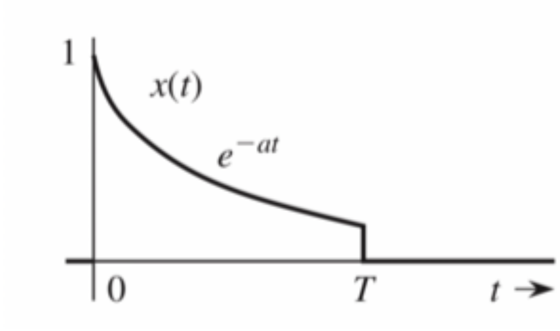
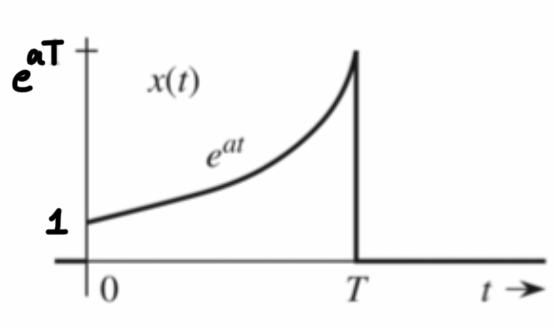
Q. Find the Fourier transforms of the following:

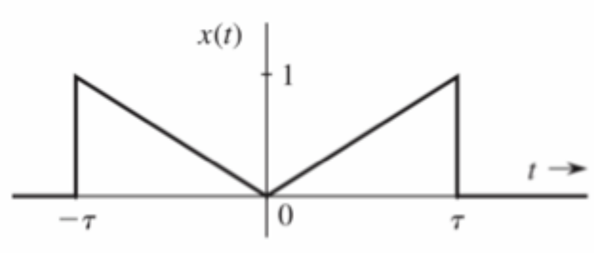


(a)



(b)

(c)



(d)

A. (a). $$X(\omega) = \int_0^T e^{-at} e^{-j\omega t}\, dt = \frac{1 - e^{-(j\omega + a)T}}{j\omega + a}$$

(b). $$X(\omega) = \int_0^T e^{at} e^{-j\omega t}\, dt = \frac{1 - e^{-(j\omega - a)T}}{j\omega - a}$$

(c). $$X(\omega) = \int_0^1 4 e^{-j\omega t}\, dt + \int_1^2 2 e^{-j\omega t}\, dt = \frac{4 - 2e^{-j\omega} - 2e^{-2j\omega}}{j\omega}$$

(d). $$X(\omega) = \int_{-\tau}^{0} -\frac{t}{\tau} e^{-j\omega t}\, dt + \int_0^{\tau} \frac{t}{\tau} e^{-j\omega t}\, dt$$

$$= \frac{2}{\tau\omega^2}\left(\cos\omega\tau + \omega\tau \sin\omega\tau - 1\right)$$

Q. If $x(t)$ is an even function of t, then show:

$$X(\omega) = 2\int_0^\infty x(t)\cos(\omega t)\,dt$$

If $x(t)$ is an odd function of t, then show:

$$X(\omega) = -2j\int_0^\infty x(t)\sin(\omega t)\,dt$$

A. $$X(\omega) = \int_{-\infty}^{\infty} x(t)\,e^{-j\omega t}\,dt = \int_{-\infty}^{\infty} x(t)\cos\omega t\,dt - j\int_{-\infty}^{\infty} x(t)\sin\omega t\,dt$$

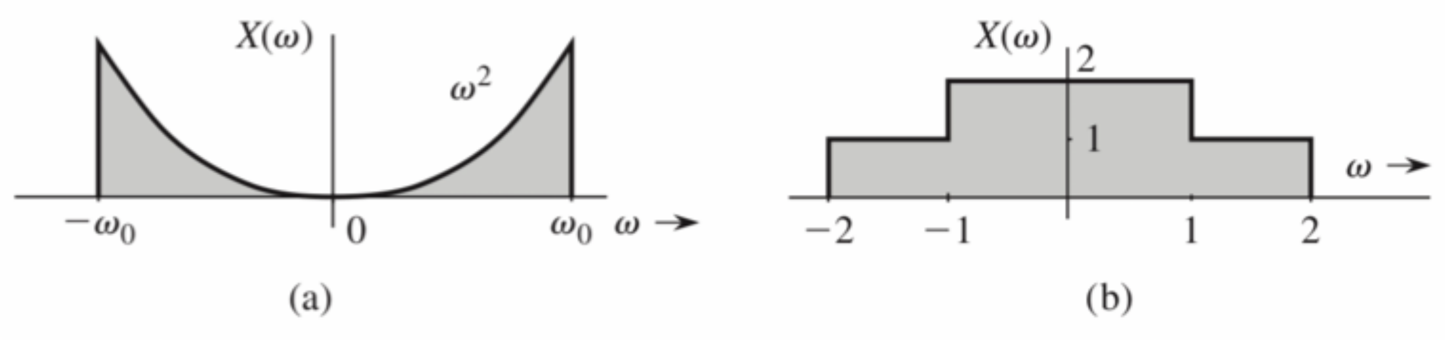
$$= \int_{-\infty}^{0} x(t)\cos\omega t\,dt + \int_{0}^{\infty} x(t)\cos\omega t\,dt - j\int_{-\infty}^{0} x(t)\sin\omega t\,dt - j\int_{0}^{\infty} x(t)\sin\omega t\,dt$$

