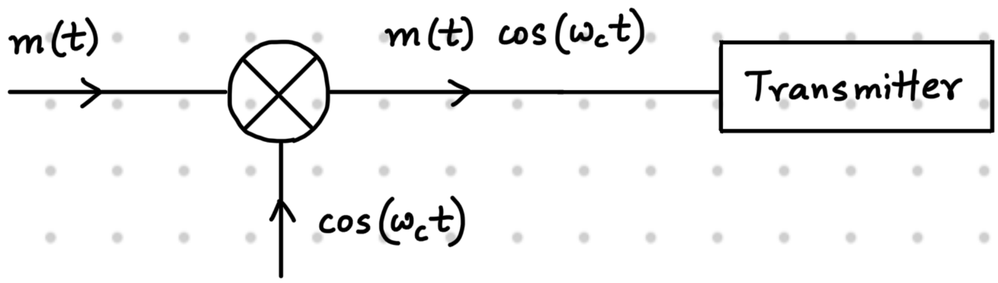
Modulation and Demodulation:

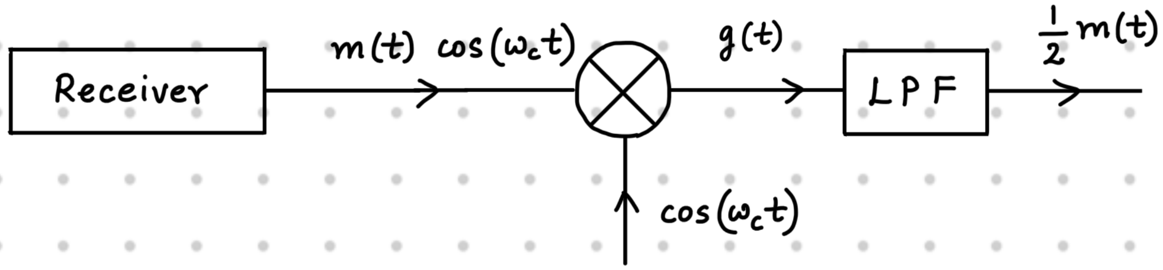
Fourier Transform is pivotal in the field of communication theory. Suppose we have a message/baseband signal $m(t)$ to be transmitted. The signal is first modulated using a carrier $\cos(\omega_c t)$ causing a spectral shift in the signal. This process is called modulation.



Hence, if $m(t) \Longleftrightarrow M(\omega)$

then $m(t)\cos(\omega_c t) \Longleftrightarrow \frac{1}{2}\left(M(\omega+\omega_c) + M(\omega-\omega_c)\right)$

In order to recover our message signal $m(t)$, we must retranslate the spectrum to its original position. This process is called as demodulation.



