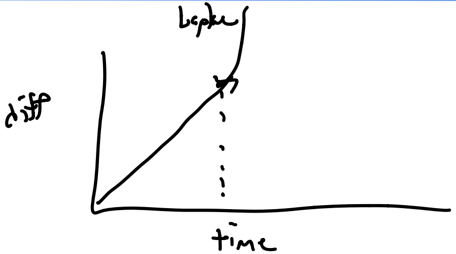
Shalinda EE2
Bruce EE2
Preetha CSE3

EE102

Lecture 14

EE102 Announcements

- Syllabus link is tinyurl.com/ucla102
- ~~My office hour meeting minutes are sent out weekly~~
- **Homeworks due on Tuesday**



Slide Credits: This lecture adapted from Prof. Jonathan Kao (UCLA) and recursive credits apply to Prof. Cabric (UCLA), Prof. Yu (Carnegie Mellon), Prof. Pauly (Stanford), and Prof. Fragouli (UCLA)

Duality

Duality of the Fourier transform

If $\mathcal{F}[f(t)] = F(j\omega)$, then

$$\boxed{F(t) \iff 2\pi f(-j\omega)}$$

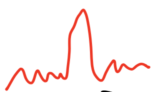
This expression may be opaque at first. What this is saying is that if I take a Fourier transform pair, I can find the dual pair by replacing all the ω's with t's in $F(j\omega)$ and all the t's with $-\omega$'s in $f(t)$. After scaling by 2π, this results in another Fourier transform pair.

Essentially, every Fourier transform pair we derive really gives us two Fourier transform pairs.

$$\mathcal{F}[f(t)] \rightarrow \int_{-\infty}^{\infty} f(t)\, e^{-j\omega t}\, dt$$

1  $\xleftrightarrow{\mathcal{F}}$ 

$f(t)$ $F(j\omega)$

2  $\xrightarrow{\mathcal{F}}$ 

$g(t) \triangleq \mathcal{F}[F(j\omega)]$

$$f(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} F(j\omega)\, e^{j\omega t}\, d\omega$$

$$2\pi f(-t) = \int_{-\infty}^{\infty} F(j\omega)\, e^{-j\omega t}\, d\omega$$

Duality

Duality of the Fourier transform (cont.)

To show this, recognize that as

$$f(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} F(j\omega) e^{j\omega t} d\omega$$

then

$$2\pi f(-t) = \int_{-\infty}^{\infty} F(j\omega) e^{-j\omega t} d\omega$$

Now, the r.h.s. of this equation is the Fourier transform of $F(j\omega)$ with the roles of ω and t reversed. Hence, $2\pi f(-t)$ is the Fourier transform of $F(j\omega)$ (!) and after we swap the ω and the t's, we arrive at the duality result.

Duality Examples

Duality examples

- Since $\mathrm{rect}(t) \iff \mathrm{sinc}(\omega/2\pi)$, then

$$\begin{aligned} \mathrm{sinc}(t/2\pi) &\iff 2\pi\mathrm{rect}(-\omega) \\ &= 2\pi\mathrm{rect}(\omega) \end{aligned}$$

  Thus, we have that $\mathrm{sinc}(t/2\pi) \iff 2\pi\mathrm{rect}(\omega)$.

- Since

$$e^{-at}u(t) \iff \frac{1}{a+j\omega}$$

  then

$$\frac{1}{a+jt} \iff 2\pi e^{a\omega}u(-\omega)$$

Dual intuition: convolution in time domain is multiplication in frequency domain. Thus, multiplication in the time domain ought be convolution in frequency domain.

Frequency domain convolution

primal dual

The frequency domain convolution theorem is that for $f_1(t) \iff F_1(j\omega)$ and $f_2(t) \iff F_2(j\omega)$, then

$$\boxed{\mathcal{F}[f_1(t)f_2(t)] = \frac{1}{2\pi}\int_{-\infty}^{\infty} F_1(j\nu)F_2(j(\omega-\nu))d\nu}$$

We typically write this as:

$$\boxed{\mathcal{F}[f_1(t)f_2(t)] = \frac{1}{2\pi}(F_1 * F_2)(j\omega)}$$

but note that the convolution is w.r.t. ω, not $j\omega$.

// cyu @ home

This means that multiplication in the time domain is convolution in the frequency domain. This proof is very similar to the time domain proof.

Modulation: duality of time-shifting

Dual intuition: Time shift in the time domain is multiplication by a complex exp. in freq domain
Thus, multiplication by a complex exp. in the freq domain ought be a shift in the freq domain.

Recall that:

Modulation //

$$\boxed{\mathcal{F}[f(t-\tau)] = e^{-j\omega\tau}F(j\omega)}$$

Using duality, we arrive at:

Dual of Modulation //

$$\mathcal{F}[f(t)e^{j\omega_0 t}] \quad = \quad F(j(\omega-\omega_0))$$

Using linearity, we also see that:

$$\begin{aligned}
\mathcal{F}[f(t)\cos(\omega_0 t)] &= \frac{1}{2}\left(F(j(\omega-\omega_0)) + F(j(\omega+\omega_0))\right) \\
\mathcal{F}[f(t)\sin(\omega_0 t)] &= \frac{1}{2j}\left(F(j(\omega-\omega_0)) - F(j(\omega+\omega_0))\right)
\end{aligned}$$

Modulation

Fourier transform of a modulated signal (cont.)

