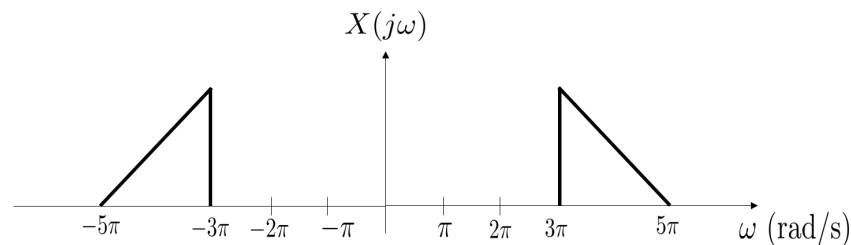


Due Tuesday, 1 June 2021, by 11:59pm to CCLE.

100 points total.

1. (16 points) **Bandpass sampling**

The figure below shows the Fourier transform of a real bandpass signal, i.e., a signal whose frequencies are not centered around the origin. We want to sample this signal. Let F_s in Hz



represent the sampling frequency.

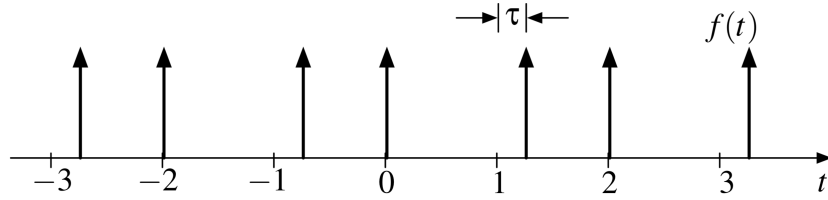
- (a) (8 points) One option is to sample this signal at the Nyquist rate. Then to recover the signal, we pass its sampled version through a low pass filter. What is the Nyquist rate of this signal? What is the cutoff frequency of the low pass filter?
- (b) (8 points) Since the signal might have high frequency components, Nyquist rate for this signal can be high. In other words, we need to have a lot of samples of the signal, which means that the sampling scheme is costly. It turns out that for this type of signal, we can sample it at a sampling frequency lower than the Nyquist rate and we can still recover the signal, however in this case, we will use a **bandpass** filter. To see this, we have the following two options for the sampling frequency:

- $F_s = 2$ Hz;
- $F_s = 2.5$ Hz;

For each case, draw the spectrum of the signal after sampling it. To recover the signal, which F_s can we use? How we should choose the frequencies of the bandpass filter? What is the minimum F_s we can use and still recover the signal?

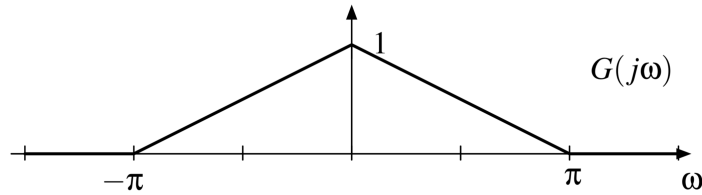
2. (24 points) **Sampling with imperfect sampler**

Imperfections in a sampler cause characteristic artifacts in the sampled signal. In this problem we will look at the case where the sample timing is non-uniform, as shown below: The



sampling function $f(t)$ has its odd samples delayed by a small time τ .

- (6 points) Write an expression for $f(t)$ in terms of two uniformly spaced sampling functions.
- (6 points) Find $F(j\omega)$, the Fourier transform of $f(t)$. Express the impulse trains as sums, and simplify.
- (6 points) Find $F(j\omega)$, for the case where $\tau = 0$, and show that this is what you expect.
- (6 points) Assume the signal we are sampling has a Fourier transform



Sketch the Fourier transform of the sampled signal. Include the baseband replica, and the replicas at $\omega = \pm\pi$. Assume that τ is small, so that $e^{j\omega\tau} \simeq 1 + j\omega\tau$.

3. (20 points) **Laplace Transform**

(a) (12 points) Find the Laplace transforms of the following signals and determine their region of convergence.

i. $f(t) = te^{-at}(\sin \omega_0 t)^2 u(t)$

ii. $f(t) = \begin{cases} 0, & 0 \leq t < 1 \\ 1, & 1 \leq t < 2 \\ e^{-3(t-2)} & 2 \leq t \end{cases}$

(b) (8 points) The Laplace transform of a causal signal $x(t)$ is given by

$$X(s) = \frac{1}{s^2 + 2s + 5}, \quad \text{ROC: } \text{Re}\{s\} > -1$$

Which of the following Fourier transforms can be obtained from $X(s)$ without actually determining the signal $x(t)$? In each case, either determine the indicated Fourier transform or explain why it cannot be determined.

i. $\mathcal{F}\{x(t)e^{-t}\}$

ii. $\mathcal{F}\{x(t)e^{3t}\}$

4. (20 points) **Inverse Laplace Transform**

Find the inverse Laplace transform $f(t)$ for each of the following $F(s)$: ($f(t)$ is a causal signal)

(a) (6 points) $F(s) = \frac{e^{-s}(s+3)}{(s-1)^2(s-2)}$

(b) (6 points) $F(s) = \frac{s+4}{s^3+4s}$

(c) (8 points) $F(s) = \frac{1}{(s+1)(s^2+2s+2)}$

5. **LTI system** (20 points)

Assume a causal LTI system \mathcal{S}_1 is described by the following differential equation:

$$\frac{d^2y(t)}{dt^2} + 3\frac{dy(t)}{dt} + 2y(t) = ax(t), \quad y(0) = 0, \quad y'(0) = 0$$

where a is a constant. Moreover, we know that when the input is e^t , the output of the system \mathcal{S}_1 is $\frac{1}{2}e^t$.

- (a) (6 points) Find the transfer function $H_1(s)$ of the system. (The answer should not be in terms of a , i.e., you should find the value of a).
- (b) (8 points) Find the output $y(t)$ when the input is $x(t) = u(t)$.
- (c) (6 points) The system \mathcal{S}_1 is linearly cascaded with another causal LTI system \mathcal{S}_2 . The system \mathcal{S}_2 is given by the following input-output pair:

$$\mathcal{S}_2 \quad \text{input : } u(t) \rightarrow \text{output : } r(t)$$

Find the overall impulse response.