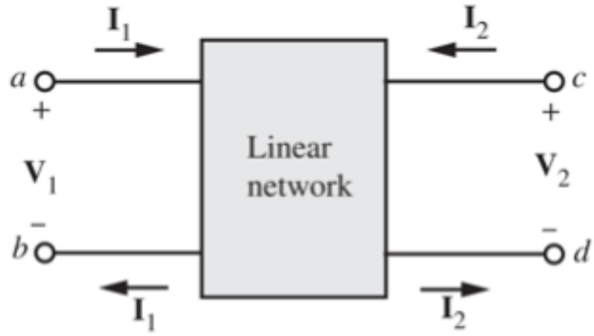
ABCD / Transmission Parameters

$(V_1, I_1) = f(V_2, -I_2)$



$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} V_2 \\ -I_2 \end{bmatrix}$$

Hence,
$$V_1 = A\,V_2 + B(-I_2)$$
$$I_1 = C\,V_2 + D(-I_2)$$

→ $-I_2$ is used as current is considered to be leaving the n/w

ABCD parameters are very useful in the analysis of circuits in cascade like transmission lines/cables.

$$A = \left.\frac{V_1}{V_2}\right|_{I_2=0}, \qquad B = \left.\frac{V_1}{-I_2}\right|_{V_2=0}$$

A → open-circuit voltage ratio

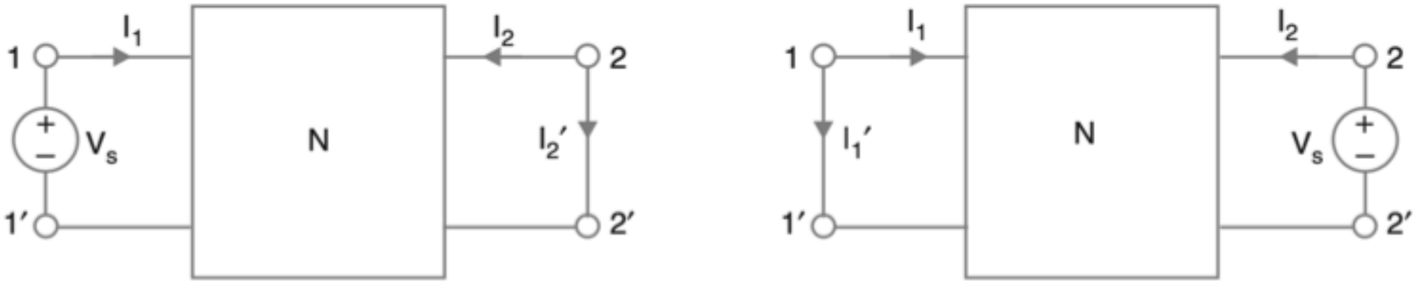
B → negative short-circuit transfer impedance

$$C = \left.\frac{I_1}{V_2}\right|_{I_2=0}, \qquad D = \left.\frac{I_1}{-I_2}\right|_{V_2=0}$$

C → open-circuit transfer admittance

D → negative short-circuit current ratio

Condition for Reciprocity & Symmetry:



$$I_2' = -I_2 = \frac{V_s}{B}$$

and $$I_1' = -I_1 = V_s\left(\frac{AD - BC}{B}\right)$$

For reciprocity,

$$I_1' = I_2' \Rightarrow \underline{\underline{AD - BC = 1}}$$

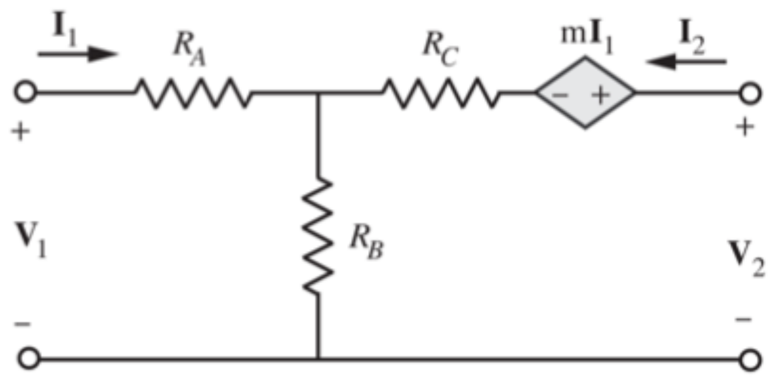
$$\text{or} \quad \begin{vmatrix} A & B \\ C & D \end{vmatrix} = 1$$

