Practice Questions

EEC201 - Signals, Systems, and Networks (Monsoon 2024)
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Fourier Series

1. Sketch the following signals and compute their Trigonometric Fourier Series coefficients:

(a) $x(t) = \begin{cases} 0 & -\pi \le t < 0 \\ t & 0 \le t < \pi \\ x(t+2\pi) & \forall t \end{cases}$

(b) $x(t) = \begin{cases} 0 & -1 \le t < \frac{-1}{2} \\ -1 & \frac{-1}{2} \le t < 0 \\ 1 & 0 \le t < 1/2 \\ 0 & \frac{1}{2} \le t < 1 \\ x(t+2) & \forall t \end{cases}$

(c) $x(t) = \begin{cases} t & \frac{-1}{2} < t < \frac{1}{2} \\ 1-t & \frac{1}{2} < t < \frac{3}{2} \\ x(t+2) & \forall t \end{cases}$

(d) $x(t) = \begin{cases} 0 & -\pi < t \le 0 \\ \sin(t) & 0 \le t < \pi \\ x(t+2\pi) & \forall t \end{cases}$

Answer:

(a) $a_n = \begin{cases} \frac{\pi}{4} & n = 0 \\ \frac{(-1)^n - 1}{\pi n^2} & n \ge 1 \end{cases}$ and $b_n = \frac{(-1)^{n+1}}{n}$

(b) $a_n = 0,\ \ b_n = \frac{2}{n\pi}\left(1 - \cos\left(\frac{n\pi}{2}\right)\right)$

(c) $a_n = \begin{cases} 0 & n = 0 \\ 0 & \text{if } n \text{ is odd} \\ \frac{4}{n^2\pi^2} & \text{if } n \text{ is even} \end{cases}$ and $b_n = 0$

(d) $a_n = \begin{cases} \frac{1}{\pi} & n = 0 \\ \frac{2}{\pi(1-n^2)} & n \ne 1 \end{cases}$ and $b_n = \begin{cases} \frac{1}{2} & n = 1 \\ 0 & n \ne 1 \end{cases}$

2. Sketch the following signals and compute their Exponential Fourier Series coefficients:

(a) $x(t) = \begin{cases} 1 & 1 \le |t| \le 3 \\ 0 & |t| \le 1 \\ x(t+10) & \forall\ t \end{cases}$

(b) $x(t) = \begin{cases} t & 0 \le t < 1/2 \\ t-1 & 1/2 \le t < 1 \\ x(t+1) & \forall t \end{cases}$

(c) $x(t) = \begin{cases} 0 & -\pi \le t < 0 \\ 1 & 0 \le t < \pi \\ x(t+2\pi) & \forall t \end{cases}$

(d) $x(t) = \begin{cases} 0 & -\pi < t < -a \\ 1 & -a < t < a \\ 0 & a < t < 2\pi \\ x(t+2\pi) & \forall t \end{cases}$

Answer:

(a) $D_n = \begin{cases} \frac{2}{5} & n = 0 \\ \frac{\sin\frac{3n\pi}{5} - \sin\frac{n\pi}{5}}{n\pi} & n \neq 0 \end{cases}$

(b) $D_n = \begin{cases} 0 & n = 0 \\ \frac{(-1)^n j}{2n\pi} & n \neq 0 \end{cases}$

(c) $D_n = \begin{cases} \frac{1}{2} & n = 0 \\ \frac{((-1)^n - 1)j}{2n\pi} & n \neq 0 \end{cases}$

(d) $D_n = \begin{cases} \frac{a}{\pi} & n = 0 \\ \frac{1}{n\pi}\sin(an) & n \neq 0 \end{cases}$

Fourier Transform

3. Compute the Fourier Transform of the following signals:

(a) $x(t) = \begin{cases} 1 & 1 \leq |t| \leq 3 \\ 0 & |t| \leq 1 \text{ and otherwise} \end{cases}$

(b) $x(t) = \begin{cases} 1 & 1 \leq |t| \leq 3 \\ -1 & |t| \leq 1 \\ 0 & \text{else} \end{cases}$

(c) $x(t) = \begin{cases} 1 - t^2 & |t| \leq 1 \\ 0 & |t| > 1 \end{cases}$

(d) $x(t) = 6e^{-a|t|}$

Answer:

(a) $X(\omega) = \dfrac{2\sin(3\omega) - 2\sin(\omega)}{\omega}$

(b) $X(\omega) = 6\text{sinc}(3\omega) - 4\text{sinc}(\omega)$ or $4\text{sinc}(\omega/2)(\cos(2\omega) - 1)$

(c) $X(\omega) = \dfrac{2\sqrt{2}}{\sqrt{\pi}} \dfrac{\sin(\omega) - \omega\cos(\omega)}{\omega^3}$

(d) $X(\omega) = \dfrac{12a}{a^2 + \omega^2}$

4. Compute the Inverse Fourier Transforms of the following functions:

(a) $X(\omega) = \dfrac{5}{6 - 5j\omega - \omega^2}$

(b) $X(\omega) = \dfrac{1}{(9 + \omega^2)(4 + \omega^2)}$

(c) $X(\omega) = \dfrac{e^{3j\omega}}{2 - j\omega}$

Answer:

(a) $x(t) = 5(e^{-2t} - e^{-3t})u(t)$

(b) $x(t) = \dfrac{e^{-2|t|}}{20} - \dfrac{e^{-3|t|}}{30}$

(c) $x(t) = e^{-2(t-3)}u(t-3)$