Assignment - 1

EEC201 - Signals, Systems, and Networks (Monsoon 2024)
Department of Electrical Engineering, IIT (ISM) Dhanbad

1. Determine the energy and power for each of the following signals:

(a) $x_1(t) = \cos^2(4t)$ (b) $x_2(t) = e^{-2t}u(t)$

(c) $x_3(t) = e^{j(2t+\pi/4)}$ (d) $x_4(t) = \dfrac{u(t) - u(t-3)}{3}$

Answer:

(a) $E \to \infty$ and $P = \dfrac{1}{2}$ (b) $E = \dfrac{1}{4}$ and $P = 0$

(c) $E \to \infty$ and $P = 1$ (d) $E = \dfrac{1}{3}$ and $P = 0$

2. Let $x(t)$ represent the signal shown below. Plot the following:

(a) $x_1(t) = x(4-t)$

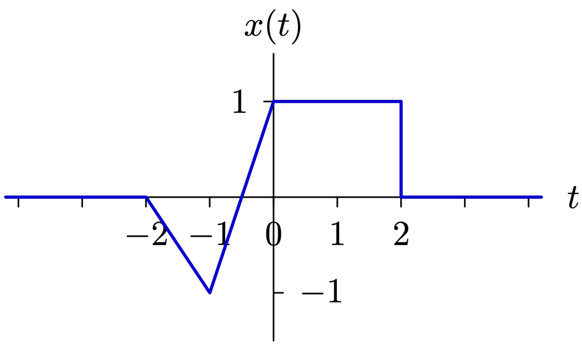
(b) $x_2(t) = x(3t+2)$

(c) $x_3(t) = 2 + x(t/2)$

(d) $x_4(t) = x(t)[\delta(t+3/2) - \delta(t-3/2)]$

(e) $x_5(t) = x_e(t) = \dfrac{x(t) + x(-t)}{2}$

(f) $x_6(t) = x_o(t) = \dfrac{x(t) - x(-t)}{2}$



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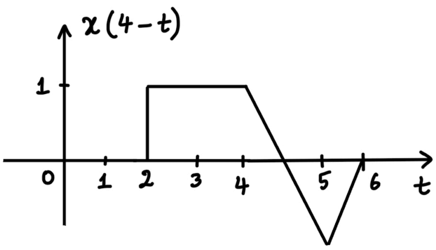

