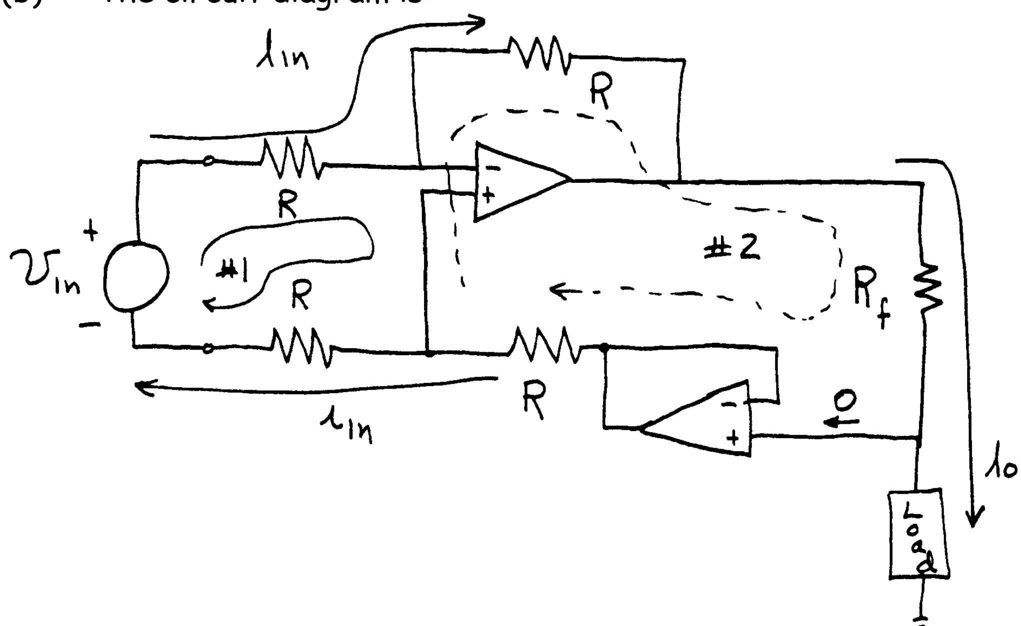
P14.25 (a)



$v_1 = 0 + Ri_o + 0 + v_2 \qquad i_o = \dfrac{v_1 - v_2}{R}$

Since i_o is independent of the load, the output impedance is infinite.

(b) The circuit diagram is:

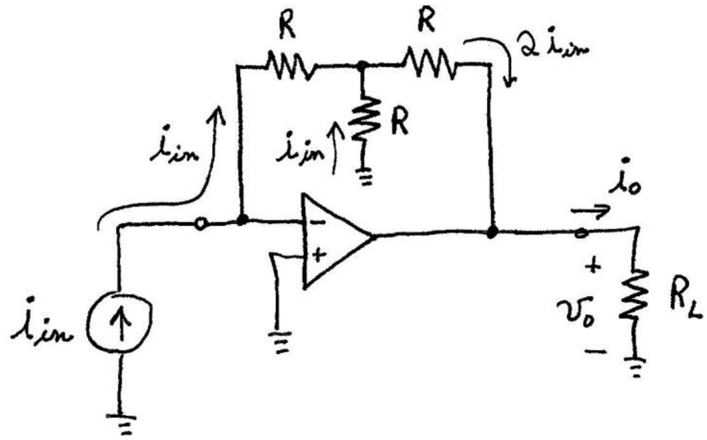


Writing KVL around loop #1, we have $v_{in} = Ri_{in} + 0 + Ri_{in}$

Writing KVL around loop #2, we have $Ri_{in} + R_f i_o + Ri_{in} = 0$

Algebra produces $i_o = -v_{in}/R_f$. Since i_o is independent of the load, the output impedance is infinite.

P14.26 (a) Using the summing-point constraint, KCL, KVL, and Ohm's law, we find the currents:



Then, we find $v_o = 3Ri_{in}$

(b) Since v_o is independent of R_L, the output behaves as a perfect voltage source, and the output impedance is zero.

(c) The input voltage is zero because of the summing-point constraint, and the input impedance is zero.

(d) This is an ideal transresistance amplifier.

P14.27 The inverting amplifier is shown in Figure 14.4 in the text and the voltage gain is $A_v = -R_2/R_1$. Thus, to achieve a voltage gain magnitude of 10, we would select the nominal values such that $R_{2nom} = 10R_{1nom}$. However for 5% tolerance resistors, we have

$$R_{1\min} = 0.95R_{1\text{nom}} \qquad R_{1\max} = 1.05R_{1\text{nom}}$$

$$R_{2\min} = 0.95R_{2\text{nom}} \qquad R_{2\max} = 1.05R_{2\text{nom}}$$

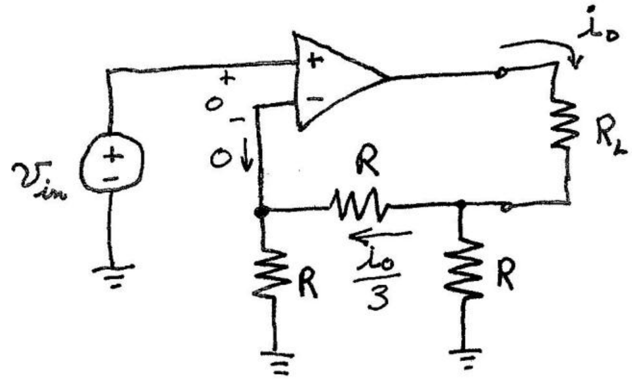
Thus we have

$$A_{v\min} = -\frac{R_{2\min}}{R_{1\max}} = -\frac{0.95R_{2\text{nom}}}{1.05R_{1\text{nom}}} = -9.0476$$

$$A_{v\max} = -\frac{R_{2\max}}{R_{1\min}} = -\frac{1.01R_{2\text{nom}}}{0.99R_{1\text{nom}}} = -11.0526$$

Thus, $|A_v| = 10$ plus or minus 5%.

P14.28 (a) Using the current-division principle, we find the currents as shown:



Then, KVL around the input loop gives $v_{in} = R(i_o / 3)$, which yields $i_o = 3v_{in} / R$.

(b) Since i_o is independent of R_L, the output behaves as a perfect current source, and the output impedance is infinite.

(c) The input current is zero because of the summing-point constraint, and the input impedance is infinite.

(d) This is an ideal transconductance amplifier.

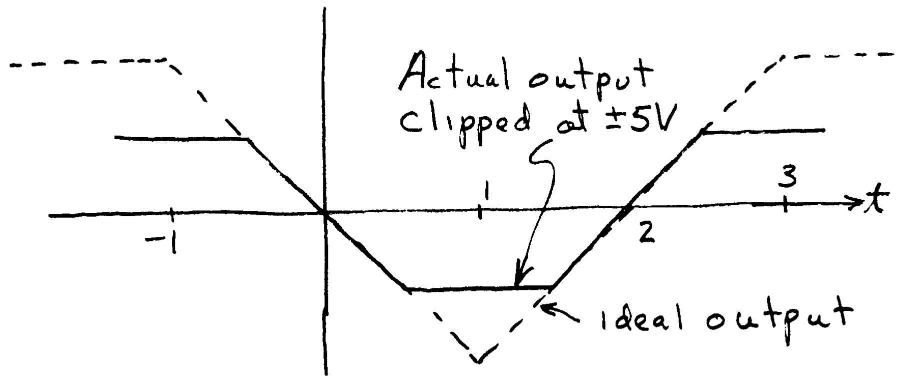
P14.29 (a) This is an inverting amplifier having $A_v = -R_2/R_1$ and $R_{in} = R_1$. The input power is $P_{in} = \frac{v_s^2}{R_{in}} = \frac{v_s^2}{R_1}$

The output power is $P_o = \frac{v_o^2}{R_L}$

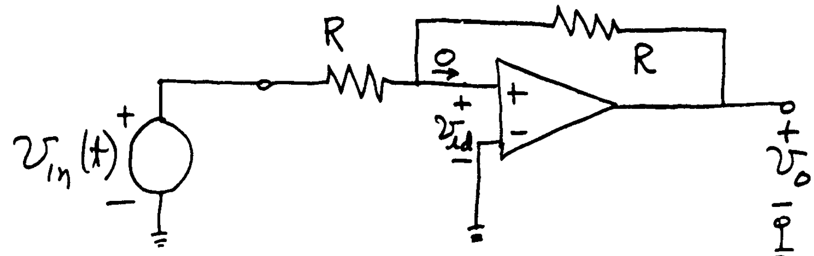
The power gain is $G = \frac{P_o}{P_{in}} = \frac{v_o^2/R_L}{v_s^2/R_1} = A_v^2 \frac{R_1}{R_L} = \frac{R_2^2}{R_1 R_L}$

(b) This is a noninverting amplifier having $i_{in} = 0$. Therefore $P_{in} = 0$, and $G = \infty$. Thus, the noninverting amplifier has the larger power gain.

P14.30* (a) This circuit has negative feedback. Assuming an ideal op amp, we have $v_o(t) = -v_{in}(t)$.



(b) This circuit has positive feedback. Therefore, the summing-point constraint does not apply.



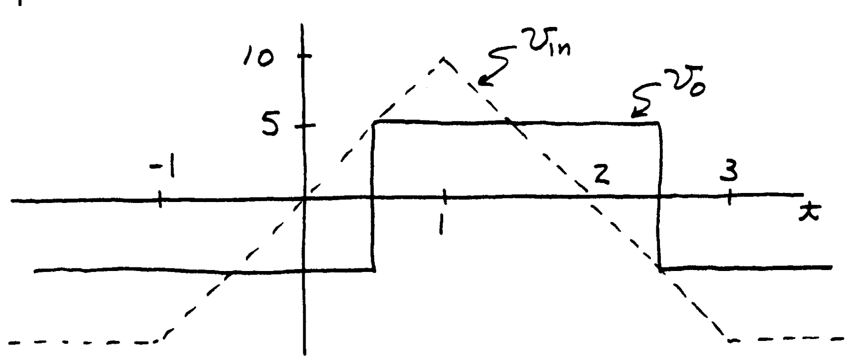
From the circuit, we can write

$$\frac{v_{id} - v_{in}}{R} + \frac{v_{id} - v_o}{R} = 0$$

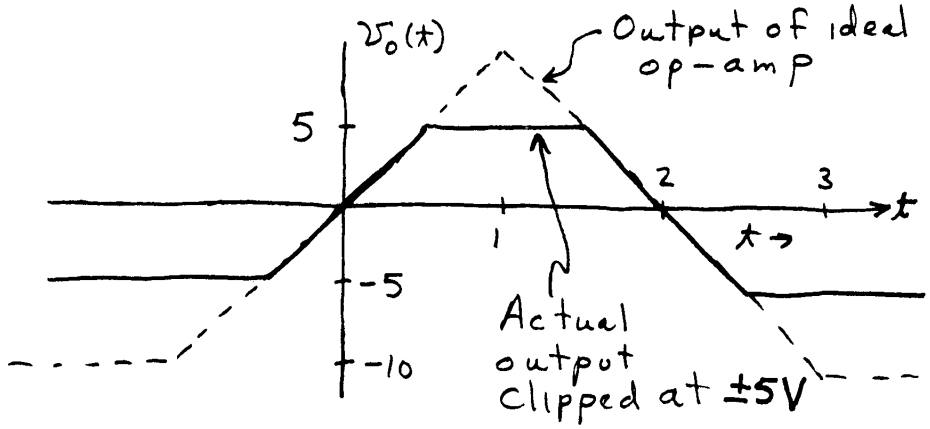
Solving for v_{id}, we have

$$v_{id} = \frac{v_o + v_{in}}{2}$$

If $v_{id} > 0$, then $v_o = +5$. On the other hand, if $v_{id} < 0$, then $v_o = -5$. The output waveform is

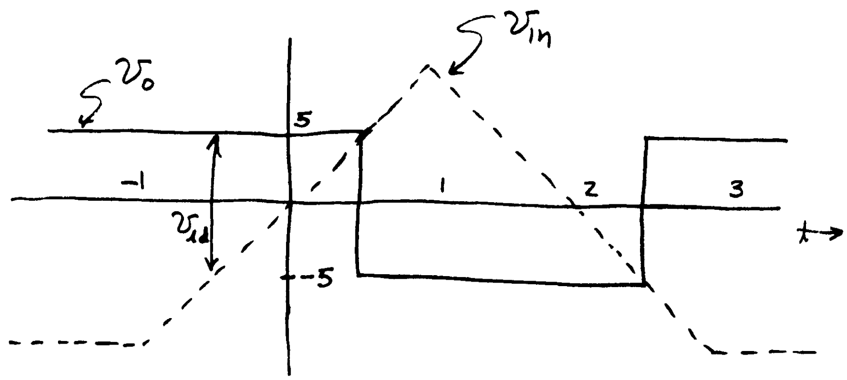


P14.31 (a) This circuit has negative feedback. It is the voltage follower and has unity gain except that the output voltage cannot exceed 5 V. The output waveform is:

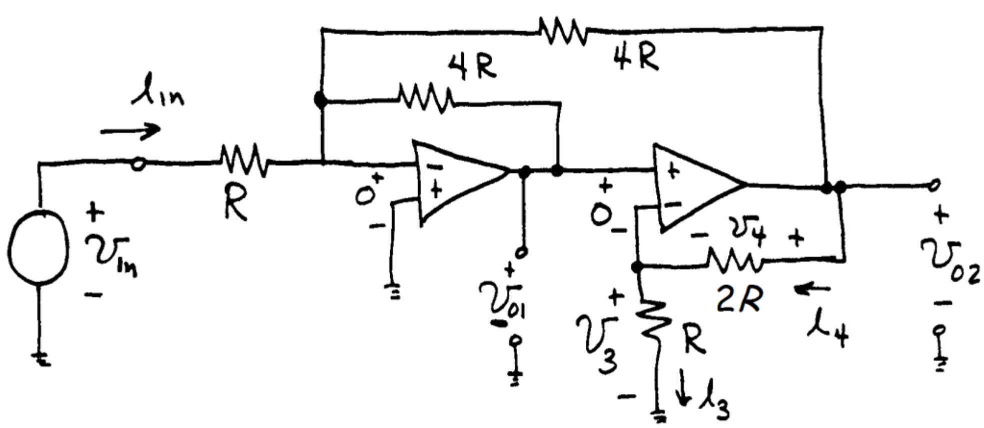


(b) This circuit has positive feedback, and v_o = +5 if the differential input voltage v_{id} is positive. On the other hand, v_o = -5 if v_{id} is negative. In this circuit, we have $v_{id} = v_o - v_{in}$

Thus, the output waveform is:



P14.32



From the circuit we can write:

$$v_{o1} = v_3$$

$$i_4 = i_3 = \frac{v_{o1}}{R}$$

Thus we have

$$v_4 = 2Ri_3 = 2v_3 = 2v_{o1}$$

$$v_{o2} = v_3 + v_4 = 3v_{o1} \qquad (1)$$

$$i_{in} + \frac{v_{o1}}{4R} + \frac{v_{o2}}{4R} = 0$$

$$i_{in} = \frac{v_{in}}{R}$$

$$\frac{v_{in}}{R} + \frac{v_{o1}}{4R} + \frac{v_{o2}}{4R} = 0 \qquad (2)$$

Using Equation (1) to substitute into Equation 2 and rearranging, we have

$$A_1 = \frac{v_{o1}}{v_{in}} = -1$$

$$A_2 = \frac{v_{o2}}{v_{in}} = \frac{3v_{o1}}{v_{in}} = 3A_1 = -3$$

P14.33 The noninverting amplifier is shown in Figure 14.11 in the text, and the voltage gain is $A_v = 1 + R_2/R_1$. Thus, to achieve a voltage gain magnitude of 10, we would select the nominal values such that $R_{2nom} = 9R_{1nom}$ However, for 5%-tolerance resistors, we have

$$R_{1\min} = 0.95R_{1nom} \qquad R_{1\max} = 1.05R_{1nom}$$

$$R_{2\min} = 0.95R_{2nom} \qquad R_{2\max} = 1.05R_{2nom}$$

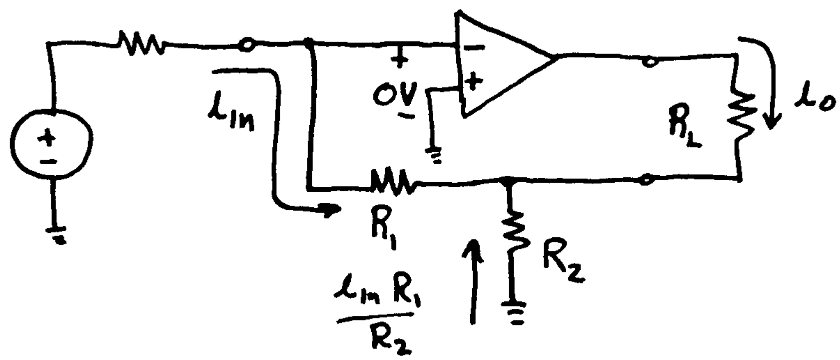
Thus we have

$$A_{v\min} = 1 + \frac{R_{2\min}}{R_{1\max}} = 1 + \frac{0.95R_{2nom}}{1.05R_{1nom}} = 9.1429$$

$$A_{v\max} = 1 + \frac{R_{2\max}}{R_{1\min}} = 1 + \frac{1.05R_{2nom}}{0.95R_{1nom}} = 10.9474$$

Thus, $|A_v| = 10 \pm 9\%$.

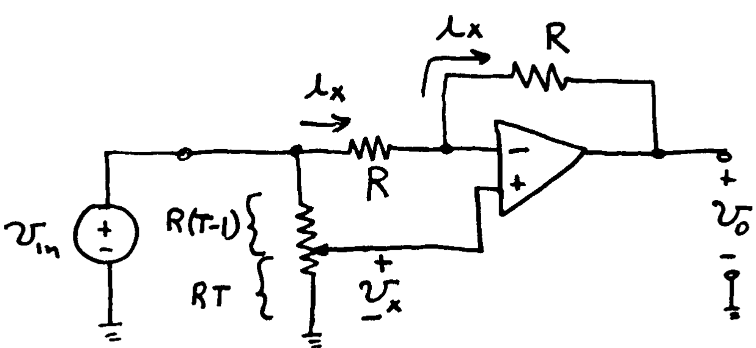
P14.34* The circuit diagram is:



$$i_o = -\left(1 + \frac{R_1}{R_2}\right) i_{in}$$

Because of the summing-point constraint, we have $v_{in} = 0$. Thus, $R_{in} = 0$. Because the output current is independent of R_L, the output impedance is infinite. In other words, looking back from the load terminals, the circuit behaves like an ideal current source.

P14.35



By the voltage-division principle, we have

$$v_x = \frac{RT}{RT + (1-T)R} v_{in} = Tv_{in}$$

Then, we can write

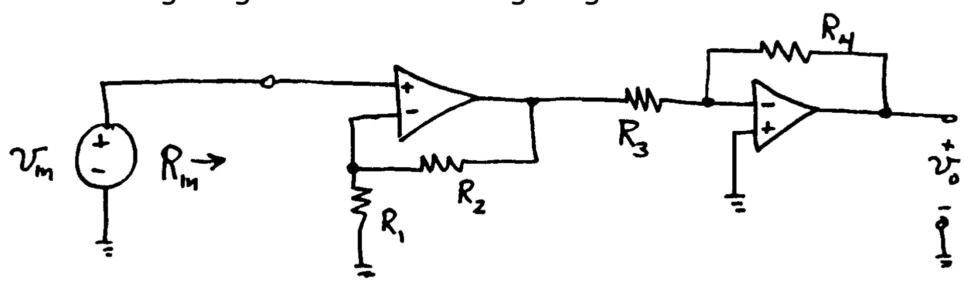
$$i_x = \frac{v_{in} - v_x}{R} = \frac{v_{in}(1-T)}{R}$$

$$\begin{aligned} v_o &= -Ri_x + v_x \\ &= -v_{in}(1-T) + Tv_{in} \\ &= v_{in}(2T - 1) \end{aligned}$$

Thus, as T varies from 0 to unity, the circuit gain varies from -1 through to 0 to +1.

P14.36 Very small resistances lead to excessively large currents, possibly exceeding the capability of the op amp, creating excessive heat or overloading the power supply.
Very large resistances lead to instability due to leakage currents over the surface of the resistors and circuit board. Stray pickup of undesired signals is also a problem in high-impedance circuits.

P14.37* To achieve high input impedance and an inverting amplifier, we cascade a noninverting stage with an inverting stage:



The overall gain is:

$$A_v = -\frac{R_1 + R_2}{R_1} \times \frac{R_4}{R_3}$$

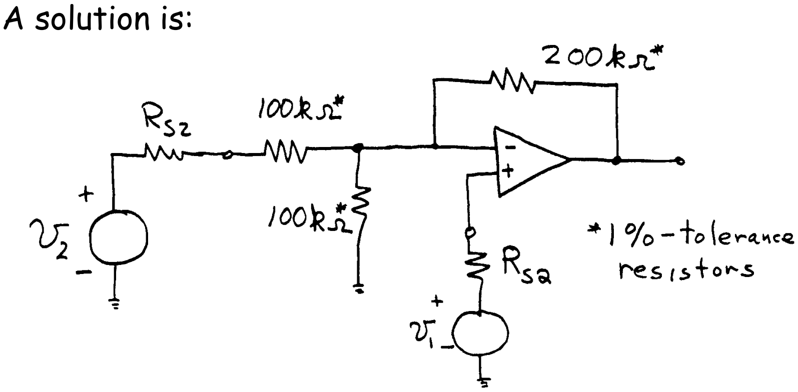
Many combinations of resistance values will achieve the given specifications. For example:

$R_1 = \infty$ and $R_2 = 0$. (Then the first stage becomes a voltage follower.) This is a particularly good choice because fewer resistors affect the overall gain, resulting in small overall gain variations.

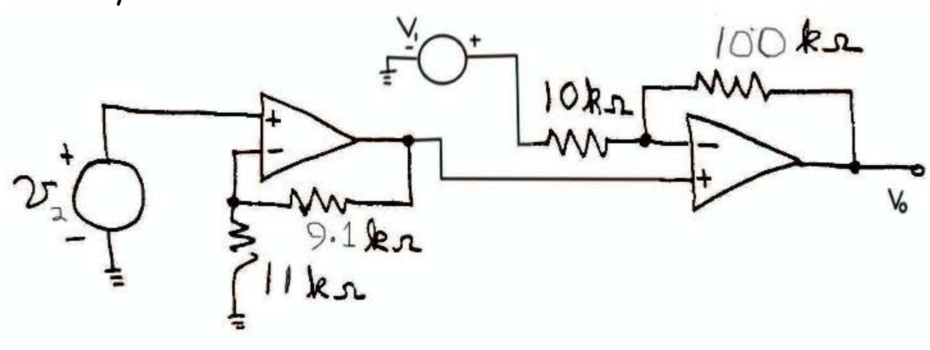
R_4 = 100 kΩ, 5% tolerance.

R_3 = 10 kΩ, 5% tolerance.

P14.38* A solution is:

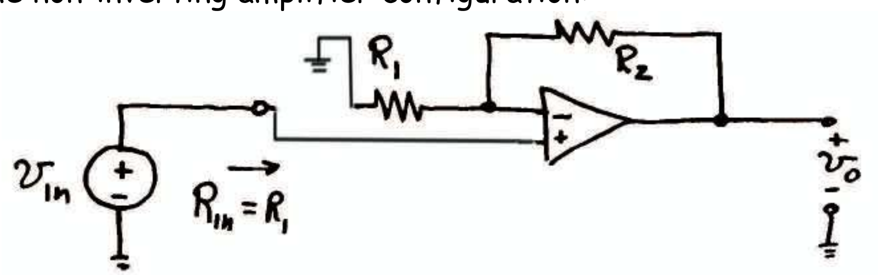


P14.39* One possibility is:



All resistors are $\pm 1\%$ tolerance.

P14.40 Use the non-inverting amplifier configuration:



Pick $R_{2nom} = 9R_{1nom}$ to achieve the desired gain magnitude.

Pick $R_{1nom} > 100$ kΩ to achieve input impedance greater than 100 kΩ.

Pick R_{1nom} and $R_{2nom} < 10$ MΩ because higher values are impractical.

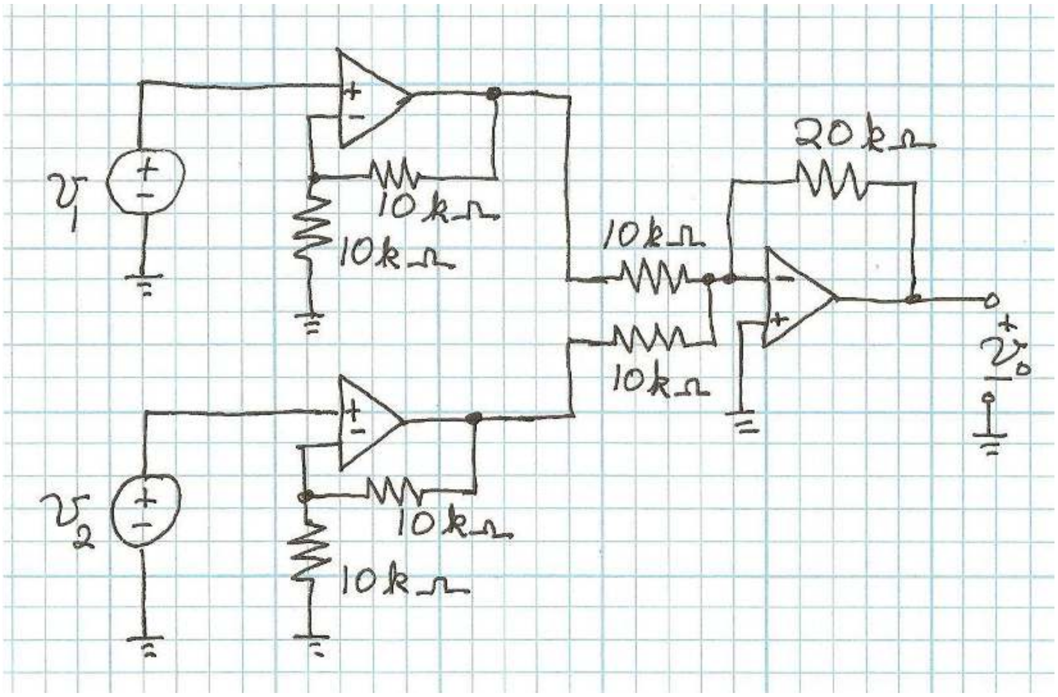
Many combinations of values will meet the specifications. For example:

(a) Use 5% tolerance resistors. R_1 = 100 kΩ and R_2 = 910 kΩ.

(b) Use 1% tolerance resistors. R_1 = 100 kΩ and R_2 = 910 kΩ.

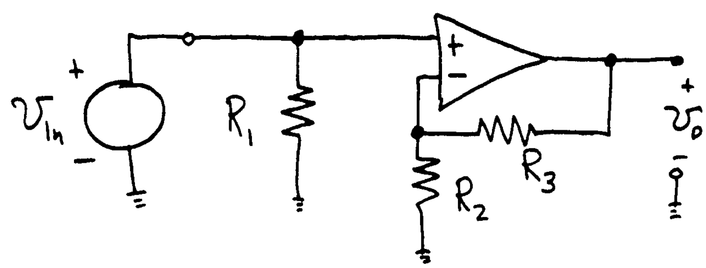
(c) R_2 = 910 kΩ 1% tolerance. R_1 = 95.3 kΩ 1% tolerance fixed resistor in series with a 10-kΩ adjustable resistor. After constructing the circuit, adjust to achieve the desired gain magnitude.

P14.41 Here is one answer in which all of the resistors have 1% tolerance:



Many other correct answers exist.

P14.42 We use a noninverting amplifier and place a resistor in parallel with the input terminals to achieve the desired input impedance.



R_1 = 2 kΩ, 1% tolerance.

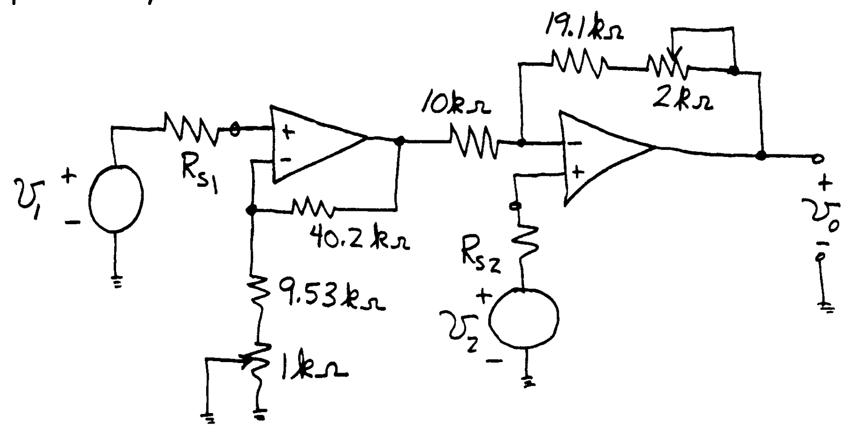
Many combinations of values for R_2 and R_3 will meet the given specifications. For example:

R_2 = 10 kΩ, 1% tolerance.

R_3 = 40.2 kΩ, 1% tolerance.

(These values result in a nominal gain of 5.02, which is within the specified range even with the resistance tolerances taken into account.)

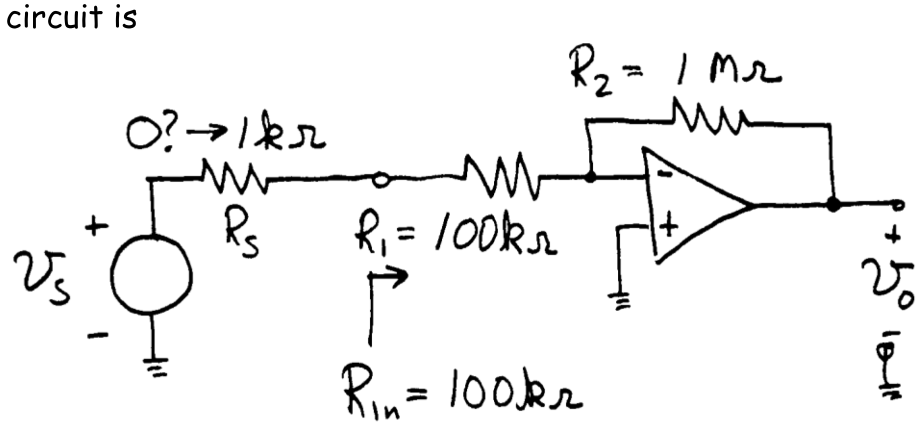
P14.43 To avoid excessive gain variations because of changes in the source resistances, we need to have input resistances that are much greater than the source resistances. Many correct answers exist. Here is one possibility:



The fixed resistors should be specified to have a tolerance of $\pm 1\%$ because they are more stable in value than 5% tolerance resistors. The adjustment procedure is:

1. Set $v_1 = 0$ and $v_2 = +1$ V. Then, adjust the 2-kΩ potentiometer to obtain $v_o = 3$ V.
2. Set $v_1 = 1$ V and $v_2 = 0$. Then, adjust the 1-kΩ potentiometer to obtain $v_o = -10$ V.

P14.44 To avoid excessive variations in $A_{vs} = v_o/v_s$ because of changes in R_s, we need to have $R_{in} >> R_s$. R_{in} =100 kΩ is sufficiently large. Thus, a suitable circuit is



R_1 and R_2 should be 1% tolerance resistors.

P14.45 Imperfections of real op amps in their linear range of operation include:

1. Finite input impedance.
2. Nonzero output impedance.
3. Finite dc open-loop gain.
4. Finite open-loop bandwidth.

P14.46* Equation 14.34 states:

$$f_t = A_{0CL} f_{BCL} = A_{0OL} f_{BOL}$$

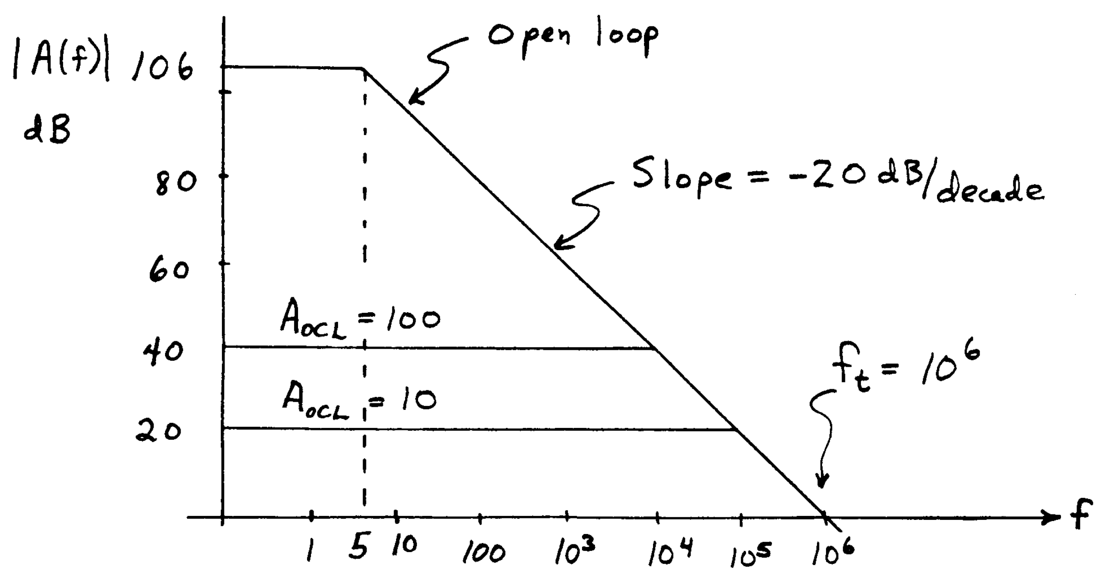
Thus, for A_{0CL} = 10, we have

$$f_{BCL} = \frac{f_t}{A_{0CL}} = \frac{15 \text{ MHz}}{10} = 1.5 \text{ MHz}$$

For A_{0CL} = 100, we have

f_{BCL} = 150 kHz

P14.47*



P14.48 Equation 14.23 gives the open-loop gain as a function of frequency:

$$A_{OL}(f) = \frac{A_{0OL}}{1 + j(f/f_{BOL})} \quad \frac{5 \ \times 10^6}{1 + j(f/25)}$$

For f = 100 Hz, we have

$$|A_{OL}(100)| = \frac{5 \times 10^6}{\sqrt{1 + (100/25)^2}} = 1.2127 \times 10^6$$

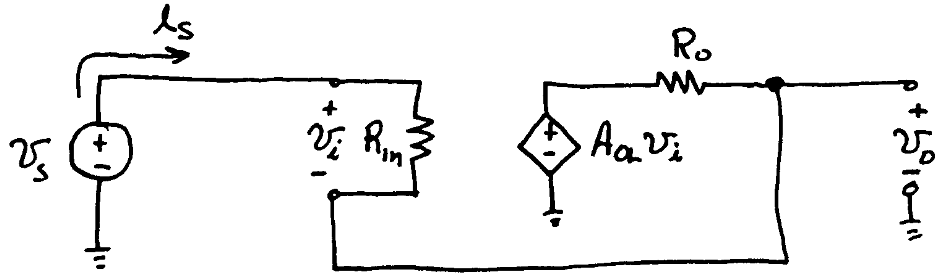
Similarly,for f = 10kHz, we have

$$|A_{OL}(f)| = \frac{5\times10^6}{\sqrt{1+(10000/25)^2}} = 12.5\times10^3$$

Similarly,for f = 5 MHz, we have

$$|A_{OL}(f)| = \frac{5\times10^6}{\sqrt{1+(5000000/25)^2}} = 250$$

P14.49 (a)



From the circuit, we can write:

$$v_s = R_{in}i_s + R_o i_s + A_{OL}(R_{in}i_s)$$
$$v_o = R_o i_s + A_{OL}(R_{in}i_s)$$

Dividing the respective sides of the previous equations yields:

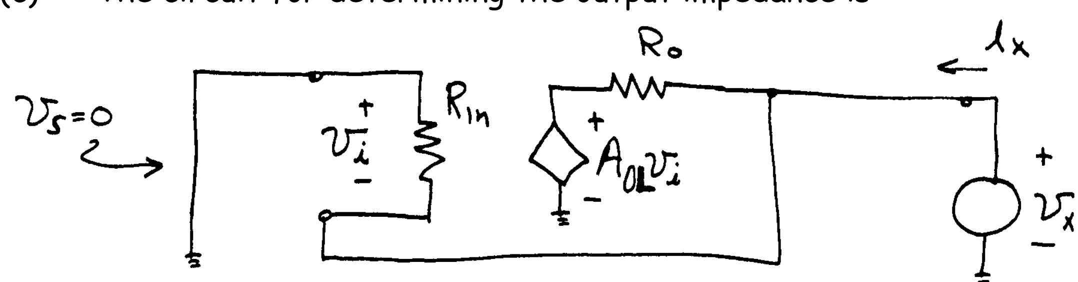
$$A_{vo} = \frac{v_o}{v_s} = \frac{R_o + A_{OL}R_{in}}{R_{in} + R_o + A_{OL}R_{in}}$$

Substituting values, we obtain:

$$A_{vo} = \frac{25 + 10^5\times10^6}{10^6 + 25 + 10^5\times10^6}$$
$$= 0.99999 \text{ (compared to unity for an ideal op amp)}$$

(b) $$Z_{in} = \frac{v_s}{i_s} = R_{in} + R_o + A_{OL}R_{in}$$
$$= 10^6 + 25 + 10^5\times10^6$$
$$= 10^{11}\ \Omega \text{ (compared to } \infty \text{ for an ideal op amp)}$$

(c) The circuit for determining the output impedance is:



$$v_i = -v_x$$
$$i_x = \frac{v_x}{R_{in}} + \frac{v_x - A_{OL}v_i}{R_o}$$
$$Z_o = \frac{v_x}{i_x} = \frac{1}{\dfrac{1}{R_{in}} + \dfrac{1 + A_{OL}}{R_o}}$$

$Z_o = 2.5 \times 10^{-4}\ \Omega$ (versus Z_o = 0 for an ideal op amp)

P14.50 Equation 14.32 gives the closed-loop gain as a function of frequency:

$$A_{CL}(f) = \frac{A_{0CL}}{1 + j(f/f_{BCL})}$$

However, the dc closed-loop gain is given as 10 so we have

$$A_{CL}(f) = \frac{10}{1 + j(f/f_{BCL})}$$

For f = 5 kHz, we have

$$|A_{CL}| = \frac{10}{\sqrt{1 + (5000/f_{BCL})^2}} = 9.5$$

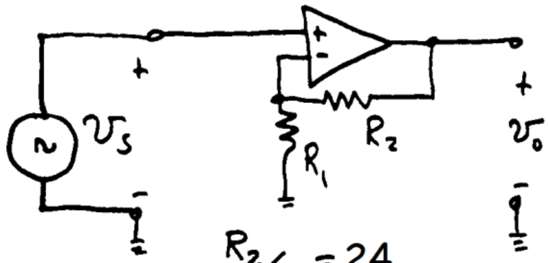
Solving, we find $f_{BCL} = 15.21$ kHz. Then, the gain bandwidth product is
$f_t = A_{0CL}f_{BCL} = 152.1\text{ kHz} = A_{0OL}f_{BOL}$

P14.51 Equation 14.32 gives the closed-loop gain as a function of frequency:

$$A_{CL}(f) = \frac{A_{0CL}}{1 + j(f/f_{BCL})}$$

The phase shift is $-\arctan(f / f_{BCL})$. Thus at 100 kHz, we have $10^\circ = \arctan[(10^5)/f_{BCL}]$which yields $f_{BCL} = 567$ kHz. Then, the gain bandwidth product is $f_t = A_{0CL}f_{BCL} = 2.83\text{ MHz} = A_{0OL}f_{BOL}$.

P14.52 Alternative 1:



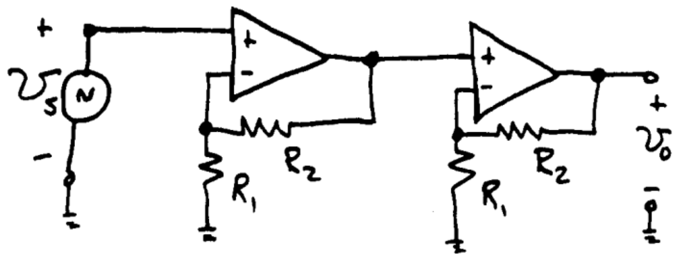
$R_2/R_1 = 24$

$$f_{BCL} = \frac{f_t}{A_{0CL}} = \frac{10^6}{25} = 40\text{ kHz}$$

$$A_{CL}(f) = \frac{25}{1 + jf/(4\times 10^4)}$$

The closed-loop bandwidth is $f_{BCL} = 40\text{ kHz}$.

Alternative 2:



$R_2/R_1 = 4$

For each stage, we have $f_{BCL} = \dfrac{f_t}{A_{0CL}} = \dfrac{10^6}{5} = 200\text{ kHz}$ and the

overall gain as a function of frequency is:

$$A_{CL}(f) = \frac{5}{1 + jf/(2\times 10^5)}$$

The overall gain is

$$A(f) = \frac{25}{\left(1 + jf/(2\times 10^5)\right)^2}$$

To find the overall 3-dB bandwidth, we have

$$\left|A(f_{3dB})\right| = \frac{25}{\sqrt{2}} = \frac{25}{1 + \left(f_{3dB}/(2\times 10^5)\right)^2}$$

Solving, we find that

$$f_{3dB} = 128.7\text{ kHz}$$

Thus, the two-stage amplifier has wider bandwidth.

