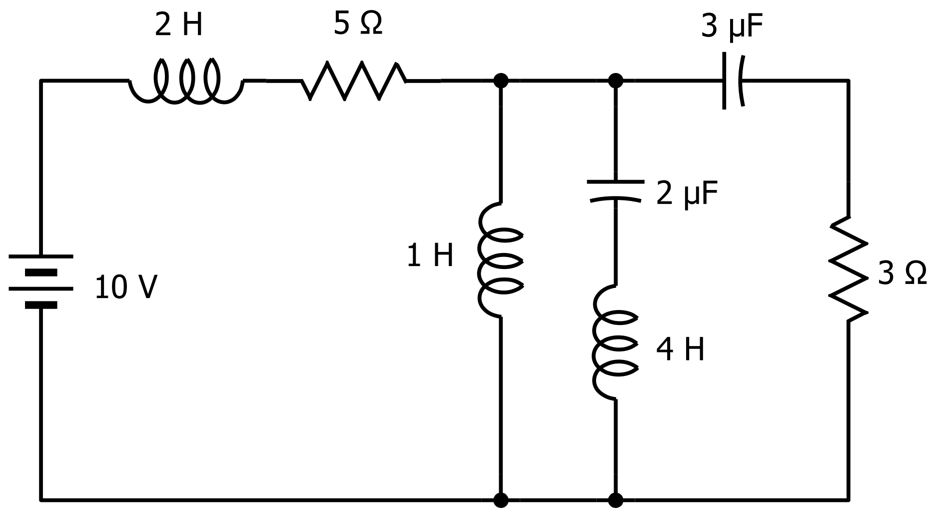
ECE3 Fall Quarter 2021
Practice Problems 3



1. This circuit has been in this condition for a "long time"; i.e., long enough for all transients to have died out.

a. What is the current through the 5 Ω resistor?
b. What is the current through the 3 Ω resistor?

$$I_{5\Omega} = \frac{10\ \text{V}}{5\ \Omega} = 2\ \text{A}$$

$$I_{3\Omega} = 0$$

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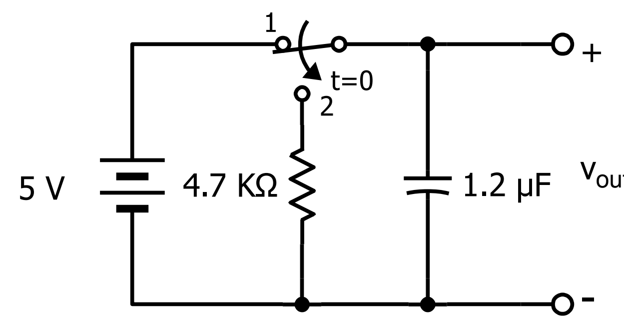
2. This is not a course in differential equations, and you will not be asked to solve a d.e. in a quiz or on the Final Exam. However, I am asking you to solve one transient problem two different ways to get an understanding of how capacitors and inductors behave at the moment of the switch changing position.



To find the trajectory followed by v_{out},
use the KCL Method:

$$i_R + i_C = 0$$

$$\frac{v_{out}}{R} + C\frac{dv_{out}}{dt} = 0$$

$$\frac{dv_{out}}{dt} + \frac{1}{RC}v_{out} = 0$$

[Separation of Vaiables] Multiply equation by $\frac{dt}{v_{out}}$:

$$\frac{dv_{out}}{v_{out}} = -\frac{1}{RC}dt$$

Integrate both sides:

$$\int \frac{dv_{out}}{v_{out}} = \left(-\frac{1}{RC}\right)\int dt \rightarrow \ln(v_{out}) + K_1 = \left(-\frac{1}{RC}\right)t + K_2 \rightarrow \ln(v_{out}) = \left(-\frac{1}{RC}\right)t + K_3$$

Raising to the power of e:

$$v_{out}(t) = e^{-\frac{t}{RC} + K_3} = \left(e^{-\frac{t}{RC}}\right)\left(e^{K_3}\right) = \left(e^{-\frac{t}{RC}}\right)K_4$$

But because we know how capacitors act, we know that $v_{out}(0^+) = v_{out}(0^-) = 5$.

$$v_{out}(0) = v_{out}(0^+) = 5 = \left(e^0\right)K_4 \rightarrow K_4 = 5$$

So $v_{out}(t) = 5\left(e^{-\frac{t}{RC}}\right) = 5\left(e^{-\frac{t}{4700 \cdot 1.2\text{e-}6}}\right) = 5\left(e^{-177t}\right)$

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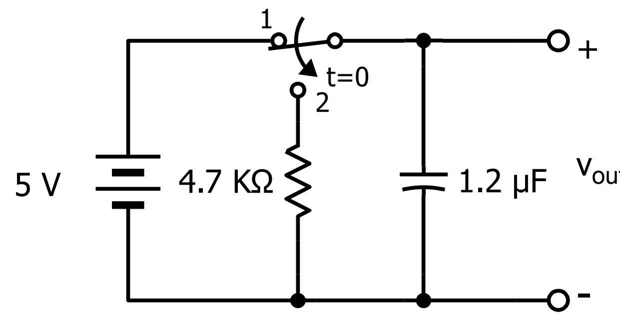
3. This is the same circuit as Problem 6. Find $v_{out}(t)$ again, this time using KVL instead of KCL.



