EE102

Lecture 7

EE102 Announcements

- Syllabus link is tiny.cc/ucla102
- CCLE difficulties, please email help@seas.ucla.edu
- **Third Homework due this Friday**
- **Note: We do not have lectures on exam days.**

Slide Credits: This lecture adapted from Prof. Jonathan Kao (UCLA) and recursive credits apply to Prof. Cabric (UCLA), Prof. Yu (Carnegie Mellon), Prof. Pauly (Stanford), and Prof. Fragouli (UCLA)

Review of Last Lecture

Last Lecture Introduced a few concepts:

- CYU: How is the Impulse Response Defined?

- CYU: Why is the Impulse Response Useful?

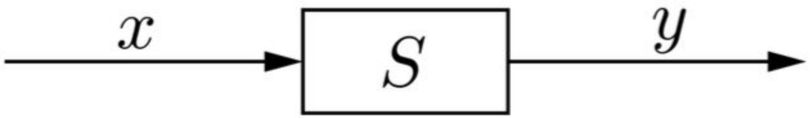
Derivation of this fact

The Convolution Integral

The Convolution Integral (Cont’d)

Convolution is what adds structure to the Black Box

A system transforms an *input signal*, $x(t)$, into an output system, $y(t)$.



Why does this work? (The Convolution Integral)

Same derivation as last lecture - shows up in other classes, even grad classes

“Gist” of convolution - smearing

Examples of Computing the Impulse Response

CYU

Notation of Convolution

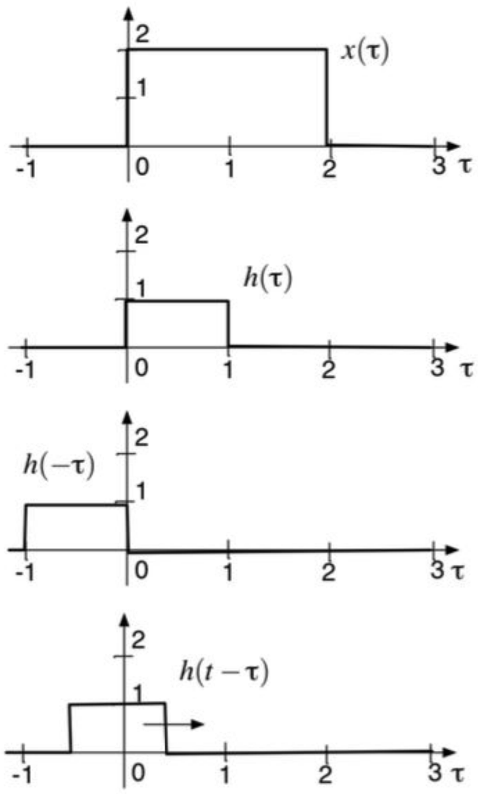
How to Compute Convolution: flip and drag

To calculate $y(t) = (x * h)(t)$,

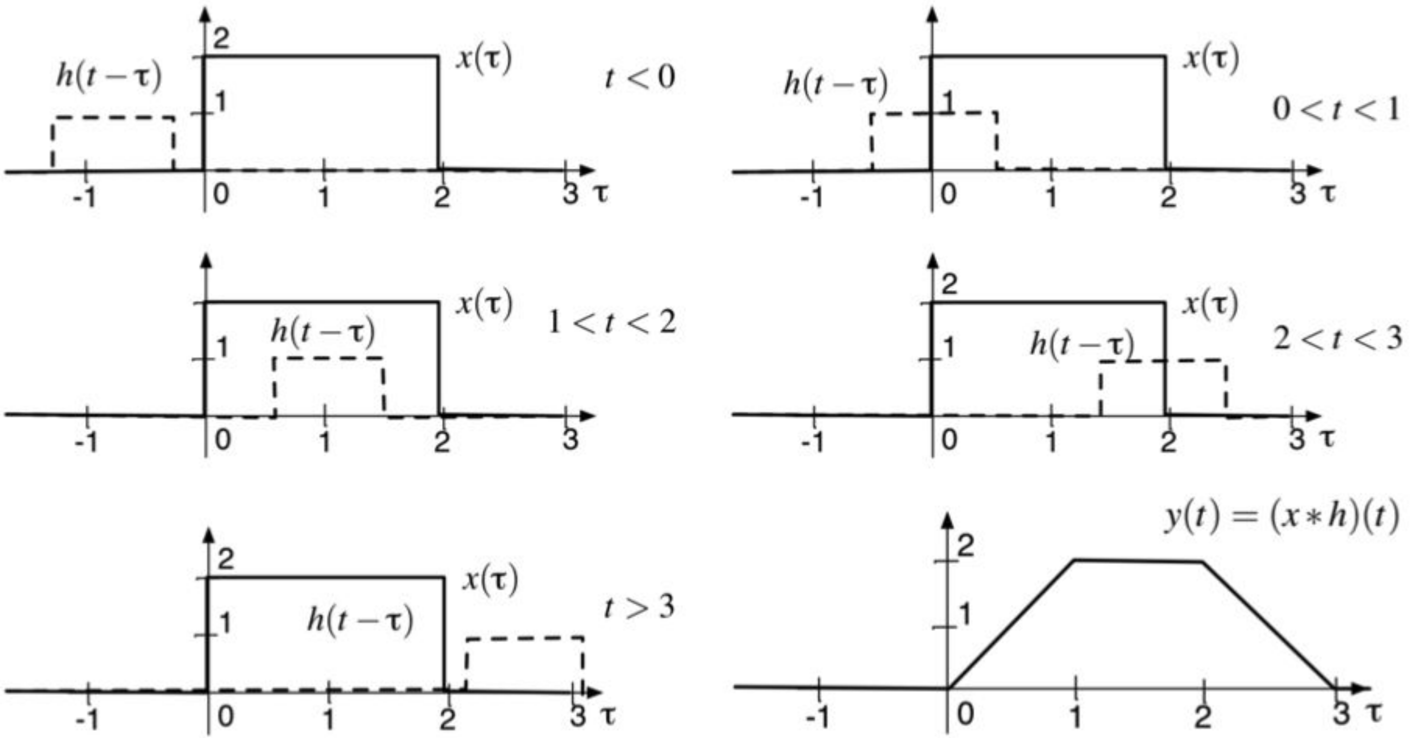
- Flip (i.e., reverse in time) the impulse response. This changes $h(\tau)$ to $h(-\tau)$.
- Begin to drag the reversed time response by some amount, t. This results in $h(t - \tau)$.
- For a given t, multiply $h(t - \tau)$ pointwise by $x(\tau)$. This produces $x(\tau)h(t - \tau)$.
- Integrate this product over τ. This produces $y(t)$ at this particular time t.

