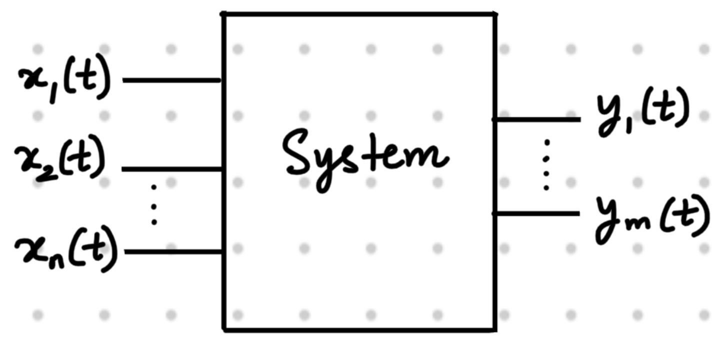
Systems

A system may consist of physical components (hardware) or simply an algorithm (software) that processes input signals (cause) in order to modify or extract information in the form of output signals (effect).

A system is characterized by its inputs, outputs, and its mathematical model (equations relating the outputs to the inputs) as shown below.



For a system with given inputs and given mathematical model, we are interested in:

1. Determining the system outputs (analysis)
2. Constructing a system which will produce a desired set of outputs for the given inputs (design or synthesis).

System Classification: Systems can be classified as

1. Linear and Nonlinear systems: A system is said to be linear if it obeys the property of superposition (additivity and homogeneity).

That is, if $x_1 \rightarrow y_1$ and $x_2 \rightarrow y_2$

then, $k_1 x_1 + k_2 x_2 \longrightarrow k_1 y_1 + k_2 y_2$

(for all constants k_1 and k_2)

Systems not obeying this property of superposition are nonlinear systems.

Note: A linear system's response can be expressed as the sum of a zero-input and a zero-state component.

$$\text{Total Response} = \underset{\text{response}}{\text{Zero-input}} + \underset{\text{response}}{\text{zero-state}}$$

can be computed by assuming the input to be zero

can be computed by assuming zero initial conditions

Note: If we express an input $f(t)$ as a sum of simpler functions, then we can determine a linear system's response by adding the system response to each of the simpler functions.

2. Time-Invariant and Time-Variant systems:

A system is time-invariant if its parameters do not change with time. That is, if the input is delayed by T seconds, then the output is the same as before but delayed by T.

In other words, if $x(t) \rightarrow y(t)$

then $x(t-\tau) \rightarrow y(t-\tau)$

A system is time-variant if it does not obey this property.

3. Instantaneous and Dynamic systems:

If the output of a system at any time t depends only on the input at the instant t, then the system is said to be instantaneous (or memoryless).

On the other hand, if the output depends on any past or future values of the input, then the system is said to be dynamic (or system with memory).

4. Causal and Noncausal systems: A system is causal if its output at any instant t_0 depends only on the input values for $t \leq t_0$. That is, only on the past and present input values, not on its future values.

A system that violates this condition of causality is called a noncausal (or anticipative) system. Note that noncausal systems are unrealizable in real time.

5. Continuous-time and Discrete-time systems:

Systems whose inputs and outputs are continuous-time signals are continuous-time systems. On the other hand, systems whose inputs and outputs are discrete-time are discrete-time systems.

Note: We can process a continuous-time signal by processing its samples with a discrete-time system.

6. Analog and Digital systems: Systems whose inputs and outputs are analog signals are analog systems, and systems with digital input and output signals are digital systems.

7. Invertible and Non invertible systems :

A system is invertible if we can obtain the input $x(t)$ back from the output $y(t)$ by some operation.

A system is non invertible if it is impossible to determine the input for a given input.

8. BIBO - stable and non BIBO-stable systems:

A system is BIBO stable if any bounded input produces bounded output otherwise it's not BIBO - stable.

