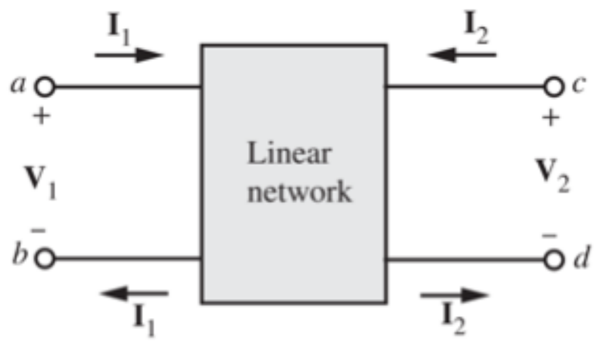
Y / Admittance / Short-circuit Parameters

$$(I_1, I_2) = f(V_1, V_2)$$



$$\Rightarrow [I] = [y][V]$$

or
$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

Hence,
$$I_1 = y_{11} V_1 + y_{12} V_2$$
$$I_2 = y_{21} V_1 + y_{22} V_2$$

Then,
$$y_{11} = \left.\frac{I_1}{V_1}\right|_{V_2=0}, \quad y_{12} = \left.\frac{I_1}{V_2}\right|_{V_1=0}$$

y_{11} → short-circuit input admittance

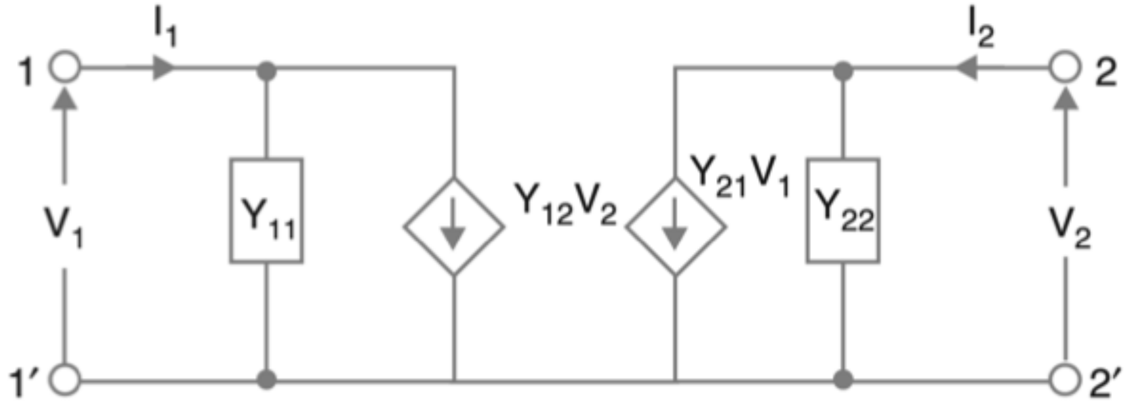
y_{12} → short-circuit reverse transfer admittance

$$y_{21} = \left.\frac{I_2}{V_1}\right|_{V_2=0}, \quad y_{22} = \left.\frac{I_2}{V_2}\right|_{V_1=0}$$

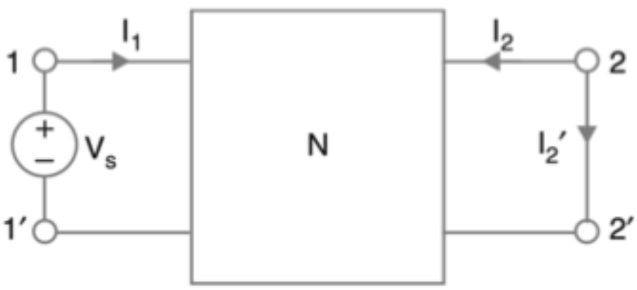
y_{21} → short-circuit forward transfer admittance

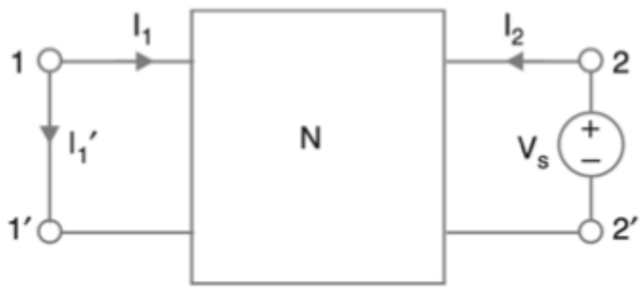
y_{22} → short-circuit output admittance

The equivalent circuit representation is given below with $Y_{12}V_2$ and $Y_{21}V_1$ as VCCS.



Condition for Reciprocity & Symmetry:





For reciprocity,

$$I_1' = I_2' \Rightarrow -V_s Y_{12} = -V_s Y_{21}$$

$$\Rightarrow \underline{\underline{Y_{12} = Y_{21}}}$$

For symmetry (the ports can be interchanged without changing the port voltages and currents)

$$\Rightarrow \left.\frac{I_1}{V_1}\right|_{V_2=0} = \left.\frac{I_2}{V_2}\right|_{V_1=0} \Rightarrow \underline{\underline{Y_{11} = Y_{22}}}$$

Matrix Partitioning for Y parameters :

For a two-port network, the KCL / Nodal equations are : $[Y][V] = [I]$

$$\begin{aligned}
y_{11} V_1 + y_{12} V_2 + \cdots + y_{1n} V_n &= I_1 \\
y_{21} V_1 + y_{22} V_2 + \cdots + y_{2n} V_n &= I_2 \\
y_{31} V_1 + y_{32} V_2 + \cdots + y_{3n} V_n &= 0 \\
&\vdots \\
y_{n1} V_1 + y_{n2} V_2 + \cdots + y_{nn} V_n &= 0
\end{aligned}$$

or

$$\Rightarrow \left[\begin{array}{cc|ccc} y_{11} & y_{12} & y_{13} & \cdots & y_{1n} \\ y_{21} & y_{22} & \cdots & \cdots & y_{2n} \\ \hline y_{31} & \vdots & & & \vdots \\ \vdots & \vdots & & & \vdots \\ y_{n1} & \vdots & \cdots & \cdots & y_{nn} \end{array}\right] \left[\begin{array}{c} V_1 \\ V_2 \\ \hline \vdots \\ \vdots \\ V_n \end{array}\right] = \left[\begin{array}{c} I_1 \\ I_2 \\ \hline 0 \\ \vdots \\ 0 \end{array}\right]$$

Hence,

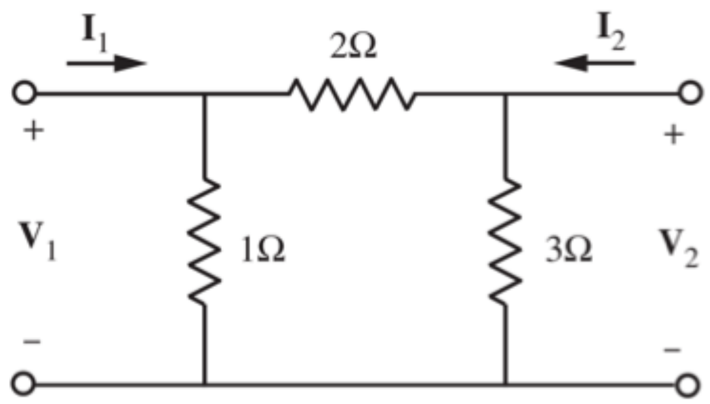
$$\left[\begin{array}{c|c} M & N \\ \hline P & Q \end{array}\right] \left[\begin{array}{c} V_1 \\ V_2 \\ \hline \vdots \\ V_n \end{array}\right] = \left[\begin{array}{c} I_1 \\ I_2 \\ \hline 0 \\ \vdots \\ 0 \end{array}\right]$$

which can be simplified as,

$$\left[M - N Q^{-1} P\right] \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

Q. Determine the Y-parameters of:

Furthermore, if a 2A source is connected at the input, find the current through a 4Ω load at the output.



