Ethan Wong

UID 305319001

ECE 102 HW3

1. a) $y(t) = \cos(t)x(t)$

Try with 2 inputs, 2 respective outputs

$y_a(t) = \cos(t)x_a(t)$ $y_b(t) = \cos(t)x_b(t)$

$x(t) = d_1 x_a(t) + d_2 x_b(t)$

$y(t) = d_1 \cos(t) x_a(t) + d_2 \cos(t) x_b(t)$

$y(t) = d_1 y_a(t) + d_2 y_b(t)$ $\boxed{\text{Linear}}$

b) $y(t) = \frac{d}{dt}\left(\frac{1}{2}x(t)^2\right)$

$y(t) = x(t) \cdot \frac{d}{dt}x(t)$

Modify the input and plug in

$y_2(t) = [ax(t)] \cdot \frac{d}{dt}[ax(t)]$

$y_2(t) = a[ax(t)] \cdot \frac{d}{dt}[x(t)]$

$[a^2 \cdot y(t)] \neq [a \cdot y(t)]$ No homogeneity → $\boxed{\text{not linear}}$

c) $y(t) = e^{x(t)}$

Modify the input and plug in

$y_1(t) = e^{d_1 x_a(t) + d_2 x_b(t)}$

$y_1(t) = \left[e^{d_1 x_a(t)}\right] \cdot \left[e^{d_2 x_b(t)}\right]$

$y_1(t) \neq d_1 e^{x_a(t)} + d_2 e^{x_b(t)}$ Not equivalent → $\boxed{\text{not linear}}$

d) $y(t) = x(t) + 2u(t+1)$

Modify the input and plug in

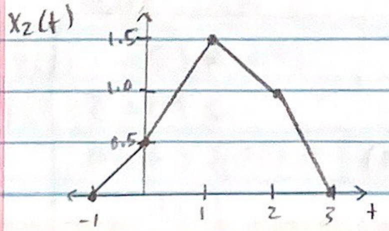
$y_1(t) = [ax(t)] + 2u(t+1)$

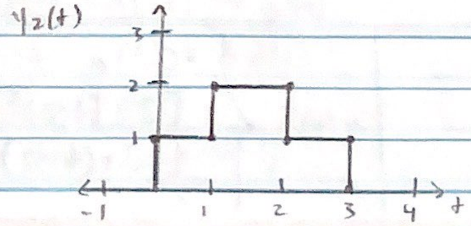
$y_1(t) \neq a(y(t))$ No homogeneity → $\boxed{\text{not linear}}$

2. a) Express $x_2(t)$ and $y_2(t)$ in terms of $x_1(t)$ and $y_1(t)$ respectively

$x_2(t) = x_1(t) + x_1(t+1)$

$y_2(t) = y_1(t) + y_1(t+1)$





b) $y_1(t) = \cos(t)u(t)$ when $x_1(t) = u(t)$

$y_2(t) = \cos(t)(u(t+1) - u(t))$ when $x_2(t) = rect(t + \frac{1}{2})$

OR $x_2(t) = u(t+1) - u(t)$

Combine both scenarious and use properties of linearity

Say: $x_a(t) = x_1(t) + x_2(t)$

$x_a(t) = u(t+1) \longrightarrow x_a(t) = x_1(t+1)$

Plug new combined input in to receive new output

$y_a(t) = [\cos(t) \cdot u(t)] + [\cos(t)(u(t+1) - u(t))]$

$y_a(t) = \cos(t)\,u(t+1)$

$$\left.\begin{array}{l} x_a(t) = x_1(t+1) \longrightarrow y_a(t) = \cos(t)u(t+1) \\ x_1(t+1) = x_1(t+1) \longrightarrow \cos(t+1)u(t+1) \end{array}\right] \text{not equivalent}$$

The time shift in the input does not produce an identical shift in the output $\boxed{\text{Not time-invariant}}$

3. a) i) $f(t) = \partial(t+1) + 2\partial(t-2)$ $\quad g(t) = e^{-t}u(t)$

$y(t) = [\partial(t+1) + 2\partial(t-2)] * [e^{-t}u(t)]$

$y(t) = [\partial(t+1) * e^{-t}u(t)] + [2\partial(t-2) * e^{-t}u(t)]$

$\boxed{y(t) = [e^{-t-1}u(t+1)] + [2e^{-t+2}u(t-2)]}$

ii) $f(t) = 2\,rect(t - \frac{3}{2})$ $\quad g(t) = 2r(t-1)rect(t - \frac{3}{2})$

