Jim: EE1

Tiffany: EE1

Tyler: CE2

Khoa: CS4

Hiya: CS2

EE102

Lecture 5

Starts @ 4:05 pm PT

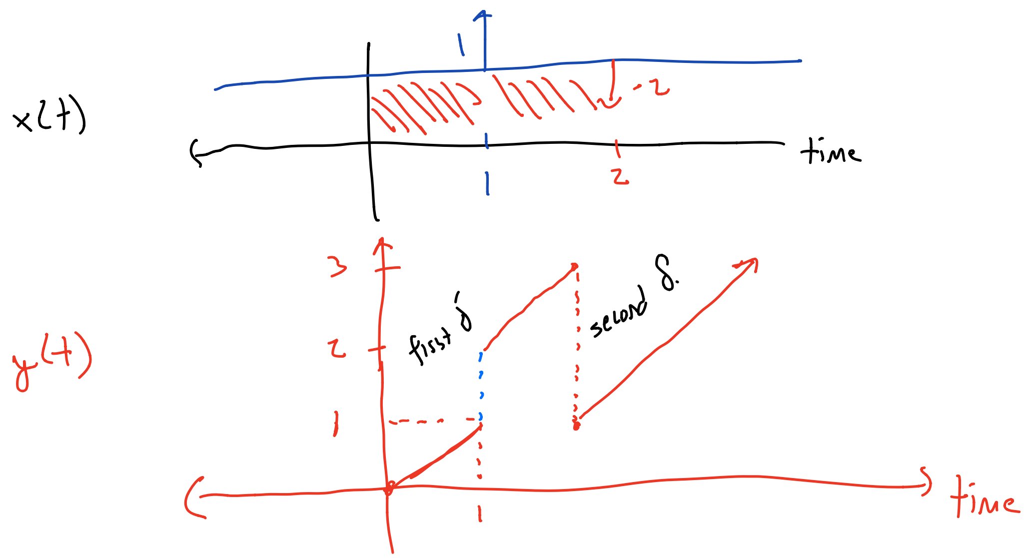
EE102 Announcements

- Syllabus link is tiny.cc/ucla102
- CCLE difficulties, please email help@seas.ucla.edu
- ~~My office hour meeting minutes are sent out weekly~~
- **Second Homework due this Friday**
- "Homework Help" Session on Tuesdays.

Slide Credits: This lecture adapted from Prof. Jonathan Kao (UCLA) and recursive credits apply to Prof. Cabric (UCLA), Prof. Yu (Carnegie Mellon), Prof. Pauly (Stanford), and Prof. Fragouli (UCLA)

CYU: Suppose $x(t) = 1 + \delta(t-1) - 2\delta(t-2)$

What does the graph of $y(t) = \int_0^t x(\tau)\, d\tau$ look like?



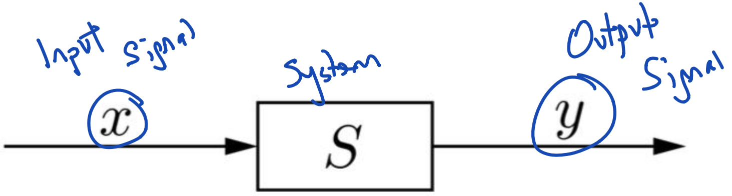
① 1st student feedback: lecture too fast.

② What have we learned?

- Signal ≈ Algebraic Function
- Prototypical Signals that Serve as Bldg. Blocks
- Today: Systems.

What is a system?

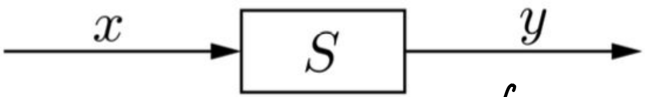
A system transforms an *input signal*, $x(t)$, into an output ~~system~~ signal, $y(t)$.



very technically, a system is a functional

- Systems, like signals, are also *functions*. However, their inputs and outputs are signals.
- Systems can have either single or multiple inputs (SI or MI, respectively) and single or multiple outputs (SO and MO). In this class, we focus on *single input, single output* systems (SISO).

