Assignment - 2

EEC201 - Signals, Systems, and Networks (Monsoon 2024)

Department of Electrical Engineering, IIT (ISM) Dhanbad

Due: 24.10.2024

1. Find the Laplace Transforms of the following:

(a) $\left(t + e^{-t} + t^2 e^{-t} + (t-4)e^{2t}\right)u(t)$ (b) $(\sin^2 t)u(t)$ (c) $\dfrac{\sin kt}{t}u(t)$

(d) $\cosh(t+\theta)u(t)$ (e) $\left(1-(1-t)e^{-3t}\right)u(t)$ (f) $|t|e^{-|t|}$

(g) $x(t) = \begin{cases} t & 0 \le t < 1 \\ e^t & t \ge 1 \\ 0 & \text{else} \end{cases}$ (h) $x(t) = \begin{cases} t-\pi & \pi \le t < 2\pi \\ 0 & \text{else} \end{cases}$ (i) $(t \sin t)u(t)$

(j) $x(t) = \begin{cases} 1 & 0 \le t < 2 \\ t^2 - 4t + 4 & t \ge 2 \\ 0 & \text{else} \end{cases}$ (k) $x(t) = \begin{cases} \cos(\pi t) & 0 < t < 4 \\ 0 & \text{else} \end{cases}$

Answer:

(a) $\dfrac{1}{s^2} + \dfrac{1}{s+1} + \dfrac{2}{(s+1)^3} + \dfrac{1}{(s-2)^2} - \dfrac{4}{s-2}$

(b) $\dfrac{2}{s(s^2+4)}$

(c) $\tan^{-1}\left(\dfrac{k}{s}\right)$

(d) $\dfrac{\sinh\theta + s\cosh\theta}{s^2-1}$

(e) $\dfrac{4s+9}{s(s+3)^2}$

(f) $\dfrac{2(s^2+1)}{(s^2-1)^2}$

(g) $\dfrac{1}{s^2} + e^{-s}\left(\dfrac{e}{s-1} - \dfrac{1}{s} - \dfrac{1}{s^2}\right)$

(h) $\dfrac{e^{-\pi s}}{s^2} - e^{-2\pi s}\left(\dfrac{\pi}{s} + \dfrac{1}{s^2}\right)$

(i) $\dfrac{2s}{(s+1)^2}$

(j) $\dfrac{1}{s} + e^{-2s}\left(\dfrac{2}{s^3} - \dfrac{1}{s}\right)$

(k) $\dfrac{s(1-e^{-4s})}{s^2+\pi^2}$

2. Find the Inverse Laplace Transforms of the following:

(a) $\dfrac{24}{s^4} - \dfrac{9}{s^2+9}$ (b) $\dfrac{8}{s^3+4s}$ (c) $\dfrac{1+e^{-2s}}{s^2+6}$ (d) $\dfrac{2s+3}{s^2+4s+13}$

(e) $\dfrac{1}{(s+1)(s^2-1)}$ (f) $\dfrac{s^2-2s}{s^4+5s^2+4}$ (g) $\dfrac{e^{-3s}}{s-2}$ (h) $\dfrac{5(s+2)^2}{s(s+1)^3}$

(i) $\dfrac{2s+5}{s^2+5s+6}e^{-2s}$ (j) $\dfrac{s}{(s-3)^5}$ (k) $\dfrac{3+e^{-(s-1)}}{s^2-2s+5}$ (l) $\dfrac{9+s}{4-s^2}$

Answer:

(a) $(4t^3 - 3\sin(3t))u(t)$

(b) $(2 - 2\cos(2t))u(t)$

(c) $\dfrac{\sin\sqrt{6}t}{\sqrt{6}}u(t) + \dfrac{\sin\left(\sqrt{6}(t-2)\right)}{\sqrt{6}}u(t-2)$

(d) $\left(2\cos 3t - \dfrac{\sin 3t}{3}\right)e^{-2t}u(t)$

(e) $\dfrac{e^t - 2te^{-t} - e^{-t}}{4}u(t)$

(f) $\dfrac{2\cos 2t + 2\sin 2t - 2\cos t - \sin t}{3}u(t)$

(g) $e^{2(t-3)}u(t-3)$

(h) $\left(20 - e^{-t}(2.5t^2 + 15t + 20)\right)u(t)$

(i) $(e^{-2(t-2)} + e^{-3(t-2)})u(t-2)$

(j) $e^{3t}\left(\dfrac{t^3}{6} + \dfrac{t^4}{8}\right)u(t)$

(k) $\dfrac{3e^t}{2}(\sin 2t)u(t) + \dfrac{e^t}{2}(\sin 2(t-1))u(t-1)$

(l) $-(\cosh 2t + 4.5\sinh 2t)$

3. Verify the Initial value theorem and the Final value theorem for the following:

(a) $e^{-t}(t+2)^2\ u(t)$ (b) $(1 + e^{-t}(\sin t + \cos t))\ u(t)$

(c) $t^2e^{-3t}\ u(t)$ (d) $u(t-T)$

Answer:

(a) $x(0^+) = 4,\ x(\infty) = 0$

(b) $x(0^+) = 2,\ x(\infty) = 1$

(c) $x(0^+) = 0,\ x(\infty) = 0$

(d) If $T > 0$: $x(0^+) = 0,\ x(\infty) = 1$

If $T < 0$: $x(0^+) = 1,\ x(\infty) = 1$

4. Solve the following differential equations:

(a) $y' + 3y = t^2 e^{-3t} + te^{-2t} + t$ with $y(0) = 1$

(b) $y'' + 4y = 0$ with $y(0) = 1,\ y'(0) = 6$

(c) $y'' + 4y' + 8y = e^{-t}$ with $y(0) = y'(0) = 0$

(d) $y''' - y' = 2$ with $y(0) = y'(0) = y''(0) = 4$

(e) $y'' + 2y' + 5y = 0$ with $y(0) = 2,\ y'(0) = 0$

(f) $y'' + 4y' + 3y = 10\cos(t)$ with $y(0) = 1,\ y'(0) = 2$

(g) $y'' + 4y' + 4y = 3te^{-2t}$ with $y(0) = 0,\ y'(0) = 1$

(h) $y'''' + y = 0$ with $y(0) = y'(0) = y''(0) = y'''(0) = 1$

Answer:

(a) $y(t) = \left(e^{-3t}\left(2 + \frac{t^3}{3}\right) + e^{-2t}(t-2) + \frac{t}{3} - \frac{1}{9}\right)u(t)$

(b) $y(t) = \cos 2t + 3\sin 2t$

(c) $\dfrac{2e^{-t} - e^{-2t}(2\cos 2t + \sin 2t)}{10}$

(d) $y(t) = 5e^{t} - e^{-t} - 2t$

(e) $y(t) = e^{-t}(2\cos 2t + \sin 2t)$

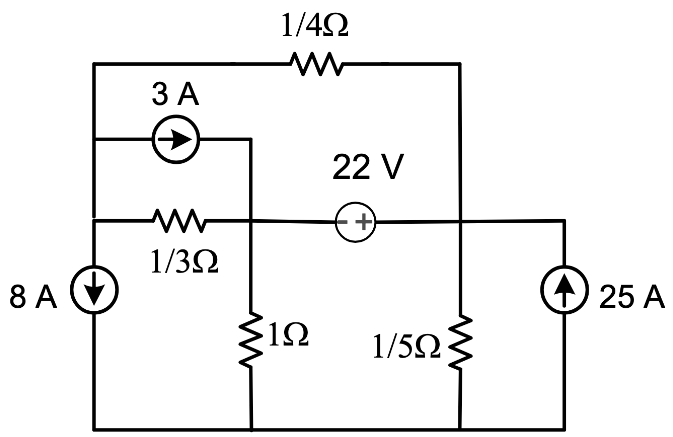
(f) $y(t) = \cos t + 2\sin t$

(g) $y(t) = e^{-2t}\left(t + \frac{t^3}{2}\right)$

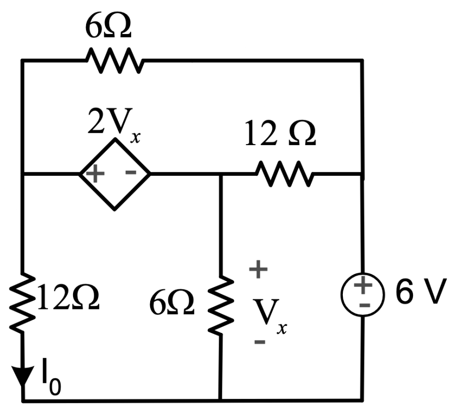
(h) $y(t) = \frac{1}{\sqrt{2}}\left(\sin\left(\frac{t}{\sqrt{2}}\right)\cosh\left(\frac{t}{\sqrt{2}}\right) - \cos\left(\frac{t}{\sqrt{2}}\right)\sinh\left(\frac{t}{\sqrt{2}}\right)\right)$

5. Using KCL/KVL and Nodal/Mesh Analysis, answer the following:

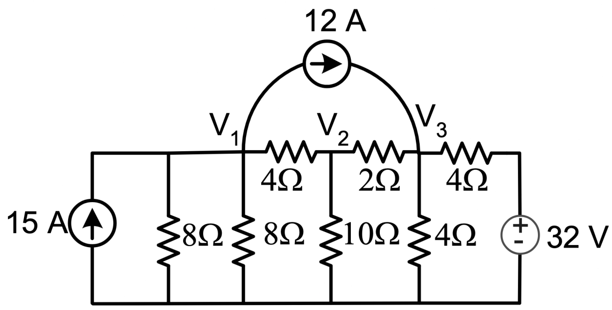
(a) What are the node voltages in this circuit?



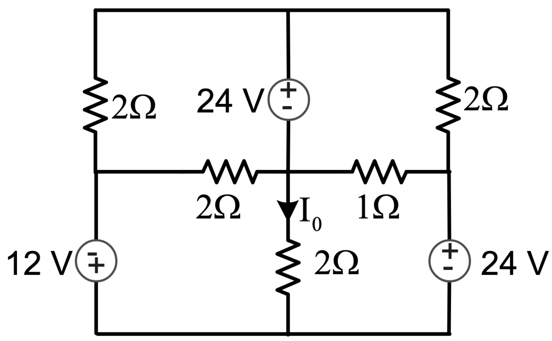
(b) Calculate the node voltages and the current I_0 in the circuit below.



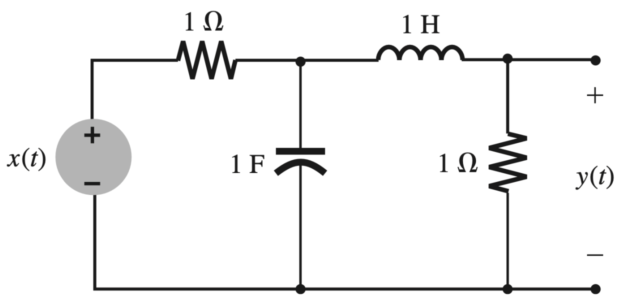
(c) What is the power dissipated by the 10 Ohm resistor?



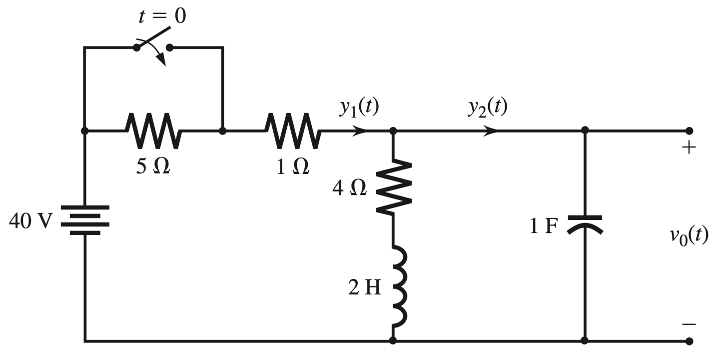
(d) What is the current I_0 in the circuit below?



