CHAPTER 1

Exercises

E1.1 Charge = Current $\times$ Time = (2 *A*) $\times$ (10 s) = 20 *C*

E1.2 $i(t) = \dfrac{dq(t)}{dt} = \dfrac{d}{dt}(0.01\sin(200t) = 0.01 \times 200\cos(200t) = 2\cos(200t)\ \ A$

E1.3 Because i_2 has a positive value, positive charge moves in the same direction as the reference. Thus, positive charge moves downward in element *C*.

Because i_3 has a negative value, positive charge moves in the opposite direction to the reference. Thus positive charge moves upward in element *E*.

E1.4 Energy = Charge $\times$ Voltage = (2 *C*) $\times$ (20 V) = 40 J

Because v_{ab} is positive, the positive terminal is *a* and the negative terminal is *b*. Thus the charge moves from the negative terminal to the positive terminal, and energy is removed from the circuit element.

E1.5 i_{ab} enters terminal *a*. Furthermore, v_{ab} is positive at terminal *a*. Thus the current enters the positive reference, and we have the passive reference configuration.

E1.6 (a) $p_a(t) = v_a(t)i_a(t) = 20t^2$

$$w_a = \int_0^{10} p_a(t)dt = \int_0^{10} 20t^2 dt = \left.\frac{20t^3}{3}\right|_0^{10} = \frac{20t^3}{3} = 6667 \text{ J}$$

(b) Notice that the references are opposite to the passive sign convention. Thus we have:

$$p_b(t) = -v_b(t)i_b(t) = 20t - 200$$

$$w_b = \int_0^{10} p_b(t)dt = \int_0^{10} (20t - 200)dt = 10t^2 - 200t\Big|_0^{10} = -1000 \text{ J}$$

E1.7 (a) Sum of currents leaving = Sum of currents entering

$i_a = 1 + 3 = 4$ A

(b) $2 = 1 + 3 + i_b \quad \Rightarrow \quad i_b = -2$ A

(c) $0 = 1 + i_c + 4 + 3 \quad \Rightarrow \quad i_c = -8$ A

E1.8 Elements *A* and *B* are in series. Also, elements *E*, *F*, and *G* are in series.

E1.9 Go clockwise around the loop consisting of elements *A*, *B*, and *C*:

$-3 - 5 + v_c = 0 \quad \Rightarrow \quad v_c = 8$ V

Then go clockwise around the loop composed of elements *C*, *D* and *E*:

$-v_c - (-10) + v_e = 0 \quad \Rightarrow \quad v_e = -2$ V

E1.10 Elements *E* and *F* are in parallel; elements *A* and *B* are in series.

E1.11 The resistance of a wire is given by $R = \dfrac{\rho L}{A}$. Using $A = \pi d^2/4$ and substituting values, we have:

$$9.6 = \frac{1.12 \times 10^{-6} \times L}{\pi(1.6 \times 10^{-3})^2/4} \quad \Rightarrow \quad L = 17.2 \text{ m}$$

E1.12 $P = V^2/R \quad \Rightarrow \quad R = V^2/P = 144\,\Omega \quad \Rightarrow \quad I = V/R = 120/144 = 0.833\text{ A}$

E1.13 $P = V^2/R \quad \Rightarrow \quad V = \sqrt{PR} = \sqrt{0.25 \times 1000} = 15.8\text{ V}$

$I = V/R = 15.8/1000 = 15.8\text{ mA}$

E1.14 Using KCL at the top node of the circuit, we have $i_1 = i_2$. Then, using KVL going clockwise, we have $-v_1 - v_2 = 0$; but $v_1 = 25$ V, so we have $v_2 = -25$ V. Next we have $i_1 = i_2 = v_2/R = -1$ A. Finally, we have $P_R = v_2 i_2 = (-25) \times (-1) = 25$ W and $P_s = v_1 i_1 = (25) \times (-1) = -25$ W.

E1.15 At the top node we have $i_R = i_s = 2$A. By Ohm's law we have $v_R = Ri_R = 80$ V. By KVL we have $v_s = v_R = 80$ V. Then $p_s = -v_s i_s = -160$ W (the minus sign is due to the fact that the references for v_s and i_s are opposite to the passive sign configuration). Also we have $P_R = v_R i_R = 160$ W.

Answers for Selected Problems

P1.7* Electrons are moving in the reference direction (i.e., from a to b).

$Q = 9\ C$

P1.9* $i(t) = 2 + 2t\ A$

P1.12* $Q = 2$ coulombs

P1.14* (a) $h = 17.6$ km
(b) $v = 587.9$ m/s
(c) The energy density of the battery is 172.8×10^3 J/kg
which is about 0.384% of the energy density of gasoline.

P1.17* $Q = 3.6 \times 10^5$ coulombs

$\text{Energy} = 4.536 \times 10^6$ joules

P1.20* (a) 30 W absorbed
(b) 30 W absorbed
(c) 60 W supplied

P1.22* $Q = 50\ C$. Electrons move from b to a.

P1.24* $\text{Energy} = 500$ kWh

$P = 694.4$ W $\quad I = 5.787\ A$

$\text{Reduction} = 8.64\%$

P1.27* (a) $P =$ 50 W taken from element A.
(b) $P =$ 50 W taken from element A.
(c) $P = 50$ W delivered to element A.

P1.34* Elements *E* and *F* are in series.

P1.36* $i_a = -2$ A. $i_c = 1$ A. $i_d = 4$ A. Elements *A* and B are in series.

P1.37*

$i_c = 1\text{ A}$ $\quad$ $i_e = 5\text{ A}$

$i_f = -3\text{ A}$ $\quad$ $i_g = -7\text{ A}$

P1.41* $v_a = -5$ V. $v_c = 10$ V. $v_b = -5$ V.

P1.42*

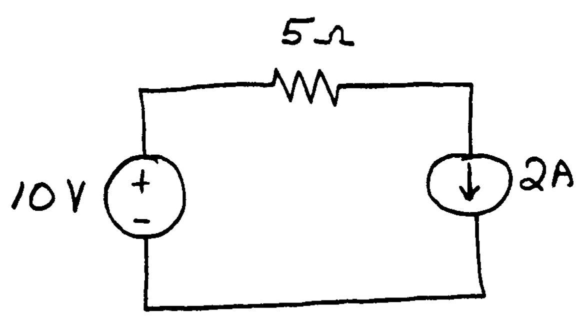
$i_c = 1\text{ A}$ $\quad$ $i_b = -2\text{ A}$

$v_b = -6\text{ V}$ $\quad$ $v_c = 4\text{ V}$

$P_A = -20\text{ W}$ $\quad$ $P_B = 12\text{ W}$

$P_C = 4\text{ W}$ $\quad$ $P_D = 4\text{ W}$

P1.52*



P1.58* $R = 100\ \Omega$; 19% reduction in power

P1.62* (a) Not contradictory.

(b) A 2-A current source in series with a 3-A current source is contradictory.

(c) Not contradictory.

(d) A 2-A current source in series with an open circuit is contradictory.

(e) A 5-V voltage source in parallel with a short circuit is contradictory.

P1.63* $i_R = 2A$

$P_{current-source} = -40$ W. Thus, the current source delivers power.

$P_R = 20$ W. The resistor absorbs power.

$P_{voltage-source} = 20$ W. The voltage source absorbs power.

P1.64* $v_x = 17.5$ V

P1.69* (a) $v_x = 10/6 = 1.667$ V

(b) $i_x = 0.5556$ A

(c) $P_{voltage-source} = -10i_x = -5.556$ W. (This represents power delivered by the voltage source.)

$P_R = 3(i_x)^2 = 0.926$ W (absorbed)

$P_{controlled-source} = 5v_x i_x = 4.63$ W (absorbed)

P1.70* The circuit contains a voltage-controlled current source. $v_s = 15$ V

Practice Test

T1.1 (a) 4; (b) 7; (c) 16; (d) 18; (e) 1; (f) 2; (g) 8; (h) 3; (i) 5; (j) 15; (k) 6; (l) 11;

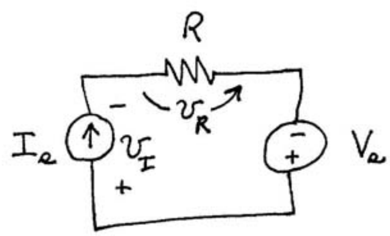
(m) 13; (n) 9; (o) 14.

T1.2 (a) The current $I_s = 3$ A circulates clockwise through the elements entering the resistance at the negative reference for v_R. Thus, we have $v_R = -I_s R = -6$ V.

(b) Because I_s enters the negative reference for V_s, we have $P_V = -V_s I_s = -30$ W. Because the result is negative, the voltage source is delivering energy.

(c) The circuit has three nodes, one on each of the top corners and one along the bottom of the circuit.

(d) First, we must find the voltage v_I across the current source. We choose the reference shown:



Then, going around the circuit counterclockwise, we have $-v_I + V_s + v_R = 0$, which yields $v_I = V_s + v_R = 10 - 6 = 4$ V. Next, the power for the current source is $P_I = I_s v_I = 12$ W. Because the result is positive, the current source is absorbing energy.

Alternatively, we could compute the power delivered to the resistor as $P_R = I_s^2 R = 18$ W. Then, because we must have a total power of zero for the entire circuit, we have $P_I = -P_V - P_R = 30 - 18 = 12$ W.

T1.3 (a) The currents flowing downward through the resistances are v_{ab}/R_1 and v_{ab}/R_2. Then, the KCL equation for node *a* (or node *b*) is

$$I_2 = I_1 + \frac{v_{ab}}{R_1} + \frac{v_{ab}}{R_2}$$

Substituting the values given in the question and solving yields $v_{ab} = -8$ V.
(b) The power for current source I_1 is $P_{I1} = v_{ab} I_1 = -8 \times 3 = -24$ W. Because the result is negative we know that energy is supplied by this current source.
The power for current source I_2 is $P_{I2} = -v_{ab} I_2 = 8 \times 1 = 8$ W. Because the result is positive, we know that energy is absorbed by this current source.
(c) The power absorbed by R_1 is $P_{R1} = v_{ab}^2 / R_1 = (-8)^2 / 12 = 5.33$ W. The power absorbed by R_2 is $P_{R2} = v_{ab}^2 / R_2 = (-8)^2 / 6 = 10.67$ W.

T1.4 (a) Applying KVL, we have $-V_s + v_1 + v_2 = 0$. Substituting values given in the problem and solving we find $v_1 = 8$ V.
(b) Then applying Ohm's law, we have $i = v_1 / R_1 = 8 / 4 = 2$ A.
(c) Again applying Ohm's law, we have $R_2 = v_2 / i = 4 / 2 = 2\ \Omega$.

T1.5 Applying KVL, we have $-V_s + v_x = 0$. Thus, $v_x = V_s = 15$ V. Next Ohm's law gives $i_x = v_x / R = 15/10 = 1.5$ A. Finally, KCL yields
$i_{sc} = i_x - av_x = 1.5 - 0.3 \times 15 = -3$ A.

CHAPTER 2

Exercises

E2.1 (a) R_2, R_3, and R_4 are in parallel. Furthermore R_1 is in series with the combination of the other resistors. Thus we have:

$$R_{eq} = R_1 + \frac{1}{1/R_2 + 1/R_3 + 1/R_4} = 3\,\Omega$$

(b) R_3 and R_4 are in parallel. Furthermore, R_2 is in series with the combination of R_3, and R_4. Finally R_1 is in parallel with the combination of the other resistors. Thus we have:

$$R_{eq} = \frac{1}{1/R_1 + 1/[R_2 + 1/(1/R_3 + 1/R_4)]} = 5\,\Omega$$

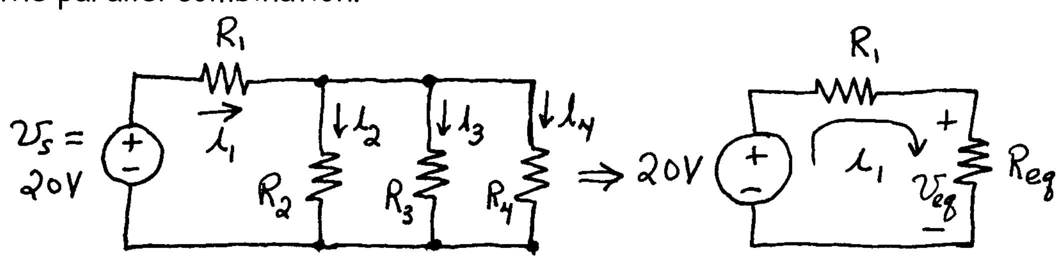
(c) R_1 and R_2 are in parallel. Furthermore, R_3, and R_4 are in parallel. Finally, the two parallel combinations are in series.

$$R_{eq} = \frac{1}{1/R_1 + 1/R_2} + \frac{1}{1/R_3 + 1/R_4} = 52.1\,\Omega$$

(d) R_1 and R_2 are in series. Furthermore, R_3 is in parallel with the series combination of R_1 and R_2.

$$R_{eq} = \frac{1}{1/R_3 + 1/(R_1 + R_2)} = 1.5\text{ k}\Omega$$

E2.2 (a) First we combine R_2, R_3, and R_4 in parallel. Then R_1 is in series with the parallel combination.

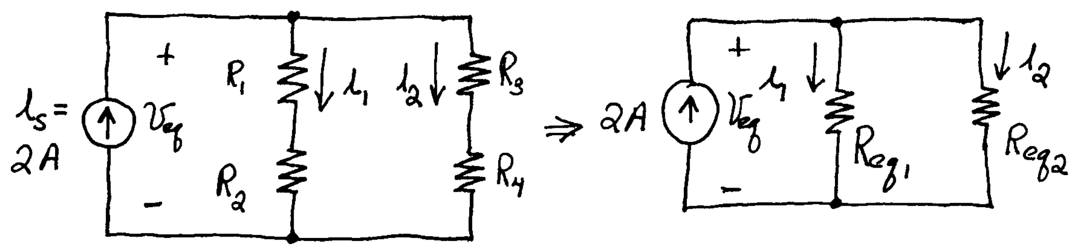


$$R_{eq} = \frac{1}{1/R_2 + 1/R_3 + 1/R_4} = 9.231\,\Omega \qquad i_1 = \frac{20\text{ V}}{R_1 + R_{eq}} = \frac{20}{10 + 9.231} = 1.04\text{ A}$$

$$v_{eq} = R_{eq} i_1 = 9.600\text{ V} \qquad i_2 = v_{eq}/R_2 = 0.480\text{ A} \qquad i_3 = v_{eq}/R_3 = 0.320\text{ A}$$

$$i_4 = v_{eq}/R_4 = 0.240\text{ A}$$

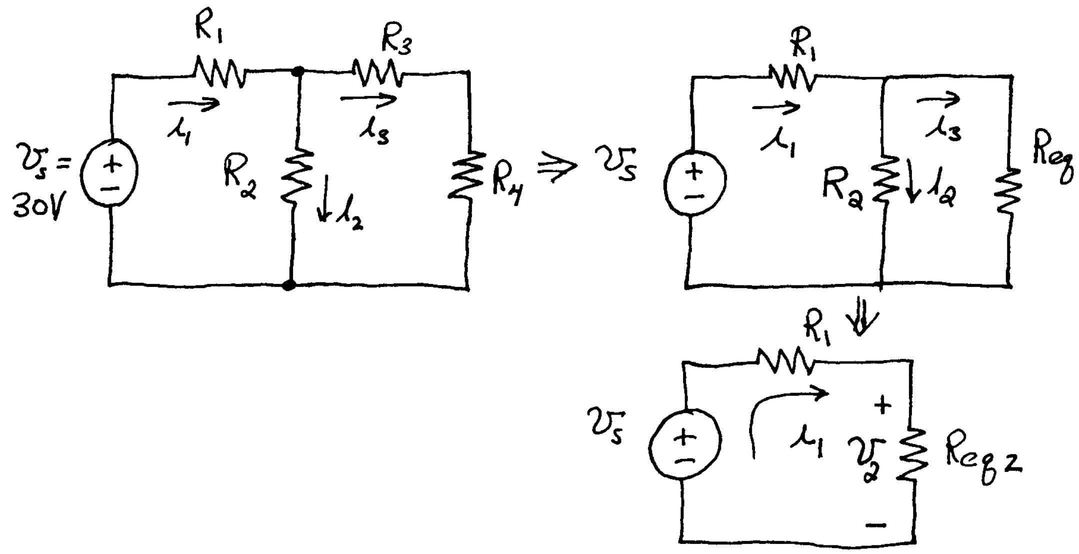
(b) R_1 and R_2 are in series. Furthermore, R_3, and R_4 are in series. Finally, the two series combinations are in parallel.



$$R_{eq1} = R_1 + R_2 = 20\,\Omega \qquad R_{eq2} = R_3 + R_4 = 20\,\Omega \quad R_{eq} = \frac{1}{1/R_{eq1} + 1/R_{eq2}} = 10\,\Omega$$

$$v_{eq} = 2 \times R_{eq} = 20\text{ V} \quad i_1 = v_{eq} / R_{eq1} = 1\,A \quad i_2 = v_{eq} / R_{eq2} = 1\,A$$

(c) R_3, and R_4 are in series. The combination of R_3 and R_4 is in parallel with R_2. Finally the combination of R_2, R_3, and R_4 is in series with R_1.



$$R_{eq1} = R_3 + R_4 = 40\,\Omega \quad R_{eq2} = \frac{1}{1/R_{eq1} + 1/R_2} = 20\,\Omega \quad i_1 = \frac{v_s}{R_1 + R_{eq2}} = 1\,A$$

$$v_2 = i_1 R_{eq2} = 20\text{ V} \quad i_2 = v_2 / R_2 = 0.5\,A \quad i_3 = v_2 / R_{eq1} = 0.5\,A$$

E2.3 (a) $v_1 = v_s \dfrac{R_1}{R_1 + R_2 + R_3 + R_4} = 10\text{ V}$. $v_2 = v_s \dfrac{R_2}{R_1 + R_2 + R_3 + R_4} = 20\text{ V}$.

Similarly, we find $v_3 = 30\text{ V}$ and $v_4 = 60\text{ V}$.

(b) First combine R_2 and R_3 in parallel: $R_{eq} = 1/(1/R_2 + 1/R_3) = 2.917\ \Omega$.
Then we have $v_1 = v_s \dfrac{R_1}{R_1 + R_{eq} + R_4} = 6.05\text{ V}$. Similarly, we find

$v_2 = v_s \dfrac{R_{eq}}{R_1 + R_{eq} + R_4} = 5.88\text{ V}$ and $v_4 = 8.07\text{ V}$.

E2.4 (a) First combine R_1 and R_2 in series: $R_{eq} = R_1 + R_2 = 30\ \Omega$. Then we have $i_1 = i_s \dfrac{R_3}{R_3 + R_{eq}} = \dfrac{15}{15+30} = 1\ A$ and $i_3 = i_s \dfrac{R_{eq}}{R_3 + R_{eq}} = \dfrac{30}{15+30} = 2\ A$.

(b) The current division principle applies to two resistances in parallel. Therefore, to determine i_1, first combine R_2 and R_3 in parallel: $R_{eq} = 1/(1/R_2 + 1/R_3) = 5\ \Omega$. Then we have $i_1 = i_s \dfrac{R_{eq}}{R_1 + R_{eq}} = \dfrac{5}{10+5} = 1\ A$.

Similarly, $i_2 = 1\ A$ and $i_3 = 1\ A$.

E2.5 Write KVL for the loop consisting of v_1, v_y, and v_2. The result is $-v_1 - v_y + v_2 = 0$ from which we obtain $v_y = v_2 - v_1$. Similarly we obtain $v_z = v_3 - v_1$.

E2.6 Node 1: $\dfrac{v_1 - v_3}{R_1} + \dfrac{v_1 - v_2}{R_2} = i_a$ Node 2: $\dfrac{v_2 - v_1}{R_2} + \dfrac{v_2}{R_3} + \dfrac{v_2 - v_3}{R_4} = 0$

Node 3: $\dfrac{v_3}{R_5} + \dfrac{v_3 - v_2}{R_4} + \dfrac{v_3 - v_1}{R_1} + i_b = 0$

E2.7 Following the step-by-step method in the book, we obtain

$$\begin{bmatrix} \dfrac{1}{R_1} + \dfrac{1}{R_2} & -\dfrac{1}{R_2} & 0 \\ -\dfrac{1}{R_2} & \dfrac{1}{R_2} + \dfrac{1}{R_3} + \dfrac{1}{R_4} & -\dfrac{1}{R_4} \\ 0 & -\dfrac{1}{R_4} & \dfrac{1}{R_4} + \dfrac{1}{R_5} \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} = \begin{bmatrix} -i_s \\ 0 \\ i_s \end{bmatrix}$$

E2.8 Instructions for various calculators vary. The MATLAB solution is given in the book following this exercise.

E2.9 (a) Writing the node equations we obtain:

Node 1: $\frac{v_1 - v_3}{20} + \frac{v_1}{5} + \frac{v_1 - v_2}{10} = 0$

Node 2: $\frac{v_2 - v_1}{10} + 10 + \frac{v_2 - v_3}{5} = 0$

Node 3: $\frac{v_3 - v_1}{20} + \frac{v_3}{10} + \frac{v_3 - v_2}{5} = 0$

(b) Simplifying the equations we obtain:

$$0.35v_1 - 0.10v_2 - 0.05v_3 = 0$$
$$-0.10v_1 + 0.30v_2 - 0.20v_3 = -10$$
$$-0.05v_1 - 0.20v_2 + 0.35v_3 = 0$$

(c) and (d) Solving using Matlab:

```
>>clear
>>G = [0.35 -0.1 -0.05; -0.10 0.30 -0.20; -0.05 -0.20 0.35];
>>I = [0; -10; 0];
>>V = G\I
V =
  -27.2727
  -72.7273
  -45.4545
>>Ix = (V(1) - V(3))/20
Ix =
   0.9091
```

E2.10 Using determinants we can solve for the unknown voltages as follows:

$$v_1 = \frac{\begin{vmatrix} 6 & -0.2 \\ 1 & 0.5 \end{vmatrix}}{\begin{vmatrix} 0.7 & -0.2 \\ -0.2 & 0.5 \end{vmatrix}} = \frac{3 + 0.2}{0.35 - 0.04} = 10.32\text{ V}$$

$$v_2 = \frac{\begin{vmatrix} 0.7 & 6 \\ -0.2 & 1 \end{vmatrix}}{\begin{vmatrix} 0.7 & -0.2 \\ -0.2 & 0.5 \end{vmatrix}} = \frac{0.7 + 1.2}{0.35 - 0.04} = 6.129\text{ V}$$

Many other methods exist for solving linear equations.

E2.11 First write KCL equations at nodes 1 and 2:

$$\text{Node 1:}\quad \frac{v_1 - 10}{2} + \frac{v_1}{5} + \frac{v_1 - v_2}{10} = 0$$

$$\text{Node 2:}\quad \frac{v_2 - 10}{10} + \frac{v_2}{5} + \frac{v_2 - v_1}{10} = 0$$

Then, simplify the equations to obtain:

$$8v_1 - v_2 = 50 \quad \text{and} \quad -v_1 + 4v_2 = 10$$

Solving manually or with a calculator, we find v_1 = 6.77 V and v_2 = 4.19 V. The MATLAB session using the symbolic approach is:

```
>> clear
[V1,V2] = solve('(V1-10)/2+(V1)/5 +(V1 - V2)/10 = 0' , ...
                '(V2-10)/10 +V2/5 +(V2-V1)/10 = 0')
V1 =
210/31
V2 =
130/31
```

Next, we solve using the numerical approach.

```
>> clear
G = [8 -1; -1 4];
I = [50; 10];
V = G\I
V =
   6.7742
   4.1935
```

E2.12 The equation for the supernode enclosing the 15-V source is:

$$\frac{v_3 - v_2}{R_3} + \frac{v_3 - v_1}{R_1} = \frac{v_1}{R_2} + \frac{v_2}{R_4}$$

This equation can be readily shown to be equivalent to Equation 2.37 in the book. (Keep in mind that v_3 = -15 V.)

E2.13 Write KVL from the reference to node 1 then through the 10-V source to node 2 then back to the reference node:

$$-v_1 + 10 + v_2 = 0$$

Then write KCL equations. First for a supernode enclosing the 10-V source, we have:

$$\frac{v_1}{R_1} + \frac{v_1 - v_3}{R_2} + \frac{v_2 - v_3}{R_3} = 1$$

Node 3:

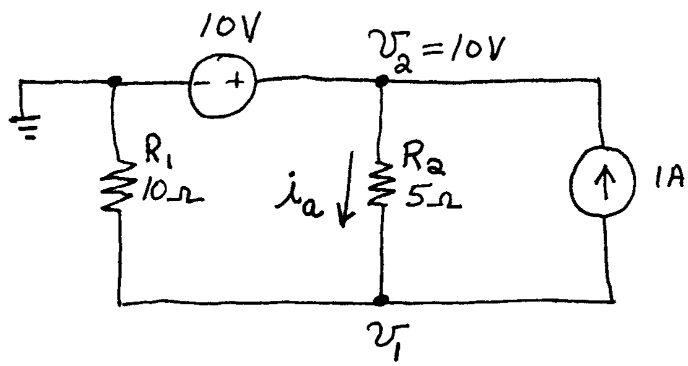
$$\frac{v_3}{R_4} + \frac{v_3 - v_1}{R_2} + \frac{v_3 - v_2}{R_3} = 0$$

Reference node:

$$\frac{v_1}{R_1} + \frac{v_3}{R_4} = 1$$

An independent set consists of the KVL equation and any two of the KCL equations.

E2.14 (a) Select the reference node at the left-hand end of the voltage source as shown at right.



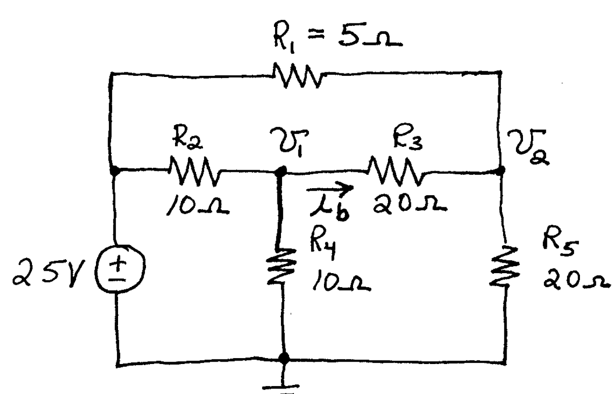
Then write a KCL equation at node 1.

$$\frac{v_1}{R_1} + \frac{v_1 - 10}{R_2} + 1 = 0$$

Substituting values for the resistances and solving, we find v_1 = 3.33 V.

Then we have $i_a = \dfrac{10 - v_1}{R_2} = 1.333$ A.

(b) Select the reference node and assign node voltages as shown.



Then write KCL equations at nodes 1 and 2.

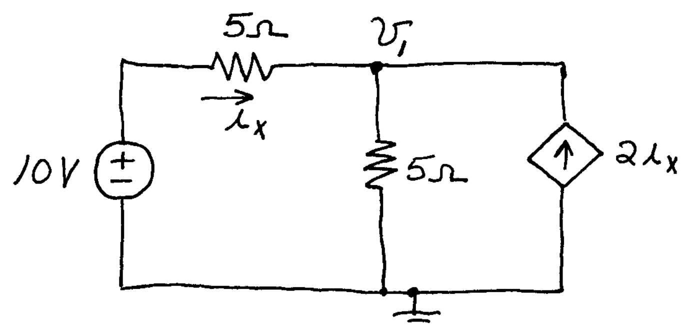
$$\frac{v_1 - 25}{R_2} + \frac{v_1}{R_4} + \frac{v_1 - v_2}{R_3} = 0$$

$$\frac{v_2 - 25}{R_1} + \frac{v_2 - v_1}{R_3} + \frac{v_2}{R_5} = 0$$

Substituting values for the resistances and solving, we find v_1 = 13.79 V and v_2 = 18.97 V. Then we have $i_b = \frac{v_1 - v_2}{R_3}$ = -0.259 A.

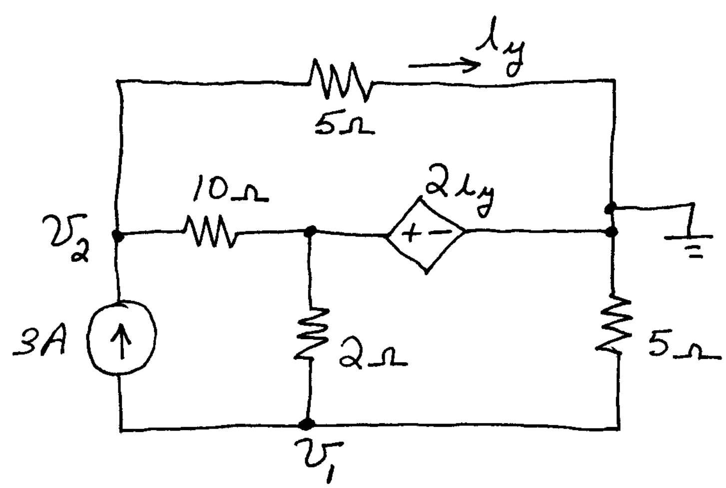
E2.15 (a) Select the reference node and node voltage as shown. Then write a KCL equation at node 1, resulting in

$$\frac{v_1}{5} + \frac{v_1 - 10}{5} - 2i_x = 0$$



Then use $i_x = (10 - v_1)/5$ to substitute and solve. We find v_1 = 7.5 V. Then we have $i_x = \frac{10 - v_1}{5} = 0.5$ A.

(b) Choose the reference node and node voltages shown:



Then write KCL equations at nodes 1 and 2:

$$\frac{v_1}{5} + \frac{v_1 - 2i_y}{2} + 3 = 0 \qquad \frac{v_2}{5} + \frac{v_2 - 2i_y}{10} = 3$$

Finally use $i_y = v_2 / 5$ to substitute and solve. This yields $v_2 = 11.54$ V and $i_y = 2.31$ A.

E2.16
```
>> clear
>> [V1 V2 V3] = solve('V3/R4 + (V3 - V2)/R3 + (V3 - V1)/R1 = 0', ...
                     'V1/R2 + V3/R4 = Is', ...
                     'V1 = (1/2)*(V3 - V1) + V2' ,'V1','V2','V3');
>> pretty(V1), pretty(V2), pretty(V3)

                R2 Is (2 R3 R1 + 3 R4 R1 + 2 R4 R3)
           ------------------------------------------------
           2 R3 R1 + 3 R4 R1 + 3 R1 R2 + 2 R4 R3 + 2 R3 R2

                R2 Is (3 R3 R1 + 3 R4 R1 + 2 R4 R3)
           ------------------------------------------------
           2 R3 R1 + 3 R4 R1 + 3 R1 R2 + 2 R4 R3 + 2 R3 R2

                 Is R2 R4 (3 R1 + 2 R3)
          ------------------------------------------------
          2 R3 R1 + 3 R4 R1 + 3 R1 R2 + 2 R4 R3 + 2 R3 R2
```

E2.17 Refer to Figure 2.33b in the book. (a) Two mesh currents flow through R_2: i_1 flows downward and i_4 flows upward. Thus the current flowing in R_2 referenced upward is i_4 - i_1. (b) Similarly, mesh current i_1 flows to the left through R_4 and mesh current i_2 flows to the right, so the total current referenced to the right is i_2 - i_1. (c) Mesh current i_3 flows downward through R_8 and mesh current i_4 flows upward, so the total current referenced downward is i_3 - i_4. (d) Finally, the total current referenced upward through R_8 is i_4 - i_3.

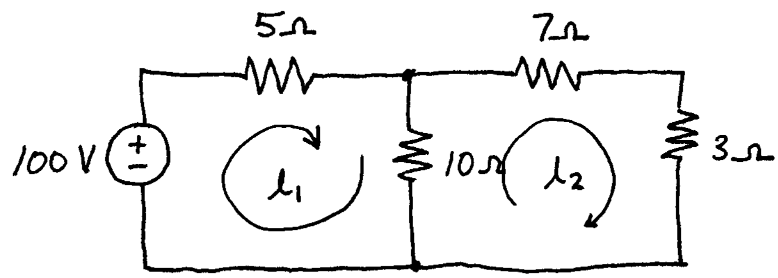
E2.18 Refer to Figure 2.33b in the book. Following each mesh current in turn, we have

$$
\begin{aligned}
&R_1 i_1 + R_2(i_1 - i_4) + R_4(i_1 - i_2) - v_A = 0 \\
&R_5 i_2 + R_4(i_2 - i_1) + R_6(i_2 - i_3) = 0 \\
&R_7 i_3 + R_6(i_3 - i_2) + R_8(i_3 - i_4) = 0 \\
&R_3 i_4 + R_2(i_4 - i_1) + R_8(i_4 - i_3) = 0
\end{aligned}
$$

In matrix form, these equations become

$$\begin{bmatrix} (R_1+R_2+R_4) & -R_4 & 0 & -R_2 \\ -R_4 & (R_4+R_5+R_6) & -R_6 & 0 \\ 0 & -R_6 & (R_6+R_7+R_8) & -R_8 \\ -R_2 & 0 & -R_8 & (R_2+R_3+R_8) \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ i_3 \\ i_4 \end{bmatrix} = \begin{bmatrix} v_A \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

E2.19 We choose the mesh currents as shown:

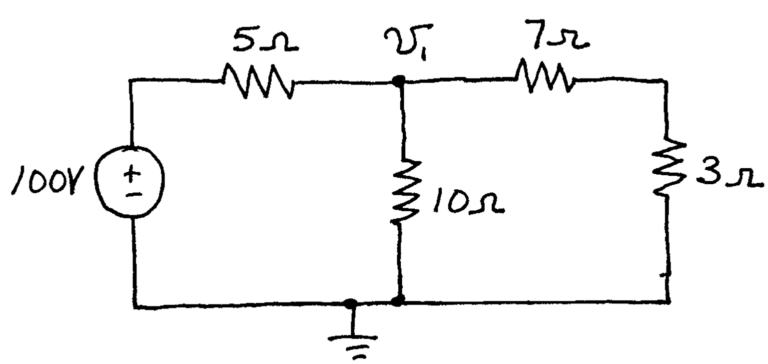


Then, the mesh equations are:

$$5i_1 + 10(i_1 - i_2) = 100 \quad \text{and} \quad 10(i_2 - i_1) + 7i_2 + 3i_2 = 0$$

Simplifying and solving these equations, we find that $i_1 = 10\text{ A}$ and $i_2 = 5\text{ A}$. The net current flowing downward through the 10-Ω resistance is $i_1 - i_2 = 5\text{ A}$.

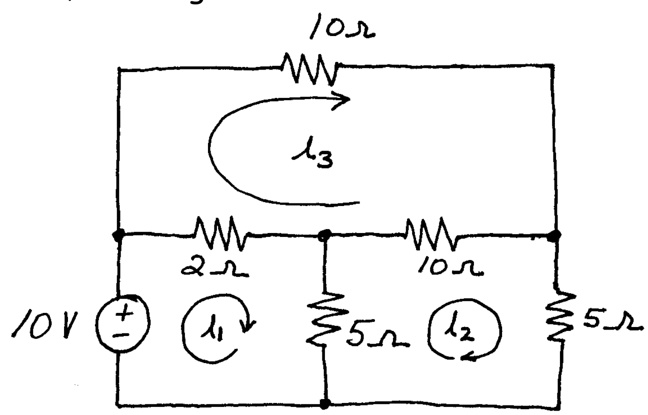
To solve by node voltages, we select the reference node and node voltage shown. (We do not need to assign a node voltage to the connection between the 7-Ω resistance and the 3-Ω resistance because we can treat the series combination as a single 10-Ω resistance.)



The node equation is $(v_1 - 10)/5 + v_1/10 + v_1/10 = 0$. Solving we find that v_1 = 50 V. Thus we again find that the current through the 10-Ω resistance is $i = v_1/10 = 5$ A.

Combining resistances in series and parallel, we find that the resistance "seen" by the voltage source is 10 Ω. Thus the current through the source and 5-Ω resistance is (100 V)/(10 Ω) = 10 A. This current splits equally between the 10-Ω resistance and the series combination of 7 Ω and 3 Ω.

E2.20 First, we assign the mesh currents as shown.



Then we write KVL equations following each mesh current:

$$2(i_1 - i_3) + 5(i_1 - i_2) = 10$$
$$5i_2 + 5(i_2 - i_1) + 10(i_2 - i_3) = 0$$
$$10i_3 + 10(i_3 - i_2) + 2(i_3 - i_1) = 0$$

Simplifying and solving, we find that i_1 = 2.194 A, i_2 = 0.839 A, and i_3 = 0.581 A. Thus the current in the 2-Ω resistance referenced to the right is i_1 - i_3 = 2.194 - 0.581 = 1.613 A.

E2.21 Following the step-by-step process, we obtain

$$\begin{bmatrix} (R_2 + R_3) & -R_3 & -R_2 \\ -R_3 & (R_3 + R_4) & 0 \\ -R_2 & 0 & (R_1 + R_2) \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ i_3 \end{bmatrix} = \begin{bmatrix} v_A \\ -v_B \\ v_B \end{bmatrix}$$

E2.22 Refer to Figure 2.39 in the book. In terms of the mesh currents, the current directed to the right in the 5-A current source is i_1, however by the definition of the current source, the current is 5 A directed to the left. Thus, we conclude that i_1 = -5 A. Then we write a KVL equation following i_2, which results in $10(i_2 - i_1) + 5i_2 = 100$.

E2.23 Refer to Figure 2.40 in the book. First, for the current source, we have

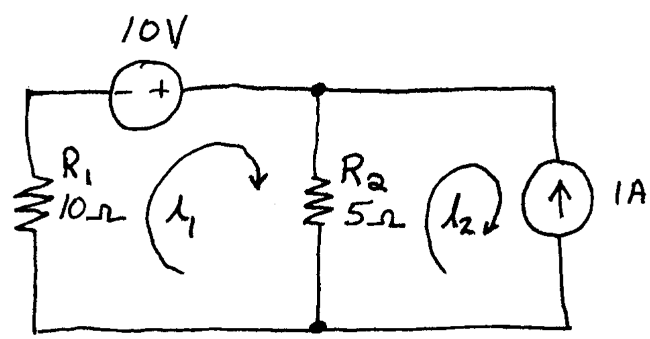
$$i_2 - i_1 = 1$$

Then, we write a KVL equation going around the perimeter of the entire circuit:

$$5i_1 + 10i_2 + 20 - 10 = 0$$

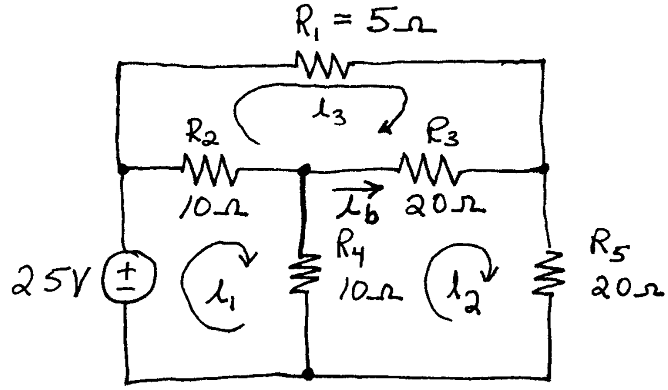
Simplifying and solving these equations we obtain i_1 = -4/3 A and i_2 = -1/3 A.

E2.24 (a) As usual, we select the mesh currents flowing clockwise around the meshes as shown. Then for the current source, we have i_2 = -1 A. This is because we defined the mesh



current i_2 as the current referenced downward through the current source. However, we know that the current through this source is 1 A flowing upward. Next we write a KVL equation around mesh 1: $10i_1 - 10 + 5(i_1 - i_2) = 0$. Solving, we find that i_1 = 1/3 A. Referring to Figure 2.30a in the book we see that the value of the current i_a referenced downward through the 5 Ω resistance is to be found. In terms of the mesh currents, we have $i_a = i_1 - i_2 = 4/3\ A$.

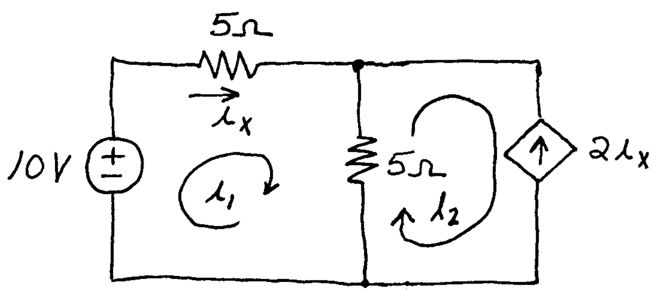
(b) As usual, we select the mesh currents flowing clockwise around the meshes as shown. Then we write a KVL equation for each mesh.



$$-25 + 10(i_1 - i_3) + 10(i_1 - i_2) = 0$$
$$10(i_2 - i_1) + 20(i_2 - i_3) + 20i_2 = 0$$
$$10(i_3 - i_1) + 5i_3 + 20(i_3 - i_2) = 0$$

Simplifying and solving, we find i_1 = 2.3276 A, i_2 = 0.9483 A, and i_3 = 1.2069 A. Finally, we have $i_b = i_2 - i_3$ = -0.2586 A.

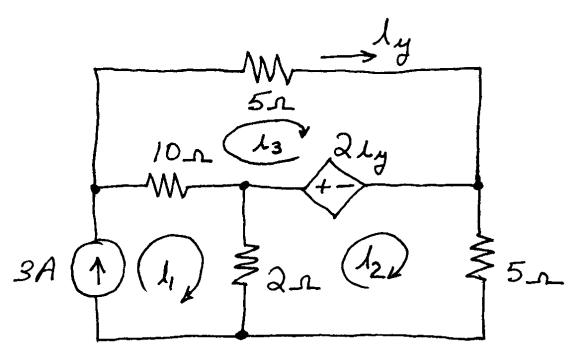
E2.25 (a) KVL mesh 1:
$-10 + 5i_1 + 5(i_1 - i_2) = 0$
For the current source:
$i_2 = -2i_x$
However, i_x and i_1 are the same current, so we also have $i_1 = i_x$.



Simplifying and solving, we find $i_x = i_1 = 0.5$ A.

(b) First for the current source, we have: $i_1 = 3$ A
Writing KVL around meshes 2 and 3, we have:

$$2(i_2 - i_1) + 2i_y + 5i_2 = 0$$
$$10(i_3 - i_1) + 5i_3 - 2i_y = 0$$

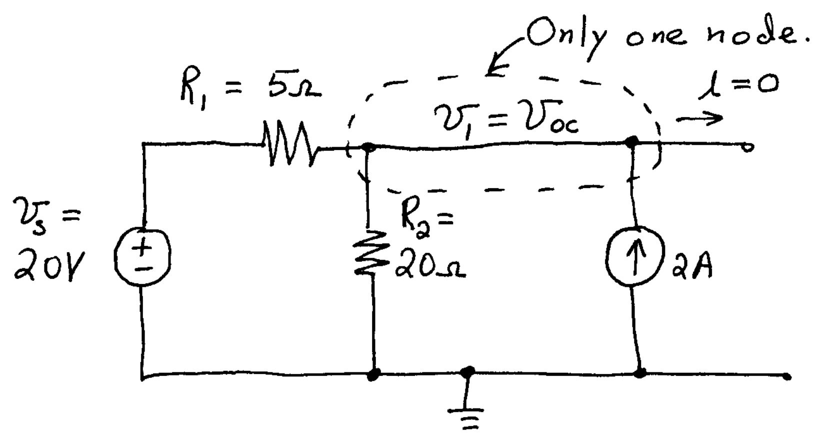


However i_3 and i_y are the same current: $i_y = i_3$. Simplifying and solving, we find that $i_3 = i_y = 2.31$ A.

E2.26 Under open-circuit conditions, 5 A circulates clockwise through the current source and the 10-Ω resistance. The voltage across the 10-Ω resistance is 50 V. No current flows through the 40-Ω resistance so the open circuit voltage is $V_t = 50$ V.

With the output shorted, the 5 A divides between the two resistances in parallel. The short-circuit current is the current through the 40-Ω resistance, which is $i_{sc} = 5\dfrac{10}{10+40} = 1$ A. Then, the Thévenin resistance is $R_t = v_{oc} / i_{sc} = 50\ \Omega$.

E2.27 Choose the reference node at the bottom of the circuit as shown:

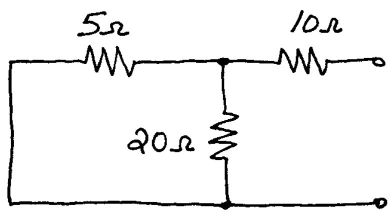


Notice that the node voltage is the open-circuit voltage. Then write a KCL equation:

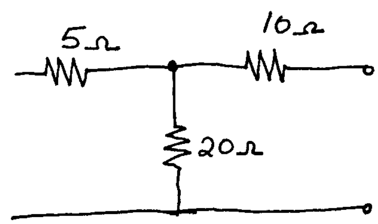
$$\frac{v_{oc} - 20}{5} + \frac{v_{oc}}{20} = 2$$

Solving we find that v_{oc} = 24 V which agrees with the value found in Example 2.17.

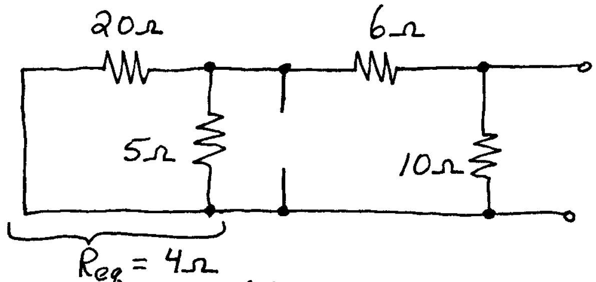
E2.28 To zero the sources, the voltage sources become short circuits and the current sources become open circuits. The resulting circuits are :



(a)



(b)



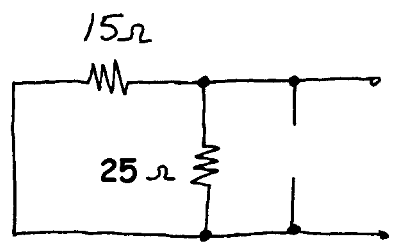
(c)

(a) $R_t = 10 + \dfrac{1}{1/5 + 1/20} = 14\ \Omega$ (b) $R_t = 10 + 20 = 30\ \Omega$

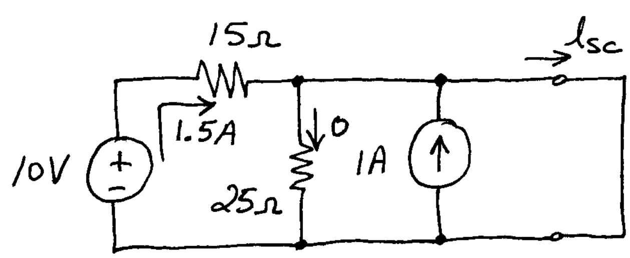
(c) $R_t = \dfrac{1}{\dfrac{1}{10} + \dfrac{1}{6 + \dfrac{1}{(1/5 + 1/20)}}} = 5\ \Omega$

E2.29 (a) Zero sources to determine Thévenin resistance. Thus

$R_t = \dfrac{1}{1/15 + 1/25} = 9.375\ \Omega.$

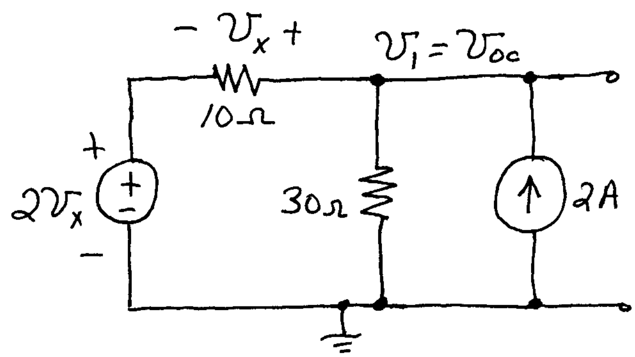


Then find short-circuit current:



$I_n = i_{sc} = 10/15 + 1 = 1.67\ A$

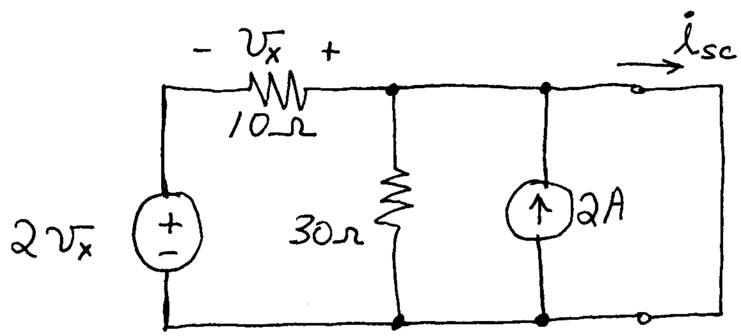
(b) We cannot find the Thévenin resistance by zeroing the sources, because we have a controlled source. Thus, we find the open-circuit voltage and the short-circuit current.



$$\frac{v_{oc} - 2v_x}{10} + \frac{v_{oc}}{30} = 2 \qquad\qquad v_{oc} = 3v_x$$

Solving, we find $V_t = v_{oc} = 30$ V.

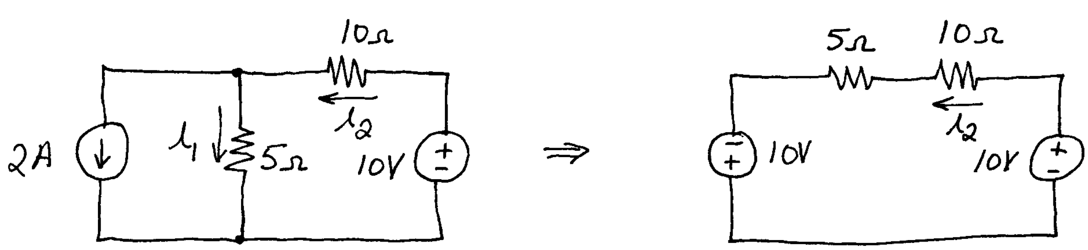
Now, we find the short-circuit current:



$$2v_x + v_x = 0 \qquad \Rightarrow \qquad v_x = 0$$

Therefore $i_{sc} = 2$ A. Then we have $R_t = v_{oc} / i_{sc} = 15\ \Omega$.

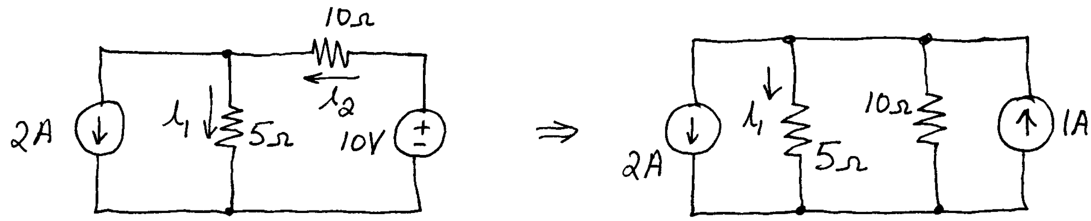
E2.30 First, we transform the 2-A source and the 5-Ω resistance into a voltage source and a series resistance:



Then we have $i_2 = \dfrac{10+10}{15} = 1.333$ A.

From the original circuit, we have $i_1 = i_2 - 2$, from which we find $i_1 = -0.667$ A.

The other approach is to start from the original circuit and transform the 10-Ω resistance and the 10-V voltage source into a current source and parallel resistance:



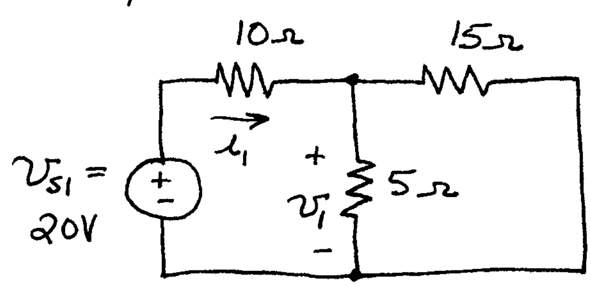
Then we combine the resistances in parallel. $R_{eq} = \dfrac{1}{1/5 + 1/10} = 3.333\ \Omega$.

The current flowing upward through this resistance is 1 A. Thus the voltage across R_{eq} referenced positive at the bottom is 3.333 V and $i_1 = -3.333/5 = -0.667$ A. Then from the original circuit we have $i_2 = 2 + i_1 = 1.333$ A, as before.

E2.31 Refer to Figure 2.62b. We have $i_1 = 15/15 = 1$ A.

Refer to Figure 2.62c. Using the current division principle, we have $i_2 = -2 \times \dfrac{5}{5+10} = -0.667$ A. (The minus sign is because of the reference direction of i_2.) Finally, by superposition we have $i_T = i_1 + i_2 = 0.333$ A.

E2.32 With only the first source active we have:

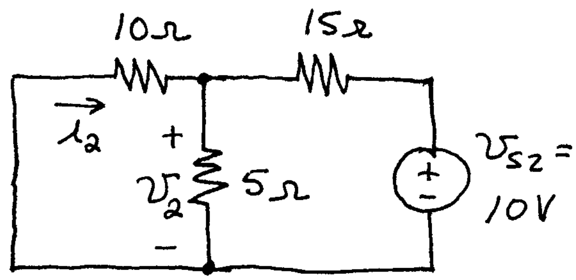


Then we combine resistances in series and parallel:

$$R_{eq} = 10 + \frac{1}{1/5 + 1/15} = 13.75\ \Omega$$

Thus, $i_1 = 20/13.75 = 1.455$ A, and $v_1 = 3.75 i_1 = 5.45$ V.

With only the second source active, we have:



Then we combine resistances in series and parallel:

$$R_{eq2} = 15 + \frac{1}{1/5 + 1/10} = 18.33\ \Omega$$

Thus, $i_s = 10/18.33 = 0.546\ A$, and $v_2 = 3.33 i_s = 1.818$ V. Then, we have $i_2 = (-v_2)/10 = -0.1818\ A$

Finally we have $v_T = v_1 + v_2 = 5.45 + 1.818 = 7.27$ V and $i_T = i_1 + i_2 = 1.455 - 0.1818 = 1.27\ A$.

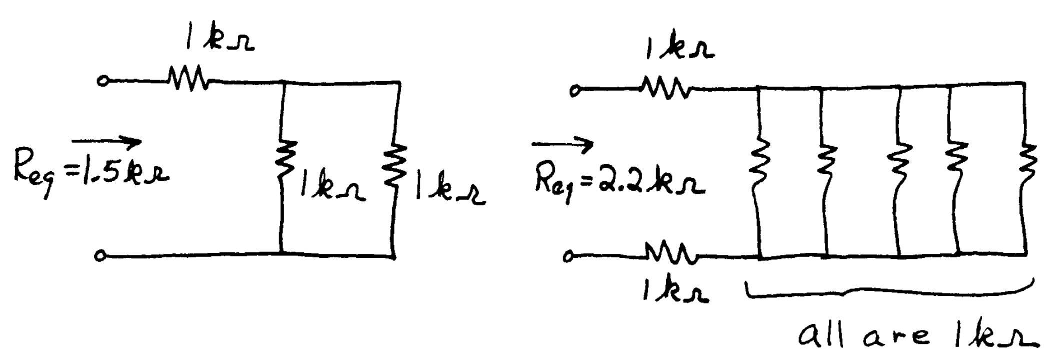
Answers for Selected Problems

P2.1* (a) $R_{eq} = 20\ \Omega$ (b) $R_{eq} = 23\ \Omega$

P2.2* $R_x = 5\ \Omega$.

P2.3* $R_{ab} = 10\ \Omega$

P2.4*



P2.5* $R_{ab} = 9.6\ \Omega$

P2.23* $i_1 = 1\ A$ $i_2 = 0.5\ A$

P2.24* $v_1 = 3\ \mathrm{V}$ $v_2 = 0.5\ \mathrm{V}$

P2.25* $v = 140\ \mathrm{V};\ i = 1\ A$

P2.34* $i_1 = 1.5\ A$ $i_2 = 0.5\ A$
$P_{4A} = 30\ \mathrm{W}$ delivering
$P_{2A} = 15\ \mathrm{W}$ absorbing
$P_{5\Omega} = 11.25\ \mathrm{W}$ absorbing
$P_{15\Omega} = 3.75\ \mathrm{W}$ absorbing

P2.35* $i_1 = 2.5\ A$ $i_2 = 0.8333\ A$

P2.36* $v_1 = 5\ \mathrm{V}$ $v_2 = 7\ \mathrm{V}$ $v_3 = 13\ \mathrm{V}$

P2.37* $i_1 = 1\ A$ $i_2 = 2\ A$

P2.38* $v = 3.333\ \mathrm{V}$

P2.43* $R_g = 25\ \mathrm{m}\Omega$

P2.48* $v_1 = 14.29\ \mathrm{V}$ $v_2 = 11.43\ \mathrm{V}$ $i_1 = 0.2857\ A$

P2.49* $v_1 = 6.667\ \mathrm{V}$ $v_2 = -3.333\ \mathrm{V}$ $i_s = -3.333\ A$

P2.56* $v_1 = 6\ \mathrm{V}$ $v_2 = 4\ \mathrm{V}$ $i_x = 0.4\ A$

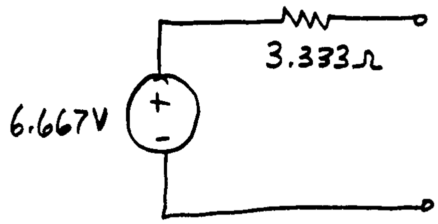
P2.57* $v_1 = 5.405\ \mathrm{V}$ $v_2 = 7.297\ \mathrm{V}$

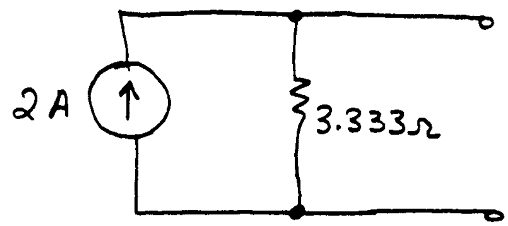
P2.65* $i_1 = 2.364\ A$ $i_2 = 1.818\ A$ $P = 4.471\ \mathrm{W}$

P2.66* $v_2 = 0.500\ \mathrm{V}$ $P = 6\ \mathrm{W}$

P2.67* $i_1 = 0.2857\ A$

P2.80*





P2.81* $R_t = 50\ \Omega$

P2.91* $R_t = 0$ $\quad P_{max} = 80\ \text{W}$

P2.94* $i_v = 2\ A$ $\quad i_c = 2\ A$ $\quad i = i_v + i_c = 4\ A$

P2.95* $i_s = -3.333\ A$

P2.103* $R_3 = 5932\ \Omega$ $\quad i_{\text{detector}} = 31.65 \times 10^{-9}\ A$

Practice Test

T2.1 (a) 6, (b) 10, (c) 2, (d) 7, (e) 10 or 13 (perhaps 13 is the better answer), (f) 1 or 4 (perhaps 4 is the better answer), (g) 11, (h) 3, (i) 8, (j) 15, (k) 17, (l) 14.

T2.2 The equivalent resistance seen by the voltage source is:

$$R_{eq} = R_1 + \frac{1}{1/R_2 + 1/R_3 + 1/R_4} = 16\ \Omega$$

$$i_s = \frac{V_s}{R_{eq}} = 6\ A$$

Then, using the current division principle, we have

$$i_4 = \frac{G_4}{G_2 + G_3 + G_4} i_s = \frac{1/60}{1/48 + 1/16 + 1/60} 6 = 1\ A$$

T2.3 Writing KCL equations at each node gives

$$\frac{v_1}{4} + \frac{v_1 - v_2}{5} + \frac{v_1 - v_3}{2} = 0$$
$$\frac{v_2 - v_1}{5} + \frac{v_2}{10} = 2$$
$$\frac{v_3}{1} + \frac{v_3 - v_1}{2} = -2$$

In standard form, we have:

$$0.95v_1 - 0.20v_2 - 0.50v_3 = 0$$
$$-0.20v_1 + 0.30v_2 = 2$$
$$-0.50v_1 + 1.50v_3 = -2$$

In matrix form, we have

$$\mathbf{GV} = \mathbf{I}$$
$$\begin{bmatrix} 0.95 & -0.20 & -0.50 \\ -0.20 & 0.30 & 0 \\ -0.50 & 0 & 1.50 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 2 \\ -2 \end{bmatrix}$$

The MATLAB commands needed to obtain the column vector of the node voltages are

```
G = [0.95 -0.20 -0.50; -0.20 0.30 0; -0.50 0 1.50]
I = [0; 2; -2]
V = G\I       % As an alternative we could use V = inv(G)*I
```

Actually, because the circuit contains only resistances and independent current sources, we could have used the short-cut method to obtain the **G** and **I** matrices.

T2.4 We can write the following equations:

KVL mesh 1: $R_1 i_1 - V_s + R_3(i_1 - i_3) + R_2(i_1 - i_2) = 0$

KVL for the supermesh obtained by combining meshes 2 and 3:

$$R_4 i_2 + R_2(i_2 - i_1) + R_3(i_3 - i_1) + R_5 i_3 = 0$$

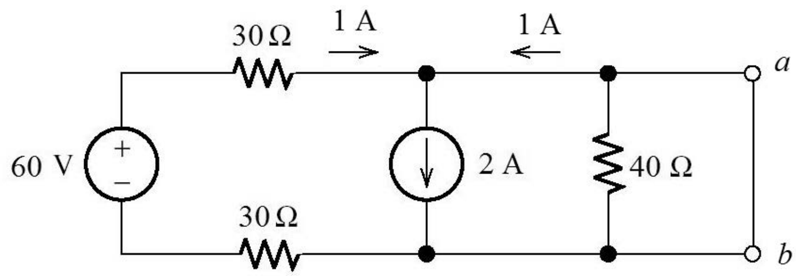
KVL around the periphery of the circuit:

$$R_1 i_1 - V_s + R_4 i_2 + R_5 i_3 = 0$$

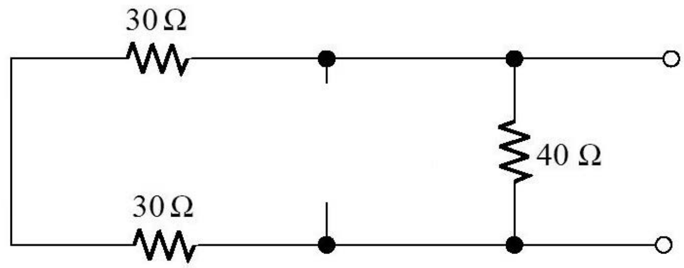
Current source: $i_2 - i_3 = I_s$

A set of equations for solving the network must include the current source equation plus two of the mesh equations. The three mesh equations are dependent and will not provide a solution by themselves.

T2.5 Under short-circuit conditions, the circuit becomes



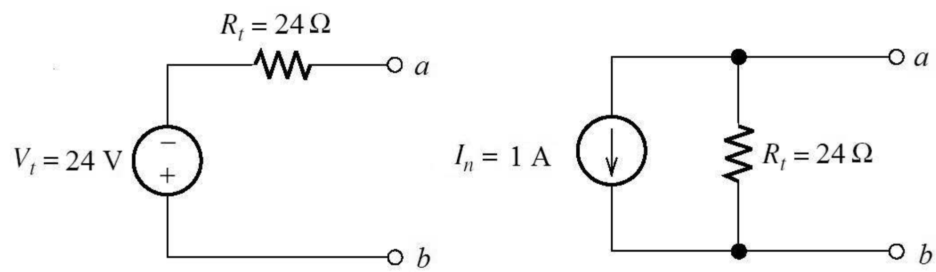
Thus, the short-circuit current is 1 A flowing out of *b* and into *a*. Zeroing the sources, we have



Thus, the Thévenin resistance is

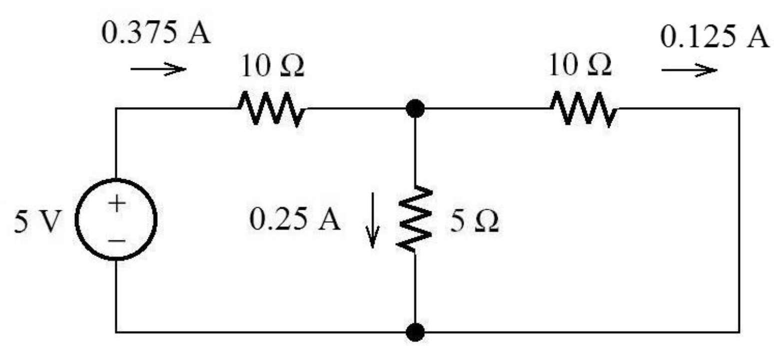
$$R_t = \frac{1}{1/40 + 1/(30+30)} = 24\ \Omega$$

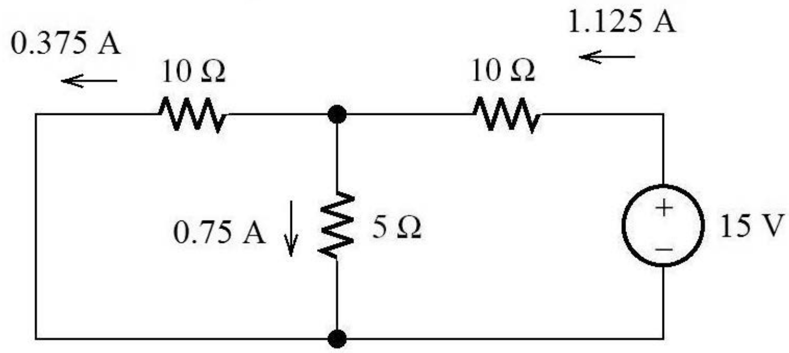
and the Thévenin voltage is $V_t = I_{sc}R_t = 24\text{ V}$. The equivalent circuits are:



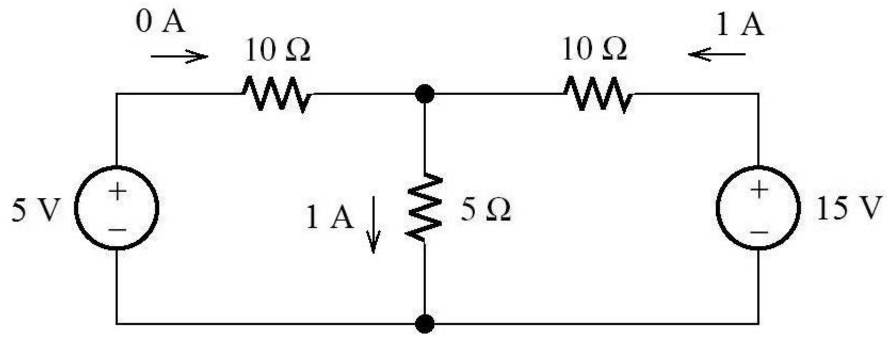
Because the short-circuit current flows out of terminal *b*, we have oriented the voltage polarity positive toward *b* and pointed the current source reference toward *b*.