To prove the modulation result, note that if $\mathcal{F}[f(t)] = F(j\omega)$ then

cyu @ home

$$
\begin{aligned}
\mathcal{F}[f(t)e^{j\omega_0 t}] &= \int_{-\infty}^{\infty} f(t)e^{j\omega_0 t}e^{-j\omega t}dt \\
&= \int_{-\infty}^{\infty} f(t)e^{-j(\omega-\omega_0)t}dt \\
&= F(j(\omega-\omega_0))
\end{aligned}
$$

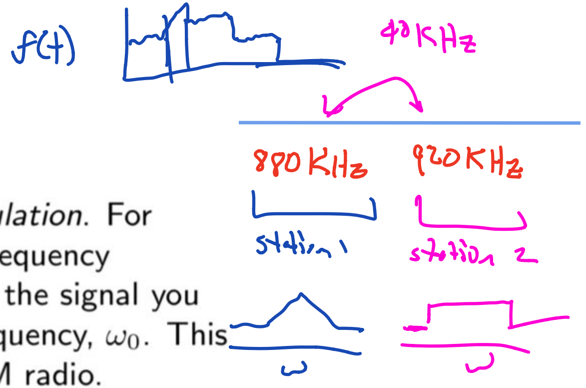
To get the cosine and sine results, we note that e.g., for cosine,

$$\cos(\omega_0 t) = \frac{1}{2}\left(e^{j\omega_0 t} + e^{-j\omega_0 t}\right)$$

From here, we can use linearity to compute the Fourier transform.

Human Ear 20 - 20,000 Hz

Modulation

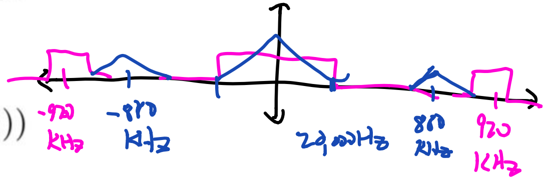


Fourier transform of a modulated signal

A major component of communications has to do with *modulation*. For example, AM and FM radio are amplitude modulation and frequency modulation respectively. AM radio involves multiplying $f(t)$, the signal you wish to transmit, with a complex exponential at a carrier frequency, ω_0. This frequency, ω_0, is the frequency you dial in your car to get AM radio.

Here are three ways to modulate a signal: If $\mathcal{F}[f(t)] = F(j\omega)$, then

$$\begin{aligned}
\mathcal{F}[f(t)e^{j\omega_0 t}] &= F(j(\omega-\omega_0)) \\
\mathcal{F}[f(t)\cos(\omega_0 t)] &= \frac{1}{2}\left(F(j(\omega-\omega_0)) + F(j(\omega+\omega_0))\right) \\
\mathcal{F}[f(t)\sin(\omega_0 t)] &= \frac{1}{2j}\left(F(j(\omega-\omega_0)) - F(j(\omega+\omega_0))\right)
\end{aligned}$$



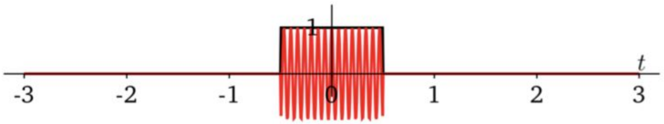
Typically, modulation is done through multiplication by $\cos(\omega_0 t)$. Modulation is dual to the time shift Fourier transform.

What modulation intuitively does is take $F(j\omega)$ and create replicas at $\pm\omega_0$.

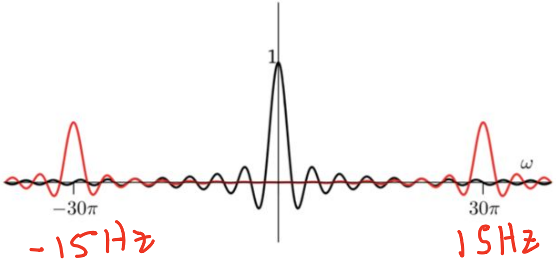
Modulation

Fourier transform of a modulated signal (cont.)

Below, we show what modulation does. We take a signal (here a rect) and multiply it by a cosine with $\omega_0 = 30\pi$. This is denoted in red in the plot below.



The spectrum takes the FT of our signal (i.e., a sinc) and creates replicas at $\pm 30\pi$.

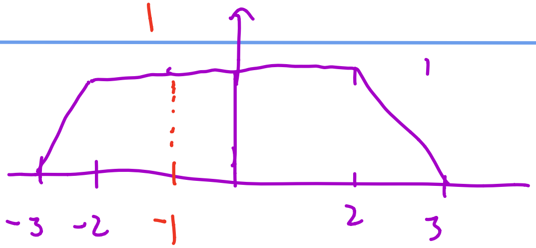


From here, you can gain some intuition for why different radio stations use different frequencies. They're given these frequencies to transmit whatever signals they like; each radio station occupies a different part of the spectrum!

