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Zeid CE2

Anthony MatE 3

Jaime CE2

Blake EE1

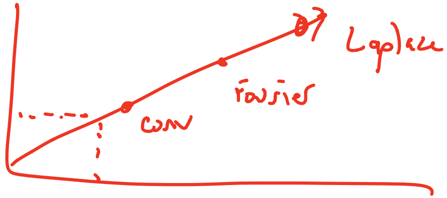
Henry MatE4

EE102

Lecture 6

EE102 Announcements

- Syllabus link is tiny.cc/ucla102
- CCLE difficulties, please email help@seas.ucla.edu
- **Second Homework due this Friday**
- Pace of the course like r(t), the ramp function!

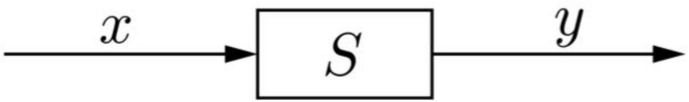


Slide Credits: This lecture adapted from Prof. Jonathan Kao (UCLA) and recursive credits apply to Prof. Cabric (UCLA), Prof. Yu (Carnegie Mellon), Prof. Pauly (Stanford), and Prof. Fragouli (UCLA)

What is a system?

A system transforms an *input signal*, $x(t)$, into an output system, $y(t)$.

$S(x) \rightarrow y$



functional

- Systems, like signals, are also *functions*. However, their inputs and outputs are signals.
- Systems can have either single or multiple inputs (SI or MI, respectively) and single or multiple outputs (SO and MO). In this class, we focus on *single input, single output* systems (SISO).

Linearity and time-invariance recap

	Linear	T.I.
$y(t) = \sqrt{x(t)}$	No	Yes
$y(t) = x(t)\, z(t)$	Yes	No
$\vdots$	$\vdots$	$\vdots$

Memory

A system has *memory* if its output depends on past or future values of the input. If the output depends only on present values of the input, the system is called *memoryless*.

// AM Radio : $y(t) = x(t)\cos(\omega_c t)$ Memoryless

// Integrator $y(t) = \int_{-\infty}^{t} x(\tau)\, d\tau$ Memory ✓

$x \rightarrow \boxed{S} \rightarrow y$

Invertibility

$y \hat{=} S(x)$

A system is called *invertible* if an input can always be exactly recovered from the output. That is, a system S is invertible if there exists an S^{inv} such that

$$\underline{x = S^{\text{inv}}(S(x))} = S^{inv}(y)$$

Is this invertible?

// sq: $y(t) = [x(t)]^2$

Not.

// diff $y(t) = \frac{dx(t)}{dt}$

// scale $y(t) = a\,x(t)$

CYU

Review

Suppose $\exists\ S$, where S is linear and has an inverse S^{inv}.

Qst: Is S^{inv} also linear? I don't know, so let me prove it.

$\Leftarrow\ x = S^{inv} S(x)$

// $S(ax + b\tilde{x}) = aS(x) + bS(\tilde{x})$

$S^{inv}(ax + b\tilde{x}) = a \cdot S^{inv}(x) + b S^{inv}(\tilde{x})$

Can you use these two equations to get visibility on the magenta equation?