Systems have Properties



$$\underbrace{[0 \quad 3 \quad 1] \rightarrow \boxed{S} \rightarrow [\infty \quad \infty \quad \infty]}_{x}$$

Unbounded Example.

① Stability

English // A system is *bounded-input, bounded-ouput* (BIBO) stable if every bounded input leads to a bounded output.

Math // $$|x(t)| < \infty \implies |y(t)| < \infty \quad \forall t$$

② Causality

English // A system is causal if its output only depends on past and present values of the input.

Math // S is causal if $y(t)$ depends only on $x(t), x(t-1), \ldots \quad \forall t.$

Systems have Properties

② Time-invariance

English ‖ A system is *time invariant* if a time shift in the input only produces an identical time shift of the output.

Math ‖ Mathematically, a system S is time-invariant if

$$y(t) = S(x(t))$$

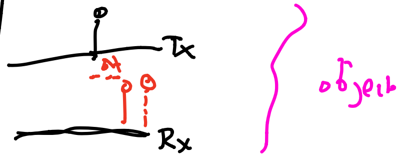
$x \rightarrow \boxed{S} \rightarrow y$

implies that

$$y(t-\tau) = S(x(t-\tau))$$

$x(t) \rightarrow \boxed{\text{Delay}} \rightarrow x(t-3) \rightarrow \boxed{S} \rightarrow y(t-3)$

$x(t) \rightarrow \boxed{S} \rightarrow y(t) \rightarrow \boxed{\text{Delay}} \rightarrow y(t-3)$



\$100 → $\boxed{\text{Spy}}$ → \$110
@ t1

\$100 → $\boxed{\text{Spy}}$ → \$109.99
@ t2

Examples of Time Invariance

// cyu: Squared System: $y(t) = [x(t)]^2$

Qst. Is the squared system time invariant?

shift input

$\overset{?}{=}$

shift output

$x \to \boxed{\text{Delay}} \to \boxed{S} \to$ Output $\quad [x(t-3)]^2$

$x \to \boxed{S} \to \boxed{\text{Delay}} \to \; : \quad y(t-3) = [x(t-3)]^2$

/ cyu: System: $y(t) = x(t)\,t$

Shift Input : $x(t-3)(t)$

shift output : $y(t-3) = x(t-3)(t-3)$

$\neq$ Not Invariant.

Qst. Is this system time invariant

/ cyu: AM Radio: $y(t) = x(t)\cos(\omega_c t)$

shift input $\rightarrow$ $x(t-3)\cos(\omega_c t)$

shift output $\rightarrow$ $y(t-3) = x(t-3)\cos(\omega_c(t-3))$

$\neq$ Not T.I.

Linearity

$x \to \boxed{S} \to y$

$ax \to \boxed{S} \to ay$

A system is *linear* if the following two properties hold:

1. **Homogeneity**: for any signal, x, and any scalar a,

$$S(ax) = aS(x)$$

2. **Superposition**: for any two signals, x and $\tilde{x}$,

$$\underbrace{S(x + \tilde{x})}_{\text{adding two inputs and applying S.}} = \underbrace{S(x) + S(\tilde{x})}_{\text{first applying S then adding}}$$

$$S(ax + b\tilde{x}) = aS(x) + bS(\tilde{x}) \quad \forall a, b, x, \tilde{x}$$

// Combines homogeneity and superposition.

Linearity Examples

AM Radio: $y(t) = x(t)\cos(\omega_c t)$

$= AM[x(t)]$

CYU: Is the AM radio linear?

Goal: Show $\underbrace{AM(ax(t) + b\tilde{x}(t))}_{LHS} = \underbrace{a\,AM(x(t)) + b\,AM(\tilde{x}(t))}_{RHS}$

LHS: $[ax(t) + b\tilde{x}(t)]\cos(\omega_c t)$

$ax(t)\cos(\omega_c t) + b\tilde{x}(t)\cos(\omega_c t)$

$a\,AM(x(t)) + b\,AM(\tilde{x}(t)) = RHS$

CYU: $y(t) = [x(t)]^2$

$= S(x)$

Show whether $S(ax(t) + b\tilde{x}(t)) = aS(x(t)) + bS(\tilde{x}(t))$

CYU @ home.

