Useful Signal Operations:

Consider a signal $f(t)$. Then, we can have the following operations:

1. Time Shifting: $\phi(t) = f(t-T)$

   where $\phi(t)$ is $f(t)$ delayed by T seconds

2. Time Scaling: $\phi(t) = f\left(\frac{t}{k}\right)$

   where $\phi(t)$ is $f(t)$ expanded in time by a factor k $(k > 1)$

3. Time Inversion: $\phi(t) = f(-t)$

   where $\phi(t)$ is $f(t)$ inverted or mirror-imaged about the vertical axis

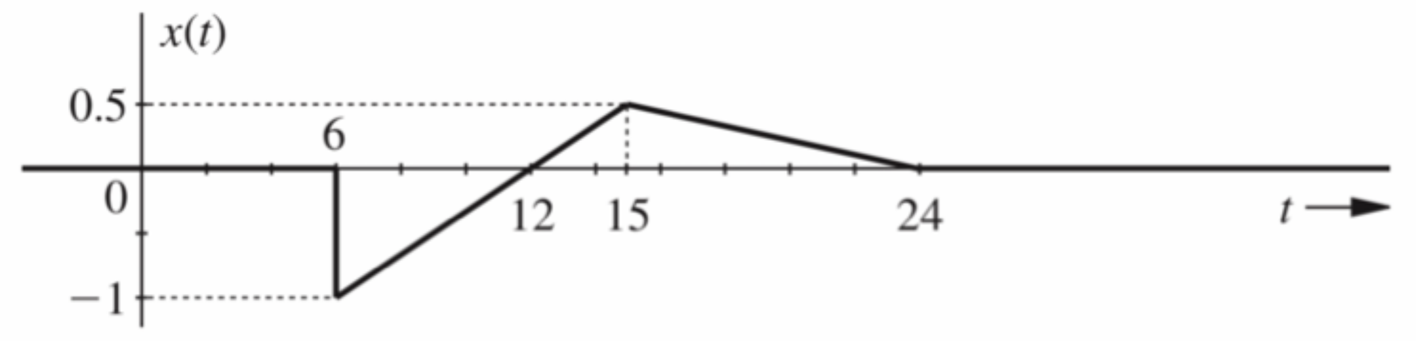
Note: Every signal $f(t)$ can be expressed as a sum of even and odd components.

$$f(t) = \underbrace{\frac{f(t) + f(-t)}{2}}_{\text{even component}} + \underbrace{\frac{f(t) - f(-t)}{2}}_{\text{odd component}}$$

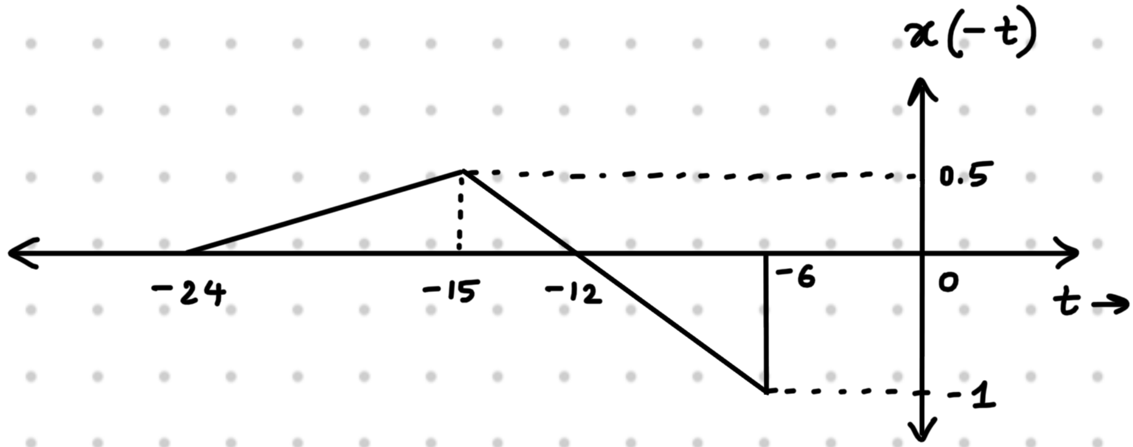
Q. For the signal $x(t)$ shown below, sketch the signals :