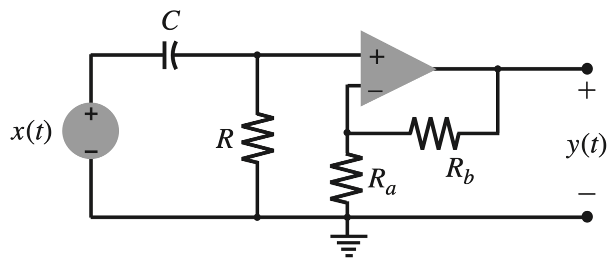
(e) If $x(t) = te^{-t}u(t)$ V, calculate $y(t)$:



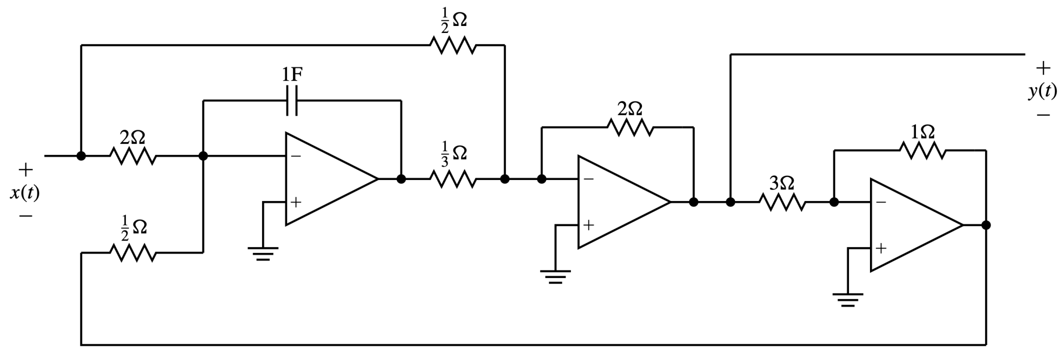
(f) If the switch is open for a long time and suddenly closed at $t = 0$, calculate $y_1(t)$ and $y_2(t)$:



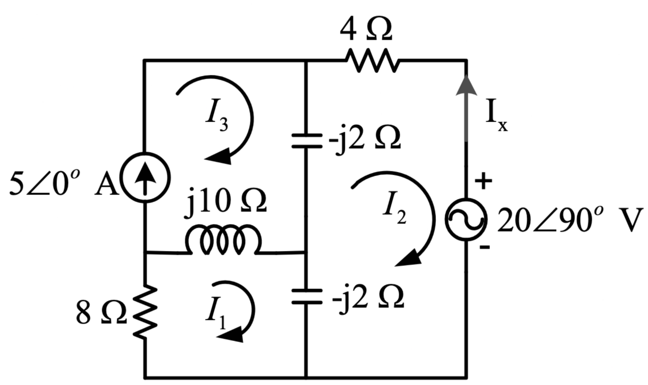
(g) What is the transfer function of this circuit?



(h) What is the transfer function of this circuit?

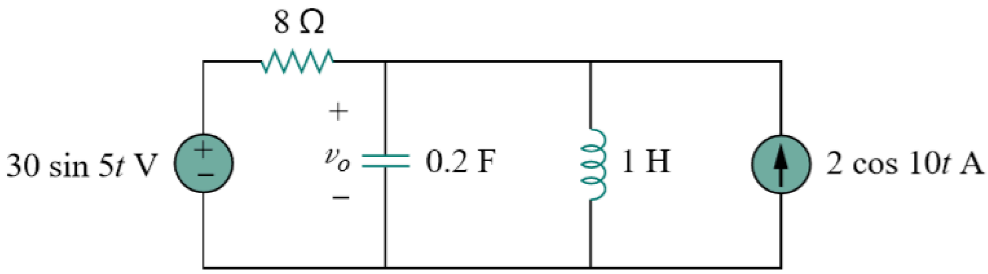


(i) What is the current I_x flowing out of the voltage source?

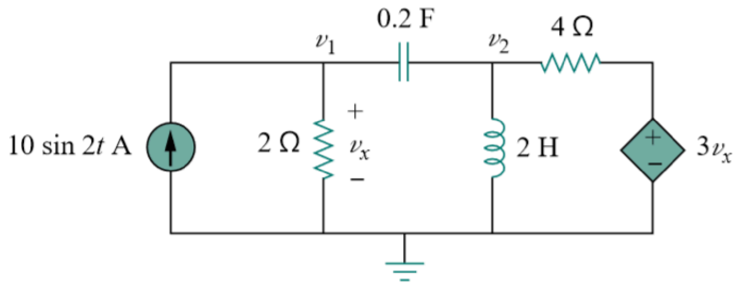


(j) What is the voltage $V_0(t)$ across the capacitor?

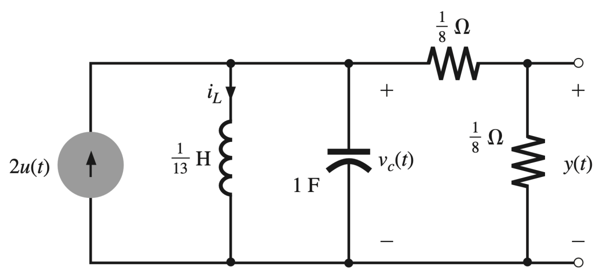
(Hint: Convert the circuit to phasor domain, solve the equations and the convert back to time domain.)



(k) What are the node voltages v_1 and v_2?

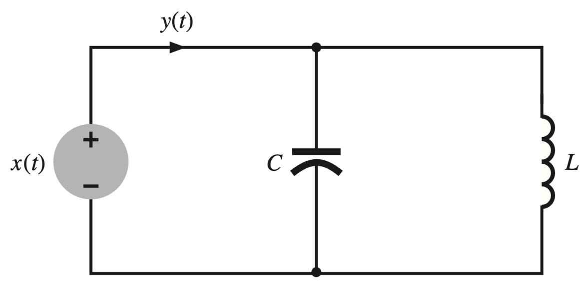


(l) If $i_L(0^-) = 1A$ and $v_C(0^-) = 3V$, calculate the voltage $y(t)$ for $t \geq 0$.

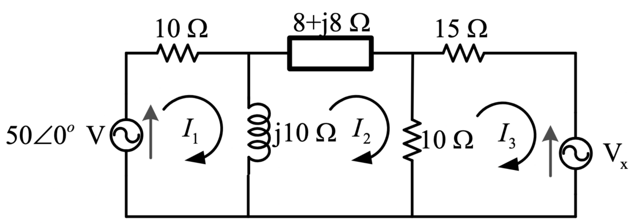


(m) Assuming zero initial conditions and $\omega_0^2 = \dfrac{1}{LC}$, calculate the current $y(t)$ if

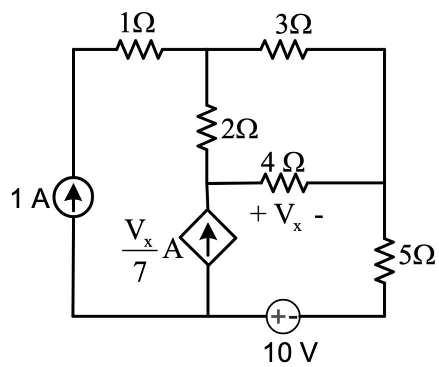
$$x(t) = (A \sin \omega_0 t + B \cos \omega_0 t)u(t)$$



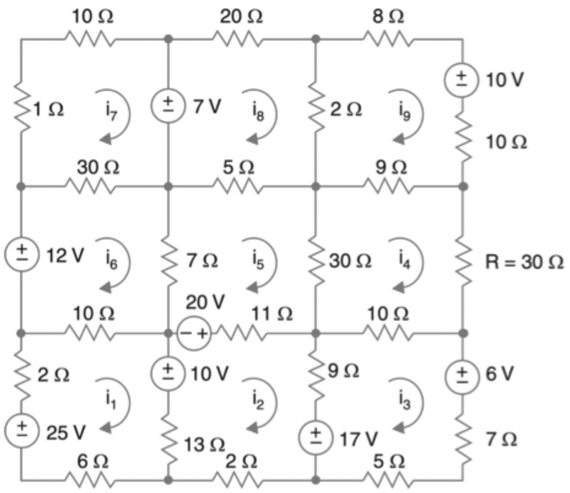
(n) What should be the value of V_x so that no current flows through the impedance $8 + 8j$?



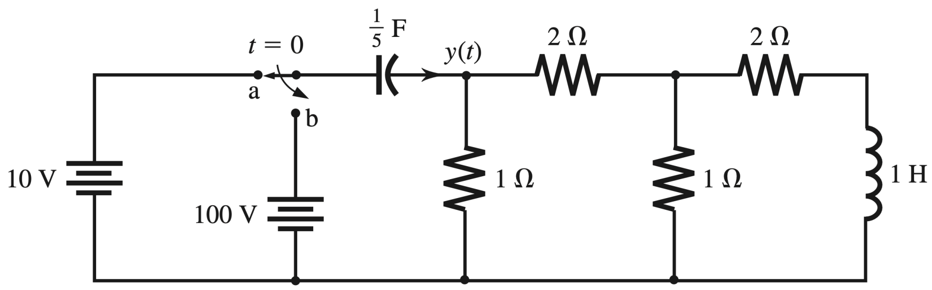
(o) What is the power dissipated or supplied by the voltage source?



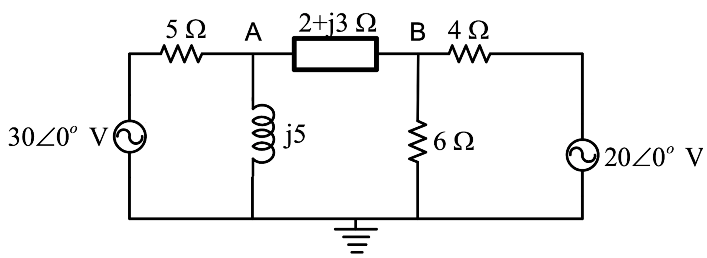
(p) What is the power dissipated by the 30 Ohm resistor?



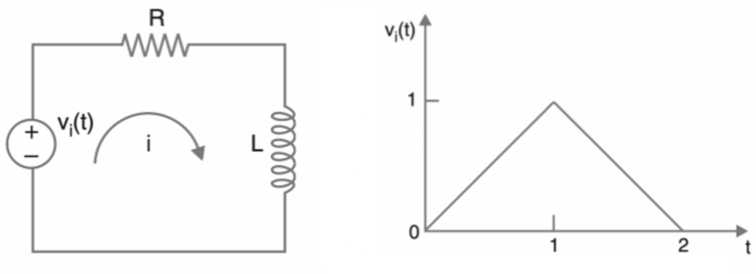
(q) If the switch is at "a" for a long time and suddenly moved to "b" at $t = 0$, calculate $y(t)$:



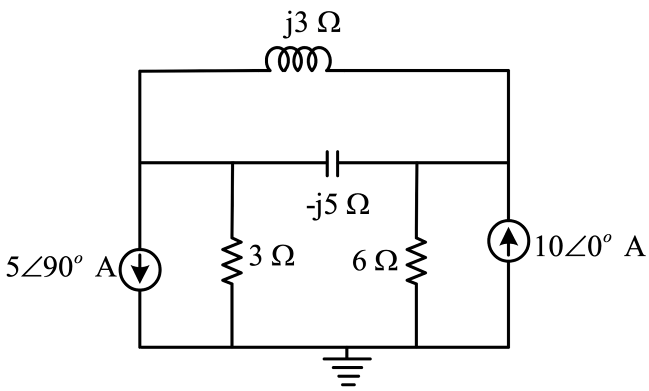
(r) What is the current flowing through the 6 Ohm resistor?



