Poles and Zeros

All the network functions we discussed are clearly the ratio of polynominals of "s" and have the general form:

$$N(s) = \frac{A(s)}{B(s)} = \frac{a_0 s^n + a_1 s^{n-1} + \cdots + a_{n-1} s + a_n}{b_0 s^m + b_1 s^{m-1} + \cdots + b_{m-1} s + b_m}$$

Suppose $A(s) = 0$ has n roots $z_1, z_2, \cdots, z_n$ and $B(s) = 0$ has m roots $p_1, p_2, \cdots, p_m$.

Then, $$N(s) = \frac{a_0}{b_0} \frac{(s-z_1)(s-z_2)\cdots(s-z_n)}{(s-p_1)(s-p_2)\cdots(s-p_m)}$$

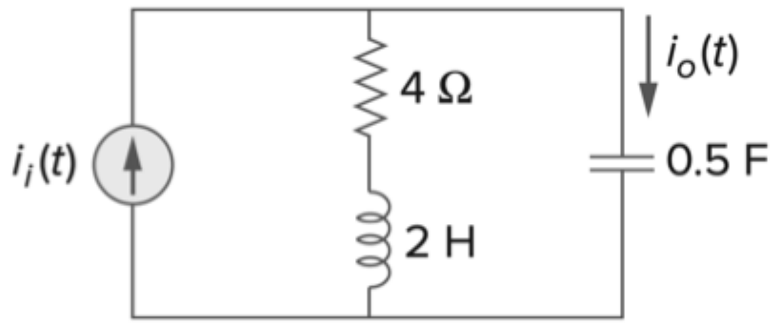
Clearly, $N(s) = 0$ for $s = z_i$ $(i = 1, 2, \cdots n)$. Such complex frequencies are known as the <u>zeros</u> of the network function.

Further, $N(s) \to \infty$ for $s = p_i$ $(i = 1, 2, \cdots m)$. Such complex frequencies are known as the <u>poles</u> of the network function.

<u>Note</u>: A network function having real and complex poles and zeros is <u>stable</u> if the real parts of the poles are negative (poles in LHP).

Q. Calculate the current gain $\frac{I_o(s)}{I_i(s)}$ of the circuit :

What are its poles and zeros ?



A. $$I_o(s) = \frac{4+2s}{4+2s+\frac{2}{s}} \cdot I_i(s)$$

$$\Rightarrow \frac{I_o(s)}{I_i(s)} = \frac{s^2+2s}{s^2+2s+1}$$

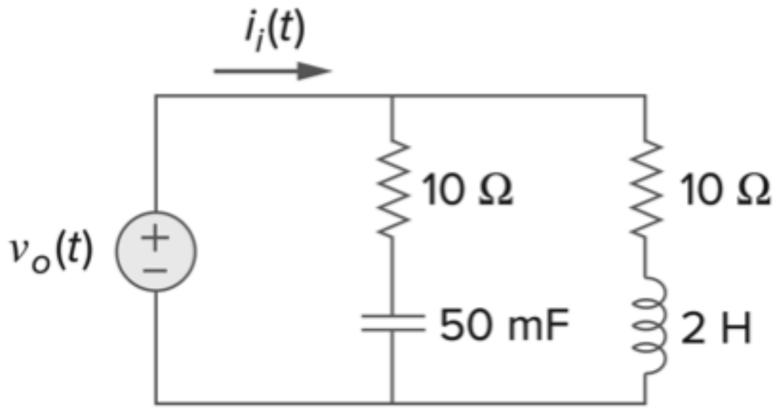
Zeros : $s^2+2s = 0 \Rightarrow z_1 = 0, \; z_2 = -2$

Poles : $s^2+2s+1 = 0 \Rightarrow p_1 = p_2 = -1$

Both the poles are in the LHP. Hence, the network function is stable.

Q. Calculate the transfer function $\frac{V_o(s)}{I_i(s)}$ of the circuit :

What are its poles and zeros ?



A. $\frac{V_o(s)}{I_i(s)}$ is simply the input impedance of this circuit.

$$\Rightarrow \quad \frac{V_o(s)}{I_i(s)} = \frac{(10+2s)\left(10+\frac{20}{s}\right)}{10+2s+10+\frac{20}{s}} = \frac{10s^2+70s+100}{s^2+10s+10}$$