(a). $x(-t)$ (c). $x(3t)$

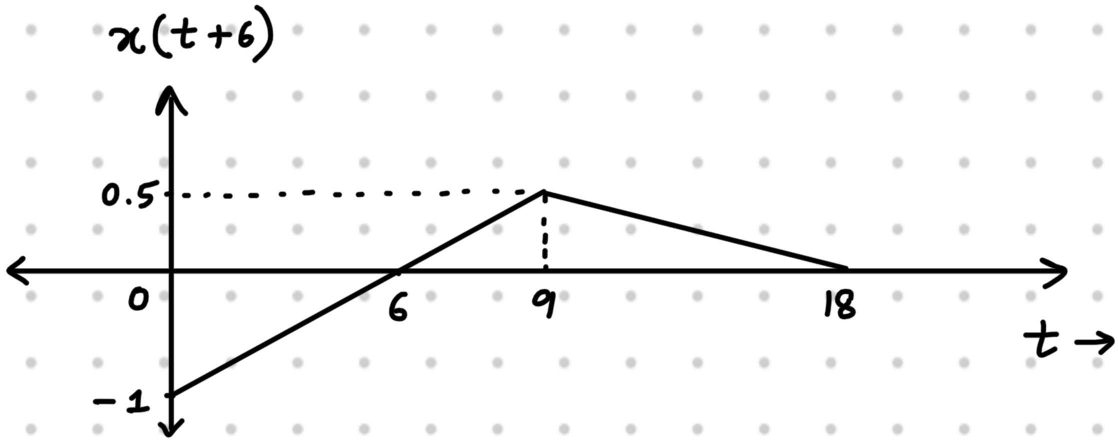
(b). $x(t+6)$ (d). $x(t/2)$



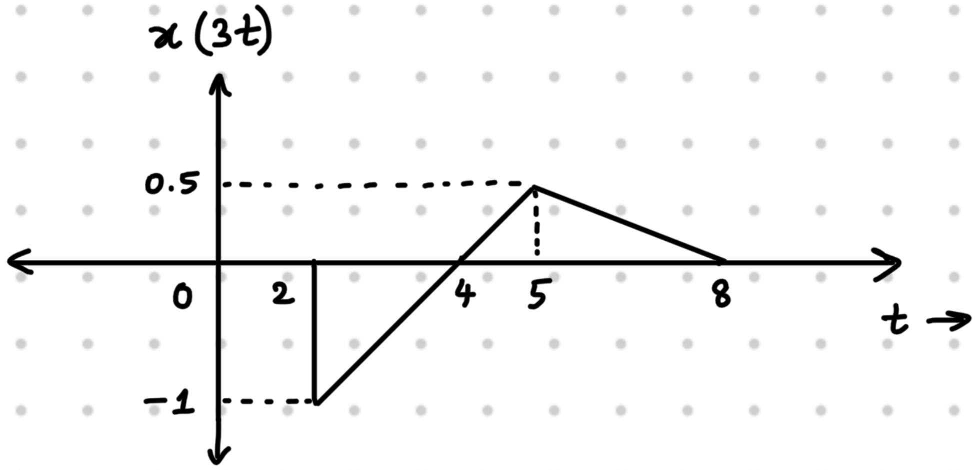
A. (a).



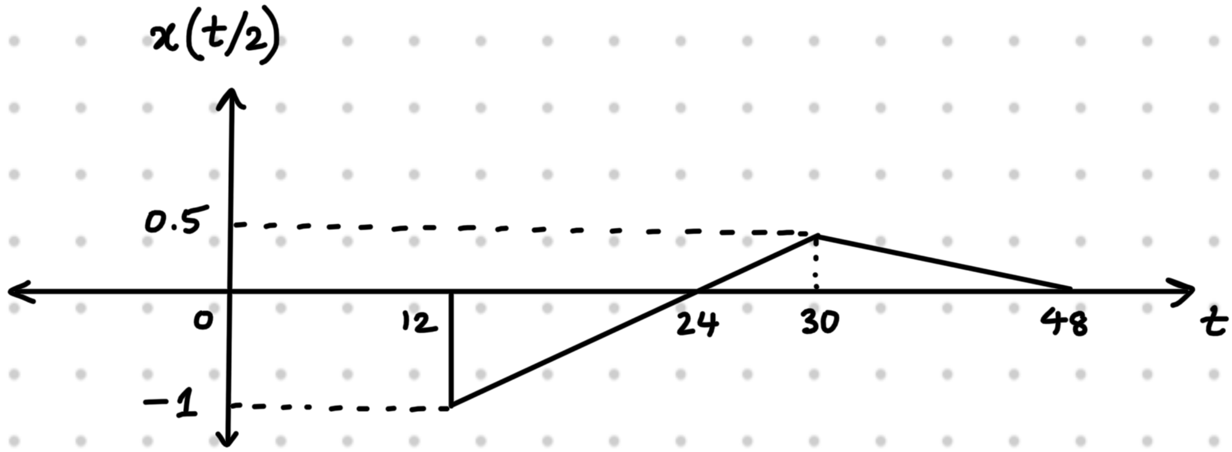
(b)



(c).



