ECE102, Spring 2021 | **Homework #4**
Signals & Systems | Prof. A. Kadambi
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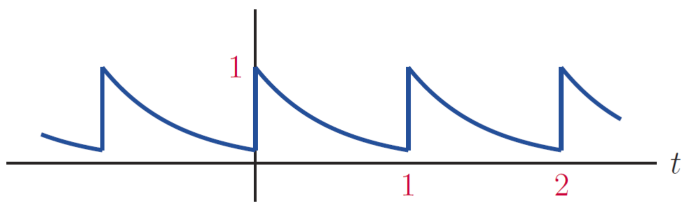
Due Friday, 7 May 2021, by 11:59pm to CCLE.
100 points total.
This homework covers questions relate to Fourier series and LTI systems.

1. (28 points) **Fourier Series**

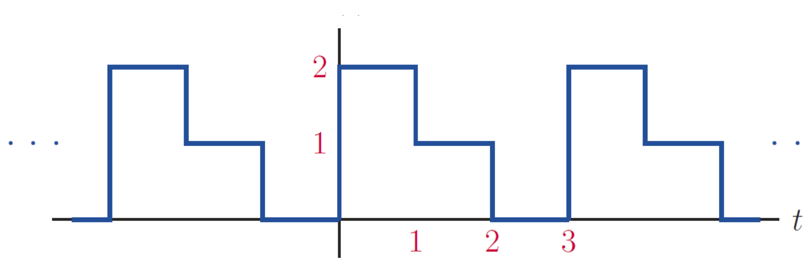
   (a) (18 points) Find the Fourier series coefficients for each of the following periodic signals:

      i. $f(t) = \cos(3\pi t) + \frac{1}{2}\sin(4\pi t)$

      ii. $f(t)$ is a periodic signal with period $T = 1$ s, where one period of the signal is defined as e^{-2t} for $0 < t < 1$ s, as shown below.



      iii. $f(t)$ is the periodic signal shown below:



   (b) (10 points) Suppose you have two periodic signals $x(t)$ and $y(t)$, of periods T_1 and T_2 respectively. Let x_k and y_k be the Fourier series coefficients of $x(t)$ and $y(t)$.

      i. If $T_1 = T_2$, express the Fourier series coefficients of $z(t) = x(t) + y(t)$ in terms of x_k and y_k.

      ii. If $T_1 = 2T_2$, express the Fourier series coefficients of $w(t) = x(t) + y(t)$ in terms of x_k and y_k.

2. (20 points) **Fourier series of transformation of signals**

   Suppose that $f(t)$ is a periodic signal with period T_0, with the following Fourier series:

$$f(t) = \sum_{k=-\infty}^{\infty} c_k e^{jk\omega_0 t}$$

   Determine the period of each of the following signals, then express its Fourier series in terms of c_k:

(a) $g(t) = f(t) + 1$

(b) $g(t) = f(-t)$

(c) $g(t) = f(at)$, where a is positive real number

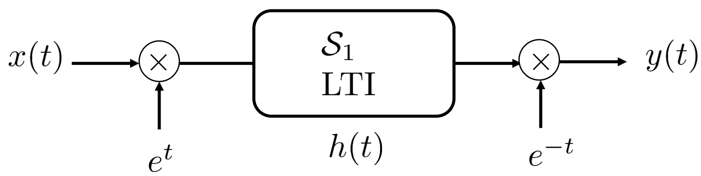
3. (10 points) **Eigenfunctions and LTI systems**

   (a) (5 points) Show that $f(t) = \cos(\omega_0 t)$ is not an eigenfunction of an LTI system.

   (b) (5 points) Show that $f(t) = t$ is not an eigenfunction of an LTI system.

4. (29 points) **LTI systems**

   Consider the following system:



   The system takes as input $x(t)$, it first multiplies the input with e^t, then sends it through an LTI system. The output of the LTI system gets multiplied by e^{-t} to form the output $y(t)$.

   (a) Show that we can write $y(t)$ as follows:

$$y(t) = \left[\left(e^t x(t)\right) * h(t)\right] e^{-t} \tag{1}$$

   (b) Use the definition of convolution to show that (1) can be equivalently written as:

$$y(t) = \int_{-\infty}^{\infty} h'(\tau) x(t - \tau) d\tau \tag{2}$$

   where $h'(t)$ is a function to define in terms of $h(t)$.

   (c) Equation (2) represents a description of the equivalent system that maps $x(t)$ to $y(t)$. Show using (2) that the equivalent system is LTI and determine its impulse response $h_{eq}(t)$ in terms of $h(t)$.

   (d) Suppose that system $\mathcal{S}_1$ is given by its step response $s(t) = r(t - 1)$. Find the impulse response $h(t)$ of $\mathcal{S}_1$. What can you say about the causality and stability of system $\mathcal{S}_1$? What can you say about the causality and stability of the overall equivalent system?

5. (13 points) **MATLAB**

   (a) (6 points) **Task 1**
   Write an m-file that takes a set of Fourier series coefficients, a fundamental frequency, and a vector of output times, and computes the truncated Fourier series evaluated at these times. The declaration and help for the m-file might be:

```
 function fn = myfs(Dn,omega0,t)
%
% fn = myfs(Dn,omega0,t)
% % Evaluates the truncated Fourier Series at times t
%
% Dn -- vector of Fourier series coefficients
%
% omega0 -- fundamental frequency
% t -- vector of times for evaluation
%
% fn -- truncated Fourier series evaluated at t
```

The output of the m-file should be

$$f_N(t) = \Sigma_{n=-N}^{N} D_n e^{j\omega_0 nt}$$

The length of the vector Dn should be $2N + 1$. You will need to calculate N from the length of Dn.

(b) (7 points) **Task 2** Verify the output of your routine by checking the Fourier series coefficients for Problem 1-a-ii. Try for $N = 10$, $N = 50$ and $N = 100$. Use the MATLAB command "subplot" to put multiple plots on a page. As usual, include both codes and plots.

(c) (7 points) **Task 3** Repeat the steps of Task 2 for the case of the signal from Problem 1-a-iii.