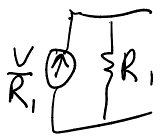
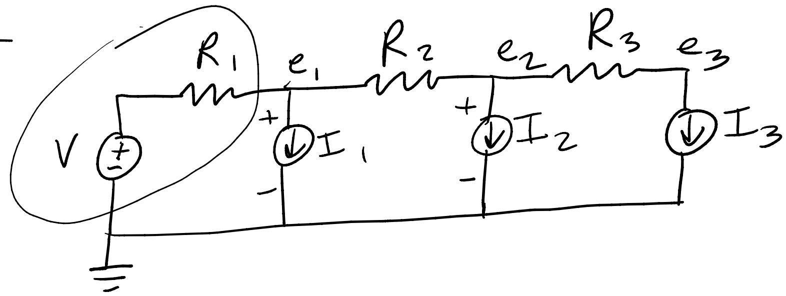
Discussion 5 Notes

Friday, April 29, 2022 6:20 PM

Midterm





$I_1, I_2, I_3 = I, 0$

On Off

$$G\begin{bmatrix} e_1 \\ e_2 \\ e_3 \end{bmatrix} = S$$

$$\underbrace{\begin{bmatrix} \frac{V}{R_1} - I_1 \\ I_2 \\ I_3 \end{bmatrix}}_{S} = \underbrace{\begin{bmatrix} \frac{1}{R_1}+\frac{1}{R_2} & -\frac{1}{R_2} & 0 \\ -\frac{1}{R_2} & \frac{1}{R_2}+\frac{1}{R_3} & -\frac{1}{R_3} \\ 0 & -\frac{1}{R_3} & \frac{1}{R_3} \end{bmatrix}}_{G} \begin{bmatrix} e_1 \\ e_2 \\ e_3 \end{bmatrix}$$

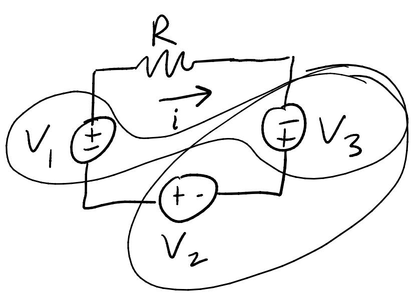
$\boxed{\text{Def: } P = VI}$

All current sources on.

$P_{Max} = 3IV$

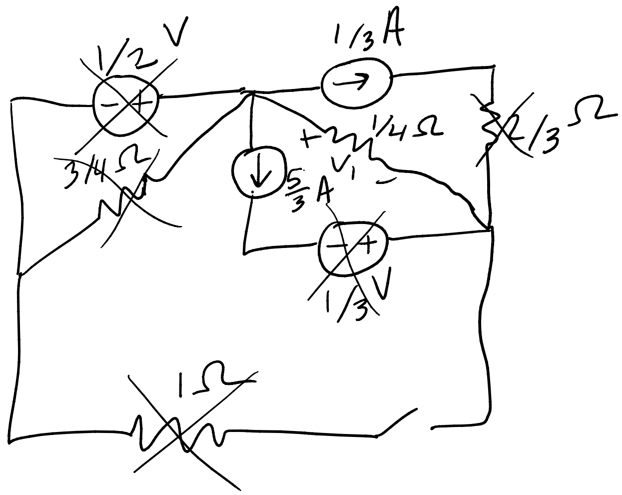
V_1 off: $i_1 = \frac{V_2+V_3}{R}$

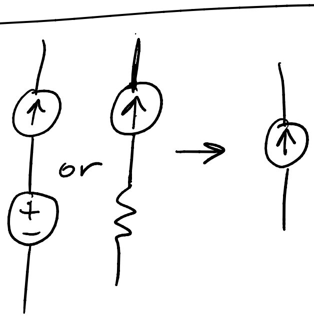
V_2 off: $i_2 = \frac{V_1+V_3}{R}$

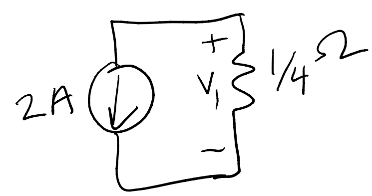


$i = \frac{V_1+V_2+V_3}{R}$

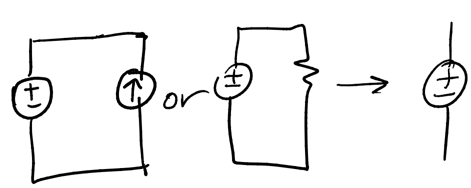
$i = i_1 + i_2 = \frac{V_1+V_2+2V_3}{R}$

