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W O N G

Homework 3

1\. a) My surname is 4 letters so assuming the different strings are also 4 letters long, 24 different strings.

b) $4! + \binom{4}{3} + \binom{4}{2} + \binom{4}{1}$

$24 + 24 + 12 + 4 = 64$ different ways

c) $1 + 4 + 6 + 4 = 15$ different ways

d) bookkeeper - 10 letters 1 b, 2 o, 2 k, 3 e, 1 p, r

~~$\frac{10!}{1!2!2!3!1!1!} = 151,200$ combinations~~

$\left[\binom{10}{9}\cdot 9!\right] + \left[\binom{10}{8}\cdot 8!\right] + \left[\binom{10}{7}\cdot 7!\right] + \left[\binom{10}{6}\cdot 6!\right] + \left[\binom{10}{5}\cdot 5!\right]$

$\left[\binom{10}{4}\cdot 4!\right] + \left[\binom{10}{3}\cdot 3!\right] + \left[\binom{10}{2}\cdot 2!\right] + \left[\binom{10}{1}\cdot 1!\right] = 10,345,621$

$1 + \binom{6}{5} + \binom{6}{4} + \binom{6}{3} + \binom{6}{2} + \binom{6}{1} = 63$

2\. $x = 2 \quad y = 2$

$4^n = \sum_{k=0}^{n} \binom{n}{k} 2^k \cdot 2^{n-k}$

$\binom{n}{k}$: # of k-combinations

for each $x_1, \ldots, x_n$, $x_i \in S$ or $x_i \notin S$ | cardinality of $\mathcal{P}(x)$ when $|x| = n$ obtained by first counting the number of k-combinations for fixed k, then the addition principle.

4^n total subsets

3\. a) 4 digit code : each digit ranges from 0-9 (10 choices)

10 digits · 10 digits · 10 digits · 10 digits = 10,000 passwords

In the worst case scenario you'd need 3,333 hours.

b) In this scenario there would be 1,000 different passwords since one digit is already known ($10 \times 10 \times 10$).

This means in the worst-case scenario it would take 333 hours to guess the correct password.

c) $4 \times 10^3 = 4,000$ possibilities

#	10	10	10	$= 10^3$
10	#	10	10	$= 10^3$
10	10	#	10	$= 10^3$
10	10	10	#	$= 10^3$

4,000 possibilities -> 1,333 hours to guess the password in worst case scenario.

* 4. $X = \{0, 1, 2, 3, 4, 5, 6\}$ 17 subsets, cardinality ≤ 3

Two subsets add up to the same number

The highest possible sum for a subset would be 15. This occurs with the subset of $\{4, 5, 6\}$. This is the largest possible sum that can be created because it has the greatest possible cardinality and the three greatest elements in X.

The lowest possible sum for a subset would be 0. This occurs with the subset of $\{0\}$. This is the smallest possible sum because that can be created because it has the smallest possible cardinality and the smallest element in X.

The total amount of "pigeonholes" would be the amant of different sums that can be created from the subsets. This means there are 16 pigeonholes, one hole from 0-15.

* Note: I am considering the empty set to have a sum of 0. I realize there are no elements in an empty set so technically you can't sum up its elements, but it allows the pigeonhole principle to be applied.

The "pigeons" would be the actual sums found by summing up the contents of 17 subsets. This means there will be 17 pigeons.

* Note: not every sum is necessarily unique

K = # of pigeonholes $K = 16$ n = # of pigeons $n = 17$

Because $K < n$, by definition of the set-theoretical pigeonhole principle, it can be concluded that at least two of the subsets will add up to the same number.

We know this applies in all cases b/c it applies at K's max value of 5.

For example:

n	n	n	n	n	n	n	n
0	1	2	3	4	5	6	7

n	n	n	n	n	n	n	n n
8	9	10	11	12	13	14	15

At least one of the pigeonholes has two pigeons.

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0 1 2 3   4 5 6 7
1 2 3 4   5 6 7 8
2 3 4 5   6 7 8 9
3 4 5 6

5. $X = \{0, 1, 2, 3, 4, 5, 6, 8, 9\}$ 7 subsets, cardinality $= 4$

The number of pigeonholes corresponds to the number of different largest elements in the 7 chosen subsets. That would mean there are 6 possible pigeonholes for 3, 4, 5, 6, 8, 9.

The number of pigeons is 7 because there are 7 possible subsets.

K = pigeonholes $= 6$
n = pigeonholes $= 7$

Because $K < n$, this is an example of the set-theoretical pigeonhole principle. It can be concluded that at least two subsets of the seven will share the same largest element.

1	2	3	4	5	6
$\boxed{n}$	$\boxed{n}$	$\boxed{n}$	$\boxed{n}$	$\boxed{n}$	$\boxed{nn}$

This hole has to double up on pigeons.