P14.53 (a) From the circuit (shown in Figure P14.53 in the text), we can write:

$$\frac{v_s + v_i}{R_1} + \frac{v_o + v_i}{R_2} + \frac{v_i}{R_{in}} = 0$$

$$\frac{v_o + v_i}{R_2} + \frac{v_o - A_{OL}v_i}{R_o} = 0$$

Algebra results in:

$$A_{vo} = \frac{v_o}{v_s} = \frac{-R_2}{R_1\left[1 + \left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_{in}}\right)\frac{R_oR_2 + R_2^2}{A_{OL}R_2 - R_o}\right]}$$

Substituting values, we obtain:

$A_{vo} = -9.9989$ (compared to -10 for an ideal op amp)

(b) From the circuit, we can write:

$$v_s = R_i i_s - v_i$$

$$v_i + (R_2 + R_o)(v_i/R_{in} + i_s) + A_{OL}v_i = 0$$

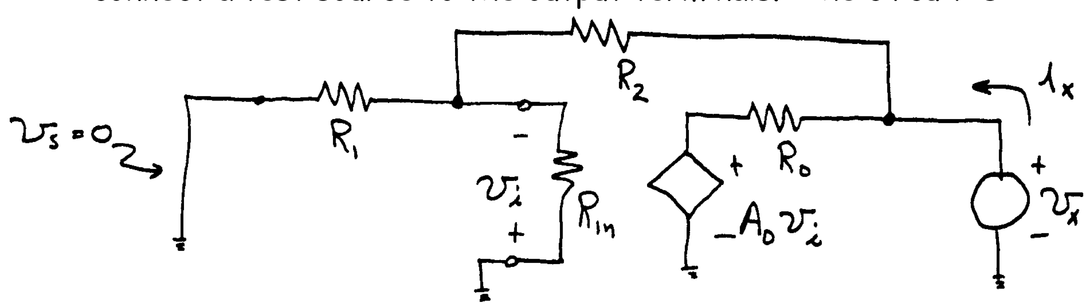
Algebra results in

$$Z_{in} = \frac{v_s}{i_s} = R_1 + \frac{R_2 + R_o}{1 + A_{OL} + (R_2 + R_o)/R_{in}}$$

Substituting values, we obtain:

$Z_{in} = 1.0001\ \mathrm{k\Omega}$ (compared to 1 kΩ for an ideal op amp

(c) To find the output impedance, we zero the input source and connect a test source to the output terminals. The circuit is:



$$v_i = -\frac{R'_{in}}{R_2 + R'_{in}}v_x \qquad \text{where } R'_{in} = \frac{1}{1/R_1 + 1/R_{in}}$$

$$i_x = \frac{v_x}{R_2 + R'_{in}} + \frac{v_x - A_{OL}v_i}{R_o}$$

$$Z_o = \frac{v_x}{i_x} = \frac{1}{\dfrac{1}{R_2 + R'_{in}} + \dfrac{1}{R_o} + \dfrac{A_{OL}R'_{in}}{R_o(R_2 + R'_{in})}}$$

Substituting values, we obtain:

$Z_o = 2.75 \times 10^{-3}\ \Omega$ versus $Z_o = 0$ for an ideal op amp

P14.54 The nonlinear limitations of real op amps include:

1. Limited output voltage magnitude.
2. Limited output current magnitude.
3. Limited rate of change of output voltage. (Slew rate.)

P14.55* (a) $f_{FP} = \dfrac{SR}{2\pi V_{om}} = \dfrac{10^7}{2\pi 10} = 159\ \text{kHz}$

(b) $V_{om} = 10\ \text{V}$. (It is limited by the maximum output voltage capability of the op amp.)

(c) In this case, the limit is due to the maximum current available from the op amp. Thus, the maximum output voltage is:

$$V_{om} = 20\ \text{mA} \times 100\ \Omega = 2\ \text{V}$$

(d) In this case, the slew-rate is the limitation.

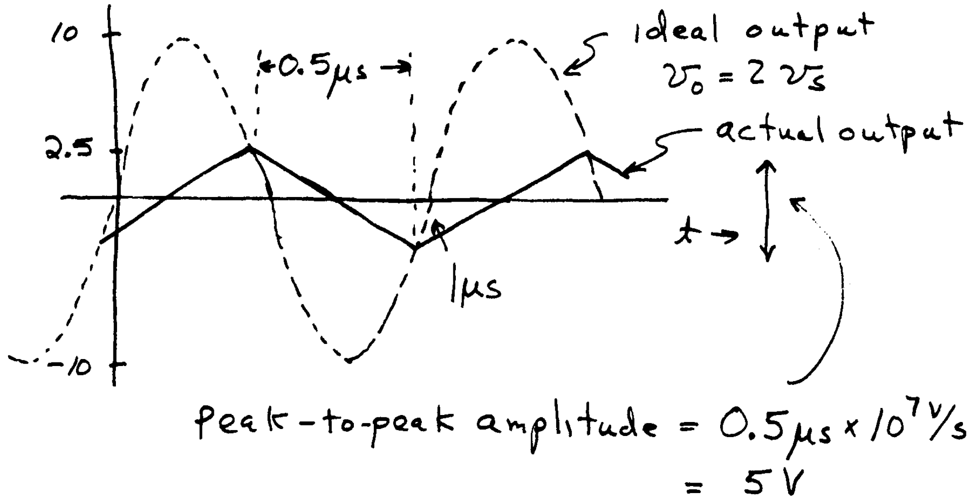
$$v_o(t) = V_{om}\sin(\omega t)$$

$$\frac{dv_o(t)}{dt} = \omega V_{om}\cos(\omega t)$$

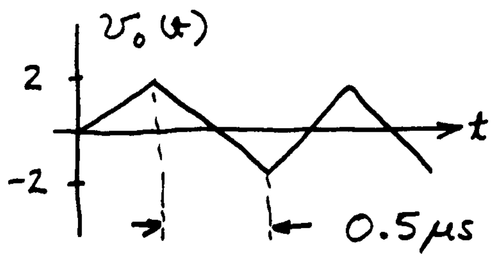
$$\left|\frac{dv_o(t)}{dt}\right|_{max} = \omega V_{om} = SR$$

$$V_{om} = \frac{SR}{\omega} = \frac{10^7}{2\pi 10^6} = 1.59\ \text{V}$$

(e)



P14.56* The output waveform is



The rate of change is:

$$SR = (4\text{ V})/(0.5\ \mu s) = 8\ \text{V}/\mu s$$

P14.57 The full-power bandwidth of an op amp is the range of frequencies for which the op amp can produce an undistorted sinusoidal output with peak amplitude equal to the guaranteed maximum output voltage.

P14.58 If the ideal output, with a sinusoidal input signal, greatly exceeds the full-power bandwidth, the output becomes a triangular waveform. The slope of the triangle is equal to the maximum slew rate in magnitude. The triangle goes from the negative peak to the positive peak in half of the period. Thus, the peak-to-peak amplitude is

$$V_{p-p} = SR \times T/2 = 10^7 \times 0.5 \times 10^{-6} = 5\ \text{V}$$

P14.59 The desired output voltage is

$$v_o(t) = V_{om}\sin(\omega t)$$

and the rate of change of the output is

$$\frac{dv_o(t)}{dt} = \omega V_{om} \cos(\omega t)$$

The maximum rate of change of the output is

$$\left|\frac{dv_o(t)}{dt}\right|_{\max} = \omega V_{om}$$

Thus, we require the slew rate to be at least as large as the maximum rate of change of the output voltage.

$$SR = \omega V_{om} = 2\pi(2\times 10^5)(5) = 7.28\ \text{V}/\mu\text{s}$$

P14.60 To avoid slew-rate distortion, the op-amp slew-rate specification must exceed the maximum rate of change of the output-voltage magnitude. For the gain and input given in the problem, the output voltage is

$$\begin{aligned} v_o(t) &= 0 \quad t \le 0 \\ &= 5t \quad t \ge 0 \end{aligned}$$

The rate of change is

$$\begin{aligned} \frac{dv_o(t)}{dt} &= 0 \quad t \le 0 \\ &= 5 \quad t \ge 0 \end{aligned}$$

The maximum value occurs at $t \ge 0$, and is 5 V/ms. Thus, the required minimum slew-rate specification is 5 V/ms.

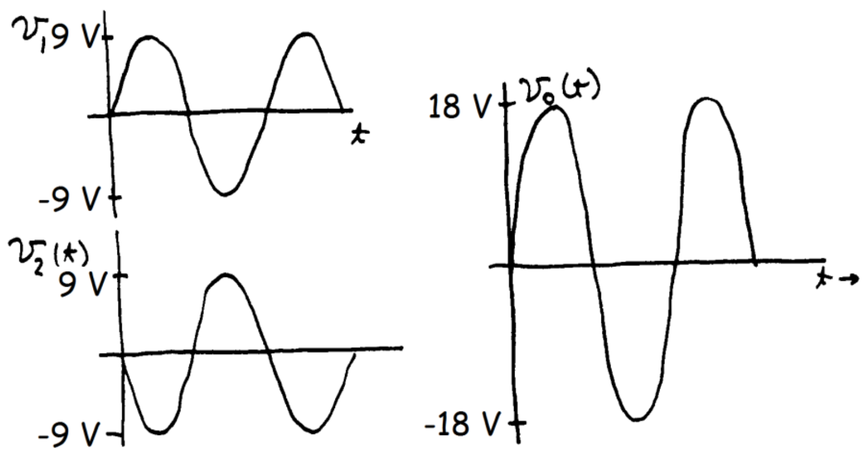
P14.61 (a) One op amp (the lower one in the Figure) is configured as an inverting amplifier with a gain of -3 and the other op amp (the top one in the figure) is configured as a noninverting amplifier with a gain +3. Thus, we can write:

$$v_2(t) = -3v_s(t)$$

$$v_1(t) = 3v_s(t)$$

$$v_o(t) = v_1(t) - v_2(t) = 6v_s(t) \qquad A_v = 6$$

(b)



(c) The peak value of $v_o(t)$ at the threshold of clipping is 26 V.

P14.62 To avoid slew-rate distortion, the op-amp slew-rate specification must exceed the maximum rate of change of the output-voltage magnitude. For a voltage follower, the gain is unity. For the input given in the problem, the output voltage is

$$v_o(t) = 0 \quad t \le 0$$
$$= t^3 \quad 0 \le t \le 2$$
$$= 8 \quad 2 \le t$$

The rate of change is

$$dv_o(t)/dt = 0 \quad t \le 0$$
$$= 3t^2 \quad 0 \le t \le 2$$
$$= 0 \quad 2 < t$$

The maximum value occurs at t = 2, and is 12 V/μs. Thus, the required minimum slew-rate specification is 12 V/μs or 12×10^6 V/s.

P14.63 (a) $$f_{FP} = \frac{SR}{2\pi V_{om}} = \frac{5 \times 10^6}{2\pi(5)} = 159.1\,\text{kHz}$$

(b) The limit on peak output voltage is due to the current limit of the op amp. Because R_2 is much greater than R_L, the current through R_2 can be neglected. Thus, we have:

$$V_{om} = 20\,\text{mA} \times R_L = 1.0\,\text{V}$$

(c) In this case, $V_{om} = 5\text{ V}$. (This is the maximum voltage that the op amp can achieve.)

(d) In this case, the slew rate limits the maximum voltage.

$$V_{om} = \frac{SR}{2\pi f} = \frac{5\times 10^6}{2\pi 10^6} = 0.796\text{ V}$$

P14.64* See Figure 14.29 in the text.

P14.65 A FET-input op amp usually has much lower values of bias current and offset current than a BJT-input op amp.

P14.66 The dc imperfections are bias current, offset current, and offset voltage. The net effect is to add a constant (dc) term to the desired output signal. Often this is undesirable.

P14.67* The worst-case outputs due to the offset voltage are:

$$V_{o,voff} = V_{off}\left(1 + \frac{R_2}{R_1}\right) = \pm 44\text{ mV}$$

For the bias current, the worst case output voltages are:

$$V_{o,bias} = R_2 I_B = 10\text{ mV and } 20\text{ mV}$$

For the offset current, the worst-case output voltages are:

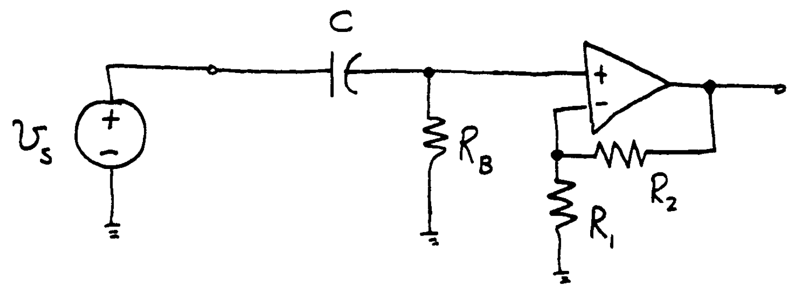
$$V_{o,ioff} = R_2\left(I_{off}/2\right) = \pm 2.5\text{ mV}$$

Due to all of the imperfections, the extreme output voltages are:

$$V_{o.\max} = 44 + 20 + 2.5 = 66.5\text{ mV}$$
$$V_{o,\min} = -44 + 10 - 2.5 = -36.5\text{ mV}$$

P14.68 The circuit shown in Figure P14.67 is a poor design because no dc path is provided for the bias current flowing into the noninverting input terminal. The bias current would charge the capacitance eventually resulting in a large voltage that would exceed the linear range of the op amp.

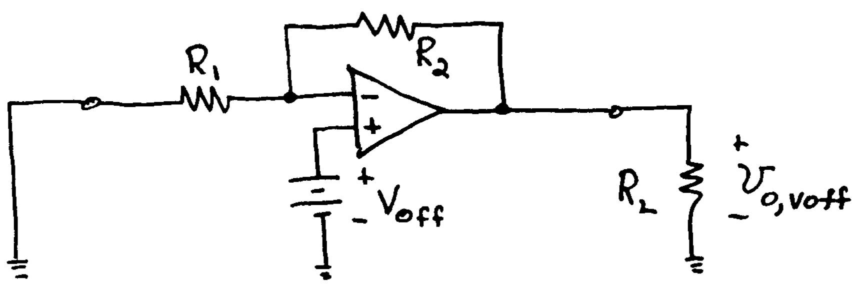
The solution is to add a resistance as shown:



To minimize the effect of the bias currents, we should select:

$$R_B = \frac{1}{1/R_1 + 1/R_2} = 5\ \text{k}\Omega$$

P14.69 (a) The circuit with the signal source zeroed and including the offset voltage source is:

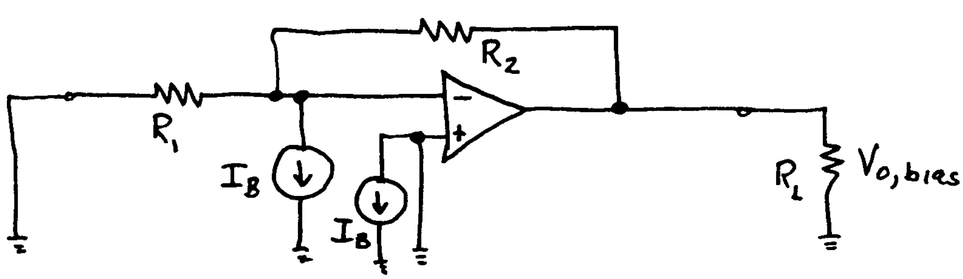


The output voltage is:

$$V_{o,voff} = (1 + R_2/R_1)V_{off} = 11\, V_{off}$$

Thus to keep $V_{o,voff}$ less than 50 mV in magnitude, we need an op amp with V_{off} that is less than 4.55 mV.

(b) The circuit with only the bias current sources is:

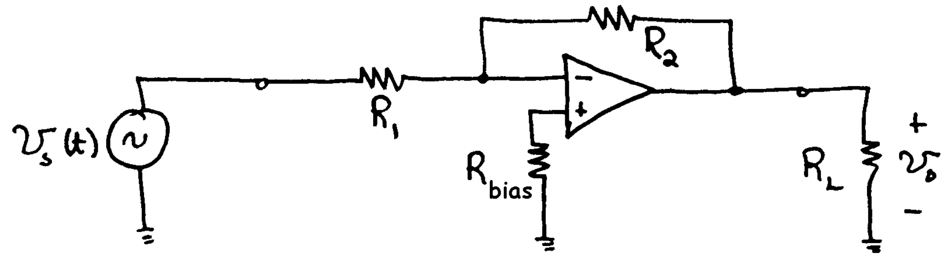


The output voltage is:

$$V_{o,bias} = R_2 I_{bias}$$

Thus, to keep $V_{o,bias}$ less than 50 mV in magnitude, we need an op amp with I_{bias} less than 0.5 μA.

(c) If we add a resistance $R_{bias} = 1/(1/R_1 + 1/R_2) = 9.09\ \text{k}\Omega$ in series with the noninverting input terminal, the effects of the bias currents will cancel. The circuit is:



(d) With the resistance of part (c) in place, the output voltage due to the offset current is:

$$V_{o,ioff} = R_2 I_{off}$$

[The derivation of this result is given earlier in the solution for Exercise 14.14(d).] Thus, to keep $V_{o,ioff}$ less than 50 mV in magnitude, we need an op amp with I_{off} less than 0.5 μA.

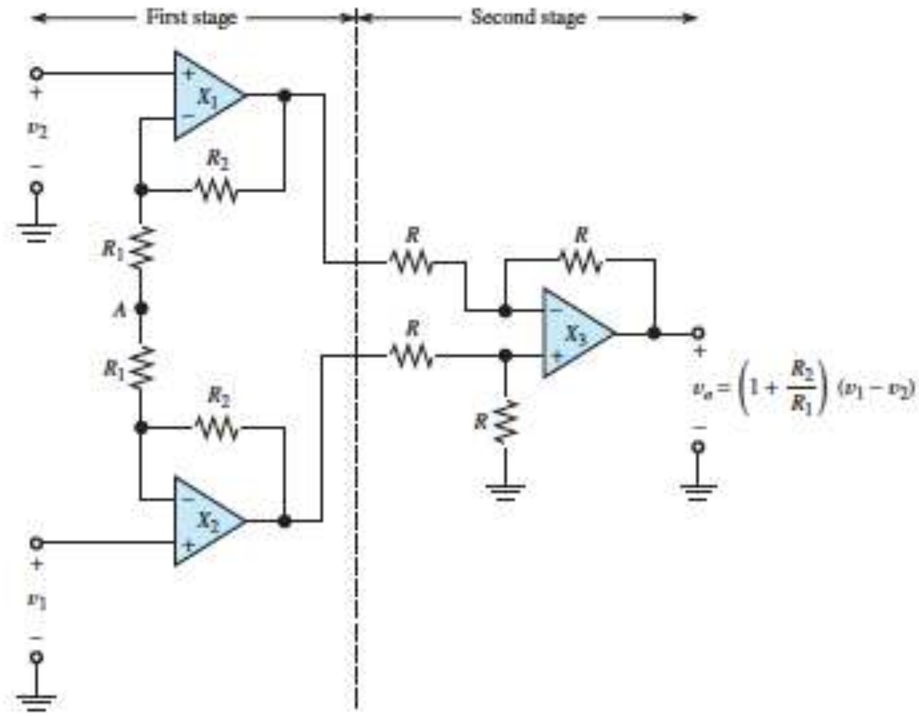
P14.70 The function of a differential amplifier is to produce an output that is proportional to the differential input component and is independent of the common-mode component of the input.

P14.71* The circuit diagram is shown in Figure 14.33 in the text. To achieve a nominal gain of 10, we need to have $R_2 = 10R_1$. Values of R_1 ranging from about 1 kΩ to 100 kΩ are practical. A good choice of values is R_1 = 10 kΩ and R_2 = 100 kΩ.

P14.72 The circuit diagram is shown in Figure 14.34 in the text. To achieve a nominal gain of 10, we need to have $R_2 = 9R_1$. Values of R_1 ranging from about 1 kΩ to 100 kΩ are good. A good choice of values is R_1 = 20 kΩ andR_2 = 180 kΩ. Any value of R in the range from 1 kΩ to 1 MΩ is acceptable.

P14.73 (a) The differential and common-mode components of the input signal are:

$$v_{id} = v_1 - v_2 = 5\sin(1200\pi t) + 5\sin(2000\pi t)$$
$$v_{icm} = \tfrac{1}{2}(v_1 + v_2) = 2.5\sin(1200\pi t) + \cos(120\pi t) - 2.5\sin(2000\pi t)$$



(b) As discussed in the book, the first-stage gain for the differential signal is $1 + R_2/R_1$ which for the values given is 3. On the other hand, the first-stage gain for the common-mode component is unity. Thus, the output voltages are:

$$v_{X1out} = 15\cos(2000\pi t) + 6\cos(120\pi t)$$

$$v_{X1out} = -15\cos(2000\pi t) + 6\cos(120\pi t)$$

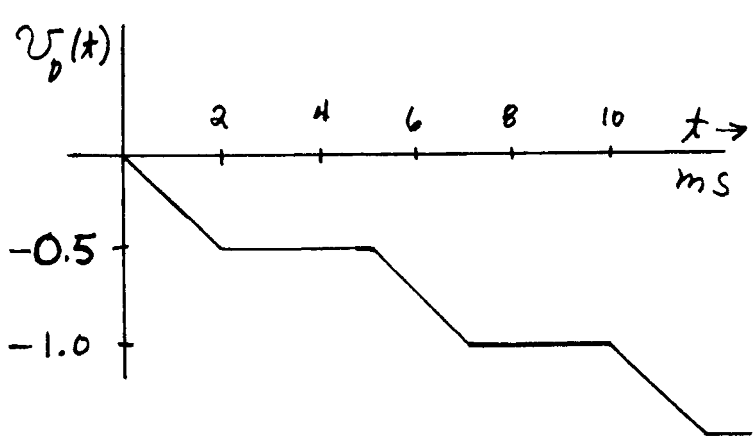
For solution P14.73(c) see end of document.

P14.74 A running time integral is the integral for which the upper limit of integration is time.

P14.75* This is an integrator circuit, and the output voltage is given by:

$$v_o(t) = -\frac{1}{RC}\int_0^t v_{in}(t)dt$$

$$v_o(t) = -50\int_0^t v_{in}(t)dt$$



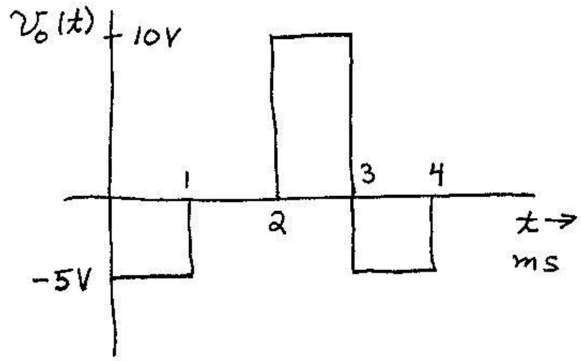
Each pulse reduces v_o by 0.5 V. Thus, 20 pulses are required to produce v_o = -10V.

P14.76 This is a differentiator circuit, and the output is given by:

$$v_o(t) = -RC\frac{dv_{in}(t)}{dt}$$

$$= -10^{-3}\frac{dv_{in}(t)}{dt}$$

A sketch of $v_o(t)$ versus is:



P14.77 Let $x(t) =$ displacement in meters. Then, we have

$$v_{in}(t) = 100x(t)$$

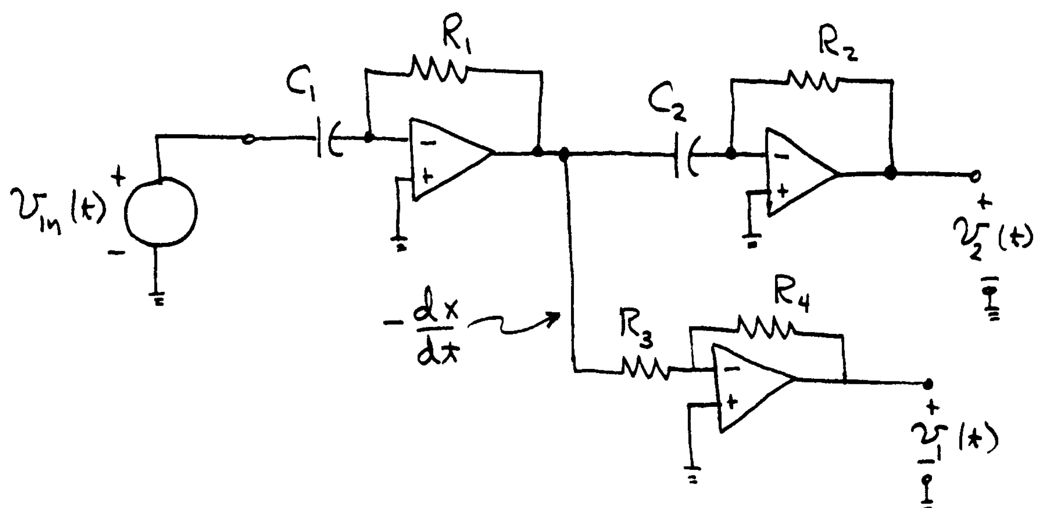
and we want

$$v_1(t) = \frac{dx(t)}{dt} = 0.01\frac{dv_{in}(t)}{dt}$$

and

$$v_2(t) = \frac{d^2x(t)}{dt^2} = \frac{dv_1(t)}{dt}$$

A circuit that produces the desired voltages is:



We need $R_1C_1 = 0.01$, $R_2C_2 = 1$, and $R_3 = R_4$. Suitable component values are:

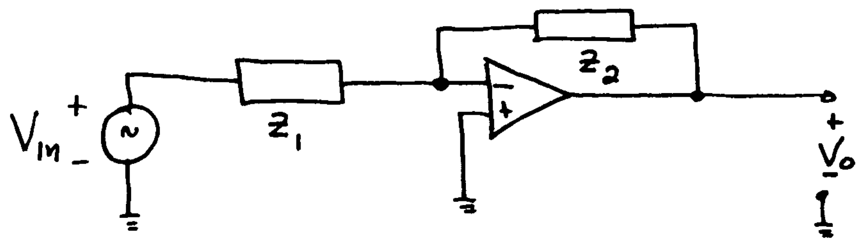
$$R_1 = R_2 = 1\ \text{M}\Omega \qquad C_1 = 0.01\ \mu\text{F}$$
$$C_2 = 1.0\ \mu\text{F} \qquad R_3 = R_4 = 10\ \text{k}\Omega$$

P14.78 Ideally, an active filter should:

1. Contain few components.
2. Have a transfer function that is insensitive to component tolerances.
3. Place modest demands on the op amp's gain–bandwidth product, output impedance, slew rate, and other specifications.
4. Be easily adjusted.
5. Require a small spread of component values.
6. Allow a wide range of useful transfer functions to be realized.

Passive filters are filters composed of passive components such as resistors and capacitors. Filters composed of op amps, resistors, and capacitors are said to be active filters . In many respects, active filters have improved performance compared to passive circuits.

P14.79* Both of the circuits are of the form:



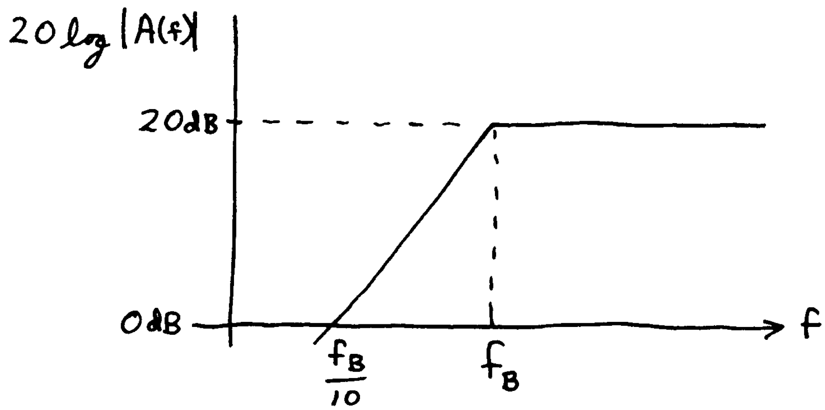
This is the inverting amplifier configuration and the gain is

$$A(f) = \frac{\mathbf{V}_o}{\mathbf{V}_{in}} = -\frac{Z_2}{Z_1}$$

(a) $$A(f) = -\frac{10R}{R + \dfrac{1}{j\omega C}} = \frac{-10}{1 - jf_B/f}$$

where $f_B = \dfrac{1}{2\pi RC}$

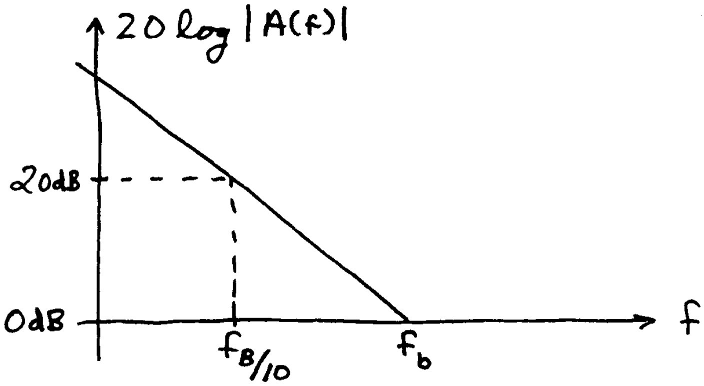
The magnitude Bode plot is:



(b) $$A(f) = -\frac{R + 1/j\omega C}{R} = -\left(1 - j\frac{f_B}{f}\right)$$

where $f_B = \dfrac{1}{2\pi RC}$

The magnitude Bode plot is:



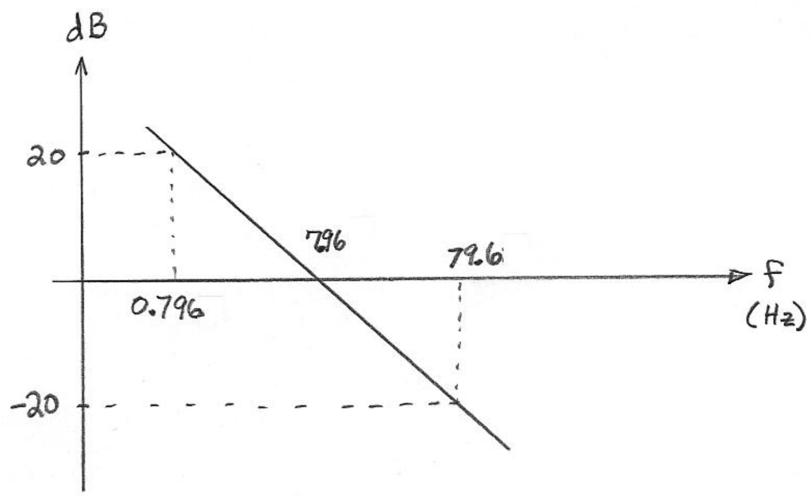
P14.80 The gain is:

$$A(f) = -\frac{\frac{1}{j\omega C}}{R} = -\frac{1}{j\omega RC} = -\frac{1}{jf/7.96}$$

In decibels, the gain magnitude is

$$20\log|A(f)| = -20\log(f/7.96)$$

The sketch is:



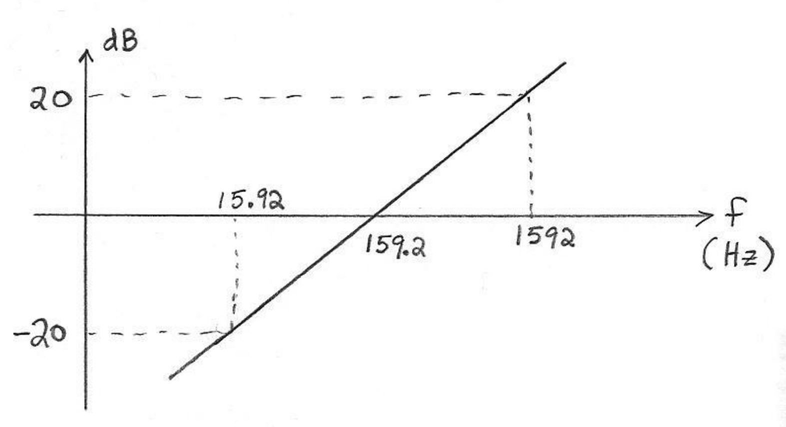
P14.81 The gain is:

$$A(f) = -\frac{R}{\frac{1}{j\omega C}} = -j\omega RC = -j(f/159.2)$$

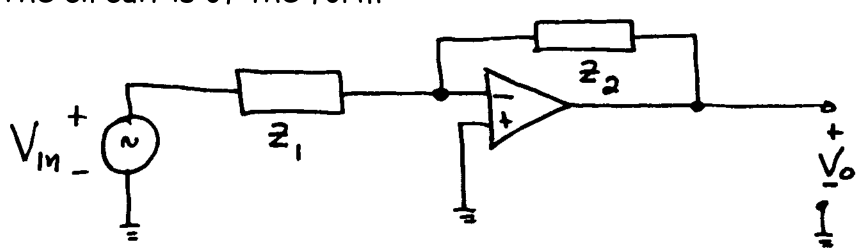
In decibels, the gain magnitude is

$$20\log|A(f)| = 20\log(f/159.2)$$

The sketch is:



P14.82 The circuit is of the form



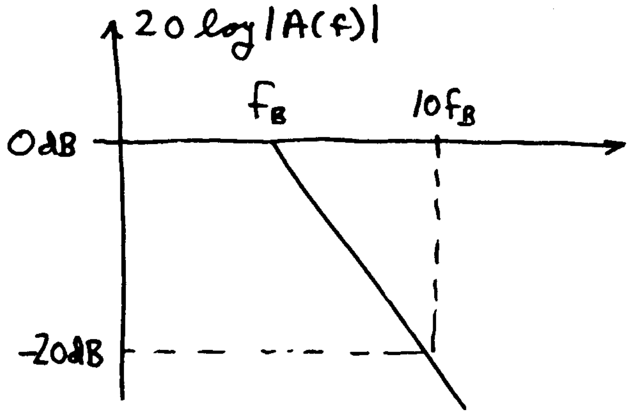
This is the inverting amplifier configuration and the gain is

$$A(f) = \frac{\mathbf{V}_o}{\mathbf{V}_{in}} = -\frac{Z_2}{Z_1}$$

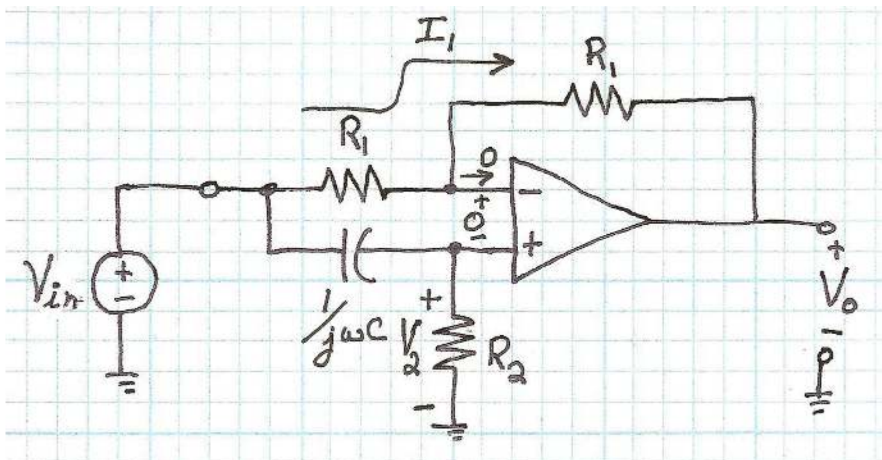
$$A(f) = -\frac{\frac{1}{1/R + j\omega C}}{R} = -\frac{1}{1 + jf/f_B}$$

where $f_B = \dfrac{1}{2\pi RC}$

The magnitude Bode plot is:



P14.83 The circuit is



Using the voltage division principle, we have

$$\mathbf{V}_2 = \mathbf{V}_{in} \frac{R_2}{R_2 + 1/(j\omega C)}$$

Also, we have

$$\mathbf{V}_o = -R_1\mathbf{I}_1 + \mathbf{V}_2 \qquad \text{and} \qquad \mathbf{I}_1 = \frac{\mathbf{V}_{in} - \mathbf{V}_2}{R_1}$$

which yield

$$\mathbf{V}_o = -R_1 \frac{\mathbf{V}_{in} - \mathbf{V}_2}{R_1} + \mathbf{V}_2 = -\mathbf{V}_{in} + 2\mathbf{V}_2$$

Then, substituting for $\mathbf{V}_2$ and algebra eventually yields

$$A(f) = -\frac{1 - jf/f_B}{1 + jf/f_B}$$

where $f_B = \dfrac{1}{2\pi R_2 C}$

From this we find $|A(f)| = 1$ and angle of $A(f) = 180° - 2\arctan(f/f_B)$. Thus, the amplitude of an input sine wave is not affected by the amplifier but the phase angle is. This is a phase shifting circuit.

Practice Test

T14.1 (a) The circuit diagram is shown in Figure 14.4 and the voltage gain is $A_v = -R_2/R_1$. Of course, you could use different resistance labels such as R_A and R_B so long as your equation for the gain is modified accordingly.

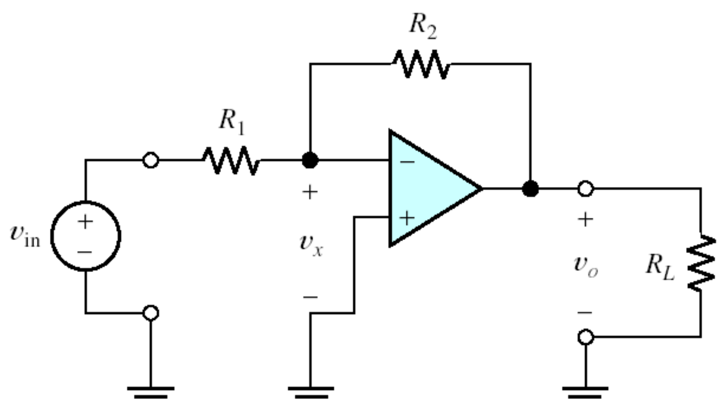


Figure 14.4 The inverting amplifier.

(b) The circuit diagram is shown in Figure 14.11 and the voltage gain is $A_v = 1 + R_2/R_1$.

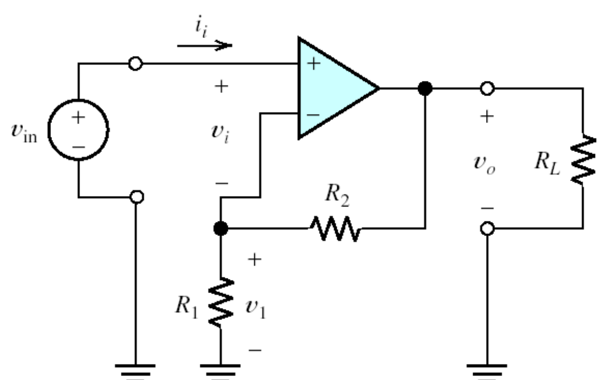
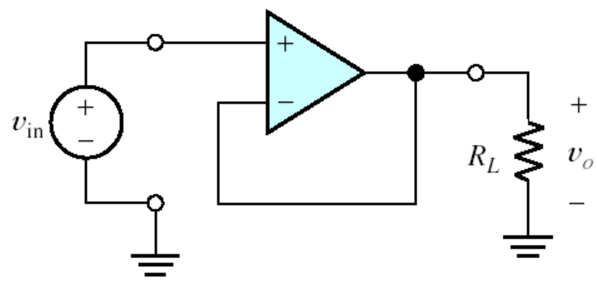


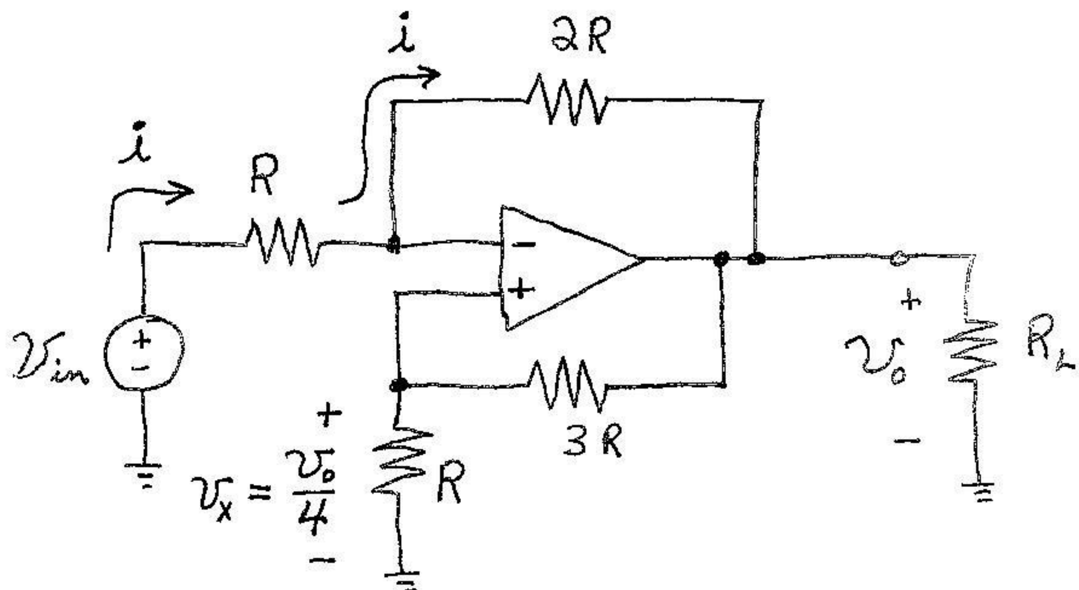
Figure 14.11 Noninverting amplifier.

(c) The circuit diagram is shown in Figure 14.12 and the voltage gain is $A_v = 1$.



T14.2 Because the currents flowing into the op-amp input terminals are zero, we can apply the voltage-division principle to determine the voltage v_x at the noninverting input with respect to ground:

$$v_x = v_o \frac{R}{R+3R} = \frac{v_o}{4}$$



This is also the voltage at the inverting input, because the voltage between the op-amp input terminals is zero. Thus, the current i is

$$i = \frac{v_{in} - v_o / 4}{R}$$

Then, we can write a voltage equation starting from the ground node, through v_o, through the $2R$ resistance, across the op-amp input terminals, and then through v_x to ground. This gives

$$-v_o - 2Ri + 0 + v_x = 0$$

Substituting for i and v_x gives:

$$-v_o - 2R\frac{v_{\text{in}} - v_o/4}{R} + 0 + \frac{v_o}{4} = 0$$

which simplifies to $v_o = -8v_{\text{in}}$. Thus, the voltage gain is $A_v = -8$.

T14.3 (a) $f_{BCL} = \dfrac{f_t}{A_{0CL}} = \dfrac{A_{0OL} f_{BOL}}{A_{0CL}} = \dfrac{2 \times 10^5 \times 5}{100} = 10\text{ kHz}$

(b) Equation 14.32 gives the closed-loop gain as a function of frequency:

$$A_{CL}(f) = \frac{A_{0CL}}{1 + j(f/f_{BCL})} = \frac{100}{1 + j(f/10^4)}$$

The input signal has a frequency of 10^5 Hz, and a phasor representation given by $\mathbf{V}_{\text{in}} = 0.05\angle 0^\circ$. The transfer function evaluated for the frequency of the input signal is

$$A_{CL}(10^5) = \frac{100}{1 + j(10^5/10^4)} = 9.95\angle -84.29^\circ$$

The phasor for the output signal is

$$\mathbf{V}_o = A_{CL}(10^5)\mathbf{V}_{\text{in}} = (9.95\angle -84.29^\circ) \times (0.05\angle 0^\circ) = 0.4975\angle -84.29^\circ$$

and the output voltage is $v_o(t) = 0.4975\cos(2\pi \times 10^5 t - 84.29^\circ)$.

