Chapter 2
Logic Operation in Spiking Neural P System with Chain Structure

Jing Luan and Xi-yu Liu

Abstract In this paper, a new P system called spiking neural P system with chain structure (SNPC, for short) has been proposed, which combines spiking neural P system (SNP, for short) with discrete Morse theory, that is to say, neural membrane cells in spiking neural P system are set on chain by discrete gradient vector path, building a SNP system with chain structure. Compared with original SNP system, the structural design of SNPC system is simpler, showing stronger parallelism, and avoiding the time-consuming phenomenon caused by the random selection of membranes in computational process of P system. The logic operation in SNPC system has been completed, compared with the implemented method in traditional P system, the efficiency of the algorithm significantly improved, showing the advantage of SNPC system.

Keywords Membrane computing · Spiking neural P system · Discrete Morse theory · Logic operation

2.1 Introduction

Natural computing is a field trying to simulate nature in the process of calculation. Membrane computing is a new branch of natural computing, which focuses natural computing at the cellular level and belongs to a kind of molecular computing.

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It regards the whole membrane as a computing unit, abstracting the chemical reactions and material flow at the cellular level as the calculating process [1]. This computing model was proposed by Paun, Romanian Academy of Sciences, in 1998, due to its maximum parallelism and well distributed manner, showing strong computational completeness and efficiency, and has aroused widespread concern and research.

According to different organizations of cells which inspire the different computing models, P systems can be classified to three main types as follows: (i) Cell-like P systems inspired from living cells; (ii) Tissue-like P systems inspired from tissues; (iii) Neural-like P systems inspired from neural systems. In this paper, we will use a class of neural-like P systems which is called spiking neural P systems (SNP systems, for short). Inspired from the biological phenomenon that neurons cooperate in the brain by exchanging spikes via synapses, Păun et al. developed the SNP systems in 2006 [4]. Similar to neural-like P systems, these systems have the network structure.

SNP system on chain is a new membrane structure, not network structure but chain structure, linking traditional membrane according to certain rules and forming a chain structure. The chain structure makes the process of membrane computing not to select next reacting membrane at random, but to react in accordance with the order of the membrane in the chain in successive. Avoid the time-consuming phenomenon resulted from randomness greatly and improve the computational efficiency of the SNP system. In this paper, based on the discrete Morse theory forming the SNP system with chain structure(SNPC, for short), the discrete gradient vector path is the standard to link the membranes, that is to say, along the discrete gradient vector path of the discrete Morse theory to form membrane structure. Morse theory had been proposed for a long time, and it is a useful tool in differential topology, used for investigating the topology of smooth manifolds, particularly for computer graphics, being the focus of the study. In the end of last century, Forman extended several aspects of the fundamental tool to discrete structures, providing an effective tool to describe the topology of the discrete objects, and made a big contribute to theory and applied mathematics. Its combinatorial aspect allows computation completely independent of a geometric realization, greatly enriching its range of applications. The literature [2] has proved the possibility of solving optimization problems with discrete Morse theory, showing the computational efficiency and power. The SNPC system, which combines SNP system with discrete Morse theory, enhances the computational power and efficiency of membrane system.

The maximum parallelism of P system makes the fulfillment of logic operation possible, and reduces the computational complexity greatly. Literature [3] has proved the possibility of performing the logic operation of the P system, and provided effective method to complete the logic operation, which is easier compared with the implementation in the general computer architecture. In this paper, we propose a new method with SNPC system to fulfill the basic logic operation (AND、OR、NOT), the efficiency of the algorithm being improved.

In the paper, in the Sect. 2.2, SNP system is introduced briefly. Then the improved SNP system, that is to say, the SNP system with chain structure and skin membrane is given in the Sect. 2.3, and the formal presentation and rules are described. In the Sect. 2.4, the method with SNPC system to solve the basic logic operations (AND, OR, NOT) is given. And then the summary and outlook of the SNPC system are introduced.

