ECE102, Spring 2021 — **Homework #5**

Signals & Systems — Prof. A. Kadambi

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Due Friday, 14 May 2021, by 11:59pm to CCLE.
100 points total.
This homework covers questions relate to Fourier series and Fourier transform.

1. (18 points) **Fourier Series**

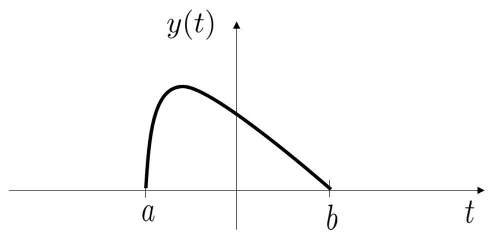
   (a) (7 points) When the periodic signal $f(t)$ is real, you have seen in class some properties of symmetry for the Fourier series coefficients of $f(t)$ (see the Lecture 11 slide titled: Fourier Series Properties: Fourier Symmetry (cont.)). How do these properties of symmetry change when $f(t)$ is imaginary (with no real component)?

   (b) (7 points) A *real* and *odd* signal $x(t)$ has the following properties:

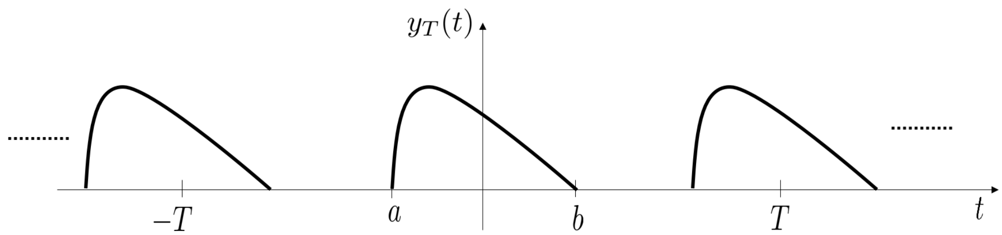
   - it is a periodic signal with period 1 s;
   - it has one positive frequency component (positive frequency component meaning c_k with k > 0);
   - it has a power of 9 (hint: consider Parseval's relation. The power of the signal in the time domain is the same as the sum of the powers of its frequency components).

   What is $x(t)$?

   (c) (4 points) Consider the signal $y(t)$ shown below and let $Y(j\omega)$ denote its Fourier transform.



   Let $y_T(t)$ denote its periodic extension:



   How can the Fourier series coefficients of $y_T(t)$ can be obtained from the Fourier transform $Y(j\omega)$ of $y(t)$? (Note that the figures given in this problem are for illustrative purposes, the question is for any arbitrary $y(t)$).

2. (32 points) **Symmetry properties of Fourier transform**

(a) (16 points) Determine which of the signals, whose Fourier transforms are depicted in Fig. 1, satisfy each of the following:

i. $x(t)$ is even
ii. $x(t)$ is odd
iii. $x(t)$ is real
iv. $x(t)$ is complex (neither real, nor pure imaginary)
v. $x(t)$ is real and even
vi. $x(t)$ is imaginary and odd
vii. $x(t)$ is imaginary and even
viii. There exists a non-zero ω_0 such that $e^{j\omega_0 t}x(t)$ is real and even

