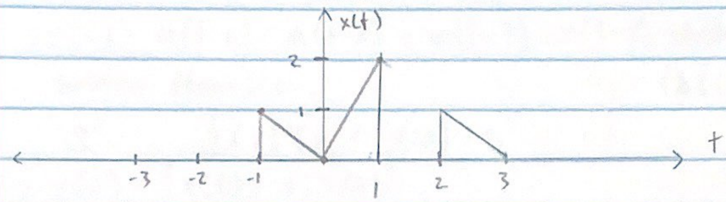
Ethan Wong
305319001

ECE 102 Homework 2

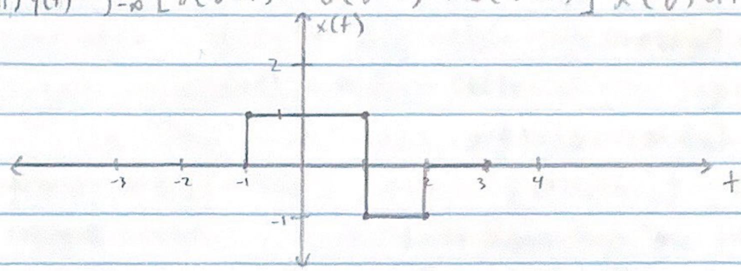
1. a) i) $y(t) = x(t)[1 - u(t-1) + u(t-2)]$

$$y(t) = \begin{cases} x(t) & 2 \le t \le 3 \text{ or } -1 \le t \le 1 \\ 0 & \text{otherwise} \end{cases}$$

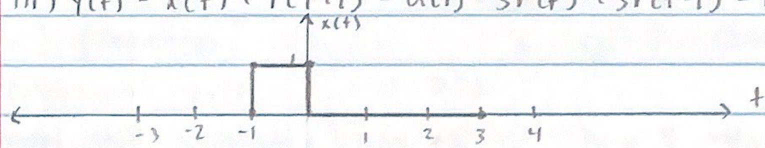


ii) $y(t) = \int_{-\infty}^{t} [\delta(\tau+1) - \delta(\tau-1) + \delta(\tau-2)]\, x(\tau)\, dt$

(sifting property)



iii) $y(t) = x(t) + r(t+1) - u(t) - 3r(t) + 3r(t-1) - r(t-3)$



b) i) $\int_{-\infty}^{\infty} f(t+1)\, \delta(t+1)\, dt$ (sifting property)

$\left[\int_{-\infty}^{\infty} \delta(t+1)\, dt\right] \cdot f(0)$

$\boxed{f(0)}$ 0

ii) $\int_{t}^{\infty} e^{-2\tau} u(\tau-1)\, d\tau$

$$\begin{cases} u(\tau-1) = 1 & t \ge 1 \\ u(\tau-1) = 0 & t < 1 \end{cases}$$

$t < 1$ $\int_{t}^{\infty} e^{-2\tau} u(\tau-1)\, d\tau$

$\int_{1}^{\infty} e^{-2\tau}\, d\tau$

$\left[\frac{-e^{-2\tau}}{2}\right]_{t}^{\infty} = \boxed{\frac{e^{-2}}{2}}$

$t \ge 1$ $\int_{t}^{\infty} e^{-2\tau} u(\tau-1)\, d\tau$

$\int_{t}^{\infty} e^{-2\tau}\, d\tau$

$\left[\frac{-e^{-2\tau}}{2}\right]_{t}^{\infty} = \boxed{\frac{e^{-2t}}{2}}$

iii) $\int_0^\infty f(t)\,[\partial(t-1) + \partial(t+1)]\,dt$

split in two $^{①}\int_0^\infty f(t)\,\partial(t+1)dt + ^{②}\int_0^\infty f(t)\,\partial(t-1)\,dt$ (sifting property)

① $\int_0^\infty f(t)\,\partial(t+1)\,dt = 0$, limit doesn't include -1 $[0,\infty)$

