Course 1 week 1 Quiz 2

$$\left.\begin{aligned} 3x-2y+z&=7 \\ x+y+z&=2 \\ 3x-2y-z&=3 \end{aligned}\right\}$$

$z = 3x-2y-3 \Rightarrow 3x-2y+3x-2y-3=7$

$x+y+3x-2y-3=2$

$\Downarrow$

$$\left.\begin{aligned} 6x-4y&=10 \\ 4x-y&=5 \end{aligned}\right\} \quad y=4x-5$$

$\Downarrow$

$6x-(16x-20)=10$

$-10x+20=10$

$x=1$

$\hookrightarrow y=-1$

$z=3+2-3=2$

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week 2 Quiz 1

$\vec{r} = \begin{pmatrix} 3 \\ -4 \\ 0 \end{pmatrix} \quad \vec{s} = \begin{pmatrix} 10 \\ 5 \\ -6 \end{pmatrix}$

$\vec{r}\cdot\vec{s} = |r|\,|s|\cos\theta$

$|s|\cos\theta = \text{proj } s \text{ onto } r = \dfrac{\vec{r}\cdot\vec{s}}{|r|} = \dfrac{30+(-20)+0}{\sqrt{9+16}} = \dfrac{10}{5} = 2$

vector proj of $\vec{s}$ onto $\vec{r} = \dfrac{\vec{r}\cdot\vec{s}}{|r||r|}\vec{r} = \dfrac{2}{5}\begin{pmatrix} 3 \\ -4 \\ 0 \end{pmatrix} = \begin{pmatrix} 6/5 \\ -8/5 \\ 0 \end{pmatrix}$

$\vec{a} = \begin{pmatrix} 3 \\ 0 \\ 4 \end{pmatrix}, \vec{b} = \begin{pmatrix} 0 \\ 5 \\ 12 \end{pmatrix}$

$|\vec{a}+\vec{b}| = \left|\begin{matrix} 3 \\ 5 \\ 16 \end{matrix}\right| = \sqrt{9+25+256} = 17.03$

$|\vec{a}|+|\vec{b}| = \sqrt{9+16}+\sqrt{25+144} = 5+13=18$

Lecture Quiz

$r_c = \begin{pmatrix} 3 \\ 4 \end{pmatrix}$ $b_2 = \begin{pmatrix} -2 \\ 4 \end{pmatrix}$

$$\frac{r_c \cdot b_2}{|b_2|^2} = \frac{-6+16}{4+16} = \frac{10}{20} = \frac{1}{2}$$

Week 2 Quiz 2

$V = \begin{pmatrix} 5 \\ -1 \end{pmatrix}$ $b_1 = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$ $b_2 = \begin{pmatrix} 1 \\ -1 \end{pmatrix}$ in standard basis

$\because$ b1 and b2 are orthogonal to each other

↳ V onto b1:

$$\frac{V \cdot b_1}{|b_1|^2} = \frac{5-1}{1+1} = 2 \Rightarrow 2b_1$$

↳ V onto b2:

$$\frac{V \cdot b_2}{|b_2|^2} = \frac{5+1}{1+1} = 3 \Rightarrow 3b_2$$

in b_1, b_2 basis

$V = 2b_1 + 3b_2 = \begin{pmatrix} 2 \\ 2 \end{pmatrix} + \begin{pmatrix} 3 \\ -3 \end{pmatrix} = \begin{pmatrix} 5 \\ -1 \end{pmatrix}$

$\downarrow$

$V_b = \begin{pmatrix} 2 \\ 3 \end{pmatrix}$

$V = \begin{pmatrix} 10 \\ -5 \end{pmatrix}$ $b_1 = \begin{pmatrix} 3 \\ 4 \end{pmatrix}$ $b_2 = \begin{pmatrix} 4 \\ -3 \end{pmatrix}$ in standard basis

$\because$ b_1 orthorgonal to b_2

↳ V onto b1

$$\frac{V \cdot b_1}{|b_1|^2} = \frac{30-20}{9+16} = \frac{10}{25} = \frac{2}{5} \Rightarrow \frac{2}{5} b_1$$

↳ V onto b2

$$\frac{V \cdot b_2}{|b_2|^2} = \frac{40+15}{16+9} = \frac{55}{25} = \frac{11}{5} \Rightarrow \frac{11}{5} b_2$$

$\frac{2}{5} b_1 + \frac{11}{5} b_2$

$\Downarrow$

$V_b = \begin{pmatrix} 2/5 \\ 11/5 \end{pmatrix}$

$V = \begin{pmatrix} 2 \\ 2 \end{pmatrix}$ $b_1 = \begin{pmatrix} -3 \\ 1 \end{pmatrix}$ $b_2 = \begin{pmatrix} 1 \\ 3 \end{pmatrix}$ in standard basis

$\because b_1 \underline{\perp} b_2$

$\therefore$ V on b_1:

$$\frac{v \cdot b_1}{|b_1|^2} = \frac{-6+2}{9+1} = \frac{-4}{10} = \frac{-2}{5} \Rightarrow -\frac{2}{5} b_1$$

V on b_2:

$$\frac{v \cdot b_2}{|b_2|^2} = \frac{2+6}{1+9} = \frac{4}{5} \Rightarrow \frac{4}{5} b_2$$

$$V_b = \begin{pmatrix} -2/5 \\ 4/5 \end{pmatrix}$$

$V = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$ $b_1 = \begin{pmatrix} 2 \\ 1 \\ 0 \end{pmatrix}$ $b_2 = \begin{pmatrix} 1 \\ -2 \\ -1 \end{pmatrix}$ $b_3 = \begin{pmatrix} -1 \\ 2 \\ -5 \end{pmatrix}$

$\because b_1 \underline{\perp} b_2 \underline{\perp} b_3$

$\therefore$ V on b_1:

$$\frac{v \cdot b_1}{|b_1|^2} = \frac{2+1}{4+1} = \frac{3}{5} \Rightarrow 3/5\, b_1$$

V on b_2:

$$\frac{v \cdot b_2}{|b_2|^2} = \frac{1-2-1}{1+4+1} = \frac{-2}{6} = -1/3 \Rightarrow -1/3\, b_2$$

V on b_3:

$$\frac{v \cdot b_3}{|b_3|^2} = \frac{-1+2-5}{1+4+25} = \frac{-4}{30} = -2/15 \Rightarrow -2/15\, b_3$$

$$V_b = \begin{pmatrix} 3/5 \\ -1/3 \\ -2/15 \end{pmatrix}$$

$V = \begin{pmatrix} 1 \\ 1 \\ 2 \\ 3 \end{pmatrix}$ $b_1 = \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix}$ $b_2 = \begin{pmatrix} 0 \\ 2 \\ -1 \\ 0 \end{pmatrix}$ $b_3 = \begin{pmatrix} 0 \\ 1 \\ 2 \\ 0 \end{pmatrix}$ $b_4 = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 3 \end{pmatrix}$

$$\frac{v \cdot b_1}{|b_1|^2} = \frac{1}{1} = 1 \Rightarrow b_1$$

$$\frac{v \cdot b_2}{|b_2|^2} = \frac{2-2}{4+1} = 0$$

$$\frac{v \cdot b_3}{|b_3|^2} = \frac{1+4}{1+4} = 1 \Rightarrow b_3$$

$$\frac{v \cdot b_4}{|b_4|^2} = \frac{9}{9} = 1 \Rightarrow b_4$$

$$V_b = \begin{pmatrix} 1 \\ 0 \\ 1 \\ 1 \end{pmatrix}$$

week2 Quiz 3

Q3

$a = q_1 b + q_2 c$

$$\begin{pmatrix} 2 \\ 2 \end{pmatrix} = q_1 \begin{pmatrix} 1 \\ -2 \end{pmatrix} + q_2 \begin{pmatrix} -1 \\ 0 \end{pmatrix}$$

$$\left.\begin{aligned} q_1 - q_2 &= 2 \\ -2q_1 &= 2 \end{aligned}\right\} \begin{aligned} q_1 &= -1 \\ q_2 &= q_1 - 2 = -3 \end{aligned}$$

Q2 $a = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$ $b = \begin{pmatrix} 2 \\ 1 \end{pmatrix}$

$x\begin{pmatrix} 1 \\ 1 \end{pmatrix} + y\begin{pmatrix} 2 \\ 1 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$ must satisfy if linearly dependent

$$\begin{pmatrix} 1 & 2 \\ 1 & 1 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \Rightarrow \left(\begin{array}{cc|c} 1 & 2 & 0 \\ 1 & 1 & 0 \end{array}\right) \Rightarrow \left(\begin{array}{cc|c} 1 & 2 & 0 \\ 0 & -1 & 0 \end{array}\right) \Rightarrow \left(\begin{array}{cc|c} 1 & 0 & 0 \\ 0 & -1 & 0 \end{array}\right) = \left(\begin{array}{cc|c} 1 & 0 & 0 \\ 0 & 1 & 0 \end{array}\right)$$

i.e. $x = y = 0 \Rightarrow$ linearly independent

Q4
$$\left(\begin{array}{ccc|c} 1 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{array}\right) \Rightarrow \left(\begin{array}{ccc|c} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{array}\right) \Rightarrow \left(\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{array}\right)$$

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week 2 Assessment

Q1 ship $= i + 2j$ current $= i + j$

$= \begin{pmatrix} 1 \\ 2 \end{pmatrix} = s$ $= \begin{pmatrix} 1 \\ 1 \end{pmatrix} = c$

ship on current : vector

$$\frac{s \cdot c}{|c|^2} c = \frac{1+2}{2} c = \frac{3}{2}\begin{pmatrix} 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 3/2 \\ 3/2 \end{pmatrix}$$