Impulse Response

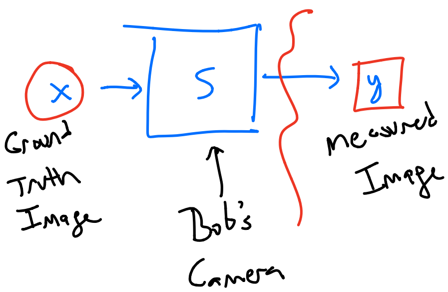
System impulse response

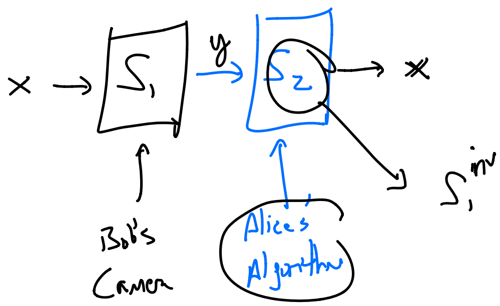
Outline

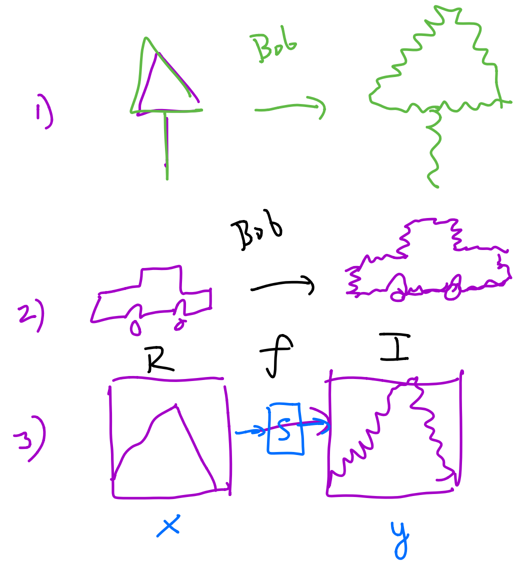
This lecture introduces time-domain analysis of systems, including the impulse response. It also discusses linear time-invariant systems. Topics include:

- Impulse response definition
- Impulse response of LTI systems
- The impulse response as a sufficient characterization of an LTI system
- Impulse response and the convolution integral

$$I = f(R)$$


x
S
y
Ground Truth Image
Bob's Camera
measured Image


x
S_1
y
S_2
x
S_1^{inv}
Bob's Camera
Alice's Algorithm


1)
Bob
2)
Bob
R
f
I
3)
S
x
y

Why do we need the impulse response?

(1) E.g. $y(t) = [x(t)]^2$ Pset 2.

(2) Real-Life, we do not know much if anything about the functional form of S.

Usually S is very complicated and not known!

Goal: Given some input, specific input x, please predict $y = S(x)$.

Magical Solution: If I know the systems response to $x = \delta(t)$, then I know its response to ANY x, $\forall x$. [Assuming S is LTI]

Types of Responses

$x \to \boxed{H} \to y$

System : H

Several special systems are characterized by their functional response.

- zero response : $H(0)$
- impulse response : $H(\delta(t))$
- step response : $H(u(t))$

// CYU: Say H is linear, what is its zero response?

$$H(ax) = aH(x) \quad \forall\, a, x$$

Suppose $a = 0$

LHS $H(0) = 0$ QED

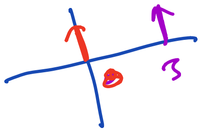
$H(0) = 0$

Impulse Response Definition

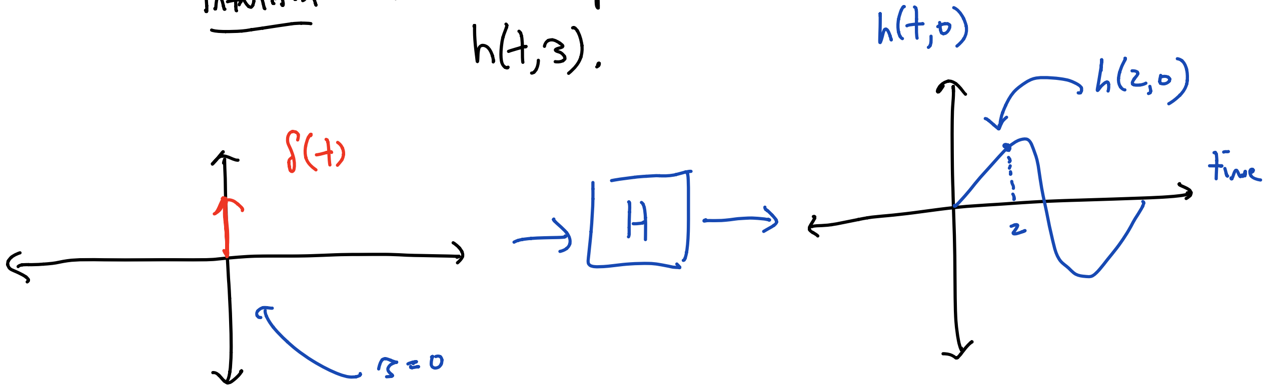
Impulse Response: Let H be a system and $y(t) = H[x(t)]$

The impulse response is:

$$h(t,\tau) = H(\delta(t-\tau))$$



Intuition: Send an impulse at time τ, the output of H is $h(t,\tau)$.



input $\rightarrow$ $x(t)$ $\quad$ $y(t)$ $\leftarrow$ output $\quad$ $h(t)$ $\leftarrow$ typically reserved for IR

$$h(t, \tau) = H(\underbrace{\delta(t - \tau)})$$

There are important things to be careful of when looking at this equation.

