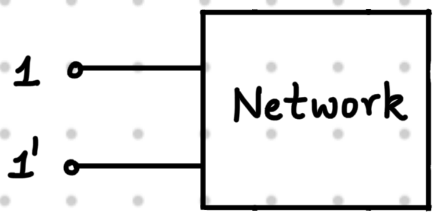
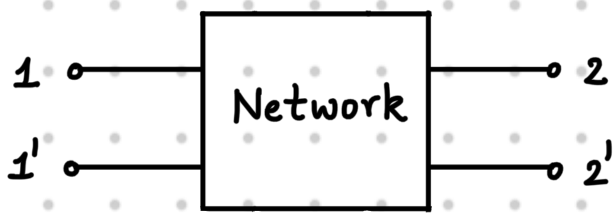
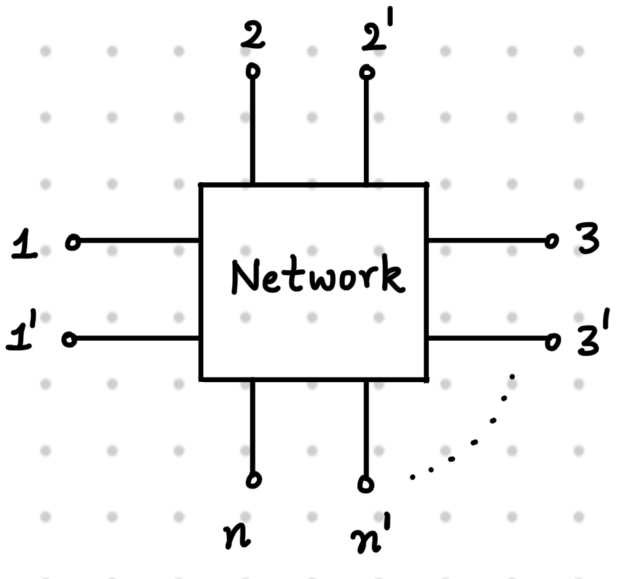
Network Functions

A **port** is a pair of terminals connecting an electrical network/circuit to an external circuit, as a point of entry or exit for charge.

A one-port network, a two-port network and a general n-port network is shown below.







(We will limit our discussion to 1-port and 2-port networks.)

The input port is generally connected to an energy source, which forms the driving force for the network. The output port is generally connected to a load.

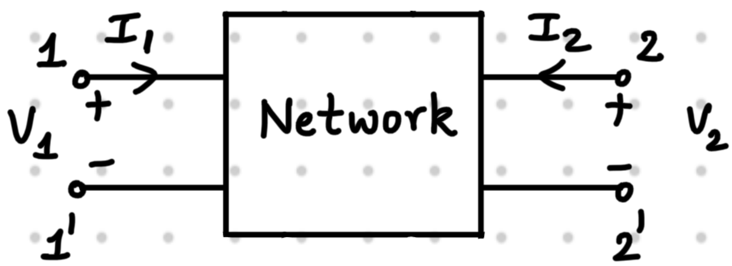
We shall now define some network functions that are commonly used in network theory.

Transform Impedance : Defined as the ratio of the voltage transform to the current transform at a port of a network with zero initial conditions and no independent sources. This quantity is also called as <u>driving - point impedance</u>.

That is,

$$Z(s) = \frac{V_1(s)}{I_1(s)}$$

at port $1-1'$



Transform Admittance : Defined as the ratio of the current transform to the voltage transform at a port of a network with zero initial conditions and no independent sources. This quantity is also called as <u>driving - point admittance</u>.

That is, $$Y(s) = \frac{I_1(s)}{V_1(s)} = \frac{1}{Z(s)}$$

at port $1-1'$

Voltage Transfer Function: Defined as the ratio of one voltage to another voltage.

Example: $$G_{21}(s) = \frac{V_2(s)}{V_1(s)}$$

Current Transfer Function: Defined as the ratio of one current to another current.

