ECE102, Spring 2021 **Midterm**

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UCLA True Bruin academic integrity principles apply.

This exam is open book. Collaboration is not allowed.

8:00 am Wednesday, 29 Apr 2021
- 8:00 am Thursday, 30 Apr 2021.

State your assumptions and reasoning.
No credit without reasoning.

Name: ____________________

Signature: ____________________

ID#: ____________________

Problem 1	_____	/ 30
Problem 2	_____	/ 22
Problem 3	_____	/ 20
Problem 4	_____	/ 28
BONUS	_____	/ 1.5 bonus points
Total	_____	/ 100 points + 1.5 bonus points

1. **Signal Properties** (30 points)

   (a) (5 points) *Complex numbers.* Find the real and imaginary parts of:

   $$x(t) = e^{5+j3t}(cos(t) + jsin(3t)), \tag{1}$$

   t is real.

   (b) (10 points)*Energy and power.* What are the energy and power of the following signals:
   (i)

   $$x(t) = Ae^{-a|t|} \quad with \quad a > 0. \tag{2}$$

   (ii)

   $$x(t) = Asin(\omega t)u(t) \quad with \quad \omega \neq 0. \tag{3}$$

(c) (5 points) Consider the following signal:

$$s(t) = \sum_{k=-\infty}^{\infty} \frac{1}{1 + (t + 5k)^4}. \tag{4}$$

Determine whether s(t) is periodic or not. If periodic, identify the fundamental period. Mathematically justify your reasoning.

(d) (5 points) Identify the even and odd parts of the following signals:

(i)

$$x(t) = sin(3t) - cos^2(2t) \tag{5}$$

(ii)

$$x(t) = t.sin(2t) \tag{6}$$

(e) (5 points) Consider the standard triangular signal as defined below:

$$\Delta(t) = \begin{cases} 1 - |t|, & |t| < 1 \\ 0 & \text{otherwise.} \end{cases} \tag{7}$$

Consider the following statement and identify whether it is true or false, with proper justification:

'The signal $s(t) = \sum_{k=-\infty}^{\infty} \Delta(t - kT)$, $T > 0$ has a fundamental period of T for all valid T.'

2. **System Properties** (22 points)

(a) (12 points) A system with input $x(t)$ and output $y(t)$ can be linear, time-invariant, causal or stable. Determine which of these properties hold for the following system. Explain your answer.

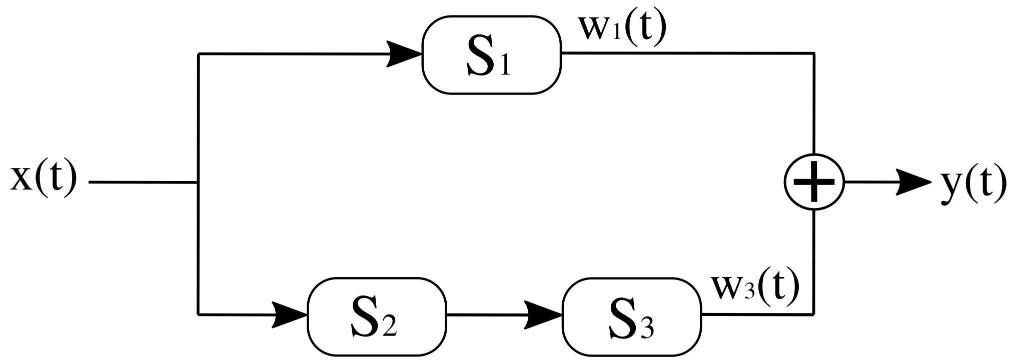
$$y(t) = \frac{d}{dt}(sin(t)x(t)) + x(3-t).$$

(b) (10 points) Determine if each of the following statements concerning LTI systems is true or false. Explain your reasoning.

i. If h(t) is the impulse response of an LTI system and h(t) is bounded, the system is stable.

ii. If h(t) is the impulse response of an LTI system and h(t) is even and nonzero, the system is causal.

