Active Filters :

Passive filters cannot generate gain greater than 1, may require bulky and expensive inductors, and cause loading.

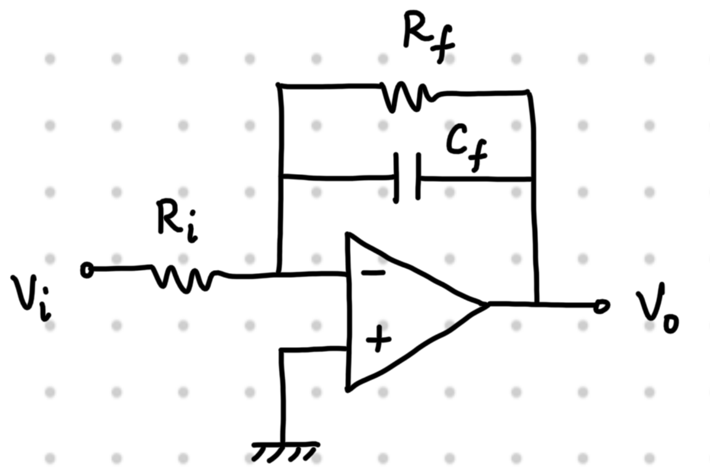
Active filters address these problems. However, they are less stable and introduce noise.

We shall now discuss the four types of filters using active components.

1. Low-Pass Filter :

An active low pass filter can be built using the circuit below.

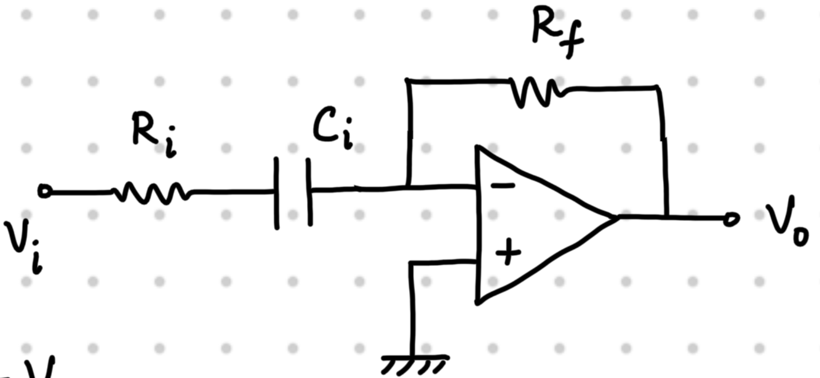
$$\frac{V_i}{R_i} = \frac{-V_o}{\dfrac{R_f \cdot \frac{1}{C_f s}}{R_f + \frac{1}{C_f s}}}$$



$$\Rightarrow \quad H(s) = \frac{V_o}{V_i} = \frac{-R_f}{R_i} \cdot \frac{1}{1 + s R_f C_f}$$

Hence, $$\left| H(j\omega) \right| = \underbrace{\frac{R_f}{R_i}}_{\text{maximum gain}} \cdot \frac{1}{\sqrt{1 + \omega^2 R_f^2 C_f^2}} \quad , \quad \underbrace{\omega_c}_{\text{corner frequency}} = \frac{1}{R_f C_f}$$

2. <u>High-Pass Filter</u>: An active high pass filter can be built using the circuit below.



$$\frac{V_i}{R_i + \frac{1}{C_i s}} = \frac{-V_o}{R_f}$$

$$\Rightarrow \quad \frac{V_i\, C_i\, s}{1 + s R_i C_i} = \frac{-V_o}{R_f}$$

$$\Rightarrow \quad H(s) = \frac{V_o}{V_i} = \frac{-R_f C_i s}{1 + s R_i C_i}$$

$$\Rightarrow \quad H(j\omega) = \frac{-j R_f C_i \omega}{1 + j\omega R_i C_i}$$

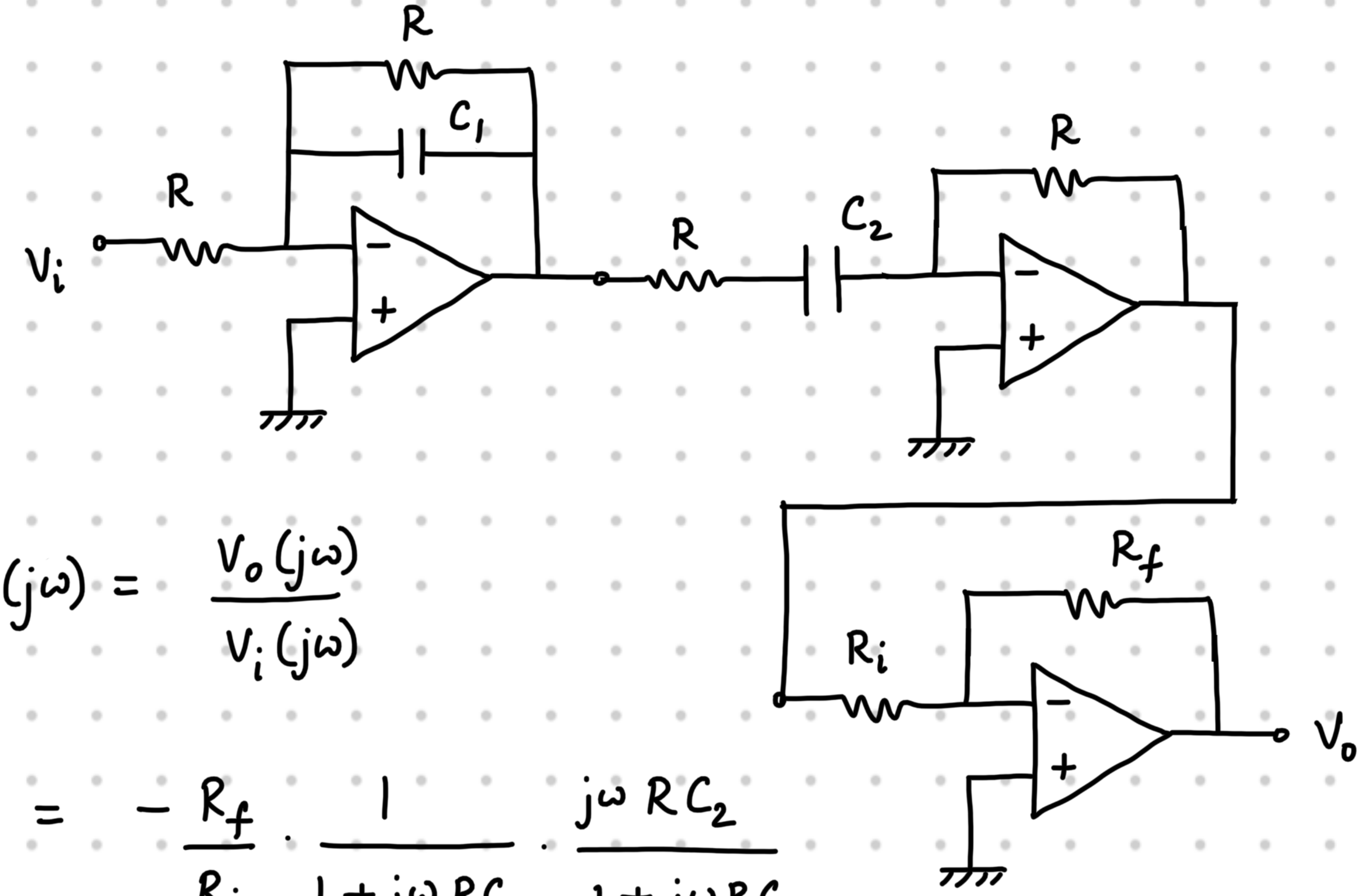
Hence, $$|H(j\omega)| = \frac{R_f C_i \omega}{\sqrt{1 + \omega^2 R_i^2 C_i^2}}, \quad \omega_c = \frac{1}{R_i C_i}$$

corner frequency

Clearly, $|H(0)| = 0$

$$|H(\infty)| = \frac{R_f}{R_i}$$

3. <u>Band-Pass Filter</u>: An active band pass filter can be built by cascading a low pass filter with a high pass filter as shown.



$$H(j\omega) = \frac{V_o(j\omega)}{V_i(j\omega)}$$

$$= -\frac{R_f}{R_i} \cdot \frac{1}{1 + j\omega RC_1} \cdot \frac{j\omega RC_2}{1 + j\omega RC_2}$$

Clearly, $H(0) = H(\infty) = 0$

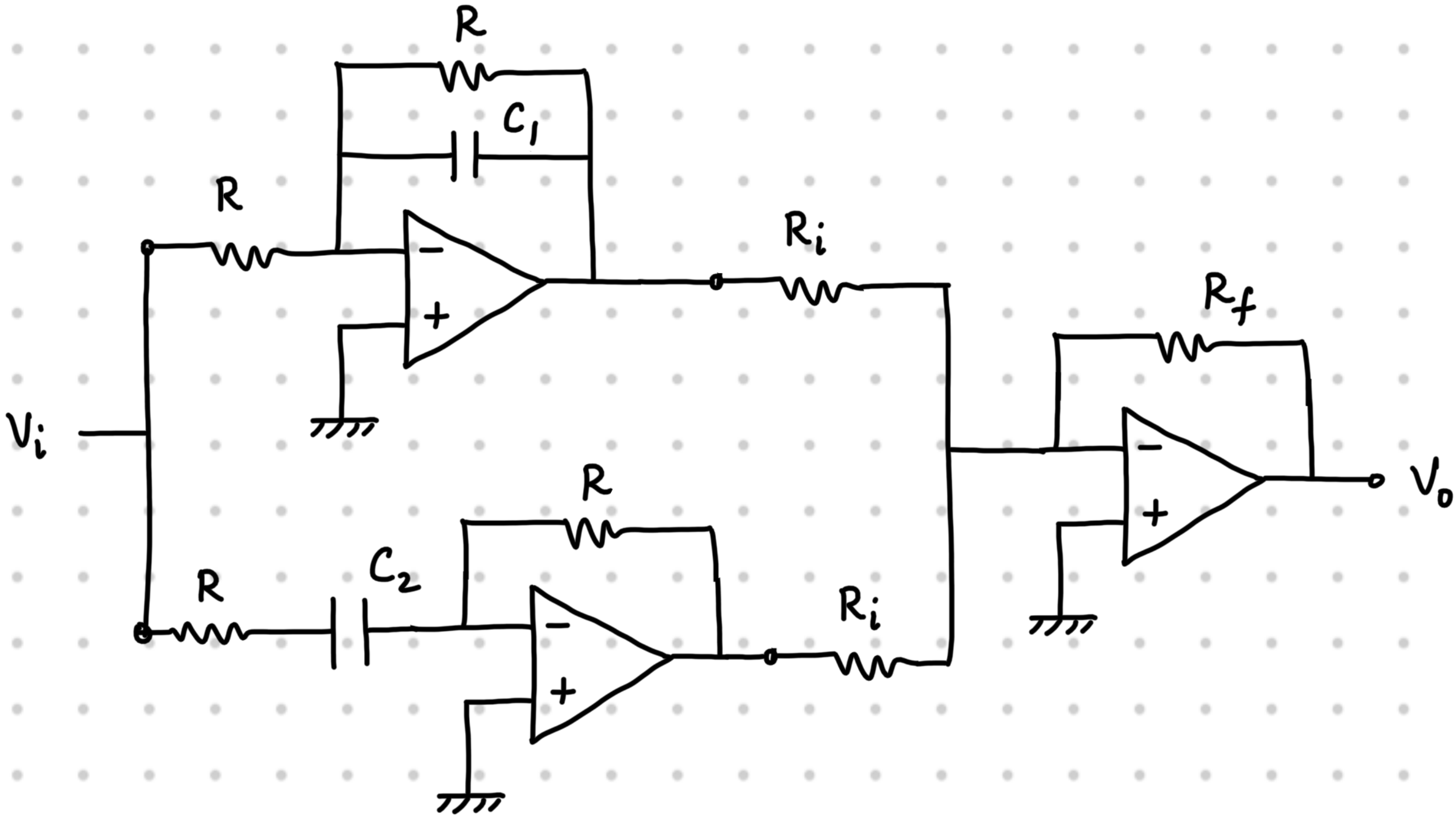
The corner frequencies are $\omega_1 = \frac{1}{RC_2}$ & $\omega_2 = \frac{1}{RC_1}$

The center-frequency is given as: $\omega_0 = \sqrt{\omega_1 \omega_2}$

The maximum gain (passband gain) is:

$$|H(j\omega_0)| = \frac{R_f}{R_i} \cdot \frac{\omega_2}{\omega_1 + \omega_2}$$

4. <u>Band-Stop Filter</u>: An active band stop filter can be built by summing a low pass filter with a high pass filter as shown.



$$H(j\omega) = \frac{V_o(j\omega)}{V_i(j\omega)} = \frac{R_f}{R_i}\left(\frac{1}{1+j\omega RC_1} + \frac{j\omega RC_2}{1+j\omega RC_2}\right)$$

Clearly, $H(0) = H(\infty) = K = \dfrac{R_f}{R_i}$

As before, $\omega_1 = \dfrac{1}{RC_2}$, $\omega_2 = \dfrac{1}{RC_1}$, $\omega_0 = \sqrt{\omega_1\omega_2}$

The minimum gain (stop band gain) is:

$$|H(j\omega_0)| = \frac{R_f}{R_i} \cdot \frac{2\omega_1}{\omega_1+\omega_2}$$