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ECE 102 HW1

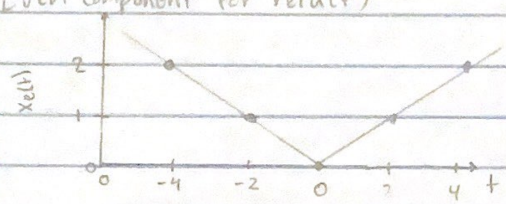
1. $x(t) = \begin{cases} t & t \geq 0 \\ 0 & t < 0 \end{cases}$

Replace odd and even

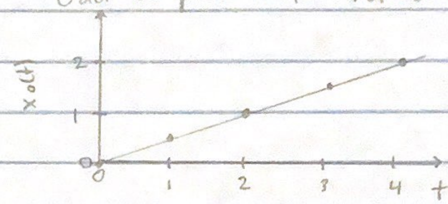
$x_e(t) = \frac{1}{2}(x(t) + x(-t)) \longrightarrow x_e(t) = \frac{1}{2}|t|$

$x_o(t) = \frac{1}{2}(x(t) - x(-t)) \longrightarrow x_o(t) = \frac{1}{2}t$

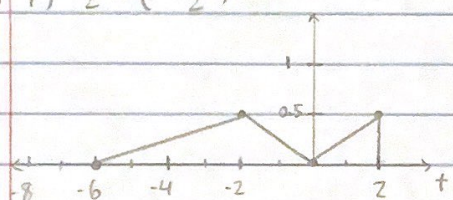
Even Component for relu(t)



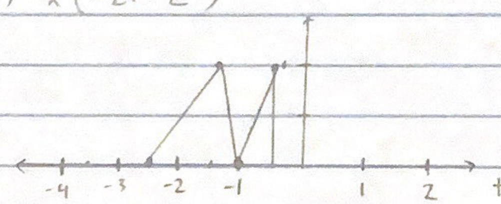
Odd Component for relu(t)



2. a) i) $\frac{1}{2}x(-\frac{t}{2})$



ii) $x(-2t-2)$



b) i) $y(t) = z(-2t-2)$ $\longrightarrow$ compress, mirror, shift

ii) $z(t) = y(-\frac{1}{2}t - 1)$ $\longrightarrow$ expand, mirror, shift

3. a) i) $x_1(t) = \sin(\frac{5t}{6} + \frac{\pi}{3})$

This signal is periodic. Its period is $2\pi \div (\frac{5}{6}) = \frac{12\pi}{5}$ sec.

The frequency is $\frac{5}{12\pi}$ Hz.

ii) $x_2(t) = \cos^2(3\pi t)$

This signal is periodic. Its period is $\frac{1}{3}$ sec. The frequency is 3 Hz.

iii) $x_3(t) = x_1(t) + x_2(t) \longrightarrow x_3(t) = \sin(\frac{5t}{6} + \frac{\pi}{3}) + \cos^2(3\pi t)$

This signal is not periodic. When graphed, we can see that. If P_1 is the period of $x_1(t)$ and P_2 is the period of $x_2(t)$, you would want a ratio where both can be multiplied by a constant to form a rational ratio. $\frac{P_1}{P_2} = \frac{x}{y}$ yields $\frac{36\pi}{5}$, which isn't rational. Therefore the signal isn't periodic.

(iv) $x_4(t) = e^t x_1(t) \rightarrow e^t \cdot \sin\left(\frac{5t}{6} + \frac{\pi}{3}\right)$

This signal is not periodic. The addition of the e^t means the signal will grow exponentially over time. This would render the signal non-periodic.

(v) $x_5(t) = e^{j(\pi t+1)} x_2(t) \rightarrow e^{j(\pi t+1)} \cdot \cos^2(3\pi t)$

$$x_5(t) = e^{j(\pi t+1)} \cdot \frac{1}{2}\left(1+\cos(6\pi t)\right)$$

$$x_5(t) = e^{j(\pi t+1)} \cdot \left(1 + \frac{1}{2}\left(e^{j6\pi t} + e^{-j6\pi t}\right)\right)$$

$$x_5(t) = \frac{1}{2} e^{j}\left(\underset{2}{2e^{j\pi t}} + \underset{2/7}{e^{j7\pi t}} + \underset{2/5}{e^{-j5\pi t}}\right)$$

This signal is periodic based on logic from iii). Period is 2 sec, frequency is $\frac{1}{2}$ Hz.

b) $y(t) = x_1(t) + x_2(t)$ $\quad$ $y(t)$ is periodic with period T_0

They do not both need to be periodic.