- The t on the left and right hand side of these equations *are not the same*!
- The t on the left hand side is the impulse response at a specific value of time.
- The t on the right hand side varies across all time.
- The output at the specific time t on the left will depend on the input at several times t on the right.

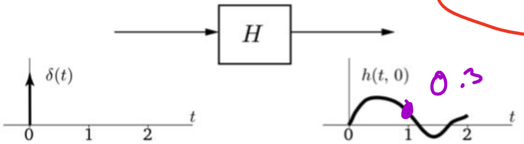
cyu @ home: Review those bullets

Notation on t

$$h(t, \tau) = H(\delta(t - \tau))$$

There are important things to be careful of when looking at this equation.

An example of these t's not being the same is shown below. In this example, let $\tau = 0$.



It may be tempting to write:

$$h(1, 0) = H(\delta(1))$$

This is wrong.

- On the left, $\delta(1) = 0$. We know if H is linear, then $H(0) = 0$, implying that $h(1, 0) = 0$.
- But in general, the impulse response can be non-zero, i.e., $h(1, 0) \neq 0$ in the above diagram, if the impulse response produces some non-zero response.

Assuming $\tau = 0$

Caveat: these two t's are different

Eqn 1 $\quad h(t, 0) = H[\delta(t)]$

NOT CORRECT

Set $t = 1$

$$\underbrace{h(1, 0)}_{\text{nonzero}} = \underbrace{H[\delta(1)]}_{\text{zero}}$$

Time invariant Impulse Response

Time Invariant H

$$h(t,3) = H(\delta(t-3)) \quad \therefore$$

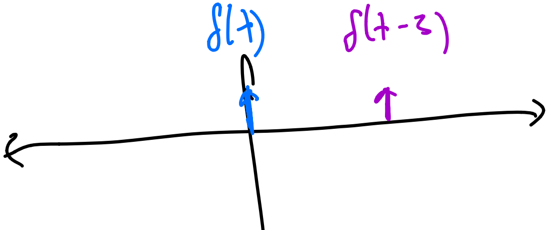
$$h(t,0) = H(\delta(t))$$

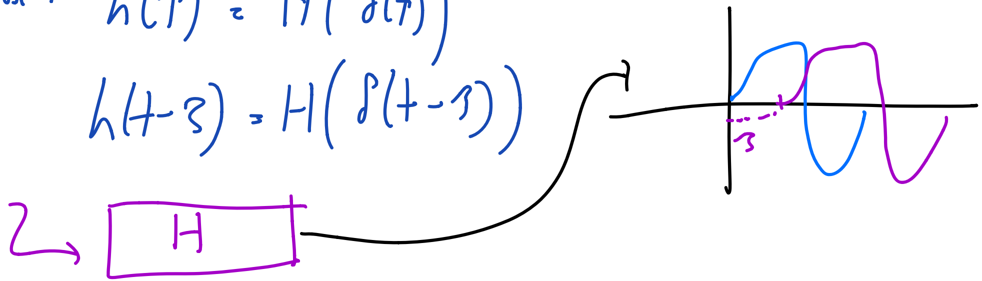
Suppose H is time invariant. $H(\delta(t-3)) = h(t-3,0)$

$$= h(t,3)$$

Suppose H is time invariant. $h(t) = H(\delta(t))$

$$h(t-3) = H(\delta(t-3))$$

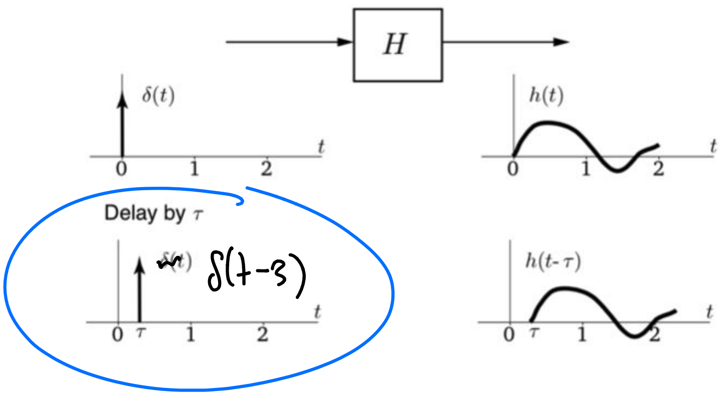




Time Invariant Impulse Response

Impulse response of a time-invariant system (cont.)

This property of the impulse response for a time-invariant system is drawn below:



Important Fact about the Impulse Response

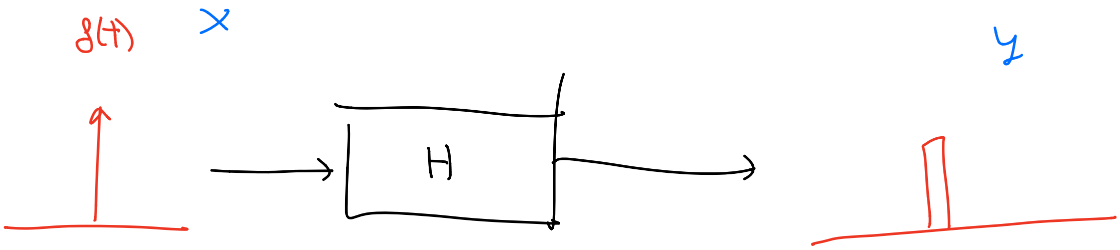
FACT: If H is an LTI (linear time-invariant system) with impulse response

$$h(t) = H(\delta(t))$$

then we can calculate $H(x(t))$ for ANY $x(t)$ **IF** we know $h(t)$.

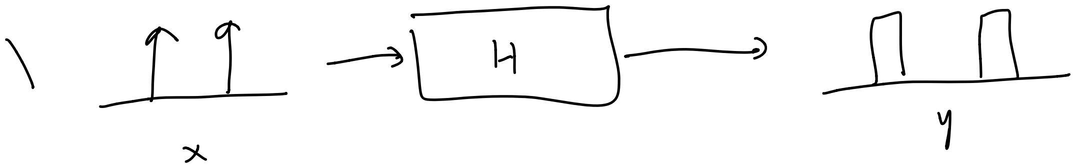
This is a ***very important* result.**

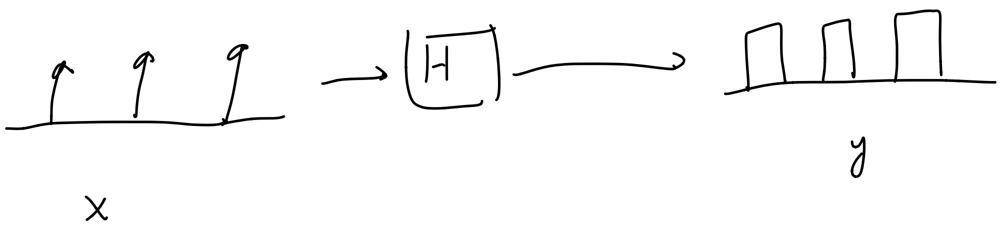
Test

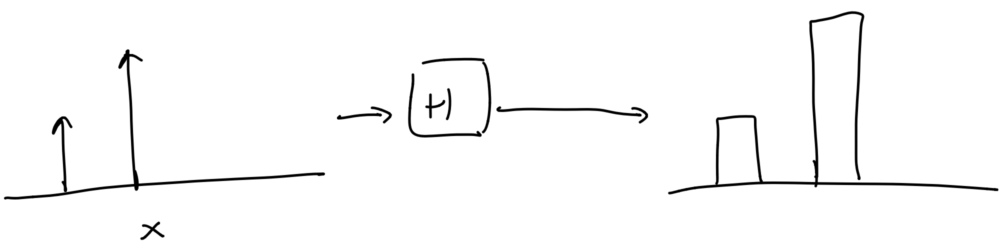


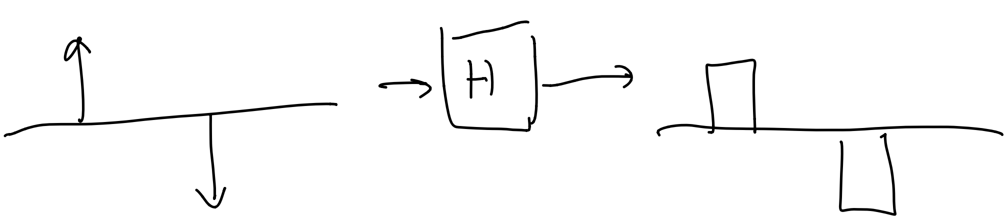
Goal: For a different x predict y

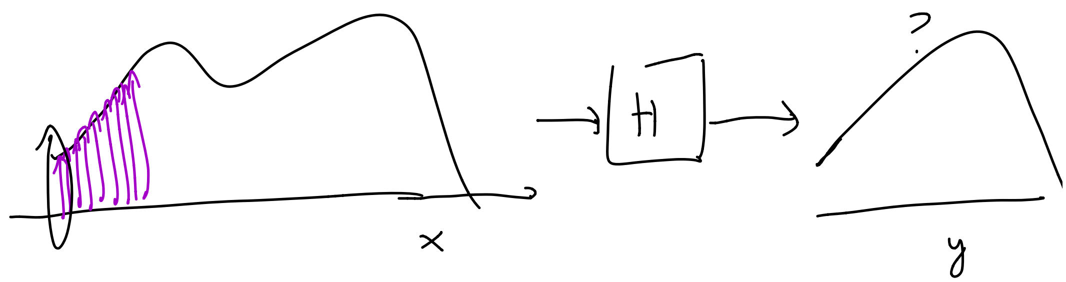
?







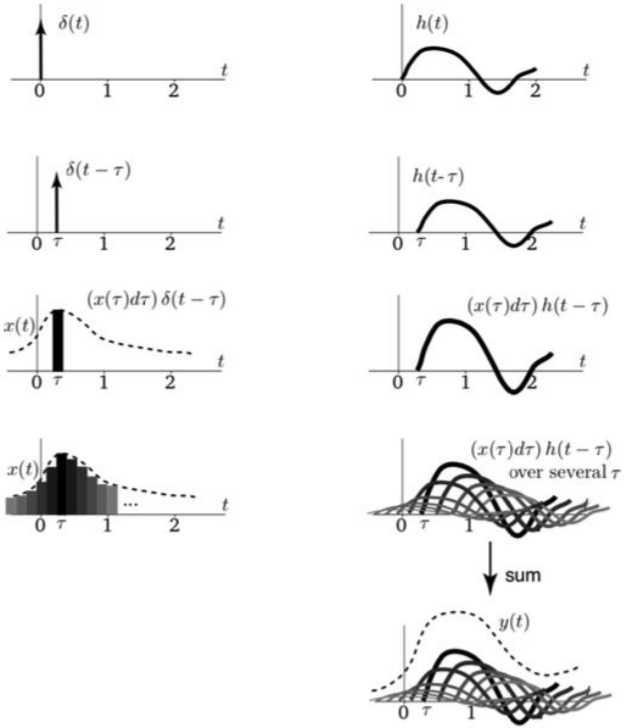



H
x
y
?

Derivation of this fact

The Convolution Integral

Intuition of What’s Going on In Convolution



Examples of Computing the Impulse Response

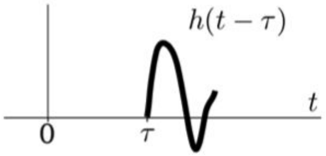
Notation of Convolution

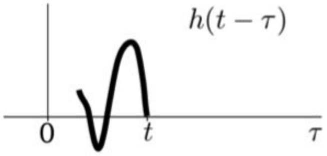