T14.4 (a) $f_{FP} = \dfrac{SR}{2\pi V_{om}} = \dfrac{20 \times 10^6}{2\pi \times 4.5} = 707.4\text{ kHz}$

(b) In this case, the limit is due to the maximum current available from the op amp. Thus, the maximum output voltage is:

$$V_{om} = 5\text{ mA} \times 200\ \Omega = 1\text{ V}$$

(The current through R_2 is negligible.)

(c) $V_{om} = 4.5\text{ V}$. (It is limited by the maximum output voltage capability of the op amp.)

(d) In this case, the slew-rate is the limitation.

$$v_o(t) = V_{om}\sin(\omega t)$$

$$\frac{dv_o(t)}{dt} = \omega V_{om}\cos(\omega t)$$

$$\left|\frac{dv_o(t)}{dt}\right|_{\max} = \omega V_{om} = SR$$

$$V_{om} = \frac{SR}{\omega} = \frac{20 \times 10^6}{2\pi \times 5 \times 10^6} = 0.637\text{ V}$$

T14.5 See Figure 14.29 for the circuit.

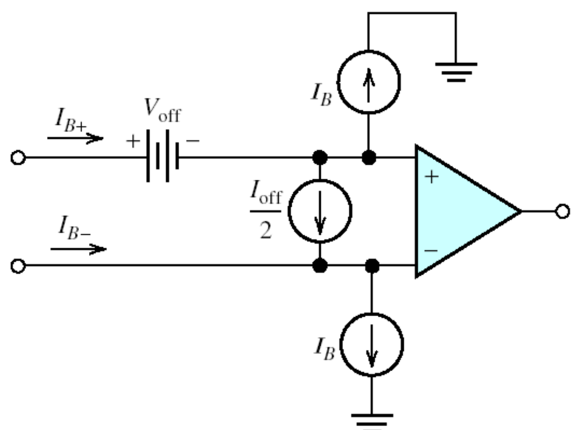


Figure 14.29 Three current sources and a voltage source model the dc imperfections of an op amp.

The effect on amplifiers of bias current, offset current, and offset voltage is to add a (usually undesirable) dc voltage to the intended output signal.

T14.6 See Figure 14.33 in the book.

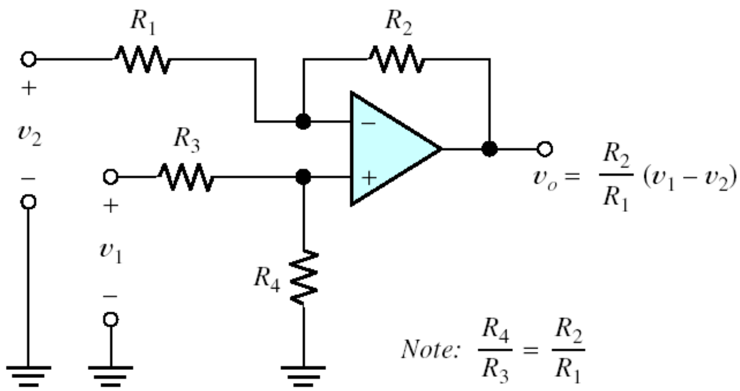


Figure 14.33 Differential amplifier.

Usually, we would have $R_1 = R_3$ and $R_2 = R_4$.

T14.7 See Figures 14.35 and 14.38 in the book:

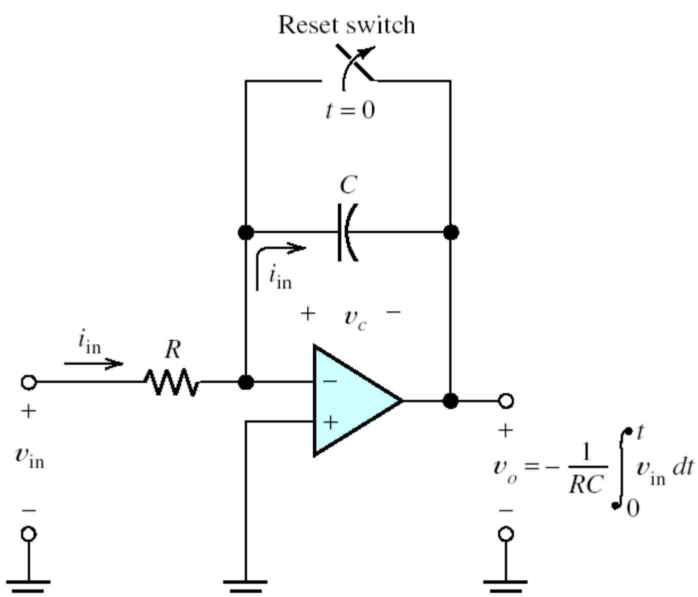


Figure 14.35 Integrator.

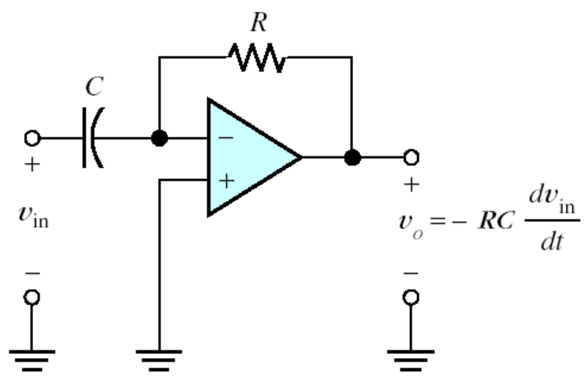


Figure 14.38 Differentiator.

T14.8 Filters are circuits designed to pass input components with frequencies in one range to the output and prevent input components with frequencies in other ranges from reaching the output.

An active filter is a filter composed of op amps, resistors, and capacitors.

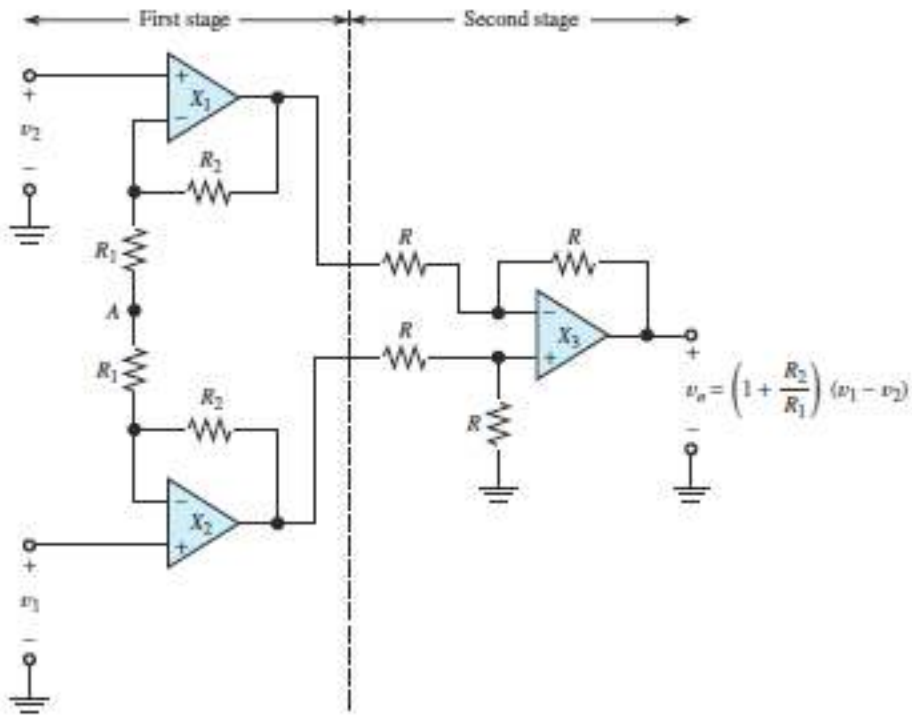
Some applications for filters mentioned in the text are:

1. In an electrocardiograph, we need a filter that passes the heart signals, which have frequencies below about 100 Hz, and rejects higher frequency noise that can be created by contraction of other muscles.

2. Using a lowpass filter to remove noise from historical phonograph recordings.

3. In digital instrumentation systems, a low pass filter is often needed to remove noise and signal components that have frequencies higher than half of the sampling frequency to avoid a type of distortion, known as aliasing, during sampling and analog-to-digital conversion.

P14.73 (b)



For the above figure R1=2kΩ R2=4kΩ R=15kΩ As discussed in the book, the first-stage gain for the differential signal is $1 + R_2/R_1$ which for the values given is 3. On the other hand, the first-stage gain for the common-mode component is unity. Thus, the output voltages are:

$v_{X1out} = 15\cos(2000\pi t) + 6\cos(120\pi t)$
$v_{X2out} = -15\cos(2000\pi t) + 6\cos(120\pi t)$

(c) The second stage is a differential amplifier, since all the resistances are equal,
$V_0 = v_{X1out} - v_{X2out} = 30\cos(2000\pi t)$

CHAPTER 15

Exercises

E15.1 If one grasps the wire with the right hand and with the thumb pointing north, the fingers point west under the wire and curl around to point east above the wire.

E15.2 If one places the fingers of the right hand on the periphery of the clock pointing clockwise, the thumb points into the clock face.

E15.3 $\mathbf{f} = q\mathbf{u} \times \mathbf{B} = (-1.602 \times 10^{-19})10^{5}\mathbf{u}_x \times \mathbf{u}_y = -1.602 \times 10^{-14}\mathbf{u}_z$

in which $\mathbf{u}_x$, $\mathbf{u}_y$, and $\mathbf{u}_z$ are unit vectors along the respective axes.

E15.4 $f = i\ell B \sin(\theta) = 10(1)0.5\sin(90^\circ) = 5\text{ N}$

E15.5 (a) $\varphi = BA = B\pi r^2 = 0.5\pi(0.05)^2 = 3.927\text{ mWb}$

$\lambda = N\varphi = 39.27$ mWb turns

(b) $e = \dfrac{d\lambda}{dt} = -\dfrac{39.27 \times 10^{-3}}{10^{-3}} = -39.27\text{ V}$

More information would be needed to determine the polarity of the voltage by use of Lenz's law. Thus the minus sign of the result is not meaningful.

E15.6 $B = \dfrac{\mu I}{2\pi r} = \dfrac{4\pi \times 10^{-7} \times 20}{2\pi 10^{-2}} = 4 \times 10^{-4}\text{ T}$

E15.7 By Ampère's law, the integral equals the sum of the currents flowing through the surface bounded by the path. The reference direction for the currents relates to the direction of integration by the right-hand rule. Thus, for each part the integral equals the sum of the currents flowing upward. Refering to Figure 15.9 in the book, we have

$$\oint_{\text{Path1}} \mathbf{H} \cdot d\ell = 10\text{ A} \qquad \oint_{\text{Path2}} \mathbf{H} \cdot d\ell = 10 - 10 = 0\text{ A} \qquad \oint_{\text{Path3}} \mathbf{H} \cdot d\ell = -10\text{ A}$$

E15.8 Refer to Figure 15.9 in the book. Conceptually the left-hand wire produces a field in the region surrounding it given by

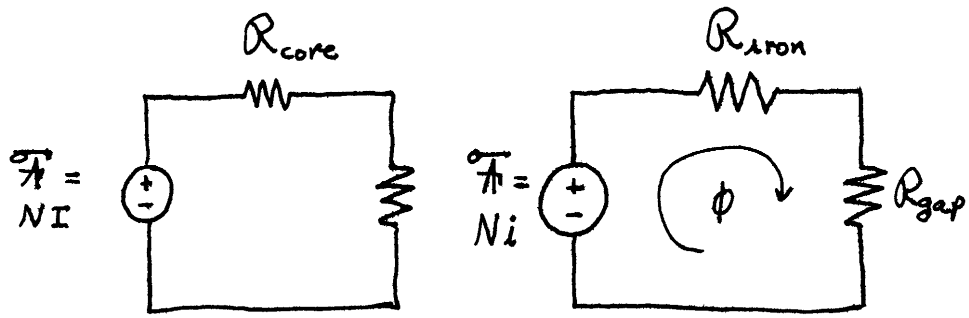
$$B = \frac{\mu I}{2\pi r} = \frac{4\pi \times 10^{-7} \times 10}{2\pi 10^{-1}} = 2 \times 10^{-5}\ \text{T}$$

By the right-hand rule, the direction of this field is in the direction of Path 1. The field in turn produces a force on the right-hand wire given by

$$f = B\ell i = 2 \times 10^{5}(1)(10) = 2 \times 10^{-4}\ \text{N}$$

By the right-hand rule, the direction of the force is such that the wires repel one another.

E15.9 The magnetic circuit is:



The reluctance of the iron is:

$$R_{\text{iron}} = \frac{\ell_{\text{iron}}}{\mu_r \mu_0 A_{\text{iron}}} = \frac{27 \times 10^{-2}}{5000 \times 4\pi \times 10^{-7} \times 4 \times 10^{-4}}$$

$$R_{\text{iron}} = 107.4 \times 10^{3}$$

The reluctance of the air gap is:

$$R_{\text{gap}} = \frac{\ell_{\text{gap}}}{\mu_0 A_{\text{gap}}} = \frac{10^{-2}}{4\pi \times 10^{-7} \times 9 \times 10^{-4}}$$

$$R_{\text{gap}} = 8.842 \times 10^{6}$$

Then we have

$$\varphi = B_{\text{gap}} A_{\text{gap}} = 0.5 \times 9 \times 10^{-4} = 0.45\ \text{mWb}$$

$$i = \frac{(R_{\text{iron}} + R_{\text{gap}})\varphi}{N} = \frac{(107.4 \times 10^{3} + 8.842 \times 10^{6})(0.45 \times 10^{-3})}{1000} = 4.027\ \text{A}$$

E15.10 Refer to Example 15.6 in the book. Neglecting the reluctance of the iron, we have:

$$R_c = 0$$

$$R_a = \frac{\ell_{gap}}{\mu_0 A_a} = \frac{1 \times 10^{-2}}{4\pi \times 10^{-7} \times 9 \times 10^{-4}} = 8.842 \times 10^{6}$$

$$R_b = \frac{\ell_{gap}}{\mu_0 A_b} = \frac{0.5 \times 10^{-2}}{4\pi \times 10^{-7} \times 6.25 \times 10^{-4}} = 6.366 \times 10^6$$

$$\varphi_a = \frac{Ni}{R_a} = \frac{500 \times 2}{8.842 \times 10^6} = 113.1\ \mu\text{Wb}$$

$$B_a = \frac{\varphi_a}{A_a} = \frac{113.1 \times 10^{-6}}{9 \times 10^{-4}} = 0.1257\ \text{T}$$

compared to 0.1123 T found in the example for an error of 11.9%.

$$\varphi_b = \frac{Ni}{R_b} = \frac{500 \times 2}{6.366 \times 10^6} = 157.1\ \mu\text{Wb}$$

$$B_b = \frac{\varphi_b}{A_b} = \frac{157.1 \times 10^{-6}}{6.25 \times 10^{-4}} = 0.2513\ \text{T}$$

compared to 0.2192T found in the Example for an error of 14.66%.

E15.11

$$\varphi_2 = \frac{N_2 i_2}{R} = \frac{200 i_2}{10^7} = 2 \times 10^{-5} i_2$$

$$\lambda_{12} = N_1 \varphi_2 = 200 \times 10^{-5} i_2$$

$$M = \frac{\lambda_{12}}{i_2} = 2\ \text{mH}$$

E15.12 By the right-hand rule, clockwise flux is produced by i_1 and counterclockwise flux is produced by i_2. Thus the currents produce opposing fluxes.

If a dot is placed on the top terminal of coil 1, current entering the dot produces clockwise flux. Current must enter the bottom terminal of coil 2 to produce clockwise flux. Thus the corresponding dot should be on the bottom terminal of coil 2.

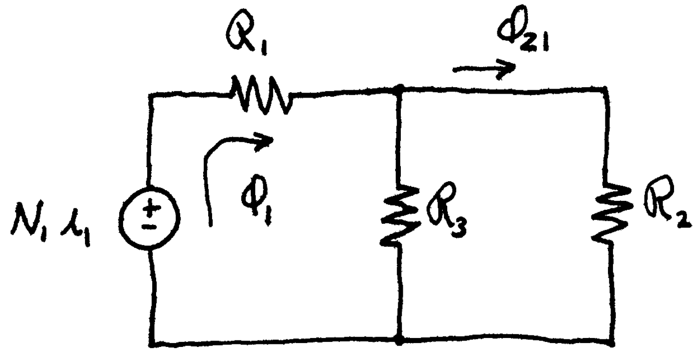
The voltages are given by Equations 15.36 and 15.37 in which we choose the minus signs because the currents produce opposing fluxes. Thus we have

$$e_1 = L_1 \frac{di_1}{dt} - M \frac{di_2}{dt} \quad \text{and} \quad e_2 = -M \frac{di_1}{dt} + L_2 \frac{di_2}{dt}$$

E15.13 (a) Using the right-hand rule, we find that the fluxes produced by i_1 and i_2 aid in path 1, aid in path 2, and oppose in path 3.

If a dot is placed on the top terminal of coil 1, the corresponding dot should be on the top terminal of coil 2, because then currents entering the dotted terminals produce aiding flux linkages of the coils.

(b) For $i_2 = 0$, the magnetic circuit is:



Then the reluctance seen by the source is

$$R_{total} = R_1 + \frac{1}{1/R_2 + 1/R_3} = 1.5 \times 10^6$$

$$\varphi_1 = \frac{N_1 i_1}{R_{total}} \qquad \lambda_{11} = N_1 \varphi_1 = \frac{N_1^2 i_1}{R_{total}}$$

$$L_1 = \frac{\lambda_{11}}{i_1} = \frac{N_1^2}{R_{total}} = 6.667 \text{ mH}$$

The flux φ_1 splits equally between paths 2 and 3. Thus we have $\varphi_{21} = \varphi_1 / 2$. Then

$$\lambda_{21} = N_2 \varphi_{21} = \frac{N_1 N_2 i_1}{2R_{total}} \quad \text{and} \quad M = \frac{\lambda_{21}}{i_1} = \frac{N_1 N_2}{2R_{total}} = 10 \text{ mH}$$

Similarly, we find $L_2 = 60$ mH.

(c) Because the currents produce aiding flux linkages, the mutual term carries a + sign.

E15.14 The energy lost per cycle is $W_{cycle} = (40 \text{ J/m}^3) \times (200 \times 10^{-6} \text{ m}^3) = 8 \text{ mJ}$, and the power loss is $P = W_{cycle} f = 8 \times 10^{-3} \times 60 = 0.48$ W.

E15.15 $H_{gap} = \frac{NI}{\ell_{gap}} = \frac{1000}{0.5\times 10^{-2}} = 200\times 10^{3}$ A/m

$B_{gap} = \mu_0 H_{gap} = 0.2513$ T

$W = W_v \times \text{Volume} = \frac{B_{gap}^2}{2\mu_0}(2\times 10^{-2}\times 3\times 10^{-2}\times 0.5\times 10^{-2}) = 0.0754$ J

E15.16 Refer to Figure 15.26c in the book.

$$\mathbf{I}_2 = \frac{V_s'}{R_s' + Z_L} = \frac{100\angle 0^\circ}{10 + 10 + j20} = 3.536\angle -45^\circ$$

$$\mathbf{V}_2 = Z_L \mathbf{I}_2 = (10 + j20)\mathbf{I}_2 = 79.06\angle 18.43^\circ \text{ V}$$

$$P_L = I_{2\text{rms}}^2 R_L = \left(\frac{3.536}{\sqrt{2}}\right)^2 (10) = 62.51 \text{ W}$$

E15.17 $R_L' = \left(\frac{N_1}{N_2}\right)^2 R_L = \left(\frac{1}{4}\right)^2 400 = 25\ \Omega$

$\mathbf{I}_1 = \frac{100\angle 0^\circ}{R_s + R_L'} = 1.538\angle 0^\circ$

$\mathbf{I}_2 = \left(\frac{N_1}{N_2}\right)\mathbf{I}_1 = 0.3846\angle 0^\circ$

$\mathbf{V}_2 = R_L \mathbf{I}_2 = 153.8\angle 0^\circ$

$P_L = R_L' I_{1\text{rms}}^2 = R_L I_{2\text{rms}}^2 = 29.60$ W

E15.18 For maximum power transfer, we need

$$R_s = R_L'$$

However we have

$$R_L' = \left(\frac{N_1}{N_2}\right)^2 R_L = \left(\frac{N_1}{N_2}\right)^2 400$$

Thus we have

$$R_s = 40 = \left(\frac{N_1}{N_2}\right)^2 400$$

Solving we find

$$\frac{N_1}{N_2} = \frac{1}{\sqrt{10}}$$

Problems

P15.1 According to Faraday's law of magnetic induction, a voltage is induced in a coil equal to the time rate of change of flux linkages.

Lenz's law states that the polarity of the induced voltage is such that the current flowing through a resistance placed across the terminals of the coil tends to oppose the change in flux linkages.

P15.2 Ampère's law states that the integral of magnetic field intensity around a closed path is equal to the sum of the currents flowing through any surface bounded by the path. In equation form:

$$\oint \mathbf{H} \cdot d\ell = \sum i$$

If one encircles the path of integration with the fingers of the right hand pointing in the direction of integration, the thumb points in the positive reference direction for the currents.

P15.3 (a) If one grasps a current-carrying wire using the right hand with the thumb pointing in the direction of the current, the fingers encircle the wire in the direction of the magnetic field.

(b) If one grasps a current-carrying coil using the right hand with the fingers encircling the coil in the direction of the current flow, the thumb points in the direction of the magnetic field inside the coil.

P15.4 Electrical charge in motion is the fundamental cause of magnetic fields.

P15.5* By Lenz's law, the polarity of the induced voltage is such that the current flowing through a resistance placed across the terminals of the coil tends to oppose the change in flux linkages. As the magnet approaches, the coil would produce a field pointing toward the magnet. This requires a current from a to b through the coil. Thus, we find that b must be positive so v_{ab} is negative.

P15.6* We have $B = \dfrac{\mu I}{2\pi r} = \dfrac{4\pi \times 10^{-7} \times 10}{2\pi r} = 0.1 \times 3 \times 10^{-5}$ T. Solving, we find $r = 66.67$ cm.

Some ways to reduce the effects of electrical wiring on a navigation compass are to keep the wires far from the compass, to keep the return conductor close to the wire so the magnetic field of the two currents tend to cancel, and to use ac currents where possible so the forces are averaged by the inertia of the compass.

P15.7* Equation 15.4 in the text states:

$$f = i\ell B \sin\theta$$

Since the wire is perpendicular to the field, $\theta = 90^\circ$. Solving for the magnetic flux density and substituting values, we have:

$$i = \frac{f}{B\ell} = \frac{3}{0.6 \times 1} = 5\ Amp$$

P15.8 For any closed path inside the pipe, we have

$$\oint \mathbf{H} \cdot d\ell = \sum i = 0$$

Thus, we conclude that the magnetic field is zero inside the pipe. Outside the pipe the analysis is the same as for a long wire.

P15.9 Each part of the loop may experience force due to the field of another part of the loop. However, according to Newton's third law of motion, for every action there is an equal and opposite reaction. Thus, the net force is zero.

P15.10 Rearranging Equation 15.2, we have $B = \dfrac{f}{qu\sin(\theta)}$. However, the units of force are $\text{kg} \cdot \text{m/s}^2$ because force equals mass times acceleration. Furthermore, $\sin(\theta)$ is unitless. Thus, the units of B become

$$\frac{\text{kg} \cdot \text{m/s}^2}{C \cdot \text{m/s}} = \frac{\text{kg}}{C \cdot \text{s}}$$

The units for H are

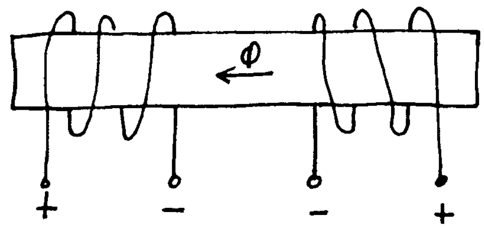
$$\frac{\text{A}}{\text{m}} = \frac{C}{\text{m} \cdot \text{s}}$$

From equation 15.10, we have $\mu = B/H$ and the units of μ are

$$\frac{\text{kg}/C \cdot \text{s}}{C/\text{m} \cdot \text{s}} = \frac{\text{kg} \cdot \text{m}}{C^2}$$

P15.11 Using the right-hand rule, we find that the north magnetic poles are at the left-hand ends of the coils. The coils attract one another.

P15.12 Using Lenz's law, we find that the polarities of the induced voltages are:



P15.13 Each wire produces a field in the region surrounding it given by

$$B = \frac{\mu I}{2\pi r} = \frac{4\pi \times 10^{-7} \times 10}{2\pi 10^{-2}} = 2 \times 10^{-4}\ \text{T}$$

The field in turn produces a force on the other wire given by

$$f = B\ell i = 2 \times 10^{-4}(0.5)(10) = 10^{-3}\ \text{N}$$

Using the right hand rule, we can determine the direction of the field at the second wire due to the current in the first wire. Then using $\mathbf{f} = i\ell \times \mathbf{B}$, we can determine the direction of the force on the second wire. The direction of the forces is such that the wires attract one another.

P15.14* The flux linking the coil is the product of the coil area and the flux density.

$$\varphi = BA = B\pi r^2 = 2 \times \pi \times 0.2^2 = 0.2531\ \text{Wb}$$

The flux linkages are given by:

$$\lambda = N\varphi = 5 \times 0.2513 = 1.2565\ \text{Wb turns}$$

Thus the induced voltage is

$$e = \frac{d\lambda}{dt} = \frac{\Delta\lambda}{\Delta t} = \frac{1.2565}{10 \times 10^{-3}} = 125.65\ \text{V}$$

P15.15* $\mu = \dfrac{B}{H} = \dfrac{0.1}{50} = 2 \times 10^{-3}\ \text{Wb/Am}$

$$\mu_r = \frac{\mu}{\mu_0} = \frac{2 \times 10^{-3}}{4\pi \times 10^{-7}} = 1592$$

P15.16 Between $t = 0$ and $t = 4$, i_2 is increasing and voltage is induced in coil 1 such that i_1 is positive. After $t = 4$, i_2 and the magnetic field are constant so i_1 becomes zero.

At $t = 1$, i_1 is positive and i_2 is negative resulting in north-seeking poles at the left-hand ends of both coils. Thus, we have a force of attraction.

At $t = 2$, i_2 is zero, so the force is zero.

At $t = 3$, i_1 and i_2 are both positive resulting in north-seeking poles at the outer ends of the coils. Thus, we have a repelling force.

At $t = 5$, i_1 is zero, so the force is zero.

P15.17 From Equation 15.18, we have:

$$\phi = \frac{\mu N I r^2}{2R} = \frac{\mu(200)[0.05\sin(200t)](10^{-2})^2}{2(10^{-1})}$$
$$= 0.005\mu\sin(200t)$$

However,

$$e = N\frac{d\phi}{dt} = 200\mu\cos(200t)$$

Since the voltage is known to be $e = 0.5\cos(200t)$, we have:

$$0.5 = 200\mu$$
$$\mu = 0.0025$$
$$\mu_r = \frac{\mu}{\mu_0} = \frac{0.0025}{4\pi \times 10^{-7}} = 1989$$

P15.18 The flux linking the coil is the product of the coil area and the flux density.

$$\varphi = BA = B\pi r^2 = 0.5\sin(377t)\pi \times 0.1^2 = 0.0157\sin(377t)$$

The flux linkages are given by

$$\lambda = N\varphi = 15.7\sin(377t)\ Wb\ \text{turns}$$

The induced voltage is:

$$e = \frac{d\lambda}{dt} = 5.919 \times 10^3\cos(377t)\,\text{V}$$

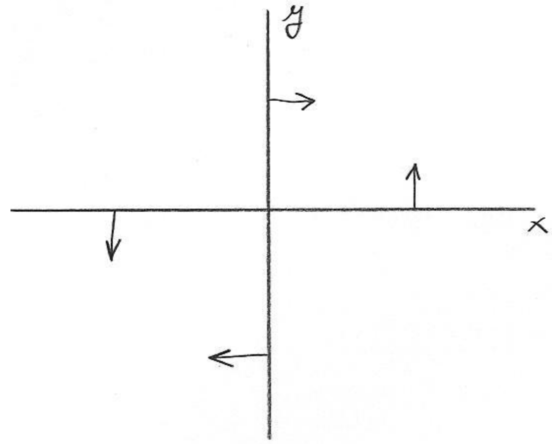
Maximum value of flux linkage $= 15.7\ Wb$ turns

Maximum value of induced voltage $= 5191\ V$

P15.19 Solving Equation 15.9 for the conductor length, we have:

$$\ell = \frac{e}{Bu} = \frac{120}{0.5 \times 30} = 8\text{ m}$$
$$N = \frac{\ell}{0.1} = 80\text{ conductors}$$

P15.20 (a) The forces are indicated



(b) $$T = \int_{-\infty}^{\infty} f y dy = 2\int_{0}^{\infty} \frac{\mu I_x}{2\pi y} I_y y dy = \infty$$

P15.21 (a) Equal and opposite forces are exerted on the top and bottom sides of the loop. Thus, we only need to consider the forces on the vertical sides of the loop. The net force is

$$f = \frac{\mu I_1}{2\pi r_1} I_2 \ell - \frac{\mu I_1}{2\pi r_2} I_2 \ell = \frac{\mu I_1}{2\pi} I_2 \ell \left(\frac{1}{r_1} - \frac{1}{r_2} \right)$$

(b) $$f = \frac{4\pi \times 10^{-7}(10)}{2\pi}(10)(0.1)\left(\frac{1}{0.01} - \frac{1}{0.1} \right) = 180 \times 10^{-6} \text{ N}$$

(c) The loop is attracted by the wire.

P15.22 (a) $$\phi = \int_A \mathbf{B} \cdot \mathbf{dA} = \int_0^{\ell} \int_{r_1}^{r_2} \frac{\mu i(t)}{2\pi r} dr dz = \ell \frac{\mu i(t)}{2\pi} \ln\left(\frac{r_2}{r_1} \right)$$

(b) $$v_{ab}(t) = \frac{d\phi}{dt} = \ell \frac{\mu}{2\pi} \ln\left(\frac{r_2}{r_1} \right) \frac{di(t)}{dt}$$

(c) $i(t) = 10\sqrt{2}\cos(120\pi t)$

$$v_{ab}(t) = -0.1 \frac{4\pi \times 10^{-7}}{2\pi} \ln\left(\frac{0.1}{0.01} \right) 1200\pi\sqrt{2}\sin(120\pi t)$$

$$= -173.6\sqrt{2}\sin(120\pi t)\ \mu\text{V}$$

Thus, we have $V_{ab} = 173.6\ \mu\text{V rms}$.

P15.23 $$e = 120\sqrt{2}\cos(120\pi t) \qquad (1)$$

Since,

$$e = N\frac{d\varphi}{dt}$$

we must have:

$$\varphi = \varphi_{\max}\sin(120\pi t)$$

$$e = 120\pi N\varphi_{\max}\cos(120\pi t) \qquad (2)$$

Equating the amplitudes of the expressions given in (1) and (2), we have:

$$120\sqrt{2} = 120\pi N\phi_{\max}$$

$$\varphi_{\max} = \frac{\sqrt{2}}{N\pi} = 900\times 10^{-6}\text{ Wb}$$

$$\varphi_{\text{rms}} = \frac{\varphi_{\text{peak}}}{\sqrt{2}} = 637\times 10^{-6}\text{ Wb}$$

P15.24* Without the gap, the reluctance of the left-hand leg becomes:

$$R_a = \frac{\ell_a}{\mu_r\mu_0 A_{\text{core}}} = \frac{30\times 10^{-2}}{1000\times 4\pi\times 10^{-7}\times 4\times 10^{-4}}$$

$$R_a = 596.8\times 10^3$$

As in Example 15.6, we have:

$$R_c = 1.989\times 10^5$$

$$R_b = 6.953\times 10^6$$

$$R_{\text{total}} = R_c + \frac{1}{1/R_a + 1/R_b} = 748.5\times 10^3$$

$$\phi_c = \frac{Ni}{R_{\text{total}}} = \frac{500\times 2}{748.5\times 10^3} = 1.336\times 10^{-3}\text{ Wb}$$

$$\varphi_b = \varphi_c\frac{R_a}{R_a + R_b} = 105.6\ \mu\text{Wb}$$

This compares to $\varphi_b = 137.0\ \mu\text{Wb}$ in Example 15.6. The flux in the right-hand leg is less after the gap in the left-hand leg is filled because the flux takes the lower reluctance path.

P15.25* $R_{gap} = \dfrac{\ell_{gap}}{\mu_{gap}A} = R_{core} = \dfrac{\ell_{core}}{\mu_{core}A}$

Rearranging, we find that

$$\mu_{core} = \frac{\ell_{core}\,\mu_{gap}}{\ell_{gap}}$$

$$\mu_o \mu_r = \frac{\ell_{core}\,\mu_o}{\ell_{gap}}$$

$$\mu_r = \frac{\ell_{core}}{\ell_{gap}} = \frac{500}{1} = 500$$

P15.26* Magnetomotive force $F = Ni$ in a magnetic circuit is analogous to a voltage source in an electrical circuit. Reluctance R is analogous to electrical resistance. Magnetic flux φ is analogous to electrical current.

P15.27 Reluctance is given by Equation 15.21:

$$R = \frac{\ell}{\mu A}$$

Thus if the length of the magnetic path is doubled, the reluctance is doubled. If the cross sectional area is doubled, the reluctance is halved. If the permeability is doubled, the reluctance is halved.

P15.28 Reluctance is given by Equation 15.21:

$$R = \frac{\ell}{\mu A}$$

Furthermore, Equation 15.11 gives the units of μ as Weber/(Ampere meter). Thus, the units of reluctance are:

$$\frac{\text{meter}}{(\text{Weber/Ampe re meter})(\text{meter})^2} = \frac{\text{Ampere}}{\text{Weber}}$$

Using the fact that Ampere = Coulomb/second, the units of reluctance become:

$$\frac{\text{Coulomb}}{\text{Weber second}}$$

However, since we have $e = \dfrac{d\lambda}{dt}$, in which λ has a unit of webers, we conclude that:

$$\text{Weber} = \text{Volt seconds}$$

However,

$$\text{Volt} = \text{Joule/Coul omb} = \frac{\text{kilogram meter}^2}{\text{second}^2\text{Coulomb}}$$

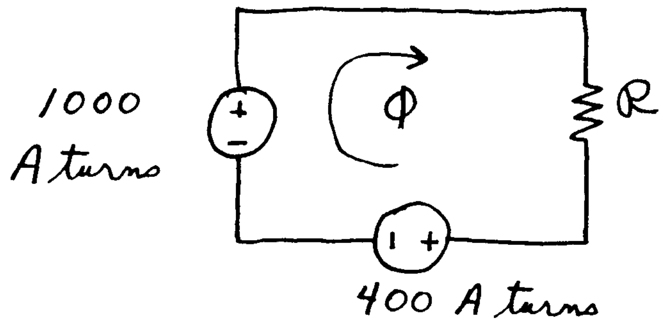
and we have:

$$\text{Weber} = \frac{\text{kilogram meter}^2}{\text{second Coulomb}}$$

Thus, the units of reluctance become:

$$\frac{\text{Coulomb}^2}{\text{kilogram meter}^2}$$

P15.29 The magnetic circuit is:



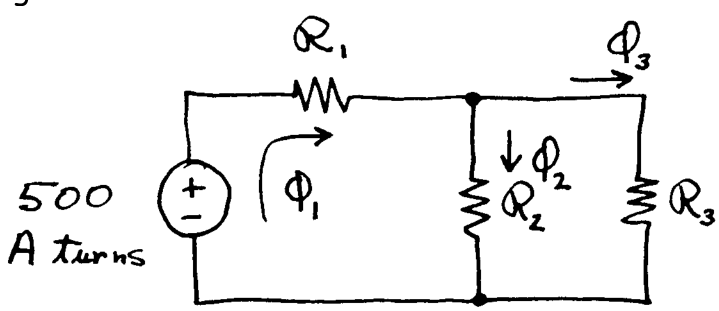
The reluctance is:

$$R = \frac{\ell}{\mu_r \mu_0 A_{core}} = \frac{36 \times 10^{-2}}{10^3 \times 4\pi \times 10^{-7} \times 9 \times 10^{-4}} = 318.3 \times 10^3$$

$$\varphi = \frac{1000 - 400}{R} = 1.885 \times 10^{-3} \text{ Wb}$$

$$B = \frac{\varphi}{A} = \frac{1.885 \times 10^{-3}}{9 \times 10^{-4}} = 2.094 \text{ T}$$

P15.30 The magnetic circuit is:



The permeability is:

$$\mu = \mu_r \mu_0 = 5000 \times 4\pi \times 10^{-7} = 6.283 \times 10^{-3}$$

The reluctances are given by Equation 15.21:

$$R = \frac{\ell}{\mu A}$$

$$R_1 = \frac{20\times10^{-2}}{6.283\times10^{-3}\times4\times10^{-4}} = 79.58\times10^3$$
$$R_2 = 31.83\times10^3$$
$$R_3 = 79.58\times10^3$$

The equivalent reluctance seen by the source is:

$$R_{eq} = R_1 + \frac{1}{1/R_2 + 1/R_3} = 102.3\times10^3$$

The fluxes are:

$$\phi_1 = \frac{F}{R_{eq}} = \frac{500}{102.3\times10^3} = 4.887\times10^{-3}\text{ Wb}$$
$$\varphi_2 = \varphi_1\frac{R_3}{R_3+R_2} = 3.490\times10^{-3}\text{ Wb}$$
$$\varphi_3 = \varphi_1\frac{R_2}{R_3+R_2} = 1.396\times10^{-3}\text{ Wb}$$

P15.31 The reluctance of each of the two gaps is:

$$R_{gap} = \frac{\ell_{gap}}{\mu_0 A} = \frac{\ell_{gap}}{4\pi\times10^{-7}\times20\times10^{-4}}$$
$$\varphi = BA = 1\times20\times10^{4} = 2\times10^{-3}\text{ Wb}$$

The total reluctance of the two gaps is $2R$.

$$F = 2R\phi = 2\times\ell_{gap}\times397.9\times10^6\times2\times10^{-3}$$

$$F = NI = 100\times10 = 2\times\ell_{gap}\times397.9\times10^6\times2\times10^{-3}$$

$$\ell_{gap} = \frac{1}{4\times397.9} \cong 0.6\text{ mm}$$

P15.32 With the gap, the reluctance of the right-hand leg becomes:

$$R_3 = \frac{19.5\times10^{-2}}{6.283\times10^{-3}\times4\times10^{-4}} + \frac{0.5\times10^{-2}}{4\pi\times10^{-7}\times(2.5)^2\times10^{-4}}$$
$$= 6.442\times10^6$$

The equivalent reluctance seen by the source is:

$$R_{eq} = R_1 + \frac{1}{1/R_2 + 1/R_3} = 111.3 \times 10^3$$

$$\varphi_1 = \frac{F}{R_{eq}} = \frac{500}{111.3 \times 10^3} = 4.494 \times 10^{-3} \text{ Wb}$$

$$\varphi_2 = \varphi_1 \frac{R_3}{R_3 + R_2} = 4.472 \times 10^{-3} \text{ Wb}$$

$$\varphi_3 = \varphi_1 \frac{R_2}{R_3 + R_2} = 0.022 \times 10^{-3} \text{ Wb}$$

The flux in the right-hand leg is much smaller after the gap is cut into it, and most of the flux takes the lower reluctance path through the center leg.

P15.33* The flux must pass through two gaps, one at the left-hand of the end of the plunger and the other in the center of the plunger. The area of the gap surrounding the center of the plunger is approximately:

$$A = \pi(d + \ell_g)L$$

Thus, the reluctance is given by:

$$R_{\text{center}} = \frac{\ell_g}{\mu_0 \pi (d + \ell_g) L}$$

The reluctance of the gap at the left-hand end of the plunger is:

$$R_{\text{left}} = \frac{x}{\mu_0 \pi (d/2)^2}$$

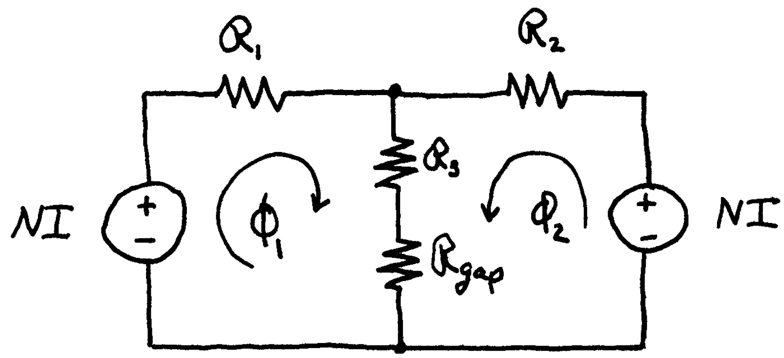
The total reluctance is:

$$R = R_{\text{center}} + R_{\text{left}}$$
$$= \frac{\ell_g}{\mu_0 \pi (d + \ell_g) L} + \frac{x}{\mu_0 \pi (d/2)^2}$$

The flux is given by

$$\phi = \frac{NI}{R} = \frac{NI}{\dfrac{\ell_g}{\mu_0 \pi (d + \ell_g) L} + \dfrac{x}{\mu_0 \pi (d/2)^2}}$$

P15.34 The magnetic circuit is:



We have

$$R_1 = R_2 = \frac{\ell_1}{\mu_0 \mu_r A_{core}} = \frac{14 \times 10^{-2}}{4\pi \times 10^{-7} \times 2500 \times 4 \times 10^{-4}} = 111.4 \times 10^3$$

$$R_3 = \frac{\ell_3}{\mu_0 \mu_r A_{core}} = \frac{3.9 \times 10^{-2}}{4\pi \times 10^{-7} \times 2500 \times 4 \times 10^{-4}} = 31.04 \times 10^3$$

$$R_{gap} = \frac{\ell_{gap}}{\mu_0 A_{gap}} = \frac{10^{-3}}{4\pi \times 10^{-7} \times 4.41 \times 10^{-4}} = 1.804 \times 10^6$$

The flux through the air gap is

$$\varphi_1 + \varphi_2 = B_{gap} A_{gap} = 0.25 \times 4.41 \times 10^{-4} = 110.3\ \mu\text{Wb}$$

By symmetry, we have $\varphi_1 = \varphi_2 = 55.13\ \mu\text{Wb}$.

