

# Formulae

## · Trigonometric Identities:

$$e^{\pm jx} = \cos x \pm j \sin x, \quad \sin(2x) = 2 \sin x \cos x, \quad \cos(2x) = 2 \cos^2 x - 1$$

$$\sin(x \pm y) = \sin x \cos y \pm \cos x \sin y, \quad \cos(x \pm y) = \cos x \cos y \mp \sin x \sin y$$

## · Calculus:

$$(u(x)v(x))' = u(x)v'(x) + u'(x)v(x), \quad \int u(x)v'(x)dx = u(x)v(x) - \int u'(x)v(x)dx$$

## · Convolution:

$$x(t) * h(t) = \int_{-\infty}^{\infty} x(\tau)h(t-\tau)d\tau = \int_{-\infty}^{\infty} x(t-\tau)h(\tau)d\tau$$

## · Laplace Transform:

$$X(s) = \int_{-\infty}^{\infty} x(t)e^{-st}dt, \quad x(t) = \frac{1}{2\pi j} \int_{c-j\infty}^{c+j\infty} X(s)e^{st}ds$$

$$\delta(t) \iff 1, \quad u(t) \iff \frac{1}{s}, \quad t^n u(t) \iff \frac{n!}{s^{n+1}}, \quad e^{at}u(t) \iff \frac{1}{s-a}, \quad t^n e^{at}u(t) \iff \frac{n!}{(s-a)^{n+1}}$$

$$(\cos bt)u(t) \iff \frac{s}{s^2+b^2}, \quad (\sin bt)u(t) \iff \frac{b}{s^2+b^2}, \quad (e^{-at}\cos bt)u(t) \iff \frac{s+a}{(s+a)^2+b^2}$$

$$(e^{-at}\sin bt)u(t) \iff \frac{b}{(s+a)^2+b^2}, \quad (re^{-at}\cos(bt+\theta))u(t) \iff \frac{(r\cos\theta)s + (a\cos\theta - br\sin\theta)}{s^2+2as+(a^2+b^2)}$$

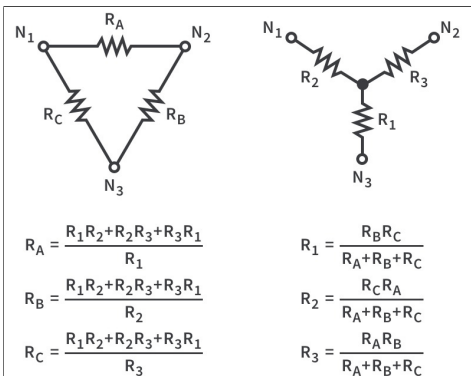
## · Laplace Transform Properties:

$$x(t) \iff X(s), \quad x(t-t_0)u(t-t_0) \iff e^{-st_0}X(s), \quad x(t)e^{s_0 t} \iff X(s-s_0), \quad x(at) \iff \frac{1}{a}X\left(\frac{s}{a}\right)$$

$$x_1(t) * x_2(t) \iff X_1(s) \cdot X_2(s), \quad x_1(t) \cdot x_2(t) = \frac{1}{2\pi j} X_1(s) * X_2(s), \quad -tx(t) \iff \frac{dX(s)}{ds}$$

$$\frac{d^n x}{dt^n} \iff s^n X(s) - \sum_{k=1}^n s^{n-k} x^{(k-1)}(0^-), \quad \int_{-\infty}^t x(\tau)d\tau \iff \frac{X(s)}{s} + \frac{1}{s} \int_{-\infty}^{0^-} x(t)dt, \quad \frac{x(t)}{t} \iff \int_s^{\infty} X(z)dz$$

$$\text{IVT: } x(0^+) = \lim_{s \rightarrow \infty} sX(s) \quad (n > m), \quad \text{FVT: } x(\infty) = \lim_{s \rightarrow 0} sX(s) \quad (\text{poles of } X(s) \text{ in LHP})$$



