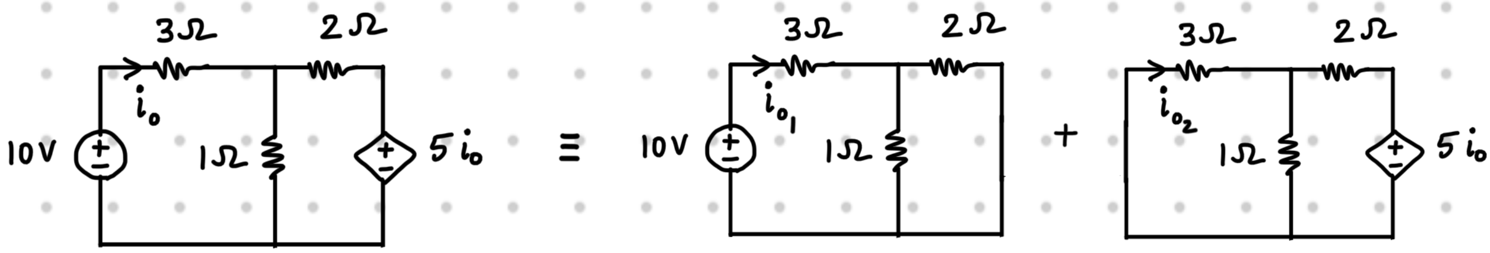
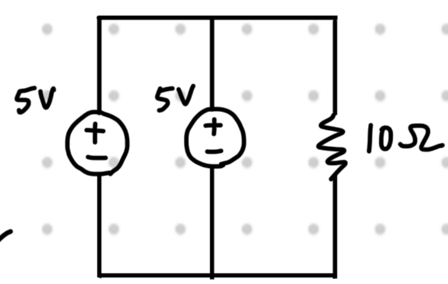
Superposition of Dependent Sources:

Although many textbooks do not apply the superposition theorem to dependent sources, Prof. Marshall Leach, a late-professor of Georgia Tech has long argued that the use of superposition of dependent sources often leads to simpler solutions.

To apply superposition to dependent sources, the controlling variable must not be set to zero when a source is deactivated.

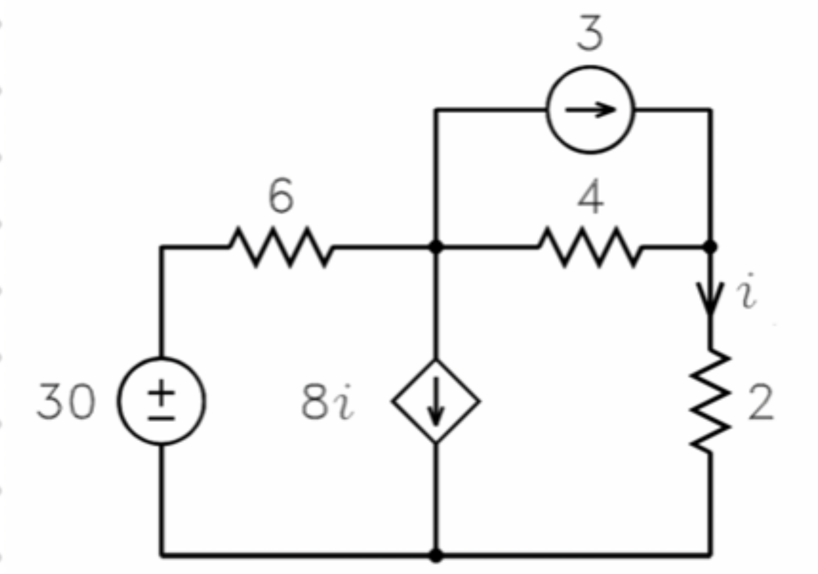


Note: Irrespective of dependent or independent sources, superposition theorem cannot be applied to circuits in which when all sources but one are deactivated, the circuit must not contain any node at which the voltage is indeterminate and any branch in which the current is indeterminate.



cannot apply superposition theorem

Q. Using Superposition theorem, find the current i :



A. We will apply superposition to both dependent and independent sources.

Response due to 30V alone :

$$i_1 = \frac{30}{12} A = \frac{5}{2} A$$

Response due to 3A alone :