For symmetry,

$$\left.\frac{V_1}{I_1}\right|_{I_2=0} = \left.\frac{V_2}{I_2}\right|_{I_1=0}$$

$$\Rightarrow \frac{A}{C} = \frac{D}{C} \quad \text{or} \quad \underline{\underline{A = D}}$$

Q. Determine the ABCD-parameters of:



A. Writing the KVL equations:

$$I_1 (R_A + R_B) + I_2 R_B = V_1$$

$$I_1 (m + R_B) + (R_B + R_C) I_2 = V_2$$

Hence,

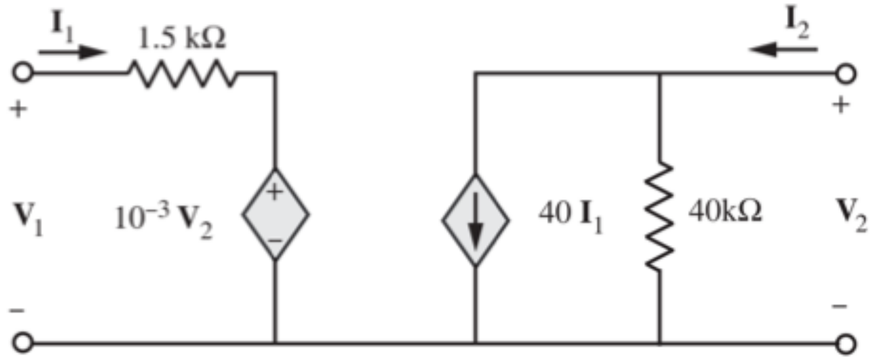
$$A = \left.\frac{V_1}{V_2}\right|_{I_2=0} = \frac{R_A + R_B}{m + R_B}$$

$$B = \left.\frac{V_1}{-I_2}\right|_{V_2=0} = \frac{R_A R_B + R_B R_C + R_A R_C - m R_B}{m + R_B}$$

$$C = \left.\frac{I_1}{V_2}\right|_{I_2=0} = \frac{1}{m + R_B}$$

$$D = \left.\frac{I_1}{-I_2}\right|_{V_2=0} = \frac{R_B + R_C}{m + R_B}$$

Q. Determine the ABCD-parameters of :



A.

$$1.5 \times 10^3 I_1 + 10^{-3} V_2 = V_1$$

$$40 I_1 + \frac{1}{40} \cdot 10^{-3} V_2 = I_2$$

Hence,

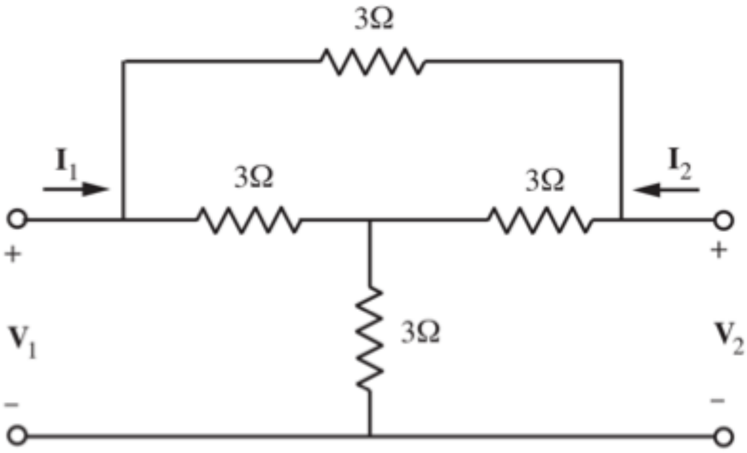
$$A = \left.\frac{V_1}{V_2}\right|_{I_2=0} = 6.25 \times 10^{-5}$$