T2.6 With one source active at a time, we have





Then, with both sources active, we have



We see that the 5-V source produces 25% of the total current through the 5-Ω resistance. However, the power produced by the 5-V source with both sources active is zero. Thus, the 5-V source produces 0% of the power delivered to the 5-Ω resistance. Strange, but true! Because power is a nonlinear function of current (i.e., $P = Ri^2$), the superposition principle does not apply to power.

CHAPTER 3

Exercises

E3.1 $v(t) = q(t)/C = 10^{-6}\sin(10^5 t)/(2\times 10^{-6}) = 0.5\sin(10^5 t)\text{ V}$

$$i(t) = C\frac{dv}{dt} = (2\times 10^{-6})(0.5\times 10^5)\cos(10^5 t) = 0.1\cos(10^5 t)\ A$$

E3.2 Because the capacitor voltage is zero at t = 0, the charge on the capacitor is zero at t = 0.

$$q(t) = \int_0^t i(x)dx + 0$$

$$= \int_0^t 10^{-3}dx = 10^{-3}t \text{ for } 0 \le t \le 2\text{ ms}$$

$$= \int_0^{2E-3} 10^{-3}dx + \int_{2E-3}^{t} -10^{-3}dx = 4\times 10^{-6} - 10^{-3}t \text{ for } 2\text{ ms} \le t \le 4\text{ ms}$$

$$v(t) = q(t)/C$$

$$= 10^4 t \text{ for } 0 \le t \le 2\text{ ms}$$

$$= 40 - 10^4 t \text{ for } 2\text{ ms} \le t \le 4\text{ ms}$$

$$p(t) = i(t)v(t)$$

$$= 10t \text{ for } 0 \le t \le 2\text{ ms}$$

$$= -40\times 10^{-3} + 10t \text{ for } 2\text{ ms} \le t \le 4\text{ ms}$$

$$w(t) = Cv^2(t)/2$$

$$= 5t^2 \quad \text{for } 0 \le t \le 2\text{ ms}$$

$$= 0.5\times 10^{-7}(40 - 10^4 t)^2 \text{ for } 2\text{ ms} \le t \le 4\text{ ms}$$

in which the units of charge, electrical potential, power, and energy are coulombs, volts, watts and joules, respectively. Plots of these quantities are shown in Figure 3.8 in the book.

E3.3 Refer to Figure 3.10 in the book. Applying KVL, we have

$$v = v_1 + v_2 + v_3$$

Then using Equation 3.8 to substitute for the voltages we have

$$v(t) = \frac{1}{C_1}\int_0^t i(t)dt + v_1(0) + \frac{1}{C_2}\int_0^t i(t)dt + v_2(0) + \frac{1}{C_3}\int_0^t i(t)dt + v_3(0)$$

This can be written as

$$v(t) = \left(\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}\right)\int_0^t i(t)dt + v_1(0) + v_2(0) + v_3(0) \qquad (1)$$

Now if we define

$$\frac{1}{C_{eq}} = \left(\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}\right) \quad \text{and} \quad v(0) = v_1(0) + v_2(0) + v_3(0)$$

we can write Equation (1) as

$$v(t) = \frac{1}{C_{eq}}\int_0^t i(t)dt + v(0)$$

Thus the three capacitances in series have an equivalent capacitance given by Equation 3.25 in the book.

E3.4 (a) For series capacitances:

$$C_{eq} = \frac{1}{1/C_1 + 1/C_2} = \frac{1}{1/2 + 1/1} = 2/3\ \mu\text{F}$$

(b) For parallel capacitances:

$$C_{eq} = C_1 + C_2 = 1 + 2 = 3\ \mu\text{F}$$

E3.5 From Table 3.1 we find that the relative dielectric constant of polyester is 3.4. We solve Equation 3.26 for the area of each sheet:

$$A = \frac{Cd}{\varepsilon} = \frac{Cd}{\varepsilon_r \varepsilon_0} = \frac{10^{-6} \times 15 \times 10^{-6}}{3.4 \times 8.85 \times 10^{-12}} = 0.4985\ \text{m}^2$$

Then the length of the strip is

$$L = A/W = 0.4985/(2 \times 10^{-2}) = 24.93\ \text{m}$$

E3.6 $$v(t) = L\frac{di(t)}{dt} = (10 \times 10^{-3})\frac{d}{dt}\left[0.1\cos\left(10^4 t\right)\right] = -10\sin\left(10^4 t\right)\ \text{V}$$

$$w(t) = \tfrac{1}{2}Li^2(t) = 5 \times 10^{-3} \times \left[0.1\cos\left(10^4 t\right)\right]^2 = 50 \times 10^{-6}\cos^2\left(10^4 t\right)\ \text{J}$$

E3.7

$$i(t) = \frac{1}{L}\int_0^t v(x)dx + i(0) = \frac{1}{150 \times 10^{-6}}\int_0^t v(x)dx$$

$$= 6667\int_0^t 7.5 \times 10^6 x dx = 25 \times 10^9 t^2 \text{ V for } 0 \le t \le 2\ \mu s$$

$$= 6667\int_0^{2E-6} 7.5 \times 10^6 x dx = 0.1 \text{ V for } 2\mu s \le t \le 4\ \mu s$$

$$= 6667\left(\int_0^{2E-6} 7.5 \times 10^6 x dx + \int_{4E-6}^{t} (-15)dx\right) = 0.5 - 10^5 t \text{ V for } 4\mu s \le t \le 5\ \mu s$$

A plot of $i(t)$ versus t is shown in Figure 3.19b in the book.

E3.8 Refer to Figure 3.20a in the book. Using KVL we can write:

$$v(t) = v_1(t) + v_2(t) + v_3(t)$$

Using Equation 3.28 to substitute, this becomes

$$v(t) = L_1\frac{di(t)}{dt} + L_2\frac{di(t)}{dt} + L_3\frac{di(t)}{dt} \qquad (1)$$

Then if we define $L_{eq} = L_1 + L_2 + L_3$, Equation (1) becomes:

$$v(t) = L_{eq}\frac{di(t)}{dt}$$

which shows that the series combination of the three inductances has the same terminal equation as the equivalent inductance.

E3.9 Refer to Figure 3.20b in the book. Using KCL we can write:

$$i(t) = i_1(t) + i_2(t) + i_3(t)$$

Using Equation 3.32 to substitute, this becomes

$$i(t) = \frac{1}{L_1}\int_0^t v(t)dt + i_1(0) + \frac{1}{L_2}\int_0^t v(t)dt + i_2(0) + \frac{1}{L_3}\int_0^t v(t)dt + i_3(0)$$

This can be written as

$$v(t) = \left(\frac{1}{L_1} + \frac{1}{L_2} + \frac{1}{L_3}\right)\int_0^t v(t)dt + i_1(0) + i_2(0) + i_3(0) \qquad (1)$$

Now if we define

$$\frac{1}{L_{eq}} = \left(\frac{1}{L_1} + \frac{1}{L_2} + \frac{1}{L_3}\right) \quad \text{and} \quad i(0) = i_1(0) + i_2(0) + i_3(0)$$

we can write Equation (1) as

$$i(t) = \frac{1}{L_{eq}}\int_0^t v(t)dt + i(0)$$

Thus, the three inductances in parallel have the equivalent inductance shown in Figure 3.20b in the book.

E3.10 Refer to Figure 3.21 in the book.

(a) The 2-H and 3-H inductances are in series and are equivalent to a 5-H inductance, which in turn is in parallel with the other 5-H inductance. This combination has an equivalent inductance of 1/(1/5 + 1/5) = 2.5 H. Finally the 1-H inductance is in series with the combination of the other inductances so the equivalent inductance is 1 + 2.5 = 3.5 H.

(b) The 2-H and 3-H inductances are in series and have an equivalent inductance of 5 H. This equivalent inductance is in parallel with both the 5-H and 4-H inductances. The equivalent inductance of the parallel combination is 1/(1/5 + 1/4 + 1/5) = 1.538 H. This combination is in series with the 1-H and 6-H inductances so the overall equivalent inductance is 1.538 + 1 + 6 = 8.538 H.

E3.11 The MATLAB commands including some explanatory comments are:

```
% We avoid using i alone as a symbol for current because
% we reserve i for the square root of -1 in MATLAB. Thus, we
% will use iC for the capacitor current.
syms t iC qC vC % Define t, iC, qC and vC as symbolic objects.
iC = 0.5*sin((1e4)*t);
ezplot(iC, [0 3*pi*1e-4])
qC=int(iC,t,0,t); % qC equals the integral of iC.
figure % Plot the charge in a new window.
ezplot(qC, [0 3*pi*1e-4])
vC = 1e7*qC;
figure % Plot the voltage in a new window.
ezplot(vC, [0 3*pi*1e-4])
```

The plots are very similar to those of Figure 3.5 in the book. An m-file (named Exercise_3_11) containing these commands can be found in the MATLAB folder on the OrCAD disk.

E.12 The MATLAB commands including some explanatory comments are:

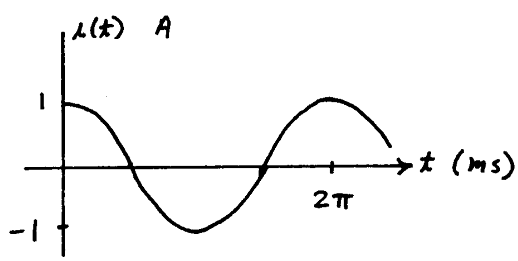
```
% We avoid using i by itself as a symbol for current because
% we reserve i for the square root of -1 in MATLAB. Thus, we
% will use iC for the capacitor current.
syms t vC iC pC wC % Define t, vC, iC, pC and wC as symbolic objects.
vC = 1000*t*(heaviside(t)- heaviside(t-1)) + ...
    1000*(heaviside(t-1) - heaviside(t-3)) + ...
    500*(5-t)*(heaviside(t-3) - heaviside(t-5));
ezplot(vC, [0 6])
iC = (10e-6)*diff(vC, 't'); % iC equals C times the derivative of vC.
figure % Plot the current in a new window.
ezplot(iC, [0 6])
pC = vC*iC;
figure % Plot the power in a new window.
ezplot(pC, [0 6])
wC = (1/2)*(10e-6)*vC^2;
figure % Plot the energy in a new window.
ezplot(wC, [0 6])
```

The plots are very similar to those of Figure 3.6 in the book. An m-file (named Exercise_3_12) containing these commands can be found in the MATLAB folder on the OrCAD disk.

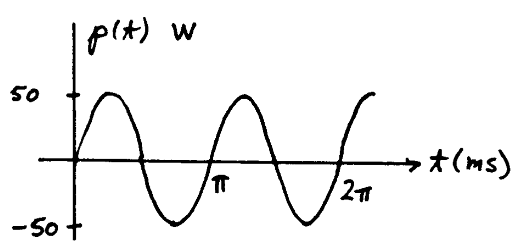
Answers for Selected Problems

P3.5* $\Delta t = 2000 \text{ s}$

P3.6*
$$i(t) = C\frac{dv}{dt}$$
$$= 10^{-5}\frac{d}{dt}(100\sin 1000t)$$
$$= \cos(1000t)$$

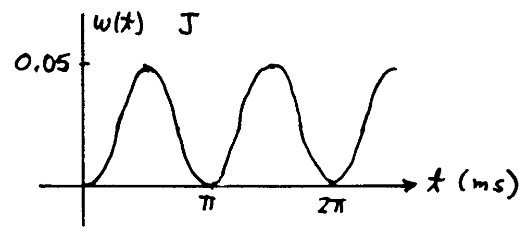


$$p(t) = v(t)i(t)$$



$$= 100\cos(1000t)\sin(1000t)$$
$$= 50\sin(2000t)$$

$$w(t) = \frac{1}{2}C[v(t)]^2$$
$$= 0.05\sin^2(1000t)$$



P3.7* At $t = 0$, we have $p(0) = -60$ mW. Because the power has a negative value, the capacitor is delivering energy.

At $t = 1$ s, we have $p(1) = 120$ mW. Because the power is positive, we know that the capacitor is absorbing energy.

P3.8* $V = 51.8$ kV

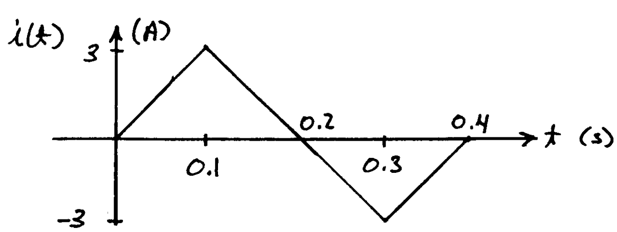
P3.24* (a) $C_{eq} = 2\ \mu$F
(b) $C_{eq} = 8\ \mu$F

P3.25* $C = 198\ \mu$F
$I_{battery} = 19.8\ \mu A$
Ampere-hour rating of the battery is 0.867 Ampere hours

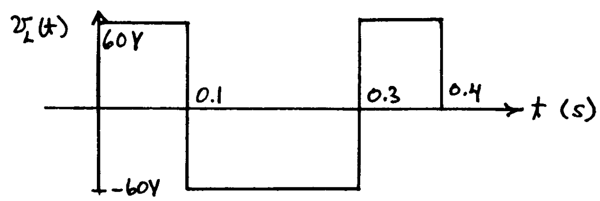
P3.31* $C = 0.398\ \mu$F

P3.32* $W_1 = 500\ \mu$J $\quad C_2 = 500$ pF
$V_2 = 2000$ V $\quad W_2 = 1000\ \mu$J
The additional energy is supplied by the force needed to pull the plates apart.

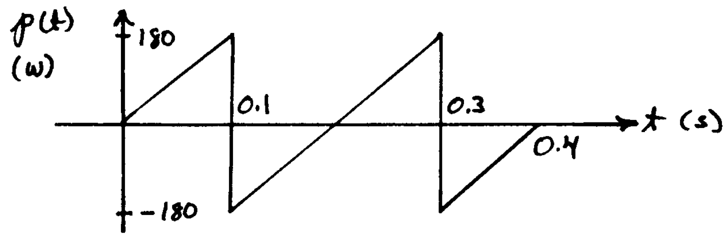
P3.43* $L = 2$ H



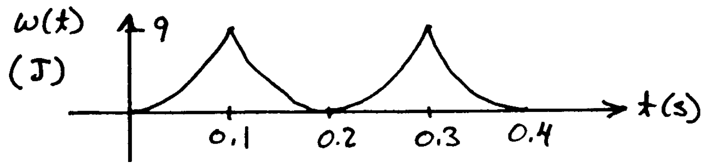
$$v_L(t) = L\frac{di_L(t)}{dt}$$



$$p(t) = v_L(t)i_L(t)$$



$$w(t) = \frac{1}{2}L[i_L(t)]^2$$



P3.44* $t_x = 1\ \mu s$

P3.45* $v_L = 10\text{ V}$

P3.60* (a) $L_{eq} = 3\text{ H}$

(b) $L_{eq} = 6\text{ H}$

P3.61* $i_1(t) = \frac{L_2}{L_1 + L_2}i(t) = \frac{2}{3}i(t)$

$i_2(t) = \frac{L_1}{L_1 + L_2}i(t) = \frac{1}{3}i(t)$

P3.72* (a) $L_{eq} = L_1 + 2M + L_2$

(b) $L_{eq} = L_1 - 2M + L_2$

Practice Test

T3.1

$$v_{ab}(t) = \frac{1}{C}\int_0^t i_{ab}(t)dt + v_C(0) = 10^5\int_0^t 0.3\exp(-2000t)dt$$

$$v_{ab}(t) = -15\exp(-2000t)\Big|_0^t$$

$$v_{ab}(t) = 15 - 15\exp(-2000t) \text{ V}$$

$$w_C(\infty) = \frac{1}{2}Cv_C^2(\infty) = \frac{1}{2}10^{-5}(15)^2 = 1.125 \text{ mJ}$$

T3.2 The 6-μF and 3-μF capacitances are in series and have an equivalent capacitance of

$$C_{eq1} = \frac{1}{1/6 + 1/3} = 2 \ \mu\text{F}$$

C_{eq1} is in parallel with the 4-μF capacitance, and the combination has an equivalent capacitance of

$$C_{eq2} = C_{eq1} + 4 = 6 \ \mu\text{F}$$

C_{eq2} is in series with the 12-μF and the combination, has an equivalent capacitance of

$$C_{eq3} = \frac{1}{1/12 + 1/6} = 4 \ \mu\text{F}$$

Finally, C_{eq3} is in parallel with the 1-μF capacitance, and the equivalent capacitance is

$$C_{eq} = C_{eq3} + 1 = 5 \ \mu\text{F}$$

T3.3

$$C = \frac{\varepsilon_r\varepsilon_0 A}{d} = \frac{80 \times 8.85 \times 10^{-12} \times 2 \times 10^{-2} \times 3 \times 10^{-2}}{0.1 \times 10^{-3}} = 4248 \text{ pF}$$

T3.4

$$v_{ab}(t) = L\frac{di_{ab}}{dt} = 2 \times 10^{-3} \times 0.3 \times 2000\cos(2000t) = 1.2\cos(2000t) \text{ V}$$

The maximum value of $\sin(2000t)$ is unity. Thus the peak current is 0.3 A, and the peak energy stored is

$$w_{peak} = \frac{1}{2}Li_{peak}^2 = \frac{1}{2} \times 2 \times 10^{-3}(0.3)^2 = 90 \ \mu\text{J}$$

T3.5 The 2-H and 4-H inductances are in parallel and the combination has an equivalent inductance of

$$L_{eq1} = \frac{1}{1/2 + 1/4} = 1.333 \text{ H}$$

Also, the 3-H and 5-H inductances are in parallel, and the combination has an equivalent inductance of

$$L_{eq2} = \frac{1}{1/3 + 1/5} = 1.875 \text{ H}$$

Finally, L_{eq1} and L_{eq2} are in series. The equivalent inductance between terminals a and b is

$$L_{eq} = L_{eq1} + L_{eq2} = 3.208 \text{ H}$$

T3.6 For these mutually coupled inductances, we have

$$v_1(t) = L_1 \frac{di_1(t)}{dt} - M \frac{di_2(t)}{dt}$$
$$v_2(t) = -M \frac{di_1(t)}{dt} + L_2 \frac{di_2(t)}{dt}$$

in which the currents are referenced into the positive polarities. Thus the currents are

$$i_1(t) = 2\cos(500t) \quad \text{and} \quad i_2(t) = -2\exp(-400t)$$

Substituting the inductance values and the current expressions we have

$$v_1(t) = -40 \times 10^{-3} \times 1000 \sin(500t) - 20 \times 10^{-3} \times 800 \exp(-400t)$$
$$v_1(t) = -40 \sin(500t) - 16 \exp(-400t) \text{ V}$$

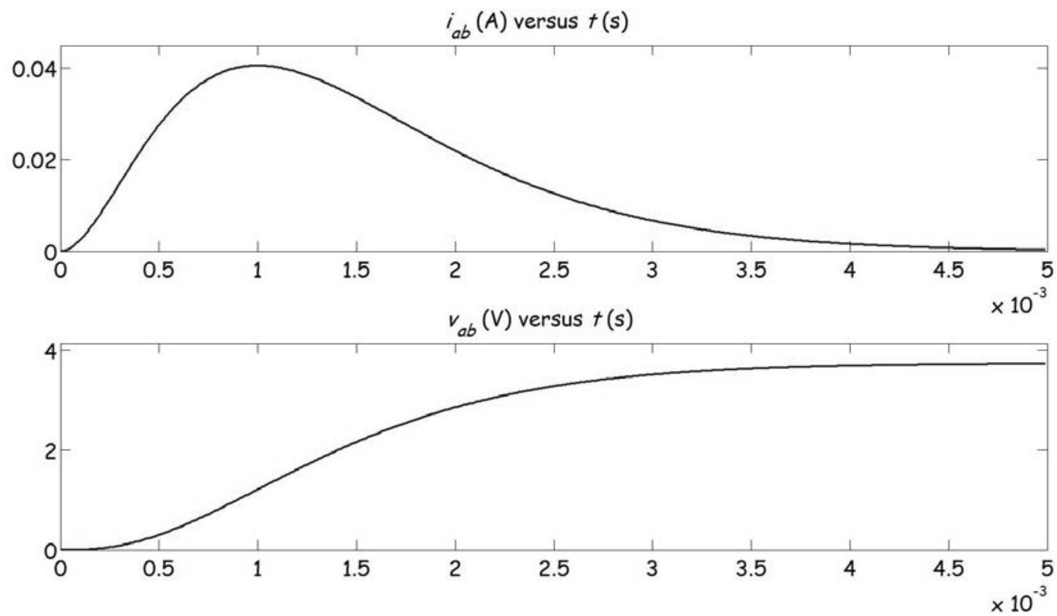
$$v_2(t) = 20 \times 10^{-3} \times 1000 \sin(500t) - 30 \times 10^{-3} \times 800 \exp(-400t)$$
$$v_2(t) = 20 \sin(500t) - 24 \exp(-400t) \text{ V}$$

T3.7 One set of commands is

```
syms vab iab t
iab = 3*(10^5)*(t^2)*exp(-2000*t);
vab = (1/20e-6)*int(iab,t,0,t)
subplot(2,1,1)
ezplot(iab, [0 5e-3]), title('\iti_a_b\rm (A) versus \itt\rm (s)')
subplot(2,1,2)
ezplot(vab, [0 5e-3]), title('\itv_a_b\rm (V) versus \itt\rm (s)')
```

The results are

$$v_{ab} = \frac{15}{4} - \frac{15}{4}\exp(-2000t) - 7500\exp(-2000t) - 7.5\times 10^{6} t^{2} \exp(-2000t)$$



CHAPTER 4

Exercises

E4.1 The voltage across the circuit is given by Equation 4.8:

$$v_C(t) = V_i \exp(-t / RC)$$

in which V_i is the initial voltage. At the time $t_{1\%}$ for which the voltage reaches 1% of the initial value, we have

$$0.01 = \exp(-t_{1\%} / RC)$$

Taking the natural logarithm of both sides of the equation, we obtain

$$\ln(0.01) = -4.605 = -t_{1\%} / RC$$

Solving and substituting values, we find $t_{1\%}$ = 4.605RC = 23.03 ms.

E4.2 The exponential transient shown in Figure 4.4 is given by

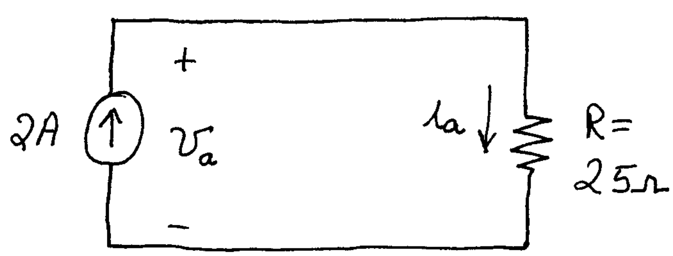
$$v_C(t) = V_s - V_s \exp(-t / \tau)$$

Taking the derivative with respect to time, we have

$$\frac{dv_C(t)}{dt} = \frac{V_s}{\tau} \exp(-t / \tau)$$

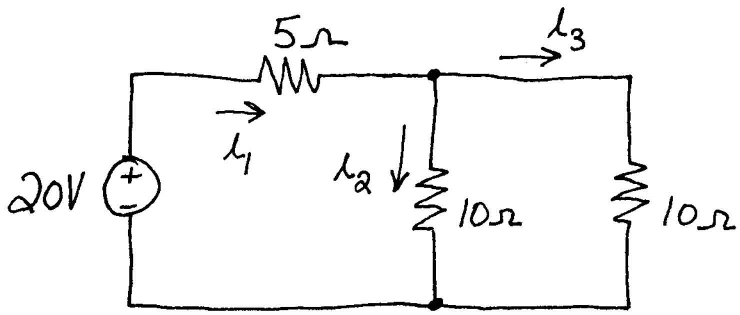
Evaluating at t = 0, we find that the initial slope is V_S / τ. Because this matches the slope of the straight line shown in Figure 4.4, we have shown that a line tangent to the exponential transient at the origin reaches the final value in one time constant.

E4.3 (a) In dc steady state, the capacitances act as open circuits and the inductances act as short circuits. Thus the steady-state (i.e., t approaching infinity) equivalent circuit is:



From this circuit, we see that $i_a = 2$ A. Then ohm's law gives the voltage as $v_a = Ri_a = 50$ V.

(b) The dc steady-state equivalent circuit is:



Here the two 10-Ω resistances are in parallel with an equivalent resistance of 1/(1/10 + 1/10) = 5 Ω. This equivalent resistance is in series with the 5-Ω resistance. Thus the equivalent resistance seen by the source is 10 Ω, and $i_1 = 20/10 = 2$ A. Using the current division principle, this current splits equally between the two 10-Ω resistances, so we have $i_2 = i_3 = 1$ A.

E4.4 (a) $\tau = L/R_2 = 0.1/100 = 1\text{ ms}$

(b) Just before the switch opens, the circuit is in dc steady state with an inductor current of $V_s/R_1 = 1.5$ A. This current continues to flow in the inductor immediately after the switch opens so we have $i(0+) = 1.5$ A. This current must flow (upward) through R_2 so the initial value of the voltage is $v(0+) = -R_2 i(0+) = -150$ V.

(c) We see that the initial magnitude of $v(t)$ is ten times larger than the source voltage.

(d) The voltage is given by

$$v(t) = -\frac{V_s L}{R_1 \tau}\exp(-t/\tau) = -150\exp(-1000t)$$

Let us denote the time at which the voltage reaches half of its initial magnitude as t_H. Then we have

$$0.5 = \exp(-1000t_H)$$

Solving and substituting values we obtain

$$t_H = -10^{-3}\ln(0.5) = 10^{-3}\ln(2) = 0.6931\text{ ms}$$

E4.5 First we write a KCL equation for $t \geq 0$.

$$\frac{v(t)}{R} + \frac{1}{L}\int_0^t v(x)dx + 0 = 2$$

Taking the derivative of each term of this equation with respect to time and multiplying each term by R, we obtain:

$$\frac{dv(t)}{dt} + \frac{R}{L}v(t) = 0$$

The solution to this equation is of the form:

$$v(t) = K\exp(-t/\tau)$$

in which $\tau = L/R = 0.2\text{ s}$ is the time constant and K is a constant that must be chosen to fit the initial conditions in the circuit. Since the initial (t = 0+) inductor current is specified to be zero, the initial current in the resistor must be 2 A and the initial voltage is 20 V:

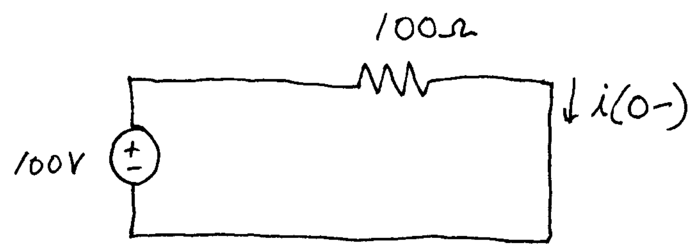
$$v(0+) = 20 = K$$

Thus, we have

$$v(t) = 20\exp(-t/\tau) \qquad i_R = v/R = 2\exp(-t/\tau)$$

$$i_L(t) = \frac{1}{L}\int_0^t v(x)dx = \frac{1}{2}\Big[-20\tau\exp(-x/\tau)\Big]_0^t = 2 - 2\exp(-t/\tau)$$

E4.6 Prior to t = 0, the circuit is in DC steady state and the equivalent circuit is



Thus we have i(0-) = 1 A. However the current through the inductor cannot change instantaneously so we also have i(0+) = 1 A. With the switch open, we can write the KVL equation:

$$\frac{di(t)}{dt} + 200i(t) = 100$$

The solution to this equation is of the form

$$i(t) = K_1 + K_2\exp(-t/\tau)$$

in which the time constant is $\tau = 1/200 = 5\text{ ms}$. In steady state with the switch open, we have $i(\infty) = K_1 = 100/200 = 0.5\text{ A}$. Then using the initial current, we have $i(0+) = 1 = K_1 + K_2$, from which we determine that $K_2 = 0.5$. Thus we have

$$i(t) = 1.0 \text{ A for } t < 0$$
$$= 0.5 + 0.5\exp(-t/\tau) \text{ for t} > 0.$$

$$v(t) = L\frac{di(t)}{dt}$$
$$= 0 \text{ V for } t < 0$$
$$= -100\exp(-t/\tau) \text{ for t} > 0.$$

E4.7 As in Example 4.4, the KVL equation is

$$Ri(t) + \frac{1}{C}\int_0^t i(x)dx + v_C(0+) - 2\cos(200t) = 0$$

Taking the derivative and multiplying by C, we obtain

$$RC\frac{di(t)}{dt} + i(t) + 400C\sin(200t) = 0$$

Substituting values and rearranging the equation becomes

$$5\times 10^{-3}\frac{di(t)}{dt} + i(t) = -400\times 10^{-6}\sin(200t)$$

The particular solution is of the form

$$i_p(t) = A\cos(200t) + B\sin(200t)$$

Substituting this into the differential equation and rearranging terms results in

$$5\times 10^{-3}\left[-200A\sin(200t) + 200B\cos(200t)\right] + A\cos(200t) + B\sin(200t)$$
$$= -400\times 10^{-6}\sin(200t)$$

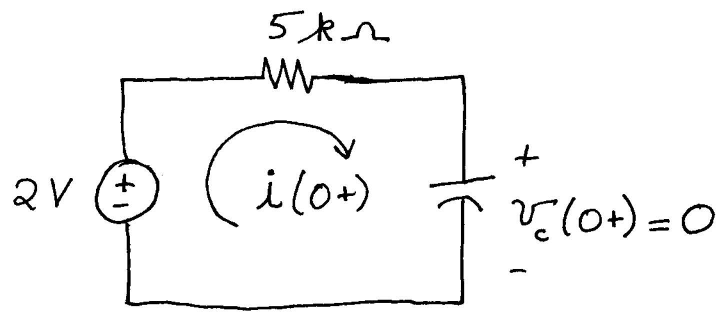
Equating the coefficients of the cos and sin terms gives the following equations:

$$-A + B = -400\times 10^{-6} \quad \text{and } B + A = 0$$

from which we determine that $A = 200\times 10^{-6}$ and $B = -200\times 10^{-6}$. Furthermore, the complementary solution is $i_C(t) = K\exp(-t/\tau)$, and the complete solution is of the form

$$i(t) = 200\cos(200t) - 200\sin(200t) + K\exp(-t/\tau) \ \ \mu A$$

At t = 0+, the equivalent circuit is



from which we determine that $i(0+) = 2/5000 = 400\ \mu A$. Then evaluating our solution at t = 0+, we have $i(0+) = 400 = 200 + K$, from which we determine that $K = 200$ μA. Thus the complete solution is

$$i(t) = 200\cos(200t) - 200\sin(200t) + 200\exp(-t/\tau)\ \ \mu A$$

E4.8 The KVL equation is

$$Ri(t) + \frac{1}{C}\int_0^t i(x)dx + v_C(0+) - 10\exp(-t) = 0$$

Taking the derivative and multiplying by C, we obtain

$$RC\frac{di(t)}{dt} + i(t) + 10C\exp(-t) = 0$$

Substituting values and rearranging, the equation becomes

$$2\frac{di(t)}{dt} + i(t) = -20\times 10^{-6}\exp(-t)$$

The particular solution is of the form

$$i_p(t) = A\exp(-t)$$

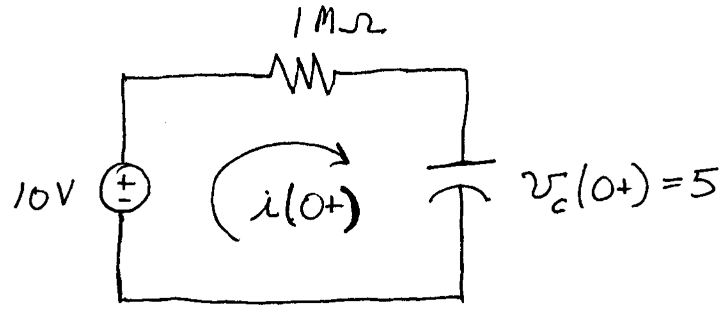
Substituting this into the differential equation and rearranging terms results in

$$-2A\exp(-t) + A\exp(-t) = -20\times 10^{-6}\exp(-t)$$

Equating the coefficients gives $A = 20\times 10^{-6}$. Furthermore, the complementary solution is $i_c(t) = K\exp(-t/2)$, and the complete solution is of the form

$$i(t) = 20\exp(-t) + K\exp(-t/2)\ \ \mu A$$

At t = 0+, the equivalent circuit is



from which we determine that $i(0+) = 5/10^6 = 5\ \mu A$. Then evaluating our solution at t = 0+, we have $i(0+) = 5 = 20 + K$, from which we determine that $K = -15\ \mu A$. Thus the complete solution is

$$i(t) = 20\exp(-t) - 15\exp(-t/2)\ \ \mu A$$

E4.9 (a) $\omega_0 = \dfrac{1}{\sqrt{LC}} = \dfrac{1}{\sqrt{10^{-3} \times 10^{-7}}} = 10^5 \quad \alpha = \dfrac{1}{2RC} = 2 \times 10^5 \quad \zeta = \dfrac{\alpha}{\omega_0} = 2$

(b) At t = 0+, the KCL equation for the circuit is

$$0.1 = \frac{v(0+)}{R} + i_L(0+) + Cv'(0+) \qquad (1)$$

However, $v(0+) = v(0-) = 0$, because the voltage across the capacitor cannot change instantaneously. Furthermore, $i_L(0+) = i_L(0-) = 0$, because the current through the inductance cannot change value instantaneously. Solving Equation (1) for $v'(0+)$ and substituting values, we find that $v'(0+) = 10^6$ V/s.

(c) To find the particular solution or forced response, we can solve the circuit in steady-state conditions. For a dc source, we treat the capacitance as an open and the inductance as a short. Because the inductance acts as a short $v_p(t) = 0$.

(d) Because the circuit is overdamped ($\zeta > 1$), the homogeneous solution is the sum of two exponentials. The roots of the characteristic solution are given by Equations 4.72 and 4.73:

$$s_1 = -\alpha - \sqrt{\alpha^2 - \omega_0^2} = -373.2 \times 10^3$$

$$s_2 = -\alpha + \sqrt{\alpha^2 - \omega_0^2} = -26.79 \times 10^3$$

Adding the particular solution to the homogeneous solution gives the general solution:

$v(t) = K_1 \exp(s_1 t) + K_2 \exp(s_2 t)$

Now using the initial conditions, we have

$v(0+) = 0 = K_1 + K_2 \qquad v'(0+) = 10^6 = K_1 s_1 + K_2 s_2$

Solving we find $K_1 = -2.887$ and $K_2 = 2.887$. Thus the solution is:

$v(t) = 2.887[\exp(s_2 t) - \exp(s_1 t)]$

E4.10 (a) $\omega_0 = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{10^{-3} \times 10^{-7}}} = 10^5 \qquad \alpha = \frac{1}{2RC} = 10^5 \qquad \zeta = \frac{\alpha}{\omega_0} = 1$

(b) The solution for this part is the same as that for Exercise 4.9b in which we found that $v'(0+) = 10^6$ V/s.

(c) The solution for this part is the same as that for Exercise 4.9c in which we found $v_p(t) = 0$.

(d) The roots of the characteristic solution are given by Equations 4.72 and 4.73:

$s_1 = -\alpha - \sqrt{\alpha^2 - \omega_0^2} = -10^5 \qquad s_2 = -\alpha + \sqrt{\alpha^2 - \omega_0^2} = -10^5$

Because the circuit is critically damped $(\zeta = 1)$, the roots are equal and the homogeneous solution is of the form:

$v(t) = K_1 \exp(s_1 t) + K_2 t \exp(s_1 t)$

Adding the particular solution to the homogeneous solution gives the general solution:

$v(t) = K_1 \exp(s_1 t) + K_2 t \exp(s_1 t)$

Now using the initial conditions we have

$v(0+) = 0 = K_1 \qquad v'(0+) = 10^6 = K_1 s_1 + K_2$

Solving we find $K_2 = 10^6$ Thus the solution is:

$v(t) = 10^6 t \exp(-10^5 t)$

E4.11 (a) $\omega_0 = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{10^{-3} \times 10^{-7}}} = 10^5 \qquad \alpha = \frac{1}{2RC} = 2 \times 10^4 \qquad \zeta = \frac{\alpha}{\omega_0} = 0.2$

(b) The solution for this part is the same as that for Exercise 4.9b in which we found that $v'(0+) = 10^6$ V/s.

(c) The solution for this part is the same as that for Exercise 4.9c in which we found $v_p(t) = 0$.

(d) Because we have $(\zeta < 1)$, this is the underdamped case and we have

$$\omega_n = \sqrt{\omega_0^2 - \alpha^2} = 97.98 \times 10^3$$

Adding the particular solution to the homogeneous solution gives the general solution:

$$v(t) = K_1 \exp(-\alpha t)\cos(\omega_n t) + K_2 \exp(-\alpha t)\sin(\omega_n t)$$

Now using the initial conditions we have

$$v(0+) = 0 = K_1 \qquad v'(0+) = 10^6 = -\alpha K_1 + \omega_n K_2$$

Solving we find $K_2 = 10.21$ Thus the solution is:

$$v(t) = 10.21\exp(-2\times 10^4 t)\sin(97.98\times 10^3 t)\ \text{V}$$

E4.12 The commands are:

```
syms ix t R C vCinitial w
ix = dsolve('(R*C)*Dix + ix = (w*C)*2*cos(w*t)', 'ix(0)=-vCinitial/R');
ians =subs(ix,[R C vCinitial w],[5000 1e-6 1 200]);
pretty(vpa(ians, 4))
ezplot(ians,[0 80e-3])
```

An m-file named Exercise_4_12 containing these commands can be found in the MATLAB folder on the OrCAD disk.

E4.13 The commands are:

```
syms vc t
% Case I R = 300:
vc = dsolve('(1e-8)*D2vc + (1e-6)*300*Dvc+ vc =10', ...
            'vc(0) = 0','Dvc(0)=0');
vpa(vc,4)
ezplot(vc, [0 1e-3])
hold on % Turn hold on so all plots are on the same axes
% Case II R = 200:
vc = dsolve('(1e-8)*D2vc + (1e-6)*200*Dvc+ vc =10',...
            'vc(0) = 0','Dvc(0)=0');
vpa(vc,4)
ezplot(vc, [0 1e-3])
% Case III R = 100:
vc = dsolve('(1e-8)*D2vc + (1e-6)*100*Dvc+ vc =10',...
            'vc(0) = 0','Dvc(0)=0');
vpa(vc,4)
```

ezplot(vc, [0 1e-3])

An m-file named Exercise_4_13 containing these commands can be found in the MATLAB folder on the OrCAD disk.

Answers for Selected Problems

P4.2* The leakage resistance must be greater than 11.39 MΩ.

P4.3* $v_C(t) = 10 - 20\exp\left(-t/(2\times 10^{-3})\right)$ V $\qquad t_0 = 2\ln(2) = 1.386$ ms

P4.4* $t_2 = 0.03466$ seconds

P4.5* $R = 4.328\ \text{M}\Omega$

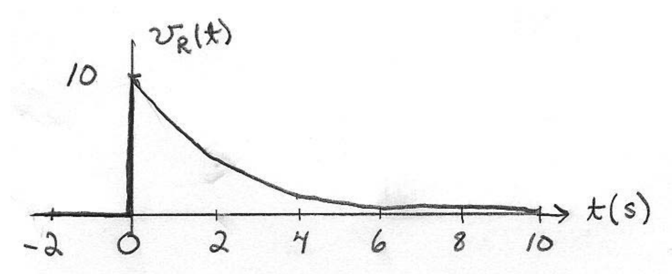
P4.6* $v(t) = V_1 \exp[-(t - t_1)/RC]$ for $t \geq t_1$

P4.21* $i_1 = 0 \qquad i_3 = i_2 = 2$ A

P4.22* $v_{C,\text{steady state}} = 10$ V $\qquad t_{99} = 46.05$ ms

P4.23*

$$v_R(t) = 0 \quad t < 0$$
$$= 10\exp(-t/\tau) \quad \text{V} \quad \text{for } t \geq 0$$



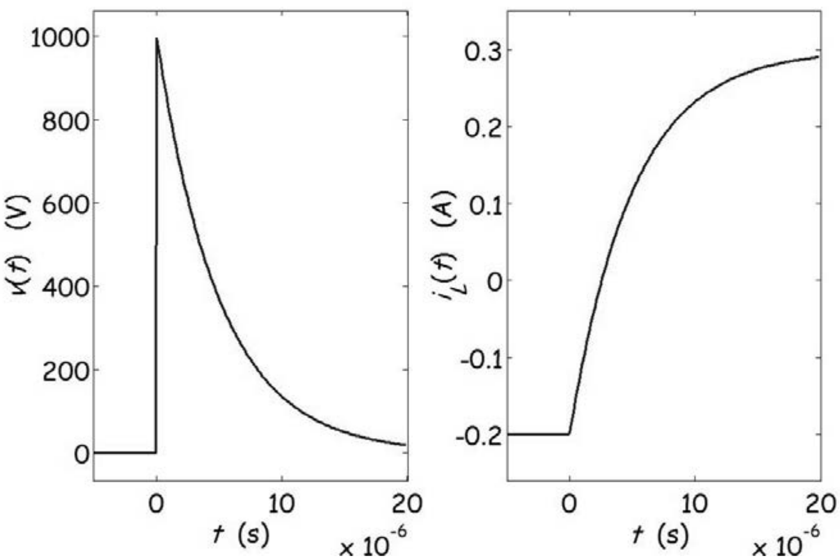
P4.33*

$$i(t) = 0 \qquad \text{for } t < 0$$
$$= 1 - \exp(-20t) \ \text{A} \quad \text{for } t \geq 0$$

P4.34* $i_L(t) = 0.3 - 0.5\exp\left(-2 \times 10^5 t\right) \quad A \quad \text{for } t > 0$

$v(t) = 0 \text{ for } t < 0$

$\quad = 1000\exp\left(-2 \times 10^5 t\right) \; A \quad \text{for } t > 0$



P4.36* $R \leq 399.6 \; \mu\Omega$

P4.45* $i_L(t) = -\exp(-t) + \exp\left(-Rt/L\right) \text{ for } \; t \geq 0$

P4.46* $v_C(t) = 10^6 \exp(-t) - 10^6 \exp(-3t) \quad t > 0$

P4.47* $v(t) = 25\exp(-t/\tau) + 25\cos(10t) - 25\sin(10t) \quad t \geq 0$

P4.61* $s_1 = -0.2679 \times 10^4$

$s_2 = -3.732 \times 10^4$

$v_C(t) = 50 - 53.87\exp(s_1 t) + 3.867\exp(s_2 t)$

P4.62* $s_1 = -10^4$

$v_C(t) = 50 - 50\exp(s_1 t) - \left(50 \times 10^4\right) t \exp(s_1 t)$

P4.63* $\alpha = 0.5 \times 10^4$

$\omega_n = 8.660 \times 10^3$

$$v_C(t) = 50 - 50\exp(-\alpha t)\cos(\omega_n t) - (28.86)\exp(-\alpha t)\sin(\omega_n t)$$

Practice Test

T4.1 (a) Prior to the switch opening, the circuit is operating in DC steady state, so the inductor acts as a short circuit, and the capacitor acts as an open circuit.

$i_1(0-) = 10/1000 = 10\ \text{mA}$ $\quad$ $i_2(0-) = 10/2000 = 5\ \text{mA}$

$i_3(0-) = 0$ $\quad$ $i_L(0-) = i_1(0-) + i_2(0-) + i_3(0-) = 15\ \text{mA}$

$v_C(0-) = 10\ \text{V}$

(b) Because infinite voltage or infinite current are not possible in this circuit, the current in the inductor and the voltage across the capacitor cannot change instantaneously. Thus, we have $i_L(0+) = i_L(0-) = 15\ \text{mA}$ and $v_C(0+) = v_C(0-) = 10\ \text{V}$. Also, we have $i_1(0+) = i_L(0+) = 15\ \text{mA}$, $i_2(0+) = v_C(0+)/5000 = 2\,\text{mA}$, and $i_3(0+) = -i_2(0+) = -2\ \text{mA}$.

(c) The current is of the form $i_L(t) = A + B\exp(-t/\tau)$. Because the inductor acts as a short circuit in steady state, we have

$i_L(\infty) = A = 10/1000 = 10\ \text{mA}$

At $t = 0+$, we have $i_L(0+) = A + B = 15\,\text{mA}$, so we find $B = 5\ \text{mA}$.

The time constant is $\tau = L/R = 2\times10^{-3}/1000 = 2\times10^{-6}\ \text{s}$.

Thus, we have $i_L(t) = 10 + 5\exp(-5\times10^{5}t)\ \text{mA}$.

(d) This is a case of an initially charged capacitance discharging through a resistance. The time constant is $\tau = RC = 5000\times10^{-6} = 5\times10^{-3}\ \text{s}$. Thus we have $v_C(t) = V_i\exp(-t/\tau) = 10\exp(-200t)\ \text{V}$.

T4.2 (a) $2\dfrac{di(t)}{dt} + i(t) = 5\exp(-3t)$

(b) The time constant is $\tau = L/R = 2\ \text{s}$ and the complementary solution is of the form $i_c(t) = A\exp(-0.5t)$.

(c) The particular solution is of the form $i_p(t) = K\exp(-3t)$. Substituting into the differential equation produces

$$-6K\exp(-3t) + K\exp(-3t) \equiv 5\exp(-3t)$$

from which we have $K = -1$.

(d) Adding the particular solution and the complementary solution, we have

$$i(t) = A\exp(-0.5t) - \exp(-3t)$$

However, the current must be zero in the inductor prior to $t = 0$ because of the open switch, and the current cannot change instantaneously in this circuit, so we have $i(0+) = 0$. This yields $A = 1$. Thus, the solution is

$$i(t) = \exp(-0.5t) - \exp(-3t) \text{ A}$$

T4.3 (a) Applying KVL to the circuit, we obtain

$$L\frac{di(t)}{dt} + Ri(t) + v_C(t) = 15 \qquad (1)$$

For the capacitance, we have

$$i(t) = C\frac{dv_C(t)}{dt} \qquad (2)$$

Using Equation (2) to substitute into Equation (1) and rearranging, we have

$$\frac{d^2v_C(t)}{dt^2} + (R/L)\frac{dv_C(t)}{dt} + (1/LC)v_C(t) = 15/LC \qquad (3)$$

$$\frac{d^2v_C(t)}{dt^2} + 2000\frac{dv_C(t)}{dt} + 25\times 10^6 v_C(t) = 375\times 10^6$$

(b) We try a particular solution of the form $v_{Cp}(t) = A$, resulting in $A = 15$. Thus, $v_{Cp}(t) = 15$. (An alternative method to find the particular solution is to solve the circuit in dc steady state. Since the capacitance acts as an open circuit, the steady-state voltage across it is 15 V.)

(c) We have

$$\omega_0 = \frac{1}{\sqrt{LC}} = 5000 \text{ and } \alpha = \frac{R}{2L} = 1000$$

Since we have $\alpha < \omega_0$, this is the underdamped case. The natural frequency is given by:

$$\omega_n = \sqrt{\omega_0^2 - \alpha^2} = 4899$$

The complementary solution is given by:

$$v_{Cc}(t) = K_1\exp(-1000t)\cos(4899t) + K_2\exp(-1000t)\sin(4899t)$$

(d) The complete solution is

$$v_C(t) = 15 + K_1 \exp(-\alpha t)\cos(\omega_n t) + K_2 \exp(-\alpha t)\sin(\omega_n t)$$

The initial conditions are

$$v_C(0) = 0 \quad \text{and} \quad i(0) = 0 = C\frac{dv_C(t)}{dt}\Big|_{t=0}$$

Thus, we have

$$v_C(0) = 0 = 15 + K_1$$

$$\frac{dv_C(t)}{dt}\Big|_{t=0} = 0 = -\alpha K_1 + \omega_n K_2$$

Solving, we find $K_1 = -15$ and $K_2 = -3.062$. Finally, the solution is

$$v_C(t) = 15 - 15\exp(-1000t)\cos(4899t) - (3.062)\exp(-1000t)\sin(4899t) \text{ V}$$

T4.4 One set of commands is

```
syms vC t
S = dsolve('D2vC + 2000*DvC + (25e6)*vC = 375e6',...
           'vC(0) = 0, DvC(0) = 0');
simple(vpa(S,4))
```

These commands are stored in the m-file named T_4_4 on the OrCAD disk.

CHAPTER 5

Exercises

E5.1 (a) We are given $v(t) = 150\cos(200\pi t - 30^\circ)$. The angular frequency is the coefficient of t so we have $\omega = 200\pi$ radian/s. Then

$$f = \omega / 2\pi = 100\text{ Hz} \qquad T = 1/f = 10\text{ ms}$$

$$V_{rms} = V_m / \sqrt{2} = 150/\sqrt{2} = 106.1\text{ V}$$

Furthermore, $v(t)$ attains a positive peak when the argument of the cosine function is zero. Thus keeping in mind that ωt has units of radians, the positive peak occurs when

$$\omega t_{\max} = 30 \times \frac{\pi}{180} \quad \Rightarrow \quad t_{\max} = 0.8333\text{ ms}$$

(b) $P_{avg} = V_{rms}^2 / R = 225\text{ W}$

(c) A plot of $v(t)$ is shown in Figure 5.4 in the book.

E5.2 We use the trigonometric identity $\sin(z) = \cos(z - 90^\circ)$. Thus

$$100\sin(300\pi t + 60^\circ) = 100\cos(300\pi t - 30^\circ)$$

E5.3 $\omega = 2\pi f \cong 377$ radian/s $\qquad T = 1/f \cong 16.67$ ms $\qquad V_m = V_{rms}\sqrt{2} \cong 155.6$ V

The period corresponds to 360° therefore 5 ms corresponds to a phase angle of $(5/16.67) \times 360^\circ = 108^\circ$. Thus the voltage is

$$v(t) = 155.6\cos(377t - 108^\circ)$$

E5.4 (a)

$$\mathbf{V}_1 = 10\angle 0^\circ + 10\angle -90^\circ = 10 - j10 \cong 14.14\angle -45^\circ$$

$$10\cos(\omega t) + 10\sin(\omega t) = 14.14\cos(\omega t - 45^\circ)$$

(b)

$$\mathbf{I}_1 = 10\angle 30^\circ + 5\angle -60^\circ \cong 8.660 + j5 + 2.5 - j4.330$$

$$\cong 11.16 + j0.670 \cong 11.18\angle 3.44^\circ$$

$$10\cos(\omega t + 30^\circ) + 5\sin(\omega t + 30^\circ) = 11.18\cos(\omega t + 3.44^\circ)$$

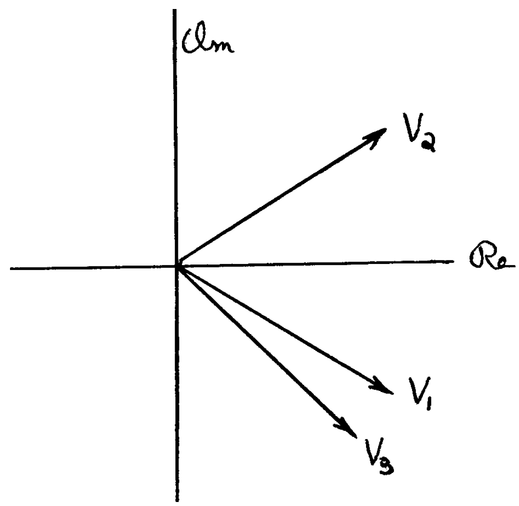
(c)

$$\mathbf{I}_2 = 20\angle 0^\circ + 15\angle -60^\circ \cong 20 + j0 + 7.5 - j12.99$$

$$\cong 27.5 - j12.99 \cong 30.41\angle -25.28^\circ$$

$$20\sin(\omega t + 90^\circ) + 15\cos(\omega t - 60^\circ) = 30.41\cos(\omega t - 25.28^\circ)$$

E5.5 The phasors are $\mathbf{V}_1 = 10\angle -30^\circ$ $\mathbf{V}_2 = 10\angle +30^\circ$ and $\mathbf{V}_3 = 10\angle -45^\circ$



v_1 lags v_2 by 60° (or we could say v_2 leads v_1 by 60°)

v_1 leads v_3 by 15° (or we could say v_3 lags v_1 by 15°)

v_2 leads v_3 by 75° (or we could say v_3 lags v_2 by 75°)

E5.6 (a) $Z_L = j\omega L = j50 = 50\angle 90^\circ$ $\mathbf{V}_L = 100\angle 0^\circ$

$\mathbf{I}_L = \mathbf{V}_L / Z_L = 100 / j50 = 2\angle -90^\circ$

(b) The phasor diagram is shown in Figure 5.11a in the book.

E5.7 (a) $Z_C = 1/j\omega C = -j50 = 50\angle -90^\circ$ $\mathbf{V}_C = 100\angle 0^\circ$

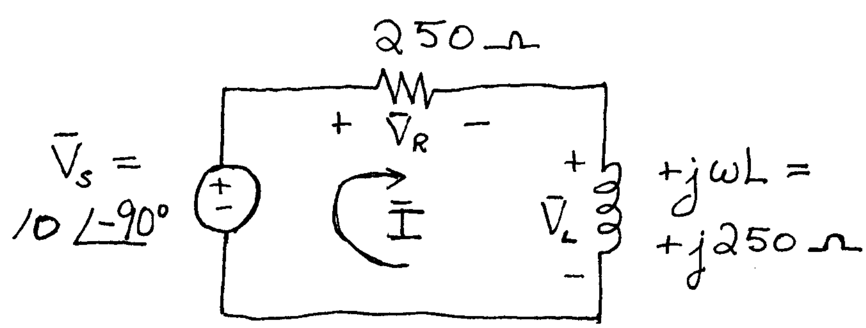
$\mathbf{I}_C = \mathbf{V}_C / Z_C = 100/(-j50) = 2\angle 90^\circ$

(b) The phasor diagram is shown in Figure 5.11b in the book.

E5.8 (a) $Z_R = R = 50 = 50\angle 0^\circ$ $\mathbf{V}_R = 100\angle 0^\circ$ $\mathbf{I}_R = \mathbf{V}_R / R = 100/(50) = 2\angle 0^\circ$

(b) The phasor diagram is shown in Figure 5.11c in the book.

E5.9 (a) The transformed network is:



$$\mathbf{I} = \frac{\mathbf{V}_s}{Z} = \frac{10\angle -90^\circ}{250 + j250} = 28.28\angle -135^\circ \text{ mA}$$

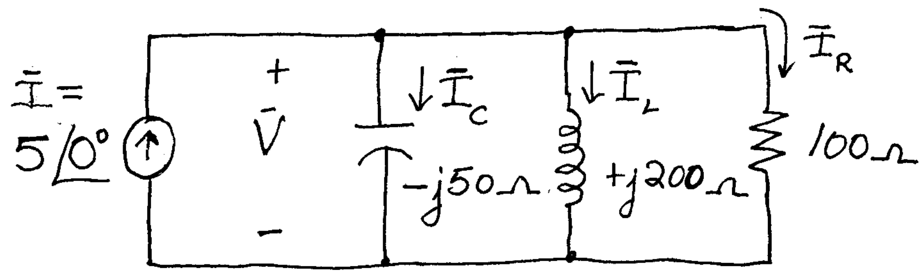
$i(t) = 28.28\cos(500t - 135^\circ)$ mA

$\mathbf{V}_R = R\mathbf{I} = 7.07\angle -135^\circ \qquad \mathbf{V}_L = j\omega L\mathbf{I} = 7.07\angle -45^\circ$

(b) The phasor diagram is shown in Figure 5.17b in the book.

(c) $i(t)$ lags $v_s(t)$ by 45°.

E5.10 The transformed network is:



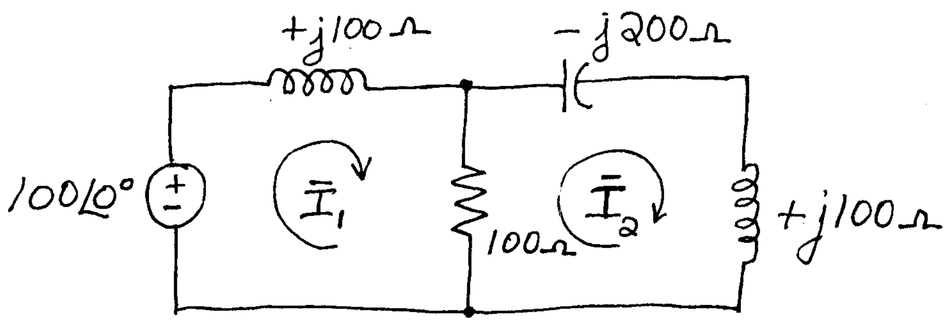
$$Z = \frac{1}{1/100 + 1/(-j50) + 1/(+j200)} = 55.47\angle -56.31^\circ \ \Omega$$

$\mathbf{V} = Z\mathbf{I} = 277.4\angle -56.31^\circ$ V $\qquad \mathbf{I}_C = \mathbf{V}/(-j50) = 5.547\angle 33.69^\circ$ A

$\mathbf{I}_L = \mathbf{V}/(j200) = 1.387\angle -146.31^\circ$ A

$\mathbf{I}_R = \mathbf{V}/(100) = 2.774\angle -56.31^\circ$ A

E5.11 The transformed network is:



We write KVL equations for each of the meshes:

$$j100\mathbf{I}_1 + 100(\mathbf{I}_1 - \mathbf{I}_2) = 100$$

$$-j200\mathbf{I}_2 + j100\mathbf{I}_2 + 100(\mathbf{I}_2 - \mathbf{I}_1) = 0$$

Simplifying, we have

$$(100 + j100)\mathbf{I}_1 - 100\mathbf{I}_2 = 100$$

$$-100\mathbf{I}_1 + (100 - j100)\mathbf{I}_2 = 0$$

Solving we find $\mathbf{I}_1 = 1.414\angle -45^\circ$ A and $\mathbf{I}_2 = 1\angle 0^\circ$ A. Thus we have $i_1(t) = 1.414\cos(1000t - 45^\circ)$ A and $i_2(t) = \cos(1000t)$.

E5.12 (a) For a power factor of 100%, we have $\cos(\theta) = 1$, which implies that the current and voltage are in phase and $\theta = 0$. Thus, $Q = P\tan(\theta) = 0$. Also $I_{rms} = P/[V_{rms}\cos(\theta)] = 5000/[500\cos(0)] = 10$ A. Thus we have $I_m = I_{rms}\sqrt{2} = 14.14$ and $\mathbf{I} = 14.14\angle 40^\circ$.

(b) For a power factor of 20% lagging, we have $\cos(\theta) = 0.2$, which implies that the current lags the voltage by $\theta = \cos^{-1}(0.2) = 78.46^\circ$. Thus, $Q = P\tan(\theta) = 24.49$ kVAR. Also, we have $I_{rms} = P/[V_{rms}\cos(\theta)] = 50.0$ A. Thus we have $I_m = I_{rms}\sqrt{2} = 70.71$ A and $\mathbf{I} = 70.71\angle -38.46^\circ$.

(c) The current ratings would need to be five times higher for the load of part (b) than for that of part (a). Wiring costs would be lower for the load of part (a).

E5.13 The first load is a 10 μF capacitor for which we have

$Z_C = 1/(j\omega C) = 265.3\angle -90^\circ\ \Omega \quad \theta_C = -90^\circ \quad I_{Crms} = V_{rms}/|Z_C| = 3.770$ A

$P_C = V_{rms}I_{Crms}\cos(\theta_C) = 0 \quad Q_C = V_{rms}I_{Crms}\sin(\theta_C) = -3.770$ kVAR

The second load absorbs an apparent power of $V_{rms}I_{rms} = 10$ kVA with a power factor of 80% lagging from which we have $\theta_2 = \cos^{-1}(0.8) = 36.87^\circ$. Notice that we select a positive angle for θ_2 because the load has a lagging power factor. Thus we have $P_2 = V_{rms}I_{2rms}\cos(\theta_2) = 8.0$ kW and $Q_2 = V_{rms}I_{2rms}\sin(\theta) = 6$ kVAR.

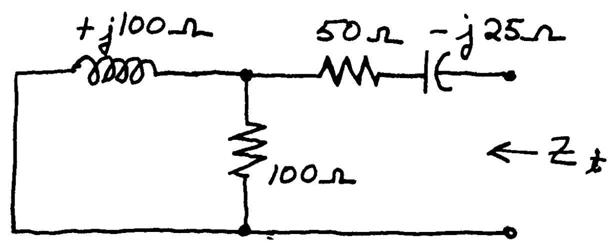
Now for the source we have:

$P_s = P_C + P_2 = 8$ kW $\quad Q_s = Q_C + Q_2 = 2.23$ kVAR

$V_{rms}I_{srms} = \sqrt{P_s^2 + Q_s^2} = 8.305$ kVA $\quad I_{srms} = V_{rms}I_{srms}/V_{rms} = 8.305$ A

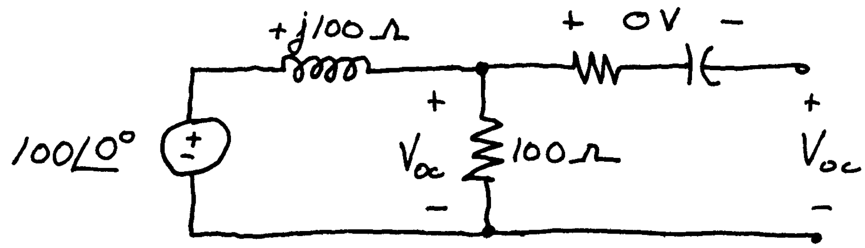
power factor $= P_s/(V_{rms}I_{srms}) \times 100\% = 96.33\%$

E5.14 First, we zero the source and combine impedances in series and parallel to determine the Thévenin impedance.



$$Z_t = 50 - j25 + \frac{1}{1/100 + 1/j100} = 50 - j25 + 50 + j50$$
$$= 100 + j25 = 103.1\angle 14.04^\circ$$

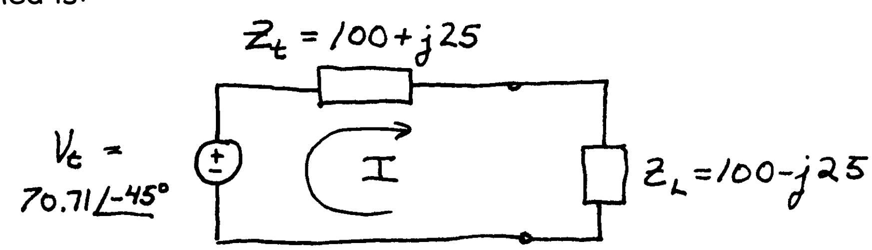
Then we analyze the circuit to determine the open-circuit voltage.



$$\mathbf{V}_t = \mathbf{V}_{oc} = 100 \times \frac{100}{100 + j100} = 70.71\angle -45^\circ$$

$$\mathbf{I}_n = V_t / Z_t = 0.6858\angle -59.04^\circ$$

E5.15 (a) For a complex load, maximum power is transferred for $Z_L = Z_t^* = 100 - j25 = R_L + jX_L$. The Thévenin equivalent with the load attached is:



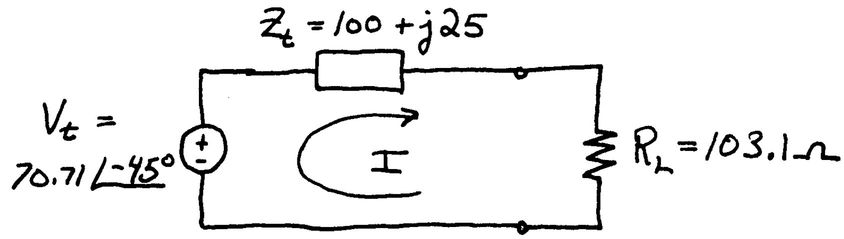
The current is given by

$$\mathbf{I} = \frac{70.71\angle -45^\circ}{100 + j25 + 100 - j25} = 0.3536\angle -45^\circ$$

The load power is

$$P_L = R_L I_{rms}^2 = 100(0.3536/\sqrt{2})^2 = 6.25\ \text{W}$$

(b) For a purely resistive load, maximum power is transferred for $R_L = |Z_t| = \sqrt{100^2 + 25^2} = 103.1\ \Omega$. The Thévenin equivalent with the load attached is:



The current is given by

$$\mathbf{I} = \frac{70.71\angle -45^\circ}{103.1 + 100 - j25} = 0.3456\angle -37.98^\circ$$

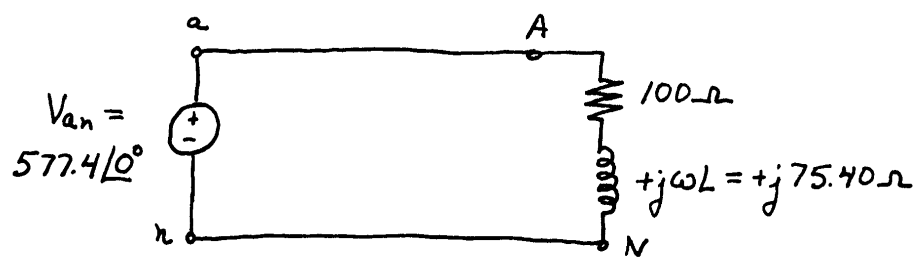
The load power is

$$P_L = R_L I_{rms}^2 = 103.1(0.3456/\sqrt{2})^2 = 6.157\ \text{W}$$

E5.16 The line-to-neutral voltage is $1000/\sqrt{3} = 577.4$ V. No phase angle was specified in the problem statement, so we will assume that the phase of $\mathbf{V}_{an}$ is zero. Then we have

$$\mathbf{V}_{an} = 577.4\angle 0^\circ \qquad \mathbf{V}_{bn} = 577.4\angle -120^\circ \qquad \mathbf{V}_{cn} = 577.4\angle 120^\circ$$

The circuit for the *a* phase is shown below. (We can consider a neutral connection to exist in a balanced Y-Y connection even if one is not physically present.)



The *a*-phase line current is

$$\mathbf{I}_{aA} = \frac{\mathbf{V}_{an}}{Z_L} = \frac{577.4\angle 0^\circ}{100 + j75.40} = 4.610\angle -37.02^\circ$$

The currents for phases *b* and *c* are the same except for phase.

$$\mathbf{I}_{bB} = 4.610\angle -157.02^\circ \qquad \mathbf{I}_{cC} = 4.610\angle 82.98^\circ$$

$$P = 3\frac{V_Y I_L}{2}\cos(\theta) = 3\frac{577.4 \times 4.610}{2}\cos(37.02^\circ) = 3.188\ \text{kW}$$

$$Q = 3\frac{V_Y I_L}{2}\sin(\theta) = 3\frac{577.4 \times 4.610}{2}\sin(37.02^\circ) = 2.404\ \text{kVAR}$$

E5.17 The a-phase line-to-neutral voltage is

$$\mathbf{V}_{an} = 1000 / \sqrt{3}\angle 0^\circ = 577.4\angle 0^\circ$$

The phase impedance of the equivalent Y is $Z_y = Z_\Delta / 3 = 50/3 = 16.67\ \Omega$.