CYU:

Compute $\int_{-\infty}^{\infty} e^{-j\omega} X(j\omega)\, d\omega$

Given $x(t)$



$$x(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} X(j\omega)\, e^{j\omega t}\, dt$$

$$x(t)\Big|_{-1} = \frac{1}{2\pi}\int_{-\infty}^{\infty} X(j\omega)\, e^{-j\omega}\, dt$$

$$2\pi\, x(-1) = \int_{-\infty}^{\infty} X(j\omega)\, e^{-j\omega}\, dt$$

2π

Time-reversal

Fourier transform of a time-reversed signal

If $\mathcal{F}[f(t)] = F(j\omega)$, then

$$\boxed{\mathcal{F}[f(-t)] = F(-j\omega)}$$

To show this, apply the time-scaling result with $a = -1$.

Time-reversal

Time reversal example

Find the Fourier transform of $f(t) = e^{-a|t|}$ (for $a > 0$) without doing integration.

We know that

$$e^{-at}u(t) \Longleftrightarrow \frac{1}{a + j\omega}$$

// cyu
@ home

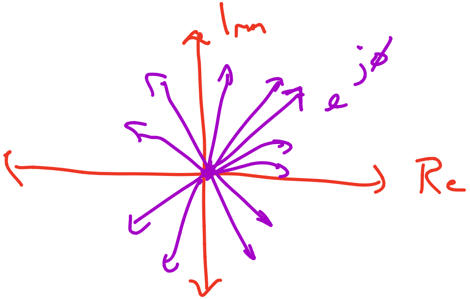
Time-reversal

CYU: What is the Fourier Transform of "1"

CYU @ home: $F(j\omega) = \int_{-\infty}^{\infty} f(t)\, e^{-j\omega t}\, dt$

Suppose $f(t) = 1$.

Answer: $\int_{-\infty}^{\infty} e^{-j\omega t}\, dt = \cancel{\left.\frac{e^{-j\omega t}}{-j\omega}\right|_{-\infty}^{\infty}}$



$1 \longleftrightarrow 2\pi \cdot \delta(\omega)$

Other Fourier Transforms

$$\boxed{\delta(t) \iff 1}$$

$$\boxed{\delta(t-\tau) \iff e^{-j\omega\tau}}$$

$$\boxed{1 \iff 2\pi\delta(\omega)}$$

$$\boxed{u(t) \iff \pi\delta(\omega) + \frac{1}{j\omega}}$$

$$\boxed{e^{j\omega_0 t} \iff 2\pi\delta(\omega-\omega_0)}$$

$$\boxed{\cos(\omega_0 t) \iff \pi(\delta(\omega-\omega_0)+\delta(\omega+\omega_0))}$$

$$\boxed{\sin(\omega_0 t) \iff j\pi(\delta(\omega+\omega_0)-\delta(\omega-\omega_0))}$$

CYU: Fourier transform of Step Function

What is the F.T. of Heaviside Step Fn $\quad u(t) = \begin{cases} 1 & t \geq 0 \\ 0 & \text{o.w.} \end{cases}$

Answer: $\quad \mathcal{F}[u(t)] = \int_{-\infty}^{\infty} u(t)\, e^{-j\omega t}\, dt$

$$= \int_{0}^{\infty} e^{-j\omega t}\, dt$$

$$= \left.\frac{e^{-j\omega t}}{-j\omega}\right|_{0}^{\infty}$$

doesn't converge

Introduce a new Idea: "limiting F.T."

Fourier transform of Step Function

Limiting Fourier transforms

When the Fourier transform integral doesn't converge, and there's not a "trick" we can use, an alternative approach is to use limiting Fourier transforms.

In this approach, we represent the signal as a limit of a sequence of signals for which the Fourier transforms do exist. i.e., consider $f_n(t)$ which does have a Fourier transform. If

$$f(t) = \lim_{n \to \infty} f_n(t)$$

then we also have that

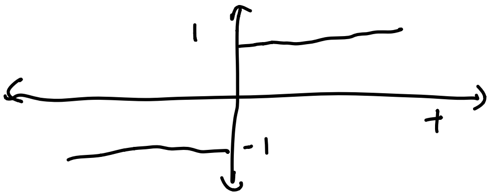
$$F(j\omega) = \lim_{n \to \infty} F_n(j\omega)$$

if the limit makes sense.

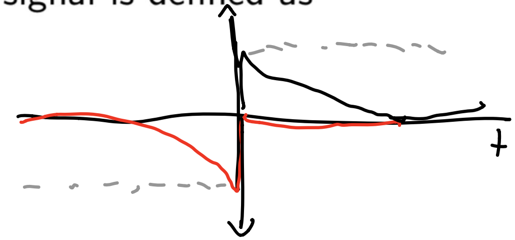
Fourier Transform of a Step Function

Limiting Fourier transform example

Consider the Fourier transform of $f(t) = \mathrm{sign}(t)$. This signal is defined as



$$f(t) = \begin{cases} 1, & t > 0 \\ 0, & t = 0 \\ -1, & t < 0 \end{cases}$$



We previously derived the Fourier transform for $e^{-at}u(t)$. We can use this signal to make a limiting approximation to $\mathrm{sign}(t)$ by setting

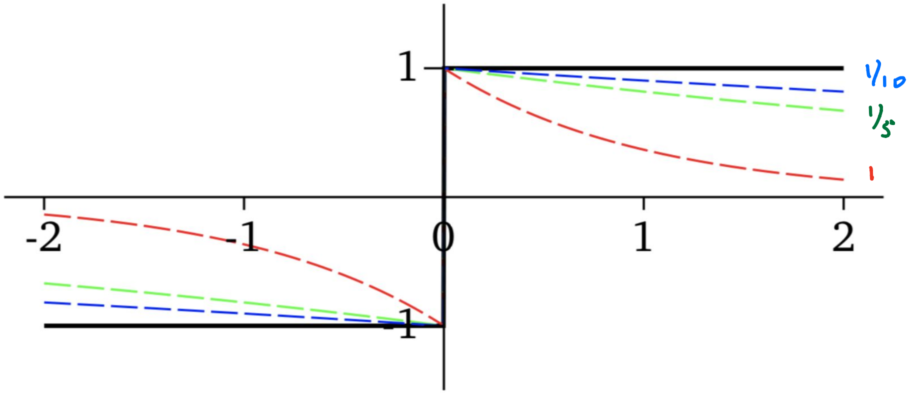
$$f_a(t) = e^{-at}u(t) - e^{at}u(-t)$$

This approximation is shown on the next slide.

Fourier Transform of a Step Function

Limiting Fourier transform example (cont.)

Below we show $f_a(t)$ for $a = 1$ (red), $a = 1/5$ (green), and $a = 1/10$ (blue).



As $a \to 0$, then, $f_a(t) \to \operatorname{sign}(t)$.

Fourier Transform of a Step Function

Hence, we can compute the Fourier transform, $F_a(j\omega) = \mathcal{F}[f_a(t)]$, and then compute the Fourier transform of $\text{sign}(t)$ as the limit of $F_a(j\omega)$ as $a \to 0$.

1st goal: find $f_a(t)$

2nd goal: Compute $F_a(t)$

3rd goal: $\lim_{a \to 0} F_a(t)$

$$
\begin{aligned}
F_a(j\omega) &= \mathcal{F}[f_a(t)] \\
&= \mathcal{F}\left[e^{-at}u(t) - e^{at}u(-t)\right] \\
&= \mathcal{F}\left[e^{-at}u(t)\right] - \mathcal{F}\left[e^{at}u(-t)\right] \\
&= \frac{1}{a+j\omega} - \frac{1}{a-j\omega} \\
&= \frac{-2j\omega}{a^2+\omega^2}
\end{aligned}
$$

When $\omega = 0$, then $F_a(j\omega) = 0$ for any $a \neq 0$. Otherwise, if $\omega \neq 0$, then

Side Note $\frac{1}{j} \overset{\Delta}{=} -j$

$$
\begin{aligned}
\lim_{a\to 0} F_a(j\omega) &= \lim_{a\to 0} \frac{-2j\omega}{a^2+\omega^2} \\
&= \frac{-2j\omega}{\omega^2} \\
&= \frac{2}{j\omega}
\end{aligned}
$$

Fourier Transform of a Step Function

Limiting Fourier transform example (cont.)

With this, we can state that

$$\boxed{\operatorname{sign}(t) \iff \begin{cases} \frac{2}{j\omega}, & \omega \neq 0 \\ 0, & \omega = 0 \end{cases}}$$

DC component as a sanity check

Fourier Transform of a Step Function

The step function can be written in terms of the sign function, i.e.,

$$u(t) = \frac{1}{2} + \frac{1}{2}\text{sign}(t)$$

Therefore,

$$\begin{aligned}
\mathcal{F}[u(t)] &= \mathcal{F}\left[\frac{1}{2} + \frac{1}{2}\text{sign}(t)\right] \\
&= \frac{1}{2}2\pi\delta(\omega) + \frac{1}{2}\left(\frac{2}{j\omega}\right) \\
&= \pi\delta(\omega) + \frac{1}{j\omega}
\end{aligned}$$

$-j \triangleq \frac{1}{j}$

@ home

Note that the second term is zero at $\omega = 0$, and so the spectrum of $u(t)$ is $\pi\delta(\omega)$ at $\omega = 0$. Thus,

$$\boxed{u(t) \iff \pi\delta(\omega) + \frac{1}{j\omega}}$$

Fourier Transform of Integral (handwritten)

$$\mathcal{F}\left[\int_{-\infty}^{t} f(t)\,dt\right] = \mathcal{F}\left[f(t) * u(t)\right]$$

$$= \mathcal{F}\left[f(t)\right] \cdot \mathcal{F}\left[u(t)\right]$$

$$= F(j\omega)\left[\pi\delta(\omega) + \frac{1}{j\omega}\right]$$

$$= \boxed{\pi F(0)\,\delta(\omega) + \frac{F(j\omega)}{j\omega}}$$

Fourier Transform of an Integral (slide)

All Properties

1. **Linearity:** $$\boxed{\mathcal{F}[af_1(t) + bf_2(t)] = a\mathcal{F}[f_1(t)] + b\mathcal{F}[f_2(t)]}$$

2. **Time scaling:** $$\boxed{\mathcal{F}[f(at)] = \frac{1}{|a|}F\left(j\frac{\omega}{a}\right)}$$

3. **Time reversal:** $$\boxed{\mathcal{F}[f(-t)] = F(-j\omega)}$$

4. **Complex conjugate:** $$\boxed{f^*(t) \iff F^*(-j\omega)}$$

5. **Duality:** $$\boxed{F(t) \iff 2\pi f(-j\omega)}$$

6. **Time-shifting:** $$\boxed{\mathcal{F}[f(t-\tau)] = e^{-j\omega\tau}F(j\omega)}$$

