Q1(a) # of numbers from 1 to 1000 divisible by 7 = $\left\lfloor \frac{1000}{7} \right\rfloor = 142$.

So probability random number drawn is divisible by 7 is $\frac{142}{1000}$.

b) # of numbers divisible by: ① 4 : $\left\lfloor \frac{1000}{4} \right\rfloor = 250$ 6: $\left\lfloor \frac{1000}{6} \right\rfloor = 166$ Both 4 & 6: $\left\lfloor \frac{1000}{12} \right\rfloor = 83$ (since LCM of 4 & 6 is 12).

so # of numbers from 1 to 1000 divisible by 4 or 6: $250 + 166 - 83 = 333$.

so probability random number drawn is divisible by 4 or 6 is $\frac{333}{1000}$.

Q 2a) In a room of 25 people, probability that at least 2 share a birthday

$= 1 -$ (probability that everyone does not share a birthday) $= 1 - \frac{365 \times 364 \times \ldots \times 341}{365^{25}}$

b) Probability that at least 2 people share last 4 digits of their phone number

$= 1 -$ (probability no 2 people share last 4 digits) $= 1 - \frac{10000 \times 9999 \times \ldots \times 9901}{10000^{100}}$

Q3a) # of ~~unique~~ letters = 26, # of ~~unique~~ digits = 10.

Total # of possible license plates = $\cancel{26 \times 26 \times 26 \times 10 \times 10 \times 10}$ $26 \times 25 \times 24 \times 10 \times 9 \times 8 = 11\ 232\ 000$

26 options for first letter, 25 for second, 24 for third
10 options for first number, 9 for second, 8 for third

b) # of license plates like $\underbrace{\text{U W \_}}_{\text{letters}}$ $\underbrace{\text{\_ \_ \_}}_{\text{numbers}}$ $= \cancel{26 \times 10 \times 10 \times 10}$ $24 \times 10 \times 9 \times 8$

# of license plates like $\underbrace{\text{\_ U W}}_{\text{letters}}$ $\underbrace{\text{\_ \_ \_}}_{\text{numbers.}}$ $= \cancel{26 \times 10 \times 10 \times 10}$ $24 \times 10 \times 9 \times 8$

Total # of license plates containing "UW" in order $= \cancel{2 \times 26 \times 10 \times 10 \times 10}$ $2 \times 24 \times 10 \times 9 \times 8$.

Q4a) Unique orders $= 3 \times 4 \times 4 \times 12 = 576$.

b) Unique orders $= 3 \times 4 \times \binom{4}{2} \times \binom{12}{2} = 3 \times 4 \times 6 \times 66 = 4752$

c) Unique orders $= 3 \times 4 \times 4 \times \left(\binom{12}{0} + \binom{12}{1} + \ldots + \binom{12}{12} \right) = 3 \times 4 \times 4 \times 2^{12}$ (ie each topping is either included or not).

d) # of combinations with no pineapple $= 3 \times 4 \times 4 \times 2^{11}$ (pineapple not selected in all scenarios).

Probability friend eat pizza $= \frac{3 \times 4 \times 4 \times 2^{11}}{3 \times 4 \times 4 \times 2^{12}} = \frac{1}{2}$.

Q 5a) Size of outcome space $\Omega = 6 \times 6 = 36$

since each die has 6 faces.

b) Subset of size 1 from Ω from 36 possible outcomes will be a single outcome of rolling the 2 dice.

$\Rightarrow$ so, the number of subsets of size 1 is simply size of Ω, 36.

c) # of possible subsets of Ω of size k is simply choosing k outcomes from 36 possible ones

so $\binom{36}{k}$.

d) Fix outcome (1,1) to be selected, need 2 more outcomes.

so subsets of size 3 with (1,1) included $= \binom{35}{2}$.

Subsets of size 3 that do not include (1,1) = (subsets of size 3) − (subsets of size 3 with (1,1) included)

$= \binom{36}{3} - \binom{35}{2} = \binom{35}{3}$