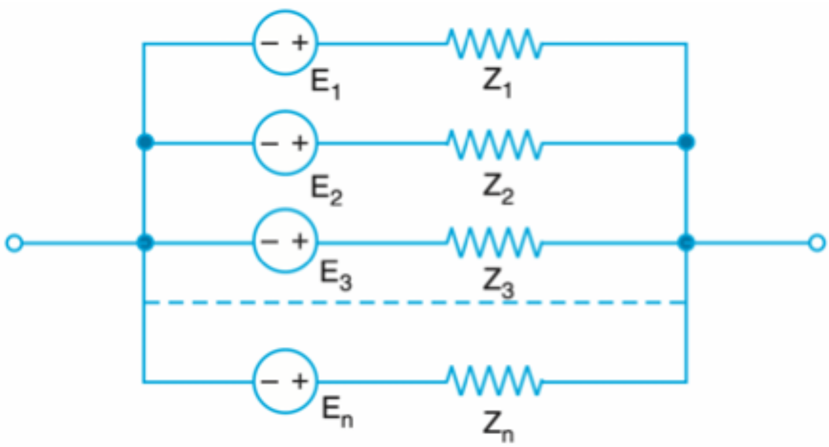
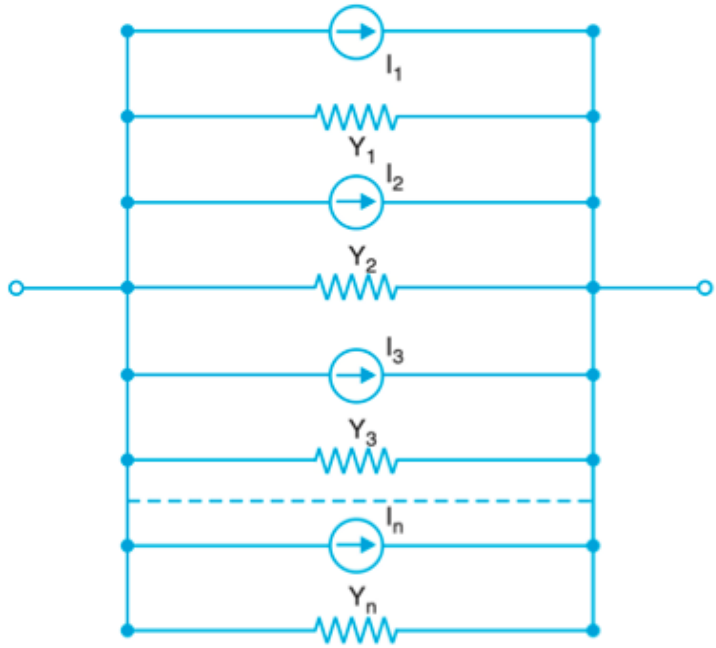
Millman's Theorem:

Part I: Any circuit containing multiple voltage sources, each one in series with its own resistance can be replaced by an equivalent voltage source in series with an equivalent resistance.

Let E_i $(i = 1, 2, \ldots, n)$ be the open-circuit voltages of n voltage sources having internal impedances Z_i in series, respectively, as shown in the figure.



