

SDS 321 Worksheet 1

1. What do you know about sets?
 - a. $\{1,2\} \cup \{2,3\} =$
 - b. $\{1,2\} \cap \{2,3\} =$
 - c. True or False: $\{\{1,2\}, \{3\}, \{4,5\}\}$ is a partition of $\{1,2,3,4,5\}$
2. Express the following in terms of the events A, B and C using the operations of complement, union and intersection:
 - at least one of the events A, B, C occurs
 - events A and B occur, but not C
3. Which of these numbers cannot be a probability?
 - a) - 0.00001
 - b) 0.5
 - c) 1.001
 - d) 0
 - e) 1
4. Let A and B be two sets. Under what conditions is the set $A \cap (A \cup B)^c$ empty?
5. Is it possible to have the following: $P(E) = .3$, $P(F) = .4$, and $P(E \cup F) = .5$? Explain.
6. Given $P(A) = 0.55$, $P(B^c) = 0.35$, and $P(A \cup B) = 0.75$, find $P(B)$ and $P(A \cap B)$.
7. Given that $P(A^c) = 0.5$, $P(B) = 0.4$, and $P(A \cap B) = 0.1$, determine $P(A \cup B)$.
8. Give a mathematical derivation of the formula
$$P((A \cap B^c) \cup (A^c \cap B)) = P(A) + P(B) - 2P(A \cap B)$$
9. Show that $P(A \cup B \cup C) =$
$$P(A) + P(B) + P(C) - P(B \cap C) - P(A \cap B) - P(A \cap C) + P(A \cap B \cap C)$$
10. Consider two rolls of a fair four-sided die. Let X be the outcome on the first roll and Y be the outcome on the second. Determine:
 - a. $P((X, Y) \text{ is } (1,1) \text{ or } (1,2)) =$
 - b. $P(\{X = 2\}) =$
 - c. $P(X+Y \text{ is even}) =$

d. $P(\min(X, Y) = 1) =$

e. $P(\min(X, Y) > 1) =$

11. If you toss a fair coin until you first see a head, letting your sample space be the number of tosses to reach a head, then the sample space S will be $S = \{1, 2, \dots\}$ with $P(n) = (1/2)^n$, $n = 1, 2, \dots$

Find $P(\text{the number of tosses before seeing a head is even})$.

Note:

$$a + ar + ar^2 + ar^3 + ar^4 + \dots = \sum_{k=0}^{\infty} ar^k = \frac{a}{1-r}, \text{ for } |r| < 1.$$