That is, if $|x(t)| < \infty \Rightarrow |y(t)| < \infty$

Q. For the systems described by the equations below, with the input $x(t)$ and output $y(t)$, determine which of the systems are linear :

(a). $$\frac{dy}{dt} + 3t\,y(t) = t^2 x(t)$$

(b). $$\frac{dy}{dt} + 2\,y(t) = x^2(t)$$

(c). $$y(t) = \int_{-\infty}^{t} x(\tau)\,d\tau$$

(d). $$\frac{dy}{dt} + 2\,y(t) = x(t)\,\frac{dx}{dt}$$

A. (a). Suppose $x_1(t)$ produces $y_1(t)$ and $x_2(t)$ produces $y_2(t)$.

That is, $$\frac{dy_1}{dt} + 3t\,y_1(t) = t^2 x_1(t)$$

and $$\frac{dy_2}{dt} + 3t\,y_2(t) = t^2 x_2(t)$$

For linearity, both homogeneity and additivity properties have to be satisfied.

Check if $k_1 x_1 + k_2 x_2$ produces $k_1 y_1 + k_2 y_2$:

$$\Rightarrow \quad k_1\left(\frac{dy_1}{dt} + 3t\, y_1(t)\right) + k_2\left(\frac{dy_2}{dt} + 3t\, y_2(t)\right)$$
$$= k_1\, t^2 x_1(t) + k_2\, t^2 x_2(t)$$

$$\Rightarrow \quad \frac{d}{dt}(k_1 y_1 + k_2 y_2) + 3t\,(k_1 y_1 + k_2 y_2)$$
$$= t^2\,(k_1 x_1(t) + k_2 x_2(t))$$

Hence, an input of $k_1 x_1 + k_2 x_2$ produced an output of $k_1 y_1 + k_2 y_2$. Therefore, the system is linear.

(b). Suppose $x_1(t)$ produces $y_1(t)$

That is, $\dfrac{dy_1}{dt} + 2\, y_1(t) = x_1^2(t)$

Check if $k_1 x_1$ produces $k_1 y_1$:

$$\Rightarrow \quad k_1^2\left(\frac{dy_1}{dt} + 2\, y_1(t)\right) = k_1^2\, x_1^2(t)$$

$$\Rightarrow \quad \frac{d}{dt}(k_1^2 y_1) + 2\,(k_1^2 y_1(t)) = (k_1 x_1(t))^2$$

Hence, $k_1 x_1$ produces $k_1^2 y_1$ instead of $k_1 y_1$.

Therefore, this system is not linear.

(c). Suppose $x_1(t)$ produces $y_1(t)$ and

$x_2(t)$ produces $y_2(t)$.

That is,

$$y_1(t) = \int_{-\infty}^{t} x_1(\tau)\,d\tau \quad \text{and} \quad y_2(t) = \int_{-\infty}^{t} x_2(\tau)\,d\tau$$

Then,

$$k_1 y_1(t) + k_2 y_2(t) = k_1 \int_{-\infty}^{t} x_1(\tau)\,d\tau + k_2 \int_{-\infty}^{t} x_2(\tau)\,d\tau$$

$$= \int_{-\infty}^{t} \Big(k_1 x_1(\tau) + k_2 x_2(\tau)\Big)\,d\tau$$

Hence, the system is linear.

(d). Suppose $x_1(t)$ produces $y_1(t)$

That is, $\dfrac{dy_1}{dt} + 2\,y_1(t) = x_1(t)\,\dfrac{dx_1}{dt}$

$$\Rightarrow \quad k_1^2\left(\frac{dy_1}{dt} + 2\,y_1(t)\right) = k_1^2\,x_1(t)\,\frac{dx_1}{dt}$$

$$\Rightarrow \quad \frac{d}{dt}\left(k_1^2 y_1\right) + 2\left(k_1^2 y_1(t)\right) = k_1 x_1(t)\cdot\frac{d(k_1 x_1)}{dt}$$

Hence, $k_1 x_1$ produces $k_1^2 y_1$ instead of $k_1 y_1$.

Therefore, this system is not linear.