Violins and the Cold: A Deeper Look into Harmonic Structure

By Ethan Lee

Many musicians despise playing their instruments in unpreferable temperature conditions, often citing that either their instruments might get hurt, or that they will sound worse. While the former complaint is an undisputed point of contention for those looking to perform outdoors, let us explore the latter idea: do instruments truly give off a different sound in various weather conditions, or is the difference simply in the holder of the instrument playing with shivers or cold fingers? This brings up another thought – how does one alter the sound of his instrument? Outside of the human playing it differently, there are also ways to alter the instrument to make it sound different. For one, there are concert mutes, which are often used on brass and stringed instruments. These mutes can dampen the sound coming from the instruments, altering their timbre. This muting effect can also be achieved by simply holding the bridge of a stringed instrument. However, these alterations are limited in that they can only mute down sound, rather than opening it up. Let us explore here whether or not heat can alter the sound of a violin.

Figure 1. From left to right: Violin 1, Violin 2, Violin 3, Viola.

In this experiment, there were 4 instruments used to establish differences in heat. Among these were three violins and one viola. The cheapest violin, Violin 1, is a factory violin which was ordered from Amazon. It was ordered on June 16th, 2021 and was promptly delivered 3 days later. Requiring construction, the violin has a very bland and muted sound, clearly made in an inexpensive factory for violin making. Violin 2 is an inexpensive violin which was purchased in Violin Outlet Las Vegas. That being said, coming out of a violin shop it was still somewhat pricey, as violin shops ensure quality products to their customers. Therefore, the violin sounds much nicer under the ear, but it does not produce a significantly louder tone. Violin 3 is similar to Violin 2, as it was bought from the same shop. Albeit a little pricier, it is worth it due to its much louder tone compared to Violin 2. In regards to this violin, I have heard multiple professional violinists

Violin	From Where?	Price	Grant?
Violin 1	Amazon	<$50	Yes
Violin 2	Violin Outlet	$200	Yes
Violin 3	Violin Outlet	$400	No
Viola	Pawn Shop	$250-$750	Yes
Violin 4	Metzler Violin	>$10,000	No

Figure 2. Table of instruments used along with prices. Viola was bought from a pawn shop which sold for $250, but market value is $750. Some instruments were bought using a grant from Pomona College, and those are listed above as well.

mention that this violin sounds much better than its price. The viola is a used product, but it is a bit more expensive than all the other violins combined. It has a nice luscious sound, but it also will give off different harmonics than the violins simply because its body is bigger. It vibrates in different ways to create sound, and also has different strings.

However, simply describing how these violins sound does not quantify "better" sounding instruments (I am putting "better" in quotation marks here because it is hard to argue that one sound is objectively greater than another). The method that will be used here to quantify quality of sound is to look at the magnitude (which, for our purposes, is equivalent to amplitude) of each harmonic from a sound waveform. This was chosen because of Ohm's acoustic law, which states that a musical sound is perceived by the ear as a set of a number of constituent pure harmonic tones. The phases of the sine waves don't have any effect on the timbre, meaning that we only need to know the amplitudes of the component harmonics; we don't need to know anything about their phases. In this case, the G3 note (196 Hz) is used due to the abundant amount of data it provides, since there are more harmonics which are perceived by the ear. In order to obtain the magnitudes from each harmonic, the Fourier transform must be applied.

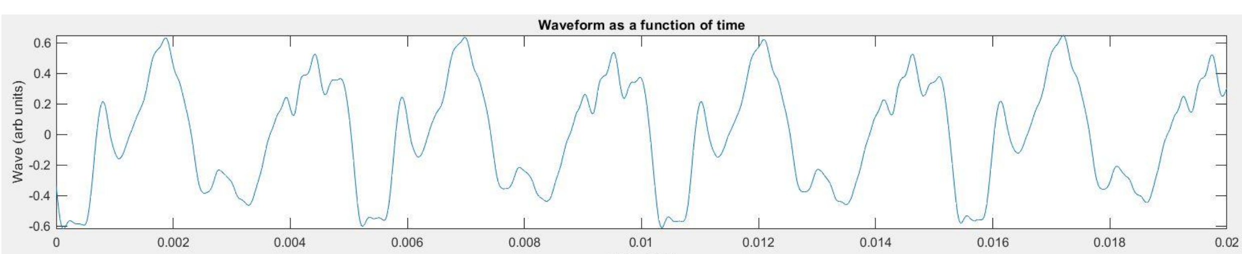


Figure 3. Waveform vs time graph from the first control trial of Violin 3. Is also referred to as f(t). Run in MATLAB code written by Alma Zook.

