Fabrizio	CS2
Muneeb	EE2
Zeid	CE2
Ryan	CS2

EE102

Lecture 7

EE102 Announcements

- Syllabus link is tiny.cc/ucla102
- CCLE difficulties, please email help@seas.ucla.edu
- **Third Homework due this Friday**
- **Note: We do not have lectures on exam days.**

Slide Credits: This lecture adapted from Prof. Jonathan Kao (UCLA) and recursive credits apply to Prof. Cabric (UCLA), Prof. Yu (Carnegie Mellon), Prof. Pauly (Stanford), and Prof. Fragouli (UCLA)

Review of Last Lecture

Last Lecture Introduced a few concepts:

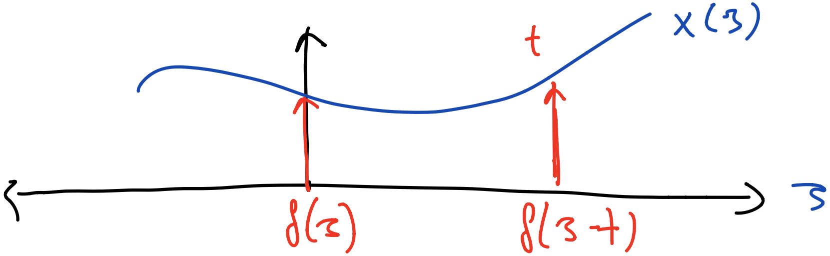
- CYU: How is the Impulse Response Defined?

$$x \rightarrow \boxed{H} \rightarrow y \qquad x = \delta(t)$$

- CYU: Why is the Impulse Response Useful?

IR lets you predict the response of an LTI system, not just to the impulse (of course!), but to ANY input

Derivation of this fact



① Calculate $x(0)$ using δ's: $x(\tau)\,\delta(\tau) = \underbrace{x(0)\,\delta(\tau)}_{}$

↳ Integrate this, you get $x(0)$.

② Calculate $x(t)$ using δ's: $x(\tau)\,\delta(\tau - t) = \underbrace{x(t)\,\delta(\tau - t)}_{}$

↳ Integrate to get $x(t)$

The Convolution Integral

$$\int_{-\infty}^{\infty} x(\tau)\,\delta(\tau-t)\,d\tau = \int_{-\infty}^{\infty} x(t)\,\delta(\tau-t)\,d\tau$$

$$= x(t)\underbrace{\int_{-\infty}^{\infty} \delta(\tau-t)\,d\tau}_{=1}$$

$$= x(t)$$

// Convolution Integral

$$x(t) = \int_{-\infty}^{\infty} x(\tau)\,\delta(\tau-t)\,d\tau$$

$$\boxed{= \int_{-\infty}^{\infty} x(\tau)\,\delta(t-\tau)\,d\tau}$$

Canonical form of Convolution Integral.

The Convolution Integral (Cont'd)

$h(t) \triangleq H(\delta(t))$

① If we have $h(t)$ then given any $x(t)$ we can get $y(t)$

$$x(t) \longrightarrow \boxed{H} \longrightarrow y(t)$$

LTI

// Goal

$$x(t) \longrightarrow \boxed{h(t)} \longrightarrow y(t)$$

$$y(t) = \int_{-\infty}^{\infty} x(\tau)\, h(t-\tau)\, d\tau \quad //$$

$$y(t) = x(t) \underset{=}{*} h(t)$$

convolution

Ⓐ Given a system H and x, predict y

Ⓑ

$$\underline{y(t)} = H(x(t))$$

$$= H\left(\int_{-\infty}^{\infty} x(\tau)\, \delta(t-\tau)\, d\tau\right)$$

Linearity

$$= \int_{-\infty}^{\infty} x(\tau)\, H(\delta(t-\tau))\, d\tau$$

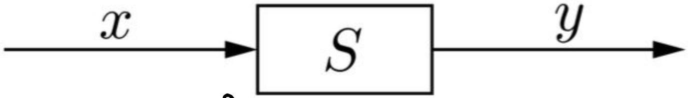
Time Invariance

$$= \int_{-\infty}^{\infty} x(\tau)\, h(t-\tau)\, d\tau$$

Convolution is what adds structure to the Black Box

A system transforms an *input signal*, $x(t)$, into an output system, $y(t)$.