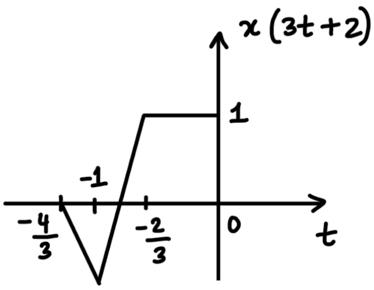

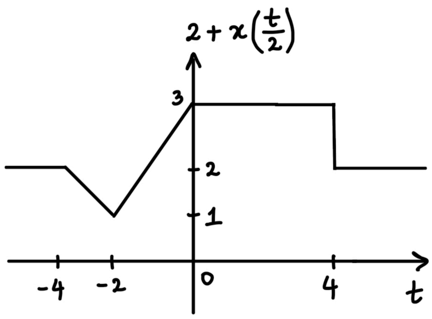

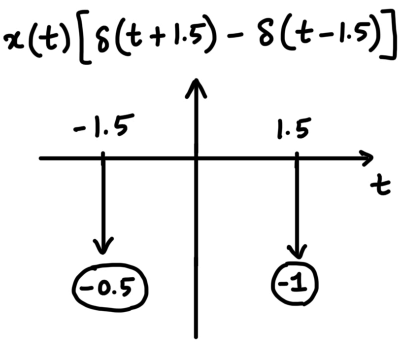

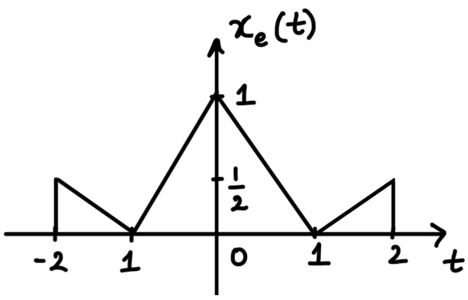

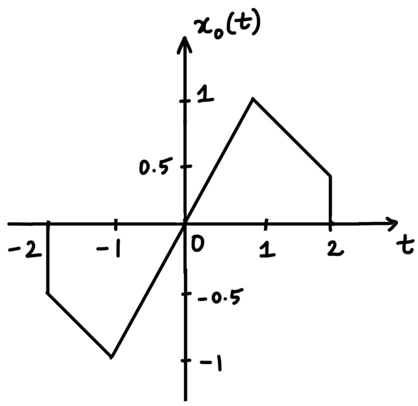

3. Let $x(t)$ be a signal with $x(t) = 0$ for $t < 3$. For each signal given below, determine the values of t for which the signal is guaranteed to be zero:

(a) $x_1(t) = x(-t)$

(b) $x_2(t) = x(1-t) + x(2-t)$

(c) $x_3(t) = x(1-t)\ x(2-t)$

Answer:

(a) We know that $x(t)$ is zero for $t < 3$. Then, $x(-t)$ will be zero for $t > -3$.

(b) We know that $x(1-t)$ is zero for $t > -2$. Similarly, $x(2-t)$ is zero for $t > -1$.
Therefore, $x(1-t) + x(2-t)$ will be zero for $t > -1$.

(c) $x(1-t)$ is zero for $1 - t < 3 \Rightarrow t > -2$. Similarly, $x(2-t)$ is zero for $t > -1$.
Therefore, $x(1-t)\ x(2-t)$ will be zero for $t > -2$.

4. Simplify and/or evaluate the following expressions:

(a) $\dfrac{(x^2+1)\ \delta(x-1)}{x^2+7}$ (b) $\cos(2\pi t)\big(\dot{u}(t) + \delta(t+1/4)\big)$ (c) $\dfrac{(x^4 - x^2 + 6)\ \delta(2x+3)}{e^{x^2} + 5x}$

(d) $\displaystyle\int_{-\infty}^{\infty} (\tau^2+1)\ \delta(\tau-2)\ \mathrm{d}\tau$ (e) $\displaystyle\int_{-\infty}^{\infty} e^{t-1} \cos\left(\frac{\pi(t-5)}{2}\right)\ \delta(t-3)\ \mathrm{dt}$

(f) $\displaystyle\int_{-\infty}^{\infty} \sin(\pi t)\ \delta(2t-3)\ \mathrm{dt}$ (g) $\displaystyle\int_{-\infty}^{\infty} \sin(2\pi t)\ \delta(4t-1)\ \mathrm{dt}$

Answer:

(a) $\dfrac{\delta(x-1)}{4}$ (b) $\delta(t)$ (c) $4.433\ \delta(2x+3)$ (d) 5

(e) $-e^2$ (f) $\dfrac{-1}{2}$ (g) $\dfrac{1}{4}$

5. Determine if each system given below is linear. Explain your reasoning.

(a) $y(t) = tu(t)x(t)$ (b) $y(t) = \dfrac{x(t)-2}{3}$ (c) $y(t) = 3\big[x(4t+3)\big]$

(d) $y(t) = \dfrac{x^2(t)}{1+x(t)}$ (e) $y(t) = x(t-10)$ (f) $y(t) = x(t) + t$

Answer:

(a) Linear
(b) Not-linear
(c) Linear
(d) Not-linear
(e) Linear
(f) Not-linear

6. Determine if each system given below is time-invariant. Explain your reasoning.

(a) $y(t) = tu(t)x(t)$ (b) $y(t) = \dfrac{d^5x(t)}{dt^5}$ (c) $y(t) = x(2t+5)$

(d) $y(t) = e^{4x(t)}$ (e) $y(t) = \dfrac{x(t)}{1+x(t)}$ (f) $y(t) = \ln(x(t)) + 4$

Answer:

(a) Not time-invariant
(b) Time-invariant
(c) Not time-invariant
(d) Time-invariant
(e) Time-invariant
(f) Time-invariant

7. Determine if each system given below is memoryless <u>and/or</u> causal. Explain your reasoning.

(a) $y(t) = x(t^2)$ (b) $y(t) = 30$ (c) $y(t) = x(u(t))$

(d) $y(t) = \dfrac{x^7(t) + x^6(t) + x^5(t)}{1 + x^2(t) + x^9(t)}$ (e) $y(t) = \dfrac{dx(t)}{dt} + x^2(t)$

Answer:

(a) Not memoryless and Not Causal
(b) Memoryless and Causal
(c) Not memoryless and Not Causal
(d) Memoryless and Causal
(e) Not memoryless and Not Causal

8. Determine if each system given below is BIBO stable. Explain your reasoning.

(a) $y(t) = u(t)x(t)$ (b) $y(t) = e^{x(t)}$ (c) $y(t) = \dfrac{x^2(t) + x^4(t)}{1 + x(t)}$

(d) $y(t) = \ln(x(t))$ (e) $y(t) = |x(t) - 4|$

Answer:

(a) BIBO stable
(b) BIBO stable
(c) Not BIBO stable
(d) Not BIBO stable
(e) BIBO stable

9. Find the zero-input response of the following LTIC systems with their initial conditions described below. Furthermore, investigate the asymptotic (internal) and BIBO (external) stabilities of the systems.

(a) $(D+5)y(t) = x(t)$ with the initial condition $y(0) = 5$.

(b) $(D^2+2D)y(t) = (5D+2)x(t)$ with the initial conditions $y(0) = 1$ and $\dot{y}(0) = 4$.

(c) $(D^2+6D+9)y(t) = (3D+5)x(t)$ with the initial conditions $y(0) = 3$ and $\dot{y}(0) = -7$.

(d) $(D+1)(D^2+5D+6)y(t) = Dx(t)$ with the initial conditions $y(0) = 2$, $\dot{y}(0) = -1$, and $\ddot{y}(0) = 5$.

Answer:

(a) $5e^{-5t} \quad (t \geq 0) \implies$ Asymptotically and BIBO stable.

(b) $3 - 2e^{-2t} \quad (t \geq 0) \implies$ Marginally stable and BIBO unstable.

(c) $(3+2t)e^{-3t} \quad (t \geq 0) \implies$ Asymptotically and BIBO stable.

(d) $6e^{-t} - 7e^{-2t} + 3e^{-3t} \quad (t \geq 0) \implies$ Asymptotically and BIBO stable.

10. Find the zero-state response of the following LTIC systems with the unit impulse response and input described below. Essentially, compute the convolution $x(t) * h(t)$.

(a) $h(t) = u(t) - u(t-1)$ and $x(t) = u(t) - u(t-1)$

(b) $h(t) = e^{-3t}u(t)$ and $x(t) = (e^{-3t} - e^{-4t})u(t)$

(c) $h(t) = -\delta(t) + 2e^{-t}u(t)$ and $x(t) = e^{t}u(-t)$

(d) $h(t) = e^{-t}u(t)$ and $x(t) = \begin{cases} 1 & |t| \leq 1 \\ 0 & \text{else} \end{cases}$

(e) $h(t) = e^{-t}u(t)$ and $x(t) = 4e^{-2t}\cos(3t)u(t)$

(f) $h(t) = u(t)$ and $x(t) = \begin{cases} 2 - |t| & |t| \leq 2 \\ 0 & \text{else} \end{cases}$

Answer:

(a) $y(t) = \begin{cases} t & 0 \leq t \leq 1 \\ 2-t & 1 \leq t \leq 2 \\ 0 & \text{else} \end{cases}$ (b) $y(t) = (te^{-3t} - e^{-3t} + e^{-4t})u(t)$ (c) $y(t) = e^{-t}u(t)$

(d) $y(t) = \begin{cases} 0 & t \leq -1 \\ 1 - e^{-(t+1)} & -1 \leq t \leq 1 \\ e^{-t+1} - e^{-t-1} & 1 \leq t \end{cases}$ (e) $y(t) = \dfrac{4}{\sqrt{10}}u(t)\left(\tan^{-1}(3)e^{-t} - e^{-2t}\cos(3t + \tan^{-1}(3))\right)$

(f) $y(t) = \begin{cases} 0 & t \leq -2 \\ 0.5(t+2)^2 & -2 \leq t \leq 0 \\ 0.5(4+4t-t^2) & 0 \leq t \leq 2 \\ 4 & 2 \leq t \end{cases}$