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EE102

Lecture 4

EE102 Announcements

- Syllabus link is ~~tinyurl.com/ucla102~~ tiny.cc/ucla102
- CCLE difficulties, please email help@seas.ucla.edu
- ~~My office hour meeting minutes are sent out weekly~~
- **First Homework due this Friday**

Slide Credits: This lecture adapted from Prof. Jonathan Kao (UCLA) and recursive credits apply to Prof. Cabric (UCLA), Prof. Yu (Carnegie Mellon), Prof. Pauly (Stanford), and Prof. Fragouli (UCLA)

Sidebar: Regarding Periodic Signals

The sum or product of a periodic signal is itself periodic if:

x_1 : period T_1 $\longrightarrow$ $\exists\, T_1 \quad f(t+T_1) = f(t) \quad \forall\, t.$

x_2 : period T_2

when is z periodic and what's the period?

cyu $z = x_1 + x_2 \quad \ldots$

$\square$ We have established that z is periodic if $\exists\, T$ s.t.

$$T = k_1 T_1 = k_2 T_2 .$$

$$\frac{T_1}{T_2} = \frac{k_2}{k_1}$$

cyu: See if you can find k_2, k_1 that make z aperiodic

CYU: Come up with an x_1, x_2 s.t. sum is aperiodic.

irrational number

$$\frac{T_1}{T_2} = c\pi$$

$$\cos(\pi t) + \cos(2t) = z$$

aperiodic

Signal Models

$\omega = 2\pi f$

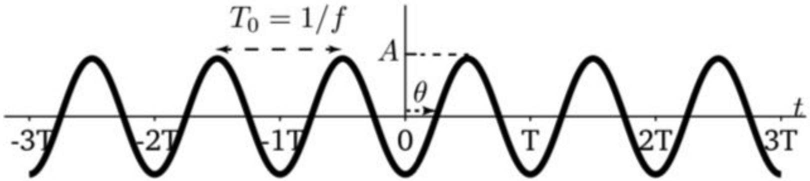
Real sinusoids (cont.)

$f = \frac{\omega}{2\pi}$, $T_0 = \frac{1}{f} = \frac{2\pi}{\omega}$

Fundamental Frequency

We illustrate a sinusoid signal below:

$$x(t) = A\cos(\omega t - \theta)$$



Signal Models

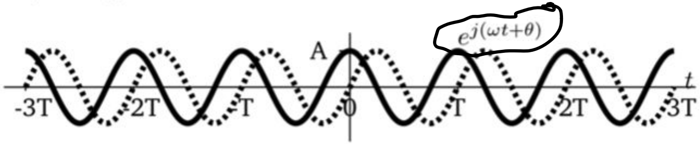
Eulers

Complex sinusoids

The complex sinusoid is given by:

$$Ae^{j(\omega t+\theta)} = A\cos(\omega t+\theta) + jA\sin(\omega t+\theta)$$

We draw complex signals with dotted lines.



The real part of the complex sinusoid (solid line) is:

$$\Re\left(Ae^{j(\omega t+\theta)}\right) = A\cos(\omega t+\theta)$$

The imaginary part of the complex sinusoid (dotted line) is:

$$\Im\left(Ae^{j(\omega t+\theta)}\right) = A\sin(\omega t+\theta)$$

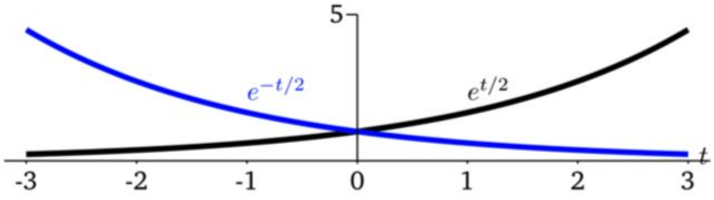
Signal Models

Exponential

An exponential signal is given by

$$x(t) = e^{\sigma t}$$

- If $\sigma > 0$, this signal grows with increasing t (black signal in plot below). This is called exponential growth.
- If $\sigma < 0$, this signal decays with increasing t (blue signal in plot below). This is called exponential decay.



