

· *Trigonometric Identities:*

$$e^{\pm jx} = \cos x \pm j \sin x, \quad \sin(2x) = 2 \sin x \cos x, \quad \cos(2x) = 2 \cos^2 x - 1$$

$$\sin(x \pm y) = \sin x \cos y \pm \cos x \sin y, \quad \cos(x \pm y) = \cos x \cos y \mp \sin x \sin y$$

· *Calculus:*

$$(u(x)v(x))' = u(x)v'(x) + u'(x)v(x), \quad \int u(x)v'(x)dx = u(x)v(x) - \int u'(x)v(x)dx$$

· *Convolution:*

$$x(t) * h(t) = \int_{-\infty}^{\infty} x(\tau)h(t-\tau)d\tau = \int_{-\infty}^{\infty} x(t-\tau)h(\tau)d\tau$$

· *Trigonometric Fourier Series:*

$$x(t) = a_0 + \sum_{n=1}^{\infty} a_n \cos(n\omega_0 t) + \sum_{n=1}^{\infty} b_n \sin(n\omega_0 t)$$

$$a_0 = \frac{1}{T_0} \int_{T_0} x(t)dt, \quad a_n = \frac{2}{T_0} \int_{T_0} x(t)\cos(n\omega_0 t)dt, \quad b_n = \frac{2}{T_0} \int_{T_0} x(t)\sin(n\omega_0 t)dt$$

· *Compact Trigonometric Fourier Series:*

$$x(t) = C_0 + \sum_{n=1}^{\infty} C_n \cos(n\omega_0 t + \theta_n), \quad C_0 = a_0, \quad C_n = \sqrt{a_n^2 + b_n^2}, \quad \theta_n = \tan^{-1}(-b_n/a_n)$$

· *Exponential Fourier Series:*

$$x(t) = \sum_{n=-\infty}^{\infty} D_n e^{jn\omega_0 t}, \quad D_n = \frac{1}{T_0} \int_{T_0} x(t)e^{-jn\omega_0 t}dt$$

· *Fourier Series Properties:*

$$x(t) \iff D_n, \quad kx(t) \iff kD_n, \quad x(-t) \iff D_{-n}$$

$$x(t-t_0) \iff D_n e^{-jn\omega_0 t_0}, \quad x(t)e^{jn_0\omega_0 t} \iff D_{n-n_0}, \quad x^*(t) \iff D_{-n}^*$$

· *Fourier Transform:*

$$X(\omega) = \int_{-\infty}^{\infty} x(t)e^{-j\omega t}dt, \quad x(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} X(\omega)e^{j\omega t}d\omega$$

$$\delta(t) \iff 1, \quad 1 \iff 2\pi\delta(\omega), \quad u(t) \iff \pi\delta(\omega) + \frac{1}{j\omega}, \quad e^{j\omega_0 t} \iff 2\pi\delta(\omega - \omega_0)$$

$$\text{rect}\left(\frac{t}{\tau}\right) \iff \tau \text{sinc}\left(\frac{\omega\tau}{2}\right), \quad \Delta\left(\frac{t}{\tau}\right) \iff \frac{\tau}{2} \text{sinc}^2\left(\frac{\omega\tau}{4}\right), \quad \sum_{n=-\infty}^{\infty} \delta(t - nT_0) = \frac{2\pi}{T_0} \sum_{n=-\infty}^{\infty} \delta\left(\omega - \frac{2\pi n}{T_0}\right)$$

· *Fourier Transform Properties:*

$$x(t) \iff X(\omega), \quad x(t-t_0) \iff e^{-j\omega t_0} X(\omega), \quad x(t)e^{j\omega_0 t} \iff X(\omega - \omega_0)$$

$$X(t) \iff 2\pi x(-\omega), \quad x(at) \iff \frac{1}{|a|} X\left(\frac{\omega}{a}\right), \quad x^*(t) \iff X^*(-\omega), \quad \frac{d^n x}{dt^n} \iff (j\omega)^n X(\omega)$$

$$\int_{-\infty}^t x(\tau)d\tau \iff \pi X(0)\delta(\omega) + \frac{X(\omega)}{j\omega}, \quad x_1(t) * x_2(t) \iff X_1(\omega) \cdot X_2(\omega), \quad x_1(t) \cdot x_2(t) = \frac{1}{2\pi} X_1(\omega) * X_2(\omega)$$

· *Parseval's Theorem:*

$$P_x = \frac{1}{T_0} \int_{T_0} |x(t)|^2 dt = a_0^2 + \frac{1}{2} \sum_{n=1}^{\infty} (a_n^2 + b_n^2) = C_0^2 + \frac{1}{2} \sum_{n=1}^{\infty} C_n^2 = \sum_{n=-\infty}^{\infty} |D_n|^2$$

$$E_x = \int_{-\infty}^{\infty} |x(t)|^2 dt = \frac{1}{2\pi} \int_{-\infty}^{\infty} |X(\omega)|^2 d\omega$$