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Math 61 HW #4

1. i) $s_0 = 1$

ii) $S_{n+1} = 2(n+1)S_n$ for every $n \geq 0$

$S_1 = 2(0+1) \cdot 1 = 2$ $[2 \cdot 1 \cdot 1]$

$S_2 = 2(1+1) \cdot 2 = 8$ $[2 \cdot 2 \cdot 2]$

$S_3 = 2(2+1) \cdot 2^3 = 48$ $[2^4 \cdot 3]$

$S_4 = 2(3+1) \cdot 2^4 \cdot 3 = 384$ $[2^5 \cdot 4 \cdot 3]$

$S_5 = 2(4+1) \cdot 2^5 \cdot 4 \cdot 3 = 3840$ $[2^6 \cdot 5 \cdot 4 \cdot 3]$

$$\boxed{S_n = 2^{n+1} \cdot \frac{n!}{2} \quad \text{when } n \geq 1}$$

basis step

- $S_1 = 2 \cdot 1 \cdot 1$ as defined by our sequence and seen above

when $n = 1 \rightarrow 2^{1+1} \cdot \frac{1!}{2} = 2^2 \cdot \frac{1}{2} = 4 \cdot \frac{1}{2} = 2$ ✓

$2 = 2$ ✓ $(n=1) = S_1$ ✓

Inductive step

We will assume that

$S_n = 2^{n+1} \cdot \frac{n!}{2}$ for an n when $n \geq 1$

$$\begin{aligned} S_{n+1} &= 2(n+1)S_n \\ &= 2(n+1)(2^{n+1} \cdot \tfrac{n!}{2}) \\ &= 2^{n+2}(n+1) \cdot \tfrac{n!}{2} \\ &= 2^{n+2} \cdot \tfrac{(n+1)!}{2} \end{aligned}$$

By showing this we see that

$S_{n+1} = 2^{(n+1)+1} \cdot \frac{(n+1)!}{2} = 2^{n+2} \cdot \frac{(n+1)!}{2}$ ✓

This proof by induction shows that the explicit formula mentioned above is correct.

★ 2. i) $s_0 = 4$, $s_1 = 10$

ii) $s_{n+1} = 2s_n + 8s_{n-1}$ for all $n \geq 1$

2^{nd} order linear homogenous recursive relation that has constant coefficients. Use theorem covered in lecture to solve

$s_n = 2s_{n-1} + 8s_{n-2}$ for all $n \geq 2$

$t^n = 2t^{n-1} + 8t^{n-2}$

$t^n - 8t^{n-2} - 2t^{n-1} = 0$

$t^2 - 2t - 8 = 0$

$(t+2)(t-4) = 0$

$t = -2 \quad t = 4$

$S_n = 4^n \quad T_n = -2^n$ → next define a general solution

- $aS + bT$ will be solutions (as long as a and b are real numbers) if S and T are solutions.

SUBSTITUTE

- $a4^n + b(-2)^n$ is a solution! for $a, b \in \mathbb{R}$

PLUG IN

$S_0 = 4$

$4 = a(4)^0 + b(-2)^0$

$S_0 = 4$ → $4 = a + b$ → $4 = a + b$

$10 = a \cdot (4)^1 + b \cdot (-2)^1$

$S_1 = 10$ → $10 = 4a - 2b$ → $2b = 4a - 10$

$b = 2a - 5$

↓

$4 = a + b$

$b = 2a - 5$

$4 = a + (2a - 5)$

$4 = 3a - 5$

$9 = 3a$

$a = 3 \qquad b = 1$

$\boxed{\text{Solution: } S_n = 3 \cdot (4)^n + (-2)^n}$