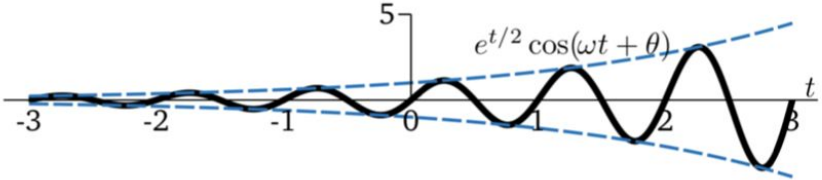
Signal Models

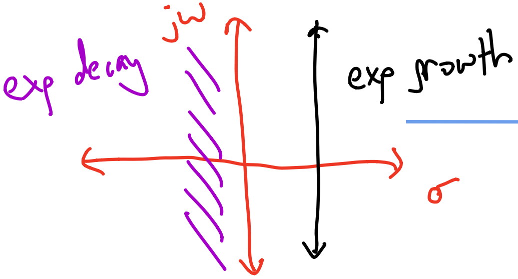
Damped or growing sinusoids

A damped or growing sinusoid is denoted

$$x(t) = e^{\sigma t} \cos(\omega t + \theta)$$

The sinusoid will grow exponentially if $\sigma > 0$ and decay exponentially if $\sigma < 0$.



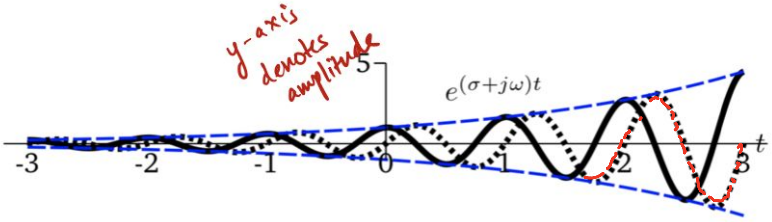


Complex exponential

A complex sinusoid is denoted

$$x(t) = e^{(\sigma + j\omega)t}$$

It is a combination of the complex sinusoid and an exponential. All prior signals are special cases of the complex exponential signal.



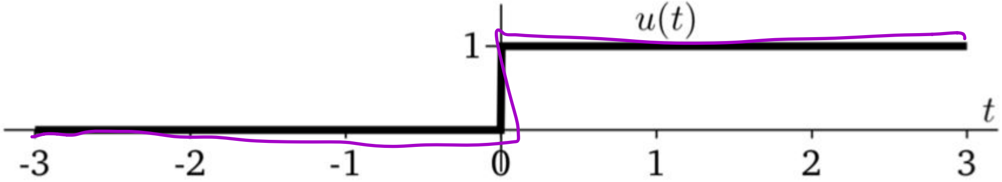
It is helpful to think of σ and $j\omega$ in the complex plane. σ is the x-axis and $j\omega$ is the y-axis. Then complex exponentials in the left complex plane are decreasing signals and those in the right are increasing signals.

Heaviside Step Function

The unit step function, denoted $u(t)$ in this class, is given by

$$u(t) = \begin{cases} 1, & t \geq 0 \\ 0, & t < 0 \end{cases}$$

It is also called the Heavyside step function. Drawn below:



