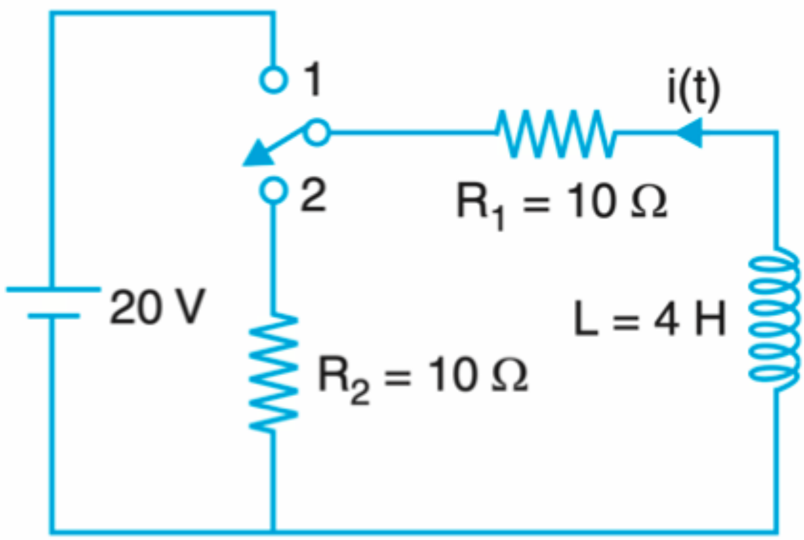
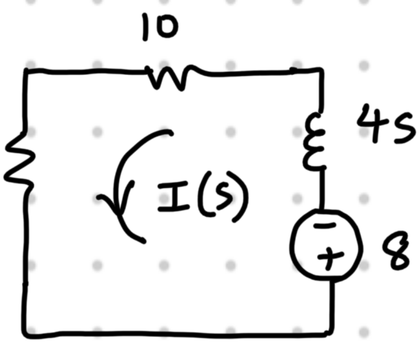
Q. In the circuit shown below, the switch is thrown from position 1 to 2 at time $t=0$ after being in steady-state for long time. Find the energy dissipated in R_1 from $t=0$ to $t = 200$ ms.



A. Writing the KVL equation in the Laplace domain,



$$(20 + 4s)\, I(s) = -4(2)$$

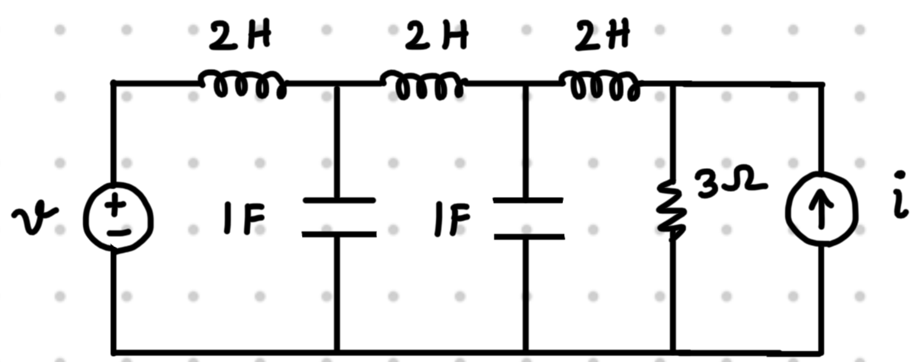
$$\Rightarrow \quad I = \frac{-8}{4s+20} = \frac{-2}{s+5}$$

$$\Rightarrow \quad i(t) = \mathcal{L}^{-1}\left(\frac{-2}{s+5}\right) = -2e^{-5t} \text{ A}$$

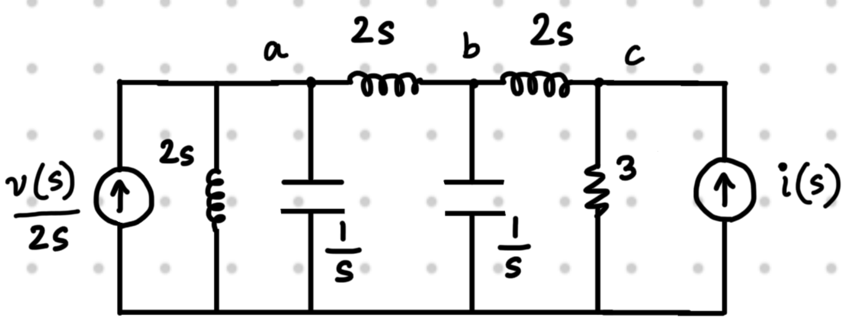
$$\text{Energy dissipated by } 10\,\Omega \text{ resistor} = \int_{t_1}^{t_2} i^2 R_1\, dt$$

$$= 40 \left.\frac{e^{-10t}}{-10}\right|_0^{0.2} \simeq 3.46 \text{ J}$$

Q. For the network shown below, write the KCL and KVL matrix equations in the Laplace transformed domain. (Assume zero initial conditions)

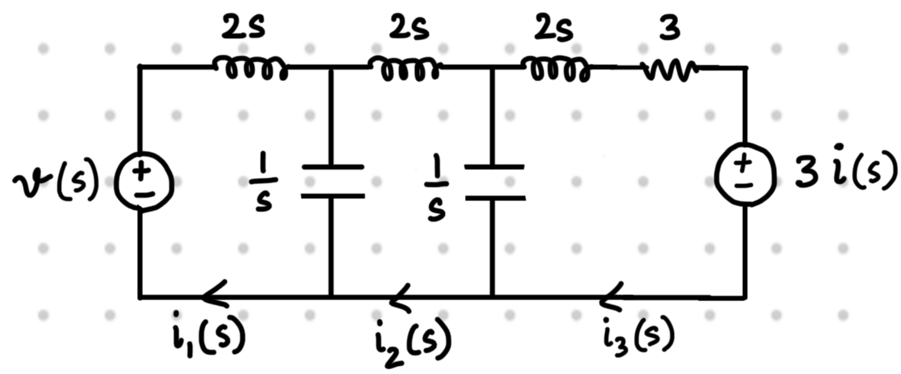


A. The KCL equations of the network.



$$\begin{bmatrix} \frac{1}{2s} + \frac{1}{2s} + s & -\frac{1}{2s} & 0 \\ -\frac{1}{2s} & \frac{1}{2s} + \frac{1}{2s} + s & -\frac{1}{2s} \\ 0 & -\frac{1}{2s} & \frac{1}{2s} + \frac{1}{3} \end{bmatrix} \begin{bmatrix} v_a(s) \\ v_b(s) \\ v_c(s) \end{bmatrix} = \begin{bmatrix} \frac{v(s)}{2s} \\ 0 \\ i(s) \end{bmatrix}$$

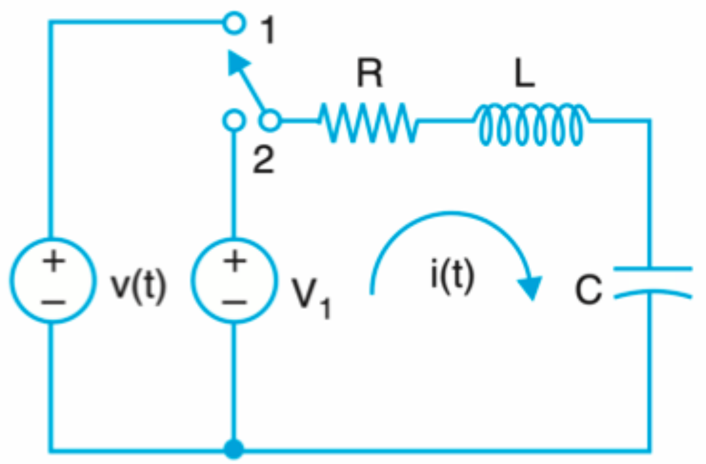
The KVL equations of the network:



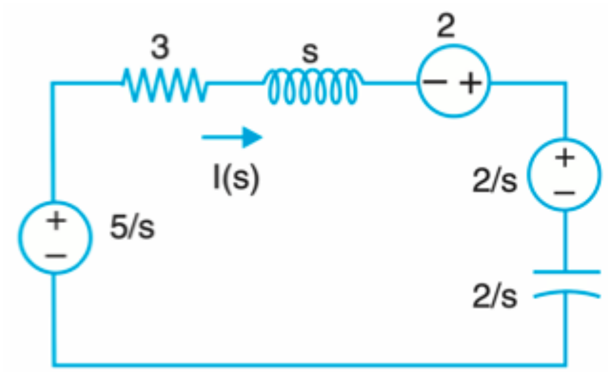
$$\begin{bmatrix} 2s + \frac{1}{s} & -\frac{1}{s} & 0 \\ -\frac{1}{s} & 2s + \frac{2}{s} & -\frac{1}{s} \\ 0 & -\frac{1}{s} & 3 + 2s + \frac{1}{s} \end{bmatrix} \begin{bmatrix} i_1(s) \\ i_2(s) \\ i_3(s) \end{bmatrix} = \begin{bmatrix} v(s) \\ 0 \\ -3\,i(s) \end{bmatrix}$$

Q. In the circuit shown below, the switch is thrown from position 1 to 2 at time $t=0$.

Given $i_L(0^-) = 2A$, $v_c(0^-) = 2V$, find the current $i(t)$ for $t>0$. Use $L = 1H$, $R = 3\Omega$, $C = 0.5F$ and $V_1 = 5V$.



A. Transforming the circuit into Laplace domain and writing KVL,



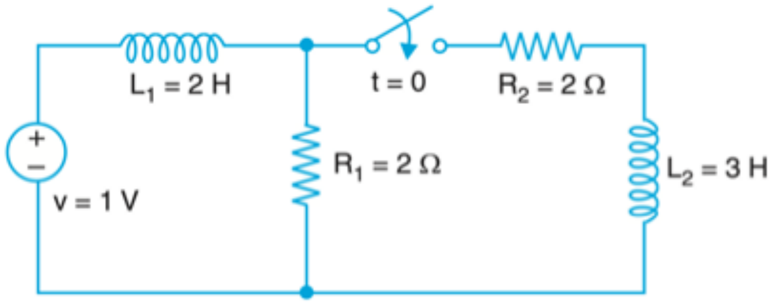
$$\left(3 + s + \frac{2}{s}\right) I = \left(\frac{5}{s} + 2 - \frac{2}{s}\right)$$

$$\Rightarrow \quad I = \frac{2s+3}{s^2+3s+2} = \frac{2s+3}{(s+2)(s+1)}$$

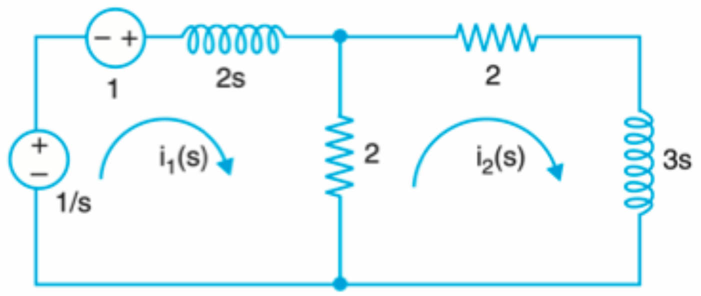
$$\Rightarrow \quad i(t) = \mathcal{L}^{-1}\left(\frac{2s+3}{(s+2)(s+1)}\right) = e^{-t} + e^{-2t} \quad ; \; t \geq 0$$

Q. In the circuit shown below, the switch is closed at time $t = 0$.

Determine the current in the 3H inductor.



A. Writing the KVL equations for the transformed network in the matrix form,



$$\begin{bmatrix} 2+2s & -2 \\ -2 & 4+3s \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} \frac{1}{s} + 1 \\ 0 \end{bmatrix}$$

Using Cramer's rule,

$$I_2 = \frac{1}{2s} - \frac{2}{5s + \frac{5}{3}} - \frac{1}{10(s+2)}$$

Therefore, $i_2(t) = \left(\frac{1}{2} - \frac{1}{10} e^{-2t} - \frac{2}{5} e^{-\frac{t}{3}} \right) u(t)$