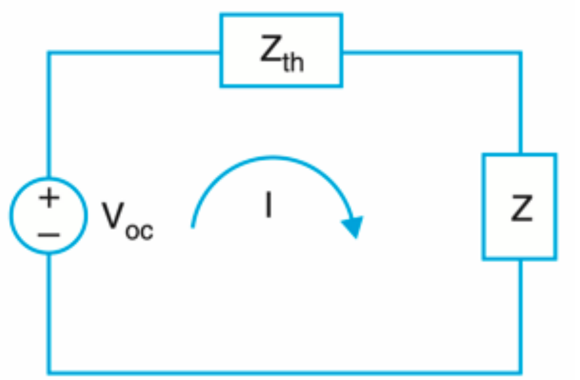
Compensation Theorem :

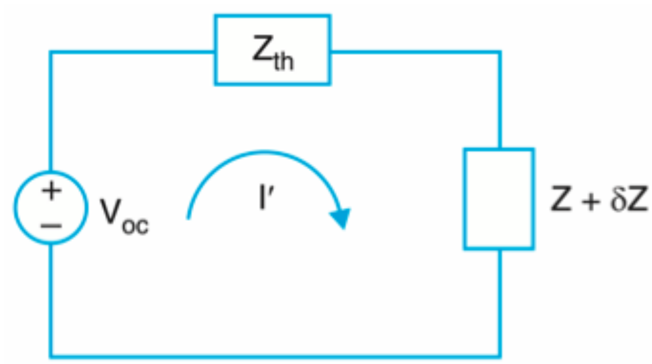
In a linear network N, if the current in a branch is I and the impedance Z of the branch is changed by δZ, then the incremental voltage and current in each branch of the network is that voltage or current that would be produced by an opposing voltage source of value $V_c = I\,\delta Z$ introduced in the altered branch after the modification.

Consider the circuit shown below:



We have,

$$I = \frac{V_{oc}}{Z + Z_{th}}$$



Let δZ be the change in Z, Then

$$I' = \frac{V_{oc}}{Z + \delta Z + Z_{th}}$$

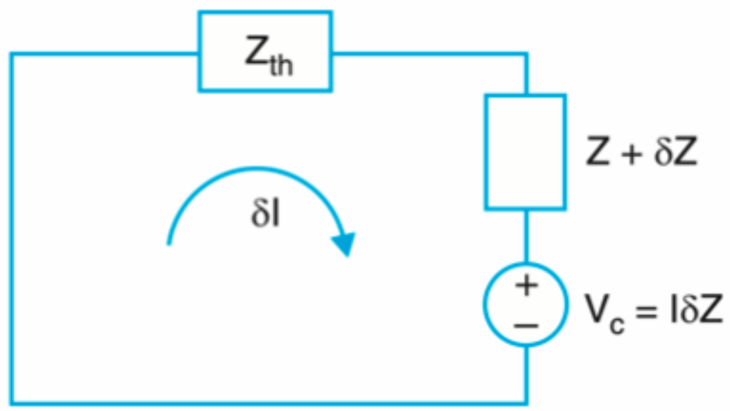
$$\Rightarrow \quad \delta I = I' - I = \frac{V_{oc}}{Z + \delta Z + Z_{th}} - \frac{V_{oc}}{Z + Z_{th}}$$

$$\Rightarrow \quad \delta I = \frac{-V_{oc}}{Z + Z_{th}} \cdot \frac{\delta Z}{Z + \delta Z + Z_{th}}$$

$$= \frac{-I\,\delta Z}{Z + \delta Z + Z_{th}} = \frac{-V_c}{Z + \delta Z + Z_{th}}$$

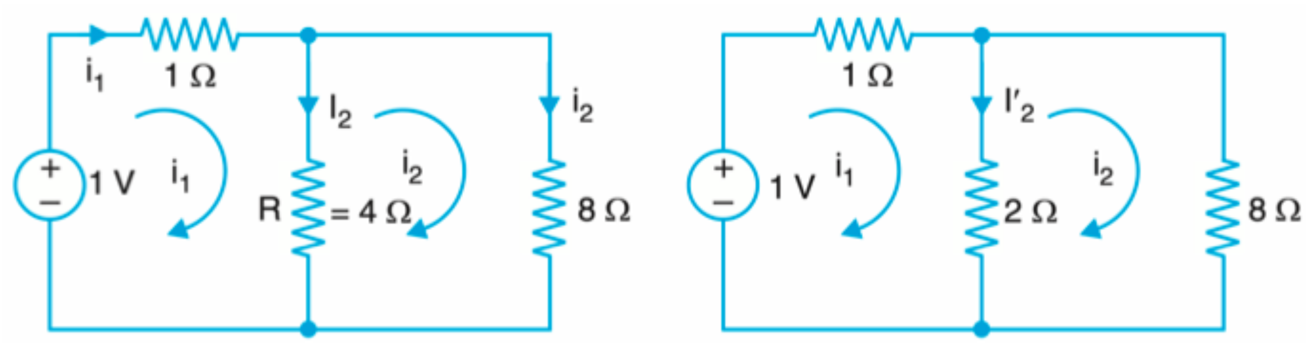
where $V_c = I\,\delta Z$

Hence, this shows that the change in current δI due to a change in any branch in a linear network can be calculated



by determining the current in that branch in a network obtained from the original network by nulling all the independent sources and placing a voltage source called the compensation source in series with the branch whose value is $V_c = I\,\delta Z$ as shown above. Note that the direction of V_c is opposite to that of I.