Thus the line current is

$$\mathbf{I}_{aA} = \frac{\mathbf{V}_{an}}{Z_y} = \frac{577.4\angle 0^\circ}{16.67} = 34.63\angle 0^\circ\ A$$

Similarly, $\mathbf{I}_{bB} = 34.63\angle -120^\circ\ A$ and $\mathbf{I}_{cC} = 34.63\angle 120^\circ\ A$.

Finally, the power is

$$P = 3(I_{aA}/\sqrt{2})^2 R_y = 30.00\ \text{kW}$$

E5.18 Writing KCL equations at nodes 1 and 2 we obtain

$$\frac{\mathbf{V}_1}{100 + j30} + \frac{\mathbf{V}_1 - \mathbf{V}_2}{50 - j80} = 1\angle 60^\circ$$

$$\frac{\mathbf{V}_2}{j50} + \frac{\mathbf{V}_2 - \mathbf{V}_1}{50 - j80} = 2\angle 30^\circ$$

In matrix form, these become

$$\begin{bmatrix} \left(\frac{1}{100 + j30} + \frac{1}{50 - j80}\right) & -\frac{1}{50 - j80} \\ -\frac{1}{50 - j80} & \left(\frac{1}{j50} + \frac{1}{50 - j80}\right) \end{bmatrix} \begin{bmatrix} \mathbf{V}_1 \\ \mathbf{V}_2 \end{bmatrix} = \begin{bmatrix} 1\angle 60^\circ \\ 2\angle 30^\circ \end{bmatrix}$$

The MATLAB commands are

```
Y = [(1/(100+j*30)+1/(50-j*80))  (-1/(50-j*80));...
     (-1/(50-j*80))  (1/(j*50)+1/(50-j*80))];
I = [pin(1,60); pin(2,30)];
V = inv(Y)*I;
pout(V(1))
pout(V(2))
```

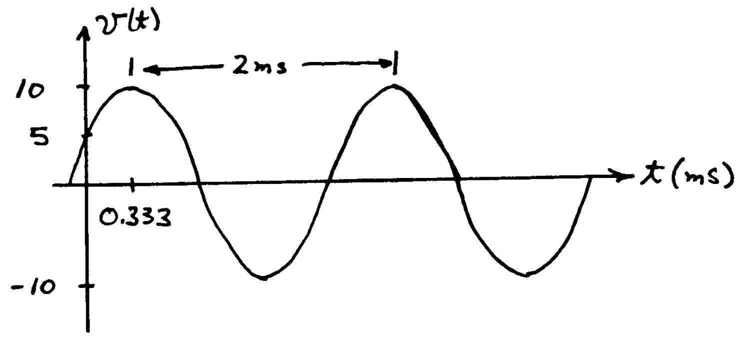
The results are

$$\mathbf{V}_1 = 79.98\angle 106.21^\circ \quad \text{and} \quad \mathbf{V}_2 = 124.13\angle 116.30^\circ$$

Answers for Selected Problems

P5.4*

$$\omega = 1000\pi \text{ rad/s}$$
$$f = 500 \text{ Hz}$$
$$\text{phase angle} = \theta = -60^\circ = -\pi/3 \text{ radians}$$
$$T = 2 \text{ ms}$$
$$V_{rms} = 7.071 \text{ V}$$
$$P = 1 \text{ W}$$
$$t_{peak} = 0.3333 \text{ ms}$$



P5.6* $v(t) = 28.28\cos(2\pi 10^4 t - 72^\circ)$ V

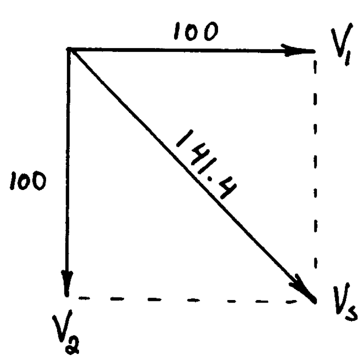
P5.12* $V_{rms} = 10.61$ V

P5.13* $V_{rms} = 3.808$ A

P5.23* $5\cos(\omega t + 75^\circ) - 3\cos(\omega t - 75^\circ) + 4\sin(\omega t) = 3.763\cos(\omega t + 82.09^\circ)$

P5.24*

$$v_s(t) = 141.4\cos(\omega t - 45^\circ)$$



$\mathbf{V}_2$ lags $\mathbf{V}_1$ by 90°
$\mathbf{V}_s$ lags $\mathbf{V}_1$ by 45°
$\mathbf{V}_s$ leads $\mathbf{V}_2$ by 45°

P5.25*

$$v_1(t) = 10\cos(400\pi t + 30^\circ)$$
$$v_2(t) = 5\cos(400\pi t + 150^\circ)$$
$$v_3(t) = 10\cos(400\pi t + 90^\circ)$$

$v_1(t)$ lags $v_2(t)$ by 120°

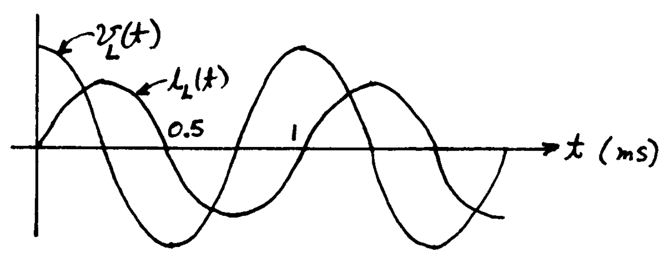
$v_1(t)$ lags $v_3(t)$ by 60°

$v_2(t)$ leads $v_3(t)$ by 60°

P5.35*

$$Z_L = 200\pi\angle 90^\circ$$
$$\mathbf{V}_L = 10\angle 0^\circ$$
$$\mathbf{I}_L = (1/20\pi)\angle -90^\circ$$
$$i_L(t) = (1/20\pi)\cos(2000\pi t - 90^\circ) = (1/20\pi)\sin(2000\pi t)$$

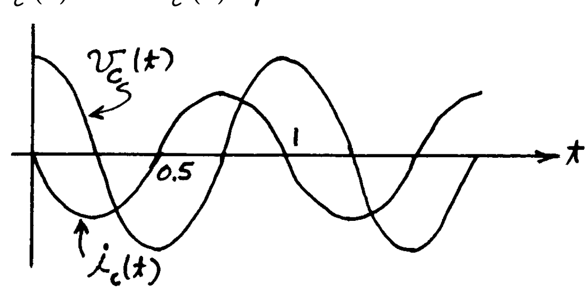
$i_L(t)$ lags $v_L(t)$ by 90°



P5.37*

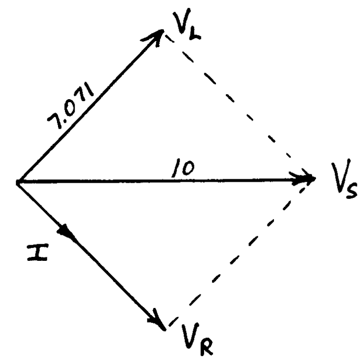
$$Z_C = 15.92\angle -90^\circ\ \Omega$$
$$\mathbf{V}_C = 10\angle 0^\circ$$
$$\mathbf{I}_C = \mathbf{V}_C / Z_C = 0.6283\angle 90^\circ$$
$$i_C(t) = 0.6283\cos(2000\pi t + 90^\circ) = -0.6283\sin(2000\pi t)$$

$i_C(t)$ leads $v_C(t)$ by 90°



P5.42*

$$\mathbf{I} = 70.71\angle -45^\circ \text{ mA}$$
$$\mathbf{V}_R = 7.071\angle -45^\circ \text{ V}$$
$$\mathbf{V}_L = 7.071\angle 45^\circ \text{ V}$$

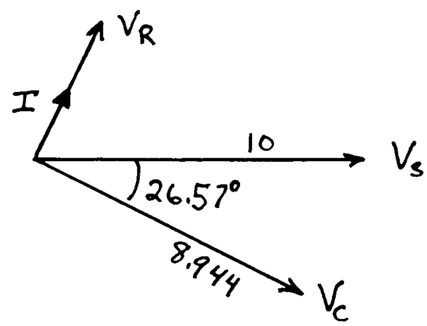


$\mathbf{I}$ lags $\mathbf{V}_s$ by 45°

P5.44*

$$\mathbf{I} = 4.472\angle 63.43^\circ \text{ mA}$$
$$\mathbf{V}_R = 4.472\angle 63.43^\circ \text{ V}$$
$$\mathbf{V}_C = 8.944\angle -26.57^\circ \text{ V}$$

$\mathbf{I}$ leads $\mathbf{V}_s$ by 63.43°



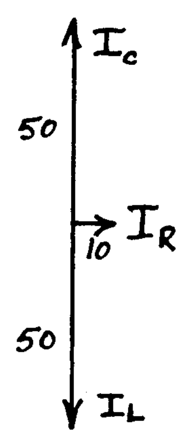
P5.46* $\omega = 500: \quad Z = 158.1\angle -71.57^\circ$

$\omega = 1000: \quad Z = 50\angle 0^\circ$

$\omega = 2000: \quad Z = 158.1\angle 71.57^\circ$

P5.49*

$$\mathbf{I}_R = 10\angle 0^\circ \text{ mA}$$
$$\mathbf{I}_L = 50\angle -90^\circ \text{ mA}$$
$$\mathbf{I}_C = 50\angle 90^\circ \text{ mA}$$

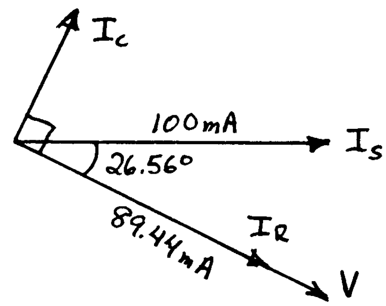


The peak value of $i_L(t)$ is five times larger than the source current!

P5.52*

$$\mathbf{V} = 8.944\angle -26.56^\circ \text{ V}$$
$$\mathbf{I}_R = 89.44\angle -26.56^\circ \text{ mA}$$
$$\mathbf{I}_C = 44.72\angle 63.44^\circ \text{ mA}$$

$\mathbf{V}$ lags $\mathbf{I}_s$ by 26.56°



P5.67*

$$\mathbf{I} = 15.11\angle 20.66^\circ$$
$$P = 10 \text{ kW}$$
$$Q = -3.770 \text{ kVAR}$$
$$\text{Apparent power} = 10.68 \text{ kVA}$$
$$\text{Power factor} = 93.57\% \text{ leading}$$

P5.69* This is a capacitive load.

$P = 22.5$ kW

$Q = -11.25$ kVAR

power factor $= 89.44\%$

apparent power $= \sqrt{P^2 + Q^2} = 25.16$ KVA

P5.78*

$$P_s = 22 \text{ kW}$$
$$Q_s = 13.84 \text{ kVAR}$$
$$\text{Apparent power} = 26 \text{ kVA}$$
$$\text{Power factor} = 84.62\% \text{ lagging}$$

P5.83* (a)

$$\mathbf{I} = 400\sqrt{2}\angle -75.52^\circ$$

(b)

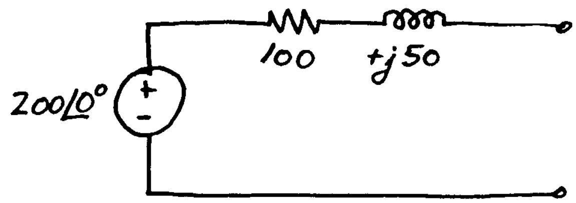
$$C = 1027\,\mu\text{F}$$

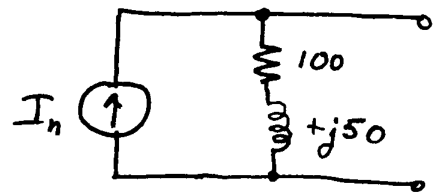
The capacitor must be rated for at least 387.3 kVAR.

$$\mathbf{I} = 100\angle 0^\circ$$

(c) The line current is smaller by a factor of 4 with the capacitor in place, reducing I^2R losses in the line by a factor of 16.

P5.87* (a) $\mathbf{I}_n = 1.789\angle -26.57^\circ$





(b) $P_{load} = 50 \text{ W}$

(c) $P_{load} = 47.21 \text{ W}$

P5.91* $R_{load} = 12.5\ \Omega$
$C_{load} = 106.1\ \mu\text{F}$

P5.95* $Z_\Delta = 70.29\angle -62.05^\circ\ \Omega$

P5.96*
$V_L = 762.1 \text{ V rms}$
$I_L = 14.67 \text{ A rms}$
$P = 19.36 \text{ kW}$

P5.99*
$\mathbf{I}_{aA} = 59.87\angle 0^\circ$
$\mathbf{V}_{An} = 322.44\angle -21.80^\circ$
$\mathbf{V}_{AB} = 558\angle 8.20^\circ$
$\mathbf{I}_{AB} = 34.56\angle 30^\circ$
$P_{load} = 26.89 \text{ kW}$
$P_{line} = 5.38 \text{ kW}$

P5.105* $\mathbf{V}_1 = 9.402\angle 29.58^\circ$
$\mathbf{V}_2 = 4.986\angle 111.45^\circ$

P5.107* $\mathbf{I}_1 = 1.372\angle 120.96^\circ$
$\mathbf{I}_2 = 1.955\angle 136.22^\circ$

Practice Test

T5.1 $I_{rms} = \sqrt{\frac{1}{T}\int_0^T i^2(t)dt} = \sqrt{\frac{1}{3}\int_0^2 (3t)^2 dt} = \sqrt{t^3\Big|_0^2} = \sqrt{8} = 2.828\ A$

$P = I_{rms}^2 R = 8(50) = 400\ \ \text{W}$

T5.2 $\mathbf{V} = 5\angle -45^\circ + 5\angle -30^\circ = 3.5355 - j3.5355 + 4.3301 - j2.5000$

$\mathbf{V} = 7.8657 - j6.0355 = 9.9144\angle -37.50^\circ$

$v(t) = 9.914\cos(\omega t - 37.50^\circ)$

T5.3 (a) $V_{1rms} = \frac{15}{\sqrt{2}} = 10.61\,\text{V}$

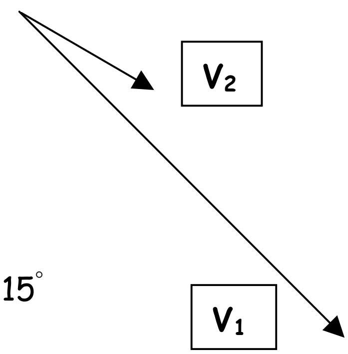
(b) $f = 200\ \text{Hz}$

(c) $\omega = 400\pi$ radians/s

(d) $T = 1/f = 5\ \text{ms}$

(e) $\mathbf{V}_1 = 15\angle -45^\circ$ and $\mathbf{V}_2 = 5\angle -30^\circ$

V$_1$ lags **V**$_2$ by 15° or **V**$_2$ leads **V**$_1$ by 15°



T5.4 $\mathbf{I} = \frac{\mathbf{V}_s}{R + j\omega L - j/\omega C} = \frac{10\angle 0^\circ}{10 + j15 - j5} = \frac{10\angle 0^\circ}{14.14\angle 45^\circ} = 0.7071\angle -45^\circ\ \ A$

$\mathbf{V}_R = 10\mathbf{I} = 7.071\angle -45^\circ\,\text{V}$ $\qquad \mathbf{V}_L = j15\mathbf{I} = 10.606\angle 45^\circ\ \ \text{V}$

$\mathbf{V}_C = -j5\mathbf{I} = 5.303\angle -135^\circ\ \ \text{V}$

T5.5 $\mathbf{S} = \frac{1}{2}\mathbf{VI}^* = \frac{1}{2}(440\angle 30^\circ)(25\angle 10^\circ) = 5500\angle 40^\circ = 4213 + j3535\ \ \text{VA}$

$P = \text{Re}(\mathbf{S}) = 4213\,\text{W}$

$Q = \text{Im}(\mathbf{S}) = 3535\,\text{VAR}$

Apparent power $= |\mathbf{S}| = 5500\,\text{VA}$

Power factor $= \cos(\theta_V - \theta_I) = \cos(40^\circ) = 76.6\%$ lagging

T5.6 We convert the delta to a wye and connect the neutral points with an ideal conductor.

$Z_Y = Z_\Delta / 3 = 2 + j8/3$

$Z_{total} = Z_{line} + Z_Y = 0.3 + j0.4 + 2 + j2.667 = 2.3 + j3.067$

$$Z_{total} = 3.833\angle 53.13°$$
$$\mathbf{I}_{aA} = \frac{\mathbf{V}_{an}}{Z_{total}} = \frac{208\angle 30°}{3.833\angle 53.13°} = 54.26\angle -23.13° \text{ A}$$

T5.7 The mesh equations are:

$$j10\mathbf{I}_1 + 15(\mathbf{I}_1 - \mathbf{I}_2) = 10\angle 45°$$
$$-j5\mathbf{I}_2 + 15(\mathbf{I}_2 - \mathbf{I}_1) = -15$$

In matrix form these become

$$\begin{bmatrix} (15 + j10) & -15 \\ -15 & (15 - j5) \end{bmatrix} \begin{bmatrix} \mathbf{I}_1 \\ \mathbf{I}_2 \end{bmatrix} = \begin{bmatrix} 10\angle 45° \\ -15 \end{bmatrix}$$

The commands are:

```
Z = [(15+j*10)  -15; -15  (15-j*5)]
V = [pin(10,45); -15]
I = inv(Z)*V
pout(I(1))
pout(I(2))
```

CHAPTER 6

Exercises

E6.1 (a) The frequency of $v_{in}(t) = 2\cos(2\pi \cdot 2000t)$ is 2000 Hz. For this frequency $H(f) = 2\angle 60^\circ$. Thus, $\mathbf{V}_{out} = H(f)\mathbf{V}_{in} = 2\angle 60^\circ \times 2\angle 0^\circ = 4\angle 60^\circ$ and we have $v_{out}(t) = 4\cos(2\pi \cdot 2000t + 60^\circ)$.

(b) The frequency of $v_{in}(t) = \cos(2\pi \cdot 3000t - 20^\circ)$ is 3000 Hz. For this frequency $H(f) = 0$. Thus, $\mathbf{V}_{out} = H(f)\mathbf{V}_{in} = 0 \times 2\angle 0^\circ = 0$ and we have $v_{out}(t) = 0$.

E6.2 The input signal $v(t) = 2\cos(2\pi \cdot 500t + 20^\circ) + 3\cos(2\pi \cdot 1500t)$ has two components with frequencies of 500 Hz and 1500 Hz. For the 500-Hz component we have:

$$\mathbf{V}_{out,1} = H(500)\mathbf{V}_{in} = 3.5\angle 15^\circ \times 2\angle 20^\circ = 7\angle 35^\circ$$

$$v_{out,1}(t) = 7\cos(2\pi \cdot 500t + 35^\circ)$$

For the 1500-Hz component:

$$\mathbf{V}_{out,2} = H(1500)\mathbf{V}_{in} = 2.5\angle 45^\circ \times 3\angle 0^\circ = 7.5\angle 45^\circ$$

$$v_{out,2}(t) = 7.5\cos(2\pi \cdot 1500t + 45^\circ)$$

Thus the output for both components is

$$v_{out}(t) = 7\cos(2\pi \cdot 500t + 35^\circ) + 7.5\cos(2\pi \cdot 1500t + 45^\circ)$$

E6.3 The input signal $v(t) = 1 + 2\cos(2\pi \cdot 1000t) + 3\cos(2\pi \cdot 3000t)$ has three components with frequencies of 0, 1000 Hz and 3000 Hz.

For the dc component, we have

$$v_{out,1}(t) = H(0) \times v_{in,1}(t) = 4 \times 1 = 4$$

For the 1000-Hz component, we have:

$$\mathbf{V}_{out,2} = H(1000)\mathbf{V}_{in,2} = 3\angle 30^\circ \times 2\angle 0^\circ = 6\angle 30^\circ$$

$$v_{out,1}(t) = 6\cos(2\pi \cdot 1000t + 30^\circ)$$

For the 3000-Hz component:

$$\mathbf{V}_{out,3} = H(3000)\mathbf{V}_{in,3} = 0 \times 3\angle 0^\circ = 0$$

$$v_{out,3}(t) = 0$$

Thus, the output for all three components is

$$v_{out}(t) = 4 + 6\cos(2\pi \cdot 1000t + 30^\circ)$$

E6.4 Using the voltage-division principle, we have:

$$\mathbf{V}_{\text{out}} = \mathbf{V}_{\text{in}} \times \frac{R}{R + j2\pi fL}$$

Then the transfer function is:

$$H(f) = \frac{\mathbf{V}_{\text{out}}}{\mathbf{V}_{\text{in}}} = \frac{R}{R + j2\pi fL} = \frac{1}{1 + j2\pi fL/R} = \frac{1}{1 + jf/f_B}$$

E6.5 From Equation 6.9, we have $f_B = 1/(2\pi RC) = 200\ \text{Hz}$, and from Equation 6.9, we have $H(f) = \dfrac{\mathbf{V}_{\text{out}}}{\mathbf{V}_{\text{in}}} = \dfrac{1}{1 + jf/f_B}$.

For the first component of the input, the frequency is 20 Hz,
$H(f) = 0.995\angle -5.71^\circ$, $\mathbf{V}_{\text{in}} = 10\angle 0^\circ$, and $\mathbf{V}_{\text{out}} = H(f)\mathbf{V}_{\text{in}} = 9.95\angle -5.71^\circ$
Thus the first component of the output is

$$v_{\text{out},1}(t) = 9.95\cos(40\pi t - 5.71^\circ)$$

For the second component of the input, the frequency is 500 Hz,
$H(f) = 0.371\angle -68.2^\circ$, $\mathbf{V}_{\text{in}} = 5\angle 0^\circ$, and $\mathbf{V}_{\text{out}} = H(f)\mathbf{V}_{\text{in}} = 1.86\angle -68.2^\circ$
Thus the second component of the output is

$$v_{\text{out},2}(t) = 1.86\cos(40\pi t - 68.2^\circ)$$

For the third component of the input, the frequency is 10 kHz,
$H(f) = 0.020\angle -88.9^\circ$, $\mathbf{V}_{\text{in}} = 5\angle 0^\circ$, and $\mathbf{V}_{\text{out}} = H(f)\mathbf{V}_{\text{in}} = 0.100\angle -88.9^\circ$
Thus the third component of the output is

$$v_{\text{out},2}(t) = 0.100\cos(2\pi \times 10^4 t - 88.9^\circ)$$

Finally, the output with for all three components is:

$$v_{\text{out}}(t) = 9.95\cos(40\pi t - 5.71^\circ) + 1.86\cos(40\pi t - 68.2^\circ)$$
$$+ 0.100\cos(2\pi \times 10^4 t - 88.9^\circ)$$

E6.6 $\left|H(f)\right|_{\text{dB}} = 20\log\left|H(f)\right| = 20\log(50) = 33.98\ \text{dB}$

E6.7 (a) $\left|H(f)\right|_{\text{dB}} = 20\log\left|H(f)\right| = 15\ \text{dB}$

$\log\left|H(f)\right| = 15/20 = 0.75$

$H(f) = 10^{0.75} = 5.623$

(b) $\left|H(f)\right|_{\text{dB}} = 20\log\left|H(f)\right| = 30 \text{ dB}$

$\log\left|H(f)\right| = 30/20 = 1.5$

$H(f) = 10^{1.5} = 31.62$

E6.8 (a) $1000 \times 2^2 = 4000 \text{ Hz}$ is two octaves higher than 1000 Hz.

(b) $1000/2^3 = 125$ Hz is three octaves lower than 1000 Hz.

(c) $1000 \times 10^2 = 100$ kHz is two decades higher than 1000 Hz.

(d) $1000/10 = 100$ Hz is one decade lower than 1000 Hz.

E6.9 (a) To find the frequency halfway between two frequencies on a logarithmic scale, we take the logarithm of each frequency, average the logarithms, and then take the antilogarithm. Thus

$$f = 10^{[\log(100)+\log(1000)]/2} = 10^{2.5} = 316.2 \text{ Hz}$$

is half way between 100 Hz and 1000 Hz on a logarithmic scale.

(b) To find the frequency halfway between two frequencies on a linear scale, we simply average the two frequencies. Thus (100 + 1000)/2 = 550 Hz is halfway between 100 and 1000 Hz on a linear scale.

E6.10 To determine the number of decades between two frequencies we take the difference between the common (base-ten) logarithms of the two frequencies. Thus 20 Hz and 15 kHz are $\log(15 \times 10^3) - \log(20) = 2.875$ decades apart.

Similarly, to determine the number of octaves between two frequencies we take the difference between the base-two logarithms of the two frequencies. One formula for the base-two logarithm of z is

$$\log_2(z) = \frac{\log(z)}{\log(2)} \cong 3.322\log(z)$$

Thus the number of octaves between 20 Hz and 15 kHz is

$$\frac{\log(15 \times 10^3)}{\log(2)} - \frac{\log(20)}{\log(2)} = 9.551$$

E6.11 The transfer function for the circuit shown in Figure 6.17 in the book is

$$H(f) = \frac{\mathbf{V}_{\text{out}}}{\mathbf{V}_{\text{in}}} = \frac{1/(j2\pi fC)}{R + 1/(j2\pi fC)} = \frac{1}{1 + j2\pi RCf} = \frac{1}{1 + jf/f_B}$$

in which $f_B = 1/(2\pi RC) = 1000$ Hz. Thus the magnitude plot is approximated by 0 dB below 1000 Hz and by a straight line sloping downward at 20 dB/decade above 1000 Hz. This is shown in Figure 6.18a in the book.

The phase plot is approximated by 0° below 100 Hz, by -90° above 10 kHz and by a line sloping downward between 0° at 100 Hz and -90° at 10 kHz. This is shown in Figure 6.18b in the book.

E6.12 Using the voltage division principle, the transfer function for the circuit shown in Figure 6.19 in the book is

$$H(f) = \frac{\mathbf{V}_{\text{out}}}{\mathbf{V}_{\text{in}}} = \frac{R}{R + 1/(j2\pi fC)} = \frac{j2\pi RC}{1 + j2\pi RCf} = \frac{j(f/f_B)}{1 + j(f/f_B)}$$

in which $f_B = 1/(2\pi RC)$.

E6.13 Using the voltage division principle, the transfer function for the circuit shown in Figure 6.22 in the book is

$$H(f) = \frac{\mathbf{V}_{\text{out}}}{\mathbf{V}_{\text{in}}} = \frac{j2\pi fL}{R + j2\pi fL} = \frac{j2\pi fL/R}{1 + j2\pi fL/R} = \frac{j(f/f_B)}{1 + j(f/f_B)}$$

in which $f_B = R/(2\pi L)$.

E6.14 A first-order filter has a transfer characteristic that decreases by 20 dB/decade below the break frequency. To attain an attenuation of 50 dB the signal frequency must be 50/20 = 2.5 decades below the break frequency. 2.5 decades corresponds to a frequency ratio of $10^{2.5} = 316.2$. Thus to attenuate a 1000 Hz signal by 50 dB the high-pass filter must have a break frequency of 316.2 kHz. Solving Equation 6.22 for capacitance and substituting values, we have

$$C = \frac{1}{2\pi f_B R} = \frac{1}{2\pi \times 1000 \times 316.2 \times 10^3} = 503.3\,\text{pF}$$

E6.15 $C = \dfrac{1}{\omega_0^2 L} = \dfrac{1}{(2\pi f_0)^2 L} = \dfrac{1}{(2\pi 10^6)^2 10 \times 10^{-6}} = 2533\,\text{pF}$

$R = \omega_0 L / Q_s = 1.257\ \Omega$

$B = f_0 / Q_s = 20\,\text{kHz}$

$f_L \cong f_0 - B/2 = 990\,\text{kHz}$

$f_H \cong f_0 + B/2 = 1010\,\text{kHz}$

E6.16 At resonance we have

$$\mathbf{V}_R = \mathbf{V}_s = 1\angle 0^\circ$$

$$\mathbf{V}_L = j\omega_0 L\mathbf{I} = j\omega_0 L\mathbf{V}_s / R = jQ_s\mathbf{V}_s = 50\angle 90^\circ \text{ V}$$

$$\mathbf{V}_C = (1/j\omega_0 C)\mathbf{I} = (1/j\omega_0 C)\mathbf{V}_s / R = -jQ_s\mathbf{V}_s = 50\angle -90^\circ \text{ V}$$

E6.17 $$L = \frac{1}{\omega_0^2 C} = \frac{1}{(2\pi f_0)^2 C} = \frac{1}{(2\pi \times 5 \times 10^6)^2 470 \times 10^{-12}} = 2.156\ \mu\text{H}$$

$$Q_s = f_0 / B = (5 \times 10^6)/(200 \times 10^3) = 25$$

$$R = \frac{1}{\omega_0 C Q_s} = \frac{1}{2\pi \times 5 \times 10^6 \times 470 \times 10^{-12} \times 25} = 2.709\ \Omega$$

E6.18 $$f_0 = \frac{1}{2\pi\sqrt{LC}} = 711.8\text{ kHz} \qquad Q_p = \frac{R}{\omega_0 L} = 22.36 \qquad B = f_0 / Q_p = 31.83\text{ kHz}$$

E6.19 $$Q_p = f_0 / B = 50 \qquad L = \frac{R}{\omega_0 Q_p} = 0.3183\ \mu\text{H} \qquad C = \frac{Q_p}{\omega_0 R} = 795.8\text{ pF}$$

E6.20 A second order lowpass filter with f_0 = 5 kHz is needed. The circuit configuration is shown in Figure 6.34a in the book. The normalized transfer function is shown in Figure 6.34c. Usually we would want a filter without peaking and would design for Q = 1. Given that L = 5 mH, the other component values are

$$R = \frac{2\pi f_0 L}{Q} = 157.1\ \Omega \qquad C = \frac{1}{(2\pi f_0)^2 L} = 0.2026\ \mu\text{F}$$

The circuit is shown in Figure 6.39 in the book.

E6.21 We need a bandpass filter with $f_L = 45$ kHz and $f_H = 55$ kHz. Thus we have

$$f_0 \cong \frac{f_L + f_H}{2} = 50\text{ kHz} \qquad B = f_H - f_L = 10\text{ kHz} \qquad Q = f_0 / B = 5$$

$$R = \frac{2\pi f_0 L}{Q} = 62.83\ \Omega \qquad C = \frac{1}{(2\pi f_0)^2 L} = 10.13\text{ nF}$$

The circuit is shown in Figure 6.40 in the book.

E6.22 The files Example_6_8 and Example_6_9 can be found in the MATLAB folder on the OrCAD disk. The results should be similar to Figures 6.42 and 6.44.

E6.23 (a) Rearranging Equation 6.56, we have

$$\frac{\tau}{T} = \frac{a}{1-a} = \frac{0.9}{1-0.9} = 0.9$$

Thus we have $\tau = 9T$.

(b) From Figure 6.49 in the book we see that the step response of the digital filter reaches 0.632 at approximately $n = 9$. Thus the speed of response of the RC filter and the corresponding digital filter are comparable.

E6.24 Writing a current equation at the node joining the resistance and capacitance, we have

$$\frac{y(t)}{R} + C\frac{d[y(t)-x(t)]}{dt} = 0$$

Multiplying both sides by R and using the fact that the time constant is $\tau = RC$, we have

$$y(t) + \tau\frac{dy(t)}{dt} - \tau\frac{dx(t)}{dt} = 0$$

Next we approximate the derivatives as

$$\frac{dx(t)}{dt} \cong \frac{\Delta x}{\Delta t} = \frac{x(n)-x(n-1)}{T} \quad \text{and} \quad \frac{dy(t)}{dt} \cong \frac{\Delta y}{\Delta t} = \frac{y(n)-y(n-1)}{T}$$

which yields

$$y(n) + \tau\frac{y(n)-y(n-1)}{T} - \tau\frac{x(n)-x(n-1)}{T} = 0$$

Solving for $y(n)$, we obtain

$$y(n) = a_1 y(n-1) + b_0 x(n) + b_1 x(n-1)$$

in which

$$a_1 = b_0 = -b_1 = \frac{\tau / T}{1 + \tau / T}$$

E6.25 (a) Solving Equation 6.58 for d and substituting values, we obtain

$$d = \frac{f_s}{2f_{notch}} = \frac{10^4}{2\times 500} = 10$$

(b) Repeating for f_{notch} = 300 Hz, we have

$$d = \frac{f_s}{2f_{notch}} = \frac{10^4}{2 \times 300} = 16.67$$

However, d is required to be an integer value so we cannot obtain a notch filter for 300 Hz exactly for this sampling frequency. (Possibly other more complex filters could provide the desired performance.)

Answers for Selected Problems

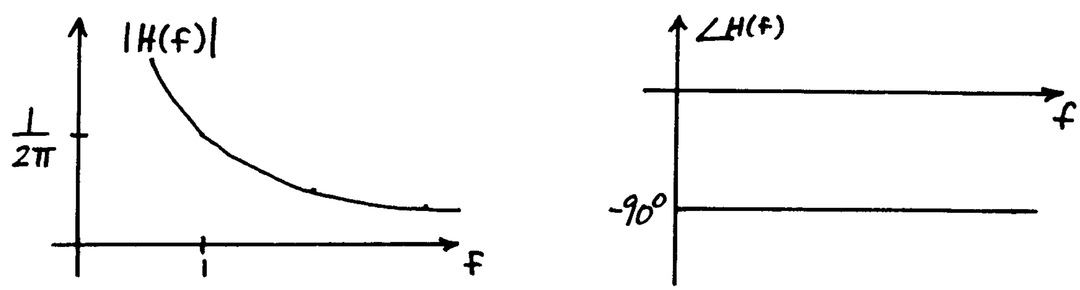
P6.8* $v_{out}(t) = 10 + 3.5\cos(2\pi 2500t - 15^\circ) + 2.5\cos(2\pi 7500t - 135^\circ)$

P6.11* $H(5000) = 0.5\angle 45^\circ$

P6.12* $f = 250\ \text{Hz}$ $\qquad H(250) = \frac{\mathbf{V}_{out}}{\mathbf{V}_{in}} = 3\angle -45^\circ$

P6.13* $v_o(t) = 2$

P6.14* $H(f) = \frac{-j}{2\pi f}$

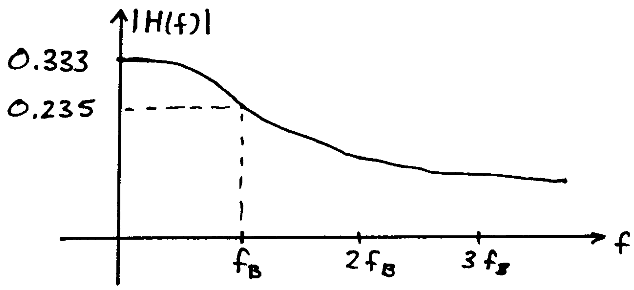


P6.23* For $\angle H(f) = -1^\circ$, we have $f = 0.01746 f_B$.
For $\angle H(f) = -10^\circ$, we have $f = 0.1763 f_B$.
For $\angle H(f) = -89^\circ$, we have $f = 57.29 f_B$.

P6.25* $v_{out}(t) = 4.472\cos(500\pi t - 26.57^\circ) + 3.535\cos(1000\pi t - 45^\circ) + 2.236\cos(2000\pi t - 63.43^\circ)$

P6.30* $f_B = 11.94\text{ Hz}$

$$\frac{\mathbf{V}_{out}}{\mathbf{V}_{in}} = \frac{1/3}{1 + j(f/f_B)}$$

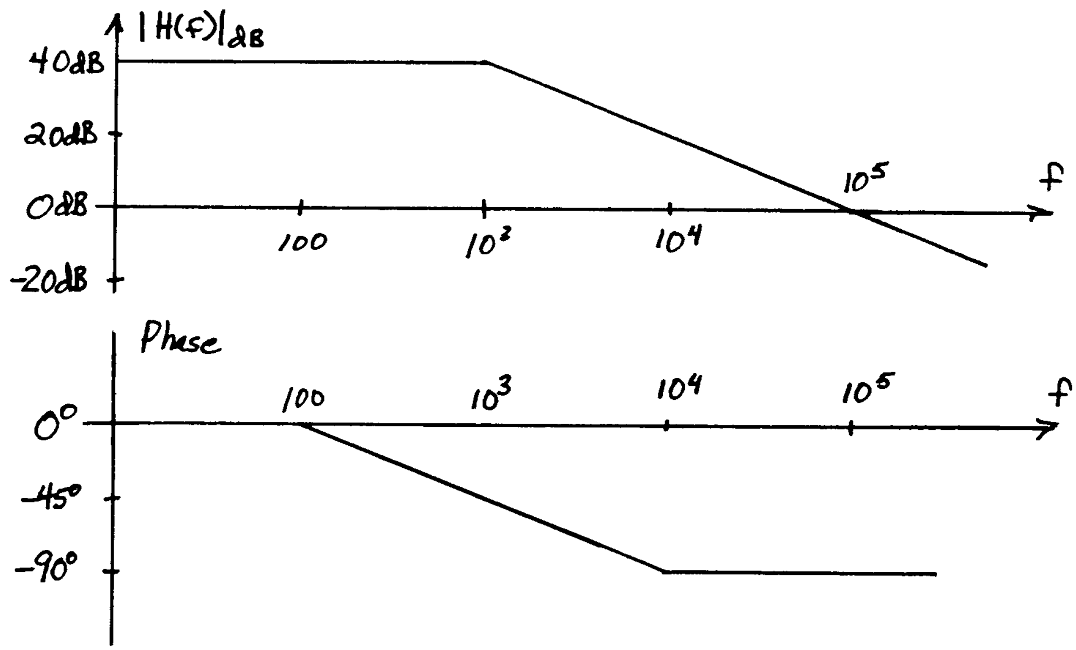


P6.40* (a) $|H(f)| = 0.3162$ (b) $|H(f)| = 3.162$

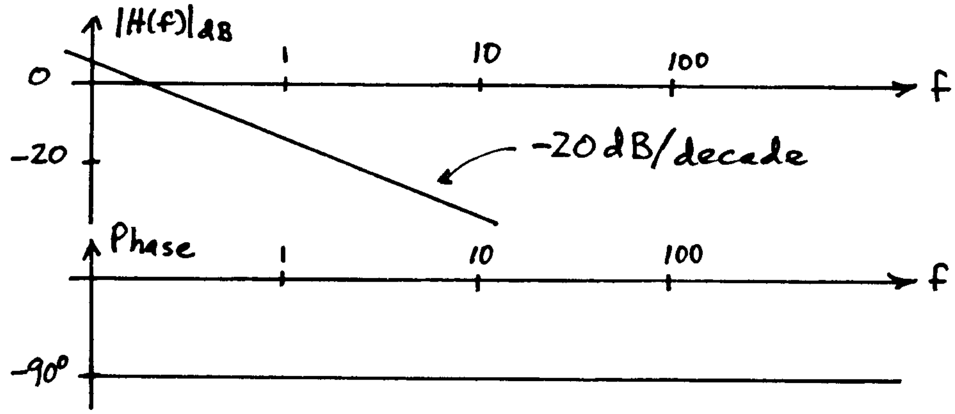
P6.41* (a) 547.7 Hz (b) 1550 Hz

P6.46* (a) $H(f) = \dfrac{1}{[1 + j(f/f_B)]^2}$ (b) $f_{3dB} = 0.6436 f_B$

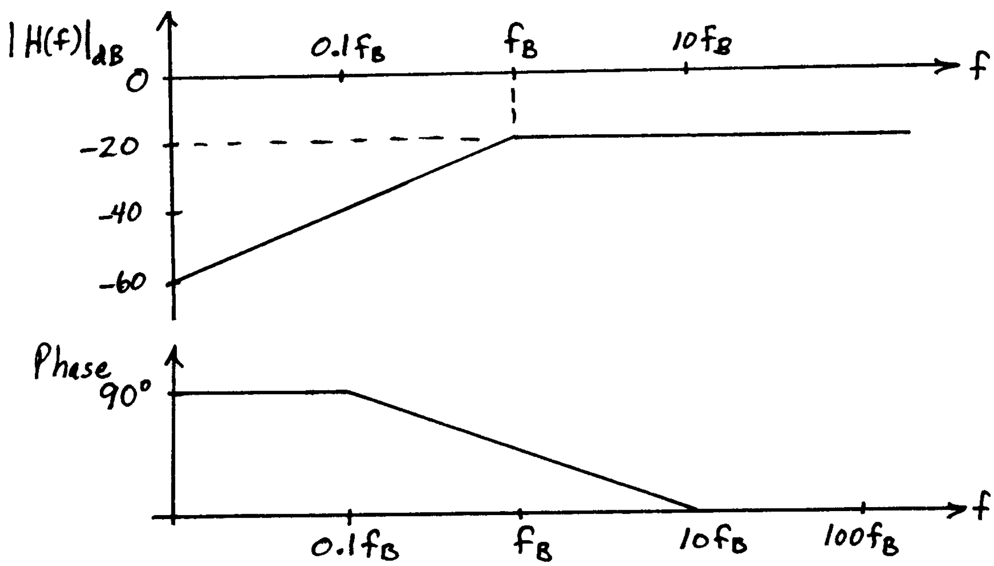
P6.52*



P6.60*



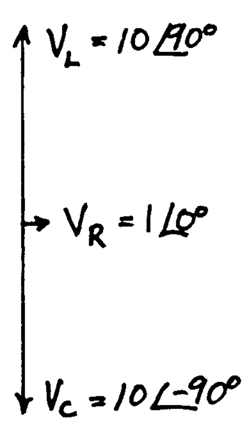
P6.64*



P6.65* $v_{out}(t) = 3.536\cos(2000\pi t + 45^{\circ})$

P6.72*

$$f_0 = 1.125\text{ MHz}$$
$$Q_s = 10$$
$$B = 112.5\text{ kHz}$$
$$f_H \cong 1.181\text{ MHz}$$
$$f_L \cong 1.069\text{ MHz}$$



P6.75* $L = 79.57\ \mu\text{H}$ $\quad \mathbf{V}_C = 20\angle -90^\circ$

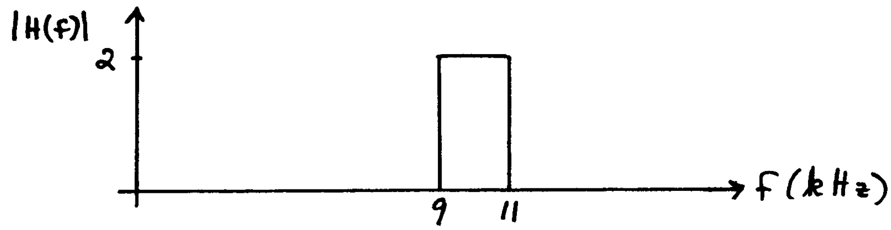
$C = 318.3\ \text{pF}$

P6.79* $f_0 = 1.592\ \text{MHz}$

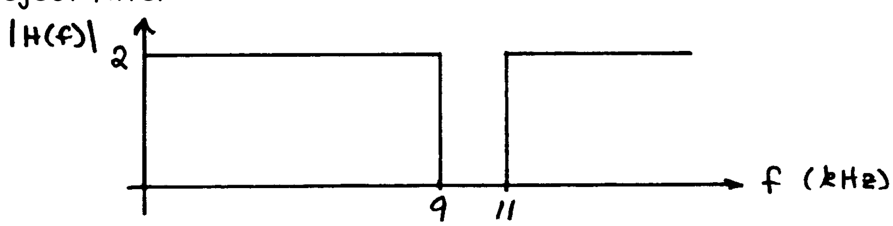
$Q_p = 10.00$

$B = 159.2\ \text{kHz}$

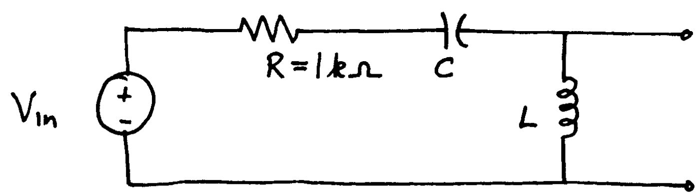
P6.84* Bandpass filter:



Band-reject filter:



P6.88*



$L = 1.592\ \text{mH}$ $\qquad C = 1592\ \text{pF}$

P6.104* $L = \dfrac{Q_s}{\omega_0}$ and $C = \dfrac{1}{\omega_0 Q_s}$

$$y(n) = \frac{\omega_0 T + 2Q_s}{Q_s + \omega_0^2 T^2 Q_s + \omega_0 T} y(n-1) - \frac{Q_s}{Q_s + \omega_0^2 T^2 Q_s + \omega_0 T} y(n-2)$$
$$+ \frac{\omega_0 T}{Q_s + \omega_0^2 T^2 Q_s + \omega_0 T}[x(n) - x(n-1)]$$

Practice Test

T6.1 All real-world signals (which are usually time-varying currents or voltages) are sums of sinewaves of various frequencies, amplitudes, and phases. The transfer function of a filter is a function of frequency that shows how the amplitudes and phases of the input components are altered to produce the output components.

T6.2 Applying the voltage-division principle, we have:

$$H(f) = \frac{\mathbf{V}_{out}}{\mathbf{V}_{in}} = \frac{j2\pi fL}{R + j2\pi fL} = \frac{j2\pi fL/R}{1 + j2\pi fL/R}$$
$$= \frac{j(f/f_B)}{1 + j(f/f_B)}$$

in which $f_B = R/2\pi L = 1000\text{ Hz}$. The input signal has components with frequencies of 0 (dc), 500 Hz, and 1000 Hz. The transfer function values for these frequencies are: $H(0) = 0$, $H(500) = 0.4472\angle 63.43°$, and $H(1000) = 0.7071\angle 45°$. Applying the transfer function values to each of the input components, we have
$H(0) \times 3 = 0$, $H(500) \times 4\angle 0° = 1.789\angle 63.43°$, and
$H(1000) \times 5\angle -30° = 3.535\angle 15°$. Thus, the output is

$$v_{out}(t) = 1.789\cos(1000\pi t - 63.43°) + 3.535\cos(2000\pi t + 15°)$$

T6.3 (a) The slope of the low-frequency asymptote is +20 dB/decade.
(b) The slope of the high-frequency asymptote is zero.
(c) The coordinates at which the asymptotes meet are 20log(50) = 34 dB and 200 Hz.
(d) This is a first-order highpass filter.
(e) The break frequency is 200 Hz.

T6.4 (a) $f_0 = \dfrac{1}{2\pi\sqrt{LC}} = 1125\,\text{Hz}$

(b) $Q_s = \dfrac{2\pi f_0 L}{R} = 28.28$

(c) $B = \dfrac{f_0}{Q_s} = 39.79\,\text{Hz}$

(d) At resonance, the impedance equals the resistance, which is $5\,\Omega$.

(e) At dc, the capacitance becomes an open circuit so the impedance is infinite.

(f) At infinite frequency the inductance becomes an open circuit, so the impedance is infinite.

T6.5 (a) $f_0 = \dfrac{1}{2\pi\sqrt{LC}} = 159.2\,\text{kHz}$

(b) $Q_p = \dfrac{R}{2\pi f_0 L} = 10.00$

(c) $B = \dfrac{f_0}{Q_p} = 15.92\,\text{kHz}$

(d) At resonance, the impedance equals the resistance which is $10\,\text{k}\Omega$.

(e) At dc, the inductance becomes a short circuit, so the impedance is zero.

(f) At infinite frequency the capacitance becomes a short circuit, so the impedance is zero.

T6.6 (a) This is a first-order circuit because there is a single energy-storage element (L or C). At very low frequencies, the capacitance approaches an open circuit, the current is zero, $\mathbf{V}_{out} = \mathbf{V}_{in}$ and $|H| = 1$. At very high frequencies, the capacitance approaches a short circuit, $\mathbf{V}_{out} = 0$, and $|H| = 0$. Thus, we have a first-order lowpass filter.

(b) This is a second-order circuit because there are two energy-storage elements (L or C). At very low frequencies, the capacitance approaches an open circuit, the inductance approaches a short circuit, the current is zero, $\mathbf{V}_{out} = \mathbf{V}_{in}$ and $|H| = 1$. At very high frequencies, the inductance approaches an open circuit, the capacitance approaches a short circuit, $\mathbf{V}_{out} = 0$, and $|H| = 0$. Thus we have a second-order lowpass filter.

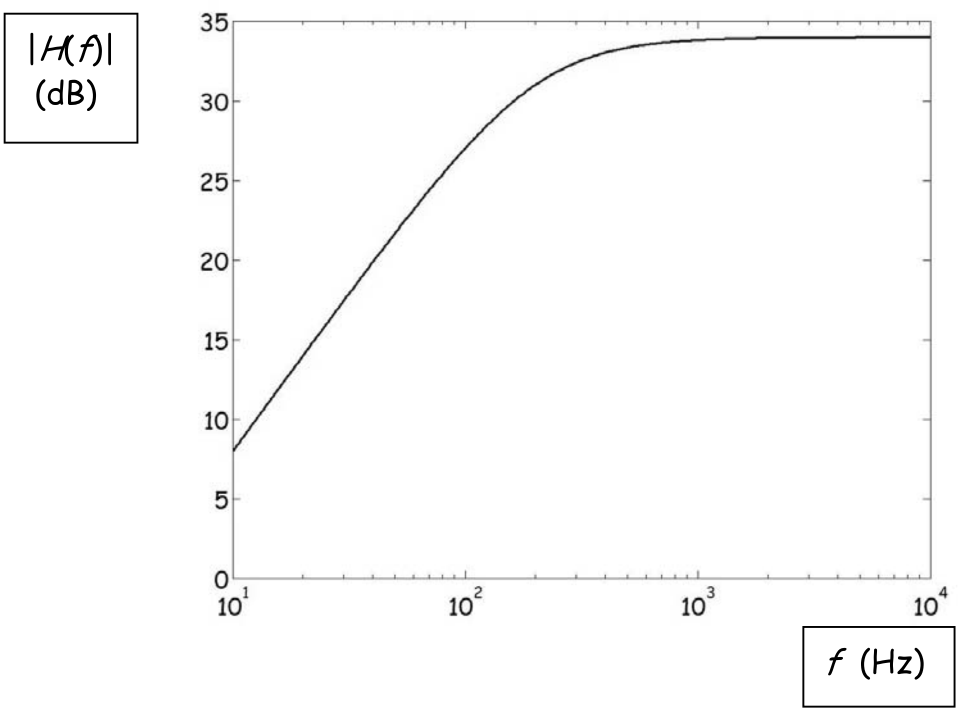
(c) This is a second-order circuit because there are two energy-storage elements (*L* or *C*). At very low frequencies, the inductance approaches a short circuit, $\mathbf{V}_{out} = \mathbf{V}_{in}$ and $|H| = 1$. At very high frequencies, the capacitance approaches a short circuit, $\mathbf{V}_{out} = \mathbf{V}_{in}$ and $|H| = 1$. At the resonant frequency, the *LC* combination becomes an open circuit, the current is zero, $\mathbf{V}_{out} = 0$, and $|H| = 0$. Thus, we have a second-order band-reject (or notch) filter.

(d) This is a first-order circuit because there is a single energy-storage element (*L* or *C*). At very low frequencies, the inductance approaches a short circuit, $\mathbf{V}_{out} = 0$, and $|H| = 0$. At very high frequencies the inductance approaches an open circuit, the current is zero, $\mathbf{V}_{out} = \mathbf{V}_{in}$ and $|H| = 1$. Thus we have a first-order highpass filter.

T6.7 One set of commands is:

```
f = logspace(1,4,400);
H = 50*i*(f/200)./(1+i*f/200);
semilogx(f,20*log10(abs(H)))
```

Other sets of commands are also correct. You can use MATLAB to see if your commands give a plot equivalent to:



CHAPTER 7

Exercises

E7.1 (a) For the whole part, we have:

	Quotient	Remainders
23/2	11	1
11/2	5	1
5/2	2	1
2/2	1	0
1/2	0	1

Reading the remainders in reverse order, we obtain:

$23_{10} = 10111_2$

For the fractional part we have

$2 \times 0.75 = 1 + 0.5$

$2 \times 0.50 = 1 + 0$

Thus we have

$0.75_{10} = 0.110000_2$

Finally, the answer is $23.75_{10} = 10111.11_2$

(b) For the whole part we have:

	Quotient	Remainders
17/2	8	1
8/2	4	0
4/2	2	0
2/2	1	0
1/2	0	1

Reading the remainders in reverse order we obtain:

$17_{10} = 10001_2$

For the fractional part we have

$2 \times 0.25 = 0 + 0.5$

$2 \times 0.50 = 1 + 0$

Thus we have

$0.25_{10} = 0.010000_2$

Finally, the answer is $17.25_{10} = 10001.01_2$

(c) For the whole part we have:

	Quotient	Remainders
4/2	2	0
2/2	1	0
1/2	0	1

Reading the remainders in reverse order we obtain:

$4_{10} = 100_2$

For the fractional part, we have

$2 \times 0.30 = 0 + 0.6$

$2 \times 0.60 = 1 + 0.2$

$2 \times 0.20 = 0 + 0.4$

$2 \times 0.40 = 0 + 0.8$

$2 \times 0.80 = 1 + 0.6$

$2 \times 0.60 = 1 + 0.2$

Thus we have

$0.30_{10} = 0.010011_2$

Finally, the answer is $4.3_{10} = 100.010011_2$

E7.2 (a) $1101.111_2 = 1\times2^3 + 1\times2^2 + 0\times2^1 + 1\times2^0 + 1\times2^{-1} + 1\times2^{-2} + 1\times2^{-3} = 13.875_{10}$

(b) $100.001_2 = 1\times2^2 + 0\times2^1 + 0\times2^0 + 0\times2^{-1} + 0\times2^{-2} + 1\times2^{-3} = 4.125_{10}$

E7.3 (a) Using the procedure of Exercise 7.1, we have

$97_{10} = 1100001_2$

Then adding two leading zeros and forming groups of three bits we have

$001\ 100\ 001_2 = 141_8$

Adding a leading zero and forming groups of four bits we obtain

$0110\ 0001 = 61_{16}$

(b) Similarly

$229_{10} = 11100101_2 = 345_8 = E5_{16}$

E7.4 (a) $72_8 = 111\ 010 = 111010_2$

(b) $FA6_{16} = 1111\ 1010\ 0110 = 111110100110_2$

E7.5 $197_{10} = 0001\ 1001\ 0111 = 000110010111_{BCD}$

E7.6 To represent a distance of 20 inches with a resolution of 0.01 inches, we need 20/0.01 = 2000 code words. The number of code words in a Gray code is 2^L in which L is the length of the code words. Thus we need L = 11, which produces 2048 code words.

E7.7 (a) First we convert to binary

$22_{10} = 16 + 4 + 2 = 10110_2$

Because an eight-bit representation is called for, we append three leading zeros. Thus +22 becomes

00010110

in two's complement notation.

(b) First we convert +30 to binary form

$30_{10} = 16 + 8 + 4 + 2 = 11110_2$

Attaching leading zeros to make an eight-bit result we have

$30_{10} = 00011110_2$

Then we take the ones complement and add 1 to find the two's complement:

one's complement:	11100001
add 1	<u>+1</u>
	11100010

Thus the eight-bit two's complement representation for -30_{10} is 11100010.

E7.8 First we convert 19_{10} and -4_{10} to eight-bit two's complement form then we add the results.

19	00010011
<u>-4</u>	<u>111111100</u>
15	00001111

Notice that we neglect the carry out of the left-most bit.

E7.9 See Tables 7.3 and 7.4 in the book.

E7.10 See Table 7.5 in the book.

E7.11 (a) To apply De Morgan's laws to the expression

$AB + \bar{B}C$

first we replace each logic variable by its inverse

$$\overline{A}\,\overline{B} + B\overline{C}$$

then we replace AND operations by OR operations and vice versa

$$(\overline{A} + \overline{B})(B + \overline{C})$$

finally we invert the entire expression so we have

$$D = AB + \overline{B}C = \overline{(\overline{A} + \overline{B})(B + \overline{C})}$$

(b) Following the steps of part (a) we have

$$\overline{[F(G + \overline{H}) + F\overline{G}]}$$

$$\overline{[\overline{F}(\overline{G} + H) + \overline{F}G]}$$

$$\overline{[(\overline{F} + \overline{G}H)(\overline{F} + G)]}$$

$$E = \overline{[F(G + \overline{H}) + F\overline{G}]} = [(\overline{F} + \overline{G}H)(\overline{F} + G)]$$

E7.12 For the AND gate we use De Morgan's laws to write

$$AB = \overline{(\overline{A} + \overline{B})}$$

See Figure 7.21 in the book for the logic diagrams.

E7.13 The truth table for the exclusive-OR operation is

A	B	$A \oplus B$
0	0	0
0	1	1
1	0	1
1	1	0

Focusing on the rows in which the result is 1, we can write the SOP expression

$$A \oplus B = \overline{A}B + A\overline{B}$$

The corresponding logic diagram is shown in Figure 7.25a in the book.

Focusing on the rows in which the result is 0, we can write the POS expression

$$A \oplus B = (A + B)(\overline{A} + \overline{B})$$

The corresponding logic diagram is shown in Figure 7.25b in the book.

E7.14 The truth table is shown in Table 7.7 in the book. Focusing on the rows in which the result is 1, we can write the SOP expression

$$A = \sum m(3, 6, 7, 8, 9, 12)$$
$$= \overline{F}\,\overline{D}GR + \overline{F}\,DG\overline{R} + \overline{F}DGR + F\overline{D}\,\overline{G}\,\overline{R} + F\overline{D}\,\overline{G}R + FD\overline{G}\,\overline{R}$$

Focusing on the rows in which the result is 0, we can write the POS expression

$$A = \prod M(0, 1, 2, 4, 5, 10, 11, 13, 14, 15)$$
$$= (F + D + G + R)(F + D + G + \overline{R})(F + D + \overline{G} + R)\cdots(\overline{F} + \overline{D} + \overline{G} + \overline{R})$$

E7.15 The Truth table is shown in Table 7.8.

E7.16 (a) $\overline{A}\,\overline{B}C\overline{D}$

(b) $\overline{A}\,\overline{B}\,\overline{C}\,D$

E7.17 See Figure 7.34 in the book.

E7.18 See Figure 7.35 in the book.

E7.19 Because S is high at $t = 0$, Q is high and remains so until R becomes high at $t = 3$. Q remains low until S becomes high at $t = 7$. Then Q remains high until R becomes high at $t = 11$.

E7.20 See Table 7.9.

E7.21 See Figure 7.49 in the book.

Answers for Selected Problems

P7.1* 1. When noise is added to a digital signal, the logic levels can still be exactly determined, provided that the noise is not too large in amplitude. Usually, noise cannot be completely removed from an analog signal.

2. Component values in digital circuits do not need to be as precise as in analog circuits.

3. Very complex digital logic circuits (containing millions of components) can be economically produced. Analog circuits often call for large capacitances and/or precise component values that are impossible to manufacture as large-scale integrated circuits.

P7.6 (a)* 5.625
(f)* 21.375

P7.7 (c)* $9.75_{10} = 1001.11_2 = 1001.01110101_{BCD}$

P7.9 (a)* 1101.11 + 101.111 = 10011.101

P7.10 (a)* $10010011.0101_{BCD} + 00110111.0001_{BCD} = 93.5_{10} + 37.1_{10} = 130.6_{10} = 000100110000.0110_{BCD}$

P7.11 (d)* $313.0625_{10} = 100111001.0001_2 = 471.04_8 = 139.1_{16}$

P7.12 (c)* 75 = 01001011
(d)* -87 = 10101001

P7.17*

```
0000
0001
0011
0010
0110
0111
0101
0100
1100
1101
1111
1110
1010
1011
1001
1000
```

P7.18 (a)* $FA5.6_{16} = 4005.375_{10}$

(b)* $725.3_8 = 469.375_{10}$

P7.19 (a)* $11101000 \Rightarrow \overbrace{00010111}^{\text{One's Complement}} \Rightarrow \overbrace{00011000}^{\text{Two's Complement}}$

P7.20 (c)*

```
  33        00100001
 -37        11011011
  -4        11111100
```

P7.23* If the variables in a logic expression are replaced by their inverses, the AND operation is replaced by OR, the OR operation is replaced by AND, and the entire expression is inverted, the resulting logic expression yields the same values as before the changes. In equation form, we have:

$$ABC = \overline{\overline{A} + \overline{B} + \overline{C}} \qquad (A + B + C) = \overline{\overline{A}\,\overline{B}\,\overline{C}}$$

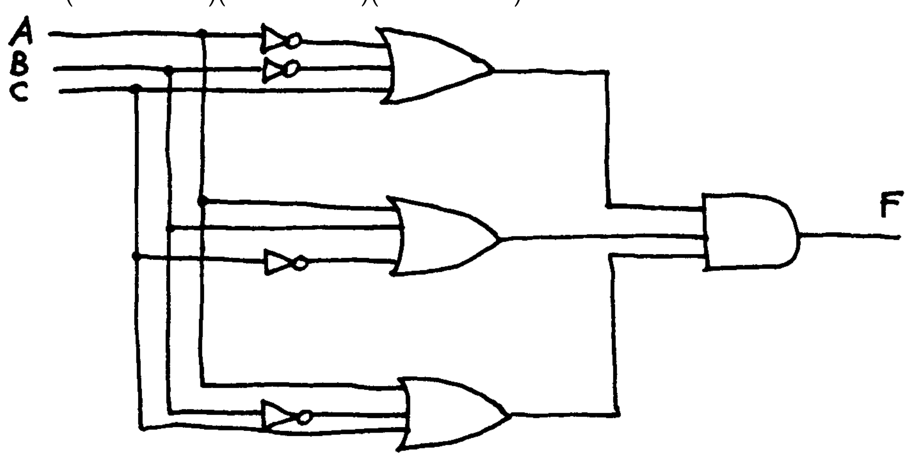
P7.26 (b)* $E = AB + A\overline{B}C + \overline{C}D$

A	B	C	D	E
0	0	0	0	0
0	0	0	1	1
0	0	1	0	0
0	0	1	1	0
0	1	0	0	0
0	1	0	1	1
0	1	1	0	0
0	1	1	1	0
1	0	0	0	0
1	0	0	1	1
1	0	1	0	1
1	0	1	1	1
1	1	0	0	1
1	1	0	1	1
1	1	1	0	1
1	1	1	1	1

P7.28*

A	B	C	$(A+B)(A+C)$	$A+BC$
0	0	0	0	0
0	0	1	0	0
0	1	0	0	0
0	1	1	1	1
1	0	0	1	1
1	0	1	1	1
1	1	0	1	1
1	1	1	1	1

P7.32 (c)* $F = (\overline{A} + \overline{B} + C)(A + B + \overline{C})(A + \overline{B} + C)$



P7.33 (d)* $F = (A + B + C)(A + \overline{B} + C)(\overline{A} + B + \overline{C}) = \overline{\overline{A}\,\overline{B}\,\overline{C} + \overline{A}\,B\overline{C} + A\overline{B}\,C}$

(e)* $F = ABC + A\overline{B}\,C + \overline{A}\,B\overline{C} = \overline{(\overline{A} + \overline{B} + \overline{C})(\overline{A} + B + \overline{C})(A + \overline{B} + C)}$

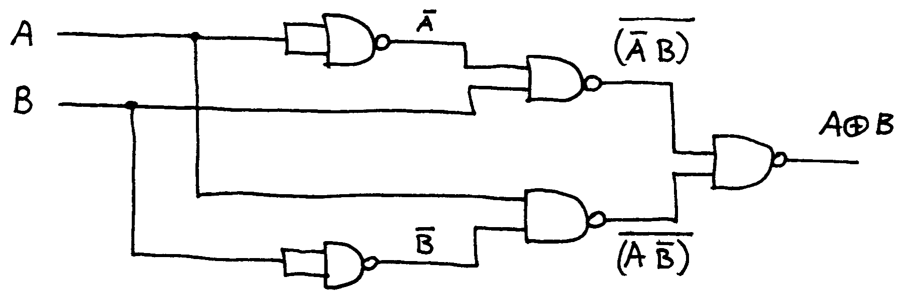
P7.40* $F = \overline{A}\,\overline{B}\,\overline{C} + \overline{A}B\overline{C} + A\overline{B}C + ABC$

$= \sum m(0,2,5,7)$

$F = (A + B + \overline{C})(A + \overline{B} + \overline{C})(\overline{A} + B + C)(\overline{A} + \overline{B} + C)$

$= \prod M(1,3,4,6)$

P7.49*

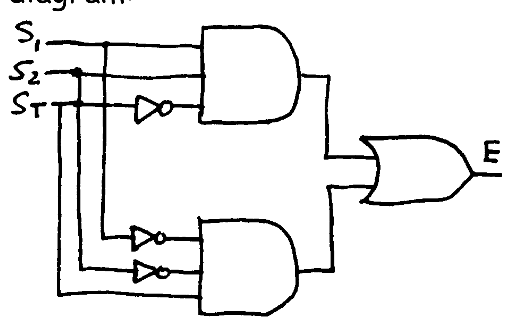


P7.52* (a)

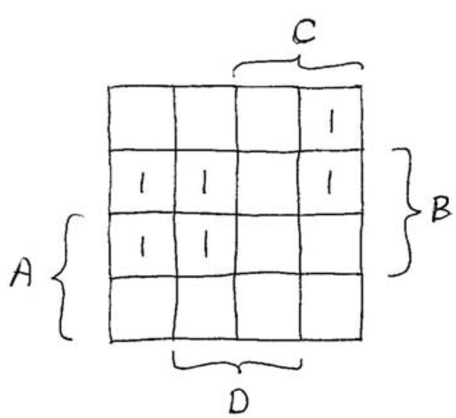
S_1	S_2	S_T	E
0	0	0	0
0	0	1	1
0	1	0	0
0	1	1	0
1	0	0	0
1	0	1	0
1	1	0	1
1	1	1	0

(b) $E = \sum m(1,6) = \overline{S_1}\,\overline{S_2}S_T + S_1S_2\overline{S_T}$

(c) Circuit diagram:



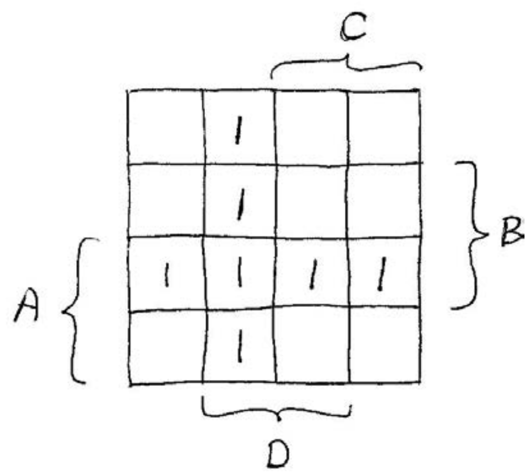
P7.53* (a) The Karnaugh map is:



(b) $F = B\overline{C} + \overline{A}\,C\overline{D}$

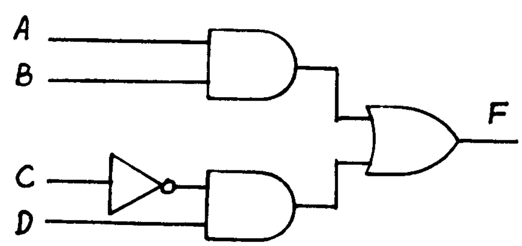
(c) Inverting the map, and writing the minimum SOP expression yields $\overline{F} = AC + \overline{B}\,\overline{C} + CD$. Then applying DeMorgan's laws gives
$F = (\overline{A} + \overline{C})(B + C)(\overline{C} + \overline{D})$

P7.58* (a) The Karnaugh map is:



(b) $F = AB + \overline{C}D$

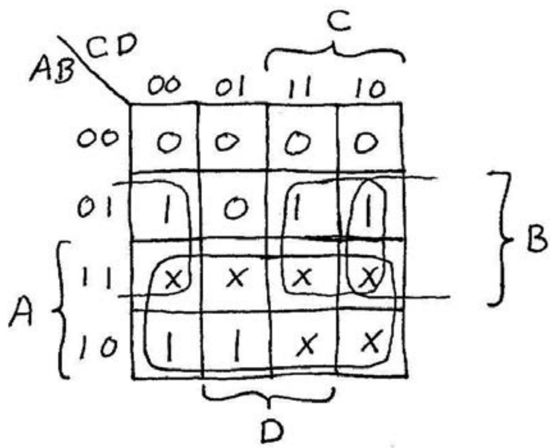
(c) The circuit is:



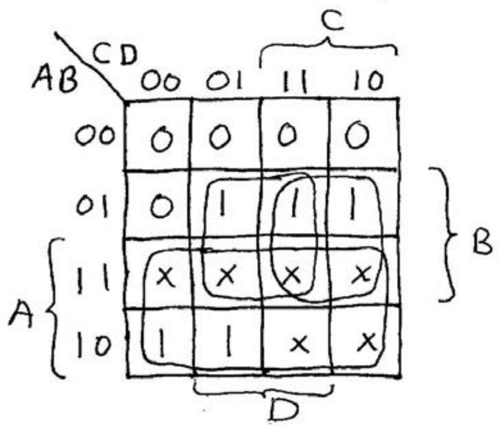
(d) Inverting the map, and writing the minimum SOP expression yields
$\overline{F} = \overline{A}\,C + \overline{A}\,\overline{D} + \overline{B}\,C + \overline{B}\,\overline{D}$. Then, applying DeMorgan's laws gives

$F = (A + \overline{C})(A + D)(B + \overline{C})(B + D)$

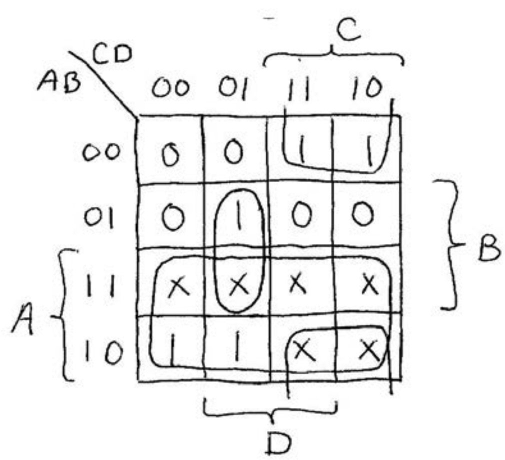
P7.71* (a) $F = A + BC + B\overline{D}$



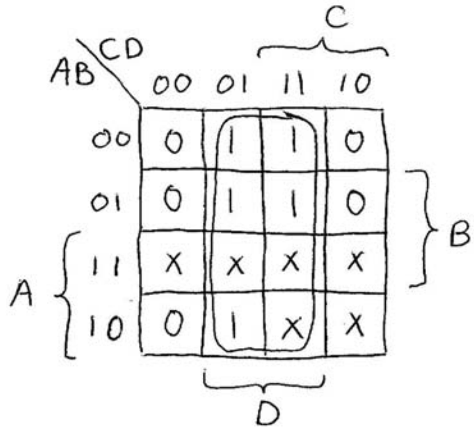
(b) $G = A + BD + BC$



(c) $H = A + \overline{B}C + B\overline{C}D$



(d) $I = D$



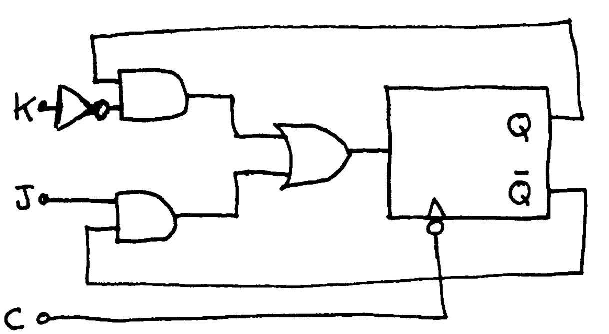
P7.81* The successive states are:

Q_0	Q_1	Q_2
1	0	0
0	1	0
1	0	1
1	1	0
1	1	1
0	1	1
0	0	1

(repeats)

Thus, the register returns to the initial state after seven shifts.

