Network Analysis

The purpose of network theory and network analysis is to be able to predict the value of voltage or current at any point and at any instant of time, in an inter-connection of electrical devices when the voltage or current at some other point is known.

Electric network theory, like all scientific theories, attempts to do this by setting up a mathematical model. This model will not only help us to understand the natural phenomena of current and voltage but also predict the behavior of the model under conditions which we establish.

The first step in establishing a model is to conduct experiments to establish universal relationships among the measurable quantities. The general conclusions drawn are then regarded as "laws," and are stated in terms of the variables of the mathematical model.

Kirchhoff's Current Law: The algebraic sum of the branch currents at a node is zero at all instants of time.

If a network has N_b branches and N_v nodes, then

$$\sum_{j=1}^{N_b} a_{kj}\, i_j(t) = 0 \quad ; \quad k = 1, 2, \dots, N_v$$

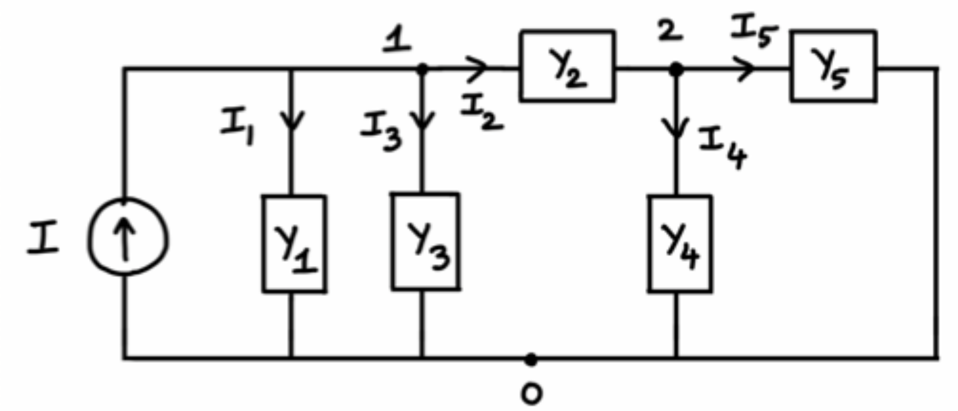
where

$$a_{kj} = \begin{cases} 1 & \text{if branch } j \text{ is connected to node } k \text{ and its current reference is directed away from the node} \\ -1 & \text{if branch } j \text{ is connected to node } k \text{ and its current reference is directed towards the node} \\ 0 & \text{if branch } j \text{ is not connected to node } k \end{cases}$$

Note: KCL equations are linearly dependent. Expressed in terms of the number of nodes, we can say that at most $N_v - 1$ of the equations are independent.

Generalized Nodal Analysis:

Suppose we have a network as shown below.



Let us choose the reference or datum node marked 0 in the figure. Clearly, there are three nodes (including 0) between elements in the network. Hence, it is possible to write $N_v - 1 = 2$ nodal equations in terms of the potentials E_1 and E_2.

Using KCL, at node 1,

$$I - I_1 - I_2 - I_3 = 0$$

which, in terms of potentials and admittances, is

$$I - Y_1 E_1 - Y_3 E_1 - Y_2 (E_1 - E_2) = 0$$

$$\Rightarrow (Y_1 + Y_2 + Y_3) E_1 + (-Y_2) E_2 = I$$

Similarly, at node 2, $I_2 - I_5 - I_4 = 0$

$$\Rightarrow (-Y_2) E_1 + (Y_2 + Y_4 + Y_5) E_2 = 0$$

Hence, we can write,

$$\begin{bmatrix} Y_1 + Y_2 + Y_3 & -Y_2 \\ -Y_2 & Y_2 + Y_4 + Y_5 \end{bmatrix} \begin{bmatrix} E_1 \\ E_2 \end{bmatrix} = \begin{bmatrix} I \\ 0 \end{bmatrix}$$

Using Cramer's rule,

$$E_1 = \frac{1}{\Delta} \begin{vmatrix} I & -Y_2 \\ 0 & Y_2 + Y_4 + Y_5 \end{vmatrix}$$

$$E_2 = \frac{1}{\Delta} \begin{vmatrix} Y_1 + Y_2 + Y_3 & I \\ -Y_2 & 0 \end{vmatrix}$$

where $\Delta = \begin{vmatrix} Y_1 + Y_2 + Y_3 & -Y_2 \\ -Y_2 & Y_2 + Y_4 + Y_5 \end{vmatrix}$

Hence, the generalized node equations can be written as:

$$[Y][E] = [I]$$

where the square matrix Y is called the admittance matrix, E is the column matrix of the node voltages with respect to the reference node and I is the column matrix of input currents.

Considering a generalised network with $n+1$ nodes (including the reference node), we can write node equations in matrix form of order $(n \times n)$ using KCL, as

$$\begin{bmatrix} Y_{11} & Y_{12} & \cdots & Y_{1n} \\ Y_{21} & Y_{22} & \cdots & Y_{2n} \\ \vdots & \vdots & & \vdots \\ Y_{n1} & Y_{n2} & \cdots & Y_{nn} \end{bmatrix} \begin{bmatrix} E_1 \\ E_2 \\ \vdots \\ E_n \end{bmatrix} = \begin{bmatrix} I_1 \\ I_2 \\ \vdots \\ I_n \end{bmatrix}$$

The admittance of all branches connected to node j are summed up and denoted by Y_{jj}, called the self-admittance of node j.

All admittances connected to node j and k are summed up and denoted by Y_{jk}. This Y_{jk} is written on the left-hand side of the equation with a negative sign. If no admittance is connected between nodes j and k, then Y_{jk} is zero.