Say that $\quad x_1(t) = e(t) + z(t)$

$\quad x_2(t) = z(t) - e(t)$

And assume $z(t)$ is periodic but $e(t)$ is not periodic.

This means neither $x_1(t)$ nor $x_2(t)$ will be periodic.

$y(t) = [e(t) + z(t)] + [z(t) - e(t)] = 2z(t)$ $\quad$ periodic ✓

This $y(t)$ is periodic then — $x_1(t)$ and $x_2(t)$ don't have to be periodic.

If it was $y(t) = x_1(t) \cdot x_2(t)$:

Say that $\quad x_1(t) = z(t) \div e(t)$

$\quad x_2(t) = z(t) \cdot e(t)$

$y(t) = [z(t) \div e(t)] \cdot [z(t) \cdot e(t)] = z^2(t)$

Since $z(t)$ was defined to be a periodic function, $z^2(t)$ would also be periodic. Thus, in a similar manner as above, $x_1(t)$ and $x_2(t)$ do not need to be periodic for $y(t)$ to be periodic.

4. a) i) $x(t) = e^{-|t|}$

$E = \int_{-\infty}^{\infty} |e^{-|t|}|^2 dt$

$E = \int_{-\infty}^{0} e^{2t} dt + \int_{0}^{\infty} e^{-2t} dt$

$E = 2\int_{0}^{\infty} e^{-2t} dt = -e^{-2t} \Big]_0^{\infty}$ $\boxed{E = 1}$

This is an energy signal, its power is 0.

ii) $x(t) = 1 + e^{-|t|}$

$E = \int_{-\infty}^{\infty} |1 + e^{-|t|}| dt$

$E = \int_{-\infty}^{\infty} 1 + 2e^{-|t|} + e^{-2|t|} dt$ $E = \infty$ [diverges]

This is not an energy signal. To determine the power:

$P = \lim_{T \to \infty} \frac{1}{2T} \int_{-T}^{T} (1 + e^{-|t|})^2 dt$

$P = 2 \cdot \lim_{T \to \infty} \frac{1}{2T} \int_{0}^{T} (1 + e^{-t})^2 dt$

$P = 2 \cdot \lim_{T \to \infty} \frac{1}{2T} \int_{0}^{T} (1 + 2e^{-t} + e^{-2t}) dt$

$P = 2 \cdot \lim_{T \to \infty} \frac{1}{2T} \cdot T$ $\boxed{P = 1}$

b) if $x(t)$ is even and $y(t)$ is odd, then $x(t)\, y(t)$ is an odd signal

Even: $x(t) = x(-t)$ Odd - $y(t) = y(-t)$

$x(-t) \cdot y(-t) = x(t) \cdot [-y(t)] = -x(t) \cdot y(t)$

$x(-t) \cdot y(-t) = -[x(t) \cdot y(t)]$

This shows that $x(t) \cdot y(t)$ will be an odd signal.

if $z(t)$ is odd, show for any $T > 0$ we have:

$$\int_{-T}^{T} z(t)\, dt = 0$$

Split in two: $\int_{-T}^{0} z(t) dt + \int_{0}^{T} z(t) dt = \int_{-T}^{T} z(t) dt$

Substitution

$t = -x$ $-\int_{T}^{0} z(-x) dx + \int_{0}^{T} z(t) dt = \int_{-T}^{T} z(t) dt$

Flip $\int_{0}^{T} z(-x) dx + \int_{0}^{T} z(t) dt = \int_{-T}^{T} z(t) dt$

$z(t)$ is odd $\rightarrow$ $-z(x) = z(-x)$

$\int_{0}^{T} z(t) dt - \int_{0}^{T} z(x) dx = 0$

$\int_{-T}^{T} z(t) dt = 0$

$E_x = E_{x_e} + E_{x_o}$ $\qquad$ $x(t)$ real signal

$E_x = \int_{-\infty}^{\infty} |x(t)|^2 dt$

$E_x = \int_{-\infty}^{\infty} | x_e(t) + x_o(t)|^2 dt$

$E_x = \int_{-\infty}^{\infty} x_e^2(t) + x_o^2(t) + \underbrace{2x_e(t)x_o(t)}_{\downarrow} dt$

This signal is odd, so it is equal to zero. This was proved earlier.

