Kirchhoff's Voltage Law: The algebraic sum of all branch voltages around any closed loop of a network is zero at all instants of time.

If a network has N_b branches and N_m loops, then

$$\sum_{j=1}^{N_b} b_{kj} v_j(t) = 0 \quad ; \quad k = 1, 2, \ldots, N_m$$

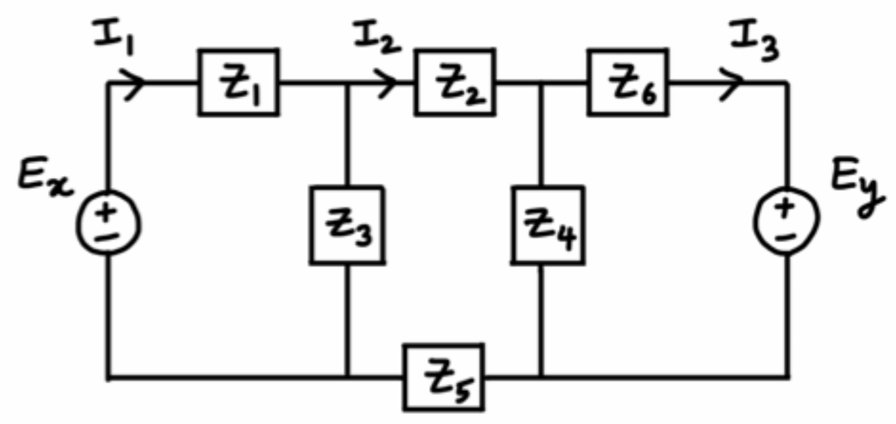
where

$$b_{kj} = \begin{cases} 1 & \text{if branch } j \text{ is in loop } k \text{ and its voltage reference is at the tail of the loop orientation arrow} \\ -1 & \text{if branch } j \text{ is in loop } k \text{ and its voltage reference is at the head of the loop orientation arrow} \\ 0 & \text{if branch } j \text{ is not in loop } k \end{cases}$$

Note: Fundamental theorem of network topology states that if a network contains N_b branches and N_v nodes, then the number of independent meshes/loops is $N_m = N_b - (N_v - 1)$. Hence, we will have at least N_m KVL equations.

Generalized Mesh Analysis:

Suppose we have a network shown below where three mesh or loop currents I_1, I_2 and I_3 are assumed and given reference directions.



Using KVL, we can write in mesh 1,

$$I_1 Z_1 + (I_1 - I_2) Z_3 - E_x = 0$$

$$\Rightarrow \quad (Z_1 + Z_3) I_1 + (-Z_3) I_2 + (0) I_3 = E_x$$

Similarly, in mesh 2,

$$(-Z_3) I_1 + (Z_2 + Z_3 + Z_4 + Z_5) I_2 + (-Z_4) I_3 = 0$$

and mesh 3,

$$(0) I_1 + (-Z_4) I_2 + (Z_4 + Z_6) I_3 = -E_y$$

Hence,

$$\begin{bmatrix} Z_1 + Z_3 & -Z_3 & 0 \\ -Z_3 & Z_2 + Z_3 + Z_4 + Z_5 & -Z_4 \\ 0 & -Z_4 & Z_4 + Z_6 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} E_x \\ 0 \\ -E_y \end{bmatrix}$$

Using Cramer's rule, we can find mesh currents,

$$I_1 = \frac{1}{\Delta}\begin{vmatrix} E_x & -Z_3 & 0 \\ 0 & Z_2+Z_3+Z_4+Z_5 & -Z_4 \\ -E_y & -Z_4 & Z_4+Z_6 \end{vmatrix}$$

$$I_2 = \frac{1}{\Delta}\begin{vmatrix} Z_1+Z_3 & E_x & 0 \\ -Z_3 & 0 & -Z_4 \\ 0 & -E_y & Z_4+Z_6 \end{vmatrix}$$

$$I_3 = \frac{1}{\Delta}\begin{vmatrix} Z_1+Z_3 & -Z_3 & E_x \\ -Z_3 & Z_2+Z_3+Z_4+Z_5 & 0 \\ 0 & -Z_4 & -E_y \end{vmatrix}$$

where $\Delta = \begin{vmatrix} Z_1+Z_3 & -Z_3 & 0 \\ -Z_3 & Z_2+Z_3+Z_4+Z_5 & -Z_4 \\ 0 & -Z_4 & Z_4+Z_6 \end{vmatrix}$

Hence, the generalized mesh equations can be written as:

$$[Z]\,[I] = [E]$$

where the square matrix Z is called the impedance matrix, I is the column matrix of the mesh currents, and E is the column matrix of input voltages.