$h(t)$



If LTI

Then, convolution is the operation that maps x to y.

$$y = x * h$$

Why does this work? (The Convolution Integral)

Same derivation as last lecture - shows up in other classes, even grad classes

without loss of generality $\quad x \rightarrow \boxed{H} \rightarrow y$

$$y(t) = H[x(t)]$$

$$y(t) = H\left[\int_{-\infty}^{\infty} x(\tau)\,\delta(t-\tau)\,d\tau\right]$$

Linearity of H

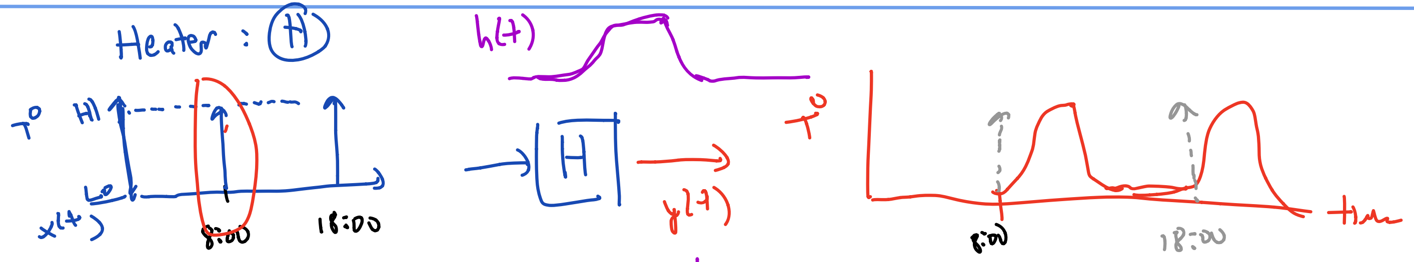
$$y(t) = \int_{-\infty}^{\infty} x(\tau)\,H(\delta(t-\tau))\,d\tau$$

Shift Invariance

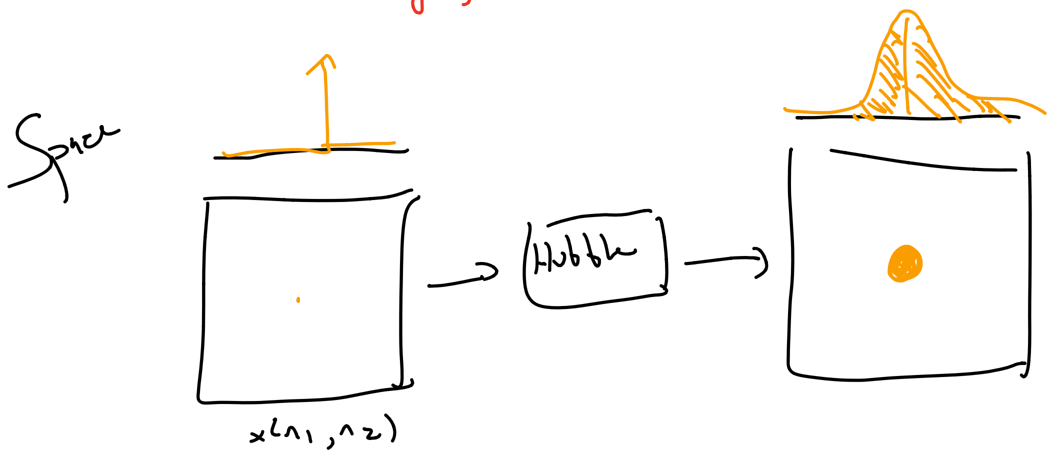
$$y(t) = \int_{-\infty}^{\infty} x(\tau)\,h(t-\tau)\,d\tau$$

$$y(t) = x(t) * h(t)$$

"Gist" of convolution - smearing



$y(t) = x(t) * h(t)$



Examples of Computing the Impulse Response

$$y(t) = \int_{-\infty}^{t} x(\tau)\, d\tau \qquad \text{(A)}$$

LTI System the Integrator

$x \to \boxed{I} \to y$

$y = x * h$

What is the impulse response of this system?