$E_x = \int_{-\infty}^{\infty} x_e^2(t)dt + \int_{-\infty}^{\infty} x_o^2(t)dt$ ✓

$E_x = E_{x_e} + E_{x_o}$

5. a) i) $\cos^2(\theta) + \sin^2(\theta) = 1$

Eulers: $e^{j\theta} = \cos(\theta) + j\sin(\theta)$ $\qquad$ $e^{-j\theta} = \cos(\theta) - j\sin(\theta)$

$[\cos(\theta) + j\sin(\theta)] \cdot [\cos(\theta) - j\sin(\theta)] = \cos^2(\theta) + \sin^2(\theta)$

$\cos^2(\theta) + \sin^2(\theta) = e^{j\theta} \cdot e^{-j\theta} = 1$

ii) $\cos(\theta + \psi) = \cos(\theta)\cos(\psi) - \sin(\theta)\sin(\psi)$

$\cos(\theta)\cos(\psi) = \frac{1}{4}\left[e^{j(\theta+\psi)} + e^{-j(\theta+\psi)} + e^{j(\theta-\psi)} + e^{j(\psi-\theta)}\right]$

$\sin(\theta)\sin(\psi) = -\frac{1}{4}\left[-e^{j(\theta+\psi)} - e^{-j(\theta+\psi)} + e^{j(\theta-\psi)} + e^{j(\psi-\theta)}\right]$

cancel out $\qquad$ cancel out

$\frac{1}{4}\left[e^{j(\theta+\psi)} + e^{-j(\theta+\psi)}\right] + \frac{1}{4}\left[e^{j(\theta+\psi)} + e^{-j(\theta+\psi)}\right]$

$= \frac{1}{2}\left[e^{j(\theta+\psi)} + e^{-j(\theta+\psi)}\right]$

$\cos(\theta + \psi) = \frac{1}{2}\left[e^{j(\theta+\psi)} + e^{-j(\theta+\psi)}\right]$ ✓

5 b) $x(t) = (1-\sqrt{3}j)e^{j(t+2)}$ $\qquad y(t) = \frac{1}{1+j}$

i) $x(t) = (1-\sqrt{3}j)e^{j(t+2)}$

$x(t) = (1-\sqrt{3}j)(\cos(t+2) + j\sin(t+2))$

$$\boxed{x(t) = \underbrace{\cos(t+2) - \sqrt{3}\sin(t+2)}_{\text{real}} + \underbrace{[\sin(t+2) - \sqrt{3}\cos(t+2)]}_{\text{imaginary}}j}$$

$y(t) = \frac{1}{1+j}$

$y(t) = \frac{1}{1+j}\left(\frac{1-j}{1-j}\right) = \frac{1-j}{(1-j)(1+j)}$

$$\boxed{y(t) = \underbrace{\tfrac{1}{2}}_{\text{real}} - \underbrace{\tfrac{1}{2}j}_{\text{imaginary}}}$$

ii) $x(t) = (1-\sqrt{3}j)e^{j(t+2)}$ $\qquad \cos(\frac{\pi}{3}) - \sin(\frac{\pi}{3})$

$x(t) = (\cos(\frac{\pi}{3}) - \sin(\frac{\pi}{3}))e^{j(t+2)}$ $\qquad \frac{1}{2} \quad - \quad \frac{\sqrt{3}}{2}$

$x(t) = 2e^{-j\frac{\pi}{3}} \cdot e^{j(t+2)}$

$x(t) = 2e^{j(t+2-\frac{\pi}{3})}$

$$\boxed{\begin{array}{l}\text{The magnitude is } 2. \\ \text{The phase is } (t+2-\frac{\pi}{3}) \text{ rad}\end{array}}$$

$y(t) = \frac{1}{1+j}$

$y(t) = \frac{1}{\sqrt{2}} \cdot \left[\frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2}j\right]$