(d).



Q. Suppose an energy signal $x(t)$ has energy $E[x(t)]$. Let T be a nonzero, finite, real-valued constant.

Prove the following:

(a). $E[x(-t)] = E[x(t)]$

(b). $E[-x(t)] = E[x(t)]$

(c). $E[x(t-T)] = E[x(t)]$

(d). $E[T\,x(t)] = T^2\, E[x(t)]$

(e). $E[x(Tt)] = \frac{1}{|T|} E[x(t)]$

(f). $E[x(at-b)] = \frac{1}{|a|} E[x(t)]$

A. (a). $E[x(-t)] = \int_{-\infty}^{\infty} (x(-t))^2 dt = \int_{-\infty}^{\infty} x^2(\tau)\, d\tau$

$= E[x(t)]$

(b). $E[-x(t)] = \int_{-\infty}^{\infty} (-x(t))^2\, dt = \int_{-\infty}^{\infty} (x(t))^2 dt$

$= E[x(t)]$

(c). $$E\left[x(t-T)\right] = \int_{-\infty}^{\infty} \left(x(t-T)\right)^2 dt = \int_{-\infty}^{\infty} \left(x(\tau)\right)^2 d\tau$$

$$= E\left[x(t)\right]$$

(d). $$E\left[T x(t)\right] = \int_{-\infty}^{\infty} \left(T x(t)\right)^2 dt = T^2 E\left[x(t)\right]$$

(e). $$E\left[x(Tt)\right] = \int_{-\infty}^{\infty} \left(x(Tt)\right)^2 dt = \int_{-\infty}^{\infty} \frac{1}{|T|} \left(x(\tau)\right)^2 d\tau$$

$$= \frac{1}{|T|} E\left[x(t)\right]$$

(f). $$E\left[x(at-b)\right] = \int_{-\infty}^{\infty} \left(x(at-b)\right)^2 dt = \int_{-\infty}^{\infty} \frac{1}{|a|} \left(x(\tau)\right)^2 d\tau$$

$$= \frac{1}{|a|} E\left[x(t)\right]$$

Q. Show that :

(a). $\delta(t)$ is an even function of t.

(b). $\delta(at) = \frac{1}{|a|}\delta(t)$

(c). $\int_{-\infty}^{\infty} \dot{\delta}(t)\,\phi(t)\,dt = -\dot{\phi}(0)$

where $\phi(t)$ and $\dot{\phi}(t)$ are continuous at $t=0$

and $\phi(t) \to 0$ as $t \to \pm\infty$.

A. (a). $\int_{-\infty}^{\infty} x(t)\,\delta(-t)\,dt = \int_{-\infty}^{\infty} x(-\tau)\,\delta(\tau)\,d\tau = x(0)$

$$\Rightarrow \int_{-\infty}^{\infty} x(t)\,\delta(t)\,dt = \int_{-\infty}^{\infty} x(t)\,\delta(-t)\,dt = x(0)$$

$$\Rightarrow \delta(-t) = \delta(t)$$

(b). $\int_{-\infty}^{\infty} x(t)\,\delta(at)\,dt = \frac{1}{|a|}\int_{-\infty}^{\infty} x\left(\frac{\tau}{a}\right)\delta(\tau)\,d\tau = \frac{x(0)}{|a|}$

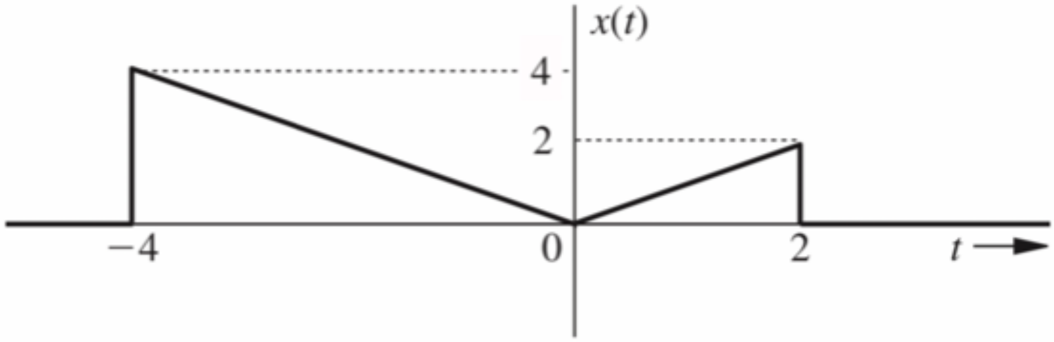
$$= \frac{1}{|a|}\int_{-\infty}^{\infty} x(t)\,\delta(t)\,dt \Rightarrow \delta(at) = \frac{\delta(t)}{|a|}$$

