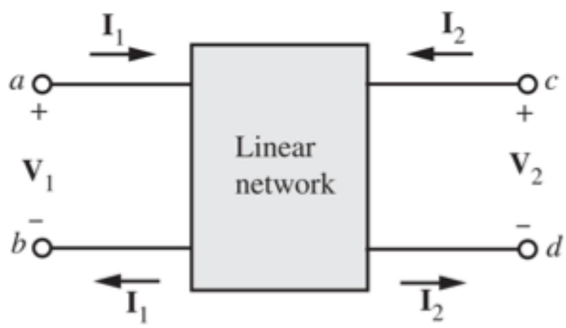
Two-Port Networks

Networks can be classified according to the number of terminals that are available for external connection. If there are n pairs of terminals, we refer to the network as an n-port.

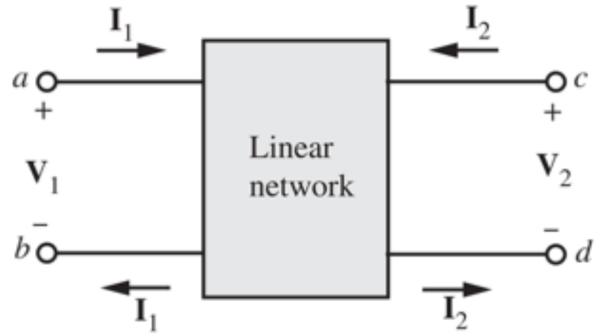
The two-port network is the most common and the most important of this class of networks, and is widely used to model transistors, opamps, transformers, and transmission lines.

A two-port network is commonly represented as shown in the figure.



The parameters of a two-port network completely describes its behaviour in terms of the voltage and current at each port. Thus, knowing the parameters of a two-port network permits us to describe its operation when it is connected into a larger network. We shall now discuss the four popular types of two-ports parameters.

Z / Impedance / Open-circuit Parameters



We assume that there are no independent sources or nonzero initial conditions within the linear two-port network.

Hence, $(V_1, V_2) = f(I_1, I_2)$

$$\Rightarrow [V] = [Z][I]$$

or

$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

Clearly, $Z_{11} = \left.\dfrac{V_1}{I_1}\right|_{I_2=0}$, $Z_{12} = \left.\dfrac{V_1}{I_2}\right|_{I_1=0}$

Z_{11} → open-circuit input impedance

Z_{12} → open-circuit reverse transfer impedance

$$Z_{21} = \left.\frac{V_2}{I_1}\right|_{I_2=0} , \quad Z_{22} = \left.\frac{V_2}{I_2}\right|_{I_1=0}$$

Z_{21} → open-circuit forward transfer impedance

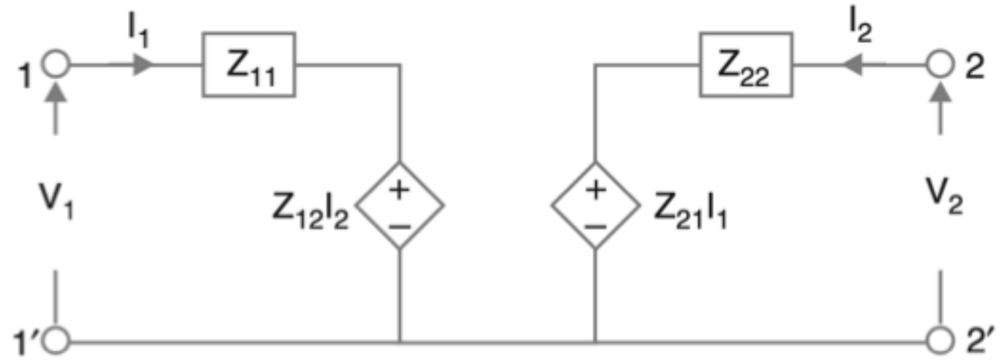
Z_{22} → open-circuit output impedance

Hence, to determine Z_{11} and Z_{21}, the output port is open circuited and the input port is excited with a voltage source $V_1 = V_s$. Calculating I_1 and V_2 will determine Z_{11} and Z_{21}. Similarly, to determine Z_{12} and Z_{22}, the input port is open circuited and the output port is excited with the same voltage source $V_2 = V_s$. Calculating I_2 and V_1 will determine Z_{12} and Z_{22}.