2.2 Spiking Neural P System

2.2.1 Preliminary of SNP

SNP system is a new computing device in the field of membrane computing. This new P system is proposed in 2006 by Ionescu et al. for the first time, which simulates the biological phenomena of collaborative of the neurons through synapses and processing spike [4].

Unlike the P system proposed before, SNP system has its own unique composition and operation mode: its basic calculating unit is neuron, and is indicated by the nodes in the directed graph commonly, the arc between nodes indicating synaptic. There is a certain number of a in each neuron, which represents spike. The rules in the SNP system are mainly two, that is to say, firing rules and forgotten rules. The firing rules make neuron send information to other neurons with spikes, and then other neurons can receive these spikes after a certain period of time. The forgotten rules are used to eliminate a certain number of spikes in the neurons.

There are two main calculation modes in SNP system, generating and accepting modes. In the generating mode, the membrane system is regarded as a generator, due to not need to input characters to the system. For the accepting mode, in contrast to the generating mode, the SNP system can omit the output neurons, and the data read from the environment through the input neurons [5].

2.2.2 The Structure of SNP

In fact, SNP system is a special kind of Tissue-like and Neural-like P system, which can be obtained by simulating the function of spike neurons. There are some common definitions and symbols of SNP system. For the alphabet V, V^* is the collection of all finite strings on the V, where λ denotes the empty string. V^+ denote all non-empty finite string of V. If $V = \{a\}$ (here call V single-letter collection), $\{a\}^*$, $\{a\}^+$ are respectively abbreviated as a^*, a^+.

The regular expression over the alphabet V is defined as followed:

(1) λ and $a \in V$ are regular expression;
(2) if E_1 and E_2 are regular expressions over V, then $E_1 + E_2$, E_1E_2 and E_1^* are regular expressions over V;

(3) nothing else is a regular expression over V.

A language L over V is a set of strings over V, with each regular expression E, we associate a language L(E), defined as followed:

(1) $L(\lambda) = \{\lambda\}$, and for each $a \in V$, $L(a) = \{a\}$;
(2) for any expression E_1, E_2 over V, $L(E_1 \cup E_2) = L(E_1) \cup L(E_2)$, $L(E_1 E_2) = L(E_1) L(E_2)$, $L(E_1^+) = L(E_1)^+$.
   A spiking neural P system of degree $m \geq 1$ is a construct of the form $\Pi = (O, \sigma_1, \ldots, \sigma_m, syn, in, out)$, where

   (i) $O = \{a\}$is the singleton alphabet (a is called spike);
   (ii) $\sigma_1, \ldots, \sigma_m$ are neurons, of the form $\sigma_i = (n_i, R_i)$, $1 \leq i \leq m$, where:

(1) $n_i \geq 0$ is the initial number of spikes contained by the neuron σ_i;
(2) R_i is a finite set of rules of the following two forms:
(i) firing rules: $E/a^c \rightarrow a^p; d$, $c \geq 1$, $p \geq 1$, $d \geq 0$, where E is a regular expression over a, d is the delay time, that is, the interval between using the rule and releasing the spike, and $c \geq p$;
(ii) forgetting rules: $E'/a^s \rightarrow \lambda$, $s \geq 1$, where E' is a regular expression over a, with the restriction that, for Ri in type i) each rule $E/a^c \rightarrow a^p; d$, satisfies $L(E) \cap L(E') = \varphi$;
(3) syn $\{1, 2, \ldots, m\} \times \{1, 2, \ldots, m\}$indicates the synapses between neurons, for any $1 \leq i \leq m$, there is $(i, i) \notin syn$;
(4) in, out $\in \{1, 2, \ldots, m\}$indicate that the input and output neuron.

2.3 Spiking Neural P Chain System with Skin

P chain system is a new model of P system, which constructs membrane on the chain structure by some rules, getting P system with chain structure. Its outstanding advantage is that the communication between membranes is not at random, but in accordance with the order of the membrane in the chain in successive, successfully avoiding the time-consuming phenomenon caused by the random. In this paper, the rules of constructing the membrane structure we choose are combined with the discrete Morse theory, forming the membrane system over the discrete gradient vector path, that is to say, each cell on the discrete gradient vector path represents a membrane, and part or all of the path are considered as a membrane system. The discrete gradient vector path is a path satisfied:

(1) simplices $a_0^{(p)}, \beta_0^{(P+1)}, a_1^{(p)}, \beta_1^{(P+1)}, \ldots, \beta_r^{(P+1)}, a_{r+1}^{(p)}$, such that for each $i = 0, \ldots, r$, $\{a_r, \beta_r\} \in V$ and $\beta_i \succ a_{i+1} \neq a_i$,
(2) if $r \geq 0$ and $a_0 = a_{r+1}$, such a path is a nontrivial closed path,

(3) a discrete vector field V on M is a collection of pairs $\{a^{(p)}, \beta^{(P+1)}\}$ of simplicies of M with $a \prec \beta$ such that each simplex is in at most one pair of V,
(4) a discrete vector field V is the gradient vector field of a discrete Morse function if and only if there are no nontrivial closed V-paths.

The chain structure spiking neural P system with skin is a kind of enrichment and expansion of P chain system and SNP system. It simulates the transmitting spike function of neurons and owns a chain structure. The SNPC system with skin of degree m ≥ 1 is a construct of the form:

$\Pi = (O, \sigma_0, \sigma_1, \ldots, \sigma_m$, syn, in, out),

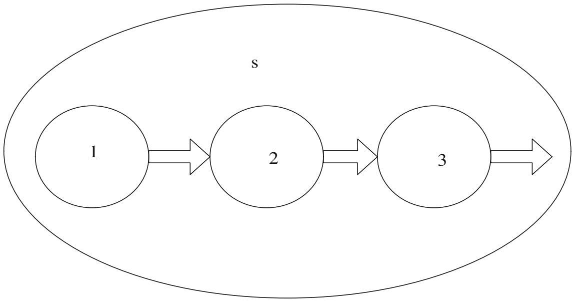
Where:

(1) O = {a}is the singleton alphabet (a is called spike);
(2) $\sigma_0, \sigma_1, \ldots, \sigma_m$ are m + 1 neurons; 0 ≤ i ≤ m, σ_0 representing skin membrane, $\sigma_1, \ldots, \sigma_m$ are m cells abstracting from discrete gradient vector path, satisfied $\sigma_0 \prec \sigma_1 \prec \ldots \prec \sigma_m$. Neurons σ_i is the form of $\sigma_i = (n_i, R_i)$, where:
(1) $n_i \geq 0$ is the initial number of spikes contained by the neuron σ_i;
(2) R_i is a finite set of rules of the following two forms:
(i) $C/a^c \rightarrow a^p$; d, c ≥ 1, p ≥ 1, d ≥ 0, where C is the constraint condition of the rule, d is the delay time, that is, the interval between using the rule and releasing the spike, and c ≥ p;
(ii) forgetting rules: $C'/a^s \rightarrow \lambda$, s ≥ 1, where C′ is the constraint condition of the rule, with the restriction that, for R_i in type (i) each rule $C/a^c \rightarrow a^p$; d, satisfies $C \cap C' = \varphi$;
(3) syn = {0, 1, 2, …, m} indicates the link relationship between neurons, for any 1 ≤ i ≤ m, there is $\sigma_0 \prec \sigma_1 \prec \ldots \prec \sigma_m$;
(4) in, out ∈{1, 2, …, m} indicate that the input and output neuron.

The rules (i) are used as followed: When restrain C is met, if there are k spikes in neuron σ_i and k ≥ c, the rule $C/a^c \rightarrow a^p$; d can be inspired in neuron σ_i. When this rule is used in neuron σ_i, it will consume c spikes (remaining k-c spikes); at the same time, generate p spikes after d unit time, and immediately sent p spikes to its follow-up neuron. What's more, during the time from using this rule to transmission the generating spikes, the neuron is closed, that is, don't receive any spikes. If the neuron fires the rule $C/a^c \rightarrow a^p$; d, d ≥ 1 at the t step, the neuron is closed during the step t, t + 1,…,t + d − 1. When a neuron is closed, all the rules over it can't be used, only if the neuron is open, the rules over it can be inspired. If a neuron sends spikes to a closed neuron, the closed neuron can't receive the spikes, and these spikes will disappear naturally. For the output neuron, it can send spikes to the environment.

The rules (ii) are used as followed: When restrain C′is met, if there are k′ spikes in neuron σ_i and k′ ≥ s, the forgetting rule $C'/a^s \rightarrow \lambda$ can be inspired in neuron σ_i, and all the firing rules over the neuron can't be used. When this rule is used in neuron σ_i, it will consume s spikes, but don't generate new spikes.

Fig. 2.1 The SNPC system with skin



The diagram of SNPC system with skin as shown in Fig. 2.1.

In Fig. 2.1, there are four membranes, and the outmost membrane is the skin, whose two main functions is that, receive the objects from chain structure membrane system and send the objects to the chain structure membrane system followed a certain rules, and whose existence makes the communication between the nonadjacent neurons be possible. There are three membranes in the chain structure membrane system, where membrane 1 is the input membrane, whose objects can be generated by the reaction in the membrane, or can be from the skin membrane, and membrane 3 is the output membrane, whose objects are sent to the skin or environment directly.

2.4 Logic Operation in SNPC

The logic operation is the basic operations of other complex logic processes, and literature [3] has proved the possibility of completion logic operation in P system. In this paper, based on that, we prove the feasibility of SNPC system to perform basic logic operations. When performing Logic operation by SNPC system, we assume that there are enough spikes in environment can be sent into the system by input membrane (the first membrane), and use spike a、a´ and null string λ to represent TRUE or FALSE.

2.4.1 Logic NOT

Logic NOT operation meets $\neg T = F, \neg F = T$, here in SNPC system, using spiking a or anti-spike a´ representing TRUE or FALSE and they are opponent. The application NOT operation in SNPC system, the membrane system required design as followed:

$\Pi(\text{Not}) = (O, \sigma_0, \sigma_1, \sigma_2, \text{syn}, \text{in}, \text{out})$

Where:

$$O = \{a,\ a'\};$$
$$\sigma_1 = \{n_1,\ R_1\},\ \sigma_2 = \{n_2,\ R_2\};$$
$$n_1 = \{0|\varphi\},\ n_2 = \{2|\ a'\};\ R_1 = \{\ a \to a\},\ R_2 = \{\{\ a'a \to \lambda;\ a' \to a'\}\};$$

in = 1, out = 2.

Here, rules R_2 are chained rules, and it is applied as followed which refers to the literature [7]: There is a chained rules R_2 which contains two vectors of rules. In the membrane, if the first rule of chained rules R is applied, then in the next step the rest of the rules from R will be applied in order in consecutive steps. However, if a rule from an already started vector of chained rules R can't be applied, then the execution of R is dropped, that is, for the current application of R, the remaining rules are not executed anymore.

As shown in Fig. 2.2, there are three membranes, a skin and two chain structure membranes. In membrane σ_1, whose function is sending objects a to the system, there are one rules a → a. In the whole P system, when all rules which can be inspired are applied once, we call it one step. In other words, in each step, all rules which can be applied have to be applied to all possible objects. In the aspect of a representing TRUE, then a´ is FALSE, at the beginning, in membrane σ_1, when there is spike a from environment to membrane 1, rules a → a will be executed. Then import a (TRUE) to membrane σ_2, then chained rule $R_2 = \{\{a'a \to \lambda; a' \to a'\}\}$ will be inspired, consuming one a´ due to rule a´a → λ and producing one a´ due to rule a´ → a´, and send a´ (FALSE) out as the output. Else if a represents FALSE, and a´ is TRUE, sending a (FALSE) into membrane σ_2, will export a´ (TRUE) as the output. The SNPC system completes the function of logic NOT operation.