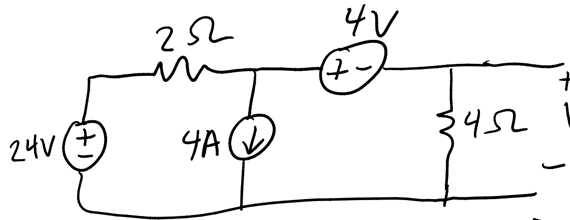




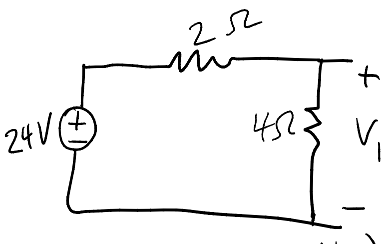


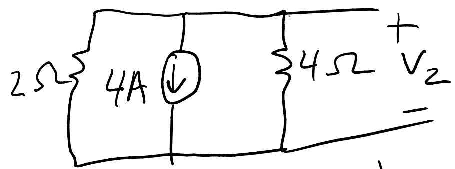
$V_1 = -2\left(\frac{1}{4}\right) = -\frac{1}{2}$

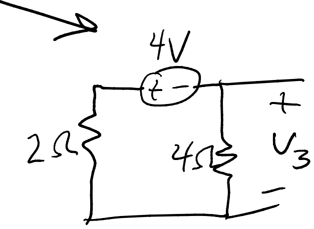




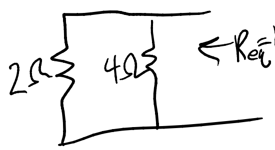
$$V_0 = V_1 + V_2 + V_3 = 16 - \frac{16}{3} - \frac{8}{3} = 8$$





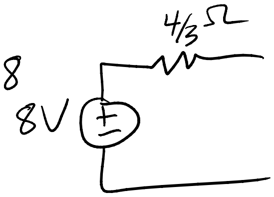


$$V_1 = 24\left(\frac{4}{6}\right) = 16\,V \qquad V_2 = -4\left(\frac{2(4)}{2+4}\right) = -\frac{16}{3} \qquad V_3 = -4\left(\frac{4}{6}\right) = -\frac{8}{3}$$

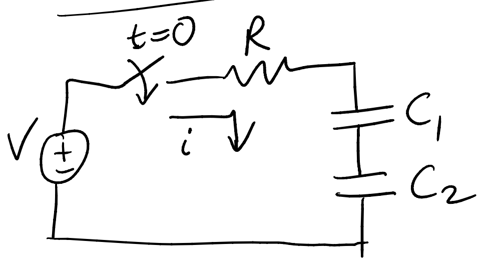


$\leftarrow R_{eq} = R_{Th} = \frac{2(4)}{6} = \frac{4}{3}$

$V_0 = V_{Th} = 8$



1st and 2nd Order Circuits



$$V = iR + \frac{1}{C_1}\int i\,dt + \frac{1}{C_2}\int i\,dt$$

$$0 = \frac{di}{dt}R + \left(\frac{1}{C_1} + \frac{1}{C_2}\right)i$$

$$i = Ke^{st} \qquad \frac{di}{dt} = Kse^{st}$$

$\xrightarrow{t=0}$ $i = K$, $i = \frac{V}{R}$

$$RKse^{st} + \left(\frac{1}{C_1} + \frac{1}{C_2}\right)Ke^{st} = 0$$

$$Ke^{st}\left(Rs + \left(\frac{1}{C_1} + \frac{1}{C_2}\right)\right) = 0$$

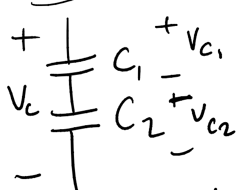
$$\frac{V}{R}e^{st}\left(Rs + \left(\frac{C_2 + C_1}{C_1 C_2}\right)\right) = 0$$

$$Ve^{st}\underbrace{\left(s + \left(\frac{C_1 + C_2}{RC_1C_2}\right)\right)}_{} = 0$$