$$B = \left.\frac{V_1}{-I_2}\right|_{V_2=0} = -37.5\ \Omega$$

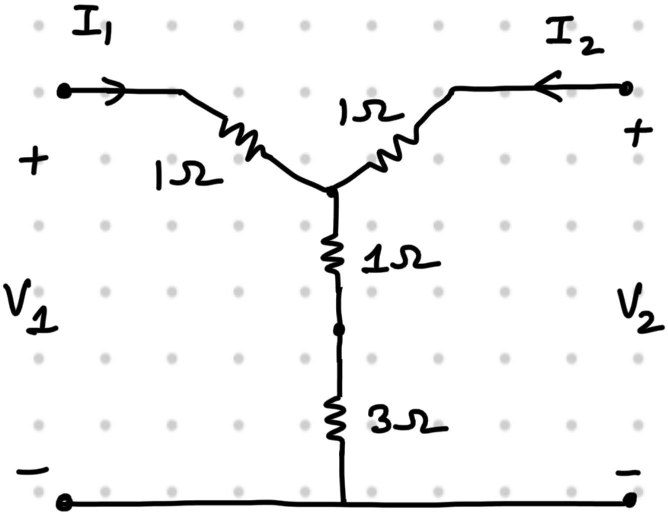
$$C = \left.\frac{I_1}{V_2}\right|_{I_2=0} = -6.25 \times 10^{-7}\ \Omega^{-1}$$

$$D = \left.\frac{I_1}{-I_2}\right|_{V_2=0} = -\frac{1}{40}$$

Q. Determine the ABCD-parameters of :



A. We can simplify this network using $\Delta - Y$ transformation first :



Now, the KVL equations :

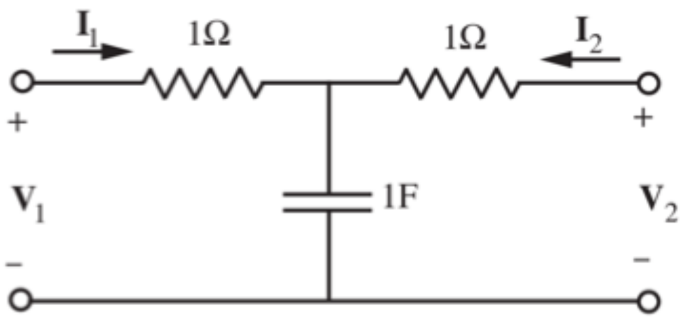
$$\begin{bmatrix} 5 & 4 \\ 4 & 5 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

Hence,

$$A = \left.\frac{V_1}{V_2}\right|_{I_2=0} = \frac{5}{4} \quad , \quad B = \left.\frac{V_1}{-I_2}\right|_{V_2=0} = \frac{9}{4}\,\Omega$$

$$C = \left.\frac{I_1}{V_2}\right|_{I_2=0} = \frac{1}{4}\,\Omega^{-1} \quad , \quad D = \left.\frac{I_1}{-I_2}\right|_{V_2=0} = \frac{5}{4}$$

Q. Determine the ABCD - parameters (s-domain) of :



A. Writing the KVL equations :

$$\begin{bmatrix} 1+\frac{1}{s} & \frac{1}{s} \\ \frac{1}{s} & 1+\frac{1}{s} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

Hence,

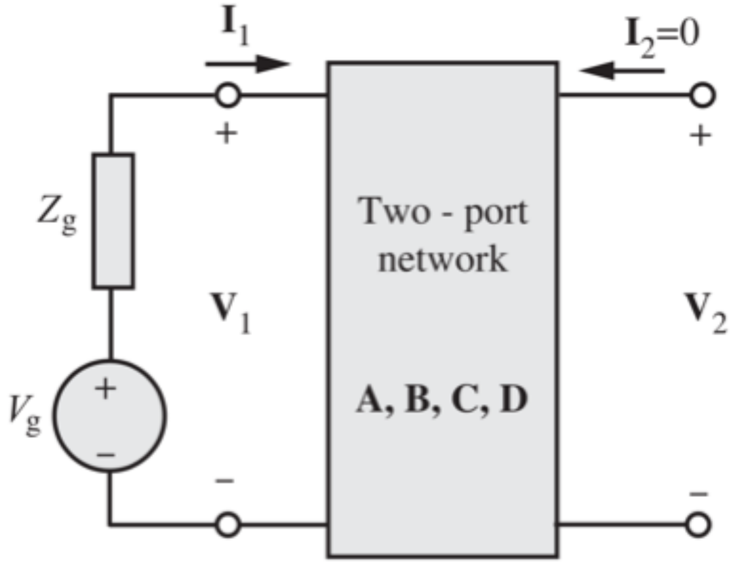
$$A = \left.\frac{V_1}{V_2}\right|_{I_2=0} = s + 1$$