How to Compute Convolution: flip and drag

$$y(t) = \int_{-\infty}^{\infty} x(\tau)h(t-\tau)d\tau$$

Let's break this integral down piece by piece.

- The term $h(t-\tau)$, w.r.t. t, is the impulse response delayed to time τ.
- However, our integral is over τ, and so we should consider how h varies with τ.
- The term $h(t-\tau)$, w.r.t. τ, tells us that we should first delay the signal to time t and then reverse the signal. This operation, which we colloquially call "flipping," is illustrated below.

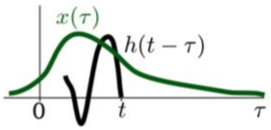


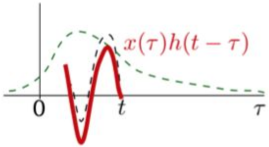


How to Compute Convolution: flip and drag

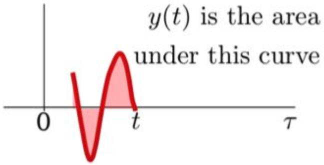
$$y(t) = \int_{-\infty}^{\infty} x(\tau)h(t-\tau)d\tau$$

- Next, convolution tells us to multiply $h(t-\tau)$, our flipped impulse response, with $x(\tau)$ and do it for all τ. This means we simply multiply $x(\tau)$ and $h(t-\tau)$ together pointwise. This is illustrated below in red.