Q2 ball $= \begin{pmatrix} 2 \\ 1 \end{pmatrix} = b$ wind $= \begin{pmatrix} 3 \\ -4 \end{pmatrix} = w$

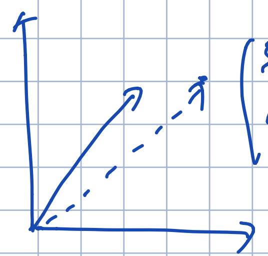
b on w : scalar

$$\frac{b \cdot w}{|w|} = \frac{6-4}{\sqrt{9+16}} = \frac{2}{5}$$

Q3 $v = \begin{pmatrix} -4 \\ -3 \\ 8 \end{pmatrix}$ $b_1 = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$ $b_2 = \begin{pmatrix} -2 \\ 1 \\ 0 \end{pmatrix}$ $b_3 = \begin{pmatrix} -3 \\ -6 \\ 5 \end{pmatrix}$

$$\left.\begin{array}{l} \dfrac{v \cdot b_1}{|b_1|^2} = \dfrac{-4-6+24}{1+4+9} = \dfrac{14}{14} = 1 \Rightarrow b_1 \\[2ex] \dfrac{v \cdot b_2}{|b_2|^2} = \dfrac{8-3}{5} = 1 \Rightarrow b_2 \\[2ex] \dfrac{v \cdot b_3}{|b_3|^2} = \dfrac{12+18+40}{9+36+25} = \dfrac{70}{70} = 1 \Rightarrow b_3 \end{array}\right\} v_b = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$$

Q5



$\begin{pmatrix} 3 \\ 2 \\ 4 \end{pmatrix}$

2hr later, movement $2\begin{pmatrix} -1 \\ 2 \\ -3 \end{pmatrix} = \begin{pmatrix} -2 \\ 4 \\ -6 \end{pmatrix}$

$$pos = \begin{pmatrix} 3 \\ 2 \\ 4 \end{pmatrix} + \begin{pmatrix} -2 \\ 4 \\ -6 \end{pmatrix} = \begin{pmatrix} 1 \\ 6 \\ -2 \end{pmatrix}$$

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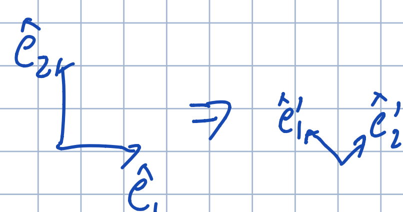
week 3 Quiz 1

Q1 $Ar = \begin{bmatrix} 1/2 & -1 \\ 0 & 3/4 \end{bmatrix} \begin{bmatrix} 3 \\ 2 \end{bmatrix} = \begin{bmatrix} 3/2 - 2 \\ 6/4 \end{bmatrix} = \begin{bmatrix} -1/2 \\ 3/2 \end{bmatrix}$

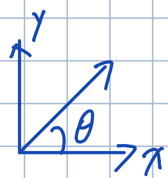
Q2 $s = \begin{bmatrix} 1/2 & -1 \\ 0 & 3/4 \end{bmatrix} \begin{bmatrix} -2 \\ 4 \end{bmatrix} = \begin{bmatrix} -1-4 \\ 3 \end{bmatrix} = \begin{bmatrix} -5 \\ 3 \end{bmatrix}$

Q3
$$\left.\begin{array}{l} \begin{bmatrix} -1/2 & 1/2 \\ 1/2 & 1/2 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} -1/2 \\ 1/2 \end{bmatrix} \\[2ex] \begin{bmatrix} -1/2 & 1/2 \\ 1/2 & 1/2 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 1/2 \\ 1/2 \end{bmatrix} \end{array}\right\}$$



Q4 rotation matrix

$$\begin{bmatrix}\cos\theta & -\sin\theta \\ \sin\theta & \cos\theta\end{bmatrix}$$



$$\hookrightarrow \begin{bmatrix}\cos 45 & -\sin 45 \\ \sin 45 & \cos 45\end{bmatrix} = \begin{bmatrix}\sqrt{2}/2 & -\sqrt{2}/2 \\ \sqrt{2}/2 & \sqrt{2}/2\end{bmatrix}$$

or 30° rotation

Q5

$$\begin{bmatrix}1 & 0 \\ 0 & 8\end{bmatrix}\begin{bmatrix}1 & 0 \\ -1/2 & 1\end{bmatrix} = \begin{bmatrix}1 & 0 \\ -4 & 8\end{bmatrix}$$

Quiz 2

Q3

$$\begin{bmatrix}1 & 3/2 & 1/2 \\ 0 & 1 & 1 \\ 2 & 8 & 13\end{bmatrix}\begin{bmatrix}a \\ b \\ c\end{bmatrix} = \begin{bmatrix}9/4 \\ -1/2 \\ 2\end{bmatrix}$$

$$\hookrightarrow \left[\begin{array}{ccc|c}1 & 3/2 & 1/2 & 9/4 \\ 0 & 1 & 1 & -1/2 \\ 0 & 5 & 12 & -5/2\end{array}\right] \Rightarrow \left[\begin{array}{ccc|c}1 & 3/2 & 1/2 & 9/4 \\ 0 & 1 & 1 & -1/2 \\ 0 & 0 & 7 & 0\end{array}\right]$$

$$\hookrightarrow \left[\begin{array}{ccc|c}1 & 3/2 & 1/2 & 9/4 \\ 0 & 1 & 1 & -1/2 \\ 0 & 0 & 1 & 0\end{array}\right]$$

Q4

$$\left[\begin{array}{ccc|c}1 & 3/2 & 1/2 & 9/4 \\ 0 & 1 & 0 & -1/2 \\ 0 & 0 & 1 & 0\end{array}\right] \Rightarrow \left[\begin{array}{ccc|c}1 & 3/2 & 0 & 9/4 \\ 0 & 1 & 0 & -1/2 \\ 0 & 0 & 1 & 0\end{array}\right]$$

$$\hookrightarrow \left[\begin{array}{ccc|c}1 & 0 & 0 & 3 \\ 0 & 1 & 0 & -1/2\end{array}\right.$$

$$\left[\begin{array}{ccc|c} 0 & 0 & 1 & 0 \end{array}\right]$$

Q5

$$\begin{bmatrix} 1 & 1 & 1 \\ 3 & 2 & 1 \\ 2 & 1 & 2 \end{bmatrix}\begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} 15 \\ 28 \\ 23 \end{bmatrix}$$

$$\Rightarrow \left[\begin{array}{ccc|c} 1 & 1 & 1 & 15 \\ 3 & 2 & 1 & 28 \\ 0 & -1 & 0 & -7 \end{array}\right] \Rightarrow \left[\begin{array}{ccc|c} 1 & 1 & 1 & 15 \\ 3 & 2 & 1 & 28 \\ 1 & 0 & 1 & 8 \end{array}\right]$$

$$\hookrightarrow \left[\begin{array}{ccc|c} 1 & 1 & 1 & 15 \\ 0 & -1 & -2 & -17 \\ 1 & 0 & 1 & 8 \end{array}\right] \Rightarrow \left[\begin{array}{ccc|c} 1 & 1 & 1 & 15 \\ 0 & -1 & -2 & -17 \\ 0 & -1 & 0 & -7 \end{array}\right]$$

$$\Rightarrow \left[\begin{array}{ccc|c} 1 & 1 & 1 & 15 \\ 0 & 1 & 2 & 17 \\ 0 & -1 & 0 & -7 \end{array}\right] \Rightarrow \left[\begin{array}{ccc|c} 1 & 1 & 1 & 15 \\ 0 & 1 & 2 & 17 \\ 0 & 0 & 2 & 10 \end{array}\right]$$

$$\Rightarrow \left[\begin{array}{ccc|c} 1 & 1 & 1 & 15 \\ 0 & 1 & 2 & 17 \\ 0 & 0 & 1 & 5 \end{array}\right]$$

Q6

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 15 \\ 0 & 1 & 0 & 7 \\ 0 & 0 & 1 & 5 \end{array}\right] \Rightarrow \left[\begin{array}{ccc|c} 1 & 1 & 0 & 10 \\ 0 & 1 & 0 & 7 \\ 0 & 0 & 1 & 5 \end{array}\right]$$

$$\hookrightarrow \left[\begin{array}{ccc|c} 1 & 0 & 0 & 3 \\ 0 & 1 & 0 & 7 \\ 0 & 0 & 1 & 5 \end{array}\right]$$

Q7 $\begin{bmatrix} 1 & 1 & 1 \\ 3 & 2 & 1 \\ 2 & 1 & 2 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ $A^{+} = B$

$A A^{-1} = I$

↳ $\leftarrow \begin{bmatrix} 1 & 1 & 1 \\ 3 & 2 & 1 \\ 2 & 1 & 2 \end{bmatrix} \begin{bmatrix} A^{-1} \end{bmatrix} = I$

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week 4 Quiz 1

Q1 $A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 0 & 1 \end{bmatrix}$ $B = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 1 \end{bmatrix}$

$$C_{21} = A_{2j} B_{j1} = A_{21}B_{11} + A_{22}B_{21} + A_{23}B_{31}$$
$$= 4 \cdot 1 + 0 + 1 \cdot 1$$
$$= 5$$

Q2 $A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 0 & 1 \end{bmatrix}$ $B = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 1 \end{bmatrix}$

$$C = AB = \begin{bmatrix} 1+3 & 1+2 & 2+3 \\ 4+1 & 4 & 1 \end{bmatrix} = \begin{bmatrix} 4 & 3 & 5 \\ 5 & 4 & 1 \end{bmatrix}$$

Q3 $\begin{bmatrix} 2 & 4 & 5 & 6 \end{bmatrix} \begin{bmatrix} 1 \\ 3 \\ 2 \\ 1 \end{bmatrix} = 2 + 12 + 10 + 6 = 30$