The Fourier transform follows the principle that a waveform vs time function is the sum of many different sin functions. Figure 3 exemplifies this idea, zooming into the waveform as a function of time to see that the graph has a period, implying the presence of some sine function. Therefore, The function f(t) can be written as a superposition of sine functions:

$$f(t) = \sum_{n=1}^{\infty} A_n \sin n\omega_1 t$$

Here, A = amplitude of the nth harmonic, n = 1,2,3..., ω_1 = frequency of the fundamental or n = 1 harmonic. $\omega_1 = 2\pi/T$ where T is the period of the sound wave.

Then, multiply each side by sin (nωt) {the reasoning behind this will be explained soon}:

$$f(t) \sin m\omega_1 t = \sum_{n=1}^{\infty} A_n (\sin n\omega_1 t)\,(\sin m\omega_1 t)$$

Take the integral with respect to time of each side:

$$\int_0^{T=2\pi/\omega_1} f(t) \sin m\omega_1 t \; dt = \int_0^{T=2\pi/\omega_1} \sum_{n=1}^{\infty} A_n (\sin n\omega_1 t)\,(\sin m\omega_1 t) \; dt$$

Integration is regarded as a process of addition; summation and integration are both fancy forms of addition. Due to the commutative property of addition, the integration and summation processes may be swapped. The amplitude A is also taken out of the integral since it is simply a constant.

$$\int_0^{T=2\pi/\omega_1} f(t) \sin m\omega_1 t \; dt = \sum_{n=1}^{\infty} \int_0^{T=2\pi/\omega_1} A_n (\sin n\omega_1 t)(\sin m\omega_1 t)\, dt$$

The reason why it was important to multiply each side by the sine function was because the total sine function becomes an orthogonal function. An orthogonal function is defined by the property given by the expression below:

$$\int_0^{2\pi} A_n (\sin n\omega_1 t)(\sin m\omega_1 t)\, dt = \begin{cases} 0, & n \neq m \\ \pi, & n = m \end{cases}$$

After calculating out integral function, we end up with:

$$\int_{0}^{T=2\pi/\omega_1} f(t)\sin m\omega_1 t \; dt = \pi \sum_{n=1}^{\infty} A_n \delta_{nm} \; where \; \delta_{nm} = \begin{cases} 0, & n \neq m \\ 1, & n = m \end{cases}$$

There is only one value of n where n is equal to m, so the summation essentially disappears, replacing n with m. Now, with an easily calculable equation, we can solve for the amplitude:

$$A_n = \frac{2}{T} \int_{0}^{T=2\pi/\omega_1} f(t)\sin m\omega_1 t \; dt$$

And so this value would become the magnitude of a peak in Figure 4.

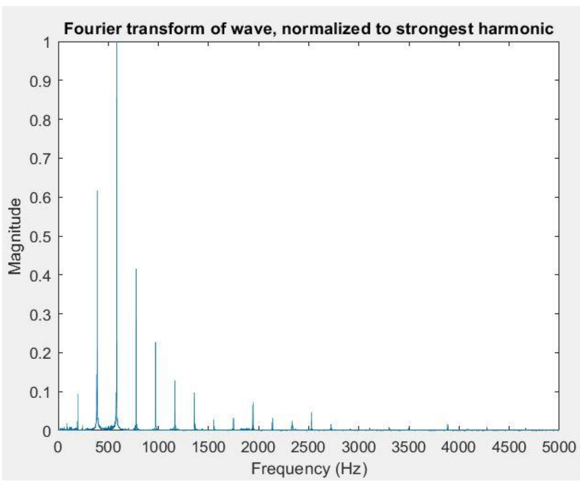


Figure 4. Graph of the fourier transformation of the waveform vs time graph of Violin 1's third control trial. Was fourier transformed using the FFT (Fast Fourier Transform). Run in MATLAB written by Alma Zook.

In order to quantify the quality of sound, let us use the magnitude of each harmonic calculated using the Fourier transform. However, in this case, the Fast Fourier Transform (FFT) is used. While it is faster, it can only be used if data is taken in even intervals (in this case, it is trivial so the FFT will always work).

I have still left the quality of sound relative to the magnitude of each harmonic undefined; there is no harmonic structure which is simply perfect. However, to gauge what an undeniably better harmonic structure looks like, we can look into the structures of a significantly more expensive violin, Violin 4 as referenced in Figure 2.

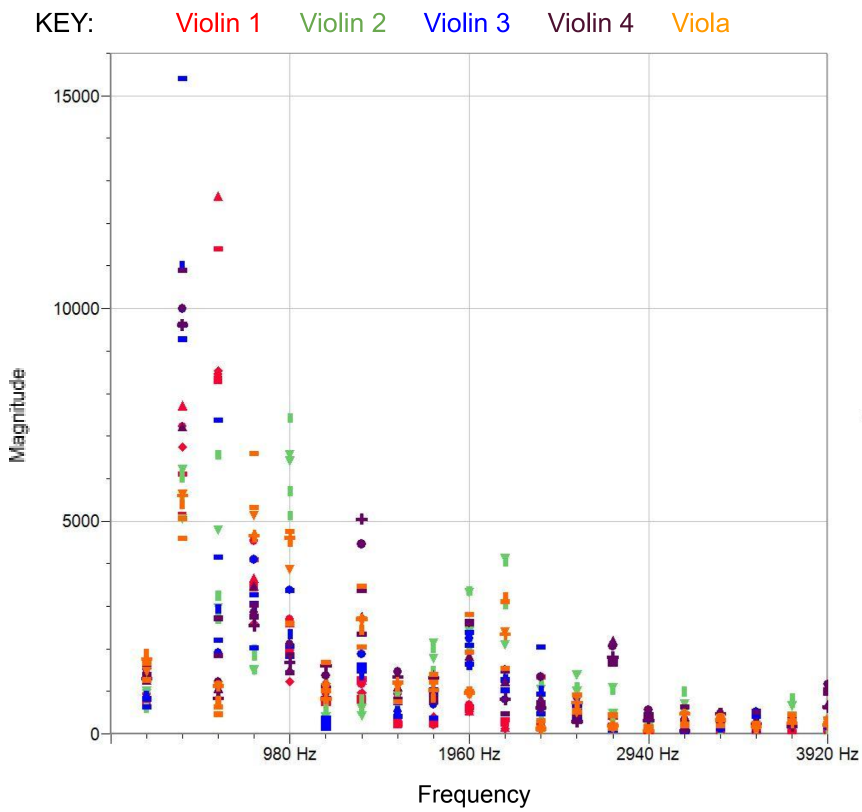


Figure 5. Graph of magnitudes of harmonics for all instruments used in this experiment. A higher magnitude equates to a stronger harmonic. For Violin 1, the third harmonic seems to be strongest relative to the others. For Violin 2, the fifth harmonic seems to be strongest relative to the others. For Violin 3, the second harmonic seems to be strongest relative to the others. For Violin 4, the seventh, thirteenth, and fifteenth harmonics seem to be strongest relative to the others. The fourth harmonic on the viola seems to be strongest relative to the others. Obtained using MATLAB code by Alma Zook and Logger Pro.

As can be seen in Figure 5, each instrument seems to have at least one harmonic in which its magnitude outshines the others. The important instruments to highlight here are the viola and the expensive violin, Violin 4. The viola is fascinating because it will naturally have a different sound as mentioned before. As can be seen in Figure 5, some harmonic magnitudes seem to separate from the rest of the set, most notably the second and third harmonics looking like low outliers while the first and fourth are higher than the rest. However, no quantitative measurements will be made here since it is not the focus of this experiment. The other instrument's harmonic spectrum that I want to discuss is that of Violin 4. While the magnitudes of the lower frequency harmonics are average at best, the violin truly shines due to its high magnitudes in the higher frequency harmonics. All four other instruments each have harmonics whose magnitudes are clearly above the rest, the difference being that the frequencies of these harmonics are all below 1000 Hz. Since there seems to be such a disparity between these instruments, it is easy to hypothesize that a higher relative magnitude within some higher frequency harmonics can potentially predict a "better" sound. Throughout the rest of this paper, this will be my metric in deciding if the quality of a sound increased or decreased in changing temperatures.

Next, it is important to outline the procedures used to obtain the sorts of data that can be seen in Figures 2, 3, and 4. Each sound clip that was inputted into the program was a two second clip of bowed violin playing of the G string. This recording was done in my house with a FIFINE K678 microphone connected to a laptop running audacity. Since harmonics do not scale evenly with sound intensity, it is important to regulate how loudly I play. I achieved this by placing tapes where I stand and where the microphone is to regulate how far away I am from the microphone, then using an iPhone app, Decibel X, I was able to see how loudly I am playing. In this experiment, I attempted to play the instrument at around 70 decibels for 3+ seconds for each trial, doing 5 or more trials for each instrument. Then, I can move to Audacity and cut each audio file into 2 second snippets so I can plug it into MATLAB. The environment's background noise generally hovered around 30 db, and the recordings were done at night in an open space on wood flooring.

Figure 6. Image of Violin 2 chilling in a freezer. This is also the same location where violin bows were chilled. The violin is being covered in a paper towel because some moisture from the air in this freezer makes the violin damp, and the experiment might be affected by this moisture.

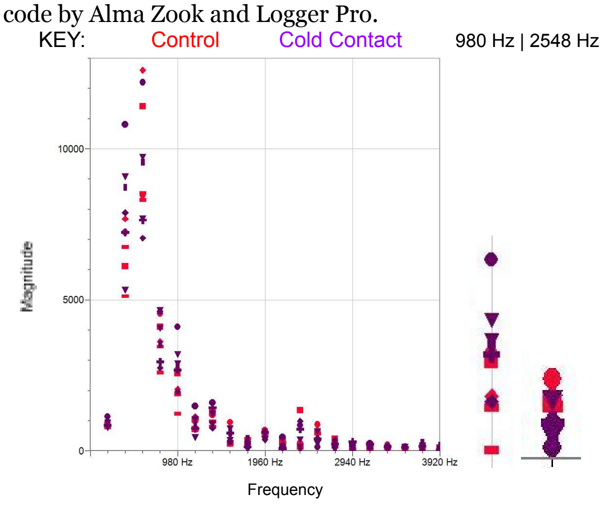
In this experiment, I am measuring if temperature changes will affect how the instrument sounds. Therefore, I need some way of making it warmer and colder. In creating a cold environment, there were two different methods: chilling the violin bow, and chilling the violin itself. Both were chilled to 255.372 K (0 F) in a freezer as shown in Figure 6, then were taken out quickly to be played; in the case of the violin itself, I had to take some time in between to tune. The reason why I am chilling the bow is to attempt to create a colder contact point between the violin string and the bow itself, as it might have some potential effect on the sound. In chilling the violin itself the goal is to chill the violin wood as to attempt to affect the vibrations of the violin body. This same effect was attempted with heat, but it was not as effective as by the time this experiment took place the outside weather in Las Vegas cooled down to around 311 K (100 F), which did not seem to provide a significant amount of heat. That being said, there are still some trials with a violin bow which was exposed to heat.

Before I dive into results, I want to mention my method used to calculate how significant my values are. I am using a two sample t-test for the difference of means to determine whether or not one sample set's magnitude is significantly greater than another. In this experiment, I will be using an α value of .05 to determine whether or not my results are significant; if the probability that the two sets are of the same set is less than .05, then that would confirm that there was a significant difference between the two sets of data.

Instrument	Conditions	Date(s) (2021)
Violin 1	Control	8/10
Violin 1	Cold Bow	8/4, 8/6
Violin 1	Cold Violin	8/18, 8/19
Violin 2	Control	8/10
Violin 2	Cold Bow	8/4, 8/6
Violin 2	Cold Violin	***
Violin 3	Control	8/3, 8/5
Violin 3	Hot Bow	8/13, 8/15, 8/16
Violin 4	Control	8/3, 8/5
Viola	Control	8/10
Viola	Cold Bow	8/4, 8/6

Figure 7. Table of dates of each recording session; all recording sessions were indoors in a controlled environment.

Figure 8. Graph of magnitudes of harmonics for Violin 1 control trial vs. cold contact trial. On the right, zoom in on the 5th and 13th harmonics. Obtained using MATLAB code by Alma Zook and Logger Pro.



It might seem pretty trivial that some heat might affect the violin's sound, but this experiment would suggest otherwise. While some harmonics remained of roughly the same magnitude, there were also some harmonics which were significantly stronger or weaker in magnitude than the control harmonics. Let us begin by taking a look at what cooling down the point of contact can do.