7. **Derivative:** $$\boxed{\mathcal{F}[f'(t)] = j\omega F(j\omega)}$$

8. **Convolution:** $$\boxed{\mathcal{F}[(f_1 * f_2)(t)] = F_1(j\omega)F_2(j\omega)}$$

All Properties

9. Parseval's theorem:

$$\int_{-\infty}^{\infty} |f(t)|^2 \, dt = \frac{1}{2\pi} \int_{-\infty}^{\infty} |F(j\omega)|^2 \, d\omega$$

10. Multiplication:

$$\mathcal{F}[f_1(t) f_2(t)] = \frac{1}{2\pi} (F_1 * F_2)(j\omega)$$

11. Modulation:

$$\mathcal{F}[f(t) e^{j\omega_0 t}] \quad = \quad F(j(\omega - \omega_0))$$

12. Integral:

$$\int_{-\infty}^{t} f(\tau) d\tau \iff \pi F(0)\delta(\omega) + \frac{F(j\omega)}{j\omega}$$

FT Pairs

$$\mathrm{rect}(t/T) \iff T\,\mathrm{sinc}(\omega T/2\pi)$$

$$e^{-at}u(t) \iff \frac{1}{a+j\omega}$$

$$\mathcal{F}\left[e^{a|t|}\right] = \frac{2a}{a^2+\omega^2}$$

$$\mathrm{sinc}(t/2\pi) \iff 2\pi\mathrm{rect}(\omega)$$

$$\Delta(t) \iff \mathrm{sinc}^2(\omega/2\pi)$$

$$\mathcal{F}[\mathrm{sinc}^2(t)] = \Delta(\omega/2\pi)$$

$$\boxed{\delta(t) \iff 1}$$

$$\boxed{\delta(t-\tau) \iff e^{-j\omega\tau}}$$

$$\boxed{1 \iff 2\pi\delta(\omega)}$$

$$\boxed{u(t) \iff \pi\delta(\omega)+\frac{1}{j\omega}}$$

$$\boxed{e^{j\omega_0 t} \iff 2\pi\delta(\omega-\omega_0)}$$

$$\boxed{\cos(\omega_0 t) \iff \pi(\delta(\omega-\omega_0)+\delta(\omega+\omega_0))}$$

$$\boxed{\sin(\omega_0 t) \iff j\pi(\delta(\omega+\omega_0)-\delta(\omega-\omega_0))}$$

$x \to \boxed{h} \to y$

Frequency Response

Motivation for this lecture

★★ Very Important ★★

We previously discussed the impulse response, $h(t)$, which is the output of a system when the input is an impulse, $\delta(t)$. We saw that $h(t)$ characterized any LTI system, as for any LTI system with input, $x(t)$, we could calculate the output as

$$
\begin{aligned}
y(t) &= (x * h)(t) \\
&= \int_{-\infty}^{\infty} x(\tau)h(t-\tau)d\tau
\end{aligned}
$$

A complication we discussed is that computing the output this way requires evaluating a convolution integral, which can be difficult and time-consuming.

Frequency Response

Motivation for this lecture (cont.)

But now, equipped with the convolution theorem, why not just take the Fourier transform of both sides? This turns the convolution into multiplication.

$$Y(j\omega) = H(j\omega)X(j\omega)$$

where $X(j\omega)$ is the Fourier transform of the input, $Y(j\omega)$ is the Fourier transform of the output, and $H(j\omega)$ is the *frequency response*, i.e., the Fourier transform of the impulse response.

"Frequency Response fn" OR

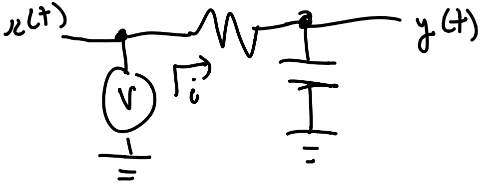
"Transfer Function"

$\mathcal{F}[y'] = j\omega Y(j\omega)$

Example: RC Circuit

$x(t) \leftrightarrow X(j\omega)$

$y(t) \leftrightarrow Y(j\omega)$



(1) $x - y = iR$

(2) $i = C\frac{dy}{dt}$

$x - y = \frac{dy}{dt} RC$ Solve through diff eqns.

$\mathcal{F}(x - y) = \mathcal{F}\left(\frac{dy}{dt} RC\right)$

$X(j\omega) - Y(j\omega) = j\omega Y(j\omega) RC$

$X(j\omega) = Y(j\omega)[1 + j\omega RC]$

$H(j\omega) = \frac{1}{1 + j\omega RC}$

$Y(j\omega) = H(j\omega) X(j\omega)$

Goal: $Y(j\omega) = X(j\omega) H(j\omega)$

or another way to look @ it $H(j\omega) = \frac{Y(j\omega)}{X(j\omega)}$

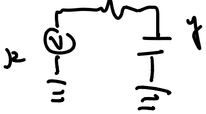
Example: RC Circuit

2nd goal : Calculate $|H(j\omega)|$

$$H(j\omega) = \frac{1}{1+j\omega RC} = \frac{1}{1+j\omega RC} \cdot \left(\frac{1-j\omega RC}{1-j\omega RC}\right) = \frac{1-j\omega RC}{1+\omega^2 R^2 C^2} \Rightarrow a+bj$$