(c). $\int_{-\infty}^{\infty} \dot{\delta}(t)\,\phi(t)\,dt = \phi(t)\,\delta(t)\Big|_{-\infty}^{\infty} - \int_{-\infty}^{\infty} \dot{\phi}(t)\,\delta(t)\,dt$

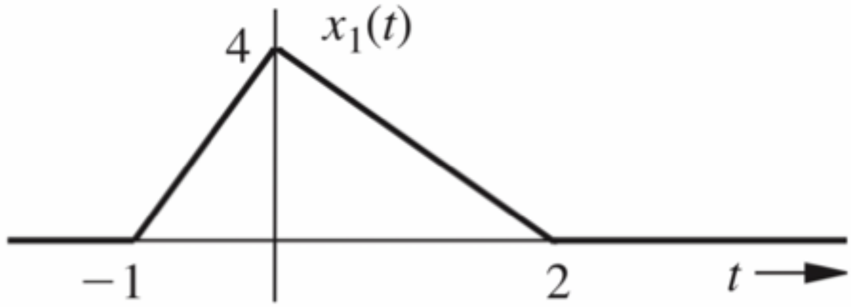
$$= 0 - \int_{-\infty}^{\infty} \dot{\phi}(t)\,\delta(t)\,dt = -\dot{\phi}(0)$$

Q. Sketch :

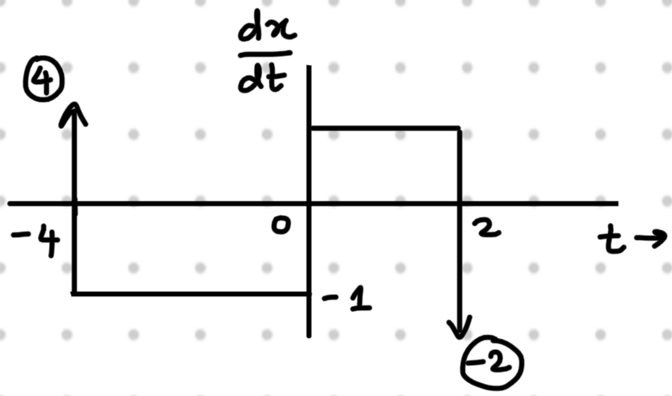
(a). $\frac{dx}{dt}$ for the signal $x(t)$ and



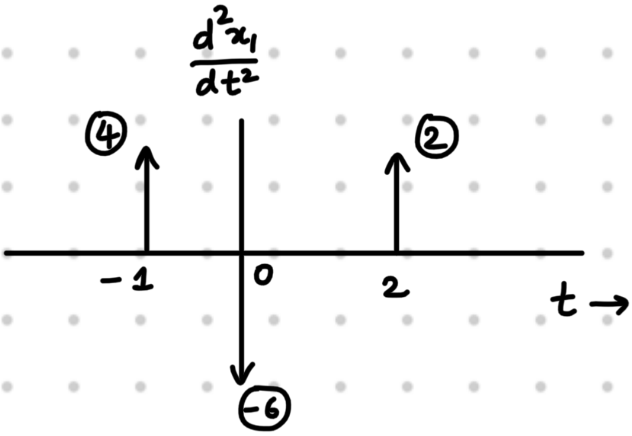
(b). $\frac{d^2x_1}{dt^2}$ for the signal $x_1(t)$



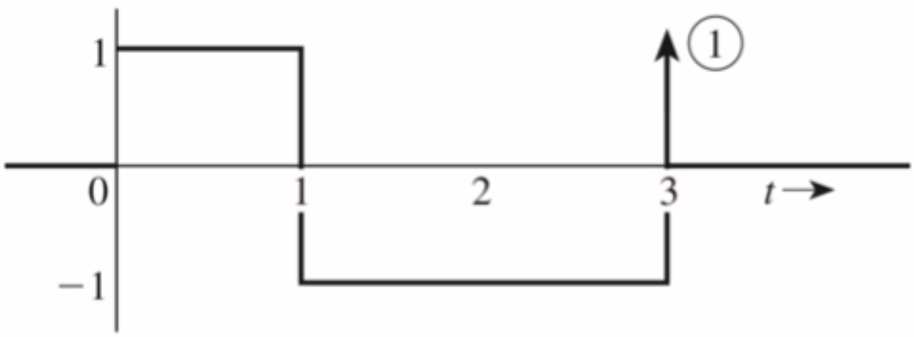
A. (a).