$$B = \left.\frac{V_1}{-I_2}\right|_{V_2=0} = (s+2)\ \Omega$$

$$C = \left.\frac{I_1}{V_2}\right|_{I_2=0} = s\ \Omega^{-1}$$

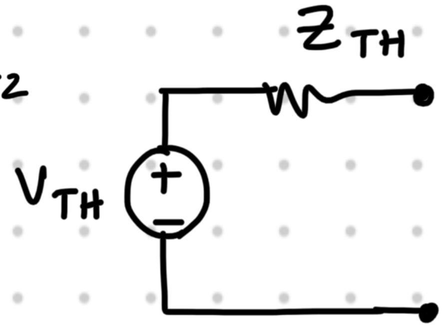
$$D = \left.\frac{I_1}{-I_2}\right|_{V_2=0} = s + 1$$

Q. Determine the Thevenin equivalent circuit at the output of :



A. We have, $V_1 = AV_2 - BI_2$

$I_1 = CV_2 - DI_2$



To find V_{TH} : $I_2 = 0$

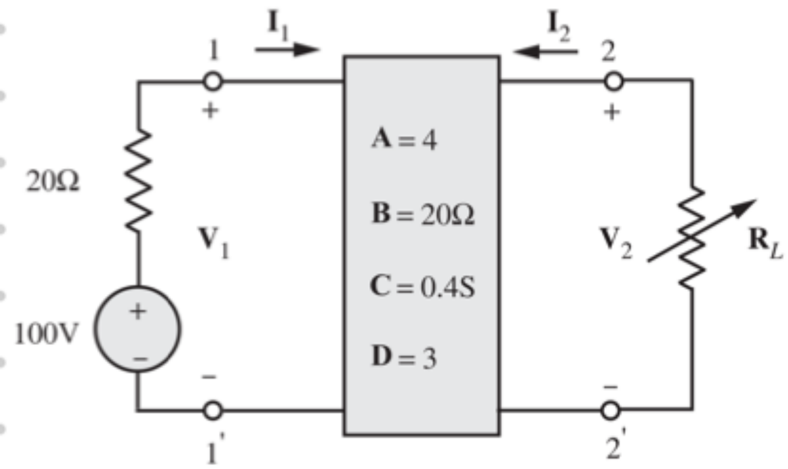
$$\Rightarrow V_{TH} = V_2 = \frac{V_1}{A} = \frac{V_g - I_1 Z_g}{A} = \frac{V_g - CV_2 Z_g}{A}$$

Hence, $$V_{TH} = V_2 = \frac{V_g}{A + CZ_g}$$

To find Z_{TH} : $V_1 = -I_1 Z_g$

$$\Rightarrow Z_{TH} = \frac{V_2}{I_2} = \frac{B + DZ_g}{A + CZ_g}$$

Q. Determine the maximum power transferred in :

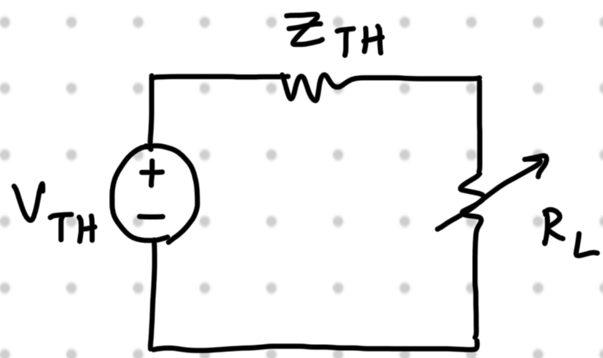


A. $V_1 = 4V_2 - 20 I_2$

$I_1 = 0.4 V_2 - 3 I_2$

For maximum power transfer: $R_L = Z_{TH}$

$$V_{TH} = \frac{100}{4 + (0.4)20}$$



$\Rightarrow \quad V_{TH} \approx 8.33\,V$

$$Z_{TH} = \frac{20 + 3(20)}{4 + 0.4(20)} \approx 6.66\,\Omega = R_L$$

Hence, $P_{max} = \dfrac{V_{TH}^2}{4R_L} \approx 2.6\,W$