② $\int_0^\infty f(t)\,\partial(t-1)dt$

$\int_0^\infty f(1)\,\partial(t-1)\,dt$

$f(1)$ $0 + f(1) = \boxed{f(1)}$ 2

c) $\partial(bt) = \frac{1}{b}\partial(t)$

Think of delta function as: $\partial(t) = \lim_{x \to 0} \text{rect}_x(t)$

$\partial(bt) = \lim_{x \to 0} \text{rect}_x(bt)$

Now say we declare $x' = \frac{1}{b}x$

$\text{rect}_x(bt) = \frac{1}{b}[\text{rect}_{x'}(t)]$

In this way we can prove that:

$\lim_{x \to 0} \text{rect}_x(bt) = \lim_{x' \to 0} \frac{1}{b}[\text{rect}_{x'}(t)]$

$\partial(bt) = \frac{1}{b}\partial(t)$ ✓

(I'm using x instead of Δ because Δ makes me think of the unit triangle and I got confused)

2. a) i) This looks like 2 unit triangles added together. Both triangles had to be shifted and scaled before adding.

$\boxed{x(t) = [2\Delta(t-2)] + [\Delta(t-1)]}$

ii) This looks like 2 unit triangles with a unit rectangle. The rectangle must be scaled up and the triangles shifted sideways.

$\boxed{x(t) = [\text{rect}(\tfrac{t}{2})] + [\Delta(t-0.5)] + [\Delta(t+0.5)]}$

iii) This is many unit triangles added together. The smaller middle triangle is simply $\Delta(2t)$. The taller left triangle is the sum of two smaller triangles, represented by $[2\Delta(t-\frac{3}{2})] + [2\Delta(2(t-1))]$. The larger right triangle is basically the same but with the signs flipped since it is being shifted the opposite way.

$\boxed{x(t) = [\Delta(2t)] + [2\Delta(t-\tfrac{3}{2})] + [2\Delta(2(t-1))] + [2\Delta(t+\tfrac{3}{2})] + [2\Delta(2(t+1))]}$

b) i) Scale up, right one, right two, right 3 (drop down by one)

$\boxed{x(t) = 3u(t) - u(t-1) - u(t-3) - u(t-4)}$

ii) right one, right one, right one, right two, right two, right one

$\boxed{x(t) = u(t-1) - u(t-8) - u(t-7) + u(t-2) + 4u(t-5) - 4u(t-3)}$

balance them out

3. a) i) $y(t) = |x(t)| + x(t^2)$

Delay input $y_\tau(t) = |x(t-\tau)| + x(t^2 - \tau)$

Delay output $y(t-\tau) = |x(t-\tau)| + x((t-\tau)^2)$ Time-variant

Output can depend on future values of the input

i.e. when $t=3$, need $x(9)$ to solve Not causal

$|x(t)| \leq I$ for any t, then

$|y(t)| = ||x(t) + x(t^2)| \leq |x(t)| + |x(t^2)| \leq 2I$ Bounded

$\boxed{\text{Time-variant, not causal, stable}}$

ii) $y(t) = \int_{t-T}^{t+T} x(\lambda)\, d\lambda$ → T is positive and constant

Delay input $y_\tau(t) = \int_{t-T}^{t+T} x(\lambda - \tau)\, d\lambda$

Delay output $y_\tau(t-\tau) = \int_{(t-\tau)-T}^{(t-\tau)+T} x(\lambda - \tau)\, d(\lambda - \tau)$ Time-invariant

Output can depend on future values of the input

i.e. integrating all the way to $t+T$ (future) Not causal

$|x(t)| \leq I$ for any t, then

$|y(t)| = \left|\int_{t-T}^{t+T} x(\lambda)\, d\lambda\right| \leq \int_{t-T}^{t+T} |x(\lambda)|\, d\lambda \leq \int_{t-T}^{t+T} B_x\, dx = 2TI$ Bounded

$\boxed{\text{Time-invariant, not causal, stable}}$

iii) $y(t) = (t+1)\int_{-\infty}^{t} x(\lambda)\, d\lambda$

Delay input $y_\tau(t) = (t+1)\int_{-\infty}^{t} x(\lambda - \tau)\, d\lambda$

Delay output $y(t-\tau) = (t-\tau+1)\int_{-\infty}^{t-\tau} x(\lambda)\, d\lambda$ Time-variant