$$|H(j\omega)|^2 = H(j\omega) \cdot H^*(j\omega) = \frac{1}{1+\omega^2 R^2 C^2}$$

$$\boxed{|H(j\omega)| = \frac{1}{\sqrt{1+\omega^2 R^2 C^2}}}$$



Example: RC Circuit

$$|H(j\omega)| = \frac{1}{\sqrt{1+\omega^2R^2C^2}}$$

$$|Y(j\omega)| = |H(j\omega)||X(j\omega)|$$

Frequency Response

- In addition to *frequency response*, $H(j\omega)$ is sometimes called the *transfer function* of the system.
- The reason its called frequency response is that $H(j\omega)$ describes how the input is changed at every single frequency.
- In particular, the frequency response scales the amplitude response by $|H(j\omega)|$, i.e.,
$$|Y(j\omega)| = |H(j\omega)||X(j\omega)|$$
- The frequency response shifts the phase response by $\angle H(j\omega)$, i.e.,

$$\angle Y(j\omega) = \angle H(j\omega) + \angle X(j\omega)$$

Frequency Response

To see this, note that if the input to a system is a complex exponential, $e^{j\omega_0 t}$ (recall, these are the eigenfunctions of an LTI system), then

$$\begin{aligned} X(j\omega) &= \mathcal{F}\left[e^{j\omega_0 t}\right] \\ &= 2\pi\delta(\omega - \omega_0) \end{aligned}$$

Therefore, the output is

$$\begin{aligned} Y(j\omega) &= H(j\omega)(2\pi\delta(\omega - \omega_0)) \\ &= H(j\omega_0)(2\pi\delta(\omega - \omega_0)) \end{aligned}$$

Frequency Response

This means that

$$\begin{aligned} y(t) &= \mathcal{F}^{-1}[Y(j\omega)] \\ &= \mathcal{F}^{-1}[H(j\omega_0)(2\pi\delta(\omega - \omega_0))] \\ &= H(j\omega_0)e^{j\omega_0 t} \\ &= |H(j\omega_0)|\, e^{j(\omega_0 t + \angle H(j\omega_0))} \end{aligned}$$

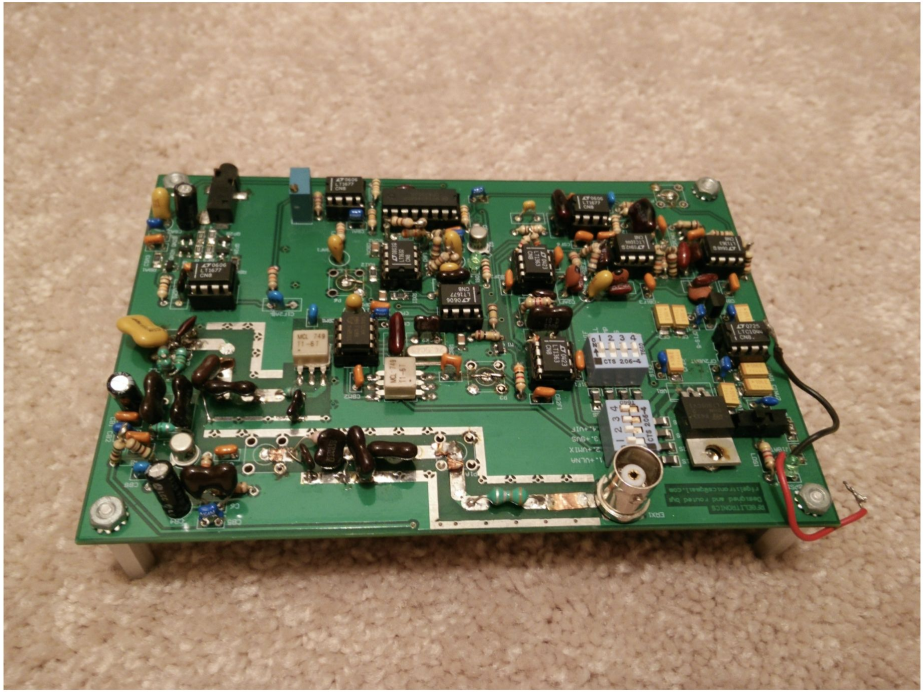
To summarize here, we input a sinusoidal input, $x(t) = e^{j\omega_0 t}$ to a LTI system, and saw that the output was