Q5 $\begin{bmatrix} 2 & -1 \\ 0 & 3 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} 0 & 1 & 4 & -1 \\ -2 & 0 & 0 & 2 \end{bmatrix}$

$$= \begin{bmatrix} 2 & 2 & 8 & -4 \\ -6 & 0 & 0 & 6 \\ 0 & 1 & 4 & -1 \end{bmatrix}$$

Q6 $D = ABC$

$A: 5\times3 \quad B: 3\times7 \quad C: 7\times4$

$D \Rightarrow 5\times7 \cdot 7\times4 \Rightarrow 5\times4$

Quiz 2

Q1 $\vec{r}' = \vec{r} + x\hat{s}$

$s_3 = \hat{s}\cdot\hat{e}_3$

$r'\cdot\hat{e}_3 = 0$

$\vec{r}\cdot\hat{e}_3 + xs_3 = 0$

$xs_3 = -\vec{r}\cdot\hat{e}_3$

$x = \dfrac{-\vec{r}\cdot\hat{e}_3}{s_3}$

$$\vec{r}' = \vec{r} + \frac{-\vec{r}\cdot\hat{e}_3}{s_3}\hat{s}$$

$$= \vec{r} - \frac{\vec{r}\cdot\hat{e}_3}{s_3}\hat{s}$$

Q3 $r_i' = (I_{ij} - s_i I_{3j}/s_3) r_j$

$= A_{ij} r_j$

$I_{3j} r_j = [\hat{e}_3]_j$

$A_{11} = (I_{11} - s_1 I_{31}/s_3) = 1$

$A_{12} = I_{12} - s_1 I_{32}/s_3 = 0$

$I_{ij} = \delta_{ij}$

$A_{13} = I_{13} - s_1 I_{33}/s_3 = -s_1/s_3$

$$A_{21} = I_{21} - S_2 I_{31}/S_3 = 0$$

$$A_{22} = I_{22} - S_2 I_{32}/S_3 = 1$$

$$A_{23} = I_{23} - S_2 I_{33}/S_3 = -S_2/S_3$$

$$\begin{bmatrix} 1 & 0 & -S_1/S_3 \\ 0 & 1 & -S_2/S_3 \end{bmatrix}$$

//

Q5

$$A = \begin{bmatrix} 1 & 0 & -S_1/S_3 \\ 0 & 1 & -S_2/S_3 \end{bmatrix} \qquad \hat{S} = \begin{bmatrix} 4/13 \\ -3/13 \\ -12/13 \end{bmatrix}$$

$$\hookrightarrow \begin{bmatrix} 1 & 0 & 1/3 \\ 0 & 1 & -1/4 \end{bmatrix}$$

$$\Rightarrow A\,v = \begin{bmatrix} 1 & 0 & 1/3 \\ 0 & 1 & -1/4 \end{bmatrix} \begin{bmatrix} 6 \\ 2 \\ 3 \end{bmatrix}$$

$$= \begin{bmatrix} 6+1 \\ 2 - 3/4 \end{bmatrix} = \begin{bmatrix} 7 \\ 1.25 \end{bmatrix}$$

$$A\,R = \begin{bmatrix} 1 & 0 & 1/3 \\ 0 & 1 & -1/4 \end{bmatrix} \begin{bmatrix} 5 & -1 & -3 & 7 \\ 4 & -4 & 1 & -2 \\ 9 & 3 & 0 & 12 \end{bmatrix}$$

2×3 $\qquad$ 3×4 $\qquad$ 2×4

$$= \begin{bmatrix} 5+3 & -1+1 & -3 & 7+4 \\ 4 - 9/4 & -4 - 3/4 & 1 & -2-3 \end{bmatrix} = \begin{bmatrix} 8 & 0 & -3 & 11 \\ 1.75 & -4.75 & 1 & -5 \end{bmatrix}$$

week 4 Coding Gram-Schmidt decomposition

$$\begin{bmatrix} A_{00} & A_{01} & A_{02} & A_{03} \\ A_{10} & A_{11} & A_{12} & A_{13} \\ A_{20} & A_{21} & A_{22} & A_{23} \\ A_{30} & A_{31} & A_{32} & A_{33} \end{bmatrix}$$

basis vector

$\vec{v}_0 / |\vec{v}_0| = \hat{e}_0$

① 0^{th} col: divide by its mod

② 1^{st} col: substract off overlap from 0^{th} new vector

$$v_1 - \frac{v_1 \cdot \hat{e}_0}{\cancel{|\hat{e}_0|}} \frac{\hat{e}_0}{\cancel{|\hat{e}_0|}} = v_1 - (v_1 \cdot \hat{e}_0)\hat{e}_0$$

→ if anything left → linear independent → normalize

else fill zero

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week 5 Quiz 1

Q1 $A = \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix}$

$\det[A - \lambda I] = 0$

$$\det\left(\begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix} - \begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix}\right) = 0$$

$$\det\begin{pmatrix} 1-\lambda & 0 \\ 0 & 2-\lambda \end{pmatrix} = 0$$

$$(1-\lambda)(2-\lambda) = 0$$

$$\lambda = 1, \lambda = 2$$

Q2 $(A - \lambda I)x = 0$

$$\left[\begin{pmatrix} 1 & 0 \\ 0 & 2 \end{pmatrix} - \begin{pmatrix} \lambda & 0 \\ 0 & \lambda \end{pmatrix}\right]\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = 0$$

$$\begin{pmatrix} 1-\lambda & 0 \\ 0 & 2-\lambda \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = 0$$

① $\lambda=1$ $\quad \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix}=0$

$$\begin{pmatrix} 0 \\ x_2 \end{pmatrix}=0 \quad \Rightarrow \begin{pmatrix} 0 \\ t \end{pmatrix}$$

② $\lambda=2$ $\quad \begin{pmatrix} -1 & 0 \\ 0 & 0 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix}=0$

$$\begin{pmatrix} x_1 \\ 0 \end{pmatrix}=0 \quad \Rightarrow \begin{pmatrix} t \\ 0 \end{pmatrix}$$

Q3 $\quad A=\begin{bmatrix} 3 & 4 \\ 0 & 5 \end{bmatrix}$ $\qquad (A-\lambda I)x=0$

$$\det(A-\lambda I)=0$$

$$\det\begin{pmatrix} 3-\lambda & 4 \\ 0 & 5-\lambda \end{pmatrix} = (3-\lambda)(5-\lambda)=0$$

$$\lambda=3 \text{ or } \lambda=5$$

Q4 $\quad (A-\lambda I)x=0$

$$\begin{pmatrix} 3-\lambda & 4 \\ 0 & 5-\lambda \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix}=0$$

$\lambda=3:$ $\quad \begin{pmatrix} 0 & 4 \\ 0 & 2 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix}=0$

$$\begin{pmatrix} 4x_2 \\ 2x_2 \end{pmatrix}=0$$

$$\begin{pmatrix} 2x_2 \\ x_2 \end{pmatrix}=0$$

$\lambda=5:$ $\quad \begin{pmatrix} -2 & 4 \\ 0 & 0 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix}=0$

$$\begin{pmatrix} -2x_1+4x_2 \\ 0 \end{pmatrix}=0$$

$$\begin{pmatrix} x_1+2x_2 \end{pmatrix}=0$$

Q5 $A = \begin{bmatrix} 1 & 0 \\ -1 & 4 \end{bmatrix}$

$$\det(A - \lambda I) = \begin{pmatrix} 1-\lambda & 0 \\ -1 & 4-\lambda \end{pmatrix} = (1-\lambda)(4-\lambda) = 0$$

$$\lambda = 1, \lambda = 4$$

Q6 $(A - \lambda I)x = 0$

$$\begin{pmatrix} 1-\lambda & 0 \\ -1 & 4-\lambda \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = 0$$

$\lambda = 1$, $\begin{pmatrix} 0 & 0 \\ -1 & 3 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = 0 \Rightarrow \begin{pmatrix} 0 \\ -x_1 + 3x_2 \end{pmatrix} = 0$

$\lambda = 4$, $\begin{pmatrix} -3 & 0 \\ -1 & 0 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = 0 \Rightarrow \begin{pmatrix} -3x_1 \\ -x_1 \end{pmatrix} = 0$

$$\begin{pmatrix} 3x_1 \\ x_1 \end{pmatrix} = 0$$

Q7 $A = \begin{bmatrix} -3 & 8 \\ 2 & 3 \end{bmatrix}$

$$\det(A - I\lambda) = \begin{pmatrix} -3-\lambda & 8 \\ 2 & 3-\lambda \end{pmatrix} = (-3-\lambda)(3-\lambda) - 16 = 0$$

$$\hookrightarrow (3+\lambda)(3-\lambda) = -16$$

$$9 - \lambda^2 = -16$$

$$\lambda^2 = 25$$

$$\lambda = \pm 5$$

Q8 $(A - I\lambda)x = 0$

$$\begin{pmatrix} -3-\lambda & 8 \\ 2 & 3-\lambda \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = 0$$

$\lambda = 5$: $\begin{pmatrix} -8 & 8 \\ 2 & -2 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = 0 \Rightarrow \begin{pmatrix} -8x_1 + 8x_2 \\ 2x_1 - 2x_2 \end{pmatrix} = 0 \Rightarrow \begin{pmatrix} -4x_1 + 4x_2 \\ x_1 - x_2 \end{pmatrix} = 0$

$\lambda = -5$: $\begin{pmatrix} +2 & 8 \\ 2 & -8 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = 0 \Rightarrow \begin{pmatrix} 2x_1 + 8x_2 \\ 2x_1 - 8x_2 \end{pmatrix} = 0 \Rightarrow \begin{pmatrix} x_1 + 4x_2 \\ x_1 - 4x_2 \end{pmatrix} = 0$

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week 5 Quiz 2

Q_1 $D = C^{-1}TC = \begin{pmatrix} 2 & -1 \\ -1 & 1 \end{pmatrix}\begin{pmatrix} 6 & -1 \\ 2 & 3 \end{pmatrix}\begin{pmatrix} 1 & 1 \\ 1 & 2 \end{pmatrix}$

$C = \begin{pmatrix} 1 & 1 \\ 1 & 2 \end{pmatrix}$ $= \begin{pmatrix} 10 & -5 \\ -4 & 4 \end{pmatrix}\begin{pmatrix} 1 & 1 \\ 1 & 2 \end{pmatrix}$

$C^{-1} = \begin{pmatrix} 2 & -1 \\ -1 & 1 \end{pmatrix}$ $= \begin{pmatrix} 5 & 0 \\ 0 & 4 \end{pmatrix}$

Q_2 $C = \begin{pmatrix} 7 & 1 \\ -3 & 0 \end{pmatrix}$ $T = \begin{pmatrix} 2 & 7 \\ 0 & -1 \end{pmatrix}$

$C^{-1} = \frac{1}{3}\begin{pmatrix} 0 & -1 \\ 3 & 7 \end{pmatrix}$

$D = C^{-1}TC = \frac{1}{3}\begin{pmatrix} 0 & -1 \\ 3 & 7 \end{pmatrix}\begin{pmatrix} 2 & 7 \\ 0 & -1 \end{pmatrix}\begin{pmatrix} 7 & 1 \\ -3 & 0 \end{pmatrix}$

$= \frac{1}{3}\begin{pmatrix} 0 & 1 \\ 6 & 14 \end{pmatrix}\begin{pmatrix} 7 & 1 \\ -3 & 0 \end{pmatrix}$

$= \frac{1}{3}\begin{pmatrix} -3 & 0 \\ 0 & 6 \end{pmatrix}$

$= \begin{pmatrix} -1 & 0 \\ 0 & 2 \end{pmatrix}$

Q_3 $C = \begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix}$ $T = \begin{pmatrix} 1 & 0 \\ 2 & -1 \end{pmatrix}$

$C^{-1} = \begin{pmatrix} 1 & 0 \\ -1 & 1 \end{pmatrix}$

$D = C^{-1}TC = \begin{pmatrix} 1 & 0 \\ -1 & 1 \end{pmatrix}\begin{pmatrix} 1 & 0 \\ 2 & -1 \end{pmatrix}\begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix}$

$= \begin{pmatrix} 1 & 0 \\ 1 & -1 \end{pmatrix}\begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix}$

$$= \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

Q5 T^3

$$= \begin{pmatrix} 1 & 1 \\ 1 & 2 \end{pmatrix} \begin{pmatrix} \overset{125}{5^3} & 0 \\ 0 & \underset{64}{4^3} \end{pmatrix} \begin{pmatrix} 2 & -1 \\ -1 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} 125 & 64 \\ 125 & 128 \end{pmatrix} \begin{pmatrix} 2 & -1 \\ -1 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} 186 & -61 \\ 122 & 3 \end{pmatrix}$$

---

week5 final Quiz

Q6 $A = \begin{bmatrix} 3/2 & -1 \\ -1/2 & 1/2 \end{bmatrix}$

$A - \lambda I = 0$

$$\begin{pmatrix} 3/2 - \lambda & -1 \\ -1/2 & 1/2 - \lambda \end{pmatrix} \Rightarrow (3/2 - \lambda)(1/2 - \lambda) = 1/2$$

$$\frac{3}{4} - 3/2\lambda - 1/2\lambda + \lambda^2 = 1/2$$

$$3 - 6\lambda - 2\lambda + 4\lambda^2 = 2$$

$$4\lambda^2 - 8\lambda + 1 = 0$$

$$\lambda^2 - 2\lambda + 1/4 = 0$$

Q7 $\lambda = \dfrac{2 \pm \sqrt{4 - 4 \cdot 1/4}}{2}$

$$= \frac{2 \pm \sqrt{3}}{2}$$

$$= 1 \pm \sqrt{3}/2$$

Q8 $(A - \lambda I)x = 0$

$\lambda = 1 + \frac{\sqrt{3}}{2}$: $\left(\begin{pmatrix} 3/2 & -1 \\ -1/2 & 1/2 \end{pmatrix} - \begin{pmatrix} 1+\frac{\sqrt{3}}{2} & 0 \\ 0 & \frac{1+\sqrt{3}}{2} \end{pmatrix} \right) \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = 0$

Q9 $C = \begin{pmatrix} -1-\sqrt{3} & -1+\sqrt{3} \\ 1 & 1 \end{pmatrix}$

$A = \begin{bmatrix} 3/2 & -1 \\ -1/2 & 1/2 \end{bmatrix}$

Course 2 week 1

Quiz 2

Q2 $A(x) = (x+2)(3x-3)$

$A'(x) = (3x-3) + (x+2)3$

$= 3x - 3 + 3x + 6$

$= 6x + 3$

Q3 $f(x) = x^3 \sin(x)$

$f'(x) = 3x^2 \sin(x) + x^3 \cos(x)$

Q4 $f(x) = e^x/x = e^x x^{-1}$

$f'(x) = e^x x^{-1} - e^x x^{-2}$

$= \frac{e^x}{x} - \frac{e^x}{x^2}$

Q6 $f(x) = xe^x \cos(x)$

$f'(x) = e^x \cos(x) + xe^x \cos(x) - xe^x \sin(x)$

$= e^x (\cos(x) + x\cos(x) - x\sin(x))$

Quiz 3

Q2 $f(x) = e^{x^2-3}$

$= 2x e^{x^2-3}$

Q3 $f(x) = \sin^3(x)$

$g(h) = h^3$

$h(x) = \sin x$

$\frac{dh}{dx} = \frac{dh}{dx} \frac{dg}{h} = \cos x \cdot 3h^2$

$= \cos x \cdot 3 \sin^2 x$

Q4

$\frac{d}{dx}(\tan x) = \sec^2 x$

Q5 $f(x) = e^{\sin(x^2)}$

$f(g) = e^g$

$g(h) = \sin(h)$

$h(x) = x^2$

$\frac{df}{dx} = \frac{df}{dg} \frac{dg}{dh} \frac{dh}{dx}$

$= e^g \cos(h)\, 2x$

$= e^g \cos(x^2)\, 2x$

$= \left(e^{\sin(x^2)}\right) \cos(x^2)\, 2x$

final quiz

Q1 $f(x) = x^{3/2} + \pi x^2 + \sqrt{7}$ @ $x=2$

$f'(x) = \frac{3}{2} x^{1/2} + 2\pi x$

$= \frac{3}{2}\sqrt{2} + 4\pi$

Q2 $f(x) = x^3 \cos(x) e^x$

$f'(x) = 3x^2 \cos(x) e^x - x^3 \sin(x) e^x + x^3 \cos(x) e^x$

Q3 $f(x) = e^{(x+1)^2}$

$f'(x) = 2(x+1) e^{(x+1)^2}$

Q4 $f(x) = x^2 \cos(x^3)$

$f'(x) = 2x \cos(x^3) - x^2 3x^2 \sin(x^3)$

Q5 $f(x) = \sin(x) e^{\cos(x)}$ @ $x = \pi$

$f'(x) = \cos(x) e^{\cos(x)} - \sin(x) e^{\cos(x)} \sin(x)$

$= -1 e^{-1}$

$= -\frac{1}{e}$

---

week 2 Quiz 2

Q1 $f(x,y) = \pi x^3 + xy^2 + my^4$

$\frac{\partial f}{\partial x} = 3\pi x^2 + y^2$

$\frac{\partial f}{\partial y} = 2xy + 4my^3$

Q2 $f(x,y,z) = x^2 y + y^2 z + z^2 x$

$$\frac{\partial f}{\partial x} = 2xy + z^2$$

$$\frac{\partial f}{\partial y} = x^2 + 2yz$$

$$\frac{\partial f}{\partial z} = y^2 + 2zx$$

Q3 $f(x, y, z) = e^{2x} \sin(y) z^2 + \cos(z) e^x e^y$

$$\frac{\partial f}{\partial x} = 2e^{2x} \sin(y) z^2 + \cos(z) e^x e^y$$

$$\frac{\partial f}{\partial y} = e^{2x} \cos(y) z^2 + \cos(z) e^x e^y$$

$$\frac{\partial f}{\partial z} = e^{2x} \sin(y) 2z - \sin(z) e^x e^y$$

Q4 $f(x,y) = \frac{\sqrt{x}}{y}$ $\quad x(t) = t$

$y(t) = \sin(t)$

$$\frac{df}{dt} = \frac{\partial f}{\partial x}\frac{dx}{dt} + \frac{\partial f}{\partial y}\frac{dy}{dt}$$

$$= \left(\frac{1}{2}\frac{1}{y\sqrt{x}}\right)(1) + \left(-\frac{\sqrt{x}}{y^2}\right)(\cos(t))$$

$$= \frac{1}{2y\sqrt{x}} - \frac{\sqrt{x}}{y^2}\cos(t)$$

$$= \frac{1}{2\sin(t)\sqrt{t}} - \frac{\sqrt{t}}{\sin^2(t)}\cos(t)$$

Quiz 2

Q1 $f(x,y) = x^2 y + \frac{3}{4}xy + 10$

$$\left.\begin{aligned} \frac{\partial f}{\partial x} &= 2xy + \tfrac{3}{4}y \\ \frac{\partial f}{\partial y} &= x^2 + \tfrac{3}{4}x \end{aligned}\right\} J = \left[2xy + \tfrac{3}{4}y,\ x^2 + \tfrac{3}{4}x\right]$$

Q2 $f(x,y) = e^{x}\cos(y) + xe^{3y} - 2$

$$\left.\begin{aligned}\frac{\partial f}{\partial x} &= e^{x}\cos(y) + e^{3y}\\ \frac{\partial f}{\partial y} &= -e^{x}\sin(y) + 3xe^{3y}\end{aligned}\right\} J = \left[e^{x}\cos(y) + e^{3y},\ -e^{x}\sin(y) + 3xe^{3y}\right]$$

Q3 $f(x,y,z) = e^{x}\cos(y) + x^2y^2z^2$

$$\left.\begin{aligned}\frac{\partial f}{\partial x} &= e^{x}\cos(y) + 2xy^2z^2\\ \frac{\partial f}{\partial y} &= -e^{x}\sin(y) + 2x^2yz^2\\ \frac{\partial f}{\partial z} &= 2x^2y^2z\end{aligned}\right\} J$$

Q4 $f(x,y,z) = x^2 + 3e^{y}e^{z} + \cos(x)\sin(z)$

$$\left.\begin{aligned}\frac{\partial f}{\partial x} &= 2x - \sin(x)\sin(z)\\ \frac{\partial f}{\partial y} &= 3e^{y}e^{z}\\ \frac{\partial f}{\partial z} &= 3e^{y}e^{z} + \cos(x)\cos(z)\end{aligned}\right\} J(0,0,0) = [0, 3, 4]$$

Q5 $f(x,y,z) = xe^{y}\cos(z) + 5x^2\sin(y)e^{z}$

$$\left.\begin{aligned}\frac{\partial f}{\partial x} &= e^{y}\cos(z) + 10x\sin(y)e^{z}\\ \frac{\partial f}{\partial y} &= xe^{y}\cos(z) + 5x^2\cos(y)e^{z}\\ \frac{\partial f}{\partial z} &= -xe^{y}\sin(z) + 5x^2\sin(y)e^{z}\end{aligned}\right\} \begin{aligned}&J(0,0,0)\\ &= [1+0, 0, 0]\\ &= [1, 0, 0]\end{aligned}$$

Hessians Quiz

Q1: $f(x,y) = x^3 y + x + 2y$

$$J = \left[\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}\right] = \left[3x^2 y + 1, x^3 + 2\right]$$

$$H = \begin{bmatrix} 6xy & 3x^2 \\ 3x^2 & 0 \end{bmatrix}$$

Q2: $f(x,y) = e^x \cos(y)$

$$J = \left[e^x \cos(y), -e^x \sin(y)\right]$$

$$H = \begin{bmatrix} e^x \cos(y) & -e^x \sin(y) \\ -e^x \sin(y) & -e^x \cos(y) \end{bmatrix}$$

Q3: $f(x,y) = \frac{x^2}{2} + xy + \frac{y^2}{2}$

$$J = \left[x + y, x + y\right]$$

$$H = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$$

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week2 final Quiz

Q1 $f(x,y,z) = x^2 \cos(y) + e^z \sin(y)$

$$J = \left[2x \cos(y), -x^2 \sin(y) + e^z \cos(y), e^z \sin(y)\right]$$

$(x,y,z) = (\pi, \pi, 1)$

$$J = \left[-2\pi, -e, 0\right]$$

Q2 $u(x,y) = x^2 y - \cos(x)\sin(y)$

$v(x,y) = e^{x+y}$

$$J = \begin{bmatrix} 2xy + \sin(x)\sin(y) & x^2 - \cos(x)\cos(y) \\ e^{x+y} & e^{x+y} \end{bmatrix}$$

$(0,\pi)$: $\begin{bmatrix} 0 & 1 \\ e^{\pi} & e^{\pi} \end{bmatrix}$

Q3 $f(x,y) = x^3\cos(y) - x\sin(y)$

$J = [\,3x^2\cos(y) - \sin(y),\ -x^3\sin(y) - x\cos(y)\,]$

$$H = \begin{bmatrix} 6x\cos(y) & -3x^2\sin(y) - \cos(y) \\ -3x^2\sin(y) - \cos(y) & -x^3\cos(y) + x\sin(y) \end{bmatrix}$$

Q4 $f(x,y,z) = xy + \sin(y)\sin(z) + z^3e^x$

$J = [\,y + z^3e^x,\ x + \cos(y)\sin(z),\ \sin(y)\cos(z) + 3z^2e^x\,]$

$$H = \begin{bmatrix} z^3e^x & 1 & 3z^2e^x \\ 1 & -\sin(y)\sin(z) & \cos(y)\cos(z) \\ 3z^2e^x & \cos(y)\cos(z) & -\sin(y)\sin(z) + 6ze^x \end{bmatrix}$$

Q5 $f(x,y,z) = xy\cos(z) - \sin(x)e^yz^3$

$J = [\ y\cos(z) - \cos(x)e^yz^3,\ x\cos(z) - \sin(x)e^yz^3,$

$-xy\sin(z) - 3\sin(x)e^y z^2 \Big]$

$$H = \begin{matrix} \sin(x)e^y z^3 & \cos(z)-\cos(x)e^y z^3 & -y\sin(z)-3z^2\cos(x)e^y \\ \cos(z)-\cos(x)e^y z^3 & -\sin(x)e^y z^3 & -x\sin(z)-3z^2\sin(x)e^y \\ -y\sin(z)-3\cos(x)e^y z^2 & -x\sin(z)-3\sin(x)e^y z^2 & -xy\cos(z)-6z\sin(x)e^y \end{matrix}$$

$(0,0,0)$ $$H = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

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week 3 Quiz 1

① $f(\vec{x}) = f(x_1, x_2) = x_1^2 x_2^2 + x_1 x_2$

$x_1(t) = 1 - t^2$

$x_2(t) = 1 + t^2$

$$\frac{df}{dt} = \frac{\partial f}{\partial \vec{x}} \cdot \frac{d\vec{x}}{dt}$$

$$\frac{\partial f}{\partial \vec{x}} = \left[\frac{\partial f}{\partial x_1}, \frac{\partial f}{\partial x_2}\right] = \left[2x_1 x_2^2 + x_2,\ 2x_1^2 x_2 + x_1\right]$$

$$\frac{d\vec{x}}{dt} = \begin{bmatrix} \frac{dx_1}{dt} \\ \frac{dx_2}{dt} \end{bmatrix} = \begin{bmatrix} -2t \\ 2t \end{bmatrix}$$

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② $f(\vec{x}) = f(x_1, x_2, x_3) = x_1^3 \cos(x_2) e^{x_3}$

$x_1(t) = 2t$

$x_2(t) = 1 - t^2$

$x_3(t) = e^t$

$$\frac{df}{dt} = \frac{\partial f}{\partial \vec{x}} \cdot \frac{d\vec{x}}{dt}$$

$$\frac{\partial f}{\partial \vec{x}} = \left[\frac{\partial f}{\partial x_1}, \frac{\partial f}{\partial x_2}, \frac{\partial f}{\partial x_3}\right]$$

$$= \left[3x_1^2\cos(x_2)e^{x_3}, -x_1^3\sin(x_2)e^{x_3}, x_1^3\cos(x_2)e^{x_3}\right]$$

$$\frac{d\vec{x}}{dt} = \begin{bmatrix} dx_1/dt \\ dx_2/dt \\ dx_3/dt \end{bmatrix} = \begin{bmatrix} 2 \\ -2t \\ e^t \end{bmatrix}$$

---

③ $f(\vec{x}) = f(x_1, x_2) = x_1^2 - x_2^2$

$x_1(u_1, u_2) = 2u_1 + 3u_2$

$x_2(u_1, u_2) = 2u_1 - 3u_2$

$u_1(t) = \cos(t/2)$

$u_2(t) = \sin(2t)$

$$\frac{df}{dt} = \frac{\partial f}{\partial \vec{x}} \frac{\partial \vec{x}}{\partial \vec{u}} \frac{d\vec{u}}{dt}$$

$$\frac{\partial f}{\partial \vec{x}} = \left[\frac{\partial f}{\partial x_1}, \frac{\partial f}{\partial x_2}\right] = \left[2x_1, -2x_2\right]$$

$$\frac{\partial \vec{x}}{\partial \vec{u}} = \begin{bmatrix} \frac{\partial x_1}{\partial u_1} & \frac{\partial x_1}{\partial u_2} \\ \frac{\partial x_2}{\partial u_1} & \frac{\partial x_2}{\partial u_2} \end{bmatrix} = \begin{bmatrix} 2 & 3 \\ 2 & -3 \end{bmatrix}$$

$$\frac{d\vec{u}}{dt} = \begin{bmatrix} \frac{du_1}{dt} \\ \frac{du_2}{dt} \end{bmatrix} = \begin{bmatrix} -\frac{1}{2}\sin(t/2) \\ 2\cos(2t) \end{bmatrix}$$