Output does not depend on future values, only care about

values up until time t Causal

This system is not bounded due to it integrating from negative infinity

$\boxed{\text{Time-variant, causal, unstable}}$

iv) $y(t) = 1 + e^{x(t)}$

Delay input : $y_\tau(t) = 1 + e^{x(t-\tau)}$

Delay output : $y(t-\tau) = 1 + e^{x(t-\tau)}$ Time invariant

Output does not depend on any future values, only care about values up until time t Causal

$|x(t)| \le I$, then

$|y(t)| \le e^{I}$

Exponent will be finite as long as input is finite → bounded

$\boxed{\text{Time-invariant, Causal, Stable}}$

v) $y(t) = \frac{1}{1+x^2(t)}$

Delay input : $y_\tau(t) = \frac{1}{1+x^2(t-\tau)}$

Delay output : $y(t-\tau) = \frac{1}{1+x^2(t-\tau)}$ Time invariant

Output does not depend on any future values, only care about present value t Causal

No matter what the input is, the output will always be a positive fraction → $0 \le y(t) \le 1$ → Bounded

$\boxed{\text{Time-invariant, Causal, stable}}$

b) $S_1 : w(t) = x\left(\frac{t}{2}\right)$ $S_2 : z(t) = \int_{-\infty}^{t} w(\tau)d\tau$ $S_3 : y(t) = S_3(z(t))$

$y(t) = \int_{-\infty}^{t-1} x(\tau)d\tau$

Translate $z(t)$ $z(t) = \int_{-\infty}^{t} w(\tau)d\tau \rightarrow \int_{-\infty}^{t} x\left(\frac{\tau}{2}\right)d\tau$

Substitution $\tilde{\tau} = \frac{1}{2}\tau$ $d\tilde{\tau} = \frac{1}{2}d\tau$

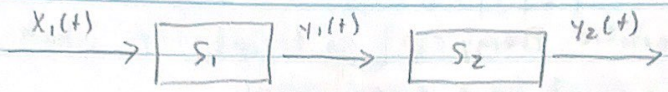
$z(t) = 2\int_{-\infty}^{t/2} x(\tilde{\tau})d\tilde{\tau}$

Match y $z(2t) = 2\int_{-\infty}^{t} x(\tilde{\tau})d\tilde{\tau}$

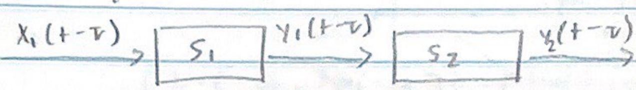
$z(2t-2) = 2\int_{-\infty}^{t-1} x(\tilde{\tau})d\tilde{\tau}$

$\boxed{y(t) = \frac{z}{2}(2t-2)}$

c) i) Original Diagram

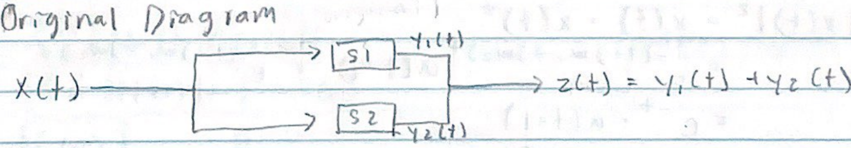


Delayed Diagram

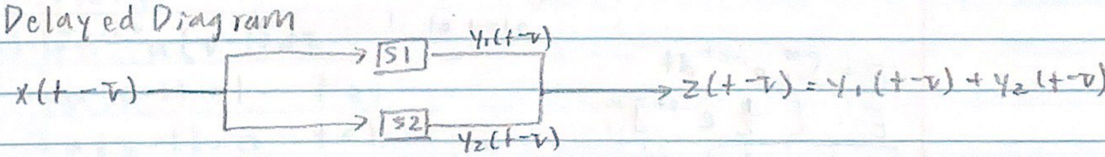


Because both S1 and S2 are time-invariant, the series cascade of both S1 and S2 is time-invariant. As seen in the "delayed" diagram, the first input to the series is delayed. The output of that first system is also delayed. Again, inputting a delayed system into the second time-invariant system yields an identical shift in the output. Because we see in the delayed diagram that an input of $x_1(t-\tau)$ yields $y_2(t-\tau)$, we can conclude that the series cascade is time-invariant as there are corresponding shifts between the original and delayed diagrams.

ii) Original Diagram



Delayed Diagram



As seen in the above diagrams, adding a delay to the initial input $x(t)$ so that it is $x(t-\tau)$ makes the systems yield $y_1(t-\tau)$ and $y_2(t-\tau)$. The final output of the parallel cascade is equal to $z(t-\tau)$. The shift in the input leads to an identical shift in the output, so the parallel cascade is time-invariant.