This technique is referred to as the "flip-and-drag" technique.

How to Compute Convolution: flip and drag

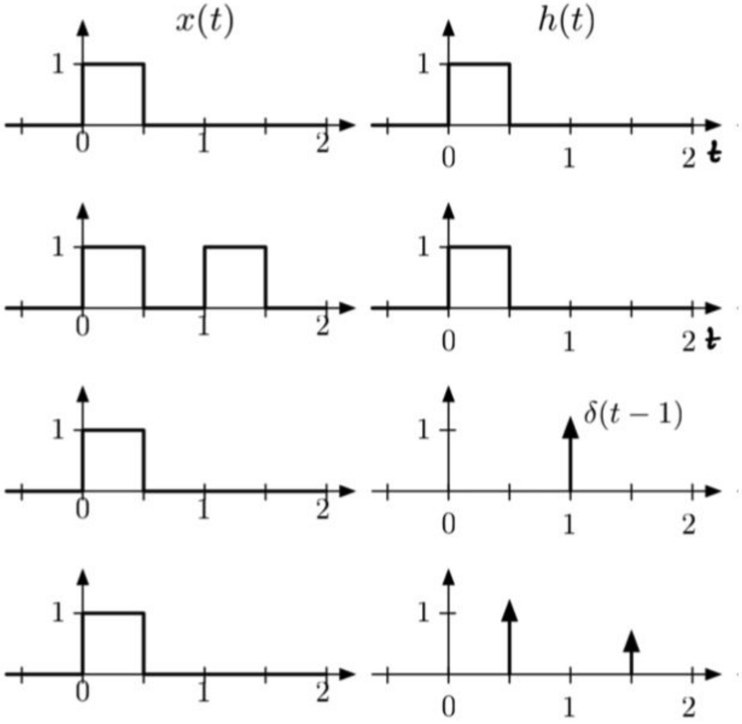


How to Compute Convolution: flip and drag

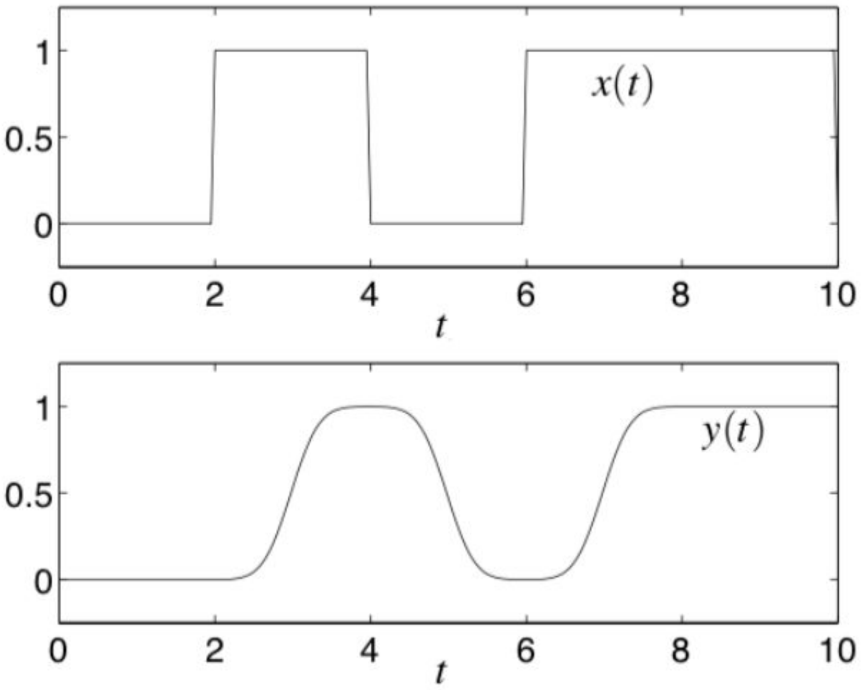


How to Compute Convolution: flip and drag

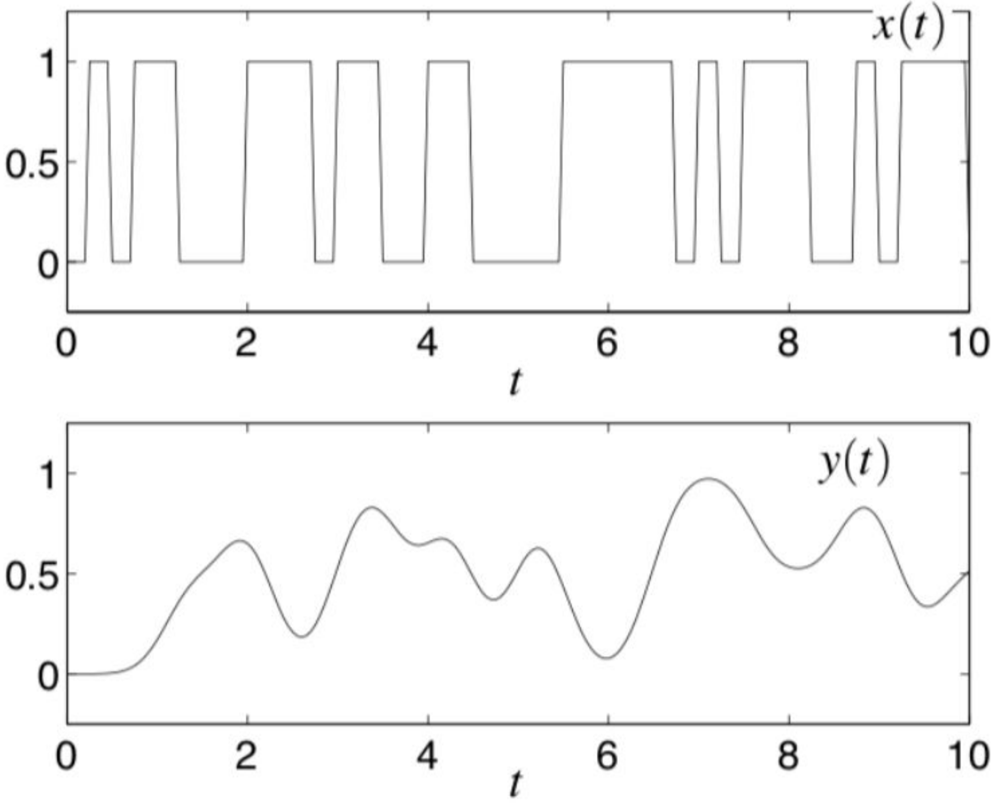
Examples: Try these:



Example: Noisy Communication



Example Noisy Communication



Causal Convolution

Convolution for a causal system

In a causal system, $h(t) = 0$ for $t < 0$. (Why? Hint: what happens if $h(t) \neq 0$ for some $t < 0$?)

This means that $h(t - \tau) = 0$ if $\tau > t$. Hence, there is no need to integrate if τ exceeds t, since $h(t - \tau) = 0$. We can use this to simplify the convolution integral.

$$
\begin{aligned}
y(t) &= \int_{-\infty}^{\infty} x(\tau)h(t-\tau)d\tau \\
&= \int_{-\infty}^{t} x(\tau)h(t-\tau)d\tau
\end{aligned}
$$

This equation tells us that only past and present values of $x(\tau)$ contribute to $y(t)$.

Properties of Convolution

Commutativity $\boxed{(x * h)(t) = (h * x)(t)}$

Associativity $\boxed{(f * (g * h))(t) = ((f * g) * h)(t)}$

Distributivity $\boxed{f * (g + h) = f * g + f * h}$

Linearity $h * (\alpha x_1 + \beta x_2) = \alpha(h * x_1) + \beta(h * x_2)$

Time-invariance

CYU Question from HW