CYU: Is this causal

Answer: yes

LiDAR → Tx

Unit Rectangle

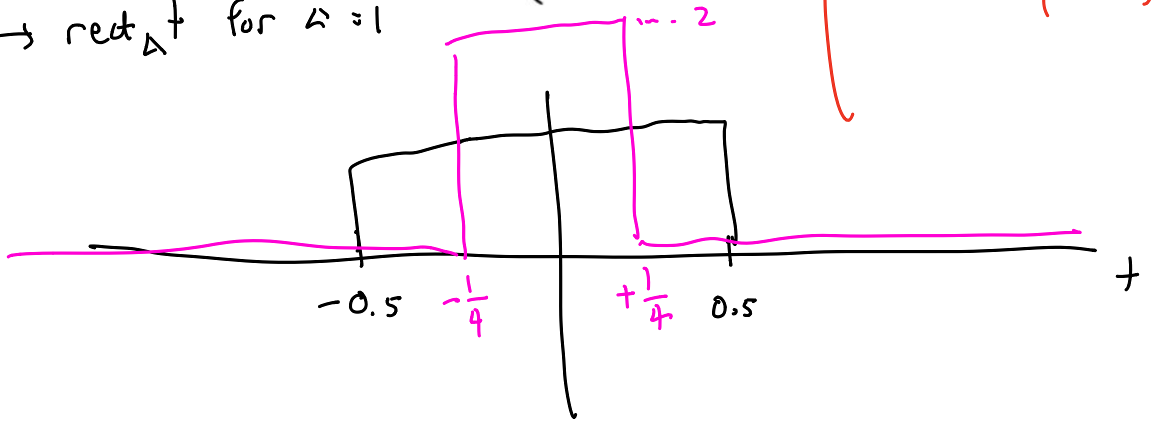
"Boxer"?

rect(t) width of 1

$$\mathrm{rect}(t) = \begin{cases} 1, & |t| \le 1/2 \\ 0, & \text{else} \end{cases}$$

$$\mathrm{rect}_\Delta(t) = \begin{cases} \frac{1}{\Delta}, & |t| \le \frac{\Delta}{2} \\ 0, & \text{o.w.} \end{cases}$$

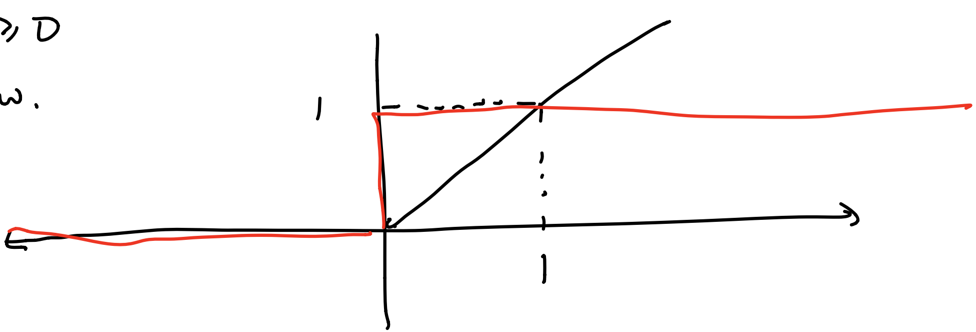
rect(t) → $\mathrm{rect}_\Delta t$ for $\Delta = 1$



$\mathrm{rect}_{\Delta=\frac{1}{2}}(t)$?

Unit Ramp Function "ReLU"

$$r(t) = \begin{cases} t, & t \geq 0 \\ 0, & \text{o.w.} \end{cases}$$



CYU: How can I express $r(t)$ in terms of the building blocks I've learned?

$$r(t) = \int_{-\infty}^{t} u(\tau)\, d\tau$$

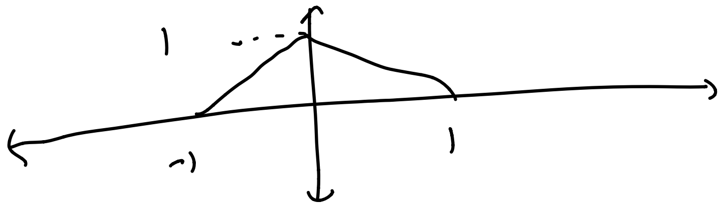
τ: dummy variable.