Then, using Norton's theorem, we can have

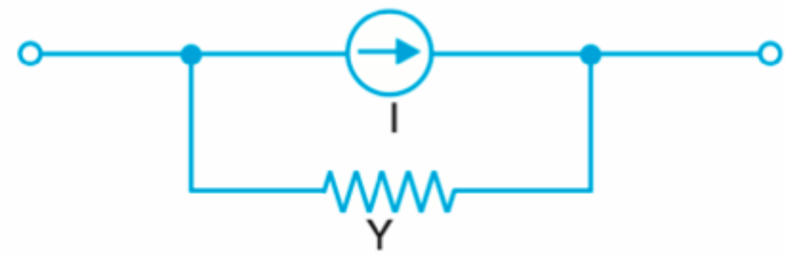


$$I_i = \frac{E_i}{Z_i}$$

and $$Y_i = \frac{1}{Z_i}$$

$$i = 1, 2, \ldots, n$$

Next, summing up all the current sources and the admittances,



$$I = \sum_{i=1}^{n} I_i$$

$$I = \frac{\sum_{i=1}^{n} E_i}{\sum_{i=1}^{n} Z_i} = \sum_{i=1}^{n} E_i Y_i$$

and $$Y = \sum_{i=1}^{n} Y_i = \frac{1}{\sum_{i=1}^{n} Z_i}$$

Finally, we can write the Thevenin's equivalent of the circuit,

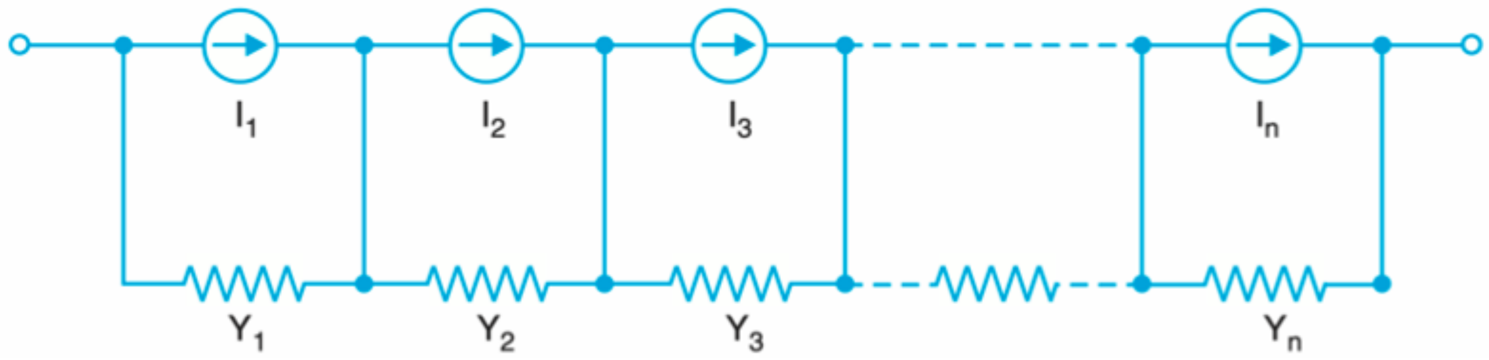
$$E = IZ$$

Hence, $$E = \frac{I}{Y} = \frac{\sum_{i=1}^{n} E_i Y_i}{\sum_{i=1}^{n} Y_i}$$

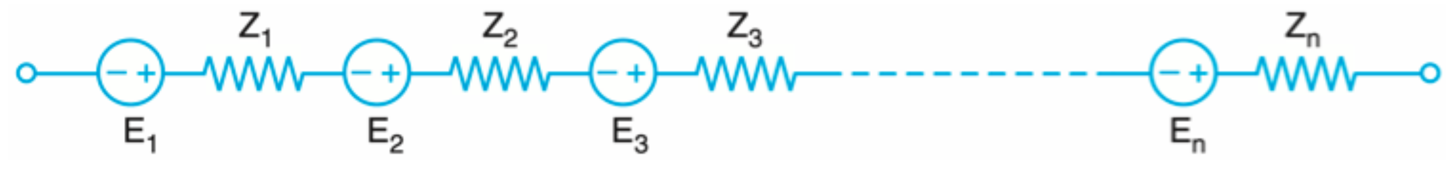
and $$Z = \frac{1}{Y} = \frac{1}{\sum_{i=1}^{n} Y_i}$$

Part II: Any circuit containing multiple current sources, each one in parallel with its own admittance can be replaced by an equivalent current source in parallel with an equivalent admittance.

Let I_i $(i = 1, 2, \ldots, n)$ be the n current sources having internal admittances Y_i in parallel as shown.



