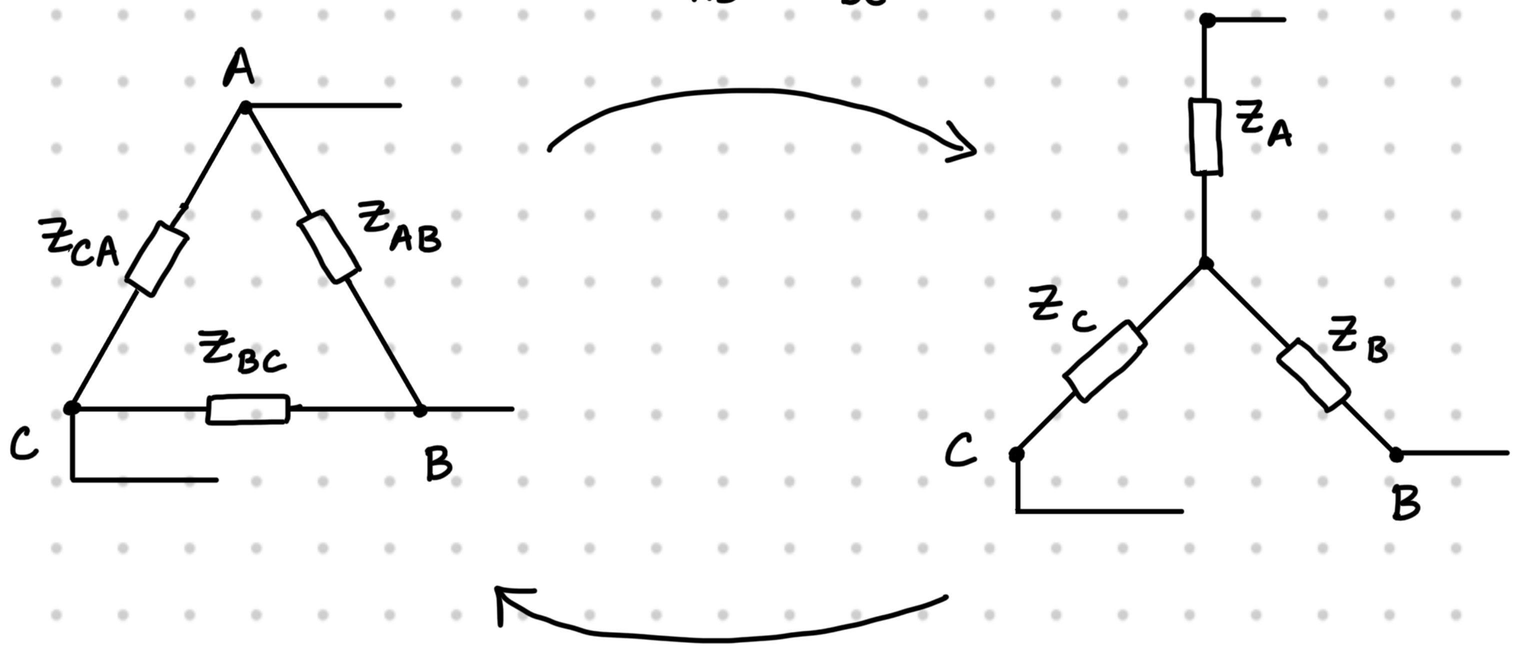
Delta - Star Transformation

Star / Delta connections are arrangements of passive elements (R, L, C) such that the formed shapes are neither connected in series nor parallel.

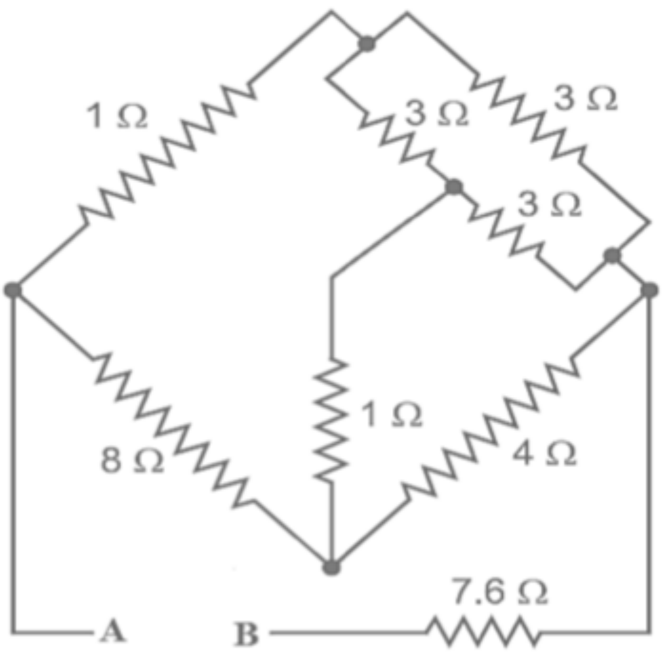
Delta-to-star and Star-to-Delta transformations help simplify the circuit for network analysis.

