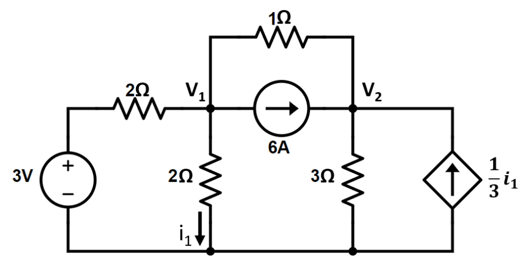
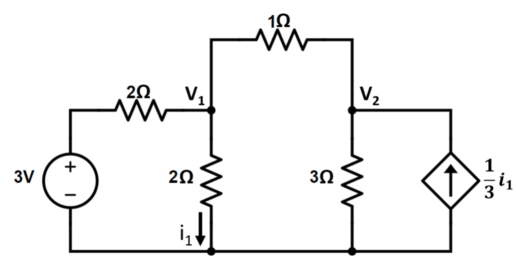
ECE100: Homework 3, Part 2 Solutions

Q1: Use the superposition principle in the circuit in the figure below to find the power consumed by that 2Ω resistor which has current i_1 flowing through it (as labeled in the circuit).



The response due to the voltage source:



Using KCL:

$$\frac{3 - v_1}{2} = \frac{v_1}{2} + (v_1 - v_2)$$

$$\left(\frac{1}{3}\right)\left(\frac{v_1}{2}\right) = \frac{v_2}{3} + (v_2 - v_1)$$

Simplifying and solving:

$$\frac{3}{2} - \frac{v_1}{2} = \frac{v_1}{2} + v_1 - v_2$$

$$\frac{3}{2} + v_2 = 2v_1$$

$$v_1 = \frac{1}{2}v_2 + \frac{3}{4}$$

$$\frac{v_1}{6} = \frac{4}{3}v_2 - v_1$$

$$\frac{7}{6}v_1 = \frac{4}{3}v_2$$

$$v_2 = \frac{7}{8}v_1$$

$$v_1 = \frac{7}{16}v_1 + \frac{3}{4}$$

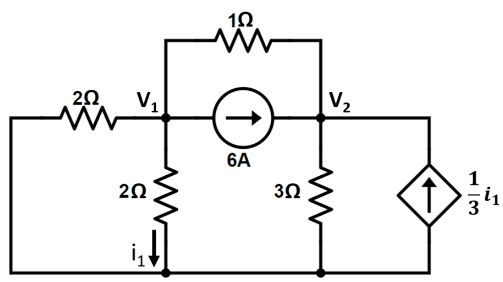
$$\frac{9}{16}v_1 = \frac{3}{4}$$

$$v_1 = \frac{4}{3}$$

$$v_2 = \frac{7}{8} * \frac{4}{3} = \frac{7}{6}$$

$$i_1 = \frac{v_1}{2} = \frac{2}{3}$$

The response due to the current source:



Using KCL:

$$0 = v_1 + 6 + (v_1 - v_2)$$

$$6 + \frac{1}{6}v_1 = \frac{v_2}{3} + (v_2 - v_1)$$

Simplifying and solving:

$$v_2 = 2v_1 + 6$$

$$6 + \frac{7}{6}v_1 = \frac{4}{3}v_2$$

$$6 + \frac{7}{6}v_1 = \frac{8}{3}v_1 + 8$$

$$-2 = \frac{9}{6}v_1$$

$$v_1 = -\frac{4}{3}$$

$$i_1 = \frac{v_1}{2} = -\frac{2}{3}$$

Combining the responses:

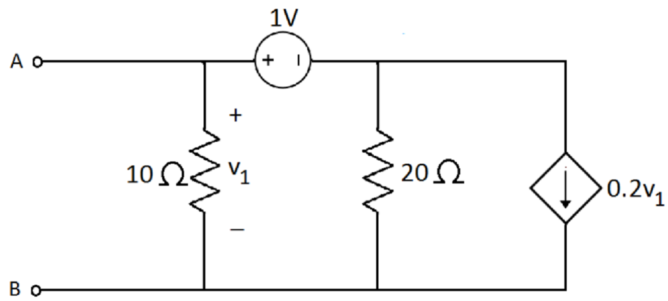
$$i_1 = i_{vsrc} + i_{isrc} = \frac{2}{3} - \frac{2}{3} = 0$$

$$P = IV = I^2R = 0$$

Q2: Consider the circuit in the figure below. A resistor R is connected between terminals A and B of the circuit. Find the power dissipated in the resistor R when:

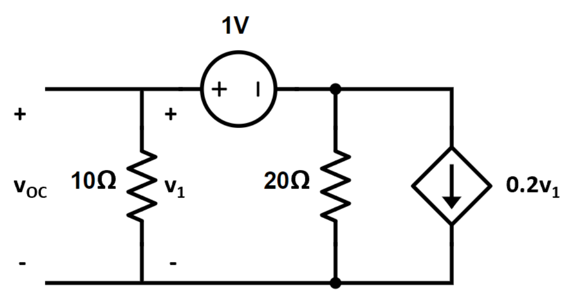
R = 10Ω

R = 20Ω



[Hint: Use Norton's theorem to solve this problem.]

Finding v_{OC} and i_{SC}.

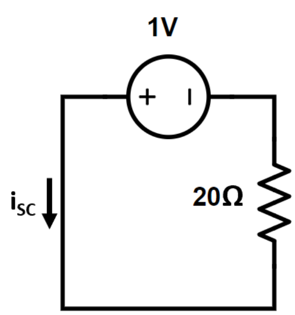


$$v_{OC} = v_1$$

Using KVL:

$$1 = v_1 + 20\left(\frac{v_1}{10} + 0.2v_1\right) = v_1 + 2v_1 + 4v_1 = 7v_1$$

$$v_1 = \frac{1}{7} = v_{OC}$$



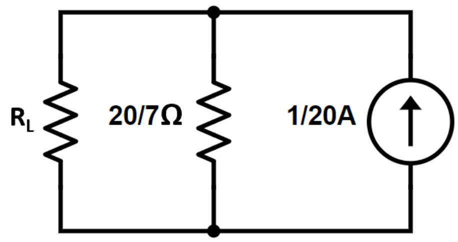
Since the voltage across the 10Ω resistor is 0, the dependent current source will have a value of 0.

$$i_{SC} = \frac{1}{20}$$

Thus, the Norton equivalent values:

$$i_N = \frac{1}{20} A$$

$$R_N = \frac{v_{OC}}{i_{SC}} = \frac{20}{7}\Omega$$



To find the power over R_L:

$$V = \frac{1}{20}\left(\frac{\frac{20}{7}R_L}{\frac{20}{7}+R_L}\right) = \frac{\frac{1}{7}R_L}{\frac{20}{7}+R_L} = \frac{R_L}{20+7R_L}$$

$$P = IV = \frac{V^2}{R_L} = \frac{1}{R_L}\left(\frac{R_L}{20+7R_L}\right)^2 = \frac{R_L}{(20+7R_L)^2}$$

Power for $R_L = 10\Omega$:

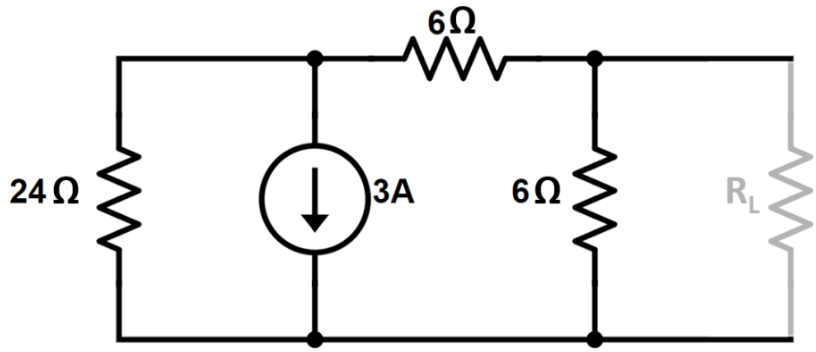
$$P = \frac{10}{8100} = \frac{1}{810} = 1.23\text{mW}$$

Power for $R_L = 20\Omega$:

$$P = \frac{20}{25600} = \frac{1}{1280} = 781\mu\text{W}$$

Q3: Problem 2.90 from the book. (Copied below)

Find the maximum power that can be delivered to a resistive load by the circuit shown in the figure below. For what value of load resistance (R_L) is the power maximum?