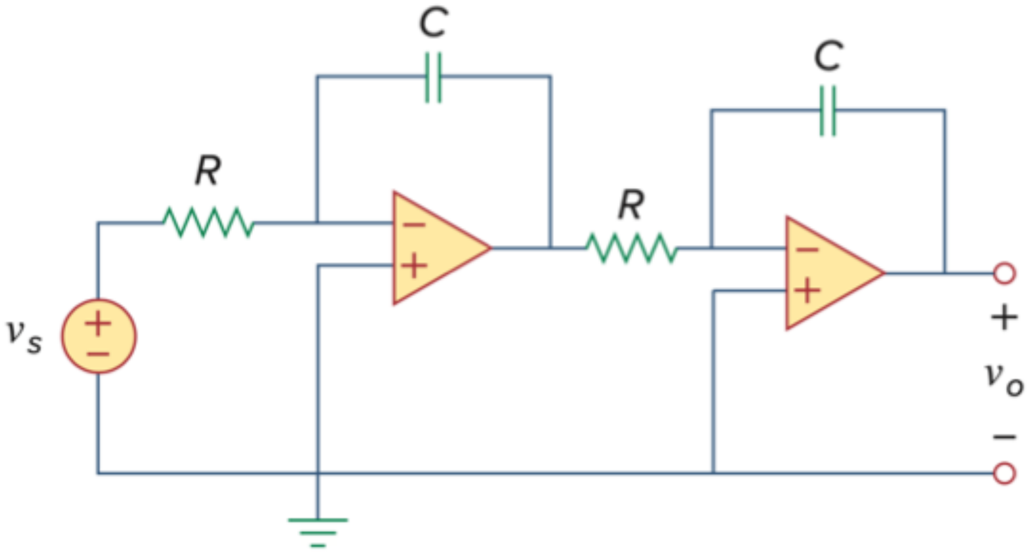
Zeros : $10s^2+70s+100 = 0 \Rightarrow z_1 = -2 \, , \; z_2 = -5$

Poles : $s^2+10s+10 = 0 \Rightarrow p_1 = -5+\sqrt{15}$

$p_2 = -5-\sqrt{15}$

Both the poles are in the LHP. Hence, the network function is stable.

Q. Is this op amp circuit stable ?



A. $$\frac{V_s(s)}{R} = -\frac{V_{o_1}(s)}{\frac{1}{Cs}}$$

and $$\frac{V_{o_1}(s)}{R} = -\frac{V_o(s)}{\frac{1}{Cs}}$$

$$\Rightarrow \quad \frac{V_o(s)}{V_s(s)} = \frac{1}{s^2R^2C^2}$$

(unstable as repeated poles on the imaginary axis)

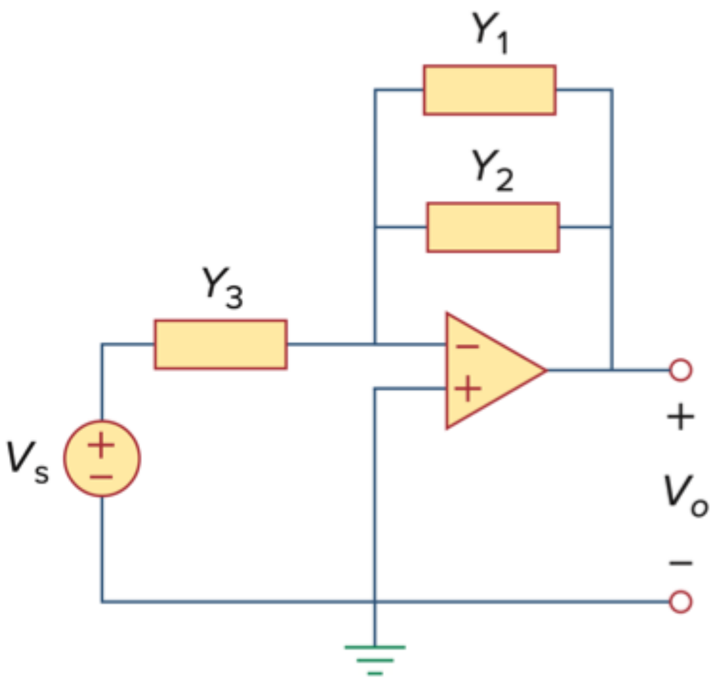
Also, $$\frac{V_o(s)}{V_s(s)} \Longleftrightarrow \frac{t\,u(t)}{R^2C^2}$$ (unstable)

Q. Let $Y_1 = sC_1$, $Y_2 = \frac{1}{R_1}$, $Y_3 = sC_2$.

Choose $R_1 = 1\ k\Omega$,

determine C_1 & C_2 such

that $\frac{V_o(s)}{V_s(s)} = \frac{-s}{s+10}$.



A. $$\frac{V_s(s)}{\frac{1}{C_2 s}} = -V_o(s)\left(\frac{1}{R_1} + \frac{1}{\frac{1}{C_1 s}}\right)$$

$$\Rightarrow \quad \frac{V_o(s)}{V_s(s)} = \frac{-s\left(\frac{C_2}{C_1}\right)}{s + \frac{1}{R_1 C_1}} = \frac{-s}{s+10}$$

Hence, $C_1 = C_2$ and $\frac{1}{10^3 C_1} = 10$

$$\Rightarrow \quad C_1 = C_2 = 100\ \mu F$$