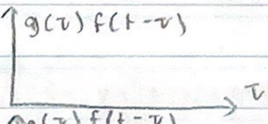
$y(t) = [2\,rect(t - \frac{3}{2})] * [2r(t-1)\,rect(t - \frac{3}{2})]$

$y(t) = \int_{-\infty}^{\infty} g(\tau) f(t-\tau) \partial\tau$

Use intervals $t \leq 2$, $2 < t \leq 3$, $3 < t \leq 4$, $t > 4$

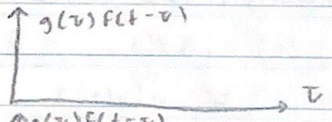
$t \leq 2$

No overlap



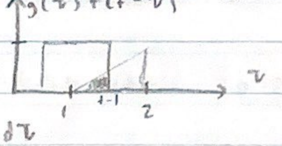
$t > 4$

No overlap



$2 < t \leq 3$

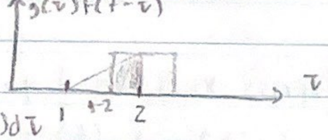
Overlap



$\int_1^{t-1} 4(\tau - 1)\partial\tau$

$3 < t \leq 4$

Overlap



$\int_{t-2}^{2} 4(\tau - 1)\partial\tau$

Combine all overlappings together

$$\boxed{y(t) = \begin{cases} 0 & t \leq 2 \\ [2 \cdot (t-2)^2] & 2 < t \leq 3 \\ [-2 \cdot (t-2) \cdot (t-4)] & 3 < t \leq 4 \\ 0 & t > 4 \end{cases}}$$

b) i) $y(t) = \int_{t-T}^{t} x(\tau)\, d\tau$

$h(t) = \int_{t-T}^{t} \partial(\tau)\, d\tau$

$\boxed{h(t) = u(t) - u(t-T)}$

ii) $y(t) = \sum_{n=-\infty}^{\infty} x(t - nT_s)$

$\boxed{h(t) = \sum_{n=-\infty}^{\infty} \partial(t - nT_s)}$

c) i) $[\partial(t-3) + \partial(t+2)] \cdot [e^{3t} u(-t) + \partial(t+2) + 2]$ → Expand

$e^{3(t-3)} u(-t+3) + e^{3(t+2)} u(-t-2) + \partial(t-1) + \partial(t+4) + 2 + 2$

$\boxed{e^{3(t-3)} u(-t+3) + e^{3(t+2)} u(-t-2) + \partial(t-1) + \partial(t+4) + 4}$

