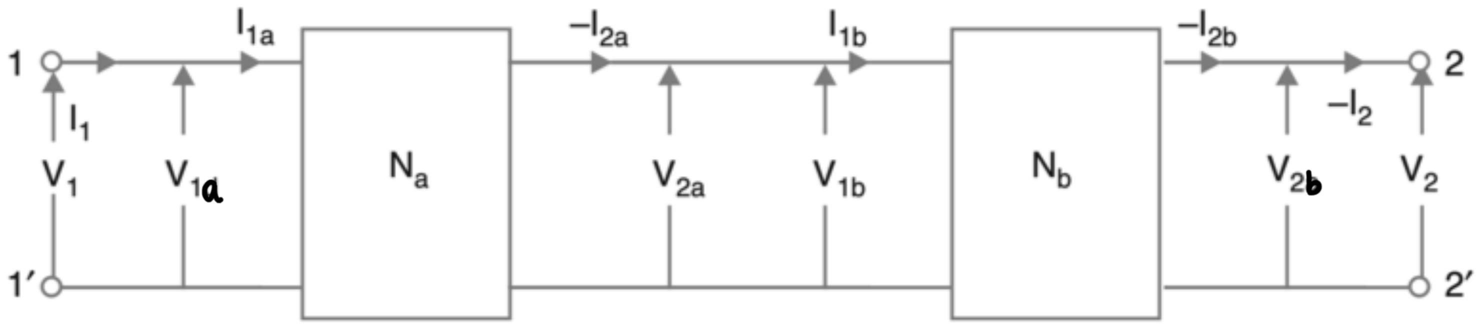
Interconnection of Two-Port Networks

A complex two-port network can be designed by interconnecting simple two-port networks.

We will study three types of connections: Cascade, Series, and Parallel.

Cascade Connection:

Two networks are said to be connected in cascade if the output port of the first becomes the input port of the second as shown below.



Using ABCD / transmission parameters:

for N_a: $$\begin{bmatrix} V_{1a} \\ I_{1a} \end{bmatrix} = \begin{bmatrix} A_a & B_a \\ C_a & D_a \end{bmatrix} \begin{bmatrix} V_{2a} \\ -I_{2a} \end{bmatrix}$$

and for N_b : $\begin{bmatrix} V_{1b} \\ I_{1b} \end{bmatrix} = \begin{bmatrix} A_b & B_b \\ C_b & D_b \end{bmatrix} \begin{bmatrix} V_{2b} \\ -I_{2b} \end{bmatrix}$

Now, for the entire network,

$$\text{let} \quad \begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} V_2 \\ -I_2 \end{bmatrix}$$

$$\text{but} \quad \begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} V_{1a} \\ I_{1a} \end{bmatrix} = \begin{bmatrix} A_a & B_a \\ C_a & D_a \end{bmatrix} \begin{bmatrix} V_{2a} \\ -I_{2a} \end{bmatrix}$$

$$= \begin{bmatrix} A_a & B_a \\ C_a & D_a \end{bmatrix} \begin{bmatrix} V_{1b} \\ I_{1b} \end{bmatrix}$$

$$= \begin{bmatrix} A_a & B_a \\ C_a & D_a \end{bmatrix} \begin{bmatrix} A_b & B_b \\ C_b & D_b \end{bmatrix} \begin{bmatrix} V_{2b} \\ -I_{2b} \end{bmatrix}$$

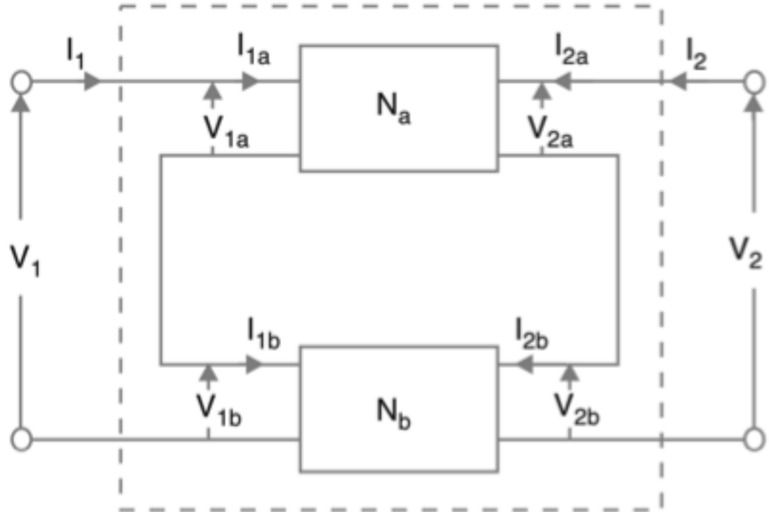
$$= \begin{bmatrix} A_a & B_a \\ C_a & D_a \end{bmatrix} \begin{bmatrix} A_b & B_b \\ C_b & D_b \end{bmatrix} \begin{bmatrix} V_2 \\ -I_2 \end{bmatrix}$$

Hence, $\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} A_a & B_a \\ C_a & D_a \end{bmatrix} \begin{bmatrix} A_b & B_b \\ C_b & D_b \end{bmatrix}$

Therefore, the overall transmission parameter matrix is the product of the transmission parameter matrices of each network in the cascade.

Series Connection :

Two networks are said to be connected in series if they are connected as shown below.



Using Z-parameters will be helpful in characterizing series connections.

$$\begin{bmatrix} V_{1a} \\ V_{2a} \end{bmatrix} = \begin{bmatrix} Z_{11a} & Z_{12a} \\ Z_{21a} & Z_{22a} \end{bmatrix} \begin{bmatrix} I_{1a} \\ I_{2a} \end{bmatrix}$$

$$\begin{bmatrix} V_{1b} \\ V_{2b} \end{bmatrix} = \begin{bmatrix} Z_{11b} & Z_{12b} \\ Z_{21b} & Z_{22b} \end{bmatrix} \begin{bmatrix} I_{1b} \\ I_{2b} \end{bmatrix}$$

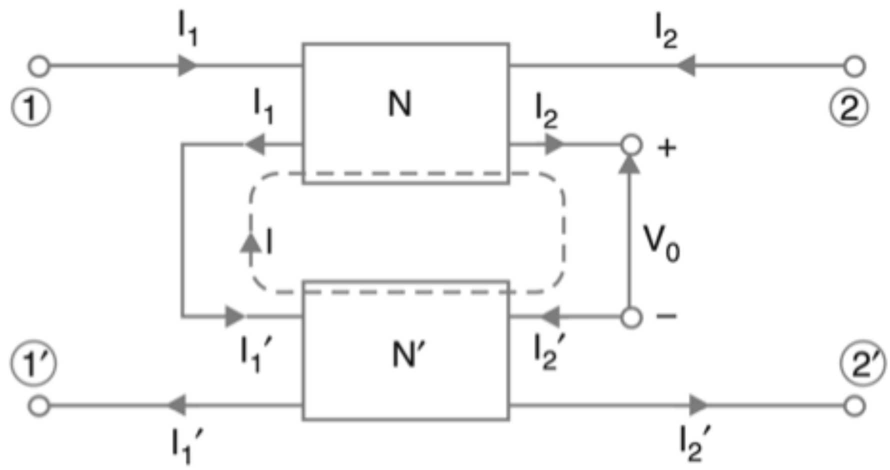
Clearly, $V_1 = V_{1a} + V_{1b} = (Z_{11a} + Z_{11b}) I_1 + (Z_{12a} + Z_{12b}) I_2$

$$V_2 = V_{2a} + V_{2b} = (Z_{21a} + Z_{21b}) I_1 + (Z_{22a} + Z_{22b}) I_2$$

Hence, $\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} z_{11a} + z_{11b} & z_{12a} + z_{12b} \\ z_{21a} + z_{21b} & z_{22a} + z_{22b} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$

Therefore, the overall impedance parameter matrix is the sum of the impedance parameter matrices of each network connected in series

