2. Analysis of Electrical Networks :

Electrical networks may be analyzed by writing the integro-differential equation(s) of the system and then solving these equations by the Laplace transform. However, it is also possible to analyze electrical networks directly without having to write the integro-differential equations.

For this purpose, we need to represent a network in the "frequency domain" where all the voltages and currents are represented by their Laplace transforms.

$$v(t) = iR \quad \Rightarrow \quad V(s) = I(s)\,R$$

$$v(t) = L\frac{di}{dt} \quad \Rightarrow \quad V(s) = L\left(s\,I(s) - i(0^-)\right)$$

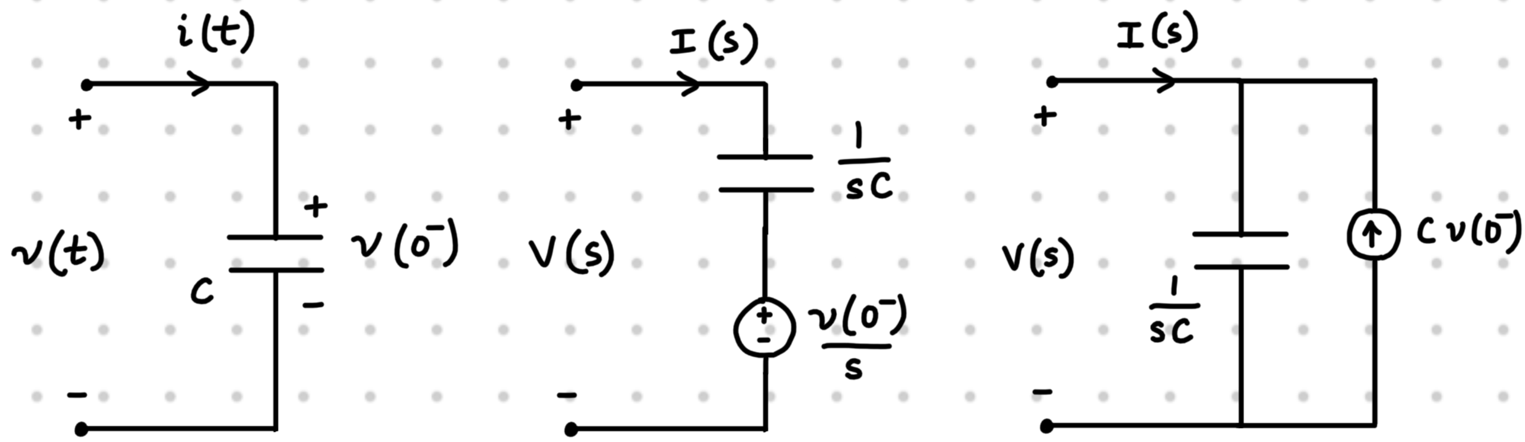
$$i(t) = C\frac{dv}{dt} \quad \Rightarrow \quad I(s) = C\left(s\,V(s) - v(0^-)\right)$$

$$\sum_{j=1}^{k} v_j(t) = 0 \quad \Rightarrow \quad \sum_{j=1}^{k} V_j(s) = 0$$

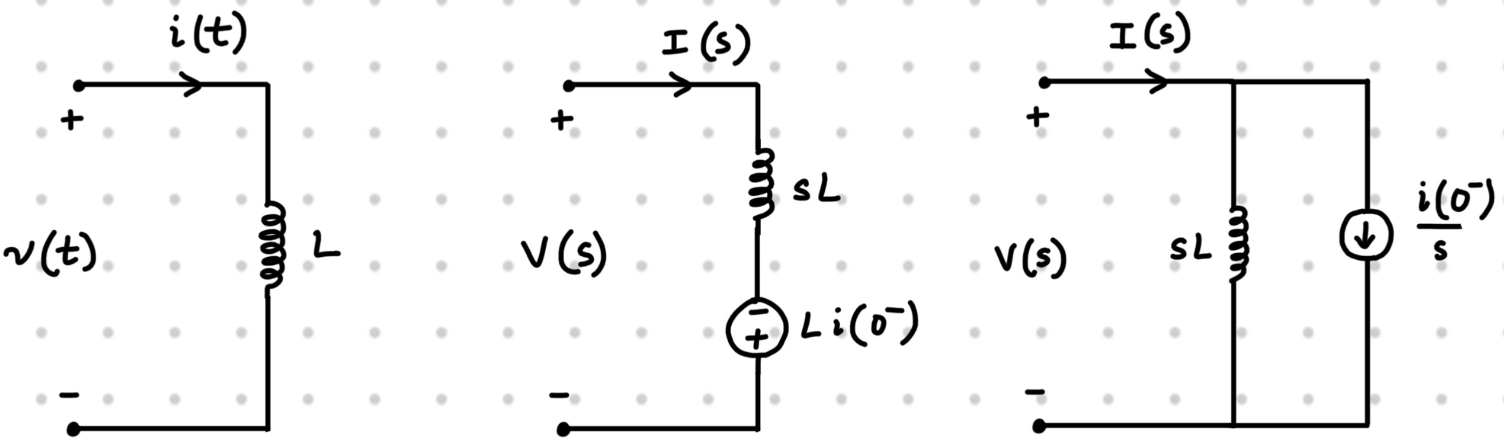
$$\sum_{j=1}^{m} i_j(t) = 0 \quad \Rightarrow \quad \sum_{j=1}^{m} I_j(s) = 0$$

Initial condition generators

Equivalent representation of a Capacitor with initial voltage:

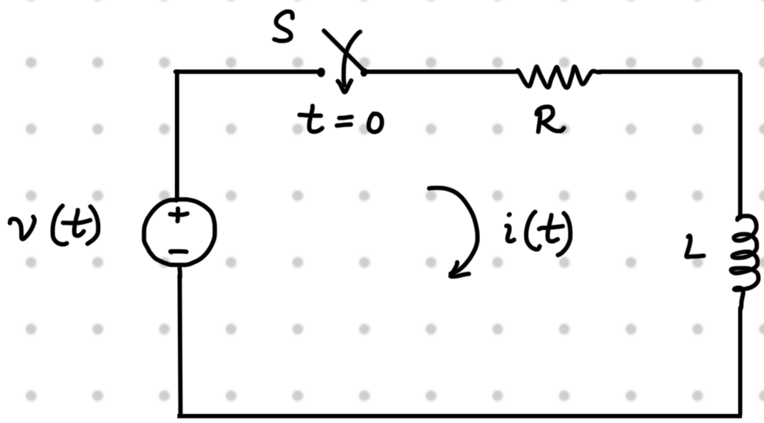


Equivalent representation of an Inductor with initial current:



Series R-L circuit

Let the switch S be closed at time $t=0$ for the RL circuit shown below.



Applying Kirchoff's voltage law,

$$v(t) = R\,i(t) + L\frac{di}{dt}$$

Taking Laplace transform,

$$V(s) = R\,I(s) + L\left\{s\,I(s) - i(0^-)\right\}$$

With $i(0^-) = i(0^+) = 0$,

$$\Rightarrow \quad V(s) = (R + sL)\,I(s)$$

Step Response :

If $v(t) = V_0\,u(t)$, then $V(s) = \dfrac{V_0}{s}$

$$\Rightarrow \quad \frac{V_0}{s} = (R + sL)\,I(s)$$

$$\Rightarrow \quad I(s) = \frac{V_0}{L} \cdot \frac{1}{s\left(s + \frac{R}{L}\right)} = \frac{V_0}{R}\left(\frac{1}{s} - \frac{1}{s + \frac{R}{L}}\right)$$

Therefore, $$i(t) = \frac{V_0}{R}\left(1 - e^{\frac{-Rt}{L}}\right) u(t)$$

Impulse Response:

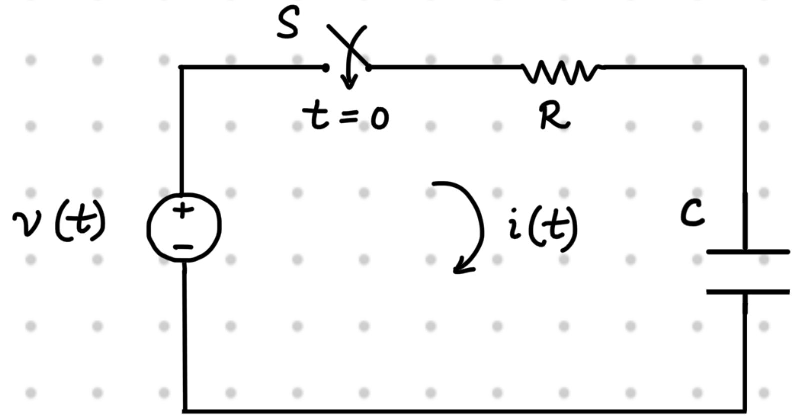
If $v(t) = \delta(t) \Rightarrow V(s) = 1$

$$\Rightarrow \quad 1 = (R + sL)\, I(s)$$

$$\Rightarrow \quad I(s) = \frac{1}{L}\left(\frac{1}{s + R/L}\right) \Rightarrow i(t) = \frac{1}{L} e^{\frac{-Rt}{L}} u(t)$$

Series R-C circuit

Let the switch S be closed at time $t = 0$ for the RC circuit shown below.



