Interrelationships between the Parameters

$Z \rightarrow Y$:

We know $$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

and $$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

Hence, $$\begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} \begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} = 1$$

$$\Rightarrow \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} = \begin{bmatrix} \frac{Y_{22}}{\Delta Y} & \frac{-Y_{12}}{\Delta Y} \\ \frac{-Y_{21}}{\Delta Y} & \frac{Y_{11}}{\Delta Y} \end{bmatrix}$$

$\rightarrow Y_{11} Y_{22} - Y_{12} Y_{21}$

$Z \longrightarrow ABCD$:

We know $$V_1 = A V_2 - B I_2$$
$$I_1 = C V_2 - D I_2$$

Solving the equations, $$V_1 = \frac{A}{C} I_1 + \left(\frac{AD - BC}{C}\right) I_2$$

and $$V_2 = \frac{1}{C} I_1 + \frac{D}{C} I_2$$

$$\Rightarrow Z_{11} = \frac{A}{C}, \quad Z_{12} = \frac{AD-BC}{C}, \quad Z_{21} = \frac{1}{C}, \quad Z_{22} = \frac{D}{C}$$

$Z \longrightarrow H$:

We know $V_1 = h_{11} I_1 + h_{12} V_2$

$I_2 = h_{21} I_1 + h_{22} V_2$

Solving the equations, $V_1 = \frac{\Delta h}{h_{22}} I_1 + \frac{h_{12}}{h_{22}} I_2$

and $V_2 = -\frac{h_{21}}{h_{22}} I_1 + \frac{1}{h_{22}} I_2$

$\Rightarrow z_{11} = \frac{\Delta h}{h_{22}}, \quad z_{12} = \frac{h_{12}}{h_{22}}, \quad z_{21} = \frac{-h_{21}}{h_{22}}, \quad z_{22} = \frac{1}{h_{22}}$

In a similar way, we can convert the other parameters and construct the following table:

	z	y	T	h
z	$\begin{bmatrix} z_{11} & z_{12} \\ z_{21} & z_{22} \end{bmatrix}$	$\begin{bmatrix} \frac{y_{22}}{\Delta y} & \frac{-y_{12}}{\Delta y} \\ \frac{-y_{21}}{\Delta y} & \frac{y_{11}}{\Delta y} \end{bmatrix}$	$\begin{bmatrix} \frac{A}{C} & \frac{\Delta T}{C} \\ \frac{1}{C} & \frac{D}{C} \end{bmatrix}$	$\begin{bmatrix} \frac{\Delta h}{h_{22}} & \frac{h_{12}}{h_{22}} \\ \frac{-h_{21}}{h_{22}} & \frac{1}{h_{22}} \end{bmatrix}$
y	$\begin{bmatrix} \frac{z_{22}}{\Delta z} & \frac{-z_{12}}{\Delta z} \\ \frac{-z_{21}}{\Delta z} & \frac{z_{11}}{\Delta z} \end{bmatrix}$	$\begin{bmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{bmatrix}$	$\begin{bmatrix} \frac{D}{B} & \frac{-\Delta T}{B} \\ \frac{-1}{B} & \frac{A}{B} \end{bmatrix}$	$\begin{bmatrix} \frac{1}{h_{11}} & \frac{-h_{22}}{h_{11}} \\ \frac{h_{21}}{h_{11}} & \frac{\Delta h}{h_{11}} \end{bmatrix}$
T	$\begin{bmatrix} \frac{z_{11}}{z_{21}} & \frac{\Delta z}{z_{21}} \\ \frac{1}{z_{21}} & \frac{z_{22}}{z_{21}} \end{bmatrix}$	$\begin{bmatrix} \frac{-y_{22}}{y_{21}} & \frac{-1}{y_{21}} \\ \frac{-\Delta y}{y_{21}} & \frac{-y_{11}}{y_{21}} \end{bmatrix}$	$\begin{bmatrix} A & B \\ C & D \end{bmatrix}$	$\begin{bmatrix} \frac{-\Delta h}{h_{21}} & \frac{-h_{11}}{h_{21}} \\ \frac{-h_{22}}{h_{21}} & \frac{-1}{h_{21}} \end{bmatrix}$
h	$\begin{bmatrix} \frac{\Delta z}{z_{22}} & \frac{z_{12}}{z_{22}} \\ \frac{-z_{21}}{z_{22}} & \frac{1}{z_{22}} \end{bmatrix}$	$\begin{bmatrix} \frac{1}{y_{11}} & \frac{-y_{12}}{y_{11}} \\ \frac{y_{21}}{y_{11}} & \frac{\Delta y}{y_{11}} \end{bmatrix}$	$\begin{bmatrix} \frac{B}{D} & \frac{\Delta T}{D} \\ -\frac{1}{D} & \frac{C}{D} \end{bmatrix}$	$\begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix}$

$\Delta z = z_{11}z_{22} - z_{12}z_{21}, \Delta y = y_{11}y_{22} - y_{12}y_{21}, \Delta h = h_{11}h_{22} - h_{12}h_{21}, \Delta T = AD - BC$

Q. The following measurements were made of a linear passive two port network.

Fill in the blanks of the table.