3. **System Response of LTI system** (20 points)



We know that Systems 1, 2 and 3 are all LTI systems, which are used in parts a through c. $w_1(t)$ and $w_3(t)$ are the outputs of Systems 1 and 3, respectively. Let $h_1(t)$, $h_2(t)$ and $h_3(t)$ represent the impulse response for System 1, 2 and 3, respectively.
For parts (a) through (c), we have prior knowledge of Subsystem 3

$$h_3(t) = \int_{-\infty}^{t-1} \delta(\tau - 2) d\tau.$$

For parts (a) through (c), we also have prior knowledge of the input/output mapping of the entire system

$$y(t) = 3x(t-1) + \int_{-\infty}^{t+\frac{9}{2}} x(\tau) d\tau.$$

(a) (6 points) What is the impulse response of the entire system (i.e. S_{eq})? What is the step response of S_{eq}?

(b) (8 points) Find $h_1(t)$ and $h_2(t)$ that satisfies the input and output relationship that is given. It might be useful to determine the values of $w_1(t)$ and $w_3(t)$ first.

(c) (6 points) Using the solution from parts a and b, is System 1 Causal/Stable? Is the other subsystem (Cascaded Systems 2 and 3) Causal/Stable? Finally, is S_{eq} Causal/Stable?

4. **Convolution** (28 points)

(a) (6 points) Let $x(t) = 0$ for $t > T_2^x$ and $t < -T_1^x$, and $h(t) = 0$ for $t > T_2^h$ and $t < -T_1^h$. Note that $T_1^x, T_2^x, T_1^y, T_2^y$ are all non-negative. Find the range of t for which $y(t) = x(t) * h(t)$ is non zero, in general.

(b) (6 points) Let $x(t) * y(t) = z(t)$.
Prove the following property: $x(t - T_1) * y(t - T_2) = z(t - (T_1 + T_2))$, for all T_1, T_2.

(c) (8 points) Consider the following definition of $rect(t)$:

$$\text{rect}(t) = \begin{cases} 1, & |t| \leq \frac{1}{2} \\ 0 & \text{otherwise.} \end{cases} \tag{8}$$

For any $a, b > 0$ and $b > a$, find $y(t) = rect(t/(2a)) * rect(t/(2b))$ Show work (e.g. how you arrive at the x coordinates of your final solution) and plot $y(t)$.

(d) (8 points) Let $x(t) = e^{-3t}u(t-3)$ and $h(t) = rect((t-1)/2)$ ($rect(t)$ is defined as shown in Problem 4.c). Compute the output $y(t) = x(t) * h(t)$.

5. Bonus (1.5 points)

(This question was used in an industry interview to use system modeling of a real-world problem.)

Self-driving cars have devices on top of the car known as LiDAR (light detection and ranging). LiDAR uses the *time of flight* principle to obtain the 3D shape of the surrounding environment. LiDAR works as follows: a packet of photons gets sent to the scene - it bounces off an object a range of z meters away and returns to the LiDAR unit. Since we know how fast the photons travel, we can compute the range of the object.

First, recall that the round-trip distance is

$$distance = velocity \times t_d. \tag{9}$$

where distance is the round-trip distance of travel (from the source, to the object and back) and t_d is the time the trip takes to occur. We are interested in the range of the object which is

$$range = distance \times 0.5 = velocity \times t_d \times 0.5. \tag{10}$$

In the specific case of sending a laser pulse to the target, we know the speed of photons is $c = 3 \times 10^8$ meters per second. Therefore, the range is computed as:

$$range = c \times t_d \times 0.5, \tag{11}$$

and we only need to measure the time delay of the pulse to obtain the range. Using the ideas from this class, this can be written as a system.

$$\delta(t) \rightarrow \boxed{\delta\left(t - \frac{2 \times range}{c}\right)} \rightarrow y, \tag{12}$$

Here the laser pulse that we sent is $\delta(t)$, and it goes into a system that introduces a delay to yield the measurement. This is an idealized model of LiDAR that does not involve any multipath and assumes the laser can emit a Dirac pulse.

(a) (0.5 points) Please draw the equivalent system diagram if there is a transparent window at $range_1$ and a brick wall behind it at $range_2$. We will assume that the impulse response of the transparent window and brick wall can be modeled as Dirac Delta functions.

(b) (0.5 points) Please draw the equivalent system diagram if the brick wall in part (a) is changed to a block of wax. Please justify and/or explain your answer.

(c) (0.5 points) Now we will assume that the laser is a cheap laser. It cannot fire a short pulse that mimics a Dirac. In fact, it's more like your room lightbulb which takes a finite time to turn on. Using your understanding of signals and systems, explain concisely how the output y will be changed and why this could be a problem in the multipath case.

Space for Solution 5: