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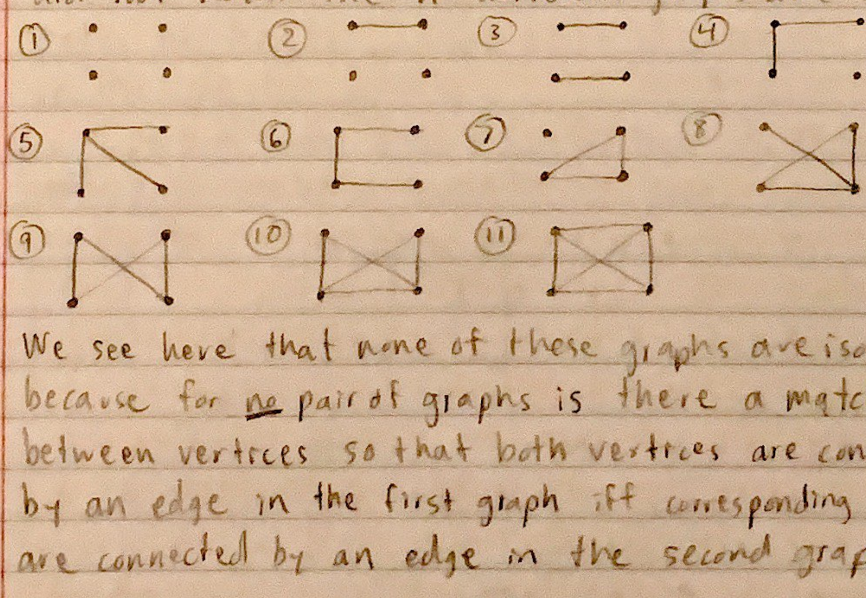
Math HW #6

1. Dijkstra's algorithm could be helpful for a delivery driver who wants to finish her deliveries as quickly as possible. For example, a GPS typically uses Dijkstra's algoritm. A road would count as an edge in a graph. The amount of time needed to traverse the road would be the weight of the edge. The vertices would be the locations that are receiving the deliveries. The GPS would look at traffic in real time to construct a weighted graph. Dijkstra's algorithm would be used to create a path connecting all the delivery locations together so the path has the least weight - meaning the path will be the quickest way to finish all the deliveries.

* 3. There are 11 different non-isomorphic graphs with 4 vertices. Given a graph $G = (V, E)$ and $G' = (V', E')$. A graph isomorphism from G to G' is a bijection $\varphi : V \to V'$ such that $\{v, w\} \in E_G \longleftrightarrow \{\varphi(v), \varphi(w)\} \in E_{G'}$

A non-isomorphic graph would mean the above conditions did not hold. The 11 different graphs are as follows:



We see here that none of these graphs are isomorphic because for no pair of graphs is there a matching between vertices so that both vertices are connected by an edge in the first graph iff corresponding vertices are connected by an edge in the second graph.

Graph 1: 4 vertices degree 0
Graph 2: 2 vertices degree 1, 2 vertices degree 0
Graph 3: 4 vertices degree 1
Graph 4: 1 vertex degree 2, 2 vertices degree 1, 1 vertex degree 0
Graph 5: 1 vertex degree 3, 3 vertices degree 1
Graph 6: 2 vertices degree 2, 2 vertices degree 1
Graph 7: 3 vertices degree 2, 1 vertex degree 0
Graph 8: 1 vertex degree 3, 2 vertices degree 2, 1 vertex degree 1
Graph 9: 4 vertices degree 2
Graph 10: 2 vertices degree 3, 2 vertices degree 1
Graph 11: 4 vertices degree 3

As we can tell, there are no graphs with the same types of vertices. This shows all 11 graphs are nonisomorphic.

4. A maximal planar graph is a planar graph that can't have more edges added to it without making it non-planar. This means each face of a maximal planar graph will be bound by 3 edges, and each edge is a boundary between 2 faces. This means $3F = 2E$.

We can substitute this into Euler's Formula

$$V = 2 + E - F$$

$$V = 2 + E - \frac{2E}{3}$$

$$3V = 6 + E \longrightarrow 3V - 6 = E$$

Planar graphs must have equal to or less edges than a max planar graph with the same amount of vertices

Therefore: $E \leq 3V - 6$

The statement $E \leq 3V - 6$ is great for demonstrating whether or not a graph is planar. For example, we can look at K_5. K_5 has 5 vertices and 10 edges. This means that: $10 \leq 3(5) - 6$

This is clearly not true as 10 is not less than or equal to 9. This proves K_5 is not planar.