ii) $\frac{d}{\partial t}[(u(t) - u(t+1)) \star u(t-2)]$

$u(t) \star u(t) = \int_{-\infty}^{\infty} u(\tau) u(t-\tau)\, d\tau$

$= u(t) \cdot \int_0^t 1\, d\tau$

$= t \cdot u(t) \longrightarrow r(t)$

Expand $\frac{d}{\partial t}[(u(t) - u(t+1)) \star u(t-2)]$

$\frac{d}{\partial t}[r(t-2) - r(t-1)]$

$\boxed{u(t-2) - u(t-1)}$

d) i) x(t) and h(t) both odd

$y(t) = x(t) \star h(t)$ → y(t) is an even function

$y(t) = \int_{-\infty}^{\infty} x(\tau) h(t-\tau)\, d\tau$

Substitute τ with $(\tau' = -\tau)$

$y(t) = \int_{-\infty}^{\infty} x(-\tau') \cdot h(t + \tau')\, d\tau'$

Switch signs of x(t) and h(t) → can do this b/c they are odd

$y(t) = \int_{-\infty}^{\infty} -x(\tau') \cdot -h(-t - \tau')\, d\tau'$

$y(t) = \int_{-\infty}^{\infty} x(\tau') \cdot h(-t - \tau')\, d\tau'$

$y(t) = y(-t)$ → $\boxed{\text{True : } y(t) \text{ is an even function}}$

ii) $y(t) = x(t) \star h(t)$

say $h(t) = \partial(t)$

$h(2t) = \frac{1}{2}\partial(t)$

$y(t) = u(t) \star \partial(t)$

$y(t) = u(t)$

$y(2t) = h(2t) \star x(2t)$

$x(t) = u(t)$

$x(2t) = u(2t) \rightarrow u(t)$

$y(2t) = u(t) \star \frac{1}{2}\partial(t)$

$y(2t) = \frac{1}{2} u(t)$

$\boxed{\text{False : } y(t) \neq y(2t) \text{ in this scenario}}$

4. $S_1 : y(t) = \int_{-\infty}^{t} x(\tau - t_0)\,d\tau$

$S_2 : h_2(t) = u(t-2)$

$S_3 : h_3(t) = u(t+3)$

a) $y(t) = \int_{-\infty}^{t} x(\tau - t_0)\,d\tau$

Substitution : $(\tau - t_0) \rightarrow \tau'$

$y(t) = \int_{-\infty}^{t-t_0} x(\tau')\,d\tau$

$y(t) = \int_{-\infty}^{t-t_0} x(\tau)\,d\tau$

$h(t) = \int_{-\infty}^{t-t_0} \delta(\tau)\,d\tau \quad \rightarrow \quad \boxed{h(t) = u(t-t_0)}$

b) $x(t) \longrightarrow \overset{h_{eq}(t)}{\boxed{S_{eq}}} \longrightarrow w(t)$

$h_{eq}(t) = -(h_2(t) * h_3(t)) + h(t)$

$h_{eq}(t) = -(u(t-2) * u(t+3)) + u(t-t_0)$

$\boxed{h_{eq}(t) = -r(t+1) + u(t-t_0)}$

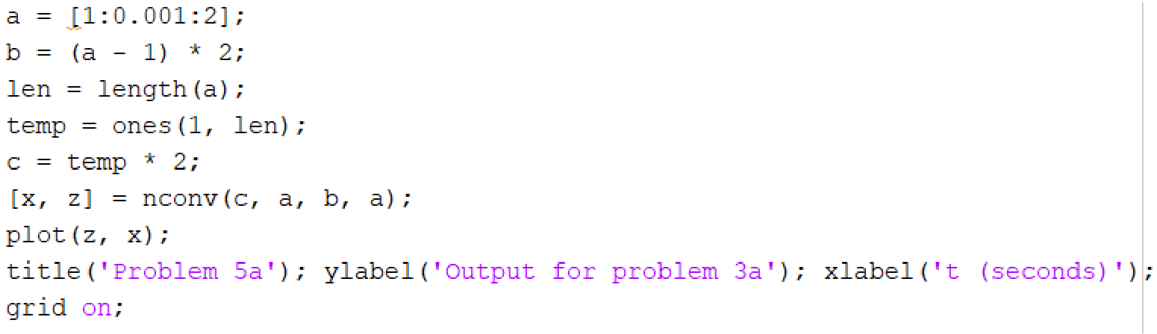
c) $x(t) = \delta(t) + \delta(t-3)$

$y(t) = x(t) * h_{eq}(t)$

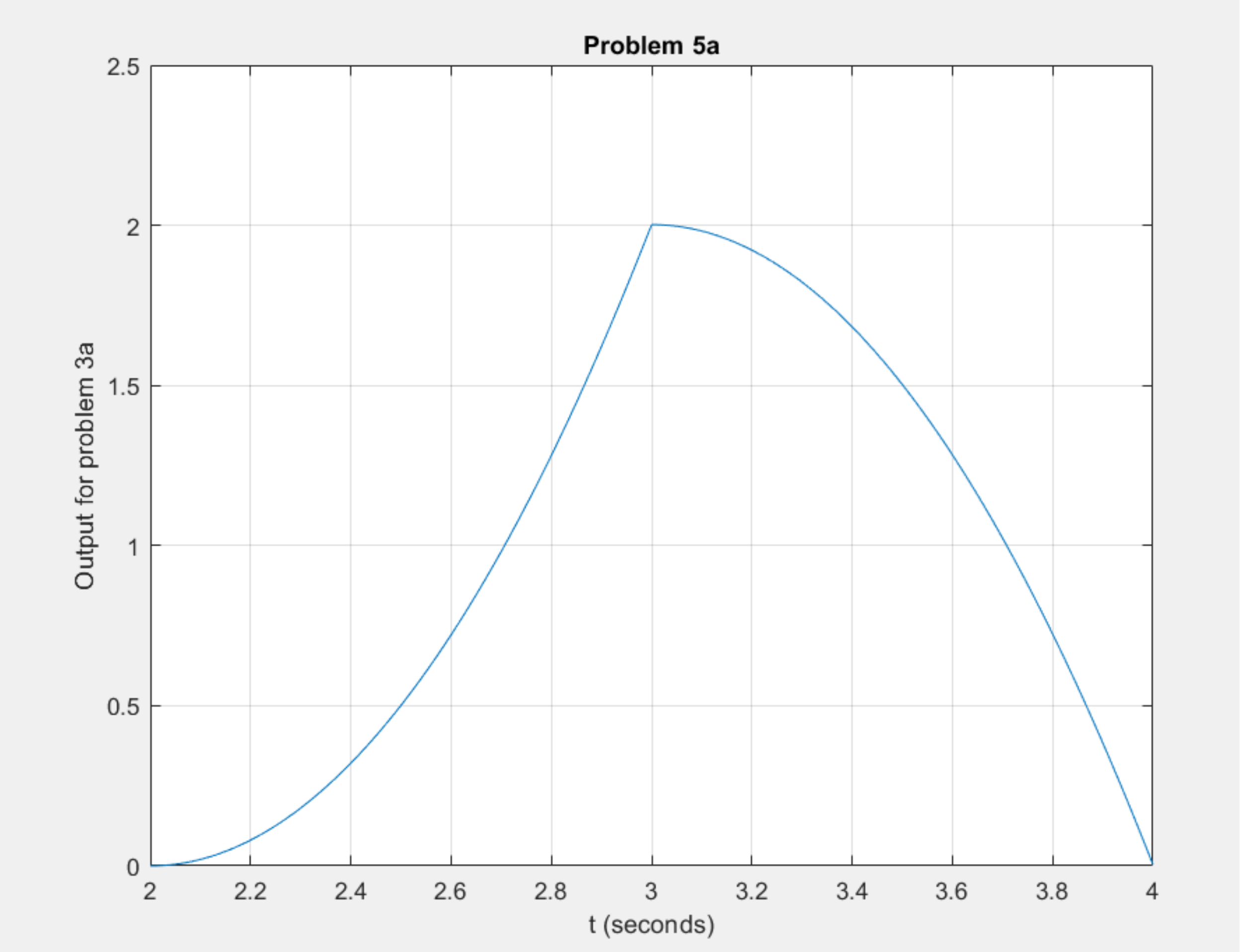
$y(t) = [\delta(t) + \delta(t-3)] * [-r(t+1) + u(t-t_0)]$

$\boxed{y(t) = -r(t+1) - r(t-2) + u(t-t_0) + u(t-t_0-3)}$

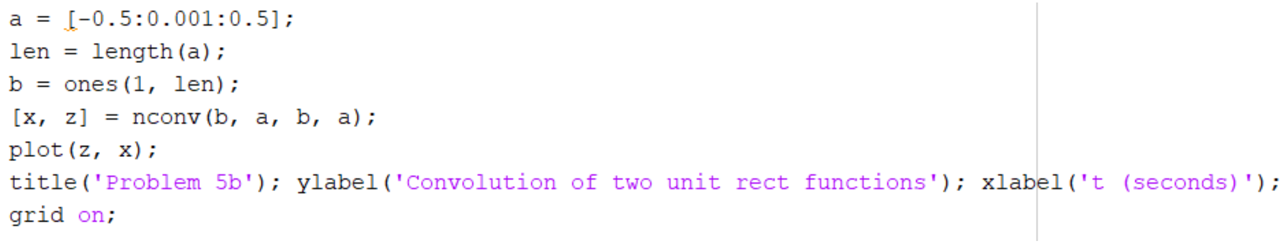
5a)



