External Watchman Route Problem Under Limited Visibility

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Abstract

The watchman route problem is to find a shortest route for a point robot such that the robot "*guards*" a polygon either internally or externally. This paper reviews some of the results in watchman route problem and discusses open problems raised in the domain of limited visibility.

Given a simple polygon P, and a moving "watchman" point robot r outside of P, the *External Watchman Route Problem* is to find a shortest closed route for r such that each point on the boundary of P is clearly seen from at least one position of r. The recent thesis of Rafa Absar [Abs05] investigated this problem for a watchman whose visibility is limited.

Guarding a polygonal linkage

A shortest external watchman route, without a given starting point, must visit at least one vertex of the polygon. In particular, the watchman route must touch the extreme points of the polygon, and follow the edges on the convex hull. Thus, we can partition the boundary of the polygon into a set of polygonal linkages: Given a simple polygon P, partition the sequence of edges such that each subsequence of edges is either entirely on the convex hull of P, or entirely disjoint from the convex hull of P except at the two end vertices.

Lemma 1. *Given a simple polygon P, if the shortest external watchman route R is a convex hull route, the edges on the convex hull are in R.*

Proof required

Thus, we only need to consider the sequence of edges that are not on the convex hull. The problem now reduces to finding a route inspecting one side of a polygonal linkage, where the route begins its tour at one end point of the linkage and ends at the other. For each polygonal linkage l, the region created by the Minkowski sum of the points along the linkage and visibility range d is denoted by $M(l)$. Then the following results hold.

The 1-link case

Theorem 1. *Given a 1-link $\overline{ab}$, the shortest $a - b$ path inspecting $\overline{ab}$ stays within $M(\overline{ab})$.*

Theorem 2. *Given a 1-link $\overline{ab}$, and let p be a path starting from some point in the half-circle centered at a, and ending at some point in the half-circle centered at b. If p stays in $M(\overline{ab})$ region throughout, p inspects $\overline{ab}$ completely.*

The general case

Theorem 3. *Given a polygonal linkage l, the shortest path inspecting l stays in $M(l)$.*

Theorem 4. *Given a polygonal linkage l, and let p be a path visiting circles centered at every vertex in l.If p stays in $M(l)$ region throughout, p inspects l completely.*

Using this result, we attempt to find the shortest watchman route that visits the circles centered at every vertex in the polygon. According to [Nta91], the shortest watchman route must visit these circles in the polygonal order. In the same paper, the author mentions that finding the shortest path visiting such circles involves evaluation of high order equations. Instead, he gives a PTAS using the safari route problem.

Problems

Inspecting horizontal segments

Given a set P of horizontal line segments, how can we inspect the segments in P using a watchman with limited visibility?

Note: This problem can be shown to be NP-hard. For sketch of the proof, take an arbitrary instance of geometric travelling salesman problem. Scale the position of the points large, and draw a line segment extending from each point to ϵ to the right. By setting the visibility range to zero, we now have a inspection of horizontal segments problem.

Optimum *Time Interval* Route

Given a watchman route, we wish to compute the maximum time interval that a point to be inspected experiences between sightings by the watchman. For example, consider the scissor-route for watching the convex side of the 2-link. The delay between inspection for the middle vertex is much less than for the end vertices. On the other hand, for a triangular route that follows the convex hull of the 2-link, all points experience the same delay. *Note*: If a route forms a circular route, *and* there is no intersection between the minkowski region of two non-adjacent links, then the time interval for each point is exactly the length of the tour.

This observation raises another scheme of problems to consider: Given a polygonal shape to be inspected, can we find a route that minimizes the maximum delay between inspections at any point.

Note: This problem seems easier to solve. In both scissor route and circular route, the maximum delay is the length of a closed tour. Thus, to minimize the maximum delay, one needs to minimize the length of a colosed tour, and the minimum watchman route gives us the answer.

Shortest Watchman Route with limited visibility

In [Nta91], a polynomial approximation scheme for the shortest watchman route with visibility range d is given. According to [Nta91], finding the exact shortest route that visits circles at each vertex involves evaluation of high-order equation and should be solved numerically. Instead, the author approximates the circles centered at vertices using regular k-gons and applies the safari route problem. While this method gives us a PTAS for the problem, the problem of finding the exact solution is still open. Moreover, even a constant factor approximation algorithm is unknown. Note: In [Tan04], a $\sqrt{2}$-approximation algorithm is obtained under the *infinite* visibility model.

Inspecting 2 convex polygons with limited visibility

In [GN95], a $O(n^2)$ time algorithm was introduced to compute the shortest external watchman route inspecting *two* convex polygons with *infinite* visibility. The same question under the limited visibility model is open.

Inspecting many convex polygons with limited visibility

For the case of inspecting arbitrarily many convex polygons, as well as for *internal* watchman route problem with a polygon with holes, the problem is known to be NP-hard. Often, the hardness is shown by reduction from Euclidean TSP. On the other hand, a variant of the Euclidean TSP has been studied extensively. Given a collection of n regions (neighborhoods), find a shortest tour that visits each region. As a generalization of the original Euclidean TSP, this problem is also NP-hard, but a PTAS exists. Can we use the notion of TSPN(TSP with neighborhoods) to solve the external watchman route problem with arbitrarily many convex polygons? Or, is there a solution with a parametrized complexity?

See [Abs05], [Tan04], [CN86], [NG94], [PKA05], [GN95], [Nta91].

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