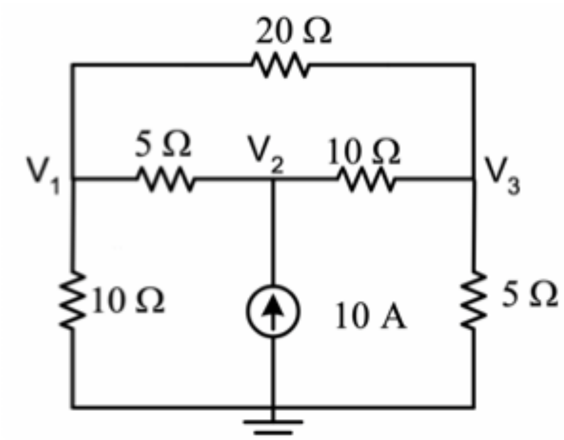
Q. Using nodal analysis, find the voltage V_1, V_2 and V_3 of the circuit shown below.



A. Using KCL,

$$\begin{bmatrix} 0.35 & -0.2 & -0.05 \\ -0.2 & 0.3 & -0.1 \\ -0.05 & -0.1 & 0.35 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 10 \\ 0 \end{bmatrix}$$

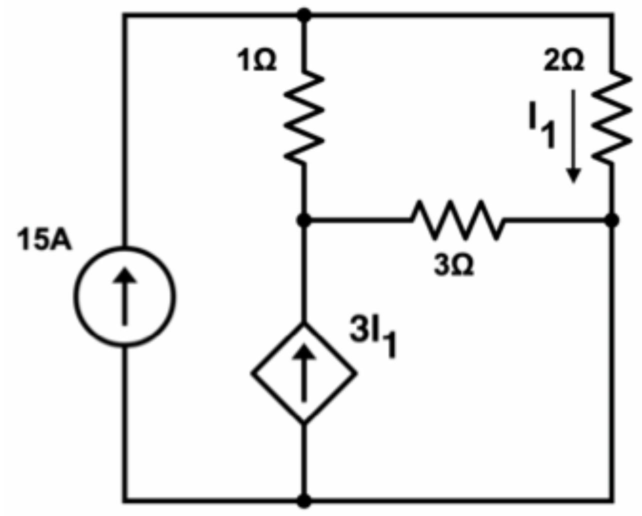
Hence,

$$V_1 = \frac{\Delta_1}{\Delta} = \frac{0.75}{0.0165} = 45.45\,V$$

$$V_2 = \frac{\Delta_2}{\Delta} = \frac{1.2}{0.0165} = 72.73\,V$$

$$V_3 = \frac{\Delta_3}{\Delta} = \frac{0.45}{0.0165} = 27.27\,V$$

Q. Using nodal analysis, find the power delivered by the dependent source.



A. Writing the KCL equations as matrices,

$$\begin{bmatrix} 1.5 & -1 \\ -1 & \frac{4}{3} \end{bmatrix} \begin{bmatrix} E_1 \\ E_2 \end{bmatrix} = \begin{bmatrix} 15 \\ \underbrace{3I_1}_{\frac{3E_1}{2}} \end{bmatrix}$$

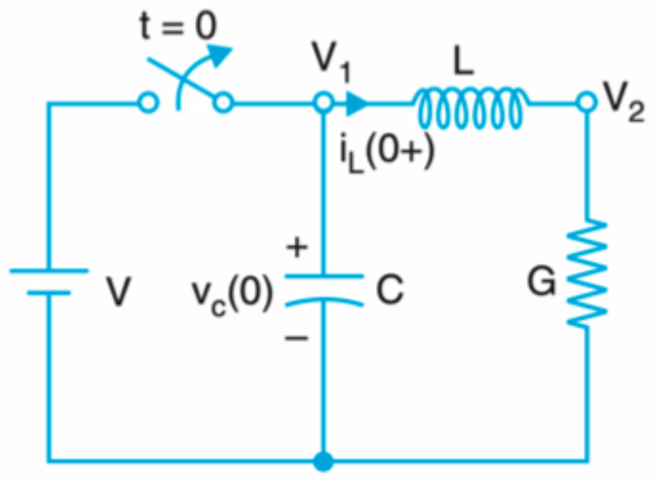
$$\Rightarrow \begin{bmatrix} 1.5 & -1 \\ -2.5 & \frac{4}{3} \end{bmatrix} \begin{bmatrix} E_1 \\ E_2 \end{bmatrix} = \begin{bmatrix} 15 \\ 0 \end{bmatrix}$$

$$\Rightarrow E_1 = \frac{\Delta_1}{\Delta} = -40\,V, \quad E_2 = \frac{\Delta_2}{\Delta} = -75\,V$$