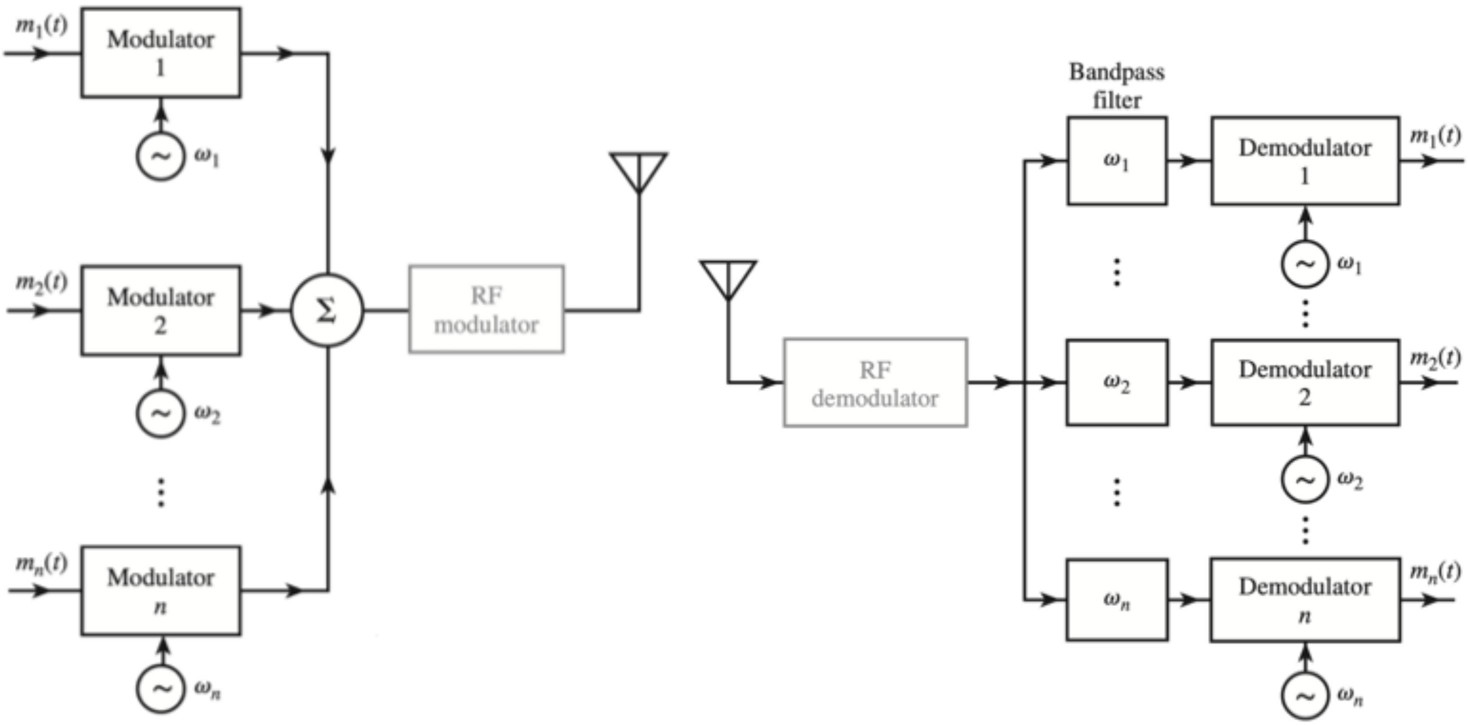
$$g(t) = m(t)\cos^2(\omega_c t) = \frac{1}{2} m(t)\left[1 + \cos 2\omega_c t\right]$$

$$\Rightarrow \quad G(\omega) = \frac{1}{2} M(\omega) + \frac{1}{4}\left[M(\omega + 2\omega_c) + M(\omega - 2\omega_c)\right]$$

Clearly, passing the signal $g(t)$ through a low pass filter will result in the output $\frac{1}{2} m(t)$.

Frequency-Division Multiplexing:

In order to transmit several signals on the same channel, we employ frequency division multiplexing. Although several signals share the band of the same channel, each signal is modulated by a different carrier frequency (subcarriers) and individually demodulated by an appropriate subcarrier to obtain the original message signals.



Q. A baseband signal $x(t) = \Delta\left(\frac{t}{2\pi}\right)$ is modulated with a carrier $\cos(10t)$. Find the Fourier Transform of the modulated signal.

A. $$y(t) = x(t) \cdot \cos(10t)$$

$$= \Delta\left(\frac{t}{2\pi}\right) \cos(10t)$$

$$= \Delta\left(\frac{t}{2\pi}\right) \left[\frac{e^{j10t} + e^{-j10t}}{2}\right]$$

