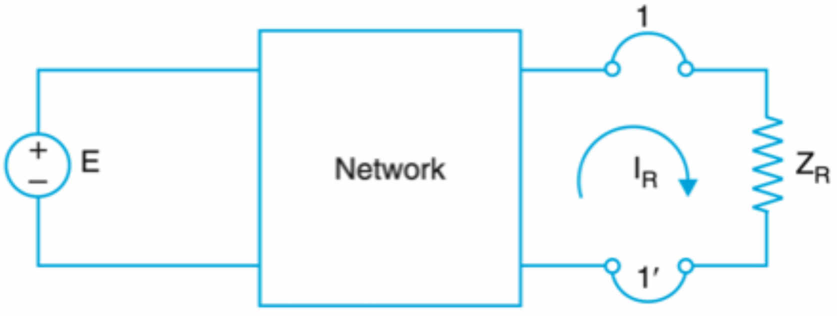
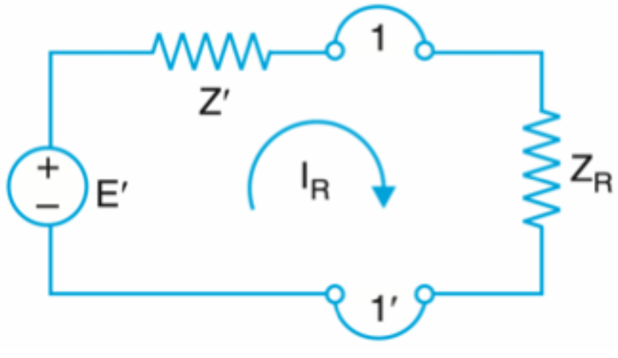
Thevenin's Theorem:

Any two terminal linear network containing energy sources and impedances can be replaced with an equivalent circuit consisting of a voltage source E' in series with an impedance Z'.

The value of E' is the open-circuit voltage between the terminals of the network and Z' is the impedance measured between the terminals of the network with all energy sources eliminated (but not their impedances).

That is, the following two terminal linear network can be replaced with a simpler equivalent of E' and Z'.

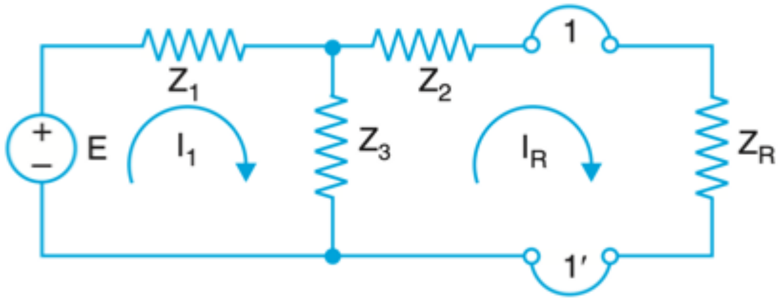




Thevenin's theorem is particularly useful when we are interested in the solution of current or voltage of a small part of the network.

Illustration of Thevenin's theorem :

Suppose we have the following circuit :

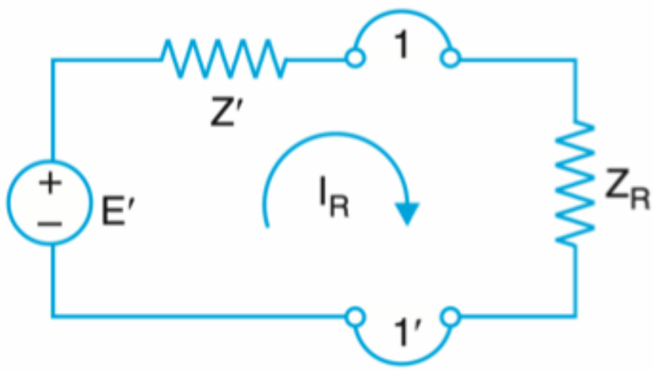


Using the mesh equations,

$$\begin{bmatrix} z_1 + z_3 & -z_3 \\ -z_3 & z_2 + z_3 + z_R \end{bmatrix} \begin{bmatrix} I_1 \\ I_R \end{bmatrix} = \begin{bmatrix} E \\ 0 \end{bmatrix}$$

$$\Rightarrow \quad I_R = \frac{E\, z_3}{z_2(z_1+z_3) + z_1 z_3 + (z_1+z_3) z_R} = \frac{E\left(\frac{z_3}{z_1+z_3}\right)}{z_2 + \frac{z_1 z_3}{z_1+z_3} + z_R}$$