$y(t) = \frac{1}{\sqrt{2}} \cdot [\cos(\frac{\pi}{4}) - \sin(\frac{\pi}{4})j]$

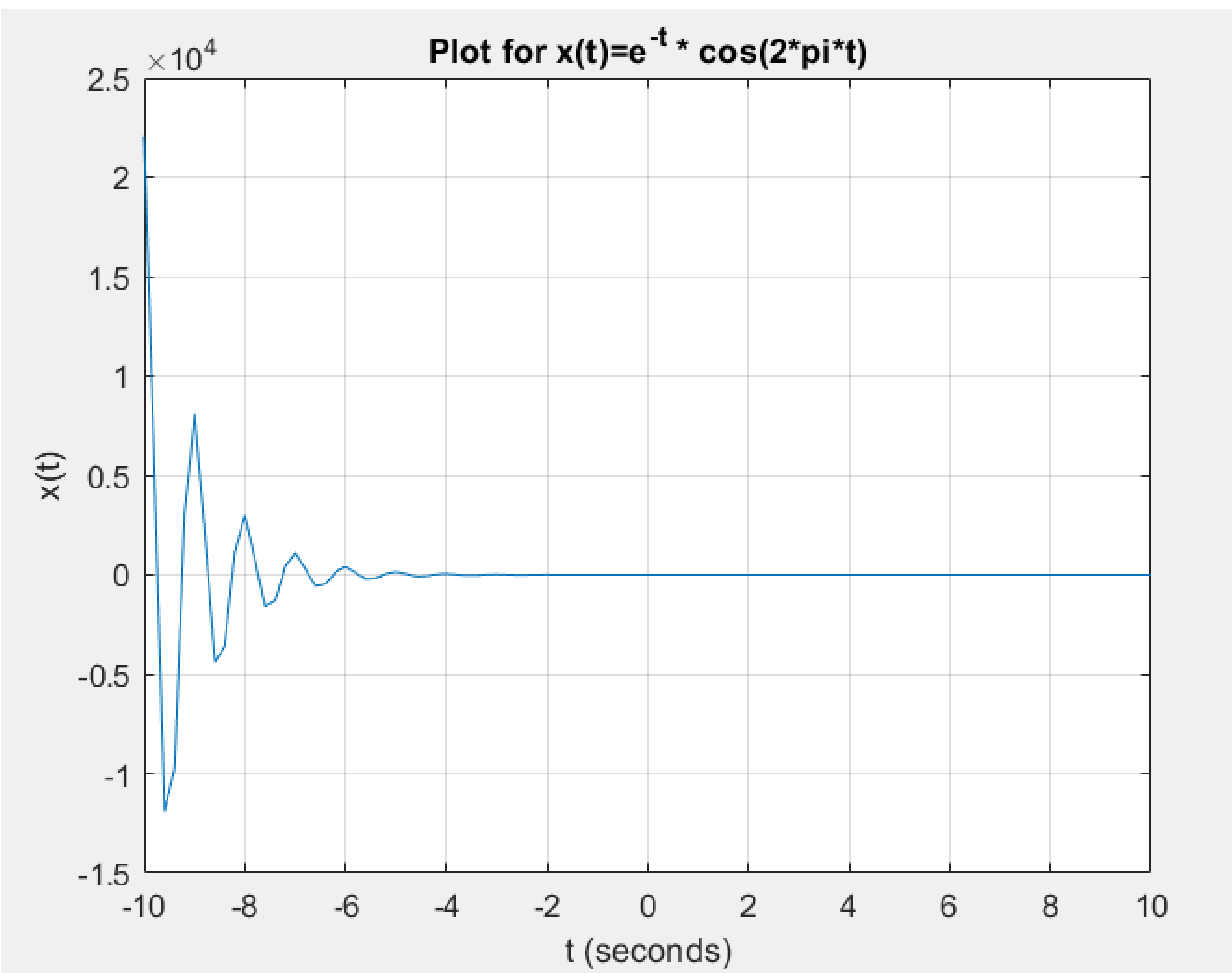
$y(t) = \frac{1}{\sqrt{2}} \cdot e^{-j\frac{\pi}{4}}$

$$\boxed{\begin{array}{l}\text{The magnitude is } \frac{1}{\sqrt{2}} \\ \text{The phase is } -\frac{\pi}{4} \text{ rad}\end{array}}$$