Finding the Thevenin Equivalent:

$$R_{Th} = \frac{6(30)}{6+30} = 5\Omega$$

To find V_{OC} we can find the equivalent resistance from the perspective of the current source to get the voltage across the current source:

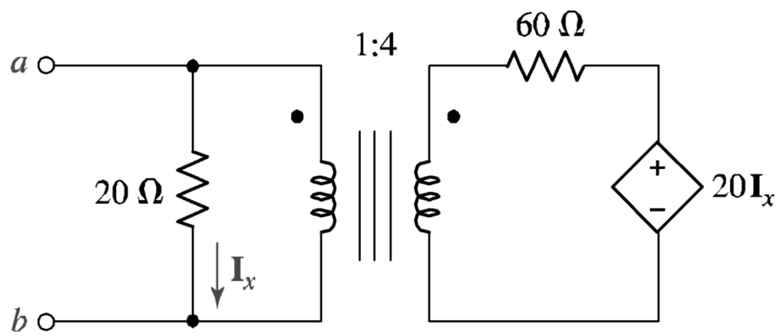
$$V_{isrc} = IR_{eq} = 3\frac{24(12)}{12+24} = 24$$

$$V_L = -V_{isrc}\left(\frac{1}{2}\right) = -12$$

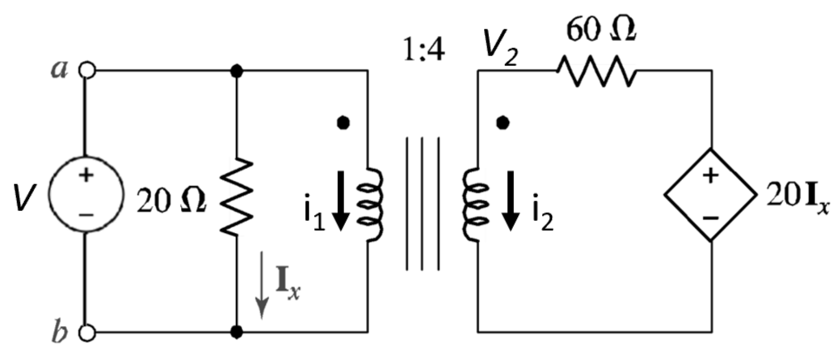
Max Power:

$$P = \frac{V^2}{R_L} = \frac{144}{5} = 28.8\text{W}$$

Q4: Find the Thevenin equivalent at terminals *a* and *b* for the network shown below.



Since there is no independent source, both $V_{OC} = 0$ and $I_{SC} = 0$. From this, we know that $V_{Th} = 0$. While we normally can find R_{Th} as $R_{Th} = \frac{V_{OC}}{I_{SC}}$, that does not work since we get the indeterminate result $\frac{0}{0}$. To find R_{Th}, we can apply a "test" voltage source across *a* and *b*. This way, we can find the current into the circuit so we can get the equivalent resistance. Note that V cannot be DC since we are working with a transformer.



Since this is an ideal transformer, we know that $V_2 = \frac{N_2}{N_1} V_1$ and $i_2 = -\frac{N_1}{N_2} i_1$

For the transformer, the voltage on the left side (v_1) is the same as the applied voltage.

$$V_1 = V$$

$$V_2 = 4V$$

$$I_x = \frac{V}{20}$$

Using KVL for the right loop:

$$V_2 + 60i_2 = 20I_x$$

$$4V + 60i_2 = \frac{20V}{20} = V$$

$$3V = -60\left(-\frac{1}{4}i_1\right) = 15i_1$$

$$i_1 = \frac{1}{5}V$$

The input current at terminal *a*:

$$i_a = I_x + i_1 = \frac{V}{20} + \frac{V}{5} = \frac{5}{20}V = \frac{1}{4}V$$

Using Ohm's Law to get the equivalent resistance:

$$R = \frac{V}{i_a} = 4\Omega$$