Linearity examples (Cont'd)

Integrator: $y(t) = \int_{-\infty}^{t} x(\tau)\, d\tau \;=\; S(x(t))$.   Q: Is the integrator linear?

Show $S\left[a x(t) + b\tilde{x}(t)\right] = a S(x(t)) + b S(\tilde{x}(t))$

LHS   RHS

LHS: $\int_{-\infty}^{t} \left[a x(\tau) + b\tilde{x}(\tau)\right] d\tau = a \int_{-\infty}^{t} x(\tau)\, d\tau + b \int_{-\infty}^{t} \tilde{x}(\tau)\, d\tau$

$= a\, S(x(t)) + b\, S(\tilde{x}(t))$

$\boxed{\text{YES!}}$ System is linear.

Linearity and time-invariance recap

	Linear?	Time-Inv?
1. $y(t) = \sqrt{x(t)}$	No	Yes
2. $y(t) = \underbrace{x(t)}_{\text{input}} \cdot z(t)$ for $z(\cdot)$ non-zero	Yes	No
3. $y(t) = x(at)$	Yes	No
4. $y(t) = x(t-3)$	Yes	Yes
5. $y(t) = x(\tau - t)$	Yes	No

Memory

A system has *memory* if its output depends on past or future values of the input. If the output depends only on present values of the input, the system is called *memoryless*.

Memory?

AM Radio: $y(t) = x(t)\cos(\omega_c t)$ memoryless

Integrator $y(t) = \int_{-\infty}^{t} x(\tau)\, d\tau$ Memory

Invertibility

$$x \to \boxed{S} \to y \to \boxed{S^{inv}} \to x$$

A system is called *invertible* if an input can always be exactly recovered from the output. That is, a system S is invertible if there exists an S^{inv} such that

$$x = S^{\text{inv}}(S(x))$$
$$= S^{inv}(y)$$

// sq. $y(t) = [x(t)]^2$ — Not Invertible b/c we cannot get back the sign

diff $y(t) = \dfrac{dx(t)}{dt}$ — Not Invertible b/c of the "+ C" constant

Scalar $y(t) = a\,x(t)$ for $a \neq 0$ — Invertible $x(t) = \dfrac{y(t)}{a}$

Homework

CYU

Impulse Response

System impulse response

This lecture introduces time-domain analysis of systems, including the impulse response. It also discusses linear time-invariant systems. Topics include:

- Impulse response definition
- Impulse response of LTI systems
- The impulse response as a sufficient characterization of an LTI system
- Impulse response and the convolution integral

Why do we need the impulse response?

Types of Responses

Impulse Response Definition

$$h(t, \tau) = H(\delta(t - \tau))$$

There are important things to be careful of when looking at this equation.

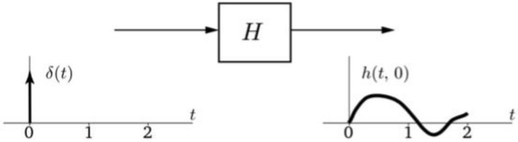
- The t on the left and right hand side of these equations *are not the same*!
- The t on the left hand side is the impulse response at a specific value of time.
- The t on the right hand side varies across all time.
- The output at the specific time t on the left will depend on the input at several times t on the right.

Notation on t

$$h(t, \tau) = H(\delta(t - \tau))$$

There are important things to be careful of when looking at this equation.

An example of these t's not being the same is shown below. In this example, let $\tau = 0$.



It may be tempting to write:

$$h(1, 0) = H(\delta(1))$$

This is wrong.

