2nd Homework Assignment
Project on Support Vector Machines

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Theoritical Background

We have the following non linear program:

$$\min\{F(x) = \frac{c^T x}{d^T x} : Ax = b;\ x \geq 0\} \tag{1}$$

Algorithm 1 Bisection Method for Optimal α

Given: interval $[L, U]$ that contains optimal α
repeat
  $\alpha := \frac{u+l}{2}$
  Solve the feasibility problem:
    $c^T x \leq \alpha d^T x$
    $d^T x > 0$
    $Ax = b$
  Adjust the bounds
  if feasible **then**
    $U := \alpha$
  else
    $L := \alpha$
  end if
until $U - L \leq \epsilon$

Problem 4

(a) Updating the Error Cache

When a Lagrange multiplier is non-bound after being optimized, its cached error is zero. The stored errors of other non-bound multipliers not involved in joint optimization are updated as follows.

$$E_k^{\text{new}} = E_k^{\text{old}} + u_k^{\text{new}} - u_k^{\text{old}} \tag{3.36}$$

$$E_k^{\text{new}} = E_k^{\text{old}} + u_k^{\text{new}} - u_k^{\text{old}} \tag{3.37}$$

For any k-th example in the training set, the difference between its new SVM output value and its old SVM output value, $u_k^{\text{new}} - u_k^{\text{old}}$, is due to the change in α_1, α_2 and the change in the threshold b.

$$u_k^{\text{new}} - u_k^{\text{old}} = y_1 \alpha_1^{\text{new}} k_{1k} + y_2 \alpha_2^{\text{new}} k_{2k} - b^{\text{new}} - \left(y_1 \alpha_1^{\text{old}} k_{1k} + y_2 \alpha_2^{\text{old}} k_{2k} - b^{\text{old}}\right) \tag{3.38}$$

Substituting equation (3.37) into equation (3.36), we have

$$E_k^{\text{new}} = E_k^{\text{old}} + y_1 \left(\alpha_1^{\text{new}} - \alpha_1^{\text{old}}\right) k_{1k} + y_2 \left(\alpha_2^{\text{new}} - \alpha_2^{\text{old}}\right) k_{2k} - \left(b^{\text{new}} - b^{\text{old}}\right) \tag{3.39}$$

References

[1] John Platt. Sequential Minimal Optimization: A Fast Algorithm for Training Support Vector Machines. Technical Report MSR-TR-98-14, Microsoft, April 1998. https://www.microsoft.com/en-us/research/publication/sequential-minimal-optimization-a-fast-algorithm-for-training-support-vector-machines/.

[2] Ginny Mak. The Implementation of Support Vector Machines Using the Sequential Minimal Optimization Algorithm. Master's thesis, McGill University, School of Computer Science, Montreal, Canada, April 2000.