4. a) $x(t) = Ae^{j\omega t} + Be^{-j\omega t}$

This signal is periodic. Energy can't go to infinity, which means that this must be a power signal.

$$P_x = \lim_{T\to\infty} \frac{1}{2T}\int_{-T}^{T} |x(t)|^2 dt$$

$$|x(t)|^2 = x(t)\cdot x(t)^*$$
$$= [Ae^{j\omega t} + Be^{-j\omega t}]\cdot[Ae^{-j\omega t} + Be^{j\omega t}]$$
$$= A^2 + B^2 + ABe^{-2j\omega t} + ABe^{2j\omega t}$$
$$= A^2 + B^2 + 2AB\cos(2\omega t)$$

$$P = \lim_{T\to\infty} \frac{1}{2T}\int_{-T}^{T} A^2 + B^2 + 2AB\cos(2\omega t)\, dt$$

$$P = \lim_{T\to\infty} \frac{1}{2T}\left[2TA^2 + 2TB^2 + \frac{AB}{\omega}\sin(2\omega t)\right]_{-T}^{T}$$

$$P = \lim_{T\to\infty} \frac{1}{2T}\left[2TA^2 + 2TB^2 + \frac{2AB}{\omega}\sin(2\omega T)\right]$$

$$P = \lim_{T\to\infty} \left[A^2 + B^2 + \frac{AB}{\omega T}\sin(2\omega T)\right]$$

$\boxed{P = A^2 + B^2}$ Power signal

b) $x(t) = e^{-(1+j\omega_1+j\omega_2)t}u(t+1)$

This signal is not periodic

Energy : $E = \int_{-\infty}^{\infty} |x(t)|^2 dt$

$$|x(t)|^2 = x(t)\cdot x(t)^*$$
$$= e^{-(1+j\omega_1+j\omega_2)t}u(t+1) \cdot e^{-(1-j\omega_1-j\omega_2)t}$$
$$= e^{-t}\cdot u(t+1)$$

start at -1

$$E = \int_{-1}^{\infty} e^{-2t} dt$$

$$E = \left[-\frac{1}{2}e^{-2t}\right]_{-1}^{\infty}$$

$\boxed{E = \frac{1}{2}e^2}$ Energy Signal

$P = 0$

5a)

The decay rate is supposed to reduce the signal level to $\frac{1}{3}$ of its original value by 10 seconds.