Summing mmfs around the left-hand loop of the magnetic circuit, we have

$$NI = R_1\varphi_1 + R_3(\varphi_1 + \varphi_2) + R_{gap}(\varphi_1 + \varphi_2) = 208.5$$

Dividing by I, we determine that N = 104 turns are needed.

P15.35 The voltage across the 200-turn coil is

$$v(t) = 10\cos(10^5 t) = 200\frac{d\phi}{dt}$$

from which we deduce that

$$\phi = 5 \times 10^{-7} \sin(10^5 t)\ \text{Wb}$$

Equation 15.18 states

$$\phi = \frac{\mu N i(t) r^2}{2R}$$

Solving for the current and substituting values gives

$$i(t) = \frac{2\pi R\phi}{\mu N r^2} = \frac{2(0.05)[5 \times 10^{-7} \sin(10^5 t)\ ]}{1000 \times 4\pi 10^{-7} \times 200 \times (0.02)^2} = 497.4 \sin(10^5 t)\ \mu A$$

The voltage across the 400-turn coil is

$$v_{400}(t) = 400\frac{d\phi}{dt} = 20\cos(10^5 t)$$

P15.36* The impedance of the inductance to a sinusoidal current is:

$$X_L = \omega L = \frac{V_{\text{rms}}}{I_{\text{rms}}} = \frac{120}{2} = 60\Omega$$

For a f = 60-Hz source, we have:

$$\omega = 2\pi f = 377$$

Thus, the inductance is

$$L = \frac{X_L}{\omega} = \frac{60}{377} = 0.1592 \text{ H}$$

Rearranging Equation 15.25, we have:

$$R = \frac{N^2}{L} = \frac{1000^2}{0.1592} = 6.289 \times 10^6$$

Rearranging Equation 15.21, we have:

$$\mu = \frac{\ell}{RA} = \frac{20 \times 10^{-2}}{6.289 \times 10^6 \times 5 \times 10^{-4}} = 63.6 \times 10^{-6}$$

$$\mu_r = \frac{\mu}{\mu_0} = \frac{63.6 \times 10^{-6}}{4\pi \times 10^{-7}} = 50.61$$

P15.37* According to Equation 15.25, inductance is given by:

$$L = \frac{N^2}{R}$$

Thus, inductance is proportional to the square of the number of turns, and the inductance of the 600 turn coil is 900 mH.

P15.38 Consider a coil carrying a current i that sets up a flux φ linking the coil. The self-inductance of the coil can be defined as flux linkages divided by current:

$$L = \frac{\lambda}{i}$$

Assuming that the flux is confined to the core so that all of the flux links all of the turns,

We can write $\lambda = N$. Then, we have

$$L = \frac{N\phi}{i}$$

Substituting $\phi = Ni/R$, we obtain

$$L = \frac{N^2}{R}$$

Thus,we see that the inductance depends on the number of turns, the core dimensions, and the core material. Notice that inductance is proportional to the square of the number of turns.

When two coils are wound on the same core, some of the flux produced by one coil links the other coil.We denote the flux linkages of coil 2 caused by the current in coil 1 as λ_{21}. Correspondingly, the flux linkages of coil 1 produced by its own current are denoted as λ_{11}. Similarly, the current in coil 2 produces flux linkages λ_{22} in coil 2 and λ_{12} in coil 1.

Then, self-inductances of the coil are written as

$$L_1 = \frac{\lambda_{11}}{i_1}$$

and

$$L_2 = \frac{\lambda_{22}}{i_2}$$

The mutual inductance between the coils is

$$M = \frac{\lambda_{21}}{i_1} = \frac{\lambda_{12}}{i_2}$$

P15.39* Because coil 2 is short circuited, we have $v_2(t) = 0$. Thus, we have

$$0 = M\frac{di_1}{dt} + L_2\frac{di_2}{dt} = 0.5\frac{di_1}{dt} + 10\frac{di_2}{dt}$$

$$\frac{di_2}{dt} = -0.05\frac{di_1}{dt}$$

which implies that $i_2 = -0.05 i_1 + C$, in which C is a constant of integration to be determined later. After the switch closes, we have

$$L_1 \frac{di_1(t)}{dt} + M \frac{di_2(t)}{dt} = 12$$

Substituting results found above, this becomes

$$0.075 \frac{di_1}{dt} = 12$$

Integrating and using the fact that $i_1(0) = 0$, we have

$$i_1(t) = 160t$$

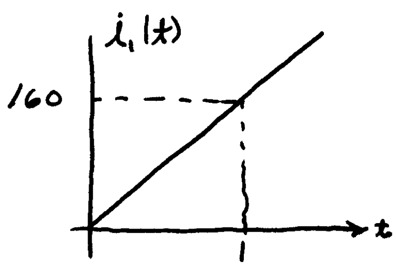
Substituting into the equation found earlier for i_2, we have

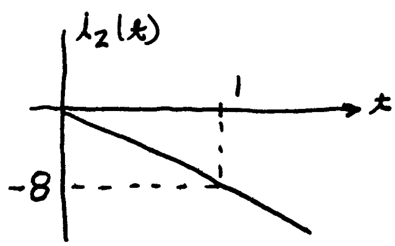
$$i_2 = -8t + C$$

However because we are given that i_2 is initially zero, we have $C = 0$ resulting in

$$i_2 = -8t$$

The sketches are:





P15.40* The voltages are given by Equations 15.36 and 15.37. We select the + signs because both currents enter dotted terminals and produce aiding fluxes.

$$e_1 = L_1 \frac{di_1}{dt} + M \frac{di_2}{dt}$$

$$e_2 = M \frac{di_1}{dt} + L_2 \frac{di_2}{dt}$$

$$e_1 = \frac{d[\cos(377t)]}{dt} + 0.5 \frac{d[0.5\cos(377t)]}{dt}$$

$$e_2 = 0.5 \frac{d[\cos(377t)]}{dt} + 2 \frac{d[0.5\cos(377t)]}{dt}$$

$$e_1 = 471.3 \sin(377t)$$

$$e_2 = 565.5 \sin(377t)$$

P15.41 The reluctance of a magnetic path is inversely proportional to the magnetic permeability of the core. Furthermore, inductance is inversely proportional to the reluctance of the core. Thus, inductance is directly proportional to permeability, and the new inductance is 200 mH.

P15.42 The reluctance of the first core is

$$R_1 = \frac{N_1^2}{L_1} = \frac{100^2}{0.1} = 10^5 \text{ A-turns/Wb}$$

When the dimensions of the core are doubled, the cross-sectional area is increased by a factor of four and the path length is doubled. Then, because reluctance is given by $R = \ell / \mu A$, we have $R_2 = R_1/2 = 5\times 10^4$ A-turns/Wb. Then, the inductance of the second coil is

$$L_2 = \frac{N_2^2}{R_2} = \frac{200^2}{5\times 10^4} = 800 \text{ mH}$$

P15.43 $v(t) = 48\sqrt{2}\cos(\omega t) = N\dfrac{d\phi}{dt}$

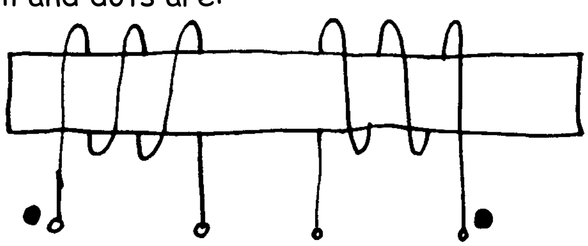
$$\phi(t) = \frac{48\sqrt{2}}{N\omega}\sin(\omega t) = 1.800\times 10^{-3}\sin(\omega t)$$

$$\phi_{peak} = 1.800\times 10^{-3}$$

$$X_L = \omega L = \frac{V_{rms}}{I_{rms}} = \frac{48}{0.05} = 960\Omega \quad L = \frac{X_L}{\omega} = 2.456\ H$$

$$R = \frac{N^2}{L} = \frac{100^{\ 2}}{2.546} = 3927.7$$

P15.44 The coil and dots are:



P15.45 The voltages are given by Equations 15.36 and 15.37. We select the + signs because both currents enter dotted terminals and produce aiding fluxes.

$$e_1 = L_1\frac{di_1}{dt} + M\frac{di_2}{dt}$$

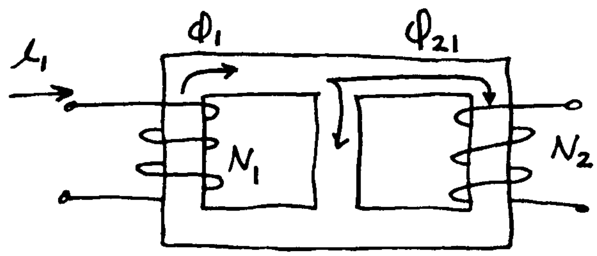
$$e_2 = M\frac{di_1}{dt} + L_2\frac{di_2}{dt}$$
$$e_1 = 0.2\frac{d[\exp(-1000t)]}{dt} + 0.1\frac{d[2\exp(-1000t)]}{dt}$$
$$e_2 = 0.1\frac{d[\exp(-1000t)]}{dt} + 0.5\frac{d[2\exp(-1000t)]}{dt}$$
$$e_1 = -400\exp(-1000t)$$
$$e_2 = -1100\exp(-1000t)$$

P15.46 According to Equations 15.34 and 15.35, we have
$$\lambda_1 = L_1 i_1 \pm M i_2$$
$$\lambda_2 = \pm M i_1 + L_2 i_2$$

When both currents enter dotted terminals, the fluxes aid and we use the + signs.
$$\lambda_1 = L_1 i_1 + M i_2 = 1(1) + 0.5(0.5) = 1.25 \text{ Weber turns}$$
$$\lambda_2 = M i_1 + L_2 i_2 = 0.5(1) + 2(0.5) = 1.5 \text{ Weber turns}$$

On the other hand, when one current enters a dotted terminal and the other enters an undotted terminal, the fluxes oppose one another, and we use the - signs.
$$\lambda_1 = L_1 i_1 - M i_2 = 1(1) - 0.5(0.5) = 0.75 \text{ Weber turns}$$
$$\lambda_2 = -M i_1 + L_2 i_2 = -0.5(1) + 2(0.5) = 0.5 \text{ Weber turns}$$

P15.47 Consider the coils shown below with current flowing only in winding 1. φ_1 is the flux produced by the current in coil 1, and $\varphi_{21} = k\varphi_1$ is the flux that links coil 2. The flux linkages of the coils are $\lambda_{11} = N_1\varphi_1$ and $\lambda_{21} = N_2 k\varphi_1$, respectively.



Using Equations 15.29 and 15.31, we have
$$L_1 = \frac{\lambda_{11}}{i_1} = \frac{N_1\varphi_1}{i_1} \qquad (1)$$

$$M = \frac{\lambda_{21}}{i_1} = \frac{N_2 k \phi_1}{i_1} \qquad (2)$$

Solving (1) for i_1 and substituting into (2), we have:

$$M = kL_1 \frac{N_2}{N_1} \qquad (3)$$

Repeating this analysis with coils 1 and 2 interchanged, results in:

$$M = kL_2 \frac{N_1}{N_2} \qquad (4)$$

Multiplying the respective sides of Equations (3) and (4), yields the desired result:

$$M^2 = k^2 L_1 L_2 \quad \text{or} \quad M = k\sqrt{L_1 L_2}$$

P15.48 Because coil 2 is open circuited, $i_2(t) = 0$. Thus, we have

$$L_1 \frac{di_1(t)}{dt} = 12$$

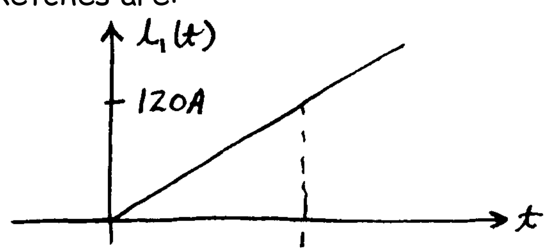
Integrating and using the fact that $i_1(0) = 0$, we have

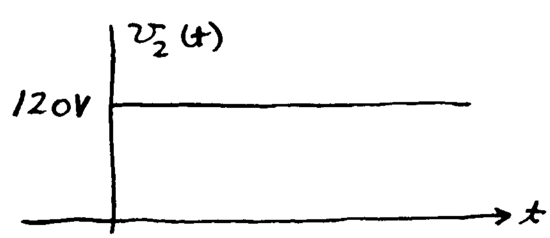
$$i_1(t) = (12/L_1)t = 120t$$

Substituting into Equation 15.37, we have:

$$v_2(t) = M\frac{di_1(t)}{dt} = 120 \text{ V}$$

The sketches are:





P15.49 According to Equation 15.25, inductance is given by:

$$N^2 = LR = 100 \times 10^{-3} \times 25 \times 10^5 = 2500 \times 10^2$$

$$N = 500 \text{ turns}$$

P15.50* Assuming constant peak flux density, power loss due to eddy currents increases with the square of frequency. Power loss due to hysteresis is directly proportional to frequency. Thus, the power loss at 400 Hz is

$$P = 1 \times \left(\frac{400}{60}\right) + 0.5 \times \left(\frac{400}{60}\right)^2$$
$$= 28.88 \text{ W}$$

P15.51* Two causes of core loss are hysteresis and eddy currents. To minimize loss due to hysteresis, we should select a material having a thin hysteresis loop (as shown in Figure 15.21 in the book). To minimize loss due to eddy currents, we laminate the core. The laminations are insulated from one another so eddy currents cannot flow between them.

If the frequency of operation is doubled, the power loss due to hysteresis doubles and the power loss due to eddy currents is quadrupled.

P15.52 For use in a permanent magnet, a material with a broad hysteresis loop and a high intercept on the B-axis is desirable so the magnetic field is strong with no applied magnetomotive force.

For use in a motor or transformer, we want a material having a hysteresis loop of small area, so that the core loss is small.

P15.53 See Figure 15.18c in the book.

P15.54 (a) $R = \frac{\ell}{\mu A} = \frac{0.2}{2000 \times 4\pi \times 10^{-7} \times 4 \times 10^{-4}} = 1.989 \times 10^5$

$$B = \frac{\mu N I}{\ell} = \frac{2000 \times 4\pi \times 10^{-7} \times 500 \times 0.1}{0.2} = 0.6283 \text{ T}$$

$$L = \frac{N^2}{R} = 1.257 \text{ H}$$

(b) $W = \frac{1}{2} L I^2 = 6.283 \text{ mJ}$

(c) $W_v = \dfrac{B^2}{2\mu} = \dfrac{0.6283^2}{2 \times 2000 \times 4\pi \times 10^{-7}} = 78.54\ \text{J/m}^3$

$W = W_v A\ell = 6.283\ \text{mJ}$

P15.55 We denote the eddy-current power loss at 60 Hz as P_e and the hysteresis power loss as P_h. Then we have

$$P_e + P_h = 3.6$$

$$\left(\frac{120}{60}\right)^2 P_e + \left(\frac{120}{60}\right) P_h = 11.2$$

Solving these equations we find P_e = 2.0 W and P_h = 1.6 W.

P15.56 The reluctance of the air gap is

$$R = \frac{\ell_g}{\mu_0 A}$$

Where $A = 6 \times 10^{-4}\ \text{m}^2$ is the area of the air gap.

The flux is given by:

$$\varphi = \frac{F}{R} = \frac{1000\,\mu_0 A}{\ell_g}$$

The flux density is:

$$B = \frac{\varphi}{A} = \frac{1000\,\mu_0}{\ell_g}$$

Equation 15.41 gives the energy stored per unit volume:

$$W_v = \frac{B^2}{2\mu_0}$$

Substituting, we have:

$$W_v = \frac{10^6\,\mu_0}{2\ell_g^2}$$

Finally, the energy stored is the product of W_v and the volume of the air gap:

$$W = W_v A \ell_g$$
$$= \frac{10^6\,\mu_0 A}{2\ell_g}$$

$$= \frac{10^6 \times 4\pi \times 10^{-7} \times 6 \times 10^{-4}}{2\ell_g}$$
$$= \frac{377.0 \times 10^{-6}}{\ell_g}$$

P15.57 As indicated by Equation 15.40, the area enclosed by the hysteresis loop is the energy dissipated per unit volume for each cycle. The area of the hysteresis loop shown in Figure P15.54 is:

$$W_v = 2 \times 30 = 60 \text{ joules/m}^3$$

To find the power loss, we multiply W_v by the volume of the core and by the frequency of operation.

$$\begin{aligned} P_{\text{hysteresis}} &= W_v \times \text{volume} \times f \\ &= 60 \times 1000 \times 10^{-6} \times 60 \\ &= 3.6 \text{ W} \end{aligned}$$

P15.58 In deriving the current and voltage relationships for an ideal transformer, we assumed that all of the flux links all of the turns of both windings, that the reluctance of the core is very small so a negligible mmf is needed to establish the flux, and that the core losses and the resistances of the coils are negligible.

P15.59* If we tried to make the 25 Ω load look like 100 Ω by adding 75 Ω in series, 75% of the power delivered by the source would be dissipated in the 75-Ω resistance. On the other hand, when using the transformer, virtually all of the power taken from the source is delivered to the load. Thus, from the standpoint of efficiency, the transformer is a much better choice.

P15.60* If residential power was distributed at 12 V (rather than 120 V) higher currents (by an order of magnitude) would be required to deliver the same amounts of power. This would require much larger wire sizes to avoid excessive power loss in the resistances of the conductors.

On the other hand, if residential power was distributed at 12 kV, greater safety hazards would result.

P15.61* (a) The dots should be placed on the top end of coil 2 and on the right-hand end of coil 3.

(b) $\mathbf{V}_2 = \frac{N_2}{N_1}\mathbf{V}_1 = 50\angle 0^\circ$ $\qquad$ $\mathbf{I}_2 = \frac{\mathbf{V}_2}{5\,\Omega} = 10\angle 0^\circ$

$\mathbf{V}_3 = \frac{N_3}{N_1}\mathbf{V}_1 = 100\angle 0^\circ$ $\qquad$ $\mathbf{I}_3 = \frac{\mathbf{V}_3}{10\,\Omega} = 10\angle 0^\circ$

(c) The total mmf is:

$$F = N_1\mathbf{I}_1 - N_2\mathbf{I}_2 - N_3\mathbf{I}_3$$

(Notice that the mmfs of $\mathbf{I}_2$ and $\mathbf{I}_3$ oppose that of $\mathbf{I}_1$.) Since we assume that the reluctance of the core is negligible, the net mmf is zero and we have:

$$N_1\mathbf{I}_1 - N_2\mathbf{I}_2 - N_3\mathbf{I}_3 = 0$$

Solving for $\mathbf{I}_1$ and substituting values, we have:

$$\mathbf{I}_1 = (N_2/N_1)\mathbf{I}_2 + (N_3/N_1)\mathbf{I}_3$$
$$= 15\angle 0^\circ$$

P15.62 For the ideal transformer, we have:

$$V_{2\mathrm{rms}} = \frac{N_2}{N_1} \times V_{1\mathrm{rms}}$$

The load current is

$$I_{2\mathrm{rms}} = \frac{V_{2\mathrm{rms}}}{R_L}$$

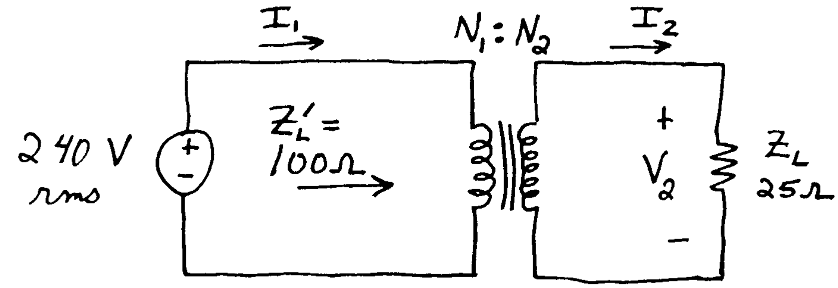
The load power is given by

$$P_L = V_{2\mathrm{rms}} I_{2\mathrm{rms}}$$

The results for the various turns ratios are:

Turns Ratio (N_1/N_2)	$V_{2\mathrm{rms}}$	$I_{2\mathrm{rms}}$	P_L
10	20 V	0.2 A	4 W
1	200 V	2.0 A	400 W
0.1	2000 V	20.0 A	40 kW

P15.63 The circuit is:



According to Equation 15.62, impedances are reflected to the primary by the square of the turns ratio.

$$Z'_L = \left(\frac{N_1}{N_2}\right)^2 Z_L$$

We have $Z_L = 25\ \Omega$ and want $Z'_L = 100\ \Omega$. Thus, we need a turns ratio $N_1/N_2 = 2$.

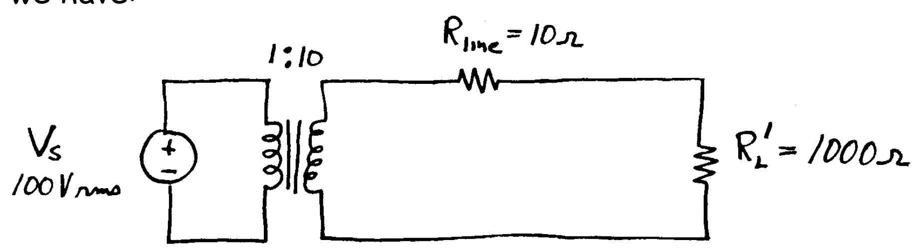
The secondary voltage and currents are:

$$V_2 = V_1(N_2/N_1) = 120 \text{ V rms}$$
$$I_1 = 240/100 = 2.4 \text{ A rms}$$
$$I_2 = 120/25 = 4.8 \text{ A rms}$$

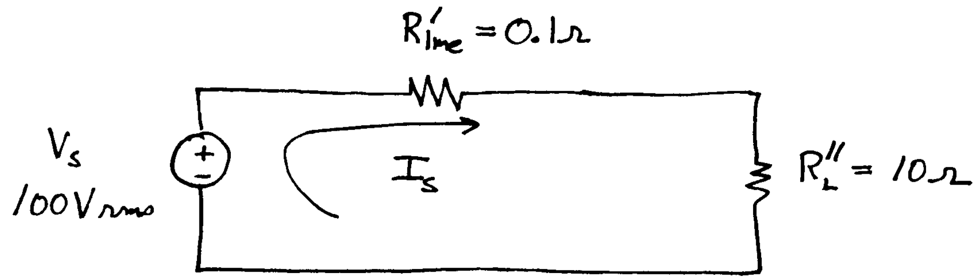
P15.64 (a)

$$I_L = V_s/(R_{\text{line}} + R_L) = 5 \text{ A rms}$$
$$P_{\text{line}} = R_{\text{line}} I_L^2 = 250 \text{ W}$$
$$P_s = V_s I_L = 500 \text{ W}$$
$$P_L = R_L I_L^2 = 250 \text{ W}$$
$$\text{Efficiency} = (P_L/P_s) \times 100\% = 50\%$$

(b) Reflecting the load to the primary of the step-down transformer, we have:



Then, reflecting both the load and line resistances to the primary of the step-up transformer, we have:



$$I_s = V_s/(R'_{\text{line}} + R''_L) = 9.901 \text{ A rms}$$
$$P_{\text{line}} = R'_{\text{line}} I_s^2 = 9.803 \text{ W}$$
$$P_L = R''_{\text{line}} I_s^2 = 980.3 \text{ W}$$
$$P_s = V_s I_L = 990.1 \text{ W}$$
$$\text{Efficiency} = (P_L/P_s) \times 100\% = 99.01\%$$

P15.65 (a) $\mathbf{I}_1 = 6\angle 30^\circ \quad \mathbf{V}_2 = 400\angle 0^\circ$

(b) $P_v = -V_{1rms} I_{1rms} \cos(\theta_1) = -\dfrac{200}{\sqrt{2}} \times \dfrac{6}{\sqrt{2}} \cos(-30) = -519.6 \text{ W}$

$P_I = V_{2rms} I_{2rms} \cos(\theta_2) = \dfrac{400}{\sqrt{2}} \times \dfrac{3}{\sqrt{2}} \cos(0 - 30) = 519.6 \text{ W}$

Power is taken from the voltage source and delivered to the current source.

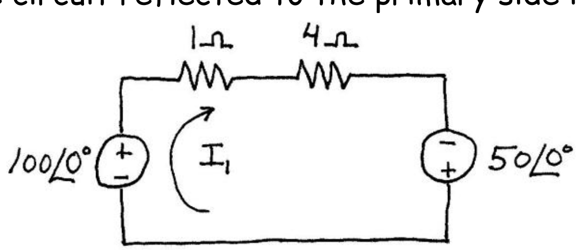
(c) $\mathbf{I}_1 = 6\angle -150^\circ \qquad \mathbf{V}_2 = 400\angle 180^\circ$

$P_v = -V_{1rms} I_{1rms} \cos(\theta_1) = -\dfrac{200}{\sqrt{2}} \times \dfrac{6}{\sqrt{2}} \cos[0 - (-150)] = 519.6 \text{ W}$

$P_I = V_{2rms} I_{2rms} \cos(\theta_2) = \dfrac{400}{\sqrt{2}} \times \dfrac{3}{\sqrt{2}} \cos(180 - 30) = -519.6 \text{ W}$

Power is taken from the current source and delivered to the voltage source.

P15.66 (a) The circuit reflected to the primary side is:



Then we have $\mathbf{I}_1 = \dfrac{100+50}{1+4} = 30\angle 0^\circ$.

(b) With the dot moved to the top of the secondary, the polarity of the right-hand voltage source is reversed, and we have

$$\mathbf{I}_1 = \frac{100-50}{1+4} = 10\angle 0^\circ$$

P15.67 (a) Let N_1 = 200 turns and N_2 = 300 turns (i.e., N_2 is the total number of turns). Then, we have:

$$v_1 = N_1 \frac{d\varphi}{dt}$$

$$v_2 = N_2 \frac{d\varphi}{dt}$$

Dividing the respective sides of these equations, we obtain:

$$\frac{v_1}{v_2} = \frac{N_1}{N_2} = \frac{2}{3}$$

(b) The net mmf is:

$$F = N_1(i_1 - i_2) - (N_2 - N_1)i_2 = 0$$

Simplifying, we have:

$$\frac{i_1}{i_2} = \frac{N_2}{N_1} = \frac{3}{2}$$

P15.68 $R_L' = 40\ \Omega$

$C_L' = 0.25\ \mu\text{F}$

P15.69 We have:

$$v_1 = N_1 \frac{d\varphi}{dt} \quad \text{and} \quad v_2 = N_2 \frac{d\varphi}{dt}$$

Dividing the respective sides of these equations, we obtain:

$$\frac{v_1}{v_2} = \frac{N_1}{N_2} = 10$$

Thus we have $\mathbf{V}_2 = 0.1\mathbf{V}_1 = 12\angle 0^\circ$ and $\mathbf{I}_2 = \mathbf{V}_2 / 6 = 2\angle 0^\circ$. Using KCL, we can write $\mathbf{I}_1 = \mathbf{I}_2 + \mathbf{I}_3$. Next because we assume that the net MMF is zero, we have $(N_1 - N_2)\mathbf{I}_1 + N_2\mathbf{I}_3 = 0$. From the previous two equations we obtain

$$\frac{\mathbf{I}_1}{\mathbf{I}_2} = \frac{N_2}{N_1}$$

which results in $\mathbf{I}_1 = 0.1\mathbf{I}_2 = 0.2\angle 0^\circ$. Finally we have

$$\mathbf{I}_3 = \mathbf{I}_1 - \mathbf{I}_2 = -1.8\angle 0^\circ$$

P15.70* The equivalent circuit of a real transformer is shown in Figure 15.28 in the book. The resistances R_1 and R_2 account for the resistance of the wires used to wind the coils of the transformer. L_1 and L_2 account for flux produced by each coil that does not link the other coil. L_m accounts for the current needed to set up the mutual flux in the core. Finally, R_c accounts for core losses due to eddy currents and hysteresis.

P15.71* We follow the method of Example 15.13. The results are:

$$\mathbf{I}_2 = 8.333\angle -36.87^\circ \text{ A rms}$$

$$\mathbf{I}_1 = \frac{N_2}{N_1}\mathbf{I}_2 = \frac{1}{33.33} \times 8.333\angle -36.87^\circ = 0.25\angle -36.87^\circ \text{ A rms}$$

$$\mathbf{V}_2 = \mathbf{V}_{\text{load}} + (R_2 + jX_2)\mathbf{I}_2$$

$$\mathbf{V}_2 = 240.88 + j0.90$$

$$\mathbf{V}_1 = \frac{N_1}{N_2}\mathbf{V}_2$$

$$\mathbf{V}_1 = 8029.4 + j30.0$$

$$\mathbf{V}_s = \mathbf{V}_1 + (R_1 + jX_1)\mathbf{I}_1$$

$$\mathbf{V}_s = 8050.6\angle 0.368^\circ$$

$$P_{\text{loss}} = \frac{V_s^2}{R_c} + I_1^2 R_1 + I_2^2 R_2$$

$$P_{\text{loss}} = 324.1 + 0.9 + 1.4$$

$$P_{\text{loss}} = 326.4 \text{ W}$$

$$P_{\text{load}} = V_{\text{load}} I_2 \times \text{power factor}$$

$$P_{\text{load}} = 2 \text{ kVA} \times 0.8 = 1600 \text{ W}$$

$$P_{\text{in}} = P_{\text{load}} + P_{\text{loss}}$$

$$P_{\text{in}} = 1926.4 \text{ W}$$

$$\text{Efficiency} = \left(1 - \frac{P_{\text{loss}}}{P_{\text{in}}}\right) \times 100\%$$

$$\text{Efficiency} = 83.06\%$$

Next, we can determine the no-load voltages. Under no-load conditions, we have:

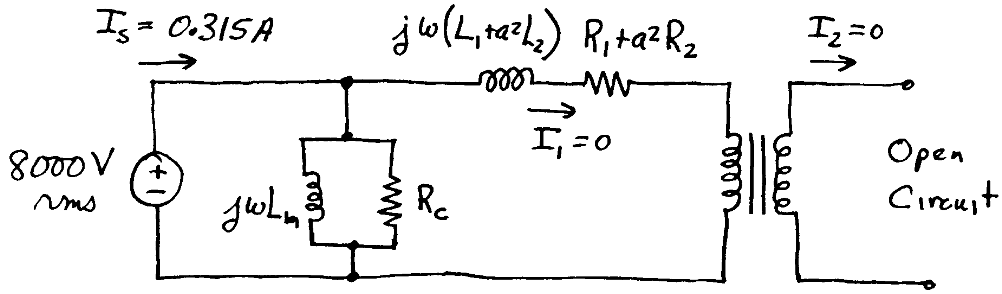
$$I_1 = I_2 = 0$$

$$V_1 = V_s = 8050.6$$

$$V_{\text{no-load}} = V_2 = V_1 \frac{N_2}{N_1} = 241.5$$

$$\text{Percent regulation} = \frac{V_{\text{no-load}} - V_{\text{load}}}{V_{\text{load}}} \times 100\% = 0.625\%$$

P15.72 The equivalent circuit is:



Because the load is an open circuit, we have $\mathbf{I}_1 = \mathbf{I}_2 = 0$. From the data given in the problem, we can determine the values of L_m and R_c. Since the power is dissipated in R_c, we have

$$R_c = \frac{V_s^2}{P} = \frac{8000^2}{360} = 177.8\ \text{k}\Omega$$

The reactive power is:

$$Q = \sqrt{(V_s I_s)^2 - P^2}$$
$$= \sqrt{(8000 \times 0.315)^2 - 360^2}$$
$$= 2494\ \text{Var}$$

However, the reactive power is absorbed in the magnetizing inductance.

$$X_m = \frac{V_s^2}{Q} = \frac{8000^2}{2494} = 25.66\ \text{k}\Omega$$

P15.73* The voltage across a transformer coil is approximately equal to

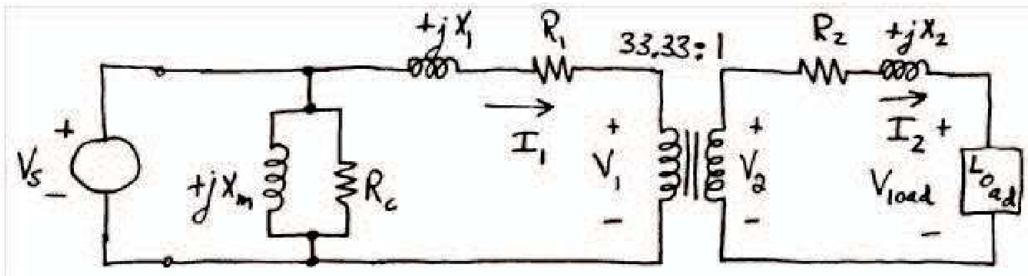
$$N\frac{d\varphi}{dt}$$

in which N is the number of turns and $\varphi = BA$ is the flux in the core. If B is reduced in magnitude, either N or the core area A would need to be increased to maintain the same voltage rating. In either case, more material (i.e., iron for the core or copper for the windings) is needed for the transformer.

On the other hand if the peak value of B is much higher than the saturation point, much more magnetizing current is required resulting in higher losses.

P15.74 In dc steady state, the magnetizing inductance L_m acts as a short circuit. Therefore, the applied dc voltage is dropped across the winding resistance R_1 and the voltage across the primary winding of the ideal transformer part of the model is zero. Consequently, the output voltage is zero.

P15.75 We follow the method of Example 15.13. The equivalent circuit is:



The turns ratio is the ratio of the rated voltages.

$$\frac{N_1}{N_2} = \frac{8000 \text{ V}}{240 \text{ V}} = 33.33$$

$$\mathbf{V}_{load} = 240\angle 0^\circ \text{ V rms}$$

For rated load (40 kVA), the load current is:

$$I_2 = \frac{40 \text{ kVA}}{240 \text{ V}} = 166.67 \text{ A rms}$$

The load power factor is:

$$\text{power factor} = \cos(\theta) = 0.9$$

Solving, we find that

$$\theta = 25.84^\circ$$

Thus, the phasor load current is:

$$\mathbf{I}_2 = 166.67\angle -25.84^\circ \text{ A rms}$$

where the phase angle is negative because the load was stated to have a lagging power factor.

The primary current is related to the secondary current by the turns ratio.

$$\mathbf{I}_1 = \frac{N_2}{N_1}\mathbf{I}_2 = \frac{1}{33.33} \times 166.67\angle -25.84^\circ = 5\angle -25.84^\circ \text{A rms}$$

Next, we can compute the voltages.

$$\mathbf{V}_2 = \mathbf{V}_{load} + (R_2 + jX_2)\mathbf{I}_2$$

$$\mathbf{V}_2 = 240 + (0.04 + j0.02) \times 166.67\angle -25.84^\circ$$
$$\mathbf{V}_2 = 240 + 33.98\angle 52.85^\circ$$
$$\mathbf{V}_2 = 262.52 + j27.08 \text{ V rms}$$

The primary voltage is related to the secondary voltage by the turns ratio.

$$\mathbf{V}_1 = \frac{N_1}{N_2}\mathbf{V}_2 = 33.33 \times (262.52 + j27.08)$$
$$\mathbf{V}_1 = 8749.79 + j902.58 \text{ V rms}$$

Now, we can compute the source voltage.

$$\mathbf{V}_s = \mathbf{V}_1 + (R_1 + jX_1)\mathbf{I}_1$$
$$\mathbf{V}_s = 8749.79 + j902.58 + (20 + j100) \times 5\angle -25.84^\circ$$
$$\mathbf{V}_s = 9152\angle 8.22^\circ$$

Next, we compute the power loss in the transformer.

$$P_{\text{loss}} = \frac{V_s^2}{R_c} + I_1^2 R_1 + I_2^2 R_2$$
$$P_{\text{loss}} = 418.80 + 25 \times 20 + (166.67)^2 \times 0.04$$
$$P_{\text{loss}} = 2030 \text{ W}$$

The power delivered to the load is given by:

$$P_{\text{load}} = V_{\text{load}} I_2 \times \text{power factor}$$
$$P_{\text{load}} = 40000 \times 0.9 = 36000\text{W}$$

The input power is given by:

$$P_{\text{in}} = P_{\text{load}} + P_{\text{loss}}$$
$$P_{\text{in}} = 36{,}000 + 2030 = 38030 \text{ W}$$

At this point, we can compute the power efficiency.

$$\text{Efficiency} = \left(1 - \frac{P_{\text{loss}}}{P_{\text{in}}}\right) \times 100\%$$

$$\text{Efficiency} = \left(1 - \frac{2030}{38030}\right) \times 100\% = 94.7\%$$

Next, we can determine the no-load voltages. Under no-load conditions, we have:

$$I_1 = I_2 = 0$$

$$V_1 = V_s \quad = 9152 \text{ V}$$

$$V_{\text{no-load}} = V_2 = V_1 \frac{N_2}{N_1} = 274.6 \text{ V}$$

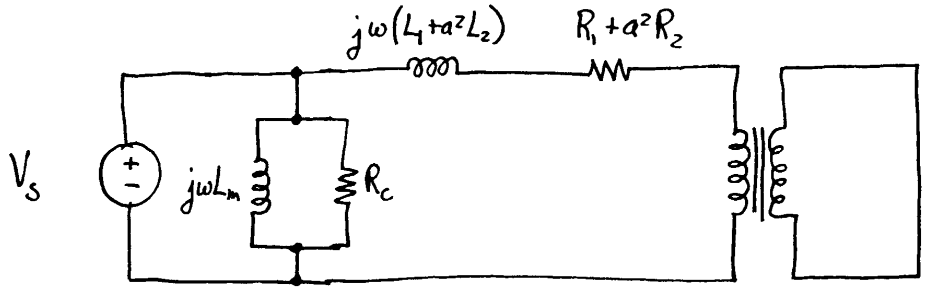
Finally, the percentage regulation is:

$$\text{Percent regulation} = \frac{V_{\text{no-load}} - V_{\text{load}}}{V_{\text{load}}} \times 100\%$$

$$\text{Percent regulation} = \frac{274.6 - 240}{240} \times 100\%$$

$$\text{Percent regulation } = 14.4\%$$

P15.76 The equivalent circuit is:



With a short-circuit load and rated current, V_s is very small and the currents through R_c and L_m are negligible. Since the power is dissipated in the resistance $R_1 + a^2R_2$, we have:

$$R_1 + a^2R_2 = \frac{P}{I_s^2} = \frac{270}{2.5^2} = 43.2\,\Omega$$

The reactive power is:

$$Q = \sqrt{(V_s I_s)^2 - P^2}$$

$$= \sqrt{(500 \times 2.5)^2 - 270^2}$$

$$= 1220 \text{ Var}$$

However, the reactive power is absorbed in the inductance $L_1 + a^2L_2$. The reactance is given by:

$$X = \frac{Q}{I_s^2} = \frac{1220}{2.5^2} = 195.3\,\Omega$$

$$L_1 + a^2L_2 = \frac{X}{\omega} = \frac{195.3}{377} = 0.518 \text{ H}$$

P15.77 The average power level is

$$P = \frac{400 \text{ kWh}}{(24\text{h/day}) \times (30 \text{ day/month})} = 555.5 \text{ W}$$

Of course, the power absorbed by a residence is not constant. Sometimes it is higher than the average and other times it is lower. Under maximum rated load, the primary and secondary currents are:

$$I_1 = \frac{20 \text{ kVA}}{2400 \text{ V}} = 8.333 \text{ A}$$

$$I_2 = \frac{20 \text{ kVA}}{240 \text{ V}} = 83.33 \text{ A}$$

Then, the powers dissipated in the winding resistances are:

$$P_{R_1} = I_1^2 R_1 = 208.33 \text{ W}$$

$$P_{R_2} = I_2^2 R_2 = 208.33 \text{ W}$$

The power absorbed by the core loss resistance R_c is

$$P_{\text{core}} = \frac{2400^2}{R_c} = 57.6 \text{ W}$$

Although the loss in the winding resistances is much higher than the core loss under maximum load, a residence operates at maximum load rarely (if ever). At the average load power, the loss in winding resistances is much smaller than the core loss. Core loss continues 24 hours per day regardless of the load. Thus, we conclude that R_c is the most significant parameter of the transformer from the standpoint of energy conservation.

P15.78 For operation with sinusoidal voltages, the flux in the core must be sinusoidal. Thus, we have

$$\varphi(t) = \varphi_{\max} \sin(\omega t)$$

The voltage across a transformer coil is then given by

$$v(t) = N\frac{d\phi(t)}{dt} = N\omega\phi_{\max}\cos(\omega t)$$

Because N and $\varphi_{\max}$ remain the same for a given transformer operated near its practical limits, the voltage ratings are nearly proportional to frequency, assuming that the insulation can withstand the higher voltages. Thus the voltage ratings at 120 Hz could be as high as 9600 V rms and 480 V rms for the primary and secondary, respectively, depending on the insulation. The resistances of the windings are nearly independent of frequency, so the power lost in the windings remains the same as the frequency is increased. We know that the core loss will increase with frequency. Thus, to maintain the same operating temperature the coil currents must be reduced somewhat. Thus the rating at 120 Hz will be higher than 10 kVA but less than 20 kVA.

Practice Test

T15.1 (a) We have $f = i\ell B\sin(\theta) = 12(0.2)0.3\sin(90^\circ) = 0.72\text{ N}$. ($\theta$ is the angle between the field and the wire.) The direction of the force is that of $i\mathbf{l} \times \mathbf{B}$ in which the direction of the vector $\mathbf{l}$ is the positive direction of the current (given as the positive x direction). Thus, the force is directed in the negative y direction.

(b) The current and the field are in the same direction so $\theta = 0$ and the force is zero. Direction does not apply for a vector of zero magnitude.