integral bounds contain time

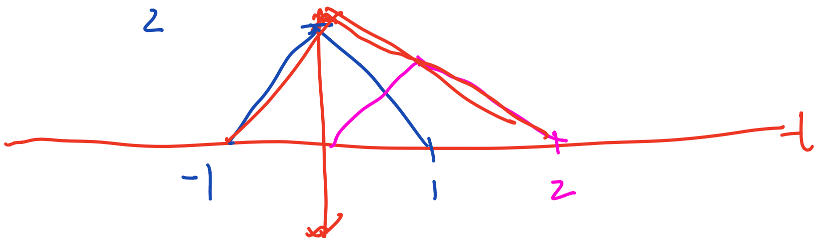
$$r(t) = t\,u(t)$$

Unit Triangle

$$\Delta(t) = \begin{cases} 1-|t|, & |t| \le 1 \\ 0, & \text{o.w.} \end{cases}$$



// $z = 2\Delta(t) + \Delta(t-1)$

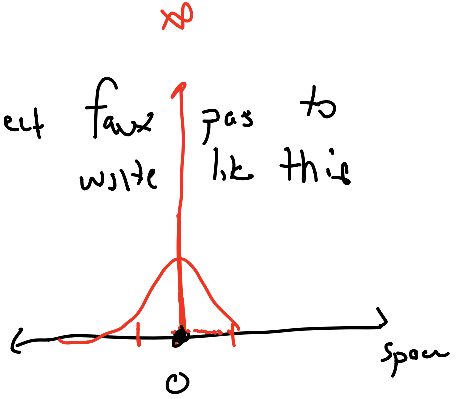


Dirac Impulse Function (Important!)

Impulse Function (Dirac Delta Fn)

$$\cancel{\delta(t) = \begin{cases} \infty, & t=0 \\ 0, & \text{o.w.} \end{cases}}$$

Incorrect faux pas to write like this



Properties of $\delta(t)$

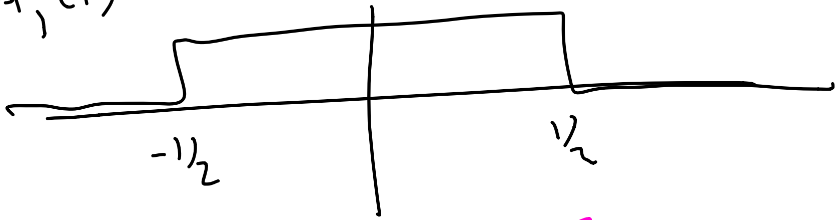
① It's very large (i.e. it approaches ∞ when $t=0$)

② It's zero everywhere $t \neq 0$

③ The area of this fn is 1.

Impulse Function (intuition)