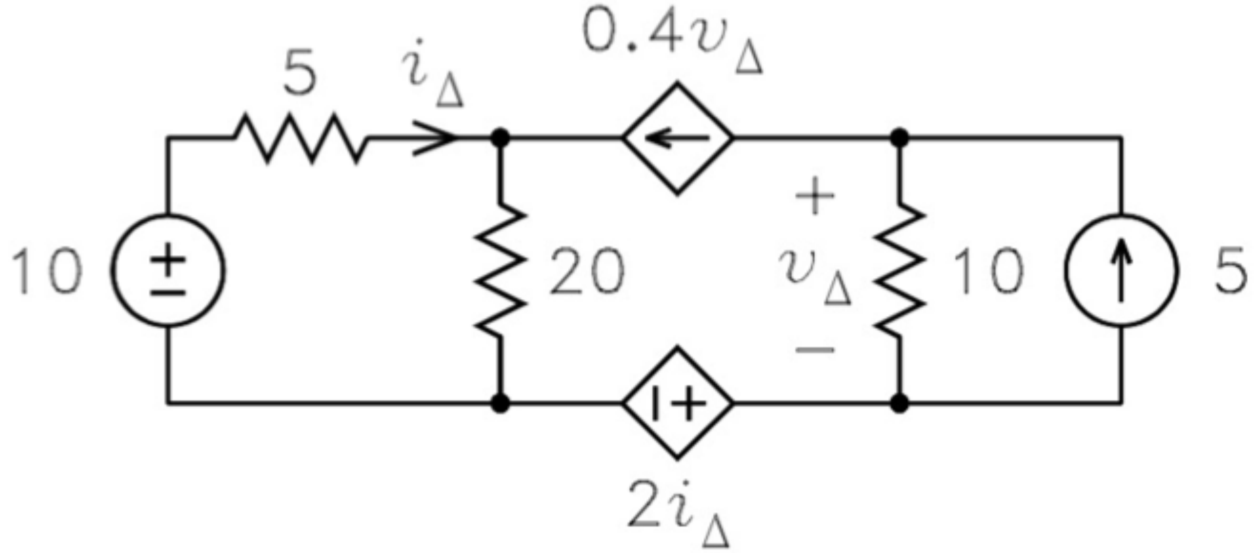
$$i_2 = \frac{4}{12} \times 3 = 1 A$$

Response due to 8i alone :

$$i_3 = \frac{-6}{6+6} \cdot 8i = -4i$$

Hence, $i = \frac{5}{2} + 1 - 4i \Rightarrow i = 0.7 A$

Q. Using Superposition theorem, find the voltage v_Δ :



A. We will apply superposition to both dependent and independent sources.

Response due to 10V alone : $v_{\Delta_1} = 0V$

Response due to 5A alone : $v_{\Delta_2} = 50V$

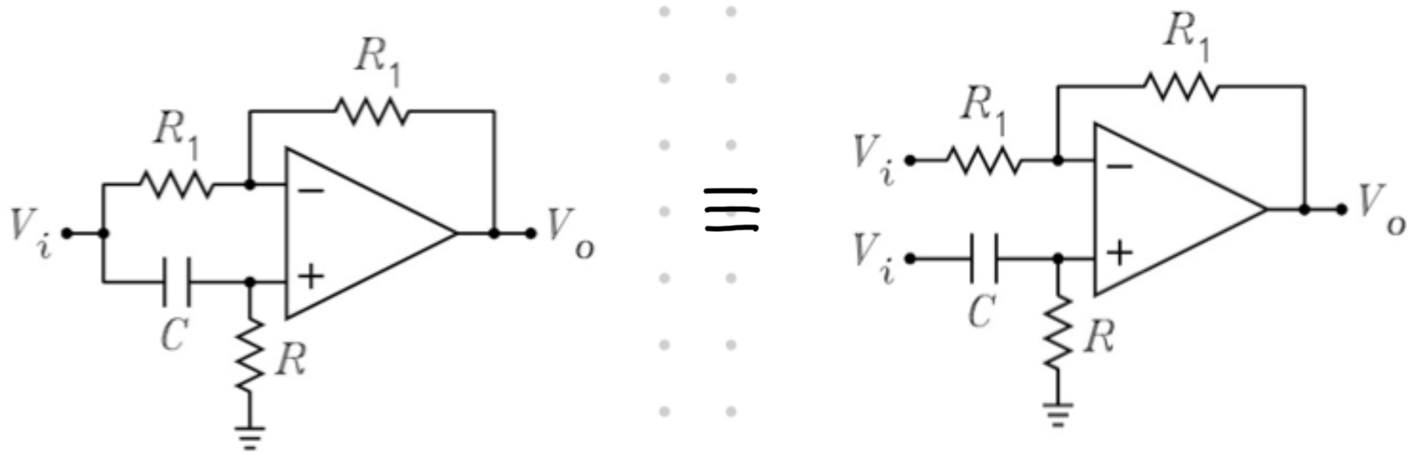
Response due to $0.4 v_\Delta$ alone :

$$v_{\Delta_3} = -(0.4 v_\Delta)10 = -4v_\Delta$$

Response due to $2i_\Delta$ alone : $v_{\Delta_4} = 0$

Hence, $v_\Delta = 50 - 4v_\Delta \Rightarrow v_\Delta = 10V$

Q. Using Superposition theorem, find the output voltage V_o :



A. Response due to "top" V_i alone :

$$\frac{V_i}{R_1} = \frac{-V_{o_1}}{R_1} \Rightarrow V_{o_1}(s) = -V_i(s)$$

Response due to "bottom" V_i alone :

