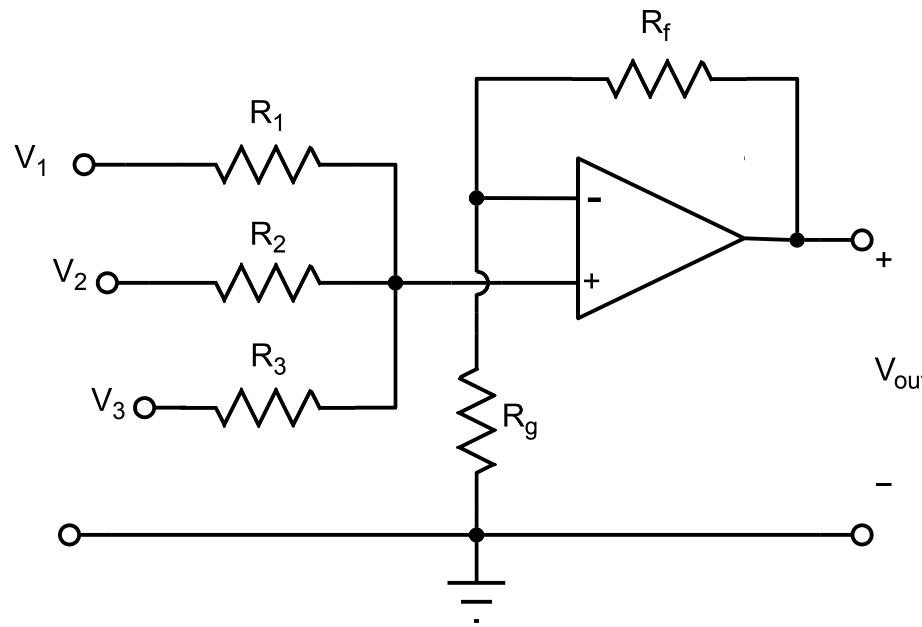
ECE3 Fall 2021
Practice Problems 7

1.

a. Find $V_{out}=f(V's, R's)$.

b. Let $R_1=R_2=R_3=R$, and $R_f=2R_g$. Find V_{out}.



① $\frac{v_p-V_1}{R_1}+\frac{v_p-V_2}{R_2}+\frac{v_p-V_3}{R_3}+i_p=0$

② $\frac{v_p-V_{out}}{R_f}+\frac{v_p}{R_g}=0$

① $v_p\left(\frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{R_3}\right)=\frac{V_1}{R_1}+\frac{V_2}{R_2}+\frac{V_3}{R_3}$

② $v_p\left(\frac{1}{R_f}+\frac{1}{R_g}\right)=\frac{V_{out}}{R_g}$

② $v_p=V_{out}\left(\frac{R_g}{R_f+R_g}\right)$

① $V_{out}\left(\frac{R_g}{R_f+R_g}\right)=\left(\frac{V_1}{R_1}+\frac{V_2}{R_2}+\frac{V_3}{R_3}\right)\left(\frac{1}{\frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{R_3}}\right)$

$$V_{out}=\left(\frac{R_f+R_g}{R_g}\right)\left(\frac{V_1}{R_1}+\frac{V_2}{R_2}+\frac{V_3}{R_3}\right)\left(\frac{R_1R_2R_3}{R_1R_2+R_1R_3+R_2R_3}\right)$$

If $R_1=R_2=R_3=R_i$ and $R_f=2R_g$,

$$V_{out}=(1+2)\left(\frac{V_1+V_2+V_3}{R_i}\right)\left(\frac{R_i^3}{3R_i^2}\right)$$

$$V_{out}=V_1+V_2+V_3$$

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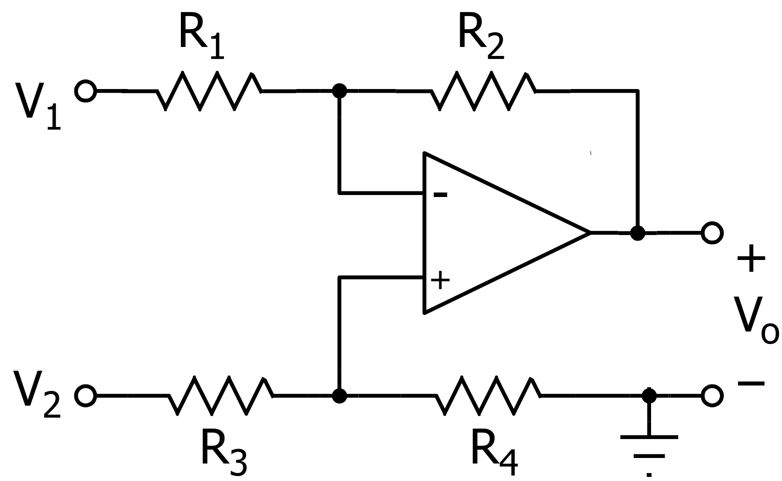
2. Let $R_4 = R_2$, and $R_3 = R_1$. Find $V_o = f(V\text{'s}, R\text{'s})$. HINT: write 2 KCL equations at the input.



$$\boxed{V_O = \frac{R_2}{R_1}(V_2 - V_1)}$$

Write 2 KCL equations:

$$\frac{v_n - V_1}{R_1} + \frac{v_n - V_o}{R_2} = 0$$

$$\frac{v_p - V_2}{R_3} + \frac{v_p}{R_4} = 0$$

But $R_3 = R_1$ and $R_4 = R_2$, so

$$\frac{v_n - V_1}{R_1} + \frac{v_n - V_o}{R_2} = 0$$

$$\frac{v_p - V_2}{R_1} + \frac{v_p}{R_2} = 0$$

v_n, v_p, and V_o are unknown, so we need a third equation.

$$v_p = v_n = v$$

$$\frac{v - V_1}{R_1} + \frac{v - V_o}{R_2} = 0 = \frac{v - V_2}{R_1} + \frac{v}{R_2}$$

$$\frac{-V_1}{R_1} - \frac{V_o}{R_2} = -\frac{V_2}{R_2}$$

$$V_o = \frac{R_2}{R_1}(V_2 - V_1)$$

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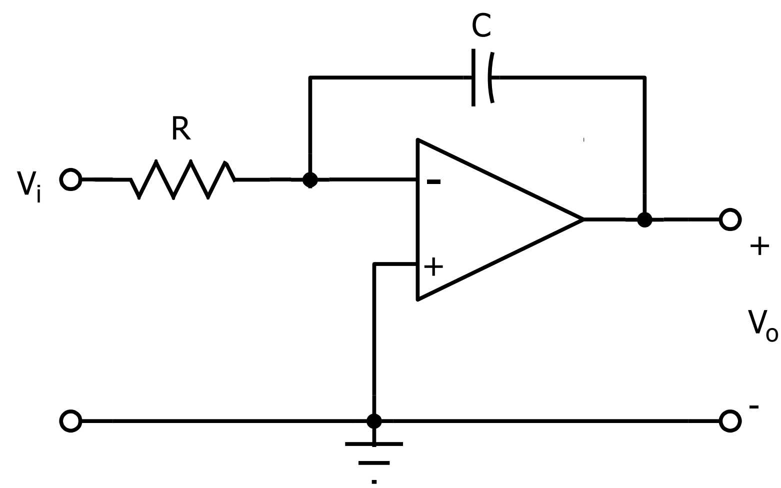
3. This is something else that op amps do well. Find V_o as a function of V, C, and R.

HINT: write a KCL equation at the inverting input.



$$\frac{0-V_i}{R}+C\frac{d(0-V_o)}{dt}=0$$

$$\frac{dV_o}{dt}=-\left(\frac{1}{RC}\right)V_i$$

$$V_o=-\left(\frac{1}{RC}\right)\int V_i\,dt$$

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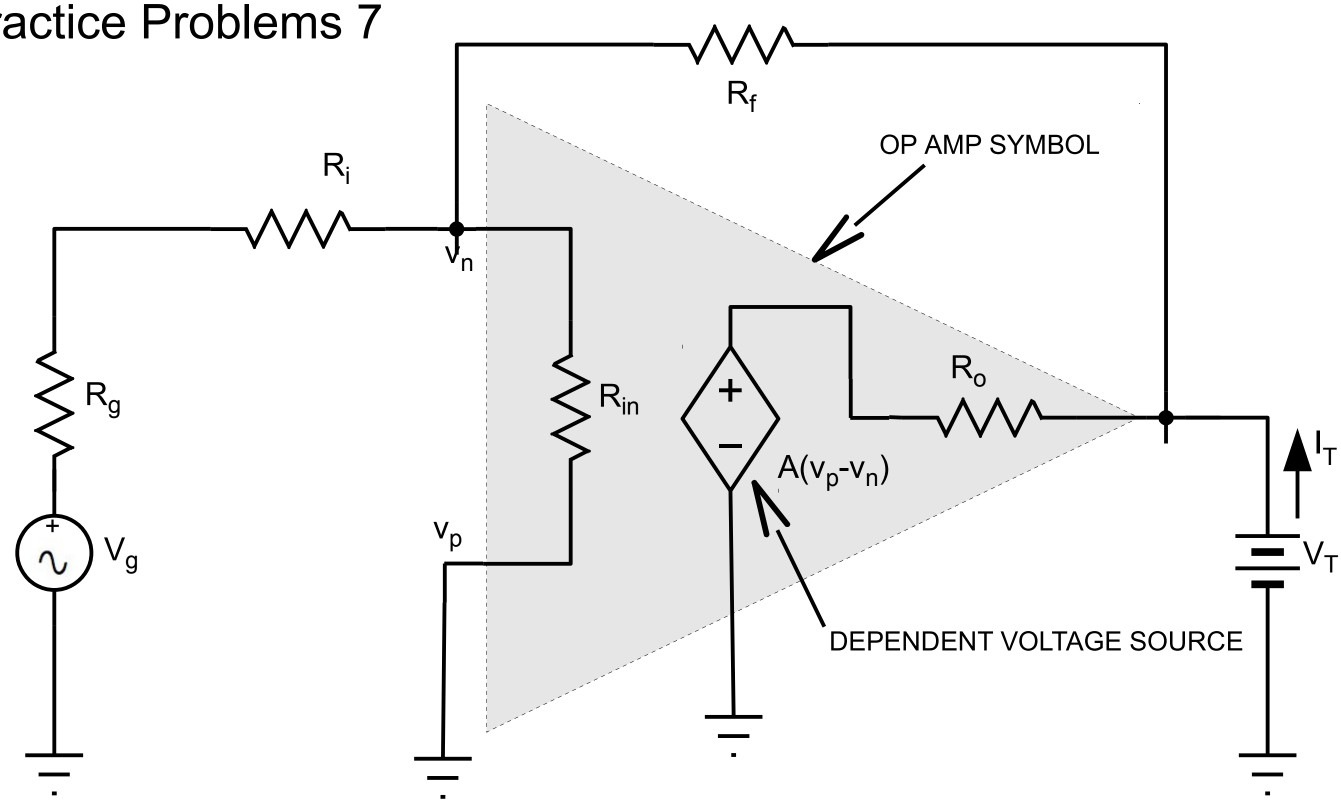
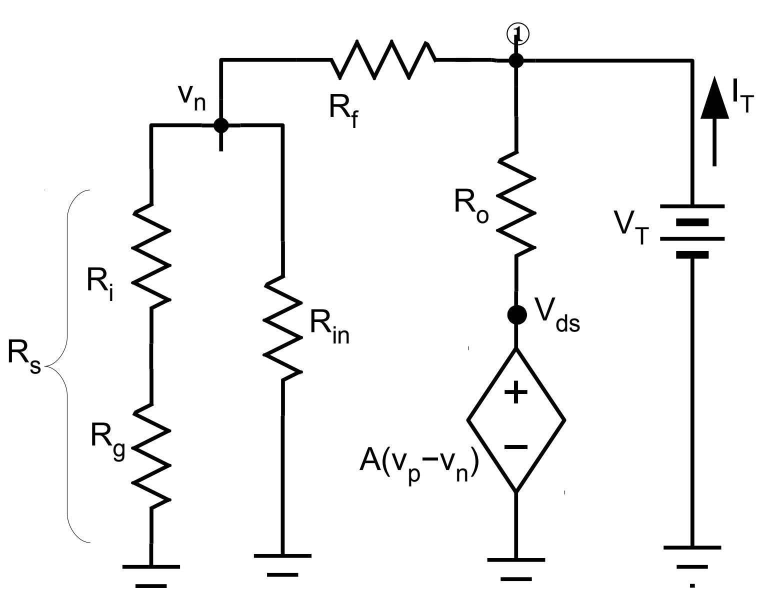


