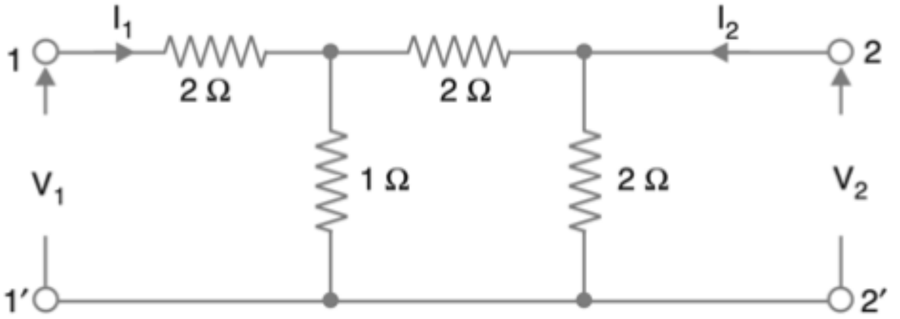
Q. Determine the Z-parameters and draw the equivalent Z-parameter circuit of:



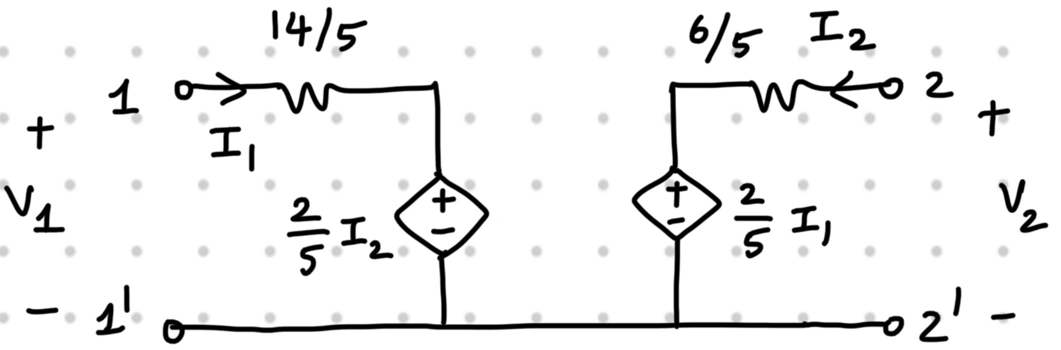
A. Writing the KVL equations,

$$\begin{bmatrix} 3 & -1 & 0 \\ -1 & 5 & 2 \\ 0 & 2 & 2 \end{bmatrix} \begin{bmatrix} I_1 \\ I_3 \\ I_2 \end{bmatrix} = \begin{bmatrix} V_1 \\ 0 \\ V_2 \end{bmatrix}$$

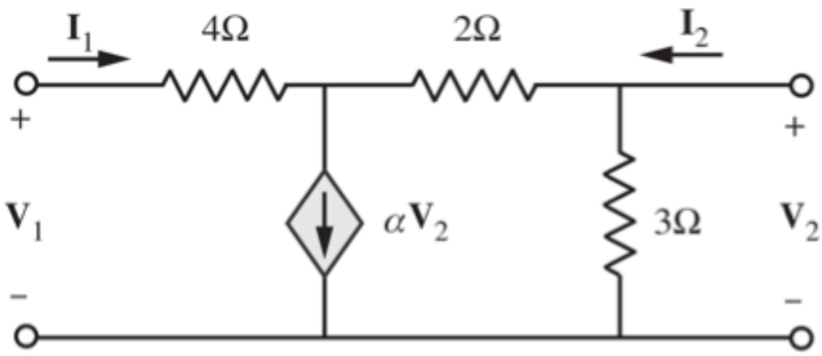
$$Z_{11} = \left.\frac{V_1}{I_1}\right|_{I_2=0} = \frac{14}{5}\ \Omega \quad , \quad Z_{21} = \left.\frac{V_2}{I_1}\right|_{I_2=0} = \frac{2}{5}\ \Omega$$

$$Z_{12} = \left.\frac{V_1}{I_2}\right|_{I_1=0} = \frac{2}{5}\ \Omega \quad , \quad Z_{22} = \left.\frac{V_2}{I_2}\right|_{I_1=0} = \frac{6}{5}\ \Omega$$

Equivalent circuit :



Q. Determine the Z-parameters of :



(use $\alpha = 4/3$)

