

## ECE 3 HWS

Capacitive Reactance:  $X_C = \frac{1}{\omega C}$

Capacitive impedance:  $Z_C = \frac{1}{j\omega C} = -jX_C$

Inductive reactance:  $X_L = \omega L$

Inductive impedance:  $Z_L = j\omega L = jX_L$

a) For DC supply,  $\omega = 0$

This means the inductive impedance is 0 because

$$Z_L = j\omega L \rightarrow Z_L = j0L \rightarrow Z_L = 0$$

This then causes a short circuit, so  $V_{out} = V_{in}$

b) When  $\omega = \infty$ , the inductive impedance is  $\infty$  because

$$Z_L = j\omega L \rightarrow Z_L = j\infty L \rightarrow Z_L = \infty$$

This means the inductor is similar to an open circuit

This then causes  $V_{out}$  to be 0  $V_{out} = 0$

$$c) V_{out}(j\omega) = \frac{V_{in}(j\omega) \cdot 2K}{4j\omega + 2K}$$

$$\frac{V_{out}(j\omega)}{V_{in}(j\omega)} = \frac{2K}{4j\omega + 2K}$$

$$\frac{V_{out}(j\omega)}{V_{in}(j\omega)} = \frac{1K}{2j\omega + 1K}$$

Low frequencies means  $\omega = 0 \rightarrow \text{plug in } 0$

$$\frac{V_{out}(j\omega)}{V_{in}(j\omega)} = \frac{1K}{1K} \rightarrow V_{in} = V_{out}$$

High frequencies means  $\omega = \infty \rightarrow \text{plug in } \infty$

$$\frac{V_{out}(j\omega)}{V_{in}(j\omega)} = \frac{1K}{\infty + 1K} \rightarrow V_{out}(j\omega) = 0$$

This filter allows low frequencies but blocks high frequencies. It is a lowpass filter.

d)  $\left| \frac{V_{out}}{V_{in}} \right|$

$$V_{out} = V_{in} \cdot \frac{R}{X_L + R} \rightarrow V_{out} = V_{in} \cdot \frac{R}{j\omega L + R}$$

$$\frac{V_{out}}{V_{in}} = \frac{R}{j\omega L + R}$$

$$\left| \frac{V_{out}}{V_{in}} \right| = \frac{R^2}{\sqrt{\omega^2 L^2 + R^2}} \quad (\text{square to get rid of } j)$$

$$\left| \frac{V_{out}}{V_{in}} \right| = \frac{R}{\sqrt{R^2 + \omega^2 L^2}} \quad \begin{matrix} R = 2k\Omega \\ L = 4H \end{matrix}$$

e) Cutoff frequency given by:  $f_c = \frac{R}{2\pi L}$

$$f_c = \frac{2000}{2\pi(4)}$$

$$f_c = 79.57 \text{ Hz}$$

f) Time constant given by:  $\tau = \frac{L}{R}$

$$\tau = \frac{4}{2000}$$

$$\tau = 0.002 \text{ seconds}$$

$$\tau = 2 \text{ ms}$$