$$y(t) = |H(j\omega_0)|\, e^{j(\omega t + \angle H(j\omega_0))}$$

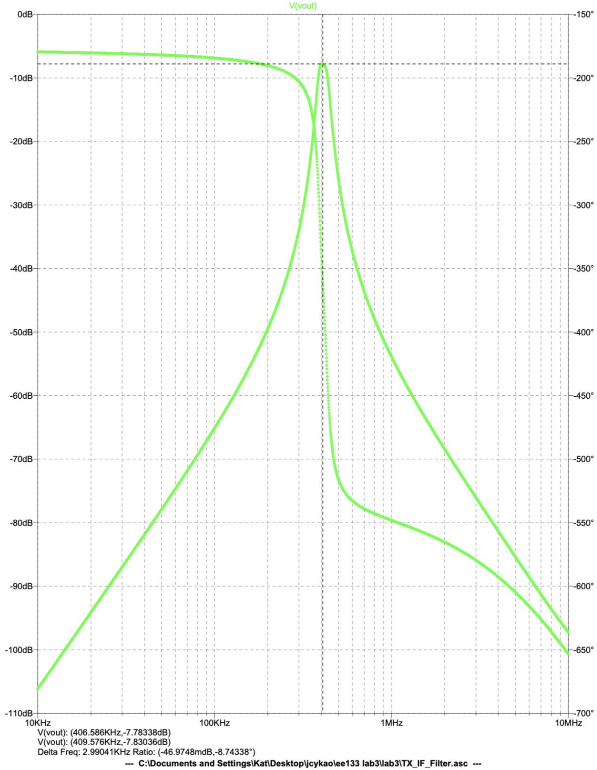
i.e., inputting a complex sinusoid to an LTI system produces an output that:

- is at the same frequency, ω_0.
- is scaled in amplitude by $|H(j\omega_0)|$.
- is phase shifted by $\angle H(j\omega_0)$.

Application: tuning circuits



Application: tuning circuits



Frequency Response Example

Consider the input:

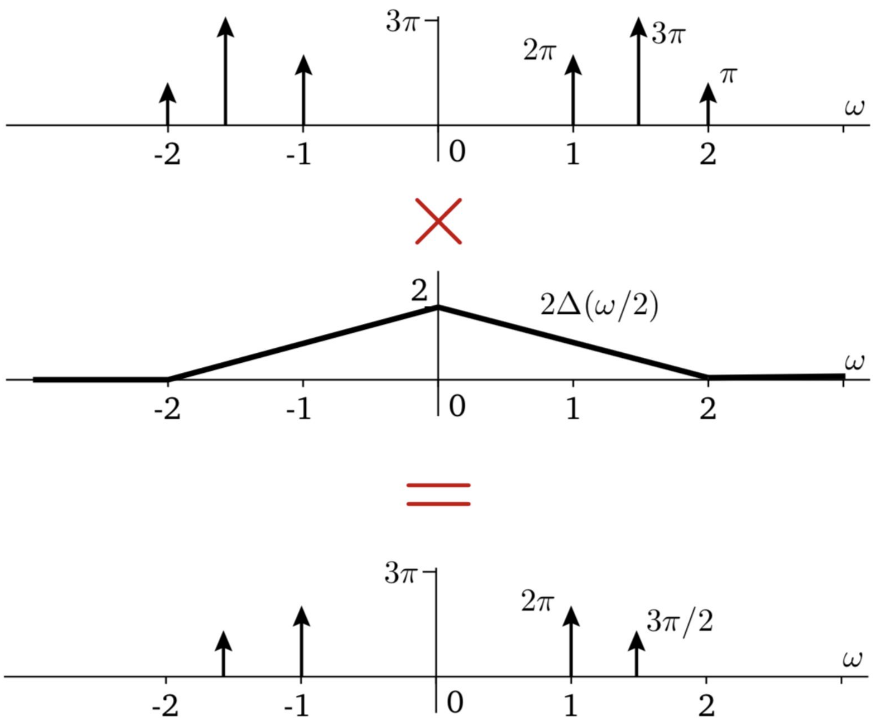
$$x(t) = 2\cos(t) + 3\cos(3t/2) + \cos(2t)$$

and system with impulse response

$$h(t) = \frac{2}{\pi}\,\mathrm{sinc}^2(t/\pi)$$

Find $y(t) = (x * h)(t)$.

Frequency Response Example



Frequency Response Example

This gives that

$$Y(j\omega) = 2\pi\left[\delta(\omega - 1) + \delta(\omega + 1)\right] + \frac{3\pi}{2}\left[\delta(\omega - 3/2) + \delta(\omega + 3/2)\right]$$

Taking the inverse Fourier transform, we get that

$$y(t) = 2\cos(t) + \frac{3}{2}\cos(3t/2)$$

Frequency Response Example 2

Let $x(t) = e^{-t}u(t)$. We input this signal into a system with impulse response:

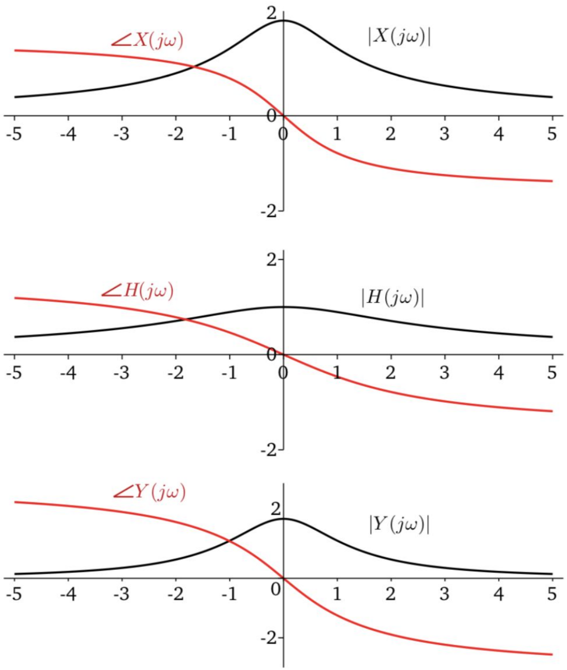
$$h(t) = 2e^{-2t}u(t)$$

What are $Y(j\omega)$ and $y(t)$?