$$Z_A = \frac{Z_{AB}\, Z_{CA}}{Z_{AB} + Z_{BC} + Z_{CA}}$$

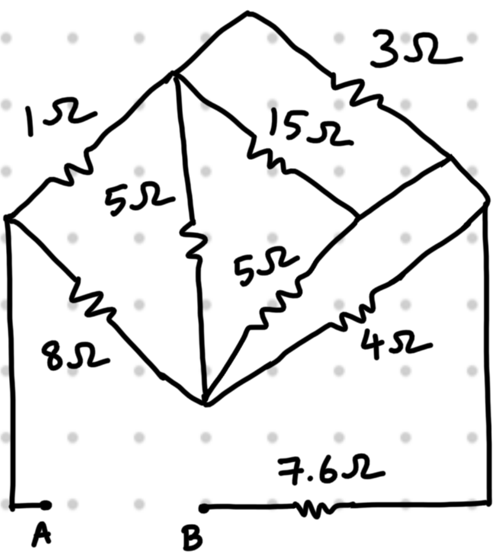


$$Z_{AB} = Z_A + Z_B + \frac{Z_A Z_B}{Z_C}$$

Q. What is the equivalent resistance across the terminals A and B ?



A. Using Star-Delta transformations,



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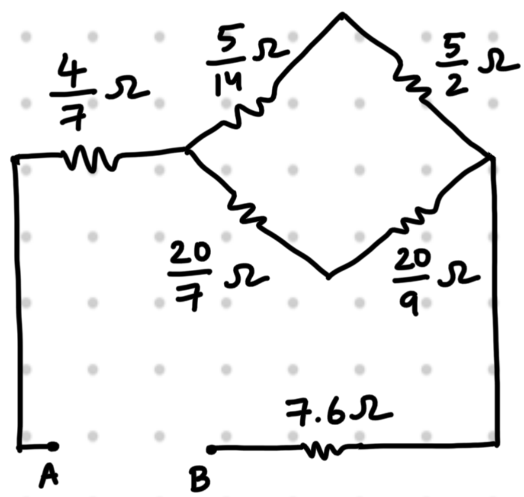
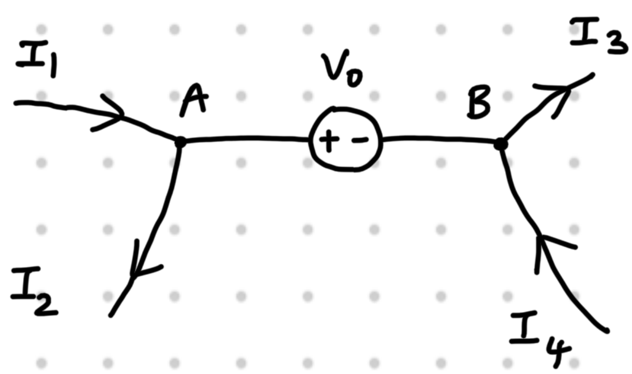


Hence, $R_{AB} = 2.4 + 7.6 = 10\,\Omega$

Supernode

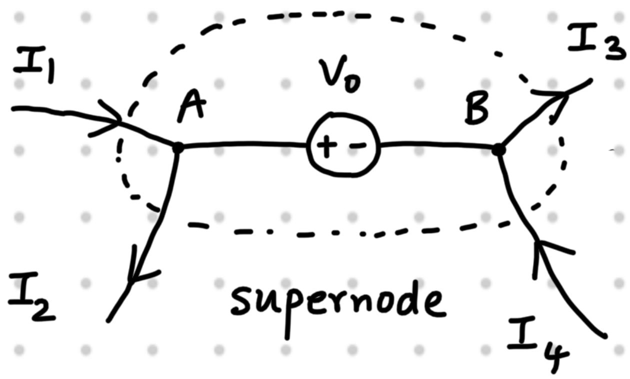
If we have floating voltage sources (not connected to the reference node), we cannot use Ohm's law to represent the current through the source. To circumvent this problem, we create a Supernode.