1) Set $x(t) = \delta(t)$
2) Calculate the output $\longrightarrow h(t)$

Plug-and-Chug

$$h(t) = \int_{-\infty}^{t} \delta(\tau)\, d\tau = u(t)$$

CW: Show that (A) and (B) are equivalent.

You can also express the output in terms of the step.

$$y(t) = x(t) * h(t) = x(t) * u(t) = \int_{-\infty}^{\infty} x(\tau)\, u(t-\tau)\, d\tau \qquad \text{(B)}$$

CYU

// $y(t) = x(t - 3')$

Qst: Calculate the impulse response and write y in terms of the convolution integral

① Get IR.

$$h(t) = \delta(t - 3')$$

② $y(t) = x(t) * h(t) = \int_{-\infty}^{\infty} x(3)\, h(t-3)\, d3 = \underbrace{\int_{-\infty}^{\infty} x(3)\, \delta(t - 3' - 3)\, d3}_{\text{sifting Property}}$

$$= x(t - 3')$$

Notation of Convolution

$$y(t) = \int_{-\infty}^{\infty} x(\tau)\, h(t-\tau)\, d\tau$$

- x is input
- h is impulse response
- y is the output

Notation

Most rigorous $\quad y(t) = (x * h)(t)$

In reality $\quad y = x * h$

$$y(t) = x(t) * h(t)$$

Block notation

$$x(t) \rightarrow \boxed{h(t)} \rightarrow y(t)$$

$$y(t) = (x * h)(t) = x * h.$$

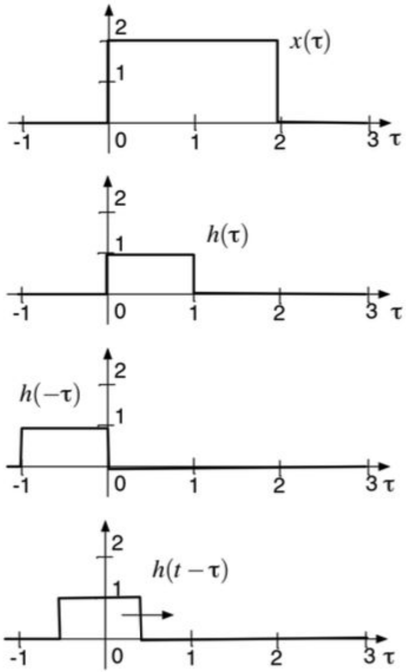
How to Compute Convolution: flip and drag