A. Writing the KCL equations,

$$\begin{bmatrix} 3/2 & -1/2 \\ -1/2 & 5/6 \end{bmatrix}\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

Hence,

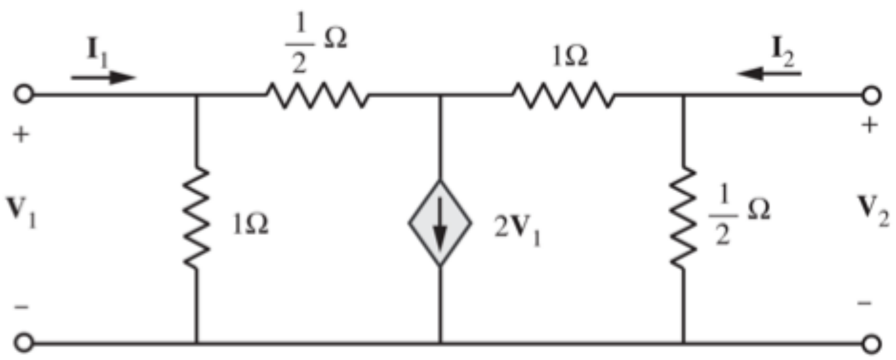
$$Y_{11} = \left.\frac{I_1}{V_1}\right|_{V_2=0} = \frac{3}{2}\,\Omega^{-1}, \quad Y_{12} = \left.\frac{I_1}{V_2}\right|_{V_1=0} = \frac{-1}{2}\,\Omega^{-1}$$

$$Y_{21} = \left.\frac{I_2}{V_1}\right|_{V_2=0} = \frac{-1}{2}\,\Omega^{-1}, \quad Y_{22} = \left.\frac{I_2}{V_2}\right|_{V_1=0} = \frac{5}{6}\,\Omega^{-1}$$

$$\Rightarrow \quad I_1 = \frac{3}{2}V_1 - \frac{1}{2}V_2 \quad \text{and} \quad I_2 = \frac{-1}{2}V_1 + \frac{5}{6}V_2$$

If $I_1 = 2\,A \Rightarrow I_2 = \frac{-2}{11}\,A$

Q. Determine the y-parameters of :



A. Writing the KCL equations,

$$3V_1 - 2V_3 = I_1$$

$$V_2 - 3V_3 = 0$$

$$3V_2 - V_3 = I_2$$

$$\Rightarrow \quad 9V_1 - 2V_2 = 3I_1$$

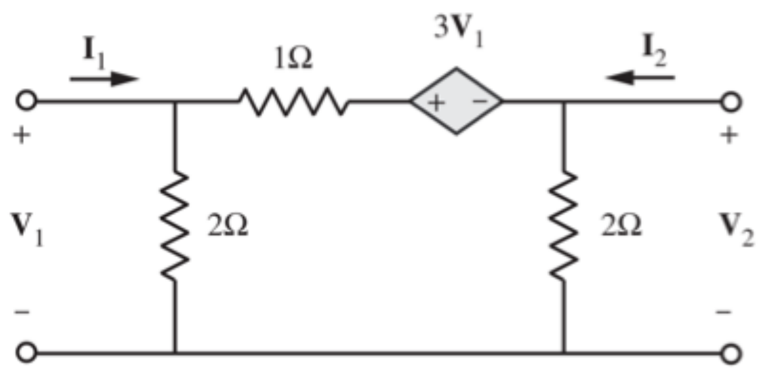
$$8V_2 = 3I_2$$

Hence,

$$y_{11} = \left.\frac{I_1}{V_1}\right|_{V_2=0} = 3\ \Omega^{-1}, \quad y_{12} = \left.\frac{I_1}{V_2}\right|_{V_1=0} = \frac{-2}{3}\ \Omega^{-1}$$

$$y_{21} = \left.\frac{I_2}{V_1}\right|_{V_2=0} = 0\ \Omega^{-1}, \quad y_{22} = \left.\frac{I_2}{V_2}\right|_{V_1=0} = \frac{8}{3}\ \Omega^{-1}$$

Q. Determine the Y-parameters of :



A. Writing the KCL equations,

$$\frac{V_1}{2} + \frac{V_2}{2} = I_1 + I_2$$

$$-\left(\frac{3V_1}{2} + V_2\right) = I_1$$

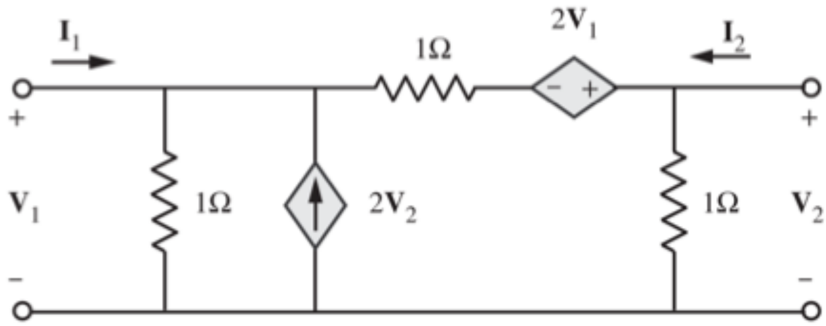
Hence,

$$\begin{bmatrix} -\frac{3}{2} & -1 \\ 2 & \frac{3}{2} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

$$Y_{11} = \left.\frac{I_1}{V_1}\right|_{V_2=0} = -\frac{3}{2}\ \Omega^{-1} \quad , \quad Y_{12} = \left.\frac{I_1}{V_2}\right|_{V_1=0} = -1\ \Omega^{-1}$$

$$Y_{21} = \left.\frac{I_2}{V_1}\right|_{V_2=0} = 2\ \Omega^{-1} \quad , \quad Y_{22} = \left.\frac{I_2}{V_2}\right|_{V_1=0} = \frac{3}{2}\ \Omega^{-1}$$

Q. Determine the Y-parameters of :



A. Writing the KCL equations,

$$V_1 - V_2 = I_1 + I_2$$

$$4V_1 - 3V_2 = I_1$$

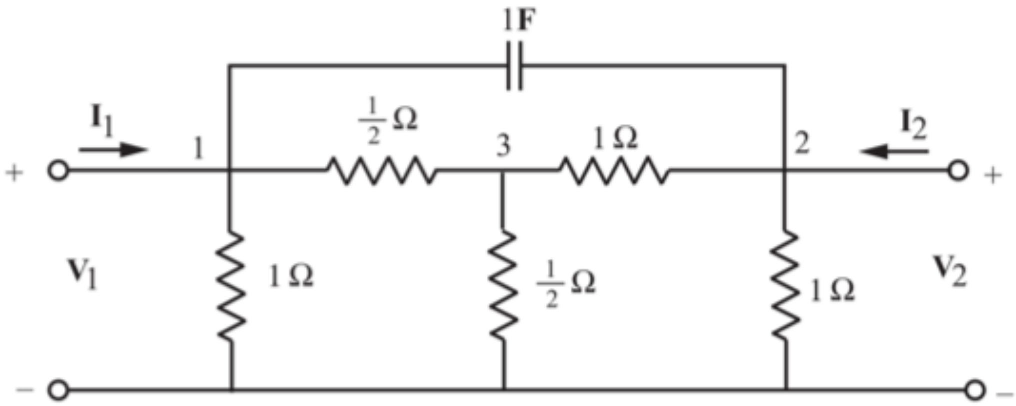
Hence,

$$\begin{bmatrix} 4 & -3 \\ -3 & 2 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

$$Y_{11} = \left.\frac{I_1}{V_1}\right|_{V_2=0} = 4\ \Omega^{-1}, \quad Y_{12} = \left.\frac{I_1}{V_2}\right|_{V_1=0} = -3\ \Omega^{-1}$$

$$Y_{21} = \left.\frac{I_2}{V_1}\right|_{V_2=0} = -3\ \Omega^{-1}, \quad Y_{22} = \left.\frac{I_2}{V_2}\right|_{V_1=0} = 2\ \Omega^{-1}$$