- Finally, to get $y(t)$ for this particular value of t, we integrate this curve over all τ. This is illustrated below.



- Now, to get $y(t)$ for all values of t, we repeat this process, "dragging" $h(t-\tau)$ across different delays t.

How to Compute Convolution: flip and drag

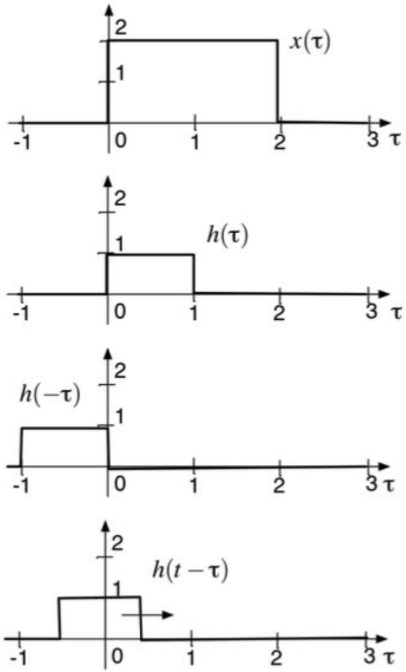
Summary of the flip and drag technique

To calculate $y(t) = (x * h)(t)$,

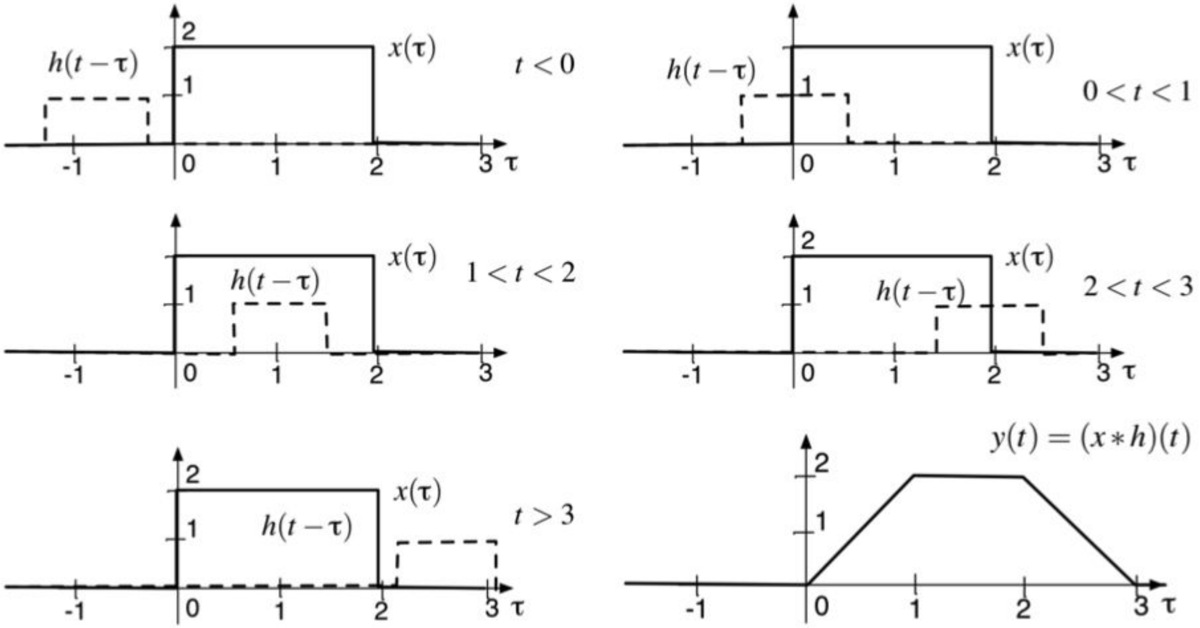
- Flip (i.e., reverse in time) the impulse response. This changes $h(\tau)$ to $h(-\tau)$.
- Begin to drag the reversed time response by some amount, t. This results in $h(t - \tau)$.
- For a given t, multiply $h(t - \tau)$ pointwise by $x(\tau)$. This produces $x(\tau)h(t - \tau)$.
- Integrate this product over τ. This produces $y(t)$ at this particular time t.

This technique is referred to as the "flip-and-drag" technique.

How to Compute Convolution: flip and drag



How to Compute Convolution: flip and drag



How to Compute Convolution: flip and drag

Examples: Try these: