EE102

Lecture 4

EE102 Announcements

- Syllabus link is <u>tinyurl.com/ucla102</u>
- CCLE difficulties, please email help@seas.ucla.edu
- My office hour meeting minutes are sent out weekly
- First Homework due this Friday

Slide Credits: This lecture adapted from Prof. Jonathan Kao (UCLA) and recursive credits apply to Prof. Cabric (UCLA), Prof. Yu (Carnegie Mellon), Prof. Pauly (Stanford), and Prof. Fragouli (UCLA)

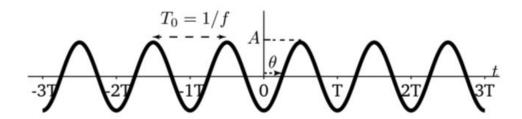
Sidebar: Regarding Periodic Signals

The sum or product of a periodic signal is itself periodic if:

Real sinusoids (cont.)

We illustrate a sinusoid signal below:

$$x(t) = A\cos(\omega t - \theta)$$

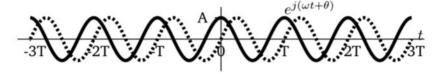


Complex sinusoids

The complex sinusoid is given by:

$$Ae^{j(\omega t + \theta)} = A\cos(\omega t + \theta) + jA\sin(\omega t + \theta)$$

We draw complex signals with dotted lines.



The real part of the complex sinusoid (solid line) is:

$$\Re\left(Ae^{j(\omega t+\theta)}\right) = A\cos(\omega t + \theta)$$

The imaginary part of the complex sinusoid (dotted line) is:

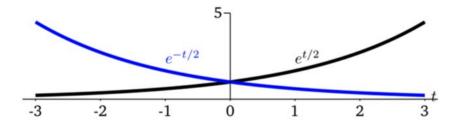
$$\Im\left(Ae^{j(\omega t+\theta)}\right) = A\sin(\omega t + \theta)$$

Exponential

An exponential signal is given by

$$x(t) = e^{\sigma t}$$

- If $\sigma > 0$, this signal grows with increasing t (black signal in plot below). This is called exponential growth.
- If $\sigma < 0$, this signal decays with increasing t (blue signal in plot below). This is called exponential decay.

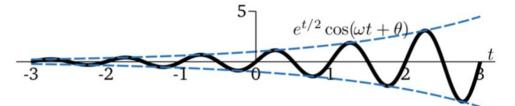


Damped or growing sinusoids

A damped or growing sinusoid is denoted

$$x(t) = e^{\sigma t} \cos(\omega t + \theta)$$

The sinusoid will grow exponentially if $\sigma > 0$ and decay exponentially if $\sigma < 0$.

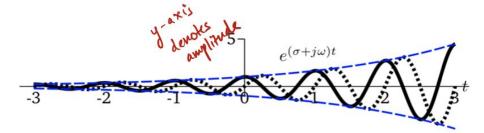


Complex exponential

A complex sinusoid is denoted

$$x(t) = e^{(\sigma + j\omega)t}$$

It is a combination of the complex sinusoid and an exponential. All prior signals are special cases of the complex exponential signal.



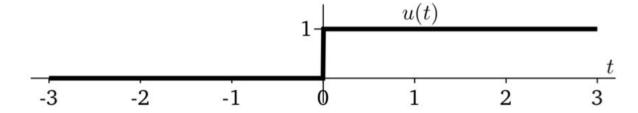
It is helpful to think of σ and $j\omega$ in the complex plane. σ is the x-axis and $j\omega$ is the y-axis. Then complex exponentials in the left complex plane are decreasing signals and those in the right are increasing signals.

Heaviside Step Function

The unit step function, denoted u(t) in this class, is given by

$$u(t) = \begin{cases} 1, & t \ge 0 \\ 0, & t < 0 \end{cases}$$

It is also called the Heavyside step function. Drawn below:



Unit Rectangle

$$rect(t) = \begin{cases} 1, & |t| \le 1/2 \\ 0, & else \end{cases}$$

Unit Ramp Function

Unit Triangle

Impulse Function (Important!)

Impulse Function (intuition)

Impulse Function Intuition

Impulse Sampling Property

Impulse Sampling Property

Impulse Sifting Property

Impulse Sifting Property

CYU: Calculate

$$\int_{-2}^{3+} f(t) \left[1 + \delta(t+1) - 3\delta(t-1) + 2\delta(t+3) \right] dt$$

CYU: Integral of an Impulse

$$\int_{-\infty}^{t} \delta(\tau)d\tau =$$

CYU (Visual)

Suppose
$$x(t) = 1 + \delta(t-1) - 2\delta(t-2)$$
 then what is $y(t) = \int_0^t x(\tau)d\tau$

Systems

A system transforms an input signal, x(t), into an output signal, y(t).

Systems, like signals, are similar to functions. However, they map a signal to another signal, so the term we might use is "operator".

For EE102, we will not nitpick this distinction and focus on SISO systems.