ECE102, Spring 2021 **Homework #1**
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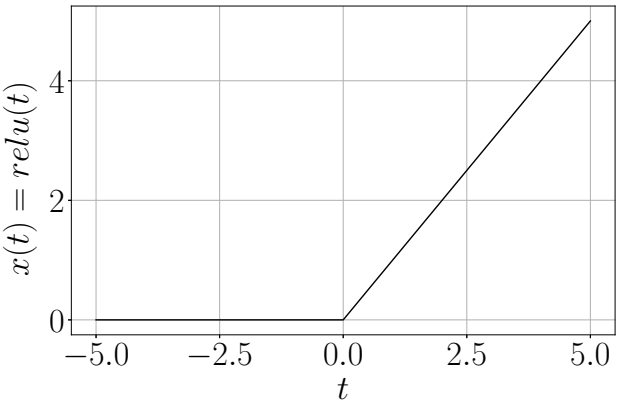
Due Friday, 9 Apr 2021, by 11:59pm to CCLE.
100 points total.

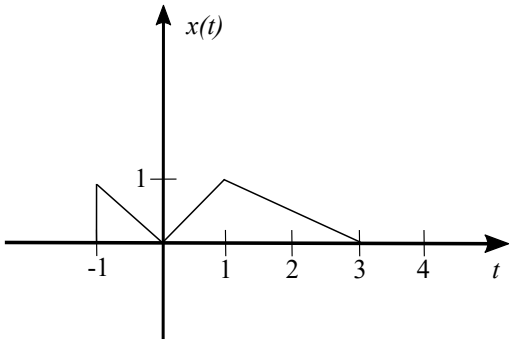
1. (10 points) **Even and odd parts.**
   Sketch and write the even and odd components of the following signal:

$$x(t) = relu(t) = \begin{cases} t & t \geq 0 \\ 0 & t < 0 \end{cases}$$



2. (15 points) **Time scaling and shifting.**

   (a) (10 points) Consider the following signal.

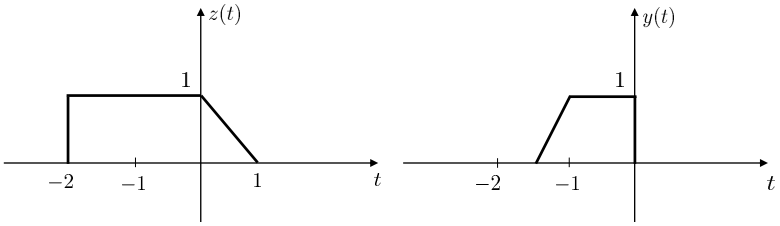


   Sketch the following:

   i. $\frac{1}{2}x(-t/2)$

   ii. $x(-2t-2)$

   (b) (5 points) The figure below shows two signals: $z(t)$ and $y(t)$. Express (i) $y(t)$ in terms of $z(t)$, and (ii) $z(t)$ in terms of $y(t)$



3. (20 points) **Periodic signals.**

   (a) (15 points) For each of the following signals, determine whether it is periodic or not. If the signal is periodic, determine the fundamental period and frequency.

      i. $x_1(t) = \sin(5t/6 + \pi/3)$
      ii. $x_2(t) = \cos^2(3\pi t)$
      iii. $x_3(t) = x_1(t) + x_2(t)$
      iv. $x_4(t) = e^t x_1(t)$
      v. $x_5(t) = e^{j(\pi t+1)} x_2(t)$

   (b) (5 points) A signal $y(t)$ is periodic with period T_0, and is the sum of two other signals.

$$y(t) = x_1(t) + x_2(t)$$

   Must $x_1(t)$ and $x_2(t)$ both be periodic?
   What if y(t) = $x_1(t) \times x_2(t)$?

4. (25 points) **Energy and power signals.**

   (a) (10 points) Determine whether the following signals are energy or power signals. If the signal is an energy signal, determine its energy. If the signal is a power signal, determine its power.

      i. $x(t) = e^{-|t|}$
      ii. $x(t) = 1 + e^{-|t|}$

   (b) (15 points) Show the following two properties:

      - If $x(t)$ is an even signal and $y(t)$ is an odd signal, then $x(t)y(t)$ is an odd signal;
      - If $z(t)$ is an odd signal, then for any $\tau > 0$ we have:

$$\int_{-\tau}^{\tau} z(t)dt = 0$$

   Use these two properties to show that the energy of $x(t)$ is the sum of the energy of its even component $x_e(t)$ and the energy of its odd component $x_o(t)$, i.e.,

$$E_x = E_{x_e} + E_{x_o}$$

   Assume $x(t)$ is a real signal.