$$s = -\left(\frac{C_1 + C_2}{RC_1C_2}\right) = -\frac{1}{R\left(\frac{C_1C_2}{C_1+C_2}\right)}$$

$$i = \frac{V}{R}e^{-\frac{t}{R\left(\frac{C_1C_2}{C_1+C_2}\right)}}$$

I shared so $Q_1 = Q_2$



$V_c = V_{c_1} + V_{c_2}$

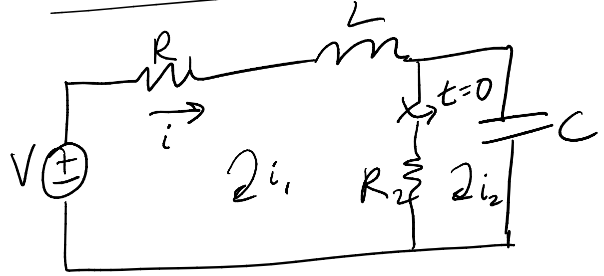
$$V_{c_1}C_1 = V_{c_2}C_2$$

$$V_{c_1} = V_{c_2}\frac{C_2}{C_1}$$

$$V_{c_1} = (V - V_{c_1})\frac{C_2}{C_1}$$

$$V_{c_1}\left(1 + \frac{C_2}{C_1}\right) = V\frac{C_2}{C_1}$$

$$V_{c_1} = V\frac{C_2}{C_1} \cdot \frac{C_1}{C_1 + C_2} = V\frac{C_2}{C_1 + C_2}$$



$i_{0^-} = 0 \qquad i_\infty = \frac{V}{R_1+R_2}$

$i_{0^+} = 0 \qquad i_{R_2 0^+} = \frac{V}{R_2}$

$i_1 = i$

$$\left(\begin{array}{l} V = i_1 R_1 + L\frac{di_1}{dt} + R_2(i_1 - i_2) \rightarrow \quad i_2 = \frac{i_1 R_1 + L\frac{di_1}{dt} + R_2 i_1 - V}{R_2} \\ R_2(i_1 - i_2) = \frac{1}{C}\int i_2\, dt \end{array}\right.$$

$$\frac{di_2}{dt} = \frac{di_1}{dt}\left(\frac{R_1}{R_2}\right) + \frac{L}{R_2}\frac{d^2 i_1}{dt^2} + \frac{di_1}{dt}$$

$$R_2\frac{di_1}{dt} - R_2\frac{di_2}{dt} = \frac{1}{C} i_2$$

$$R_2\frac{di_1}{dt} - R_2\left(\frac{di_1}{dt}\left(\frac{R_1}{R_2}\right) + \frac{L}{R_2}\frac{d^2 i_1}{dt^2} + \frac{di_1}{dt}\right) = i_1\frac{R_1}{CR_2} + \frac{L}{CR_2}\frac{di_1}{dt} + \frac{i_1}{C} - \frac{V}{CR_2}$$

$$\underbrace{\frac{L}{R_2}}_{a_1}\frac{d^2 i}{dt^2} - \underbrace{\left(\frac{R_2^2 - R_1 - R_2 - L}{R_2}\right)}_{a_2}\frac{di}{dt} + \underbrace{\frac{R_1 + R_2}{R_2 C}}_{a_3} i = 0$$

$$a_1 s^2 + a_2 s + a_3 = 0$$

$$s = \frac{-a_2 \pm \sqrt{a_2^2 - 4a_1 a_3}}{2a_1} \qquad D = a_2^2 - 4a_1 a_3$$

$$\left(\begin{array}{l} i \overset{?}{=} Ke^{st} \\ \frac{di}{dt} = Kse^{st} \\ \frac{d^2 i}{dt^2} = Ks^2 e^{st} \end{array}\right)$$

$D > 0$	$D = 0$	$D < 0$
2 real values for s s_1, s_2 $i = K_1 e^{s_1 t} + K_2 e^{s_2 t}$	1 real s s $i = K_1 e^{st} + K_2 t e^{st}$	2 complex values for s $s_1 = s_2^*$ $\quad j = \sqrt{-1}$ $s_1 = r + \omega j$ $e^{jx} = \cos(x) + j\sin(x)$ $i = e^{rt}(K_1 \cos(\omega t) + jK_2 \sin(\omega t))$