EE102

Lecture 2

EE102 Announcements

- Syllabus link is tiny.cc/ucla102
- For a class-wide bonus, each lecture, 80% of viewers should fill out the feedback
- CCLE difficulties, please email help@seas.ucla.edu
- **First Homework due Friday April 9th, 11:59pm PT.**

Slide Credits: This lecture adapted from Prof. Jonathan Kao (UCLA) and recursive credits apply to Prof. Cabric (UCLA), Prof. Yu (Carnegie Mellon), Prof. Pauly (Stanford), and Prof. Fragouli (UCLA)

Roadmap

- We will first build up basic operations and properties on signals and systems, before then building up to new topics.
- The first 2-3 weeks will feel math-heavy, but not application heavy.
- This is to build the foundation.

Concepts covered in This Lecture

Signal operations and properties

This lecture overviews several mathematical operations and properties that will provide a foundation for the rest of the class. It jumps between various topics as we need to know all of these before moving on.

- Time scaling, reversal and shifting.
- Even and odd signals
- Periodicity
- Review of sinusoids and complex numbers
- Causality
- Energy and power signals
- Euler's formula

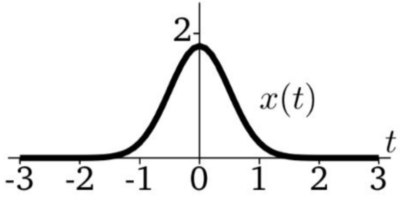
Discrete vs Continuous Signals

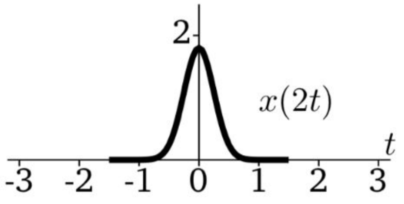
Amplitude Scaling

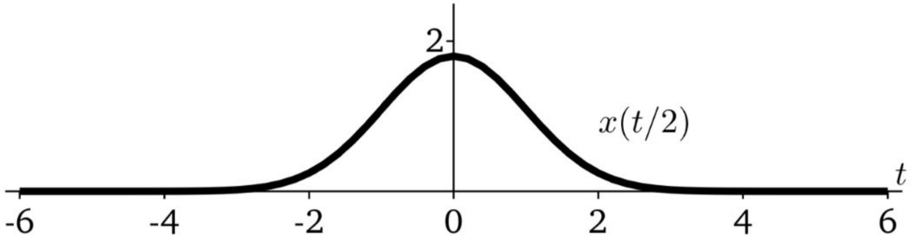
Time Scaling

Time Scaling

- If $a > 1$ then the signal is compressed in time.
- If $0 < a < 1$ then the signal is expanded in time.







As you work on examples of this, it is sometimes helpful to plug in values of t to make sure you have compressed / expanded the values correctly.

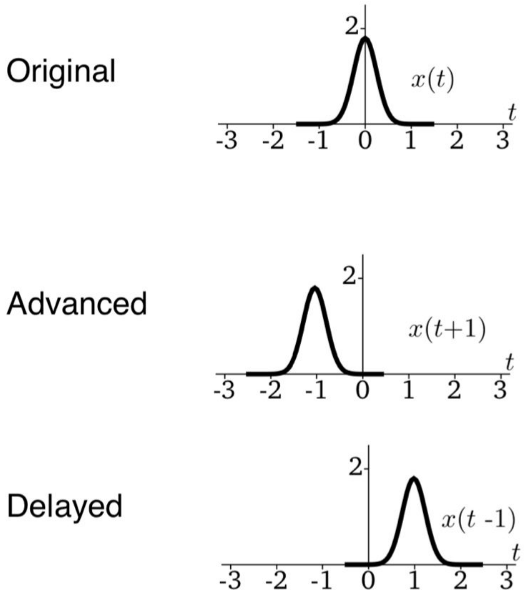
Time Reversal

Time Shifting

A signal $x(t)$ can be shifted in time by some amount $t_1 > 0$.

Time Shifting

- The signal $x(t - t_1)$ is delayed in time by t_1.
- The signal $x(t + t_1)$ is advanced in time by t_1.



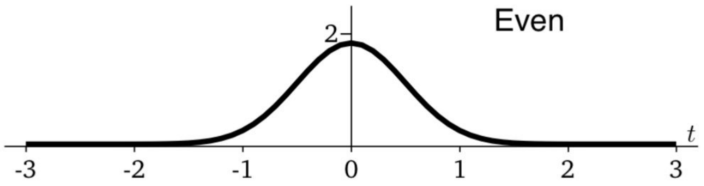
As you work on time shift examples, it may be helpful to consider when $t - t_1 = 0$.