Then, using Thevenin's theorem, we can have



$$E_i = \frac{I_i}{Y_i} \quad \text{and} \quad Z_i = \frac{1}{Y_i}$$

$$i = 1, 2, \ldots, n$$

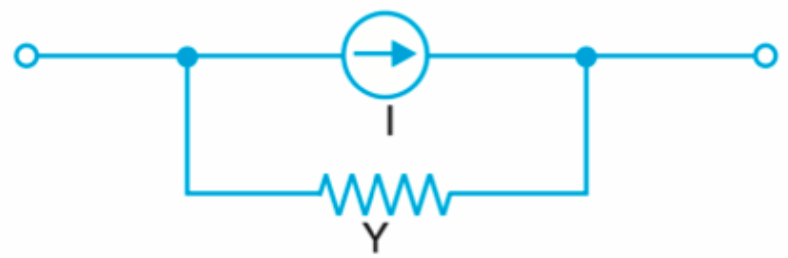
Next, summing up all the voltage sources and the impedances,

$$E = \sum_{i=1}^{n} E_i$$

$$E = \sum_{i=1}^{n} \frac{I_i}{Y_i} \quad \text{and} \quad Z = \sum_{i=1}^{n} Z_i = \sum_{i=1}^{n} \frac{1}{Y_i}$$

Finally, we can write the Norton's equivalent of the circuit,

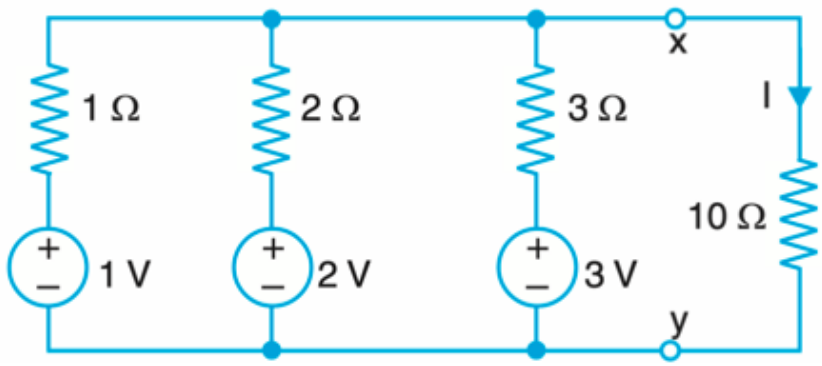
$$I = \frac{E}{Z}$$



$$I = \frac{\sum_{i=1}^{n} \frac{I_i}{Y_i}}{\sum_{i=1}^{n} \frac{1}{Y_i}} \quad \text{and} \quad Y = \frac{1}{Z} = \frac{1}{\sum_{i=1}^{n} \frac{1}{Y_i}}$$

Note: Millman's theorem is an extension of Thevenin's voltage equivalent and Norton's current-equivalent, taking a number of sources into account.

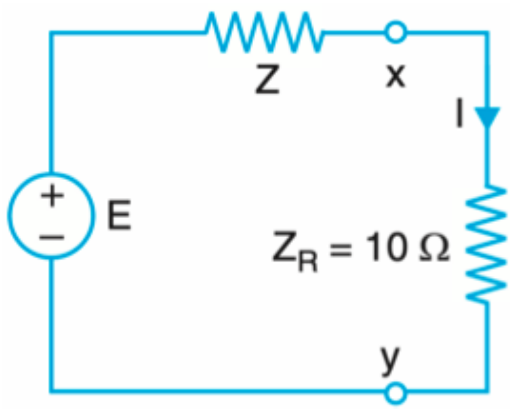
Q. Calculate the load current I in the circuit using Millman's theorem.



A. $$Z = \frac{1}{\frac{1}{Z_1} + \frac{1}{Z_2} + \frac{1}{Z_3}} = \frac{6}{11}\ \Omega$$

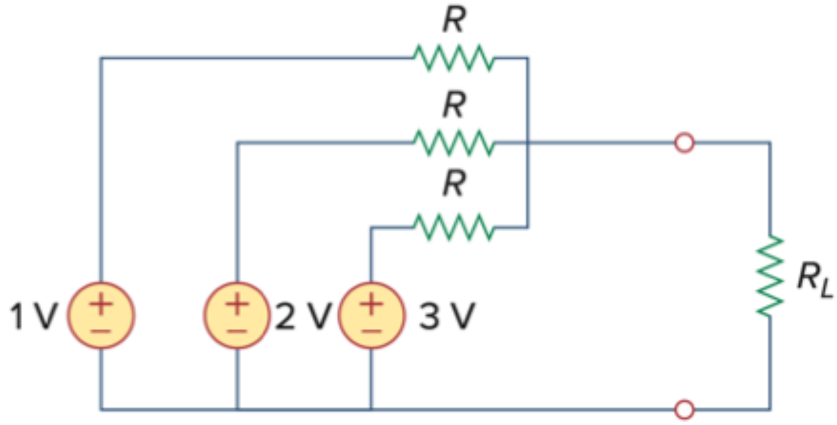
$$E = \frac{\frac{E_1}{Z_1} + \frac{E_2}{Z_2} + \frac{E_3}{Z_3}}{\frac{1}{Z_1} + \frac{1}{Z_2} + \frac{1}{Z_3}} = \frac{3}{\frac{11}{6}} = \frac{18}{11}\ V$$

Hence, from the Millman's equivalent circuit,



$$I = \frac{\frac{18}{11}}{\frac{6}{11} + 10} = \frac{9}{58}\ A$$

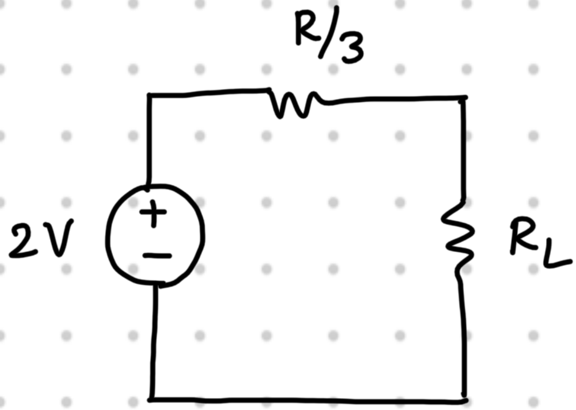
Q. Determine the value of R such that the maximum power delivered to the load R_L is 12mW.



A. Using Millman's theorem,

$$Z = \frac{1}{\frac{1}{Z_1} + \frac{1}{Z_2} + \frac{1}{Z_3}} = \frac{R}{3}$$

$$E = \frac{\frac{E_1}{Z_1} + \frac{E_2}{Z_2} + \frac{E_3}{Z_3}}{\frac{1}{Z_1} + \frac{1}{Z_2} + \frac{1}{Z_3}} = 2V$$

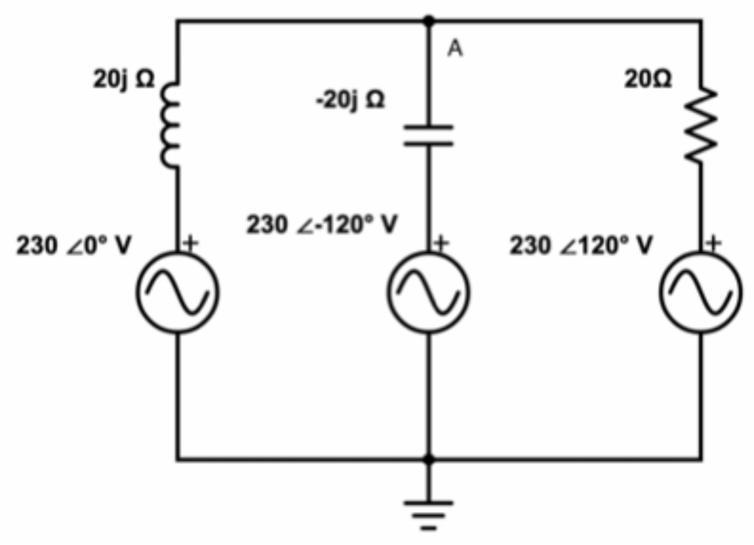


$P_{max} = 12\,mW$

$$\Rightarrow \frac{4}{4 \cdot \frac{R}{3}} = 12\,mW$$

$$\Rightarrow R = 250\,\Omega$$

Q. Using Millman's theorem, determine the voltage V_A of the network.



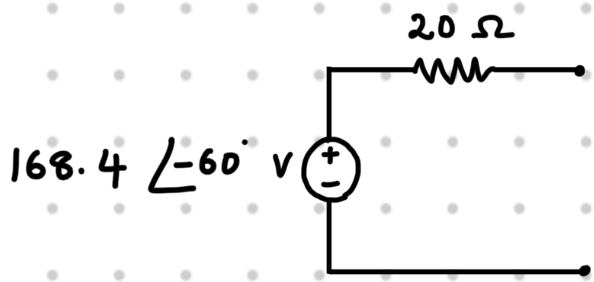
A. $Y = Y_1 + Y_2 + Y_3 = \frac{1}{20j} - \frac{1}{20j} + \frac{1}{20} = \frac{1}{20}$

$$Z = \frac{1}{Y} = 20\,\Omega$$

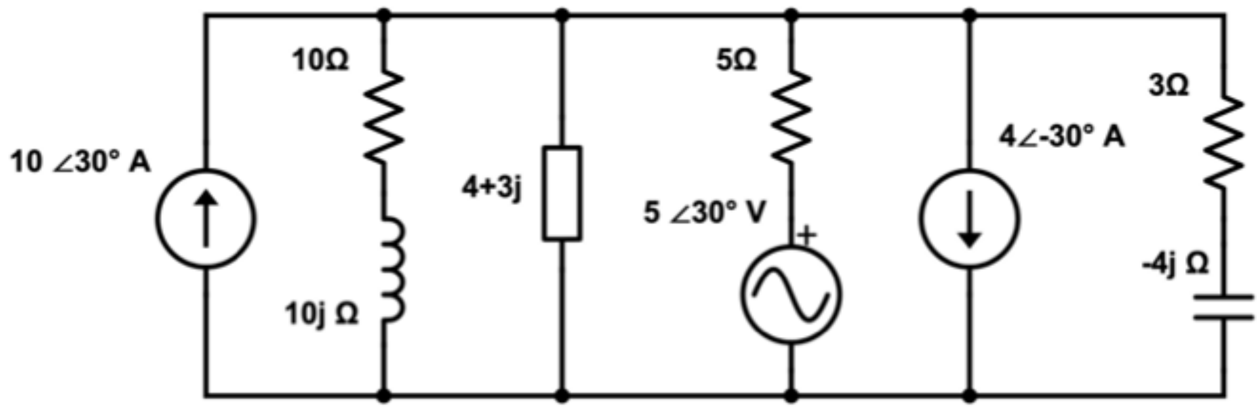
$$E = \frac{E_1 Y_1 + E_2 Y_2 + E_3 Y_3}{Y_1 + Y_2 + Y_3} = \frac{-11.5j + 11.5\angle -30^\circ + 11.5\angle 120}{0.05}$$

$$\Rightarrow E = \frac{4.2 - 7.3j}{0.05} = 168.4\angle -60^\circ\ V$$

Hence, Millman's equivalent is:



Q. Using Millman's theorem, determine the current flowing through the impedance 4+3j.

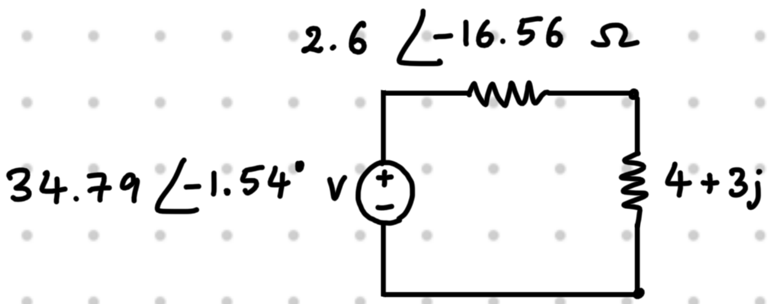


A. Replacing the current sources with voltage sources using source transformation,

$$Z = \frac{1}{\frac{1}{Z_1} + \frac{1}{Z_2} + \frac{1}{Z_3}} = \frac{1}{\frac{1}{10+10j} + \frac{1}{5} + \frac{1}{3-14j}}$$

$$\Rightarrow \quad Z = 2.6 \angle -16.56 \ \Omega$$

$$E = \frac{\frac{E_1}{Z_1} + \frac{E_2}{Z_2} + \frac{E_3}{Z_3}}{\frac{1}{Z_1} + \frac{1}{Z_2} + \frac{1}{Z_3}} = \frac{13.432 \angle 15^\circ}{0.386 \angle 16.56^\circ} = 34.79 \angle -1.54^\circ$$



Hence,

$$I = 5.061 \angle -20.72^\circ \ A$$