T15.2 The flux linking the coil is

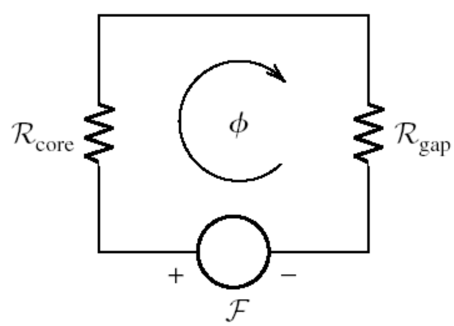
$\phi = BA = 0.7[\sin(120\pi t)](0.25)^2 = 43.75\times10^{-3}\sin(120\pi t)$ Wb

The induced voltage is

$$v = \frac{d\lambda}{dt} = N\frac{d\phi}{dt} = 10\times43.75\times10^{-3}\times120\pi\cos(120\pi t) = 164.9\cos(120\pi t)\text{ V}$$

T15.3 $e = Blu = 0.4\times0.2\times15 = 1.2\text{ V}$

T15.4 (a) The magnetic circuit is:



The permeability of the core is:

$$\mu_{core} = \mu_r\mu_0 = 1500\times4\pi\times10^{-7} = 1.885\times10^{-3}$$

The reluctance of the core is given by

$$R_{core} = \frac{\ell_{core}}{\mu_{core}A_{core}} = \frac{33.7\times10^{-2}}{1.885\times10^{-3}\times2\times10^{-2}\times3\times10^{-2}} = 298.0\times10^{3}$$

To account for fringing, we add the gap length to the width and depth of the gap.

$$R_{gap} = \frac{0.3 \times 10^{-2}}{4\pi \times 10^{-7} \times 2.3 \times 10^{-2} \times 3.3 \times 10^{-2}} = 3.145 \times 10^{6}$$

The equivalent reluctance seen by the source is:

$$R_{eq} = R_{core} + R_{gap} = 3.443 \times 10^{6}$$

The flux is :

$$\phi = \frac{F}{R_{eq}} = \frac{4 \times 350}{3.443 \times 10^{6}} = 406.6 \times 10^{-6} \text{ Wb}$$

Finally, the flux density in the gap is approximately

$$B_{gap} = \frac{\phi}{A_{gap}} = \frac{406.6 \times 10^{-6}}{2.3 \times 10^{-2} \times 3.3 \times 10^{-2}} = 0.5357 \text{ T}$$

(b) The inductance is

$$L = \frac{N^2}{R_{eq}} = \frac{350^2}{3.443 \times 10^{6}} = 35.58 \text{ mH}$$

T15.5 The two mechanisms by which power is converted to heat in an iron core are hysteresis and eddy currents. To minimize loss due to hysteresis, we choose a material for which the plot of B versus H displays a thin hysteresis loop. To minimize loss due to eddy currents, we make the core from laminated sheets or from powered iron held together by an insulating binder. Hysteresis loss is proportional to frequency and eddy-current loss is proportional to the square of frequency.

T15.6 (a) With the switch open, we have $I_{2\text{rms}} = 0$, $I_{1\text{rms}} = 0$ and the voltage across R_s is zero. Therefore, we have $V_{1\text{rms}} = 120$ V and $V_{2\text{rms}} = (N_2/N_1)V_{1\text{rms}} = 1200$ V. (The dots affect the phases of the voltages but not their rms values. Thus, $V_{2\text{rms}} = -1200$ V would not be considered to be correct.)

(b) With the switch closed, the impedance seen looking into the primary is $R'_L = (N_1/N_2)^2 R_L = 10\ \Omega$. Then, using the voltage division principle, we have $V_{1rms} = 120\dfrac{R'_L}{R_s + R'_L} = 114.3$ V. Next, $V_{2\text{rms}} = (N_2/N_1)V_{1\text{rms}} = 1143$ V.

The primary current is $I_{1rms} = 120/(10.5) = 11.43$ A. The secondary current is $I_{2\text{rms}} = (N_1/N_2)I_{1\text{rms}} = 1.143$ A.

T15.7 Core loss is nearly independent of load, while loss in the coil resistances is nearly proportional to the square of the rms load current. Thus, for a transformer that is lightly loaded most of the time, core loss is more significant. Transformer *B* would be better from the standpoint of total energy loss and operating costs.

CHAPTER 16

Exercises

E16.1 The input power to the dc motor is

$$P_{in} = V_{source} I_{source} = P_{out} + P_{loss}$$

Substituting values and solving for the source current we have

$$220 I_{source} = 50 \times 746 + 3350$$

$$I_{source} = 184.8 \text{ A}$$

Also we have

$$\eta = \frac{P_{out}}{P_{in}} \times 100\% = \frac{50 \times 746}{50 \times 746 + 3350} = 91.76\%$$

$$\text{speed regulation} = \frac{n_{no-load} - n_{full-load}}{n_{full-load}} \times 100\%$$

$$= \frac{1200 - 1150}{1150} \times 100\% = 4.35\%$$

E16.2 (a) The synchronous motor has zero starting torque and would not be able to start a high-inertia load.
(b) The series-field dc motor shows the greatest amount of speed variation with load in the normal operating range and thus has the poorest speed regulation.
(c) The synchronous motor operates at fixed speed and has zero speed regulation.
(d) The ac induction motor has the best combination of high starting torque and low speed regulation.
(e) The series-field dc motor should not be operated without a load because its speed becomes excessive.

E16.3 Repeating the calculations of Example 16.2, we have

(a) $$i_A(0+) = \frac{V_T}{R_A} = \frac{2}{0.05} = 40 \text{ A}$$

$$f(0+) = Bli_A(0+) = 2(0.3)40 = 24 \text{ N}$$

$$u = \frac{V_T}{Bl} = \frac{2}{2(0.3)} = 3.333 \text{ m/s}$$

(b) $$i_A = \frac{f_{load}}{Bl} = \frac{4}{2(0.3)} = 6.667 \text{ A}$$

$$e_A = V_T - R_A I_A = 2 - 0.05(6.667) = 1.667 \text{ V}$$
$$u = \frac{e_A}{Bl} = \frac{1.667}{2(0.3)} = 2.778 \text{ m/s}$$
$$p_m = f_{load}u = 4(2.778) = 11.11 \text{ W}$$
$$p_R = i_A^2 R = 2.222 \text{ W}$$
$$p_t = V_T i_A = 2(6.667) = 13.33 \text{ W}$$
$$\eta = \frac{p_m}{p_t} \times 100\% = \frac{11.11}{13.33} = 83.33\%$$

(c) $$i_A = \frac{f_{pull}}{Bl} = \frac{2}{2(0.3)} = 3.333 \text{ A}$$
$$e_A = V_T + R_A I_A = 2 + 0.05(3.333) = 2.167 \text{ V}$$
$$u = \frac{e_A}{Bl} = \frac{2.167}{2(0.3)} = 3.611 \text{ m/s}$$
$$p_m = f_{pull}u = 2(3.611) = 7.222 \text{ W}$$
$$p_t = V_T i_A = 2(3.333) = 6.667 \text{ W}$$
$$p_R = i_A^2 R = 0.5555 \text{ W}$$
$$\eta = \frac{p_t}{p_m} \times 100\% = \frac{6.667}{7.222} = 92.31\%$$

E16.4 Referring to Figure 16.15 we see that $E_A \cong 125$ V for $I_F = 2$ A and $n = 1200$. Then for $n = 1500$, we have
$$E_A = 125 \times \frac{1500}{1200} = 156 \text{ V}$$

E16.5 Referring to Figure 16.15 we see that $E_A \cong 145$ V for $I_F = 2.5$ A and $n = 1200$. Then for $n = 1500$, we have
$$E_A = 145 \times \frac{1500}{1200} = 181.3 \text{ V}$$
$$\omega_m = n \times \frac{2\pi}{60} = 157.1 \text{ rad/s}$$
$$T_{dev} = \frac{P_{dev}}{\omega_m} = \frac{10 \times 746}{157.1} = 47.49 \text{ Nm}$$
$$I_A = \frac{P_{dev}}{E_A} = \frac{10 \times 746}{181.3} = 41.15 \text{ A}$$
$$V_T = E_A + R_A I_A = 181.3 + 0.3(41.15) = 193.6 \text{ V}$$

E16.6 $R_{adj} = \frac{V_T - R_F I_F}{I_F} = \frac{300 - 10 \times 10}{10} = 20\ \Omega$

Because I_F remains constant the value of $K\varphi$ is the same value as in Example 16.4, which is 2.228. Furthermore the loss torque also remains constant at 11.54 Nm, and the developed torque remains at 261.5 Nm. Thus the armature current is still 117.4 A. Then we have

$$E_A = V_T - R_A I_A = 300 - 0.065(117.4) = 292.4\text{ V}$$

$$\omega_m = \frac{E_A}{K\varphi} = \frac{292.4}{2.228} = 131.2\text{ rad/s}$$

$$n_m = \omega_m \frac{60}{2\pi} = 1253\text{ rpm}$$

Thus the motor speeds up when V_T is increased.

E16.7 Following Example 16.4, we have

$$I_F = \frac{V_T}{R_F + R_{adj}} = \frac{240}{10 + 30} = 6\ A$$

Referring to Figure 16.18 we see that $E_A \cong 200$ V for $I_F = 6$ A and $n = 1200$. Thus we have

$$K\varphi = \frac{E_A}{\omega_m} = \frac{200}{1200(2\pi/60)} = 1.592$$

$$I_A = \frac{T_{dev}}{K\varphi} = \frac{261.5}{1.592} = 164.3\ A$$

$$E_A = V_T - R_A I_A = 240 - 0.065(164.3) = 229.3\text{ V}$$

$$\omega_m = \frac{E_A}{K\varphi} = \frac{229.3}{1.592} = 144.0\text{ rad/s}$$

$$n_m = \omega_m \frac{60}{2\pi} = 1376\text{ rpm}$$

$$P_{out} = T_{out}\omega_m = 36\text{ kW}$$

$$P_{in} = V_T(I_F + I_A) = 240(6 + 164.3) = 40.87\text{ kW}$$

$$\eta = \frac{P_{out}}{P_{in}} \times 100\% = 88.08\%$$

E16.8 $\omega_{m3} = \omega_{m1}\sqrt{\frac{T_{dev1}}{T_{dev3}}} = 125.7\sqrt{\frac{12}{6}} = 177.8\text{ rad/s}$

$$n_{m3} = \omega_{m3}\frac{60}{2\pi} = 1698\text{ rpm}$$

$$P_{out3} = T_{out3}\omega_{m3} = 1066\text{ W}$$

E16.9 With $R_A = 0$ and fixed V_T, the shunt motor has constant speed independent of the load torque. Thus we have

$$n_{m2} = n_{m1} = 1200 \text{ rpm}$$
$$\omega_{m2} = \omega_{m1} = 125.7 \text{ rad/s}$$
$$P_{out1} = T_{out1}\omega_{m1} = 1508 \text{ W}$$
$$P_{out2} = T_{out2}\omega_{m2} = 3016 \text{ W}$$

E16.10 Decreasing V_T decreases the field current and therefore the flux φ. In the linear portion of the magnetization curve, flux is proportional to the field current. Thus reduction of V_T leads to reduction of φ and according to Equation 16.35, the speed remains constant. (Actually, some speed variation will occur due to saturation effects.)

E16.11 The torque--speed relationship for the separately excited machine is given by Equation 16.27

$$T_{dev} = \frac{K\varphi}{R_A}(V_T - K\varphi\omega_m)$$

which plots as a straight line in the T_{dev} - ω_m plane. A family of plots for various values of V_T is shown in Figure 16.27 in the book.

E16.12 The torque--speed relationship for the separately excited machine is given by Equation 16.27

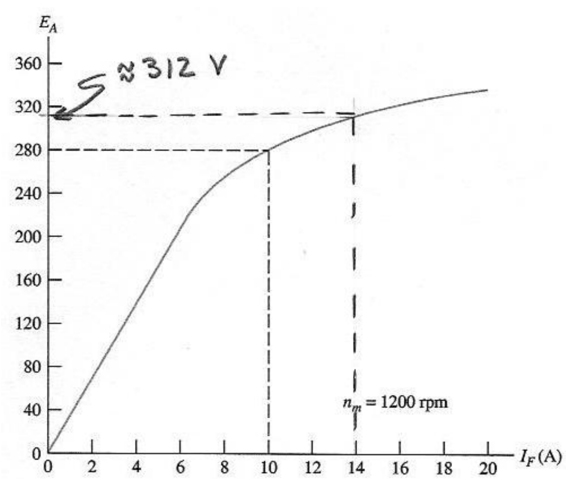
$$T_{dev} = \frac{K\varphi}{R_A}(V_T - K\varphi\omega_m)$$

which plots as a straight line in the T_{dev} - ω_m plane. As the field current is increased, the flux φ increases. A family of plots for various values of I_F and φ is shown in Figure 16.28 in the book.

E16.13

$$I_F = \frac{V_F}{R_{\text{adj}} + R_F} = \frac{140}{0+10} = 14 \text{ A}$$

$$V_{NL} = E_A = 312\frac{1000}{1200} = 260 \text{ V}$$



$$V_{FL} = E_A - R_A I_A = 260 - 200 \times 0.065 = 247 \text{ V}$$

$$\text{voltage regulation} = \frac{V_{NL} - V_{FL}}{V_{FL}} \times 100\% = \frac{260 - 247}{247} \times 100\% = 5.26\%$$

$$P_{\text{out}} = I_L V_{FL} = 200 \times 247 = 49.4 \text{ kW}$$

$$P_{\text{dev}} = P_{\text{out}} + R_A I_A^2 = 49400 + 0.065(200)^2 = 52.0 \text{ kW}$$

$$\omega_m = n_m \frac{2\pi}{60} = 104.7 \text{ rad/sec} \qquad P_{\text{in}} = \frac{P_{\text{out}}}{0.85} = \frac{49.4}{0.85} = 58.1 \text{ kW}$$

$$P_{\text{losses}} = P_{\text{in}} - P_{\text{dev}} = 58.1 - 52.0 = 6.1 \text{ kW}$$

$$T_{\text{in}} = \frac{P_{\text{in}}}{\omega_m} = \frac{58100}{104.7} = 555 \text{ nm} \qquad T_{\text{dev}} = \frac{P_{\text{dev}}}{\omega_m} = \frac{52000}{104.7} = 497 \text{ nm}$$

Problems

P16.1 The two types of windings found in electrical machines are field windings and armature windings. Field windings establish the magnetic field in the machine are not needed in permanent-magnet machines because the magnets provide the field.

P16.2 Dc motors are advantageous in automotive applications because dc power is available from the battery. Dc motors are advantageous when speed and direction must be controlled; however, this advantage is rapidly being lost to ac motors with electronic drives.

P16.3 Dc machines contain brushes and commutators which act as mechanical switches that reverse the connections to the rotor conductors as needed to maintain a constant output voltage polarity.

P16.4 The principal parts of a rotating electrical machine include the stator, rotor, shaft, field windings, and armature windings.

P16.5 The two principal types of three-phase motors are induction motors and synchronous motors. Induction motors are far more common.

P16.6* Two disadvantages of dc motors compared to signal-phase ac induction motors for a ventilation fan, which we can expect to operate most of the time, are first that dc power is usually not readily available in a home and second that dc machines tend to require more frequent maintenance than ac induction motors.

P16.7* $\text{speed regulation} = \dfrac{n_{\text{no-load}} - n_{\text{full-load}}}{n_{\text{full-load}}} \times 100\% = 5\%$

$$\text{speed regulation} = \frac{n_{\text{no-load}} - 1760}{1760} \times 100\% = \frac{5}{100}$$

Solving, we obtain $n_{\text{no-load}} = 1848$ rpm

P16.8 The input power is the output power divided by the efficiency.

$$P_{\text{in}} = \frac{P_{\text{out}}}{\eta} = \frac{10 \times 746}{0.75} = 8146 \text{ W}$$

Solving Equation 16.1 for the line current, we have

$$I_{\text{rms}} = \frac{P_{\text{in}}}{\sqrt{3} V_{\text{rms}} \cos\theta} = \frac{8146}{\sqrt{3} \times 220 \times 0.9} = 29.1 \text{ A}$$

P16.9 $\omega_m = n_m \times \dfrac{2\pi}{60} = 1760 \times \dfrac{2\pi}{60} = 184.3 \text{ radian/s}$

$$T_{\text{out}} = \frac{P_{\text{out}}}{\omega_m} = \frac{5 \times 746}{184.3} = 20.2 \text{ Nm}$$

P16.10* At full load, we have:

$$\omega_m = n_m \times \frac{2\pi}{60} = 1750 \times \frac{2\pi}{60} = 183.3 \text{ radian/s}$$

$$T_{\text{out}} = \frac{P_{\text{out}}}{\omega_m} = \frac{25 \times 746}{183.3} = 101.8 \text{ Nm}$$

Starting with rated voltage, we have:

$$T_{\text{start}} = 2 \times T_{\text{out}} = 203.6 \text{ Nm}$$

Starting with reduced line voltage, we estimate:

$$T_{\text{start}} = 203.6\left(\frac{220}{440}\right)^2$$
$$= 50.9 \text{ Nm}$$

P16.11 Rearranging Equation 16.9 we have $f = n_s P / 120 = 1000(4)/120 = 33.33$ Hz. Then, with a two-pole motor, we would have a speed of 2000 rpm which is the highest speed attainable with a synchronous motor using this frequency. For six poles, we have a speed of 666.7 rpm, and for eight poles we have a speed of 500 rpm.

P16.12 First, we determine the value of the constant K.

$$K = \frac{T_{\text{load}}}{(\omega_m)^2} = \frac{P_{\text{load}}}{(\omega_m)^3} = \frac{0.75 \times 746}{\left(1000 \times \dfrac{2\pi}{60}\right)^3} = 487.2 \times 10^{-6}$$

The equation for the torque--speed characteristic shown in Figure P16.12 is:

$$T = 20 - 0.1\omega_m$$

Equating the motor torque to the load torque, we have:

$$K\omega_m^2 = 20 - 0.1\omega_m$$

Solving, we find the equilibrium speed as:

$$\omega_m = 124.5 \text{ radian/s}$$

(The negative root is extraneous.)

$$n_m = \omega_m \times \frac{60}{2\pi} = 1189 \text{ rpm}$$

The torque is:

$$T = 20 - 0.1(124.5) = 7.55 \text{ Nm}$$

The power is:

$$P = \omega_m T = 940.1 \text{ W}$$
$$= 1.260 \text{ hp}$$

P16.13

$$P_{\text{in, full-load}} = \sqrt{3} V_{\text{rms}} I_{\text{rms}} \cos\theta$$
$$= \sqrt{3} \times 440 \times 35 \times 0.83$$
$$= 22.14 \text{ kW}$$

$$P_{\text{loss, full-load}} = P_{\text{in, full-load}} - P_{\text{out}}$$
$$= 22.14 - 25 \times 0.746$$
$$= 3.49 \text{ kW}$$

$$\eta = \frac{P_{\text{out}}}{P_{\text{in}}} \times 100\%$$
$$= \frac{25 \times 0.746}{22.14} \times 100\%$$
$$= 84.23\%$$

$$P_{\text{in, no-load}} = \sqrt{3} V_{\text{rms}} I_{\text{rms}} \cos\theta$$
$$= \sqrt{3} \times 440 \times 6.5 \times 0.30$$
$$= 1.49 \text{ kW}$$
$$\text{speed regulation} = \frac{n_{\text{no-load}} - n_{\text{full-load}}}{n_{\text{full-load}}} \times 100\%$$
$$= \frac{1797 - 1750}{1750} \times 100\%$$
$$= 2.69\%$$

P16.14 $P_{\text{in}} = V_{rms} I_{rms} \times \text{power factor} = 440(14)0.8 = 4928 \text{ W}$

$P_{out} = 6 \times 746 = 4476 \text{ W}$ $\qquad \eta = (P_{out} / P_{in}) \times 100\% = 90.8\ \%$

$P_{\text{loss}} = P_{\text{in}} - P_{\text{out}} = 452 \text{ W}$

P16.15* $\omega_m = n_m \times \frac{2\pi}{60} = 1150 \times \frac{2\pi}{60} = 120.4 \text{ radian/s}$

$$P_{\text{out}} = T_{\text{out}} \omega_m = 15 \times 120.4 = 1806 \text{ W}$$
$$P_{\text{out}} = 2.42 \text{ hp}$$
$$P_{\text{in}} = \sqrt{3} V_{\text{rms}} I_{\text{rms}} \cos\theta$$
$$= \sqrt{3} \times 440 \times 3.4 \times 0.8$$
$$= 2073 \text{ W}$$

$$P_{\text{loss}} = P_{\text{in}} - P_{\text{out}}$$
$$= 267 \text{ W}$$
$$\eta = \frac{P_{\text{out}}}{P_{\text{in}}} \times 100\%$$
$$= 87.1\%$$

P16.16 (a) At nearly zero speed, the torque required by the load is 40 Nm but the motor produces only 20 Nm. Thus, the system will not start from a standing start.

(b) To find the speeds for which the system can run at a constant speed, we equate the load torque to the motor torque.

$$\frac{800}{20+\omega_m} = 20 - \frac{\omega_m}{10}$$

Solving, we find two roots which are $\omega_{m1} = 25.97$ rad/s and $\omega_{m2} = 154.0$ rad/s.

(c) If the speed becomes slightly less than the lower root, the load torque exceeds the motor torque and the system slows to a stop. Thus, the system is not stable when running at the lower root.

(d) If the speed becomes slightly less than the higher root, the load torque is less than the motor torque and the system speeds up. Thus, the system is stable when running at the higher root.

P16.17 (a) For no load, we have:

$$T_{\text{out}} = 0 = 10^{-2}(60\pi - \omega_m)\omega_m$$

Two speeds satisfy this equation, $\omega_m = 0$ and $\omega_m = 60\pi$.

(b) To find maximum torque, we have:

$$\frac{dT_{\text{out}}}{d\omega_m} = 0 = 10^{-2}(60\pi - 2\omega_m)$$

Solving, we find that the speed for maximum torque is:

$$\omega_m = 30\pi$$

and the maximum torque is

$$T_{\text{out,max}} = 88.8\ \text{Nm}$$

(c)
$$P_{\text{out}} = \omega_m T_{\text{out}}$$
$$= 10^{-2}(60\pi - \omega_m)\omega_m^2$$

To find the maximum output power, we have:

$$\frac{dP_{\text{out}}}{d\omega_m} = 0 = 10^{-2}\left(120\pi\omega_m - 3\omega_m^2\right)$$

Solving, we find:

$$\omega_m = 40\pi$$

(The root $\omega_m = 0$ corresponds to minimum output power.) The maximum power is:

$$P_{\text{out,max}} = 9922\ \text{W}$$
$$= 13.3\ \text{hp}$$

(d) The starting torque is zero. The motor could be started with some other source of mechanical power to get it moving.

P16.18 The no-load speed of an induction motor is very close to the synchronous speed given by Equation 16.9:

$$n_s = \frac{120f}{P}$$

in this case we have f = 60 Hz. Furthermore the number of poles must be an even integer. Thus we have a 2-pole machine with a no-load speed of approximately 3600 rpm. Because of mechanical losses, the actual no-load speed will be slightly lower, perhaps 3590 rpm. In that case the speed regulation is

$$\begin{aligned}\text{speed regulation} &= \frac{n_{\text{no-load}} - n_{\text{full-load}}}{n_{\text{full-load}}} \times 100\% \\ &= \frac{3590 - 3500}{3500} \times 100\% \\ &= 2.6\%\end{aligned}$$

P16.19 $P_{in} = P_{out} + P_{loss}$

$$2500 = 3 \times 746 + P_{loss}$$

$$P_{loss} = 2500 - 2238 = 262 \text{ W}$$

$$\eta = \frac{P_{out}}{P_{in}} \times 100\% = \frac{3 \times 746}{2500} \times 100\% = 89.52\%$$

$$\cos\theta = \frac{P_{\text{in}}}{\sqrt{3}V_{\text{rms}}I_{\text{rms}}} = \frac{2500}{\sqrt{3} \times 220 \times 10} = 0.6561$$

Thus, the power factor is 65.61%. Usually the power factor is lagging for induction motors.

P16.20* In steady-state with no load, we have $V_T = e_A = B\ell u$ and the current i_A is zero.

(a) If V_T is doubled, the steady-state no-load speed is doubled.

(b) If the resistance is doubled, the steady-state no-load speed is not changed. (However, it will take longer for the motor to achieve this speed.)

(c) If B is doubled, the steady-state no-load speed is halved.

P16.21* When the switch is closed, current flows toward the right through the sliding bar. The force on the bar is given by:

$$\mathbf{f} = i_A \mathbf{l} \times \mathbf{B}$$

Thus, the force is directed toward the bottom of the page. The starting current and starting force are:

$$i_{A,\,\text{starting}} = V_T / R_A = 5/0.1 = 50\ A$$

$$f_{\text{starting}} = i_A B\ell = 50 \times 0.75 \times 1.3 = 48.75\ \text{N}$$

Under no-load conditions in steady state, we have

$$i_A = 0$$

$$V_T = e_A = B\ell u$$

Thus, the steady-state speed is

$$u = \frac{V_T}{B\ell} = \frac{5}{1.3 \times 0.75}$$

$$= 5.13\ \text{m/s}$$

P16.22 The output power is $P_{\text{out}} = 2 \times 746 = 1492$ W

$$P_{\text{out}} = 1492 = f \times u = f \times 20$$

$$f = 74.6\ \text{N}$$

Solving Equation 16.11 for the current and substituting values, we have:

$$i_A = \frac{f}{B\ell} = \frac{74.6}{1 \times 1} = 74.6$$

$$e_A = Blu = 1 \times 1 \times 20 = 20\ \text{V}$$

$$V_T = i_A R_A + e_A$$

$$= 74.6 \times 0.5 + 20 = 57.3\ \text{V}$$

$$P_{\text{in}} = V_T i_A$$

$$= 4274.6\ \text{W}$$

$$\eta = \frac{P_{\text{out}}}{P_{\text{in}}} \times 100\%$$

$$= \frac{1492}{4274.6} \times 100\%$$

$$= 34.9\%$$

P16.23 Under starting conditions $(u = 0)$, we have $e_A = B\ell u = 0$, $i_A = V_T/R_A$, and $f = i_A B\ell$.

(a) If the source voltage V_T is doubled, the starting force doubles.

(b) If R_A is doubled, the starting force is halved.

(c) If the magnetic flux density B is doubled, the starting force is doubled.

P16.24 As in Problem 16.21, we find that the starting force is 48.75 N. Since this is greater than the load force (10 N) applied to the bar, the machine acts as a motor. In steady state, we have:

$$f = 10 = i_A B\ell$$

Solving, we find that

$$i_A = \frac{f}{B\ell} = \frac{10}{0.75 \times 1.3} = 10.26 \text{ A}$$

$$\begin{aligned} e_A &= V_T - R_A i_A \\ &= 5 - 0.1 \times 10.26 \\ &= 3.974 \text{ V} \end{aligned}$$

The steady state velocity is:

$$u = \frac{e_A}{B\ell} = \frac{3.974}{1.3 \times 0.75} = 4.076 \text{ m/s}$$

(a) The power supplied by the voltage source is:

$$\begin{aligned} P_{\text{in}} &= V_T i_A \\ &= 5 \times 10.26 \\ &= 51.28 \text{ W} \end{aligned}$$

(b) The power absorbed by the resistance is:

$$\begin{aligned} P_{R_A} &= R_A i_A^2 \\ &= 0.1 \times (10.26)^2 \\ &= 10.52 \text{ W} \end{aligned}$$

(c) The mechanical output power is:

$$\begin{aligned} P_{\text{out}} &= fu \\ &= 10 \times 4.076 \\ &= 40.76 \text{ W} \end{aligned}$$

As a check, we note that $P_{\text{in}} = P_{R_A} + P_{\text{out}}$.

P16.25 (a) When the switch closes, current flows clockwise in the circuit. By the right hand rule, the field created by the current in the rails points into the page in the vicinity of the projectile. The force on the projectile is given by $\mathbf{f} = i_A \mathbf{l} \times \mathbf{B}$. Thus, the force is directed toward the right-hand side of Figure P16.25.

(b) Equating the energy stored in the capacitor to the kinetic energy in the projectile we have

$$\tfrac{1}{2} CV^2 = \tfrac{1}{2} mu^2$$

Solving for the velocity and substituting values, we have

$$u = \sqrt{\frac{CV^2}{m}} = \sqrt{\frac{1000 \times 10^{-6} \times (10^4)^2}{3 \times 10^{-3}}} = 5773 \text{ m/s}$$

(c) The velocity attained will be less than the value computed in part (b) because of air resistance, friction, energy lost in the resistance of the circuit. Furthermore, some of the energy may remain stored in the capacitor or in the magnetic field at the instant that the projectile leaves the end of the rails.

P16.26 In this case, the machine acts as a generator. We have:

$$f = -10 = i_A B\ell$$

Solving, we find that

$$i_A = \frac{f}{B\ell} = \frac{-10}{0.75 \times 1.3} = -10.26 \text{ A}$$

$$\begin{aligned} e_A &= V_T - R_A i_A \\ &= 5 \times 0.1 \times (-10.26) \\ &= 6.026 \text{ V} \end{aligned}$$

The steady-state velocity is

$$u = \frac{e_A}{B\ell} = \frac{6.026}{1.3 \times 0.75} = 6.181 \text{ m/s}$$

(a) The power supplied to the voltage source is

$$\begin{aligned} P_{\text{out}} &= V_T i_A \\ &= 5 \times 10.26 \\ &= 51.28 \text{ W} \end{aligned}$$

(b) The power absorbed by the resistance is

$$\begin{aligned} P_{R_A} &= R_A i_A^2 \\ &= 0.1 \times (10.26)^2 \\ &= 10.52 \text{ W} \end{aligned}$$

(c) The mechanical input power is

$$P_{\text{in}} = fu$$
$$= 10 \times 6.181$$
$$= 61.81 \text{ W}$$

As a check, we note that $P_{\text{in}} = P_{R_A} + P_{\text{out}}$.

P16.27* Using the right-hand rule we see that in Figure 16.10, the north pole of the rotor is at the top of the rotor. Because the north rotor pole is attracted to the south stator pole, the torque is counterclockwise, as indicated in the figure.

In Figure 16.11, the north rotor poles are in the upper right-hand and lower left-hand portions of the rotor. South poles appear in the upper left-hand and lower right-hand parts of the rotor. Because the north rotor poles are attracted to the south stator poles, the torque is counterclockwise, as indicated in the figure.

P16.28* Converting the speed of 1500 rpm to angular velocity, we have

$$\omega_m = n_m \times \frac{2\pi}{60} = 1500 \times \frac{2\pi}{60} = 156.1 \text{ rad/sec}$$

Solving Equation 16.15 for the machine constant $K\varphi$ and substituting values, we have

$$K\varphi = \frac{E_A}{\omega_m} = \frac{240}{157.1} = 1.528$$

Then we use Equation 16.16 to compute the torque

$$T_{\text{dev}} = K\varphi I_A = 1.528 \times 20 = 30.56 \text{ Nm}$$

$$P_{\text{dev}} = \omega_m T_{\text{dev}} = 4798 \text{ W}$$

The voltage applied to the armature circuit is

$$V_T = R_A I_A + E_A$$
$$= 1.5 \times 20 + 240$$
$$= 270 \text{ V}$$

P16.29* The voltage induced in each armature conductor is given by

$$e_A = B\ell u$$

where $\ell = 0.3 \text{ m}$ is the length of the conductor, $B = 1 \text{ T}$ is the flux density, and u is the linear velocity of the conductor which is given by the product of the number of revolutions per second $n_m/60 = 20$ and the circumference of the rotor.

$$u = 20 \times 2\pi \times 0.1 = 4\pi \text{ m/s}$$

Thus, we have

$$e_A = B\ell u = 1 \times 0.3 \times 4\pi = 3.770 \text{ V}$$

The machine is designed to have $E_A \cong 240$ (because in a good design $R_A I_A$ is small and $V_T \cong E_A$). Thus, we need to have

$$N = \frac{E_A}{e_A} = \frac{240}{3.77} \cong 64$$

armature conductors connected in series.

P16.30* Because the field current is constant so is $K\varphi$. Then because the developed torque is constant, I_A is constant. For V_T = 200 V, we have

$$E_{A1} = V_{T1} - R_A I_A = 200 - 5 \times 10 = 150 \text{ V}$$

and for V_T = 300 V, we have

$$E_{A2} = V_{T2} - R_A I_A = 300 - 5 \times 10 = 250 \text{ V}$$

Then we have

$$n_2 = n_1 \frac{E_{A2}}{E_{A1}} = 1800 \frac{250}{150} = 3000 \text{ rpm}$$

P16.31 For a permanent-magnet motor there are no field losses and $K\varphi$ is constant. Under no-load conditions, we have

$$E_A = V_T - R_A I_A = 240 - 7 \times 1 = 233 \text{ V}$$

$$\omega_m = n_m \frac{2\pi}{60} = 1500 \frac{2\pi}{60} = 50\pi$$

$$K\varphi = \frac{E_A}{\omega_m} = \frac{233}{50\pi} = 1.483$$

$$T_{dev} = T_{loss} = K\varphi I_A = 1.438 \text{ Nm}$$

Under loaded conditions, we have

$$\omega_m = n_m \frac{2\pi}{60} = 1300 \frac{2\pi}{60} = 136.1 \text{ rad/s}$$

$$E_A = K\varphi\omega_m = 201.9 \text{ V}$$

$$I_A = \frac{V_T - E_A}{R_A} = \frac{240 - 201.9}{7} = 5.444 \text{ A}$$

$$P_{in} = V_T I_A = 1307 \text{ W}$$

$$P_{loss} = T_{loss}\omega_m = 1.438 \times 136.1 = 195.7 \text{ W}$$

$$P_{out} = P_{dev} - P_{loss} = E_A I_A - P_{loss} = 903.4\text{ W}$$
$$\eta = \frac{P_{out}}{P_{in}} \times 100\% = 69.1\%$$

P16.32 $$E_A = V_T - R_A I_A = 180 - 1.2 \times 10 = 168\text{ V}$$
$$\omega_m = n_m \times \frac{2\pi}{60} = 1500 \times \frac{2\pi}{60} = 156.1\text{ rad/sec}$$
$$K\varphi = \frac{E_A}{\omega_m} = \frac{168}{40\pi} = 1.337$$
$$T_{\text{dev}} = K\varphi I_A = 1.337 \times 10 = 13.37\text{ N}$$
$$P_{\text{dev}} = T_{\text{dev}}\omega_m = 1680\text{ W} = 2.252\text{ hp}$$
$$P_{\text{heat}} = I_A^2 R_A + \frac{V_F^2}{R_F}$$
$$= 10^2 \times 1.2 + \frac{180^2}{150}$$
$$= 336\text{ W}$$

P16.33 (a) The magnetic intensity in the air gap is
$$H = \frac{2NI_F}{2\ell_{\text{gap}}} = \frac{2 \times 250 \times 3}{2 \times 1.5 \times 10^{-3}} = 500 \times 10^3\text{ A/m}$$
The flux density is
$$B = \mu_0 H = 4\pi \times 10^{-7} \times 500 \times 10^3 = 0.6283\text{ T}$$

(b) The force exerted on each armature conductor is given by
$$f = I_A \ell_A B$$
where $\ell_A = 0.5\text{ m}$ is the length of an armature conductor. Thus, we have
$$f = 30 \times 0.5 \times 0.6283 = 9.425\text{ N}$$

P16.34 Under no-load conditions we have
$$E_A = V_T - R_A I_A = 480 - 0.5 \times 2 = 479\text{ V}$$
$$\omega_m = n_m \frac{2\pi}{60} = 1200 \frac{2\pi}{60} = 40\pi$$
$$K\varphi = \frac{E_A}{\omega_m} = \frac{479}{40\pi} = 3.812$$
$$T_{dev} = T_{loss} = K\varphi I_A = 1.906\text{ Nm}$$
Then with the load applied:

$$T_{dev} = T_{loss} + T_{load} = 1.906 + 50 = 51.91$$

$$I_A = \frac{T_{dev}}{K\varphi} = \frac{51.91}{3.812} = 13.62\ A$$

$$E_A = V_T - R_A I_A = 480 - 13.62 \times 2 = 452.8\ \text{V}$$

$$n_{full-load} = n_{no-load} \frac{E_{A,full-load}}{E_{A,full-load}} = 1200\,\frac{452.8}{479} = 1134\ \text{rpm}$$

$$\text{speed regulation} = \frac{n_{no-load} - n_{full-load}}{n_{full-load}} \times 100\%$$

$$= 5.82\%$$

P16.35 $$\omega_{m1} = n_{m1} \times \frac{2\pi}{60} = 1500 \times \frac{2\pi}{60} = 50\pi$$

$$\omega_{m2} = n_{m2} \times \frac{2\pi}{60} = 1800 \times \frac{2\pi}{60} = 60\pi$$

$$K\varphi = \frac{E_A}{\omega_{m1}} = \frac{240}{50\pi} = 1.528$$

$$T_{\text{dev}} = \frac{P_{\text{dev}}}{\omega_{m2}} = \frac{10 \times 746}{60\pi} = 39.58\ \text{Nm}$$

$$I_A = \frac{T_{\text{dev}}}{K\varphi} = \frac{39.58}{1.528} = 25.9\ A$$

P16.36 Equation 16.15 states

$$E_A = K\varphi\omega_m$$

With constant field current, the magnetic flux φ is constant. Therefore, the back emf E_A is proportional to machine speed ω_m (or equivalently to n_m). Thus, we have

n_m(rpm)	E_A(V)
600	120
1200	240
1500	300

P16.37 The magnetic field in the yoke is nearly constant in magnitude and direction. Thus, voltages that result in eddy currents are not induced in the yoke, and laminations are not necessary.

On the other hand as the rotor turns, the field alternates in direction through the rotor material. This induces voltages that could cause eddy currents and large power losses if the rotor was not laminated.

P16.38* (a)

$$E_A = V_T - R_A I_A = 440 - 0.1 \times 103 = 429.7\text{ V}$$

$$\omega_{m1} = n_{m1} \times \frac{2\pi}{60} = 1500 \times \frac{2\pi}{60} = 50\pi$$

$$K\phi = \frac{E_A}{\omega_{m1}} = \frac{429.7}{50\pi} = 2.736$$

$$T_{\text{dev}} = K\varphi I_A = 2.736 \times 103 = 281.8\text{ N}$$

$$P_{\text{dev}} = T_{\text{dev}}\omega_{m1} = 44.26\text{ kW} = 59.33\text{ hp}$$

$$P_{R_A} = I_A^2 R_A = 1.061\text{ kW}$$

$$P_{\text{rot}} = P_{\text{dev}} - P_{\text{out}}$$
$$= 44.26 - 50 \times 0.746 = 6.96\text{ kW} = 9.330\text{ hp}$$

(b) Since we are assuming that the rotational power loss is proportional to speed, we can write:

$$P_{\text{rot}} = 6960 \times \frac{\omega_m}{\omega_{m1}} = 44.31\omega_m\text{ W}$$

Also, we have

$$P_{\text{dev}} = K\varphi I_A \omega_m$$
$$= K\varphi \frac{V_T - E_A}{R_A}\omega_m$$
$$= K\varphi \frac{V_T - K\varphi\omega_m}{R_A}\omega_m$$

Substituting values, we have

$$P_{\text{dev}} = 12.04 \times 10^3\omega_m - 74.86\omega_m^2$$

With no load, we have $P_{\text{out}} = 0$, and

$$P_{\text{dev}} = P_{\text{rot}}$$
$$12.04 \times 10^3\omega_m - 74.86\omega_m^2 = 44.31\omega_m$$

Solving, we find

$$\omega_m = 160.24 \quad \text{or} \quad n_m = 1530\text{ rpm}$$

(The root $\omega_m = 0$ is extraneous.)

P16.39* (a) The field current is

$$I_F = \frac{V_T}{R_F + R_{\text{adj}}} = \frac{240}{240} = 1.0 \text{ A}$$

From the magnetization curve shown in Figure P16.39, we find that $E_A = 165 \text{ V}$ with $I_F = 1.0 \text{ A}$ and $n_m = 1000 \text{ rpm}$. Neglecting losses at no load, we have $I_A = 0$ and $E_A = V_T = 240 \text{ V}$. Since E_A is proportional to speed, the no-load speed is:

$$n_{\text{no-load}} = \frac{240}{165} \times 1000 \text{ rpm} = 1455 \text{ rpm}$$

(b) When the speed drops by 6%, we have

$$n_m = 1455 \times 0.94 = 1367 \text{ rpm}$$

$$\omega_m = 1367 \times \frac{2\pi}{60} = 143.2 \text{ radian/s}$$

$$E_A = 240 \times 0.94 = 225.6 \text{ V}$$

$$I_A = \frac{V_T - E_A}{R_A} = \frac{240 - 225.6}{1.5} = 9.6 \text{ A}$$

$$P_{\text{out}} = P_{\text{dev}} = E_A I_A = 2166 \text{ W}$$

$$T_{\text{load}} = T_{\text{dev}} = \frac{P_{\text{dev}}}{\omega_m} = 15.13 \text{ Nm}$$

$$P_F = V_T I_F = 240 \times 1.0 = 240 \text{ W} \qquad P_{R_A} = R_A I_A^2 = 138.2 \text{ W}$$

P16.40* (a) $$I_F = \frac{V_T}{R_F + R_{\text{adj}}} = \frac{200}{200} = 1.0 \text{ A}$$

We are given that $P_{\text{rot}} = 50 \text{ W}$ and $E_{A,\text{ref}} = 175 \text{ V}$ at $n_{m,\text{ref}} = 1200 \text{ rpm}$. Next, we determine the value of the machine constant $K\varphi$.

$$K\varphi = \frac{E_{A,\text{ref}}}{\omega_{m,\text{ref}}} = \frac{175}{1200 \times \frac{2\pi}{60}} = 1.393$$

We assume that the rotational power loss is proportional to speed, which is equivalent to assuming constant torque.

$$T_{\text{rot}} = \frac{P_{\text{rot}}}{\omega_{m,\text{ref}}} = \frac{50}{1200 \times \frac{2\pi}{60}} = 0.3980 \text{ Nm}$$

At no load, we have $T_{\text{out}} = 0$ and $T_{\text{dev}} = T_{\text{rot}}$.

$$I_A = \frac{T_{\text{dev}}}{K\varphi} = \frac{0.3980}{1.393} = 0.2857\ A$$

$$E_A = V_T - R_A I_A = 199.71 \text{ V}$$

$$n_{m,\text{no-load}} = n_{m,\text{ref}} \times \frac{E_A}{E_{A,\text{ref}}} = 1200 \times \frac{199.71}{175} = 1369 \text{ rpm}$$

$$\omega_{m,\text{no-load}} = n_{m,\text{no-load}} \times \frac{2\pi}{60} = 143.4 \text{ radian/s}$$

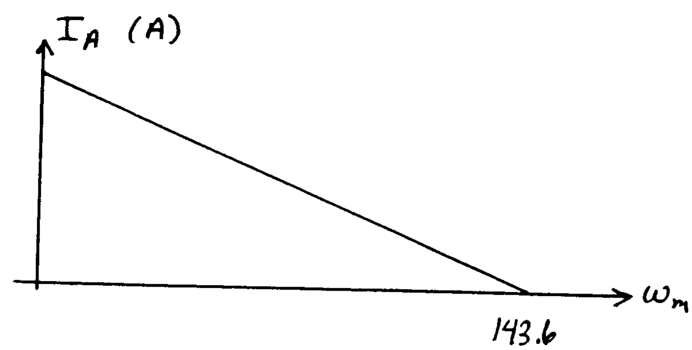
(b) We have

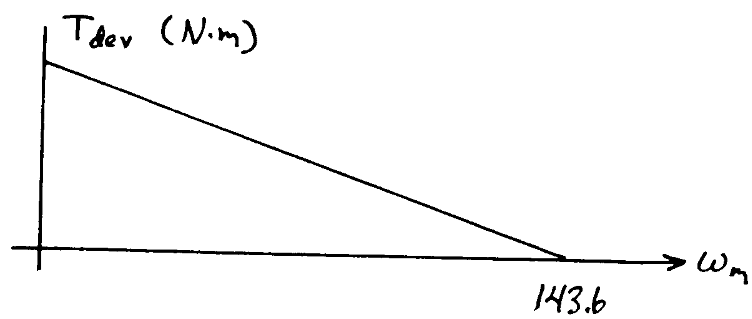
$$I_A = \frac{V_T - K\varphi\omega_m}{R_A} = 200 - 1.393\omega_m$$

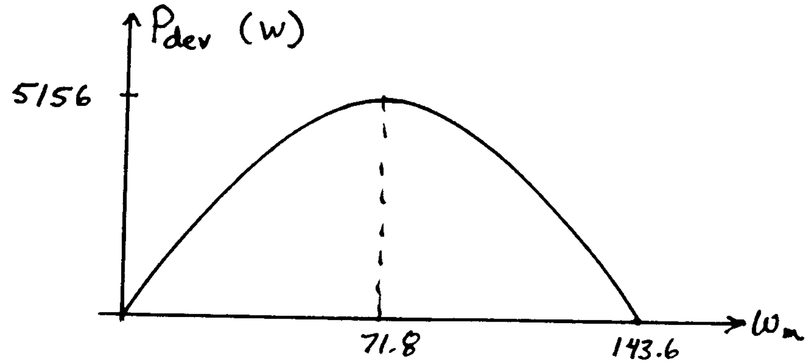
$$T_{\text{dev}} = K\varphi I_A = 278.6 - 1.940\omega_m$$

$$P_{\text{dev}} = T_{\text{dev}}\omega_m = (278.6 - 1.940\omega_m)\omega_m$$

The plots are:







P16.41* We have

$$I_A = \frac{V_T - E_A}{R_A} = \frac{V_T - K\varphi\omega_m}{R_A}$$

$$P_{\text{dev}} = E_A I_A$$

$$P_{\text{dev}} = \frac{V_T - K\varphi\omega_m}{R_A} K\varphi\omega_m$$

Substituting values, we obtain

$$5 \times 746 = 166.67\omega_m - 0.83333\omega_m^2$$

Solving, we find the two roots and the corresponding armature currents as

$$\omega_{m1} = 174.3 \text{ and } I_{A1} = 21.4 \text{ A for which } \eta = 87.2\%$$

$$\omega_{m2} = 25.67 \text{ and } I_{A2} = 141 \text{ A for which } \eta = 13.5\%$$

The first solution is more likely to fall within the rating because the efficiency for the second solution is very low.

P16.42* The magnetization curve is a plot of E_A versus the field current I_F at a stated speed. Because a permanent magnet motor does not have field current, the concept of a magnetization curve does not apply to it.

P16.43* Under no load conditions, we have:

$$E_A = V_T - R_A I_A = 12.6 - 1 \times 0.5 = 12.1 \text{ V}$$

$$P_{\text{rot}} = P_{\text{dev}} = E_A I_A = 12.1 \text{ W}$$

$$T_{\text{rot}} = \frac{P_{\text{rot}}}{\omega_{\text{m}}} = \frac{12.1}{1000 \times \frac{2\pi}{60}} = 11.55 \times 10^{-2} \text{ Nm}$$

$$K\varphi = \frac{T_{\text{dev}}}{I_A} = 11.55 \times 10^{-2}$$

With the load applied, we have:

$$n_m = 900 \text{ rpm and } \omega_{\text{m}} = 94.25 \text{ radian/s}$$

$$E_A = K\varphi\omega_m = 10.89 \text{ V}$$

$$I_A = \frac{V_T - E_A}{R_A} = 3.42 \text{ A}$$

$$P_{dev} = E_A I_A = 37.24 \text{ W}$$

$$P_{rot} = \frac{900}{1000} \times 12.1 = 10.89 \text{ W}$$

$$P_{out} = P_{dev} - P_{rot} = 26.35 \text{ W}$$

$$P_{in} = V_T I_A \cong 43 \text{ W}$$

$$\eta = \frac{P_{out}}{P_{in}} \times 100\% = 61.15\ \%$$

P16.44 With a locked rotor, $\omega_m = 0$ and $E_A = 0$. Then, we have

$$R_A = \frac{V_T}{I_{A,\text{locked}}} = \frac{12}{20} = 0.6\ \Omega$$

Next, we have the following equations:

$$T_{dev} = K_T \omega_m^2 \qquad (1)$$

(Because load torque is given to be proportional to the square of speed.)

$$T_{dev} = K\varphi I_A \qquad (2)$$

$$I_A = \frac{V_T - K\varphi\omega_m}{R_A} \qquad (3)$$

Using the given data $n_m = 800$ rpm (or equivalently, $\omega_m = 83.77$), $I_A = 3.5$ A, and $V_T = 12$ V, we can find the values of the constants:

$$K\varphi = 0.11817$$

$$K_T = 58.93 \times 10^{-6}$$

Then, we set the right hand sides of Equations (1) and (2) equal and use Equation (3) to substitute for I_A resulting in:

$$K_T \omega_m^2 = K\varphi \frac{V_T - K\varphi\omega_m}{R_A}$$

Substituting values and solving for the speed gives the answers which are:

V_T(V)	ω_m(radian/s)	n_m(rpm)
10	71.63	694
12	83.78	800
14	95.42	911

(We have discarded the extraneous negative roots of the quadratic equation.)

P16.45 (a) $P_{\text{in}} = V_T I_L = 4660\text{ W}$

(b) $P_F = V_T I_F = 300\text{ W}$

(c) $I_A = I_L - I_F = 21.8\text{ A}$

$$P_{R_A} = R_A I_A^2 = 190.1\text{ W}$$

(d) $P_{\text{rot}} = P_{\text{in}} - P_F - P_{R_A} - P_{\text{out}}$

$$= 440\text{ W}$$

(e) $$\eta = \frac{P_{\text{out}}}{P_{\text{in}}} \times 100\% = 80.0\%$$

P16.46 (a) $E_A = V_T - R_A I_A = 204.1\text{ V}$

$$K\varphi = \frac{E_A}{\omega_m} = \frac{204.1}{950(2\pi/60)} = 2.052$$

$$P_{\text{in}} = V_T I_A = 2684\text{ W}$$

$$P_{\text{out}} = 3 \times 746 = 2338\text{ W}$$

$$P_{R_A} = R_A I_A^2 = 193.5\text{ W}$$

$$P_{\text{rot}} = P_{\text{in}} - P_{\text{out}} - P_{R_A} = 152.5\text{ W}$$

$$P_{\text{dev}} = P_{\text{out}} + P_{\text{rot}} = 2490.5\text{ W}$$

$$T_{\text{dev}} = \frac{P_{\text{dev}}}{\omega_{\text{m}}} = \frac{2490.5}{950 \times \frac{2\pi}{60}} = 25.03\text{ Nm}$$

(b) $$T_{\text{rot}} = \frac{P_{\text{rot}}}{\omega_m} = \frac{152.5}{950 \times \frac{2\pi}{60}} = 1.533\text{ Nm}$$

$$I_{A,\text{ no-load}} = \frac{T_{\text{rot}}}{K\varphi} = 0.7470\text{ A}$$

$$E_{A,\text{ no-load}} = V_T - R_A I_A = 219.0\text{ V}$$

$$\omega_{m,\text{ no-load}} = \frac{E_{A,\text{ no-load}}}{K\varphi} = 106.74\text{ rad/s}$$

$$n_{m,\text{ no-load}} = 1019\text{ rpm}$$

P16.47 (a) With a locked rotor, $\omega_m = 0$ and $E_A = 0$. Then we have

$$R_A = \frac{V_T}{I_{A,\text{ locked}}} = \frac{12}{20} = 0.6\,\Omega$$

(b) and (c)

$$P_{\text{dev}} = E_A I_A = (V_T - R_A I_A) I_A$$

$$\frac{dP_{\text{dev}}}{dI_A} = 0 = V_T - 2R_A I_A$$

$$I_A = \frac{V_T}{2R_A}$$

$$P_{\text{dev, max}} = \frac{V_T^2}{4R_A}$$

V_T(V)	$P_{\text{dev, max}}$(W)
10	41.67
12	60
14	81.66

P16.48 $I_F = \frac{V_T}{R_F + R_{\text{adj}}} = \frac{440}{100} = 4.4\text{ A}$

$$I_A = I_L - I_F = 45.6\text{ A}$$

$$E_A = V_T - I_A R_A = 440 - 45.6 \times 0.05 = 437.7\text{ V}$$

$$P_{\text{dev}} = E_A I_A = 19.96\text{ kW}$$

$$T_{\text{dev}} = \frac{P_{\text{dev}}}{\omega_{\text{m}}} = \frac{19960}{1500 \times \frac{2\pi}{60}} \cong 127\text{ Nm}$$

$$P_{\text{in}} = V_T I_L = 22\text{ kW}$$

$$\eta = \frac{P_{\text{out}}}{P_{\text{in}}} \times 100\% = \frac{18 \times 746}{22000} \times 100\% \cong 61\ \%$$

P16.49 (a) I_A doubles, and the speed remains constant.

(b) I_A doubles, and the speed remains constant.

(c) The field current is cut in half, the speed remains constant, and I_A doubles.

(d) The speed increases to 2400 rpm and I_A remains constant.

P16.50 $I_F = \frac{V_T}{R_F + R_{\text{adj}}} = \frac{200}{10 + 2.5} = 16\text{ A}$

From Figure 16.19, we find that $E_{A,\text{ref}} = 320\text{ V}$ at $n_{m,\text{ref}} = 1200\text{ rpm}$. Next, we determine the value of the machine constant $K\varphi$.

$$K\varphi = \frac{E_{A,\text{ref}}}{\omega_{m,\text{ref}}} = \frac{320}{1200 \times \frac{2\pi}{60}} = 2.546$$

We assume that the rotational power loss is proportional to speed, which is equivalent to assuming constant torque.

$$T_{\text{rot}} = \frac{P_{\text{rot}}}{\omega_{m,\text{ref}}} = \frac{1000}{1200 \times \dfrac{2\pi}{60}} = 7.96\ \text{Nm}$$

$$T_{\text{dev}} = T_{\text{out}} + T_{\text{rot}} = 200 + 7.96 = 207.96\ \text{Nm}$$

$$I_A = \frac{T_{\text{dev}}}{K\varphi} = \frac{207.96}{2.546} = 81.68\ A$$

$$E_A = V_T - R_A I_A = 193.06\ \text{V}$$

$$n_m = n_{m,\text{ref}} \times \frac{E_A}{E_{A,\text{ref}}} = 1200 \times \frac{193.06}{320} = 724.0\ \text{rpm}$$

$$\omega_m = n_m \times \frac{2\pi}{60} = 75.81\ \text{radian/s}$$

$$P_{\text{out}} = T_{\text{out}}\omega_m = 15.16\ \text{kW}$$

$$P_{\text{in}} = V_T(I_A + I_F) = 19.54\ \text{kW}$$

$$\eta = \frac{P_{\text{out}}}{P_{\text{in}}} = 100\% = 77.6\%$$

P16.51 $I_F = \dfrac{V_T}{R_F + R_{\text{adj}}} = \dfrac{200}{400} = 0.5\ A$

$$I_A = I_L - I_F = 1.2 - 0.5 = 0.7\ A$$

$$E_A = V_T - R_A I_A = 200 - 0.5 \times 0.7 = 199.7\ \text{V}$$

Since $P_{\text{out}} = 0$, we have $P_{\text{rot}} = P_{\text{dev}} = E_A I_A = 139.8\ \text{W}$

P16.52 For operation at 1000 rpm, we have

$$\omega_m = n_m \frac{2\pi}{60} = 104.7\ \text{rad/s}$$

$$K\varphi = \frac{E_A}{\omega_m} = \frac{120}{104.7} = 1.146$$

The torque-speed relationship is given by Equation 16.27:

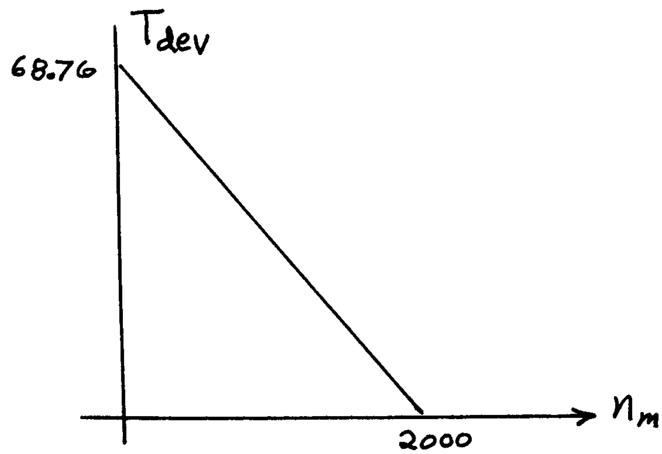
$$T_{\text{dev}} = \frac{K\varphi}{R_A}(V_T - K\varphi\omega_m) = \frac{K\varphi}{R_A}\left(V_T - K\varphi n_m \frac{2\pi}{60}\right)$$

Substituting values, we obtain

$$T_{\text{dev}} = \frac{1.146}{4}\left(240 - 1.146 n_m \frac{2\pi}{60}\right)$$

$$T_{\text{dev}} = 68.76 - 0.03438 n_m$$

A sketch of the torque-speed characteristic is:



P16.53 Under full-load in a well designed machine, I_A should be much larger than I_F because all of the field power $I_F V_T$ is converted to heat while the output power is part of the armature power $I_A V_T$. For high efficiency, the field power must be small compared to the output power. Perhaps an acceptable ratio would be I_A/I_F = 20.

P16.54 (a) The field current is

$$I_F = \frac{V_T}{R_F + R_{\text{adj}}} = \frac{240}{160} = 1.5\text{ A}$$

From the magnetization curve shown in Figure P16.54, we find that $E_A \cong 200\text{ V}$ with $I_F = 1.5\text{ A}$ and $n_m = 1000\text{ rpm}$. Neglecting losses at no load, we have $I_A = 0$ and $E_A = V_T = 240\text{ V}$. Since E_A is proportional to speed, the no-load speed is

$$n_{\text{no-load}} = \frac{240}{200} \times 1000\text{ rpm} = 1200\text{ rpm}$$

(b) When the speed drops by 3%, we have

$$n_m = 1200 \times 0.97 = 1164\text{ rpm}$$

$$\omega_m = 1164 \times \frac{2\pi}{60} = 121.9\text{ radian/s}$$

$$E_A = 240 \times 0.97 = 232.8\text{ V}$$

$$I_A = \frac{V_T - E_A}{R_A} = \frac{240 - 232.8}{2} = 3.6\ \text{A}$$

$$P_{out} = P_{dev} = E_A I_A \cong 838\ \text{W}$$

$$T_{load} = T_{dev} = \frac{P_{dev}}{\omega_m} = 6.87\ \text{nm}$$

$$P_F = V_T I_F = 240 \times 1.5 = 360\ \text{W}$$

$$P_{R_A} = R_A I_A^2 = 25.92\ \text{W}$$

P16.55* For $I_A = 40\ \text{A}$, we have:

$$\omega_m = n_m \times \frac{2\pi}{60} = 900 \times \frac{2\pi}{60} = 94.25\ \text{radian/s}$$

$$E_A = V_T - (R_F + R_A) I_A = 196\ \text{V}$$

Rearranging Equation 16.30 and substituting values, we have:

$$KK_F = \frac{E_A}{I_A \omega_m} = \frac{196}{40 \times 94.25} = 51.99 \times 10^{-3}$$

For $I_A = 20\ \text{A}$, we have:

$$\omega_m = \frac{E_A}{KK_F I_A} = \frac{208}{51.99 \times 10^{-3} \times 20} = 200.0\ \text{radian/s}$$

$$n_m = 1910\ \text{rpm}$$

P16.56 A universal motor would be a poor choice for a clock because the speed is variable with the output torque required. The clock would be very inaccurate.

A universal motor would be a poor choice for a furnace fan because it would have too short a service life because of brush and commutator wear.

A universal motor would be a good choice for a coffee grinder because of its higher power-to-mass ratio and small amount of time in use.

P16.57 Any ac motor that contains brushes and a commutator is most likely a universal motor.

P16.58 1. Higher power-to-mass ratio.
2. Higher starting torque.

3. Slows down for higher torque loads. Suitable for variable torque loads.
4. Speed can be higher.

P16.59 For $I_A = 25\,A$, we have:

$$\omega_m = n_m \times \frac{2\pi}{60} = 1200 \times \frac{2\pi}{60} = 125.7 \text{ radian/s}$$
$$E_A = V_T - (R_F + R_A)I_A = 267.5\,\text{V}$$

Rearranging Equation 16.30 and substituting values, we have:

$$KK_F = \frac{E_A}{I_A \omega_m} = \frac{267.5}{25 \times 125.7} = 85.12 \times 10^{-3}$$
$$P_{dev} = E_A I_A = 6688\text{ W}$$
$$P_{out} = P_{dev} - P_{rot} = 6338\text{ W}$$
$$T_{dev} = \frac{P_{dev}}{\omega_m} = 53.21\text{ Nm}$$
$$T_{out} = \frac{P_{out}}{\omega_m} = 50.42\text{ Nm}$$
$$T_{rot} = T_{dev} - T_{out} = 2.784\text{ Nm}$$

(Since we assume that P_{rot} is proportional to speed, T_{rot} is constant with speed.)

Now when the output torque is increased by a factor of 2, we have:

$$T_{dev} = 2 \times 50.42 + 2.784 = 103.6\text{ Nm}$$

From Equation 16.31, we have:

$$I_A = \sqrt{\frac{T_{dev}}{KK_F}} = 34.88\,A$$
$$E_A = V_T - (R_F + R_A)I_A = 262.6\,\text{V}$$
$$P_{dev} = E_A I_A = 9158\text{ W}$$
$$\omega_m = \frac{P_{dev}}{T_{dev}} = 88.39 \text{ radian/s}$$
$$n_m = 844.1\text{ rpm}$$

P16.60 For $I_A = 25\,A$, we have:

$$\omega_m = n_m \times \frac{2\pi}{60} = 1200 \times \frac{2\pi}{60} = 125.7 \text{ radian/s}$$
$$E_A = V_T - (R_F + R_A)I_A = 267.5\,\text{V}$$

Rearranging Equation 16.30 and substituting values, we have:

$$KK_F = \frac{E_A}{I_A \omega_m} = \frac{267.5}{25 \times 125.7} = 85.15 \times 10^{-3}$$

For $I_A = 10\ A$, we have:

$$\omega_m = \frac{E_A}{KK_F I_A} = \frac{275}{85.15 \times 10^{-3} \times 10} = 323.0 \text{ radian/s}$$

$$n_m = 3084 \text{ rpm}$$

P16.61 For $I_A = 40\ A$, we have:

$$\omega_m = n_m \times \frac{2\pi}{60} = 900 \times \frac{2\pi}{60} = 94.25 \text{ radian/s}$$

$$E_A = V_T - (R_F + R_A) I_A = 196 \text{ V}$$

Rearranging Equation 16.30 and substituting values, we have:

$$KK_F = \frac{E_A}{I_A \omega_m} = \frac{196}{40 \times 94.25} = 51.99 \times 10^{-3}$$

$$P_{\text{dev}} = E_A I_A = 7840 \text{ W}$$

$$P_{\text{out}} = P_{\text{dev}} - P_{\text{rot}} = 7440 \text{ W}$$

$$T_{\text{dev}} = \frac{P_{\text{dev}}}{\omega_m} = 83.18 \text{ Nm}$$

$$T_{\text{out}} = \frac{P_{\text{out}}}{\omega_m} = 78.94 \text{ Nm}$$

$$T_{\text{rot}} = T_{\text{dev}} - T_{\text{out}} = 4.24 \text{ Nm}$$

(Since we assume that P_{rot} is proportional to speed, T_{rot} is constant with speed.)

Now when the output torque is reduced by a factor of 2, we have:

$$T_{\text{dev}} = \frac{78.94}{2} + 4.24 = 43.71 \text{ Nm}$$

From Equation 16.31, we have:

$$I_A = \sqrt{\frac{T_{\text{dev}}}{KK_F}} = 29.00\ A$$

$$E_A = V_T - (R_F + R_A) I_A = 202.6 \text{ V}$$

$$P_{\text{dev}} = E_A I_A = 5875 \text{ W}$$

$$\omega_m = \frac{P_{\text{dev}}}{T_{\text{dev}}} = 134.4 \text{ radian/s}$$

$$n_m = 1284 \text{ rpm}$$

P16.62 (a) According to Equation 16.34 for a series connected motor, we have:

$$T_{\text{dev}} = \frac{KK_F V_T^2}{(R_A + R_F + KK_F\omega_m)^2}$$

However, in this problem we have $R_A + R_F = 0$, so we have:

$$T_{\text{dev}} = \frac{V_T^2}{KK_F\omega_m^2}$$

Since the rotational losses are negligible, we have $T_{\text{out}} = T_{\text{dev}}$. Thus, torque is inversely proportional to speed squared so we can write:

$$\frac{T_{\text{out1}}}{T_{\text{out2}}} = \left(\frac{n_{m2}}{n_{m1}}\right)^2$$

$$\frac{100}{300} = \left(\frac{n_{m2}}{1200}\right)^2$$

Solving, we find the speed as:

$$n_{m2} = 693 \text{ rpm}$$

(b) In theory, the no-load speed is infinite. Of course, rotational losses will limit the speed to a finite value. However, the speed can become high enough to damage the machine unless protective circuits remove electrical power when the load is removed.

P16.63 We have:

$$P_{\text{in}} = V_T I_A = 220 \times 20 = 4400 \text{ W}$$

$$E_A = V_T - (R_F + R_A)I_A = 180 \text{ V}$$

$$P_{\text{dev}} = E_A I_A = 180 \times 20 = 3600 \text{ W}$$

$$P_{\text{out}} = P_{\text{dev}} - P_{\text{rot}} = 3600 - P_{rot}$$

$$\eta = \frac{P_{\text{out}}}{P} \times 100\% = \frac{80}{100} = \frac{3600 - P_{rot}}{4400}$$

$$P_{rot} = 80 \text{ W}$$

P16.64* See Figures 16.26, 16.27 and 16.28 in the book.

P16.65* Equation 16.34 gives the developed torque of the series motor.

$$T_{\text{dev}} = \frac{KK_F V_T^2}{(R_A + R_F + KK_F\omega_m)^2}$$

Substituting $R_A + R_F = 0$ and solving for speed, we have:

$$\omega_m = \frac{V_T}{\sqrt{KK_F T_{\text{dev}}}}$$

Thus, speed for a constant torque load is proportional to the applied voltage. To achieve a speed of 1000 rpm, the average applied voltage must be:

$$V_T = \frac{1000}{1500} \times 50 = 33.33 \text{ V}$$

$$\frac{T_{\text{on}}}{T} = \frac{33.33}{50} = 0.667$$

P16.66* Operating at 1400 rpm without added resistance, we have:

$$P_{\text{dev}} = I_A E_A = \omega_m T_{\text{dev}} = 1400 \times \frac{2\pi}{60} \times 25 = 3665 \text{ W}$$

$$E_A = V_T - (R_A + R_F)I_A$$

$$I_A[V_T - (R_A + R_F)I_A] = I_A[75 - (0.1)I_A] = 3665$$

$I_A = 52.55$ A (The other root is unrealistically large for a practical machine.) Then solving Equation 16.31 for KK_F, we have

$$KK_F = \frac{T_{dev}}{I_A^2} = \frac{25}{52.55^2} = 9.056 \times 10^{-3}$$

After adding series resistance, we have:

$$T_{\text{dev}} = \frac{KK_F V_T^2}{(R_{added} + R_A + R_F + KK_F\omega_m)^2}$$

Substituting values and solving for the added resistance, we obtain:

$$R_{added} = 0.379 \ \Omega$$

P16.67 (a) $$I_F = \frac{V_T}{R_F + R_{\text{adj}}} = \frac{200}{10 + 2.5} = 16 \text{ A}$$

For this field current at a speed of 1200 rpm, from the magnetization curve we find E_A = 320 V. Thus we have

$$K\varphi = \frac{E_A}{\omega_m} = \frac{320}{1200(2\pi/60)} = 2.546$$

$$T_{rot} = \frac{P_{rot}}{\omega_m} = \frac{1000}{1200(2\pi/60)} = 7.958 \text{ Nm}$$

$$I_A = \frac{T_{dev}}{K\varphi} = \frac{T_{load} + T_{rot}}{K\varphi} = \frac{200 + 7.958}{2.546} = 81.66 \text{ A}$$

(b) With zero speed, we have E_A = 0. Thus the initial armature current is

$$I_A = \frac{V_T}{R_A} = \frac{200}{0.085} = 2353 \text{ A}$$

$$T_{\text{dev, start}} = K\varphi I_A = 2.546 \times 2353 = 5990 \text{ Nm}$$

Thus the starting current is 28.8 times larger than the steady-state value. This is why additional resistance is usually placed in series with the armature to start the shunt dc motor.

(c) For a starting current of 200 A, we require

$R_A + R_{added} = \frac{V_T}{200} = 1\,\Omega$ Solving, we have $R_{added} = 1 - 0.085 = 0.915\,\Omega$

P16.68 For this shunt-connected machine, we have

$E_A = V_T$ (Because $R_A = 0$.)

$\varphi = K_F I_F$ (Because we assume operation on the linear portion of the magnetization curve.)

$$I_F = \frac{V_T}{R_F + R_{\text{adj}}}$$

$$E_A = K\varphi\omega_m$$

From these equations, we obtain the following expression for speed:

$$\omega_m = \frac{R_F + R_{\text{adj}}}{KK_F}$$

Thus, speed (either ω_m or n_m) is proportional to the resistance $(R_F + R_{\text{adj}})$.

To achieve $n_m = 1500$ rpm, we need

$$R_F + R_{\text{adj}} = (75 + 25)\frac{1500}{1000} = 150$$

$$R_{\text{adj}} = 75\ \Omega$$

The lowest speed that can be achieved is for $R_{\text{adj}} = 0$.

$$n_{m,\min} = 1000 \times \frac{75}{100} = 750 \text{ rpm}$$

P16.69 Neglecting rotational losses, the no-load speed of a PM motor is proportional to average applied voltage. To achieve a no-load speed of 1200 rpm, the applied voltage must be:

$$V_{\text{avg}} = \frac{1200}{1800} \times 12 = 8 \text{ V}$$

$$\frac{T_{\text{on}}}{T} = \frac{8}{12} = 0.67$$

P16.70 Three methods to control the speed of dc motors and the types of motors for which each is practical are:

1. Vary the voltage supplied to the armature circuit while holding the field constant. (Separately excited motors and permanent-magnet motors.)

2. Vary the field current while holding the armature supply voltage constant. (Shunt connected and separately excited motors.)

3. Insert resistance in series with the armature circuit. (Shunt connected, separately excited, permanent magnet, and series-connected motors.)

P16.71 We have $T_{\text{dev}} = T_{load} = K_T\omega_m$ and $R_A + R_F = 0$. Substituting this into Equation 16.34 and solving for speed, we obtain:

$$\omega_m = \sqrt[3]{\frac{V_T^2}{K_T K K_F}}$$

Thus, speed is proportional to the 2/3 power of the applied voltage.

$$V_T = \left(\frac{1000}{1500}\right)^{3/2} \times 50 = 27.21\text{ V}$$

$$\frac{T_{\text{on}}}{T} = \frac{27.21}{50} = 0.5443$$

P16.72* (a) $\text{voltage regulation} = \dfrac{V_{NL} - V_{FL}}{V_{FL}} \times 100\% = \dfrac{170 - 160}{160} \times 100\% = 6.25\%$

$$R_L = \frac{V_L}{I_L} = \frac{160}{10} = 16\ \Omega \qquad R_A = \frac{V_{NL} - V_{FL}}{I_{FL}} = \frac{170 - 160}{10} = 1\ \Omega$$

$$\omega_m = n_m \frac{2\pi}{60} = 157.1\ \text{rad/s} \qquad T_{dev} = \frac{P_{dev}}{\omega_m} = \frac{E_A I_A}{\omega_m} = \frac{160 \times 10}{157.1} = 10.18\ \text{Nm}$$

(b)

$$V_{NL} = E_A = 160\frac{1200}{1500} = 128\ \text{V} \qquad I_L = \frac{E_A}{R_A + R_L} = \frac{128}{16 + 1} = 7.5\text{ A}$$

$$V_L = R_L I_L = 120\text{ V} \qquad P_{\text{dev}} = E_A I_A = 960\text{ W}$$

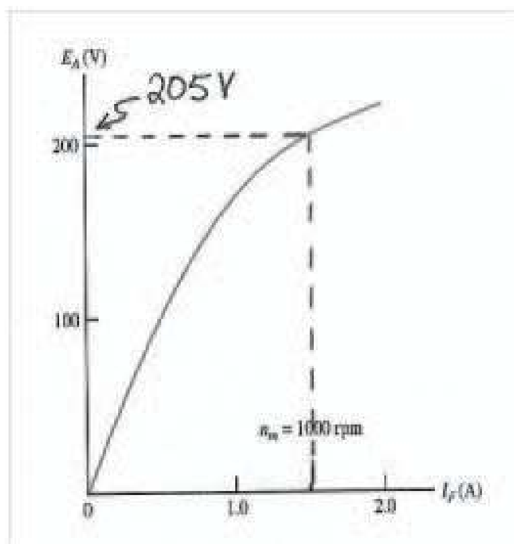
P16.73 Voltage regulation is zero for a fully compensated cumulative compound connected dc generator.

P16.74 (a) To increase the load voltage of a separately-excited generator, increase V_F, reduce R_{adj}, or increase the shaft speed.
(b) To increase the load voltage of a shunt connected generator, reduce R_{adj} or increase the shaft speed.

P16.75 1. Cumulative long-shunt compound connected.
2. Cumulative short-shunt compound connected.
3. Differential long-shunt compound connected.
4. Differential short-shunt compound connected.

P16.76 From highest to lowest voltage regulation the generators shown in Figure 16.30 are:
1. Differential compound connected
2. Shunt connected
3. Separately excited
4. Cumulative compound connected

P16.77



$$I_F = \frac{V_F}{R_{\text{adj}} + R_F} = \frac{150}{60+40} = 1.5\ A$$

$$V_{NL} = E_A = 205\frac{1200}{1000} = 246\ \text{V}$$

$$V_{FL} = E_A - R_A I_A = 246 - 1.5 \times 10 = 231\ \text{V}$$

$$\text{Voltage regulation} = \frac{V_{NL} - V_{FL}}{V_{FL}} \times 100\% = \frac{246 - 231}{231} \times 100\% = 6.5\%$$

$$P_{\text{out}} = I_L V_{FL} = 10 \times 231 = 2.31\ \text{kW}$$

$$P_{\text{dev}} = P_{\text{out}} + R_A I_A^2 = 2310 + 1.5(10)^2 = 2.46\ \text{kW}$$

$$\omega_m = n_m \frac{2\pi}{60} = 125.7 \text{ rad/sec} \qquad P_{\text{in}} = \frac{P_{\text{out}}}{0.60} = \frac{2.31}{0.60} = 3.85\,\text{kW}$$

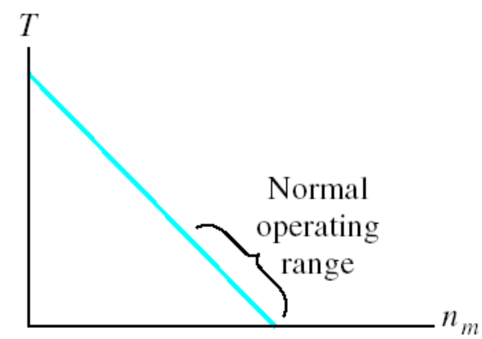
$$P_{\text{losses}} = P_{\text{in}} - P_{\text{dev}} = 3.85 - 2.46 = 1.39\text{ kW}$$

$$T_{\text{in}} = \frac{P_{\text{in}}}{\omega_m} = \frac{3850}{125.7} = 30.6\text{ nm} \qquad T_{\text{dev}} = \frac{P_{\text{dev}}}{\omega_m} = \frac{2460}{125.7} = 19.6\text{ nm}$$

Practice Test

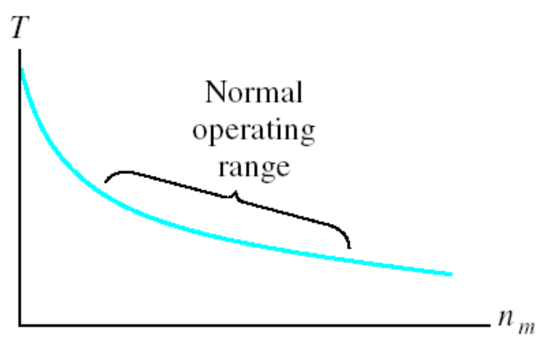
T16.1 The windings are the field winding, which is on the stator, and the armature winding, which is on the rotor. The armature current varies with mechanical load.

T16.2 See Figure 16.5(c) in the book. The speed becomes very high, and the machine can be destroyed.



(c) Shunt-connected or permanent-magnet dc motor

T16.3 See Figure 16.5(d) in the book.



(d) Series-connected dc motor or universal motor

T16.4 speed regulation $= \dfrac{n_{\text{no-load}} - n_{\text{full-load}}}{n_{\text{full-load}}} \times 100\%$

T16.5 To obtain the magnetization curve, we drive the machine at constant speed and plot the open-circuit armature voltage E_A versus field current I_F.

T16.6 Power losses in a shunt-connected dc motor are 1. Field loss, which is the power consumed in the resistances of the field circuit. 2. Armature loss, which is the power converted to heat in the armature resistance. 3. Rotational losses, which include friction, windage, eddy-current loss, and hysteresis loss.

T16.7 A universal motor is an ac motor that similar in construction to a series-connected dc motor. In principle, it can be operated from either ac or dc sources. The stator of a universal motor is usually laminated to reduce eddy-current loss. Compared to other single-phase ac motors, the universal motor has a higher power to weight ratio, produces a larger starting torque without excessive current, slows down under heavy loads so the power is more nearly constant, and can be designed to operate at higher speeds. A disadvantage of the universal motor is that it contains brushes and a commutator resulting in shorter service life.

T16.8 1. Vary the voltage supplied to the armature circuit while holding the field constant.

2. Vary the field current while holding the armature supply voltage constant.

3. Insert resistance in series with the armature circuit.

T16.9 Equation 16.15 states

$$E_A = K\varphi\omega_m$$

With constant field current, the magnetic flux φ is constant. Therefore, the back emf E_A is proportional to machine speed ω_m (or equivalently to n_m). Thus, we have

$n_m(\text{rpm})$	$E_A(\text{V})$
500	80
1500	240
2000	320

T16.10 Converting the speeds from rpm to radians/s, we have:

$$\omega_{m1} = n_{m1} \times \frac{2\pi}{60} = 1500 \times \frac{2\pi}{60} = 50\pi$$

$$\omega_{m2} = n_{m2} \times \frac{2\pi}{60} = 900 \times \frac{2\pi}{60} = 30\pi$$

Next, we can find the machine constant:

$$K\phi = \frac{E_A}{\omega_{m1}} = \frac{120}{40\pi} = \frac{3}{\pi} = 0.9549$$

The developed torque is:

$$T_{\text{dev}} = \frac{P_{\text{dev}}}{\omega_{m2}} = \frac{4 \times 746}{30\pi} = 31.66 \text{ Nm}$$

Finally, the armature current is:

$$I_A = \frac{T_{\text{dev}}}{K\phi} = \frac{31.66}{0.9549} = 33.16 \text{ A}$$

T16.11 (a) $E_A = V_T - R_A I_A = 230 \text{ V}$

$$K\varphi = \frac{E_A}{\omega_m} = \frac{230}{1200(2\pi/60)} = 1.830$$

$$P_{\text{in}} = V_T I_A = 4800 \text{ W}$$

$$P_{\text{out}} = 6 \times 746 = 4476 \text{ W}$$

$$P_{R_A} = R_A I_A^2 = 200 \text{ W}$$

$$P_{\text{rot}} = P_{\text{in}} - P_{\text{out}} - P_{R_A} = 124 \text{ W}$$

$$P_{\text{dev}} = P_{\text{out}} + P_{\text{rot}} = 4600 \text{ W}$$

$$T_{\text{dev}} = \frac{P_{\text{dev}}}{\omega_{\text{m}}} = \frac{4600}{1200 \times \frac{2\pi}{60}} = 36.60 \text{ Nm}$$

(b) $T_{\text{rot}} = \dfrac{P_{\text{rot}}}{\omega_m} = \dfrac{124}{1200 \times \frac{2\pi}{60}} = 0.9868 \text{ Nm}$

$$I_{A,\text{no-load}} = \frac{T_{\text{rot}}}{K\phi} = 0.5392 \text{ A}$$

$$E_{A,\,\text{no-load}} = V_T - R_A I_A = 239.73 \quad \text{V}$$

$$\omega_{m,\,\text{no-load}} = \frac{E_{A,\,\text{no-load}}}{K\phi} = 131.0 \text{ rad/s}$$

$$n_{m,\,\text{no-load}} = 1251 \quad \text{rpm}$$

$$\text{speed regulation} = \frac{n_{\text{no-load}} - n_{\text{full-load}}}{n_{\text{full-load}}} \times 100\% = 4.25\%$$

T16.12 For $I_A = 20$ A, we have:

$$\omega_m = n_m \times \frac{2\pi}{60} = 1000 \times \frac{2\pi}{60} = 104.7 \text{ radian/s}$$

$$E_A = V_T - (R_F + R_A)I_A = 226 \text{ V}$$

Rearranging Equation 16.30 and substituting values, we have:

$$KK_F = \frac{E_A}{I_A \omega_m} = \frac{226}{20 \times 104.7} = 0.1079$$

For $I_A = 10$ A, we have:

$$E_A = V_T - (R_F + R_A)I_A = 233 \text{ V}$$

$$\omega_m = \frac{E_A}{KK_F I_A} = \frac{233}{0.1079 \times 10} = 215.9 \text{ radian/s}$$

$$n_m = 2062 \text{ rpm}$$

CHAPTER 17

Exercises

E17.1 From Equation 17.5, we have

$$B_{gap} = Ki_a(t)\cos(\theta) + Ki_b(t)\cos(\theta - 120^\circ) + Ki_c(t)\cos(\theta - 240^\circ)$$

Using the expressions given in the Exercise statement for the currents, we have

$$B_{gap} = KI_m\cos(\omega t)\cos(\theta) + KI_m\cos(\omega t - 240^\circ)\cos(\theta - 120^\circ)$$
$$+ KI_m\cos(\omega t - 120^\circ)\cos(\theta - 240^\circ)$$

Then using the identity for the products of cosines, we obtain

$$B_{gap} = \tfrac{1}{2}KI_m[\cos(\omega t - \theta) + \cos(\omega t + \theta) + \cos(\omega t - \theta - 120^\circ)$$
$$+ \cos(\omega t + \theta - 360^\circ) + \cos(\omega t - \theta + 120^\circ)$$
$$+ \cos(\omega t + \theta - 360^\circ)]$$

However we can write

$$\cos(\omega t - \theta) + \cos(\omega t - \theta - 120^\circ) + \cos(\omega t - \theta + 120^\circ) = 0$$

$$\cos(\omega t + \theta - 360^\circ) = \cos(\omega t + \theta)$$

$$\cos(\omega t + \theta - 360^\circ) = \cos(\omega t + \theta)$$

Thus we have

$$B_{gap} = \tfrac{3}{2}KI_m\cos(\omega t + \theta)$$

which can be recognized as flux pattern that rotates clockwise.

E17.2 At 60 Hz, synchronous speed for a four-pole machine is:

$$n_s = \frac{120f}{P} = \frac{120(60)}{4} = 1800 \text{ rpm}$$

The slip is given by:

$$s = \frac{n_s - n_m}{n_s} = \frac{1800 - 1750}{1800} = 2.778\%$$

The frequency of the rotor currents is the slip frequency. From Equation

$$\omega_{\text{slip}} = s\omega$$

17.17, we have . For frequencies in the Hz, this becomes:

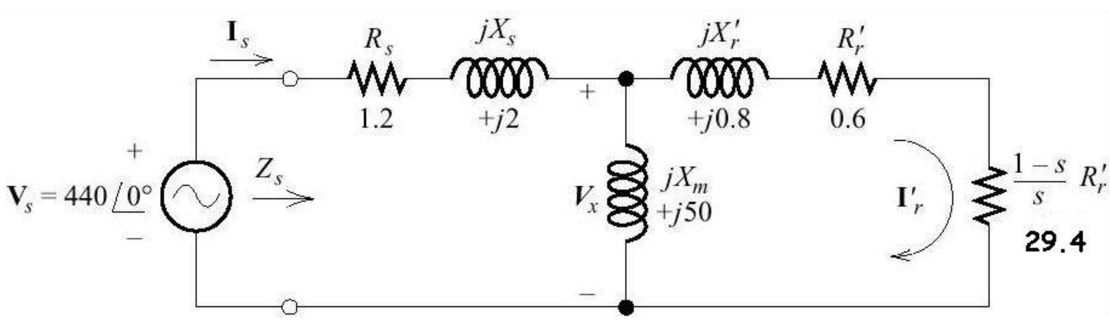
$$f_{\text{slip}} = sf = 0.02778 \times 60 = 1.667 \text{ Hz}$$

In the normal range of operation, slip is approximately proportional to output power and torque. Thus at half power, we estimate that $s = 2.778/2 = 1.389\%$. This corresponds to a speed of 1775 rpm.