Combining Operations

Combining Operations

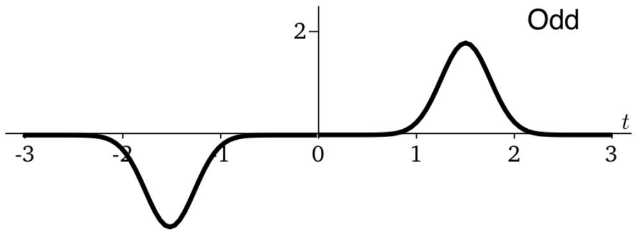
Even and Odd Signals

- An *even* signal is symmetric about $t = 0$,



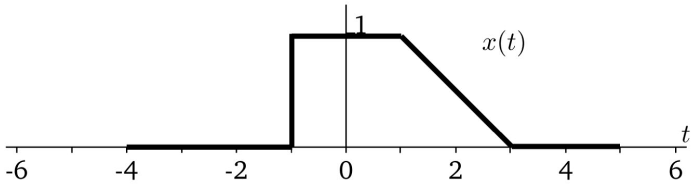
Even and Odd Signals

- An *odd* signal is antisymmetric about the origin

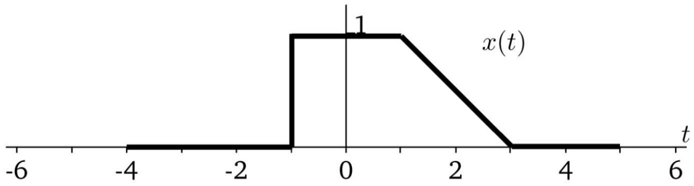


Even and Odd Decomposition

CYU Example



CYU Example (Cont'd)



Periodic Signals

The concept of periodic signals is very important in this class. Colloquially, these are signals that repeat after a given interval, T_0.

Periodic Signal Properties

Sinusoids

The most basic signal in this class is the sine or cosine wave. We'll use them *extensively* so it's worth reviewing their properties. By the end of this class, you'll be proficient at manipulating sinusoids.

A cosine is defined by:

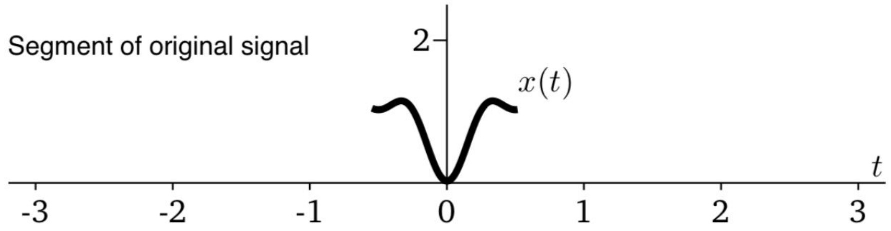
Trigonometric Rules

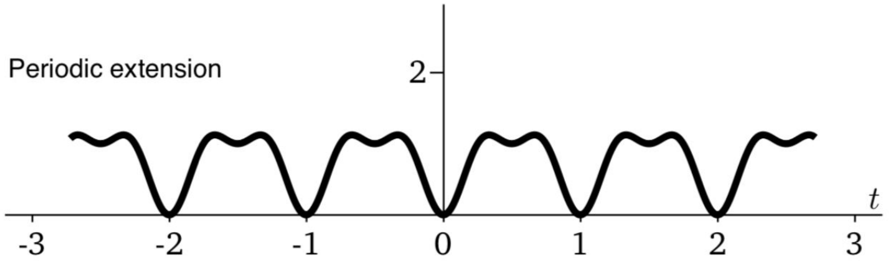
Some additional properties that you should be familiar with from trigonometry:

- $\sin(\theta) = \cos(\theta - \pi/2)$.
- Are either $\cos(\theta)$ or $\sin(\theta)$ even or odd?
- $\frac{d}{dt}\sin(\theta) = \cos(\theta)$ and $\frac{d}{dt}\cos\theta = -\sin(\theta)$.
- $\sin^2(\theta) + \cos^2(\theta) = 1$.

Periodic Extension

In this class, we will sometimes be interested in taking an aperiodic signal and making its periodic extension. What this mean is that we take some interval on this signal of length T_0 and repeat it, as illustrated below:





CYU Question

Is the sum of the following two signals periodic?

Causality

Complex Numbers Review

So far all signals we've presented are real-valued. But signals can also be complex.

