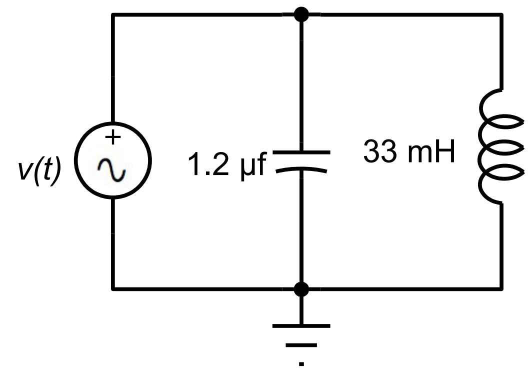
ECE3 Fall 2021
Practice Problems 5

There are several ways that capacitive and inductive impedance and reactance are used. Here is my policy:

Capacitive reactance: $X_C = 1/\omega C$
Capacitive impedance: $Z_C = 1/j\omega C = -j(1/\omega C) = -jX_C$

Inductive reactance: $X_L = \omega L$
Inductive impedance: $Z_L = j\omega L = jX_L$

ECE3 Fall 2021
Practice Problems 5



1. If *v(t)* = 20 cos(1256*t*),

   a) what is the capacitive impedance (express as jX_C)?

$$X_C = \frac{1}{j\omega C} = \frac{1}{j(1256)(1.2\text{e-}6)} = -j663\ \Omega$$

   b) what is the inductive impedance (express as jX_L)?

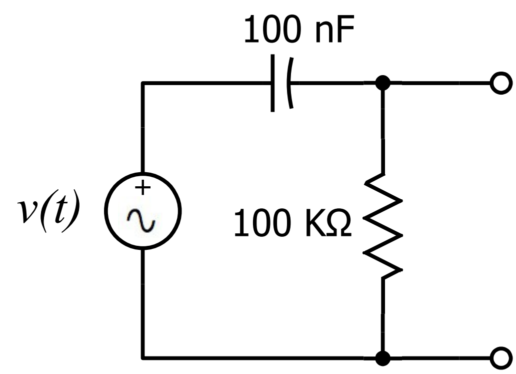
$$X_L = j\omega L = j(1256)(33\text{e-}3) = j41\ \Omega$$

   c) What is the parallel equivalent impedance?

$$X_{eqP} = \frac{(-j663)(j41)}{-j663 + j41} = \frac{27183}{-j622} = j43.7\,\Omega$$

ECE3 Fall 2021
Practice Problems 5

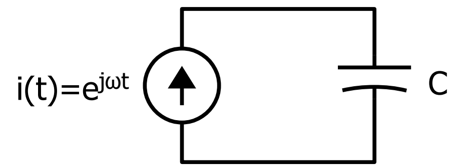
2. If $v(t)$ = 20 cos(1256t), what is the total impedance presented to the voltage source by the resistor and capacitor? Remember that $Z = R + j(X_L - X_C)$.



$$Z = 100\,K\Omega + \frac{1}{j\omega C} = 100\,K - j\frac{1}{(1256)(100\text{e-}9)} = 100\,K - j\,7.96\,K$$

ECE3 Fall 2021
Practice Problems 5

3. Prof. Pottie, in his Lecture 3 Page 8 notes, derived the expression for inductive impedance (Z_L=jωL). I repeated that derivation in this week's lecture. Now it is your turn. Derive the expression for capacitive impedance (Z_C=1/jωC).
   HINT: start by writing a KCL equation at the top of the capacitor. Also, assume that $v(0)=v_0$.



Remember that we want $Z_C = \frac{v_C(t)}{i(t)} = \frac{v_C(t)}{e^{j\omega t}}$. So we need $v_C = f(C, \omega, t)$.

Assuming that both currents at the top of the capacitor are leaving that node, and leaving off the (t) for brevity,

$-e^{j\omega t} + C\frac{dv_C}{dt} = 0$, or

(1) $C\frac{dv_C}{dt} = e^{j\omega t}$

Guess that $v_C = Ae^{st}$; then

(2) $\frac{dv_C}{dt} = Ase^{st}$

At $t = 0$, $v_C = v_0 = Ae^{s0} \Rightarrow$

(3) $A = v_0$

So now we have $v_C = v_O e^{st}$.

But we also know from Equations (1)-(3) that $C\left(Ase^{st}\right) = Cv_0 se^{st} = e^{j\omega t}$

By similarity,

(4) $s = j\omega$.

So $Cv_0 j\omega e^{j\omega t} = e^{j\omega t} \Rightarrow$

(5) $v_0 = \frac{1}{j\omega C}$

So by Equations (3), (4), and (5), our guess becomes

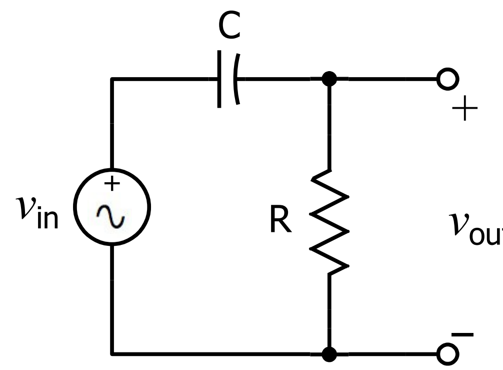
(6) $v_C = \left(\frac{1}{j\omega C}\right)e^{j\omega t}$

and we now have $v_C = f(C, \omega, t)$. So

$$Z_C = \frac{v_C(t)}{i(t)} = \frac{\left(\frac{1}{j\omega C}\right)e^{j\omega t}}{e^{j\omega t}} = \frac{1}{j\omega C} = -j\left(\frac{1}{\omega C}\right)$$

ECE3 Fall 2021
Practice Problems 5

4. In this week's lecture, you learned an intuitive way to identify the type of filter from examining its schematic. Prof. Pottie's Lecture 3, Page 10, shows an analytical method to determine the type of filter. Use Prof. Pottie's method to determine the type of filter represented by the schematic to the right. IOW, find an expression for v_{out}/v_{in} and see what happens when ω is quite small, and when ω is quite large.



$$\frac{V_{out}}{v_{in}} = \frac{R}{R+\dfrac{1}{j\omega C}} = \frac{j\omega RC}{j\omega RC+1}$$

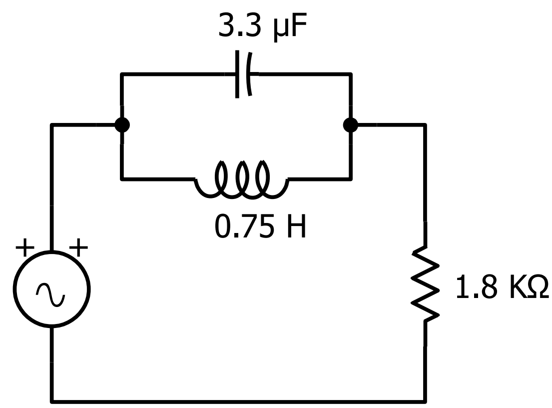
When $\omega \ll RC$, $\dfrac{v_{out}}{v_{in}} \approx 0$ (blocks low frequencies)

When $\omega \gg RC$, $\dfrac{v_{out}}{v_{in}} \approx 1$ (passes high frequencies)

So this is a high pass filter.

ECE3 Fall 2021
Practice Problems 5

5. Using the expressions for capacitive and inductive impedance, find the total impedance presented by the capacitor, inductor, and resistor to the voltage source. Set ω = 1000 rad/s.



$$Z_{tot} = Z_C \| Z_L + R$$

$$Z_C \| Z_L = \frac{Z_C Z_L}{Z_C + Z_L} + R = \frac{\dfrac{j\omega L}{j\omega C}}{\dfrac{1}{j\omega C} + j\omega L} + R = \frac{\dfrac{j\,1000 \cdot 0.75}{j\,1000 \cdot 3.3\text{e-}6}}{\dfrac{1}{j\,1000 \cdot 3.3\text{e-}6} + j\,1000 \cdot 0.75} + 1800$$

$$Z_{tot} = 1800 - j\,508$$

2. Now find the current through the voltage source if v(t) = 10 cos(1000t).

$$i_V(t) = \frac{v(t)}{Z_{tot}} = \frac{10\angle 0^\circ}{1800 - j\,508} = 5.1 + j\,1.5 \text{ mA}$$

3. Find the current through the inductor i_L.

$$i_L(t) = i_V(t)\left(\frac{Z_C}{Z_C + Z_L}\right) = (5.1 + j\,1.5)\left(\frac{-j\,303}{-j\,303 + j\,750}\right) = -3.5 - j\,1 \text{ mA}$$

4. Find the current through the capacitor i_C.

$$i_C(t) = i(t)\left(\frac{Z_L}{Z_L + Z_C}\right) = (5.1 + j\,1.5)\left(\frac{j\,750}{-j\,303 + j\,750}\right) = 8.6 + j\,2.4 \text{ mA}$$

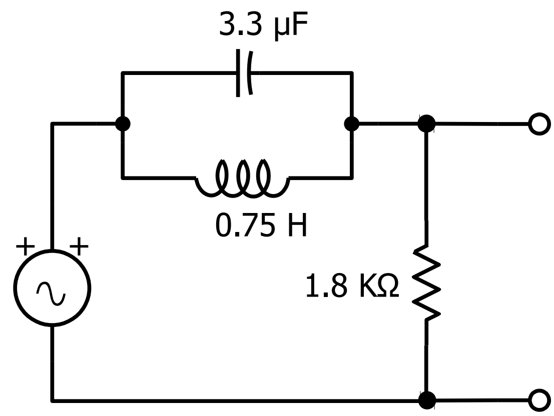
5 Add i_L and i_C. The sum should equal the current in #2.

$$i_L + i_C = -3.5 - j\,1 + 8.6 + j\,2.4 = 5.1 + j\,1.4 \text{ mA}$$

```
iT=(10+i*0)/(1800-i*508)
zL=i*1000*0.75
zC=-i/(1000*3.3e-6)
iL=iT*zC/(zL+zC)
iC=iT*zL/(zL+zC)
iC+iL
```

ECE3 Fall 2021
Practice Problems 5

6. Using the expressions for capacitive and inductive impedance, find the Thévenin Equivalent circuit looking in through the port. Set $v_s = 10\angle 0°$ and $\omega = 1000$ rad/s.



From Problem 5, $Z_L \| Z_C = -j508$

$$V_{th} = (10\angle\ 0°)\left(\frac{1800}{1800 - j508}\right) = 9.62 \angle\ 15.8°$$

Replacing the voltage source with a short, $R \| -j508$

$$Z_{th} = \frac{(1800)(-j508)}{1800 - j508} = 132.8 - j470.5$$