---

④ $f(\vec{x}) = f(x_1, x_2) = \cos(x_1)\sin(x_2)$

$x_1(u_1, u_2) = 2u_1^2 + 3u_2^2 - u_2$

$x_2(u_1, u_2) = 2u_1 - 5u_2^3$

$u_1(t) = e^{t/2}$

$u_2(t) = e^{-2t}$

$$\frac{df}{dt} = \frac{\partial f}{\partial \vec{x}} \cdot \frac{\partial \vec{x}}{\partial \vec{u}} \cdot \frac{d\vec{u}}{dt}$$

$$\frac{\partial f}{\partial \vec{x}} = \left[\frac{\partial f}{\partial x_1}, \frac{\partial f}{\partial x_2}\right] = \left[-\sin(x_1)\sin(x_2), \cos(x_1)\cos(x_2)\right]$$

$$\frac{\partial \vec{x}}{\partial \vec{u}} = \begin{bmatrix} \partial x_1/\partial u_1 & \partial x_1/\partial u_2 \\ \partial x_2/\partial u_1 & \partial x_2/\partial u_2 \end{bmatrix}$$

$$= \begin{bmatrix} 4u_1 & 6u_2 - 1 \\ 2 & -15u_2^2 \end{bmatrix}$$

$$\frac{d\vec{u}}{dt} = \begin{bmatrix} du_1/dt \\ du_2/dt \end{bmatrix} = \begin{bmatrix} \frac{1}{2}e^{t/2} \\ -2e^{-2t} \end{bmatrix}$$

---

(5) $f(\vec{x}) = f(x_1, x_2, x_3) = \sin(x_1)\cos(x_2)e^{x_3}$

$x_1(u_1, u_2) = \sin(u_1) + \cos(u_2)$

$x_2(u_1, u_2) = \cos(u_1) - \sin(u_2)$

$x_3(u_1, u_2) = e^{u_1 + u_2}$

$u_1(t) = 1 + t/2$

$u_2(t) = 1 - t/2$

$$\frac{df}{dt} = \frac{\partial f}{\partial \vec{x}} \frac{\partial \vec{x}}{\partial \vec{u}} \frac{d\vec{u}}{dt}$$

$$\frac{\partial f}{\partial \vec{x}} = \left[\frac{\partial f}{\partial x_1}, \frac{\partial f}{\partial x_2}, \frac{\partial f}{\partial x_3}\right] = \left[\cos(x_1)\cos(x_2)e^{x_3}, -\sin(x_1)\cos(x_2)e^{x_3}, \sin(x_1)\cos(x_2)e^{x_3}\right]$$

$$\frac{\partial \vec{x}}{\partial \vec{u}} = \begin{bmatrix} \partial x_1/\partial u_1 & \partial x_1/\partial u_2 \\ \partial x_2/\partial u_1 & \partial x_2/\partial u_2 \end{bmatrix} = \begin{bmatrix} \cos(u_1) & -\sin(u_2) \\ -\sin(u_1) & -\cos(u_2) \end{bmatrix}$$

$$\begin{bmatrix} \partial x_3/\partial u_1 & \partial x_3/\partial u_2 \end{bmatrix} \quad \begin{bmatrix} e^{u_1+u_2} & e^{u_1+u_2} \end{bmatrix}$$

$$\frac{d\vec{u}}{dt} = \begin{bmatrix} du_1/dt \\ du_2/dt \end{bmatrix} = \begin{bmatrix} 1/2 \\ -1/2 \end{bmatrix}$$

---

week 4 Quiz 2

(1) $f(x) = e^{x^2}$ @ $x=0$

$$g(x) = \sum_{n=0}^{\infty} \frac{1}{n!} f^{(n)}(0)\, x^n = f(0) + f'(0)x + \frac{1}{2} f''(0) x^2$$

$f(0) = 1$

$f'(0) = 2x e^{x^2} = 0$

$f''(0) = 2e^{x^2} + 2x \cdot 2x \cdot e^{x^2} = 2e^{x^2} + 4x^2 e^{x^2} = 2$

$f'''(0) = 4x e^{x^2} + 8x e^{x^2} + 4x^2 2x e^{x^2}$

$= 4x e^{x^2} + 8x e^{x^2} + 8x^3 e^{x^2}$

$= 0$

$f^{(4)}(0) = 4e^{x^2} + 8e^{x^2} + 24x^2 e^{x^2} + 8x^3 \cdot 2x e^{x^2}$

$= 12e^{x^2} + 24x^2 e^{x^2} + 16x^4 e^{x^2}$

$= 12$

$\Rightarrow g(x) = 1 + \frac{1}{2!} 2x^2 + \frac{1}{4!} \cdot 12 x^4 + \ldots$

$= 1 + x^2 + \frac{1}{2} x^4 + \ldots$

(2) $f(x) = 1/x$ @ $p=4$

$$g(x) = \sum_{n=0}^{\infty} \frac{1}{n!} f^{(n)}(p)\, (x-p)^n$$

$f(4) = 1/4$

$f'(4) = -\frac{1}{x^2} = -1/16$

$f''(4) = \frac{2}{x^3} = 2/64 = 1/32$

$\Rightarrow g(x) = \frac{1}{4} - \frac{1}{16}(x-4) + \frac{1}{64}(x-4)^2$

③ $f(x) = \ln(x)$ @ $x=10$

$f(10) = \ln(10)$

$f'(10) = \frac{1}{x} = \frac{1}{10}$

$f''(10) = -\frac{1}{x^2} = -\frac{1}{100}$

$\Rightarrow g(x) = \ln(10) + \frac{1}{10}(x-10) - \frac{1}{2}\frac{1}{100}(x-10)^2$

$\Delta\ g(2) \Rightarrow$ 0.32

④ $f(x) = \frac{1}{(1-x)^2}$ @ $x=0$

$f(0) = 1$

$f'(0) = -2(1-x)^{-3}(-1) = 2(1-x)^{-3} = 2$

$f''(0) = -6(1-x)^{-4}(-1) = 6(1-x)^{-4} = 6$

$g(x) = 1 + 2x + \frac{6}{2}x^2$

⑤ $f(x) = \sqrt{4-x}$ @ $x=0$

$f(0) = 2$

$f'(0) = \frac{1}{2}(4-x)^{-1/2}(-1) = -1/4$

$f''(0) = \frac{1}{4}(4-x)^{-3/2} = \frac{1}{4}\frac{1}{\sqrt{4^3}} = \frac{1}{32}$

$\Rightarrow g(x) = 2 - \frac{x}{4} - \frac{x^2}{64} + \dots$

---

week 4 final Quiz

Q_2 $f(x) = e^x + x + \sin(x)$ @ $x=0$

$g(x) = \sum_{n=0}^{\infty} \frac{1}{n!} f^{(n)}(p)(x-p)^n$

$f(0) = 1$

$f'(0) = e^x + 1 + \cos(x) = 1+1+1 = 3$

$f''(0) = e^x - \sin(x) = 1$

$f'''(0) = e^x - \cos(x) = 0$

$f^{(4)}(0) = e^x + \sin(x) = 1$

$\Rightarrow g(x) = 1 + 3x + \frac{1}{2}x^2 + \frac{1}{24}x^4$

$= 1 + 3x + \frac{1}{2}x^2 + \frac{1}{24}x^4$

Q3 $\quad f(x) = 2/(x^2 - x) \quad$ @ $x = 0.5$

$f(0.5) = -8$

$f'(0.5) = \frac{d}{dx} 2(x^2 - x)^{-1} = -2(x^2-x)^{-2} \cdot (2x) = -32$

$\Rightarrow g(x) = -8 - 32(x - 0.5)^2$

Q5 $\quad f(x) = e^{-2x} \quad$ @ $x = 2$

$f(2) = e^{-4}$

$f'(2) = -2e^{-2x} = -2e^{-4}$

$g(x) = e^{-4} + -2e^{-4}(x-2) + O(\Delta x^2)$

$= \frac{1}{e^4}(1 - 2(x-2)) + O(\Delta x^2)$

---

week 5 Quiz 1

① $f(x) = \frac{x^6}{6} - 3x^4 - \frac{2x^3}{3} + \frac{27x^2}{2} + 18x - 30$

$f'(x) = x^5 - 12x^3 - 2x^2 + 27x + 18$

② $x_1 = x_0 - \frac{f(1)}{f'(1)} = \qquad\qquad x_0 = 1$

$f(1) = 1/6 - 3 - 2/3 + 27/2 + 18 - 30 = -2$

$f'(1) = 1 - 12 - 2 + 27 + 18 = 32$

$x_1 = 1 - {}^{-2}/_{32} = 1.0625$

Quiz Lagrange multiplier

$Q_1 \quad g(x) = x^2 + 3(y+1)^2 - 1 = 0$

$$\frac{\partial g}{\partial x} = 2x$$

$$\frac{\partial g}{\partial y} = 6(y+1) = 6y+6$$

$Q_5 \quad f(x,y) = -e^{x-y^2+xy}$

$g(x,y) = \cosh(y) + x - 2 = 0$

$$\frac{\partial f}{\partial x} = -(1+y)e^{x-y^2+xy}$$

$$\frac{\partial f}{\partial y} = -(-2y+x)e^{x-y^2+xy}$$

---

week 6 Quiz

(0.4, 0.1)
(0.5, 0.25)
(0.6, 0.55)
(0.7, 0.75)
(0.8, 0.85)

$\bar{x} = 0.6$

$\bar{y} = 0.5$

$$m = \frac{(-0.02) + (-0.025) + 0.075 + 0.17}{0.04 + 0.01 + 0.01 + 0.04}$$

$$= \frac{0.2}{0.1} = 2$$

$c = 0.5 - 2 \times 0.6 = -0.7$

---

week Quiz

$$Q_4 \quad \frac{\partial \chi^2}{\partial a_j} = -2\sum_{i=1}^{n} \frac{y_i - f(x_i; \vec{a})}{\sigma_i^2} \frac{\partial f(x_i, \vec{a})}{\partial a_j} \qquad j = 1 \ldots n$$

$$y(x_i; a) = a_1\left(1 - e^{-a_2 x_i^2}\right), \quad \sigma_i^2 = 1$$

$$\frac{\partial f}{\partial a_1} = \left(1 - e^{-a_2 x_i^2}\right)$$

$$\frac{\partial (\chi^2)}{\partial a_1} = -2\sum_{i=1}^{n} \left(y_i - a_1\left(1 - e^{-a_2 x_i^2}\right)\right)$$

$$= a_1 - a_1 e^{-a_2 x_i^2}$$

$$\frac{df}{\partial a_2} = +a_1 x_i^2 e^{-a_2 x_i^2}$$

$$= a_1 x_i^2 e^{-a_2 x_i^2}$$

Course 2 final assignment

$$f(\vec{x}; \mu, \sigma) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left(-\frac{(x-\mu)^2}{2\sigma^2}\right)$$

$(\Delta \chi^2)$

$$J = \left[\frac{\partial(\chi)^2}{\partial \mu}, \quad \frac{\partial(\chi)^2}{\partial \sigma}\right]$$

$$\chi^2 = |y - f(x; \mu, \sigma)|^2$$

$$\frac{\partial(\chi^2)}{\partial \mu} = -2\left(y - f(x; \mu, \sigma)\right) \cdot \frac{\partial f}{\partial \mu}(x; \mu, \sigma)$$

$$\frac{\partial(\chi^2)}{\partial \sigma} = -2\left(y - f(x; \mu, \sigma)\right) \cdot \frac{\partial f}{\partial \sigma}(x; \mu, \sigma)$$

$$\frac{\partial f}{\partial \mu} = \frac{+\cancel{2}(x-\mu)}{\cancel{2}\sigma^2} \cdot f(\vec{x}; \mu, \sigma)$$

$$\frac{\partial f}{\partial \sigma} = \frac{-1}{\sigma^2\sqrt{2\pi}} \exp(\ldots) = \frac{-f(x)}{\sigma} + \frac{1}{\sigma\sqrt{2\pi}} e(\ldots) \cdot \left[+\frac{\cancel{2}(x-\mu)^2}{\cancel{2}\sigma^3}\right.$$

Course 3

week Quiz. mean

Q2 $D = \left\{ \begin{bmatrix} 1 \\ 4 \\ 7 \end{bmatrix}, \begin{bmatrix} 2 \\ 5 \\ 8 \end{bmatrix}, \begin{bmatrix} 3 \\ 6 \\ 9 \end{bmatrix} \right\}$

$$\bar{D} = \begin{bmatrix} 6/3 \\ 15/3 \\ 24/3 \end{bmatrix} = \begin{bmatrix} 2 \\ 5 \\ 8 \end{bmatrix}$$

Q_3 $D = \begin{bmatrix} 9/3 \\ 9/3 \\ 9/3 \end{bmatrix} = \begin{bmatrix} 3 \\ 3 \\ 3 \end{bmatrix} \Rightarrow \begin{bmatrix} 6 \\ 6 \\ 6 \end{bmatrix}$

Q_4 $\begin{bmatrix} 3+1 \\ 3+2 \\ 3+3 \end{bmatrix} = \begin{bmatrix} 4 \\ 5 \\ 6 \end{bmatrix}$

Q_5 $\left[\bar{x}_{n-1} \cdot (n-1) + x\right] / n$

$$= \frac{\bar{x}_{n-1}}{n}(n-1) + x/n$$

$$= \bar{x}_{n-1}\left(1 - \frac{1}{n}\right) + x/n$$

$$= \bar{x}_{n-1} - \frac{\bar{x}_{n-1}}{n} + x/n$$

$$= \bar{x}_{n-1} - \frac{1}{n}(\bar{x}_{n-1} - x)$$

$$= \bar{x}_{n-1} + \frac{1}{n}(x - \bar{x}_{n-1})$$

Quiz 2

Q_1 $D = \{1, 2, 3, 2\}$ $\bar{D} = 2$

$$Var[D] = \frac{(1-2)^2 + (3-2)^2}{4} = 0.5$$

4

Quiz 3

$D = \left\{ \begin{matrix} x \\ y \end{matrix} \begin{bmatrix} 1 \\ 2 \end{bmatrix}, \begin{bmatrix} 5 \\ 4 \end{bmatrix} \right\}$

$E[x] = 3$

$E[y] = 3$

$$Var[x] = \frac{(1-3)^2 + (5-3)^2}{2}$$

$$= \frac{4+4}{2}$$

$$= 4$$

$$Var[y] = \frac{(2-3)^2 + (4-3)^2}{2}$$

$$= 1$$

$$Cov[x, y] = E[(x - \mu_x)(y - \mu_y)]$$

	$x - \mu_x$	$y - \mu_y$
① data =	$1 - 3 = -2$	$2 - 3 = -1$
② data :	$5 - 3 = 2$	$4 - 3 = 1$

$$E[(2, 2)]$$

$$\Rightarrow 2$$

$$\Rightarrow Cov = \begin{bmatrix} 4 & 2 \\ 2 & 1 \end{bmatrix}$$

---

week 1 assignment

image_shape = (64, 64) , no. imgs = 400

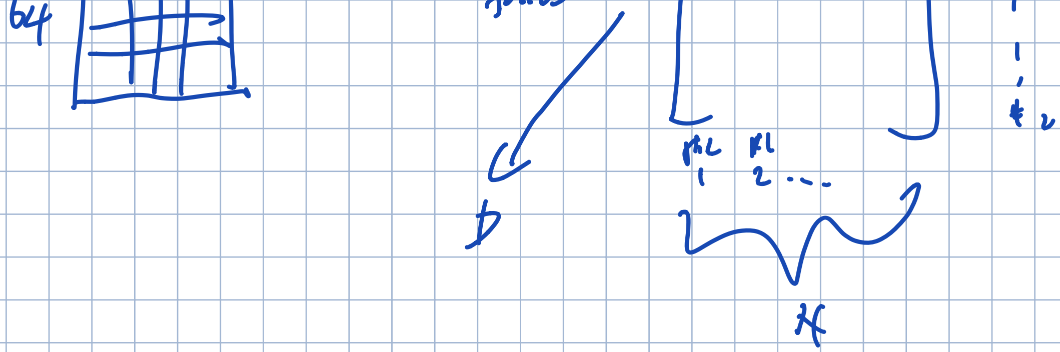
64

400

faces = 400 [

pixel #1

:



week 2 Quiz : Dot product

Q1 $x = \begin{bmatrix} 1 \\ -1 \\ 3 \end{bmatrix}$

$$\|x\| = \sqrt{1^2 + (-1)^2 + (3)^2}$$
$$= \sqrt{11}$$

Q2 $x = \begin{bmatrix} 3 \\ 4 \end{bmatrix}$ $y = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$

$\|x\| = \sqrt{9+16} = 5$

$\|y\| = \sqrt{2}$

$$\cos\alpha = \frac{x^T y}{\|x\| \|y\|} = \frac{(3 \ 4)\begin{pmatrix} 1 \\ -1 \end{pmatrix}}{5\sqrt{2}} = \frac{3-4}{5\sqrt{2}} \Rightarrow \alpha = 98.13^\circ$$
$$= 1.71 \text{ rad}$$

$x = \begin{bmatrix} 3 \\ 4 \end{bmatrix}$ $y = \begin{bmatrix} -1 \\ -1 \end{bmatrix}$

$$\cos\alpha = \frac{(3 \ 4)\begin{pmatrix} -1 \\ -1 \end{pmatrix}}{5\sqrt{2}} = \frac{-3-4}{5\sqrt{2}} \Rightarrow \alpha = ? \text{ rad}$$

Q3 $x - y = \begin{bmatrix} 2 \\ 5 \end{bmatrix}$

$$\|x-y\| = \sqrt{4+25} = 5.38$$

Q5 $x = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$ $y = \begin{bmatrix} -1 \\ 0 \\ 8 \end{bmatrix}$

$$z = x - y = \begin{pmatrix} 2 \\ 2 \\ -5 \end{pmatrix}$$

$$\|x\| = \sqrt{1^2 + 2^2 + 3^2}$$
$$= \sqrt{14}$$

$$\|z\| = \sqrt{4+4+25} = \sqrt{33}$$

$$\cos\alpha = \frac{x^T z}{\sqrt{14}\sqrt{33}} = \frac{2+4-15}{\sqrt{14}\sqrt{33}} = 114.75 = 2 \text{ rad}$$

Quiz General inner product: length / distance

① $x = \begin{bmatrix} 1 \\ -1 \\ 3 \end{bmatrix}$

$$\|x\| = \sqrt{\langle x, x \rangle} \Rightarrow \langle x, x \rangle = [1 \ -1 \ 3] \begin{bmatrix} 2 & 1 & 0 \\ 1 & 2 & -1 \\ 0 & -1 & 2 \end{bmatrix} \begin{bmatrix} 1 \\ -1 \\ 3 \end{bmatrix}$$

$$= [1 \ -4 \ 7] \begin{bmatrix} 1 \\ -1 \\ 3 \end{bmatrix}$$

$$= 1 + 4 + 21$$

$$= \sqrt{26}$$

---

② $x = \begin{bmatrix} 1/2 \\ -1 \\ -1/2 \end{bmatrix}$ $\quad y = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$

$$x - y = \begin{bmatrix} 1/2 \\ -2 \\ -1/2 \end{bmatrix}$$

$$\|x-y\|^2 = \langle x-y, x-y \rangle$$

$$= [1/2 \ -2 \ -1/2] \begin{bmatrix} 2 & 1 & 0 \\ 1 & 2 & -1 \\ 0 & -1 & 2 \end{bmatrix} \begin{bmatrix} 1/2 \\ -2 \\ -1/2 \end{bmatrix}$$

$$= [-1 \ -3 \ 1] \begin{bmatrix} 1/2 \\ -2 \\ -1/2 \end{bmatrix}$$

$$= -1/2 + 6 - 1/2$$

$$= 5$$

③ $x = \begin{bmatrix} -1 \\ 1 \end{bmatrix}$

$$\|x\| = \sqrt{\langle x, x \rangle}$$

$$\Rightarrow \langle x, x \rangle = \frac{1}{2} [-1, \ 1] \begin{bmatrix} 5 & -1 \\ -1 & 5 \end{bmatrix} \begin{bmatrix} -1 \\ 1 \end{bmatrix}$$