P7.85*



Practice Test

T7.1 (a) 12, (b) 19 (18 is incorrect because it omits the first step, inverting the variables), (c) 20, (d) 23, (e) 21, (f) 24, (g) 16, (h) 25, (i) 7, (j) 10, (k) 8, (l) 1 (the binary codes for hexadecimal symbols *A* through *F* do not occur in BCD).

T7.2 (a) For the whole part, we have:

	Quotient	Remainders
353/2	176	1
176/2	88	0
88/2	44	0
44/2	22	0
22/2	11	0
11/2	5	1
5/2	2	1
2/2	1	0
1/2	0	1

Reading the remainders in reverse order, we obtain:

$353_{10} = 101100001_2$

For the fractional part, we have

$2 \times 0.875 = 1 + 0.75$

$2 \times 0.75 = 1 + 0.5$

$2 \times 0.5 = 1 + 0$

Thus, we have

$0.875_{10} = 0.111_2$

Finally, combining the whole and fractional parts, we have

$353.875_{10} = 101100001.111_2$

(b) For the octal version, we form groups of three bits, working outward from the decimal point, and then write the octal symbol for each group.

$101\ 100\ 001.111_2 = 541.7_8$

(c) For the hexadecimal version, we form groups of four bits, working outward from the decimal point, and then write the hexadecimal symbol for each group.

$0001\ 0110\ 0001.1110_2 = 161.E_{16}$

(d) To obtain binary coded decimal, we simply write the binary equivalent for each decimal digit.

$353.875_{10} = 0011\ 0101\ 0011.1000\ 0111\ 0101_{BCD}$

T7.3 (a) Because the left-most bit is zero, this is a positive number. We simply convert from binary to decimal:

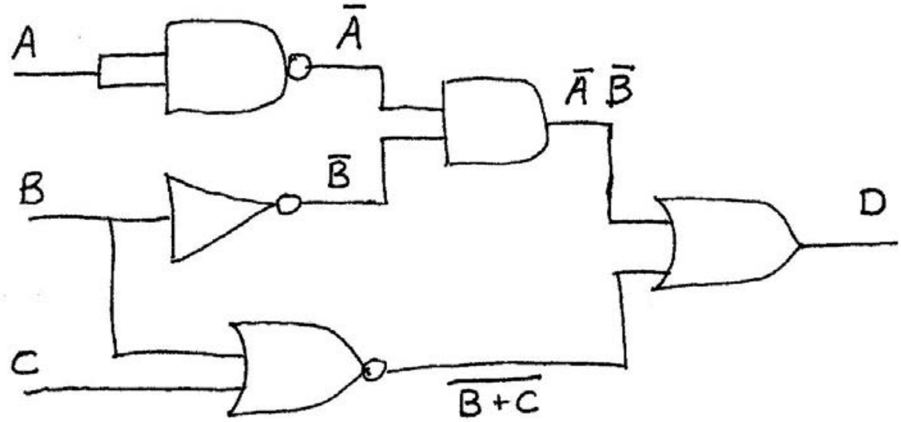
$01100001_2 = 1 \times 2^6 + 1 \times 2^5 + 1 \times 2^0 = 64 + 32 + 1 = +97_{10}$

(b) Because the left-most bit is one, this is a negative number. We form the two's complement, which is 01000110. Then, we convert from binary to decimal:

$01000110_2 = 1 \times 2^6 + 1 \times 2^2 + 1 \times 2^1 = 64 + 4 + 2 = +70_{10}$

Thus, the decimal equivalent for eight-bit signed two's complement integer 10111010 is -70.

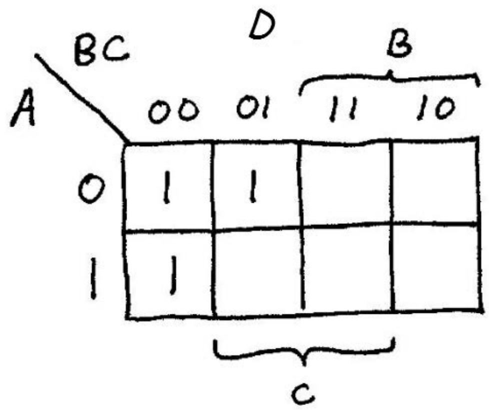
T7.4. (a) The logic expression is $D = \overline{A}\,\overline{B} + \overline{(B + C)}$.



(b) The truth table is:

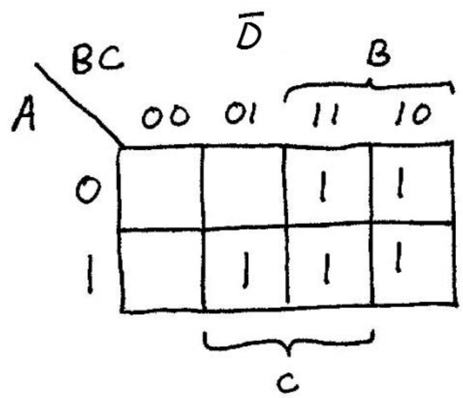
A	*B*	*C*	*D*
0	0	0	1
0	0	1	1
0	1	0	0
0	1	1	0
1	0	0	1
1	0	1	0
1	1	0	0
1	1	1	0

The Karnaugh map is:



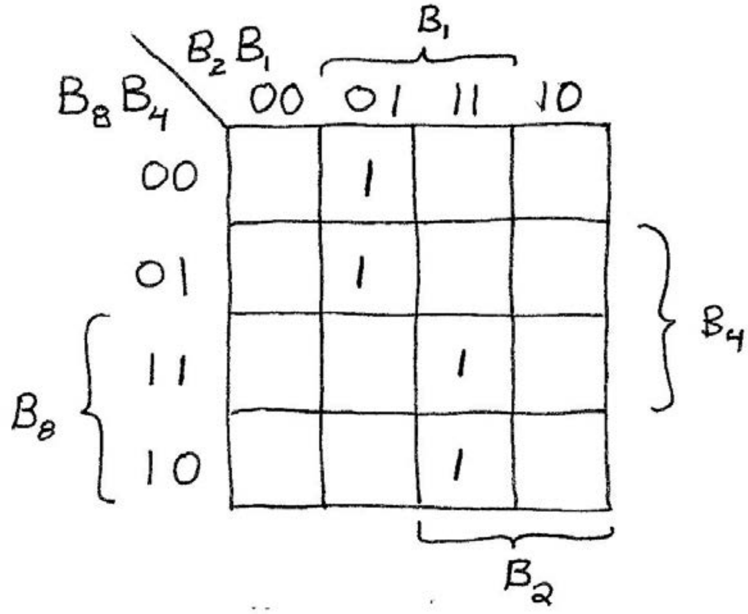
(c) The map can be covered by two 2-cubes and the minimum SOP expression is $D = \overline{A}\,\overline{B} + \overline{B}\,\overline{C}$.

(d) First, we invert the map to find:



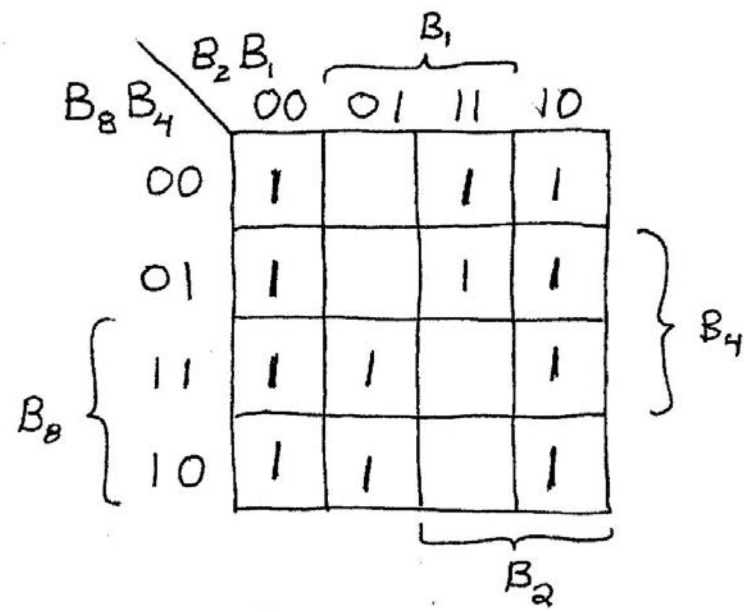
This map can be covered by one 4-cube and one 2-cube and the minimum SOP expression is $\overline{D} = B + AC$. Applying DeMorgan's laws to this yields the minimum POS expression $D = \overline{B}(\overline{A} + \overline{C})$.

T7.5 (a) The completed Karnaugh map is:



(b) The map can be covered by two 2-cubes and the minimum SOP expression is $G = B_1\overline{B_2}\ \overline{B_8} + B_1B_2B_8$.

(c) First, we invert the map to find:



This map can be covered by one 8-cube and two 4-cubes and the minimum SOP expression is $\overline{G} = \overline{B_1} + B_2\overline{B_8} + \overline{B_2}B_8$. Applying DeMorgan's laws to this yields the minimum POS expression $G = B_1(\overline{B_2} + B_8)(B_2 + \overline{B_8})$.

T7.6 Clearly, the next value for Q_0 is the NAND combination of the current values of Q_1 and Q_2. The next value for Q_2 is the present value for Q_1. Similarly, the next value for Q_1 is the present value for Q_0. Thus, the successive states (Q_0 Q_1 Q_2) of the shift register are:

100 (initial state)
110
111
011
001
100
111

The state of the register returns to its initial state after 5 shifts.

CHAPTER 8

Exercises

E8.1 The number of bits in the memory addresses is the same as the address bus width, which is 20. Thus the number of unique addresses is 2^{20} = 1,048,576 = 1024 $\times$ 1024 = 1024K.

E8.2 (8 bits/byte) $\times$ (64 Kbytes) = 8 $\times$ 64 $\times$ 1024 = 524,288 bits

E8.3 Starting from the initial situation shown in Figure 8.9a in the book, execution of the command PSHB results in:

	0907:
	0908:
SP $\rightarrow$	0909:
	090A: A2

Then, execution of the command PSHA results in:

	0907:
SP $\rightarrow$	0908:
	0909: 34
	090A: A2

Then, the PULX command reads two bytes from the stack and we have:

	0907:	X: 34A2
	0908:	
	0909: 34	
SP $\rightarrow$	090A: A2	

E8.4 Starting from the initial situation shown in Figure 8.9a in the book, execution of the command PSHX results in:

	0907:
SP $\rightarrow$	0908:
	0909: 00
	090A: 00

Then, the command PSHA results in:

SP $\rightarrow$	0907:	
	0908: 34	
	0909: 00	
	090A: 00	

Next the PULX command reads two bytes from the stack, and we have:

	0907:	X: 3400
	0908: 34	
SP $\rightarrow$	0909: 00	
	090A: 00	

E8.5 The results are given in the book.

E8.6 (a) LDAA $0202

This instruction uses extended addressing. The effective address is 0202. In Figure 8.11 we see that this location contains 1A. Thus the content of the A register after this instruction is 1A.

(b) LDAA #$43

This instruction uses immediate addressing. The effective address is the one immediately following the op code. This location contains the hexadecimal digits 43. Thus the content of the A register after this instruction is 43.

(c) LDAA $05,X

This instruction uses indexed addressing. The effective address is the content of the X register plus the offset which is 05. Thus the effective address is 0205. In Figure 8.11 we see that this location contains FF. Thus the content of the A register after this instruction is FF.

(d) LDAA $06

This instruction uses direct addressing. The effective address is 0006. In Figure 8.11 we see that this location contains 13. Thus the content of the A register after this instruction is 13.

(e) LDAA $07,X

This instruction uses indexed addressing. The effective address is the content of the X register plus the offset which is 07. Thus the effective address is 2007. In Figure 8.11 we see that this location contains 16. Thus the content of the A register after this instruction is 16.

E8.7 (a) Referring to Table 8.1 in the book, we see that CLRA is the clear accumulator A instruction with a single byte op code 4F. Furthermore execution of this command sets the Z bit of the condition code register. The BEQ $15 command occupies two memory locations with the op code 27 in the first byte and the offset of 15 in the second byte. Thus starting in location 0200, the instructions appear in memory as:

0200: 4F
0201: 27
0202: 14

(b) When the instructions are executed, the CLRA command sets the Z bit. Then if the Z-bit was clear the next instruction would be the one starting in location 0203 following the BEQ $15 command. However since the Z bit is set the next instruction is located at 0203 + 15 = 0218.

E8.8 One answer is given in the book. Of course, other correct answers exist.

E8.9 One answer is given in the book. Of course, other correct answers exist.

Answers for Selected Problems

P8.7* 262,144 bytes

P8.9* ROM is read-only memory. Some types are:

1. Mask-programmable ROM in which the data is written when the memory is manufactured.
2. Programmable read-only memory (PROM) in which data is written by special circuits that blow tiny fuses or not depending on whether the data bits are zeros or ones.
3. Erasable PROMs (EPROMs) that can be erased by exposure to ultraviolet light (through a window in the chip package) and rewritten using special circuits.
4. Electrically erasable PROMs (EEPROMs) that can be erased by applying proper voltages to the chip.

All types of ROM are nonvolatile and are used for program storage in embedded computers.

P8.13* In the ignition control system for automobiles, we need to use ROM for the programs and fixed data, because ROM is nonvolatile. Some RAM would be needed for temporary data such as air temperature and throttle setting. Presumably many units would be needed for mass production and mask programmable ROM would be least expensive.

P8.19* A digital sensor produces a logic signal or a digital word as its output. An analog sensor produces an analog output signal that varies continuously with the variable being measured.

P8.23* A D/A is a digital-to-analog converter that converts a sequence of digital words into an analog signal. They are needed when an analog actuator must be controlled by a digital computer.

P8.27* A stack is a sequence of locations in RAM used to store information such as the contents of the program counter and other registers when a subroutine is executed or when an interrupt occurs. Information is added to (pushed onto) the top of the stack and then read out (pulled off) in the reverse order that it was written. After data are pulled off the stack, they are considered to no longer exist in memory. The stack pointer is a register that keeps track of the address of the location immediately above the top of the stack.

P8.29* Initially, we have:

A: 07	0048: 00
B: A9	0049: 00
SP: 004E	004A: 00
X: 34BF	004B: 00
	004C: 00
	004D: 00
	004E: 00
	004F: 00

After the command PSHA, we have:

A: 07	0048: 00
B: A9	0049: 00
SP: 004D	004A: 00
X: 34BF	004B: 00
	004C: 00
	004D: 00
	004E: 07
	004F: 00

After the command PSHB, we have:

A: 07	0048: 00
B: A9	0049: 00
SP: 004C	004A: 00
X: 34BF	004B: 00
	004C: 00
	004D: A9
	004E: 07
	004F: 00

After the command PULA, we have:

A: A9	0048: 00
B: A9	0049: 00
SP: 004D	004A: 00
X: 34BF	004B: 00
	004C: 00
	004D: A9
	004E: 07
	004F: 00

After the command PULB, we have:

A: A9	0048: 00
B: 07	0049: 00
SP: 004E	004A: 00
X: 34BF	004B: 00
	004C: 00
	004D: A9
	004E: 07
	004F: 00

After the command PSHX, we have:

A: A9	0048: 00
B: 07	0049: 00
SP: 004C	004A: 00
X: 34BF	004B: 00
	004C: 00
	004D: 34
	004E: BF
	004F: 00

P8.31* A sequence of instructions that results in swapping the high and low bytes of the X register is:

```
PSHD    save the original contents of A and B
PSHX
PULD    move the content of X to D
PSHA
PSHB
PULX
PULD    restore contents of A and B
```

P8.32

(a)*	LDAA	$2002	Extended addressing	A:20
(c)*	LDAA	$04	Direct addressing	A:9A
(e)*	INCA		Inherent addressing	A:02
(g)*	LDAA	$2007	Extended addressing	A:F3

P8.33 *(a) The command starting at location 200A is the next to be executed after the BMI $07 command.

P8.34

	Mnemonics	Machine codes	Memory locations occupied by instruction
(a)*	CLRA	4F	1
(b)*	ADDA $4A	9B 4A	2

P8.36* (a) 48
(b) 09

P8.37* A:14 and B:E0

P8.39* * ANSWER FOR PROBLEM 8.39

```
        ORG     0200     Directive to begin in location 0200
        LDAB    #$0B     0B is hex equivalent of decimal 11
        MUL              compute product
        STD     $FF00    store result
        STOP
        END
```

P8.41* This subroutine first clears register B which will hold the quotient after the program has been executed. Then 3 is repeatedly subtracted from the content of A, and the content of B is incremented until the value in A is negative.

```
*DIVIDE BY 3
*
DIV3    CLRB
LOOP    SUBA    #$03    subtract 3 from content of A.
        BMI     END     quit if result is negative
        INCB
        JMP     LOOP
END     ADDA    #$03    restore remainder to register A
        RTS
```

Practice Test

T8.1. **a.** 11, **b.** 17, **c.** 21, **d.** 24, **e.** 27, **f.** 13, **g.** 26, **h.** 9, **i.** 20, **j.** 12, **k.** 15, **l.** 16, **m.** 8, **n.** 29, **o.** 23, **p.** 30.

T8.2. **a.** direct, 61; **b.** indexed, F3; **c.** inherent, FF; **d.** inherent, 01; **e.** immediate, 05; **f.** immediate, A1.

T8.3 Initially, we have:

A: A6	1034: 00
B: 32	1035: 00
SP: 1038	1036: 00
X: 1958	1037: 00
	1038: 00
	1039: 00
	103A: 00
	103B: 00
	103C: 00

After the command PSHX, we have:

A: A6	1034: 00
B: 32	1035: 00
SP: 1036	1036: 00
X: 1958	1037: 19
	1038: 58
	1039: 00
	103A: 00
	103B: 00
	103C: 00

After the command PSHB, we have:

A: A6	1034: 00
B: 32	1035: 00
SP: 1035	1036: 32
X: 1958	1037: 19
	1038: 58
	1039: 00
	103A: 00
	103B: 00
	103C: 00

After the command PULA, we have:

A: 32	1034: 00
B: 32	1035: 00
SP: 1036	1036: 32
X: 1958	1037: 19
	1038: 58
	1039: 00
	103A: 00
	103B: 00
	103C: 00

After the command PSHX, we have:

A: 32	1034: 00
B: 32	1035: 19
SP: 1034	1036: 58
X: 1958	1037: 19
	1038: 58
	1039: 00
	103A: 00
	103B: 00
	103C: 00

CHAPTER 9

Solutions for Exercises

E9.1 The equivalent circuit for the sensor and the input resistance of the amplifier is shown in Figure 9.2 in the book. Thus the input voltage is

$$v_{in} = v_{sensor}\frac{R_{in}}{R_{sensor} + R_{in}}$$

We want the input voltage with an internal sensor resistance of 10 kΩ to be at least 0.995 times the input voltage with an internal sensor resistance of 5 kΩ. Thus with resistances in kΩ, we have

$$v_{sensor}\frac{R_{in}}{10 + R_{in}} \geq 0.995 v_{sensor}\frac{R_{in}}{5 + R_{in}}$$

Solving, we determine that R_{in} is required to be greater than 990 kΩ.

E9.2 (a) A very precise instrument can be very inaccurate because precision implies that the measurements are repeatable, however they could have large bias errors.

(b) A very accurate instrument cannot be very imprecise. If repeated measurements vary a great deal under apparently identical conditions, some of the measurements must have large errors and therefore are inaccurate.

E9.3 $v_d = v_1 - v_2 = 5.7 - 5.5 = 0.2\text{ V} \qquad v_{cm} = \frac{1}{2}(v_1 + v_2) = \frac{1}{2}(5.5 + 5.7) = 5.6\text{ V}$

E9.4 The range of input voltages is from -5 V to +5 V or 10 V in all. We have $N = 2^k = 2^8 = 256$ zones. Thus the width of each zone is $\Delta = \frac{10}{N} = 39.1$ mV. The quantization noise is approximately $N_{qrms} \cong \frac{\Delta}{2\sqrt{3}} = 11.3$ mV.

E9.5 Look at Figure 9.14 in the book. In this case, we have f_s = 30 kHz and f = 25 kHz. Thus, the alias frequency is $f_{alias} = f_s - f$ = 5 kHz.

E9.6 The file containing the vi is named Figure 9.17.vi and can be found on the CD that accompanies this book.

Answers for Selected Problems

P9.2* The equivalent circuit of a sensor is shown in Figure 9.2 in the book. Loading effects are caused by the voltage drop across R_{sensor} that occurs when the input resistance of the amplifier draws current from the sensor. Then the input voltage to the amplifier (and therefore overall sensitivity) depends on the resistances as well as the internal voltage of the sensor. To avoid loading effects, we need to have R_{in} much greater than R_{sensor}.

P9.4* $R_{in} \geq 99\ \mathrm{k}\Omega$

P9.5* $R_{in} \leq 102\ \Omega$

P9.8* The true value lies in the range from 69.5 cm to 70.5 cm.

P9.10* (a) Instrument *B* is the most precise because the repeated measurements vary the least. Instrument *A* is the least precise.

(b) Instrument *C* is the most accurate because the maximum error of its measurements is least. Instrument *A* is the least accurate because it has the larges maximum error.

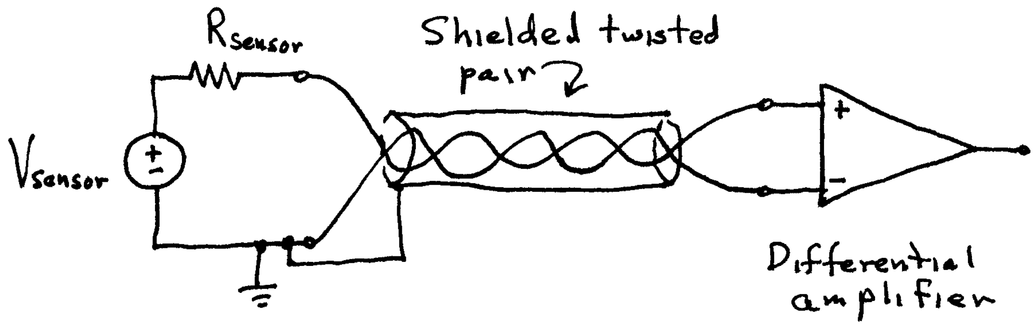
(c) Instrument *C* has the best resolution, and instrument *A* has the worst resolution.

P9.14* $v_d = v_1 - v_2 = 0.004\ \mathrm{V}$

$v_{cm} = \frac{1}{2}(v_1 + v_2) = 5\cos(\omega t)\ \mathrm{V}$

$v_o = A_d v_d = 4\ \mathrm{V}$

P9.16* To avoid ground loops, we must not have grounds at both ends of the 5-m cable. Because the sensor is grounded, we need to use a differential amplifier. To reduce interference from magnetic fields, we should use a twisted pair or coaxial cable. To reduce interference from electric fields we should choose a shielded cable and connect the shield to ground at the sensor. A schematic diagram of the sensor, cable and amplifier is:



P9.18* 60-Hz interference can be caused by magnetic fields linked with the sensor circuit. We could try a coaxial or twisted pair cable and/or move the sensor cable away from sources of 60-Hz magnetic fields such as transformers.

Another possibility is that the interference could be caused by a ground loop which we should eliminate.
Also electric field coupling is a possibility, in which case we should use a shielded cable with the shield grounded.

P9.22* Use a 10-bit converter.

P9.23* (a) $\Delta = 10/2^{12} = 2.44\ \text{mV}$

(b) $P_q = 4.967 \times 10^{-7}/R$ watts

(c) $P_s = 2/R$ watts

(d) $\text{SNR}_{\text{dB}} = 10\log\left(\dfrac{P_s}{P_q}\right) = 66.0\ \text{dB}$

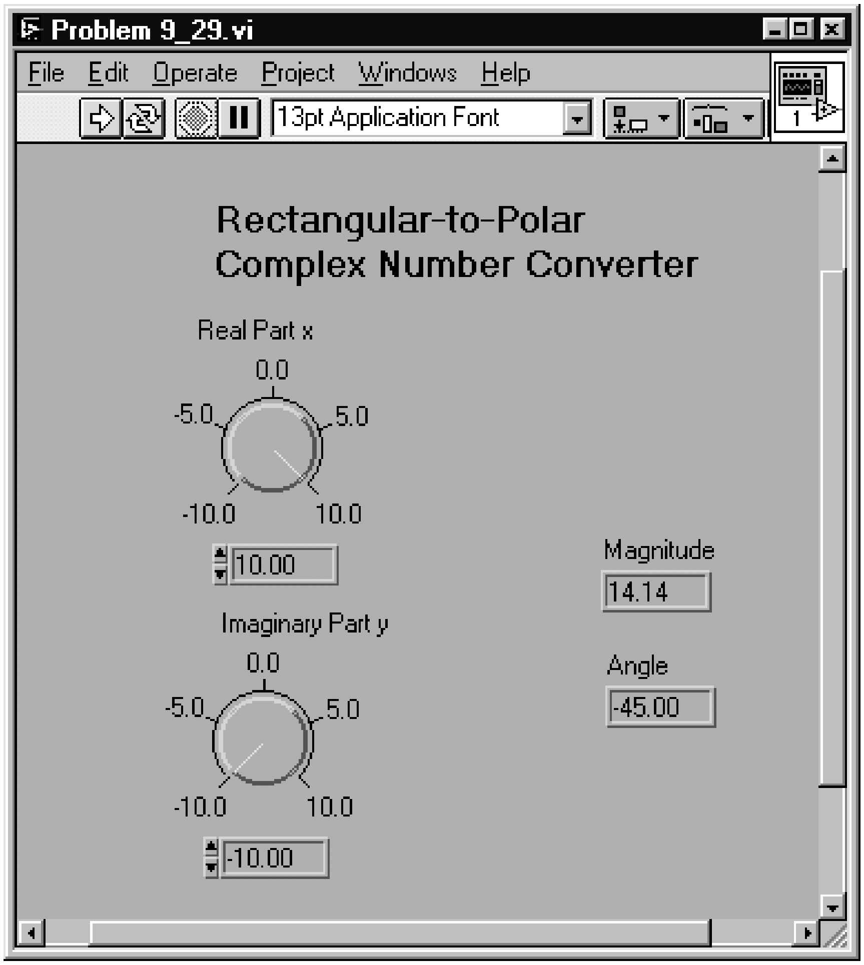
P9.24* $k = 8$

P9.25* (a) 1 kHz

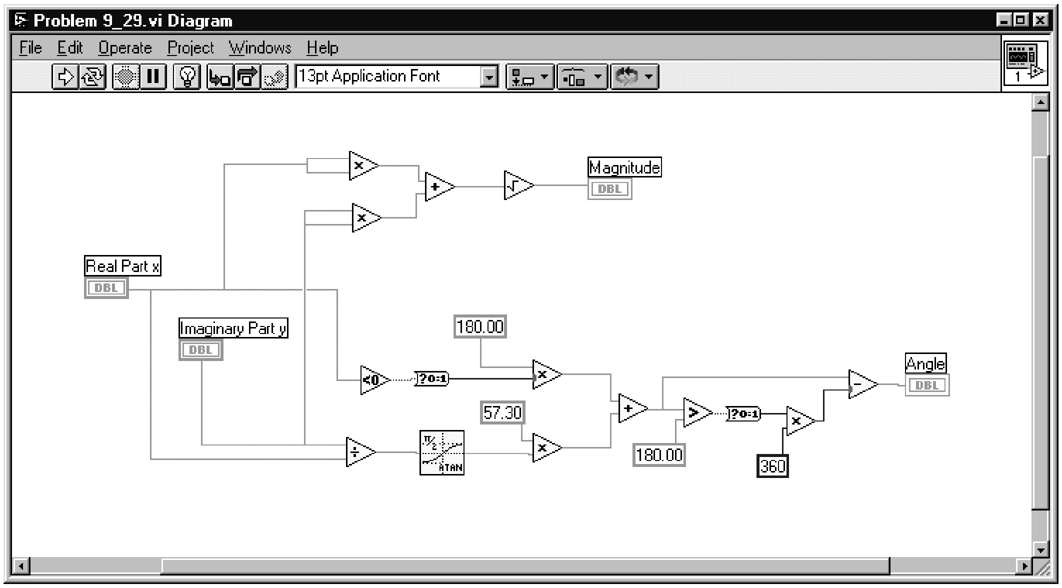
(b) 2 kHz

(c) 10 kHz

P9.29* The front panel is:



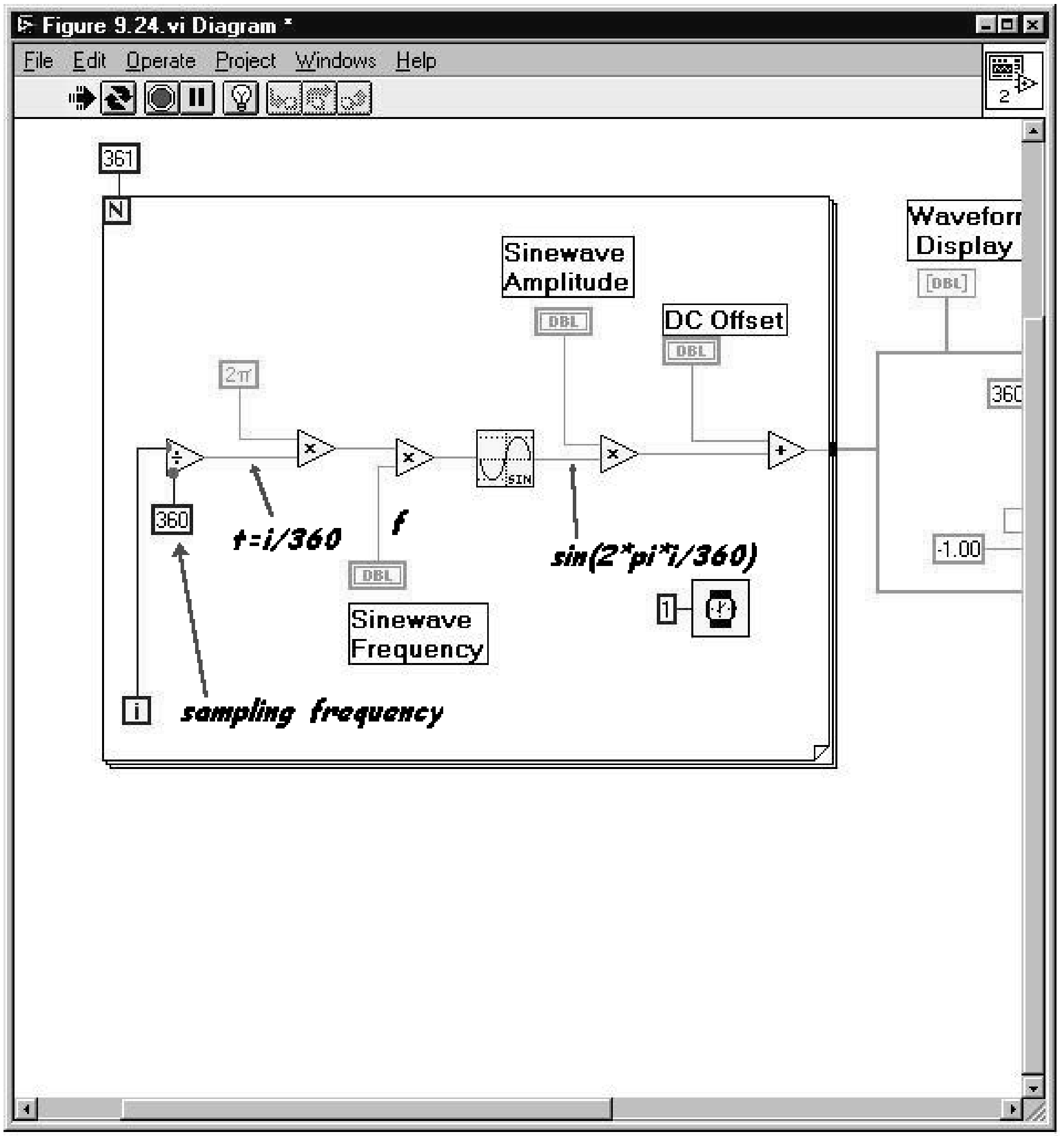
The magnitude of the complex number is computed as $A = \sqrt{x^2 + y^2}$. To find the angle, first we compute $\arctan(y / x)$ and convert the result to degrees by multiplying by 57.30. Then if x is negative, we add 180° which gives the correct angle. However we want the angle to fall between -180° and +180°. Thus if the angle is greater than 180°, we subtract 360°. The block diagram is:



P9.31* We must keep in mind that G-programs deal with sampled signals even though they may appear to be continuous in time on the displays. In the virtual instrument Figure 9.24.vi, the sampling frequency for the sinewave is 360 samples per second. Thus we can expect aliasing if the frequency is greater than 180 Hz. Referring to Figure 9.14 in the book, we see that for each frequency f, the alias frequency is

f	f_{alias}
355	5
356	4
357	3
358	2
359	1
360	0
361	1
362	2
363	3
364	4

Thus as the frequency f increases, the apparent frequency decreases as we observe on the front panel. The partial block diagram and the quantities at various points are shown below:



Practice Test

T9.1. The four main elements are sensors, a DAQ board, software, and a general-purpose computer.

T9.2. The four types of systematic (bias) errors are offset, scale error, nonlinearity, and hysteresis.

T9.3. Bias errors are the same for measurements repeated under identical conditions, while random errors are different for each measurement.

T9.4. Ground loops occur when the sensor and the input of the amplifier are connected to ground by separate connections. The effect is to add noise (often with frequencies equal to that of the power line and its harmonics) to the desired signal.

T9.5. If we are using a sensor that has one end grounded, we should choose an amplifier with a differential input to avoid a ground loop.

T9.6. Coaxial cable or shielded twisted pair cable.

T9.7. If we need to sense the open-circuit voltage, the input impedance of the amplifier should be very large compared to the internal impedance of the sensor.

T9.8. The sampling rate should be more than twice the highest frequency of the components in the signal. Otherwise, higher frequency components can appear as lower frequency components known as aliases.

CHAPTER 10

Exercises

E10.1 Solving Equation 10.1 for the saturation current and substituting values, we have

$$\begin{aligned} I_s &= \frac{i_D}{\exp(v_D / nV_T) - 1} \\ &= \frac{10^{-4}}{\exp(0.600/0.026) - 1} \\ &= 9.502 \times 10^{-15} \text{ A} \end{aligned}$$

Then for $v_D = 0.650$ V, we have

$$\begin{aligned} i_D &= I_s\left[\exp(v_D / nV_T) - 1\right] = 9.502 \times 10^{-15} \times \left[\exp(0.650/0.026) - 1\right] \\ &= 0.6841 \text{ mA} \end{aligned}$$

Similarly for $v_D = 0.700$ V, $i_D = 4.681$ mA.

E10.2 The approximate form of the Shockley Equation is $i_D = I_s \exp(v_D / nV_T)$. Taking the ratio of currents for two different voltages, we have

$$\frac{i_{D1}}{i_{D2}} = \frac{\exp(v_{D1} / nV_T)}{\exp(v_{D2} / nV_T)} = \exp\left[(v_{D1} - v_{D2}) / nV_T\right]$$

Solving for the difference in the voltages, we have:

$$\Delta v_D = nV_T \ln(i_{D1} / i_{D2})$$

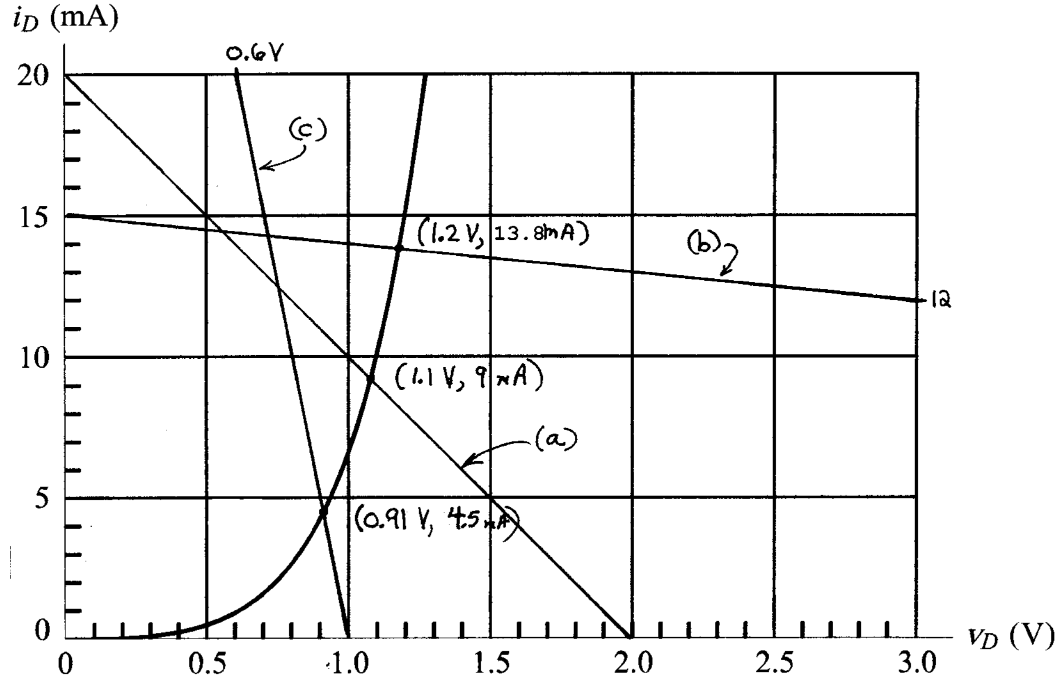
Thus to double the diode current we must increase the voltage by $\Delta v_D = 0.026 \ln(2) = 18.02$ mV and to increase the current by an order of magnitude we need $\Delta v_D = 0.026 \ln(10) = 59.87$ mV

E10.3 The load line equation is $V_{SS} = Ri_D + v_D$. The load-line plots are shown on the next page. From the plots we find the following operating points:

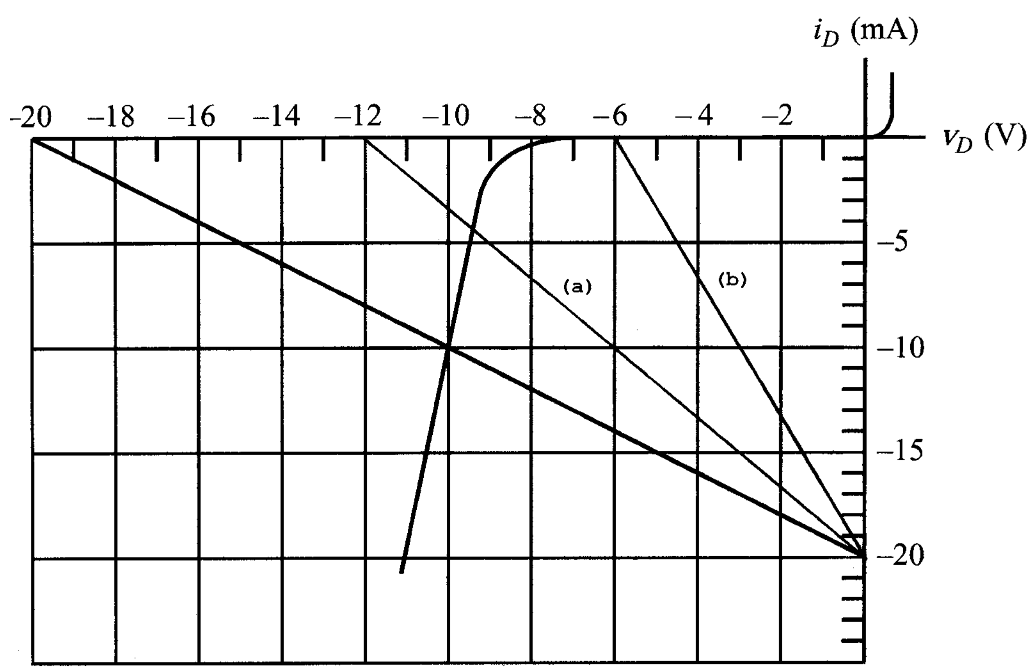
(a) $V_{DQ} = 1.1$ V $\quad I_{DQ} = 9$ mA

(b) $V_{DQ} = 1.2$ V $\quad I_{DQ} = 13.8$ mA

(c) $V_{DQ} = 0.91$ V $\quad I_{DQ} = 4.5$ mA



E10.4 Following the methods of Example 10.4 in the book, we determine that:
(a) For $R_L = 1200\ \Omega$, $R_T = 600\ \Omega$, and $V_T = 12$ V.
(b) For $R_L = 400\ \Omega$, $R_T = 300\ \Omega$, and $V_T = 6$ V.
The corresponding load lines are:

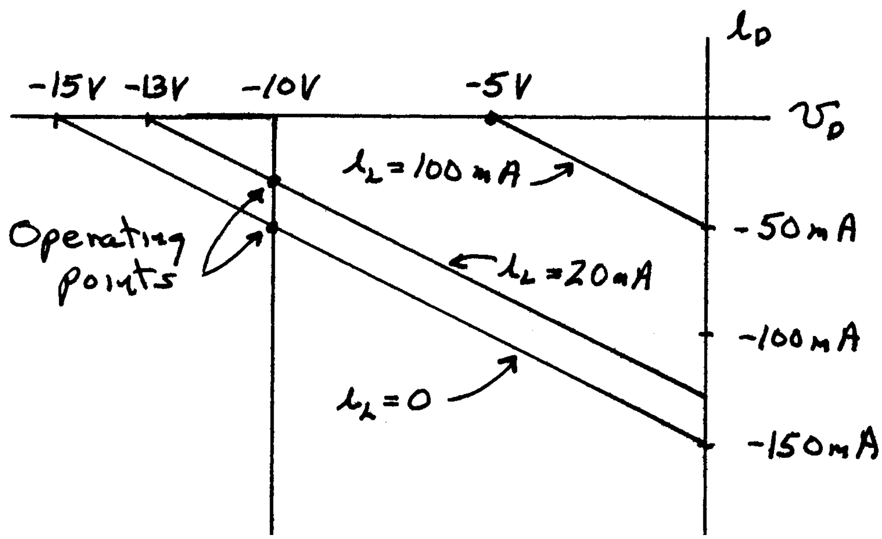


At the intersections of the load lines with the diode characteristic we find (a) $v_L = -v_D \cong 9.4$ V; (b) $v_L = -v_D \cong 6.0$ V.

E10.5 Writing a KVL equation for the loop consisting of the source, the resistor, and the load, we obtain:

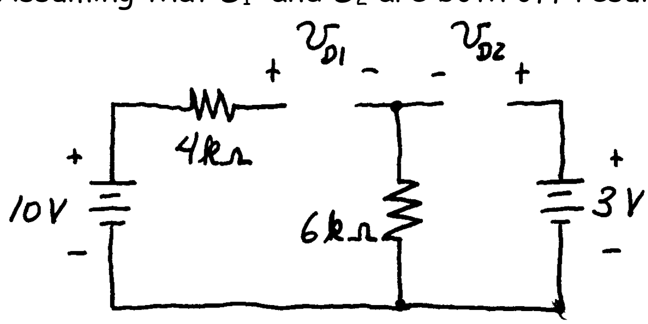
$$15 = 100(i_L - i_D) - v_D$$

The corresponding load lines for the three specified values of i_L are shown:



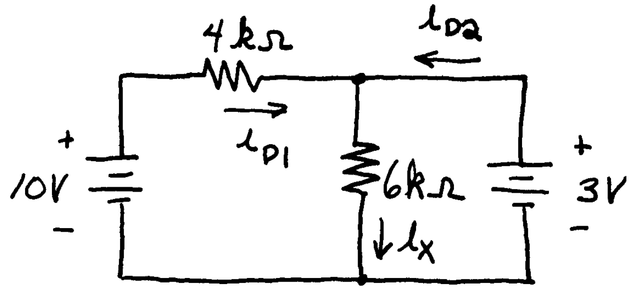
At the intersections of the load lines with the diode characteristic, we find (a) $v_o = -v_D = 10$ V; (b) $v_o = -v_D = 10$ V; (c) $v_o = -v_D = 5$ V. Notice that the regulator is effective only for values of load current up to 50 mA.

E10.6 Assuming that D_1 and D_2 are both off results in this equivalent circuit:



Because the diodes are assumed off, no current flows in any part of the circuit, and the voltages across the resistors are zero. Writing a KVL equation around the left-hand loop we obtain $v_{D1} = 10$ V, which is not consistent with the assumption that D_1 is off.

E10.7 Assuming that D_1 and D_2 are both on results in this equivalent circuit:

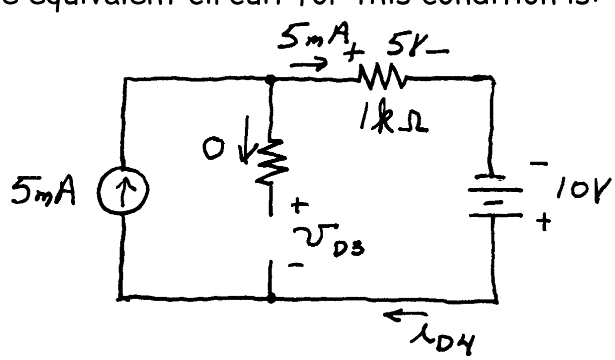


Writing a KVL equation around the outside loop, we find that the voltage across the 4-kΩ resistor is 7 V and then we use Ohm's law to find that i_{D1} equals 1.75 mA. The voltage across the 6-kΩ resistance is 3 V so i_x is 0.5 mA. Then we have $i_{D2} = i_x - i_{D1} = -1.25$ mA, which is not consistent with the assumption that D_2 is on.

E10.8 (a) If we assume that D_1 is off, no current flows, the voltage across the resistor is zero, and the voltage across the diode is 2 V, which is not consistent with the assumption. If we assume that the diode is on, 2 V appears across the resistor, and a current of 0.5 mA circulates clockwise which is consistent with the assumption that the diode is on. Thus the diode is on.

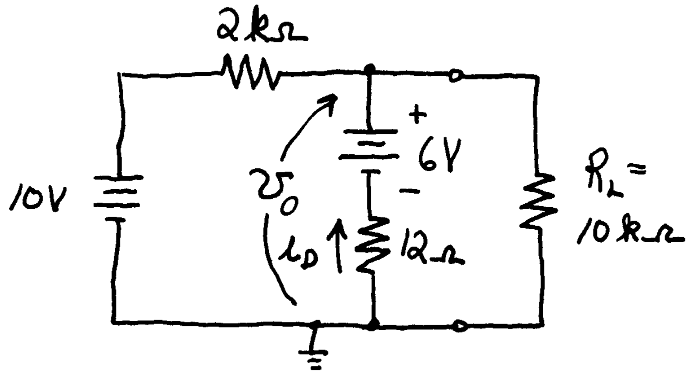
(b) If we assume that D_2 is on, a current of 1.5 mA circulates counterclockwise in the circuit, which is not consistent with the assumption. On the other hand, if we assume that D_2 is off we find that $v_{D2} = -3$ where as usual we have referenced v_{D2} positive at the anode. This is consistent with the assumption, so D_2 is off.

(c) It turns out that the correct assumption is that D_3 is off and D_4 is on. The equivalent circuit for this condition is:



For this circuit we find that $i_{D4} = 5$ mA and $v_{D3} = -5$ V. These results are consistent with the assumptions.

E10.9 (a) With R_L = 10 kΩ, it turns out that the diode is operating on line segment *C* of Figure 10.19 in the book. Then the equivalent circuit is:

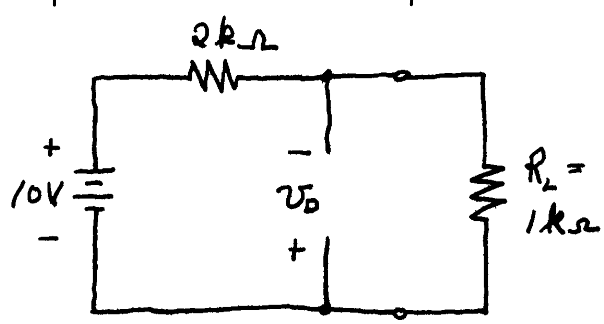


We can solve this circuit by using the node-voltage technique, treating v_o as the node voltage-variable. Notice that $v_o = -v_D$. Writing a KCL equation, we obtain

$$\frac{v_o - 10}{2000} + \frac{v_o - 6}{12} + \frac{v_o}{10000} = 0$$

Solving, we find $v_D = -v_o = -6.017$ V. Furthermore, we find that $i_D = -1.39$ mA. Since we have $v_D \le -6$ V and $i_D \le 0$, the diode is in fact operating on line segment *C*.

(b) With R_L = 1 kΩ, it turns out that the diode is operating on line segment *B* of Figure 10.19 in the book, for which the diode equivalent is an open circuit. Then the equivalent circuit is:



Using the voltage division principle, we determine that $v_D = -3.333$ V. Because we have $-6 \le v_D \le 0$, the result is consistent with the assumption that the diode operates on segment *B*.

E10.10 The piecewise linear model consists of a voltage source and resistance in series for each segment. Refer to Figure 10.18 in the book and notice that the *x*-axis intercept of the line segment is the value of the voltage source, and the reciprocal of the slope is the resistance. Now look at Figure 10.22a and notice that the intercept for segment *A* is zero and the reciprocal of the slope is (2 V)/(5 mA) = 400 Ω. Thus as shown in Figure 10.22b, the equivalent circuit for segment *A* consists of a 400-Ω resistance.

Similarly for segment *B*, the *x*-axis intercept is +1.5 V and the reciprocal slope is (0.5 mA)/(5 V) = 10 kΩ.

For segment *C*, the intercept is -5.5 V and the reciprocal slope is 800 Ω. Notice that the polarity of the voltage source is reversed in the equivalent circuit because the intercept is negative.

E10.11 Refer to Figure 10.25 in the book.

(a) The peak current occurs when the sine wave source attains its peak amplitude, then the voltage across the resistor is $V_m - V_B = 20 - 14 = 6$ V and the peak current is 0.6 *A*.

(b) Refer to Figure 10.25 in the book. The diode changes state at the instants for which $V_m \sin(\omega t) = V_B$. Thus we need the roots of $20\sin(\omega t) = 14$. These turn out to be $\omega t_1 = 0.7754$ radians and $\omega t_2 = \pi - 0.7754$ radians.

The interval that the diode is on is $t_2 - t_1 = \dfrac{1.591}{\omega} = \dfrac{1.591T}{2\pi} = 0.2532T$.

Thus the diode is on for 25.32% of the period.

E10.12 As suggested in the Exercise statement, we design for a peak load voltage of 15.2 V. Then allowing for a forward drop of 0.7 V we require $V_m = 15.9$ V. Then we use Equation 10.10 to determine the capacitance required. $C = (I_L T)/V_r = (0.1/60)/0.4 = 4167\ \mu\text{F}$.

E10.13 For the circuit of Figure 10.28, we need to allow for two diode drops. Thus the peak input voltage required is $V_m = 15 + V_r/2 + 2 \times 0.7 = 16.6$ V.

Because this is a full-wave rectifier, the capacitance is given by Equation 10.12. $C = (I_L T)/(2V_r) = (0.1/60)/0.8 = 2083\ \mu$F.

E10.14 Refer to Figure 10.31 in the book.

(a) For this circuit all of the diodes are off if $-1.8 < v_o < 10$. With the diodes off, no current flows and $v_o = v_{in}$. When v_{in} exceeds 10 V, D_1 turns on and D_2 is in reverse breakdown. Then $v_o = 9.4 + 0.6 = 10$ V. When v_{in} becomes less than -1.8 V diodes D_3, D_4, and D_5 turn on and $v_o = -3 \times 0.6 = -1.8$ V. The transfer characteristic is shown in Figure 10.31c.

(b)) For this circuit both diodes are off if $-5 < v_o < 5$. With the diodes off, no current flows and $v_o = v_{in}$.

When v_{in} exceeds 5 V, D_6 turns on and D_7 is in reverse breakdown. Then a current given by $i = \dfrac{v_{in} - 5}{2000}$ (i is referenced clockwise) flows in the circuit, and the output voltage is $v_o = 5 + 1000i = 0.5v_{in} + 2.5$ V

When v_{in} is less than -5 V, D_7 turns on and D_6 is in reverse breakdown. Then a current given by $i = \dfrac{v_{in} + 5}{2000}$ (still referenced clockwise) flows in the circuit, and the output voltage is $v_o = -5 + 1000i = 0.5v_{in} - 2.5$ V

E10.15 Answers are shown in Figure 10.32c and d. Other correct answers exist.

E10.16 Refer to Figure 10.34a in the book.

(a) If $v_{in}(t) = 0$, we have only a dc source in the circuit. In steady state, the capacitor acts as an open circuit. Then we see that D_2 is forward conducting and D_1 is in reverse breakdown. Allowing 0.6 V for the forward diode voltage the output voltage is -5 V.

(b) If the output voltage begins to fall below -5 V, the diodes conduct large amounts of current and change the voltage v_C across the capacitor. Once the capacitor voltage is changed so that the output cannot fall below -5 V, the capacitor voltage remains constant. Thus the output voltage is $v_o = v_{\text{in}} - v_C = 2\sin(\omega t) - 3$ V.

(c) If the 15-V source is replaced by a short circuit, the diodes do not conduct, $v_C = 0$, and $v_o = v_{\text{in}}$.

E10.17 One answer is shown in Figure 10.35. Other correct answers exist.

E10.18 One design is shown in Figure 10.36. Other correct answers are possible.

E10.19 Equation 10.22 gives the dynamic resistance of a semiconductor diode as $r_d = nV_T / I_{DQ}$.

I_{DQ} (mA)	r_d (Ω)
0.1	26,000
1.0	2600
10	26

E10.20 For the Q-point analysis, refer to Figure 10.42 in the book. Allowing for a forward diode drop of 0.6 V, the diode current is

$$I_{DQ} = \frac{V_C - 0.6}{R_C}$$

The dynamic resistance of the diode is

$$r_d = \frac{nV_T}{I_{DQ}}$$

the resistance R_p is given by Equation 10.23 which is

$$R_p = \frac{1}{1/R_C + 1/R_L + 1/r_d}$$

and the voltage gain of the circuit is given by Equation 10.24.

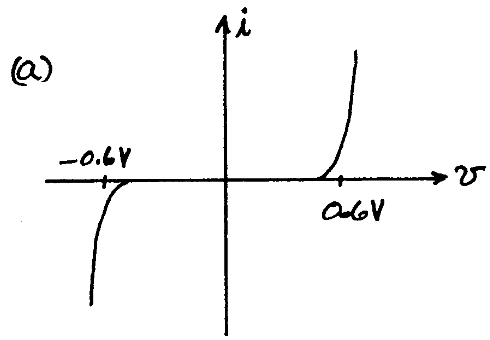
$$A_v = \frac{R_p}{R + R_p}$$

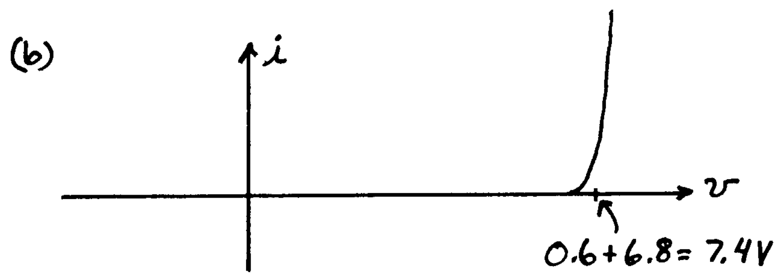
Evaluating we have

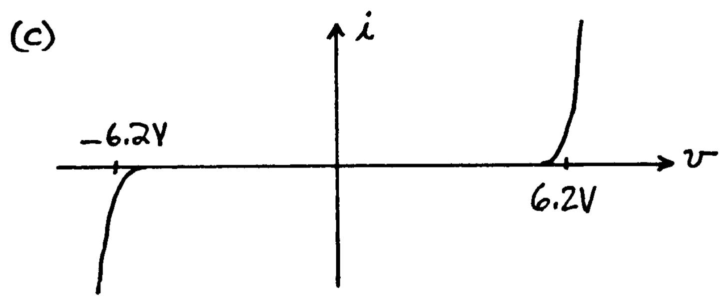
V_C (V)	1.6	10.6
I_{DQ} (mA)	0.5	5.0
r_d (Ω)	52	5.2
R_p (Ω)	49.43	5.173
A_v	0.3308	0.04919

Answers for Selected Problems

P10.6*







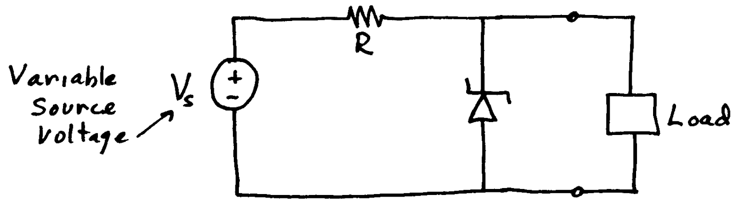
P10.8* $n = 1.336$
$I_s = 3.150 \times 10^{-11}$ A

P10.13* With $n = 1$, $v = 582$ mV.
With $n = 2$, $v = 564$ mV.

P10.15* (a) $I_A = I_B = 100$ mA
(b) $I_A = 87$ mA and $I_B = 113$ mA

P10.16* $v_x = 2.20$ V $\quad i_x = 0.80$ A

P10.26* The circuit diagram of a simple voltage regulator is:



P10.33* $i_{ab} \cong 1.4$ A $\quad v_{ab} \cong 2.9$ V

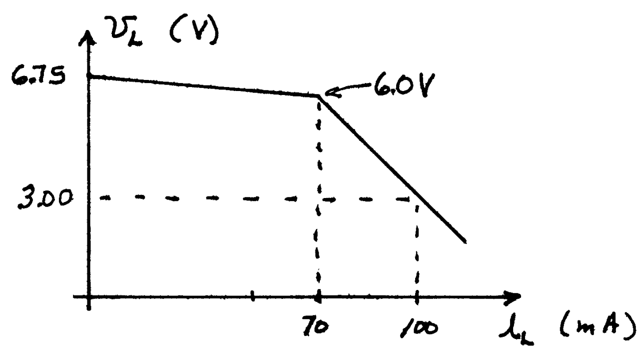
P10.37* (a) D_1 is on and D_2 is off. $V = 10$ volts and $I = 0$.

(b) D_1 is on and D_2 is off. $V = 6$ volts and $I = 6$ mA.

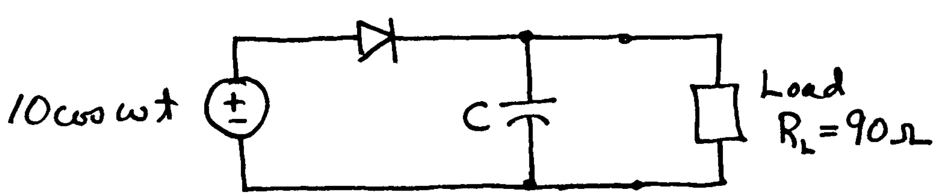
(c) Both D_1 and D_2 are on. $V = 30$ volts and $I = 33.6$ mA.

P10.46* For the circuit of Figure P10.46a, $v = 0.964$ V.
For the circuit of Figure P10.46b, $v = 1.48$ V.

P10.47*

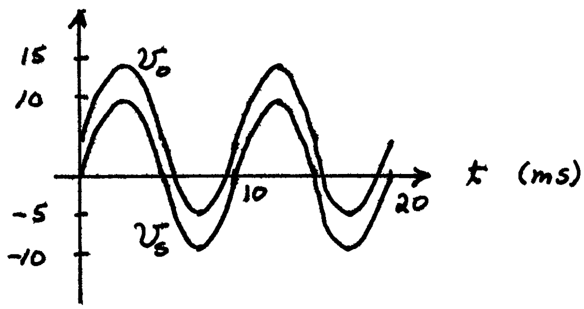


P10.54* $C = 833\ \mu$F

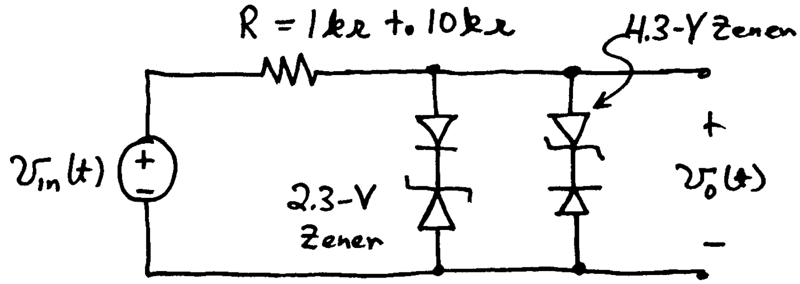


P10.58* For a half-wave rectifier, $C = 20833\ \mu$F.
For a full-wave rectifier, $C = 10416\ \mu$F.