$$= \int_{0}^{\infty} x(-t)\cos(\omega t)\,dt + \int_{0}^{\infty} x(t)\cos\omega t\,dt + j\int_{-\infty}^{0} x(-t)\sin\omega t\,dt - j\int_{0}^{\infty} x(t)\sin\omega t\,dt$$

If $x(t) = x(-t)$, $X(\omega) = 2\int_0^\infty x(t)\cos\omega t\,dt$

If $x(t) = -x(-t)$, $X(\omega) = -2j\int_0^\infty x(t)\sin\omega t\,dt$

Q. Find the inverse Fourier transforms of the following spectra:



A. (a). $$x(t) = \frac{1}{2\pi}\int_{-\omega_0}^{\omega_0} \omega^2 e^{j\omega t}\, d\omega$$

$$= \frac{(\omega_0^2 t^2 - 2)\sin \omega_0 t + 2\omega_0 t \cos \omega_0 t}{\pi t^3}$$

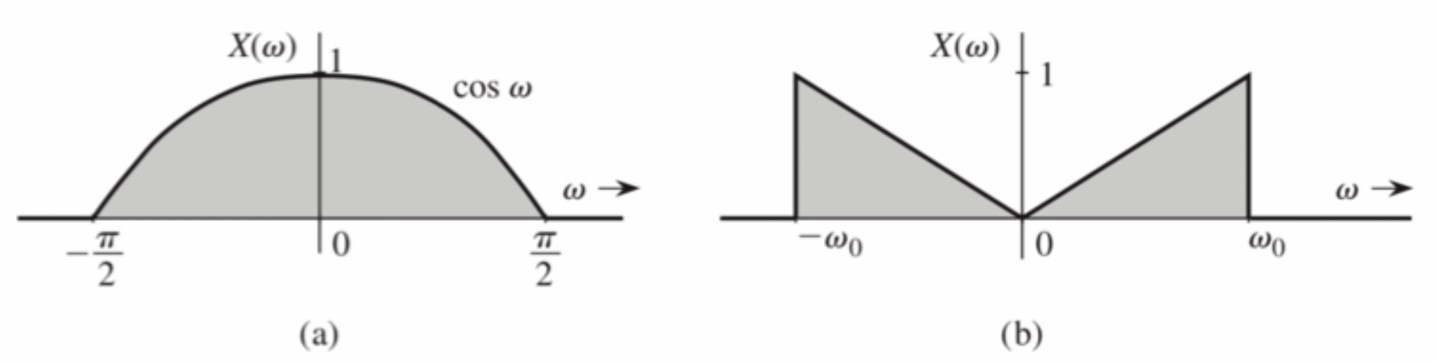
(b). $$x(t) = \frac{1}{2\pi}\left(\int_{-2}^{-1} 1\, e^{j\omega t}\, d\omega + \int_{-1}^{1} 2\, e^{j\omega t}\, d\omega + \int_{1}^{2} 1\, e^{j\omega t}\, d\omega\right)$$

$$= \frac{1}{2\pi}\left(\int_{-2}^{2} 1\, e^{j\omega t}\, d\omega + \int_{-1}^{1} 1\, e^{j\omega t}\, d\omega\right)$$

$$= \frac{1}{2\pi}\left(\left.\frac{e^{j\omega t}}{jt}\right|_{-2}^{2} + \left.\frac{e^{j\omega t}}{jt}\right|_{-1}^{1}\right)$$