Q. In the circuit below, the resistor R is changed from 4Ω to 2Ω. Verify the compensation theorem.



A. For the circuit with $R = 4\Omega$,

$$i_1 = \frac{3}{11}\,A, \quad i_2 = \frac{1}{11}\,A \Rightarrow I_1 = i_1 - i_2 = \frac{2}{11}\,A$$

For the circuit with $R = 2\Omega$,

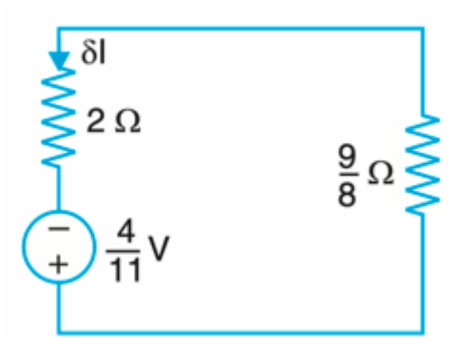
$$i_1 = \frac{10}{26}\,A, \quad i_2 = \frac{2}{26}\,A \Rightarrow I_2 = i_1 - i_2 = \frac{8}{26}\,A$$

Hence, $\delta I = I_2 - I_1 = \frac{18}{143}\,A$

Using the compensation theorem,

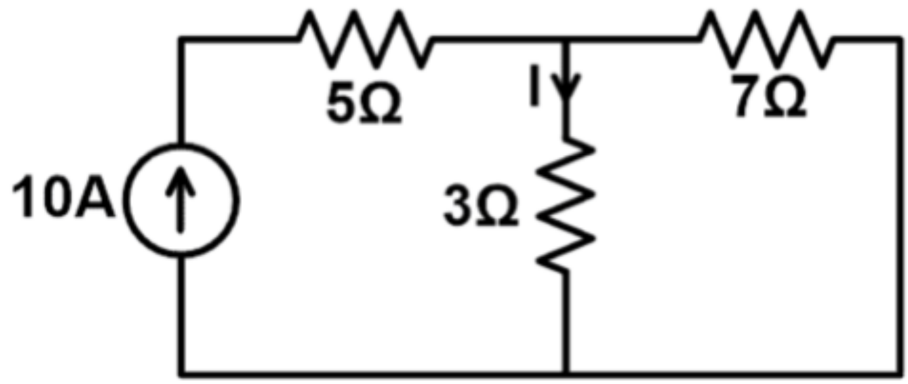
$$V_c = I_2(\delta Z) = \frac{2}{11}(-2) = \frac{-4}{11}\,V$$

Using the modified circuit,



$$\delta I = \frac{\frac{4}{11}}{2 + \frac{8}{9}} = \frac{18}{143}\,A.$$

Q. Using compensation theorem, find the change in current in the 3Ω resistor if it is replaced by a 7Ω resistor.



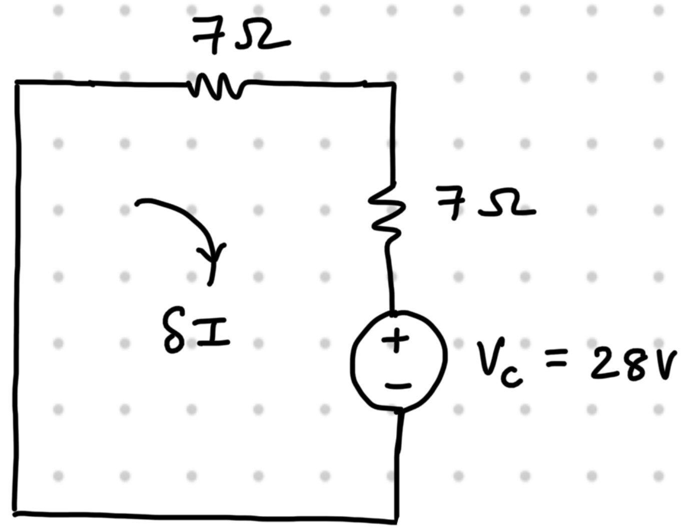
A. Across the 3Ω resistor,

$$V_{TH} = 70\,V, \quad Z_{TH} = \frac{70}{10} = 7\Omega$$

Hence,

$$V_c = \left(\frac{7}{10} \cdot 10\right) \cdot 4$$

$$= 28\,V$$



$$\delta I = \frac{-28}{14} = -2\,A$$

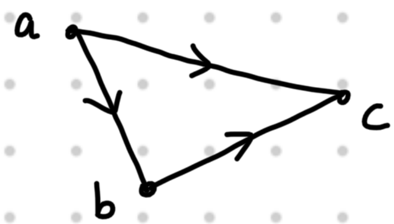
Therefore, change in current $= -2A$

and new current in $3\Omega = 7 - 2 = 5A$

Tellegen's Theorem:

At any given time, the sum of power delivered to each branch of any electric network is zero $\equiv$ Conservation of Power.