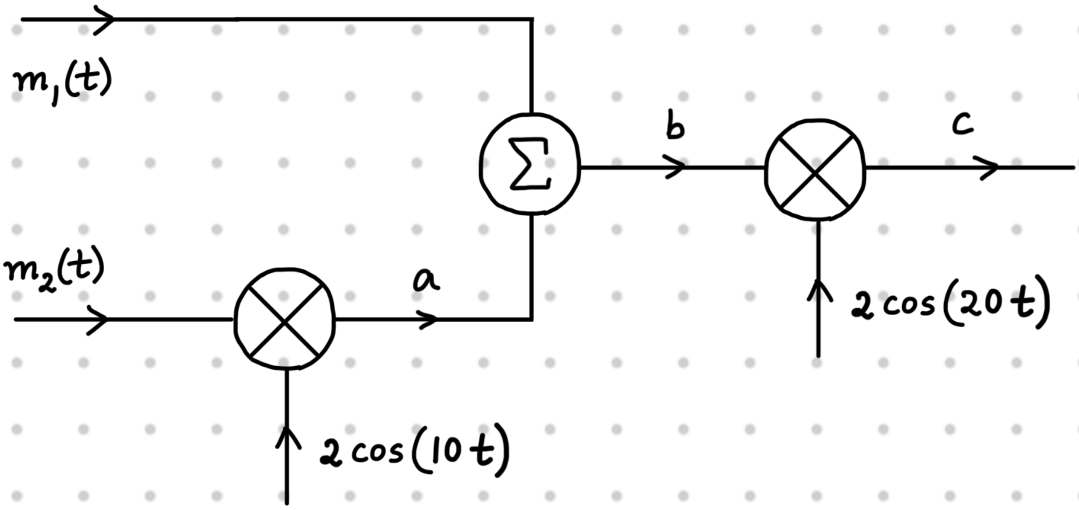
$$\Delta\left(\frac{t}{\tau}\right) \Longleftrightarrow \frac{\tau}{2} \operatorname{sinc}^2\left(\frac{\omega\tau}{4}\right)$$

$$\Delta\left(\frac{t}{2\pi}\right) \Longleftrightarrow \pi \operatorname{sinc}^2\left(\frac{\omega\pi}{2}\right)$$

Hence,

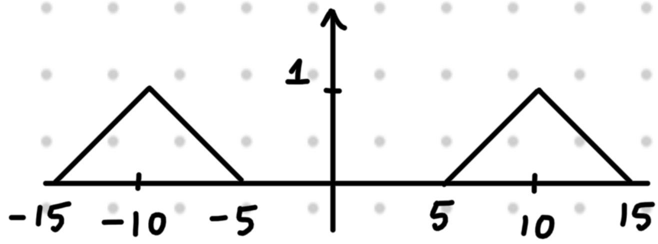
$$Y(\omega) = \frac{\pi}{2}\left[\operatorname{sinc}^2\left(\frac{\pi}{2}(\omega-10)\right) + \operatorname{sinc}^2\left(\frac{\pi}{2}(\omega+10)\right)\right]$$

Q. Message signals $m_1(t)$ and $m_2(t)$ are being transmitted simultaneously on the same channel using frequency-division multiplexing as shown below.

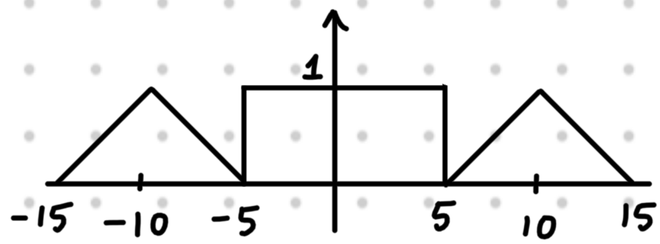


If $M_1(\omega) = \text{rect}\left(\frac{\omega}{10}\right)$ and $M_2(\omega) = \Delta\left(\frac{\omega}{10}\right)$, plot the frequency spectra of the signals at a, b, and c.

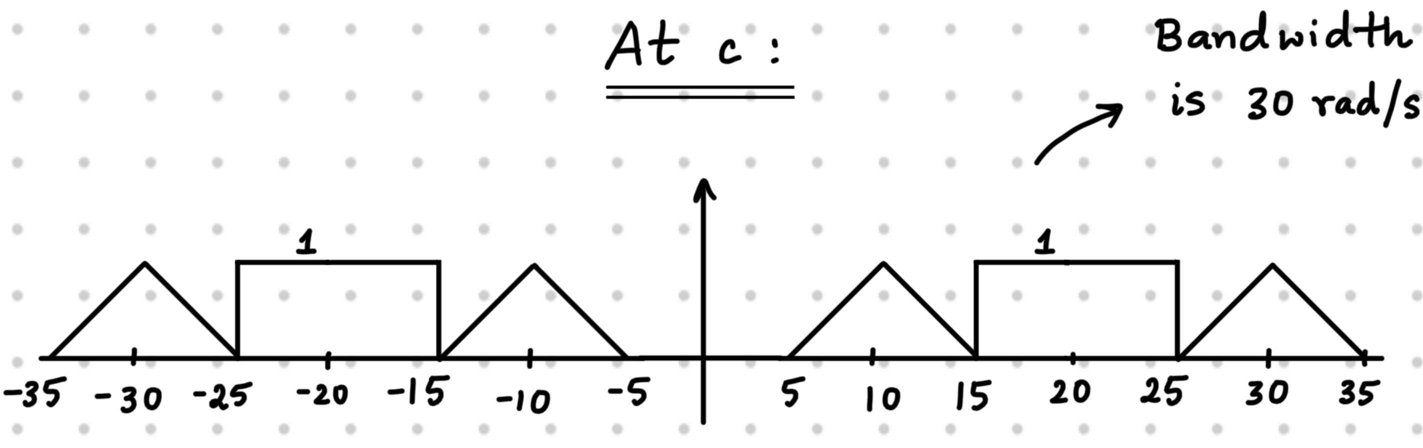
A. At a :