To calculate $y(t) = (x * h)(t)$,

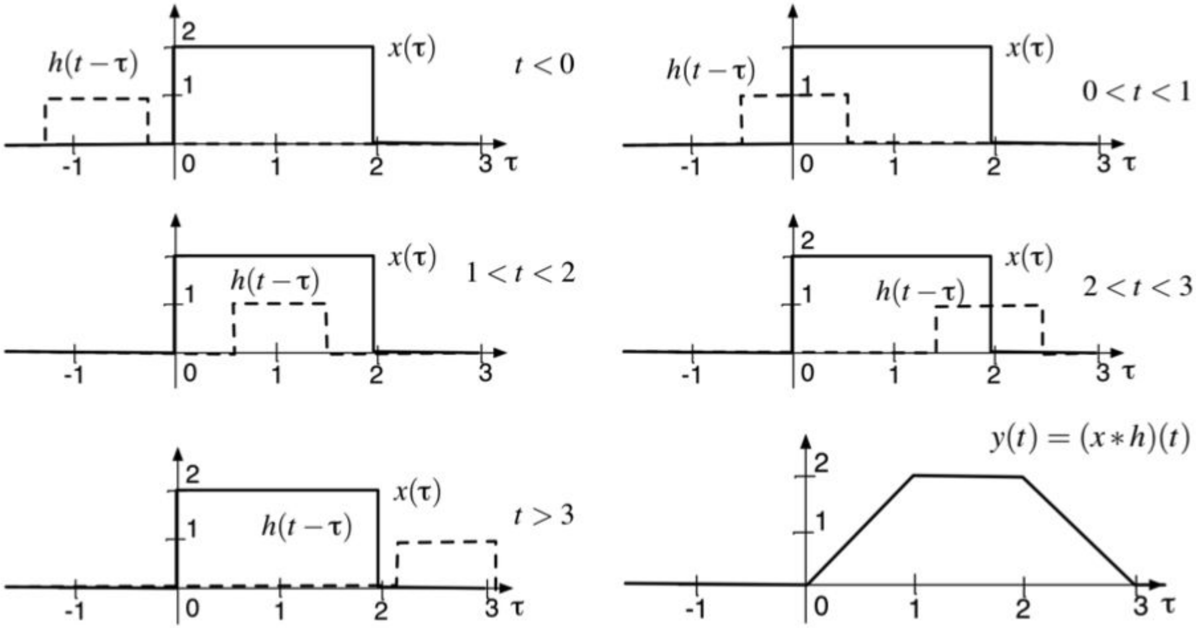
- Flip (i.e., reverse in time) the impulse response. This changes $h(\tau)$ to $h(-\tau)$.
- Begin to drag the reversed time response by some amount, t. This results in $h(t-\tau)$.
- For a given t, multiply $h(t-\tau)$ pointwise by $x(\tau)$. This produces $x(\tau)h(t-\tau)$.
- Integrate this product over τ. This produces $y(t)$ at this particular time t.

This technique is referred to as the "flip-and-drag" technique.