How does one compute the output impedance of an inverting op amp circuit (as opposed to the op amp itself)? One method is to use a slightly-more-realistic equivalent circuit (inside the dashed line of the op amp symbol), and then determine its Thévenin resistance. One can do this by applying Method B. The Thévenin resistance is the output impedance. Assume that R_g=0, R_i=10 KΩ, R_f=10 KΩ, R_o=10 Ω, R_{in}=1 MΩ, and A = 1e5, and compute the output impedance of the op amp circuit. You will see that the output resistance of the whole circuit is dramatically lower than that of the op amp itself.

WARNING: the summing point constraints do not apply to the equivalent circuit!



Dependent voltage source: $V_{ds} = A\left(v_p - v_n\right) = -Av_n$

① $-I_T + \dfrac{V_T-(-Av_n)}{R_o} + \dfrac{V_T - v_n}{R_f} = 0$

Let $R_s = R_i + R_g$

Then $v_n = V_T\left(\dfrac{R_s\|R_{in}}{R_s\|R_{in}+R_f}\right) = V_T\left(\dfrac{R_s\cdot R_{in}}{R_s\cdot R_{in}+R_f(R_s+R_{in})}\right)$

Let $K = \left(\dfrac{R_s\cdot R_{in}}{R_s\cdot R_{in}+R_f(R_s+R_{in})}\right)$

Then $v_n = V_T K$

Plug into ①:

$-I_T + \dfrac{V_T + AV_T K}{R_o} + \dfrac{V_T(1-K)}{R_f} = 0$

$V_T\left(\dfrac{1}{R_o} + \dfrac{AK}{R_o} + \dfrac{1-K}{R_f}\right) = I_T$

$R_{out} = R_{th} = \dfrac{V_T}{I_T} = \dfrac{1}{\dfrac{1}{R_o} + \dfrac{AK}{R_o} + \dfrac{1-K}{R_f}} = 0.201\text{e-}3\,\Omega << R_o = 10\ \Omega$

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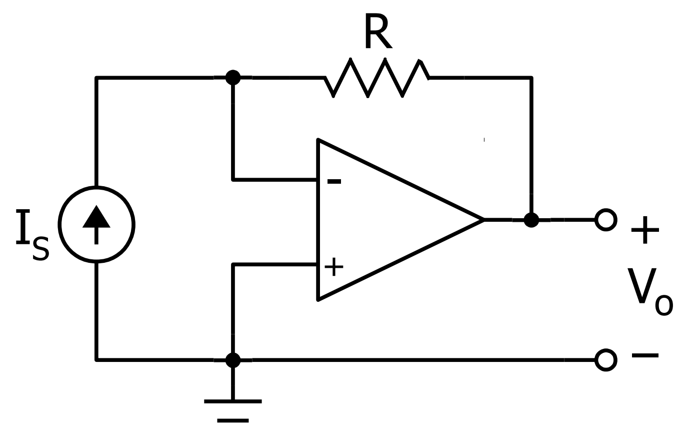
Find $V_o = f(I_S, R)$.



$V_o = - I R$

By KCL, $\frac{v_n - V_o}{R} - I = 0$

But $v_n = 0$

$\therefore \quad V_o = -IR$

This is a *transimpedance amplifier*. Check it out.