E17.3 Following the solution to Example 17.1, we have:

$$n_s = 1800 \text{ rpm}$$
$$s = \frac{n_s - n_m}{n_s} = \frac{1800 - 1764}{1800} = 0.02$$

The per phase equivalent circuit is:



$$Z_s = 1.2 + j2 + \frac{j50(0.6 + 29.4 + j0.8)}{j50 + 0.6 + 29.4 + j0.8}$$
$$= 22.75 + j15.51$$
$$= 27.53\angle 34.29^\circ$$

$$\text{power factor} = \cos(34.29^\circ) = 82.62\% \text{ lagging}$$

$$\mathbf{I}_s = \frac{\mathbf{V}_s}{Z_s} = \frac{440\angle 0^\circ}{27.53\angle 34.29^\circ} = 15.98\angle -34.29^\circ \text{ A rms}$$

For a delta-connected machine, the magnitude of the line current is

$$I_{line} = I_s\sqrt{3} = 15.98\sqrt{3} = 27.68 \text{ A rms}$$

and the input power is

$$P_{\text{in}} = 3I_sV_s\cos\theta = 17.43 \text{ kW}$$

Next, we compute $\mathbf{V}_x$ and $\mathbf{I}'_r$.

$$\mathbf{V}_x = \mathbf{I}_s \frac{j50(0.6 + 29.4 + j0.8)}{j50 + 0.6 + 29.4 + j0.8}$$
$$= 406.2 - j15.6$$
$$= 406.4\angle -2.2^\circ \quad \text{V rms}$$

$$\mathbf{I}'_r = \frac{\mathbf{V}_x}{j0.8 + 0.6 + 29.4}$$
$$= 13.54\angle -3.727^\circ \quad \text{A rms}$$

The copper losses in the stator and rotor are:

$$P_s = 3R_s I_s^2$$
$$= 3(1.2)(15.98)^2$$
$$= 919.3 \text{ W}$$

and

$$P_r = 3R'_r (I'_r)^2$$
$$= 3(0.6)(13.54)^2$$
$$= 330.0 \text{ W}$$

Finally, the developed power is:

$$P_{\text{dev}} = 3 \times \frac{1-s}{s} R'_r (I'_r)^2$$
$$= 3(29.4)(13.54)^2$$
$$= 16.17 \text{ kW}$$
$$P_{\text{out}} = P_{\text{dev}} - P_{\text{rot}} = 15.27 \text{ kW}$$

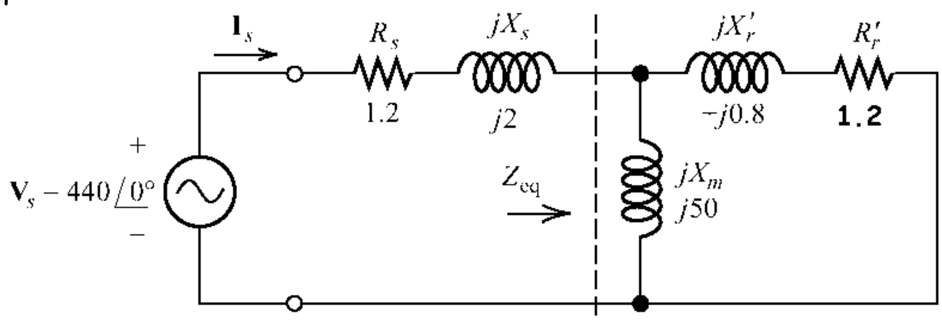
The output torque is:

$$T_{\text{out}} = \frac{P_{\text{out}}}{\omega_m} = 82.66 \text{ newton meters}$$

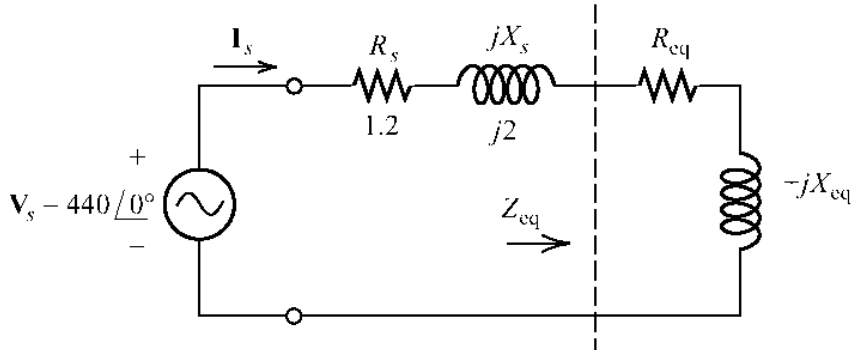
The efficiency is:

$$\eta = \frac{P_{\text{out}}}{P_{\text{in}}} \times 100\% = 87.61\%$$

E17.4 The equivalent circuit is:



(a)



$$Z_{eq} = R_{eq} + jX_{eq} = \frac{j50(1.2 + j0.8)}{j50 + 1.2 + j0.8} = 1.162 + j0.8148$$

The impedance seen by the source is:

$$\begin{aligned} Z_s &= 1.2 + j2 + Z_{eq} \\ &= 1.2 + j2 + 1.162 + j0.8148 \\ &= 3.675\angle 50.00^\circ \end{aligned}$$

Thus, the starting phase current is

$$\mathbf{I}_{s,\,\text{starting}} = \frac{\mathbf{V}_s}{Z_s} = \frac{440\angle 0^\circ}{3.675\angle 50.00^\circ}$$

$$\mathbf{I}_{s,\,\text{starting}} = 119.7\angle -50.00^\circ \text{ A rms}$$

and for a delta connection, the line current is

$$I_{line,\text{starting}} = I_{s,\text{starting}}\sqrt{3} = 119.7\sqrt{3} = 207.3 \text{ A rms}$$

The power crossing the air gap is (three times) the power delivered to the right of the dashed line in the equivalent circuit shown earlier.

$$P_{ag} = 3R_{eq}\left(I_{s,\,\text{starting}}\right)^2 = 49.95 \text{ kW}$$

Finally, the starting torque is found using Equation 17.34.

$$T_{\text{dev, starting}} = \frac{P_{ag}}{\omega_s}$$
$$= \frac{49950}{2\pi 60/2}$$
$$= 265.0 \text{ newton meters}$$

E17.5 This exercise is similar to part (c) of Example 17.4. Thus, we have

$$\frac{\sin\delta_3}{\sin\delta_1} = \frac{P_3}{P_1}$$
$$\frac{\sin\delta_3}{\sin 4.168^\circ} = \frac{200}{50}$$

which yields the new torque angle $\delta_3 = 16.90^\circ$. E_r remains constant in magnitude, thus we have

$$\mathbf{E}_{r3} = 498.9\angle -16.90^\circ \quad \text{V rms}$$
$$\mathbf{I}_{a3} = \frac{\mathbf{V}_a - \mathbf{E}_{r3}}{jX_s} = \frac{480 - 498.9\angle -16.90^\circ}{j1.4} = 103.6\angle -1.045^\circ \quad \text{A rms}$$

The power factor is $\cos(-1.045^\circ) = 99.98\%$ lagging.

E17.6 We follow the approach of Example 17.5. Thus as in the example, we have

$$I_{a1} = \frac{P_{dev}}{3V_a \cos\theta_1} = \frac{74600}{3(240)0.85} = 121.9 \text{ A}$$
$$\theta_1 = \cos^{-1}(0.85) = 31.79^\circ$$
$$\mathbf{I}_{a1} = 121.9\angle -31.79^\circ \quad \text{A rms}$$
$$\mathbf{E}_{r1} = \mathbf{V}_a - jX_s\mathbf{I}_{a1} = 416.2\angle -20.39^\circ \quad \text{V rms}$$

The phasor diagram is shown in Figure 17.24a

For 90% leading power factor, the power angle is $\theta_3 = \cos^{-1}(0.9) = 25.84^\circ$.
The new value of the current magnitude is

$$I_{a3} = \frac{P_{dev}}{3V_{a3}\cos(\theta_3)} = 115.1 \quad \text{A rms}$$

and the phasor current is

$$\mathbf{I}_{a3} = 115.1\angle 25.84^\circ \quad \text{A rms}$$

Thus we have

$$\mathbf{E}_{r3} = \mathbf{V}_a - jX_s\mathbf{I}_{a3} = 569.0\angle -14.77^\circ \quad \text{V rms}$$

The magnitude of E_r is proportional to the field current, so we have:

$$I_{f3} = I_{f1}\frac{E_{r3}}{E_{r1}} = 10 \times \frac{569.0}{416.2} = 13.67 \text{ A dc}$$

E17.7 The phasor diagram for $\delta = 90^\circ$ is shown in Figure 17.27. The developed power is given by

$$P_{\max} = 3V_a I_a \cos(\theta)$$

However from the phasor diagram, we see that

$$\cos(\theta) = \frac{E_r}{X_s I_a}$$

Substituting, we have

$$P_{\max} = \frac{3V_a E_r}{X_s}$$

The torque is

$$T_{\max} = \frac{P_{\max}}{\omega_m} = \frac{3V_a E_r}{\omega_m X_s}$$

Problems

P17.1* Using the formula for synchronous speed

$$N_S = 120\,\frac{f}{p}$$

The speed for a 6 pole motor will be 1200 rpm and for 4 pole it is 1800 rpm. Since the motor cannot operate above its synchronous speed we must chose the 4 pole motor.

P17.2* Slip is given by:

$$s = \frac{n_s - n_m}{n_s} = 1 - \frac{n_m}{n_s}$$

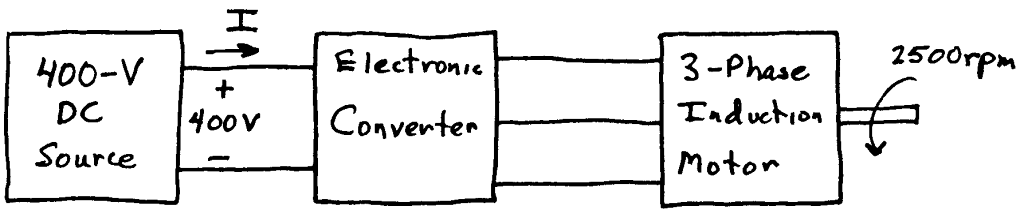
For a slip of 4% and a mechanical speed of $n_m = 2500$ rpm, the synchronous speed is:

$$n_s = \frac{n_m}{1-s} = \frac{2500}{0.96} = 2604 \text{ rpm}$$

Solving Equation 17.14 for frequency, we have:

$$f = \frac{Pn_s}{120} = \frac{4 \times 2604}{120} = 86.8\text{ Hz}$$

The block diagram of the system is shown below:



The input power to the motor is:

$$P_{\text{in, motor}} = \frac{P_{\text{out, motor}}}{\eta_{\text{motor}}} = \frac{2 \times 746}{0.80} = 1865 \text{ W}$$

The input power to the converter is:

$$P_{\text{in, converter}} = \frac{P_{\text{out, converter}}}{\eta_{\text{converter}}} = \frac{1865}{0.88} = 2119 \text{ W}$$

Finally, the current taken from the 400-V source is:

$$I = \frac{P_{\text{in, converter}}}{400} = 5.298 \text{ A}$$

P17.3* As frequency is reduced, the reactances X_s, X_m, and X'_r of the machine become smaller. (Recall that $X = \omega L$.) Thus the applied voltage must be reduced to keep the currents from becoming too large, resulting in magnetic saturation and overheating.

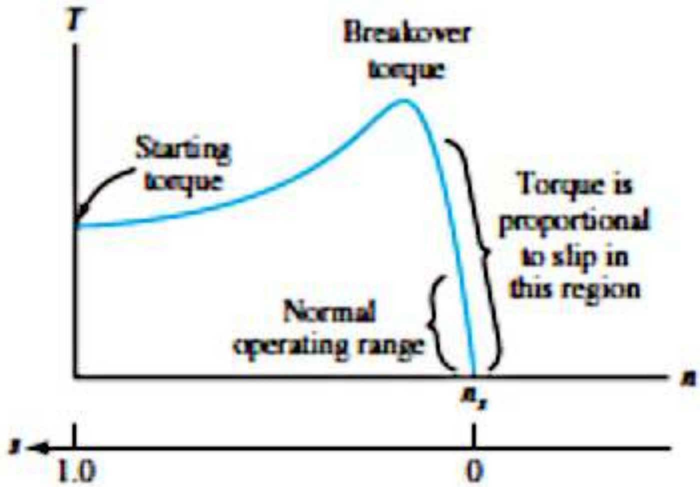
P17.4* The magnetic field is periodic with the same frequency as the source for all machines. In a two-pole machine the magnetic field makes one cycle around the air gap (from north to south and back to north). In a four-pole machine the magnetic field makes two cycles around the air gap. Similarly the field of the six-pole machine has three cycles. Thus the expressions for the fields are:

$$B_{\text{four-pole}} = B_m \cos(\omega t - 2\theta)$$

and

$$B_{\text{six-pole}} = B_m \cos(\omega t - 3\theta)$$

P17.5



First, assume that the rotor speed nm equals the synchronous speed N_S (i.e., the slip s equals zero). In this case, the relative velocity between the conductors and the field is zero (i.e., $u = 0$). Then according to Equation 17.15 from the text, the induced voltage V_c is zero. Consequently, the rotor currents are zero and the torque is zero. As the rotor slows down from synchronous speed, the stator field moves past the rotor conductors. The magnitudes of the voltages induced in the rotor conductors increase linearly with slip. For small slips, the inductive reactances of the conductors, given by sL_c, are negligible, and maximum rotor current is aligned with maximum stator field, which is the optimum situation for producing torque. Because the induced voltage is proportional to slip and the impedance is independent of slip, the currents are proportional to slip. Torque is proportional to the product of the field and the current. Hence, we conclude that torque is proportional to slip, assuming small slip. This fact is illustrated in the figure. As the motor slows further, the inductive reactance eventually dominates the denominator of Equation 17.19. Then, the magnitude of the current is nearly independent of slip. Thus, the torque tends to level out as the motor slows. Because the poles on the rotor tend to become aligned with the stator poles, the torque decreases as the motor slows to a stop. The torque for zero speed is called either the starting torque or the stall torque . The maximum torque is called either the pull-out torque or the breakover torque.

17.6

P vs ns for f = 50 Hz

P	ns (in rpm)
2	3000
4	1500
6	1000
8	750
10	600
12	500

P vs ns for f = 100 Hz

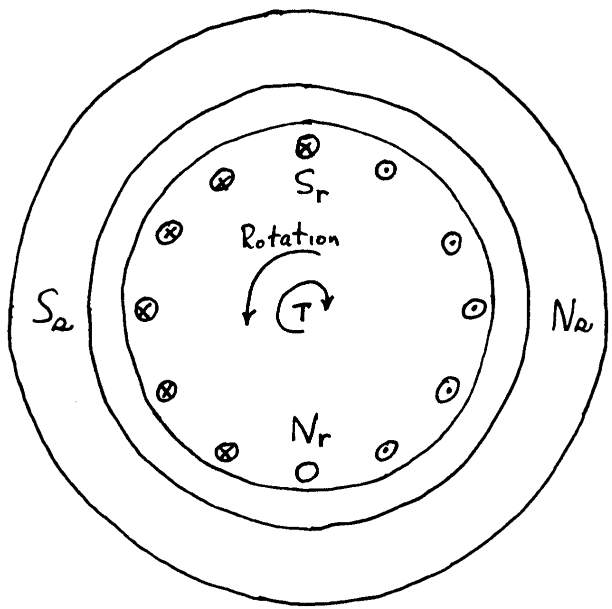
P	ns (in rpm)
2	6000
4	3000
6	2000
8	1500
10	1200
12	1000

P vs ns for f = 400 Hz

P	ns (in rpm)
2	24000
4	12000
6	8000
8	6000
10	4800
12	4000

P17.7 First consider an air-gap flux given by $B = B_m \cos(\omega t + 2\theta)$. For $t = 0$, the flux is given by $B = B_m \cos(2\theta)$. Thus, the flux completes two complete cycles as θ varies from 0 to 360° changing direction four times around the periphery of the air gap. Consequently, the motor has four poles. Similarly, a motor that has $B = B_m \cos(\omega t + 3\theta)$ is a six-pole motor. The speeds are 1500 rpm and 1000 rpm, respectively.

P17.8 In this case the developed torque opposes the direction of rotation. Power is taken from the prime mover and converted to electrical form. Thus, the machine acts as a generator.



P17.9 At 50 Hz, synchronous speed for a six-pole machine is:

$$n_s = \frac{120f}{P} = \frac{120(50)}{4} = 1500 \text{ rpm}$$

The slip is given by:

$$s = \frac{n_s - n_m}{n_s} = \frac{1500 - 1440}{1500} = 4\%$$

The frequency of the rotor currents is the slip frequency. From Equation 17.17, we have $\omega_{\text{slip}} = s\omega$. For frequencies in the Hz, this becomes:

$$f_{\text{slip}} = sf = 0.04 \times 50 = 2 \text{ Hz}$$

In the normal range of operation, slip is approximately proportional to output power and torque. Thus,

At $\frac{3}{4}$ load, $s = 4 \times 0.75 = 3\%$. This corresponds to a speed of 1455 rpm

At $\frac{1}{2}$ load, $s = 4 \times 0.50 = 2\%$. This corresponds to a speed of 1470 rpm

At $\frac{1}{4}$ load, $s = 4 \times 0.25 = 1\%$. This corresponds to a speed of 1485 rpm

P17.10 (a) To convert speed in mph to revolutions of the tire per minute, we have:

$$n_m = \text{speed in mph} \times \frac{5280 \times 12}{60 \times 20 \times \pi}$$

Thus the speed range 5 to 70 mph implies a rotational speed range of 84.0 to 1176 rpm. We assume negligible slip so this is the range of synchronous speed for the motor. Solving Equation 17.14 for frequency, we have:

$$f = \frac{Pn_s}{120}$$

From this formula, we find that the range of frequencies needed is from 2.8 to 39.2 Hz.

The final speed of 40 mph converts to 17.88 m/s. The acceleration of the vehicle is:

$$a = u/T = 17.88/10 = 1.788 \text{ m/s}^2$$

The force needed to accelerate the vehicle is:

$$f = ma = 1000 \times 1.788 = 1788 \text{ newtons}$$

Velocity is acceleration times time.

$$u = 1.788t$$

The output power needed from the motor is:

$$p(t) = fu = 1788 \times 1.788t = 3197t \text{ watts}$$

The power required from the battery is larger because of losses.

$$p_{\text{battery}} = \frac{p(t)}{0.85 \times 0.89} = 4226t \text{ watts}$$

Finally, the current taken from the 48-V battery is:

$$i(t) = \frac{p_{\text{battery}}}{48} = 88.0t \text{ amperes}$$

(b) As in part (a), the range of frequencies needed ranges from 2.8 to 39.2 Hz and the final speed of 40 mph converts to 17.88 m/s. At a speed of 40 mph the kinetic energy of the vehicle is

$$W = \tfrac{1}{2}mu^2 = 159.8 \times 10^3 \text{ J}$$

The average power required during acceleration is:

$$P = \frac{W}{T} = 15.98 \text{ kW}$$

Accounting for efficiencies, the power required from the battery is:

$$P_{\text{battery}} = \frac{P_{\text{out}}}{0.85 \times 0.89} = 21.12 \text{ kW}$$

Finally, the battery current is:

$$I = \frac{P_{\text{battery}}}{48} = 440.1 \text{ A}$$

P17.11 The total field in the machine is the sum of the fields produced by the separate windings. Thus,

$$B = B_a + B_b = Ki_a(t)\cos(\theta) + Ki_b(t)\cos(\theta - 90^\circ)$$

Substituting the expressions given for the currents, we have:

$$B = KI_m\cos(\omega t)\cos(\theta) + KI_m\cos(\omega t - 90^\circ)\cos(\theta - 90^\circ)$$

Using the trigonometric identity $\cos A \cos B = \frac{1}{2}[\cos(A - B) + \cos(A + B)]$, we have:

$$B = KI_m\cos(\omega t - \theta) + \frac{KI_m}{2}\left[\cos(\omega t + \theta) + \cos(\omega t + \theta - 180^\circ)\right]$$

However, we can write $\cos(\omega t + \theta) + \cos(\omega t + \theta - 180^\circ) = 0$ because the two terms are out of phase. Thus, we have:

$$B = KI_m\cos(\omega t - \theta)$$

According to this result, the maximum flux occurs for $\theta = \omega t$, which implies rotation of the field in the counterclockwise direction at an angular speed of ω. The maximum flux density is $B_{\max} = KI_m$.

P17.12 With zero resistance for the rotor conductors, Equation 17.19 for the rotor currents becomes

$$\mathbf{I}_c = \frac{\mathbf{V}_c}{js\omega L_c}$$

Thus, the rotor currents $\mathbf{I}_c$ lag the induced rotor voltages $\mathbf{V}_c$ by 90°. Thus, δ_{rs} becomes zero (refer to Figure 17.9 in the book), and the rotor poles lie exactly under the stator poles. Then, the developed torque is zero at all speeds. Thus using superconductors for the rotor conductors is not a good idea.

P17.13 The currents are given as

$$i_a(t) = I_m \cos(2\omega t)$$

and

$$i_b(t) = I_m \cos(2\omega t + 90^\circ)$$

The total field in the machine is the sum of the fields produced by the separate windings. Thus,

$$B = B_a + B_b = Ki_a(t)\cos(\theta) + Ki_b(t)\cos(\theta - 90^\circ)$$

Substituting the expressions given for the currents, we have:

$$B = KI_m \cos(2\omega t)\cos(\theta) + KI_m \cos(2\omega t + 90^\circ)\cos(\theta - 90^\circ)$$

Using the trigonometric identity $\cos A \cos B = \frac{1}{2}[\cos(A - B) + \cos(A + B)]$,

we have:

$$B = \frac{KI_m}{2}[\cos(2\omega t - \theta) + \cos(2\omega t - \theta + 180^\circ)] + KI_m \cos(2\omega t + \theta)$$

However, we can write $\cos(\omega t - \theta) + \cos(\omega t - \theta + 180^\circ) = 0$ because the two terms are out of phase. Thus, we have:

$$B = KI_m \cos(2\omega t + \theta)$$

According to this result, the maximum flux occurs for $\theta = -2\omega t$, which implies rotation of the field in the clockwise direction at an angular speed of 2ω.

P17.14* The synchronous speed of the machine is given by:

$$\omega_s = \frac{\omega}{(P/2)} = \frac{2\pi 60}{4/2} = 188.5 \text{ rad/s or by } n_s = \frac{120f}{P} = 1800 \text{ rpm}$$

Typically, slip is about 5% at full load. Thus, the full-load speed is:

$$n_{\text{full-load}} = 1800 \times 0.95 = 1710 \text{ rpm}$$

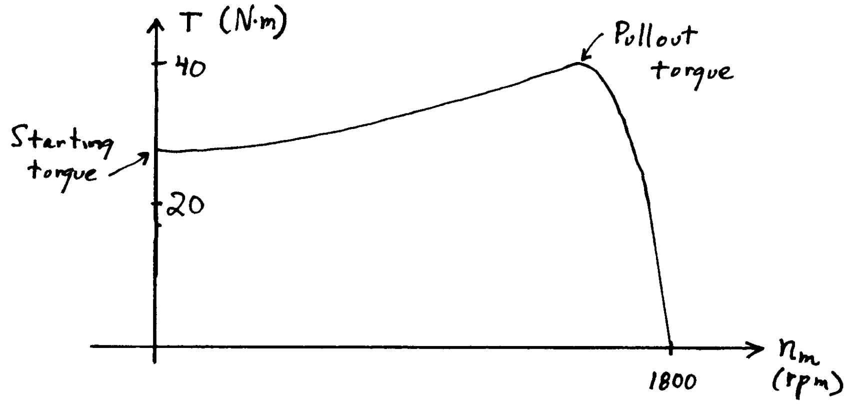
or

$$\omega_{m,\text{ full-load}} = 188.5 \times 0.95 = 179.1 \text{ rad/s}$$

The full-load torque is:

$$T_{\text{full-load}} = \frac{5 \times 746}{\omega_{m,\text{ full-load}}} = 20.8 \text{ newton meters}$$

Typically the starting torque is 1.5 times the full-load torque and the pullout torque is 2 times the full-load torque. Thus, a sketch of the torque-speed characteristic is shown below.



We estimate the efficiency of a typical machine such as this as 80%. Therefore, the input power at full load is:

$$P_{\text{in}} = \frac{P_{\text{out}}}{\eta} = \frac{5 \times 746}{0.80} = 4662 \text{ W}$$

Typically, the power factor is 75% and the line current is

$$I_{line} = \frac{P_{\text{in}}}{\sqrt{3} V \cos\theta} = \frac{4662}{\sqrt{3}(220)(0.75)} = 16.3 \text{ A rms}$$

Typically, the starting current is 5 to 7 times the full-load current, so we estimate:

$$I_{\text{starting}} = 6 \times 9.42 = 97.9 \text{ A}$$

P17.15* Following Example 17.2, we find

$$\mathbf{I}_{s,\text{ starting}} = \frac{\mathbf{V}_s}{Z_s} = \frac{220\angle 0^\circ}{3.314\angle 57.48^\circ}$$

$$\mathbf{I}_{s,\text{ starting}} = 66.39\angle -57.48^\circ$$

Because the machine is delta connected, the magnitude of the starting line current is

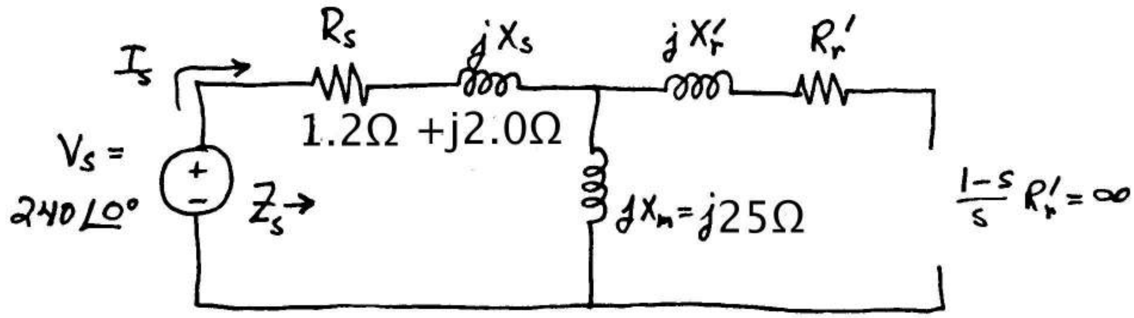
$$I_{line,\text{starting}} = I_{s,\text{starting}}\sqrt{3} = 66.39\sqrt{3} = 115.0 \text{ A rms}$$

$$P_{ag} = 3R_{eq}\left(I_{s,\text{ starting}}\right)^2$$
$$= 7.688 \text{ kW}$$

$$T_{\text{dev, starting}} = \frac{P_{ag}}{\omega_s} = \frac{7688}{2\pi 60/2}$$
$$= 40.8 \text{ newton meters}$$

Comparing these results to those of the example, we see that the starting current is reduced by a factor of 2 and the starting torque is reduced by a factor of 4. Depending on the torque--speed characteristic of the load, the system may not start.

P17.16* Neglecting rotational losses, the slip is zero with no load, and the motor runs at synchronous speed which is 1800 rpm. Then the equivalent circuit becomes:



$$Z_s = R_s + jX_s + jX_m = 1.5 + j2.0 + j25$$
$$= 27.0266\angle 87.4552^\circ \Omega$$

The power factor is:

$$\cos(87.4552^\circ) = 4.44\%$$

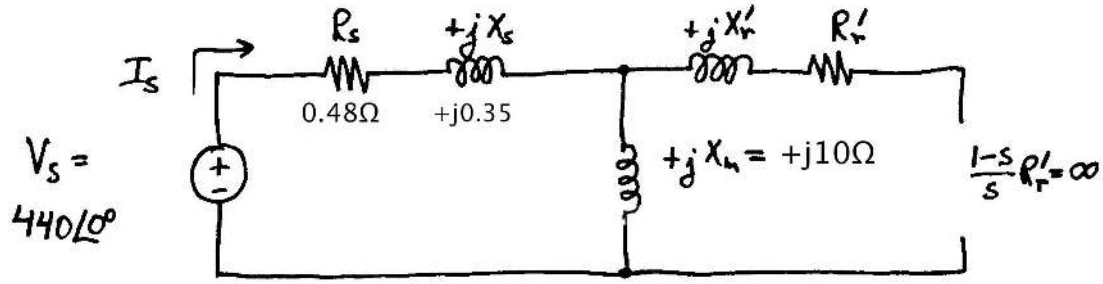
The current is:

$$\mathbf{I}_s = \frac{\mathbf{V}_s}{Z_s} = \frac{240}{27.0266\ \angle 87.4552^\circ} = 8.8801\ \angle -87.4552^\circ$$

Because the machine is delta connected, the line current magnitude is

$$I_{line} = \sqrt{3} I_s = 15.3808 \text{ A rms}$$

P17.17* Neglecting rotational losses, the slip is zero with no load, and the motor runs at synchronous speed which is 1200 rpm. Then the equivalent circuit becomes:



$$Z_s = R_s + jX_s + jX_m$$
$$= 0.48 + j0.35 + j10 = 10.3611 \angle 87.3447^\circ$$

The power factor is:

$$= \cos(87.3447^\circ) = 4.6326\%$$

The current is:

$$\mathbf{I}_s = \frac{\mathbf{V}_s}{Z_s} = \frac{440}{10.3611\angle 87.3447^\circ} = 42.4665\angle -87.3447^\circ \text{ A}$$

Because the machine is delta connected, the magnitude of the starting line current is

$$I_{line} = I_s\sqrt{3} = 42.4665\sqrt{3} = 73.5542 \text{ A rms}$$

P17.18* Because the machine is wye connected the phase voltage is:

$$V_{line}/\sqrt{3} = 440/\sqrt{3} = 254.0 \text{ V rms}$$

Refer to Figure 17.13.

$$P_{\text{in}} = 3VI\cos\theta = 3(254.0)16.8(0.80) = 10.243 \text{ kW}$$
$$P_{\text{ag}} = P_{\text{in}} - P_s = 10.243 - 0.350 = 9.893 \text{ kW}$$
$$P_{\text{dev}} = P_{\text{ag}} - P_r = 9.893 - 0.120 = 9.773 \text{ kW}$$
$$P_{\text{out}} = P_{\text{dev}} - P_{\text{rot}} = 9.773 - 0.400 = 9.373 \text{ kW}$$
$$\eta = \frac{P_{\text{out}}}{P_{\text{in}}} \times 100\% = 91.51\%$$

P17.19* First, the full-load output torque is:

$$T_{\text{out}} = \frac{P_{\text{out}}}{\omega_m} = \frac{5 \times 746}{3500(2\pi/60)} = 10.18 \text{ newton meters}$$

We assume that the developed torque is proportional to slip in the normal operating range.

$$T_{\text{dev}} = T_{\text{out}} + T_{\text{rot}} = k\frac{n_s - n_m}{n_s} = K(n_s - n_m)$$

Thus at full load, we have:

$$10.18 + T_{\text{rot}} = K(100) \qquad (1)$$

At no load, we have $T_{\text{out}} = 0$ and $(n_s - n_m) = 2$. Thus,

$$T_{\text{rot}} = K(2) \qquad (2)$$

Solving Equations (1) and (2), we obtain:

$$T_{\text{rot}} = 0.2077 \text{ newton meters}$$

$$P_{\text{rot}} = T_{\text{rot}}\omega_m = 76.12 \text{ W}$$

P17.20 In squirrel cage motor, the conductors consist of bars of aluminum that are cast into slots cut into the rotor whereas in the wound rotor, insulated wires are wound into slots in the rotor. In WRIM, external connections can be made to the windings of the wound rotor for purposes of speed control, thus it is more suited for speed control. The squirrel cage, however is the more rugged of the two types and requires lower maintenance. Thus SQUIM is the workhorse of the industry.

P17.21 Besides low cost, we usually would prefer an induction motor with:

1. High power factor.
2. High starting torque.
3. High efficiency.
4. Low starting current.
5. High pull-out torque.

P17.22 Refer to Figure 17.13. Neglecting the stator resistance R_s and rotational losses, the only loss is rotor copper loss given by:

$$P_r = 3R_r'(I_r')^2$$

The developed power and the output power are equal and given by:

$$P_{\text{out}} = P_{\text{dev}} = 3\frac{1-s}{s}R_r'(I_r')^2$$

Now the efficiency is given by:

$$\eta = \frac{P_{\text{out}}}{P_{\text{in}}} = \frac{P_{\text{out}}}{P_r + P_{\text{out}}} = \frac{\frac{1-s}{s}}{1+\frac{1-s}{s}} = 1 - s = 0.75$$

$$\eta = 75\%$$

P17.23 Synchronous speed for an 4-pole 60-Hz motor is 1800 rpm. Thus, the slip is given by:

$$s = \frac{\omega_s - \omega_m}{\omega_s} = \frac{n_s - n_m}{n_s} = \frac{1800 - 1710}{1800} = 0.05$$

Of course, the frequency of the stator currents is the line frequency or 60-Hz. The frequency of the rotor currents is the slip frequency given by:

$$f_{\text{slip}} = sf = 0.05 \times 60 = 3\,\text{Hz}$$

Refer to Figure 17.13. The developed power is:

$$P_{\text{dev}} = P_{\text{out}} + P_{\text{rot}} = (5 \times 746) + 200 = 3250\text{ W}$$

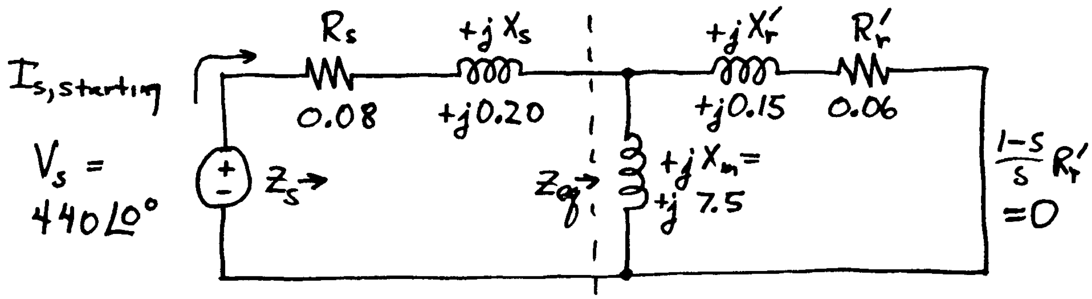
Since we have

$$P_{\text{dev}} = 3\frac{1-s}{s}R_r'(I_r')^2 \quad \text{and} \quad P_r = 3R_r'(I_r')^2$$

we can write

$$P_r = \frac{s}{1-s}P_{\text{dev}} = \frac{0.05}{1-0.05} \times 3250 = 171.0526\text{ W}$$

P17.24 This is similar to Example 17.2. The equivalent circuit is:



$$Z_{eq} = R_{eq} + jX_{eq} = \frac{j7.5(0.06 + j0.15)}{j7.5 + 0.06 + j0.15} = 0.0577 + j0.1475$$

The impedance seen by the source is:

$$Z_s = 0.08 + j0.20 + Z_{eq}$$
$$= 0.1584\angle 68.65^\circ$$

Thus, the starting current is:

$$\mathbf{I}_{s,\text{starting}} = \frac{\mathbf{V}_s}{Z_s} = \frac{440}{0.1584\angle 68.65^\circ}$$

$$\mathbf{I}_{s,\text{starting}} = 2778\angle -68.65^\circ$$

Because the machine is delta connected, the magnitude of the starting line current is

$$I_{linr,\text{starting}} = I_{s,\text{starting}}\sqrt{3} = 2778\sqrt{3} = 4812 \text{ A rms}$$

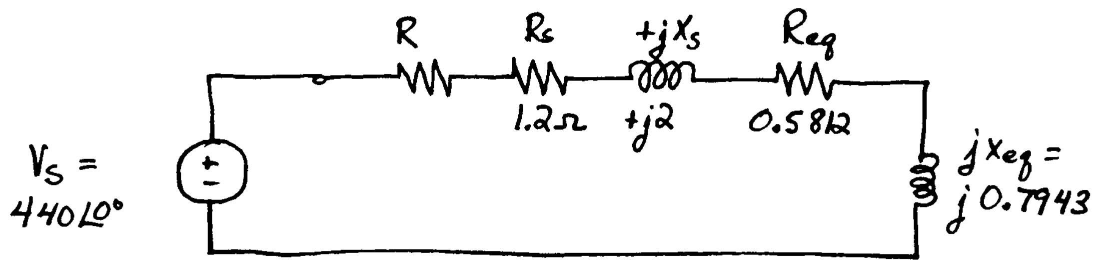
The power crossing the air gap is (three times) the power delivered to the right of the dashed line.

$$\begin{aligned} P_{ag} &= 3R_{eq}\left(I_{s,\text{ starting}}\right)^2 \\ &= 239.72 \text{ kW} \end{aligned}$$

Finally, the starting torque is found using Equation 17.34.

$$\begin{aligned} P_{\text{dev, starting}} &= \frac{P_{ag}}{\omega_s} \\ &= \frac{239.7 \times 10^3}{2\pi 60/3} \\ &= 1907 \text{ newton meters} \end{aligned}$$

P17.25 For a line current of $50\sqrt{3}$ A, the phase current is 50 A. Equivalent circuits for the machine under starting conditions are shown in Figure 17.15. With the added resistance in series, we have the circuit



The total impedance magnitude is required to be:

$$Z_{\text{total}} = \frac{V}{I} = \frac{440}{50} = 8.8\Omega$$

However, the impedance magnitude is

$$Z_{\text{total}} = \sqrt{(R + 1.2 + 0.5812)^2 + (2 + 0.7943)^2}$$

Solving for the added resistance R, we find

$$R = 6.563\ \Omega$$

As in Example 17.2, the power crossing the air gap is:

$$\begin{aligned} P_{ag} &= 3R_{eq}\left(I_{s,\text{ starting}}\right)^2 = 3(0.5812)(50)^2 \\ &= 4359 \text{ W} \end{aligned}$$

Finally, the starting torque is found using Equation 15.34.

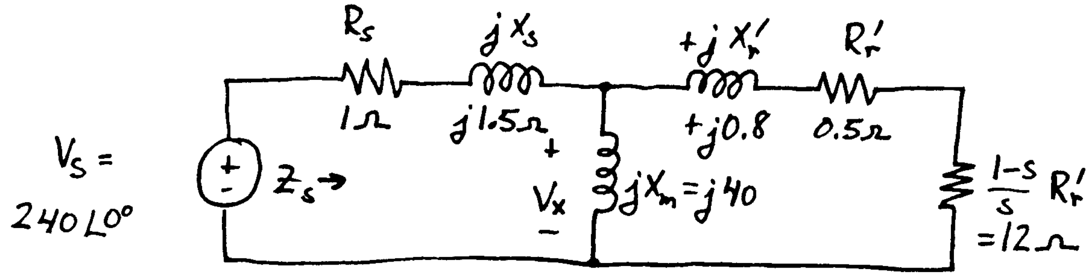
$$T_{\text{dev, starting}} = \frac{P_{ag}}{\omega_s}$$
$$= \frac{4359}{2\pi 60/2}$$
$$= 23.13 \text{ newton meters}$$

Comparing these results to those of the example, we see that the starting torque is reduced by a factor of 163.1/23.1 = 7.06. Depending on the torque--speed characteristic of the load, the system may not start.

