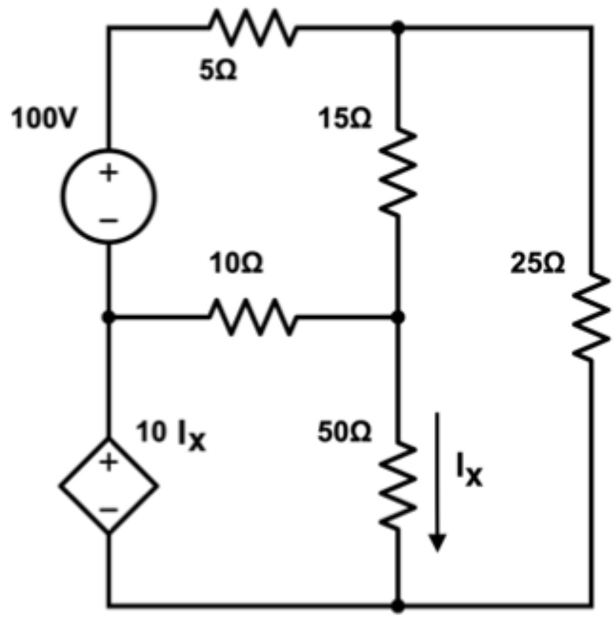
Q. What is the power delivered by the dependent voltage source?



A. Writing KVL equations:

$$\begin{bmatrix} 30 & -10 & -15 \\ -10 & 60 & -50 \\ -15 & -50 & 90 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} 100 \\ 10\, I_x \\ 0 \end{bmatrix}$$

Since $I_x = I_2 - I_3$:

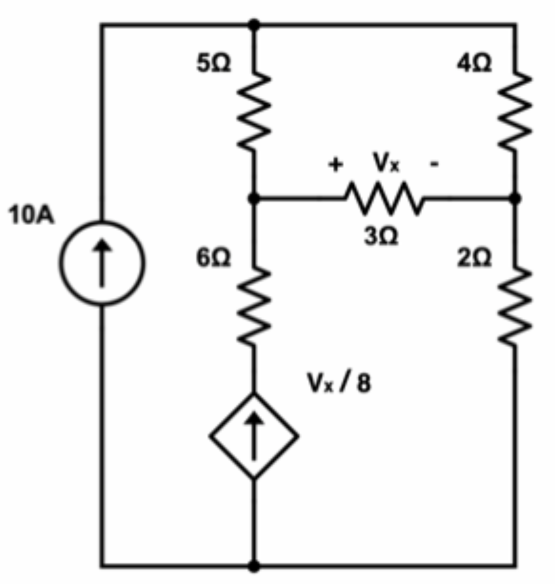
$$\begin{bmatrix} 30 & -10 & -15 \\ -10 & 50 & -40 \\ -15 & -50 & 90 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} 100 \\ 0 \\ 0 \end{bmatrix}$$