6a

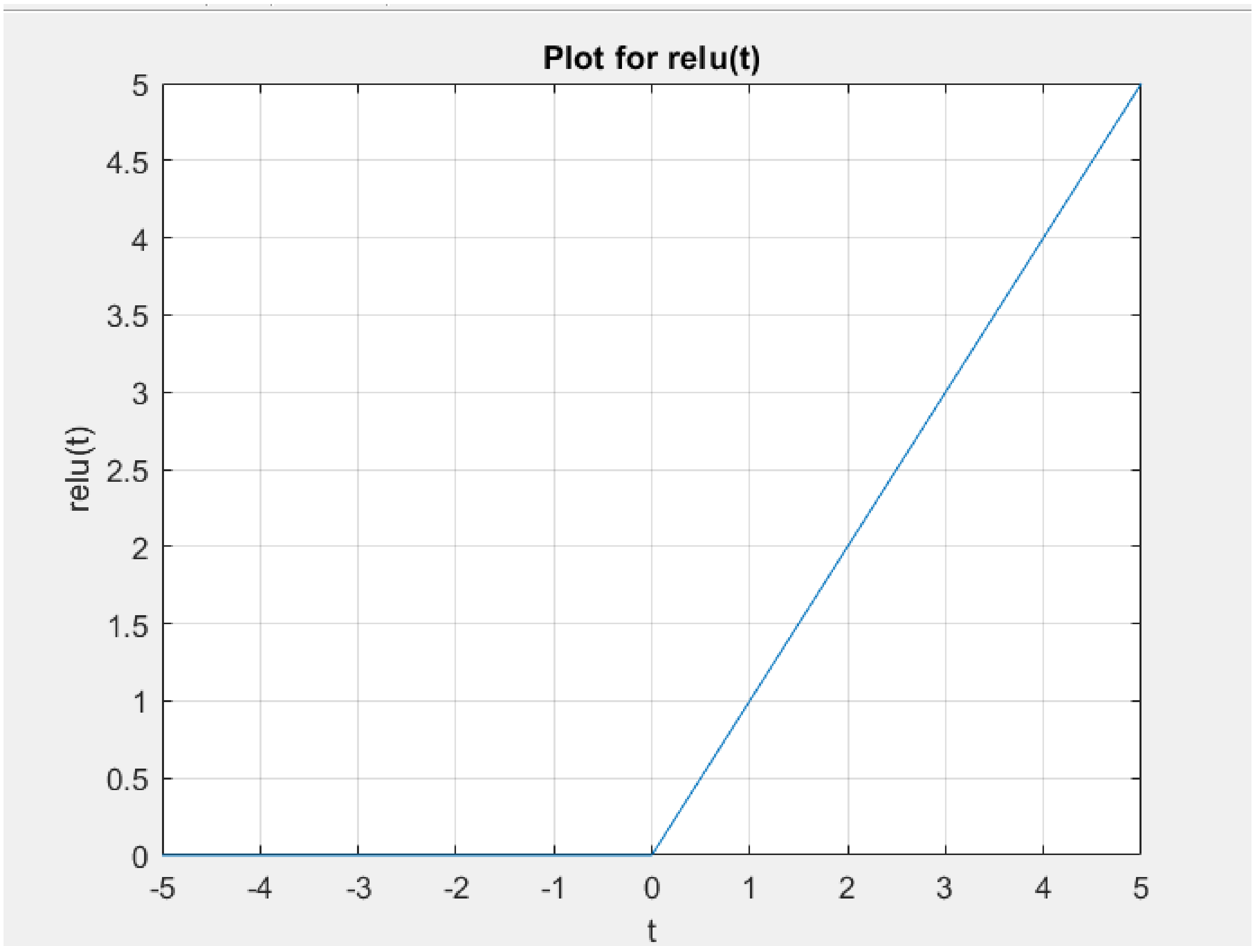
hw1a.m relu.m even.m odd.m

```
%set the domain and step size
t = -10 : 0.2 : 10;

%the function to plot
y = exp(-t).*cos(2*pi*t);

plot(t, y);
title('Plot for x(t)=e^{-t} * cos(2*pi*t)'); xlabel('t (seconds)'); ylabel('x(t)');

%enables lines for clearer plot
grid on;
```



