Q. Which of the following systems are time-invariant?

(a). $y(t) = x(t-2)$

(c). $y(t) = \int_{-5}^{5} x(\tau)\, d\tau$

(b). $y(t) = x(-t)$

(d). $y(t) = \left(\frac{dx}{dt}\right)^2$

A. (a). $x(t)$ produces $y(t)$ such that $y(t) = x(t-2)$

Now, let $x_D(t) = x(t-T)$ be the delayed input.

Then, $y_D(t) = x_D(t-2) = x(t-2-T)$

$$= y(t-T)$$

Since an input delayed by T produced an output delayed by T, the system is time invariant.

(b). $x(t)$ produces $y(t)$ such that $y(t) = x(-t)$

Now, let $x_D(t) = x(t-T)$ be the delayed input.

Then, $y_D(t) = x_D(-t) = x(-t-T)$

$$\neq x(-t+T) = y(t-T)$$

Since an input delayed by T did not produce an output delayed by T, the system is time variant.

(c). $x(t)$ produces $y(t)$ such that $y(t) = \int_{-5}^{5} x(\tau)\,d\tau$

Now, let $x_D(t) = x(t-T)$ be the delayed input.

Then, $y_D(t) = \int_{-5}^{5} x_D(\tau)\,d\tau = \int_{-5}^{5} x(\tau - T)\,d\tau = \int_{-5-T}^{5-T} x(\tau)\,d\tau$

area of $x(t)$ over $[-5-T, 5-T]$

$\neq \int_{-5}^{5} x(\tau)\,d\tau = y(t-T)$

area of $x(t)$ over $[-5, 5]$
which is a constant for all $t$

Since an input delayed by T did not produce an output delayed by T, the system is time variant.

(d). $x(t)$ produces $y(t)$ such that $y(t) = \left(\frac{dx}{dt}\right)^2$

Now, let $x_D(t) = x(t-T)$ be the delayed input.

Then, $y_D(t) = \left(\frac{dx_D}{dt}\right)^2 = \left(\frac{dx(t-T)}{dt}\right)^2$

$= y(t-T)$

Since an input delayed by T produced an output delayed by T, the system is time invariant.

Q. Which of the following systems are causal ?

(a). $y(t) = x(t-2)$ (c). $y(t) = x(at)$, $a > 1$

(b). $y(t) = x(-t)$ (d). $y(t) = x(at)$, $a < 1$

A. (a). $y(t)$ starts after the input by 2 seconds.

Hence, the system is causal.

(b). If the input starts at $t=0$, the output starts before $t=0$.

Hence, the system is not causal.

(c). If the input starts at $t>0$, the output will start before the input.

Hence, the system is not causal.

(d). If the input starts at $t<0$, the output will start before the input.

Hence, the system is not causal.

Q. Which of the following systems are invertible?

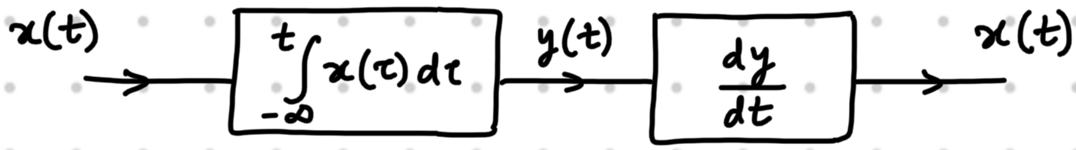
(a). $y(t) = \int_{-\infty}^{t} x(\tau)\, d\tau$

(b). $y(t) = \dfrac{dx}{dt}$

(c). $y(t) = x^n(t),\ n \in \mathbb{Z}$

(d). $y(t) = e^{x(t)},\ x(t)$ real

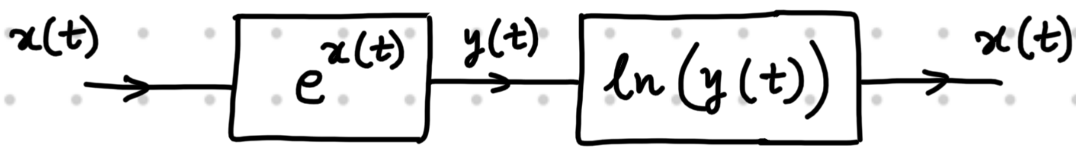
A. (a). The input can be obtained by taking the derivative. Hence, the system is invertible.