Considering a generalised network with m meshes, we can write the mesh equations in matrix form of order $(m \times m)$ using KVL, as

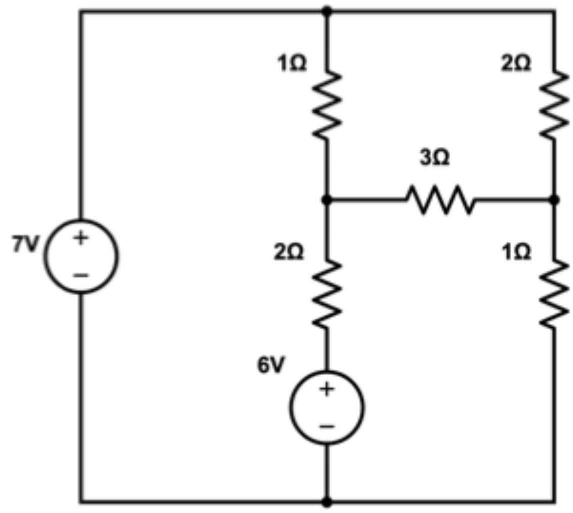
$$\begin{bmatrix} Z_{11} & Z_{12} & \cdots & Z_{1m} \\ Z_{21} & Z_{22} & \cdots & Z_{2m} \\ \vdots & \vdots & & \vdots \\ Z_{m1} & Z_{m2} & \cdots & Z_{mm} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ \vdots \\ I_m \end{bmatrix} = \begin{bmatrix} E_1 \\ E_2 \\ \vdots \\ E_m \end{bmatrix}$$

All the impedances through which the loop current i_j flows in the j^{th} loop are summed and denoted by Z_{jj}.

All the impedances through which loop currents i_j in the j^{th} loop and i_k in the k^{th} loop flow are summed up and denoted by Z_{jk}. The sign of Z_{jk} is negative if the two currents i_j and i_k through Z_{jk} are in opposite directions; otherwise the sign is positive.

E_j is the effective voltage in the j^{th} loop through which the loop current i_j flows. The sign of E_j is positive if the direction of E_j is same as that of i_j; otherwise E_j is negative.

Q. In the circuit shown, determine the mesh currents.



A. Assuming I_1, I_2, and I_3 as the currents and writing KVL equations in the matrix form:

$$\begin{bmatrix} 3 & -1 & -2 \\ -1 & 6 & -3 \\ -2 & -3 & 6 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 6 \end{bmatrix}$$

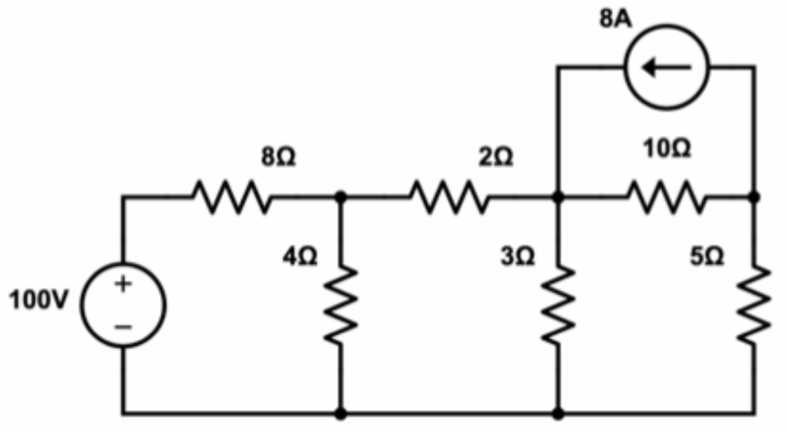
Using Cramer's rule, $I_1 = \dfrac{\Delta_1}{\Delta} = 3A$

$$I_2 = \frac{\Delta_2}{\Delta} = 2A$$

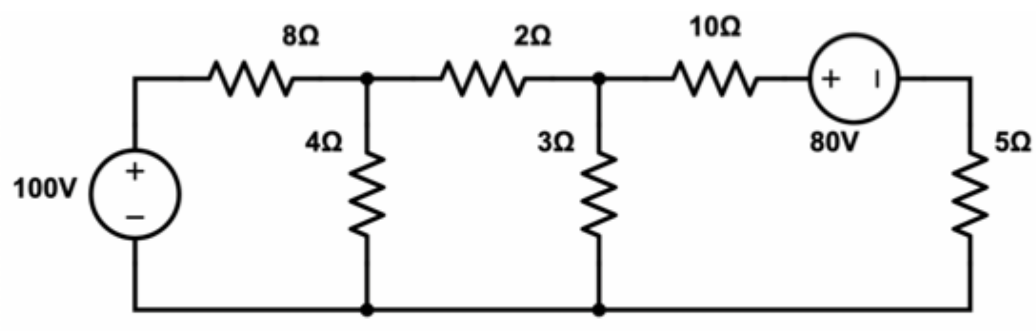
$$I_3 = \frac{\Delta_3}{\Delta} = 3A$$

Hence, $I_1 = 3A$, $I_2 = 2A$, and $I_3 = 3A$.

