Homework 5

1. Using the geometric cdf and the definition of conditional probability, prove that the geometric distribution is memoryless. In other words, if $X \sim Geom(p)$, show that

$$\mathbb{P}(X > m + n | X > m) = \mathbb{P}(X > n)$$

for any nonnegative integers m and n.

2. In every box of Animal O's cereal, there is a prize. There are four types of prizes, all equally likely: lion, giraffe, ape, and elephant. If we let X be the number of cereal boxes needed to collect all four prizes, what is $\mathbb{E}(X)$? How does this generalize to a situation where there are n different collectibles?

3. Suppose that each day, the probability of a tornado occurring near Madison is 0.03.
   (a) Suppose you wanted to model the number of tornadoes in 365 days using a Poisson distribution. What parameter would you use?
   (b) What is the probability that there are exactly 12 tornadoes in 365 days?
   (c) Do you think it is more likely that there are 11 tornadoes in 365 days, or 10 tornadoes in 365 days? Check your answer by calculating the probabilities for 11 and 10.

4. Suppose on any given day, the probability that you stub your toe is 0.08. The probability that your clumsy friend stubs their toe is 0.15. Assume days are independent.
   (a) Give the binomial distribution for X, the number of times you stub your toe in 100 days, and for Y, the number of times your friend stubs their toe in 100 days.
   (b) What is the Poisson distribution that you can use to approximate $Z = X + Y$?
   (c) Using the Poisson distribution from part (b), find $\mathbb{P}(Z = 20)$.
   (d) Find $\mathbb{V}(X) + \mathbb{V}(Y)$ using the binomial distributions from part (a). Next, find $\mathbb{V}(Z)$ using the Poisson distribution from part (b). Which expression for variance is larger? Give an intuitive explanation for why this is the case.

5. Players A and B are playing a game for \$100. They take turns flipping a fair coin, player A wins a points if the coin is heads, and B wins a point if the coin is tails. The first person to get 10 points wins the \$100.

   After player A has gotten 4 points and player B has gotten 7 points, the game is interrupted and cannot continue. The players decide to divide the \$100 based on the probability of each player winning after this point. How should the money be split?