	<b>z</b>	<b>y</b>	<b>T</b>	<b>h</b>
<b>z</b>	$\begin{bmatrix} \mathbf{z}_{11} & \mathbf{z}_{12} \\ \mathbf{z}_{21} & \mathbf{z}_{22} \end{bmatrix}$	$\begin{bmatrix} \frac{\mathbf{y}_{22}}{\Delta \mathbf{y}} & \frac{-\mathbf{y}_{12}}{\Delta \mathbf{y}} \\ \frac{-\mathbf{y}_{21}}{\Delta \mathbf{y}} & \frac{\mathbf{y}_{11}}{\Delta \mathbf{y}} \end{bmatrix}$	$\begin{bmatrix} \frac{\mathbf{A}}{\mathbf{C}} & \frac{\Delta \mathbf{T}}{\mathbf{C}} \\ \frac{1}{\mathbf{C}} & \frac{\mathbf{D}}{\mathbf{C}} \end{bmatrix}$	$\begin{bmatrix} \frac{\Delta \mathbf{h}}{\mathbf{h}_{22}} & \frac{\mathbf{h}_{12}}{\mathbf{h}_{22}} \\ \frac{-\mathbf{h}_{21}}{\mathbf{h}_{22}} & \frac{1}{\mathbf{h}_{22}} \end{bmatrix}$
<b>y</b>	$\begin{bmatrix} \frac{\mathbf{z}_{22}}{\Delta \mathbf{z}} & \frac{-\mathbf{z}_{12}}{\Delta \mathbf{z}} \\ \frac{-\mathbf{z}_{21}}{\Delta \mathbf{z}} & \frac{\mathbf{z}_{11}}{\Delta \mathbf{z}} \end{bmatrix}$	$\begin{bmatrix} \mathbf{y}_{11} & \mathbf{y}_{12} \\ \mathbf{y}_{21} & \mathbf{y}_{22} \end{bmatrix}$	$\begin{bmatrix} \frac{\mathbf{D}}{\mathbf{B}} & \frac{-\Delta \mathbf{T}}{\mathbf{B}} \\ \frac{-1}{\mathbf{B}} & \frac{\mathbf{A}}{\mathbf{B}} \end{bmatrix}$	$\begin{bmatrix} \frac{1}{\mathbf{h}_{11}} & \frac{-\mathbf{h}_{22}}{\mathbf{h}_{11}} \\ \frac{\mathbf{h}_{21}}{\mathbf{h}_{11}} & \frac{\Delta \mathbf{h}}{\mathbf{h}_{11}} \end{bmatrix}$
<b>T</b>	$\begin{bmatrix} \frac{\mathbf{z}_{11}}{\mathbf{z}_{21}} & \frac{\Delta \mathbf{z}}{\mathbf{z}_{21}} \\ \frac{1}{\mathbf{z}_{21}} & \frac{\mathbf{z}_{22}}{\mathbf{z}_{21}} \end{bmatrix}$	$\begin{bmatrix} \frac{-\mathbf{y}_{22}}{\mathbf{y}_{21}} & \frac{-1}{\mathbf{y}_{21}} \\ \frac{\mathbf{y}_{21}}{\mathbf{y}_{21}} & \frac{\mathbf{y}_{11}}{\mathbf{y}_{21}} \end{bmatrix}$	$\begin{bmatrix} \mathbf{A} & \mathbf{B} \\ \mathbf{C} & \mathbf{D} \end{bmatrix}$	$\begin{bmatrix} \frac{-\Delta \mathbf{h}}{\mathbf{h}_{21}} & \frac{-\mathbf{h}_{11}}{\mathbf{h}_{21}} \\ \frac{-\mathbf{h}_{22}}{\mathbf{h}_{21}} & \frac{-1}{\mathbf{h}_{21}} \end{bmatrix}$
<b>h</b>	$\begin{bmatrix} \frac{\Delta \mathbf{z}}{\mathbf{z}_{22}} & \frac{\mathbf{z}_{12}}{\mathbf{z}_{22}} \\ \frac{-\mathbf{z}_{21}}{\mathbf{z}_{22}} & \frac{1}{\mathbf{z}_{22}} \end{bmatrix}$	$\begin{bmatrix} \frac{1}{\mathbf{y}_{11}} & \frac{-\mathbf{y}_{12}}{\mathbf{y}_{11}} \\ \frac{\mathbf{y}_{21}}{\mathbf{y}_{11}} & \frac{\Delta \mathbf{y}}{\mathbf{y}_{11}} \end{bmatrix}$	$\begin{bmatrix} \frac{\mathbf{B}}{\mathbf{D}} & \frac{\Delta \mathbf{T}}{\mathbf{D}} \\ \frac{-1}{\mathbf{D}} & \frac{\mathbf{C}}{\mathbf{D}} \end{bmatrix}$	$\begin{bmatrix} \mathbf{h}_{11} & \mathbf{h}_{12} \\ \mathbf{h}_{21} & \mathbf{h}_{22} \end{bmatrix}$
$\Delta \mathbf{z} = \mathbf{z}_{11}\mathbf{z}_{22} - \mathbf{z}_{12}\mathbf{z}_{21}, \Delta \mathbf{y} = \mathbf{y}_{11}\mathbf{y}_{22} - \mathbf{y}_{12}\mathbf{y}_{21}, \Delta \mathbf{h} = \mathbf{h}_{11}\mathbf{h}_{22} - \mathbf{h}_{12}\mathbf{h}_{21}, \Delta \mathbf{T} = \mathbf{AD} - \mathbf{BC}$				