2.4.2 Logic AND

Logic AND operation meets $F \wedge F = F, F \wedge T = F, T \wedge T = T$, using spiking a representing TRUE and anti-spike a´ representing FALSE or the opponent case,

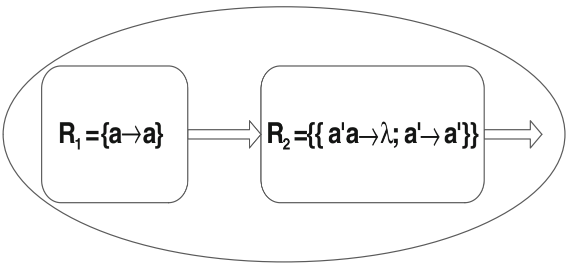
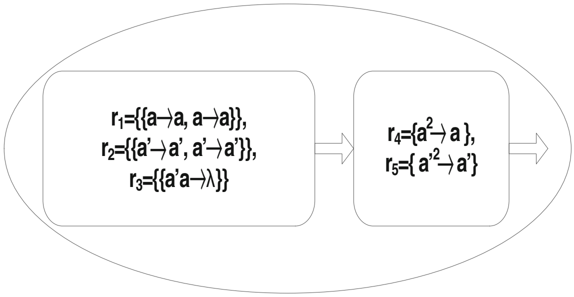


Fig. 2.2 SNPC for logic NOT operation

Fig. 2.3 SNPC for logic AND Operation



and λ here representing FALSE. The application AND operation in SNPC system, the membrane system required design as followed:

Π(AND) = (O, σ_0, σ_1, σ_2, …, σm, syn, in, out)

Where:

$$O = \{a, a'\};$$
$$\sigma_1 = \{n_1,\ R_1\},\ \sigma_2 = \{n_2,\ R_2\};\ R_1 = \{r_1,\ r_2,\ r_3\},\ R_2 = \{r_1,\ r_2\};$$
$$r_1 = \{\{a \to a,\ a \to a\}\},\ r_2 = \{\{a' \to a',\ a' \to a'\}\},\ r_3 = \{\{a'a \to \lambda\}\};$$
$$r_4 = \left\{a^2 \to a\ \right\},\ r_5 = \left\{\ a'^2 \to a'\right\};$$

in = 1, out = 2.

Here, rules r_1, r_2, r_3 are chained rules too. As shown in Fig. 2.3, there are three membranes, a skin and two chain structure membranes. In membrane σ_1, whose function is receiving the objects from environment and sending them to the system, if receive two objects a (TRUE), and chained rule r_2 is selected and executed, meaning that double a (TRUE) into membrane σ_2, then rule $a^2 \to a$ will be inspired, and send a (TRUE) out as the output. If receive two objects a´ (FALSE), and chained rule r_1 is inspired and applied, sending double a´ (FALSE) to membrane σ_2, then rule $a'^2 \to a'$ will be inspired, and send a´ (FALSE) out as the output. If receive an objects a (TRUE) and an object a´ (FALSE), rule r_3 will be executed and export null to membrane σ_2. Then the whole system will not have the output, as agreed λ representing FALSE. The SNPC complete the function of logic AND operation.

2.4.3 Logic OR

Logic OR operation meets $F \vee F = F, F \vee T = T, T \vee T = T$, using spiking a representing TRUE and anti-spike a´ representing FALSE or the opponent case. And further instructions, λ represents TRUE. The application OR operation in SNPC system, the membrane system required design as followed:

Π (OR) = (O, σ_0,σ_1,σ_2, …,σm, syn, in, out),

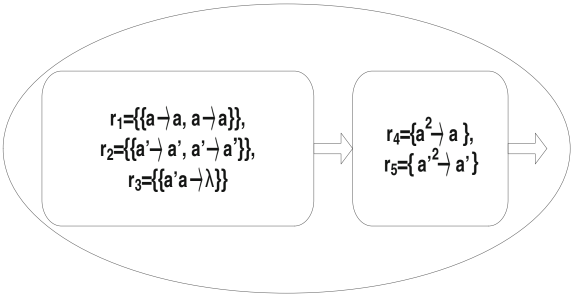


Fig. 2.4 SNPC for logic OR Operation

Where,

$$
\begin{aligned}
&\mathrm{O} = \{\mathrm{a}, \mathrm{a}'\};\\
&\sigma_1 = \{\mathrm{n}_1,\ \mathrm{R}_1\},\ \sigma_2 = \{\mathrm{n}_2,\ \mathrm{R}_2\};\ \mathrm{R}_1 = \{\mathrm{r}_1, \mathrm{r}_2, \mathrm{r}_3\},\ \mathrm{R}_2 = \{\mathrm{r}_1, \mathrm{r}_2\};\\
&\mathrm{r}_1 = \{\mathrm{a} \to \mathrm{a}, \mathrm{a} \to \mathrm{a}\},\ \mathrm{r}_2 = \{\mathrm{a}' \to \mathrm{a}', \mathrm{a}' \to \mathrm{a}'\},\ \mathrm{r}_3 = \{\mathrm{a}'\mathrm{a} \to \lambda\};\\
&\mathrm{r}_4 = \left\{\mathrm{a}^2 \to \mathrm{a}\ \right\}, r_5 = \left\{\ \mathrm{a}'^2 \to \mathrm{a}'\right\};
\end{aligned}
$$

in = 1, out = 2.

Here, rules r_1, r_2, r_3 are chained rules too. As shown in Fig. 2.4, there are three membranes, a skin and two chain structure membranes. In membrane σ_1, whose function is receiving the objects from environment and sending them to the system, if receive two objects a (TRUE), and chained rule r_2 is selected and executed, meaning that double a (TRUE) into membrane σ_2, then rule $a^2 \to a$ will be inspired, and send a (TRUE) out as the output. If receive two objects a´ (FALSE), and chained rule r1 is inspired and applied, sending double a´ (FALSE) to membrane σ_2, then rule $a^{\prime 2} \to a'$ will be inspired, and send a´ (FALSE) out as the output. If receive an object a (TRUE) and an object a´ (FALSE), rule r_3 will be executed and export null to membrane σ_2. Then the whole system will not have the output, as agreed λ representing TRUE. The SNPC complete the function of logic OR operation.

2.5 Conclusion

The membrane computing simulates the natural process of biological systems, and all the theoretical foundation based on the theory and practice of biochemistry. From the above inference process, we can see a high parallelism and distribution of computing capacity of SNPC system. Compared with method in literature [3], the method to solve basic logic operations (AND、OR、NOT) proposed in this paper is more efficient, in which the design of membrane system is simpler, and the time complexity to achieve the operation has also been further improved (with the linear time, but the efficiency of SNPC system is higher). Because a lot of computational problems are based on these basic logic operations, the results of this paper can be used as the basis of the more complex applications.

SNPC system is a new P system, which enriches the family of P system. Its chain structure makes it essentially different with other P system; the design of membrane structure of SNPC system is simpler; the SNPC system with skin further strengthen the communication of the membrane in the chain, greatly improving the operational efficiency of the whole system. In this paper, exampled as the SNPC membrane system to achieve basic logic operations, demonstrates the SNPC system computing power and efficiency. For most of the computational problems are based on basic arithmetic and Boolean computation, the basic conclusions of this paper can be used in more complex research areas, such as the construction of a computer chip, or solving some mathematical problems. Membrane computing is considered to be the most promising in solving NP problems, of course, SNPC system also can used to solve NP problems. Literature [8] proposed solving TSP problem by membrane computing, using the algorithm MCTSP, and on this basis, we can use the SNPC system to find a more efficient algorithm for solving TSP. All of these are the future direction and points of the study. Obviously, due to the current of the analog implementation of the membrane system and in reality the corresponding technical support of realizing are too little, calculation based on membrane computing models and methods are more difficult to verify, which makes to solving the problem by membrane computing relatively vague. These have also become challenges and efforts direction for the study of SNPC system.

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