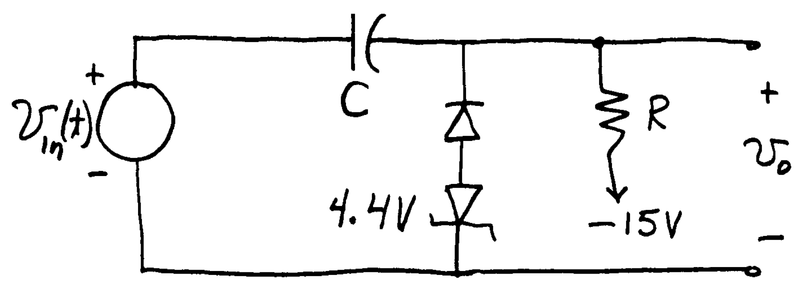
P10.70*



P10.72*

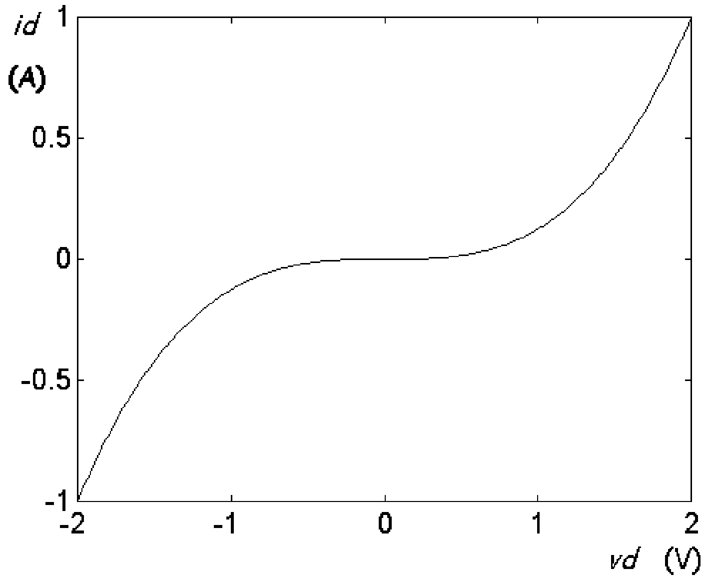


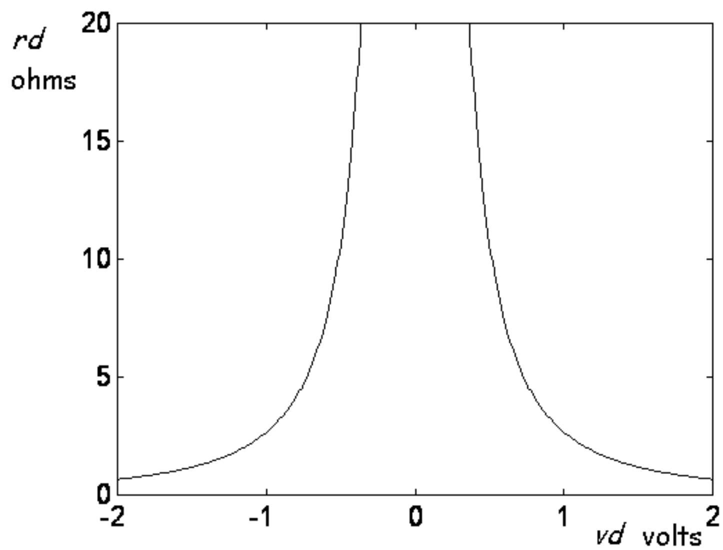
P10.75*



We must choose the time constant $RC >> T$, where T is the period of the input waveform.

P10.81*

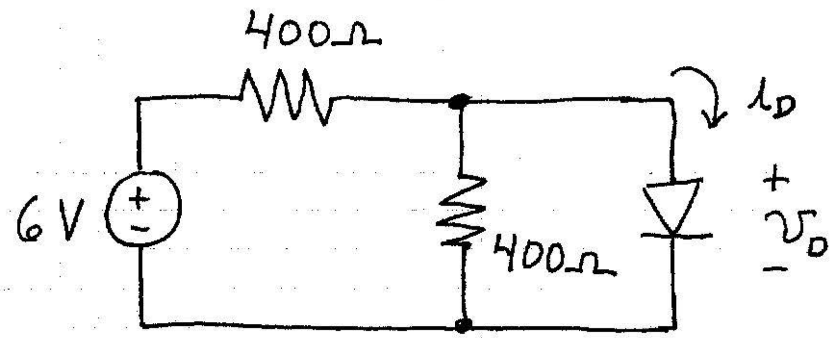




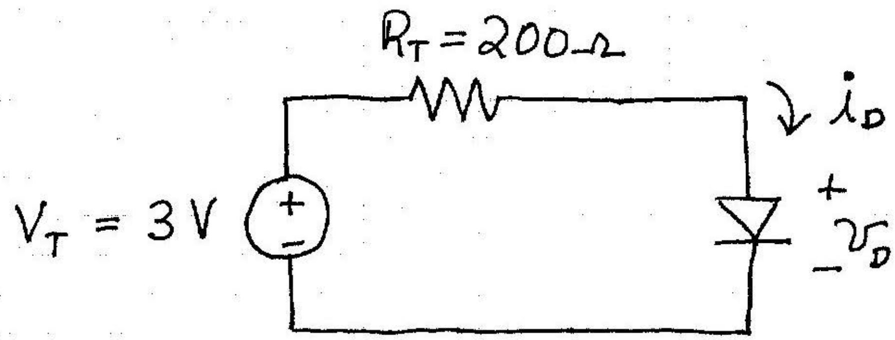
P10.85* $I_{DQ} = 100$ mA $\qquad r_D = 0.202\ \Omega$

Practice Test

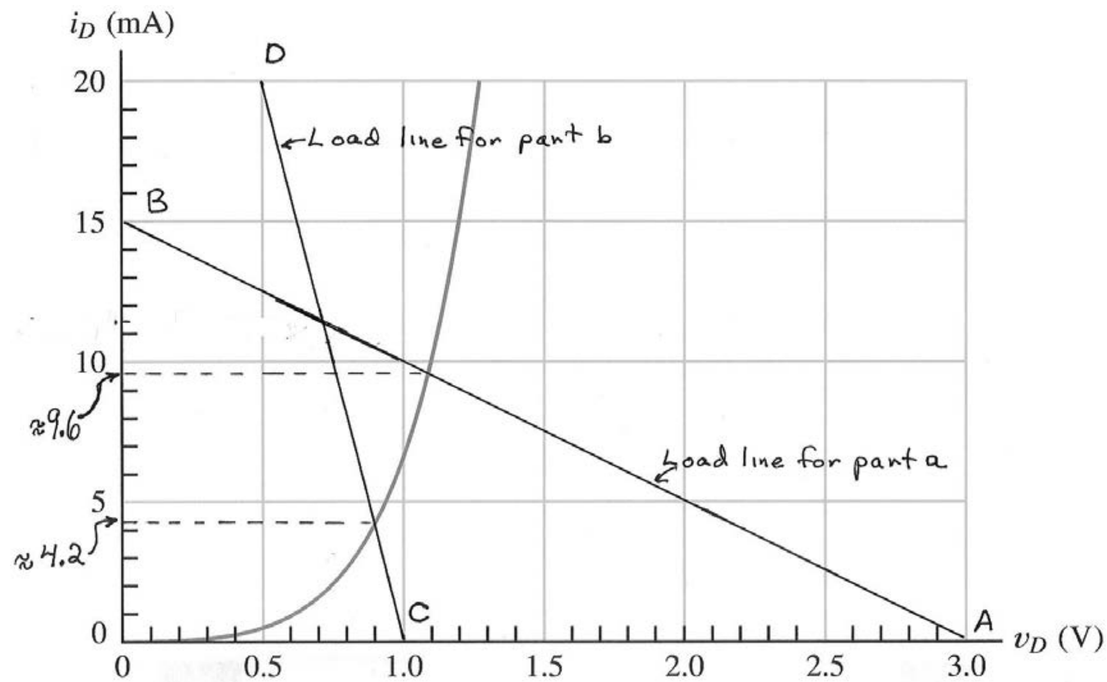
T10.1 (a) First, we redraw the circuit, grouping the linear elements to the left of the diode.



Then, we determine the Thévenin equivalent for the circuit looking back from the diode terminals.



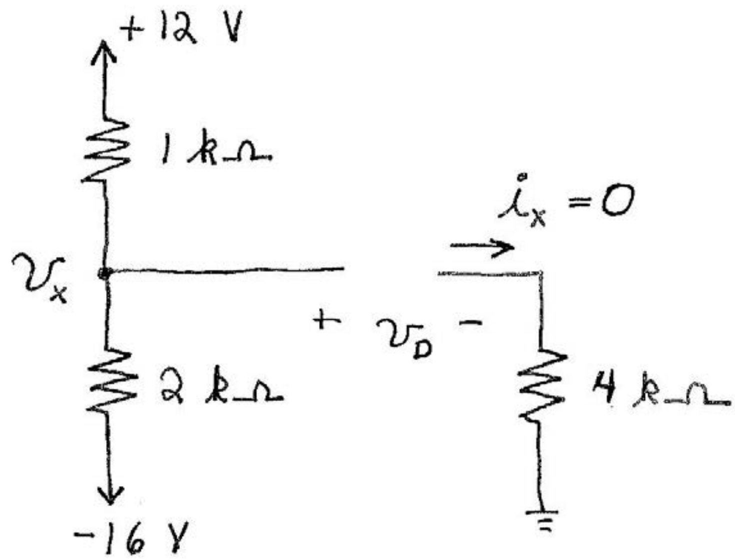
Next, we write the KVL equation for the network, which yields $V_T = R_T i_D + v_D$. Substituting the values for the Thévenin voltage and resistance, we have the load-line equation, $3 = 200 i_D + v_D$. For $i_D = 0$, we have $v_D = 3$ V which are the coordinates for Point *A* on the load line, as shown below. For $v_D = 0$, the load-line equation gives $i_D = 15$ mA which are the coordinates for Point *B* on the load line. Using these two points to plot the load line on Figure 10.8, we have



The intersection of the load line and the diode characteristic gives the current at the operating point as $i_D \cong 9.6$ mA.

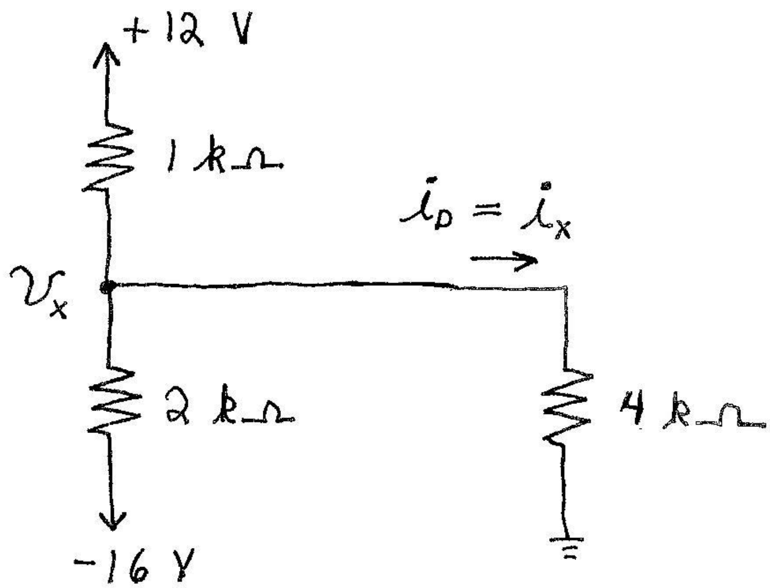
(b) First, we write the KCL equation at the top node of the network, which yields $i_D + v_D / 25 = 40$ mA. For $i_D = 0$, we have $v_D = 1$ V which are the coordinates for Point *C* on the load line shown above. For $v_D = 0$, the load-line equation gives $i_D = 40$ mA which plots off the vertical scale. Therefore, we substitute $i_D = 20$ mA, and the KCL equation then yields $v_D = 0.5$ V. These values are shown as point D. Using Points C and D we plot the load line on Figure 10.8 as shown above. The intersection of the load line and the diode characteristic gives the current at the operating point as $i_D \cong 4.2$ mA.

T10.2 If we assume that the diode is off (i.e., an open circuit), the circuit becomes



Writing a KCL equation with resistances in kΩ, currents in mA, and voltages in V, we have $\frac{v_x - 12}{1} + \frac{v_x - (-16)}{2} = 0$. Solving, we find that $v_x = 2.667$ V. However, the voltage across the diode is $v_D = v_x$, which must be negative for the diode to be off. Therefore, the diode must be on.

With the diode assumed to be on (i.e. a short circuit) the circuit becomes

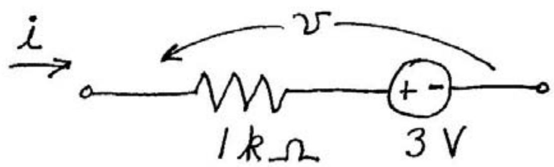


Writing a KCL equation with resistances in kΩ, currents in mA and voltages in V, we have $\frac{v_x - 12}{1} + \frac{v_x - (-16)}{2} + \frac{v_x}{4} = 0$. Solving, we find that

$v_x = 2.286$ V. Then, the current through the diode is

$i_D = i_x = \frac{v_x}{4} = 0.571$ mA. Of course, a positive value for i_D is consistent with the assumption that the diode is on.

T10.3 We know that the line passes through the points (5 V, 2 mA) and (10 V, 7 mA). The slope of the line is $-1/R = -\Delta i/\Delta v = (-5\text{ mA})/(5\text{ V})$, and we have $R = 1$ kΩ. Furthermore, the intercept on the voltage axis is at $v = 3$ V. Thus, the equivalent circuit for the device is



T10.4 The circuit diagram is:

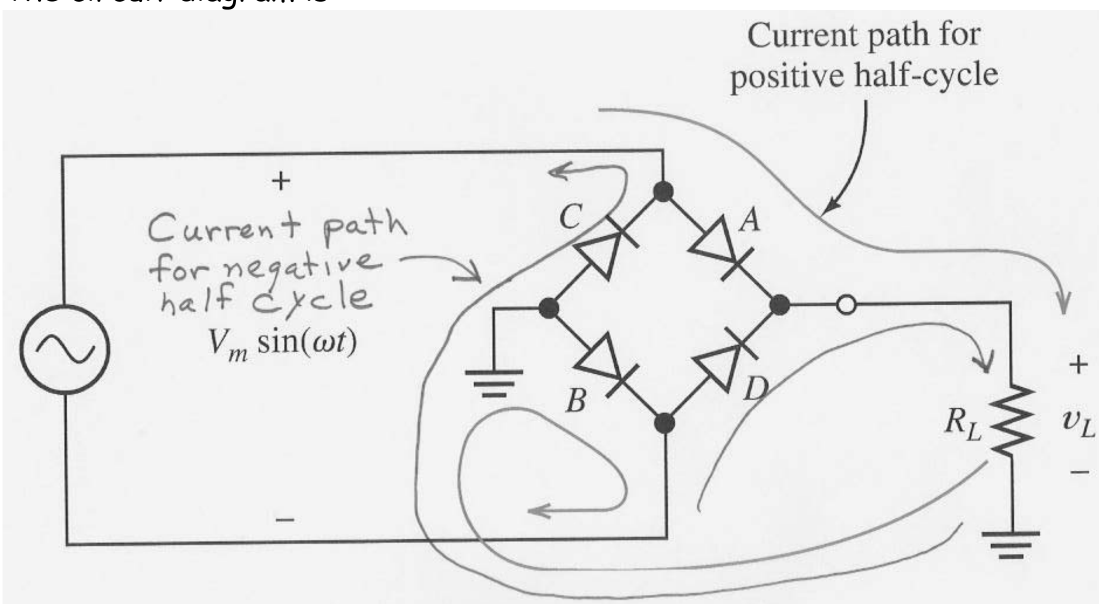
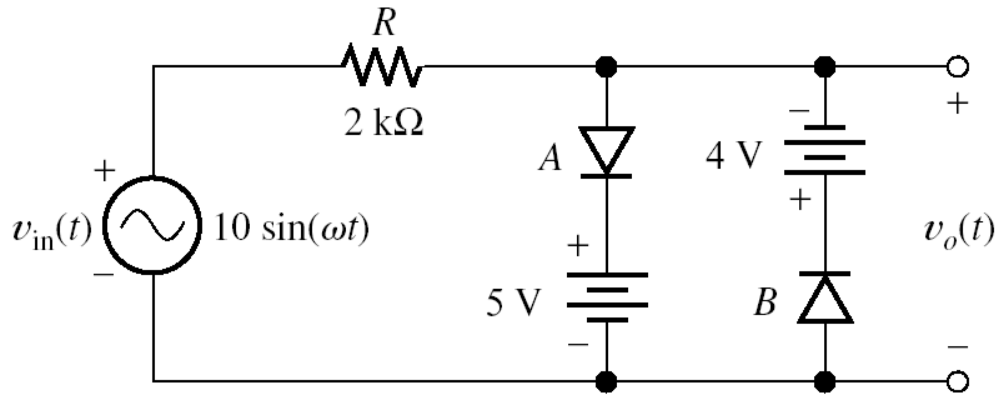


Figure 10.28 Diode-bridge full-wave rectifier.

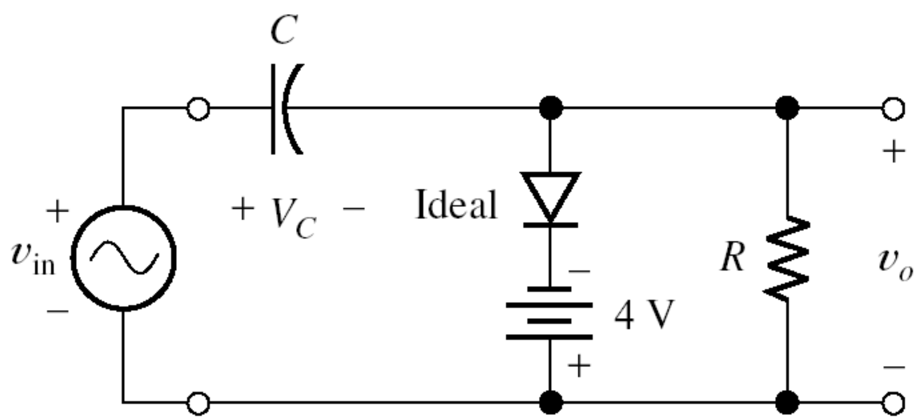
Your diagram may be correct even if it is laid out differently. Check to see that you have four diodes and that current flows from the source through a diode in the forward direction then through the load and finally through a second diode in the forward direction back to the opposite end of the source. On the opposite half cycle, the path should be through the other two diodes and through the load in the same direction as before. Notice in the diagram that current flows downward through the load on both half cycles.

T10.5 An acceptable circuit diagram is:



Your diagram may be somewhat different in appearance. For example, the 4-V source and diode *B* can be interchanged as long as the source polarity and direction of the diode don't change; similarly for the 5-V source and diode *A*. The parallel branches can be interchanged in position. The problem does not give enough information to properly select the value of the resistance, however, any value from about 1 kΩ to 1 MΩ is acceptable.

T10.6 An acceptable circuit diagram is:



The time constant *RC* should be much longer than the period of the source voltage. Thus, we should select component values so that $RC >> 0.1$ s.

T10.7 We have

$$V_T = \frac{kT}{q} = \frac{1.38 \times 10^{-23} \times 300}{1.60 \times 10^{-19}} = 25.88 \text{ mV}$$

$$r_d = \frac{nV_T}{I_{DQ}} = \frac{2 \times 25.88 \times 10^{-3}}{5 \times 10^{-3}} = 10.35\ \Omega$$

The small-signal equivalent circuit for the diode is a 10.35 Ω resistance.

CHAPTER 11

Exercises

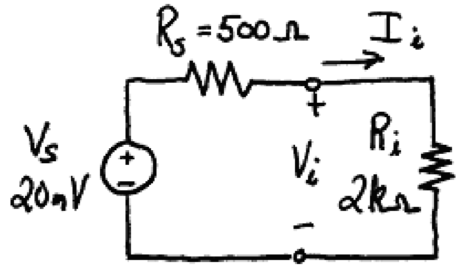
E11.1 (a) A noninverting amplifier has positive gain. Thus

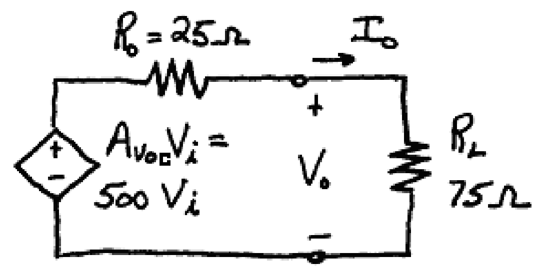
$$v_o(t) = A_v v_i(t) = 50 v_i(t) = 5.0\sin(2000\pi t)$$

(b) An inverting amplifier has negative gain. Thus

$$v_o(t) = A_v v_i(t) = -50 v_i(t) = -5.0\sin(2000\pi t)$$

E11.2





$$A_v = \frac{V_o}{V_i} = A_{voc}\frac{R_L}{R_o + R_L} = 500\frac{75}{25+75} = 375$$

$$A_{vs} = \frac{V_o}{V_s} = \frac{R_i}{R_s + R_i} A_{voc}\frac{R_L}{R_o + R_L} = \frac{2000}{500+2000} 500 \frac{75}{25+75} = 300$$

$$A_i = \frac{I_o}{I_i} = A_v\frac{R_i}{R_L} = 375 \times \frac{2000}{75} = 10^4$$

$$G = A_v A_i = 3.75 \times 10^6$$

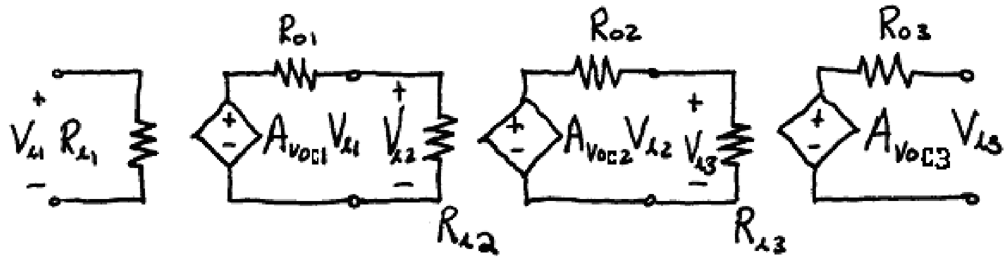
E11.3 Recall that to maximize the power delivered to a load from a source with fixed internal resistance, we make the load resistance equal to the internal (or Thévenin) resistance. Thus we make $R_L = R_o = 25\ \Omega$. Repeating the calculations of Exercise 11.2 with the new value of R_L, we have

$$A_v = \frac{V_o}{V_i} = A_{voc}\frac{R_L}{R_o + R_L} = 500\frac{25}{25+25} = 250$$

$$A_i = \frac{I_o}{I_i} = A_v\frac{R_i}{R_L} = 250 \times \frac{2000}{25} = 2 \times 10^4$$

$$G = A_v A_i = 5 \times 10^6$$

E11.4



By inspection, $R_i = R_{i1} = 1000\ \Omega$ and $R_o = R_{o3} = 30\ \Omega$.

$$A_{oc} = \frac{V_{o3}}{V_{i1}} = A_{oc1}\frac{R_{i2}}{R_{o1}+R_{i2}}A_{oc2}\frac{R_{i3}}{R_{o2}+R_{i3}}A_{oc3}$$

$$A_{oc} = \frac{V_{o3}}{V_{i1}} = 10\frac{2000}{100+2000}20\frac{3000}{200+3000}30 = 5357$$

E11.5 Switching the order of the amplifiers of Exercise 11.4 to 3-2-1, we have

$R_i = R_{i3} = 3000\ \Omega$ and $R_o = R_{o1} = 100\ \Omega$

$$A_{oc} = \frac{V_{o1}}{V_{i3}} = A_{oc3}\frac{R_{i2}}{R_{o3}+R_{i2}}A_{oc2}\frac{R_{i1}}{R_{o2}+R_{i1}}A_{oc1}$$

$$A_{oc} = \frac{V_{o1}}{V_{i3}} = 30\frac{2000}{300+2000}20\frac{1000}{200+1000}10 = 4348$$

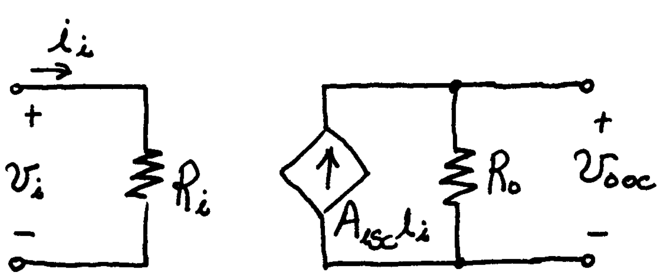
E11.6

$$P_s = (15\text{ V})\times(1.5\text{ A}) = 22.5\text{ W}$$

$$P_d = P_s + P_i - P_o = 22.5 + 0.5 - 2.5 = 20.5\text{ W}$$

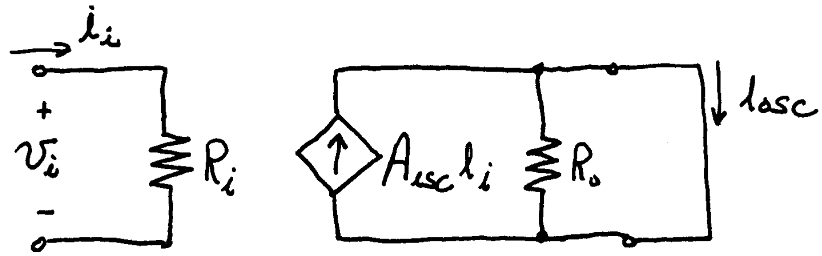
$$\eta = \frac{P_o}{P_s}\times 100\% = 11.11\%$$

E11.7 The input resistance and output resistance are the same for all of the amplifier models. Only the circuit configuration and the gain parameter are different. Thus we have $R_i = 1\text{ k}\Omega$ and $R_o = 20\ \Omega$ and we need to find the open-circuit voltage gain. The current amplifier with an open-circuit load is:



$$A_{voc} = \frac{v_{ooc}}{v_i} = \frac{A_{isc} i_i R_o}{R_i i_i} = \frac{A_{isc} R_o}{R_i} = \frac{200 \times 20}{1000} = 4$$

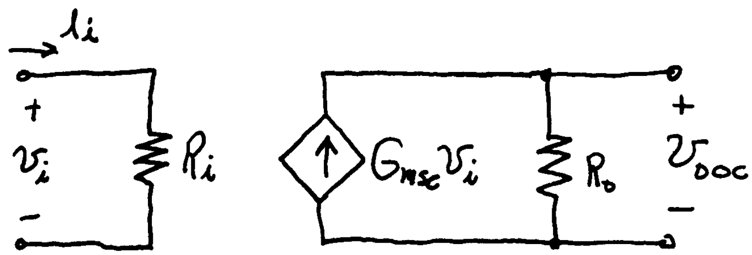
E11.8 For a transconductance-amplifier model, we need to find the short-circuit transconductance gain. The current-amplifier model with a short-circuit load is:



$$G_{msc} = \frac{i_{osc}}{v_i} = \frac{A_{isc} i_i}{R_i i_i} = \frac{A_{isc}}{R_i} = \frac{100}{500} = 0.2 \text{ S}$$

The impedances are the same for all of the amplifier models, so we have $R_i = 500\ \Omega$ and $R_o = 50\ \Omega$.

E11.9 For a transresistance-amplifier model, we need to find the open-circuit transresistance gain. The transconductance-amplifier model with an open-circuit load is:



$$R_{moc} = \frac{v_{ooc}}{i_i} = \frac{G_{msc} v_i R_o}{v_i / R_i} = G_{msc} R_o R_i = 0.05 \times 10 \times 10^6 = 500 \text{ k}\Omega$$

The impedances are the same for all of the amplifier models, so we have $R_i = 1\ \text{M}\Omega$ and $R_o = 10\ \Omega$.

E11.10 The amplifier has $R_i = 1\ \text{k}\Omega$ and $R_o = 1\ \text{k}\Omega$.

(a) We have $R_s < 10\ \Omega$ which is much less than R_i, and we also have $R_L > 100\ \text{k}\Omega$ which is much larger than R_o. Therefore for this source and load, the amplifier is approximately an ideal voltage amplifier.

(b) We have $R_s > 100\ \text{k}\Omega$ which is much greater than R_i, and we also have $R_L < 10\ \Omega$ which is much smaller than R_o. Therefore for this source and load, the amplifier is approximately an ideal current amplifier.

(c) We have $R_s < 10\ \Omega$ which is much less than R_i, and we also have $R_L < 10\ \Omega$ which is much smaller than R_o. Therefore for this source and load, the amplifier is approximately an ideal transconductance amplifier.

(d) We have $R_s > 100\ \text{k}\Omega$ which is much larger than R_i, and we also have $R_L > 100\ \text{k}\Omega$ which is much larger than R_o. Therefore for this source and load, the amplifier is approximately an ideal transresistance amplifier.

(e) Because we have $R_s \cong R_i$, the amplifier does not approximate any type of ideal amplifier.

E11.11 We want the amplifier to respond to the short-circuit current of the source. Therefore, we need to have $R_i << R_s$. Because the amplifier should deliver a voltage to the load that is independent of the load resistance, the output resistance R_o should be very small compared to the smallest load resistance. These facts (R_s very small and R_o very small) indicate that we need a nearly ideal transresistance amplifier.

E11.12 The gain magnitude should be constant for all components of the input signal, and the phase should by proportional to the frequency of each component. The input signal has components with frequencies of 500 Hz, 1000 Hz and 1500 Hz, respectively. The gain is $5\angle 30^\circ$ at a frequency of 1000 Hz. Therefore the gain should be $5\angle 15^\circ$ at 500 Hz, and $5\angle 45^\circ$ at 1500 Hz.

E11.13 We have

$$v_{\text{in}}(t) = V_m \cos(\omega t)$$

$$v_o(t) = 10 v_{\text{in}}(t - 0.01) = 10 V_m \cos[\omega(t - 0.01)] = 10 V_m \cos(\omega t - 0.01\omega)$$

The corresponding phasors are $\mathbf{V}_{\text{in}} = V_m \angle 0$ and $\mathbf{V}_{\text{o}} = 10 V_m \angle -0.01\omega$. Thus the complex gain is

$$A_v = \frac{\mathbf{V}_o}{\mathbf{V}_{\text{in}}} = \frac{10 V_m \angle -0.01\omega}{V_m \angle 0} = 10 \angle -0.01\omega$$

E11.14 $B \cong \dfrac{0.35}{t_r} = \dfrac{0.35}{66.7 \times 10^{-9}} = 5.247\text{ MHz}$

E11.15 Equation 11.13 states

Percentage tilt $\cong 200\pi f_L T$

Solving for f_L and substituting values, we obtain

$$f_L \cong \frac{\text{percentage tilt}}{200\pi T} = \frac{1}{200\pi \times 100 \times 10^{-6}} = 15.92\text{ Hz}$$

as the upper limit for the lower half-power frequency.

E11.16 (a) $v_o(t) = 100v_i(t) + v_i^2(t)$

$$= 100\cos(\omega t) + \cos^2(\omega t)$$
$$= 100\cos(\omega t) + 0.5 + 0.5\cos(2\omega t)$$

The desired term has an amplitude of V_1 = 100 and a second-harmonic distortion term with an amplitude of V_2 = 0.5. There are no higher order distortion terms so we have $D_2 = V_2 / V_1 = 0.005$ or 0.5%.

$$D = \sqrt{D_2^2 + D_3^2 + D_4^2 \ldots} = D_2 = 0.5\%$$

(b) $v_o(t) = 100v_i(t) + v_i^2(t)$

$$= 500\cos(\omega t) + 25\cos^2(\omega t)$$
$$= 500\cos(\omega t) + 12.5 + 12.5\cos(2\omega t)$$

The desired term has an amplitude of V_1 = 500 and a second-harmonic distortion term with an amplitude of V_2 = 12.5. There are no higher order distortion terms so we have $D_2 = V_2 / V_1 = 0.025$ or 2.5%.

$$D = \sqrt{D_2^2 + D_3^2 + D_4^2 \ldots} = D_2 = 2.5\%$$

E11.17 With the input terminals tied together and a 1-V signal applied, the differential signal is zero and the common-mode signal is 1 V. The common-mode gain is $A_{cm} = V_o / V_{icm} = 0.1/1 = 0.1$, which is equivalent to -20 dB. Then we have $CMRR = 20\log(|A_d|/|A_{cm}|) = 20\log(500{,}000) = 114.0\text{ dB}$.

E11.18 (a) $v_{id} = v_{i1} - v_{i2} = 1\text{ V} \qquad v_{icm} = (v_{i1} + v_{i2})/2 = 0\text{ V}$

$$v_o = A_1 v_{i1} - A_2 v_{i2} = (A_1 + A_2)/2$$
$$= A_d v_{id} + A_{cm} v_{icm} = A_d$$

Thus $A_d = (A_1 + A_2)/2$.

(b) $v_{id} = v_{i1} - v_{i2} = 0\text{ V} \qquad v_{icm} = (v_{i1} + v_{i2})/2 = 1\text{ V}$

$$v_o = A_1 v_{i1} - A_2 v_{i2} = (A_1 - A_2)$$
$$= A_d v_{id} + A_{cm} v_{icm} = A_{cm}$$

Thus $A_{cm} = A_1 - A_2$.

(c) $A_d = (A_1 + A_2)/2 = (100 + 101)/2 = 100.5$

$$A_{cm} = A_1 - A_2 = 100 - 101 = -1$$
$$\text{CMRR} = 20\log\left(\frac{|A_d|}{|A_{cm}|}\right) = 20\log\left(\frac{|A_1 + A_2|}{2|A_1 - A_2|}\right)$$
$$\text{CMRR} = 20\log\left(\frac{|A_1 + A_2|}{2|A_1 - A_2|}\right) = 20\log\left(\frac{|100 + 101|}{2|100 - 101|}\right) = 40.0\text{ dB.}$$

E11.19 Except for numerical values this Exercise is the same as Example 11.13 in the book. With equal resistances at the input terminals, the bias currents make no contribution to the output voltage. The extreme contributions to the output due to the offset voltage are

$$A_d V_{Voff} = A_d V_{off} \frac{R_{in}}{R_{in} + R_{s1} + R_{s2}}$$
$$= 500 \times (\pm 10 \times 10^{-3}) \frac{100 \times 10^3}{(100 + 50 + 50)10^3} = \pm 2.5\text{ V}$$

The extreme contributions to the output voltage due to the offset current are

$$A_d V_{Ioff} = A_d \frac{I_{off}}{2} \frac{R_{in}(R_{s1} + R_{s2})}{R_{in} + R_{s1} + R_{s2}}$$
$$= 500 \times \frac{\pm 100 \times 10^{-9}}{2} \frac{100 \times 10^3 (50 + 50) \times 10^3}{(100 + 50 + 50)10^3} = \pm 1.25\text{ V}$$

Thus, the extreme output voltages due to all sources are ± 3.75 V.

E11.20 This Exercise is similar to Example 11.13 in the book with R_{s1} = 50 kΩ and R_{s2} = 0. With unequal resistances at the input terminals, the bias currents make a contribution to the output voltage given by

$$V_{oBias} = A_d I_B \frac{R_{s1} R_{in}}{R_{s1} + R_{in}}$$
$$= 500 \times 400 \times 10^{-9} \frac{50 \times 10^3 \times 100 \times 10^3}{50 \times 10^3 + 100 \times 10^3} = +6.667\text{ V}$$

The extreme contributions to the output due to the offset voltage are

$$A_d V_{Voff} = A_d V_{off} \frac{R_{in}}{R_{in} + R_{s1} + R_{s2}}$$
$$= 500 \times (\pm 10 \times 10^{-3}) \frac{100 \times 10^3}{(100 + 50 + 0)10^3} = \pm 3.333 \text{ V}$$

The extreme contributions to the output voltage due to the offset current are

$$A_d V_{Ioff} = A_d \frac{I_{off}}{2} \frac{R_{in}(R_{s1} + R_{s2})}{R_{in} + R_{s1} + R_{s2}}$$
$$= 500 \times \frac{\pm 100 \times 10^{-9}}{2} \frac{100 \times 10^3 (50 + 0) \times 10^3}{(100 + 50 + 0)10^3} = \pm 0.8333 \text{ V}$$

Thus, the extreme output voltages due to all sources are a minimum of 2.5 V and a maximum of 10.83 V.

Answers for Selected Problems

P11.4* $A_v = 50$
$A_{vs} = 33.33$
$A_i = 1.25 \times 10^6$
$G = 62.5 \times 10^6$

P11.5* $A_i = 100$
$R_i = 200\ \Omega$

P11.10* $R_i = 666.7\ \Omega.$

P11.15* $R_{in} = 1\ \text{M}\Omega$

P11.20* $R_i = 2\ \text{k}\Omega$
$R_o = 3\ \text{k}\Omega$
$A_{voc} = 3.6 \times 10^6$

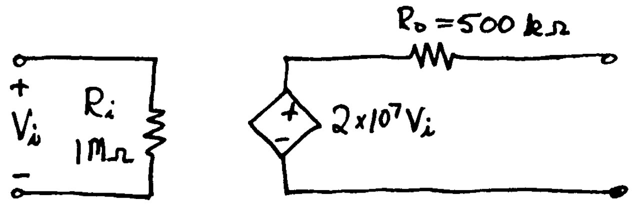
P11.22* Five amplifiers must be cascaded to attain a voltage gain in excess of 1000.

P11.25* $P_s = P_1 + P_2 + P_3 = 40 \text{ W}$

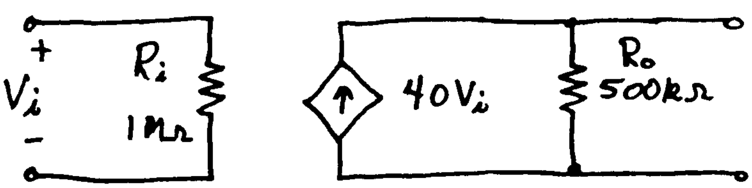
P11.32* $A_i = 2000 \qquad A_v = 500 \qquad G = 10^6 \qquad P_d = 19.87 \text{ W} \qquad \eta = 17.2\%$

P11.33* $A_{voc} = 100 \text{ V/V} \qquad G_{msc} = 0.1 \text{ S} \qquad A_{isc} = 10 \text{ A/A}$

P11.38* The voltage-amplifier model is:



The transconductance-amplifier model is:



P11.40* $A_{voc} = 100$

$A_{isc} = 500$

$R_{moc} = 100 \text{ k}\Omega$

P11.41* $A_{voc} = 20$

$A_{isc} = 100$

$G_{msc} = 0.01 \text{ S}$

P11.52* $R_x = -2.23\,\Omega$

P11.55* To sense the open-circuit voltage of a sensor, we need an amplifier with very high input resistance (compared to the Thévenin resistance of the sensor). To avoid loading effects by the variable load resistance, we need an amplifier with very low output resistance (compared to the smallest load resistance). Thus, we need a nearly ideal voltage amplifier with a gain of 1000.

P11.56* The input resistance is that of the ideal transresistance amplifier which is zero. The output resistance of the cascade is the output resistance of the ideal transconductance amplifier which is infinite. An amplifier having zero input resistance and infinite output resistance is an ideal current amplifier. Also, we have $A_{isc} = R_{moc}G_{msc}$.

P11.61* We need a nearly ideal transconductance amplifier.

$R_i = 98\ \text{k}\Omega \qquad\qquad R_o = 19.7\ \text{k}\Omega$

P11.67* The complex gain for the 1000-Hz component is

$A_v = 100\angle -20^\circ$

The complex gain for the 2000-Hz component is

$A_v = 75\angle -10^\circ$

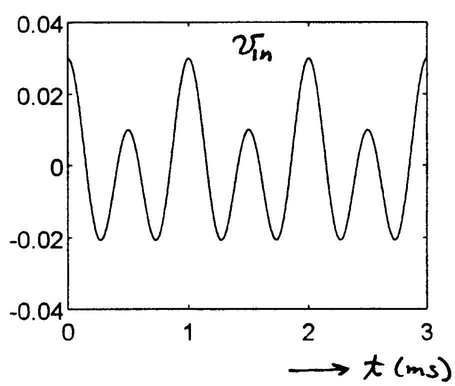
P11.68* The signal to be amplified is the short-circuit current of an electrochemical cell (or battery). This signal is dc and therefore a dc-coupled amplifier is needed.

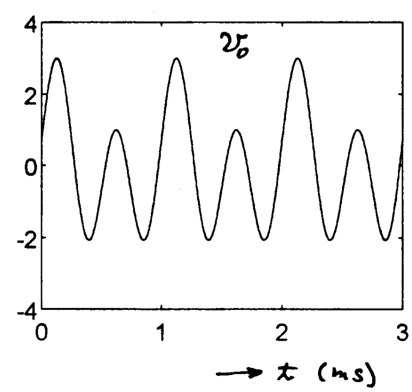
P11.70* $f_{hp} \cong 0.6436 f_B$

P11.75* The gain at 2000 Hz must be $100\angle -90^\circ$. The output signal is

$v_o(t) = 1\cos(2000\pi t - 45^\circ) + 2\cos(4000\pi t - 90^\circ)$

The plots are:

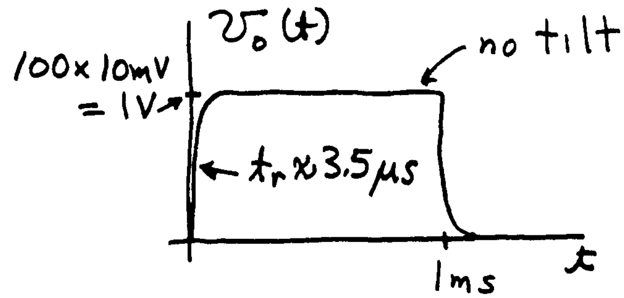




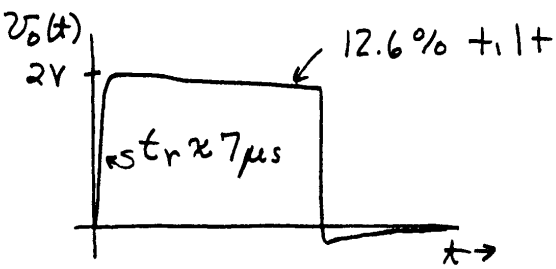
P11.82* $t_r \cong 23.3\ \mu s$

Percentage tilt $\cong 18.8\%$

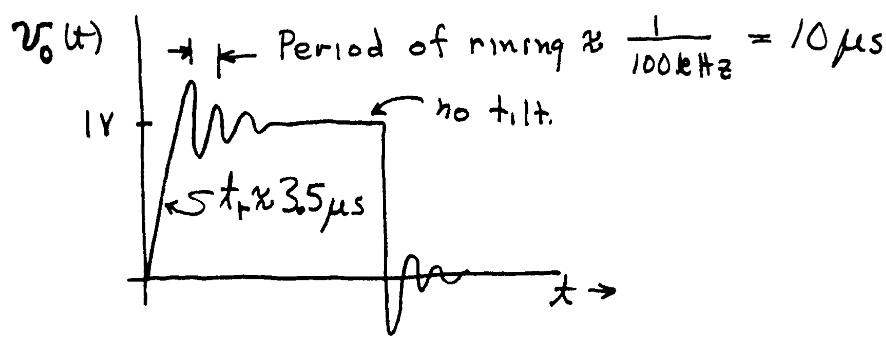
P11.83* (a)



(b)



(c)



P11.86* $D_2 = 0.02$

$D_3 = 0.01$

$D_4 = 0$

$D = 0.02236$

P11.93* $CMRR = 47.96$ dB

P11.98* The extreme values of the output voltage are:

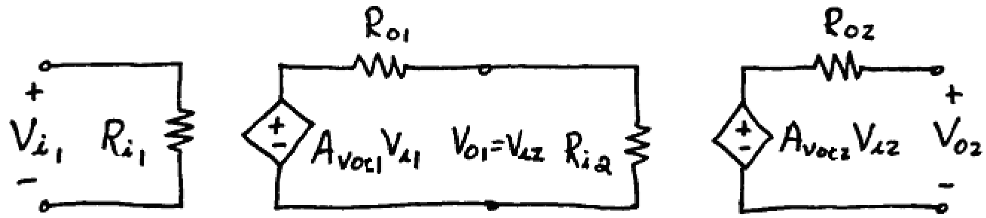
$$v_o = A_d v_{id} = \pm 5 \text{ mV}$$

If the resistors are exactly equal, then the output voltage is zero.

P11.99* The output voltage can range from -3.333 to +3.333 V.

Practice Test

T11.1 The equivalent circuit for the cascaded amplifiers is:



We can write:

$$V_{i2} = A_{oc1}V_{i1} \times \frac{R_{i2}}{R_{i2} + R_{o1}}$$

$$V_{o2} = A_{oc2}V_{i2} = A_{oc2}A_{oc1}V_{i1}\frac{R_{i2}}{R_{i2} + R_{o1}}$$

Thus, the open-circuit voltage gain is:

$$A_{oc} = \frac{V_{o2}}{V_{i1}} = A_{oc2}A_{oc1}\frac{R_{i2}}{R_{i2} + R_{o1}} = 50 \times 50\frac{60}{60 + 40} = 1500$$

The input resistance of the cascade is that of the first stage which is $R_i = 60\ \Omega$. The output resistance of the cascade is the output resistance of the last stage which is $R_o = 40\ \Omega$.

T11.2 Your answer should be similar to Table 11.1.

Table 11.1. Characteristics of Ideal Amplifiers

Amplifier Type	Input Impedance	Output Impedance	Gain Parameter
Voltage	∞	0	A_{voc}
Current	0	∞	A_{isc}
Transconductance	∞	∞	G_{msc}
Transresistance	0	0	R_{moc}

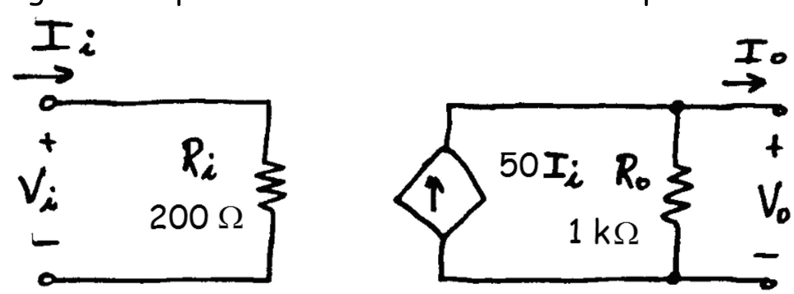
T11.3 **a.** The amplifier should sense the open-circuit source voltage, thus the input impedance should be infinite. The load current should be independent of the variable load, so the output impedance should be infinite. Thus, we need an ideal transconductance amplifier.

b. The amplifier should respond to the short-circuit source current, thus the input impedance should be zero. The load current should be independent of the variable load impedance so the output impedance should be infinite. Therefore, we need an ideal current amplifier.

c. The amplifier should sense the open-circuit source voltage, thus the input impedance should be infinite. The load voltage should be independent of the variable load, so the output impedance should be zero. Thus, we need an ideal voltage amplifier.

d. The amplifier should respond to the short-circuit source current, thus the input impedance should be zero. The load voltage should be independent of the variable load impedance, so the output impedance should be zero. Therefore, we need an ideal transresistance amplifier.

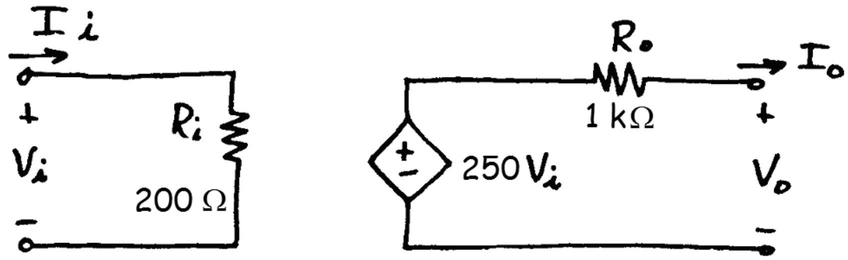
T11.4 We are given the parameters for the current-amplifier model, which is:



The open-circuit voltage gain is:

$$A_{voc} = \frac{V_{ooc}}{V_i} = \frac{50I_iR_o}{R_iI_i} = 250$$

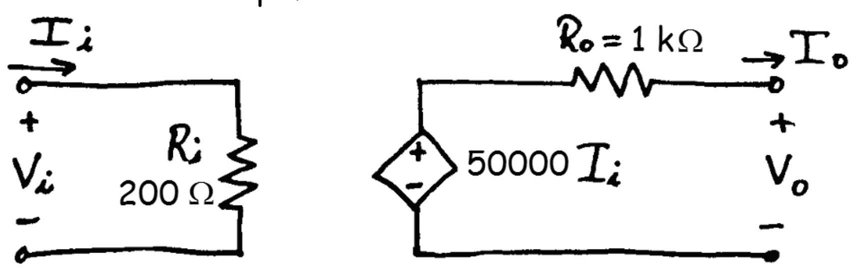
A_{voc} is unitless. (Sometimes we give the units as V/V.) The voltage-amplifier model is:



The transresistance gain is:

$$R_{moc} = \frac{V_{ooc}}{I_i} = \frac{50 I_i R_o}{I_i} = 50\ \mathrm{k}\Omega$$

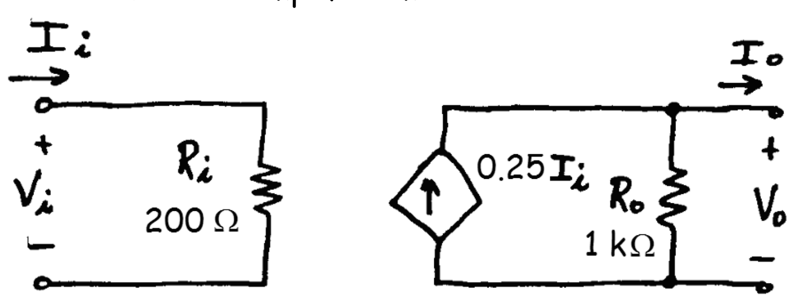
The transresistance-amplifier model is:



The transconductance gain is:

$$G_{msc} = \frac{I_{osc}}{V_i} = \frac{50 I_i}{R_i I_i} = 0.25\ \mathrm{S}$$

The transconductance-amplifier model is:



T11.5 $P_i = I_i^2 R_i = (10^{-3})^2 \times 2 \times 10^3 = 2\ \mathrm{mW}$

$P_o = (V_o)^2 / R_L = (12)^2 / 8 = 18\ \mathrm{W}$

$P_s = V_s I_s = 15 \times 2 = 30\ \mathrm{W}$

$P_d = P_s + P_i - P_o \cong 12\ \mathrm{W}$

$$\eta = \frac{P_o}{P_s} \times 100\% = 60\%$$

T11.6 To avoid linear waveform distortion, the gain magnitude should be constant and the phase response should be a linear function of frequency over the frequency range from 1 to 10 kHz. Because the gain is 100 and the peak input amplitude is 100 mV, the peak output amplitude should be 10 V. The amplifier must not display clipping or unacceptable nonlinear distortion for output amplitudes of this value.

T11.7 The principal effect of offset current, bias current, and offset voltage of an amplifier is to add a dc component to the signal being amplified.

T11.8 Harmonic distortion can occur when a pure sinewave test signal is applied to the input of an amplifier. The distortion appears in the output as components whose frequencies are integer multiples of the input frequency. Harmonic distortion is caused by a nonlinear relationship between the input voltage and output voltage.

T11.9 Common mode rejection ratio (CMRR) is the ratio of the differential gain to the common mode gain of a differential amplifier. Ideally, the common mode gain is zero, and the amplifier produces an output only for the differential signal. CMRR is important when we have a differential signal of interest in the presence of a large common-mode signal not of interest. For example, in recording an electrocardiogram, two electrodes are connected to the patient; the differential signal is the heart signal of interest to the cardiologist; and the common mode signal is due to the 60-Hz power line.

CHAPTER 12

Exercises

E12.1 (a) $v_{GS} = 1$ V and $v_{DS} = 5$ V: Because we have $v_{GS} < V_{to}$, the FET is in cutoff.

(b) $v_{GS} = 3$ V and $v_{DS} = 0.5$ V: Because $v_{GS} > V_{to}$ and $v_{GD} = v_{GS} - v_{DS} = 2.5 > V_{to}$, the FET is in the triode region.

(c) $v_{GS} = 3$ V and $v_{DS} = 6$ V: Because $v_{GS} > V_{to}$ and $v_{GD} = v_{GS} - v_{DS} = -3$ V $< V_{to}$, the FET is in the saturation region.

(d) $v_{GS} = 5$ V and $v_{DS} = 6$ V: Because $v_{GS} > V_{to}$ and $v_{GD} = v_{GS} - v_{DS} = 1$ V which is less than V_{to}, the FET is in the saturation region.

E12.2 First we notice that for $v_{GS} = 0 \text{ or } 1\text{ V}$, the transistor is in cutoff, and the drain current is zero. Next we compute the drain current in the saturation region for each value of v_{GS}:

$$K = \tfrac{1}{2}KP(W/L) = \tfrac{1}{2}(50\times10^{-6})(80/2) = 1\text{ mA/V}^2$$

$$i_D = K(v_{GS} - V_{to})^2$$

The boundary between the triode and saturation regions occurs at

$$v_{DS} = v_{GS} - V_{to}$$

v_{GS} (V)	i_D (mA)	v_{DS} at boundary
2	1	1
3	4	2
4	9	3

In saturation, i_D is constant, and in the triode region the characteristics are parabolas passing through the origin. The apex of the parabolas are on the boundary between the triode and saturation regions. The plots are shown in Figure 12.7 in the book.

E12.3 First we notice that for $v_{GS} = 0 \text{ or } -1\text{ V}$, the transistor is in cutoff, and the drain current is zero. Next we compute the drain current in the saturation region for each value of v_{GS}:

$$K = \tfrac{1}{2} KP(W/L) = \tfrac{1}{2}(25 \times 10^{-6})(200/2) = 1.25 \text{ mA/V}^2$$
$$i_D = K(v_{GS} - V_{to})^2$$

The boundary between the triode and saturation regions occurs at $v_{DS} = v_{GS} - V_{to}$.

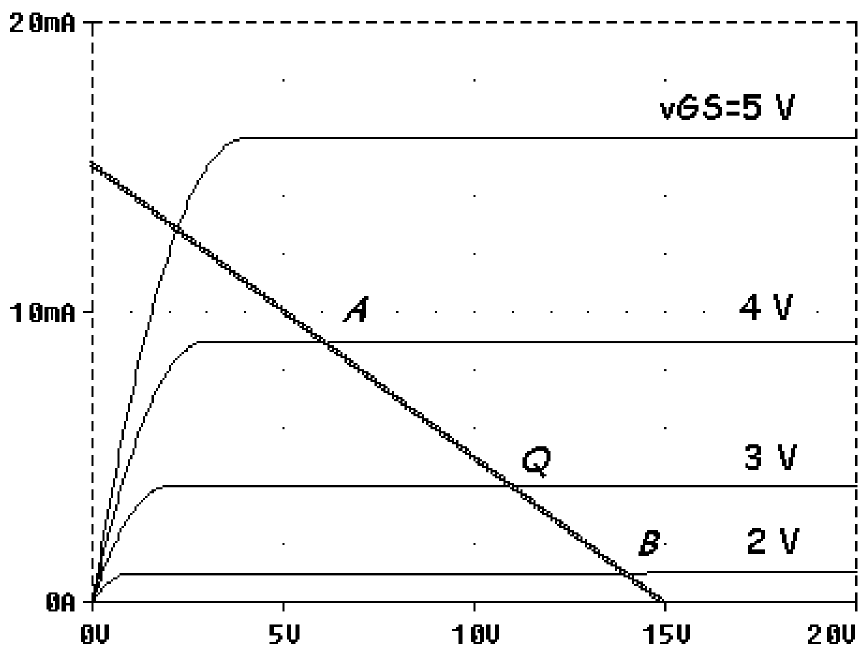
v_{GS} (V)	i_D (mA)	v_{DS} at boundary
−2	1.25	−1
−3	5	−2
−4	11.25	−3

In saturation, i_D is constant, and in the triode region the characteristics are parabolas passing through the origin. The apex of the parabolas are on the boundary between the triode and saturation regions. The plots are shown in Figure 12.9 in the book.

E12.4 We have

$$v_{GS}(t) = v_{in}(t) + V_{GG} = \sin(2000\pi t) + 3$$

Thus we have $V_{GS\max} = 4$ V, $V_{GSQ} = 3$ V, and $V_{GS\min} = 2$ V. The characteristics and the load line are:



For $v_{in} = +1$ we have $v_{GS} = 4$ and the instantaneous operating point is A. Similarly for $v_{in} = -1$ we have $v_{GS} = 2$ V and the instantaneous operating point is at B. We find $V_{DSQ} \cong 11$ V, $V_{DS\,\min} \cong 6$ V, $V_{DS\,\max} \cong 14$ V.

E12.5 First, we compute

$$V_G = V_{DD}\frac{R_2}{R_1 + R_2} = 7\text{ V}$$

and $K = \frac{1}{2}KP(W/L) = \frac{1}{2}(50\times 10^{-6})(200/10) = 0.5\,\text{mA/V}^2$

As in Example 12.2, we need to solve:

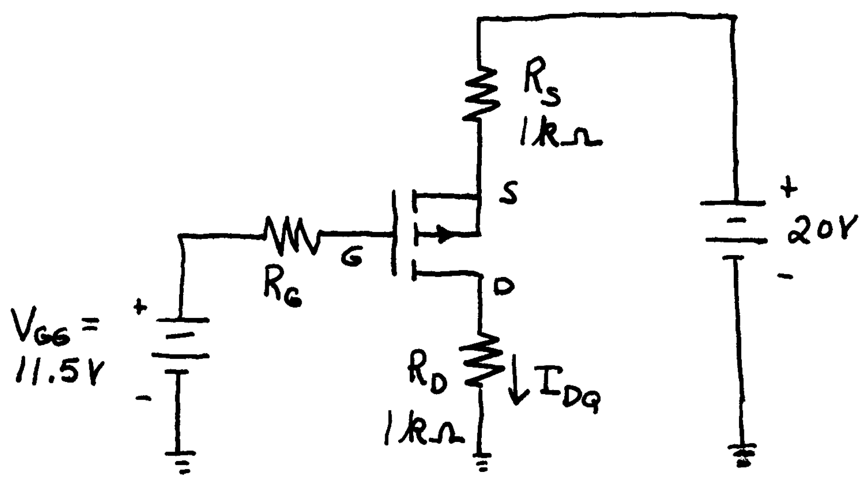
$$V_{GSQ}^2 + \left(\frac{1}{R_S K} - 2V_{to}\right)V_{GSQ} + (V_{to})^2 - \frac{V_G}{R_S K} = 0$$

Substituting values, we have

$$V_{GSQ}^2 - V_{GSQ} - 6 = 0$$

The roots are $V_{GSQ} = -2$ V and 3 V. The correct root is $V_{GSQ} = 3$ V which yields $I_{DQ} = K(V_{GSQ} - V_{to})^2 = 2$ mA. Finally, we have $V_{DSQ} = V_{DD} - R_S I_{DQ} = 16$ V.

E12.6 First, we replace the gate bias circuit with its equivalent circuit:



Then we can write the following equations:

$$K = \frac{1}{2}KP(W/L) = \frac{1}{2}(25\times 10^{-6})(400/10) = 0.5\,\text{mA/V}^2$$

$$V_{GG} = 11.5 = V_{GSQ} - R_s I_{DQ} + 20 \qquad (1)$$

$$I_{DQ} = K(V_{GSQ} - V_{to})^2 \qquad (2)$$

Using Equation (2) to substitute into Equation (1), substituting values, and rearranging, we have $V_{GSQ}^2 - 16 = 0$. The roots of this equation are $V_{GSQ} = \pm 4$ V. However $V_{GSQ} = -4$ V is the correct root for a PMOS transistor. Thus we have

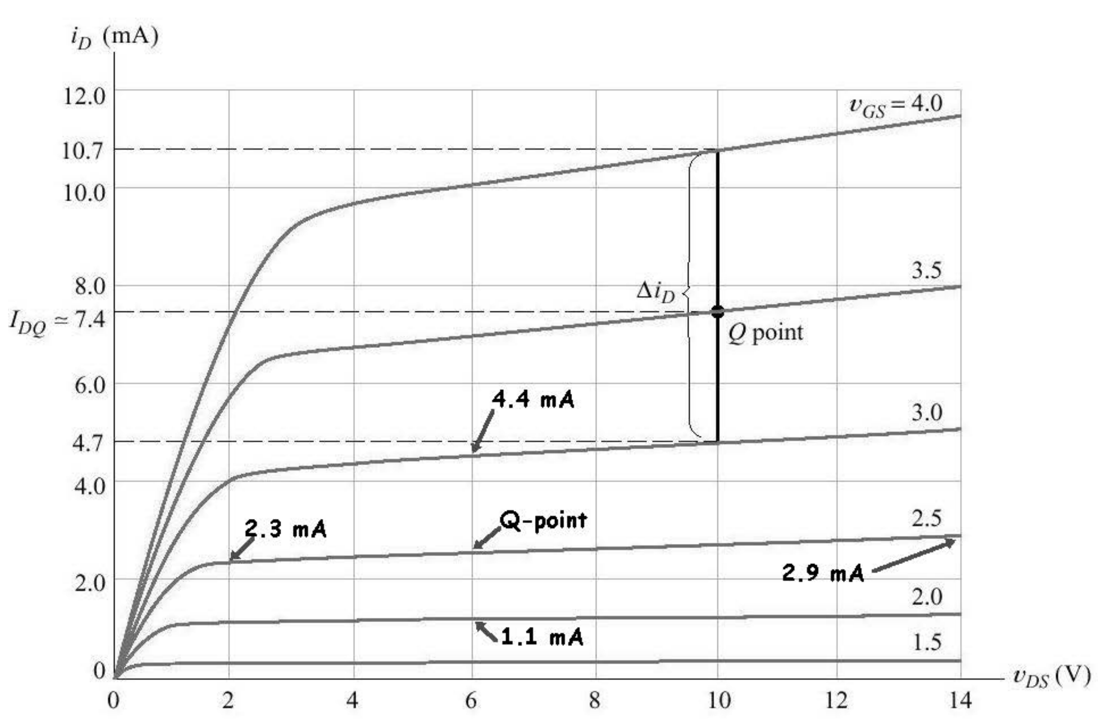
$$I_{DQ} = 4.5 \text{ mA}$$

and

$$V_{DSQ} = R_s I_{DQ} + R_D I_{DQ} - 20 = -11 \text{ V}.$$

E12.7 From Figure 12.21 at an operating point defined by $V_{GSQ} = 2.5$ V and $V_{DSQ} = 6$ V, we estimate

$$g_m = \frac{\Delta i_D}{\Delta v_{GS}} = \frac{(4.4 - 1.1)\,\text{mA}}{1\,\text{V}} = 3.3\,\text{mS}$$



$$1/r_d = \frac{\Delta i_D}{\Delta v_{GS}} \cong \frac{(2.9 - 2.3)\,\text{mA}}{(14 - 2)\,\text{V}} = 0.05 \times 10^{-3}$$

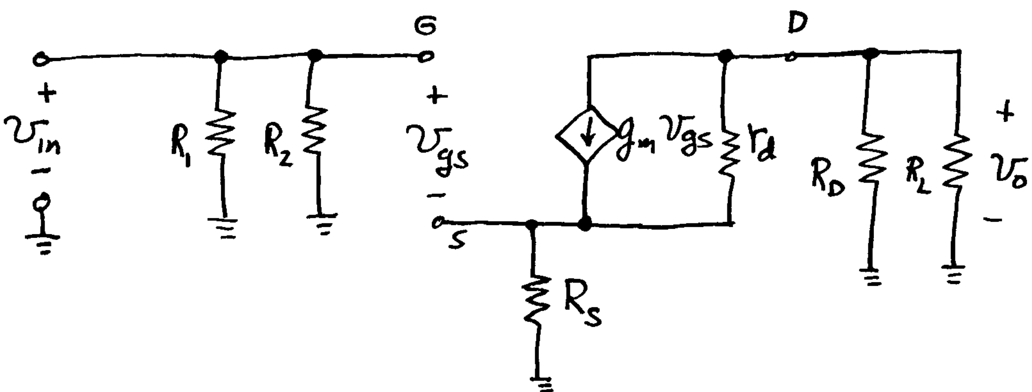
Taking the reciprocal, we find $r_d = 20$ kΩ.

E12.8 $g_m = \left.\dfrac{\partial i_D}{\partial v_{GS}}\right|_{Q-\text{point}} = \left.\dfrac{\partial}{\partial v_{GS}} K(v_{GS} - V_{to})^2\right|_{Q-\text{point}} = 2K(V_{GSQ} - V_{to})$

E12.9 $R'_L = \dfrac{1}{1/r_d + 1/R_D + 1/R_L} = R_D = 4.7\ \text{k}\Omega$

$$A_{voc} = -g_m R'_L = -(1.77\ \text{mS}) \times (4.7\ \text{k}\Omega) = -8.32$$

E12.10 For simplicity we treat r_d as an open circuit and let $R'_L = R_D || R_L$.



$$v_{in} = v_{gs} + R_s g_m v_{gs}$$

$$v_o = -R'_L g_m v_{gs}$$

$$A_v = \frac{v_o}{v_{in}} = \frac{-R'_L g_m}{1 + R'_L g_m}$$

E12.11 $R'_L = R_D \| R_L = 3.197\ \text{k}\Omega$

$$A_v = \frac{v_o}{v_{in}} = \frac{-R'_L g_m}{1 + R'_L g_m} = \frac{-(3.197\ \text{k}\Omega)(1.77\ \text{mS})}{1 + (2.7\ \text{k}\Omega)(1.77\ \text{mS})} = -0.979$$

E12.12 The equivalent circuit is shown in Figure 12.28 in the book from which we can write

$$v_{\text{in}} = 0 \qquad v_{gs} = -v_x \qquad i_x = \frac{v_x}{R_S} + \frac{v_x}{r_d} - g_m v_{gs} = \frac{v_x}{R_S} + \frac{v_x}{r_d} + g_m v_x$$

Solving, we have

$$R_o = \frac{v_x}{i_x} = \frac{1}{g_m + \dfrac{1}{R_S} + \dfrac{1}{r_d}}$$

E12.13 Refer to the small-signal equivalent circuit shown in Figure 12.30 in the book. Let $R_L' = R_D \| R_L$.

$$v_{\text{in}} = -v_{gs}$$

$$v_o = -R_L' g_m v_{gs}$$

$$A_v = v_o / v_{\text{in}} = R_L' g_m$$

$$i_{\text{in}} = v_{\text{in}} / R_s - g_m v_{gs} = v_{\text{in}} / R_s + g_m v_{\text{in}}$$

$$R_{\text{in}} = \frac{v_{\text{in}}}{i_{\text{in}}} = \frac{1}{g_m + 1/R_s}$$

If we set $v(t) = 0$, then we have $v_{gs} = 0$. Removing the load and looking back into the amplifier, we see the resistance R_D. Thus we have $R_o = R_D$.

E12.14 See Figure 12.34 in the book.

E12.15 See Figure 12.35 in the book.