Now, using the equations,

$$V_1 = Z_{11} I_1 + Z_{12} I_2$$
$$V_2 = Z_{21} I_1 + Z_{22} I_2$$

the equivalent circuit representation is given below with $Z_{12} I_2$ and $Z_{21} I_1$ as CCVS.

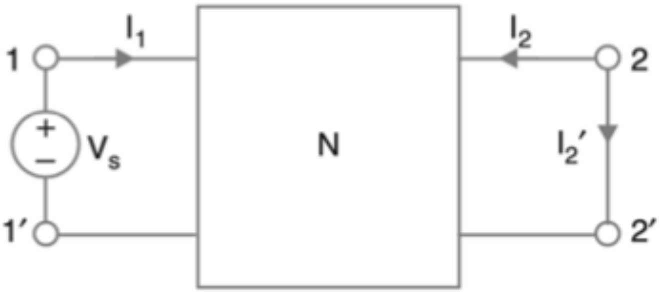


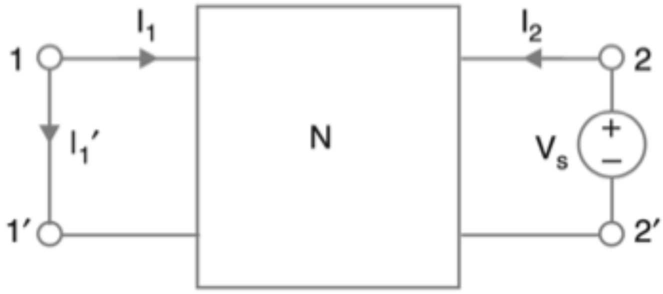
<u>Note</u>: The ports a, b, c, and d are often written as 1, 1′, 2, and 2′ respectively. We will follow both the notations.

Condition for Reciprocity & Symmetry :

A network is said to be reciprocal if the ratio of the response transform to the excitation transform is invariant to an interchange of the positions of the excitation and response in the network.

That is, $I_1' = I_2'$





From the figure,

$$I_2' = \frac{V_s \, Z_{21}}{Z_{11} Z_{22} - Z_{21} Z_{12}} \quad , \quad I_1' = \frac{V_s \, Z_{12}}{Z_{11} Z_{22} - Z_{21} Z_{12}}$$

Hence, $I_1' = I_2' \Rightarrow \underline{\underline{Z_{12} = Z_{21}}}$

Similarly, for symmetry (ports can be interchanged)

$$\left.\frac{V_s}{I_1}\right|_{I_2=0} = \left.\frac{V_s}{I_2}\right|_{I_1=0} \Rightarrow \underline{\underline{Z_{11} = Z_{22}}}$$

Matrix Partitioning for Z parameters :

For a two-port network, the KVL / Mesh equations are : $[Z][I] = [V]$

$$
\begin{aligned}
Z_{11} I_1 + Z_{12} I_2 + \cdots + Z_{1n} I_n &= V_1 \\
Z_{21} I_1 + Z_{22} I_2 + \cdots + Z_{2n} I_n &= V_2 \\
Z_{31} I_1 + Z_{32} I_2 + \cdots + Z_{3n} I_n &= 0 \\
&\vdots \\
Z_{n1} I_1 + Z_{n2} I_2 + \cdots + Z_{nn} I_n &= 0
\end{aligned}
$$

or

$$
\Rightarrow \left[\begin{array}{cc|ccc}
Z_{11} & Z_{12} & Z_{13} & \cdots & Z_{1n} \\
Z_{21} & Z_{22} & \cdots & \cdots & Z_{2n} \\
\hline
Z_{31} & \vdots & & & \vdots \\
\vdots & \vdots & & & \vdots \\
Z_{n1} & \cdots & \cdots & \cdots & Z_{nn}
\end{array}\right]
\left[\begin{array}{c}
I_1 \\ I_2 \\ \hline \vdots \\ \vdots \\ I_n
\end{array}\right]
=
\left[\begin{array}{c}
V_1 \\ V_2 \\ \hline 0 \\ \vdots \\ 0
\end{array}\right]
$$

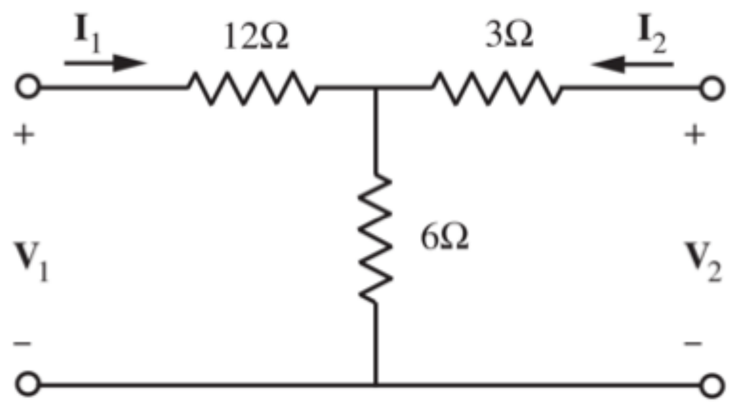
Hence,

$$
\left[\begin{array}{c|c}
M & N \\ \hline P & Q
\end{array}\right]
\left[\begin{array}{c}
I_1 \\ I_2 \\ \hline \vdots \\ I_n
\end{array}\right]
=
\left[\begin{array}{c}
V_1 \\ V_2 \\ \hline 0 \\ \vdots \\ 0
\end{array}\right]
$$

which can be simplified as,

$$
\left[M - N Q^{-1} P\right] \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}
$$

Q: Determine the Z-parameters of:



Furthermore, if

$V_1 = 15\angle 20^\circ$ V,

what is the current

through a 4Ω load connected at the output?

A. Writing the KVL equations,

$$\begin{bmatrix} 18 & 6 \\ 6 & 9 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

$$Z_{11} = \left.\frac{V_1}{I_1}\right|_{I_2=0} = 18\Omega \quad , \quad Z_{21} = \left.\frac{V_2}{I_1}\right|_{I_2=0} = 6\Omega$$

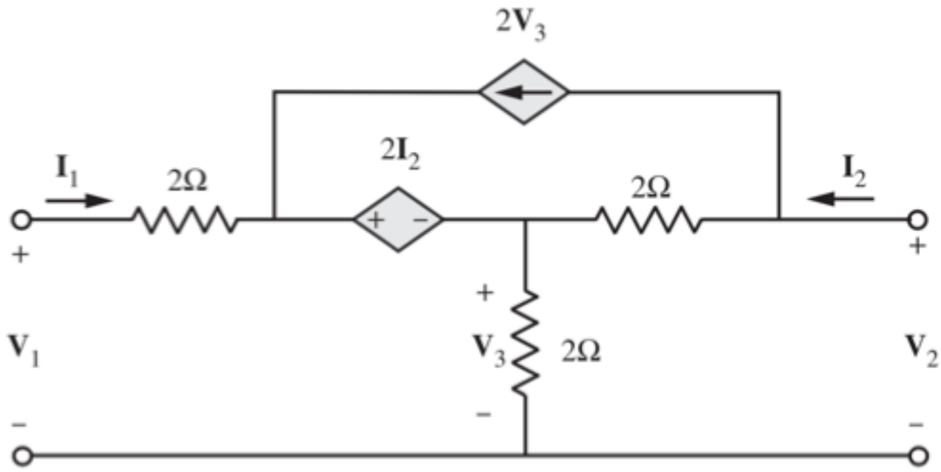
$$Z_{12} = \left.\frac{V_1}{I_2}\right|_{I_1=0} = 6\Omega \quad , \quad Z_{22} = \left.\frac{V_2}{I_2}\right|_{I_1=0} = 9\Omega$$