6b

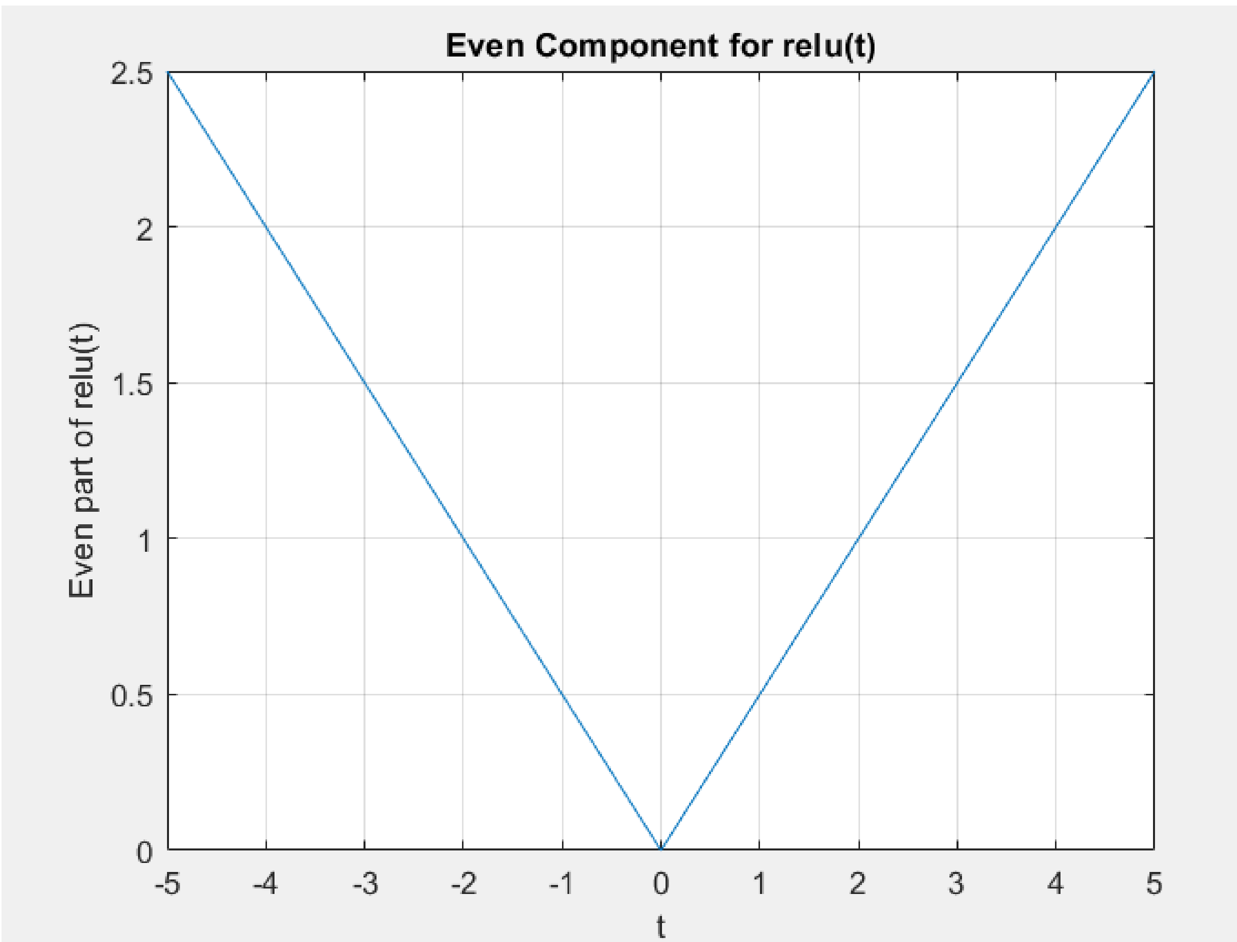
```
function output = relu(t)
output = max(0, t);
end

% run these lines in the command window to get the plot
% t = -5: 0.1 :5;
% plot(t, relu(t));
% grid on;
% title('Plot for relu(t)'); xlabel('t'); ylabel('relu(t)');
```



6c

hw1a.m relu.m even.m odd.m

```
function output = even(t, f)
output = (0.5 * f(-t)) + (0.5 * f(t));
end

% run these lines in the command window to get the plot
% t = -5: 0.1 :5;
% plot(t, even(t, @relu));
% title('Even Component for relu(t)'); xlabel('t'); ylabel('Even part of relu(t)');
% grid on;
```



hw1a.m | relu.m | even.m | odd.m

```matlab
function output = odd(t, f)
output = (0.5 * f(t)) - (0.5 * f(-t));
end

% run these lines in the command window to get the plot
% t = -5: 0.1 :5;
% plot(t, odd(t, @relu));
% title('Odd Component for relu(t)'); xlabel('t'); ylabel('Odd part of relu(t)');
% grid on;

```