Filters

Filters play an integral part in electrical circuits such as radios and TV receivers.

A filter is a circuit that is designed to pass or reject signals with desired frequencies.

There are four types of filters:

(1). <u>Low-pass filter:</u> Passes low frequencies and stops high frequencies.

(2). <u>High-pass filter:</u> Passes high frequencies and stops low frequencies.

(3). <u>Band-pass filter:</u> Passes frequencies within a band and stops frequencies outside that band.

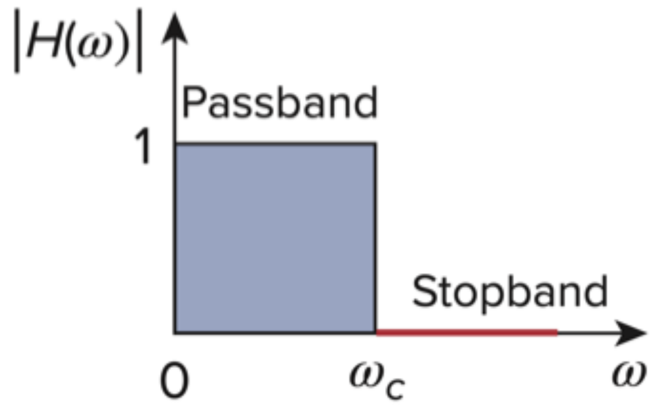
(4). <u>Band-stop filter:</u> Stops frequencies within a band and passes frequencies outside that band.

Each of these filters can be designed using <u>passive</u> components (R, L, and C) or using <u>active</u> components (opamps and transistors).

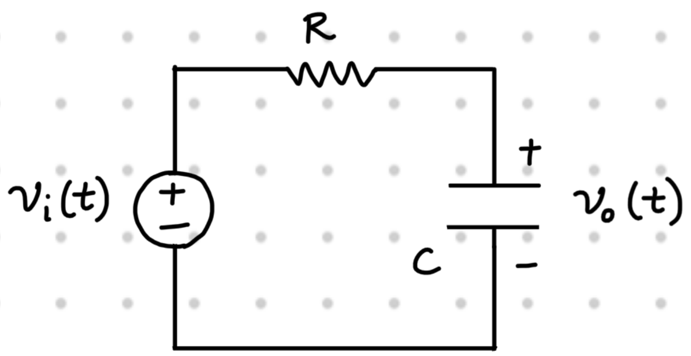
Passive Filters : We shall first discuss the four filters using passive components.

1. **Low-Pass Filter :** An ideal low-pass filter eliminates all frequencies above a designated cut-off frequency.

However, ideal filters are impossible to realize.



A simple RC/RL circuit can be used to build a practical low pass filter.



$$V_o(s) = \frac{\frac{1}{Cs}}{R + \frac{1}{Cs}} V_i(s)$$

$$\Rightarrow H(s) = \frac{V_o(s)}{V_i(s)} = \frac{1}{1 + sRC}$$

substituting $s = j\omega$, $H(j\omega) = \frac{1}{1 + j\omega RC}$

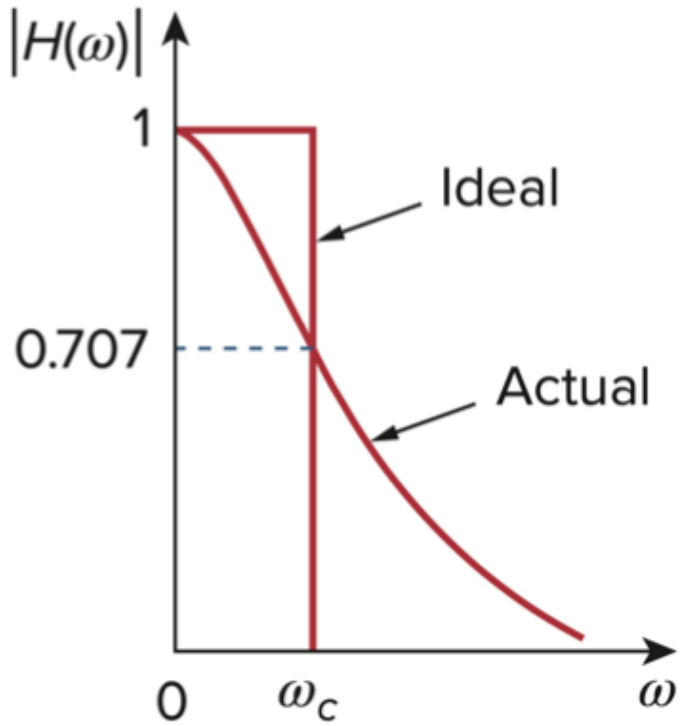
$$\Rightarrow \quad |H(j\omega)| = \frac{1}{\sqrt{1+\omega^2R^2C^2}}$$

Clearly, $H(0) = 1$, $H(\infty) = 0$

The cut-off frequency (ω_c) is defined as the frequency at which the transfer function drops in magnitude to $1/\sqrt{2}$ ($\approx 70.71\%$ of its maximum value). Hence,

$$\frac{1}{\sqrt{2}} = \frac{1}{\sqrt{1+\omega_c^2R^2C^2}}$$

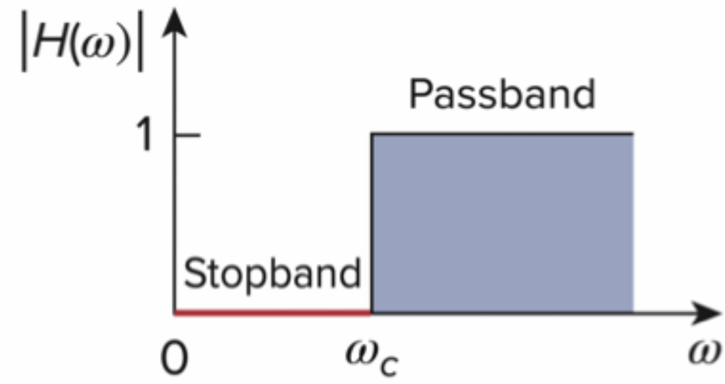
$$\Rightarrow \quad \omega_c = \frac{1}{RC}$$



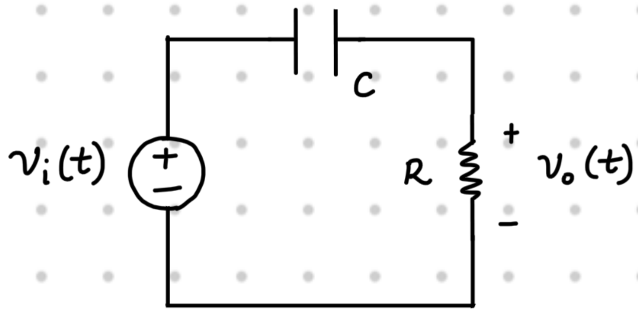
Note: In communication systems, gain is measured in decibels.

$$G_{dB} = 10 \log_{10} \frac{P_2}{P_1} = 20 \log_{10} \frac{V_2}{V_1}$$

2. <u>High-Pass Filter</u>: A high-pass filter eliminates all frequencies below a designated cut-off frequency.



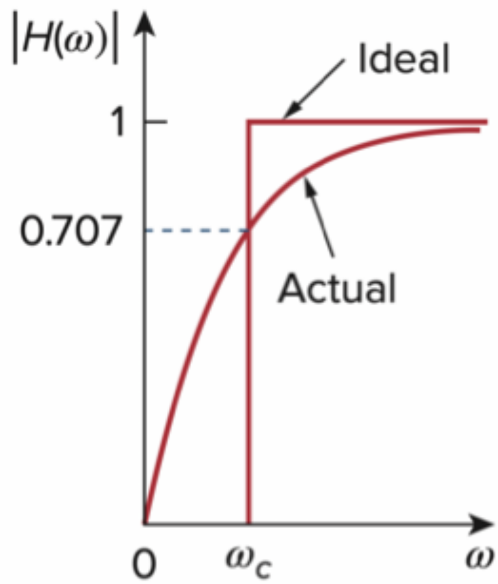
A simple CR circuit can be used to build a practical high pass filter.



$$V_o(s) = \frac{R}{R + \frac{1}{Cs}} V_i(s)$$

$$\Rightarrow \quad H(s) = \frac{V_o(s)}{V_i(s)} = \frac{sRC}{1 + sRC}$$

$$\Rightarrow \quad |H(j\omega)| = \frac{\omega RC}{\sqrt{1 + \omega^2 R^2 C^2}}$$



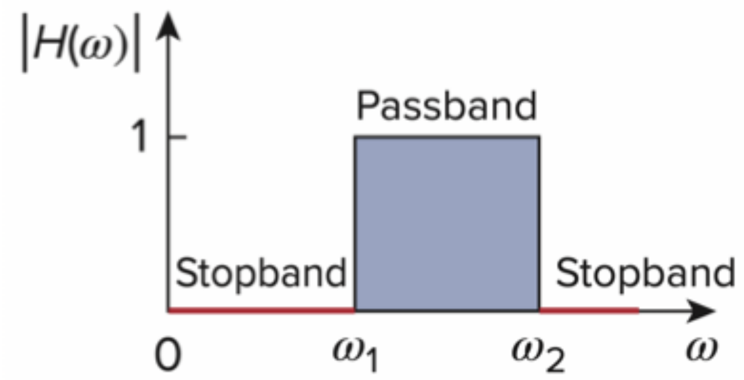
Clearly, $H(0) = 0$

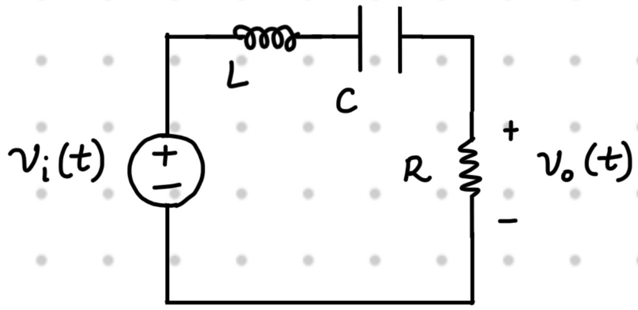
and $H(\infty) = 1$

Again, the cut-off frequency is $\omega_c = \frac{1}{RC}$.

3. Band-Pass Filter : A band-pass filter passes frequencies within a frequency band and blocks frequencies outside the band.

A series RLC circuit can be used to build a practical band-pass filter.





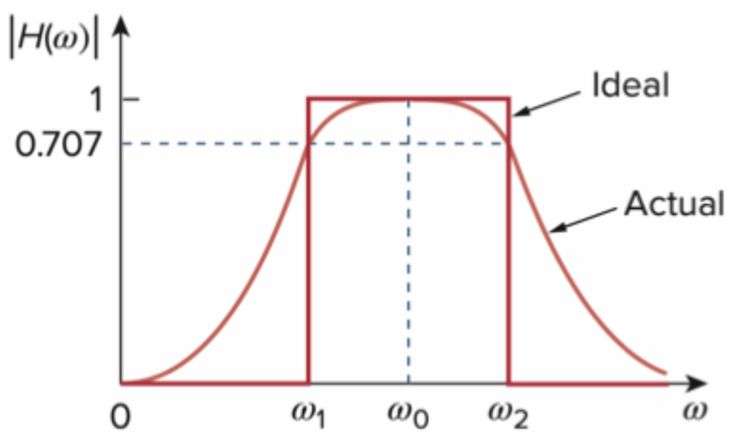
$$V_o(s) = \frac{R}{R + \frac{1}{sC} + sL} V_i(s)$$

$$\Rightarrow \quad |H(j\omega)| = \frac{R}{\sqrt{R^2 + (\omega L - 1/\omega C)^2}}$$

Clearly, $H(0) = H(\infty) = 0$

The center frequency where $|H(j\omega)| = 1$ is :

