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Proposed Research: Clustered Layouts of Graphs

The proposed research will be in the area of graph drawing, studying the combinatorial structure of complex networks and algorithms for drawing them. A tremendous amount of research has been done on drawing graphs of small size. By contrast, how to visualize large and complex graphs remains an elusive target, despite significant application areas such as social networks, communication networks, or WWW connection graphs. This is largely due to the fact that such large graphs cannot be drawn in a readable way by traditional graph drawing techniques. One approach to this problem is to exploit a hierarchy of clusters naturally arising in the graphs.

A *clustered graph* $C = (G, T)$ consists of a graph G and a recursive partitioning of the vertices of G, expressed as a rooted tree T, where the leaves of T correspond to the vertices of G. Each vertex c in T corresponds to a subset of vertices in G, forming a "cluster". Given a clustered graph, we are interested in drawing the graph so that the clusters are drawn as simple closed curves defining regions of the plane containing the associated vertices, while the edges are drawn as straight lines between the vertices. If a cluster drawing does not contain crossings between edge pairs or edge/region pairs, we say the drawing is *c-planar*.

I propose to investigate the complexity of c-planarity testing, a long-standing open problem of more than 10 years. It has been shown that if the given graph meets certain conditions, e.g., the induced subgraph within each cluster is connected, then c-planarity can be tested efficiently [1]. However, the graphs that arise in practice often do not meet such conditions, and solving the problem for general clustered graphs remains open.

Fixed Parameter Tractability [2] provides both theoertical and practical approaches to the design of algorithms for problems associated with parameters. In the case of drawing clustered graphs, the height of the cluster tree is typically small, and hence FPT could provide a useful approach. With this observation in mind, I furthermore intend to investigate parameterized complexity approaches whether or not c-planarity is in P.

Many related problems also remain unexplored. If the given graph is not c-planar, managing the edge crossings would be an important issue for visualization purposes. Crossing minimization is a well-studied topic in graph drawing (e.g., see [3, 4].), but no previous results are available for clustered graphs. Furthermore, with advances in computing hardware, the ability to deal with dynamic graphs has become important for information visualization [5]. Upon completion of the research on static graphs, I will extend and apply my algorithmic results to explore these problems.

References

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