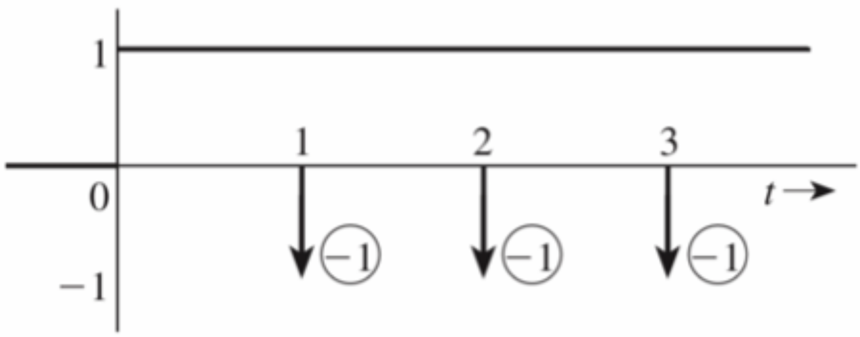
(b).



Q. Find and sketch $\int_{-\infty}^{t} x(t)\,dt$ for the following:

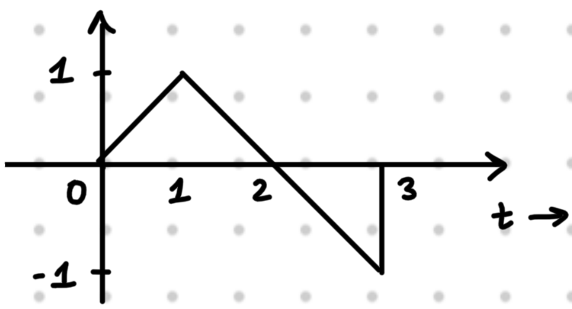


(a)

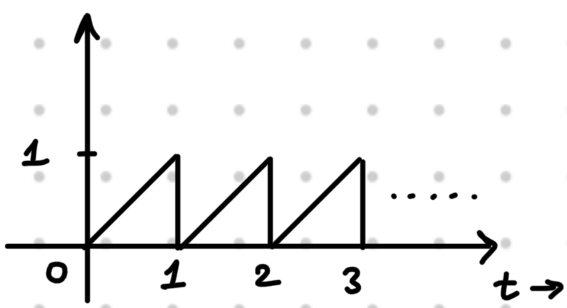


(b)

A. (a).



(b).



Q. If $x_e(t)$ and $x_o(t)$ are even and the odd components of a real signal $x(t)$, then show:

(a). $\int_{-\infty}^{\infty} x_e(t)\, x_o(t)\, dt = 0$

(b). $\int_{-\infty}^{\infty} x(t)\, dt = \int_{-\infty}^{\infty} x_e(t)\, dt$

A. (a). $\int_{-\infty}^{\infty} \frac{x(t) + x(-t)}{2} \cdot \frac{x(t) - x(-t)}{2}\, dt$

$$= \frac{1}{4} \int_{-\infty}^{\infty} \left(x^2(t) - x^2(-t) \right) dt$$

$$= \frac{1}{4} \left(\int_{-\infty}^{\infty} x^2(t)\, dt - \int_{-\infty}^{\infty} x^2(-t)\, dt \right) = 0$$

(b). $\int_{-\infty}^{\infty} x(t)\, dt = \int_{-\infty}^{\infty} \frac{x(t) + x(-t) + x(t) - x(-t)}{2}\, dt$

$$= \int_{-\infty}^{\infty} x_e(t)\, dt + \int_{-\infty}^{\infty} \frac{x(t) - x(-t)}{2}\, dt = \int_{-\infty}^{\infty} x_e(t)\, dt$$