How to Compute Convolution: flip and drag

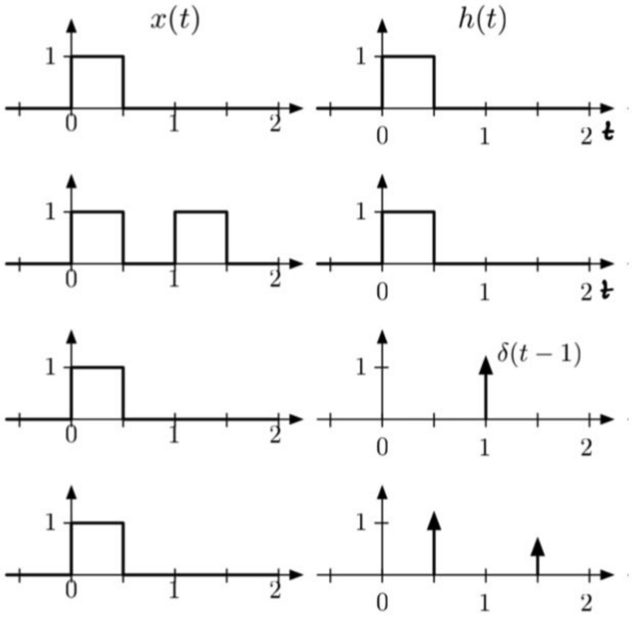


How to Compute Convolution: flip and drag

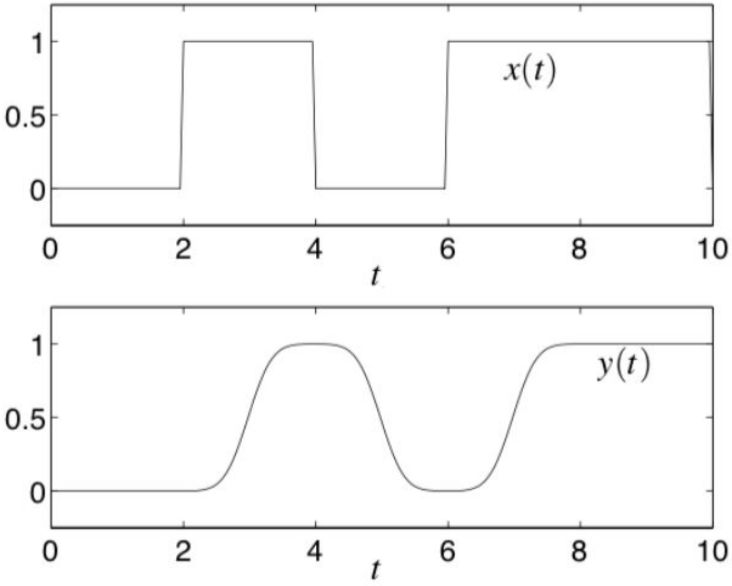


How to Compute Convolution: flip and drag

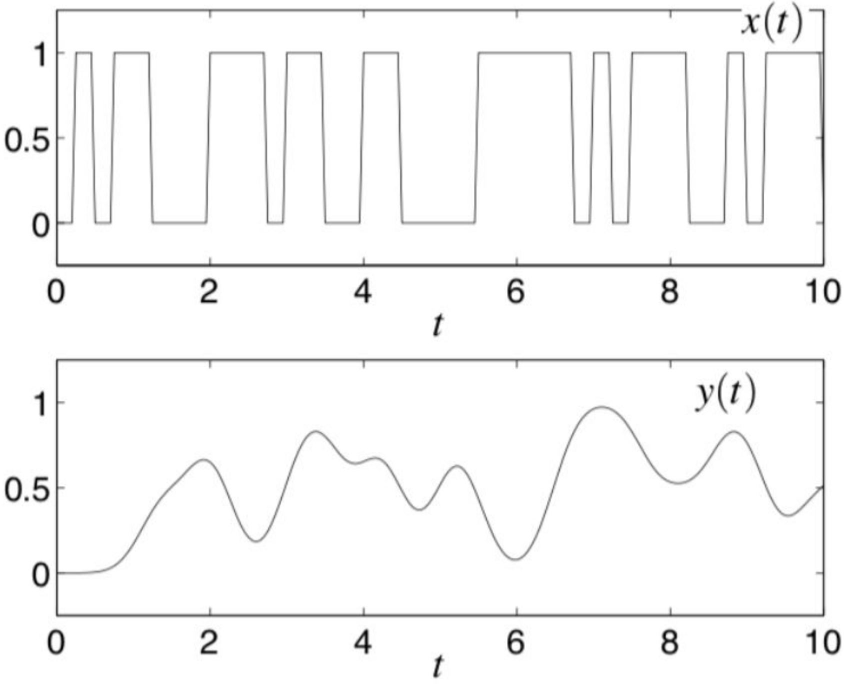
Examples: Try these:



Example: Noisy Communication



Example Noisy Communication



Causal Convolution

Convolution for a causal system

In a causal system, $h(t) = 0$ for $t < 0$. (Why? Hint: what happens if $h(t) \neq 0$ for some $t < 0$?)

This means that $h(t - \tau) = 0$ if $\tau > t$. Hence, there is no need to integrate if τ exceeds t, since $h(t - \tau) = 0$. We can use this to simplify the convolution integral.

$$
\begin{aligned}
y(t) &= \int_{-\infty}^{\infty} x(\tau)h(t - \tau)d\tau \\
&= \int_{-\infty}^{t} x(\tau)h(t - \tau)d\tau
\end{aligned}
$$

This equation tells us that only past and present values of $x(\tau)$ contribute to $y(t)$.

Properties of Convolution

Commutativity $\boxed{(x * h)(t) = (h * x)(t)}$.

Associativity $\boxed{(f * (g * h))(t) = ((f * g) * h)(t)}$

Distributivity $\boxed{f * (g + h) = f * g + f * h}$

Linearity $h * (\alpha x_1 + \beta x_2) = \alpha(h * x_1) + \beta(h * x_2)$

Time-invariance

CYU Question from HW