EEC201: Signals, Systems and Networks

Mid-Semester Examination (13.09.2024)

Department of Electrical Engineering

IIT (ISM) Dhanbad

Name: Total Marks: 32

Roll No: Time: 2 hours

General instructions:

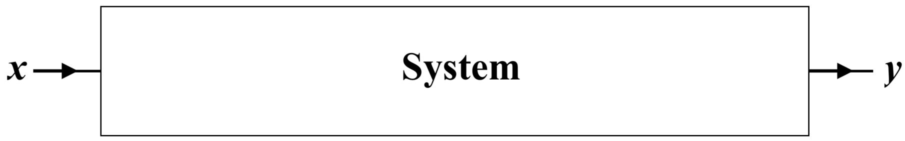
(a) Write your name and roll number on the question paper.

(b) Return the question paper along with the answer sheet.

(c) Mention clearly the steps involved in arriving at the final answer.

The Adventures of Hari and Sakshi

Hari and Sakshi are two ambitious geeks. Having enrolled in the EEC201 course, they decide to test out their analytical skills in the Networks and Systems laboratory. They come across a system with an input and an output as shown in the figure below.



Their goal is simple, they want to methodically analyse the system (that is, compute the output for a given input). Your job is to follow their conversation and help them out.

Sakshi: "In order to use the mathematical system analysis tools that we learnt in the course, we need to first check if the system is linear and time-invariant (LTI)."

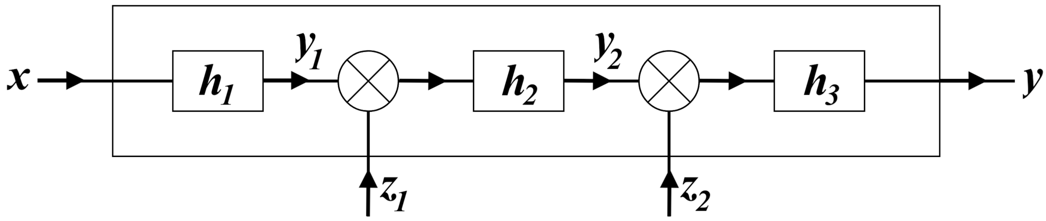
Hari: "Yes, that's correct. But before that, I would like to test you on your understanding of system classification. For the two systems given below, determine if they are Causal, Linear, Time-Invariant, and Invertible."

Q1: [3 marks] Help Sakshi fill in the table below by writing "Yes" if the property listed holds for the system and "No" if the property does not hold. Briefly justify your reasoning.

	Causal	Linear	Time-Invariant	Invertible
$y(t) = 3x(t)\sin(t)$				
$y(t) = x(t) + 2$				
$y(t) = x(\cos(t))$				

Next, Hari looks closely into the system and finds out that it is composed of three small sub-systems as shown in the figure (next page).

The input $x(t)$ is first passed through a system with unit impulse response $h_1(t)$. The output $y_1(t)$ is multiplied with a signal $z_1(t)$ using an analog multiplier and fed to the system with unit impulse response $h_2(t)$. Finally, the output $y_2(t)$ is multiplied with the signal $z_2(t)$ and passed through the system with unit impulse response $h_3(t)$ to obtain the final output $y(t)$.



Hari: "I wonder what will be the zero-state output $y_1(t)$ in terms of $x(t)$ and $h_1(t)$."

Sakshi: "Oh, that's easy. Since $x(t)$ is being fed into the system with unit impulse response $h_1(t)$, the zero-state output $y_1(t)$ will be equal to $x(t) * h_1(t)$."

Hari: "Oh, what about $y_2(t)$ then?"

Sakshi: "$y_2(t)$ will be equal to $(y_1(t) \cdot z_1(t)) * h_2(t)$."

Hari: "Great. Let's first compute $y_1(t)$."

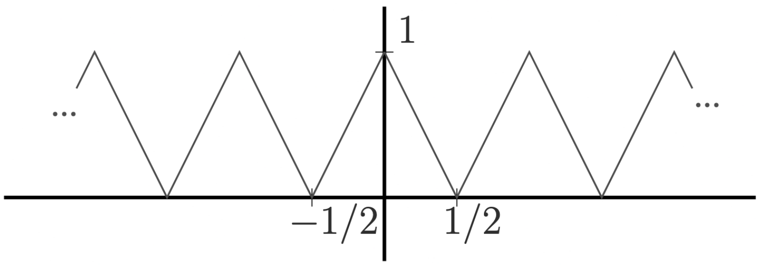
Q2: [6 marks] Using direct convolution, help Hari compute and plot the output $y_1(t)$ if

$$x(t) = \begin{cases} -1 & -1 < t < 0 \\ 1 & 0 < t < 1 \\ 0 & \text{else} \end{cases} \quad \& \quad h_1(t) = \begin{cases} t & 0 < t < 1 \\ 0 & \text{else} \end{cases}$$

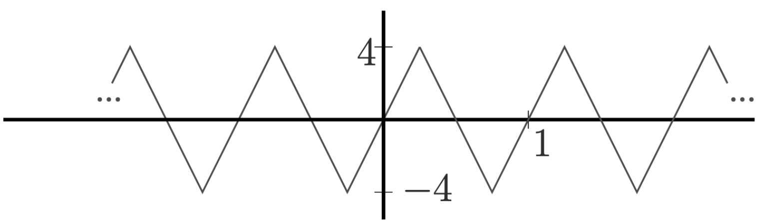
Hari: "This rigorous convolution gave me a headache, Sakshi. Is there a way to simplify this computation of the zero-state response?"

Sakshi: "Yes, breaking a periodic signal into Fourier Series and transforming an aperiodic signal using the Fourier Transform greatly simplifies the computation."

Q3(a): [5 marks] Using a function generator, Sakshi generated the signal shown below. Help Sakshi compute the signal's exponential Fourier Series coefficients (D_n). Furthermore, what is the value of $D_1 + D_2 + D_3 + D_4 + D_5$?



Q3(b): [3 marks] While Sakshi was busy computing the Fourier coefficients of the above signal, Hari was being playful with the function generator and disturbed the signal. As a result, a new signal was generated as shown below. Sakshi was furious at Hari's mischief, but immediately realized that she does not have to recalculate the Fourier coefficients from scratch. Help Sakshi compute the Fourier coefficients of the new signal (D_n') from the old signal.



Hari: "This is alright, Sakshi. But real life signals are generally aperiodic, so let us learn Fourier Transform and its applications."

Sakshi: "Sure, let us apply Fourier Transform in analysing our mysterious system. We will notice that Fourier Transform substantially decreases our computational complexity."

Q4: [12 marks] For the system discussed earlier and with the signals given below, help Hari:

$$x(t) = 2\ \text{sinc}^2(t)\cos(2t)$$

$$h_1(t) = \frac{\sin(2t)}{\pi t}, \quad h_2(t) = 2\ \text{sinc}^2\left(\frac{t}{4}\right)e^{\frac{jt}{2}}, \quad h_3(t) = \frac{1}{4\pi^2}\ \text{sinc}\left(\frac{t}{2}\right)e^{-jt}$$

$$z_1(t) = \sum_{n=-\infty}^{\infty} \delta(t - n\pi) \ \ \& \ \ z_2(t) = \sum_{k=0}^{5} \cos(kt)$$

(a) Plot $Y_1(\omega)$

(b) Plot the function $f_1(t) = y_1(t)z_1(t)$

(c) Compute $y_2(\pi)$ and $Y_2(\pi)$

(d) Plot the Fourier Transform of $f_2(t) = y_2(t)z_2(t)$

(e) Compute $\int_{-\infty}^{\infty} |y(t)|^2 dt$

(f) Plot $y(t)e^{jt}$

Sakshi: "Great job analysing our system using the Fourier Transform, Hari. Fourier Transform can also help us compute integrals that are otherwise not easily integrable."

Hari: "Oh, please give me an example."

Sakshi: "Sure, $\int_{-\infty}^{\infty} \text{sinc}(t)\ dt$ is difficult to compute in the time domain. However, we notice that if we write $X(\omega) = \int_{-\infty}^{\infty} \text{sinc}(t)e^{-j\omega t}\ dt$, then our required integral can simply be computed as $\lim_{\omega \to 0} X(\omega)$."

Q5: [3 marks] Using Sakshi's reasoning, help Hari compute the integrals

$$\int_0^{\infty} \text{sinc}(2t-1)\ dt \ \ \& \ \ \int_{-\infty}^{\infty} 4(\text{sinc}^2\omega)\ e^{j\omega}\ d\omega$$

EXTRA CREDIT

Q6: [3 marks] Compute

$$\int_{-\infty}^{\infty} \left| \left(\frac{\text{sinc}^2 t}{\pi}\right) * \left(\frac{\pi}{2}\frac{\sin(2t)}{\pi t}\frac{\sin(6t)}{\pi t}\right) \right|^2 dt$$

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