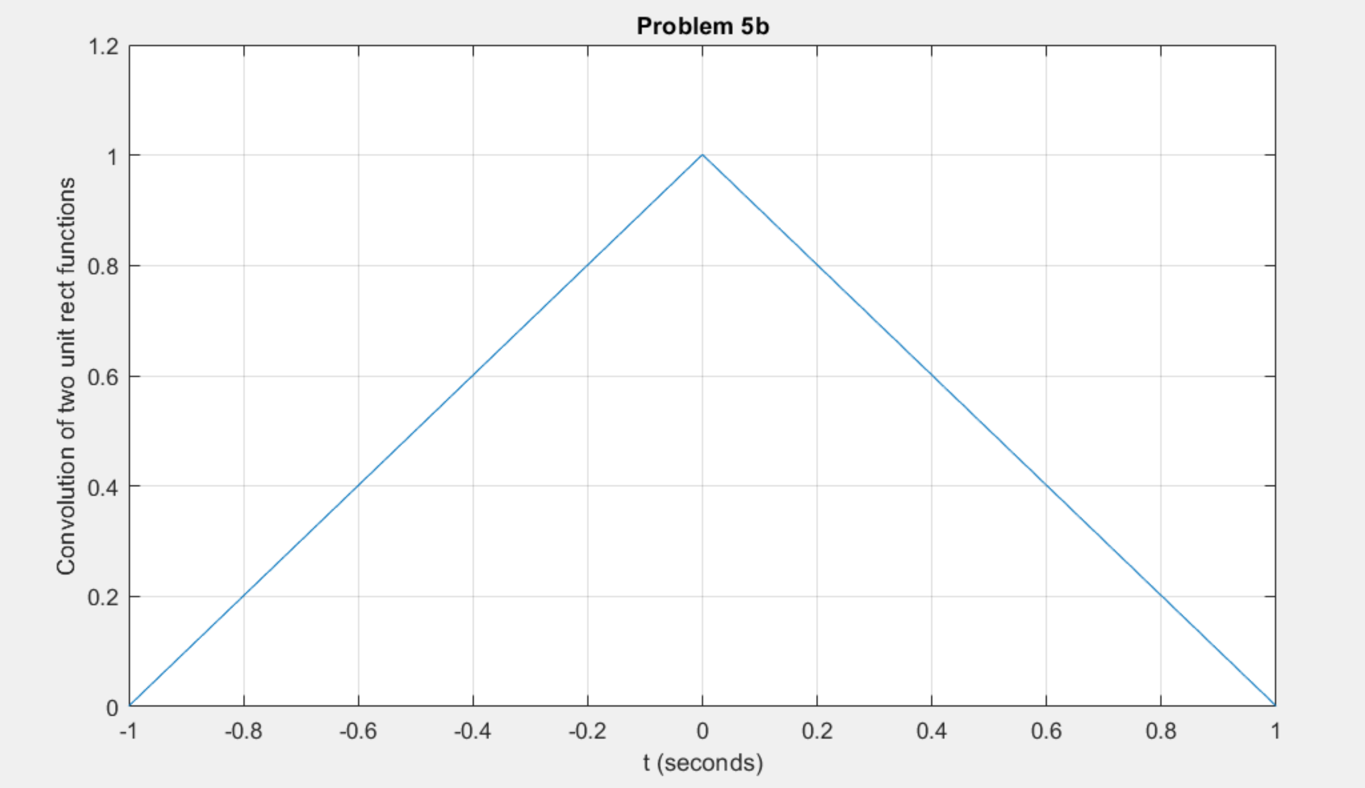
```
a = [1:0.001:2];
b = (a - 1) * 2;
len = length(a);
temp = ones(1, len);
c = temp * 2;
[x, z] = nconv(c, a, b, a);
plot(z, x);
title('Problem 5a'); ylabel('Output for problem 3a'); xlabel('t (seconds)');
grid on;
```



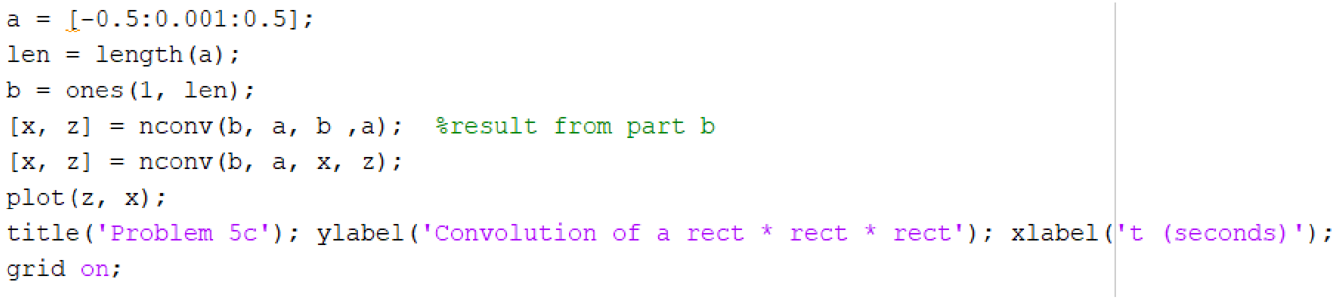
5b)



```
a = [-0.5:0.001:0.5];
len = length(a);
b = ones(1, len);
[x, z] = nconv(b, a, b, a);
plot(z, x);
title('Problem 5b'); ylabel('Convolution of two unit rect functions'); xlabel('t (seconds)');
grid on;
```



5c)



```
a = [-0.5:0.001:0.5];
len = length(a);
b = ones(1, len);
[x, z] = nconv(b, a, b ,a);  %result from part b
[x, z] = nconv(b, a, x, z);
plot(z, x);
title('Problem 5c'); ylabel('Convolution of a rect * rect * rect'); xlabel('t (seconds)');
grid on;
```