2 Let $n \in \mathbb{N}$. Let A be the adjacency matrix of the graph K_n. Derive a formula for the entries of A^i, $i \geq 1$.

If K_n is a complete graph with "n" many vertices, that means every vertex will be adjacent to each other vertex. An edge will exist between every pair of vertices in the graph as long as the vertices are distinct.

A sample adjacency matrix for one of these graphs K_n may look like this (if $n=3$)

	1	2	3
1	0	1	1
2	1	0	1
3	1	1	0

From this we can observe that an adjacency matrix of K_n will have 0's on the diagonal from upper left to bottom right because each vertex doesn't relate to itself. All other entries in the matrix are 1 because each vertex is adjacent to all the others.

The matrices A^i (when $i \geq 1$) show the adjacency matrix raised to a power. This means that the entries for these matrices represent the number of different paths of length i that exist between the vertices of the graph.

I will perform some sample calculations with the sample matrix from above ($n=3$)

$n=3 \quad i=1$
$$\begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}$$

$n=3 \quad i=2$
$$\begin{bmatrix} 2 & 1 & 1 \\ 1 & 2 & 1 \\ 1 & 1 & 2 \end{bmatrix}$$

$n=3 \quad i=3$
$$\begin{bmatrix} 2 & 3 & 3 \\ 3 & 2 & 3 \\ 3 & 3 & 2 \end{bmatrix}$$

$n=3 \quad i=4$
$$\begin{bmatrix} 6 & 5 & 5 \\ 5 & 6 & 5 \\ 5 & 5 & 6 \end{bmatrix}$$

$n=3 \quad i=5$
$$\begin{bmatrix} 10 & 11 & 11 \\ 11 & 10 & 11 \\ 11 & 11 & 10 \end{bmatrix}$$

$m = n$

$i = n$

Based on these calculations, I noticed that the diagonal entries for each matrix are always consistent. The 3 entries on the diagonal are equivalent to $(n-1) \cdot$ (the # not on the diagonal in the previous matrix). When i is odd, the numbers not on the diagonal are equal to the number on the digonal +1. When i is even, the numbers not on the diagonal are equal to the number on the diagonal -1.

More sample calculations but $n = 4$ this time

$n=4 \quad i=1$
$$\begin{bmatrix} 0 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 0 \end{bmatrix}$$

$n=4 \quad i=2$
$$\begin{bmatrix} 3 & 2 & 2 & 2 \\ 2 & 3 & 2 & 2 \\ 2 & 2 & 3 & 2 \\ 2 & 2 & 2 & 3 \end{bmatrix}$$

$n=4 \quad i=3$
$$\begin{bmatrix} 6 & 7 & 7 & 7 \\ 7 & 6 & 7 & 7 \\ 7 & 7 & 6 & 7 \\ 7 & 7 & 7 & 6 \end{bmatrix}$$

$n=4 \quad i=4$
$$\begin{bmatrix} 21 & 20 & 20 & 20 \\ 20 & 21 & 20 & 20 \\ 20 & 20 & 21 & 20 \\ 20 & 20 & 20 & 21 \end{bmatrix}$$

$n=4 \quad i=5$
$$\begin{bmatrix} 60 & 61 & 61 & 61 \\ 61 & 60 & 61 & 61 \\ 61 & 61 & 60 & 61 \\ 61 & 61 & 61 & 60 \end{bmatrix}$$

The same rules from before with $n=3$ still hold when $n=4$.

Therefore the conclusion I have reached is:

D = diagonal entries $\qquad E$ = non-diagonal entries

$D_{i=1} = 0 \qquad E_{i=1} = 1$

$D_i = E_{i-1} \cdot (n-1)$

$E_i = D_i + 1$ (when i is odd)

$E_i = D_i - 1$ (when i is even)

for $i \geq 1$

Formula for #'s not on diagonal: $E_i = \frac{1}{n}\left[(n-1)^i + (-1)^{i+1}\right]$

Formula for #'s on diagonal: $D_i = \dfrac{(n-1)\left[(n-1)^{i-1} + (-1)^i\right]}{n}$