Reciprocity Theorem :

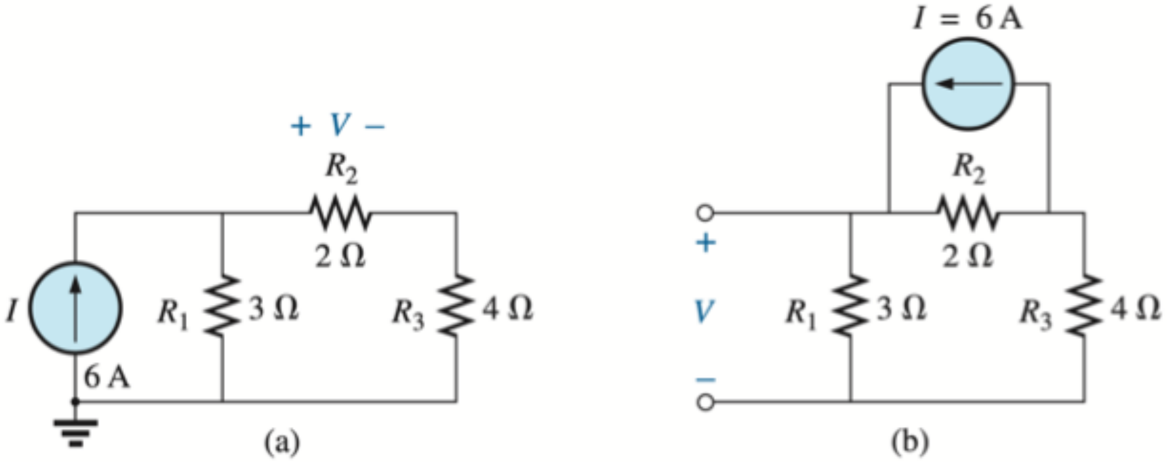
The ratio of the response transform to the excitation transform is invariant to an interchange of the position of the excitation and the response.

In other words, if we consider two loops A and B of a network N and if an ideal voltage source E in loop A produces a current I in loop B, then interchanging positions, if an identical source in loop B produces the same current I in loop A, the network is said to be reciprocal. The dual is also true.

A linear network is said to be reciprocal or bilateral if it remains invariant due to the interchange of position of cause and effect in the network.

Hence, networks for which the reciprocity theorem is true are called reciprocal networks. Any elements which violate the reciprocity theorem are called nonreciprocal elements.

Q. For the networks given below, is the reciprocity theorem satisfied ?



A. For the network (a),

$$V = 2\left(\frac{3(6)}{3+6}\right) = 4\,V$$

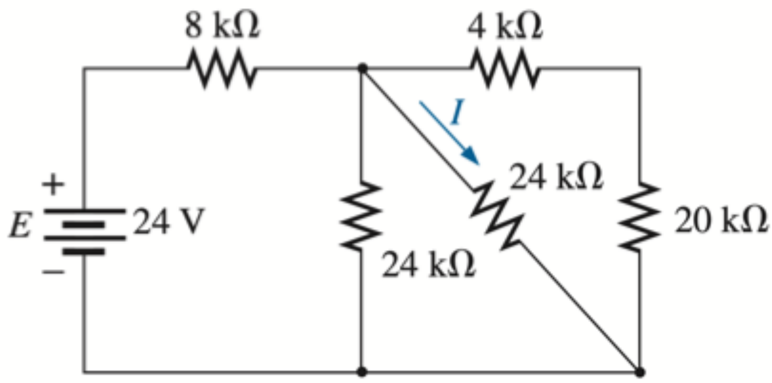
For the network (b),

$$V = 3\left(\frac{2(6)}{7+2}\right) = 4V$$

Clearly, both the voltages are equal.

Hence, the reciprocity theorem is satisfied.

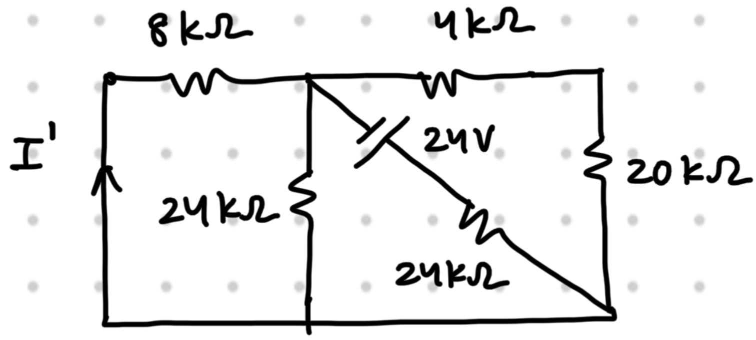
Q. For the network below, is the reciprocity theorem satisfied ?



A. For the given network,

$$I = 24 \cdot \frac{1}{\left(8 + \frac{24}{3}\right)k} \cdot \frac{1}{3} = 0.5 \text{ mA}$$

Now,



$$I' = 24\left(\frac{1}{24 + \frac{24}{5}}\right) \times \frac{12}{12+8} = 0.5 \text{mA}$$

Clearly, $I = I'$. Hence, the reciprocity theorem is satisfied.

Substitution Theorem :

If we know the voltage across any branch and the current passing through the branch, then we can replace that branch with various elements provided that the voltage and current should not change through that branch.

