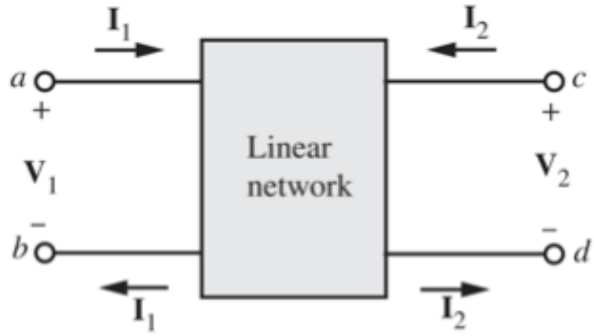
H / Hybrid Parameters

$(V_1, I_2) = f(I_1, V_2)$



$$\begin{bmatrix} V_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ V_2 \end{bmatrix}$$

Hence,

$$V_1 = h_{11} I_1 + h_{12} V_2$$

$$I_2 = h_{21} I_1 + h_{22} V_2$$

Hybrid parameters are very useful in the analysis of electronic circuits like modeling transistors.

$$h_{11} = \left. \frac{V_1}{I_1} \right|_{V_2 = 0} \quad , \quad h_{12} = \left. \frac{V_1}{V_2} \right|_{I_1 = 0}$$

h_{11}: short-circuit input impedance

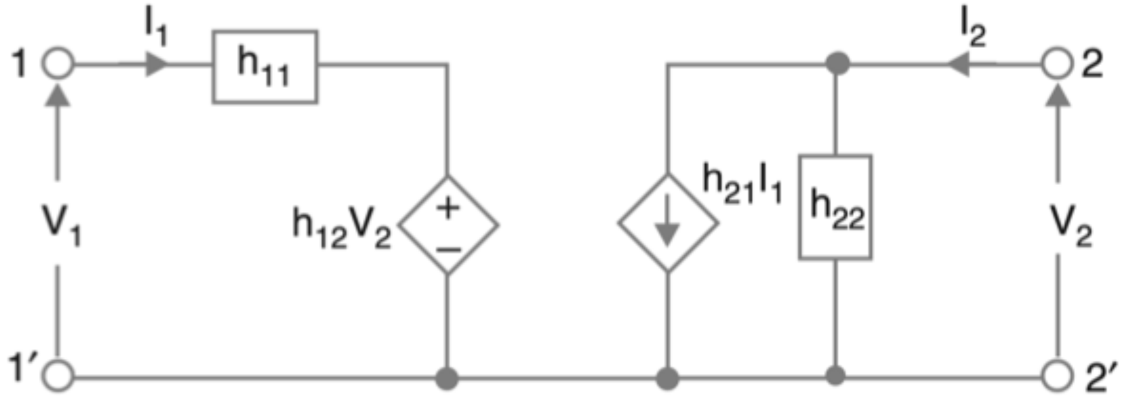
h_{12}: open-circuit reverse voltage gain

$$h_{21} = \left. \frac{I_2}{I_1} \right|_{V_2 = 0} \quad , \quad h_{22} = \left. \frac{I_2}{V_2} \right|_{I_1 = 0}$$

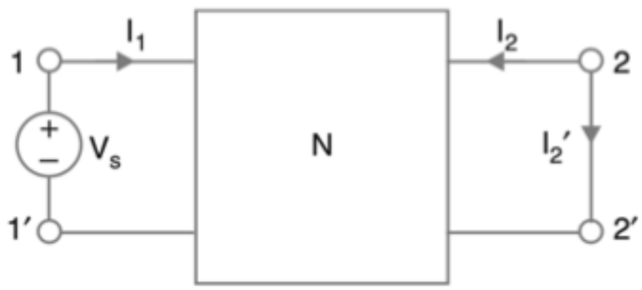
h_{21}: short-circuit forward current gain

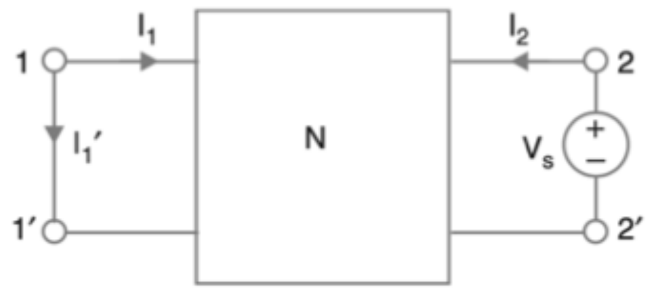
h_{22}: open-circuit output admittance

The equivalent circuit representation is given in the figure below.



Condition for Reciprocity & Symmetry:





For reciprocity,

$$I_1' = I_2' \Rightarrow -V_s \frac{h_{21}}{h_{11}} = V_s \frac{h_{12}}{h_{11}}$$

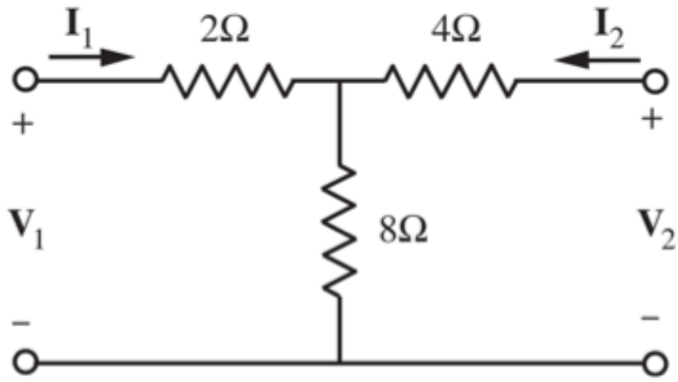
$$\Rightarrow \quad \underline{\underline{h_{21} = -h_{12}}}$$

For symmetry,

$$\left.\frac{V_1}{I_1}\right|_{I_2=0} = \left.\frac{V_2}{I_2}\right|_{I_1=0}$$

$$\Rightarrow \quad h_{11}h_{22} - h_{12}h_{21} = 1 \quad \text{or} \quad \begin{vmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{vmatrix} = 1$$

Q. Determine the H-parameters of :



A. $$\begin{bmatrix} 10 & 8 \\ 8 & 12 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

$$\Rightarrow \quad V_1 = \frac{14}{3} I_1 + \frac{2}{3} V_2$$

$$\text{and} \quad I_2 = -\frac{2}{3} I_1 + \frac{1}{12} V_2$$

Hence,

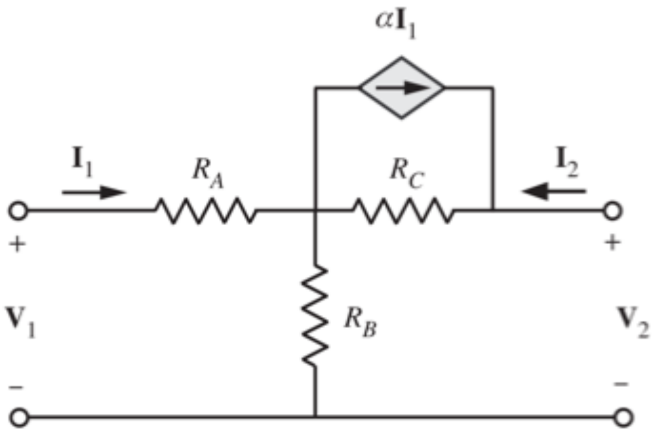
$$h_{11} = \left.\frac{V_1}{I_1}\right|_{V_2=0} = \frac{14}{3}\ \Omega$$

$$h_{12} = \left.\frac{V_1}{V_2}\right|_{I_1=0} = \frac{2}{3}$$

$$h_{21} = \left.\frac{I_2}{I_1}\right|_{V_2=0} = -\frac{2}{3}$$

$$h_{22} = \left.\frac{I_2}{V_2}\right|_{I_1=0} = \frac{1}{12}\ \Omega^{-1}$$

Q. Determine the H-parameters of :



A. We have, $I_1(R_A + R_B) + I_2 R_B = V_1$

$$I_1(R_B + \alpha R_C) + I_2(R_B + R_C) = V_2$$

Hence,

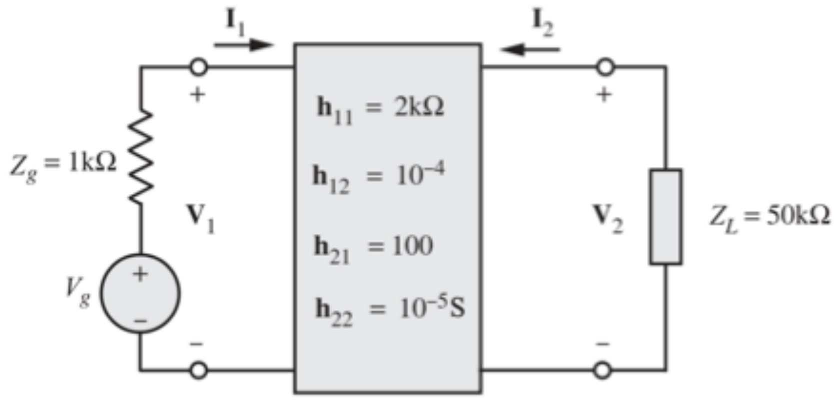
$$h_{11} = \left.\frac{V_1}{I_1}\right|_{V_2=0} = \frac{(R_A R_B + R_A R_C + R_B R_C - \alpha R_B R_C)}{(R_B + R_C)}$$

$$h_{12} = \left.\frac{V_1}{V_2}\right|_{I_1=0} = \frac{R_B}{R_B + R_C}$$

$$h_{21} = \left.\frac{I_2}{I_1}\right|_{V_2=0} = -\frac{(R_B + \alpha R_C)}{R_B + R_C}$$

$$h_{22} = \left.\frac{I_2}{V_2}\right|_{I_1=0} = \frac{1}{R_B + R_C}$$

Q. Determine $\dfrac{V_2}{V_g}$ of the circuit :



A.
$$V_1 = 2000\, I_1 + 10^{-4} V_2$$
$$I_2 = 100\, I_1 + 10^{-5} V_2$$

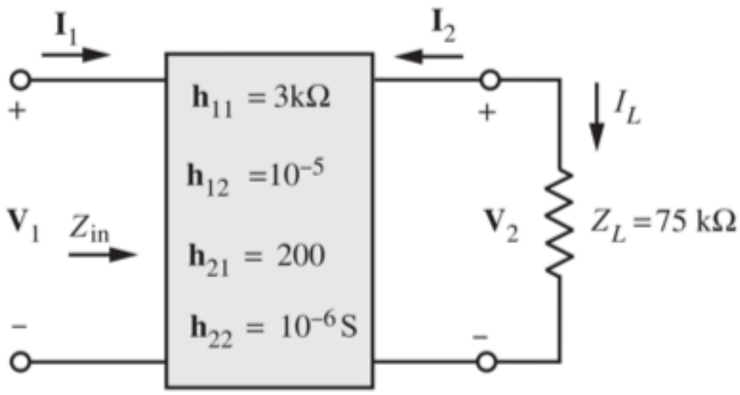
substituting $V_1 = V_g - 1000\, I_1$

and $V_2 = -50 \cdot 10^3\, I_2$

gives,
$$I_1 = \frac{3}{200} I_2$$
$$\Rightarrow \quad V_g = 40\, I_2$$

Hence, $$\frac{V_2}{V_g} = -1250$$

Q. Determine the input impedance of :



A. We have, $V_1 = 3 \times 10^3 I_1 + 10^{-5} V_2$

$$I_2 = 200 I_1 + 10^{-6} V_2$$

Also, $V_2 = -75 \times 10^3 I_2$

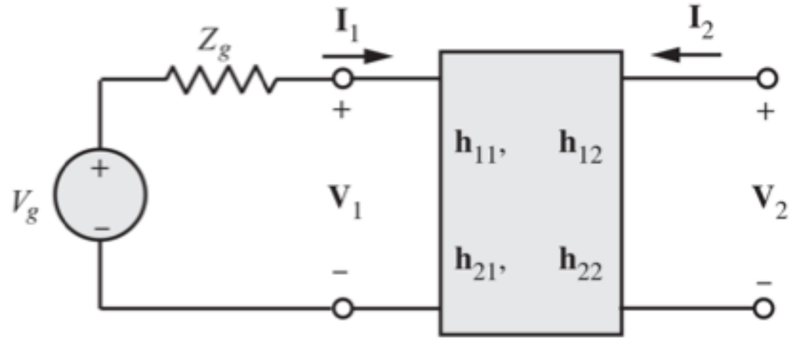
$$\Rightarrow \quad I_2 = \frac{8000}{43} I_1$$

$$\Rightarrow \quad V_2 = -\frac{60}{43} 10^7 I_1$$

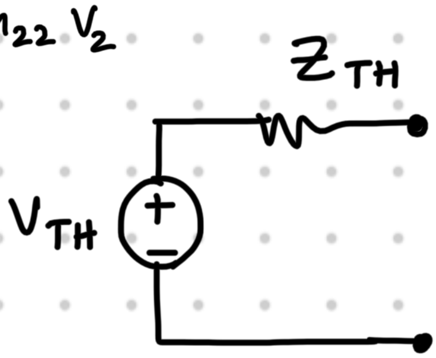
$$\Rightarrow \quad V_1 = 3000 I_1 - \frac{6000}{43} I_1 = \frac{123}{43} \cdot 10^3 I_1$$

Hence, $Z_{in} = \frac{V_1}{I_1} = \frac{123}{43} \cdot 10^3 \simeq 2.86\ k\Omega$

Q. Determine the Thevenin equivalent circuit at the output of :



A. We have,
$$V_1 = h_{11} I_1 + h_{12} V_2$$
$$I_2 = h_{21} I_1 + h_{22} V_2$$



To find V_{TH} : $I_2 = 0$

$$\Rightarrow V_{TH} = V_2 = \frac{-h_{21}}{h_{22}} I_1 = \frac{-h_{21}}{h_{22}} \left(\frac{V_g - h_{12} V_2}{Z_g + h_{11}} \right)$$

Hence, $$V_{TH} = V_2 = \frac{-h_{21} V_g}{Z_g h_{22} + h_{11} h_{22} - h_{12} h_{21}}$$

To find Z_{TH} : $V_1 = -I_1 Z_g$

$$\Rightarrow Z_{TH} = \frac{V_2}{I_2} = \frac{Z_g + h_{11}}{Z_g h_{22} + h_{11} h_{22} - h_{12} h_{21}}$$