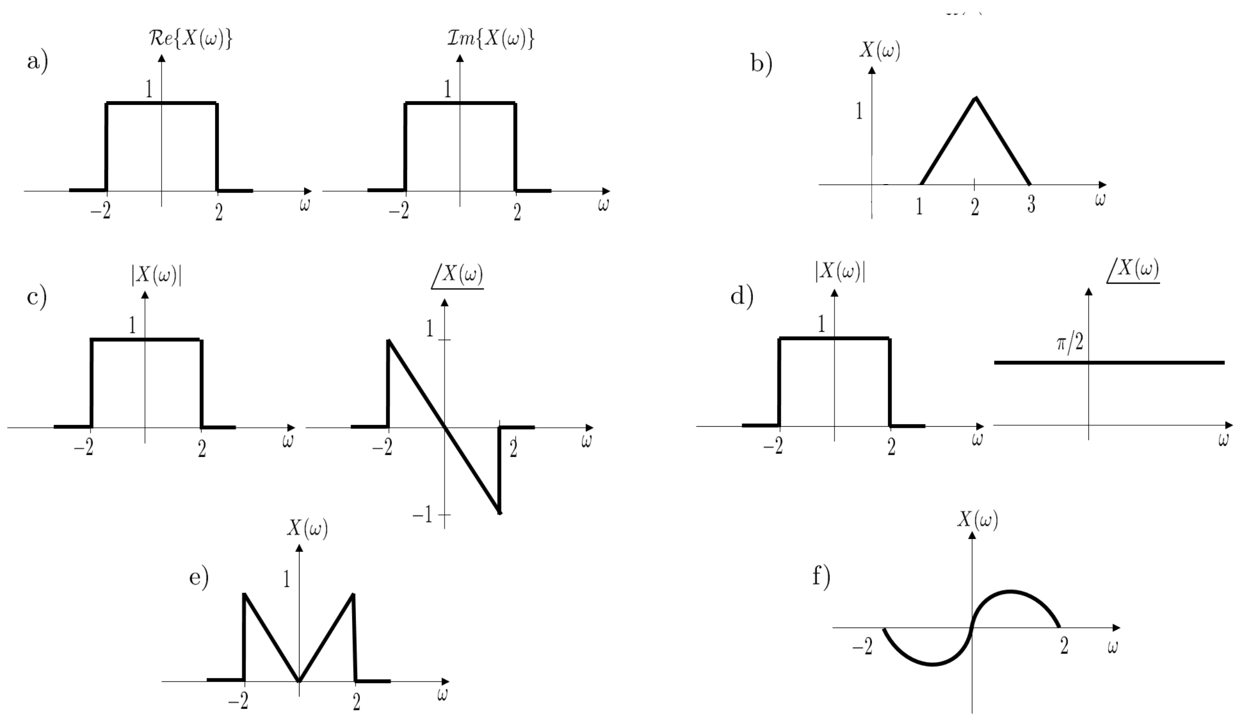


Figure 1: P2.a

(b) (8 points) Using the properties of the Fourier transform, determine whether the assertions are true or false.

i. The convolution of a real and even signal and a real and odd signal, is odd.
ii. The convolution of a signal and the same signal reversed is an even signal.

(c) (8 points) Show the following statements:

i. If $x(t) = x^*(-t)$, then $X(j\omega)$ is real.

ii. If $x(t)$ is a real signal with $X(j\omega)$ its Fourier transform, then the Fourier transforms $X_e(j\omega)$ and $X_o(j\omega)$ of the even and odd components of $x(t)$ satisfy the following:

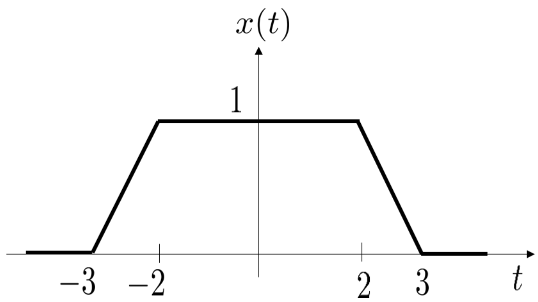
$$X_e(j\omega) = Re\{X(j\omega)\}$$

and

$$X_o(j\omega) = jIm\{X(j\omega)\}$$

3. (15 points) **Fourier transform properties**

Let $X(j\omega)$ denote the Fourier transform of the signal $x(t)$ sketched below:



Evaluate the following quantities without explicitly finding $X(j\omega)$:

(a) $\int_0^\infty X(j\omega)d\omega$
Hint: Consider the properties of $x(t)$.

(b) $X(j\omega)|_{\omega=0}$

(c) $\angle X(j\omega)$

(d) $\int_{-\infty}^{\infty} e^{-j\omega} X(j\omega)d\omega$

(e) Plot the inverse Fourier transform of $\mathcal{R}e\{e^{-3j\omega}X(j\omega)\}$
Hint: Consider the 'even and odd' properties of the Fourier transform

4. (35 points) **Fourier transform and its inverse**

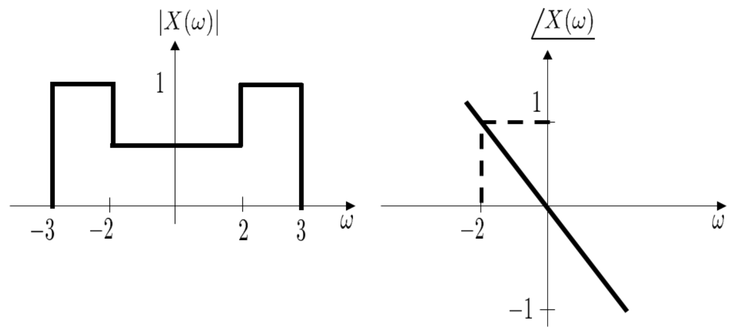
(a) (18 points) Find the Fourier transform of each of the signals given below:
Hint: you may use Fourier Transforms derived in class.

i. $x_1(t) = \begin{cases} 1 + \cos(\pi t), & |t| < 1 \\ 0, & \text{otherwise} \end{cases}$

ii. $x_2(t) = e^{(1+3j)t}u(-t+1)$

iii. $x_3(t) = 2te^{-2t}u(t)$
Hint: You can consider Fourier transform of the derivative and its dual.

(b) (7 points) Find the inverse Fourier transform of the signal shown below (note that $|X(0)| = 0.5$ and $|X(2.5)| = 1$):



(c) (10 points) Two signals $f_1(t)$ and $f_2(t)$ are defined as

$$f_1(t) = \text{sinc}(2t)$$
$$f_2(t) = \text{sinc}(t)\cos(3\pi t)$$

Let the convolution of the two signals be

$$f(t) = (f_1 * f_2)(t)$$

i. Find $F(j\omega)$, the Fourier transform of $f(t)$.

ii. Find $f(t)$.