Financial Econometrics
FINN/ECON 6219, Fall 2023

Problem Set 2
Due October 4 at the beginning of class

You may work in a group of 2 or 3 students
(submit a single set of solutions that includes the names of all group members)

Attach a copy of your Stata code at the end of your solutions

Question 1.

Download the "exchq.dta" file that is posted on the following webpage:

http://fmwww.bc.edu/ec-p/data/mills2d/mills.data.html

The file contains quarterly data on the USD/Pound exchange rate for 1972 to 1996.

a. Load the data into Stata and plot the exchange rate versus time. Label each axis appropriately and include a title. Copy and paste the plot into your solutions.
b. Let y_t denote the exchange rate for quarter t. Let $\Delta y_t = y_t - y_{t-1}$ denote the first difference of the exchange rate. Use the `regression` command to fit AR(0), AR(1), AR(2), and AR(3) models that include an intercept to Δy_t. Store the estimates produced by each model and use `estimates stats` to display the AIC and BIC for the models in a single table (i.e., used this command after fitting all four models). Make sure to fit each model using exactly the same number of observations. Copy and paste the AIC/BIC table into your solutions. Which model is selected by AIC? By BIC?
c. Use an augmented Dickey Fuller test to assess whether y_t has a unit root. The alternative for the test should be a stationary AR(2) model for y_t with a non-zero mean. Use the `regress` option for `dfuller` to display the regression table Copy and paste the regression table into your solutions. Can you reject a unit root at the 10% significant level?
d. Drop the last four observations from the data set. Let T denote the last period for this shortened sample. Use the `arima` command to an fit ARIMA(1,1,0) model to y_t. Copy and paste the ARIMA output into your solutions. Use the `tsappend` command to add four blank rows to the dataset. Use the `predict` command to construct dynamic forecasts of y_{T+1}, y_{T+2}, y_{T+3}, and y_{T+4}. Plot the forecasts versus the actual values. Label each axis appropriately and include a title. Copy and paste the plot into your solutions.

Question 2.

The program ar2sim.do is posted with the problem set. This program takes 3 arguments: the sample size, the estimated slope from a regression of Δy_t on Δy_{t-1}, and the estimated standard deviation of the regression residuals. So the command `ar2sim 100 0.9 0.5` would execute the program using 100 observations, a slope of 0.9, and a residual standard deviation of 0.5.

The program uses the parameters for the Δy_t process (an AR(1) model) to generate simulated data from the corresponding process for y_t (an AR(2) model). After generating the data, the program performs an ADF unit root test for y_t, and returns the p-value of the ADF test statistic. Use the same data as for Question 1 and complete the following.

a. Fit an AR(1) model to Δy_t using the `regression` command. Execute ar2Sim.do using the number of observations for y_t, which is 100, the estimated slope, and the estimated standard deviation of the regression residuals (the root MSE) as inputs. Round all input values to 4 decimal places. Set the seed to 6219 before you call ar2sim.do. Copy and paste the resulting regression table into your solutions. Note: your output will not match mine unless you set the seed correctly.
b. Use simulations with 5000 repetitions to look at the distribution of p-values generated by the ADF test. The syntax is `simulate pval=r(p), reps(5000): ar2sim 100 0.9 0.5`, with `0.9` and `0.5` replaced by the values from part (a). Set the seed to 6219, and find the fraction of the 5000 repetitions in which you reject a unit root at the 10% significant level. Report this fraction in your solutions. Are the results consistent with what you expected to see? Explain why or why not.
c. Note that the ADF regression

$$\Delta y_t = \gamma y_{t-1} + \beta \Delta y_{t-1} + e_t$$

can also be expressed as

$$y_t = (1 + \gamma + \beta) y_{t-1} - \beta y_{t-2} + e_t$$

The ar2sim.do program generates data from this model with $\gamma = 0$. Modify the program so that it accepts an estimate of γ as a fourth argument and generates y_t accordingly. Repeat the simulations of part (b) using the estimated values of γ, β and residual variance from part (c) of Question 1 (you will have to figure out how to find the root MSE since it is not reported by `dfuller`). Set the seed to 6219, and find the fraction of the 5000 repetitions in which you reject a unit root at the 10% significant level. Report this fraction in your solutions. Are the results consistent with what you expected to see? Explain why or why not.
d. Repeat the simulations of part (b) and (c) using 200 observations instead of 100 observations. Set the seed to 6219 in each case, and find the fraction of the 5000 repetitions in which you reject a unit root at the 10% significant level in each case. Report these fractions in your solutions. Are the results consistent with what you expected to see? Explain why or why not.