A. Writing the KVL equations :

$$4 I_1 + 3 I_2 + 5 I_3 = V_1$$

$$3 I_2 + 3 I_3 = V_2$$

$$I_1 - I_3 = \frac{4}{3} V_2$$

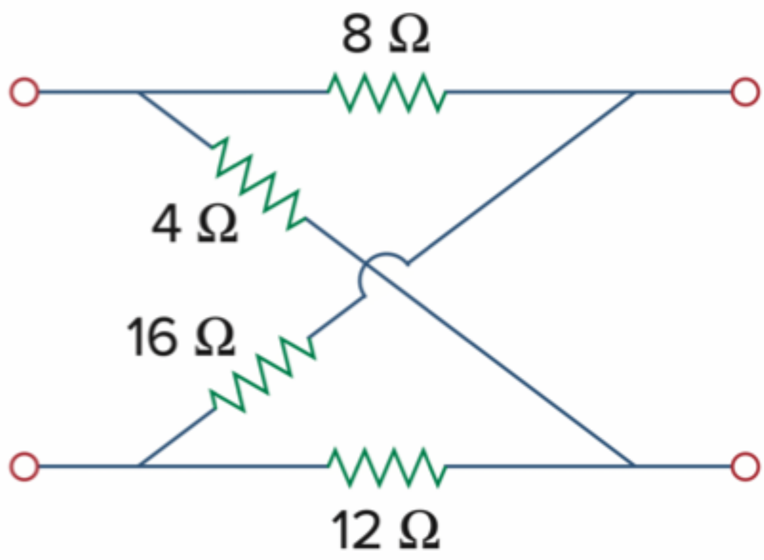
$$\Rightarrow \begin{bmatrix} 4 & 3 & 5 \\ 0 & 3 & 3 \\ 1 & 0 & -1 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} V_1 \\ V_2 \\ \frac{4}{3} V_2 \end{bmatrix}$$

$$Z_{11} = \left.\frac{V_1}{I_1}\right|_{I_2 = 0} = 5\,\Omega \quad , \quad Z_{21} = \left.\frac{V_2}{I_1}\right|_{I_2 = 0} = \frac{3}{5}\,\Omega$$

$$Z_{12} = \left.\frac{V_1}{I_2}\right|_{I_1 = 0} = -1\,\Omega \quad , \quad Z_{22} = \left.\frac{V_2}{I_2}\right|_{I_1 = 0} = \frac{3}{5}\,\Omega$$

<u>Note</u> : We can also solve this using $Z = P - Q N^{-1} M$

Q. Determine the Z-parameters of :



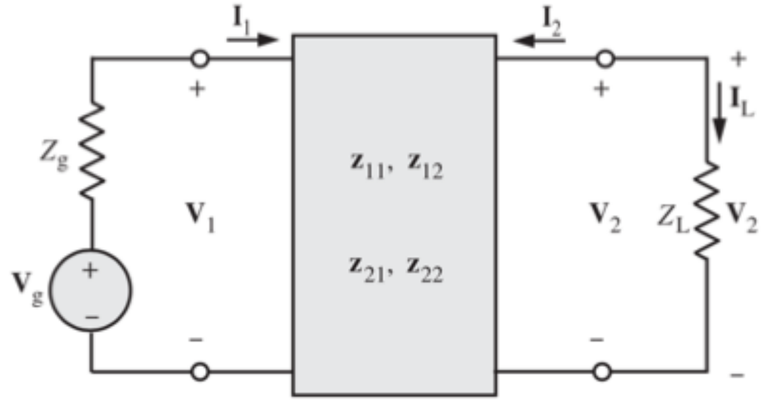
A. $$Z_{11} = \left.\frac{V_1}{I_1}\right|_{I_2=0} = \frac{(24)(16)}{40} = 9.6\,\Omega$$

$$Z_{21} = \left.\frac{V_2}{I_1}\right|_{I_2=0} = \frac{-4}{5} = -0.8\,\Omega$$

$$Z_{12} = \left.\frac{V_1}{I_2}\right|_{I_1=0} = \frac{-4}{5} = -0.8\,\Omega$$

$$Z_{22} = \left.\frac{V_2}{I_2}\right|_{I_1=0} = \frac{(12)(28)}{40} = 8.4\,\Omega$$

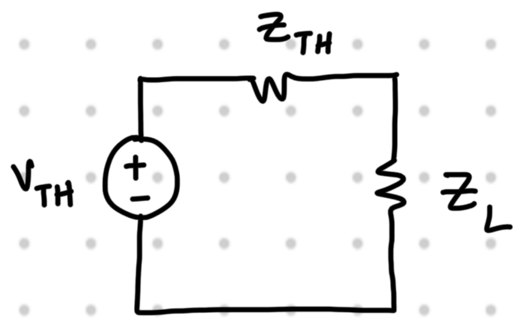
Q. Find the Thevenin equivalent with respect to port 2 of the circuit below :



A. We have, $V_1 = z_{11} I_1 + z_{12} I_2 = V_g - I_1 z_g$

$$V_2 = z_{21} I_1 + z_{22} I_2 = - I_2 z_L$$

The Thevenin equivalent of the circuit is :



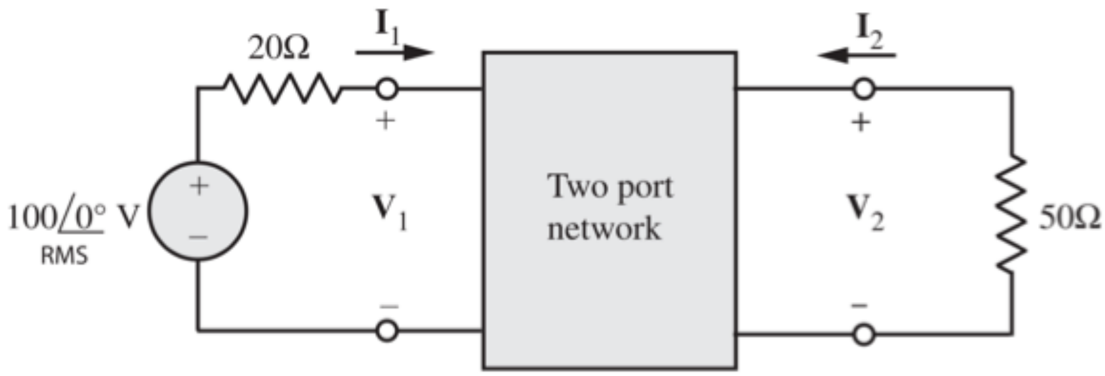
Now, $V_{TH} = V_2 \Big|_{I_2 = 0} = z_{21} I_1$

$$= z_{21} \cdot \frac{V_g}{z_{11} + z_g}$$

Further, $z_{TH} = \frac{V_2}{I_2} \Big|_{V_g = 0}$

$$= \frac{z_{11} z_{22} + z_{22} z_g - z_{12} z_{21}}{z_{11} + z_g}$$

Q. If $z = \begin{bmatrix} 40 & 10 \\ 20 & 30 \end{bmatrix}$ for the two-port network below, find the average power delivered to the $50\,\Omega$ resistor.



A. We have,

$$V_1 = 40\, I_1 + 10\, I_2$$
$$V_2 = 20\, I_1 + 30\, I_2$$

substituting $V_1 = 100\angle 0^\circ - 20\, I_1$

and $V_2 = -50\, I_2$

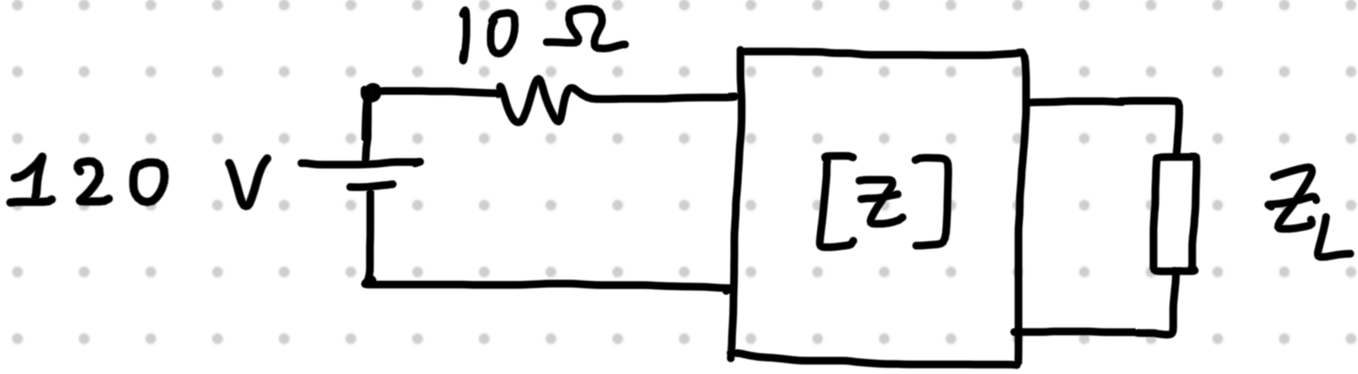
gives $I_2 = \dfrac{-10}{23} \angle 0^\circ$ A

Hence, $P_{50\Omega} = \left(\dfrac{10}{23}\right)^2 50 \approx 9.45$ W

Q. For the two-port network below,

$$[z] = \begin{bmatrix} 40 & 60 \\ 80 & 120 \end{bmatrix}$$

. Calculate maximum power transfer to Z_L.



A.

$$Z_{TH} = Z_{22} - \frac{Z_{12}\, Z_{21}}{Z_{11} + Z_s}$$

$$= 120 - \frac{80 \times 60}{40 + 10} = 24\,\Omega$$

$$V_{TH} = \frac{Z_{21}}{Z_{11} + Z_s} \cdot V_s = \frac{80}{40 + 10} \cdot 120 = 192\,V$$

Hence, maximum power transfer $= \dfrac{V_{TH}^2}{4 Z_{TH}}$

$$= 384\ W$$