$$\Rightarrow I_1 = \frac{E_1}{2} = -20\,A$$

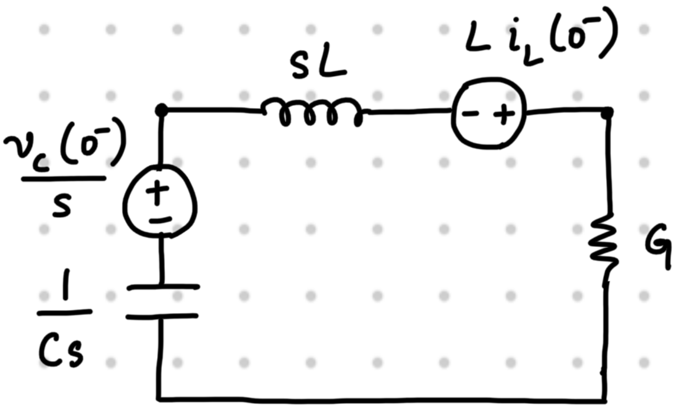
Therefore, $P_{3i_1} = (-75)(3)(-20) = 4.5\,kW$

Q. In the circuit shown below, the switch is opened at time $t = 0$.

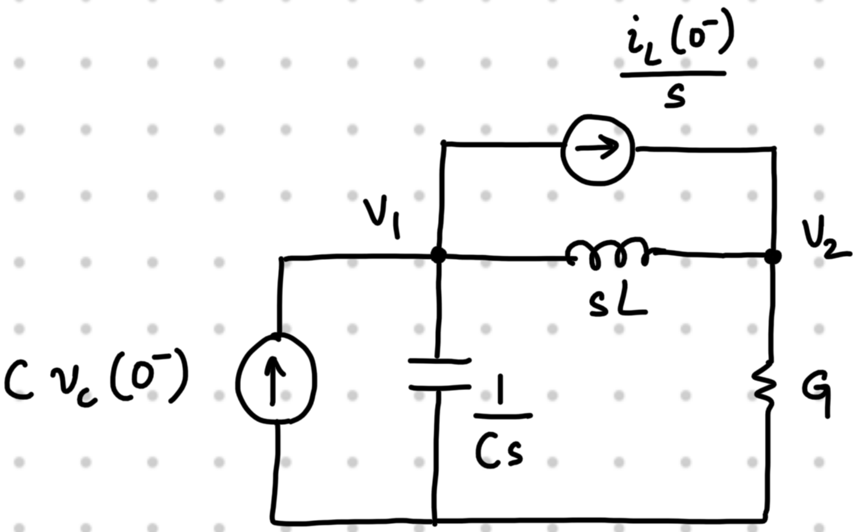


Given $i_L(0^-) = 1\,A$, $v_c(0^-) = 1\,V$, $L = 0.5\,H$, $C = 1\,F$, $G = 1$ ℧ and $V = 1\,V$, find the node voltages V_1 and V_2.

A. Transforming the network into Laplace domain :



Using source transformation,



Now, $$\begin{bmatrix} sC + \frac{1}{sL} & -\frac{1}{sL} \\ -\frac{1}{sL} & G + \frac{1}{sL} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} Cv_c(0^-) - \frac{i_L(0^-)}{s} \\ \frac{i_L(0^-)}{s} \end{bmatrix}$$

Using Cramer's rule,

$$V_1 = \frac{\begin{vmatrix} Cv_c(0^-) - \frac{i_L(0^-)}{s} & -\frac{1}{sL} \\ \frac{i_L(0^-)}{s} & G + \frac{1}{sL} \end{vmatrix}}{\begin{vmatrix} sC + \frac{1}{sL} & \frac{-1}{sL} \\ \frac{-1}{sL} & G + \frac{1}{sL} \end{vmatrix}}$$

$$V_2 = \frac{\begin{vmatrix} sC + \frac{1}{sL} & Cv_c(0^-) - \frac{i_L(0^-)}{s} \\ -\frac{1}{sL} & \frac{i_L(0^-)}{s} \end{vmatrix}}{\begin{vmatrix} sC + \frac{1}{sL} & \frac{-1}{sL} \\ \frac{-1}{sL} & G + \frac{1}{sL} \end{vmatrix}}$$

Therefore, $v_1(t) = \mathcal{L}^{-1}\left(\frac{s+1}{1+(s+1)^2}\right) = e^{-t}\cos t$

$$v_2(t) = \mathcal{L}^{-1}\left(\frac{s+2}{1+(s+1)^2}\right) = e^{-t}(\cos t + \sin t)$$