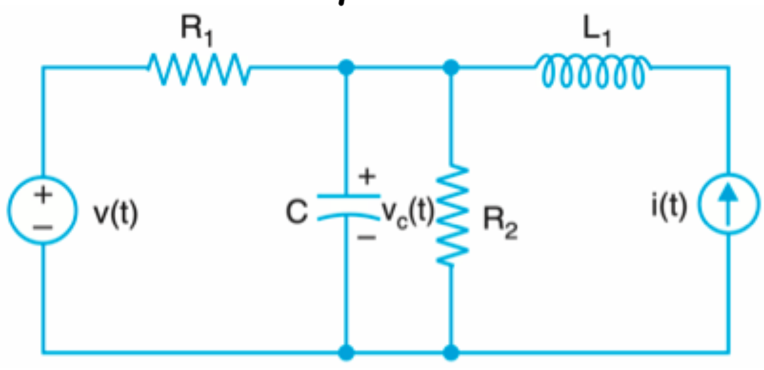
$$\frac{\left(\frac{R}{R+\frac{1}{Cs}}\right)V_i}{R_1} = \frac{V_{o_2} - \left(\frac{R}{R+\frac{1}{Cs}}\right)V_i}{R_1}$$

$$\Rightarrow \quad V_{o_2}(s) = \frac{2RCs}{1+RCs} V_i(s)$$

Hence, $$V_o(s) = \frac{RCs - 1}{RCs + 1} V_i(s)$$

<u>Note:</u> This is a first-order active all-pass filter.

Q. Using Superposition theorem, find the voltage $v_c(t)$.

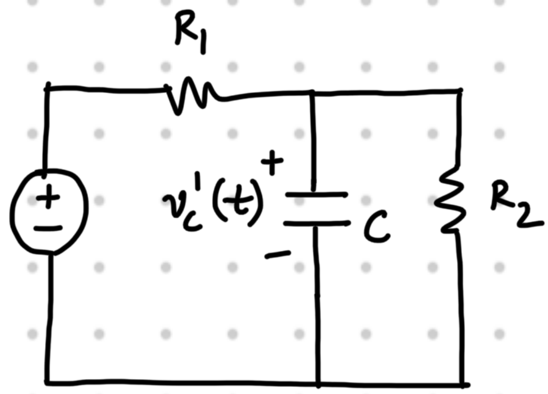


Given $R_1 = R_2 = 2\Omega$,

$C = 0.5\,F$, $v(t) = t\,u(t)$,

$i(t) = e^{-4t}\,u(t)$, and $v_c(0) = i_L(0) = 0$

A. First, response due to $v(t)$ alone :

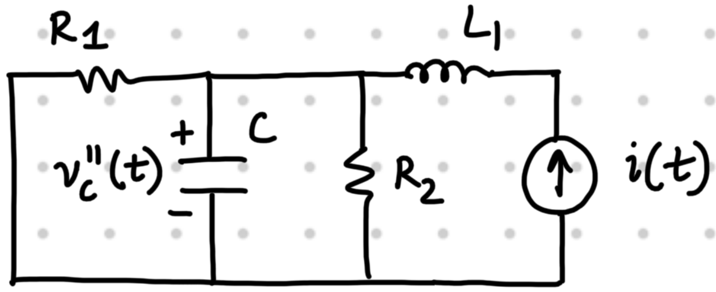


$$V_c'(s) = \frac{\dfrac{1}{\frac{1}{R_2} + sC}}{\dfrac{1}{\frac{1}{R_2}+sC} + R_1} \cdot V(s)$$

$$\Rightarrow \quad V_c'(s) = \frac{1}{s+2} \cdot V(s) = \frac{1}{s^2(s+2)}$$

$$\Rightarrow \quad v_c'(t) = \left(\frac{t}{2} + \frac{1}{4}e^{-2t} - \frac{1}{4}\right) u(t)$$

Next, response due to $i(t)$ alone :



$$V_c''(s) = \left(\frac{1}{\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{sC}} \right) I(s) = \frac{s}{(s+2)} \cdot \frac{1}{(s+4)}$$

$$\Rightarrow \quad V_c''(s) = \frac{2}{s+4} - \frac{1}{s+2}$$

$$\Rightarrow \quad v_c''(t) = \left(2e^{-4t} - e^{-2t} \right) u(t)$$

Hence, the total response due to both the sources is :

$$v_c(t) = v_c'(t) + v_c''(t)$$

$$\Rightarrow \quad v_c(t) = \left(\frac{-3}{4} e^{-2t} + 2e^{-4t} + \frac{t}{2} - \frac{1}{4} \right) u(t)$$