Q. For the circuit shown below, determine the current flowing in the 2Ω resistor.



A. Using source transformation, we can redraw the circuit as,

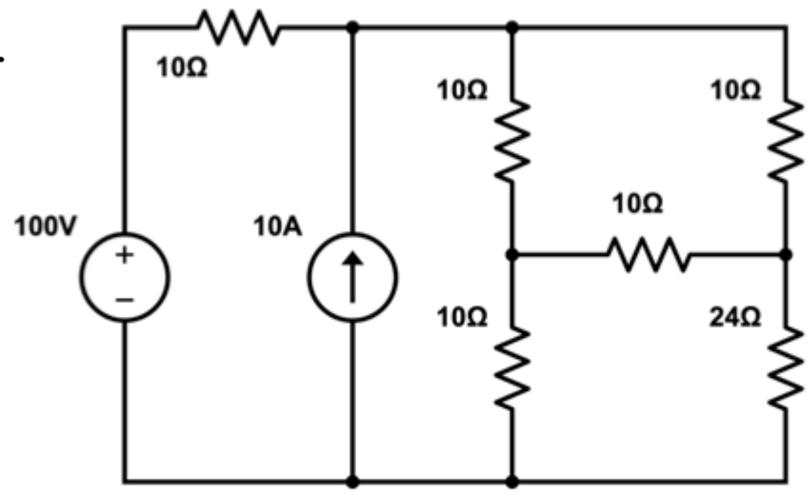


Hence, the KVL equations give us :

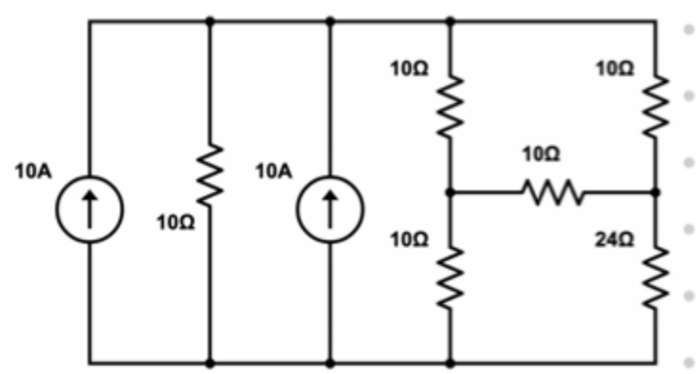
$$\begin{bmatrix} 12 & -4 & 0 \\ -4 & 9 & -3 \\ 0 & -3 & 18 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} 100 \\ 0 \\ -80 \end{bmatrix}$$

$$\Rightarrow \quad I_2 = \frac{\Delta_2}{\Delta} = \frac{4320}{1548} = 2.79\,A$$

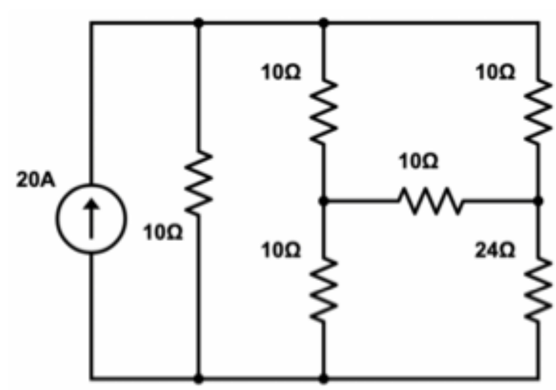
Q. Find the mesh currents of the circuit shown.



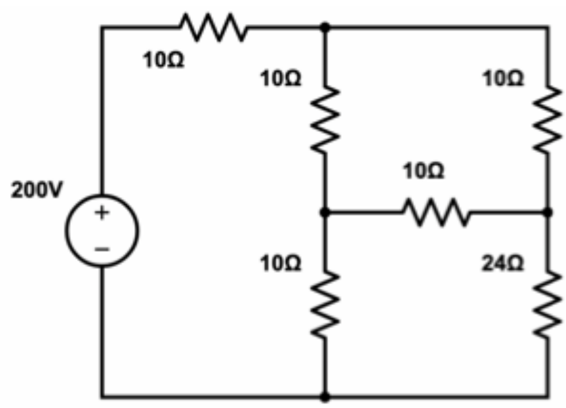
A. Using source transformation, we can redraw the circuit as,



$\Rightarrow$



$\Downarrow$



Writing the KVL equations,

$$\begin{bmatrix} 30 & -10 & -10 \\ -10 & 30 & -10 \\ -10 & -10 & 44 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} 200 \\ 0 \\ 0 \end{bmatrix}$$

Hence, $I_1 = \dfrac{\Delta_1}{\Delta} = 8.97\text{ A}$

$I_2 = \dfrac{\Delta_2}{\Delta} = 3.97\text{ A}$ and $I_3 = \dfrac{\Delta_3}{\Delta} = 2.94\text{ A}$