In general, we can represent the voltage V_{xy} across the branch $x-y$ as:

$$V_{xy} = Z_{xy} I_{xy} + E$$

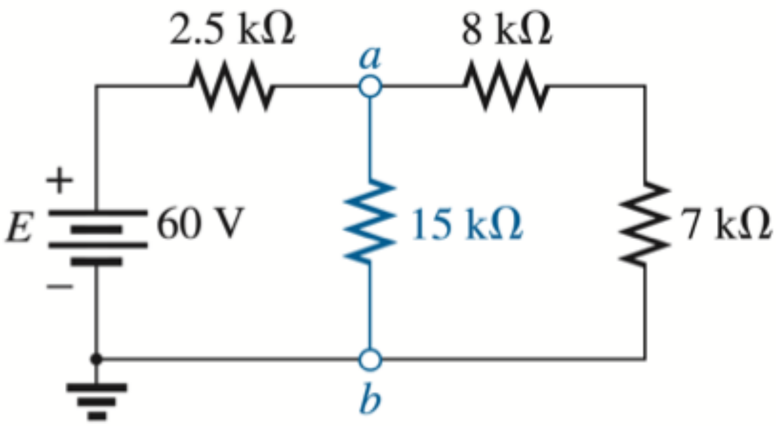
Suppose a network N has b branches. Then, we need the solution of 2b equations. Let's substitute one of the branches $x-y$ as:

$$V_{xy} = Z'_{xy} I_{xy} + E'$$

where Z'_{xy} and E' are chosen so that V_{xy} and I_{xy} are not disturbed.

After substitution, (2b - 1) branches remain unaltered. However, as the branch voltage and current of the replaced branch remain unaltered. Hence, the set of simultaneous 2b equations will be satisfied as before.

Q. Using the substitution theorem, draw three equivalent branches for the branch a–b:

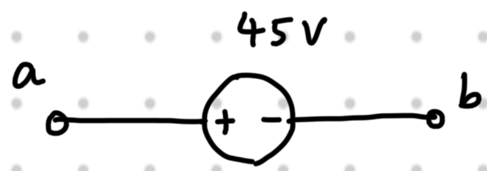


A. $$\frac{60 - V_{ab}}{2.5k} = \frac{V_{ab}}{15k} + \frac{V_{ab}}{15k}$$

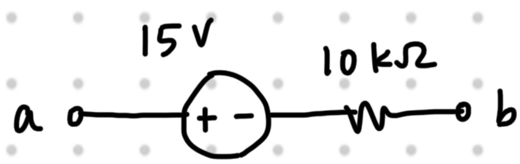
Hence, $V_{ab} = 45\,V \Rightarrow I_{ab} = 3\,mA$

Using $V_{ab} = Z_{ab} I_{ab} + E$, we can choose

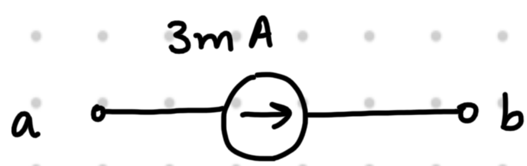
(1). $Z_{ab} = 0$ and $E = 45\,V$



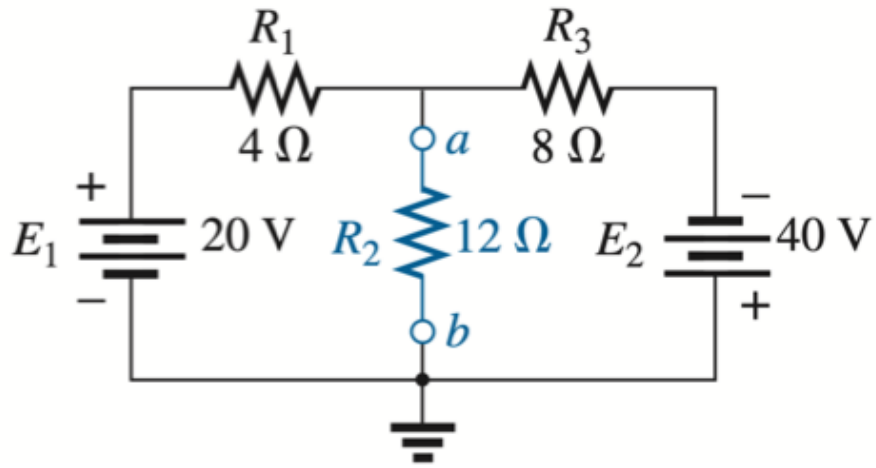
(2). $E = 15\,V$ and $Z_{ab} = 10\,k\Omega$



(3). $I_{ab} = 3\,mA$



Q. Using the substitution theorem, draw three equivalent branches for the branch a–b:



A.
$$\frac{20 - V_{ab}}{4} = \frac{V_{ab}}{12} + \frac{V_{ab} + 40}{8}$$

$$\Rightarrow \quad V_{ab} = 0\text{V} \qquad \Rightarrow \quad I_{ab} = 0\text{ A}$$

Using $V_{ab} = Z_{ab} I_{ab} + E$, we can choose

(1). $E = 0$ V

(2). $I_{ab} = 0$ A

(3). $Z_{ab} = 5\ k\Omega$
(any value)