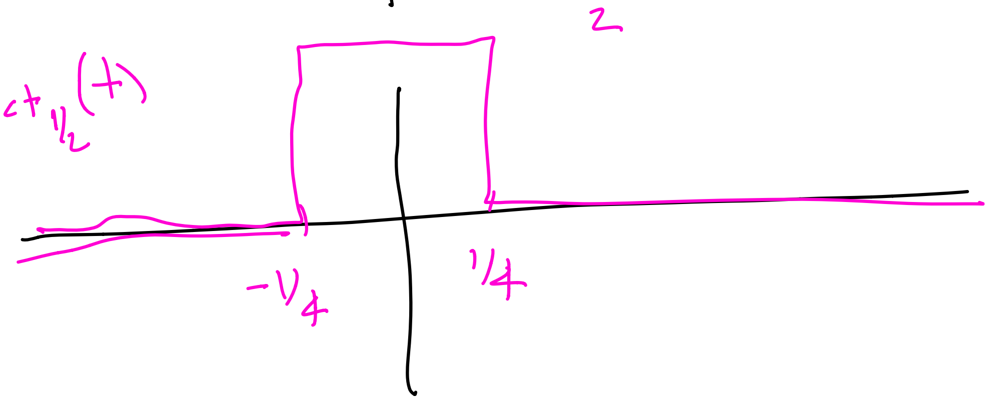
$\mathrm{rect}_1(t)$



$\delta(t)$ as being $\mathrm{rect}_{\Delta \to 0}(t)$

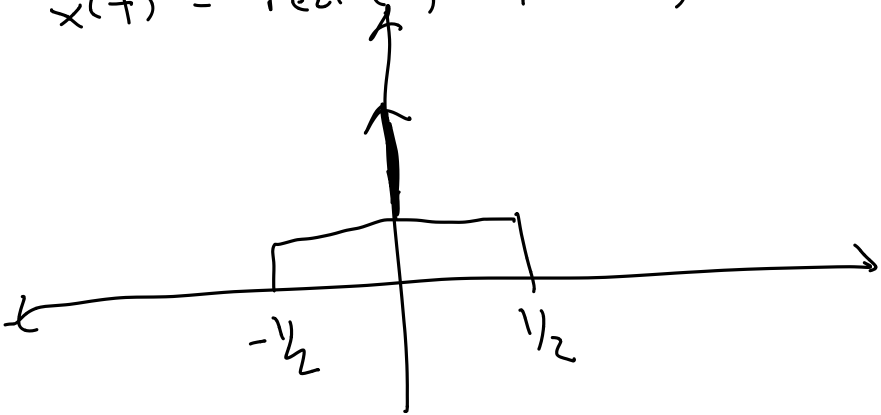
$\lim_{\Delta \to 0} \mathrm{rect}_\Delta(t)$

$\mathrm{rect}_{1/2}(t)$



Impulse Function Intuition

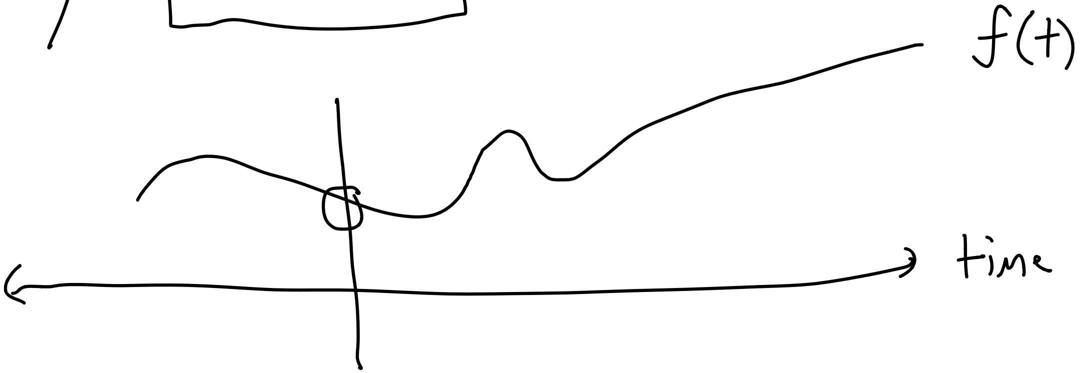
$$x(t) = \text{rect}(t) + \delta(t)$$



Impulse Sampling Property

$f(\underbrace{t=0})$

$\underbrace{f(t)\,\delta(t)} = f(0)\,\delta(t)$



$$f(t)\,\mathrm{rect}_{0.001}(t) \approx f(0)\,\mathrm{rect}_{0.001}(t)$$

Impulse Sampling Property

$$\int_{-\infty}^{\infty} f(t)\,\delta(t)\,dt = \int_{-\infty}^{\infty} f(0)\,\delta(t)\,dt$$

$$= f(0)\underbrace{\int_{-\infty}^{\infty} \delta(t)\,dt}_{1}$$

$$= f(0)$$

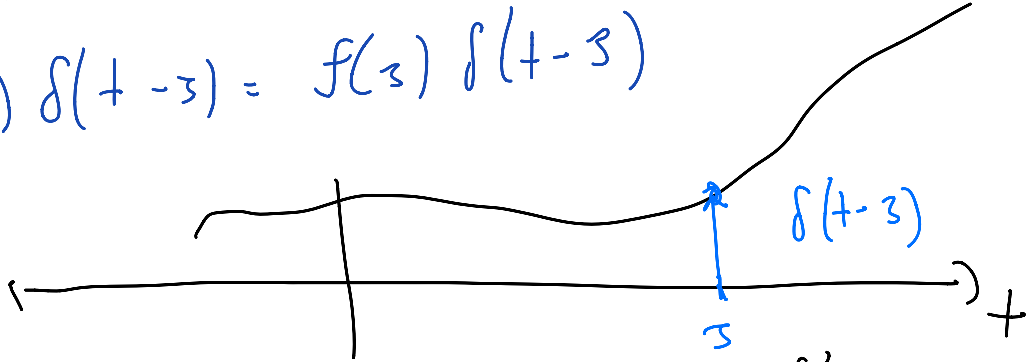
Dirac Delta has area of 1.