Sl.no	V_1	V_2	I_1	I_2
1	50	100	−1	27
2	100	50	7	24
3	200	0	–	–
4	–	–	20	0
5	–	–	10	30

A. Using $V_1 = Z_{11} I_1 + Z_{12} I_2$

$V_2 = Z_{21} I_1 + Z_{22} I_2$

$$\begin{bmatrix} -1 & 27 \\ 7 & 24 \end{bmatrix} \begin{bmatrix} Z_{11} & Z_{21} \\ Z_{12} & Z_{22} \end{bmatrix} = \begin{bmatrix} 50 & 100 \\ 100 & 50 \end{bmatrix}$$

$$\Rightarrow Z_{11} = \frac{500}{71}, \quad Z_{12} = \frac{150}{71}, \quad Z_{21} = \frac{-350}{71}, \quad Z_{22} = \frac{250}{71}$$

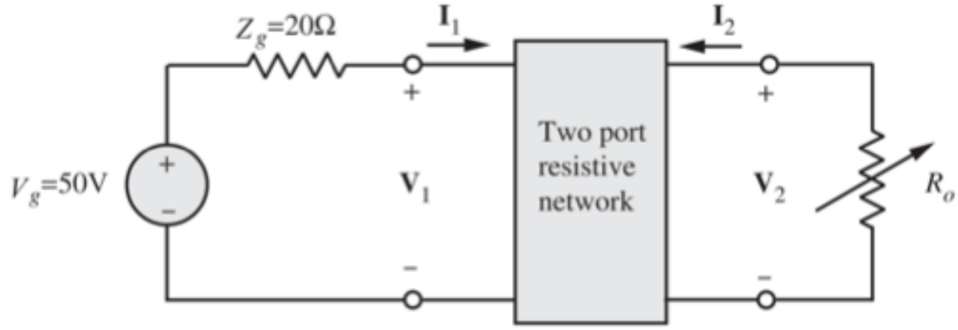
Now, filling the table :

Sl. no 3 : $I_1 = 20\,A, \quad I_2 = 28\,A$

Sl. no 4 : $V_1 = \frac{10000}{71} V, \quad V_2 = -\frac{7000}{71} V$

Sl. no 5 : $V_2 = \frac{9500}{71} V, \quad V_2 = \frac{4000}{71} V$

Q. The following measurements were done on the circuit : Determine the maximum power dissipated across R_o.



Measurement 1	Measurement 2
$V_1 = 20$ V	$V_1 = 35$ V
$I_1 = 0.8$ A	$I_1 = 1$ A
$V_2 = 0$ V	$V_2 = 15$ V
$I_2 = -0.4$ A	$I_2 = 0$ A

A. Calculating the ABCD - parameters,

$$A = \left.\frac{V_1}{V_2}\right|_{I_2=0} = \frac{7}{3}, \quad B = \left.\frac{V_1}{-I_2}\right|_{V_2=0} = 50$$

$$C = \left.\frac{I_1}{V_2}\right|_{I_2=0} = \frac{1}{15}, \quad D = \left.\frac{I_1}{-I_2}\right|_{V_2=0} = 2$$

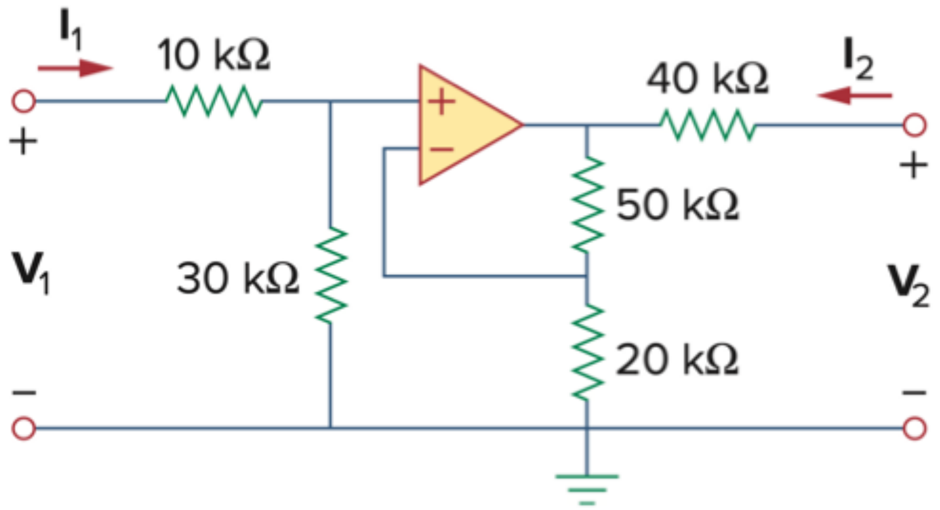
We know,

$$V_{TH} = \frac{V_g}{A + C Z_g} = \frac{50}{\frac{7}{3} + \frac{1}{15} \cdot 20} = \frac{150}{11}$$

$$Z_{TH} = \frac{B + D Z_g}{A + C Z_g} = \frac{50 + 2 \cdot 20}{\frac{7}{3} + \frac{1}{15} \cdot 20} = \frac{270}{11}$$

Hence, $P_{max} = \dfrac{V_{TH}^2}{4 Z_{TH}} \simeq 1.89$ W

Q. For the circuit shown below, calculate z-parameters and then convert them to ABCD parameters.



A. Writing the KVL equations,

$$V_1 = (10+30)\,k\Omega \times I_1$$

$$V_2 = (40\,k\Omega)\,I_2 + \frac{(50+20)\,k\Omega}{80\,k\Omega} \cdot 3V_1$$

$$\Rightarrow \quad V_2 = (105\,k\Omega)\,I_1 + (40\,k\Omega)\,I_2$$

Hence, $$[z] = \begin{bmatrix} 40 & 0 \\ 105 & 40 \end{bmatrix} k\Omega$$

Converting them into ABCD parameters:

$$[ABCD] = \begin{bmatrix} \frac{z_{11}}{z_{21}} & \frac{\Delta z}{z_{21}} \\ \frac{1}{z_{21}} & \frac{z_{22}}{z_{21}} \end{bmatrix} = \begin{bmatrix} 0.38 & 15.24\,k\Omega \\ 9.52\,\mu\Omega^{-1} & 0.38 \end{bmatrix}$$