Frequency Reponse Example 2

Frequency Response Example 2

Recall that when we multiply two complex numbers, their magnitudes multiply and their phases add. This is shown below.



Filters

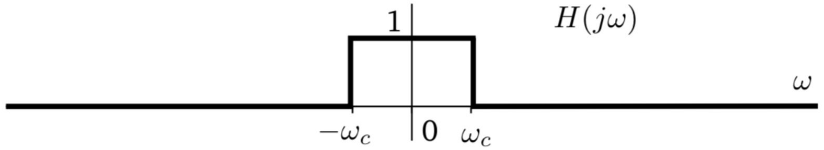
Filters are designed to extract or attenuate certain desired frequencies from a signal. For example, consider a recording of music where the microphones accidentally recorded the sopranos too loudly. It would be possible to rebalance the audio by attenuating higher frequencies in the signal.

We'll first discuss ideal filters, which only pass through certain frequencies. There are three main types of filters we'll discuss:

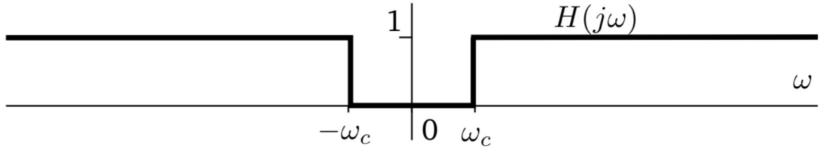
- Low pass filter: suppresses all frequencies that are higher than a specified frequency, ω_c. Its name comes from the fact that it lets frequencies less than ω_c through (i.e., low frequencies).
- High pass filter: supresses all frequencies that are lower than a specified frequency, ω_c.
- Band pass filter: supresses all frequencies outside of a range $\pm\omega_c$ around a chosen frequency ω_0.
- These three filters are illustrated on the next page.

Filter Illustration

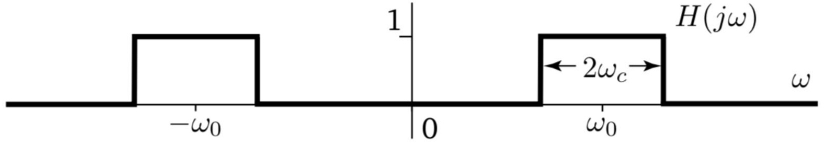
- Low pass filter:



- High pass filter:



- Band pass filter:



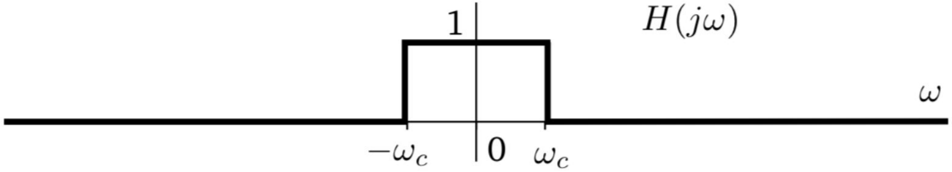
These may alternatively be written as “lowpass” or “low-pass” filter, etc.

CYU on Filters

Imagine I pass the canonical delta function through a low-pass filter. What is the output?

Ideal Low Pass Filter

The ideal low pass filter is



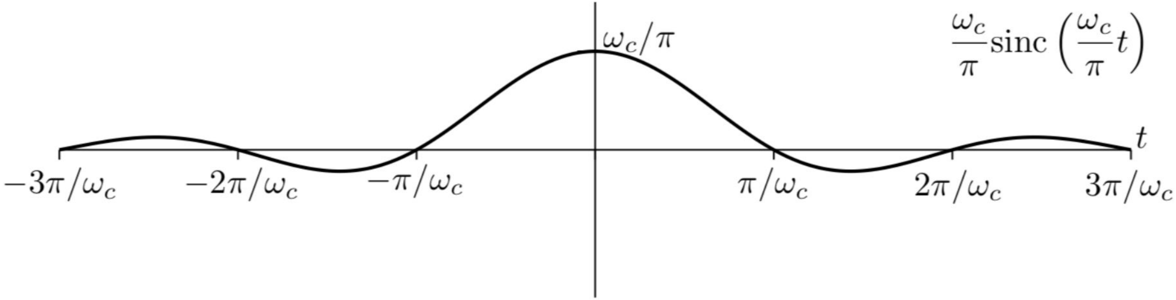
We call the region where frequencies are not suppressed (i.e., up to frequency ω_c for this ideal low pass filter) the "passband." This filter can be represented as

$$H(j\omega) = \mathrm{rect}(\omega/(2\omega_c))$$

Ideal Low Pass Filter

Ideal Low Pass Filter

Thus, the ideal low pass filter's impulse response is:



Note that we've only shown a small interval here, the sinc function is nonzero for t outside of the plotted window.

CYU from HW

The overall LTI system is described by the following differential equation:

$$\frac{d^2}{dt^2}y(t) + 5\frac{d}{dt}y(t) + 6y(t) = 3x(t)$$

i. Find the frequency response, $H(j\omega)$, of the overall system $h(t)$.