Example: $$\alpha_{21}(s) = \frac{I_2(s)}{I_1(s)}$$

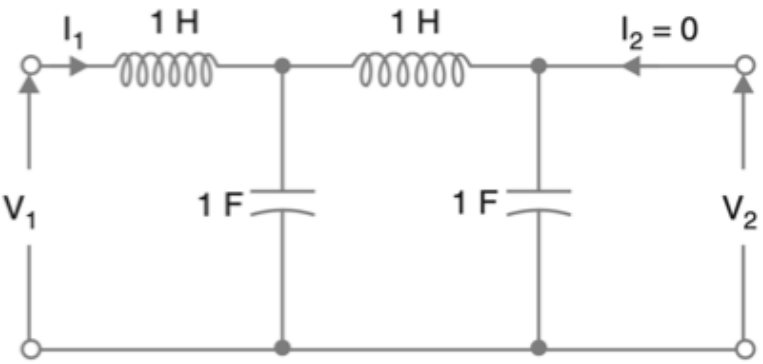
Transfer Impedance Function: Defined as the ratio of one voltage to another current.

Example: $$Z_{21}(s) = \frac{V_2(s)}{I_1(s)}$$

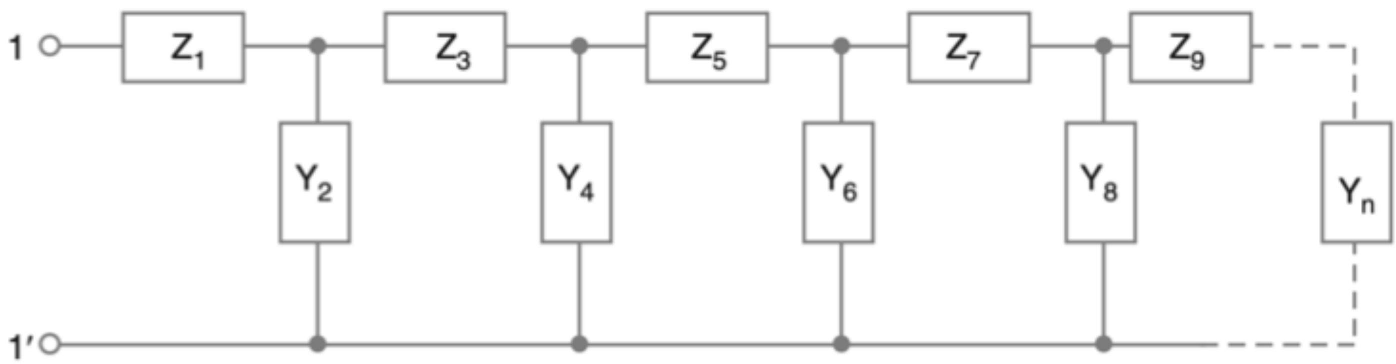
Transfer Admittance Function: Defined as the ratio of one current to another voltage.

Example: $$Y_{21}(s) = \frac{I_1(s)}{V_2(s)}$$

Q. Find the driving-point impedance at the input port of :



A. For a general ladder network, the driving-point impedance is given as:



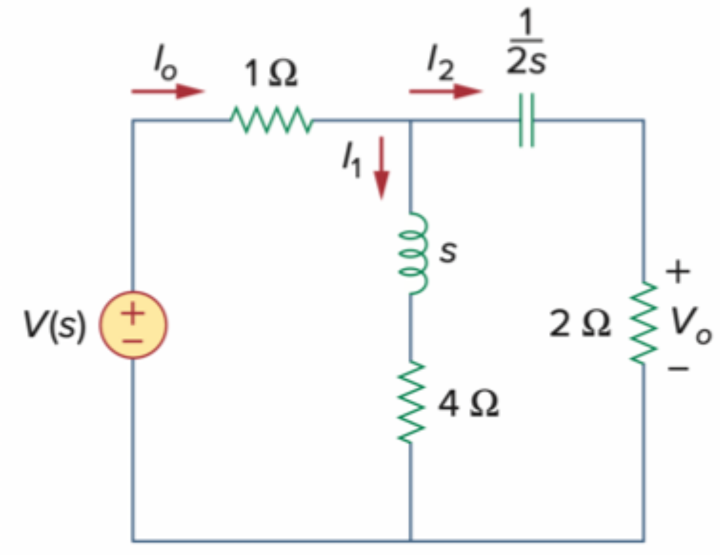
$$Z = Z_1 + \cfrac{1}{Y_2 + \cfrac{1}{Z_3 + \cfrac{1}{Y_4 + \cdots}}}$$