Violin 1, Violin 2, and the viola were used to test whether a cold point of contact affects harmonics on the violin. Let us first take a look at Violin 1. In this experiment, its harmonic structure did not change much. However, there were a couple significant changes in the 5th harmonic and the 13th harmonic. In Figure 8, it appears that the cold bow's magnitude is slightly larger than the control's magnitude on the 5th harmonic at 980 Hz. Using the t-test for difference of means, I calculated a t value of 2.24, and the probability that my t value is greater than 2.24 is .044 [$p(t > 2.24) = .0441$]; therefore, there is a 4.4% chance that the magnitude of the trials with a cold contact point is not greater than that of the control trials. Since .044 is less than our α value of .05, it is likely that the magnitude is higher for the trials with the cold contact point. There also lies a significant difference in the 13th harmonic at 2548 Hz as can be seen in this t-test (this is the form in which I will be presenting the t-test in the future):

$$t_{13} = -2.49$$
$$p(t < -2.49) = .0340$$

In this case, the control trials have a higher magnitude at 2548 Hz. Following the earlier logic that higher magnitude of higher frequency harmonics equates to better sound and vice versa, I could argue here that the quality of sound slightly went down; however these are only 2 data points so it is not very strong evidence. Violin 2 yielded quite different yet interesting results. Overall, there were a lot more significant changes, but there was not much of a pattern to these changes. Here are some t-tests of significance that I had performed on harmonics at 392 Hz, 1176 Hz, 1764 Hz, 2156 Hz, 2548 Hz, and 2940 Hz respectively:

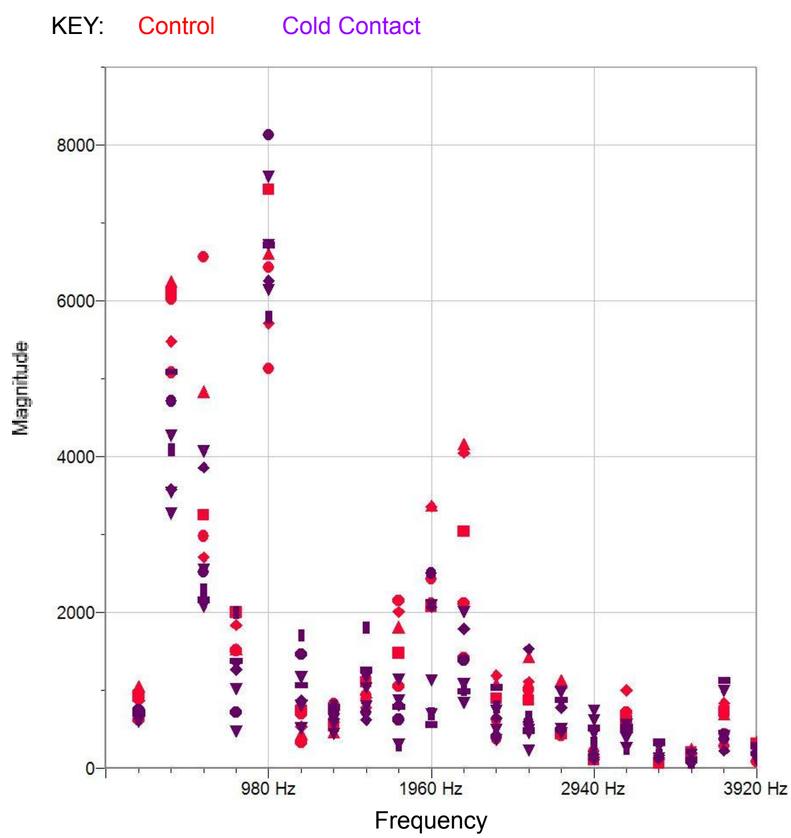


Figure 9. Graph of magnitudes of harmonics for Violin 2 control trial vs. cold contact trial. Obtained using MATLAB code by Alma Zook and Logger Pro.

$t_2 = -5.14$
$p(t < -5.14) = .00339$

$t_9 = -4.38$
$p(t < -4.38) = .00595$

$t_{11} = -2.95$
$p(t < -2.95) = .0210$

$t_{13} = -2.45$
$p(t < -2.45) = .0352$

$t_6 = 3.18$
$p(t > 3.18) = .0168$

$t_{15} = 2.88$
$p(t > 2.88) = .0225$

As can be seen, the control trial's magnitudes are higher on the 2nd, 9th, 11th, and 13th harmonics, while the trials with a cold point of contact had higher magnitudes on the 6th and 15th harmonics; all of the p values are exceeded by the α value of .05. While there are significant differences between the two sounds, it is hard to quantify which sound would be considered "better," as the differences are spread around the different harmonics; there is no certain pattern one which way. However, in the higher frequency harmonics, the control sample often had higher magnitudes than the cold contact samples, with the 15th harmonic being an exception, so there is the possibility that the 15th harmonics is an outlier for some unknown reason which can be speculated later. Lastly, let us move to the viola. The viola has some interesting harmonics similar to Violin 2;

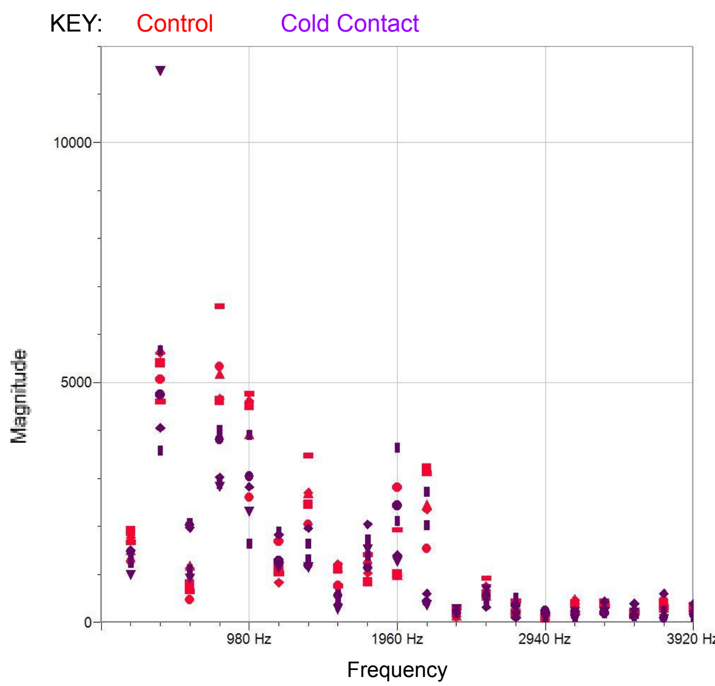


Figure 10. Graph of magnitudes of harmonics for Viola control trial vs. cold contact trial. Obtained using MATLAB code by Alma Zook and Logger Pro.

note that there is also an outlier at around a magnitude of 11500 on the second harmonic at a frequency of 392 Hz. Because of it, I am just ignoring the second harmonic since it could skew my data. With that being said, let us take a look at some significant differences between the two data sets:

$t_4 = -4.52$
$p(t < -4.52) = .00535$

$t_7 = -4.33$
$p(t < -4.33) = .00620$

$t_8 = -4.63$
$p(t < -4.63) = .00490$

$t_{11} = -2.16$
$p(t < -2.16) = .0484$

$t_3 = 2.78$
$p(t > 2.78) = .0248$

$t_9 = 2.40$
$p(t > 2.40) = .0373$

Again, all of these p values are less than the α value of .05. The viola seems to have yielded similar results to Violin 2 in that most of the harmonics would suggest that the sound got worse due to the third increasing in magnitude and the seventh, eighth, and eleventh harmonics decreasing in magnitude. However, the fourth harmonic decreased, and the ninth harmonic increased, so again there is not conclusive evidence as to whether the sound necessarily got worse, but there is more evidence suggesting so.

The Violin 3 was used to test what effects a warmer point of contact can have on a violin's sound. This was the only violin used because the weather in Las Vegas created difficulty in heating up the bow to a sufficient temperature. For one, the monsoon season caused a lot of rainy days in Las Vegas, so I could not place a violin bow outside without it getting soaked. On top of that, by the time this experiment was being performed, Las Vegas temperatures were reaching around 310 K (100 F) midday; I believe that these temperatures are not a significant enough difference between the two samples. However, there was some quite significant data to prove me wrong:

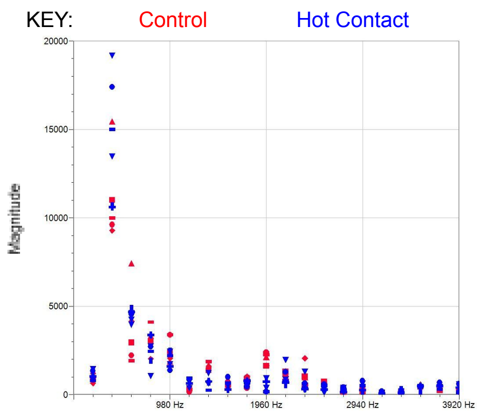


Figure 11. Graph of magnitudes of harmonics for Violin 3 control trial vs. hot contact trial. Obtained using MATLAB code by Alma Zook and Logger Pro.

$$t_7 = -5.35$$
$$p(t < -5.35) = .00535$$

$$t_{10} = -6.62$$
$$p(t < -6.62) = .00535$$

As can be seen by above, there are only two points of significant data with a p value less than the α value of .05. Both harmonics, being of high frequency, have a higher magnitude as the control sample compared to the sample with a hot contact point. This suggests that perhaps the control set sounds better than the set with a hot contact point.

Lastly, let us examine the harmonic structures of violins which have a cold body; these trials were conducted on Violin 1 and Violin 2.

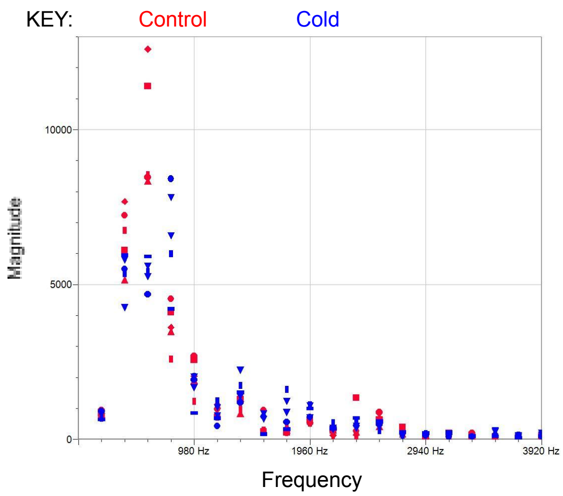


Figure 12. Graph of magnitudes of harmonics for Violin 1 control trial vs. cold body trial. Obtained using MATLAB code by Alma Zook and Logger Pro

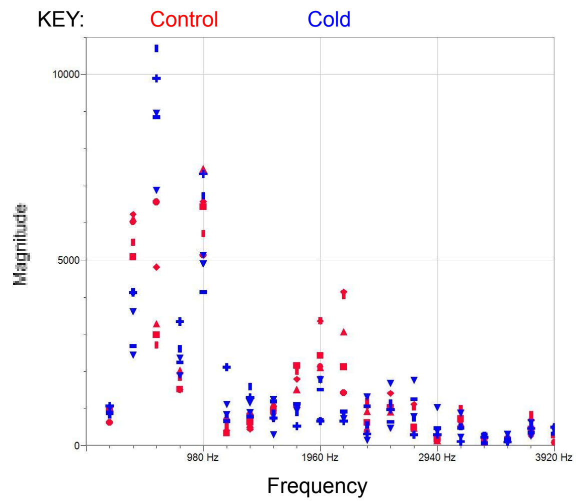


Figure 13. Graph of magnitudes of harmonics for Violin 2 control trial vs.cold body trial. Obtained using MATLAB code by Alma Zook and Logger Pro

Both violins come up with stunning results, as shown here:

Violin 1:

$$t_2 = -2.23$$
$$p(t < -2.23) = .0448$$

$$t_3 = -4.87$$

Violin 2:

$$t_2 = -5.69$$
$$p(t < -5.69) = .00236$$

$$t_3 = 5.19$$

$p(t < -4.87) = .00412$

$t_4 = 3.66$
$p(t > 3.66) = .0108$

$t_9 = 2.70$
$p(t > 2.70) = .0272$

$p(t > 5.19) = .00327$

$t_9 = -3.55$
$p(t < -3.55) = .0118$

$t_{10} = -3.60$
$p(t < -3.60) = .0113$

$t_{11} = -4.01$
$p(t < -4.01) = .00801$

Although both violins have significant results, each of the instruments suggest a different idea. Violin 1 suggested that a cold body produced better sound, as can be seen through the first couple of harmonics having higher magnitudes in the control trials while the higher frequency harmonics have higher magnitudes in the cold trials. Meanwhile, Violin 2 suggests just the opposite, displaying lower magnitudes at higher frequencies by the cold violin.

What does this all mean? For one, there was certainly a measurable change between harmonic structures based on the heat in parts of the violin. It seems that overall a change in temperature worsens the sound coming out of the violin, with exceptions. The one exception being the Violin 1 brings up many questions: were enough trials conducted? Could this be a fluke? Is Violin 1 a legitimate violin? I would argue that my results can support the idea that Violin 1 behaves much differently than the other instruments used in this experiment. For starters, Violin 1 did not have a great harmonic structure to begin with, so a lot of the harmonics simply didn't change due to the magnitudes at each frequency being so low past 2000 Hz. Violin 1 also experienced less change than the other instruments in similar trials, while also almost refuting what the data from the other instruments would suggest. However, this might not have to do with the violin itself; it is a possibility that the violin player (me) was inconsistent. While I attempted to maintain all sound to 70 db, there is likely some variability between trials which is why I included 5 or more trials of each test. In order to create a more reliable study, more trials would be needed and there would need to be a way to figure out how to maintain a steady tone on the violin (preferably with robots, who do not have the tendencies of a human to make arbitrary changes in tone). However, Violin 2 and the viola behaved similarly, so there was likely not too much difference between the playing itself and likely had more to do with the instrument. There was also an issue that popped up where a harmonic at 98 Hz would pop up as shown in Figure 14. The only explanation I can think of is that there was a piano in sight, and that since the piano's G2 does not have a dampener, it rang when the G3 note was played. However, this should not have much of an impact on the data itself, as the rest of the strings are dampened.

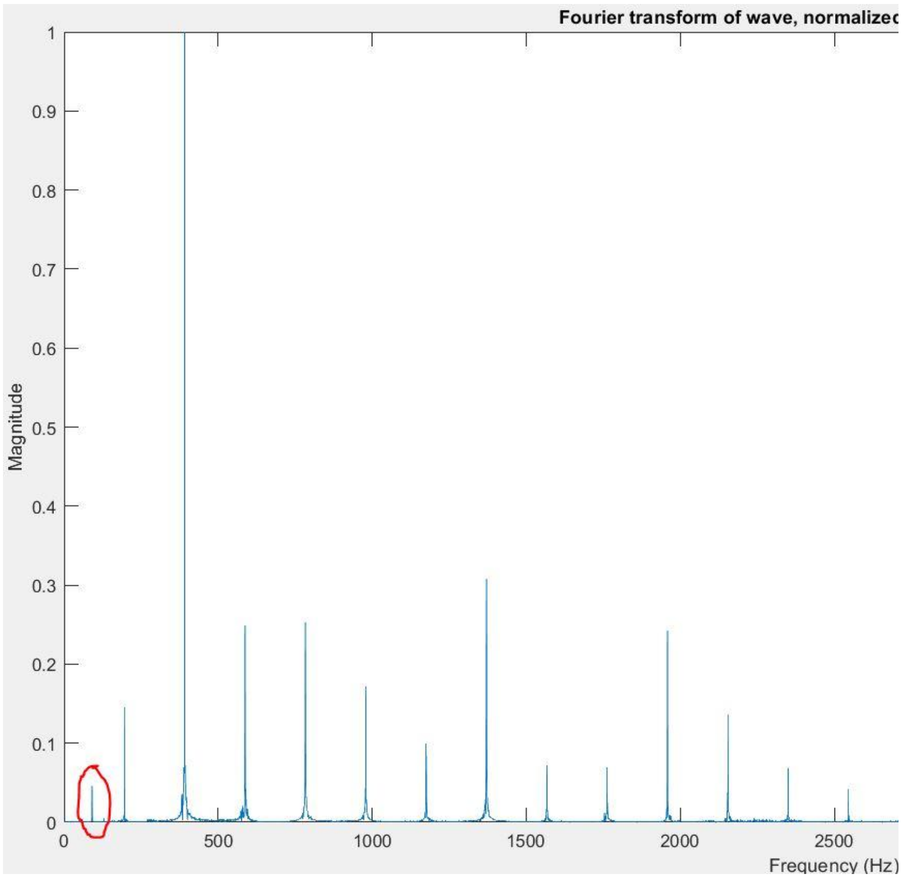


Figure 14. Graph of the fourier transformation of the waveform vs time graph of Violin 4's first control trial. Was fourier transformed using the FFT (Fast Fourier Transform). Run in MATLAB written by Alma Zook.

There might be different ways to interpret harmonic structure as well. In fact, this study in determining a "good" and "bad" sound could be incorrect in believing that each harmonic directly affects the sound. For example, what if the 14th harmonic actually had an inverse effect to what was suggested? If a higher magnitude at 2744 Hz catastrophically destroys the sound coming from the violin, then the argument for a "good" vs "bad" sound is falsified. However, an important takeaway is that I have created new data on how heat can affect the harmonic structure of a violin.

This is just one step of many to increase audio quality for outdoor performers. What if it was in fact beneficial to refrigerate an instrument to obtain a different sound in a performance? What if we can use this data and future data to create instruments to adapt to various weather conditions? All we know for sure is that there is certainly a difference made by temperature conditions that can not be overlooked.