$f(t)$ to be continuous @ $t=0$.

Impulse Sifting Property

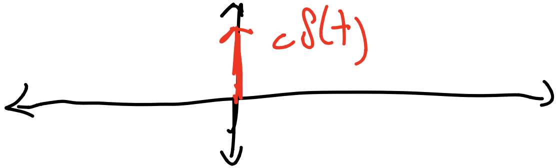
$$\int_{-\infty}^{\infty} f(t)\,\delta(t-3)\,dt = f(3)$$

$$f(t)\,\delta(t-3) = f(3)\,\delta(t-3)$$



// $c\delta(t)$

$$\int_{-\infty}^{\infty} f(t)\,c\,\delta(t)\,dt = c\,f(0)$$



Impulse Sifting Property

$$\int_{-\infty}^{\infty} \delta(t)\, dt = 1$$

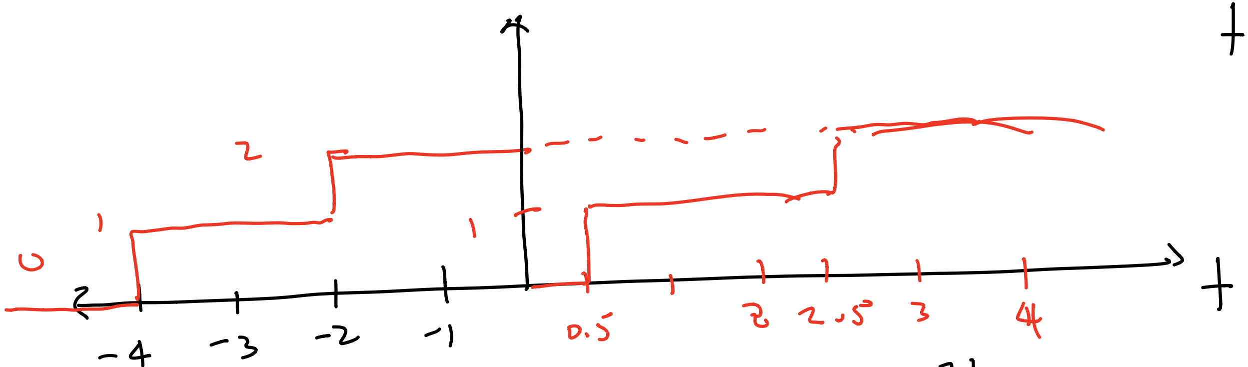
$$\int_{-\infty}^{0^-} \delta(t)\, dt = 0$$

$$\int_{-\infty}^{0^+} \delta(t)\, dt = 1$$

CYU: Calculate

$$\int_{-2}^{3+} f(t)\,[1+\delta(t+1) - 3\delta(t-1) + 2\delta(t+3)]\,dt$$

$t = -3$



$$\int_{-2}^{3+} f(t)\,dt + f(-1) - 3f(1) + \underbrace{2\int_{-2}^{3+} f(t)\,\delta(t+3)\,dt}_{0}$$

$$7 + 2 - 3 + 0 = 6$$

CYU: Integral of an Impulse

$$\int_{-\infty}^{t} \delta(\tau)d\tau = u(t) = \begin{cases} 1, & t \geq 0 \\ 0, & \text{o.w} \end{cases}$$

Integral of Dirac $\Rightarrow$ Step

Derivative of Step $\Rightarrow$ Dirac

$$\frac{d\,u(t)}{dt} = \delta(t)$$

$$\delta(t) \;\underset{\text{int}}{\downarrow}\;\overset{\text{der}}{\uparrow}\; u(t) \;\underset{\text{int}}{\downarrow}\;\overset{\text{der}}{\uparrow}\; r(t)$$

CYU (Visual)

Suppose $x(t) = 1 + \delta(t-1) - 2\delta(t-2)$ then what is $y(t) = \int_0^t x(\tau)d\tau$

Systems

A system transforms an input signal, x(t), into an output signal, y(t).

Systems, like signals, are similar to functions. However, they map a signal to another signal, so the term we might use is “operator”.

For EE102, we will not nitpick this distinction and focus on SISO systems.