Hence, for our network: $Z = s + \cfrac{1}{s + \cfrac{1}{s + \cfrac{1}{s}}}$

at input port

Hence, $$Z = \frac{s^4 + 3s^2 + 1}{s^3 + 2s}$$

Q. Calculate the transfer function $\dfrac{V_o(s)}{I_o(s)}$ of the circuit :

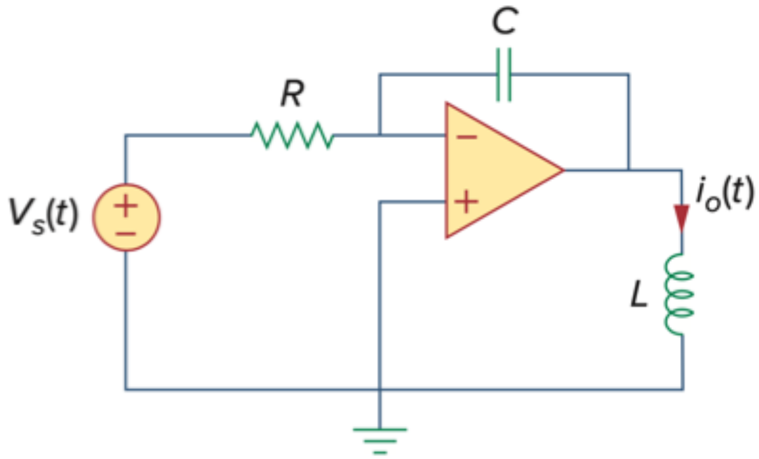


A.
$$\frac{V_o(s)}{I_o(s)} = \frac{2\,I_2(s)}{I_o(s)}$$

$$= 2\left(\frac{4+s}{4+s+\frac{1}{2s}+2}\right)$$

$$\Rightarrow \frac{V_o(s)}{I_o(s)} = \frac{4s(s+4)}{2s^2+12s+1}$$

Q. Calculate the transfer function $\dfrac{I_o(s)}{V_s(s)}$ of the circuit :

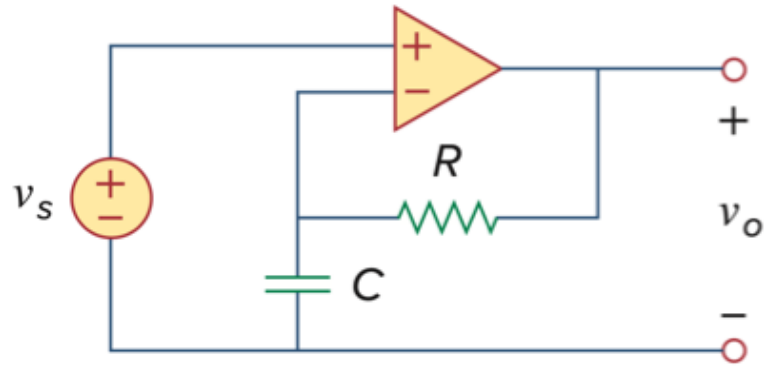


A. $$\frac{V_s(s) - 0}{R} = \frac{0 - V_o(s)}{\frac{1}{Cs}}$$

and $$I_o(s) = \frac{V_o(s)}{sL}$$

$$\Rightarrow \quad \frac{I_o(s)}{V_s(s)} = \frac{-1}{(RLCs^2)}$$

Q. Calculate the transfer function $\dfrac{V_o(s)}{V_s(s)}$ of the circuit :



A. $$\frac{V_o(s) - V_s(s)}{R} = \frac{V_s(s)}{\frac{1}{Cs}}$$

$$\Rightarrow \quad \frac{V_o(s)}{V_s(s)} = 1 + sRC$$