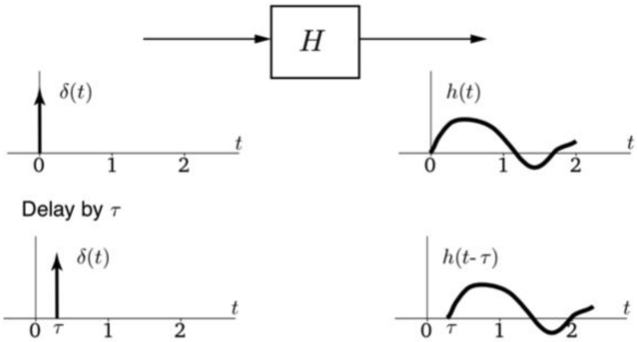
- On the left, $\delta(1) = 0$. We know if H is linear, then $H(0) = 0$, implying that $h(1, 0) = 0$.
- But in general, the impulse response can be non-zero, i.e., $h(1, 0) \neq 0$ in the above diagram, if the impulse response produces some non-zero response.

Time invariant Impulse Response

Time Invariant Impulse Response

Impulse response of a time-invariant system (cont.)

This property of the impulse response for a time-invariant system is drawn below:



ECE102, Spring 2021 **Homework #2**

Signals & Systems Prof. A. Kadambi

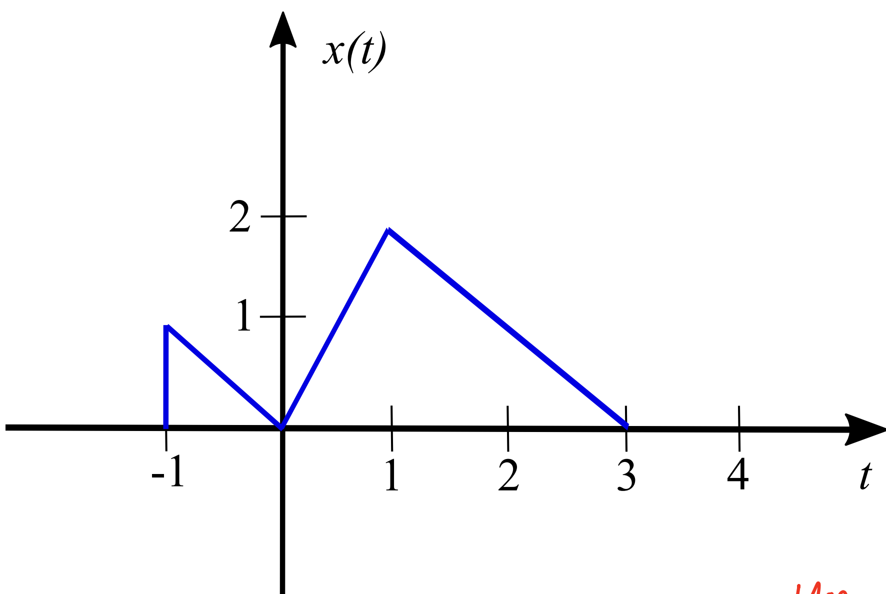
University of California, Los Angeles; Department of ECE TA: P. Chari

Due Friday, 16 Apr 2021, by 11:59pm to CCLE.

100 points total.

1. (22 points) **Elementary signals.**

   (a) (9 points) Consider the signal $x(t)$ shown below. Sketch the following:



similar to lecture

Use "sifting" property.

i. $y(t) = x(t)\,[1 - u(t-1) + u(t-2)]$

ii. $y(t) = \int_{-\infty}^{t} [\delta(\tau+1) - \delta(\tau-1) + \delta(\tau-2)]x(\tau)d\tau$

iii. $y(t) = x(t) + r(t+1) - u(t) - 3r(t) + 3r(t-1) - r(t-3)$

(b) (9 points) Evaluate these integrals:

i. $\int_{-\infty}^{\infty} f(t+1)\delta(t+1)dt$ "sifting property".

ii. $\int_{t}^{\infty} e^{-2\tau}u(\tau-1)d\tau$

iii. $\int_{0}^{\infty} f(t)(\delta(t-1)+\delta(t+1))dt$

(c) (4 points) Let b be a positive constant. Show the following property for the delta function:

$$\underbrace{\delta(bt)}_{\text{LHS}} = \underbrace{\frac{1}{b}\delta(t)}_{\text{RHS}}$$

Hint: what function is "delta-like"?