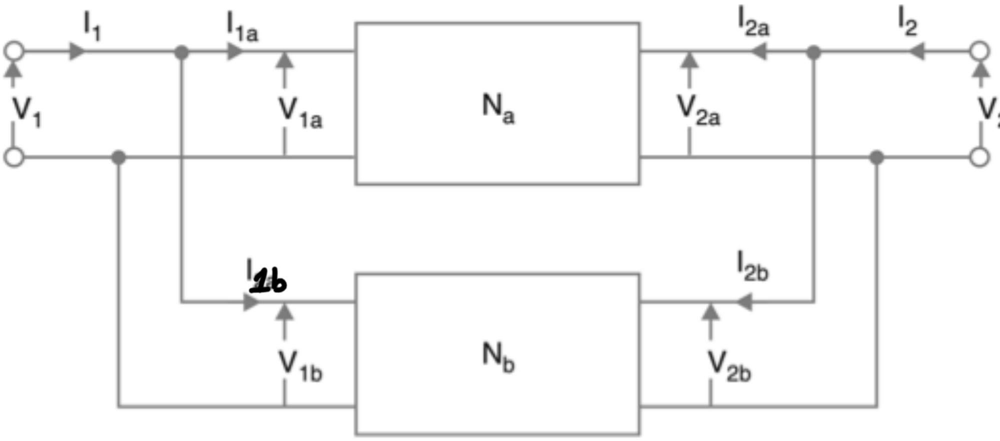
<u>Note:</u> Two networks N and N' can be connected in series only if $V_0 = 0$ as shown below.



If $V_0 \neq 0$, there will be circulating current which will imply that the current entering one terminal of port 2 will not be equal to the current leaving the other terminal.

Parallel Connection :

Two networks are said to be connected in parallel if they are connected as shown below.



Using Y-parameters will be helpful in characterizing parallel connections.

$$\begin{bmatrix} I_{1a} \\ I_{2a} \end{bmatrix} = \begin{bmatrix} y_{11a} & y_{12a} \\ y_{21a} & y_{22a} \end{bmatrix} \begin{bmatrix} V_{1a} \\ V_{2a} \end{bmatrix}$$

$$\begin{bmatrix} I_{1b} \\ I_{2b} \end{bmatrix} = \begin{bmatrix} y_{11b} & y_{12b} \\ y_{21b} & y_{22b} \end{bmatrix} \begin{bmatrix} V_{1b} \\ V_{2b} \end{bmatrix}$$

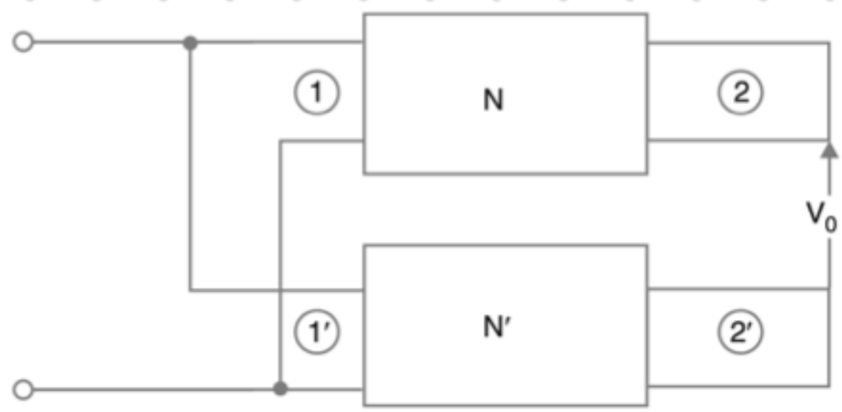
Clearly, $I_1 = I_{1a} + I_{1b} = (y_{11a} + y_{11b}) V_1 + (y_{12a} + y_{12b}) V_2$

$$I_2 = I_{2a} + I_{2b} = (y_{21a} + y_{21b}) V_1 + (y_{22a} + y_{22b}) V_2$$

Hence, $\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} y_{11a} + y_{11b} & y_{12a} + y_{12b} \\ y_{21a} + y_{21b} & y_{22a} + y_{22b} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$

Therefore, the overall admittance parameter matrix is the sum of the admittance parameter matrices of each network connected in parallel.

Note: Two networks N and N' can be connected in parallel only if $V_0 = 0$ as shown below.



In the circuit, ports 1 and 1' are connected together and ports 2 and 2' are short-circuited individually. If $V_0 \neq 0$, it will imply that the output terminals of the connected network are not short-circuited.