En as a Logarithmic Bridge

En as a Logarithmic Bridge: A Candidate for Advancing Transcendental Number Theory

Abstract:

This paper proposes the constant $E_n = \sqrt{\ln(\pi)/\ln(e/(e-1))}$ as a novel construct for advancing results in transcendental number theory. En encapsulates a fixed-point identity linking e and pi through logarithmic structure. We explore its continued fraction expansion, fixed-point behavior, relation to known theorems (Lindemann-Weierstrass, Gelfond-Schneider, Baker), and its potential to serve as a focal point in studying algebraic independence. En is analytically well-behaved, structurally novel, and invites exploration of unexplored logarithmic domains.

1. Definition and Structure of En

The constant is defined as:

$E_n = \sqrt{\ln(\pi) / \ln(e / (e - 1))}$ implying $(e / (e - 1))^{(E_n^2)} = \pi$.

The continued fraction expansion $E_n = [1;1,1,2,1,1,1,2,...]$ shows no repeating pattern, suggesting irrationality and potential transcendence.

2. Relevance in Transcendental Number Theory

The ratio of logarithms in E_n^2 closely resembles structures analyzed in theorems of Gelfond, Schneider, and Baker. While these results typically involve logarithms of algebraic numbers, En invites expansion of their methods to logarithms of transcendental arguments.

$\ln(\pi)$ and $\ln(e/(e-1))$ are believed transcendental, and their ratio is almost certainly transcendental. Proving En's transcendence would imply a polynomial relationship between e and pi, contradicting current conjectures of their algebraic independence.

3. Reformulation of Independence Problems

En allows restatement of algebraic independence conjectures. If $\ln(\pi)$ and $\ln(e/(e-1))$ are linearly independent over Q, then E_n^2 must be irrational. Assuming $E_n^2 = p/q$ in Q, then $(e/(e - 1))^{(p/q)} = \pi$, implying a polynomial relation-a contradiction.

4. Analytic and Structural Insights

- Fractal Analogy
- Fixed-Point Dynamics
- Iterative Integration with Limit Convergence:

$$\pi = \lim_{n\to\infty} (1 + (E_n^2 * \ln(E_n))/n)^n$$

$$e = \lim_{n\to\infty} (1 + (E_n - 1)/n)^{(n / (E_n - 1))}$$

- Recursive Definition of En:

$$E_n(n) = \sqrt{\ln(\pi) / \ln(E_n(n-1) * (\ln(E_n(n-1)) - \ln(E_n(n-1) - 1)))}$$

- Symbolic Irreducibility

5. Symbolic Identity Notice and Ethical Licensing

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6. Acknowledgments

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Keywords: En, transcendence, logarithmic ratio, e/pi independence, continued fraction, symbolic drift, fixed-point recursion, bifurcation analysis