$I_1 \approx 6.06\,A$, $I_2 \approx 3.63\,A$, $I_3 \approx 3.03\,A$

Therefore $P_{10 I_x} = 10\,(3.63 - 3.03) \cdot 3.63$

$\approx 21.8\ W$

Q. Find the mesh currents of the circuit shown.



A. We have, $I_1 = 10\,A$

$$V_x = 3(I_3 - I_2)$$

$$\frac{V_x}{8} = I_3 - I_1$$

Also, the KVL equation for mesh II,

$$5(I_2 - I_1) + 4I_2 + 3(I_2 - I_3) = 0$$

Simplifying the equations, we get

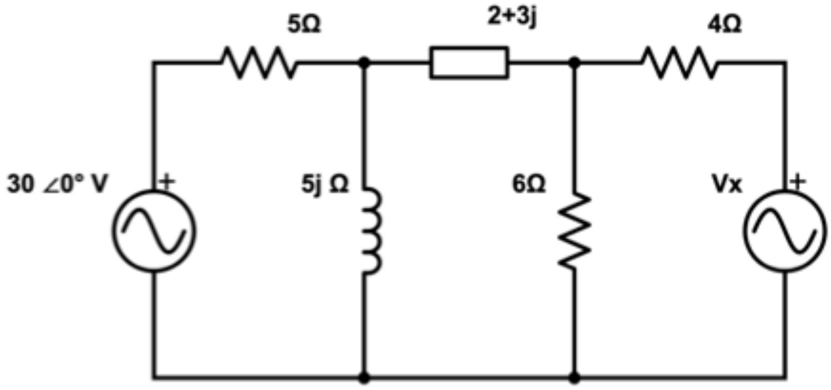
$$0.375\,I_2 + 0.625\,I_3 = 10$$

$$12\,I_2 - 3\,I_3 = 50$$

$$\Rightarrow \begin{bmatrix} 0.375 & 0.625 \\ 12 & -3 \end{bmatrix} \begin{bmatrix} I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} 10 \\ 50 \end{bmatrix}$$

$$\Rightarrow I_2 = 7.1\,A \quad \text{and} \quad I_3 = 11.74\,A$$

Q. What should be the value of V_x such that no current flows through the impedance $2+3j$?



A. Writing KVL equations as matrices :

$$\begin{bmatrix} 5+5j & -5j & 0 \\ -5j & 8+8j & -6 \\ 0 & -6 & 10 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} 30 \\ 0 \\ -V_x \end{bmatrix}$$

Solving for I_2 and equating $I_2 = 0$:

$$I_2 = \frac{\begin{vmatrix} 5+5j & 30 & 0 \\ -5j & 0 & -6 \\ 0 & -V_x & 10 \end{vmatrix}}{\begin{vmatrix} 5+5j & -5j & 0 \\ -5j & 8+j8 & -6 \\ 0 & -6 & 10 \end{vmatrix}} = \frac{(5+j5)(-6V_x) + j1500}{\Delta}$$

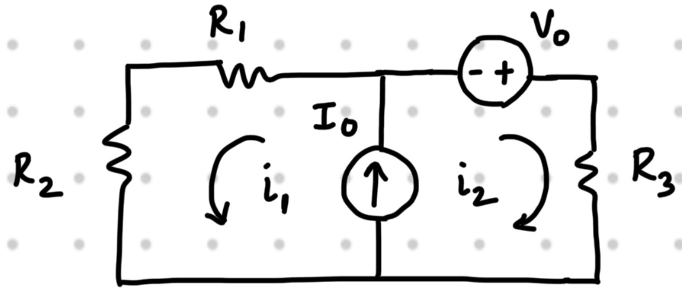
$$I_2 = 0 \Rightarrow V_x = 25(1+j)\,V \quad \text{or} \quad 35.35\angle 45^\circ\,V$$

Supermesh

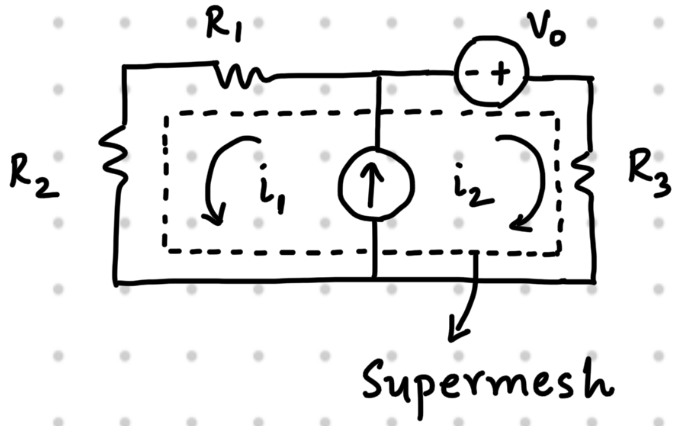
If we have current sources shared with multiple meshes, we cannot use Ohm's law to represent the voltages through the current sources. To circumvent this problem, we create a **Supermesh**.

Example: Suppose we have a current source which is part of two meshes as shown.

Clearly, we cannot apply KVL around either mesh.