(b). Not invertible. Example: Two distinct constants will produce $y(t) = 0$.

(c). Not invertible. Example: $x(t)$ and $-x(t)$ will both produce $y(t) = x^n(t)$ for even values of $n$.

(d). The input can be obtained by taking the logarithm. Hence, the system is invertible.



Q. Suppose $y(t) = 0.5 \int_{-\infty}^{\infty} x(\tau)\left[\delta(t-\tau) - \delta(t+\tau)\right] d\tau$

where $x(t)$ and $y(t)$ are the input and output of the system.

What does this system do?

(a). Is this system linear?

(b). Is this system time-invariant?

(c). Is this system invertible?

(d). Is this system causal?

(e). Is this system memoryless?

(f). Is this system BIBO stable?

A. $y(t)$ can be simplified as:

$$y(t) = 0.5\left(x(t) - x(-t)\right) = x_{odd}(t)$$

Hence, this system computes the odd part of $x(t)$.

(a). An input of $k_1 x_1 + k_2 x_2$ will produce an output of $k_1 y_1 + k_2 y_2$. Therefore, the system is linear.

(b). Suppose $x(t) = \begin{cases} t & t \in [-1, 1] \\ 0 & \text{otherwise} \end{cases}$

Then, $y(t) = \begin{cases} t & t \in [-1, 1] \\ 0 & \text{otherwise} \end{cases}$

However, $x_D(t) = x(t-1)$ will produce

$$y_D(t) = t \neq t - 1 = y_D(t).$$

Hence, the system is not time invariant.

(c). $x_1(t) = \sin t$ and $x_2(t) = \sin t + \cos t$

both yield the same output $y(t) = \sin t$.

Hence, the output cannot uniquely determine the input. Therefore, the system is not invertible.

(d). The output depends on the future values of the input (for $t < 0$). Hence, the system is not causal.

(e). The output depends on the past/future input values. Hence, the system is not memoryless.

(f). A bounded input guarantees a bounded output. Thus, the system is BIBO stable.

Q. Suppose $y(t+1) = \begin{cases} -2x(t) & \text{when } x(t) \geq 0 \\ 0 & \text{otherwise} \end{cases}$

where $x(t)$ and $y(t)$ are the input and output of the system.

(a). Is this system linear?

(b). Is this system time-invariant?

(c). Is this system invertible?

(d). Is this system causal?

(e). Is this system memoryless?

(f). Is this system BIBO stable?

A. (a). Suppose $x_1(t) = 1$, then $y_1(t+1) = -2$

but $x_2(t) = -k x_1 = -k$ (for some $k > 0$)

produces $y_2(t+1) = 0 \neq -k\, y_1(t+1)$

Hence, the system is not linear.

(b). Let $x_D(t) = x(t-T)$ be the delayed input.

Then, if $x_D(t) \geq 0$, $y_D(t+1) = -2\, x_D(t)$

$= -2\, x(t-T) = y(t+1-T)$

if $x_D(t) \leq 0$, $y_D(t+1) = 0 = y(t+1-T)$

Since an input delayed by T produced an output delayed by T, the system is time invariant.

(c). $x_1(t) = -1$ and $x_2(t) = -2$ both yield the same output $y(t+1) = 0$.

Hence, the output cannot uniquely determine the input. Therefore, the system is not invertible.

(d). The output depends solely on the input one second in the past and no future values of the input are needed to determine the output. Hence, the system is causal.

(e). The output depends on the past input values. Hence, the system is not memoryless.

(f). If $|x(t)| \leq M_x < \infty$,

then $|y(t+1)| \leq |-2x(t)| \leq 2M_x < \infty$.

Hence, a bounded input guarantees a bounded output. Thus, the system is BIBO stable.