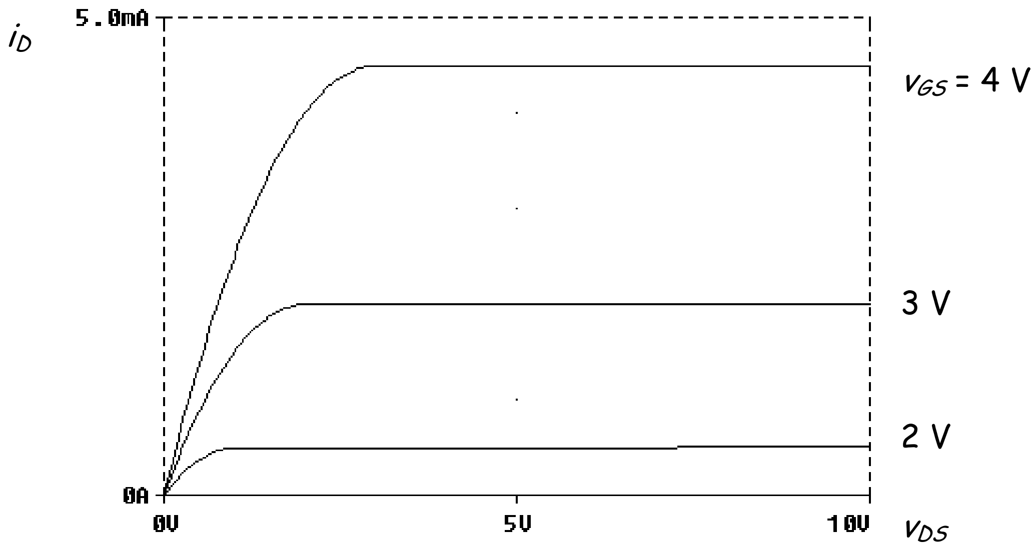
Answers for Selected Problems

P12.3* (a) Saturation i_D = 2.25 mA

(b) Triode i_D = 2 mA

(c) Cutoff i_D = 0

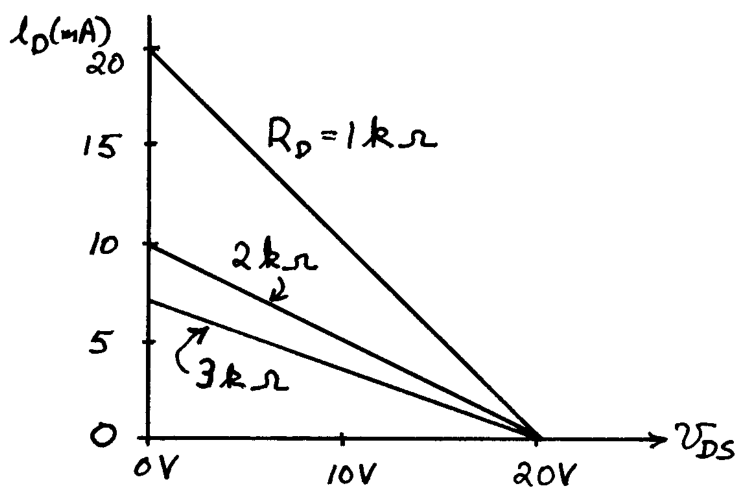
P12.4*



P12.11* V_{to} = 1.5 V
K = 0.8 mA/V^2

P12.15* $v_{GS} = -2.5$ V

P12.17*



The load line rotates around the point (V_{DD}, 0) as the resistance changes.

P12.19* The gain is zero.

P12.21* R_{Dmax} = 3.778 kΩ

P12.27* I_{DQ} = 3.432 mA
V_{DSQ} = 16.27 V

P12.28* $R_S = 3\ \text{k}\Omega$
$R_2 = 2\ \text{M}\Omega$

P12.29* $R_s = 400\,\Omega \quad R_1 = 2.583\text{M}\Omega.$

P12.34* V_{DSQ} = V_{GSQ} = 5.325 V
I_{DQ} = 4.675 mA

P12.40* $g_m = 2KV_{DSQ}$

P12.41* $r_d = \dfrac{1}{2K(V_{GSQ} - V_{to} - V_{DSQ})}$

P12.50* (a) $V_{GSQ} = 3\ \text{V}$
$I_{DQ} = 10\ \text{mA}$
$g_m = 0.01\ \text{S}$

(b) $A_v = -5$
$R_{in} = 255\ \text{k}\Omega$
$R_o = 1\ \text{k}\Omega$

P12.53* $R_o = \dfrac{1}{1/R_D + g_m} = 253\,\Omega$

P12.56* R_S = 3.382 kΩ

$A_v = 0.6922$

$R_{in} = 666.7\text{k}\Omega$

$R_o = 386.9\,\Omega$

Practice Test

T12.1 Drain characteristics are plots of i_D versus v_{DS} for various values of v_{GS}.

First, we notice that for $v_{GS} = 0.5\text{ V}$, the transistor is in cutoff, and the drain current is zero, because v_{GS} is less than the threshold voltage V_{to}. Thus, the drain characteristic for $v_{GS} = 0.5\text{ V}$ lies on the horizontal axis.

Next, we compute the drain current in the saturation region for $v_{GS} = 4\text{ V}$.

$$K = \tfrac{1}{2} KP(W/L) = \tfrac{1}{2}(80\times 10^{-6})(100/4) = 1\text{ mA/V}^2$$

$$i_D = K(v_{GS} - V_{to})^2 = K(4-1)^2 = 9\text{ mA} \text{ for } v_{DS} > v_{GS} - V_{to} = 3\text{ V}$$

Thus, the characteristic is constant at 9 mA in the saturation region.

The transistor is in the triode region for $v_{DS} < v_{GS} - V_{to} = 3\text{ V}$, and the drain current (in mA) is given by

$$i_D = K[2(v_{GS} - V_{to})v_{DS} - v_{DS}^2] = 6v_{DS} - v_{DS}^2$$

with v_{DS} in volts. This plots as a parabola that passes through the origin and reaches its apex at $i_D = 9\text{ mA}$ and $v_{DS} = 3\text{ V}$.

The drain characteristic for $v_{GS} = 4$ V is identical to that of Figure 12.11 in the book.

T12.2 We have $v_{GS}(t) = v_{in}(t) + V_{GG} = \sin(2000\pi t) + 3\text{ V}$. Thus, we have $V_{GS\max} = 4$ V, $V_{GSQ} = 3$ V, and $V_{GS\min} = 2$ V. Writing KVL around the drain circuit, we have

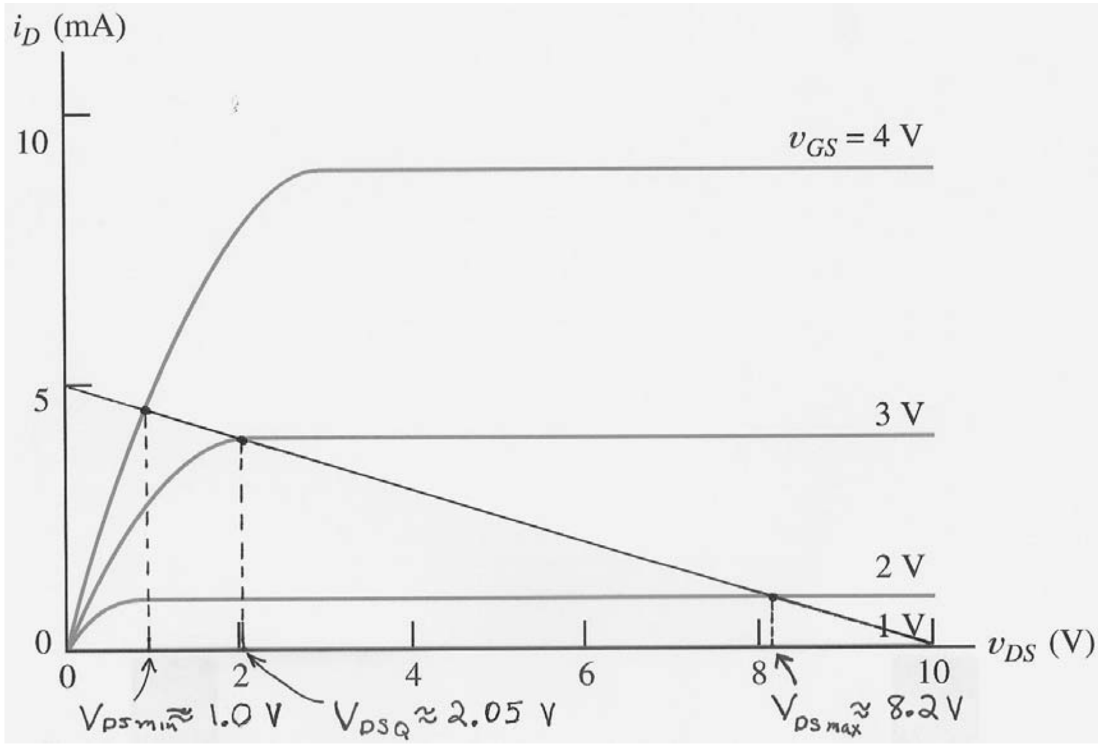
$$V_{DD} = R_D i_D + v_{DS}$$

With voltages in volts, currents in mA, and resistances in kΩ, this becomes

$$10 = 2i_D + v_{DS}$$

which is the equation for the load line.

The characteristics and the load line are:



The results of the load-line analysis are $V_{DS\min} \cong 1.0$ V, $V_{DSQ} \cong 2.05$ V, and $V_{DS\max} \cong 8.2$ V.

T12.3 Because the gate current is zero, we can apply the voltage division principle to determine the voltage at the gate with respect to ground.

$$V_G = \frac{10\ \text{k}\Omega}{(10+30)\ \text{k}\Omega} \times 12 = 3\ \text{V}$$

For the transistor, we have

$$K = \tfrac{1}{2} KP(W/L) = \tfrac{1}{2}(80 \times 10^{-6})(100/4) = 1\ \text{mA/V}^2$$

Because the drain voltage is 12 V, which is higher than the gate voltage, we conclude that the transistor is operating in the saturation region. Thus, we have

$$I_{DQ} = K(V_{GSQ} - V_{to})^2$$

$$I_{DQ} = (V_{GSQ} - 1)^2 = 0.5\ \text{mA}$$

Solving, we have $V_{GSQ} = 1.707$ V or $V_{GSQ} = 0.293$ V. However, V_{GSQ} must be larger than V_{to} for current to flow, so the second root is extraneous. Then, the voltage across R_S is $V_S = V_G - V_{GSQ} = 1.293$ V. The current through R_S is I_{DQ}. Thus, the required value is $R_S = 1.293/0.5 = 2.586$ kΩ.

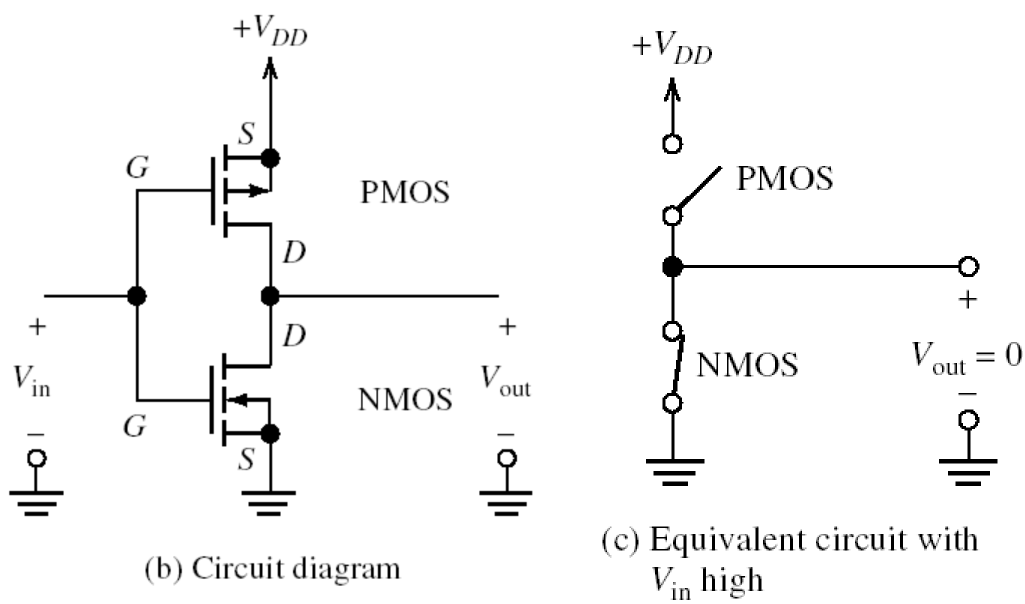
T12.4 This transistor is operating with constant v_{DS}. Thus, we can determine g_m by dividing the peak ac drain current by the peak ac gate-to-source voltage.

$$g_m = \left.\frac{\Delta i_D}{\Delta v_{GS}}\right|_{v_{DS}=V_{DSQ}} = \frac{0.05\ \text{mA}}{0.02\ \text{V}} = 2.5\,\text{mS}$$

The Q-point is $V_{DSQ} = 5\,\text{V}$, $V_{GSQ} = 2\,\text{V}$, and $I_{DQ} = 0.5\,\text{mA}$.

T12.5 (a) A dc voltage source is replaced with a short circuit in the small-signal equivalent. (b) A coupling capacitor becomes a short circuit. (c) A dc current source is replaced with an open circuit, because even if an ac voltage appears across it, the current through it is constant (i.e., zero ac current flows through a dc current source).

T12.6 See Figure 12.31(b) and (c) in the text.



(b) Circuit diagram

(c) Equivalent circuit with V_{in} high

CHAPTER 13

Exercises

E13.1 The emitter current is given by the Shockley equation:

$$i_E = I_{ES}\left[\exp\left(\frac{v_{BE}}{V_T}\right) - 1\right]$$

For operation with $i_E >> I_{ES}$, we have $\exp\left(\frac{v_{BE}}{V_T}\right) >> 1$, and we can write

$$i_E \cong I_{ES}\exp\left(\frac{v_{BE}}{V_T}\right)$$

Solving for v_{BE}, we have

$$v_{BE} \cong V_T \ln\left(\frac{i_E}{I_{ES}}\right) = 26\ln\left(\frac{10^{-2}}{10^{-14}}\right) = 718.4\text{ mV}$$

$$v_{BC} = v_{BE} - v_{CE} = 0.7184 - 5 = -4.2816\text{ V}$$

$$\alpha = \frac{\beta}{\beta + 1} = \frac{50}{51} = 0.9804$$

$$i_C = \alpha i_E = 9.804\text{ mA}$$

$$i_B = \frac{i_C}{\beta} = 196.1\ \mu\text{A}$$

E13.2 $\beta = \dfrac{\alpha}{1 - \alpha}$

α	β
0.9	9
0.99	99
0.999	999

E13.3 $i_B = i_E - i_C = 0.5\text{ mA}$ $\quad \alpha = i_C / i_E = 0.95$ $\quad \beta = i_C / i_B = 19$

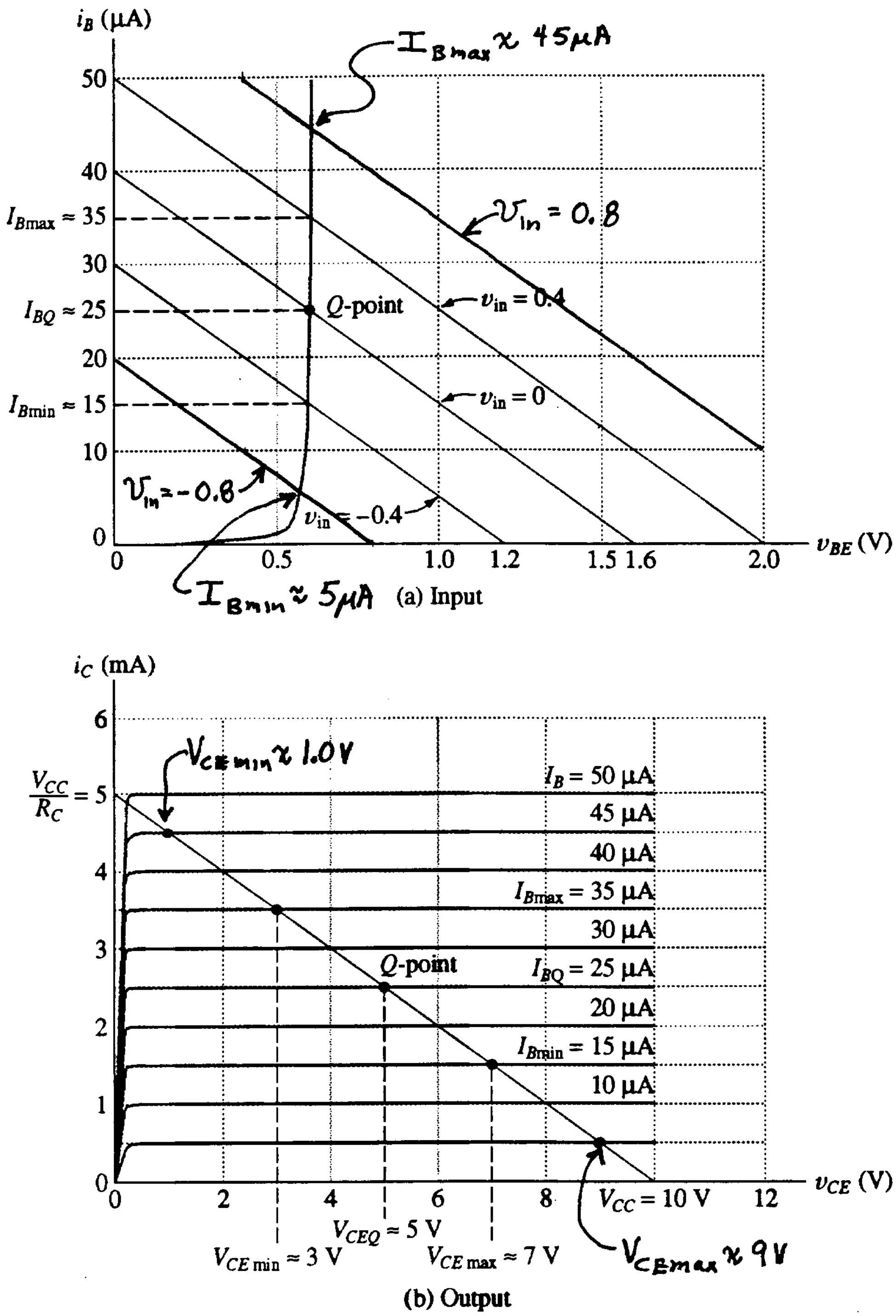
E13.4 The base current is given by Equation 13.8:

$$i_B = (1 - \alpha)I_{ES}\left[\exp\left(\frac{v_{BE}}{V_T}\right) - 1\right] = 1.961\times 10^{-16}\left[\exp\left(\frac{v_{BE}}{0.026}\right) - 1\right]$$

which can be plotted to obtain the input characteristic shown in Figure 13.6a. For the output characteristic, we have $i_C = \beta i_B$ provided that

$v_{CE} \geq$ approximately 0.2 V. For $v_{CE} \leq 0.2$ V, i_C falls rapidly to zero at $v_{CE} = 0$. The output characteristics are shown in Figure 13.6b.

E13.5 The load lines for $v_{in} = 0.8$ V and -0.8 V are shown:

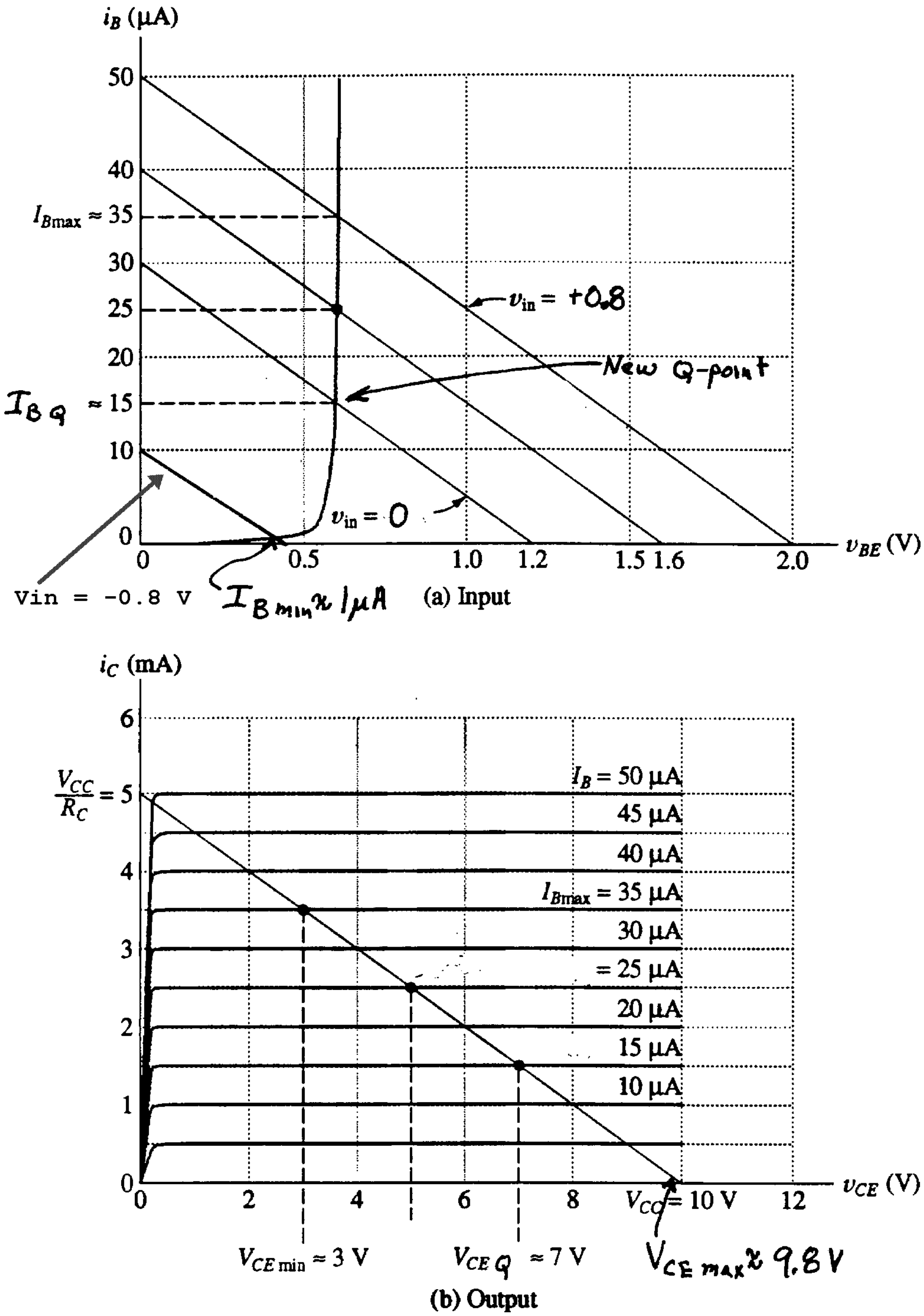


(a) Input

(b) Output

As shown on the output load line, we find $V_{CE\max} \cong 9$ V, $V_{CEQ} \cong 5$ V, and $V_{CE\min} \cong 1.0$ V.

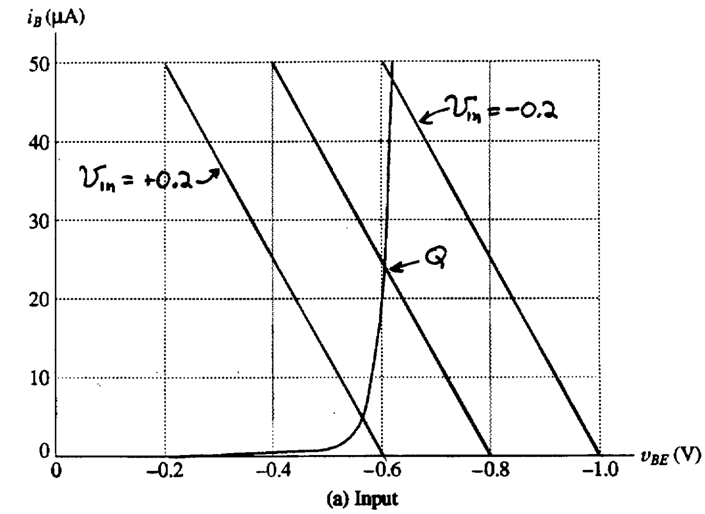
E13.6 The load lines for the new values are shown:



As shown on the output load line, we have
$V_{CE\max} \cong 9.8$ V, $V_{CEQ} \cong 7$ V, and $V_{CE\min} \cong 3.0$ V.

E13.7 Refer to the characteristics shown in Figure 13.7 in the book. Select a point in the active region of the output characteristics. For example, we could choose the point defined by $v_{CE} = -6$ V and $i_C = 2.5$ mA at which we find $i_B = 50\ \mu$A. Then we have $\beta = i_C / i_B = 50$. (For many transistors the value found for β depends slightly on the point selected.)

E13.8 (a) Writing a KVL equation around the input loop we have the equation for the input load lines: $0.8 - v_{\text{in}}(t) - 8000 i_B + v_{BE} = 0$ The load lines are shown:

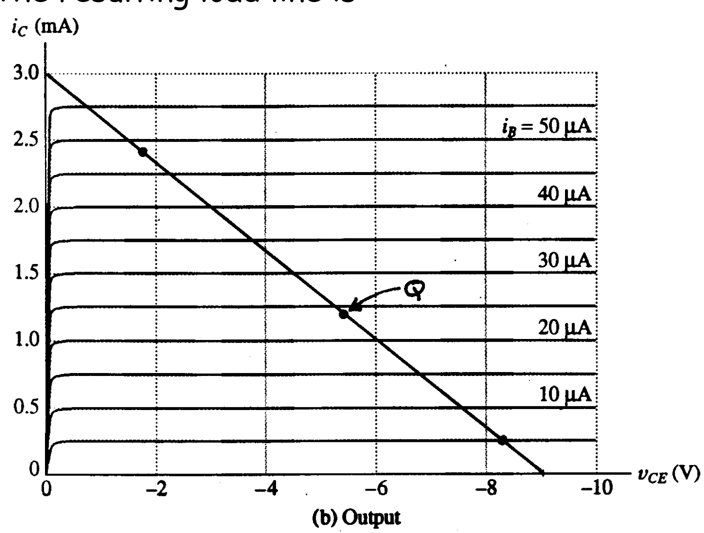


(a) Input

Then we write a KCL equation for the output circuit:

$$9 + 3000 i_C = v_{CE}$$

The resulting load line is:



(b) Output

From these load lines we find

$I_{B\max} \cong 48\ \mu A,\ I_{BQ} \cong 24\ \mu A,\ I_{B\min} \cong 5\ \mu A,$

$V_{CE\max} \cong -1.8\ \text{V},\ V_{CEQ} \cong -5.3\ \text{V},\ V_{CE\min} \cong -8.3\ \text{V}$

(b) Inspecting the load lines, we see that the maximum of v_{in} corresponds to $I_{B\min}$ which in turn corresponds to $V_{CE\min}$. Because the maximum of v_{in} corresponds to minimum V_{CE}, the amplifier is inverting. This may be a little confusing because V_{CE} takes on negative values, so the minimum value has the largest magnitude.

E13.9 (a) Cutoff because we have $V_{BE} < 0.5$ V and $V_{BC} = V_{BE} - V_{CE} = -4.5$ V which is less than 0.5 V.

(b) Saturation because we have $I_C < \beta I_B$.

(c) Active because we have $I_B > 0$ and $V_{CE} > 0.2$ V.

E13.10 (a) In this case ($\beta = 50$) the BJT operates in the active region. Thus the equivalent circuit is shown in Figure 13.18d. We have

$$I_B = \frac{V_{CC} - 0.7}{R_B} = 71.5\ \mu A \qquad I_C = \beta I_B = 3.575\ \text{mA}$$

$$V_{CE} = V_{CC} - R_C I_C = 11.43\ \text{V}$$

Because we have $V_{CE} > 0.2$, we are justified in assuming that the transistor operates in the active region.

(b) In this case ($\beta = 250$) ,the BJT operates in the saturation region. Thus the equivalent circuit is shown in Figure 13.18c. We have

$$V_{CE} = 0.2\ \text{V} \quad I_B = \frac{V_{CC} - 0.7}{R_B} = 71.5\ \mu A \quad I_C = \frac{V_{CC} - 0.2}{R_C} = 14.8\ \text{mA}$$

Because we have $\beta I_B > I_C$, we are justified in assuming that the transistor operates in the saturation region.

E13.11 For the operating point to be in the middle of the load line, we want $V_{CE} = V_{CC}/2 = 10$ V and $I_C = \dfrac{V_{CC} - V_{CE}}{R_C} = 2$ mA . Then we have

(a) $I_B = I_C / \beta = 20\ \mu A \quad R_B = \dfrac{V_{CC} - 0.7}{I_B} = 965\ \text{k}\Omega$

(b) $I_B = I_C / \beta = 6.667\ \mu A \quad R_B = \dfrac{V_{CC} - 0.7}{I_B} = 2.985\ \text{M}\Omega$

E13.12 Notice that a *pnp* BJT appears in this circuit.

(a) For $\beta = 50$, it turns out that the BJT operates in the active region.

$$I_B = \frac{20 - 0.7}{R_B} = 19.3\ \mu A \qquad I_C = \beta I_B = 0.965\ \text{mA}$$

$$V_{CE} = R_C I_C - 20 = -10.35\ \text{V}$$

(b) For $\beta = 250$, it turns out that the BJT operates in the saturation region.

$$V_{CE} = -0.2\ \text{V} \quad I_B = \frac{20 - 0.7}{R_B} = 19.3\ \mu A \quad I_C = \frac{20 - 0.2}{R_C} = 1.98\ \text{mA}$$

Because we have $\beta I_B > I_C$, we are assured that the transistor operates in the active region.

E13.13 $V_B = V_{CC}\dfrac{R_2}{R_1 + R_2} = 5\ \text{V} \qquad I_B = \dfrac{V_B - V_{BE}}{R_B + (\beta + 1)R_E}$

$I_C = \beta I_B \qquad V_{CE} = V_{CC} - R_C I_C - R_E(I_C + I_B)$

β	I_B (μA)	I_C (mA)	V_{CE} (V)
100	32.01	3.201	8.566
300	12.86	3.858	7.271

For the larger values of R_1 and R_2 used in this Exercise, the ratio of the collector currents for the two values of β is 1.205, whereas for the smaller values of R_1 and R_2 used in Example 13.7, the ratio of the collector currents for the two values of β is 1.0213. In general in the four-resistor bias network smaller values for R_1 and R_2 lead to more nearly constant collector currents with changes in β.

E13.14

$$R_B = \frac{1}{1/R_1 + 1/R_2} = 3.333\ \text{k}\Omega \qquad V_B = V_{CC}\frac{R_2}{R_1 + R_2} = 5\ \text{V}$$

$$I_{BQ} = \frac{V_B - V_{BE}}{R_B + (\beta + 1)R_E} = 14.13\ \mu A \quad I_{CQ} = \beta I_{BQ} = 4.239\ \text{mA}$$

$$r_\pi = \frac{\beta V_T}{I_{CQ}} = \frac{300(26\ \text{mV})}{4.238\ \text{mA}} = 1840\ \Omega$$

$$R_L' = \frac{1}{1/R_L + 1/R_C} = 666.7\ \Omega \qquad A_v = -\frac{\beta R_L'}{r_\pi} = -108.7$$

$$A_{voc} = \frac{R_L \beta}{r_\pi} = -163.0 \qquad Z_{in} = \frac{1}{1/R_1 + 1/R_2 + 1/r_\pi} = 1186\ \Omega$$

$$A_i = A_v \frac{Z_{in}}{R_L} = -64.43 \qquad G = A_v A_i = 7004$$

$$Z_o = R_C = 1\ \text{k}\Omega$$

$$v_o = A_v v_{\text{in}} = A_v v_s \frac{Z_{\text{in}}}{Z_{\text{in}} + R_s} = -76.46 \sin(\omega t)$$

E13.15 First, we determine the bias point:

$$R_B = \frac{1}{1/R_1 + 1/R_2} = 50.00\ \text{k}\Omega \qquad V_B = V_{CC} \frac{R_2}{R_1 + R_2} = 10\ \text{V}$$

$$I_{BQ} = \frac{V_B - V_{BE}}{R_B + (\beta + 1)R_E} = 14.26\ \mu A \qquad I_{CQ} = \beta I_{BQ} = 4.279\ \text{mA}$$

Now we can compute r_π and the ac performance.

$$r_\pi = \frac{\beta V_T}{I_{CQ}} = \frac{300(26\ \text{mV})}{4.279\ \text{mA}} = 1823\ \Omega \qquad R_L' = \frac{1}{1/R_L + 1/R_E} = 666.7\ \Omega$$

$$A_v = \frac{R_L'(\beta + 1)}{r_\pi + (\beta + 1)R_L'} = 0.9910 \qquad A_{voc} = \frac{R_E(\beta + 1)}{r_\pi + (\beta + 1)R_E} = 0.9970$$

$$Z_{in} = \frac{1}{1/R_B + 1/[r_\pi + (\beta + 1)R_L']} = 40.10\ \text{k}\Omega \qquad A_i = A_v \frac{Z_{in}}{R_L} = 39.74$$

$$G = A_v A_i = 39.38 \qquad R_s' = \frac{1}{1/R_B + 1/R_s} = 8.333\ \text{k}\Omega$$

$$Z_o = \frac{1}{\dfrac{(\beta + 1)}{R_s' + r_\pi} + \dfrac{1}{R_E}} = 33.18\ \Omega$$

Answers for Selected Problems

P13.6* $i_E = 9.3\ \text{mA}$

$\alpha = 0.9677$

$\beta = 30$

P13.7* $v_{BE} \cong 658.5\text{ mV}$

$v_{BC} = -9.341\text{ V}$

$\alpha = 0.9901$

$i_C = 9.901\text{ mA}$

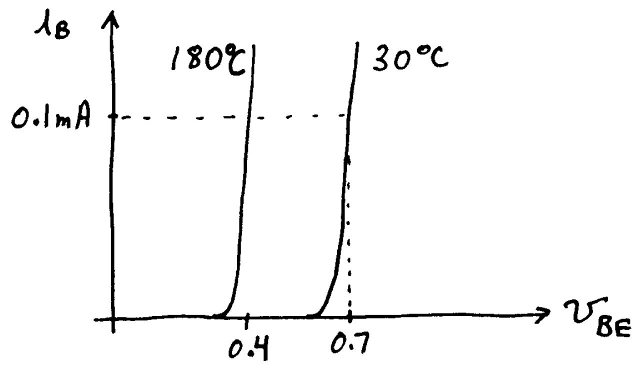
$i_B = 99.01\ \mu A$

P13.16* $I_{ESeq} = 2 \times 10^{-13}$ **A**

$\beta_{eq} = 100$

P13.18* At $180^\circ C$ and $i_B = 0.1\text{ mA}$, the base-to-emitter voltage is approximately:

$$v_{BE} = 0.7 - 0.002(180 - 30) = 0.4\text{ V}$$



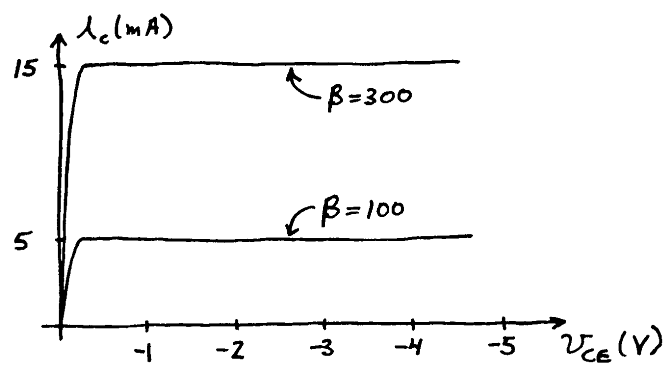
P13.19* $\beta = 400$

$\alpha = 0.9975$

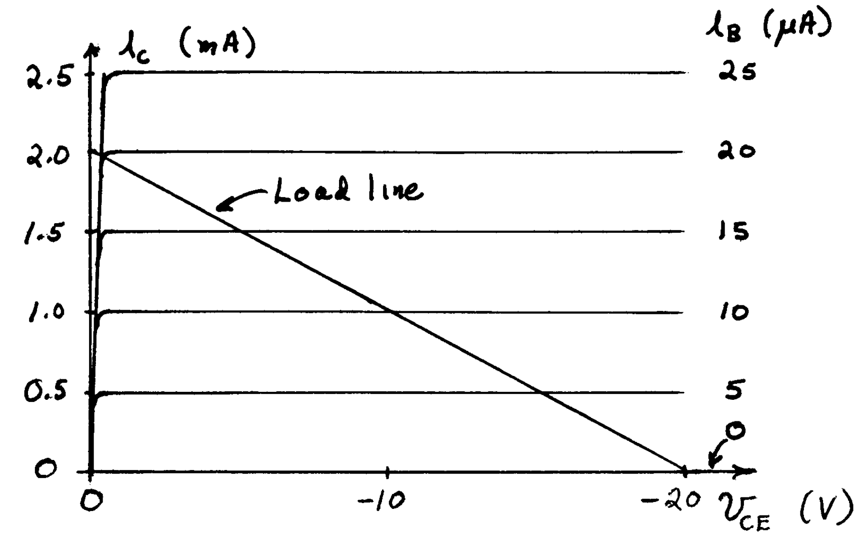
P13.24* $V_{CE\max} = 18.4\text{ V}, V_{CEQ} = 15.6\text{ V}$, and $V_{CE\min} = 12\text{ V}$

$|A_v| = 16$

P13.28*



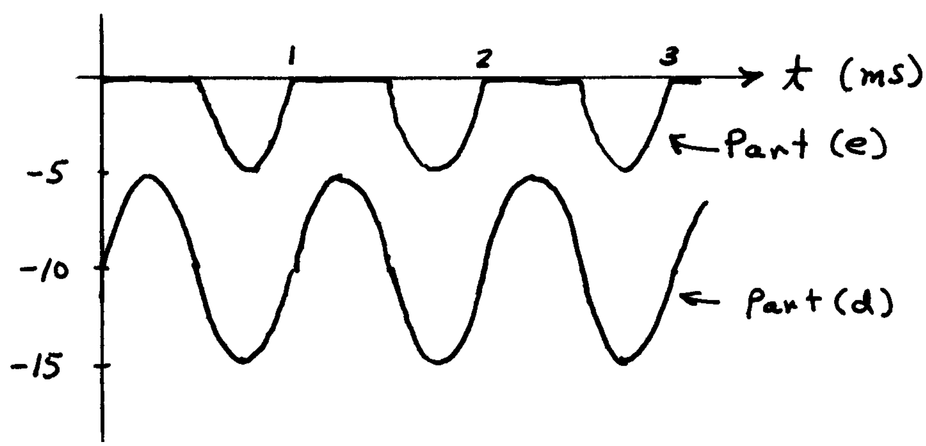
P13.29* (a) and (b)



(c) $I_{C\min} = 0.5\,\text{mA}, I_{CQ} = 1.0\,\text{mA}$, and $I_{C\max} = 1.5\,\text{mA}$

$V_{CE\min} = -15\,\text{V}, V_{CEQ} \cong -10\,\text{V},\ V_{CE\max} \cong -5\,\text{V}$

(d) and (e) The sketches of $v_{CE}(t)$ are:



P13.36* In the active region, the base-collector junction is reverse biased and the base-emitter junction is forward biased.

In the saturation region, both junctions are forward biased.

In the cutoff region, both junctions are reverse biased. (Actually, cutoff applies for slight forward bias of the base-emitter junction as well, provided that the base current is negligible.)

P13.41* 1. Assume operation in saturation, cutoff, or active region.

2. Use the corresponding equivalent circuit to solve for currents and voltages.

3. Check to see if the results are consistent with the assumption made in step 1. If so, the circuit is solved. If not, repeat with a different assumption.

P13.44* The results are given in the table:

Circuit	β	Region of operation	I_C (mA)	V_{CE} (volts)
(a)	100	active	1.93	10.9
(a)	300	saturation	4.21	0.2
(b)	100	active	1.47	5.00
(b)	300	saturation	2.18	0.2
(c)	100	cutoff	0	15
(c)	300	cutoff	0	15
(d)	100	active	6.5	8.5
(d)	300	saturation	14.8	0.2

P13.47* $R_B = 31.5\ \text{k}\Omega$ and $R_E = 753\ \Omega$

P13.49* $I_{C\max} = 0.952\ \text{mA}$

$I_{C\min} = 0.6667\ \text{mA}$

P13.56* $r_\pi = \dfrac{1581}{\sqrt{I_{CQ}}}$

For $I_{CQ} = 1$ mA, we obtain $r_\pi = 50\ \text{k}\Omega$.

P13.63*

	High impedance amplifier (Problem 13.57)	Low impedance amplifier (Problem 13.56)
I_{CQ}	0.0393 mA	3.93 mA
r_π	66.2 kΩ	662 Ω
A_v	-75.5	-75.5
A_{voc}	-151	-151
Z_{in}	54.8 kΩ	548 Ω
A_i	-41.4	-41.4
G	3124	3124
Z_o	100 kΩ	1 kΩ

P13.67* $I_{CQ} = \beta I_{BQ} = 6.41\text{ mA}$

$r_\pi = 405\ \Omega$

$A_v = 0.98$

$A_{voc} = 0.996$

$Z_{in} = 4.36\text{ k}\Omega$

$A_i = 8.61$

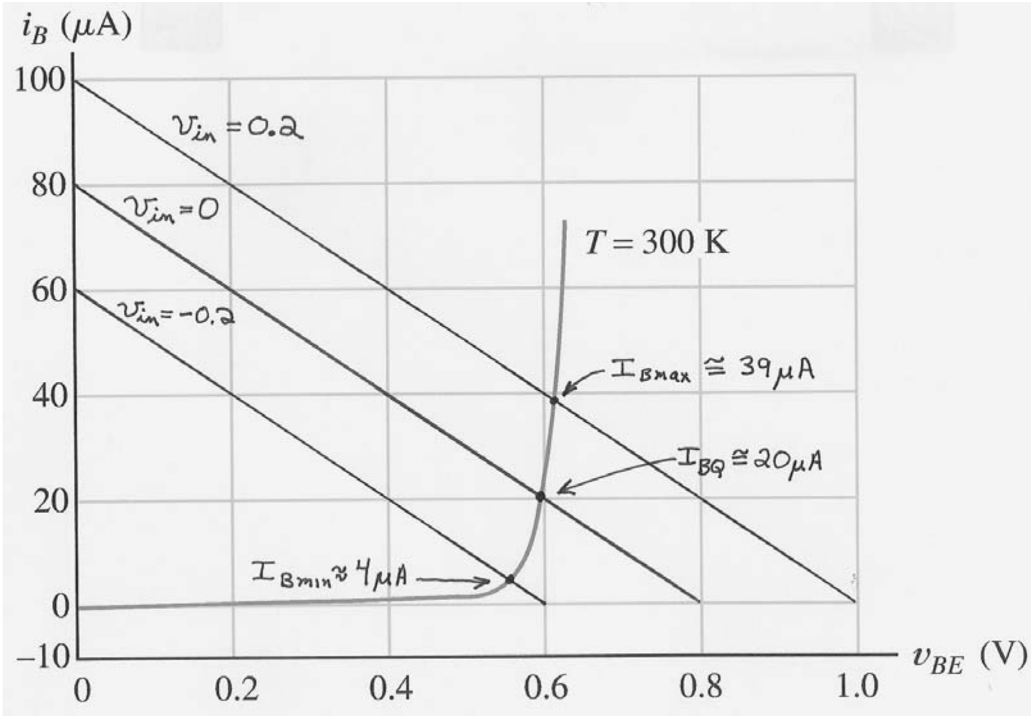
$G = 8.51$

$Z_o = 12.1\ \Omega$

Practice Test

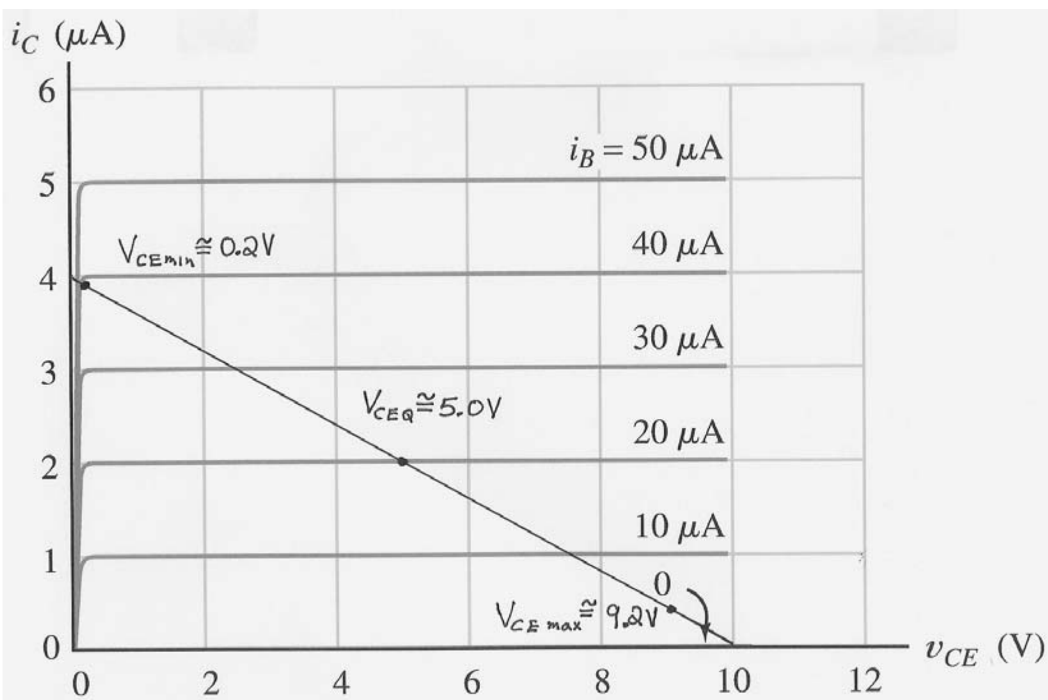
T13.1 **a.** 3, **b.** 2, **c.** 5, **d.** 7 and 1 (either order), **e.** 10, **f.** 7, **g.** 1, **h.** 7, **i.** 15, **j.** 12, **k.** 19.

T13.2 First, we construct the load lines on the input characteristics for $v_{in} = 0$, -0.2 V, and $+0.2$ V:



(a) Input characteristic

At the intersections of the characteristic with the load lines, we find the minimum, *Q*-point, and maximum values of the base current as shown. Then, we construct the load line on the collector characteristics:



(b) Output characteristics

Interpolating between collector characteristics when necessary, we find $V_{CE\min} \cong 0.2$ V, $V_{CEQ} \cong 5.0$ V, and $V_{CE\max} \cong 9.2$ V.

T13.3 $\alpha = \dfrac{I_{CQ}}{I_{EQ}} = \dfrac{1.0}{1.04} = 0.9615 \quad I_{BQ} = I_{EQ} - I_{CQ} = 0.04 \text{ mA}$

$$\beta = \frac{I_{CQ}}{I_{BQ}} = \frac{\alpha}{1-\alpha} = 25 \qquad r_\pi = \frac{\beta V_T}{I_{CQ}} = \frac{25 \times 0.026}{0.001} = 650\ \Omega$$

The small-signal equivalent circuit is shown in Figure 13.26.

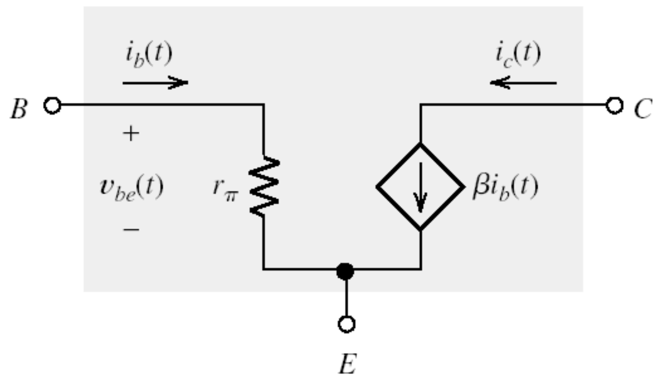
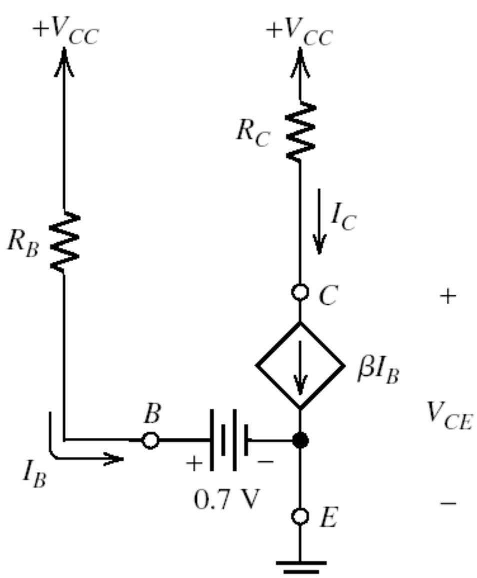


Figure 13.26 Small-signal equivalent circuit for the BJT.

T13.4 (a) It turns out that, in this case ($\beta = 50$), the BJT operates in the active region. The equivalent circuit is:



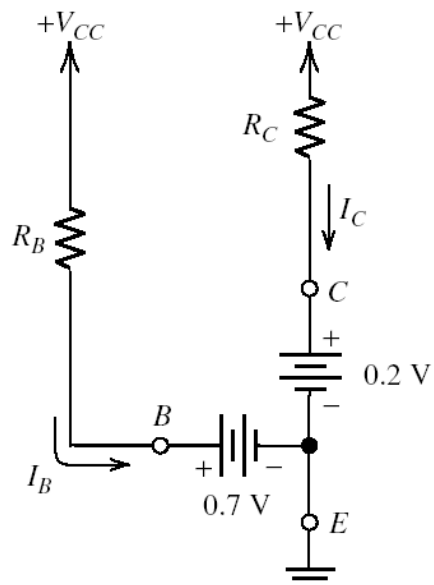
in which we have $V_{CC} = 9$ V, $R_C = 4.7$ kΩ, and $R_B = 470$ kΩ.
We have

$$I_B = \frac{V_{CC} - 0.7}{R_B} = 17.66\ \mu A \qquad I_C = \beta I_B = 0.8830\ \text{mA}$$

$$V_{CE} = V_{CC} - R_C I_C = 4.850\ \text{V}$$

Because we have $V_{CE} > 0.2$, we are justified in assuming that the transistor operates in the active region.

(b) In this case ($\beta = 250$), the BJT operates in the saturation region. The equivalent circuit is:

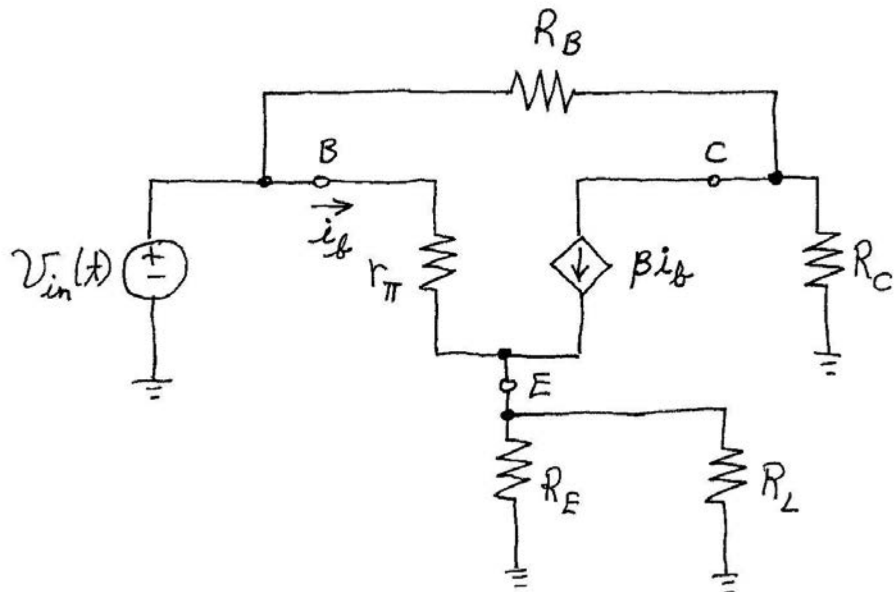


We have

$$V_{CE} = 0.2\ \text{V} \quad I_B = \frac{V_{CC} - 0.7}{R_B} = 17.66\ \mu A \quad I_C = \frac{V_{CC} - 0.2}{R_C} = 1.872\ \text{mA}$$

Because we have $\beta I_B > I_C$, we are justified in assuming that the transistor operates in the saturation region.

T13.5 We need to replace V_{CC} by a short circuit to ground, the coupling capacitances with short circuits, and the BJT with its equivalent circuit. The result is:



T13.6 This problem is similar to parts of Example 13.8.

$$r_\pi = \frac{\beta V_T}{I_{CQ}} = \frac{120(26\,\text{mV})}{4\,\text{mA}} = 780\ \Omega$$

$$R_L' = \frac{1}{1/R_L + 1/R_C} = 1.579\ \text{k}\Omega$$

$$A_v = -\frac{\beta R_L'}{r_\pi} = -243.0$$

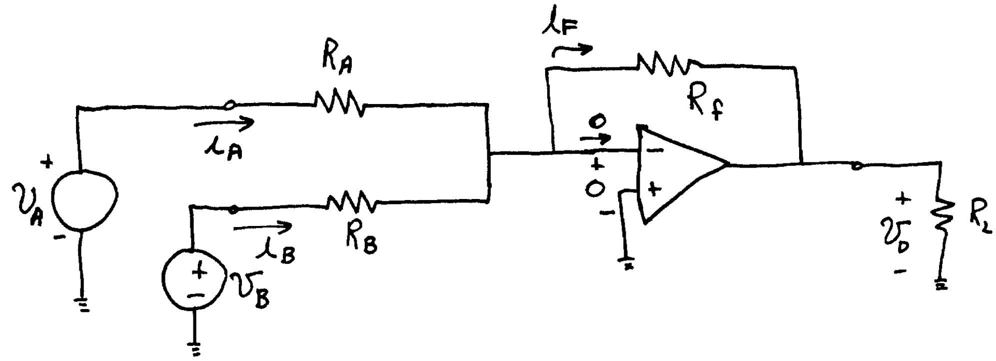
$$R_B = \frac{1}{1/R_1 + 1/R_2} = 31.97\ \text{k}\Omega$$

$$Z_{in} = \frac{1}{1/R_B + 1/r_\pi} = 761.4\ \Omega$$

CHAPTER 14

Exercises

E14.1



(a) $i_A = \dfrac{v_A}{R_A}$ $\quad i_B = \dfrac{v_B}{R_B}$ $\quad i_F = i_A + i_B = \dfrac{v_A}{R_A} + \dfrac{v_B}{R_B}$

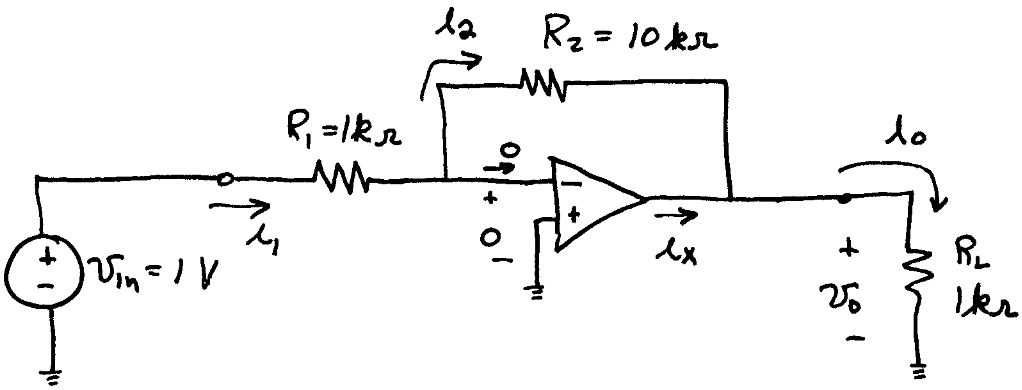
$$v_o = -R_F i_F = -R_F\left(\frac{v_A}{R_A} + \frac{v_B}{R_B}\right)$$

(b) For the v_A source, $R_{inA} = \dfrac{v_A}{i_A} = R_A$.

(c) Similarly $R_{inB} = R_B$.

(d) In part (a) we found that the output voltage is independent of the load resistance. Therefore, the output resistance is zero.

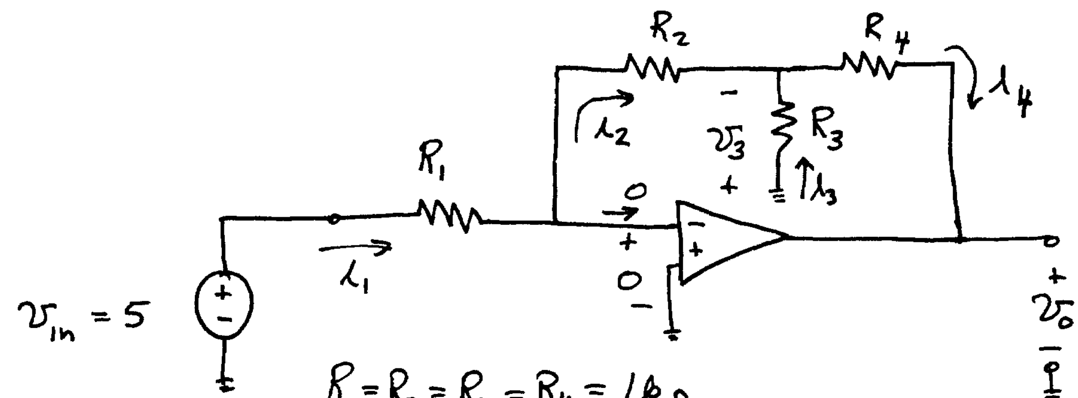
E14.2 (a)



$i_1 = \dfrac{v_{in}}{R_1} = 1\,\text{mA} \quad i_2 = i_1 = 1\,\text{mA} \quad v_o = -R_2 i_2 = -10\,\text{V}$

$i_o = \dfrac{v_o}{R_L} = -10\text{ mA} \qquad i_x = i_o - i_2 = -11\text{ mA}$

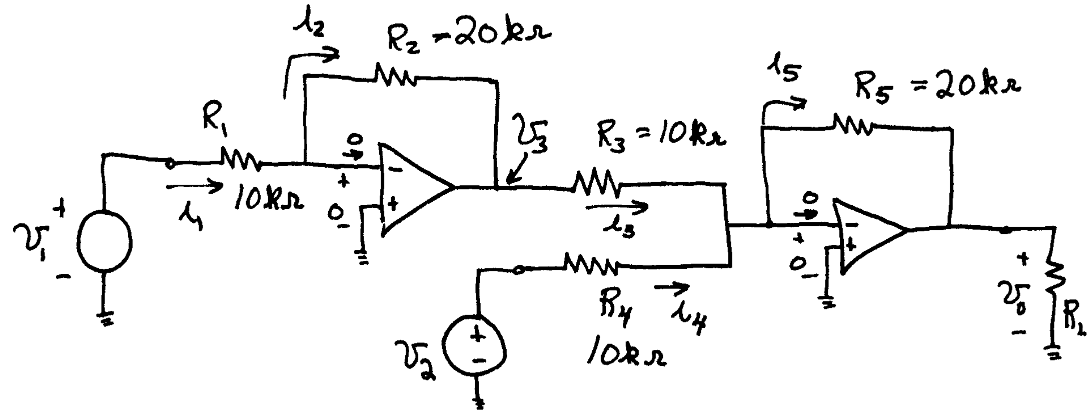
(b)



$i_1 = \dfrac{v_{in}}{R_1} = 5\text{ mA} \qquad i_2 = i_1 = 5\text{ mA} \qquad v_3 = R_2 i_2 = 5\text{ V}$

$i_3 = \dfrac{v_3}{R_3} = 5\text{ mA} \qquad i_4 = i_2 + i_3 = 10\text{ mA} \qquad v_o = -R_4 i_4 - R_2 i_2 = -15\text{ V}$

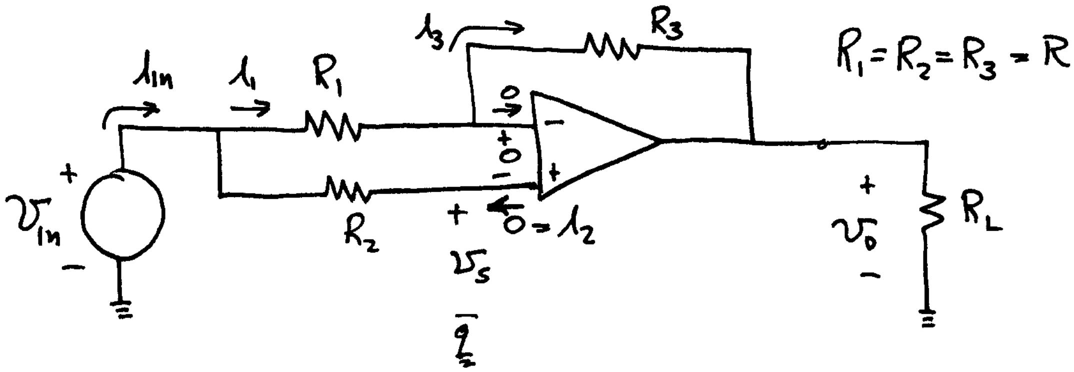
E14.3



Direct application of circuit laws gives $i_1 = \dfrac{v_1}{R_1}$, $i_2 = i_1$, and $v_3 = -R_2 i_2$. From the previous three equations, we obtain $v_3 = -\dfrac{R_2}{R_1} v_1 = -2v_1$. Then applying circuit laws gives $i_3 = \dfrac{v_3}{R_3}$, $i_4 = \dfrac{v_2}{R_4}$, $i_5 = i_3 + i_4$, and $v_o = -R_5 i_5$.

These equations yield $v_o = -\frac{R_5}{R_3}v_3 - \frac{R_5}{R_4}v_2$. Then substituting values and using the fact that $v_3 = -2v_1$, we find $v_o = 4v_1 - 2v_2$.

E14.4 (a)

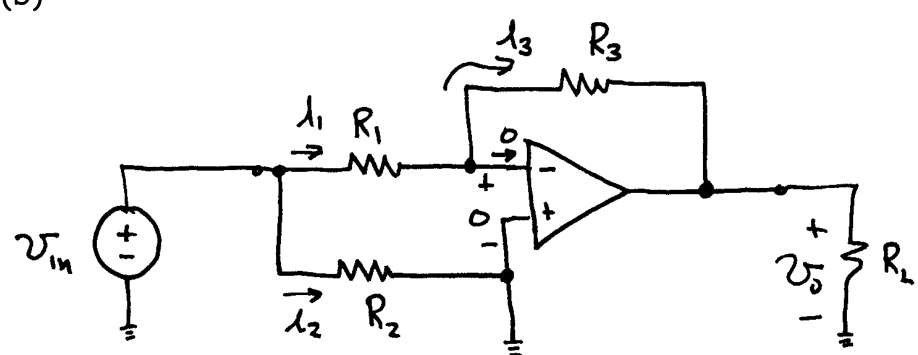


$v_s = v_{in} + R_2 i_2 = v_{in}$ (Because of the summing-point restraint, $i_2 = 0$.)

$i_1 = \frac{v_{in} - v_s}{R_1} = 0$ (Because $v_s = v_{in}$.) $\qquad i_{in} = i_1 - i_2 = 0$

$i_3 = i_1 = 0 \qquad v_o = R_3 i_3 + v_s = v_{in}$ Thus, $A_v = \frac{v_o}{v_{in}} = +1$ and $R_{in} = \frac{v_{in}}{i_{in}} = \infty$.

(b)

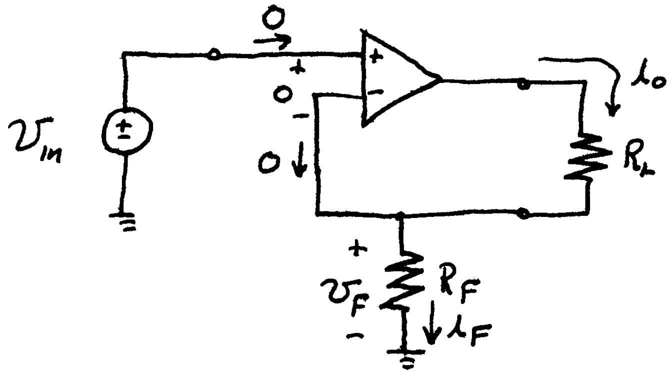


(Note: We assume that $R_1 = R_2 = R_3$.)

$i_1 = \frac{v_{in}}{R_1} = \frac{v_{in}}{R} \qquad i_2 = \frac{v_{in}}{R_2} = \frac{v_{in}}{R} \qquad i_{in} = i_1 + i_2 = \frac{2v_{in}}{R} \qquad R_{in} = \frac{R}{2}$

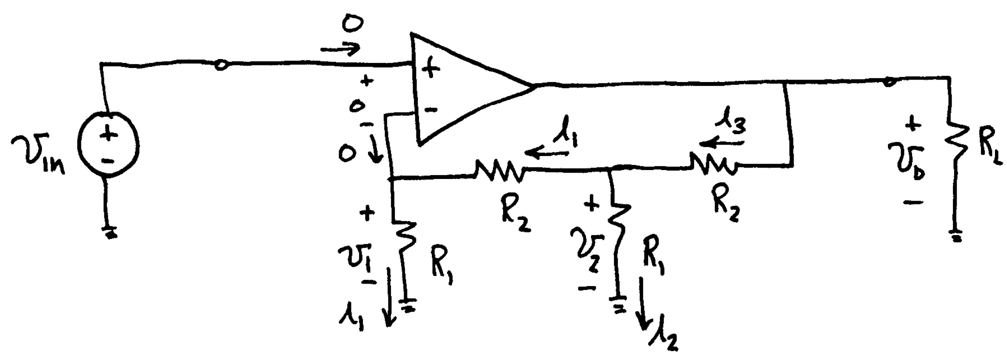
$i_3 = i_1 = \frac{v_{in}}{R_1} \qquad v_o = -R_3 i_3 = -\frac{R_3}{R_1}v_{in} = -v_{in} \qquad A_v = \frac{v_o}{v_{in}} = -1$

E14.5



From the circuit, we can write $v_F = v_{\text{in}}$, $i_F = \dfrac{v_F}{R_F}$, and $i_o = i_F$. From these equations, we find that $i_o = \dfrac{v_{\text{in}}}{R_F}$. Then because i_o is independent of R_L, we conclude that the output impedance of the amplifier is infinite. Also R_{in} is infinite because i_{in} is zero.

E14.6 (a)



$v_1 = v_{\text{in}}$ $\quad i_1 = \dfrac{v_1}{R_1}$ $\quad v_2 = R_2 i_1 + R_1 i_1$ $\quad i_2 = \dfrac{v_2}{R_1}$ $\quad i_3 = i_1 + i_2$ $\quad v_o = R_2 i_3 + v_2$

Using the above equations we eventually find that

$$A_v = \frac{v_o}{v_{\text{in}}} = 1 + 3\frac{R_2}{R_1} + \left(\frac{R_2}{R_1}\right)^2$$

(b) Substituting the values given, we find $A_v = 131$.

(c) Because $i_{\text{in}} = 0$, the input resistance is infinite.