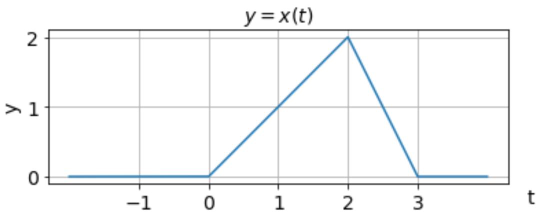
$$\delta(t) = \lim_{\Delta \to 0} \text{rect}_{\Delta}(t)$$

LHS

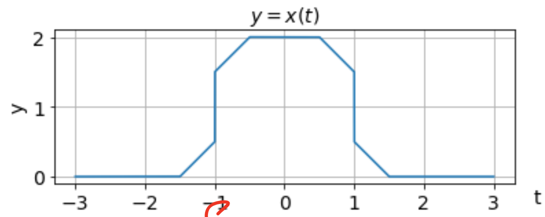
$$\delta(bt) = \lim_{\Delta \to 0} \text{rect}_{\Delta}(bt)$$

2. (23 points) **Expression for signals.**

(a) (15 points) Write the following signals as a combination (sums or products) of unit triangles $\Delta(t)$ and unit rectangles $\mathrm{rect}(t)$.

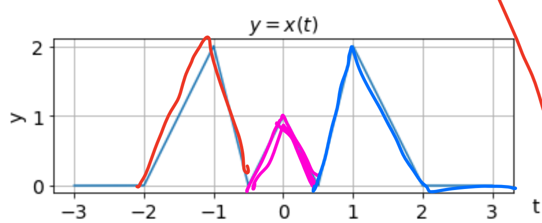


a(i)



a(ii)

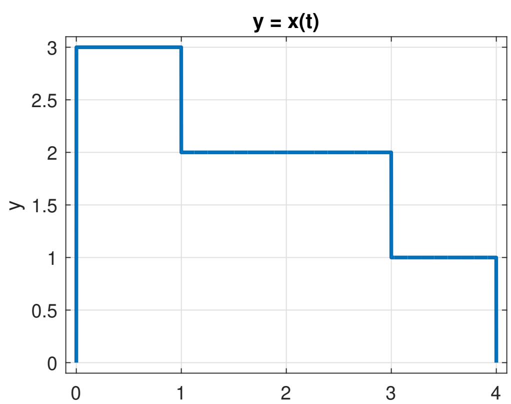
Hint: Sum of Two tringles and c rect



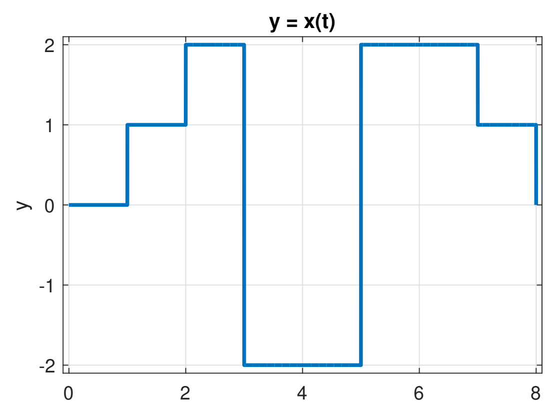
a(iii)

$\Delta(2t)$ + red + blue

(b) (8 points) Express each of the signals shown below as sums of scaled and time shifted unit-step functions.



b(i)



b(ii)

3. (30 points) **System properties.**

(a) (20 points) A system with input $x(t)$ and output $y(t)$ can be time-invariant, causal or stable. Determine which of these properties hold for each of the following systems. Explain your answer.

Time Inv

Causality

Stability

i. $y(t) = |x(t)| + x(t^2)$

ii. $y(t) = \int_{t-T}^{t+T} x(\lambda)d\lambda$, where T is positive and constant.