$$= \frac{1}{2} [-6, \ 6] \begin{bmatrix} -1 \\ 1 \end{bmatrix}$$

$$= 12/2 = 6$$

$$\|x\| = \sqrt{6}$$

---

④ $x = \begin{bmatrix} 4 \\ 2 \\ 1 \end{bmatrix}$ $\quad y = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$

$$x - y = \begin{bmatrix} 4 \\ 1 \\ 1 \end{bmatrix}$$

$$\|x - y\| = \sqrt{\langle x-y, x-y \rangle}$$

$$\langle x-y, x-y \rangle = [4 \quad 1 \quad 1] \begin{bmatrix} 2 & 1 & 0 \\ 1 & 2 & -1 \\ 0 & -1 & 2 \end{bmatrix} \begin{bmatrix} 4 \\ 1 \\ 1 \end{bmatrix}$$

$$= [9 \quad 5 \quad 1] \begin{bmatrix} 4 \\ 1 \\ 1 \end{bmatrix}$$

$$= 36 + 5 + 1$$

$$= 42$$

$$\Rightarrow \|x - y\| = \sqrt{42} = 6.48$$

---

⑤ $x = \begin{bmatrix} -1 \\ -1 \\ -1 \end{bmatrix}$ $\quad \|x\| = \sqrt{1+1+1}$

$$= \sqrt{3}$$

Quiz

① $x = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$ $y = \begin{bmatrix} -1 \\ 1 \end{bmatrix}$

$$\cos w = \frac{\langle x, y \rangle}{\|x\| \, \|y\|}$$

$$\langle x, y \rangle = x^T \begin{bmatrix} 2 & -1 \\ -1 & 4 \end{bmatrix} y = \begin{bmatrix} 1 & 1 \end{bmatrix} \begin{bmatrix} 2 & -1 \\ -1 & 4 \end{bmatrix} \begin{bmatrix} -1 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 3 \end{bmatrix} \begin{bmatrix} -1 \\ 1 \end{bmatrix} = 2$$

$$\|x\| = \sqrt{\langle x, x \rangle} \Rightarrow \begin{bmatrix} 1 & 1 \end{bmatrix} \begin{bmatrix} 2 & -1 \\ -1 & 4 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 3 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix} = 4 \Rightarrow 2$$

$$\|y\| = \sqrt{\langle y, y \rangle} = \begin{bmatrix} -1 & 1 \end{bmatrix} \begin{bmatrix} 2 & -1 \\ -1 & 4 \end{bmatrix} \begin{bmatrix} -1 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} -3 & 5 \end{bmatrix} \begin{bmatrix} -1 \\ 1 \end{bmatrix} = 9 \Rightarrow 3$$

$$\cos w = 1/3$$

$$w = \cos^{-1} 1/3 \approx 70^\circ$$

② $x = \begin{bmatrix} 0 \\ -1 \end{bmatrix}$ $y = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$

$$\cos w = \frac{\langle x, y \rangle}{\|x\| \, \|y\|}$$

$\langle x,y\rangle = \begin{bmatrix} 0 & -1 \end{bmatrix} \begin{bmatrix} 1 & -1/2 \\ -1/2 & 5 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix}$

$= \begin{bmatrix} 1/2 & -5 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix} = -4\frac{1}{2}$

$\langle x,x\rangle = \begin{bmatrix} 0 & -1 \end{bmatrix} \begin{bmatrix} 1 & -1/2 \\ -1/2 & 5 \end{bmatrix} \begin{bmatrix} 0 \\ -1 \end{bmatrix}$

$= \begin{bmatrix} 1/2 & -5 \end{bmatrix} \begin{bmatrix} 0 \\ -1 \end{bmatrix} = 5 \Rightarrow \|x\| = \sqrt{5}$

$\langle y,y\rangle = \begin{bmatrix} 1 & 1 \end{bmatrix} \begin{bmatrix} 1 & -1/2 \\ -1/2 & 5 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 1/2 & 4\frac{1}{2} \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix}$

$= 5 \Rightarrow \sqrt{5} = \|y\|$

$\cos \omega = \dfrac{-4\frac{1}{2}}{5} \qquad \omega = 154^\circ$

4) $x = \begin{bmatrix} 1 \\ 1 \end{bmatrix} \qquad y = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$

$\langle x,y\rangle = \begin{bmatrix} 1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 5 \end{bmatrix} \begin{bmatrix} 1 \\ -1 \end{bmatrix}$

$= \begin{bmatrix} 1 & 5 \end{bmatrix} \begin{bmatrix} 1 \\ -1 \end{bmatrix}$

$= -4$

$\langle x,x\rangle = \begin{bmatrix} 1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 5 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix}$

$= \begin{bmatrix} 1 & 5 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix}$

$= 6 \Rightarrow \|x\| = \sqrt{6}$

$\langle y,y\rangle = \begin{bmatrix} 1 & -1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 5 \end{bmatrix} \begin{bmatrix} 1 \\ -1 \end{bmatrix}$

$= \begin{bmatrix} 1 & -5 \end{bmatrix} \begin{bmatrix} 1 \\ -1 \end{bmatrix}$

$= 6 \qquad \|y\| = \sqrt{6}$

$\cos w = \dfrac{-4}{6} \Rightarrow w = 131.8^\circ$

(15) $x = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \quad y = \begin{bmatrix} 2 \\ -1 \\ 0 \end{bmatrix}$

$\langle x, y \rangle = [1 \; 1 \; 1] \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & -1 \\ 0 & -1 & 3 \end{bmatrix} \begin{bmatrix} 2 \\ -1 \\ 0 \end{bmatrix}$

$= [1 \; 1 \; 2] \begin{bmatrix} 2 \\ -1 \\ 0 \end{bmatrix} = 1$

$\langle x, x \rangle = [1 \; 1 \; 1] \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & -1 \\ 0 & -1 & 3 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$

$= [1 \; 1 \; 2] \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$

$= 1 + 1 + 2 = 4 \Rightarrow \|x\| = 2$

$\langle y, y \rangle = [2 \; -1 \; 0] \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & -1 \\ 0 & -1 & 3 \end{bmatrix} \begin{bmatrix} 2 \\ -1 \\ 0 \end{bmatrix}$

$= [2 \; -2 \; 1] \begin{bmatrix} 2 \\ -1 \\ 0 \end{bmatrix} = 4 + 2 = 6 \Rightarrow \|y\| = \sqrt{6}$

$\cos w = \dfrac{1}{2\sqrt{6}} \Rightarrow w = 78^\circ$

Week 3: Quiz 1

Q1 $\quad \pi_u(x) = \dfrac{\langle b, x \rangle}{\|b\|^2} b \qquad b = \begin{bmatrix} 1 \\ 2 \\ 2 \end{bmatrix}$

$\searrow \dfrac{bb^T}{\|b\|^2} x$

$\|b\|^2 = \langle b, b \rangle$

$[1 \; 2 \; 2] \begin{bmatrix} 1 \\ 2 \\ 2 \end{bmatrix} = 1 + 4 + 4 = 9$

$$bb^T = \begin{bmatrix}1\\2\\2\end{bmatrix}\begin{bmatrix}1 & 2 & 2\end{bmatrix} = \begin{bmatrix}1 & 2 & 2\\2 & 4 & 4\\2 & 4 & 4\end{bmatrix}$$

$$\text{matrix} = \frac{1}{9}\begin{bmatrix}1 & 2 & 2\\2 & 4 & 4\\2 & 4 & 4\end{bmatrix}$$

Q2.

$$\frac{1}{25}\begin{bmatrix}9 & 0 & 12\\0 & 0 & 0\\12 & 0 & 16\end{bmatrix}\begin{bmatrix}1\\1\\1\end{bmatrix}$$

$$= \frac{1}{25}\begin{bmatrix}9+12\\0\\12+16\end{bmatrix} = \frac{1}{25}\begin{bmatrix}21\\0\\28\end{bmatrix}$$

Q3:

$$\frac{1}{9}\begin{bmatrix}5\\10\\10\end{bmatrix} - \begin{bmatrix}1\\1\\1\end{bmatrix}$$

$$d = \begin{bmatrix}-4/5\\1/9\\1/9\end{bmatrix}$$

$$\Rightarrow \|d\| = \sqrt{\frac{16}{25} + \frac{1}{81} + \frac{1}{81}}$$

Week 4 Quiz

① $g(t) = \vec{x} = \begin{pmatrix}t\cos t\\t\sin t\end{pmatrix}$ $t \in \mathbb{R}$

$f(x) = \exp(x_1 x_2^2)$ $x = \begin{bmatrix}x_1\\x_2\end{bmatrix} \in \mathbb{R}^2$

$h = f(g(t))$

$$\frac{dh}{dt} = \frac{dh}{dg}\frac{dg}{dt}$$

f

$$\frac{dg}{dt} = \begin{bmatrix}\cos t - t\sin t\\ \sin t + t\cos t\end{bmatrix}$$

df

$$\frac{df}{dx} = \left[\frac{df}{dx_1} \quad \frac{df}{dx_2}\right]$$

$$= \left[x_2^2 \exp(x_1 x_2^2) \quad 2x_1 x_2 \exp(x_1 x_2^2)\right]$$

$f = \cos(t^2)$

$t = x^3$

$$\frac{df}{dx} = \frac{df}{dt}\frac{dt}{dx}$$

$$(2t) - \sin(t^2) \cdot 3x^2$$

$$-6x^5 \sin t^2$$