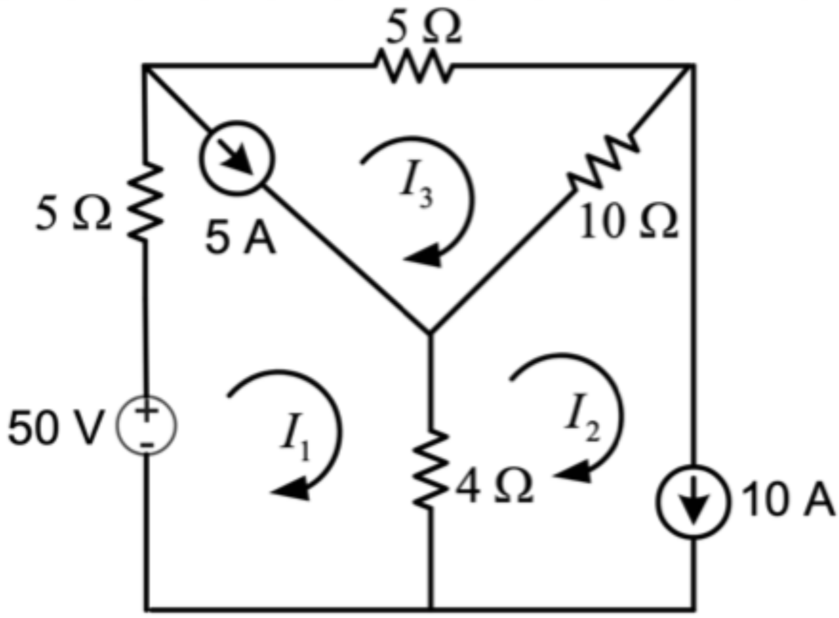
To solve this problem, we form a supermesh and write one KVL equation for the supermesh.



That is, $$V_0 - i_2 R_3 + i_1 (R_1 + R_2) = 0$$

Hence, this is simply a shortcut of writing one KVL equation instead of two KVL equations.

Q. Find the loop currents in the circuit shown.



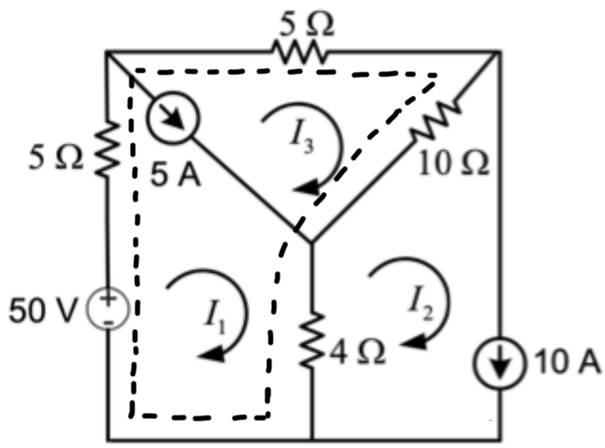
A. Clearly, $I_2 = 10\,A$. Also, $I_1 - I_3 = 5\,A$.

The KVL equation of Supermesh 1-3:

$$5I_1 + 5I_3 + 10(I_3 - I_2) + 4(I_1 - I_2) - 50 = 0$$

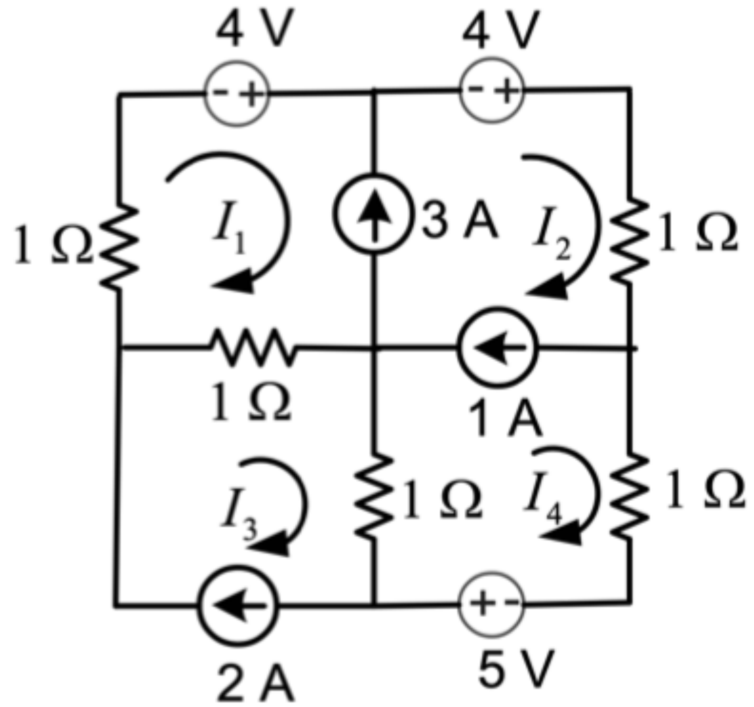
$$\Rightarrow \quad 9I_1 + 15I_3 = 190$$

$$\Rightarrow \quad \begin{bmatrix} 1 & -1 \\ 9 & 15 \end{bmatrix} \begin{bmatrix} I_1 \\ I_3 \end{bmatrix} = \begin{bmatrix} 5 \\ 190 \end{bmatrix}$$



Hence, $I_1 \simeq 11.05\,A$, $I_2 = 10\,A$ and $I_3 \simeq 6.05\,A$

Q. Find the loop currents in the circuit shown.



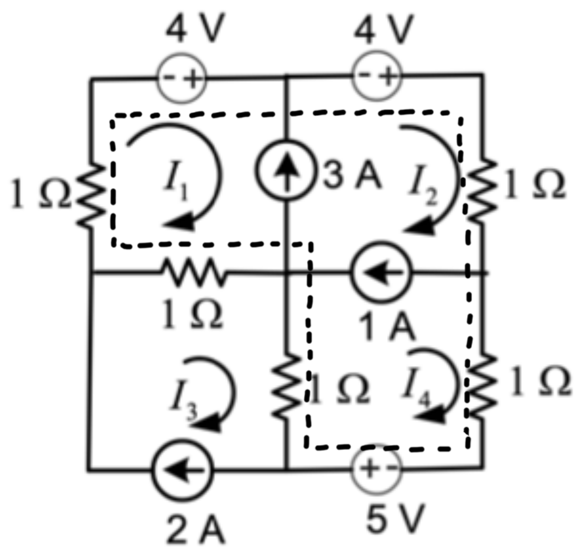
A. Clearly, $I_3 = 2A$. Now, we have three unknowns, so we need three equations.

$$I_2 - I_1 = 3A \quad , \quad I_2 - I_4 = 1A$$

The KVL equation of Supermesh 1-2-4:

$$4 + 4 - I_2 - I_4 + 5 - (I_4 - I_3) - (I_1 - I_3) - I_1 = 0$$

$$\Rightarrow \begin{bmatrix} -1 & 1 & 0 \\ 0 & 1 & -1 \\ 2 & 1 & 2 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_4 \end{bmatrix} = \begin{bmatrix} 3 \\ 1 \\ 17 \end{bmatrix}$$



Hence, $I_1 = 2A$, $I_2 = 5A$,

$I_3 = 2A$ and $I_4 = 4A$.

Summary of Mesh Analysis :

Mesh Analysis is a systematic application of KVL, where we generate a system of equations with mesh currents as the unknown variables.

The number of independent equations equals the number of unknown mesh currents.

We determine the mesh currents by solving the matrix $[Z][I] = [E]$ and use Ohm's law to find node voltages.

Step-by-step Procedure

Step 1 : Simplify the circuit as much as possible using transformations.

Step 2 : Identify and label all mesh currents.

Step 3 : Assign and label polarities of voltages.

Step 4 : Apply KVL in each mesh.

Step 5 : Solve the Matrix equation(s).