P17.26 Following the solution to Example 17.1, we have:

$$n_s = 1800 \text{ rpm}$$
$$s = \frac{n_s - n_m}{n_s} = \frac{1800 - 1728}{1800} = 0.04$$

The per-phase equivalent circuit is:



$$Z_s = 1.0 + j1.5 + \frac{j40(0.5 + 12 + j0.8)}{j40 + 0.5 + 12 + j0.8}$$
$$= 11.98 + j5.649$$
$$= 13.25\angle 25.24^\circ$$

$$\text{power factor} = \cos(25.24^\circ) = 90.45\% \text{ lagging}$$

$$\mathbf{I}_s = \frac{\mathbf{V}_s}{Z_s} = \frac{240\angle 0^\circ}{13.25\angle 25.24^\circ} = 18.11\angle -25.24^\circ$$
$$P_{in} = 3I_s V_s \cos\theta = 11.80 \text{ kW}$$

Next, we compute $\mathbf{V}_x$ and $\mathbf{I}_r'$.

$$\mathbf{V}_x = \mathbf{I}_s \frac{j40(0.5 + 12 + j0.8)}{j40 + 0.5 + 12 + j0.8}$$
$$= 212.0 - j16.85$$
$$= 212.7\angle -4.545^\circ$$

$$\mathbf{I}'_r = \frac{\mathbf{V}_x}{j0.8 + 0.5 + 12}$$
$$= 16.98\angle -8.207^\circ$$

The copper losses in the stator and rotor are:

$$P_s = 3R_s I_s^2$$
$$= 3(1)(18.11)^2$$
$$= 984.5 \text{ W}$$

and

$$P_r = 3R'_r (I'_r)^2$$
$$= 3(0.5)(16.98)^2$$
$$= 432.5 \text{ W}$$

Finally, the developed power is:

$$P_{\text{dev}} = 3 \times \frac{1-s}{s} R'_r (I'_r)^2$$
$$= 3(12)(16.98)^2$$
$$= 10.38 \text{ kW}$$
$$P_{\text{out}} = P_{\text{dev}} - P_{\text{rot}} = 10.18 \text{ kW}$$

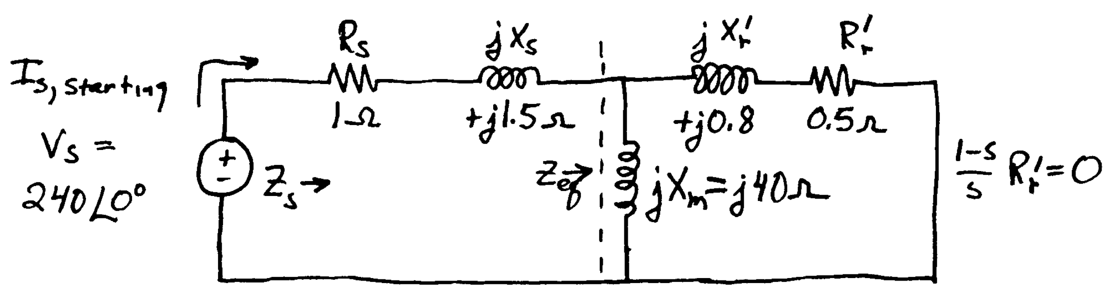
The output torque is:

$$T_{\text{out}} = \frac{P_{\text{out}}}{\omega_m} = 56.25 \text{ newton meters}$$

The efficiency is:

$$\eta = \frac{P_{\text{out}}}{P_{\text{in}}} \times 100\% = 86.29\%$$

P17.27 This is similar to Example 17.2. The equivalent circuit is:



$$Z_{eq} = R_{eq} + jX_{eq} = \frac{j40(0.5 + j0.8)}{j40 + 0.5 + j0.8} = 0.4805 + j0.7902$$

The impedance seen by the source is:

$$\begin{aligned} Z_s &= 1.0 + j1.5 + Z_{eq} \\ &= 1.0 + j1.5 + 0.4805 + j0.7902 \\ &= 2.727\angle 57.12^\circ \end{aligned}$$

Thus, the starting current is:

$$\mathbf{I}_{s,\,\text{starting}} = \frac{\mathbf{V}_s}{Z_s} = \frac{240\angle 0^\circ}{2.727\angle 57.12^\circ}$$

$$\mathbf{I}_{s,\,\text{starting}} = 88.01\angle -57.12^\circ$$

Because the machine is delta connected, the magnitude of the starting line current is

$$I_{linr,\text{starting}} = I_{s,\text{starting}}\sqrt{3} = 88.01\sqrt{3} = 152.4 \text{ A rms}$$

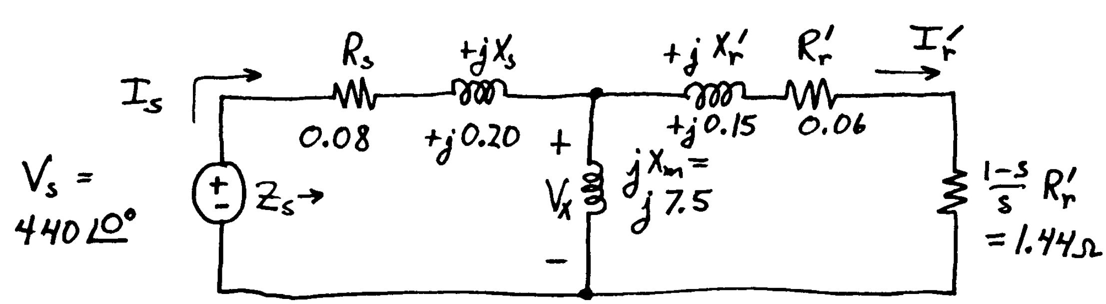
The power crossing the air gap is (three times) the power delivered to the right of the dashed line.

$$\begin{aligned} P_{ag} &= 3R_{eq}\left(I_{s,\,\text{starting}}\right)^2 \\ &= 11.17 \text{ kW} \end{aligned}$$

Finally, the starting torque is found using Equation 17.34.

$$\begin{aligned} T_{\text{dev, starting}} &= \frac{P_{ag}}{\omega_s} \\ &= \frac{11170}{2\pi 60/2} \\ &= 59.26 \text{ newton meters} \end{aligned}$$

P17.28 This is similar to Example 17.1. The per-phase equivalent circuit is:



$$Z_s = 0.08 + j0.20 + \frac{j7.5(0.06 + 1.44 + j0.15)}{j7.5 + 0.06 + 1.44 + j0.15}$$

$$= 1.468 + j0.6193$$

$$= 1.594\angle 22.87^\circ$$

$$\text{power factor} = \cos(22.87^\circ) = 92.14\%\ \text{lagging}$$

$$\mathbf{I}_s = \frac{\mathbf{V}_s}{Z_s} = \frac{440\angle 0^\circ}{1.594\angle 22.87^\circ} = 276.0\angle -22.87^\circ$$

$$P_{\text{in}} = 3I_sV_s\cos\theta = 335.8\ \text{kW}$$

Next, we compute $\mathbf{V}_x$ and $\mathbf{I}'_r$.

$$\mathbf{V}_x = \mathbf{I}_s \frac{j7.5(0.06 + 1.44 + j0.15)}{j7.5 + 0.06 + 1.44 + j0.15}$$

$$= 398.2 - j42.30$$

$$= 400.4\angle -6.063$$

$$\mathbf{I}'_r = \frac{\mathbf{V}_x}{0.06 + 1.44 + j0.15}$$

$$= 265.6 - j11.77$$

The copper losses in the stator and rotor are:

$$P_s = 3R_sI_s^2$$

$$= 18.30\ \text{kW}$$

and

$$P_r = 3R'_r(I'_r)^2 = 12.70\ \text{kW}$$

Finally, the developed power is:

$$P_{\text{dev}} = 3 \times \frac{1-s}{s} R'_r(I'_r)^2$$

$$= 304.8\ \text{kW}$$

$$P_{\text{out}} = P_{\text{dev}} - P_{\text{rot}} = 304.8 - 2.0$$

$$= 302.8\ \text{kW}$$

The output torque is:

$$T_{\text{out}} = \frac{P_{\text{out}}}{\omega_m}$$

$$= 2410\ \text{newton meters}$$

The efficiency is:

$$\eta = \frac{P_{\text{out}}}{P_{\text{in}}} \times 100\% = 90.17\%$$

P17.29 Because the machine is delta connected, the magnitude of the phase current is:

$$I_s = I_{\text{line}} / \sqrt{3} = 6/\sqrt{3} = 3.4641 \text{ A rms}$$

Then, we have

$$P_{\text{in}} = 3VI\cos\theta = 3(220)3.4641(0.9) = 2057.6764 \text{ W}$$

$$P_{\text{out}} = 5 \times 746 = 3030 \text{ W}$$

$$\text{efficiency} = \frac{P_{\text{out}}}{P_{\text{in}}} \times 100\% = 67.9101\%$$

P17.30 The no-load speed is approximately 1800 rpm. Thus, the synchronous speed appears to be 1800 rpm, and we have a four-pole motor.

Steady-state operation is at the intersection of the torque-speed characteristic of the motor and that of the load. Thus, we have $T_{\text{out}} = 25$ newton meters and $n_m = 1400$ rpm. The slip is:

$$s = \frac{n_s - n_m}{n_s} = \frac{1800 - 1400}{1800} = 22.22\%$$

$$\omega_m = n_m \frac{2\pi}{60} = 146.6 \text{ radian/s}$$

$$P_{\text{out}} = T_{\text{out}}\omega_m = 3665 \text{ W}$$

Since we are assuming $P_{\text{rot}} = 0$, we have

$$P_{\text{dev}} = P_{\text{out}} = 3\frac{1-s}{s} R_r'(I_r')^2$$

Also, we have

$$P_r = 3R_r'(I_r')^2$$

Thus,

$$P_r = P_{\text{out}} \times \frac{s}{1-s} = 3665\frac{0.2222}{1-0.2222} = 1047 \text{ W}$$

P17.31 As an engineering estimate, we take the difference between the motor torque and the load torque as approximately 25 newton meters over the speed range of interest. Thus, the angular acceleration of the system is:

$$\frac{d\omega_m}{dt} = \frac{T_{\text{motor}} - T_{\text{load}}}{\text{rotational inertia}} = \frac{25}{5} = 5 \text{ radian/s}^2$$

$n_m = 1000$ rpm corresponds to $\omega_m = 104.7$. Thus, the time required is approximately:

$$T_{\text{run-up}} = \frac{104.7}{5} = 21 \text{ seconds}$$

P17.32 Under the conditions given, we have

$$s = \frac{n_s - n_m}{n_s} = \frac{1800 - 1750}{1800} = 0.02778$$

out $=$

$$T_{\text{out}} = \frac{P_{\text{out}}}{\omega_m} = \frac{1492}{1750(2\pi/60)} = 8.142 \text{ newton meters}$$

As an approximation, we assume that the output torque is proportional to slip. Thus, we have:

$$T_{\text{out}} = Ks$$

Substituting the above values, we obtain $K = 8.142/0.02778 = 293.1$.

Now according to Equation 17.34, we have $T_{\text{dev}} = \dfrac{P_{\text{ag}}}{\omega_s}$.

However, the air-gap power is proportional to the square of the source voltage. Thus, the developed torque is proportional to source voltage squared. Neglecting rotational loss, the output torque is equal to the developed torque. Thus, output torque is proportional to the square of the source voltage.

Thus for operation at 220 V, we have:

$$K' = K(220/240)^2 = 246.3$$

Then the slip is:

$$s' = \frac{T_{\text{out}}}{K'} = \frac{8.142}{246.3} = 0.03306$$

and the new speed is $n_m = n_s(1 - s') = 1740.5 \text{ rpm}$.

P17.33* Yes. It is self-starting.

P17.34* (a) Field current remains constant. The field circuit is independent of the ac source and the load.

(b) Mechanical speed remains constant assuming that the pull-out torque has not been exceeded.

(c) Output torque increases by a factor of $1/0.8 = 1.25$

(d) Armature current increases in magnitude.

(e) Power factor decreases and becomes lagging.

(f) Torque angle increases.

P17.35* $\omega_s = \dfrac{\omega}{(P/2)} = \dfrac{2\pi 60}{10/2} = 75.40 \text{ rad/s}$ $\qquad\qquad n_s = 720 \text{ rpm}$

$$T_{\text{dev, rated}} = \frac{P_{\text{dev, rated}}}{\omega_s} = \frac{100 \times 746}{75.40} = 989.4 \text{ newton meters}$$

According to Equation 17.37, we have:

$$T_{\text{dev}} = KB_rB_{\text{total}} \sin\delta$$

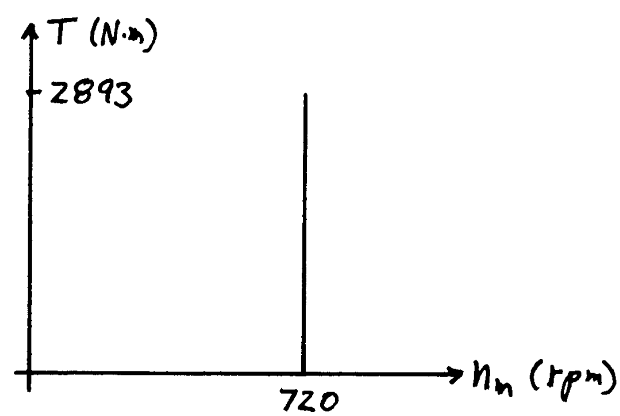
We define $K_t = KB_rB_{\text{total}}$. Then from the rated operating conditions, we find:

$$K_t = \frac{T_{\text{dev}}}{\sin\delta} = \frac{989.4}{\sin(20^\circ)} = 2893$$

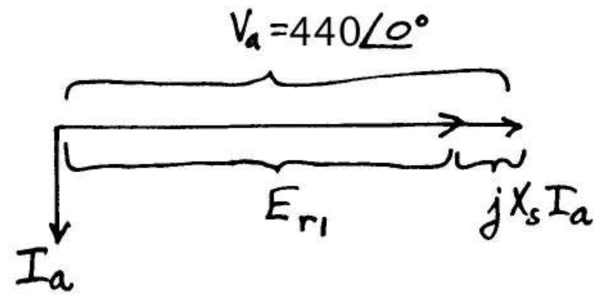
The pullout torque occurs for $\delta = 90^\circ$. Thus we have:

$$T_{\max} = K_t \sin(90^\circ) = 2893 \text{ newton meters}$$

The torque speed characteristic is:



P17.36* For zero developed power, the torque angle is zero. The phasor diagram is:



$$\mathbf{E}_{r1} = \mathbf{V}_a - jX_s\mathbf{I}_{a1} = 440 - j8(j18) = 296 \text{ V}$$

To achieve zero armature current, we must have $\mathbf{E}_{r2} = \mathbf{V}_a = 440$ V. The magnitude of $\mathbf{E}_r$ is proportional to the field current. Thus we have:

$$I_{f2} = I_{f1}\frac{E_{r2}}{E_{r1}} = 7.5\frac{440}{296} = 11.1486 \text{ A}$$

P17.37* (a) The speed of a synchronous machine is related to the frequency of the armature voltages by Equation 17.14:

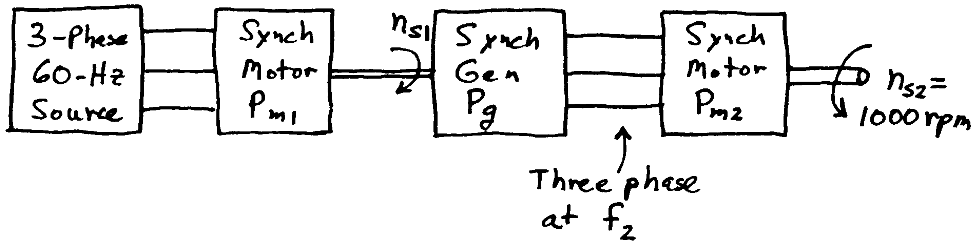
$$n_s = \frac{120f}{P}$$

The frequency of the voltage applied to the 12-pole motor is 60 Hz and the speed is 600 rpm. Thus, the generator is driven at 600 rpm and the voltages induced in its armature have a frequency of:

$$f_{gen} = \frac{n_s P_{gen}}{120} = \frac{600 \times 10}{120} = 50 \text{ Hz}$$

Thus, the two machines can be used to convert 60-Hz power into 50-Hz power.

(b) 1000 rpm is not the synchronous speed for any 60-Hz motor. Thus, we will try to convert 60-Hz into some other frequency for which 1000 rpm is a synchronous speed. The block diagram of this system is:



The speeds of motor 1 and the generator are the same, so we have:

$$n_{s1} = \frac{120 \times 60}{P_{m1}} = \frac{120 f_2}{P_g}$$

The frequencies are the same for the generator and motor 2, so we can write:

$$f_2 = \frac{n_{s2} P_{m2}}{120} = \frac{1000 P_{m2}}{120}$$

Substituting and rearranging, we have:

$$\frac{72}{10} = \frac{P_{m1} P_{m2}}{P_g}$$

Of course, the number of poles on each machine must be an even integer. One solution is:

$$P_g = 10 \qquad P_{m1} = 12 \qquad \text{and} \qquad P_{m2} = 6$$

for which $f_2 = 50\ \text{Hz}$.

Another solution is:

$$P_g = 10 \qquad P_{m1} = 6 \qquad \text{and} \qquad P_{m2} = 12$$

for which $f_2 = 100\ \text{Hz}$.

P17.38 See Figure 17.23 in the text for the V curves. As indicated by the diagram, in case of magnetic saturation, i.e. high field current, the flux is not able to increase despite as increase in the field current, thus back emf and thus the armature current remains constant.

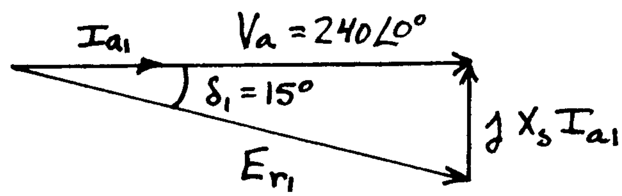
P17.39 A synchronous capacitor is an overexcited three-phase synchronous motor operating with no load. It acts as a source of reactive power and can correct the overall power factor of an industrial plant, thereby reducing energy costs.

P17.40 Two situations in which a synchronous motor is a better choice than an induction motor are:

1. A very constant speed is required as the load varies.

2. It is desirable to take advantage of the power-factor correction capability of the synchronous motor.

P17.41 Refer to Figure 17.22 for the phasor diagrams with constant developed power and variable field current. The phasor diagram for the initial operating conditions is:

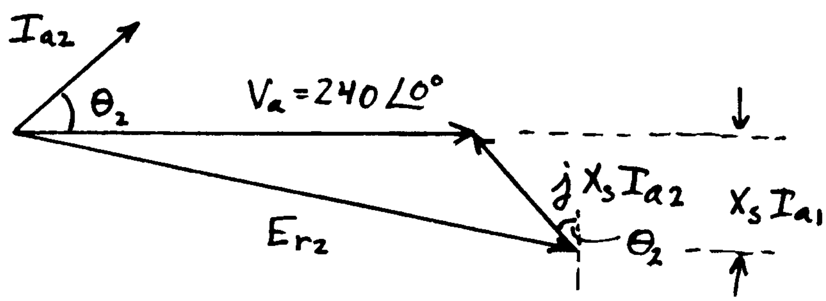


$$E_{r1} = \frac{V_a}{\cos \delta_1} = \frac{240}{\cos(15^\circ)} = 248.47$$

$$X_s I_{a1} = E_{r1} \sin\delta_1 = 64.31$$

$$E_{r2} = 1.2 \times E_{r1} = 298.16$$

The phasor diagram for the second operation condition is:



Notice that the vertical component of $jX_s\mathbf{I}_a$ is the same in both diagrams. Now we can write:

$$(V_a + X_s I_{a1}\tan\theta_2)^2 + (X_s I_{a1})^2 = (E_{r2})^2$$
$$(240 + 64.31\tan\theta_2)^2 + (64.31)^2 = (298.16)^2$$

Solving, we find $\theta_2 = 38.51^\circ$ and the power factor is $\cos\theta_2 = 78.24\%$ leading. Finally, we have:

$$\sin\delta_2 = \frac{X_s I_{a1}}{E_{r2}} = \frac{64.31}{298.16}$$

which yields:

$$\delta_2 = 12.46^\circ$$

P17.42 Synchronous speed for the machine is:

$$\omega_s = \frac{\omega}{(P/2)} = \frac{2\pi 60}{6/2} = 125.7 \text{ rad/s}$$
$$n_s = 1200 \text{ rpm}$$
$$T_{dev} = \frac{P_{dev}}{\omega_s} = 29.68 \text{ Nm}$$

According to Equation 17.37, we have:

$$T_{\text{dev}} = KB_r B_{\text{total}} \sin\delta$$

We define $K_t = KB_r B_{\text{total}}$. Then from the initial operating conditions, we find:

$$K_t = \frac{T_{\text{dev}}}{\sin\delta} = \frac{29.68}{\sin(5^\circ)} = 340.6$$

Now when the torque doubles, we have

$$\sin\delta = \frac{T_{\text{dev}}}{K_t} = \frac{2\times 29.68}{340.6}$$

which yields $\delta = 10.04^\circ$.

The pullout torque occurs for $\delta = 90^\circ$. Thus, we have:

$$T_{\max} = K_t \sin(90^\circ) = 340.6 \text{ newton meters}$$
$$P_{\text{dev,max}} = T_{\max}\omega_s = 42.82 \text{ kW or } 57.39 \text{ hp}$$

P17.43 (a) Output power remains constant.
(b) Mechanical speed remains constant.

(c) Output torque remains constant.

(d) Armature current increases in magnitude and its phase leads the source voltage.

(e) Power factor decreases and becomes leading. Reactive power is produced by the machine.

(f) Torque angle decreases.

P17.44 Because the developed power includes the losses, we have:

$$P_{in} = P_{\text{dev}} = 100 \times 746 = 74600 \text{ W} = 3I_{a1}V_a \cos\theta_1$$

Solving, we have

$$I_{a1} = \frac{P_{in}}{3V_a \cos\theta_1} = \frac{74600}{3(240)0.85} = 121.9 \text{ A}$$

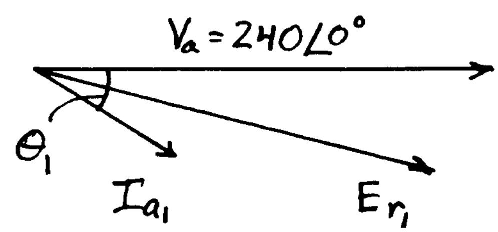
$$\theta_1 = \cos^{-1}(0.85) = 31.79^\circ$$

Thus,

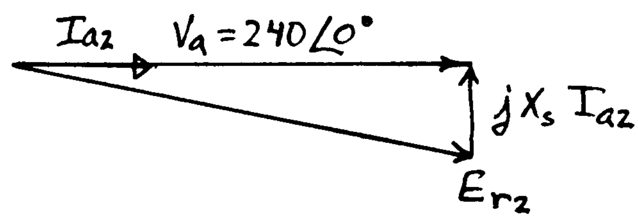
$$\mathbf{I}_{a1} = 121.9\angle -31.79^\circ$$

$$\mathbf{E}_{r1} = \mathbf{V}_a = jX_s\mathbf{I}_{a1} = 207.9 - j51.81 = 214.3\angle -13.99^\circ$$

The phasor diagram is:



For 100% power factor, the phasor diagram becomes:



Notice that (refer to Figure 17.22) the imaginary (i.e., vertical) component of $\mathbf{E}_r$ is the same in both diagrams. Thus we have:

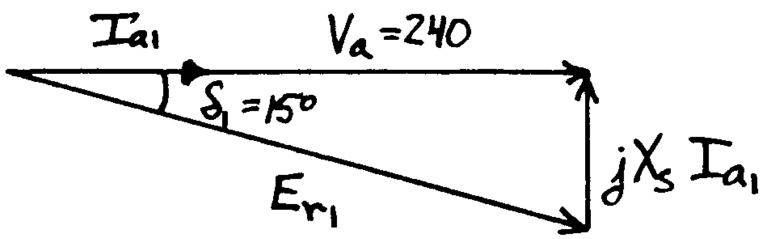
$$X_s \mathbf{I}_{a2} = \mathbf{E}_{r1} \sin(13.99^\circ) = 51.81$$

$$\mathbf{E}_{r2} = \sqrt{(V_a)^2 + (X_s I_{a2})^2} = \sqrt{240^2 + (51.81)^2} = 245.5$$

The magnitude of E_r is proportional to the field current, so we have:

$$I_{f2} = I_{f1}\frac{E_{r2}}{E_{r1}} = 10 \times \frac{245.5}{214.3} = 11.46 \text{ A}$$

P17.45 The developed power is $50 \times 746 = 37300$ W. Neglecting losses, this is the input power. On a per phase basis, we have $P_{in1} = 37300/3 = V_a I_{a1} \cos\theta_1 = 240 I_{a1} \cos 0^\circ$. This yields $I_{a1} = 51.81$ A. As a phasor $\mathbf{I}_{a1} = 51.81\angle 0^\circ$. The phasor diagram is:

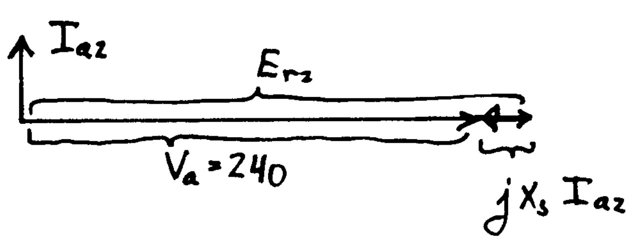


$$E_{r1} = \frac{V_a}{\cos\delta_1} = \frac{240}{\cos 15^\circ} = 248.47$$

$$X_s I_a = V_a \tan\delta_1 = 64.31$$

$$X_s = \frac{X_s I_a}{I_a} = \frac{64.31}{51.81} = 1.241\ \Omega$$

The phasor diagram with the load removed is:



Notice that E_r is constant in magnitude. Also with zero developed power, the torque angle is zero. We have

$$X_s \mathbf{I}_a = \mathbf{E}_{r2} - \mathbf{V}_a = 248.47\angle 0 - 240\angle 0 = 8.47$$

$$\mathbf{I}_a = \frac{8.47}{j1.241} = 6.821\angle 90^\circ$$

Thus, $\theta_2 = 90^\circ$, and the power factor is zero.

P17.46 (a) $\omega_s = \dfrac{\omega}{(P/2)} = \dfrac{2\pi 60}{6/2} = 125.66 \text{ rad/s}$ $\qquad n_s = 1200 \text{ rpm}$

$$T_{\text{dev}} = \frac{P_{\text{dev}}}{\omega_s} = \frac{50 \times 746}{125.66} = 296.8 \text{ newton meters}$$

(b) $P_{in} = P_{\text{dev}} = 50 \times 746 = 3I_a V_a \cos\theta = 3I_a 240(0.9)$

Thus, $I_a = 57.56$.

$\theta = \cos^{-1}(0.9) = 25.84^\circ$.

Thus, $\mathbf{I}_a = 57.56\angle 25.84^\circ$

$\mathbf{E}_{r1} = \mathbf{V}_a - jX_s\mathbf{I}_{a1} = 240 - j0.5\mathbf{I}_a = 240 - j0.5(51.80 + j25.21)$

$\mathbf{E}_{r1} = 252.6 - j25.90 = 253.9\angle -5.85^\circ$

Thus, the torque angle is $\delta_1 = 5.85^\circ$.

(c) To double the power, we must double the torque. According to Equation 17.37, the developed torque is proportional to $\sin\delta$. Thus,

$$\frac{\sin\delta_2}{\sin\delta_1} = 2$$

which yields the new torque angle $\delta_2 = 11.77^\circ$. E_r remains constant in magnitude, thus we have

$$\mathbf{E}_{r2} = 253.9\angle -11.77^\circ$$

$$\mathbf{I}_a = \frac{\mathbf{V}_a - \mathbf{E}_{r2}}{jX_s} = \frac{240 - 253.9\angle -11.77^\circ}{j0.5} = 105.0\angle 9.38^\circ$$

The power factor is $\cos(9.38^\circ) = 98.66\%$ leading.

P17.47 Referring to the V-curves shown in Figure 17.23, we see that unity power factor occurs when the armature current is minimized. Thus, we would adjust the rheostat in the field circuit to obtain minimum armature current. The setting required would change with load.

P17.48 The machine has copper-losses in the armature windings and rotational losses. (We do not consider the power that must be supplied to the field circuit.) With no load and current adjusted for minimum armature current, the machine operates with unity power factor. For a delta connection, the phase current is the line current divided by $\sqrt{3}$. The input power is:

$$P_{\text{in, no-load}} = 3(480)9.5 = 13.68\text{ kW}$$

The copper loss is:

$$P_{\text{copper,no-load}} = 3(0.05)(9.5)^2 = 13.5\text{ W}$$

Thus, the rotational loss is:

$$P_{\text{rot}} = P_{\text{in, no-load}} - P_{\text{copper,no-load}} = 13.68\text{ kW}$$

We assume that the rotational loss is independent of the load. At full-load, we have

$$P_{\text{out}} + P_{\text{rot}} + 3(I_a)^2 R_a = 3V_a I_a \cos(\theta) = P_{\text{in}}$$

$$200(746) + 13680 + 0.15(I_a)^2 = 3(480)I_a(0.9)$$

Solving for I_a, we have

$$I_a = 127.6 \text{ or } 8510$$

The appropriate root is $I_a = 127.6\text{ A}$. Thus, we have:

$$P_{\text{in}} = 3V_a I_a \cos(\theta) = 165.4\text{ kW}$$

$$\eta = \frac{P_{\text{out}}}{P_{\text{in}}} \times 100\% = 90.22\%$$

P17.49* (a)

$$P_{\text{in}} = \frac{P_{\text{out}}}{\eta} = \frac{746}{0.8} = 932.5\text{ W}$$

$$\text{power factor} = \cos\theta = \frac{P_{\text{in}}}{VI} = \frac{932.5}{120(10.2)} = 76.2\%$$

Of course, the power factor is lagging for an induction motor.

(b)

$$\begin{aligned} Z &= (V/I)\angle\cos^{-1}(\text{power factor}) \\ &= 120/10.2\angle\cos^{-1}(0.762) \\ &= 11.76\angle 40.36^\circ\ \Omega \end{aligned}$$

(c) Since the motor runs just under 1800 rpm, evidently we have a four-pole motor.

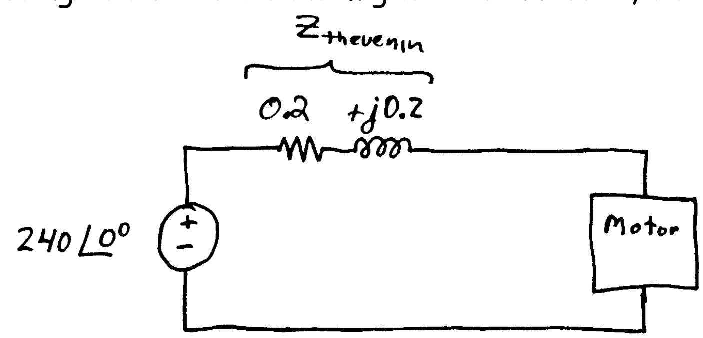
P17.50* Under full load, the current of the motor is

$$I_{\text{full}} = \frac{2 \times 746}{0.8(0.75)240} = 10.36 \text{ A}$$

We estimate the starting current as

$$I_{\text{start}} = 6 \times I_{\text{full}} = 62.2 \text{ A}$$

Neglecting other loads that might be connected, the equivalent circuit is



During starting, the voltage across the motor is

$$\mathbf{V}_{\text{motor}} = 240 - (0.2 + j0.2) \times 62.2\angle\theta_{\text{start}}$$

The phase angle of the starting current is unknown. However, we can anticipate that the motor appears inductive and θ_{start} is negative. In the worst case, we would have

$$\theta_{\text{start}} = -\tan^{-1}(0.2/0.2) = -45^\circ$$

Then, voltage drops from 240 V to $240 - \sqrt{(0.2)^2 + (0.2)^2} \times 62.2 =$ 222.4 V when the motor starts. The percentage drop in voltage is 7.33%. This would cause a noticeable momentary dimming of the lights in the farm house.

P17.51 The phase angle of $\mathbf{I}_m$ is $\theta_m = -\arctan\frac{8}{6} = -53.13^\circ$.

Thus, the phase of $\mathbf{I}_a$ needs to be $\theta_m + 90^\circ = 36.87^\circ$.

The imaginary part of Z_a is

$$9 - \frac{1}{\omega C} = 12\tan\theta_a = -9$$

Of course, $\omega = 2\pi 50$, and we find

$$C = 176.88\ \mu\text{F}$$

P17.52 At full load, the slip is

$$s_{\text{full}} = (3600 - 3500)/3600 = 2.778\%$$

At no load, the slip is

$$s_{no} = (3600 - 3595)/3600 = 0.1388\%$$

Thus, we can write

$$0.02778K_1 - K_2 = 0.5$$

$$0.001388K_1 - K_2 = 0$$

Solving, we find $K_1 = 18.95$ and $K_2 = 0.02630$.

Now, for 0.2 hp out

$$18.95s - 0.02630 = 0.2$$

Finally, the speed for 0.2 hp out is

$$n = 3600(1 - s) = 3557 \text{ rpm}$$

P17.53 To reverse the direction of rotation, reverse the connections to either the main winding or the starting winding (but not both).

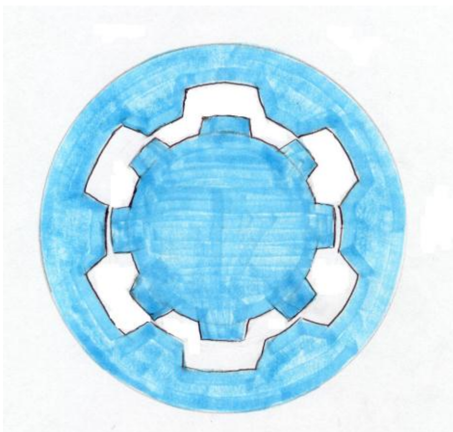
P17.54 A universal motor would be better than an induction motor in a portable vacuum cleaner because the universal motor gives a higher power to weight ratio.

An induction motor would be the better choice for a fan in a heating system because the life of a universal motor is relatively short compared to that of an induction motor.

The induction motor would be better for a refrigerator compressor, again because of longer service life.

For a variable speed hand-held drill, we should choose the universal motor because it gives higher power for a given weight.

P17.55 A sketch of a stepper motor cross section with 6 stator poles and 8 rotor poles is:



The rotation is 15° per step.

P17.56 The various manufacturers of stepper motors and BLDC are:-

1. Powertec Motors
2. Crompton and Greaves

P17.57 Stepper Motors can be of the following Types

1. Variable Reluctance Stepper Motor
2. Permanent-Magnet Stepper Motor
3. Hybrid Stepper Motor

Brushless dc motors are essentially permanent-magnet stepping motors equipped with position sensors (either Hall effect or optical) and enhanced control units. As in the stepper motor, power is applied to one stator winding at a time. When the position sensor indicates that the rotor has approached alignment with the stator field, the controller electronically switches power to the next stator winding so that smooth motion continues. By varying the amplitude and duration of the pulses applied to the stator windings, speed can be readily controlled. The result is a motor that can operate from a dc source with characteristics similar to those of a conventional shunt dc motor.

Brushless dc motors are used primarily in low---power applications. Their advantages include relatively high efficiency, long service life with little maintenance, freedom from radio interference, ability to operate in explosive chemical environments, and capability for very high speeds (50,000 rpm or more).

Practice Test

T17.1 (a) The magnetic field set up in the air gap of a four-pole three-phase induction motor consists of four magnetic poles spaced 90° from one another in alternating order (i.e., north-south-north-south). The field points from the stator toward the rotor under the north poles and in the opposite direction under the south poles. The poles rotate with time at synchronous speed around the axis of the motor.

(b) The air gap flux density of a two-pole machine is given by Equation 17.12 in the book:

$$B_{gap} = B_m \cos(\omega t - \theta)$$

in which B_m is the peak field intensity, ω is the angular frequency of the three-phase source, and θ is angular displacement around the air gap. This describes a field having two poles: a north pole corresponding to $\omega t - \theta = 0$ and a south pole corresponding to $\omega t - \theta = \pi$. The location of either pole moves around the gap at an angular speed of $\omega_s = \omega$.

For a four pole machine, the field has four poles rotating at an angular speed of $\omega_s = \omega/2$ and is given by

$$B_{gap} = B_m \cos(\omega t - \theta / 2)$$

in which B_m is the peak field intensity, ω is the angular frequency of the three-phase source, and θ denotes angular position around the gap.

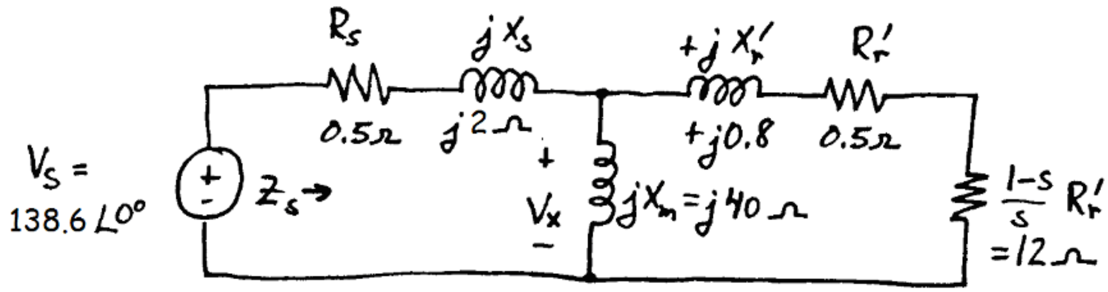
T17.2 Five of the most important characteristics for an induction motor are:

1. Nearly unity power factor.
2. High starting torque.
3. Close to 100% efficiency.
4. Low starting current.
5. High pull-out torque.

T17.3 An eight-pole 60-Hz machine has a synchronous speed of $n_s = 900$ rpm, and the slip is:

$$s = \frac{n_s - n_m}{n_s} = \frac{900 - 864}{900} = 0.04$$

Because the machine is wye connected, the phase voltage is $240/\sqrt{3} = 138.6$ V. (We assume zero phase for this voltage.)The per phase equivalent circuit is:



Then, we have

$$Z_s = 0.5 + j2 + \frac{j40(0.5 + 12 + j0.8)}{j40 + 0.5 + 12 + j0.8}$$
$$= 11.48 + j6.149 \quad \Omega$$
$$= 13.03\angle 28.17^\circ \quad \Omega$$

$$\text{power factor} = \cos(28.17^\circ) = 88.16\%\ \text{lagging}$$

$$\mathbf{I}_s = \frac{\mathbf{V}_s}{Z_s} = \frac{138.6\angle 0^\circ}{13.03\angle 28.17^\circ} = 10.64\angle -28.17^\circ \quad \text{A rms}$$

$$P_{in} = 3I_s V_s \cos\theta = 3.898 \text{ kW}$$

For a wye-connected motor, the phase current and line current are the same. Thus, the line current magnitude is 10.64 A rms.

Next, we compute $\mathbf{V}_x$ and $\mathbf{I}_r'$.

$$\mathbf{V}_x = \mathbf{I}_s \frac{j40(0.5 + 12 + j0.8)}{j40 + 0.5 + 12 + j0.8}$$
$$= 123.83 \ - j16.244$$
$$= 124.9\angle -7.473^\circ$$

$$\mathbf{I}_r' = \frac{\mathbf{V}_x}{j0.8 + 0.5 + 12}$$
$$= 9.971\angle -11.14^\circ$$

The copper losses in the stator and rotor are:

$$\begin{aligned} P_s &= 3R_s I_s^2 \\ &= 3(0.5)(10.64)^2 \\ &= 169.7\ \text{W} \end{aligned}$$

and

$$\begin{aligned} P_r &= 3R_r'(I_r')^2 \\ &= 3(0.5)(9.971)^2 \\ &= 149.1\ \text{W} \end{aligned}$$

Finally, the developed power is:

$$\begin{aligned} P_{\text{dev}} &= 3 \times \frac{1-s}{s} R_r'(I_r')^2 \\ &= 3(12)(9.971)^2 \\ &= 3.579\ \text{kW} \end{aligned}$$

$$P_{\text{out}} = P_{\text{dev}} - P_{\text{rot}} = 3.429\ \text{kW}$$

The output torque is:

$$T_{\text{out}} = \frac{P_{\text{out}}}{\omega_m} = 37.90\ \text{newton meters}$$

The efficiency is:

$$\eta = \frac{P_{\text{out}}}{P_{\text{in}}} \times 100\% = 87.97\%$$

T17.4 At 60 Hz, synchronous speed for an eight-pole machine is:

$$n_s = \frac{120f}{P} = \frac{120(60)}{8} = 900\ \text{rpm}$$

The slip is given by:

$$s = \frac{n_s - n_m}{n_s} = \frac{900 - 850}{900} = 5.56\%$$

The frequency of the rotor currents is the slip frequency. From Equation 17.17, we have $\omega_{\text{slip}} = s\omega$. For frequencies in the Hz, this becomes:

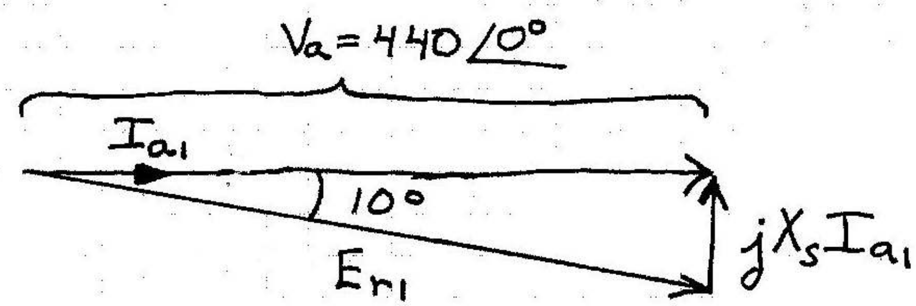
$$f_{\text{slip}} = sf = 0.05555 \times 60 = 3.333\ \text{Hz}$$

In the normal range of operation, slip is approximately proportional to output power and torque. Thus at 80% of full power, we estimate that $s = 0.8 \times 0.05555 = 0.04444$. This corresponds to a speed of 860 rpm.

T17.5 The stator of a six-pole synchronous motor contains a set of windings (collectively known as the armature) that are energized by a three-phase ac source. These windings produce six magnetic poles spaced 60° from one another in alternating order (i.e., north-south-north-south-north-south). The field points from the stator toward the rotor under the north stator poles and in the opposite direction under the south stator poles. The poles rotate with time at synchronous speed (1200 rpm) around the axis of the motor.

The rotor contains windings that carry dc currents and set up six north and south magnetic poles evenly spaced around the rotor. When driving a load, the rotor spins at synchronous speed with the north poles of the rotor lagging slightly behind and attracted by the south poles of the stator. (In some cases, the rotor may be composed of permanent magnets.)

T17.6 Figure 17.22 in the book shows typical phasor diagrams with constant developed power and variable field current. The phasor diagram for the initial operating conditions is:



Notice that because the initial power factor is unity, we have $\theta_1 = 0°$ and $\mathbf{I}_{a1}$ is in phase with $\mathbf{V}_a$. Also, notice that $jX_s\mathbf{I}_{a1}$ is at right angles to $\mathbf{I}_{a1}$. Now, we can calculate the magnitudes of $\mathbf{E}_{r1}$ and of $X_s\mathbf{I}_{a1}$.

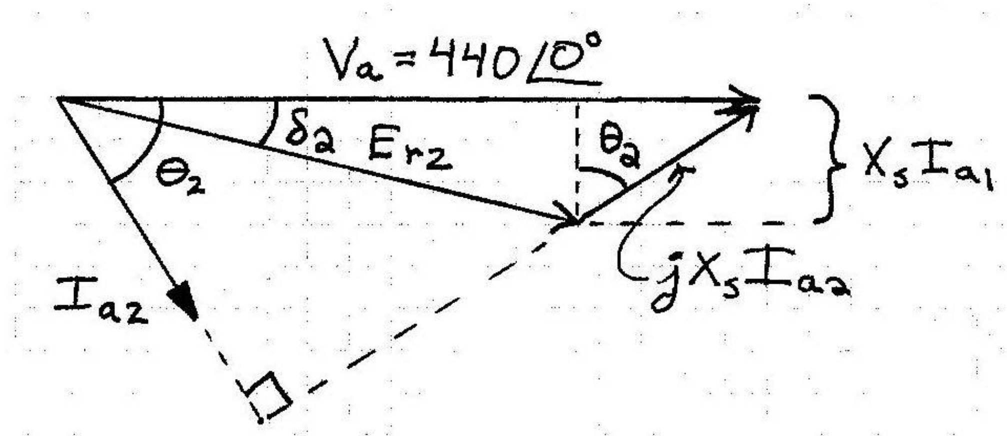
$$E_{r1} = \frac{V_a}{\cos\delta_1} = \frac{440}{\cos(10^\circ)} = 446.79 \text{ V}$$

$$X_s I_{a1} = E_{r1}\sin\delta_1 = 77.58 \text{ V}$$

Then, the field current is reduced until the magnitude of $\mathbf{E}_{r2}$ is 75% of its initial value.

$$E_{r2} = 0.75 \times E_{r1} = 335.09 \text{ V}$$

The phasor diagram for the second operating condition is:



Because the torque and power are constant, the vertical component of $jX_s\mathbf{I}_a$ is the same in both diagrams as illustrated in Figure 17.22 in the book. Thus, we have:

$$\sin\delta_2 = \frac{X_s I_{a1}}{E_{r2}} = \frac{77.58}{335.09}$$

which yields:

$$\delta_2 = 13.39^\circ$$

(Another solution to the equation is $\delta_2 = 166.61^\circ$, but this does not correspond to a stable operating point.)

Now, we can write:

$$(V_a - X_s I_{a1}\tan\theta_2)^2 + (X_s I_{a1})^2 = (E_{r2})^2$$

$$(440 - 77.58\tan\theta_2)^2 + (77.58)^2 = (335.09)^2$$

Solving, we find $\theta_2 = 55.76^\circ$, and the power factor is $\cos\theta_2 = 56.25\%$ lagging.