iii. $y(t) = (t+1)\int_{-\infty}^{t} x(\lambda)d\lambda$

iv. $y(t) = 1 + e^{x(t)}$

v. $y(t) = \frac{1}{1+x^2(t)}$

Suppose $\exists B$ s.t. $|x(t)| < B < \infty$

$$|y(t)| = \big||x(t)| + x(t^2)\big| \leq |x(t)| + |x(t^2)| \leq 2B$$

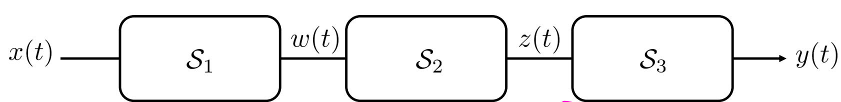
(b) (6 points) Consider the following three systems:

$$\mathcal{S}_1 : w(t) = x(t/2)$$

$$\mathcal{S}_2 : z(t) = \int_{-\infty}^{t} w(\tau)d\tau$$

$$\mathcal{S}_3 : y(t) = \mathcal{S}_3(z(t))$$

The three systems are connected in series as illustrated here:



$y(t) = f[z(t)]$

Choose the third system $\mathcal{S}_3$, such that overall system is equivalent to the following system:

$$y(t) = \int_{-\infty}^{t-1} x(\tau)d\tau$$

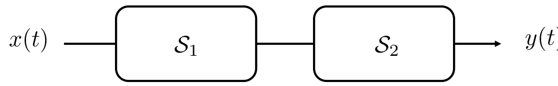
(c) (4 points) In part (b), you saw an example of three systems connected in series. In general, systems can be interconnected in series or in parallel to form what we call cascaded systems. The figure below shows the difference between a series cascade and a parallel cascade. *Note that parts (b) and (c) are unrelated.*

i. (2 points) Show that the series cascade of any two time-invariant systems is also time-invariant.

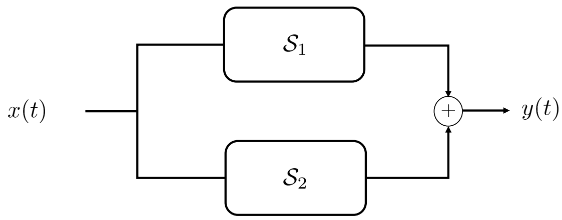
ii. (2 points) Show that the parallel cascade of any two time-invariant systems is also time-invariant.

iii. (*Optional*) Can you think of two **time-variant** systems, whose series cascade is **time-invariant**? Can you think of two **time-variant** systems, whose parallel cascade is **time-invariant**?

4. (10 points) **Power and energy of complex signals**



(a) Series Cascade



(b) Parallel Cascade

(a) (5 points) Is $x(t) = Ae^{j\omega t} + Be^{-j\omega t}$ a power or energy signal? A and B are both real numbers, not necessarily equal. If it is an energy signal, compute its energy. If it is a power signal, compute its power. (*Hint: Use the fact that the square magnitude of a complex number v is: $|v|^2 = v^*v$, where v^* is the complex conjugate of the complex number v.*)

(b) (5 points) Is $x(t) = e^{-(1+j\omega_1+j\omega_2)t}u(t+1)$ an energy signal or power signal? Again, if it is an energy signal, compute its energy. If it is a power signal, compute its power.

5. (15 points) **MATLAB**

(a) (5 points) **Task 1**

A complex sinusoid is denoted:

$$y(t) = e^{(\sigma+j\omega)t}$$

First compute a vector representing time from 0 to 10 seconds in about 500 steps (You can use `linspace`). Use this vector to compute a complex sinusoid with a period of 2 seconds, and a decay rate that reduces the signal level at 10 seconds to 1/3 its original value. What σ and ω did you choose? If your complex exponential is y, plot:

```
>> plot(y);
```

What is MATLAB doing here?

(b) (5 points) **Task 2**

Use the `real()` and `imag()` MATLAB functions to extract the real and imaginary parts of the complex exponential, and plot them as a function of time (plot them separately, you can use `subplot` for this task). This should look more reasonable. Label your axes, and check that your signal has the required period and decay rate.

(c) (5 points) **Task 3**

Use the `abs()` and `angle()` functions to plot the magnitude and phase angle of the complex exponential (plot them in the same figure). Scale the `angle()` plot by dividing it by `2*pi` so that it fits well on the same plot as the `abs()` plot (i.e. plot the angle in cycles, instead of radians, the function `angle(x)` returns the angle in radians).