Now, using Thevenin's theorem, we can replace the above network with :



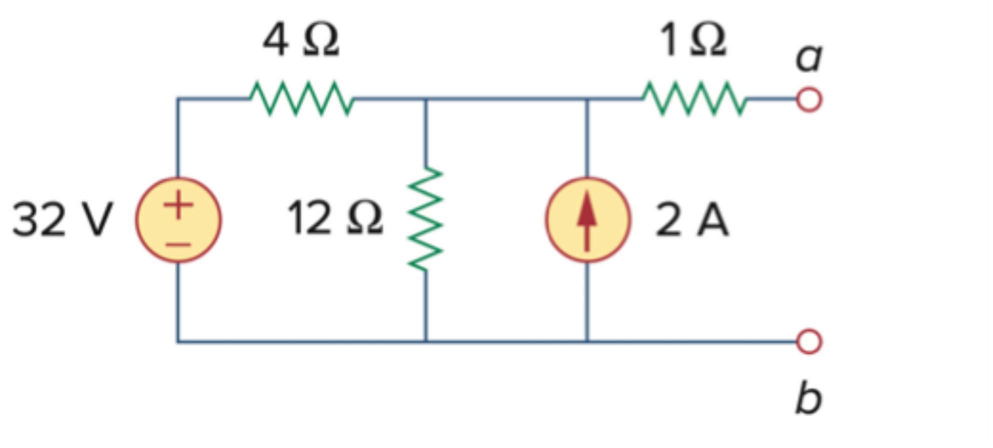
where $E' = \dfrac{E\, z_3}{z_1 + z_3}$

and $z' = z_2 + \dfrac{z_1 z_3}{z_1 + z_3}$

$$\Rightarrow \quad I_R = \frac{E'}{z' + z_R} = \frac{E\left(\frac{z_3}{z_1+z_3}\right)}{z_2 + \frac{z_1 z_3}{z_1+z_3} + z_R}$$

Which is same as calculated earlier.

Q. For the given circuit, find the Thevenin voltage (V_{TH}) and Thevenin impedance (Z_{TH}):



A. The open circuit voltage V_{ab} can be computed using superposition theorem.

$$V_{TH} = V_{ab} = \frac{12}{16} \cdot 32 + \frac{12(4)}{12+4} \cdot 2 = \underline{\underline{30\ V}}$$

The input impedance :

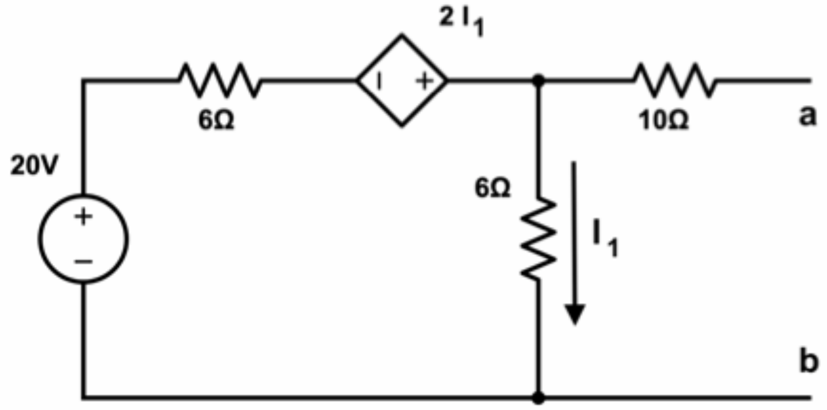
$$Z_{TH} = (4 \parallel 12) + 1 = \underline{\underline{4\Omega}}$$

We can also calculate Z_{TH} using:

$$I_{sc} = I_{ab} = \frac{32}{4 + \frac{12}{13}} \times \frac{12}{13} + \frac{3}{4} \times 2 = \frac{15}{2}\ A$$

Hence, $$Z_{TH} = \frac{V_{TH}}{I_{sc}} = \frac{30}{\frac{15}{2}} = \underline{\underline{4\Omega}}$$