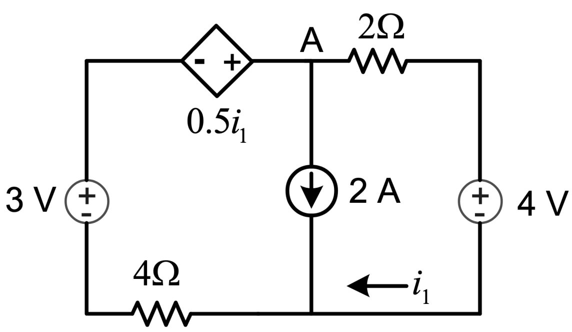
(s) Assuming zero initial conditions and R = 2 and L = 2, calculate the current in the circuit:



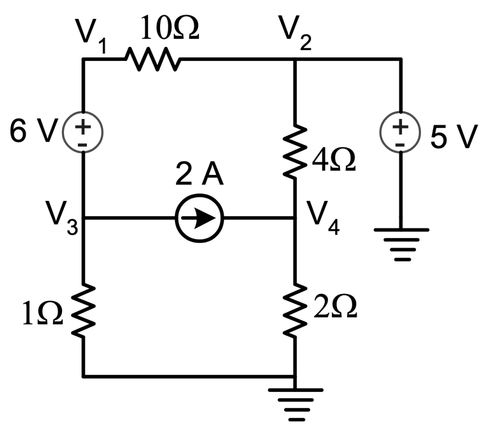
(t) What is the voltage across the 6 Ohm resistor?



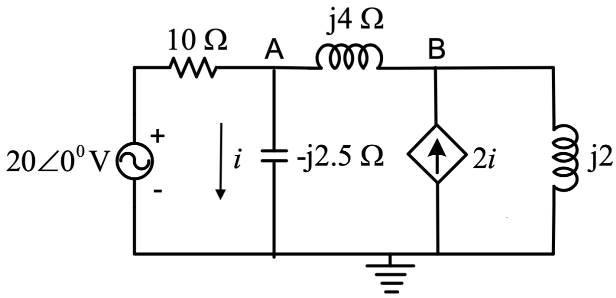
(u) Calculate the current flowing through the 4 Ohm resistor:



(v) Calculate all the node voltages in the circuit below:



(w) Calculate the current i in the circuit below:



Answer:

(a) $V_1 = -4.5V,\ V_2 = -15.5V,\ V_3 = 6.5V$

(b) $V_1 = 4.5V,\ V_2 = 1.5V,\ V_3 = 6V,\ I_0 = 0.375A$

(c) $P = 58.6\ W$

(d) $I_0 = 0\ A$

(e) $(te^{-t} - e^{-t}\sin t)u(t)\ V$

(f) $y_1(t) = (8 + 17.9e^{-1.5t}\cos(0.5t - 26.56))u(t)\ A$

$y_2(t) = 28.28e^{-1.5t}\cos(0.5t - 45))u(t)\ A$

(g) $H(s) = \dfrac{s}{s+a}\left(1 + \dfrac{R_b}{R_a}\right)$

(h) $H(s) = \dfrac{3-4s}{4+s}$

(i) $I_x = 6.12\ \angle 144.78\ A$

(j) $V_0(t) = 4.63\sin(5t - 81.12) + 1.05\cos(10t - 86.24))\ V$

(k) $v_1(t) = 20.96\sin(2t + 58)\ V$

$v_2(t) = 44.11\sin(2t + 41)\ V$

(l) $y(t) = 1.72e^{-2t}\cos(3t + 29))u(t)\ V$

(m) $y(t) = \dfrac{A}{L\omega_0}u(t) + \dfrac{B}{L\omega_0^2}\delta(t)$

(n) $V_x = 88.4\ \angle 45\ V$

(o) $P = 48.66\ W$

(p) $P = 0.01\ W$

(q) $(121.6e^{-6.5t} - 1.6e^{-2.8t})u(t)$

(r) $I = 2.64\ \angle 10\ A$

(s) $i(t) = \left(\dfrac{t - 1 + e^{-t}}{2}\right)(u(t) - 2u(t-1) + u(t-2))\ A$

(t) $V = 34.4\ \angle 24\ V$

(u) $I = 2 - 1.636 = 0.364\ A$

(v) $V_1 = 4.37V,\ V_2 = 5V,\ V_3 = -1.63V,\ V_4 = 4.33V$

(w) $I = 0.8\ \angle 92\ A$