KVL: $v_c(t) + Ri(t) = 0$

$$\frac{1}{C}\int i(t)\,dt + Ri(t) = 0$$

$$\frac{1}{RC} i(t) + \frac{di(t)}{dt} = 0 \quad \text{(constant will be added later)}$$

[Separation of Vaiables] Same process as Problem 2

$$\frac{di(t)}{i(t)} = -\frac{1}{RC}dt$$

$$i(t) = e^{-\frac{t}{RC}+K} = \left(e^{-\frac{t}{RC}}\right)\left(e^{K}\right) = \left(e^{-\frac{t}{RC}}\right)K_1$$

But because we know how capacitors act, we know that $v_{out}(0^+) = v_{out}(0^-) = 5$.

So $i(0) = i(0^+) = \dfrac{5}{4700} = 1.06\text{e-}3\left(e^{0}\right)K_1 \rightarrow K_1 = 1.06\text{e-}3$

So $i(t) = (1.06\text{e-}3)\left(e^{-\frac{t}{RC}}\right) = (1.06\text{e-}3)\left(e^{-\frac{t}{4700\cdot 1.2\text{e-}6}}\right) = (1.06\text{e-}3)\left(e^{-177t}\right)$

$\therefore \quad v_c(t) = i(t)R = \left(1.06\text{e-}3\, e^{-177t}\right)(4.7\text{e}3) = 5\,e^{-177t}$

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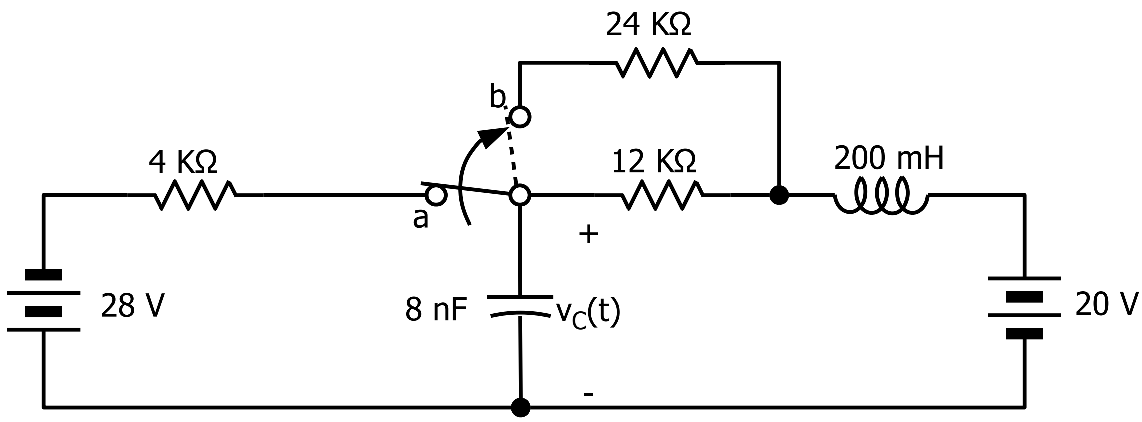

4. This is my favorite transient problem, because it requires you to use so many things that you have learned earlier in the class. You will probably see a variant of this on the Final Exam, so study it carefully. The switch has been in position a for a <u>long time</u>. All transients have died out. At t = 0, the switch moves instantaneously to position b.

   a. At $t=0^-$ (the last instant that the switch is in position a), what is the current through the capacitor? 0 A

   b. At $t=0^-$ (the last instant that the switch is in position a), what is the voltage across the capacitor? $-28+(4K\Omega)(3mA)-v_C(0^-)=0$; $v_C(0^-)=-16$ V

   c. At $t=0^+$ (the first instant that the switch is in position b), the current through the capacitor is the same as in Part a. True $\boxed{\text{False}}$

   d. At $t=0^+$ (the first instant that the switch is in position b), the voltage across the inductor is the same as at $t=0^-$. True $\boxed{\text{False}}$

   e. At $t=0^+$ (the first instant that the switch is in position b), what is the voltage across the inductor?

   KVL CCW: $-20+v_L(0^+)+(8K\Omega)(3mA)+v_C(0^+)=0$; $v_L(0^+)=+12$ V; + at right end