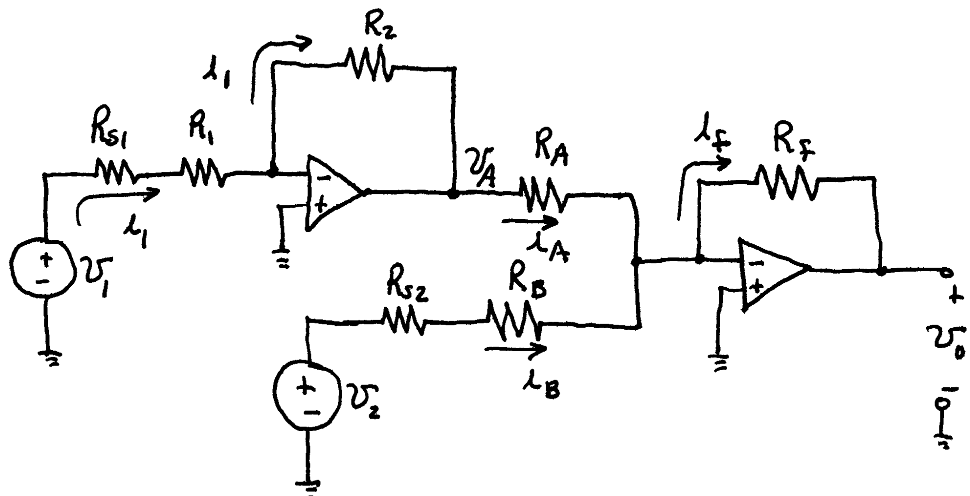
(d) Because $v_o = A_v v_{in}$ is independent of R_L, the output resistance is zero.

E14.7 We have $A_{vs} = -\dfrac{R_2}{R_s + R_1}$ from which we conclude that

$$A_{vs\max} = -\frac{R_{2\max}}{R_{s\min} + R_{1\min}} = -\frac{499 \times 1.01}{0 + 49.9 \times 0.99} = -10.20$$

$$A_{vs\min} = -\frac{R_{2\min}}{R_{s\max} + R_{1\max}} = -\frac{499 \times 0.99}{0.500 + 49.9 \times 1.01} = -9.706$$

E14.8



Applying basic circuit principles, we obtain:

$$i_1 = \frac{v_1}{R_1 + R_{s1}} \qquad v_A = -R_2 i_1 \qquad i_A = \frac{v_A}{R_A}$$

$$i_B = \frac{v_2}{R_B + R_{s2}} \qquad i_f = i_A + i_B \qquad v_o = -R_f i_f$$

From these equations, we eventually find

$$v_o = \frac{R_2}{R_{s1} + R_1}\frac{R_f}{R_A}v_1 - \frac{R_f}{R_{s2} + R_B}v_2$$

E14.9 Many correct answers exist. A good solution is the circuit of Figure 14.11 in the book with $R_2 \cong 19R_1$. We could use standard 1%-tolerance resistors with nominal values of $R_1 = 1\text{ k}\Omega$ and $R_2 = 19.1\text{ k}\Omega$.

E14.10 Many correct answers exist. A good solution is the circuit of Figure 14.18 in the book with $R_1 \geq 20R_s$ and $R_2 \cong 25(R_1 + R_s)$. We could use standard 1%-tolerance resistors with nominal values of $R_1 = 20\,\text{k}\Omega$ and $R_2 = 515\ \text{k}\Omega$.

E14.11 Many correct selections of component values can be found that meet the desired specifications. One possibility is the circuit of Figure 14.19 with:

$R_1 =$ a 453-kΩ fixed resistor in series with a 100-kΩ trimmer (nominal design value is 500 kΩ)

R_B is the same as R_1

$R_2 = 499\ \text{k}\Omega$

$R_A = 1.5\ \text{M}\Omega$

$R_f = 1.5\ \text{M}\Omega$

After constructing the circuit we could adjust the trimmers to achieve the desired gains.

E14.12 $f_{BCL} = \dfrac{f_t}{A_{0CL}} = \dfrac{A_{0OL} f_{BOL}}{A_{0CL}} = \dfrac{10^5 \times 40}{100} = 40\,\text{kHz}$ The corresponding Bode plot is shown in Figure 14.22 in the book.

E14.13 (a) $f_{FP} = \dfrac{SR}{2\pi V_{om}} = \dfrac{5 \times 10^6}{2\pi(4)} = 198.9\,\text{kHz}$

(b) The input frequency is less than f_{FP} and the current limit of the op amp is not exceeded, so the maximum output amplitude is 4 V.

(c) With a load of 100 Ω the current limit is reached when the output amplitude is 10 mA × 100 Ω = 1 V. Thus the maximum output amplitude without clipping is 1 V.

(d) In deriving the full-power bandwidth we obtained the equation:

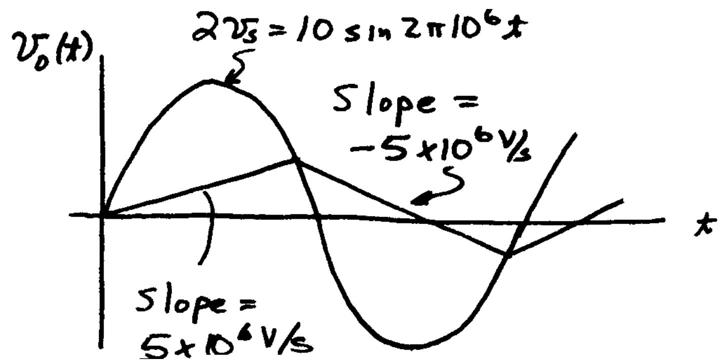
$$2\pi f V_{om} = SR$$

Solving for V_{om} and substituting values, we have

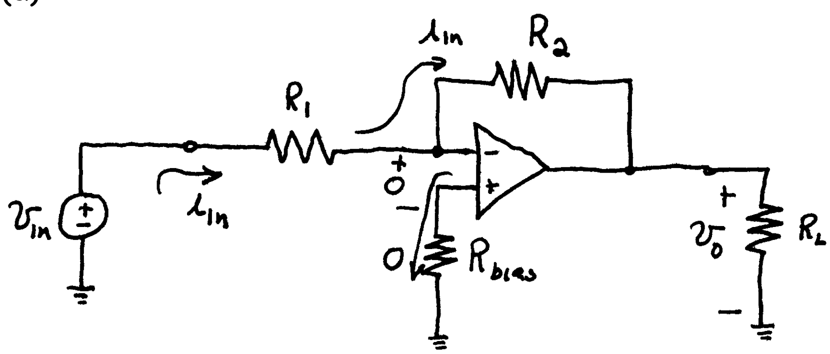
$$V_{om} = \frac{SR}{2\pi f} = \frac{5 \times 10^6}{2\pi 10^6} = 0.7958\ \text{V}$$

With this peak voltage and R_L = 1 kΩ, the current limit is not exceeded.

(e) Because the output, assuming an ideal op amp, has a rate of change exceeding the slew-rate limit, the op amp cannot follow the ideal output, which is $v_o(t) = 10\sin(2\pi 10^6 t)$. Instead, the output changes at the slew-rate limit and the output waveform eventually becomes a triangular waveform with a peak-to-peak amplitude of SR $\times$ ($T/2$) = 2.5 V.

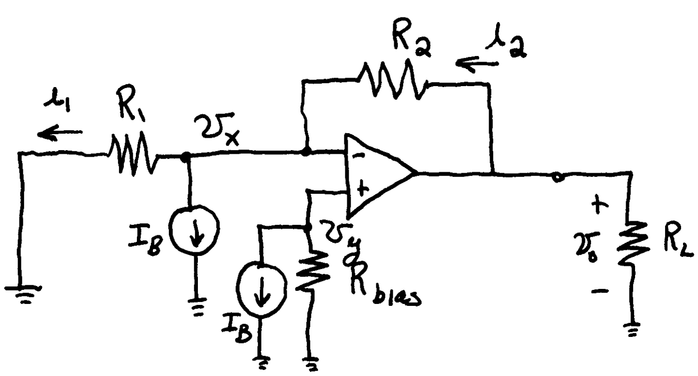


E14.14 (a)



Applying basic circuit laws, we have $i_{in} = \frac{v_{in}}{R_1}$ and $v_o = -R_2 i_{in}$. These equations yield $A_v = \frac{v_o}{v_{in}} = -\frac{R_2}{R_1}$.

(b)



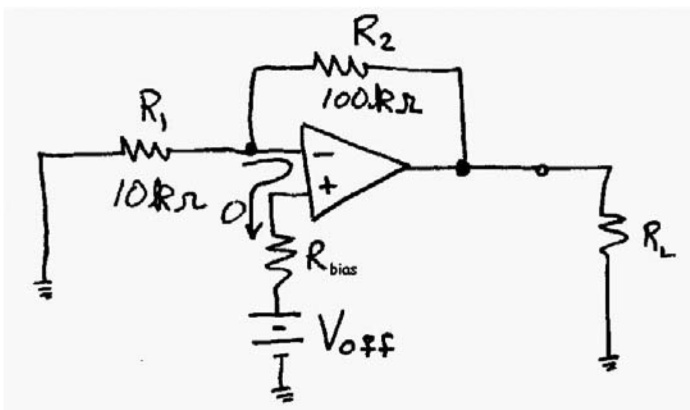
Applying basic circuit principles, algebra, and the summing-point restraint, we have

$$v_x = v_y = -R_{bias} I_B \qquad i_1 = \frac{v_x}{R_1} = -\frac{R_{bias}}{R_1} I_B = -\frac{R_2}{R_1 + R_2} I_B$$

$$i_2 = I_B + i_1 = \left(1 - \frac{R_2}{R_1 + R_2}\right) I_B = \frac{R_1}{R_1 + R_2} I_B$$

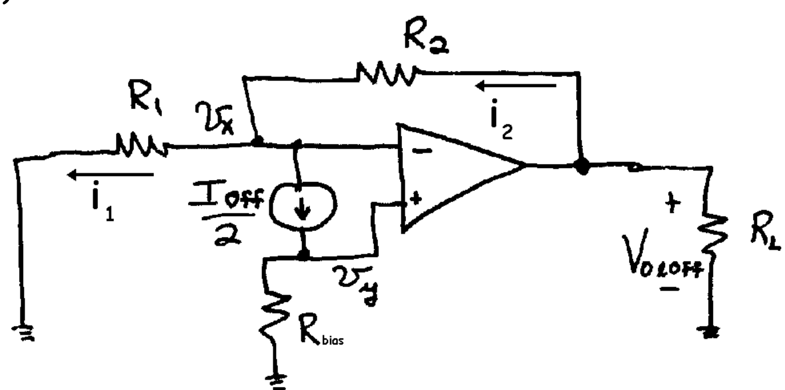
$$v_o = R_2 i_2 + v_x = R_2 \frac{R_1}{R_1 + R_2} I_B - R_{bias} I_B = 0$$

(c)



The drop across R_{bias} is zero because the current through it is zero. For the source V_{off} the circuit acts as a noninverting amplifier with a gain $A_v = 1 + \frac{R_2}{R_1} = 11$. Therefore, the extreme output voltages are given by $v_o = A_v V_{\text{off}} = \pm 33\,\text{mV}$.

(d)



Applying basic circuit principles, algebra, and the summing-point restraint, we have

$$v_x = v_y = R_{bias}\frac{I_{\text{off}}}{2} \qquad i_1 = \frac{v_x}{R_1} = \frac{R_{bias}}{R_1}\frac{I_{\text{off}}}{2} = \frac{R_2}{R_1+R_2}\frac{I_{\text{off}}}{2}$$

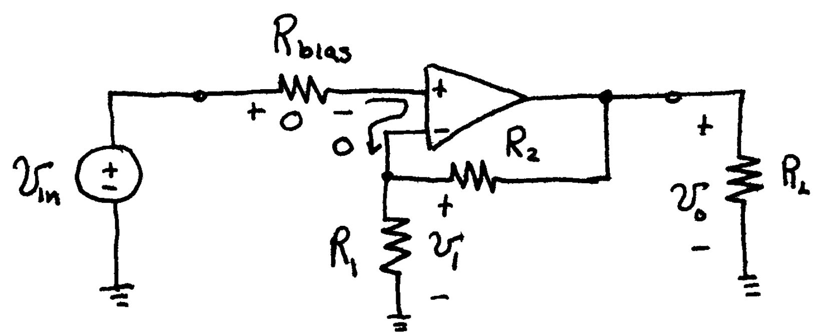
$$i_2 = \frac{I_{\text{off}}}{2} + i_1 = \left(1 + \frac{R_2}{R_1+R_2}\right)\frac{I_{\text{off}}}{2} = \frac{R_1+2R_2}{R_1+R_2}\frac{I_{\text{off}}}{2}$$

$$v_o = R_2 i_2 + v_x = R_2\frac{R_1+2R_2}{R_1+R_2}\frac{I_{\text{off}}}{2} + R_{bias}\frac{I_{\text{off}}}{2} = R_2 I_{off}$$

Thus the extreme values of v_o caused by I_{off} are $V_{o,\text{Ioff}} = \pm 4$ mV.

(e) The cumulative effect of the offset voltage and offset current is that V_o ranges from -37 to +37 mV.

E14.15 (a)



Because of the summing-point constraint, no current flows through R_{bias} so the voltage across it is zero. Because the currents through R_1 and R_2 are the same, we use the voltage division principle to write

$$v_1 = v_o\frac{R_1}{R_1+R_2}$$

Then using KVL we have

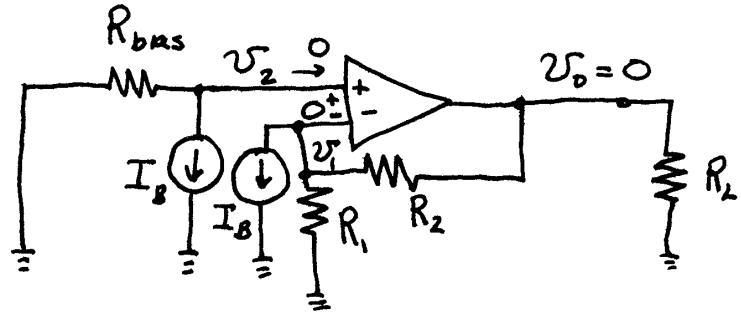
$$v_{\text{in}} = 0 + v_1$$

These equations yield

$$A_v = \frac{v_o}{v_{\text{in}}} = 1 + \frac{R_2}{R_1}$$

Assuming an ideal op amp, the resistor R_{bias} does not affect the gain since the voltage across it it zero.

(b) The circuit with the signal set to zero and including the bias current sources is shown.

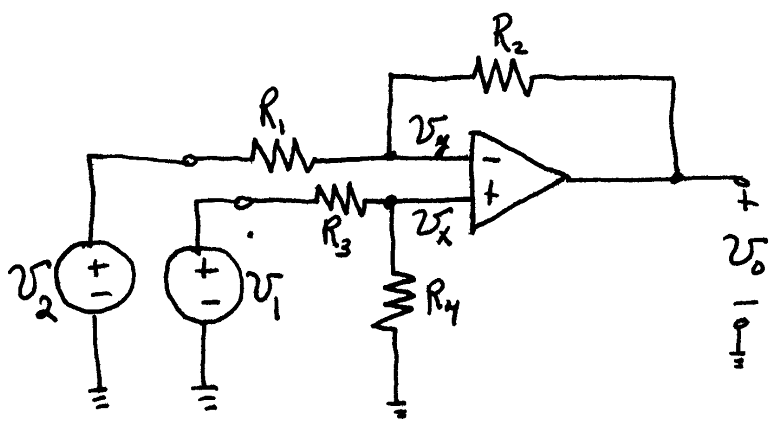


We want the output voltage to equal zero. Using Ohm's law, we can write $v_2 = -R_{\text{bias}} I_B$. Then writing a current equation at the inverting input, we have $I_B + \frac{v_1}{R_1} + \frac{v_1}{R_2} = 0$. Finally, because of the summing-point restraint, we have $v_2 = v_1$. These equations eventually yield

$$R_{\text{bias}} = \frac{1}{1/R_1 + 1/R_2}$$

as the condition for zero output due to the bias current sources.

E14.16



Because no current flows into the op-amp input terminals, we can use the voltage division principle to write

$$v_x = v_1 \frac{R_4}{R_3 + R_4}$$

Because of the summing-point restraint, we have

$$v_x = v_y = v_1 \frac{R_4}{R_3 + R_4}$$

Writing a KCL equation at the inverting input, we obtain

$$\frac{v_y - v_2}{R_1} + \frac{v_y - v_o}{R_2} = 0$$

Substituting for v_y and solving for the output voltage, we obtain

$$v_o = v_1 \frac{R_4}{R_3 + R_4} \frac{R_1 + R_2}{R_1} - v_2 \frac{R_2}{R_1}$$

If we have $R_4 / R_3 = R_2 / R_1$, the equation for the output voltage reduces to

$$v_o = \frac{R_2}{R_1}(v_1 - v_2)$$

E14.17 (a) $v_o(t) = -\frac{1}{RC}\int_0^t v_{in}(t)dt = -1000\int_0^t v_{in}(t)dt$

$$= -1000\int_0^t 5dt = -5000t \quad \text{for } 0 \le t \le 1\,\text{ms}$$

$$= -1000\left(\int_0^{1\,\text{ms}} 5dt + \int_{1\,\text{ms}}^{t} -5dt\right) = -10 + 5000t \quad \text{for } 1\ \text{ms} \le t \le 3\,\text{ms}$$

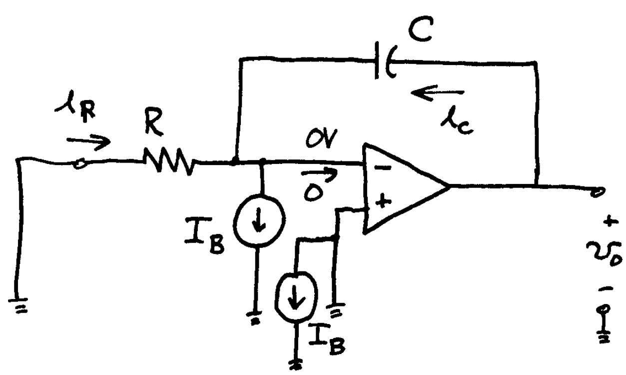
and so forth. A plot of $v_o(t)$ versus t is shown in Figure 14.37 in the book.

(b) A peak-to-peak amplitude of 2 V implies a peak amplitude of 1 V. The first (negative) peak amplitude occurs at $t = 1$ ms. Thus we can write

$$-1 = -\frac{1}{RC}\int_0^{1\,\text{ms}} v_{in}dt = -\frac{1}{10^4 C}\int_0^{1\,\text{ms}} 5dt = -\frac{1}{10^4 C} \times 5 \times 10^{-3}$$

which yields $C = 0.5\ \mu$F.

E14.18 The circuit with the input source set to zero and including the bias current sources is:



Because the voltage across R is zero, we have $i_C = I_B$, and we can write

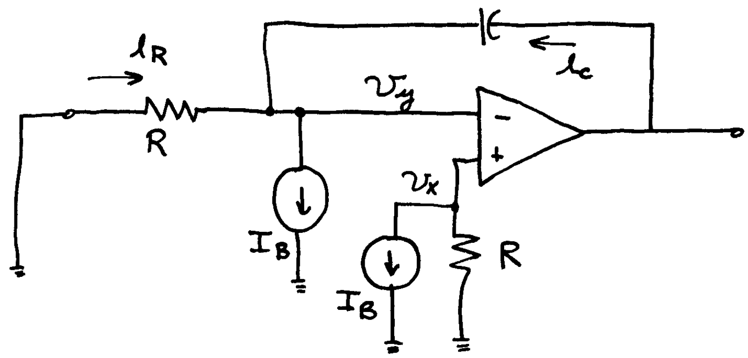
$$v_o = \frac{1}{C}\int_0^t i_C dt = \frac{1}{C}\int_0^t I_B dt = \frac{100 \times 10^{-9} t}{C}$$

(a) For C = 0.01 μF we have $v_o(t) = 10t$ V.

(b) For C = 1 μF we have $v_o(t) = 0.1t$ V.

Notice that larger capacitances lead to smaller output voltages.

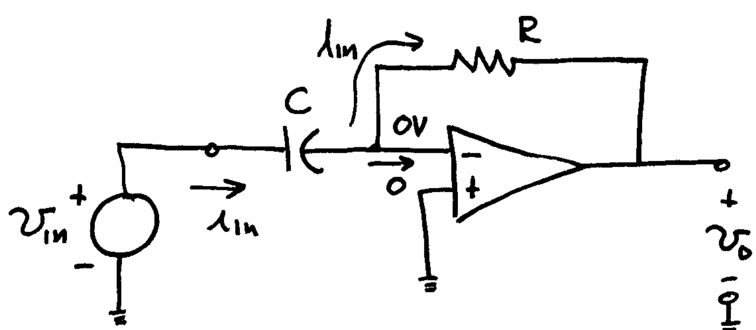
E14.19



$v_y = v_x = -I_B R_B \quad i_R = -v_y / R_B = I_B \quad i_C = i_R + I_B = 0$

Because $i_C = 0$, we have $v_C = 0$, and $v_o = v_y = -I_B R = 1$ mV.

E14.20



$$i_{\text{in}} = C\frac{dv_{\text{in}}}{dt} \qquad v_o(t) = -Ri_{\text{in}} = -RC\frac{dv_{\text{in}}}{dt}$$

E14.21 The transfer function in decibels is

$$|H(f)|_{dB} = 20\log\left[\frac{H_0}{\sqrt{1+(f/f_B)^{2n}}}\right]$$

For $f >> f_B$, we have

$$|H(f)|_{dB} \cong 20\log\left[\frac{H_0}{\sqrt{(f/f_B)^{2n}}}\right] = 20\log|H_0| + 20n\log(f_B) - 20n\log(f)$$

This expression shows that the gain magnitude is reduced by $20n$ decibels for each decade increase in f.

E14.22 Three stages each like that of Figure 14.40 must be cascaded. From Table 14.1, we find that the gains of the stages should be 1.068, 1.586, and 2.483. Many combinations of component values will satisfy the requirements of the problem. A good choice for the capacitance value is 0.01 μF, for which we need $R = 1/(2\pi C f_B) = 3.183\ \text{k}\Omega$. Also $R_f = 10\ \text{k}\Omega$ is a good choice.

Answers for Selected Problems

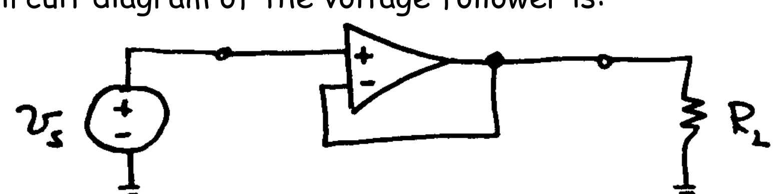
P14.4* $v_{id} = v_1 - v_2 = \cos(2000\pi t)$ $\qquad v_{icm} = \frac{1}{2}(v_1 + v_2) = 20\cos(120\pi t)$

P14.6* The steps in analysis of an amplifier containing an ideal op amp are:

1. Verify that negative feedback is present.
2. Assume that the differential input voltage and the input currents are zero.
3. Apply circuit analysis principles including Kirchhoff's and Ohm's laws to write circuit equations. Then solve for the quantities of interest.

P14.10* $A_v = -8$

P14.17* The circuit diagram of the voltage follower is:



Assuming an ideal op amp, the voltage gain is unity, the input impedance is infinite, and the output impedance is zero.

P14.18* If the source has non-zero series impedance, loading (reduction in voltage) will occur when the load is connected directly to the source. On the other hand, the input impedance of the voltage follower is very high (ideally infinite) and loading does not occur. If the source impedance is very high compared to the load impedance, the voltage follower will deliver a much larger voltage to the load than direct connection.

P14.21* $v_o = \left(\dfrac{R_1 + R_2}{R_1}\right)\dfrac{v_A R_B + v_B R_A}{R_A + R_B}$

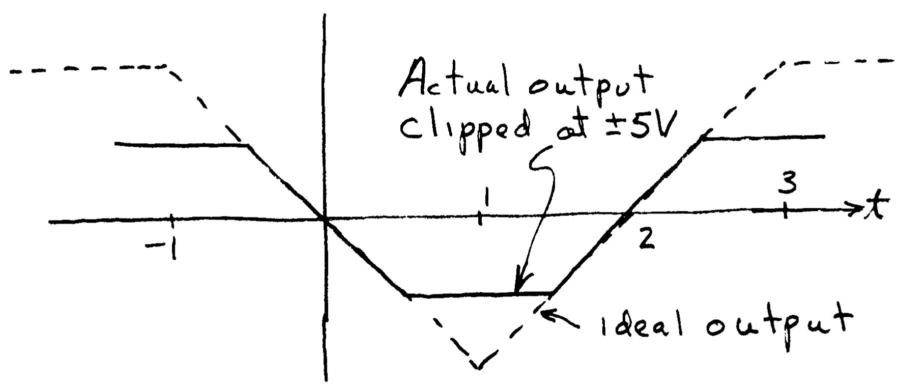
P14.24* (a) $v_o = -R_f i_{in}$

(b) Since v_o is independent of R_L, the output behaves as a perfect voltage source, and the output impedance is zero.

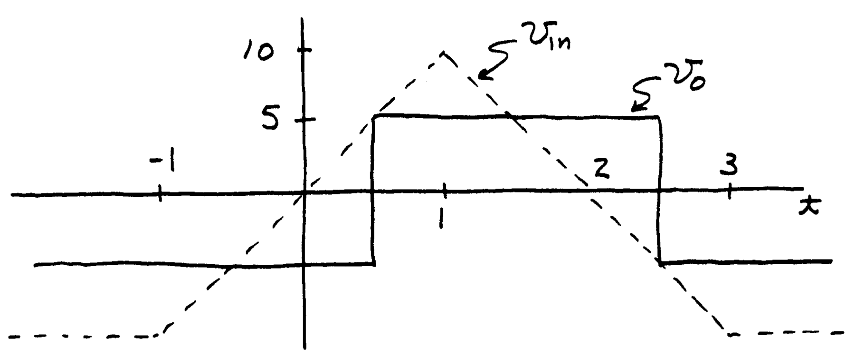
(c) The input voltage is zero because of the summing-point constraint, and the input impedance is zero.

(d) This is an ideal transresistance amplifier.

P14.28* (a)



(b)

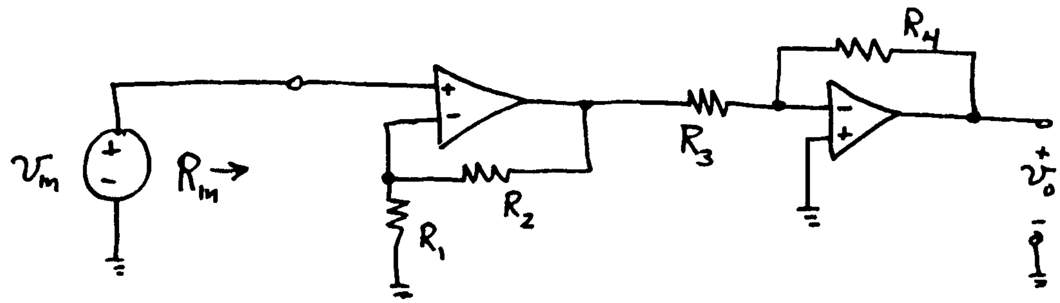


P14.32* $i_o = -\left(1 + \frac{R_1}{R_2}\right) i_{in}$

$R_{in} = 0$

The output impedance is infinite.

P14.36*



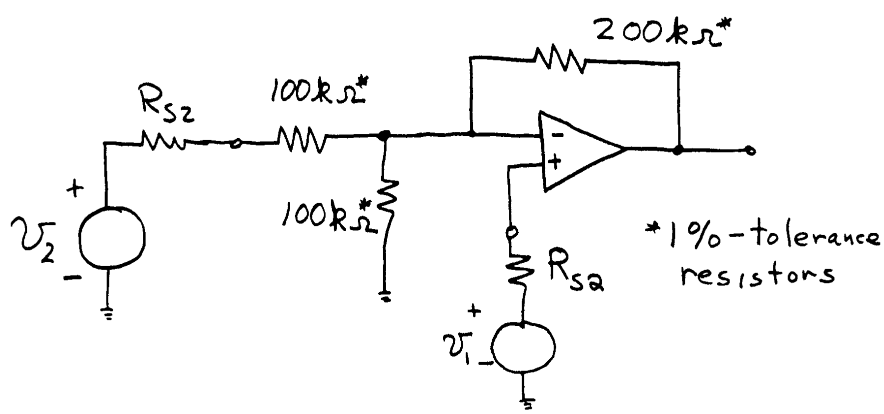
Many combinations of resistance values will achieve the given specifications. For example:

$R_1 = \infty$ and $R_2 = 0$. (Then the first stage becomes a voltage follower.) This is a particularly good choice because fewer resistors affect the overall gain, resulting in small overall gain variations.

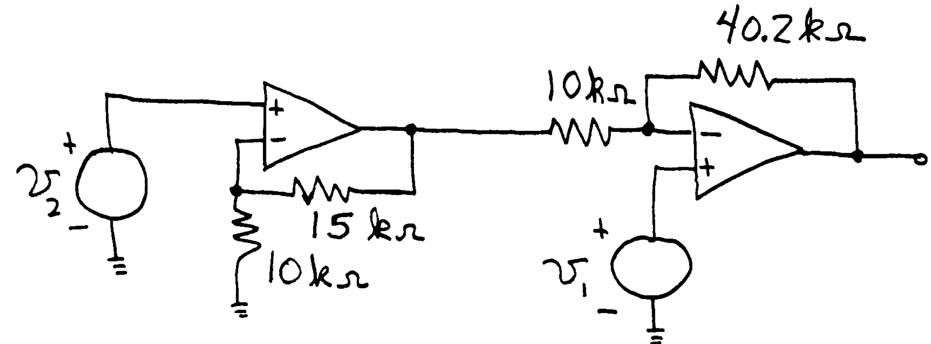
R_4 = 100 kΩ, 5% tolerance.

R_3 = 10 kΩ, 5% tolerance.

P14.37* *A* solution is:



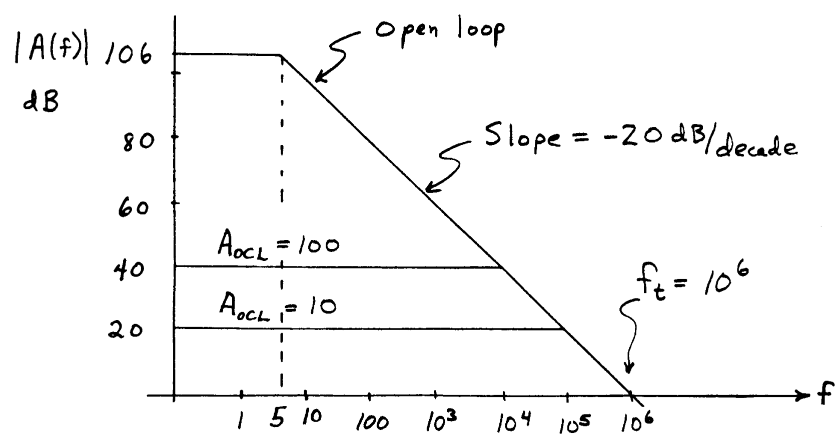
P14.41*



All resistors are $\pm 1\%$ tolerance.

P14.45* For $A_{0CL} = 10$, $f_{BCL} = 1.5\text{ MHz}$.
For $A_{0CL} = 100$, $f_{BCL} = 150$ kHz.

P14.52*



P14.57* (a) $f_{FP} = \dfrac{SR}{2\pi V_{om}} = \dfrac{10^7}{2\pi 10} = 159\text{ kHz}$

(b) $V_{om} = 10\text{ V}$. (It is limited by the maximum output voltage capability of the op amp.)

(c) In this case, the limit is due to the maximum current available from the op amp. Thus, the maximum output voltage is:

$$V_{om} = 20\text{ mA} \times 100\ \Omega = 2\text{ V}$$

(d) In this case, the slew-rate is the limitation.

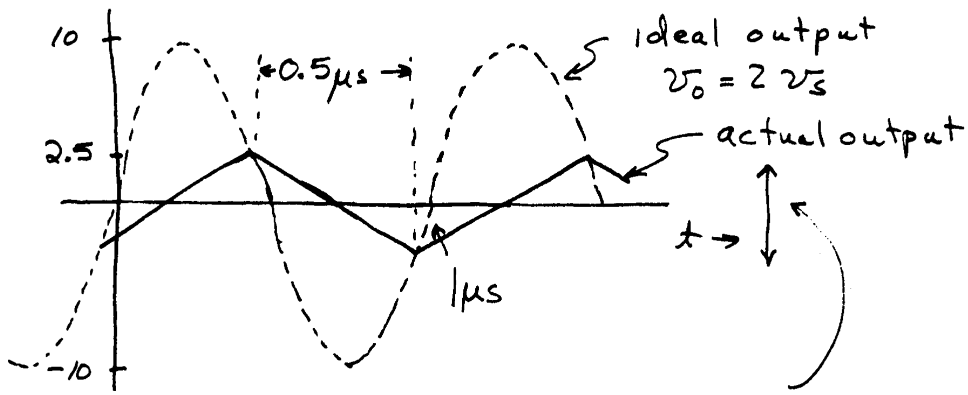
$$v_o(t) = V_{om}\sin(\omega t)$$

$$\frac{dv_o(t)}{dt} = \omega V_{om} \cos(\omega t)$$

$$\left|\frac{dv_o(t)}{dt}\right|_{\max} = \omega V_{om} = SR$$

$$V_{om} = \frac{SR}{\omega} = \frac{10^7}{2\pi 10^6} = 1.59\ \text{V}$$

(e)



peak-to-peak amplitude = $0.5\mu s \times 10^7$ V/s
= 5 V

P14.60* $SR = (4\ \text{V})/(0.5\ \mu\text{s}) = 8\ \text{V}/\mu\text{s}$

P14.63* See Figure 14.29 in the text.

P14.66* $V_{o,voff} = \pm 44\ \text{mV}$

$V_{o,bias} = 10\ \text{mV}$ and $20\ \text{mV}$

$V_{o,ioff} = \pm 2.5\ \text{mV}$

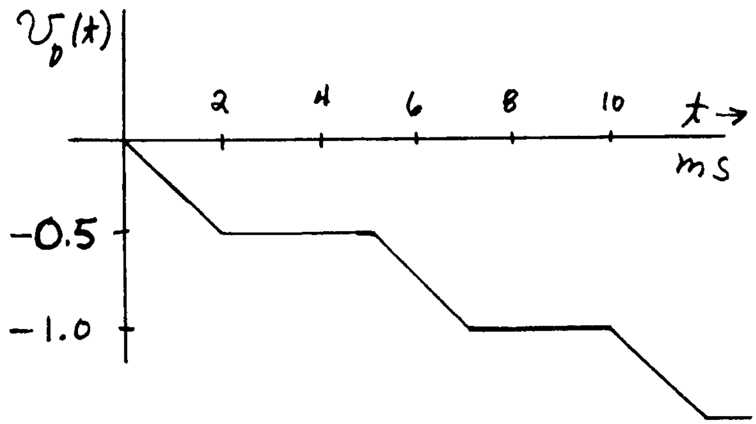
Due to all of the imperfections, the extreme output voltages are:

$V_{o.\max} = 44 + 20 + 2.5 = 66.5\ \text{mV}$

$V_{o,\min} = -44 + 10 - 2.5 = -36.5\ \text{mV}$

P14.70* The circuit diagram is shown in Figure 14.33 in the text. To achieve a nominal gain of 10, we need to have $R_2 = 10R_1$. Values of R_1 ranging from about 1 kΩ to 100 kΩ are practical. A good choice of values is R_1 = 10 kΩ and R_2 = 100 kΩ.

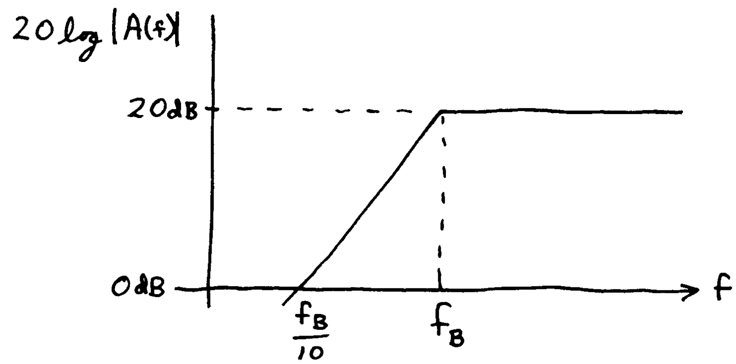
P14.74*



20 pulses are required to produce v_o = -10V.

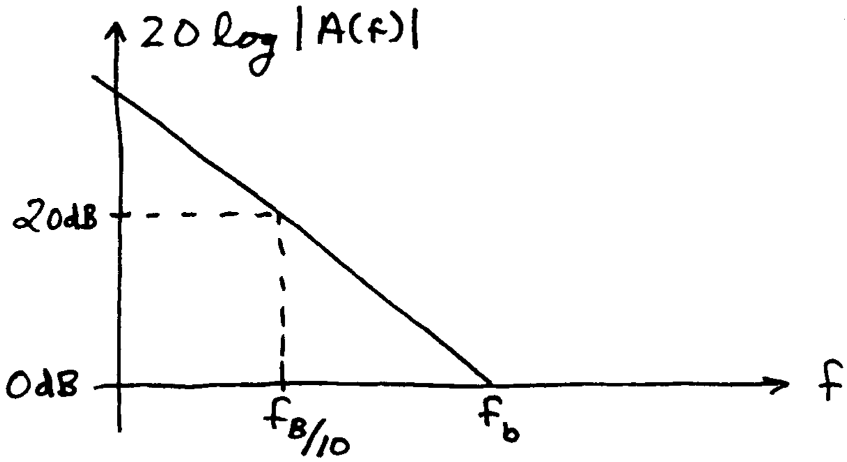
P14.78* (a) $A(f) = \dfrac{-10}{1 - jf_B/f}$

where $f_B = \dfrac{1}{2\pi RC}$



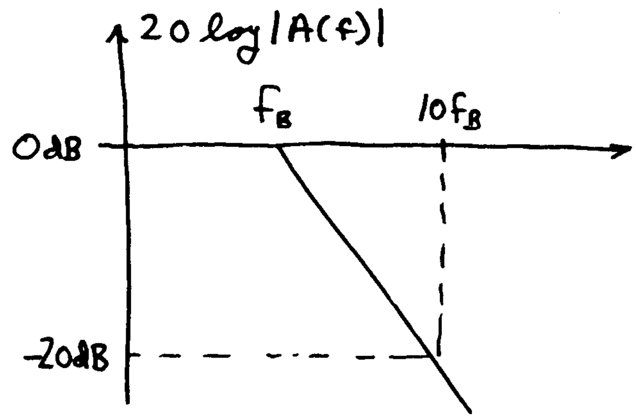
(b) $A(f) = -\dfrac{R + 1/j\omega C}{R} = -\left(1 - j\dfrac{f_B}{f}\right)$

where $f_B = \dfrac{1}{2\pi RC}$



(c) $A(f) = -\dfrac{\dfrac{1}{1/R + j\omega C}}{R} = -\dfrac{1}{1 + jf/f_B}$

where $f_B = \dfrac{1}{2\pi RC}$



Practice Test

T14.1 (a) The circuit diagram is shown in Figure 14.4 and the voltage gain is $A_v = -R_2/R_1$. Of course, you could use different resistance labels such as R_A and R_B so long as your equation for the gain is modified accordingly.

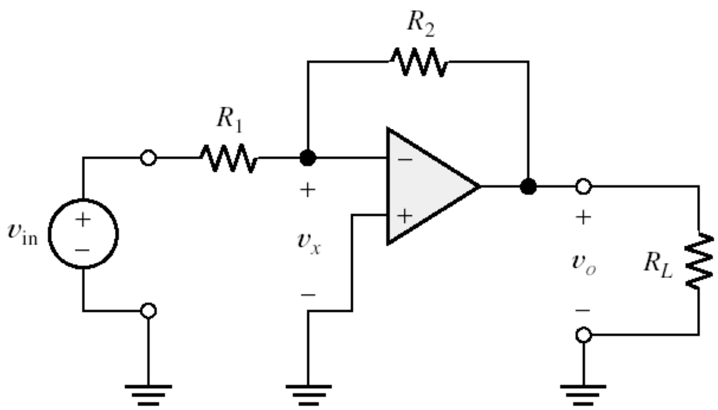


Figure 14.4 The inverting amplifier.

(b) The circuit diagram is shown in Figure 14.11 and the voltage gain is $A_v = 1 + R_2/R_1$.

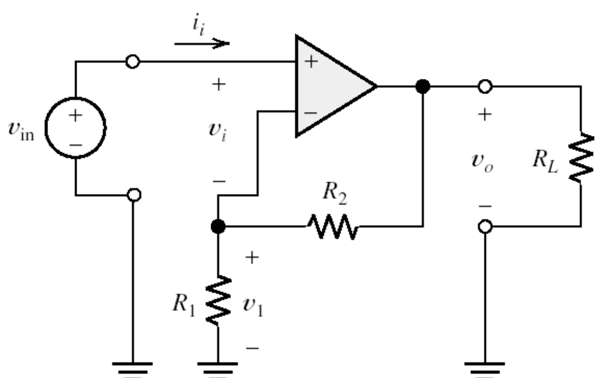
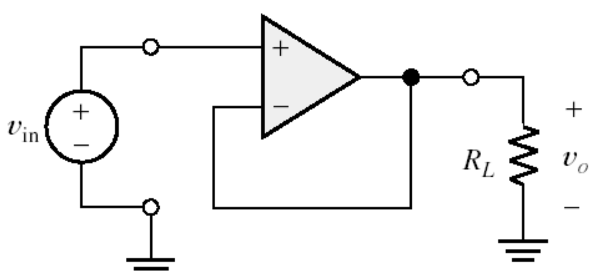


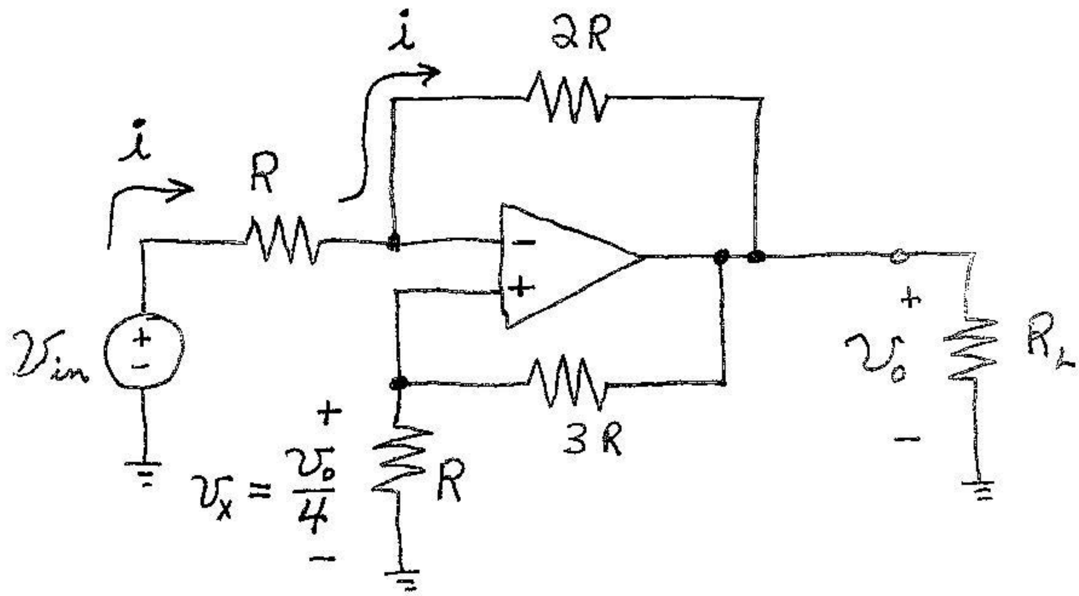
Figure 14.11 Noninverting amplifier.

(c) The circuit diagram is shown in Figure 14.12 and the voltage gain is $A_v = 1$.



T14.2 Because the currents flowing into the op-amp input terminals are zero, we can apply the voltage-division principle to determine the voltage v_x at the noninverting input with respect to ground:

$$v_x = v_o \frac{R}{R + 3R} = \frac{v_o}{4}$$



This is also the voltage at the inverting input, because the voltage between the op-amp input terminals is zero. Thus, the current i is

$$i = \frac{v_{\text{in}} - v_o / 4}{R}$$

Then, we can write a voltage equation starting from the ground node, through v_o, through the $2R$ resistance, across the op-amp input terminals, and then through v_x to ground. This gives

$$-v_o - 2Ri + 0 + v_x = 0$$

Substituting for i and v_x gives:

$$-v_o - 2R \frac{v_{\text{in}} - v_o / 4}{R} + 0 + \frac{v_o}{4} = 0$$

which simplifies to $v_o = -8v_{\text{in}}$. Thus, the voltage gain is $A_v = -8$.

T14.3 (a) $f_{BCL} = \dfrac{f_t}{A_{0CL}} = \dfrac{A_{0OL} f_{BOL}}{A_{0CL}} = \dfrac{2 \times 10^5 \times 5}{100} = 10\,\text{kHz}$

(b) Equation 14.32 gives the closed-loop gain as a function of frequency:

$$A_{CL}(f) = \frac{A_{0CL}}{1 + j(f/f_{BCL})} = \frac{100}{1 + j(f/10^4)}$$

The input signal has a frequency of 10^5 Hz, and a phasor representation given by $\mathbf{V}_{\text{in}} = 0.05\angle 0^\circ$. The transfer function evaluated for the frequency of the input signal is

$$A_{CL}(10^5) = \frac{100}{1 + j(10^5/10^4)} = 9.95\angle -84.29^\circ$$

The phasor for the output signal is

$$\mathbf{V}_o = A_{CL}(10^5)\mathbf{V}_{\text{in}} = (9.95\angle -84.29^\circ) \times (0.05\angle 0^\circ) = 0.4975\angle -84.29^\circ$$

and the output voltage is $v_o(t) = 0.4975 \cos(2\pi \times 10^5 t - 84.29^\circ)$.

T14.4 (a) $f_{FP} = \dfrac{SR}{2\pi V_{om}} = \dfrac{20 \times 10^6}{2\pi \times 4.5} = 707.4\,\text{kHz}$

(b) In this case, the limit is due to the maximum current available from the op amp. Thus, the maximum output voltage is:

$$V_{om} = 5\,\text{mA} \times 200\,\Omega = 1\,\text{V}$$

(The current through R_2 is negligible.)

(c) $V_{om} = 4.5\,\text{V}$. (It is limited by the maximum output voltage capability of the op amp.)

(d) In this case, the slew-rate is the limitation.

$$v_o(t) = V_{om}\sin(\omega t)$$

$$\frac{dv_o(t)}{dt} = \omega V_{om}\cos(\omega t)$$

$$\left|\frac{dv_o(t)}{dt}\right|_{\max} = \omega V_{om} = \text{SR}$$

$$V_{om} = \frac{SR}{\omega} = \frac{20 \times 10^6}{2\pi \times 5 \times 10^6} = 0.637\,\text{V}$$

T14.5 See Figure 14.29 for the circuit.

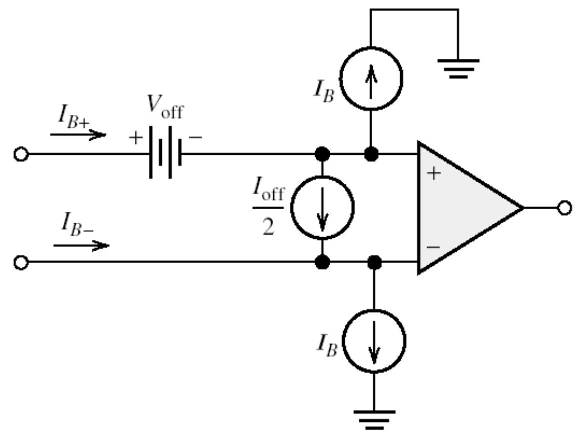


Figure 14.29 Three current sources and a voltage source model the dc imperfections of an op amp.

The effect on amplifiers of bias current, offset current, and offset voltage is to add a (usually undesirable) dc voltage to the intended output signal.

T14.6 See Figure 14.33 in the book.

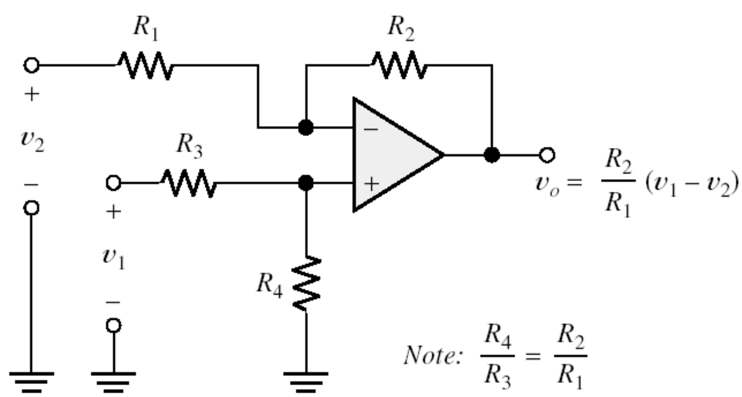


Figure 14.33 Differential amplifier.

Usually, we would have $R_1 = R_3$ and $R_2 = R_4$.

T14.7 See Figures 14.35 and 14.38 in the book:

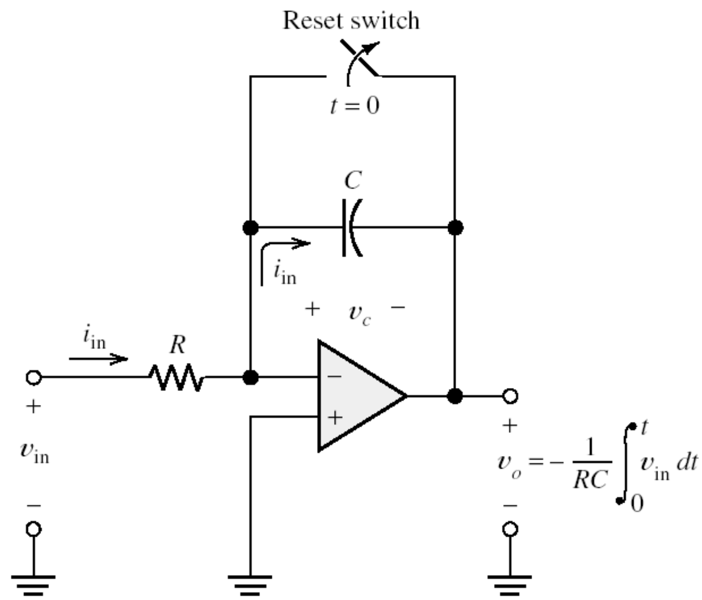


Figure 14.35 Integrator.

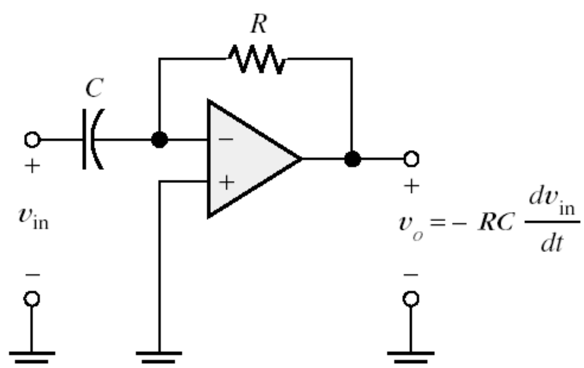


Figure 14.38 Differentiator.

T14.8 Filters are circuits designed to pass input components with frequencies in one range to the output and prevent input components with frequencies in other ranges from reaching the output.

An active filter is a filter composed of op amps, resistors, and capacitors.

Some applications for filters mentioned in the text are:

1. In an electrocardiograph, we need a filter that passes the heart signals, which have frequencies below about 100 Hz, and rejects higher frequency noise that can be created by contraction of other muscles.

2. Using a lowpass filter to remove noise from historical phonograph recordings.

3. In digital instrumentation systems, a low pass filter is often needed to remove noise and signal components that have frequencies higher than half of the sampling frequency to avoid a type of distortion, known as aliasing, during sampling and analog-to-digital conversion.

CHAPTER 15

Exercises

E15.1 If one grasps the wire with the right hand and with the thumb pointing north, the fingers point west under the wire and curl around to point east above the wire.

E15.2 If one places the fingers of the right hand on the periphery of the clock pointing clockwise, the thumb points into the clock face.

E15.3 $\mathbf{f} = q\mathbf{u} \times \mathbf{B} = (-1.602 \times 10^{-19})10^5 \mathbf{u}_x \times \mathbf{u}_y = -1.602 \times 10^{-14} \mathbf{u}_z$

in which $\mathbf{u}_x$, $\mathbf{u}_y$, and $\mathbf{u}_z$ are unit vectors along the respective axes.

E15.4 $f = i\ell B \sin(\theta) = 10(1)0.5 \sin(90^\circ) = 5 \text{ N}$

E15.5 (a) $\varphi = BA = B\pi r^2 = 0.5\pi(0.05)^2 = 3.927 \text{ mWb}$

$\lambda = N\varphi = 39.27 \text{ mWb turns}$

(b) $e = \dfrac{d\lambda}{dt} = -\dfrac{39.27 \times 10^{-3}}{10^{-3}} = -39.27 \text{ V}$

More information would be needed to determine the polarity of the voltage by use of Lenz's law. Thus the minus sign of the result is not meaningful.

E15.6 $B = \dfrac{\mu I}{2\pi r} = \dfrac{4\pi \times 10^{-7} \times 20}{2\pi 10^{-2}} = 4 \times 10^{-4} \text{ T}$

E15.7 By Ampère's law, the integral equals the sum of the currents flowing through the surface bounded by the path. The reference direction for the currents relates to the direction of integration by the right-hand rule. Thus, for each part the integral equals the sum of the currents flowing upward. Refering to Figure 15.9 in the book, we have

$$\oint_{\text{Path 1}} \mathbf{H} \cdot d\ell = 10 \text{ A} \qquad \oint_{\text{Path 2}} \mathbf{H} \cdot d\ell = 10 - 10 = 0 \text{ A} \qquad \oint_{\text{Path 3}} \mathbf{H} \cdot d\ell = -10 \text{ A}$$

E15.8 Refer to Figure 15.9 in the book. Conceptually the left-hand wire produces a field in the region surrounding it given by

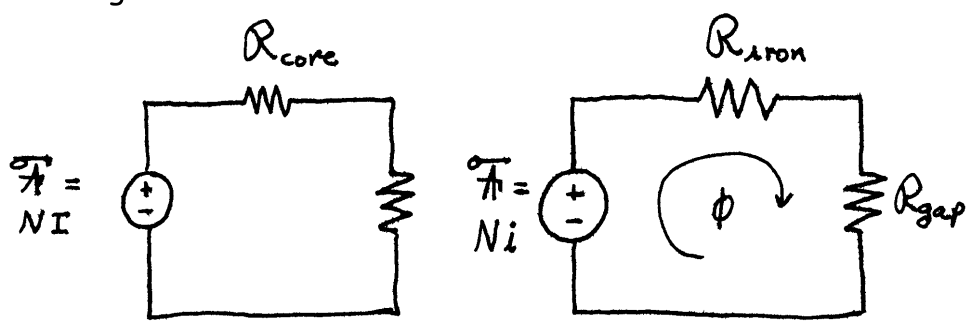
$$B = \frac{\mu I}{2\pi r} = \frac{4\pi \times 10^{-7} \times 10}{2\pi 10^{-1}} = 2 \times 10^{-5} \text{ T}$$

By the right-hand rule, the direction of this field is in the direction of Path 1. The field in turn produces a force on the right-hand wire given by

$$f = B\ell i = 2 \times 10^{5}(1)(10) = 2 \times 10^{-4} \text{ N}$$

By the right-hand rule, the direction of the force is such that the wires repel one another.

E15.9 The magnetic circuit is:



The reluctance of the iron is:

$$R_{\text{iron}} = \frac{\ell_{\text{iron}}}{\mu_r \mu_0 A_{\text{iron}}} = \frac{27 \times 10^{-2}}{5000 \times 4\pi \times 10^{-7} \times 4 \times 10^{-4}}$$

$$R_{\text{iron}} = 107.4 \times 10^{3}$$

The reluctance of the air gap is:

$$R_{\text{gap}} = \frac{\ell_{\text{gap}}}{\mu_0 A_{\text{gap}}} = \frac{10^{-2}}{4\pi \times 10^{-7} \times 9 \times 10^{-4}}$$

$$R_{\text{gap}} = 8.842 \times 10^{6}$$

Then we have

$$\varphi = B_{\text{gap}} A_{\text{gap}} = 0.5 \times 9 \times 10^{-4} = 0.45 \text{ mWb}$$

$$i = \frac{(R_{\text{iron}} + R_{\text{gap}})\varphi}{N} = \frac{(107.4 \times 10^{3} + 8.842 \times 10^{6})(0.45 \times 10^{-3})}{1000} = 4.027 \text{ A}$$

E15.10 Refer to Example 15.6 in the book. Neglecting the reluctance of the iron, we have:

$$R_c = 0$$

$$R_a = \frac{\ell_{gap}}{\mu_0 A_a} = \frac{1 \times 10^{-2}}{4\pi \times 10^{-7} \times 9 \times 10^{-4}} = 8.842 \times 10^{6}$$

$$R_b = \frac{\ell_{gap}}{\mu_0 A_b} = \frac{0.5\times10^{-2}}{4\pi\times10^{-7}\times6.25\times10^{-4}} = 6.366\times10^{6}$$

$$\varphi_a = \frac{Ni}{R_a} = \frac{500\times2}{8.842\times10^{6}} = 113.1\ \mu\text{Wb}$$

$$B_a = \frac{\varphi_a}{A_a} = \frac{113.1\times10^{-6}}{9\times10^{-4}} = 0.1257\ \text{T}$$

compared to 0.1123 T found in the example for an error of 11.9%.

$$\varphi_b = \frac{Ni}{R_b} = \frac{500\times2}{6.366\times10^{6}} = 157.1\ \mu\text{Wb}$$

$$B_b = \frac{\varphi_b}{A_b} = \frac{157.1\times10^{-6}}{6.25\times10^{-4}} = 0.2513\ \text{T}$$

compared to 0.2192T found in the Example for an error of 14.66%.

E15.11

$$\varphi_2 = \frac{N_2 i_2}{R} = \frac{200 i_2}{10^7} = 2\times10^{-5}\, i_2$$

$$\lambda_{12} = N_1\varphi_2 = 200\times10^{-5}\, i_2$$

$$M = \frac{\lambda_{12}}{i_2} = 2\ \text{mH}$$

E15.12 By the right-hand rule, clockwise flux is produced by i_1 and counterclockwise flux is produced by i_2. Thus the currents produce opposing fluxes.

If a dot is placed on the top terminal of coil 1, current entering the dot produces clockwise flux. Current must enter the bottom terminal of coil 2 to produce clockwise flux. Thus the corresponding dot should be on the bottom terminal of coil 2.

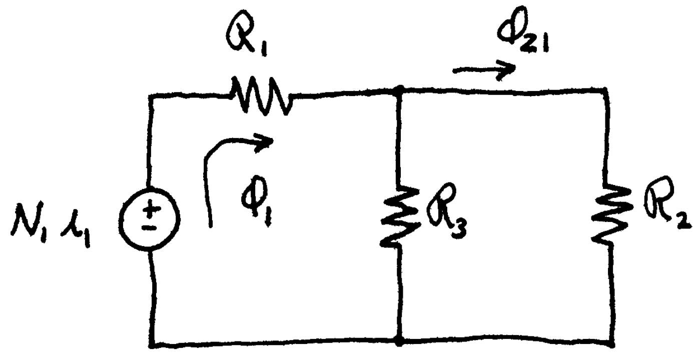
The voltages are given by Equations 15.36 and 15.37 in which we choose the minus signs because the currents produce opposing fluxes. Thus we have

$e_1 = L_1\frac{di_1}{dt} - M\frac{di_2}{dt}$ and $e_2 = -M\frac{di_1}{dt} + L_2\frac{di_2}{dt}$

E15.13 (a) Using the right-hand rule, we find that the fluxes produced by i_1 and i_2 aid in path 1, aid in path 2, and oppose in path 3.

If a dot is placed on the top terminal of coil 1, the corresponding dot should be on the top terminal of coil 2, because then currents entering the dotted terminals produce aiding flux linkages of the coils.

(b) For $i_2 = 0$, the magnetic circuit is:



Then the reluctance seen by the source is

$$R_{total} = R_1 + \frac{1}{1/R_2 + 1/R_3} = 1.5 \times 10^6$$

$$\varphi_1 = \frac{N_1 i_1}{R_{total}} \qquad \lambda_{11} = N_1 \varphi_1 = \frac{N_1^2 i_1}{R_{total}}$$

$$L_1 = \frac{\lambda_{11}}{i_1} = \frac{N_1^2}{R_{total}} = 6.667 \text{ mH}$$

The flux φ_1 splits equally between paths 2 and 3. Thus we have $\varphi_{21} = \varphi_1 / 2$. Then

$$\lambda_{21} = N_2 \varphi_{21} = \frac{N_1 N_2 i_1}{2R_{total}} \quad \text{and} \quad M = \frac{\lambda_{21}}{i_1} = \frac{N_1 N_2}{2R_{total}} = 10 \text{ mH}$$

Similarly, we find $L_2 = 60 \text{ mH}$.

(c) Because the currents produce aiding flux linkages, the mutual term carries a + sign.

E15.14 The energy lost per cycle is $W_{cycle} = (40 \text{ J/m}^3) \times (200 \times 10^{-6} \text{ m}^3) = 8 \text{ mJ}$, and the power loss is $P = W_{cycle} f = 8 \times 10^{-3} \times 60 = 0.48$ W.

E15.15 $H_{gap} = \dfrac{NI}{\ell_{gap}} = \dfrac{1000}{0.5\times 10^{-2}} = 200\times 10^{3}$ A/m

$B_{gap} = \mu_0 H_{gap} = 0.2513$ T

$W = W_v \times \text{Volume} = \dfrac{B_{gap}^2}{2\mu_0}(2\times 10^{-2}\times 3\times 10^{-2}\times 0.5\times 10^{-2}) = 0.0754$ J

E15.16 Refer to Figure 15.26c in the book.

$$\mathbf{I}_2 = \frac{V_s'}{R_s' + Z_L} = \frac{100\angle 0^\circ}{10+10+j20} = 3.536\angle -45^\circ$$

$$\mathbf{V}_2 = Z_L \mathbf{I}_2 = (10 + j20)\mathbf{I}_2 = 79.06\angle 18.43^\circ \text{ V}$$

$$P_L = I_{2\text{rms}}^2 R_L = \left(\frac{3.536}{\sqrt{2}}\right)^2 (10) = 62.51 \text{ W}$$

E15.17 $R_L' = \left(\dfrac{N_1}{N_2}\right)^2 R_L = \left(\dfrac{1}{4}\right)^2 400 = 25\ \Omega$

$$\mathbf{I}_1 = \frac{100\angle 0^\circ}{R_s + R_L'} = 1.538\angle 0^\circ$$

$$\mathbf{I}_2 = \left(\frac{N_1}{N_2}\right)\mathbf{I}_1 = 0.3846\angle 0^\circ$$

$$\mathbf{V}_2 = R_L \mathbf{I}_2 = 153.8\angle 0^\circ$$

$$P_L = R_L' I_{1\text{rms}}^2 = R_L I_{2\text{rms}}^2 = 29.60 \text{ W}$$

E15.18 For maximum power transfer, we need

$$R_s = R_L'$$

However we have

$$R_L' = \left(\frac{N_1}{N_2}\right)^2 R_L = \left(\frac{N_1}{N_2}\right)^2 400$$

Thus we have

$$R_s = 40 = \left(\frac{N_1}{N_2}\right)^2 400$$

Solving we find

$$\frac{N_1}{N_2} = \frac{1}{\sqrt{10}}$$

Answers for Selected Problems

P15.5* r = 66.67 cm

P15.6* v_{ab} is negative

P15.11* $B = 0.6$ T

P15.15* $\phi = 0.0314$ Wb

$\lambda = 0.157$ Wb turns

$e = 157$ V

P15.16* $\mu_r = 1592$

P15.24* Magnetomotive force $F = Ni$ in a magnetic circuit is analogous to a voltage source in an electrical circuit. Reluctance R is analogous to electrical resistance. Magnetic flux φ is analogous to electrical current.

P15.25* $\ell_{core} = 500$ cm

P15.28* $\phi_b = 105.6\ \mu$Wb

P15.34* $\phi = \dfrac{NI}{\dfrac{\ell_g}{\mu_0 \pi (d + \ell_g) L} + \dfrac{x}{\mu_0 \pi (d/2)^2}}$

P15.37* 800 mH

P15.38* $L = 0.3183$ H

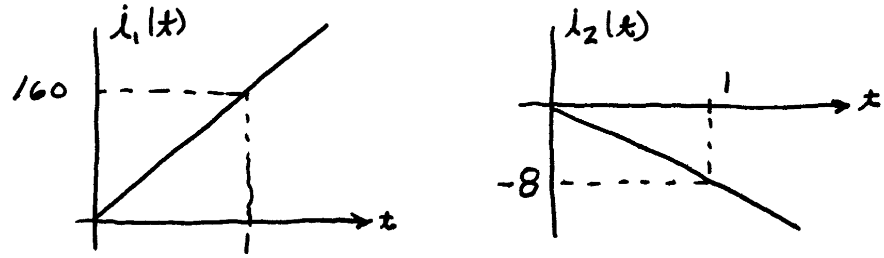
$R = 78.54 \times 10^4$

$\mu_r = 405.3$

P15.45* $e_1 = 471.3 \sin(377t)$

$e_2 = 565.5 \sin(377t)$

P15.48*



P15.52* Two causes of core loss are hysteresis and eddy currents. To minimize loss due to hysteresis, we should select a material having a thin hysteresis loop (as shown in Figure 15.21 in the book). To minimize loss due to eddy currents, we laminate the core. The laminations are insulated from one another so eddy currents cannot flow between them.

If the frequency of operation is doubled, the power loss due to hysteresis doubles and the power loss due to eddy currents is quadrupled.

P15.53* $P = 28.88$ W

P15.59* If residential power was distributed at 12 V (rather than 120 V) higher currents (by an order of magnitude) would be required to deliver the same amounts of power. This would require much larger wire sizes to avoid excessive power loss in the resistances of the conductors.

On the other hand, if residential power was distributed at 12 kV, greater safety hazards would result.

P15.62* If we tried to make the 25 Ω load look like 100 Ω by adding 75 Ω in series, 75% of the power delivered by the source would be dissipated in the 75-Ω resistance. On the other hand, when using the transformer, virtually all of the power taken from the source is delivered to the load. Thus, from the standpoint of efficiency, the transformer is a much better choice.

P15.67* (a) The dots should be placed on the top end of coil 2 and on the right-hand end of coil 3.

(b) $\mathbf{V}_2 = 50\angle 0^\circ$ $\quad\quad \mathbf{I}_2 = 10\angle 0^\circ$

$\mathbf{V}_3 = 100\angle 0^\circ$ $\quad\quad \mathbf{I}_3 = 10\angle 0^\circ$

(c) $N_1\mathbf{I}_1 - N_2\mathbf{I}_2 - N_3\mathbf{I}_3 = 0$

$\mathbf{I}_1 = 15\angle 0^\circ$

P15.70* The equivalent circuit of a real transformer is shown in Figure 15.28 in the book. The resistances R_1 and R_2 account for the resistance of the wires used to wind the coils of the transformer. L_1 and L_2 account for flux produced by each coil that does not link the other coil. L_m accounts for the current needed to set up the mutual flux in the core. Finally, R_c accounts for core losses due to eddy currents and hysteresis.

