Q1a) 6 possible outcomes for die, 2 possible outcomes for coin.

Total outcomes / elts of Ω = $6\times 2 = 12$.

* We define (a,b) as a possible outcome where a is outcome for die and b is outcome for coin.

Example of element in Ω (3, H), ie 3 for die, Head for coin.

b) Let A be the event that the coin lands on heads.

* for (a,b)
a can be 1 to 6
b can be H: Heads or T: Tails.

$A = \{(1,H), (2,H), (3,H), (4,H), (5,H), (6,H)\}$

Let B be the event that the die roll is even.

$B = \{(2,H), (2,T), (4,H), (4,T), (6,H), (6,T)\}$.

Let C be the event that the die roll is > 3 and the coin lands on tails.

$C = \{(4,T), (5,T), (6,T)\}$

c) $P(A) = \frac{|A|}{12} = \frac{6}{12} = \frac{1}{2}$ $P(B) = \frac{|B|}{12} = \frac{6}{12} = \frac{1}{2}$ $P(C) = \frac{|C|}{12} = \frac{3}{12} = \frac{1}{4}$

Q2) Let G: genius & C: loves chocolate.

$P(G) = 0.6$, $P(C) = 0.7$, $\cancel{P(A \cap B)}$ $P(G \cap C) = 0.4$.

Want to find $P((G \cup C)^c)$.

$P(G \cup C) = P(G) + P(C) - P(G \cap C) = 0.6 + 0.7 - 0.4 = 0.9$.

So, $P((G \cup C)^c) = 1 - P(G \cup C) = 1 - 0.9 = 0.1$

ie the probability that a random student is neither a genius or loves chocolate is 0.1 or 10%.

Q3)

a) Yes. If coin lands on tail twice, it does not land on head both times, so it can be represented as $\{0\}$ which is a subset of Ω.

b) Yes. this event can be represented as $\{(H,T), \text{or } (T,H)\}$. These are 2 elements in Ω where 1 toss is heads and other is tails, regardless of order, hence can be $\{1\}$, a subset of Ω

c). No. This event $\{(H,T)\}$ cannot be represented unambiguously as a subset of Ω. While it is true that 1 total head occured in this event, having 1 total head in Ω does not imply that (H,T) happened. So, it is not equivalent, we need to differentiate the order of occurence, which isn't possible with set notation in this case.

d) Yes. This can be represented as $\{1,2\}$, a subset of Ω. This event includes $\{(H,T), (T,H), (H,H)\}$, all ~~elements~~ possible outcomes where at least 1 head appears.

Q4) Total outcomes $= 6 \times 6 = 36$.

(# choices for 1st dice) $\times$ (# choices for 2nd dice) = Probability.

a) Probability $= \frac{2}{6} \times \frac{2}{6} = \frac{4}{36} = \frac{1}{9}$

b) Probability $= \frac{3}{6} \times \frac{3}{6} = \frac{1}{4}$

c) Probability $= \frac{|\text{Possible outcomes}|}{36} = \frac{|\{(1,3),(2,3),(3,3),(3,2),(3,1)\}|}{36} = \frac{5}{36}$

d)

x	Probability max is less than or equal to x	Probability max is exactly x
1	$(\frac{1}{6})^2 = \frac{1}{36}$	$\frac{1}{36}$
2	$(\frac{2}{6})^2 = \frac{4}{36}$	Possible outcomes $= \lvert\{\{(1,2),(2,1)\},(2,2)\}\rvert = 3$ so $\cancel{\frac{3}{36}}\ \frac{3}{36}$
3	$(\frac{3}{6})^2 = \frac{9}{36}$	from (c), so $\frac{5}{36}$
4	$(\frac{4}{6})^2 = \frac{16}{36}$	# choices for 1st dice, given 2nd is 4; # choices for 2nd dice, give 1st is 4; Possible outcomes $= 4+4-1 = 7$ — repeat (4,4); so $\frac{7}{36}$
5	$(\frac{5}{6})^2 = \frac{25}{36}$	$\frac{5+5-1}{36} = \frac{9}{36}$
6	$(\frac{6}{6})^2 = 1$	$\frac{6+6-1}{36} = \frac{11}{36}$

e) $P(1)+P(2)+P(3)+P(4)+P(5)+P(6)$ should equal 1, as this covers all possible outcomes in the event space, where the maximum of both dice is the range of possible values of the dice $1 \leq x \leq 6$.

To check: $\frac{1}{36} + \frac{3}{36} + \frac{5}{36} + \frac{7}{36} + \frac{9}{36} + \frac{11}{36} = \frac{36}{36} = 1$.

Q5) $P(A \cup B \cup C) = P(A)+P(B)+P(C) - P(A \cap B) - P(A \cap C) - P(B \cap C) + P(A \cap B \cap C)$.

With $P(A)+P(B)+P(C)$, we have overlaps, with $2 \times P(A \cap B)$, $2 \times P(B \cap C)$ and $2 \times P(A \cap C)$. deducting each of the intersection removes duplicates of each intersection of any 2 sets. However, this removes all of the center portion (overlap of all 3 sets). So, we need to add it back to get a ~~copy~~ single instance of each area, ~~these specifically~~ to get $P(A \cup B \cup C)$.

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