At b :

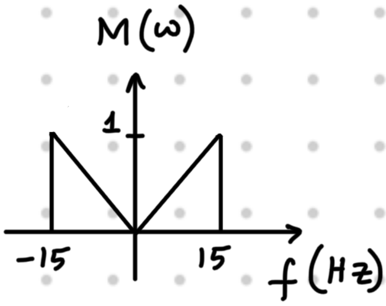


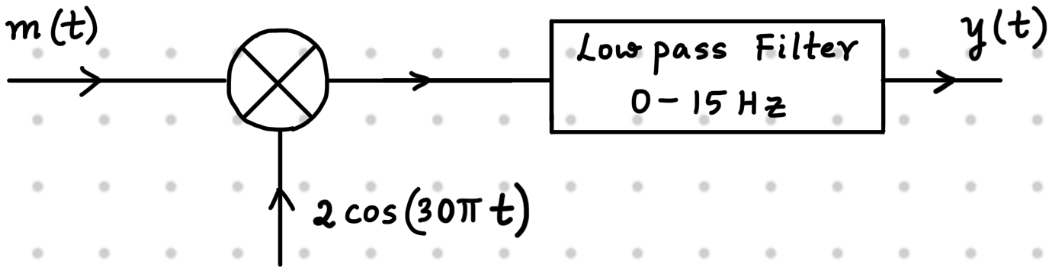
At c :



Q. A scrambler is a device that encodes a message by inverting the signal making it incomprehensible without a descrambler.

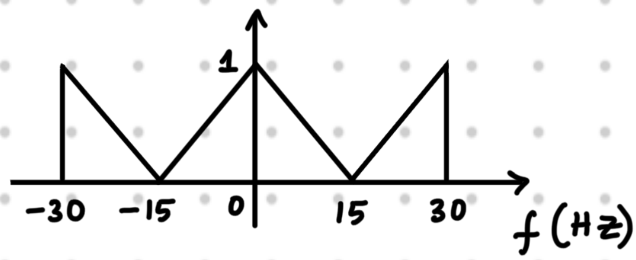
Suppose we have a message $m(t) \Longleftrightarrow M(\omega)$ which is passed through a scrambler as shown.



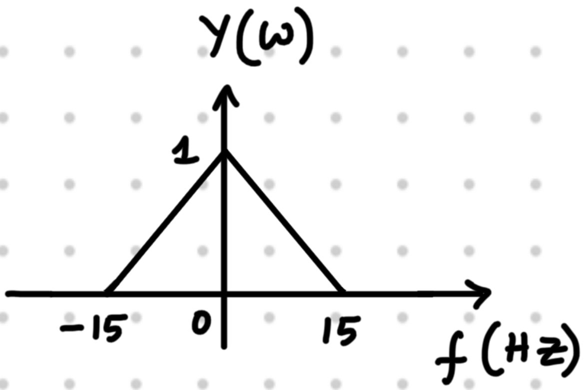


Plot the frequency spectrum $Y(\omega)$ vs f (Hz).

A. $m(t) \cdot 2\cos(30\pi t) \Longleftrightarrow$



After passing it through an LPF, we get



Clearly, the scrambler has inverted $M(\omega)$, with higher frequencies shifted to lower frequencies and vice-versa.

We can get back $M(\omega)$ from $Y(\omega)$ by passing it through the same scrambler.