5. (16 points) **Euler's identity and complex numbers.**

   (a) (8 points) Use Euler's formula to prove the following identities:

      i. $\cos^2(\theta) + \sin^2(\theta) = 1$

      ii. $\cos(\theta + \psi) = \cos(\theta)\cos(\psi) - \sin(\theta)\sin(\psi)$

   (b) (8 points) $x(t) = (1 - \sqrt{3}j)e^{j(t+2)}$ and $y(t) = \frac{1}{1+j}$.

      i. Compute the real and imaginary parts of $x(t)$ and $y(t)$.

      ii. Compute the magnitude and phase of $x(t)$ and $y(t)$.

6. (14 points) **MATLAB tasks**
   For this question, please include all relevant code in text format. For plots, please include axis labels and preferably include a grid.

   (a) (5 points) **Task 1**
   Plot the waveform

   $$x(t) = e^{-t}\cos(2\pi t)$$

   for $-10 \leq t \leq 10$, with a step size of 0.2.

   (b) (4 points) **Task 2**
   Create a function `relu(t)` that implements the function from Question 1. You will need to create a file called "relu.m" containing:

```
function out = relu(t)
out = 0; %replace this line with the appropriate implementation of the
%relu function.
end
```

   Then plot the function for $-5 \leq t \leq 5$, with a step size of 0.1.

   (c) (5 points) **Task 3**
   Create functions `even(t, f)` and `odd(t, f)` that take inputs time `t` and function (handle) `f` that compute the respective even and odd parts of `f(t)` at points t.

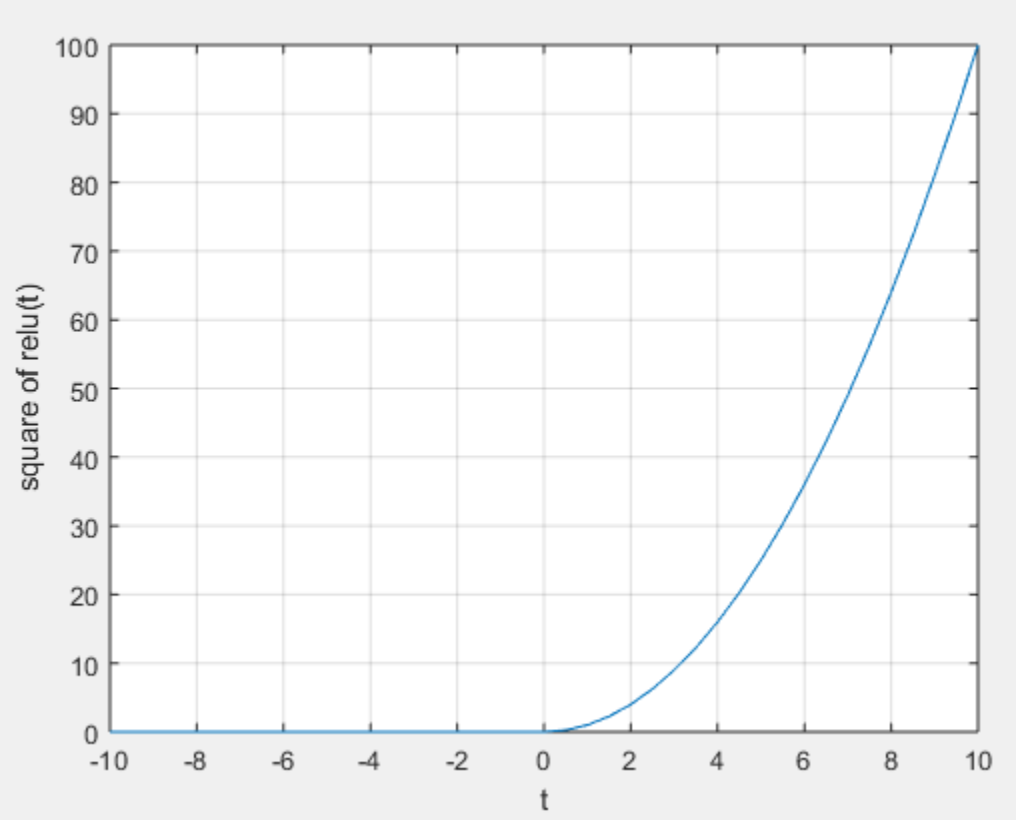
   For example, the square of a function could be implemented in a file square.m as:

```
function out = square(t, f)
out = f(t).^2;
end
```

   and run as:

```
t = -10:0.5:10;
y = square(t, @relu);
```

   where `@relu` is called a function handle of the function `relu`, and is necessary for passing a function as input to another function.
   Running `plot(t, y); grid;` yields the result:



For this question, plot the even and odd components of $relu(t)$ for $-5 \leq t \leq 5$, with a step size of 0.1 using the functions `even(t, f)` and `odd(t, f)`. Feel free to also define and play around with arbitrary functions to look at their even and odd components.