I_j denotes the value of the current source connected to node j. The sign of I_j is positive if it is flowing towards node j; otherwise it is negative.

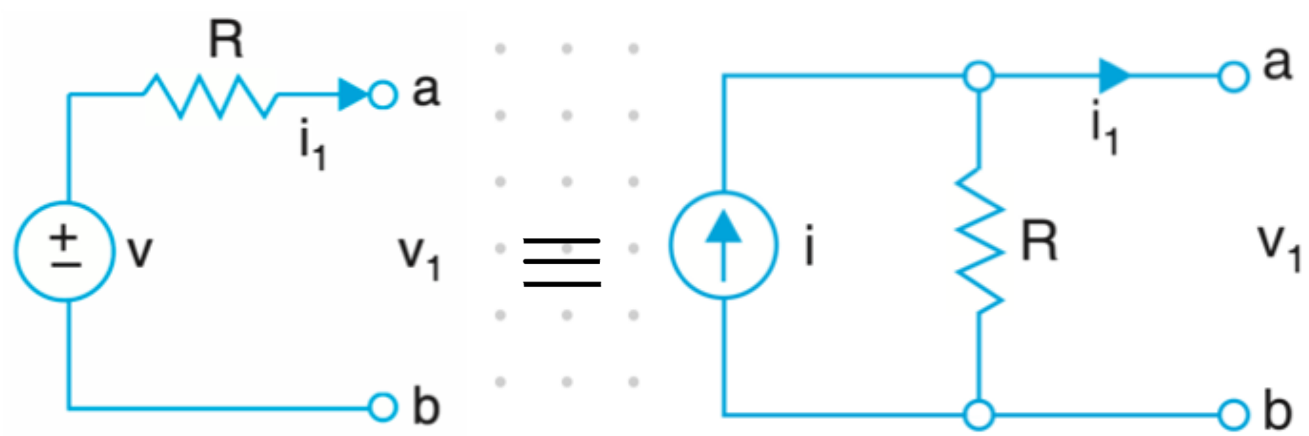
Source Transformation:

An ideal voltage source is one which gives a constant voltage irrespective of the current drawn from it.

An ideal current source is one which gives a constant current irrespective of the voltage across it.

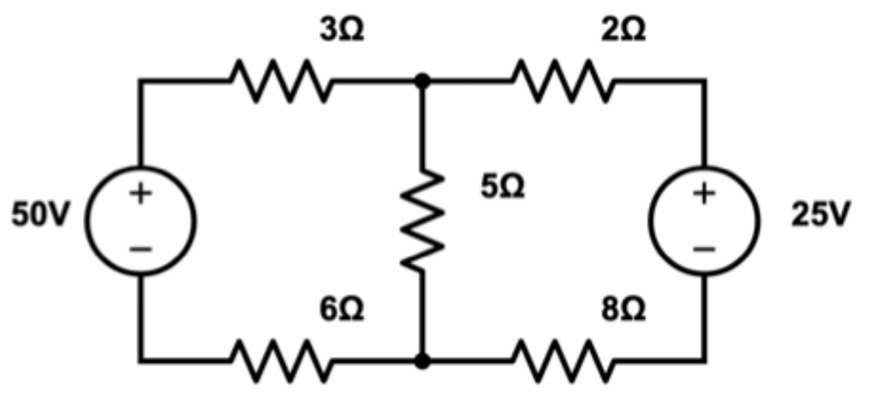
The technique of transforming a source from one form into the other is known as the Source Transformation technique.

A voltage source can be converted into an equivalent current source and vice-versa using the following transformation.

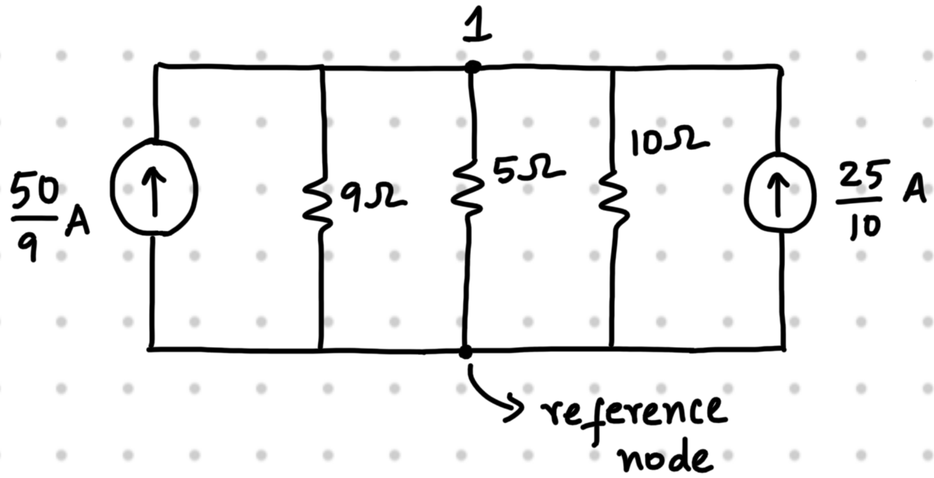


where $v = iR$

Q. Using nodal analysis, find the voltage across the 5Ω resistor.



A. Using source transformation,



Now, $[Y][E] = [I]$

$$\Rightarrow \left(\frac{1}{9} + \frac{1}{5} + \frac{1}{10}\right) E_1 = \left(\frac{50}{9} + \frac{25}{10}\right)$$

$$\Rightarrow E_1 \approx 19.6 \text{ V}$$