- A complex signal is one that takes the form:

$$z(t) = x(t) + jy(t)$$

where $x(t)$ and $y(t)$ are real-valued signals and $j = \sqrt{-1}$.

Complex Numbers Review

Because complex numbers play a large role in this class, we'll briefly review them.

- A complex number is formed from two real numbers, x and y, via:

  $$z = x + jy$$

  with $j = \sqrt{-1}$. Hence, a complex number is simply an ordered pair of real numbers, (x, y).
- $x = \Re(z)$ is called the *real* part of z. (In this class we will also write $x = \mathrm{Re}(z)$.)
- $y = \Im(z)$ is called the *imaginary* part of z. (In this class we will also write $y = \mathrm{Im}(z)$.)

- An aside: why do EE's use j as the imaginary number, while mathematicians and scientists commonly use i?

Complex Numbers Review

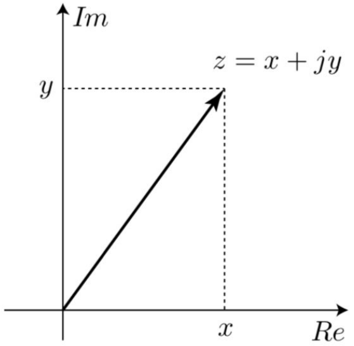
Polar representation of complex numbers

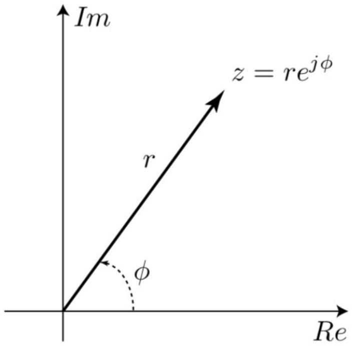
The same complex number can be written in polar form,

$$\begin{aligned} z &= x + jy \\ &= re^{j\phi} \end{aligned}$$

where

- r is the *modulus* or *magnitude* of z.
- ϕ is the *angle* or *phase* of z.
- $e^{j\phi} = \cos(\phi) + j\sin(\phi)$. We will sometimes write this as $\exp(j\phi)$. (More on this below.)





Complex Numbers Review

Cartesian vs polar coordinates

$$
\begin{aligned}
z &= x + jy \\
&= re^{j\phi}
\end{aligned}
$$

Here, the same intuitions from Cartesian and polar coordinates hold.

- $x = r\cos(\phi)$
- $y = r\sin(\phi)$
- $r = \sqrt{x^2 + y^2}$
- $\phi = \arctan y/x$

Euler's identity

Relating terms in our Cartesian and polar coordinate representation of complex numbers, we arrive at Euler's formula:

$$\begin{aligned} z &= x + jy \\ &= re^{j\phi} \end{aligned}$$

This tells us that, for $r = 1$,

$$\boxed{e^{j\phi} = \cos(\phi) + j\sin(\phi)}$$

Aside: this leads to one of the most elegant equations in mathematics:

$$e^{i\pi} + 1 = 0$$

With five terms, it incorporates Euler's constant (e), pi (π), the imaginary number (i), the multiplicative identity (1) and the additive identity (0).

CYU Question

Complex Conjugate

Some complex relations

Here are a few relations.

- **Complex conjugate**. If $z = x + jy$, then z^*, the complex conjugate of z, is
$$z^* = x - jy$$
- **Modulus and complex conjugate**. The following relation holds:
$$|z|^2 = z^* z = z z^*$$
This is because
$$\begin{aligned} zz^* &= (x + jy)(x - jy) \\ &= x^2 + y^2 \\ &= r^2 \end{aligned}$$
where $r = \sqrt{x^2 + y^2}$ as on the last slide.
- **Inverse of j**. Since $j^2 = -1$, we have that $-j = \frac{1}{j}$.

Signal Energy and Power

Signal power has units of Watts (Joules per time). Hence, to get the total energy of a signal, $x(t)$, across all time, we integrate the power.

$$\boxed{E_x = \lim_{T \to \infty} \int_{-T}^{T} |x(t)|^2 \, dt}$$

(We incorporate the absolute value, $|\cdot|$, in case $x(t)$ is a complex signal, reviewed in the next slides.) Like signal power, signal energy is usually not a *actual* energy.

We can also calculate the *average power* of the signal by calculating:

$$\boxed{P_x = \lim_{T \to \infty} \frac{1}{2T} \int_{-T}^{T} |x(t)|^2 \, dt}$$

Can we simplify this expression to obtain the power of a periodic signal?