Q. For the given circuit, find the Thevenin equivalent across the terminals a-b.



A. Thevenin voltage across AB :

$$V_{TH} = 6 I_1 = 6\left(\frac{20}{10}\right) = 12\text{ V}$$

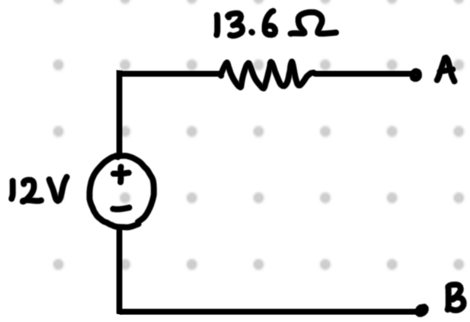
Short circuit current at terminals AB :

writing KVL equations :

$$\left.\begin{aligned} 12 I_x - 2I_1 - 6I_y - 20 &= 0 \\ -6I_x + 16 I_y &= 0 \end{aligned}\right\} \Rightarrow \begin{aligned} I_x &= 2.353\text{ A} \\ I_y = I_{sc} &= 0.882\text{ A} \end{aligned}$$

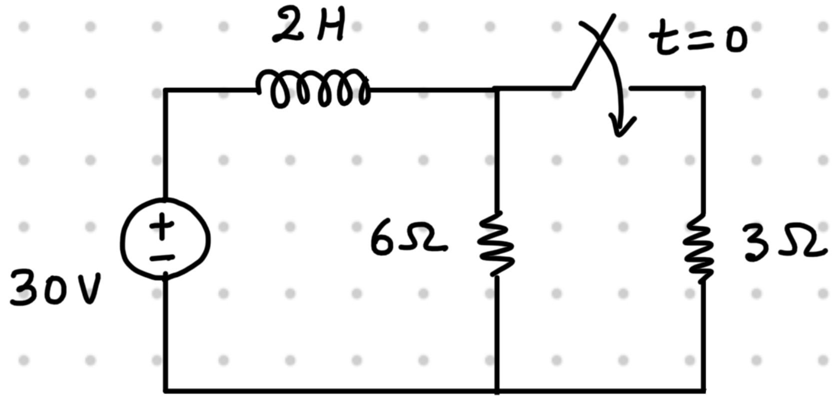
Thevenin's resistance : $R_{TH} = \dfrac{V_{TH}}{I_{sc}} = 13.6\ \Omega$

Hence, Thevenin's equivalent is :



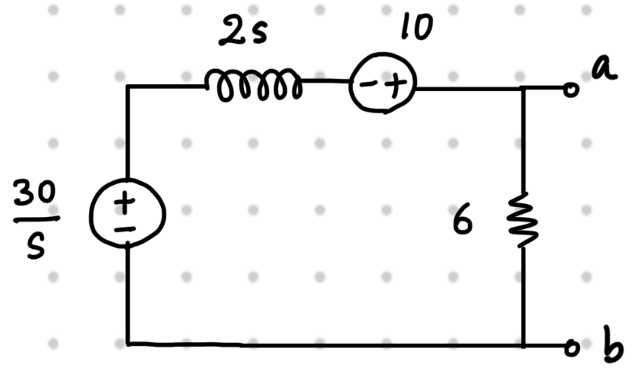
Q. In the circuit below, the switch is open for a long time and closed at $t=0$.

Find the current flowing through the $3\,\Omega$ resistor.



(use Thevenin's equivalent approach)

A. $i_L(0^-) = 5\text{A}$

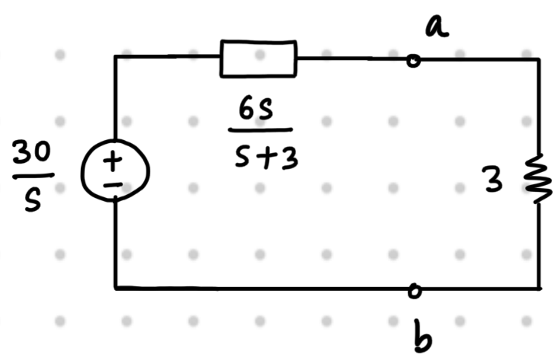


$V_{TH} = V_{ab} =$

$$6\,\frac{\frac{30}{s}+10}{6+2s} = \frac{30}{s}$$

$$I_{sc} = \frac{\frac{30}{s}+10}{2s} = \frac{5(s+3)}{s^2} \Rightarrow Z_{TH} = \frac{V_{TH}}{I_{sc}} = \frac{6s}{s+3}$$

Hence, $I_{3\Omega} = \dfrac{\frac{30}{s}}{\frac{6s}{s+3}+3}$



$$= \frac{10}{s} - \frac{20}{3}\cdot\frac{1}{s+1}$$

$$\Rightarrow \quad i_{3\Omega}(t) = \left(10 - \frac{20}{3}e^{-t}\right)u(t)$$