Q. Determine the Y-parameters (s-domain) of :



A. Writing the KCL equations,

$$\begin{bmatrix} 3+s & -s & -2 \\ -s & 2+s & -1 \\ -2 & -1 & 5 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix} = \begin{bmatrix} I_1 \\ I_2 \\ 0 \end{bmatrix}$$

Simplifying,

$$\left(s+\frac{11}{5}\right) V_1 - \left(s+\frac{2}{5}\right) V_2 = I_1$$

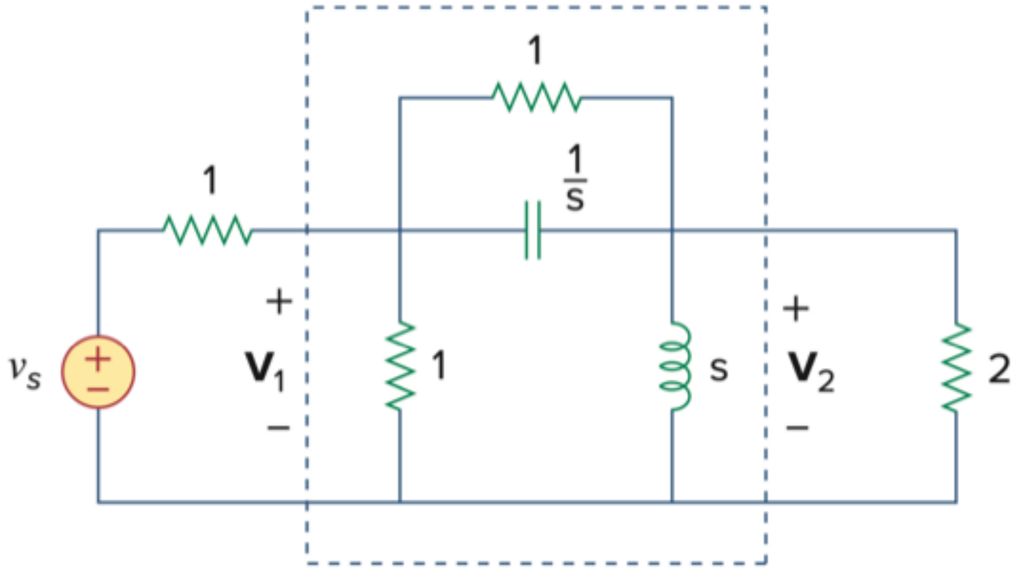
$$-(5s+2) V_1 + (5s+9) V_2 = 5 I_2$$

$$Y_{11} = \left.\frac{I_1}{V_1}\right|_{V_2=0} = s+2.2 \quad , \quad Y_{12} = \left.\frac{I_1}{V_2}\right|_{V_1=0} = -(s+0.4)$$

$$Y_{21} = \left.\frac{I_2}{V_1}\right|_{V_2=0} = -(s+0.4) \quad , \quad Y_{22} = \left.\frac{I_2}{V_2}\right|_{V_1=0} = s+1.8$$

Note : We can also solve this using $Z = P - QN^{-1}M$

Q. Determine the y-parameters (s-domain) of :



A. $Y_{11} = \left.\frac{I_1}{V_1}\right|_{V_2=0} = s+2,$

$Y_{12} = \left.\frac{I_1}{V_2}\right|_{V_1=0} = -(s+1),$

$Y_{21} = \left.\frac{I_2}{V_1}\right|_{V_2=0} = -(s+1),$

$Y_{22} = \left.\frac{I_2}{V_2}\right|_{V_1=0} = \frac{s^2+s+1}{s}$

Hence, $[Y] = \begin{bmatrix} s+2 & -(s+1) \\ -(s+1) & \frac{s^2+s+1}{s} \end{bmatrix}$