EE102

Lecture 5

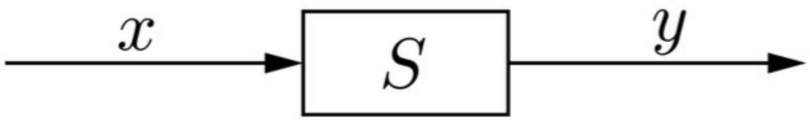
EE102 Announcements

- Syllabus link is tiny.cc/ucla102
- CCLE difficulties, please email help@seas.ucla.edu
- My office hour meeting minutes are sent out weekly
- **Second Homework due this Friday**

Slide Credits: This lecture adapted from Prof. Jonathan Kao (UCLA) and recursive credits apply to Prof. Cabric (UCLA), Prof. Yu (Carnegie Mellon), Prof. Pauly (Stanford), and Prof. Fragouli (UCLA)

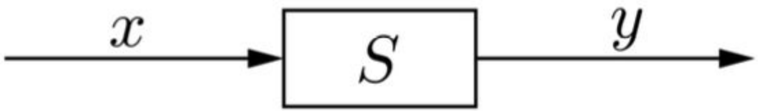
What is a system?

A system transforms an *input signal*, $x(t)$, into an output system, $y(t)$.



- Systems, like signals, are also *functions*. However, their inputs and outputs are signals.
- Systems can have either single or multiple inputs (SI or MI, respectively) and single or multiple outputs (SO and MO). In this class, we focus on *single input, single output* systems (SISO).

Systems have Properties



Stability

A system is *bounded-input, bounded-ouput* (BIBO) stable if every bounded input leads to a bounded output.

$$|x(t)| < \infty \implies |y(t)| < \infty$$

Causality

A system is causal if its output only depends on past and present values of the input.

Systems have Properties

Time-invariance

A system is *time invariant* if a time shift in the input only produces an identical time shift of the output.

Mathematically, a system S is time-invariant if

$$y(t) = S(x(t))$$

implies that

$$y(t - \tau) = S(x(t - \tau))$$

Examples of Time Invariance

Linearity

A system is *linear* if the following two properties hold:

1. **Homogeneity**: for any signal, x, and any scalar a,

$$S(ax) = aS(x)$$

2. **Superposition**: for any two signals, x and $\tilde{x}$,

$$S(x + \tilde{x}) = S(x) + S(\tilde{x})$$

Linearity Examples

Linearity examples (Cont'd)

Linearity and time-invariance recap

Memory

A system has *memory* if its output depends on past or future values of the input. If the output depends only on present values of the input, the system is called *memoryless*.

Invertibility

A system is called *invertible* if an input can always be exactly recovered from the output. That is, a system S is invertible if there exists an S^{inv} such that

$$x = S^{\text{inv}}(S(x))$$

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Impulse Response

System impulse response

This lecture introduces time-domain analysis of systems, including the impulse response. It also discusses linear time-invariant systems. Topics include:

- Impulse response definition
- Impulse response of LTI systems
- The impulse response as a sufficient characterization of an LTI system
- Impulse response and the convolution integral

Why do we need the impulse response?

Types of Responses

Impulse Response Definition

$$h(t, \tau) = H(\delta(t - \tau))$$

There are important things to be careful of when looking at this equation.

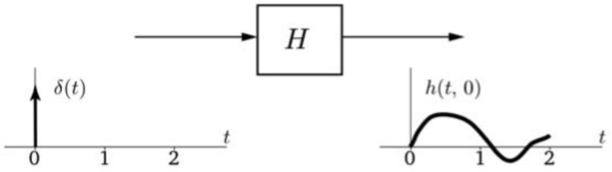
- The t on the left and right hand side of these equations *are not the same*!
- The t on the left hand side is the impulse response at a specific value of time.
- The t on the right hand side varies across all time.
- The output at the specific time t on the left will depend on the input at several times t on the right.

Notation on t

$$h(t, \tau) = H(\delta(t - \tau))$$

There are important things to be careful of when looking at this equation.

An example of these t's not being the same is shown below. In this example, let $\tau = 0$.



It may be tempting to write:

$$h(1, 0) = H(\delta(1))$$

This is wrong.

- On the left, $\delta(1) = 0$. We know if H is linear, then $H(0) = 0$, implying that $h(1, 0) = 0$.
- But in general, the impulse response can be non-zero, i.e., $h(1, 0) \neq 0$ in the above diagram, if the impulse response produces some non-zero response.

Time invariant Impulse Response

Time Invariant Impulse Response

Impulse response of a time-invariant system (cont.)

This property of the impulse response for a time-invariant system is drawn below: