Network Theorems

In large and complex networks, generalized methods of network analysis tend to become laborious and time-consuming. In such cases, it is would be useful to replace the network which is not of interest by a simpler equivalent.

Network theorems help in simplifying computations, providing good insight into the problems, and presenting simple conclusions.

Superposition Theorem:

The response of a linear network to a number of excitations applied simultaneously is equal to the sum of the responses of the network when each excitation is applied individually.

Consider a linear network N having m independent meshes. The mesh equations are:

$$\underset{m \times m}{[Z]}\ \underset{m \times 1}{[I]} = \underset{m \times 1}{[E]}$$

$$\Rightarrow \quad [I] = [Z]^{-1}[E] = [P][E]$$

$$\Rightarrow \quad I_i = \sum_{j=1}^{m} P_{ij}\, E_j \quad ; \quad i = 1, 2, \ldots, m$$

Hence, the resulting response I_i is obtained by adding the individual responses $P_{ij}\, E_j$.

On a similar line, let a linear network N have $(n+1)$ nodes. The node equations are:

$$[Y][E] = [I] \;\Rightarrow\; [E] = [Y]^{-1}[I] = [Q][I]$$

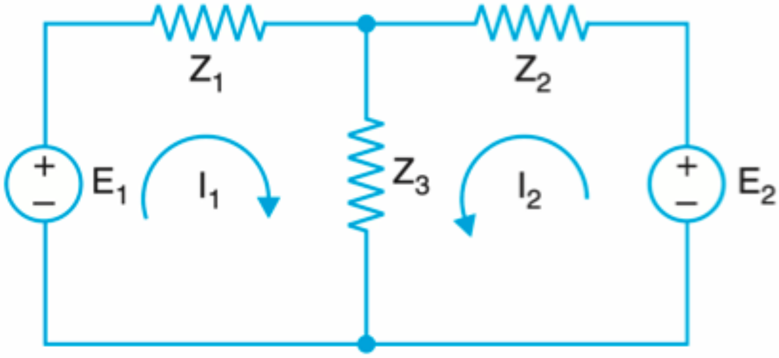
$$\Rightarrow \quad E_i = \sum_{j=1}^{n} Q_{ij}\, I_j \;, \quad i = 1, 2, \ldots, n$$

Hence, the individual responses $Q_{ij}\, I_j$ are added together to give the resulting response E_i, which is same as obtained by solving simultaneous equations.

<u>Note:</u> The superposition principle is intimately tied with the idea of linearity; we cannot have one without the other. We can say that linearity is necessary and sufficient for the validity of the principle of superposition.

Illustration of Superposition Principle:

Suppose we have a network with two sources as shown.



Then, we can write the KVL equations:

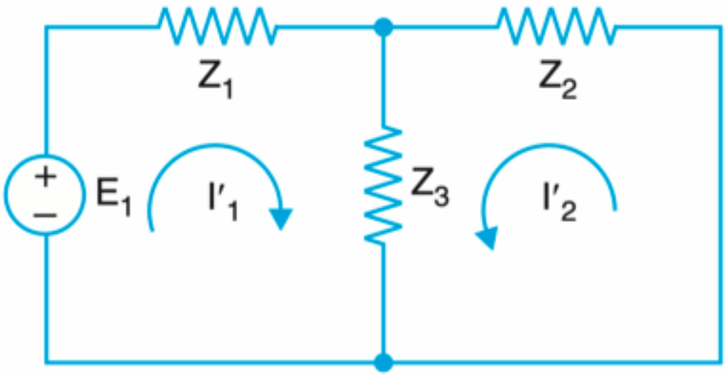
$$\begin{bmatrix} z_1 + z_3 & z_3 \\ z_3 & z_2 + z_3 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} E_1 \\ E_2 \end{bmatrix}$$

$$\Rightarrow \quad I_1 = \frac{z_2 + z_3}{z_1 z_2 + z_2 z_3 + z_3 z_1} E_1 - \frac{z_3}{z_1 z_2 + z_2 z_3 + z_3 z_1} E_2$$

and $$I_2 = \frac{-z_3}{z_1 z_2 + z_2 z_3 + z_3 z_1} E_1 + \frac{z_1 + z_3}{z_1 z_2 + z_2 z_3 + z_3 z_1} E_2$$

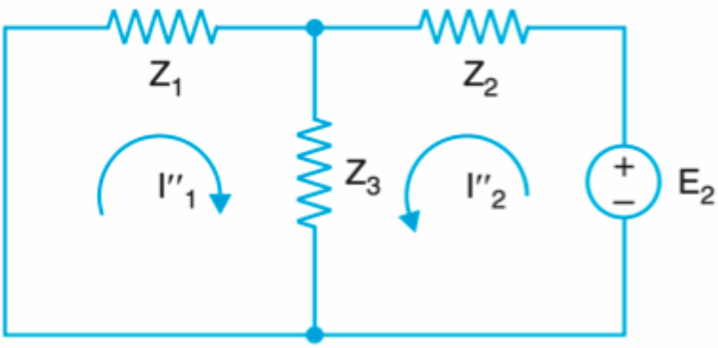
Now, if E_1 alone is active,

$$\begin{bmatrix} z_1 + z_3 & z_3 \\ z_3 & z_2 + z_3 \end{bmatrix} \begin{bmatrix} I_1' \\ I_2' \end{bmatrix} = \begin{bmatrix} E_1 \\ 0 \end{bmatrix}$$



$$\Rightarrow I_1' = \frac{Z_2 + Z_3}{Z_1Z_2 + Z_2Z_3 + Z_3Z_1} E_1 \quad \text{and} \quad I_2' = \frac{-Z_3}{Z_1Z_2 + Z_2Z_3 + Z_3Z_1} E_1$$

Similarly, if E_2 alone is active,



$$\begin{bmatrix} Z_1 + Z_3 & Z_3 \\ Z_3 & Z_2 + Z_3 \end{bmatrix} \begin{bmatrix} I_1'' \\ I_2'' \end{bmatrix} = \begin{bmatrix} 0 \\ E_2 \end{bmatrix}$$

$$\Rightarrow I_1'' = \frac{-Z_3}{Z_1Z_2 + Z_2Z_3 + Z_3Z_1} E_2 \quad \text{and} \quad I_2'' = \frac{Z_1 + Z_3}{Z_1Z_2 + Z_2Z_3 + Z_3Z_1} E_2$$

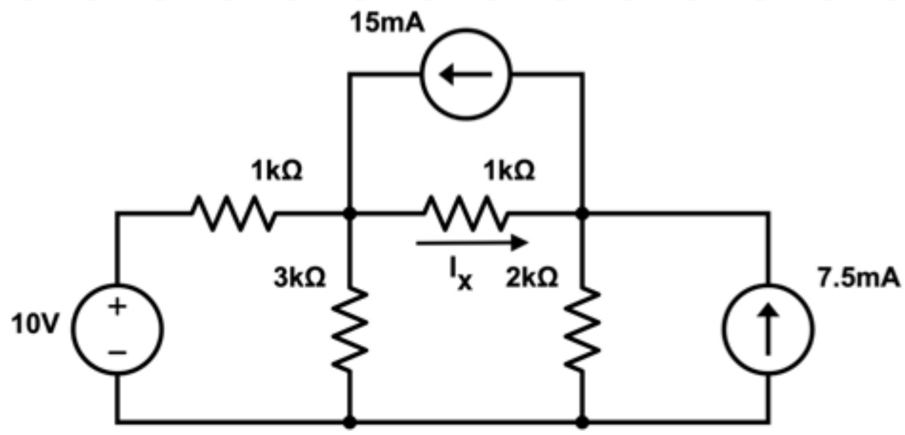
Using the superposition principle, we can find the total response as the sum of responses with E_1 and E_2 alone. That is,

$$I_1' + I_1'' = \frac{Z_2 + Z_3}{Z_1Z_2 + Z_2Z_3 + Z_3Z_1} E_1 - \frac{Z_3}{Z_1Z_2 + Z_2Z_3 + Z_3Z_1} E_2$$

$$I_2' + I_2'' = \frac{-Z_3}{Z_1Z_2 + Z_2Z_3 + Z_3Z_1} E_1 + \frac{Z_1 + Z_3}{Z_1Z_2 + Z_2Z_3 + Z_3Z_1} E_2$$

Hence, $I_1' + I_1'' = I_1$ and $I_2' + I_2'' = I_2$

Q. Using Superposition theorem, find the current I_x.



A. Response due to 10V source alone :

$$I_{x_1} = \frac{1}{2} \cdot \frac{10}{2.5\,k\Omega} = 2\,mA$$

Response due to 15mA source alone :

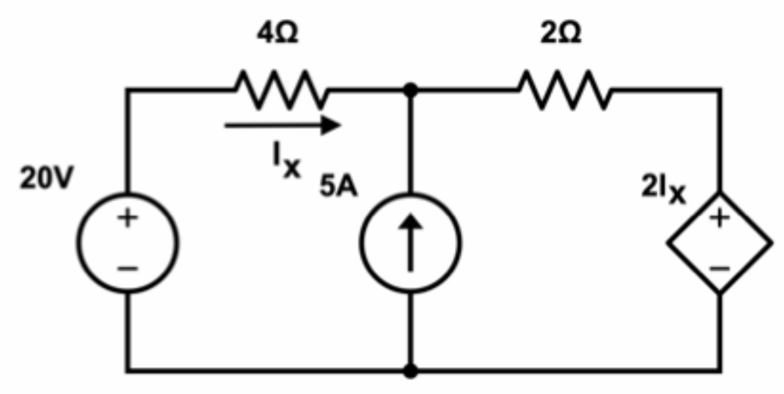
$$I_{x_2} = 15\,mA \cdot \frac{2.75\,k\Omega}{3.75\,k\Omega} = 11\,mA$$

Response due to 7.5mA source alone :

$$I_{x_3} = -7.5\,mA \cdot \frac{2\,k\Omega}{3.75\,k\Omega} = -4\,mA$$

Hence, the total current $I_x = 9\,mA$.

Q. Using Superposition theorem, find the current I_x in the circuit.



A. Response due to 20V source alone :

$$20 - 2I_{x_1} = 6I_{x_1} \Rightarrow I_{x_1} = 2.5\,A$$

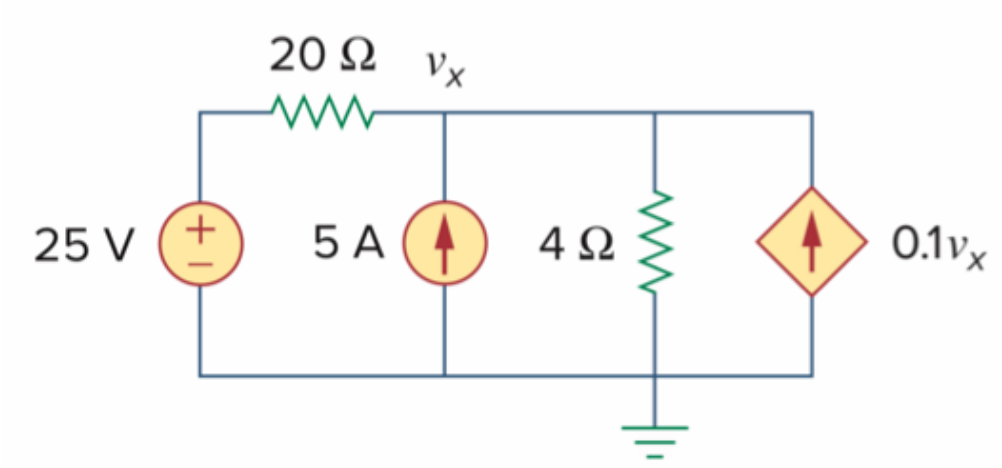
Response due to 5A source alone :

$$2I_{x_2} + 2(5 + I_{x_2}) + 4I_{x_2} = 0$$

$$\Rightarrow 8I_{x_2} = -10 \Rightarrow I_{x_2} = -1.25\,A$$

Hence, $I_x = 2.5 - 1.25 = 1.25\,A$

Q. Using Superposition theorem, find the voltage v_x :



A. Response due to 25 V source alone :

$$\frac{25 - V_{x_1}}{20} + 0.1\, V_{x_1} = \frac{V_{x_1}}{4}$$

$$\Rightarrow \quad V_{x_1} = \frac{25}{4}\ \text{V}$$

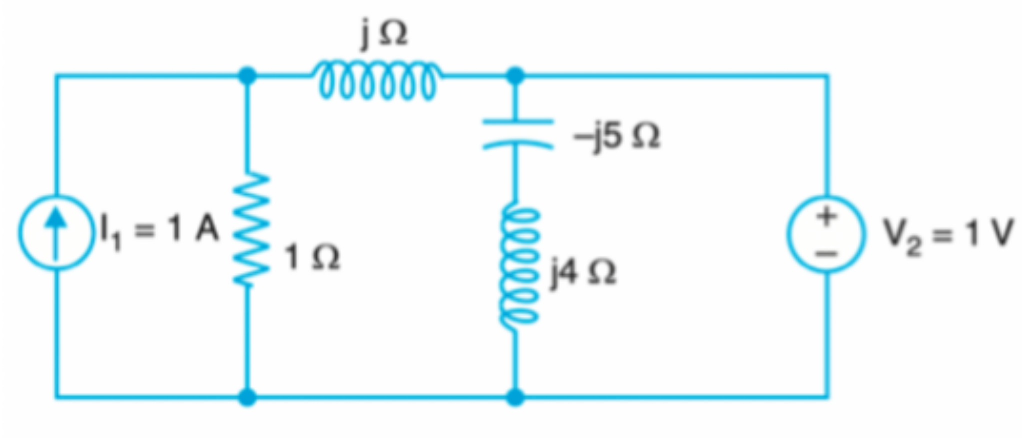
Response due to 5A source alone :

$$5 + 0.1\, V_{x_2} = \frac{V_{x_2}}{20} + \frac{V_{x_2}}{4}$$

$$\Rightarrow \quad V_{x_2} = 25\ \text{V}$$

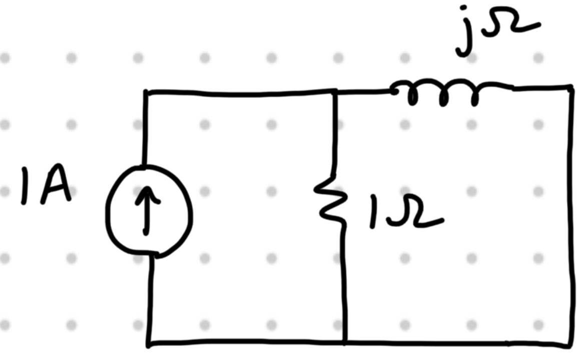
Hence, $V_x = 25 + \frac{25}{4} = 31.25\ \text{V}$

Q. Using Superposition theorem, find the voltage across the resistor.



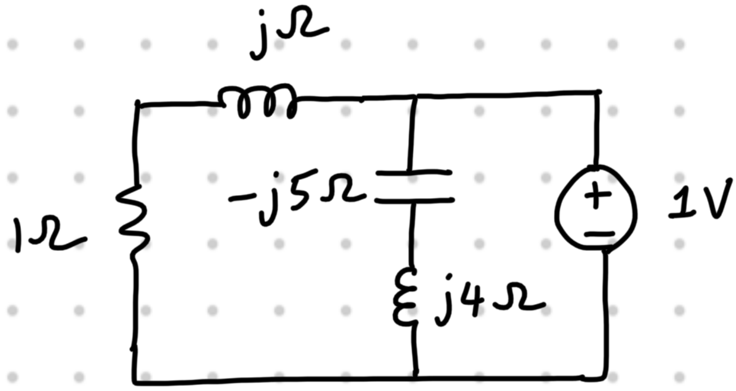
A. Response due to 1A alone:

$$V_{R_1} = \frac{j}{1+j}$$



Response due to 1V alone:

$$V_{R_2} = \frac{1}{1+j}$$



Hence, the net voltage $= V_{R_1} + V_{R_2} = 1\,V$