$$= \frac{\sin 2t + \sin t}{\pi t}$$

Q. Find the inverse Fourier transforms of the following spectra:



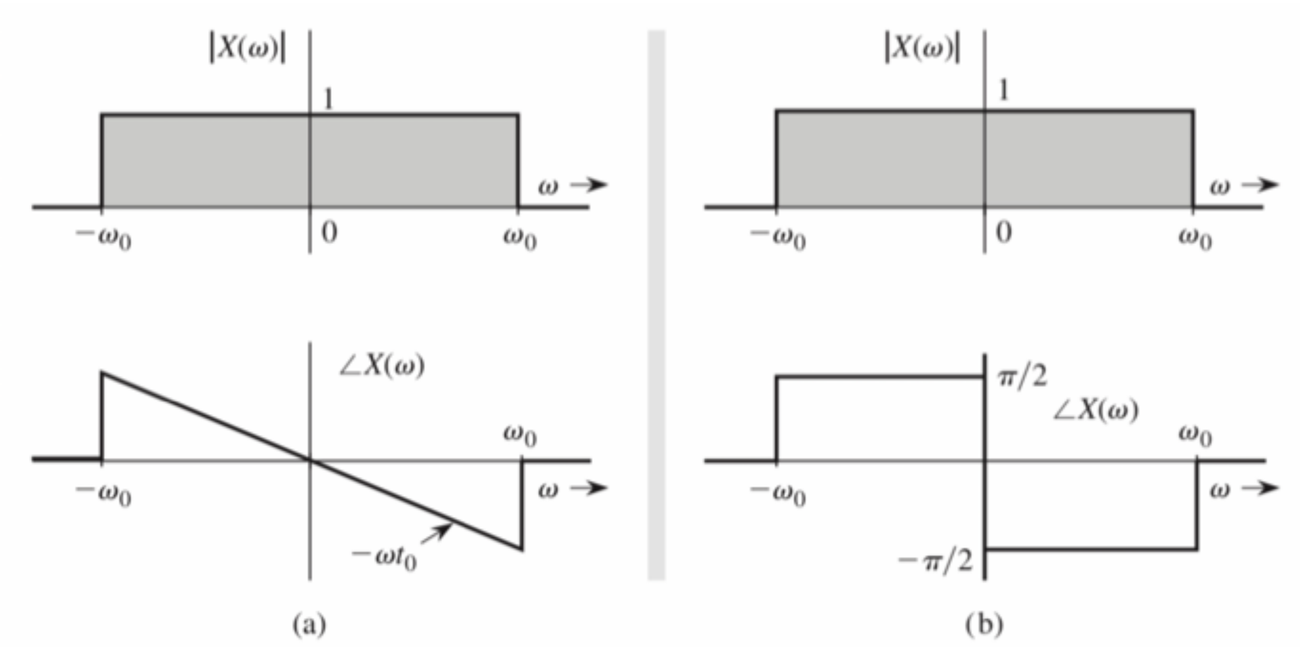
A. (a). $$x(t) = \frac{1}{2\pi} \int_{-\pi/2}^{\pi/2} (\cos\omega)\, e^{j\omega t}\, d\omega$$

$$= \frac{1}{\pi(1-t^2)} \cos\left(\frac{\pi t}{2}\right)$$

(b). $$x(t) = \frac{1}{2\pi}\left(\int_{-\omega_0}^{0} \frac{-\omega}{\omega_0} e^{j\omega t}\, d\omega + \int_{0}^{\omega_0} \frac{\omega}{\omega_0} e^{j\omega t}\, d\omega \right)$$

$$= \frac{1}{\pi \omega_0 t^2}\left(\cos(\omega_0 t) + \omega_0 t \sin(\omega_0 t) - 1 \right)$$

Q. Find the inverse Fourier transforms of the following spectra :



A. (a). $$x(t) = \frac{1}{2\pi}\int_{-\omega_0}^{\omega_0} e^{-j\omega t_0}\, e^{j\omega t}\, d\omega = \frac{1}{2\pi}\int_{-\omega_0}^{\omega_0} e^{j\omega(t-t_0)}\, d\omega$$

$$= \frac{1}{2\pi j(t-t_0)}\, e^{j\omega(t-t_0)}\Big|_{-\omega_0}^{\omega_0} = \frac{\sin\left(\omega_0(t-t_0)\right)}{\pi\,(t-t_0)}$$

(b). $$x(t) = \frac{1}{2\pi}\left[\int_{-\omega_0}^{0} j\, e^{j\omega t}\, d\omega + \int_{0}^{\omega_0} -j\, e^{j\omega t}\, d\omega\right]$$

$$= \frac{1}{2\pi t}\, e^{j\omega t}\Big|_{-\omega_0}^{0} - \frac{1}{2\pi t}\, e^{j\omega t}\Big|_{0}^{\omega_0} = \frac{1-\cos\omega_0 t}{\pi t}$$