Then, $$v(t) = R\, i(t) + \frac{1}{C}\int_{-\infty}^{t} i(t)\, dt$$

$$\Rightarrow \quad v(t) = R\, i(t) + \frac{1}{C}\int_0^t i(t)\,dt + \frac{1}{C}\int_{-\infty}^{0} i(t)\,dt$$

Taking Laplace transform,

$$V(s) = R\, I(s) + \frac{1}{C}\left[\frac{I(s)}{s} + \frac{q(0^-)}{s}\right]$$

With $q(0^-) = q(0^+) = 0$

$$\Rightarrow \quad V(s) = \left(R + \frac{1}{sC}\right) I(s)$$

Step Response :

If $v(t) = V_0\, u(t)$, then $V(s) = \frac{V_0}{s}$

$$\Rightarrow \frac{V_0}{s} = \left(R + \frac{1}{sC}\right) I(s) \quad \Rightarrow \quad I(s) = \frac{V_0/R}{s + \frac{1}{RC}}$$

Therefore, $i(t) = \frac{V_0}{R}\, e^{-t/RC}\, u(t)$

Impulse Response :

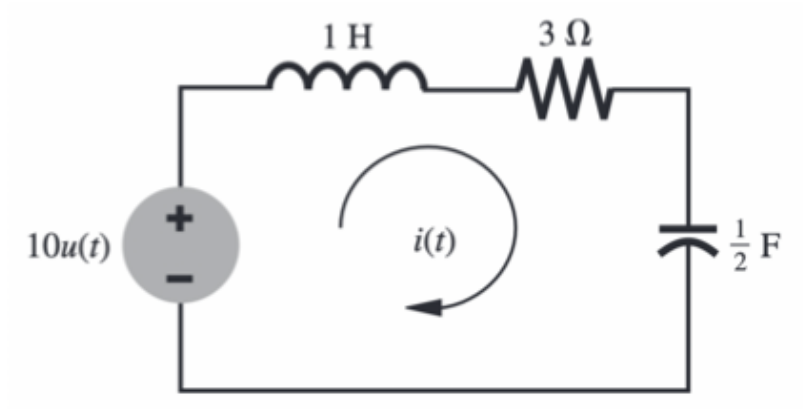
If $v(t) = \delta(t) \Rightarrow V(s) = 1$

$$\Rightarrow 1 = \left(R + \frac{1}{sC}\right) I(s) \quad \Rightarrow \quad I(s) = \frac{s}{R\left(s + \frac{1}{RC}\right)}$$

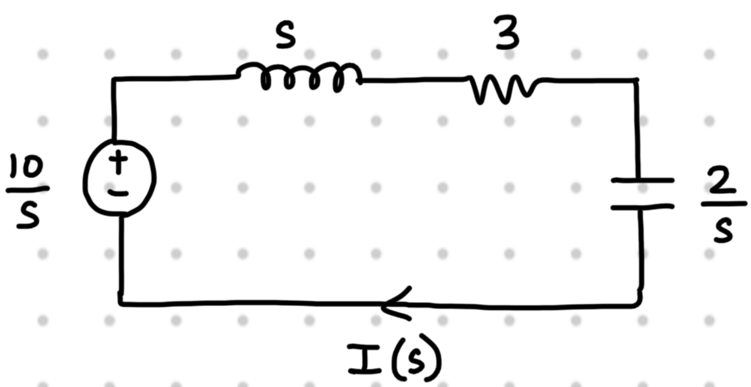
$$\Rightarrow \quad I(s) = \frac{1}{R}\left[1 - \frac{1}{RC}\,\frac{1}{(s + 1/RC)}\right]$$

Therefore, $i(t) = \frac{1}{R}\left[\delta(t) - \frac{1}{RC}\, e^{-t/RC}\, u(t)\right]$

Q. Find the loop current $i(t)$ in the circuit shown below if all the initial conditions are zero.



A. The equivalent circuit in the s-domain is:



The total impedance in the loop is:

$$Z(s) = s + 3 + \frac{2}{s} = \frac{s^2+3s+2}{s}$$

$$\Rightarrow \quad I(s) = \frac{V(s)}{Z(s)} = \frac{10/s}{\frac{s^2+3s+2}{s}} = \frac{10}{s+1} - \frac{10}{s+2}$$

$$\Rightarrow \quad i(t) = 10\left(e^{-t} - e^{-2t}\right)u(t)$$