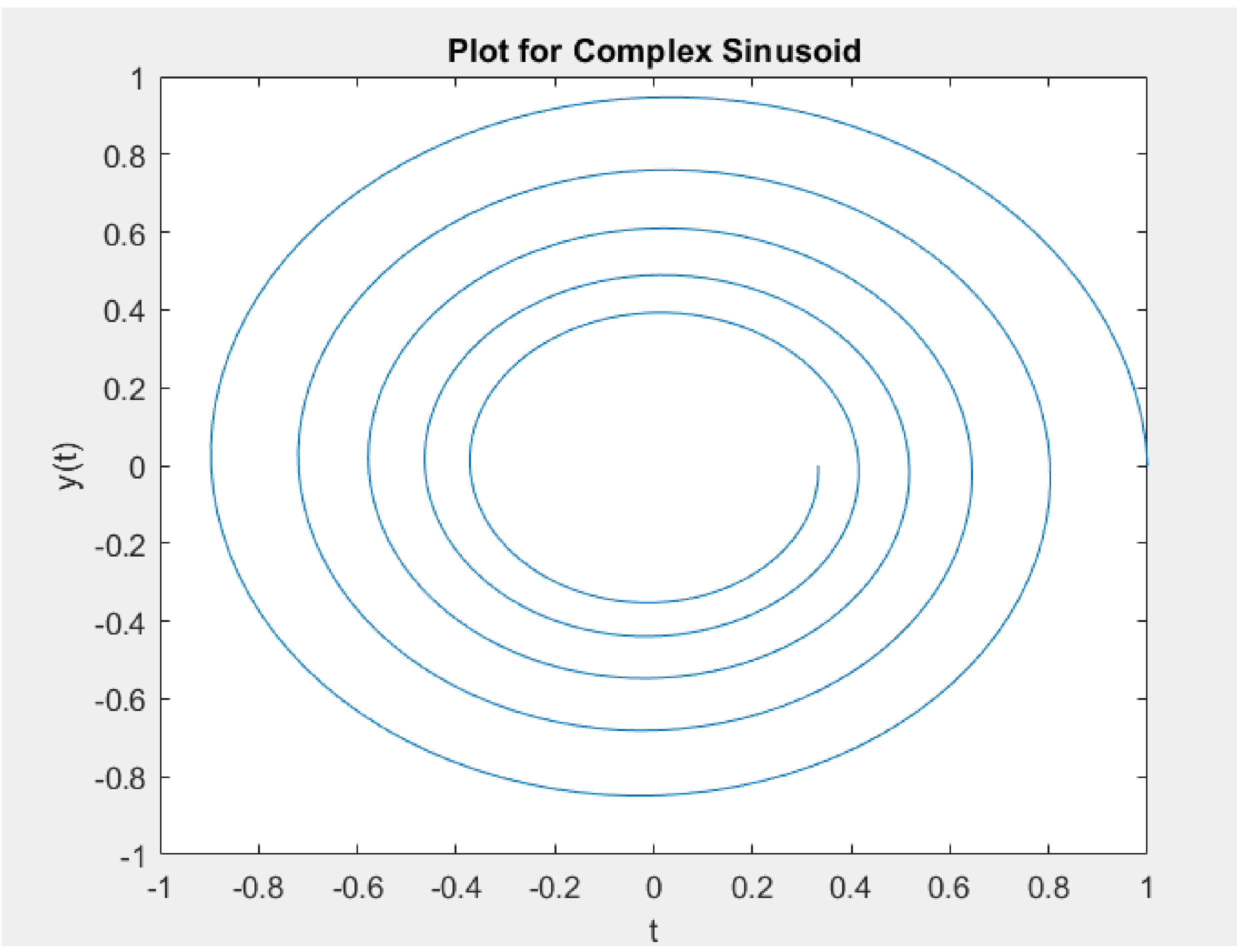
$$e^{10\sigma} = \frac{1}{3}$$

$$\sigma = -\ln(3) \div 10$$

If the period should be two seconds, that means $\omega = \pi$ because $T = 2\pi/\omega$.

hw2a.m

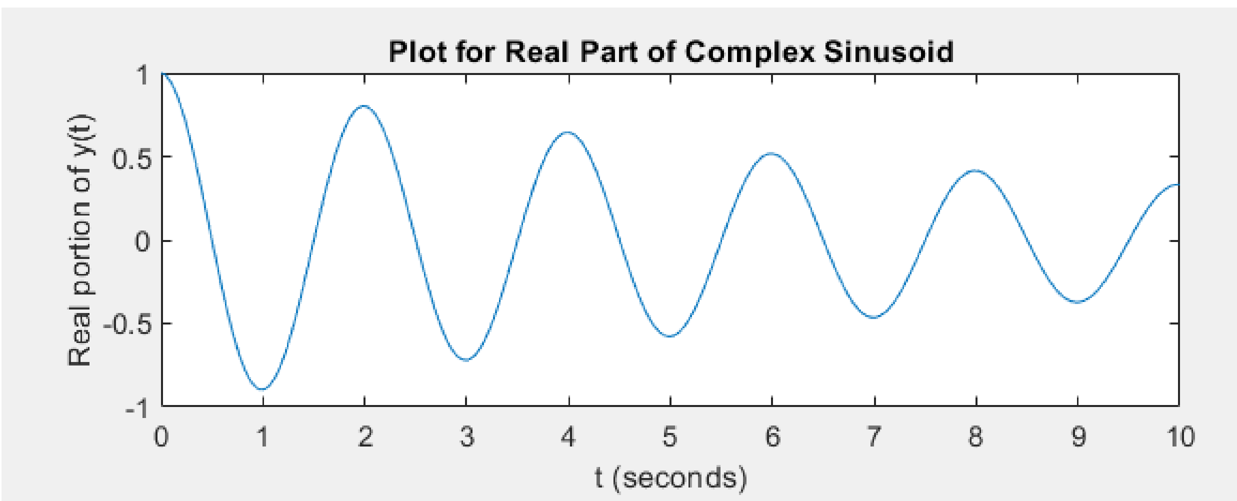
```
o = pi;
s = -log(3) / 10;
t = linspace(0, 10, 500);
y = exp(t * (s + 1i * o));
plot(y);
title('Plot for Complex Sinusoid'); xlabel('t'); ylabel('y(t)');
```