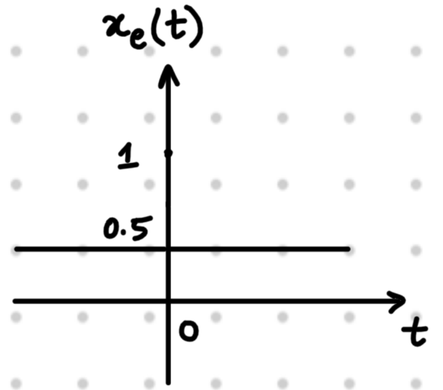
Q. Find and sketch the odd and the even components of the following signals :

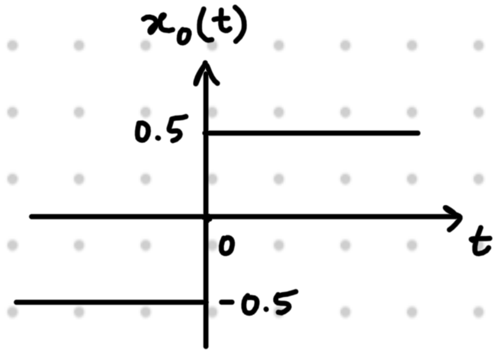
(a) $u(t)$ (c) $\sin \omega_0 t$

(b) $t\,u(t)$ (d) $\cos \omega_0 t$

A. (a). $$x_e(t) = \frac{u(t) + u(-t)}{2} = \begin{cases} 1 & t = 0 \\ 0.5 & t \neq 0 \end{cases}$$

$$x_o(t) = \frac{u(t) - u(-t)}{2} = \begin{cases} 0 & t = 0 \\ 0.5 & t > 0 \\ -0.5 & t < 0 \end{cases}$$

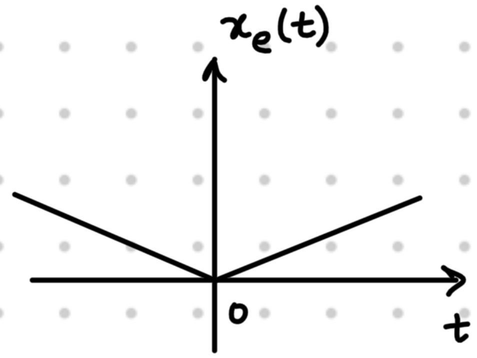


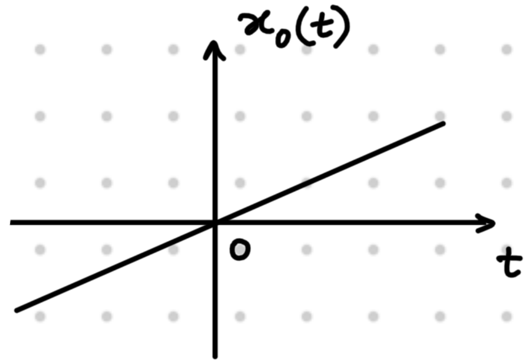


(b). $$x_e(t) = \frac{t\,u(t) + -t\,u(-t)}{2} = \frac{t}{2}\left(u(t) - u(-t)\right)$$

$$= \begin{cases} \frac{t}{2} & t \geq 0 \\ -\frac{t}{2} & t \leq 0 \end{cases} = \frac{|t|}{2}$$

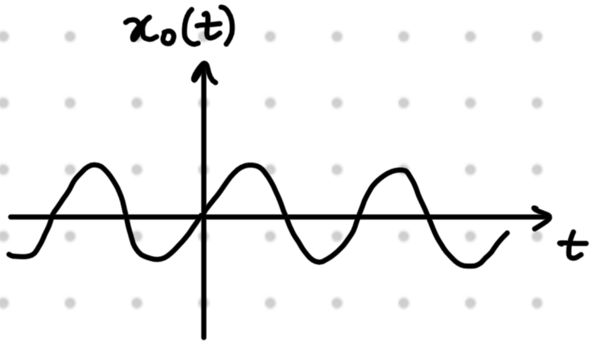
$$x_o(t) = \frac{t\,u(t) - (-t)u(-t)}{2} = \frac{t}{2}\left(u(t) + u(-t)\right) = \frac{t}{2}$$





(c). $x_e(t) = \dfrac{\sin \omega_0 t + -\sin \omega_0 t}{2} = 0$

$x_o(t) = \dfrac{\sin \omega_0 t - (-\sin \omega_0 t)}{2} = \sin \omega_0 t$



(d). $x_e(t) = \dfrac{\cos \omega_0 t + \cos(-\omega_0 t)}{2} = \cos \omega_0 t$

$x_o(t) = \dfrac{\cos \omega_0 t - \cos(-\omega_0 t)}{2} = 0$