Now, for a 4Ω load: $15\angle 20^\circ \text{ V} = 18\, I_1 + 6\, I_2$

$$-4\, I_2 = 6\, I_1 + 9\, I_2$$

Hence, $I_2 = -0.45\angle 20^\circ$ A

Q. Determine the Z-parameters of :



A. Writing the KVL equations :

$$4 I_1 + 4 I_2 = V_1$$

$$2 I_1 + 4 I_2 - 2 I_3 = V_2$$

$$I_3 = 4 (I_1 + I_2)$$

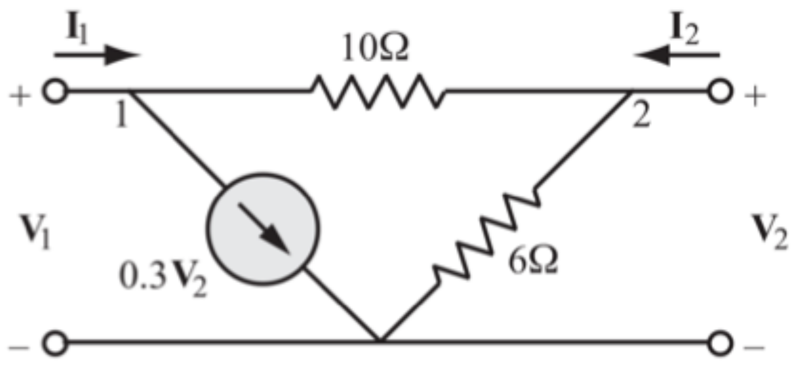
$$\Rightarrow \begin{bmatrix} 4 & 4 & 0 \\ 2 & 4 & -2 \\ 4 & 4 & -1 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} V_1 \\ V_2 \\ 0 \end{bmatrix}$$

or $$\begin{bmatrix} 4 & 4 \\ -6 & -4 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

Hence, $z_{11} = 4\Omega$, $z_{21} = -6\Omega$

$z_{12} = 4\Omega$, $z_{22} = -4\Omega$

Q. Determine the Z-parameters of :



A. Writing the KVL equations :

$$6 I_2 + 16 I_3 = V_1$$

$$6 I_2 + 6 I_3 = V_2$$

$$I_1 - I_3 = 0.3 V_2$$

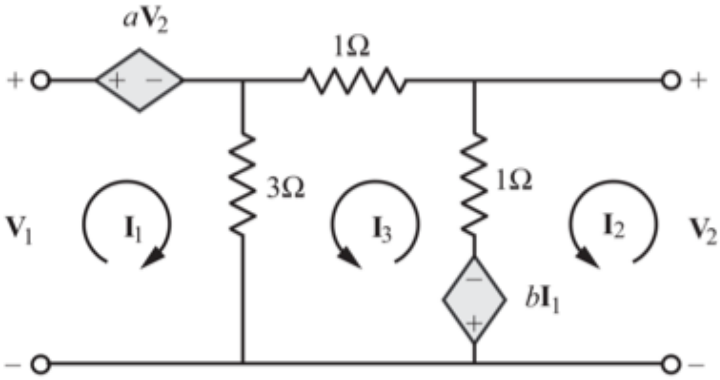
$$Z_{11} = \left.\frac{V_1}{I_1}\right|_{I_2 = 0} = \frac{40}{7} \ \Omega$$

$$Z_{21} = \left.\frac{V_2}{I_1}\right|_{I_2 = 0} = \frac{15}{7} \ \Omega$$

$$Z_{12} = \left.\frac{V_1}{I_2}\right|_{I_1 = 0} = -\frac{30}{7} \ \Omega$$

$$Z_{22} = \left.\frac{V_2}{I_2}\right|_{I_1 = 0} = \frac{15}{7} \ \Omega$$

Q. Determine $\frac{a}{b}$ for reciprocity :



A. Writing the KVL equations :

$$3I_1 + 3I_3 = V_1 - aV_2$$

$$(3+b)I_1 - I_2 + 5I_3 = 0$$

$$-bI_1 + I_2 - I_3 = V_2$$

or $$(3-3b)I_1 + 3I_2 = V_1 + (3-a)V_2$$

$$(3-4b)I_1 + 4I_2 = 5V_2$$

Now, $$z_{12} = \left.\frac{V_1}{I_2}\right|_{I_1=0} = \frac{4a+3}{5}$$

$$z_{21} = \left.\frac{V_2}{I_1}\right|_{I_2=0} = \frac{3-4b}{5}$$

For reciprocity, $z_{12} = z_{21} \Rightarrow \frac{a}{b} = -1$