We can easily verify this for a simple three node network:



$$(V_a - V_b)\, i_{ab} + (V_b - V_c)\, i_{bc} + (V_a - V_c)\, i_{ac} = 0$$

Consider an arbitrary network with b branches and n nodes. Let the branch voltages $v_1, v_2, \cdots, v_b$ satisfy all the constraints imposed by KVL and the branch currents $i_1, i_2, \cdots, i_b$ satisfy all the constraints imposed by KCL. Then the node voltages of the network $e_1, e_2, \ldots, e_p, \ldots, e_q, \cdots e_n$ are uniquely specified.

Now, let us assume that k^{th} branch connects the nodes p and q and take node 1 as the datum node $(e_1 = 0)$.

Then, $v_k i_k = (e_p - e_q)\, i_{pq}$

For all the b branches,

$$\Rightarrow \sum_{k=1}^{b} v_k i_k = \frac{1}{2} \sum_{p=1}^{n} \sum_{q=1}^{n} (e_p - e_q) i_{pq}$$

$$\Rightarrow \sum_{k=1}^{b} v_k i_k = \frac{1}{2} \sum_{p=1}^{n} e_p \sum_{q=1}^{n} i_{pq} - \frac{1}{2} \sum_{q=1}^{n} e_q \sum_{p=1}^{n} i_{pq}$$

According to KCL, $\sum_{q=1}^{n} i_{pq} = \sum_{p=1}^{n} i_{pq} = 0$

Therefore, $\sum_{k=1}^{b} v_k i_k = 0$

Hence, the sum of the power delivered to all branches of a network is zero.

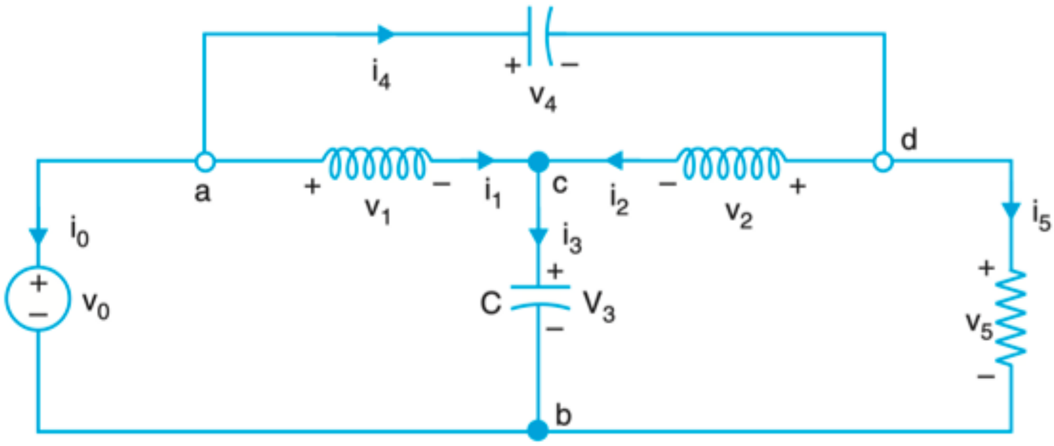
Note: Consider two networks N_1 and N_2, having the same graph with the same reference directions assigned to the branches of these networks, but with different element values and kinds. Let v_{k_1} and i_{k_1} be the voltages and currents in N_1 and v_{k_2} and i_{k_2} the voltages and currents in N_2. With all currents and voltages satisfying Kirchhoff's laws, then by

Tellegen's theorem: $\sum_{k=1}^{b} v_{k_1} i_{k_2} = 0 = \sum_{k=1}^{b} v_{k_2} i_{k_1}$

Q. Verify Tellegen's theorem for the network shown below. Given :

$v_0 = 20V$, $v_1 = 16V$, $v_2 =$ ___, $v_3 =$ ___, $v_4 =$ ___, $v_5 = 6V$

$i_0 = -16A$, $i_1 =$ ___, $i_2 = 2A$, $i_3 =$ ___, $i_4 = 4A$, $i_5 =$ ___ .



A. First, using KCL and KVL find the missing voltages and currents.

Hence,

$v_0 = 20V$, $v_1 = 16V$, $v_2 = \underline{2V}$, $v_3 = \underline{4V}$, $v_4 = \underline{14V}$, $v_5 = 6V$

$i_0 = -16A$, $i_1 = \underline{12A}$, $i_2 = 2A$, $i_3 = \underline{14A}$, $i_4 = 4A$, $i_5 = \underline{2A}$.

Now,

$$\sum_{k=0}^{5} v_k i_k = 20(-16) + 16(12) + 2(2) + 4(14) + 14(4) + 6(2)$$

$$= 0$$