Example: Suppose we have a voltage source between the nodes A and B as shown.



Clearly, we cannot write an Ohm's law equation for the current through the source.

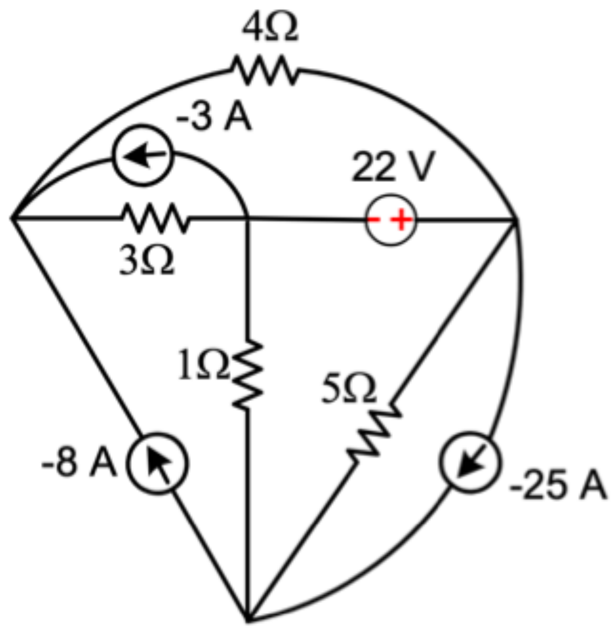
To solve this problem, we form a supernode and write one KCL equation for the nodes A & B.



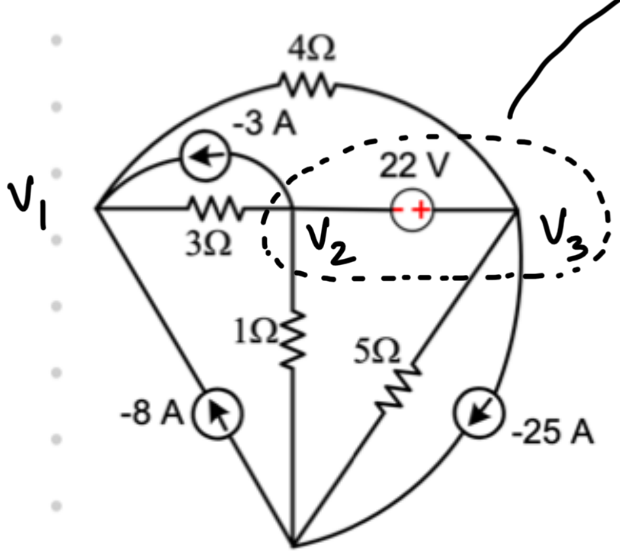
That is, $I_1 + I_4 - I_2 - I_3 = 0$

Hence, this is simply a shortcut of writing one KCL equation instead of two KCL equations.

Q. Find the node voltages for the circuit shown.



A. There is a DC voltage source between two non-reference nodes. As a result, we cannot write an equation for the current through the source. To circumvent this problem, we create a Supernode.



Now, $V_3 = V_2 + 22$

and

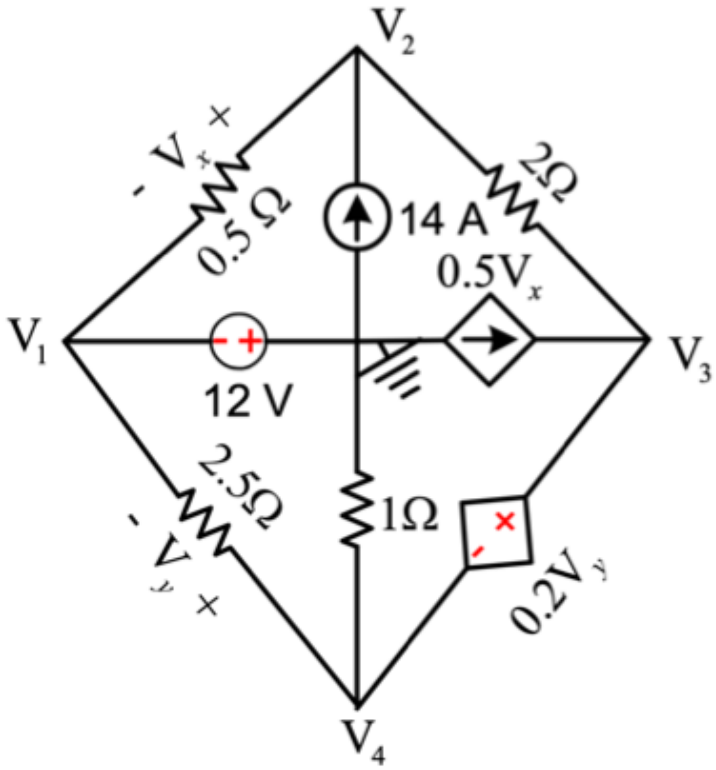
$$\frac{V_1 - V_3}{4} + \frac{V_1 - V_2}{3} = -11$$

