Alabama State Redistricting Project
Team Name: Roll Tide
Team Members: Alan Meenan, Banner Penwell, Ethan Frazier

Project Overview

We are Team Roll Tide and are tasked to help create and evaluate a new redistricting plan for the state of Alabama to reflect the latest demographic changes. Alabama currently has 7 districts that have laws and regulations the team needed to keep in mind when formulating a new redistricting plan. According to information found on ncsl.org, a summary of the laws/criteria for the state of Alabama include:

1. Population Equality: Each district should have close to the same population.
2. Racial and Language Minority Protections: Depending on the population percentage of minorities, there should be a district that aims to group these minorities together.
3. Compactness and Contiguity: Each district must be connected and touching in every part of the district. It is preferred that the districts are in a compact shape.
4. Political Subdivisions and Communities of Interest: Try not to draw such congressional districts to favor a certain party or discriminate against another.
5. Consider Existing Districts: It is preferred that you keep the current districts in mind when creating new district plans.

We aim to create a feasible congressional districting plan that abides by all the criteria listed above by creating and solving an integer program to find an optimal solution to this problem.

Integer Program Model

With these given requirements our team set out to create an Integer Program model that would satisfy the many constraints and criteria the state of Alabama has. After researching which model we wanted to use and trying different things, we decided on the Hess's Model which is based off of Moment of Inertia and can be seen below in computation and the corresponding description beside it.

Variables:

$$x_{ij} \begin{Bmatrix} 1 \; if \; county \; i \; is \; assigned \; to \; a \; district \; centered \; at \; county \; j \\ 0 \; otherwise \end{Bmatrix}$$

Parameters:

L is the lower bound for the population of the district

U is the upper bound for the population of the district

w_{ij} is the penalty for the moment-of-inertia = $p_i d_{ij}^2$ where p_i is the population of county i and d_{ij} is the distance for county i to district center county j

E is all set of edges for each county i that touches a county j

V is a set of all counties in Alabama

f_{ij}^{v} = the amount of flow, originating at district center v, that is sent across edge {i, j} (from i to j)

M = number of counties – number of districts + 1

Objective:

min=$\sum_{i \in V} * \sum_{j \in V} wijxij$		Minimize the moment of inertia Distance^2 * population * district for district i assigned to the center of the district tract j

subject to:

$\sum_{j \in V} xij = 1$	$\forall i \in V$	each county i is assigned to one district centered at county j
$\sum_{j \in V} xjj = k$		k districts are chosen
L*$x_{jj} \leq \sum_{i \in V} p_i x_{ij} \leq$ U*x_{jj}	$\forall j \in V$	each district falls between the lower bound L and the upper bound U
$\sum_{u \in N(i)}(f_{ui}^{j} - f_{iu}^{j})$ = xij	$\forall i \in$ V \ {j}, $\forall j \in$ V	if vertex i is assigned to center j, then i consumes one unit of flow; otherwise, it does not
$\sum_{u \in N(i)} f_{ui}^{j} \leq$ (M-1)* x_{ij}	$\forall i \in$ V \ {j}, $\forall j \in$ V	i can receive flow from top j only if i is assigned to center j
$\sum_{u \in N(i)} f_{ui}^{j}$ = 0	$\forall j \in$ V	j cannot receive flow of its own type
$f_{ij}^{v}, f_{ji}^{v} \geq 0$	$\forall \{i, j\} \in E, \forall v \in V$	cannot send negative flow
$x_{ij} \leq x_{jj}$	$\forall i, j \in V$	county i cannot be centered at county j if county j is not a center
$x_{ij} \in \{0,1\}$	$\forall i, j \in V.$	x is binary

<u>Experiments</u>

Many different codes were written and executed during the creation of this districting plan. All codes and experiments were run on a laptop with a processor speed of 2.6 GHz and 32 GB of RAM. To execute these codes, we used Python running Gurobi Optimizer version 10.0.0.

However, we have run into many computational errors. The model that we found success with was a Hess's Moment of Inertia model working on the county level. Doing this we were able to get an optimal value of 6593085384.64 people * miles^2 in a run time of 155 seconds. We found this with a 0.00% MIP gap. However, in this model having a single black majority district was infeasible. We originally wanted to add a second black majority district but had to remove this constraint for the purposes of this project. We were saddened by this but computations on the tract level were not manageable with the equipment we had at our disposal. We attempted a Min-Weighted-Cut-Edges model on the tract and then county level and could not get a result in a timely manner. We attempted the Min-Weighted-Cut-Edges model adding a constraint saying only the Mobile and Jefferson County could be broken up with no success due to run time. We also tried a Moment-of-Inertia model on the tract level and with all the calculations of the distances, we ran out of RAM. With all this in mind, we had to cut constraints and started looking on a county level instead. This is how we concluded the Hess's Moment-of-Inertia model on the county level was the best for our circumstance due to the fact that it was the only model that gave a timely solution. The final code used for this project and the other attempts are included in the Git Repository listed below.
https://github.com/ethfraz/Alabama_Redistricting

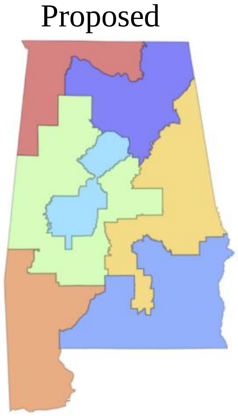
Evaluation

We have met many limitations when making our districting plan. With the number of tracts and districts in Alabama, our laptops could not handle the calculations in a timely manner. With what we were able to calculate we were able to follow the following criteria.

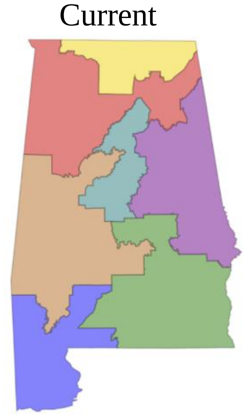
1. Population Equality: 1% district population deviation
2. Compactness and Contiguity: All districts are contiguous and compact

We originally wanted to strive to have two black-majority districts. However, we hit a roadblock with this the hardware we had at our disposal. We know that with better equipment we could run the other codes we created to find a better solution that satisfies every criterion we stated earlier. Our proposed districting plan came with the limits of computation and thus failed to meet all listed criterion. Below is our proposed plan next to the current plan they have in place.

Proposed



Current



Conclusion

To reiterate our problem statement, we aimed to create a feasible congressional districting plan that abides by all the criteria state and federal legislature has set by creating and solving an integer program to find an optimal solution to this problem. This being said, we were not able to meet all of the criteria, and we produced a solution that was feasible to us and our equipment. We recommend keeping the current district plan for the time being. However, we do not believe that this is the best solution overall. We believe it will take a deep tract level look to be able to come up with a districting plan that meets all the criteria while adding a second black majority district. We did not have the computational power to do this, but we know that this is the best solution.