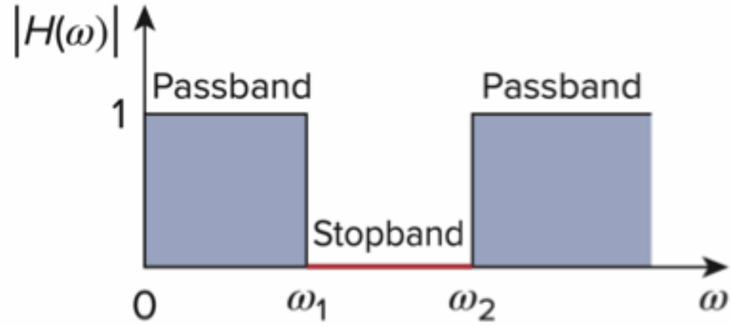
$$\omega_o = \frac{1}{\sqrt{LC}}$$

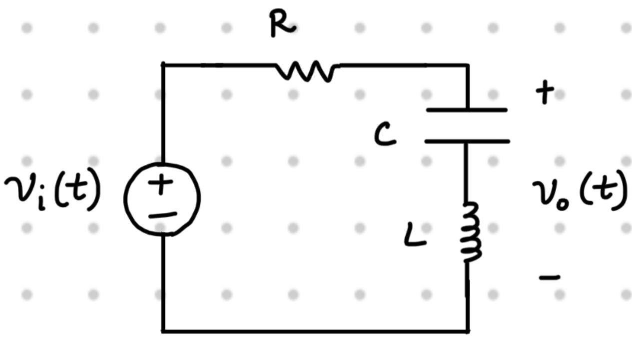


and $B = \omega_2 - \omega_1$ is called as the bandwidth of passband.

4. **Band-Stop Filter :** A band-stop filter stops frequencies within a frequency band and passes frequencies outside the band.

A series RLC circuit can be used to build a practical band-stop filter.





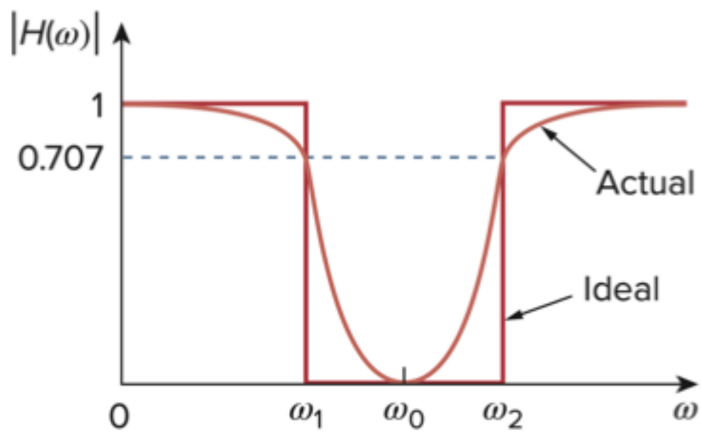
$$V_o(s) = \frac{sL + \frac{1}{sC}}{R + \frac{1}{sC} + sL} V_i(s)$$

$$\Rightarrow \left| H(j\omega) \right| = \frac{(\omega L - 1/\omega C)}{\sqrt{R^2 + (\omega L - 1/\omega C)^2}}$$

Clearly, $H(0) = H(\infty) = 1$

The center frequency where $|H(j\omega)| = 0$ is :

$$\omega_0 = \frac{1}{\sqrt{LC}}$$



and $B = \omega_2 - \omega_1$ is called as the bandwidth of rejection.

<u>NOTE</u> : As discussed in the class, ideal filter characteristics will force the impulse response $h(t)$ to be non-zero for $t<0$.

We will now show that for a system to be causal, $h(t)$ should be equal to zero for $t<0$.

$$y(t) = x(t) * h(t) = \int_{-\infty}^{\infty} x(\tau)\, h(t-\tau)\, d\tau$$

$$\Rightarrow \quad y(t_0) = \int_{-\infty}^{\infty} x(\tau)\, h(t_0-\tau)\, d\tau$$

$$\Rightarrow y(t_0) = \int_{-\infty}^{t_0} x(\tau)\, h(t_0-\tau)\, d\tau + \int_{t_0}^{\infty} x(\tau)\, h(t_0-\tau)\, d\tau$$

For causality, $y(t_0)$ should only depend on $x(t)$ for $t < t_0$

$\Rightarrow$ This term should be zero.

$\Rightarrow \quad h(t_0-\tau) < 0 \quad$ for $\quad \tau > t_0$

substituting $t_0 - \tau = t$ gives

$$h(t) < 0 \quad \text{for} \quad t < 0$$

<u>QED</u>.