$$= -3 - 25 + \frac{V_2}{1} + \frac{V_3}{5}$$

Solving the equations, $V_1 \simeq 1.1\,V$

$V_2 \simeq 10.5\,V$ and $V_3 \simeq 32.5\,V$

Q. Find the node voltages for the circuit shown.



A. Clearly, $V_1 = -12V$

and $V_3 - V_4 = 0.2\left(V_4 - V_1\right)$

KCL for node 2: $\dfrac{V_2 - V_1}{0.5} + \dfrac{V_2 - V_3}{2} - 14 = 0$

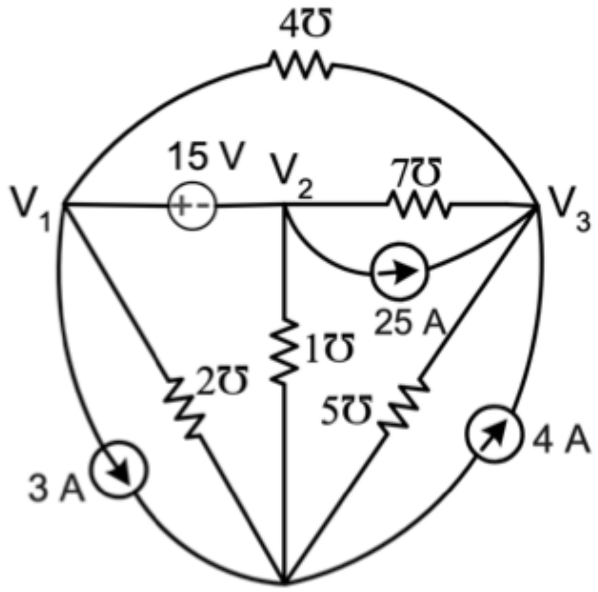
KCL for supernodes 3 & 4:

$$\frac{V_3 - V_2}{2} + \frac{V_4 - V_1}{2.5} + \frac{V_4}{1} - 0.5\left(V_2 - V_1\right) = 0$$

Solving the equations,

$V_2 = -4V$, $V_3 = 0V$ and $V_4 = -2V$

Q. Find the node voltages for the circuit shown.



A. $V_1 = V_2 + 15\,V$

KCL for node 3 :

$$4V_1 + 7V_2 - 16V_3 = -29$$

KCL for supernodes 1 & 2 :

$$6V_1 + 8V_2 - 11V_3 = -28$$

Hence,

$$\begin{bmatrix} 1 & -1 & 0 \\ 4 & 7 & -16 \\ 6 & 8 & -11 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix} = \begin{bmatrix} 15 \\ -29 \\ -28 \end{bmatrix}$$

$\Rightarrow$ $V_1 \approx 6.17\,V$, $V_2 \approx -8.82\,V$ and

$$V_3 \approx -0.5\,V$$

Summary of Nodal Analysis :

Nodal Analysis is a systematic application of KCL, where we generate a system of equations with node voltages as the unknown variables.

The number of independent equations equals the number of unknown node voltages.

We determine the node voltages by solving the matrix $[Y][E] = [I]$ and use Ohm's law to find branch currents.

Step-by-step Procedure

Step 1 : Simplify the circuit as much as possible using transformations.

Step 2 : Identify and label all node voltages.

Step 3 : Assign and label polarities of currents.

Step 4 : Apply KCL at each node.

Step 5 : Solve the Matrix equation(s).