P15.73* $\text{Efficiency} = 83.06\%$

$\text{Percent regulation} = 0.625\%$

P15.77* The voltage across a transformer coil is approximately equal to

$$N\frac{d\varphi}{dt}$$

in which N is the number of turns and $\varphi = BA$ is the flux in the core. If B is reduced in magnitude, either N or the core area A would need to be increased to maintain the same voltage rating. In either case, more material (i.e., iron for the core or copper for the windings) is needed for the transformer.

On the other hand if the peak value of B is much higher than the saturation point, much more magnetizing current is required resulting in higher losses.

Practice Test

T15.1 (a) We have $f = i\ell B\sin(\theta) = 12(0.2)0.3\sin(90^\circ) = 0.72\,\text{N}$. ($\theta$ is the angle between the field and the wire.) The direction of the force is that of $i\mathbf{l} \times \mathbf{B}$ in which the direction of the vector $\mathbf{l}$ is the positive direction of the current (given as the positive x direction). Thus, the force is directed in the negative y direction.

(b) The current and the field are in the same direction so $\theta = 0$ and the force is zero. Direction does not apply for a vector of zero magnitude.

T15.2 The flux linking the coil is

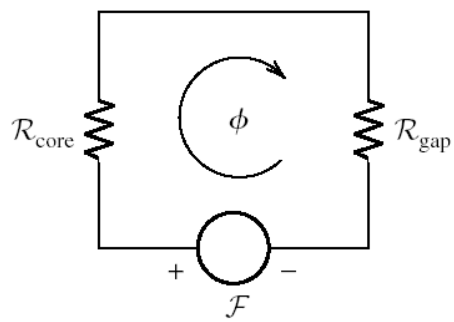
$$\phi = BA = 0.7[\sin(120\pi t)](0.25)^2 = 43.75\times10^{-3}\sin(120\pi t)\ \text{Wb}$$

The induced voltage is

$$v = \frac{d\lambda}{dt} = N\frac{d\phi}{dt} = 10\times43.75\times10^{-3}\times120\pi\cos(120\pi t) = 164.9\cos(120\pi t)\ \text{V}$$

T15.3 $e = Blu = 0.4\times0.2\times15 = 1.2\ \text{V}$

T15.4 (a) The magnetic circuit is:



The permeability of the core is:

$$\mu_{core} = \mu_r\mu_0 = 1500\times4\pi\times10^{-7} = 1.885\times10^{-3}$$

The reluctance of the core is given by

$$R_{core} = \frac{\ell_{core}}{\mu_{core}A_{core}} = \frac{33.7\times10^{-2}}{1.885\times10^{-3}\times2\times10^{-2}\times3\times10^{-2}} = 298.0\times10^3$$

To account for fringing, we add the gap length to the width and depth of the gap.

$$R_{gap} = \frac{0.3\times10^{-2}}{4\pi\times10^{-7}\times2.3\times10^{-2}\times3.3\times10^{-2}} = 3.145\times10^6$$

The equivalent reluctance seen by the source is:

$$R_{eq} = R_{core} + R_{gap} = 3.443\times10^6$$

The flux is :

$$\phi = \frac{F}{R_{eq}} = \frac{4\times350}{3.443\times10^6} = 406.6\times10^{-6}\ \text{Wb}$$

Finally, the flux density in the gap is approximately

$$B_{gap} = \frac{\phi}{A_{gap}} = \frac{406.6\times10^{-6}}{2.3\times10^{-2}\times3.3\times10^{-2}} = 0.5357\ \text{T}$$

(b) The inductance is

$$L = \frac{N^2}{R_{eq}} = \frac{350^2}{3.443 \times 10^6} = 35.58\,\text{mH}$$

T15.5 The two mechanisms by which power is converted to heat in an iron core are hysteresis and eddy currents. To minimize loss due to hysteresis, we choose a material for which the plot of B versus H displays a thin hysteresis loop. To minimize loss due to eddy currents, we make the core from laminated sheets or from powered iron held together by an insulating binder. Hysteresis loss is proportional to frequency and eddy-current loss is proportional to the square of frequency.

T15.6 (a) With the switch open, we have $I_{2\text{rms}} = 0$, $I_{1\text{rms}} = 0$ and the voltage across R_s is zero. Therefore, we have $V_{1\text{rms}} = 120$ V and $V_{2\text{rms}} = (N_2/N_1)V_{1\text{rms}} = 1200$ V. (The dots affect the phases of the voltages but not their rms values. Thus, $V_{2\text{rms}} = -1200$ V would not be considered to be correct.)

(b) With the switch closed, the impedance seen looking into the primary is $R_L' = (N_1/N_2)^2 R_L = 10\,\Omega$. Then, using the voltage division principle, we have $V_{1rms} = 120\dfrac{R_L'}{R_s + R_L'} = 114.3$ V. Next, $V_{2\text{rms}} = (N_2/N_1)V_{1\text{rms}} = 1143$ V.

The primary current is $I_{1rms} = 120/(10.5) = 11.43$ A. The secondary current is $I_{2\text{rms}} = (N_1/N_2)I_{1\text{rms}} = 1.143$ A.

T15.7 Core loss is nearly independent of load, while loss in the coil resistances is nearly proportional to the square of the rms load current. Thus, for a transformer that is lightly loaded most of the time, core loss is more significant. Transformer B would be better from the standpoint of total energy loss and operating costs.

CHAPTER 16

Exercises

E16.1 The input power to the dc motor is

$$P_{in} = V_{source} I_{source} = P_{out} + P_{loss}$$

Substituting values and solving for the source current we have

$$220 I_{source} = 50 \times 746 + 3350$$

$$I_{source} = 184.8 \text{ A}$$

Also we have

$$\eta = \frac{P_{out}}{P_{in}} \times 100\% = \frac{50 \times 746}{50 \times 746 + 3350} = 91.76\%$$

$$\text{speed regulation} = \frac{n_{no-load} - n_{full-load}}{n_{full-load}} \times 100\%$$

$$= \frac{1200 - 1150}{1150} \times 100\% = 4.35\%$$

E16.2 (a) The synchronous motor has zero starting torque and would not be able to start a high-inertia load.

(b) The series-field dc motor shows the greatest amount of speed variation with load in the normal operating range and thus has the poorest speed regulation.

(c) The synchronous motor operates at fixed speed and has zero speed regulation.

(d) The ac induction motor has the best combination of high starting torque and low speed regulation.

(e) The series-field dc motor should not be operated without a load because its speed becomes excessive.

E16.3 Repeating the calculations of Example 16.2, we have

(a) $$i_A(0+) = \frac{V_T}{R_A} = \frac{2}{0.05} = 40 \text{ A}$$

$$f(0+) = Bl i_A(0+) = 2(0.3)40 = 24 \text{ N}$$

$$u = \frac{V_T}{Bl} = \frac{2}{2(0.3)} = 3.333 \text{ m/s}$$

(b) $$i_A = \frac{f_{load}}{Bl} = \frac{4}{2(0.3)} = 6.667 \text{ A}$$

$$e_A = V_T - R_A I_A = 2 - 0.05(6.667) = 1.667 \text{ V}$$
$$u = \frac{e_A}{Bl} = \frac{1.667}{2(0.3)} = 2.778 \text{ m/s}$$
$$p_m = f_{load} u = 4(2.778) = 11.11 \text{ W}$$
$$p_R = i_A^2 R = 2.222 \text{ W}$$
$$p_t = V_T i_A = 2(6.667) = 13.33 \text{ W}$$
$$\eta = \frac{p_m}{p_t} \times 100\% = \frac{11.11}{13.33} = 83.33\%$$

(c) $$i_A = \frac{f_{pull}}{Bl} = \frac{2}{2(0.3)} = 3.333 \text{ A}$$
$$e_A = V_T + R_A I_A = 2 + 0.05(3.333) = 2.167 \text{ V}$$
$$u = \frac{e_A}{Bl} = \frac{2.167}{2(0.3)} = 3.611 \text{ m/s}$$
$$p_m = f_{pull} u = 2(3.611) = 7.222 \text{ W}$$
$$p_t = V_T i_A = 2(3.333) = 6.667 \text{ W}$$
$$p_R = i_A^2 R = 0.5555 \text{ W}$$
$$\eta = \frac{p_t}{p_m} \times 100\% = \frac{6.667}{7.222} = 92.31\%$$

E16.4 Referring to Figure 16.15 we see that $E_A \cong 125$ V for $I_F = 2$ A and $n = 1200$. Then for $n = 1500$, we have
$$E_A = 125 \times \frac{1500}{1200} = 156 \text{ V}$$

E16.5 Referring to Figure 16.15 we see that $E_A \cong 145$ V for $I_F = 2.5$ A and $n = 1200$. Then for $n = 1500$, we have
$$E_A = 145 \times \frac{1500}{1200} = 181.3 \text{ V}$$
$$\omega_m = n \times \frac{2\pi}{60} = 157.1 \text{ rad/s}$$
$$T_{dev} = \frac{P_{dev}}{\omega_m} = \frac{10 \times 746}{157.1} = 47.49 \text{ Nm}$$
$$I_A = \frac{P_{dev}}{E_A} = \frac{10 \times 746}{181.3} = 41.15 \text{ A}$$
$$V_T = E_A + R_A I_A = 181.3 + 0.3(41.15) = 193.6 \text{ V}$$

E16.6 $$R_{adj} = \frac{V_T - R_F I_F}{I_F} = \frac{300 - 10 \times 10}{10} = 20\ \Omega$$

Because I_F remains constant the value of $K\varphi$ is the same value as in Example 16.4, which is 2.228. Furthermore the loss torque also remains constant at 11.54 Nm, and the developed torque remains at 261.5 Nm. Thus the armature current is still 117.4 A. Then we have

$$E_A = V_T - R_A I_A = 300 - 0.065(117.4) = 292.4\ \text{V}$$

$$\omega_m = \frac{E_A}{K\varphi} = \frac{292.4}{2.228} = 131.2\ \text{rad/s}$$

$$n_m = \omega_m \frac{60}{2\pi} = 1253\ \text{rpm}$$

Thus the motor speeds up when V_T is increased.

E16.7 Following Example 16.4, we have

$$I_F = \frac{V_T}{R_F + R_{adj}} = \frac{240}{10 + 30} = 6\ A$$

Referring to Figure 16.18 we see that $E_A \cong 200$ V for $I_F = 6$ A and $n = 1200$. Thus we have

$$K\varphi = \frac{E_A}{\omega_m} = \frac{200}{1200(2\pi / 60)} = 1.592$$

$$I_A = \frac{T_{dev}}{K\varphi} = \frac{261.5}{1.592} = 164.3\ A$$

$$E_A = V_T - R_A I_A = 240 - 0.065(164.3) = 229.3\ \text{V}$$

$$\omega_m = \frac{E_A}{K\varphi} = \frac{229.3}{1.592} = 144.0\ \text{rad/s}$$

$$n_m = \omega_m \frac{60}{2\pi} = 1376\ \text{rpm}$$

$$P_{out} = T_{out}\omega_m = 36\ \text{kW}$$

$$P_{in} = V_T(I_F + I_A) = 240(6 + 164.3) = 40.87\ \text{kW}$$

$$\eta = \frac{P_{out}}{P_{in}} \times 100\% = 88.08\%$$

E16.8 $$\omega_{m3} = \omega_{m1}\sqrt{\frac{T_{dev1}}{T_{dev3}}} = 125.7\sqrt{\frac{12}{6}} = 177.8\ \text{rad/s}$$

$$n_{m3} = \omega_{m3} \frac{60}{2\pi} = 1698\ \text{rpm}$$

$$P_{out3} = T_{out3}\omega_{m3} = 1066\ \text{W}$$

E16.9 With $R_A = 0$ and fixed V_T, the shunt motor has constant speed independent of the load torque. Thus we have

$$n_{m2} = n_{m1} = 1200 \text{ rpm}$$
$$\omega_{m2} = \omega_{m1} = 125.7 \text{ rad/s}$$
$$P_{out1} = T_{out1}\omega_{m1} = 1508 \text{ W}$$
$$P_{out2} = T_{out2}\omega_{m2} = 3016 \text{ W}$$

E16.10 Decreasing V_T decreases the field current and therefore the flux φ. In the linear portion of the magnetization curve, flux is proportional to the field current. Thus reduction of V_T leads to reduction of φ and according to Equation 16.35, the speed remains constant. (Actually, some speed variation will occur due to saturation effects.)

E16.11 The torque--speed relationship for the separately excited machine is given by Equation 16.27

$$T_{dev} = \frac{K\varphi}{R_A}(V_T - K\varphi\omega_m)$$

which plots as a straight line in the T_{dev} - ω_m plane. A family of plots for various values of V_T is shown in Figure 16.27 in the book.

E16.12 The torque--speed relationship for the separately excited machine is given by Equation 16.27

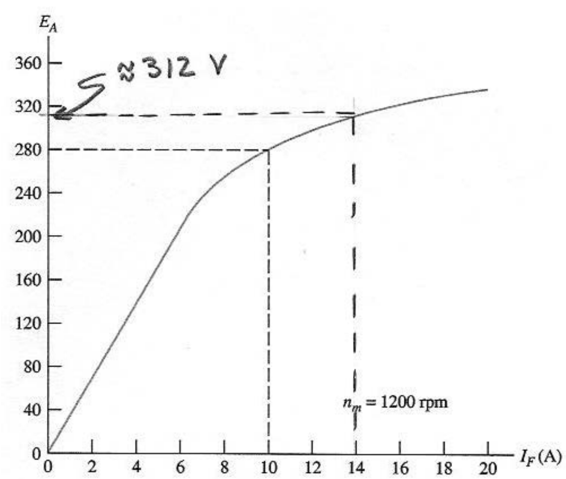
$$T_{dev} = \frac{K\varphi}{R_A}(V_T - K\varphi\omega_m)$$

which plots as a straight line in the T_{dev} - ω_m plane. As the field current is increased, the flux φ increases. A family of plots for various values of I_F and φ is shown in Figure 16.28 in the book.

E16.13

$$I_F = \frac{V_F}{R_{\text{adj}} + R_F} = \frac{140}{0+10} = 14 \text{ A}$$

$$V_{NL} = E_A = 312\frac{1000}{1200} = 260 \text{ V}$$



$$V_{FL} = E_A - R_A I_A = 260 - 200 \times 0.065 = 247 \text{ V}$$

$$\text{voltage regulation} = \frac{V_{NL} - V_{FL}}{V_{FL}} \times 100\% = \frac{260 - 247}{247} \times 100\% = 5.26\%$$

$$P_{out} = I_L V_{FL} = 200 \times 247 = 49.4 \text{ kW}$$

$$P_{dev} = P_{out} + R_A I_A^2 = 49400 + 0.065(200)^2 = 52.0 \text{ kW}$$

$$\omega_m = n_m \frac{2\pi}{60} = 104.7 \text{ rad/sec} \qquad P_{in} = \frac{P_{out}}{0.85} = \frac{49.4}{0.85} = 58.1 \text{ kW}$$

$$P_{losses} = P_{in} - P_{dev} = 58.1 - 52.0 = 6.1 \text{ kW}$$

$$T_{in} = \frac{P_{in}}{\omega_m} = \frac{58100}{104.7} = 555 \text{ nm} \qquad T_{dev} = \frac{P_{dev}}{\omega_m} = \frac{52000}{104.7} = 497 \text{ nm}$$

Answers for Selected Problems

P16.5* Two disadvantages of dc motors compared to signal-phase ac induction motors for a ventilation fan, which we can expect to operate most of the time, are first that dc power is usually not readily available in a home and second that dc machines tend to require more frequent maintenance than ac induction motors.

P16.8* $\text{speed regulation} = 2.27\%$

P16.11* $T_{start} = 50.9 \text{ Nm}$

P16.17* $P_{out} = 2.42 \text{ hp}$

$P_{loss} = 267 \text{ W}$

$\eta = 87.1\%$

P16.20* (a) If V_T is doubled, the steady-state no-load speed is doubled.

(b) If the resistance is doubled, the steady-state no-load speed is not changed. (However, it will take longer for the motor to achieve this speed.)

(c) If B is doubled, the steady-state no-load speed is halved.

P16.23* $f_{\text{starting}} = 48.75\text{ N}$

$u = 5.13\text{ m/s}$

P16.27* Using the right-hand rule we see that in Figure 16.10, the north pole of the rotor is at the top of the rotor. Because the north rotor pole is attracted to the south stator pole, the torque is counterclockwise, as indicated in the figure.

In Figure 16.11, the north rotor poles are in the upper right-hand and lower left-hand portions of the rotor. South poles appear in the upper left-hand and lower right-hand parts of the rotor. Because the north rotor poles are attracted to the south stator poles, the torque is counterclockwise, as indicated in the figure.

P16.30* $T_{\text{dev}} = 19.10\text{ Nm}$ $\quad P_{\text{dev}} = 2400\text{ W}$ $\quad V_T = 253\text{ V}$

P16.33* $N \cong 64$

P16.36* $n_2 = 1600\text{ rpm}$

P16.39* (a) $P_{\text{dev}} = 44.26\text{ kW} = 59.33\text{ hp}$

$P_{R_A} = 1.061\text{ kW}$

$P_{\text{rot}} = 6.96\text{ kW} = 9.330\text{ hp}$

(b) $n_m = 1530\text{ rpm}$

P16.42* (a) The field current is

$$I_F = \frac{V_T}{R_F + R_{\text{adj}}} = \frac{240}{240} = 1.0\text{ A}$$

From the magnetization curve shown in Figure P16.35, we find that $E_A = 165\text{ V}$ with $I_F = 1.0\text{ A}$ and $n_m = 1000\text{ rpm}$. Neglecting losses at no load, we have $I_A = 0$ and $E_A = V_T = 240\text{ V}$. Since E_A is proportional to speed, the no-load speed is:

$$n_{\text{no-load}} = \frac{240}{165} \times 1000\text{ rpm} = 1455\text{ rpm}$$

(b) $I_A = 9.6\text{ A}$

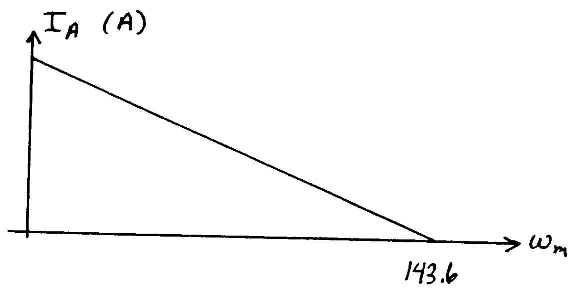
$T_{\text{load}} = 15.13\text{ Nm}$

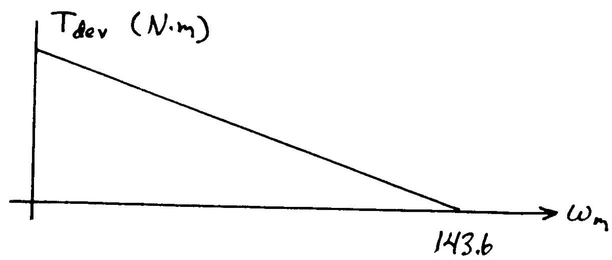
$P_F = 240$ W

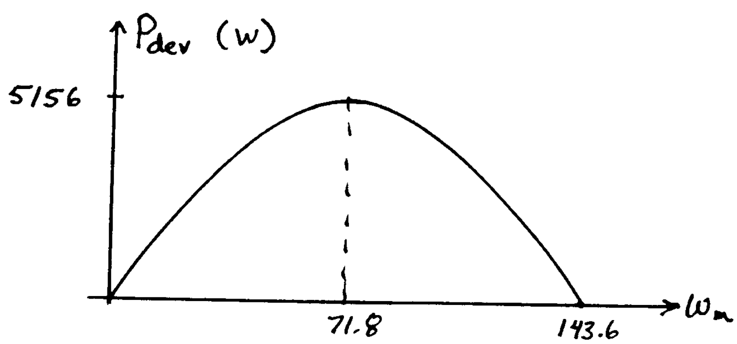
$P_{R_A} = 138.2$ W

P16.45* (a) $n_{m,\text{no-load}} = 1369$ rpm

(b)







P16.48* $\omega_{m1} = 174.3$ and $I_{A1} = 21.4$ A for which $\eta = 87.2\%$

$\omega_{m2} = 25.67$ and $I_{A2} = 141$ A for which $\eta = 13.5\%$

The first solution is more likely to fall within the rating because the efficiency for the second solution is very low.

P16.51* The magnetization curve is a plot of E_A versus the field current I_F at a stated speed. Because a permanent magnet motor does not have field current, the concept of a magnetization curve does not apply to it.

P16.54* $P_{\text{out}} = 30.87 \text{ W}$

$\eta = 74.83\%$

P16.57* $n_m = 1910 \text{ rpm}$

P16.65* See Figures 16.26, 16.27 and 16.28 in the book.

P16.68* $V_T = 33.33 \text{ V}$

$$\frac{T_{\text{on}}}{T} = 0.667$$

P16.71* $R_{added} = 0.379 \ \Omega$

P16.77* $\text{voltage regulation} = 6.667\%$ $R_L = 7.5\,\Omega$ $R_A = 0.5\,\Omega$

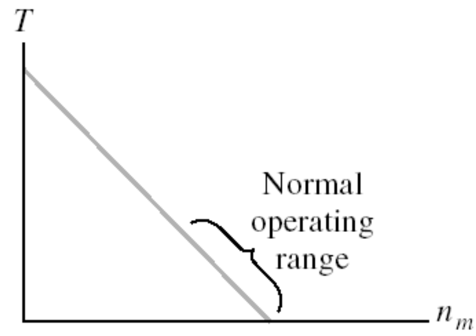
$T_{dev} = 19.10 \text{ Nm}$

P16.78* $I_L = 15 \ A$ $V_L = 112.5 \text{ V}$ $P_{\text{dev}} = 1800 \text{ W}$

Practice Test

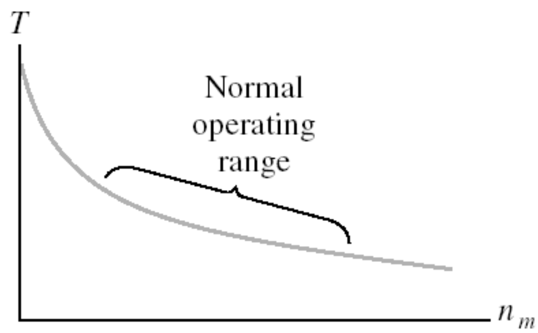
T16.1 The windings are the field winding, which is on the stator, and the armature winding, which is on the rotor. The armature current varies with mechanical load.

T16.2 See Figure 16.5(c) in the book. The speed becomes very high, and the machine can be destroyed.



(c) Shunt-connected or permanent-magnet dc motor

T16.3 See Figure 16.5(d) in the book.



(d) Series-connected dc motor or universal motor

T16.4 $$\text{speed regulation} = \frac{n_{\text{no-load}} - n_{\text{full-load}}}{n_{\text{full-load}}} \times 100\%$$

T16.5 To obtain the magnetization curve, we drive the machine at constant speed and plot the open-circuit armature voltage E_A versus field current I_F.

T16.6 Power losses in a shunt-connected dc motor are 1. Field loss, which is the power consumed in the resistances of the field circuit. 2. Armature loss, which is the power converted to heat in the armature resistance. 3. Rotational losses, which include friction, windage, eddy-current loss, and hysteresis loss.

T16.7 A universal motor is an ac motor that similar in construction to a series-connected dc motor. In principle, it can be operated from either ac or dc sources. The stator of a universal motor is usually laminated to reduce eddy-current loss. Compared to other single-phase ac motors, the universal motor has a higher power to weight ratio, produces a larger starting torque without excessive current, slows down under heavy loads so the power is more nearly constant, and can be designed to operate at higher speeds. A disadvantage of the universal motor is that it contains brushes and a commutator resulting in shorter service life.

T16.8 1. Vary the voltage supplied to the armature circuit while holding the field constant.
2. Vary the field current while holding the armature supply voltage constant.
3. Insert resistance in series with the armature circuit.

T16.9 Equation 16.15 states

$$E_A = K\varphi\omega_m$$

With constant field current, the magnetic flux φ is constant. Therefore, the back emf E_A is proportional to machine speed ω_m (or equivalently to n_m). Thus, we have

n_m(rpm)	E_A(V)
500	80
1500	240
2000	320

T16.10 Converting the speeds from rpm to radians/s, we have:

$$\omega_{m1} = n_{m1} \times \frac{2\pi}{60} = 1200 \times \frac{2\pi}{60} = 40\pi$$

$$\omega_{m2} = n_{m2} \times \frac{2\pi}{60} = 900 \times \frac{2\pi}{60} = 30\pi$$

Next, we can find the machine constant:

$$K\phi = \frac{E_A}{\omega_{m1}} = \frac{120}{40\pi} = \frac{3}{\pi} = 0.9549$$

The developed torque is:

$$T_{\text{dev}} = \frac{P_{\text{dev}}}{\omega_{m2}} = \frac{4 \times 746}{30\pi} = 31.66 \text{ Nm}$$

Finally, the armature current is:

$$I_A = \frac{T_{\text{dev}}}{K\phi} = \frac{31.66}{0.9549} = 33.16 \text{ A}$$

T16.11 (a) $E_A = V_T - R_A I_A = 230\text{ V}$

$$K\varphi = \frac{E_A}{\omega_m} = \frac{230}{1200(2\pi/60)} = 1.830$$

$$P_{\text{in}} = V_T I_A = 4800\text{ W}$$

$$P_{\text{out}} = 6 \times 746 = 4476\text{ W}$$

$$P_{R_A} = R_A I_A^2 = 200\text{ W}$$

$$P_{\text{rot}} = P_{\text{in}} - P_{\text{out}} - P_{R_A} = 124\text{ W}$$

$$P_{\text{dev}} = P_{\text{out}} + P_{\text{rot}} = 4600\text{ W}$$

$$T_{\text{dev}} = \frac{P_{\text{dev}}}{\omega_{\text{m}}} = \frac{4600}{1200 \times \frac{2\pi}{60}} = 36.60\text{ Nm}$$

(b) $$T_{\text{rot}} = \frac{P_{\text{rot}}}{\omega_m} = \frac{124}{1200 \times \frac{2\pi}{60}} = 0.9868\text{ Nm}$$

$$I_{A,\text{ no-load}} = \frac{T_{\text{rot}}}{K\phi} = 0.5392\text{ A}$$

$$E_{A,\text{ no-load}} = V_T - R_A I_A = 239.73 \quad \text{V}$$

$$\omega_{m,\text{ no-load}} = \frac{E_{A,\text{ no-load}}}{K\phi} = 131.0 \text{ rad/s}$$

$$n_{m,\text{ no-load}} = 1251 \quad \text{rpm}$$

$$\text{speed regulation} = \frac{n_{\text{no-load}} - n_{\text{full-load}}}{n_{\text{full-load}}} \times 100\% = 4.25\%$$

T16.12 For $I_A = 20\text{ A}$, we have:

$$\omega_{\text{m}} = n_m \times \frac{2\pi}{60} = 1000 \times \frac{2\pi}{60} = 104.7 \text{ radian/s}$$

$$E_A = V_T - (R_F + R_A)I_A = 226\text{ V}$$

Rearranging Equation 16.30 and substituting values, we have:

$$KK_F = \frac{E_A}{I_A \omega_m} = \frac{226}{20 \times 104.7} = 0.1079$$

For $I_A = 10\text{ A}$, we have:

$$E_A = V_T - (R_F + R_A)I_A = 233\text{ V}$$

$$\omega_{\text{m}} = \frac{E_A}{KK_F I_A} = \frac{233}{0.1079 \times 10} = 215.9 \text{ radian/s}$$

$$n_m = 2062\text{ rpm}$$

CHAPTER 17

Exercises

E17.1 From Equation 17.5, we have

$$B_{gap} = Ki_a(t)\cos(\theta) + Ki_b(t)\cos(\theta - 120^\circ) + Ki_c(t)\cos(\theta - 240^\circ)$$

Using the expressions given in the Exercise statement for the currents, we have

$$B_{gap} = KI_m\cos(\omega t)\cos(\theta) + KI_m\cos(\omega t - 240^\circ)\cos(\theta - 120^\circ)$$
$$+ KI_m\cos(\omega t - 120^\circ)\cos(\theta - 240^\circ)$$

Then using the identity for the products of cosines, we obtain

$$B_{gap} = \tfrac{1}{2}KI_m[\cos(\omega t - \theta) + \cos(\omega t + \theta) + \cos(\omega t - \theta - 120^\circ)$$
$$+ \cos(\omega t + \theta - 360^\circ) + \cos(\omega t - \theta + 120^\circ)$$
$$+ \cos(\omega t + \theta - 360^\circ)]$$

However we can write

$$\cos(\omega t - \theta) + \cos(\omega t - \theta - 120^\circ) + \cos(\omega t - \theta + 120^\circ) = 0$$

$$\cos(\omega t + \theta - 360^\circ) = \cos(\omega t + \theta)$$

$$\cos(\omega t + \theta - 360^\circ) = \cos(\omega t + \theta)$$

Thus we have

$$B_{gap} = \tfrac{3}{2}KI_m\cos(\omega t + \theta)$$

which can be recognized as flux pattern that rotates clockwise.

E17.2 At 60 Hz, synchronous speed for a four-pole machine is:

$$n_s = \frac{120f}{P} = \frac{120(60)}{4} = 1800\text{ rpm}$$

The slip is given by:

$$s = \frac{n_s - n_m}{n_s} = \frac{1800 - 1750}{1800} = 2.778\%$$

The frequency of the rotor currents is the slip frequency. From Equation

17.17, we have $\omega_{\text{slip}} = s\omega$. For frequencies in the Hz, this becomes:

$$f_{\text{slip}} = sf = 0.02778 \times 60 = 1.667 \text{ Hz}$$

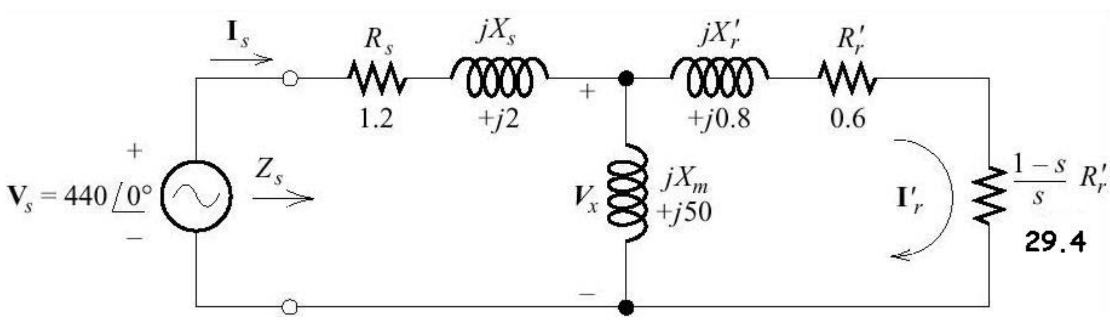
In the normal range of operation, slip is approximately proportional to output power and torque. Thus at half power, we estimate that $s = 2.778/2 = 1.389\%$. This corresponds to a speed of 1775 rpm.

E17.3 Following the solution to Example 17.1, we have:

$$n_s = 1800 \text{ rpm}$$

$$s = \frac{n_s - n_m}{n_s} = \frac{1800 - 1764}{1800} = 0.02$$

The per phase equivalent circuit is:



$$Z_s = 1.2 + j2 + \frac{j50(0.6 + 29.4 + j0.8)}{j50 + 0.6 + 29.4 + j0.8}$$

$$= 22.75 + j15.51$$

$$= 27.53\angle 34.29^\circ$$

$$\text{power factor} = \cos(34.29^\circ) = 82.62\% \text{ lagging}$$

$$\mathbf{I}_s = \frac{\mathbf{V}_s}{Z_s} = \frac{440\angle 0^\circ}{27.53\angle 34.29^\circ} = 15.98\angle -34.29^\circ \text{ A rms}$$

For a delta-connected machine, the magnitude of the line current is

$$I_{line} = I_s\sqrt{3} = 15.98\sqrt{3} = 27.68 \text{ A rms}$$

and the input power is

$$P_{\text{in}} = 3I_sV_s\cos\theta = 17.43 \text{ kW}$$

Next, we compute $\mathbf{V}_x$ and $\mathbf{I}_r'$.

$$\begin{aligned}\mathbf{V}_x &= \mathbf{I}_s \frac{j50(0.6+29.4+j0.8)}{j50+0.6+29.4+j0.8} \\ &= 406.2 - j15.6 \\ &= 406.4\angle -2.2^\circ \quad \text{V rms}\end{aligned}$$

$$\begin{aligned}\mathbf{I}_r' &= \frac{\mathbf{V}_x}{j0.8+0.6+29.4} \\ &= 13.54\angle -3.727^\circ \quad \text{A rms}\end{aligned}$$

The copper losses in the stator and rotor are:

$$\begin{aligned}P_s &= 3R_s I_s^2 \\ &= 3(1.2)(15.98)^2 \\ &= 919.3\ \text{W}\end{aligned}$$

and

$$\begin{aligned}P_r &= 3R_r'(I_r')^2 \\ &= 3(0.6)(13.54)^2 \\ &= 330.0\ \text{W}\end{aligned}$$

Finally, the developed power is:

$$\begin{aligned}P_{\text{dev}} &= 3 \times \frac{1-s}{s} R_r'(I_r')^2 \\ &= 3(29.4)(13.54)^2 \\ &= 16.17\ \text{kW}\end{aligned}$$

$$P_{\text{out}} = P_{\text{dev}} - P_{\text{rot}} = 15.27\ \text{kW}$$

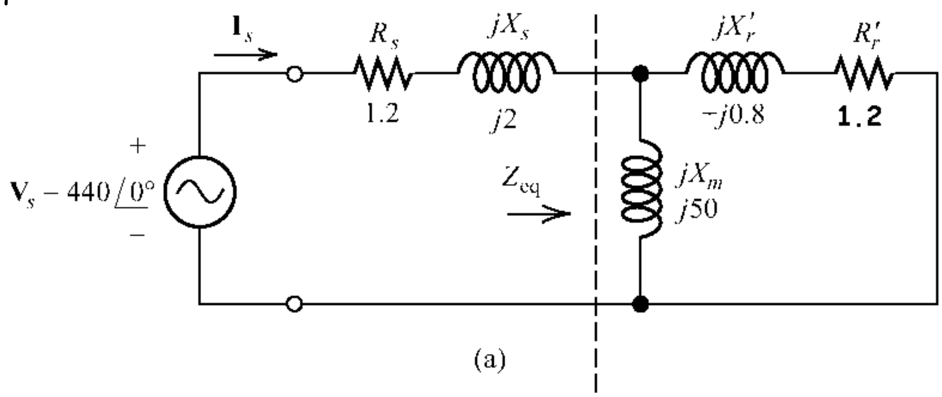
The output torque is:

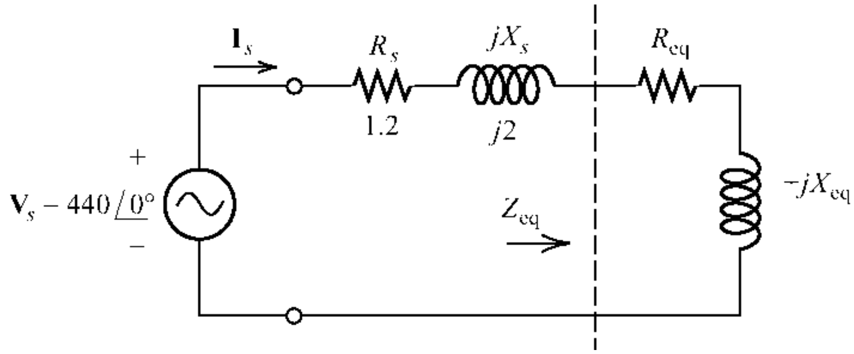
$$T_{\text{out}} = \frac{P_{\text{out}}}{\omega_m} = 82.66\ \text{newton meters}$$

The efficiency is:

$$\eta = \frac{P_{\text{out}}}{P_{\text{in}}} \times 100\% = 87.61\%$$

E17.4 The equivalent circuit is:





$$Z_{eq} = R_{eq} + jX_{eq} = \frac{j50(1.2 + j0.8)}{j50 + 1.2 + j0.8} = 1.162 + j0.8148$$

The impedance seen by the source is:

$$\begin{aligned} Z_s &= 1.2 + j2 + Z_{eq} \\ &= 1.2 + j2 + 1.162 + j0.8148 \\ &= 3.675\angle 50.00^\circ \end{aligned}$$

Thus, the starting phase current is

$$\mathbf{I}_{s,\text{ starting}} = \frac{\mathbf{V}_s}{Z_s} = \frac{440\angle 0^\circ}{3.675\angle 50.00^\circ}$$

$$\mathbf{I}_{s,\text{ starting}} = 119.7\angle -50.00^\circ \text{ A rms}$$

and for a delta connection, the line current is

$$I_{line,\text{starting}} = I_{s,\text{starting}}\sqrt{3} = 119.7\sqrt{3} = 207.3 \text{ A rms}$$

The power crossing the air gap is (three times) the power delivered to the right of the dashed line in the equivalent circuit shown earlier.

$$P_{ag} = 3R_{eq}\left(I_{s,\text{ starting}}\right)^2 = 49.95 \text{ kW}$$

Finally, the starting torque is found using Equation 17.34.

$$T_{\text{dev, starting}} = \frac{P_{ag}}{\omega_s}$$
$$= \frac{49950}{2\pi 60/2}$$
$$= 265.0 \text{ newton meters}$$

E17.5 This exercise is similar to part (c) of Example 17.4. Thus, we have

$$\frac{\sin\delta_3}{\sin\delta_1} = \frac{P_3}{P_1}$$
$$\frac{\sin\delta_3}{\sin 4.168^\circ} = \frac{200}{50}$$

which yields the new torque angle $\delta_3 = 16.90^\circ$. E_r remains constant in magnitude, thus we have

$$\mathbf{E}_{r3} = 498.9\angle -16.90^\circ \quad \text{V rms}$$
$$\mathbf{I}_{a3} = \frac{\mathbf{V}_a - \mathbf{E}_{r3}}{jX_s} = \frac{480 - 498.9\angle -16.90^\circ}{j1.4} = 103.6\angle -1.045^\circ \quad \text{A rms}$$

The power factor is $\cos(-1.045^\circ) = 99.98\% \text{ lagging}$.

E17.6 We follow the approach of Example 17.5. Thus as in the example, we have

$$I_{a1} = \frac{P_{dev}}{3V_a \cos\theta_1} = \frac{74600}{3(240)0.85} = 121.9 \text{ A}$$
$$\theta_1 = \cos^{-1}(0.85) = 31.79^\circ$$
$$\mathbf{I}_{a1} = 121.9\angle -31.79^\circ \quad \text{A rms}$$
$$\mathbf{E}_{r1} = \mathbf{V}_a - jX_s\mathbf{I}_{a1} = 416.2\angle -20.39^\circ \quad \text{V rms}$$

The phasor diagram is shown in Figure 17.24a

For 90% leading power factor, the power angle is $\theta_3 = \cos^{-1}(0.9) = 25.84^\circ$. The new value of the current magnitude is

$$I_{a3} = \frac{P_{dev}}{3V_{a3}\cos(\theta_3)} = 115.1 \quad \text{A rms}$$

and the phasor current is

$$\mathbf{I}_{a3} = 115.1\angle 25.84^\circ \quad \text{A rms}$$

Thus we have

$$\mathbf{E}_{r3} = \mathbf{V}_a - jX_s\mathbf{I}_{a3} = 569.0\angle -14.77^\circ \quad \text{V rms}$$

The magnitude of E_r is proportional to the field current, so we have:

$$I_{f3} = I_{f1}\frac{E_{r3}}{E_{r1}} = 10\times\frac{569.0}{416.2} = 13.67 \text{ A dc}$$

E17.7 The phasor diagram for $\delta = 90^\circ$ is shown in Figure 17.27. The developed power is given by

$$P_{\max} = 3V_a I_a \cos(\theta)$$

However from the phasor diagram, we see that

$$\cos(\theta) = \frac{E_r}{X_s I_a}$$

Substituting, we have

$$P_{\max} = \frac{3V_a E_r}{X_s}$$

The torque is

$$T_{\max} = \frac{P_{\max}}{\omega_m} = \frac{3V_a E_r}{\omega_m X_s}$$

Answers for Selected Problems

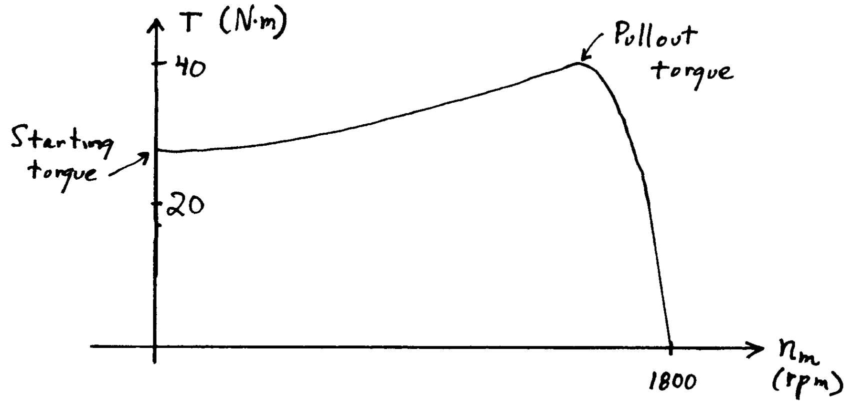
P17.1* $P = 8$ pole motor
$s = 5.55\%$

P17.7* $f = 86.8$ Hz
$I = 5.298$ A

P17.10* As frequency is reduced, the reactances X_s, X_m, and X_r' of the machine become smaller. (Recall that $X = \omega L$.) Thus the applied voltage must be reduced to keep the currents from becoming too large, resulting in magnetic saturation and overheating.

P17.13* $B_{\text{four-pole}} = B_m \cos(\omega t - 2\theta)$
$B_{\text{six-pole}} = B_m \cos(\omega t - 3\theta)$

P17.16*



$I_{line} = 16.3$ A rms

Typically the starting current is 5 to 7 times the full-load current.

P17.20* $I_{line,\text{starting}} = 115.0$ A rms

$T_{\text{dev, starting}} = 40.8$ newton meters

Comparing these results to those of the example, we see that the starting current is reduced by a factor of 2 and the starting torque is reduced by a factor of 4.

P17.23* Neglecting rotational losses, the slip is zero with no load, and the motor runs at synchronous speed which is 1800 rpm.

The power factor is 2.409%.

$I_{line} = 10$ A rms

P17.26* The motor runs at synchronous speed which is 1200 rpm.

The power factor is 1.04%.

$I_{line} = 98.97$ A rms

P17.29* $P_{\text{ag}} = 9.893$ kW

$P_{\text{dev}} = 9.773$ kW

$P_{\text{out}} = 9.373$ kW

$\eta = 91.51\%$

P17.32* $P_{\text{rot}} = 76.12$ W

P17.35* 1. Use an electronic system to convert 60-Hz power into three-phase ac of variable frequency. Start with a frequency of one hertz or less and then gradually increase the frequency.

2. Use a prime mover to bring the motor up to synchronous speed before connecting the source.

3. Start the motor as an induction motor relying on the amortisseur conductors to produce torque.

P17.38* (a) Field current remains constant. The field circuit is independent of the ac source and the load.

(b) Mechanical speed remains constant assuming that the pull-out torque has not been exceeded.

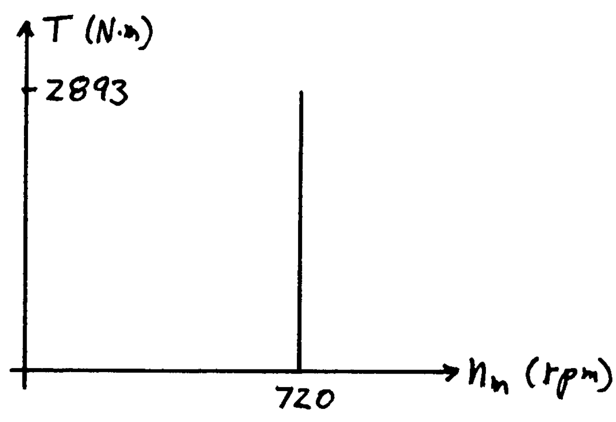
(c) Output torque increases by a factor of 1/0.75 = 1.333.

(d) Armature current increases in magnitude.

(e) Power factor decreases and becomes lagging.

(f) Torque angle increases.

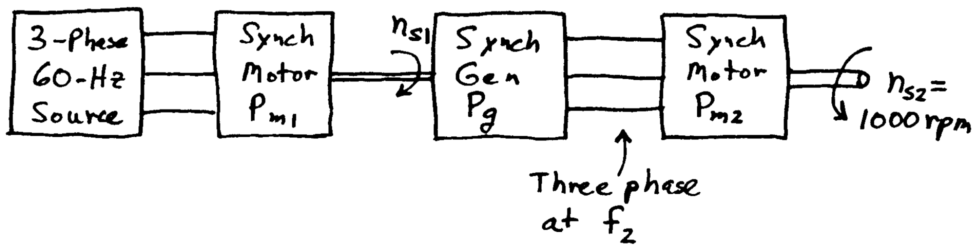
P17.41*



P17.44* $I_{f2} = 5.93\,A$

P17.47* (a) $f_{gen} = 50\text{ Hz}$

(b)



One solution is:

$P_g = 10 \qquad P_{m1} = 12 \qquad$ and $\qquad P_{m2} = 6$

for which $f_2 = 50\text{ Hz}$.

Another solution is:

$P_g = 10 \qquad P_{m1} = 6 \qquad$ and $\qquad P_{m2} = 12$

for which $f_2 = 100\text{ Hz}$.

P17.50* (a) power factor $= 76.2\%$ lagging

(b) $Z = 11.76\angle 40.36^\circ\ \Omega$

(c) Since the motor runs just under 1800 rpm, evidently we have a four-pole motor.

P17.53* The percentage drop in voltage is 7.33%.

Practice Test

T17.1 (a) The magnetic field set up in the air gap of a four-pole three-phase induction motor consists of four magnetic poles spaced 90° from one another in alternating order (i.e., north-south-north-south). The field points from the stator toward the rotor under the north poles and in the opposite direction under the south poles. The poles rotate with time at synchronous speed around the axis of the motor.

(b) The air gap flux density of a two-pole machine is given by Equation 17.12 in the book:

$$B_{gap} = B_m \cos(\omega t - \theta)$$

in which B_m is the peak field intensity, ω is the angular frequency of the three-phase source, and θ is angular displacement around the air gap. This describes a field having two poles: a north pole corresponding to $\omega t - \theta = 0$ and a south pole corresponding to $\omega t - \theta = \pi$. The location of either pole moves around the gap at an angular speed of $\omega_s = \omega$.

For a four pole machine, the field has four poles rotating at an angular speed of $\omega_s = \omega/2$ and is given by

$$B_{gap} = B_m \cos(\omega t - \theta / 2)$$

in which B_m is the peak field intensity, ω is the angular frequency of the three-phase source, and θ denotes angular position around the gap.

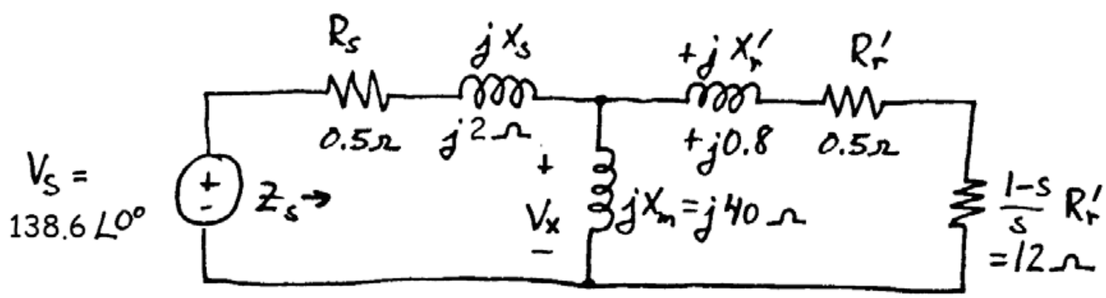
T17.2 Five of the most important characteristics for an induction motor are:

1. Nearly unity power factor.
2. High starting torque.
3. Close to 100% efficiency.
4. Low starting current.
5. High pull-out torque.

T17.3 An eight-pole 60-Hz machine has a synchronous speed of $n_s = 900$ rpm, and the slip is:

$$s = \frac{n_s - n_m}{n_s} = \frac{900 - 864}{900} = 0.04$$

Because the machine is wye connected, the phase voltage is $240 / \sqrt{3} = 138.6$ V. (We assume zero phase for this voltage.)The per phase equivalent circuit is:



Then, we have

$$Z_s = 0.5 + j2 + \frac{j40(0.5 + 12 + j0.8)}{j40 + 0.5 + 12 + j0.8}$$

$$= 11.48 + j6.149 \quad \Omega$$

$$= 13.03\angle 28.17^\circ \quad \Omega$$

$$\text{power factor} = \cos(28.17^\circ) = 88.16\% \text{ lagging}$$

$$\mathbf{I}_s = \frac{\mathbf{V}_s}{Z_s} = \frac{138.6\angle 0^\circ}{13.03\angle 28.17^\circ} = 10.64\angle -28.17^\circ \quad \text{A rms}$$

$$P_{\text{in}} = 3I_s V_s \cos\theta = 3.898 \text{ kW}$$

For a wye-connected motor, the phase current and line current are the same. Thus, the line current magnitude is 10.64 A rms.

Next, we compute $\mathbf{V}_x$ and $\mathbf{I}'_r$.

$$\mathbf{V}_x = \mathbf{I}_s \frac{j40(0.5 + 12 + j0.8)}{j40 + 0.5 + 12 + j0.8}$$

$$= 123.83 - j16.244$$

$$= 124.9\angle -7.473^\circ$$

$$\mathbf{I}'_r = \frac{\mathbf{V}_x}{j0.8 + 0.5 + 12}$$

$$= 9.971\angle -11.14^\circ$$

The copper losses in the stator and rotor are:

$$P_s = 3R_s I_s^2$$

$$= 3(0.5)(10.64)^2$$

$$= 169.7 \text{ W}$$

and

$$P_r = 3R'_r(I'_r)^2$$

$$= 3(0.5)(9.971)^2$$

$$= 149.1 \text{ W}$$

Finally, the developed power is:

$$P_{\text{dev}} = 3 \times \frac{1-s}{s} R'_r(I'_r)^2$$

$$= 3(12)(9.971)^2$$

$$= 3.579 \text{ kW}$$

$$P_{\text{out}} = P_{\text{dev}} - P_{\text{rot}} = 3.429 \text{ kW}$$

The output torque is:

$$T_{\text{out}} = \frac{P_{\text{out}}}{\omega_m} = 37.90 \text{ newton meters}$$

The efficiency is:

$$\eta = \frac{P_{\text{out}}}{P_{\text{in}}} \times 100\% = 87.97\%$$

T17.4 At 60 Hz, synchronous speed for an eight-pole machine is:

$$n_s = \frac{120f}{P} = \frac{120(60)}{8} = 900\,\text{rpm}$$

The slip is given by:

$$s = \frac{n_s - n_m}{n_s} = \frac{900 - 850}{900} = 5.56\%$$

The frequency of the rotor currents is the slip frequency. From Equation 17.17, we have $\omega_{\text{slip}} = s\omega$. For frequencies in the Hz, this becomes:

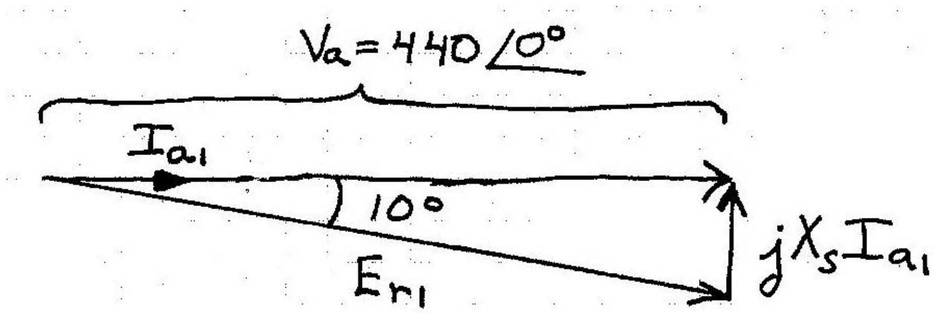
$$f_{\text{slip}} = sf = 0.05555 \times 60 = 3.333\,\text{Hz}$$

In the normal range of operation, slip is approximately proportional to output power and torque. Thus at 80% of full power, we estimate that $s = 0.8 \times 0.05555 = 0.04444$. This corresponds to a speed of 860 rpm.

T17.5 The stator of a six-pole synchronous motor contains a set of windings (collectively known as the armature) that are energized by a three-phase ac source. These windings produce six magnetic poles spaced 60° from one another in alternating order (i.e., north-south-north-south-north-south). The field points from the stator toward the rotor under the north stator poles and in the opposite direction under the south stator poles. The poles rotate with time at synchronous speed (1200 rpm) around the axis of the motor.

The rotor contains windings that carry dc currents and set up six north and south magnetic poles evenly spaced around the rotor. When driving a load, the rotor spins at synchronous speed with the north poles of the rotor lagging slightly behind and attracted by the south poles of the stator. (In some cases, the rotor may be composed of permanent magnets.)

T17.6 Figure 17.22 in the book shows typical phasor diagrams with constant developed power and variable field current. The phasor diagram for the initial operating conditions is:



Notice that because the initial power factor is unity, we have $\theta_1 = 0^\circ$ and $\mathbf{I}_{a1}$ is in phase with $\mathbf{V}_a$. Also, notice that $jX_s\mathbf{I}_{a1}$ is at right angles to $\mathbf{I}_{a1}$. Now, we can calculate the magnitudes of $\mathbf{E}_{r1}$ and of $X_s\mathbf{I}_{a1}$.

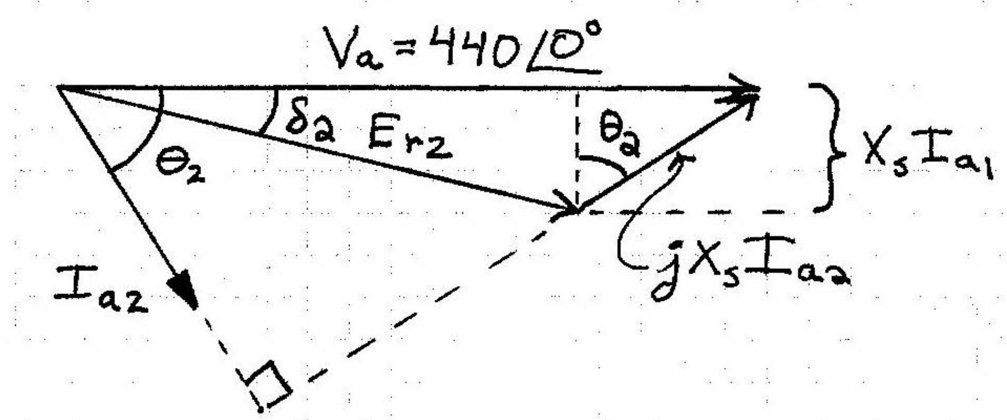
$$E_{r1} = \frac{V_a}{\cos\delta_1} = \frac{440}{\cos(10^\circ)} = 446.79\ \text{V}$$

$$X_s I_{a1} = E_{r1}\sin\delta_1 = 77.58\ \text{V}$$

Then, the field current is reduced until the magnitude of $\mathbf{E}_{r2}$ is 75% of its initial value.

$$E_{r2} = 0.75 \times E_{r1} = 335.09\ \text{V}$$

The phasor diagram for the second operating condition is:



Because the torque and power are constant, the vertical component of $jX_s\mathbf{I}_a$ is the same in both diagrams as illustrated in Figure 17.22 in the book. Thus, we have:

$$\sin\delta_2 = \frac{X_s I_{a1}}{E_{r2}} = \frac{77.58}{335.09}$$

which yields:

$$\delta_2 = 13.39^\circ$$

(Another solution to the equation is $\delta_2 = 166.61^\circ$, but this does not correspond to a stable operating point.)

Now, we can write:

$$(V_a - X_s I_{a1}\tan\theta_2)^2 + (X_s I_{a1})^2 = (E_{r2})^2$$

$$(440 - 77.58\tan\theta_2)^2 + (77.58)^2 = (335.09)^2$$

Solving, we find $\theta_2 = 55.76°$, and the power factor is $\cos\theta_2 = 56.25\%$ lagging.

APPENDIX A

Exercises

EA.1 Given $Z_1 = 2 - j3$ and $Z_2 = 8 + j6$, we have:

$$Z_1 + Z_2 = 10 + j3$$

$$Z_1 - Z_2 = -6 - j9$$

$$Z_1 Z_2 = 16 - j24 + j12 - j^2 18 = 34 - j12$$

$$Z_1 / Z_2 = \frac{2 - j3}{8 + j6} \times \frac{8 - j6}{8 - j6} = \frac{16 - j12 - j24 + j^2 18}{100} = -0.02 - j0.36$$

EA.2
$$Z_1 = 15\angle 45^\circ = 15\cos(45^\circ) + j15\sin(45^\circ) = 10.6 + j10.6$$
$$Z_2 = 10\angle -150^\circ = 10\cos(-150^\circ) + j10\sin(-150^\circ) = -8.66 - j5$$
$$Z_3 = 5\angle 90^\circ = 5\cos(90^\circ) + j5\sin(90^\circ) = j5$$

EA.3 Notice that Z_1 lies in the first quadrant of the complex plane.
$$Z_1 = 3 + j4 = \sqrt{3^2 + 4^2}\angle \arctan(4/3) = 5\angle 53.13^\circ$$

Notice that Z_2 lies on the negative imaginary axis.
$$Z_2 = -j10 = 10\angle -90^\circ$$

Notice that Z_3 lies in the third quadrant of the complex plane.
$$Z_3 = -5 - j5 = \sqrt{5^2 + 5^2}\angle(180^\circ + \arctan(-5/-5)) = 7.07\angle 225^\circ = 7.07\angle -135^\circ$$

EA.4 Notice that Z_1 lies in the first quadrant of the complex plane.
$$Z_1 = 10 + j10 = \sqrt{10^2 + 10^2}\angle \arctan(10/10) = 14.14\angle 45^\circ = 14.14\exp(j45^\circ)$$

Notice that Z_2 lies in the second quadrant of the complex plane.
$$Z_2 = -10 + j10 = \sqrt{10^2 + 10^2}\angle(180^\circ + \arctan(-10/10))$$
$$= 14.14\angle 135^\circ = 14.14\exp(j135^\circ)$$

EA.5 $Z_1Z_2 = (10\angle 30^\circ)(20\angle 135^\circ) = (10\times 20)\angle(30^\circ + 135^\circ) = 200\angle(165^\circ)$

$Z_1 / Z_2 = (10\angle 30^\circ)/(20\angle 135^\circ) = (10/20)\angle(30^\circ - 135^\circ) = 0.5\angle(-105^\circ)$

$$\begin{aligned} Z_1 - Z_2 &= (10\angle 30^\circ) - (20\angle 135^\circ) = (8.66 + j5) - (-14.14 + j14.14) \\ &= 22.8 - j9.14 = 24.6\angle -21.8^\circ \end{aligned}$$

$$\begin{aligned} Z_1 + Z_2 &= (10\angle 30^\circ) + (20\angle 135^\circ) = (8.66 + j5) + (-14.14 + j14.14) \\ &= -5.48 + j19.14 = 19.9\angle 106^\circ \end{aligned}$$

Problems

PA.1 Given $Z_1 = 2 + j3$ and $Z_2 = 4 - j3$, we have:

$$Z_1 + Z_2 = 6 + j0$$

$$Z_1 - Z_2 = -2 + j6$$

$$Z_1 Z_2 = 8 - j6 + j12 - j^2 9 = 17 + j6$$

$$Z_1 / Z_2 = \frac{2 + j3}{4 - j3} \times \frac{4 + j3}{4 + j3} = \frac{-1 + j18}{25} = 0.04 + j0.72$$

PA.2 Given that $Z_1 = 1 - j2$ and $Z_2 = 2 + j3$, we have:

$$Z_1 + Z_2 = 3 + j1$$

$$Z_1 - Z_2 = -1 - j5$$

$$Z_1 Z_2 = 2 + j3 - j4 - j^2 6 = 8 - j1$$

$$Z_1 / Z_2 = \frac{1 - j2}{2 + j3} \times \frac{2 - j3}{2 - j3} = \frac{-4 - j7}{13} = -0.3077 - j0.5385$$

PA.3 Given that $Z_1 = 10 + j5$ and $Z_2 = 20 - j20$, we have:

$$Z_1 + Z_2 = 30 - j15$$

$$Z_1 - Z_2 = -10 + j25$$

$$Z_1 Z_2 = 200 - j200 + j100 - j^2 100 = 300 - j100$$

$$Z_1 / Z_2 = \frac{10 + j5}{20 - j20} \times \frac{20 + j20}{20 + j20} = \frac{100 + j300}{800} = 0.125 + j0.375$$

PA.4 (a) $Z_a = 5 - j5 = 7.071\angle -45^\circ = 7.071\exp(-j45^\circ)$

(b) $Z_b = -10 + j5 = 11.18\angle 153.43^\circ = 11.18\exp(j153.43^\circ)$

(c) $Z_c = -3 - j4 = 5\angle -126.87^\circ = 5\exp(-j126.87^\circ)$

(d) $Z_d = -j12 = 12\angle -90^\circ = 12\exp(-j90^\circ)$

PA.5 (a) $Z_a = 5\angle 45^\circ = 5\exp(j45^\circ) = 3.536 + j3.536$

(b) $Z_b = 10\angle 120^\circ = 10\exp(j120^\circ) = -5 + j8.660$

(c) $Z_c = 15\angle -90^\circ = 15\exp(-j90^\circ) = -j15$

(d) $Z_d = -10\angle 60^\circ = 10\exp(-j120^\circ) = -5 - j8.660$

PA.6 (a) $Z_a = 5e^{j30^\circ} = 5\angle 30^\circ = 4.330 + j2.5$

(b) $Z_b = 10e^{-j45^\circ} = 10\angle -45^\circ = 7.071 - j7.071$

(c) $Z_c = 100e^{j135^\circ} = 100\angle 135^\circ = -70.71 + j70.71$

(d) $Z_d = 6e^{j90^\circ} = 6\angle 90^\circ = j6$

PA.7 (a) $Z_a = 5 + j5 + 10\angle 30^\circ = 13.66 + j10$

(b) $Z_b = 5\angle 45^\circ - j10 = 3.536 - j6.464$

(c) $Z_c = \dfrac{10\angle 45^\circ}{3 + j4} = \dfrac{10\angle 45^\circ}{5\angle 53.13^\circ} 2\angle -8.13^\circ = 1.980 - j0.283$

(d) $Z_d = \dfrac{15}{5\angle 90^\circ} = 3\angle -90^\circ = -j3$

APPENDIX C

PC.1 Because the capacitor voltage is zero at $t = 0$, the charge on the capacitor is zero at $t = 0$. Then using Equation 3.5 in the text, we have

$$q(t) = \int_0^t i(t)dx + 0$$

$$= \int_0^t 3dx = 3t$$

For $t = 2\ \mu\text{s}$, we have

$$q(3) = 3 \times 2 \times 10^{-6} = 6\ \mu\text{C}$$

PC.2 Refer to Figure PC.2 in the book. Combining the 10-Ω resistance and the 20-Ω resistance we obtain a resistance of 6.667 Ω, which is in series with the 5-Ω resistance. Thus, the total resistance seen by the 15-V source is 5 + 6.667 = 11.667 Ω. The source current is 15/11.667 = 1.286 A. The current divides between the 10-Ω resistance and the 20-Ω resistance. Using Equation 2.27, the current through the 10-Ω resistance is

$$i_{10} = \frac{20}{20+10} \times 1.286 = 0.8572\ \text{A}$$

Finally, the power dissipated in the 10-Ω resistance is

$$P_{10} = 10 i_{10}^2 = 7.346\ \text{W}$$

PC.3 The equivalent capacitance of the two capacitors in series is given by

$$C_{eq} = \frac{1}{1/C_1 + 1/C_2} = 4\mu\text{F}$$

The charge supplied by the source is

$$q = C_{eq}V = 200 \times 4 \times 10^{-6} = 800\mu\text{C}$$

PC.4 The input power to the motor is the output power divided by efficiency

$$P_{in} = \frac{P_{out}}{\eta} = \frac{2 \times 746}{0.80} = 1865 \text{ W}$$

However the input power is also given by

$$P_{in} = V_{rms} I_{rms} \cos(\theta)$$

in which $\cos(\theta)$ is the power factor. Solving for the current, we have

$$I_{rms} = \frac{P_{in}}{V_{rms}\cos(\theta)} = \frac{1865}{220 \times 0.75} = 11.30 \text{ A}$$

PC.5 $Z = R + j\omega L - \dfrac{j}{\omega C} = 30 + j40 - j80 = 30 - j40 = 50\angle -53.1^\circ$

Thus the impedance magnitude is 50 Ω.

PC.6 We have

$$\text{Apparent power} = V_{rms} I_{rms}$$

Also, the power factor is $\cos(\theta) = 0.6$ from which we find that $\theta = 53.13^\circ$. (We selected the positive angle because the power factor is stated to be lagging.) Then we have

$$Q = V_{rms} I_{rms} \sin(\theta) = (\text{Apparent power}) \times \sin(\theta) = 2000 \times 0.8 = 1600 \text{ VAR}$$

PC.7 For practical purposes, the capacitor is totally discharged after twenty time constants and all of the initial energy stored in the capacitor has been delivered to the resistor. The initial stored energy is

$$W = \tfrac{1}{2} C V^2 = \tfrac{1}{2} \times 150 \times 10^{-6} \times 100^2 = 0.75 \text{ J}$$

PC.8 $\omega = 2\pi f = 120\pi$

$$Z = R + j\omega L - \frac{j}{\omega C} = 50 + j56.55 - j106.10 = 50 - j49.55 = 70.39\angle -44.74^\circ$$

$$I_{rms} = \frac{V_{rms}}{|Z|} = \frac{110}{70.39} = 1.563 \text{ A}$$

PC.9 See Example 4.2 in the book. In this case, we have $K_2 = K_1 = V_S/R = 1$ A and $\tau = L/R = 0.5$ s. Then the current is given by

$$i(t) = 1 - \exp(-t/\tau) = 1 - \exp(-2t)$$

PC.10 We have $V_{BC} = -V_{CB} = -50 \text{ V}$ and $V_{AB} = V_{AC} - V_{BC} = 200 - (-50) = 250$. The energy needed to move the charge from point *B* to point *A* is $W = QV_{AB} = 0.2(250) = 50 \text{ J}$.