b)

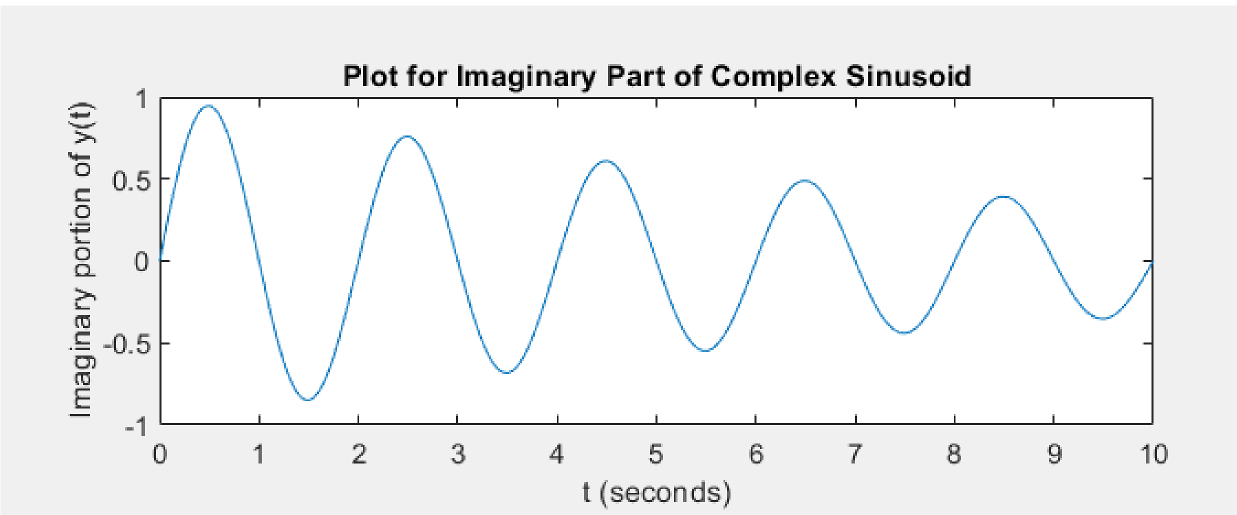
Real Part of Complex Sinusoid

```
o = pi;
s = -log(3) / 10;
t = linspace(0, 10, 500);
y = exp(t * (s + 1i * o));
subplot(2, 1, 1);
plot(t, real(y));
title('Plot for Real Part of Complex Sinusoid');
xlabel('t (seconds)'); ylabel('Real portion of y(t)');
```



Imaginary Part of Complex Sinusoid

```
o = pi;
s = -log(3) / 10;
t = linspace(0, 10, 500);
y = exp(t * (s + 1i * o));
subplot(2, 1, 2);
plot(t, imag(y));
title('Plot for Imaginary Part of Complex Sinusoid');
xlabel('t (seconds)'); ylabel('Imaginary portion of y(t)');
```



c)

Magnitude and Phase Angle of the Complex Exponential

```
o = pi;
s = -log(3) / 10;
t = linspace(0, 10, 500);
y = exp(t * (s + 1i * o));
plot(t, angle(y)/(2*pi), 'green', t, abs(y),'blue');
grid on;
xlabel('t (seconds)'); ylabel('Magnitude and phase angle for y(t)');
```