# Analysis Techniques

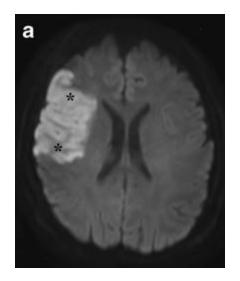
Ellie Thompson and Anna Schroder

#### Overview

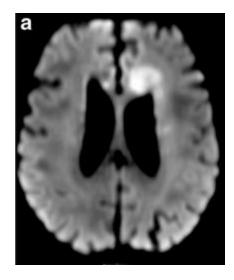
- Lecture
  - Introduction: using analysis techniques to gain biomedical insights from diffusion MRI
  - Diffusion tensor
  - Compartment models
  - Generating fODFs for tractography with constrained spherical deconvolution
- Coding Practicals
  - Practical 1: Diffusion tensor
  - Practical 2: Ball-and-stick model
  - Practical 3: Constrained Spherical Deconvolution
- Summary

# Introduction

### What can we do with diffusion MRI?



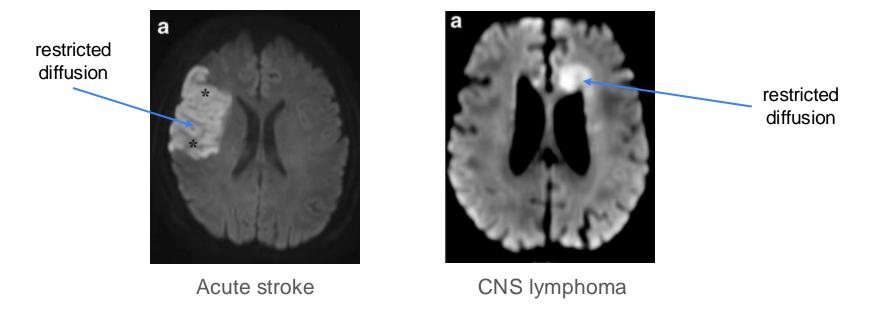
Acute stroke



CNS lymphoma

Pre-processed scans provide some clinical insight, however they lack **quantitative biomarkers** or **complex microstructural** information

### What can we do with diffusion MRI?



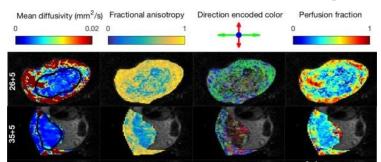
Pre-processed scans provide some clinical insight, however they lack **quantitative biomarkers** or **complex microstructural** information

We can use analysis techniques to gain

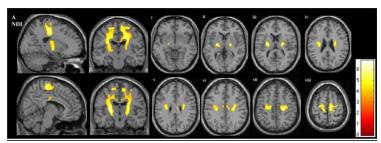
biomedical insight from dMRI

# Key applications

#### Microstructure modelling

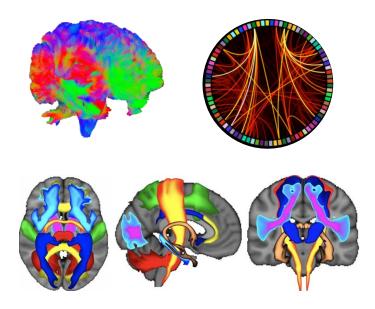


Slator PJ, el al. Placenta microstructure and microcirculation imaging with diffusion MRI. (2018)



Broad RJ, et al. Neurite orientation and dispersion density imaging (NODDI) detects cortical and corticospinal tract degeneration in ALS (2019)

#### **Mapping white matter connections**

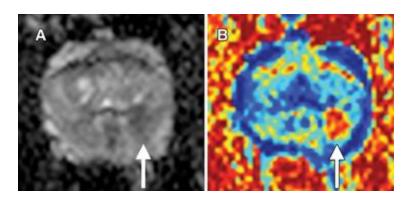


Warrington et al. Concurrent mapping of brain ontogeny and phylogeny within a common space: Standardized tractography and applications. (2022)

# We can use analysis techniques to gain **biomedical insight** from dMRI

e.g. VERDICT-MRI: non-invasive histology for prostate cancer

Apparent diffusion coefficient (ADC) map



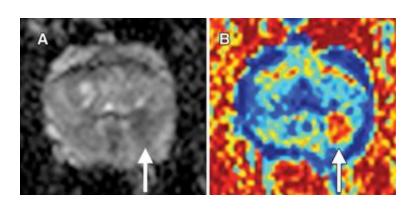
VERDICT intracellular volume fraction map

Images in a 57-year-old man with targeted biopsy-proven Gleason 3+4 prostate cancer.

# We can use analysis techniques to gain **biomedical insight** from dMRI

e.g. VERDICT-MRI: non-invasive histology for prostate cancer

Apparent diffusion coefficient (ADC) map

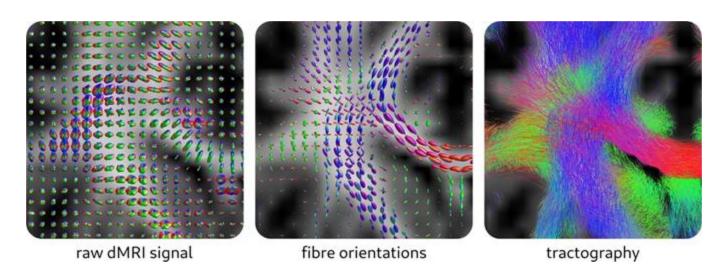


VERDICT intracellular volume fraction map

"New scanning technique reduces unnecessary biopsies by 90% meaning thousands of men could be spared pain and anxiety" - prostate cancer UK

# We can use analysis techniques to gain **biomedical insight** from dMRI

e.g. modelling fibre orientations for tractography

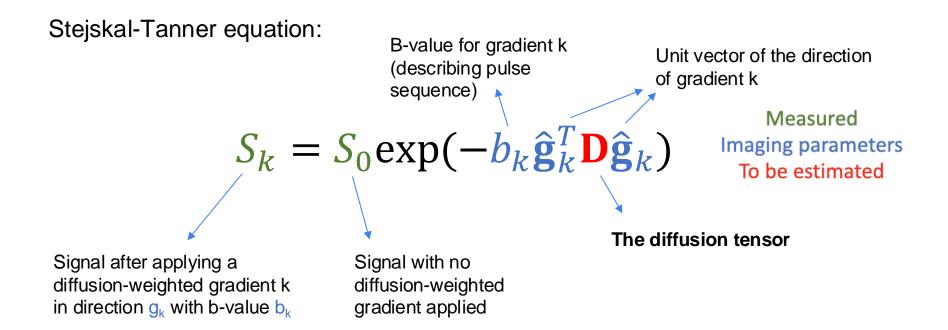


J-D Tournier, Diffusion MRI in the brain – Theory and concepts, Progress in Nuclear Magnetic Resonance Spectroscopy, 112–113, 2019, Pages 1-16,

The Diffusion Tensor

#### The Diffusion Tensor

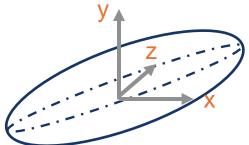
The diffusion tensor characterises Gaussian diffusion in 3D in each voxel



### What is the diffusion tensor?

3x3 positive-definite symmetric matrix characterising displacement/diffusion in
 3D

$$\mathbf{D} = \begin{pmatrix} D_{xx} & D_{xy} & D_{xz} \\ D_{xy} & D_{yy} & D_{yz} \\ D_{xz} & D_{yz} & D_{zz} \end{pmatrix}$$

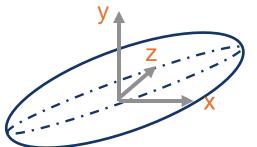


- $D_{xx}$ ,  $D_{yy}$ ,  $D_{zz}$ : diffusion along 3 orthogonal axes (always positive)
- $D_{xy}$ ,  $D_{xz}$ ,  $D_{yz}$ : correlation between displacements along these orthogonal axes (positive or negative)

### What is the diffusion tensor?

3x3 positive-definite symmetric matrix characterising displacement/diffusion in
 3D

$$\boldsymbol{D} = \begin{pmatrix} D_{xx} & D_{xy} & D_{xz} \\ D_{xy} & D_{yy} & D_{yz} \\ D_{xz} & D_{yz} & D_{zz} \end{pmatrix}$$

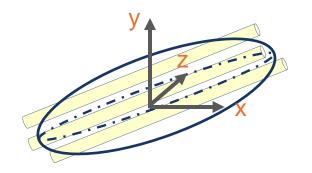


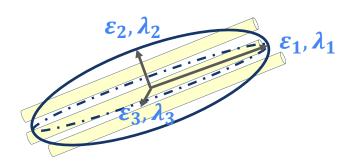
- $D_{xx}$ ,  $D_{yy}$ ,  $D_{zz}$ : diffusion along 3 orthogonal axes (always positive)
- $D_{xy}$ ,  $D_{xz}$ ,  $D_{yz}$ : correlation between displacements along these orthogonal axes (positive or negative)

# Ellipsoid from DT

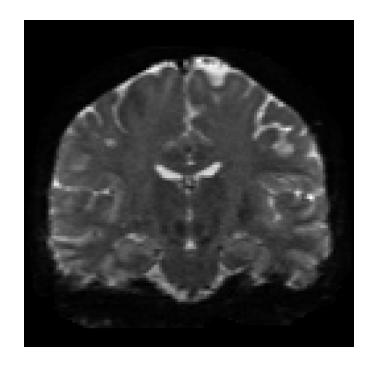
We can decompose the Diffusion tensor into eigen-values and eigen-vectors

$$\boldsymbol{D} = \begin{pmatrix} D_{xx} & D_{xy} & D_{xz} \\ D_{xy} & D_{yy} & D_{yz} \\ D_{xz} & D_{yz} & D_{zz} \end{pmatrix} = \begin{bmatrix} \varepsilon_1 & \varepsilon_2 & \varepsilon_3 \end{bmatrix} \begin{bmatrix} \lambda_1 & 0 & 0 \\ 0 & \lambda_2 & 0 \\ 0 & 0 & \lambda_3 \end{bmatrix} \begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \varepsilon_3 \end{bmatrix}$$

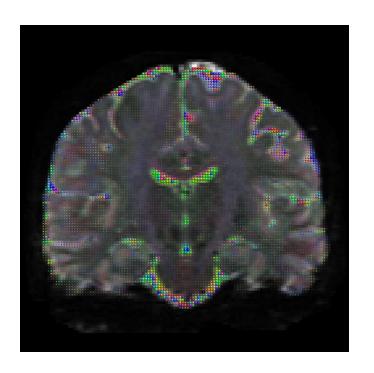




# Diffusion Tensor in real data

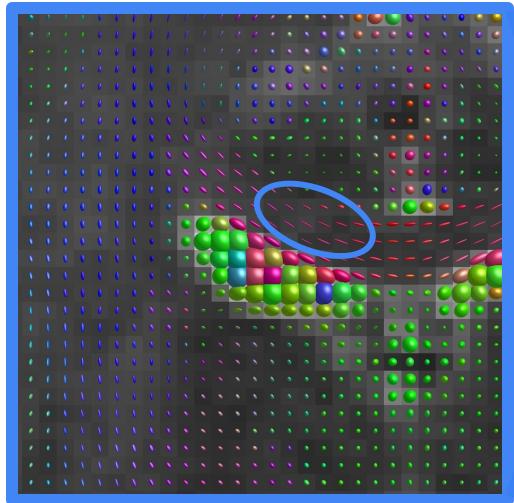


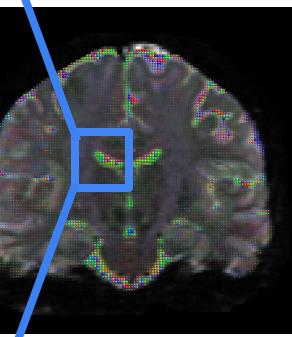
B0 diffusion signal



Diffusion tensors

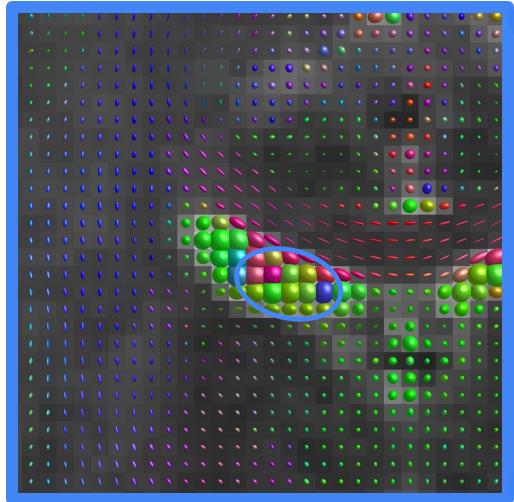


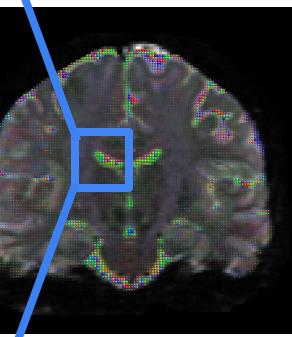




Diffusion tensors

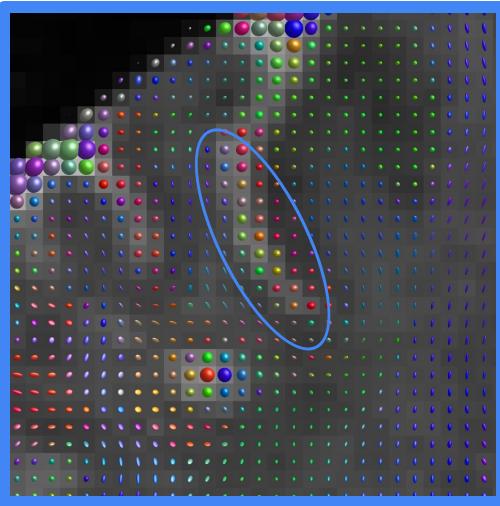


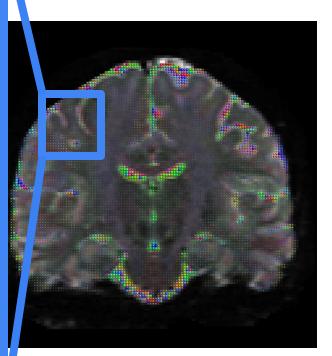




Diffusion tensors



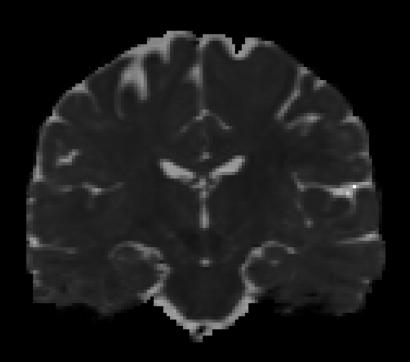


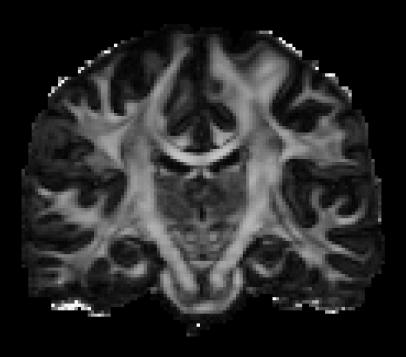


Diffusion tensors



# Scalar measures derived from the diffusion tensor





Axial Diffusivity,

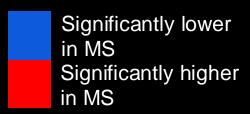
Radial Diffusivity

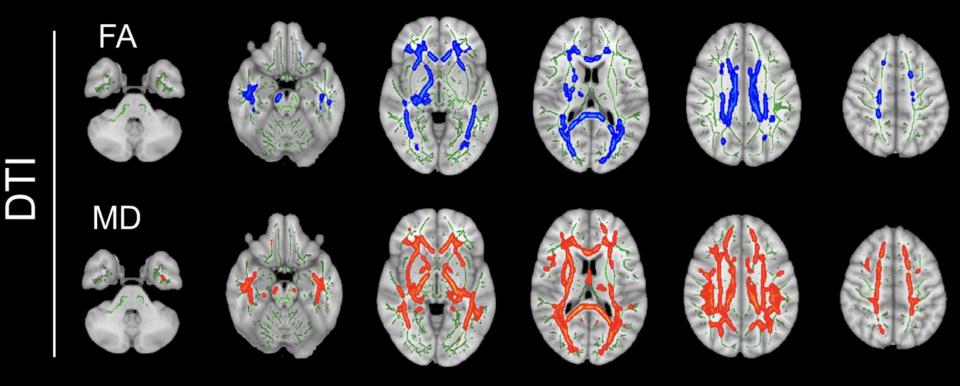
...

Mean Diffusivity (MD) = mean of the tensor eigenvalues

Fractional Anisotropy (FA) = normalised variance of tensor eigenvalues

# Measures derived from the diffusion tensor





S. Kato, et al. *Journal of the*Neurological Sciences 436 (2022)

# Fitting the diffusion tensor from an image

- Fitting the 6 variables of the diffusion tensor (Dxx, Dxy, Dxz, Dyy, Dyz, Dzz) from the diffusion image requires a minimum of 6 diffusion directions (+S<sub>0</sub>)
- Methods:
  - least squared (linear, weighted, non-linear)
  - Bayesian MCMC

# Fitting the diffusion tensor from an image

- Fitting the 6 variables of the diffusion tensor (Dxx, Dxy, Dxz, Dyy, Dyz, Dzz)
   from the diffusion image requires a minimum of 6 diffusion directions (+S<sub>0</sub>)
- Methods:
  - least squared (linear, weighted, non-linear)
  - Bayesian MCMC
- Minimise the following function:

$$f = \sum_{k} ||S_k - S_0 \exp(-b_k \widehat{\boldsymbol{g}}_k^T \boldsymbol{D} \widehat{\boldsymbol{g}}_k)||$$

# Compartment Models

# Compartment models

- Different tissues characterised by different biophysical models
- The signal is represented as a liner combination of the different compartments
- Exchange between compartments assumed to be negligible

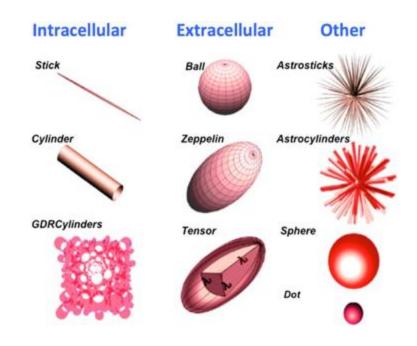
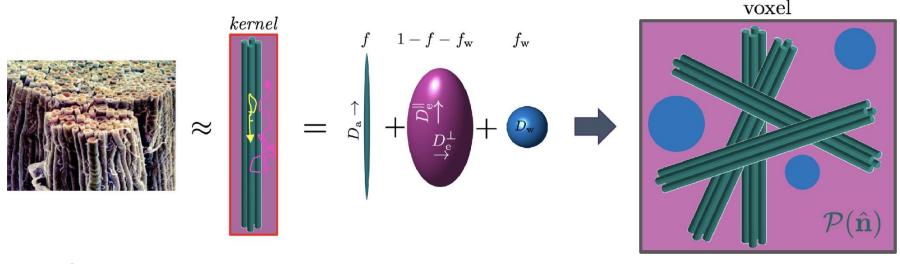


Figure from: Ferizi, U., Schneider, T., Panagiotaki, E., Nedjati-Gilani, G., Zhang, H., Wheeler-Kingshott, C.A.M. and Alexander, D.C. (2014), A ranking of diffusion MRI compartment models with in vivo human brain data. Magn. Reson. Med., 72: 1785-1792. https://doi.org/10.1002/mrm.25080

### The standard model for dMRI



- Stick compartment: axons
- Zeppelin compartment: hindered diffusion in the extra-axonal space
- Free water compartment: partial volume contributions from the CSF

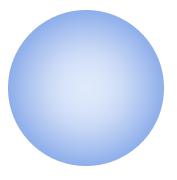
Novikov DS, Fieremans E, Jespersen SN, Kiselev VG. Quantifying brain microstructure with diffusion MRI: Theory and parameter estimation. NMR Biomed. 2019 Apr;32(4):e3998. doi: 10.1002/nbm.3998. Epub 2018 Oct 15.

Coelho S, Baete SH, Lemberskiy G, Ades-Aron B, Barrol G, Veraart J, Novikov DS, Fieremans E. Reproducibility of the Standard Model of diffusion in white matter on clinical MRI systems. Neuroimage. 2022 Aug 15;257:119290. doi: 10.1016/j.neuroimage.2022.119290. Epub 2022 May 8.

# A simple compartment model: Ball and Stick Model



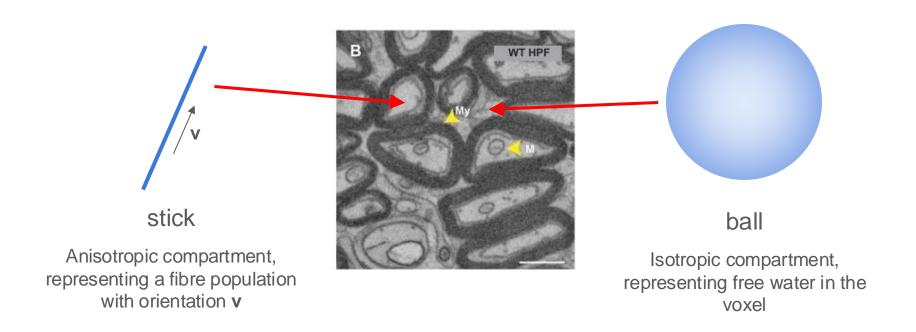
Anisotropic compartment, representing a fibre population with orientation **v** 



ball

Isotropic compartment, representing free water in the voxel

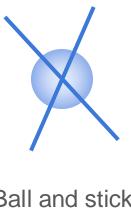
# A simple compartment model: Ball and Stick Model

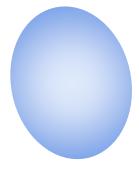


Behrens, T.E.J., Woolrich, M.W., Jenkinson, M., Johansen-Berg, H., Nunes, R.G., Clare, S., Matthews, P.M., Brady, J.M. and Smith, S.M. (2003), Characterization and propagation of uncertainty in diffusion-weighted MR imaging. Magn. Reson. Med., 50: 1077-1088. https://doi.org/10.1002/mrm.10609

This allows us to model crossing fibre populations, which is not possible with DTI



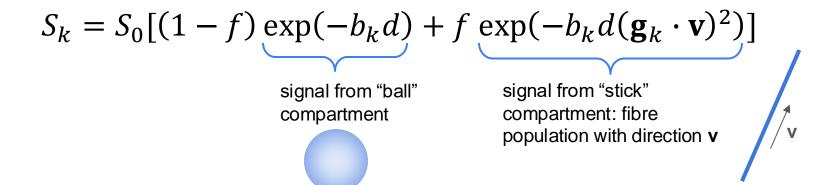




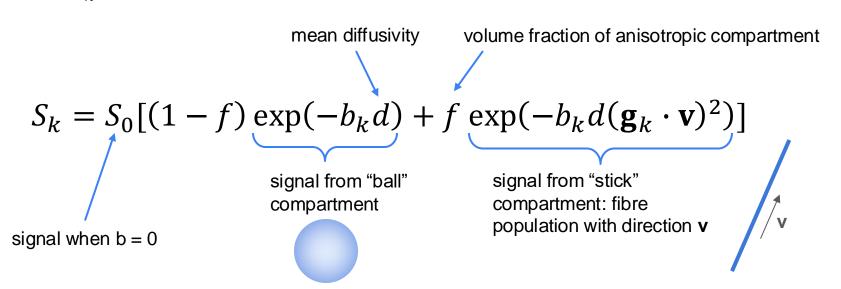
Ball and stick model

Tensor

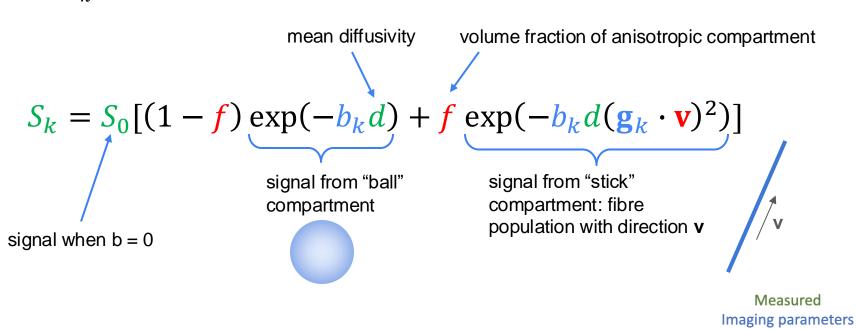
signal measured for a diffusion weighted gradient with direction  $\mathbf{g}_k$  and b-value  $b_k$ :



signal measured for a diffusion weighted gradient with direction  $\mathbf{g}_k$  and b-value  $b_k$ :



signal measured for a diffusion weighted gradient with direction  $\mathbf{g}_k$  and b-value  $b_k$ :

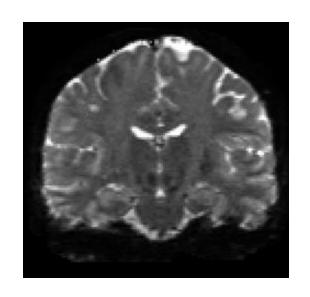


To be estimated

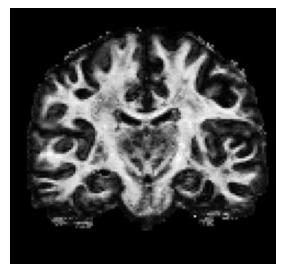
# Ball and Stick Model – extending to multiple fibre populations

$$S_k = S_0[\left(1 - \sum_{i=1}^N f_i\right) \underbrace{\exp(-b_k d)}_{\text{signal from "ball"}} + \sum_{i=1}^N f_i \underbrace{\exp(-b_k d(\mathbf{g}_k \cdot \mathbf{v}_i)^2)}_{\text{signal from "stick"}}]_{\text{signal from "stick"}}$$

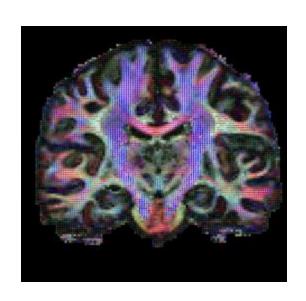
## Ball and Stick Model in real data



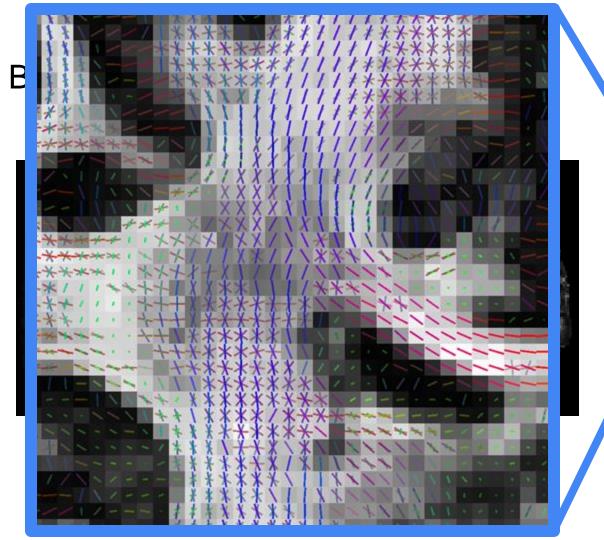
B0 diffusion signal

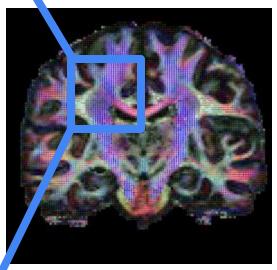


Fibre volume fraction



Sticks visualised as vectors





Sticks visualised as vectors

# More advanced compartment models

#### White matter

- CHARMED: Composite hindered and restricted model of diffusion. Assaf and Basser (2005) Neurolmage
- NODDI: Neurite orientation dispersion and density imaging. Zhang et al (2012) Neurolmage

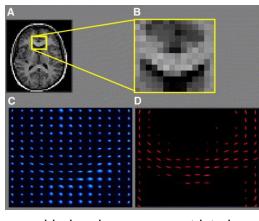
#### Grey matter

- SANDI: Soma and neurite density imaging.
   Palombo et al (2020) NeuroImage
- NEXI: neurite exchange imaging. Jelescu et al (2022) Neurolmage

#### Tumours

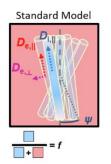
 VERDICT: Vascular, Extracellular, and Restricted Diffusion for Cytometry in Tumours. Panagiotaki et al (2014), Cancer Res

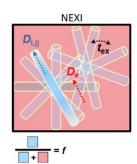
#### CHARMED

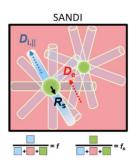


hindered

restricted





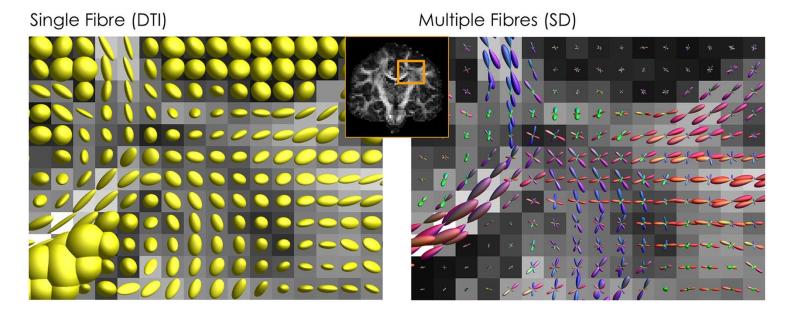


# functions (fODFs) with constrained spherical deconvolution

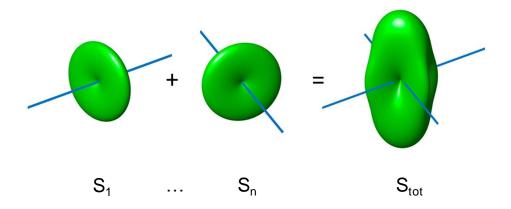
Generating fibre orientation distribution

## Spherical Deconvolution

- Method for obtaining a continuous fibre orientation distribution function: fODF
- Provides fibre orientations for tractography

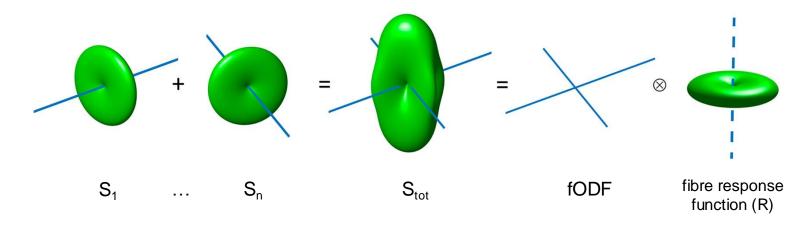


## **Spherical Deconvolution**



 The diffusion-weighted signal is a linear combination of signals from the different fibre populations in a voxel

## Spherical Deconvolution

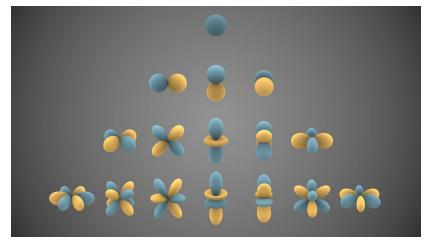


- The diffusion-weighted signal is a linear combination of signals from the different fibre populations in a voxel
- The measured signal is the convolution of the fibre response function (signal from a single, coherently-oriented fibre bundle) with the fODF
- The fODF is obtained by deconvolving the measured signal with the fibre response function

## Spherical Harmonic Basis

• The fODF is typically expressed as a weighted sum of spherical harmonics up to degree  $l_{max}$ :

$$f(\theta,\phi) = \sum_{l}^{l_{\max}} \sum_{m=-l}^{l} c_{lm} Y_{lm}(\theta,\phi)$$
 coefficients to spherical harmonics

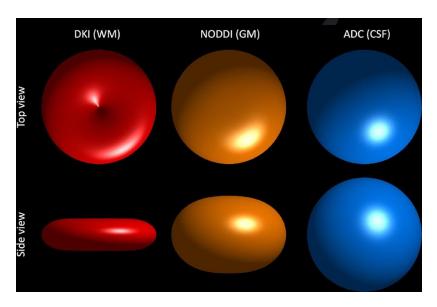


By Inigo.quilez - Own work, CC BY-SA 3.0, https://commons.wikimedia.org/w/index.php?curid=32782753

- compact representation reduces noise
- convolution is a simple product in spherical harmonic domain computationally efficient

## Step 1: Estimating the fibre response function

- The DW signal that would be acquired for single coherent fibre population
- Model based approaches: eg. axially symmetric tensor
- Direct empirical measurements from single-fibre voxels
- Response function can be derived for each tissue type (multi-tissue)

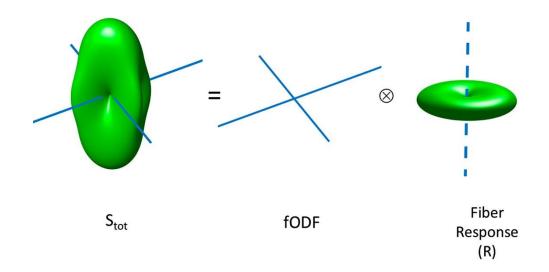


Examples of tissue-specific response functions derived from different models

Spherical deconvolution with tissue-specific response functions and multi-shell diffusion MRI to estimate multiple fiber orientation distributions (mFODs)":https://www.sciencedirect.com/science/article/pii/S1053811920306923

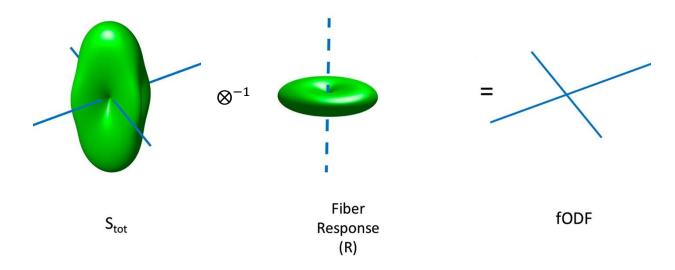
## Step 2: Using the fibre response function to determine the fODF

 We can deconvolve the measured signal with the response function, R, to obtain the fODF



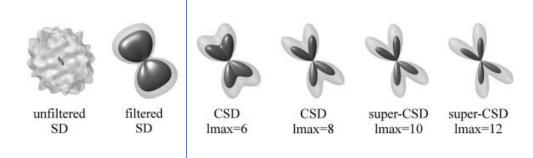
## Step 2: Using the fibre response function to determine the fODF

 We can deconvolve the measured signal with the response function, R, to obtain the fODF



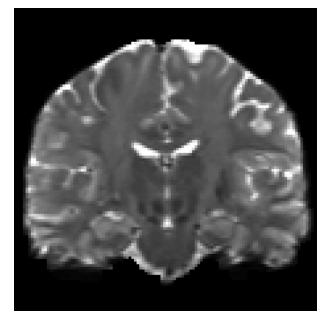
## **Constrained** Spherical Deconvolution

- The spherical deconvolution operation is ill-posed and susceptible to noise
- Tournier et al (2007) introduced a non-negativity constraint to the reconstructed fODF
- This drastically improves the robustness to noise and improves angular resolution

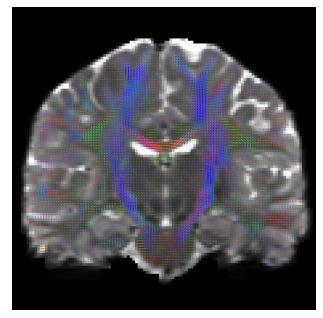


Tournier, J. D., Calamante, F., & Connelly, A. (2007). Robust determination of the fibre orientation distribution in diffusion MRI: non-negativity constrained super-resolved spherical deconvolution. NeuroImage, 35(4), 1459–1472. https://doi.org/10.1016/j.neuroimage.2007.02.016

### CSD in real data

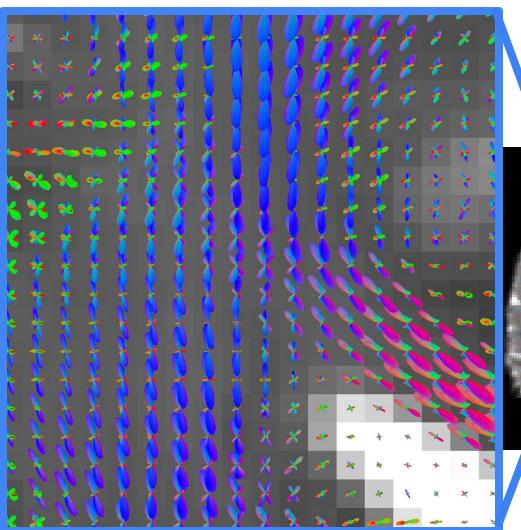


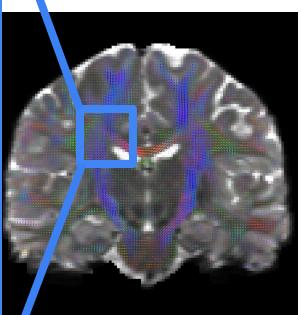
B0 diffusion signal



**fODFs** 







fODFs



**Coding Practicals** 

## Coding practicals

- Three coding practicals:
  - Diffusion tensor
  - Ball-and-stick model
  - Spherical deconvolution
- Estimate model parameters in each voxel of the FiberCup dataset<sup>1</sup>

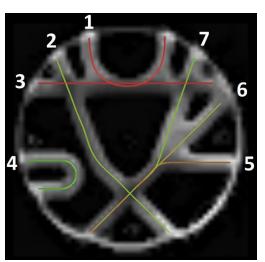


https://github.com/ethompson93/dmri\_analysis\_techniques

<sup>&</sup>lt;sup>1</sup>Poupon, C., Laribiere, L., Tournier, G., Bernard, J., Fournier, D., Fillard, P., Descoteaux, M., et al. (2010). <u>A Diffusion Hardware Phantom Looking Like a Coronal Brain Slice</u>. *Proceedings of the International Society for Magnetic Resonance in Medicine*.

## FiberCup Data

- Built as part of a MICCAI challenge and mimics a coronal section (3 slices) of the brain
- Diffusion MRI simulated from known ground truth fibre bundles



Poupon, C., Laribiere, L., Tournier, G., Bernard, J., Fournier, D., Fillard, P., Descoteaux, M., et al. (2010). A Diffusion Hardware Phantom Looking Like a Coronal Brain Slice. Proceedings of the International Society for Magnetic Resonance in Medicine.

Fillard, P., Descoteaux, M., Goh, A., Gouttard, S., Jeurissen, B., Malcolm, J., Ramirez-Manzanares, A., et al. (2011). Quantitative Evaluation of 10 Tractography Algorithms on a Realistic Diffusion MR Phantom. Neurolmage, 56(1), 234-220.

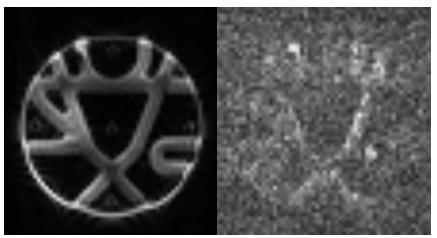
## FiberCup Data

- Built as part of a MICCAI challenge and mimics a coronal section (3 slices) of the brain
- Diffusion MRI simulated from known ground truth fibre bundles

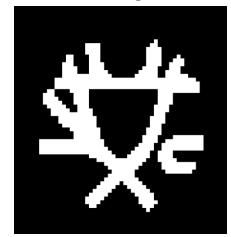
#### Files:

- Diffusion weighted images "fibrecup.nii.gz"
- White matter mask "wm\_mask.nii.gz"
- B-values and b-vectors in the 'grad.txt' file

### fibrecup.nii.gz



wm\_mask.nii.gz

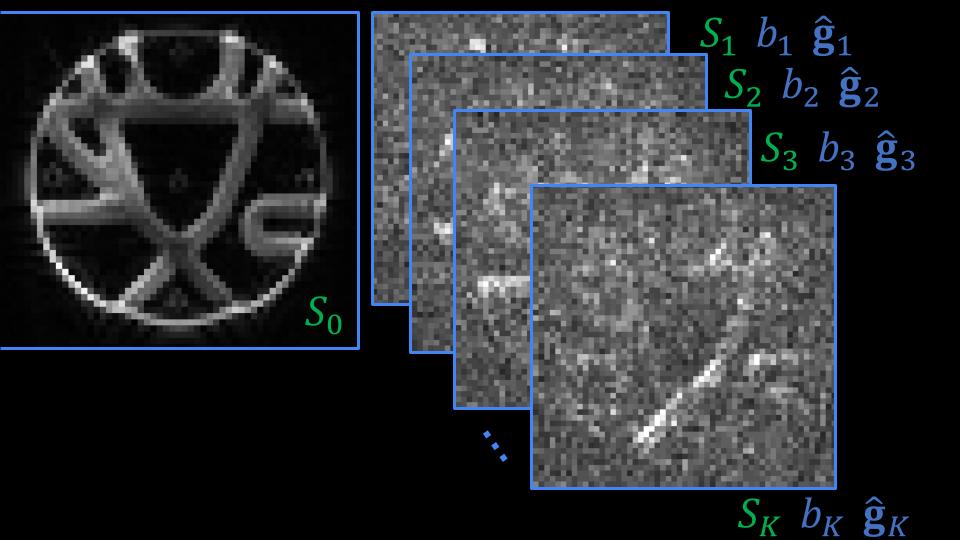


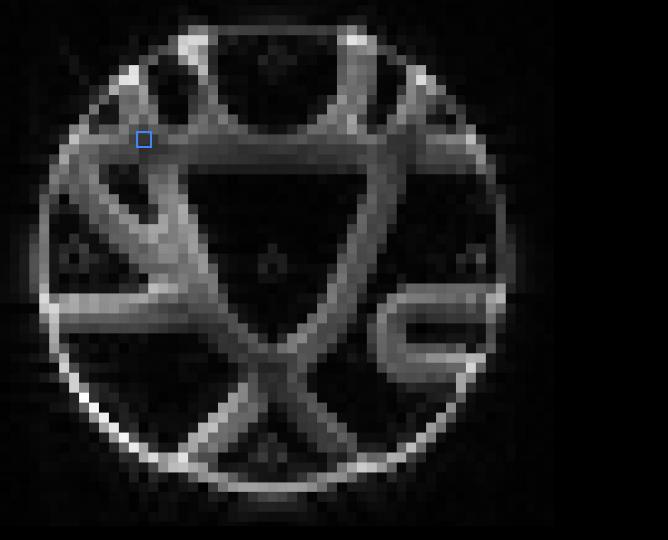
Poupon, C., Laribiere, L., Tournier, G., Bernard, J., Fournier, D., Fillard, P., Descoteaux, M., et al. (2010). A Diffusion Hardware Phantom Looking Like a Coronal Brain Slice. Proceedings of the International Society for Magnetic Resonance in Medicine.

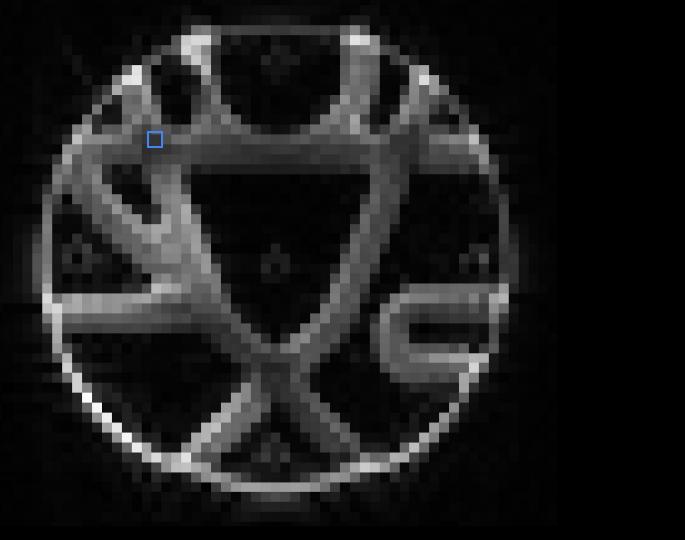
Fillard, P., Descoteaux, M., Goh, A., Gouttard, S., Jeurissen, B., Malcolm, J., Ramirez-Manzanares, A., et al. (2011). Quantitative Evaluation of 10 Tractography Algorithms on a Realistic Diffusion MR Phantom. Neurolmage, 56(1), 234-220.

## Summary of the practical – overall picture + specific aims of the tasks

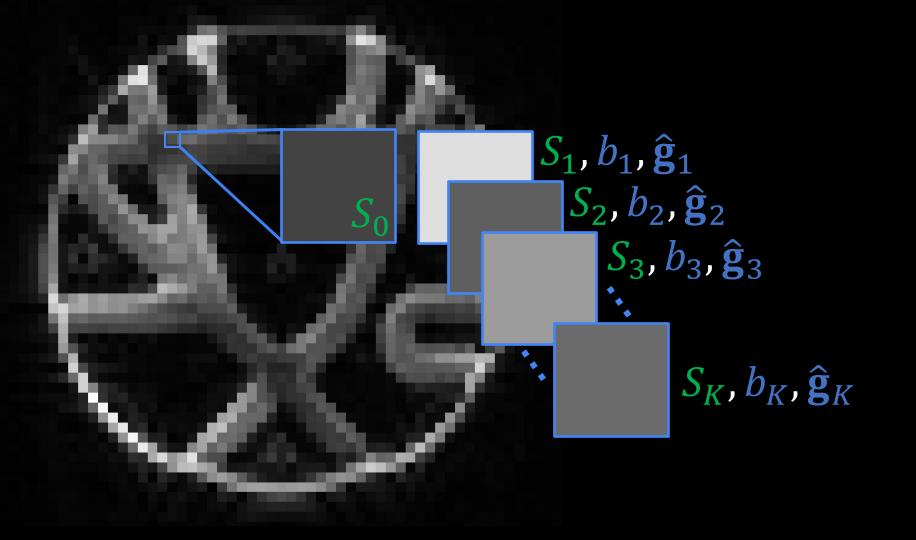
- Aim: fit parameters of interest
  - $\circ$  Diffusion tensor:  $D_{xx}$ ,  $D_{yy}$ ,  $D_{zz}$ ,  $D_{xy}$ ,  $D_{xz}$ ,  $D_{yz}$
  - $\circ$  Ball and stick: stick volume fraction f, and direction in terms of  $\theta$ ,  $\phi$
- How? Find parameter values for each voxel that minimise error between model and measurements
- We'll use non-linear least squares scipy optimise least-squares
- In each voxel we use measurements from all gradient directions to fit the model

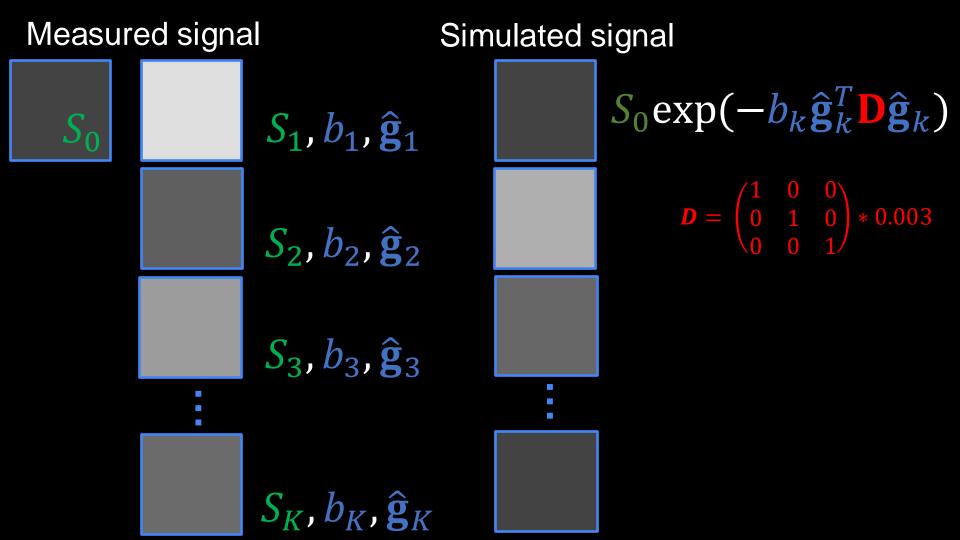












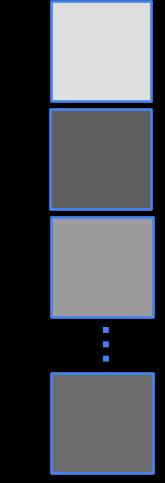
Measured signal Simulated signal  $S_0 \exp(-b_k \hat{\mathbf{g}}_k^T \mathbf{D} \hat{\mathbf{g}}_k)$  $S_1, b_1, \hat{\mathbf{g}}_1$  $\mathbf{D} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} * 0.003$  $S_2, b_2, \hat{\mathbf{g}}_2$ SSE  $S_3, b_3, \hat{\mathbf{g}}_3$ SciPy  $[S_K,b_K,\widehat{\mathbf{g}}_K]$ 

Measured signal Simulated signal  $S_0 \exp(-b_k \hat{\mathbf{g}}_k^T \mathbf{D} \hat{\mathbf{g}}_k)$  $S_1, b_1, \hat{\mathbf{g}}_1$  $\mathbf{D} = \begin{pmatrix} 0.5 & 0.1 & 0 \\ 0.1 & 1 & 0.2 \\ 0 & 0.2 & 1 \end{pmatrix} * 0.003$  $S_2, b_2, \hat{\mathbf{g}}_2$ SSE  $S_3, b_3, \hat{\mathbf{g}}_3$ SciPy  $S_K, b_K, \widehat{\mathbf{g}}_K$ ,

Measured signal Simulated signal  $S_0 \exp(-b_k \hat{\mathbf{g}}_k^T \mathbf{D} \hat{\mathbf{g}}_k)$  $S_1, b_1, \hat{\mathbf{g}}_1$  $\mathbf{D} = \begin{pmatrix} 0.8 & 0.1 & 0.3 \\ 0.1 & 0.5 & 0.1 \\ 0.3 & 0.1 & 0.6 \end{pmatrix} * 0.003$  $S_2, b_2, \hat{\mathbf{g}}_2$ SSE  $S_3, b_3, \hat{\mathbf{g}}_3$ SciPy  $S_K, b_K, \widehat{\mathbf{g}}_K$ 

## Simulated signal

 $S_0 \exp(-b_k \hat{\mathbf{g}}_k^T \mathbf{D} \hat{\mathbf{g}}_k)$ 



 $S_0 \exp(-b_1 \hat{\mathbf{g}}_1^T \mathbf{D} \hat{\mathbf{g}}_1)$  $S_0 \exp(-b_2 \hat{\mathbf{g}}_2^T \mathbf{D} \hat{\mathbf{g}}_2)$  $S_0 \exp(-b_3 \hat{\mathbf{g}}_3^T \mathbf{D} \hat{\mathbf{g}}_3)$ 

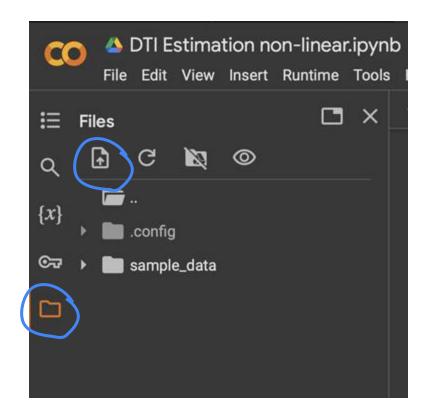
Simulated signal

 $S_0 \exp(-b_K \hat{\mathbf{g}}_K^T \mathbf{D} \hat{\mathbf{g}}_K)$ 

## Get into groups! (2 or 3 people)

## Uploading the data

- Upload to "files"



## Diffusion Tensor: Coding Exercise

Estimate the diffusion tensor in each voxel in a phantom dataset

$$S_k = S_0 \exp(-b_k \widehat{\boldsymbol{g}}_k^T \boldsymbol{D} \widehat{\boldsymbol{g}}_k)$$

Measured
Imaging parameters
To be estimated

$$\boldsymbol{D} = \begin{pmatrix} D_{xx} & D_{xy} & D_{xz} \\ D_{xy} & D_{yy} & D_{yz} \\ D_{xz} & D_{yz} & D_{zz} \end{pmatrix}$$



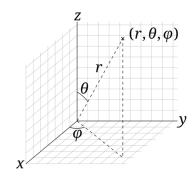
https://github.com/ethompson93/dmri\_analysis\_techniques

## Ball and Stick Model: Coding Exercise

$$S_k = S_0[(1-f)\exp(-b_k d) + f\exp(-b_k d(\mathbf{g}_k \cdot \mathbf{v})^2)]$$
signal from "ball" signal from "stick" compartment compartment with direction  $\mathbf{v}$ 

- We want to estimate f and v in each voxel
- We express  $\mathbf{v}$  in spherical coordinates  $(\theta, \phi)$  for efficiency
- Need to convert to Cartesian coordinates for dot product with g:

$$x = \sin \theta \cos \varphi$$
$$y = \sin \theta \sin \varphi$$
$$z = \cos \theta$$



## CSD coding exercise

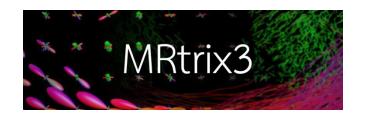
- We will use DIPY, an open-source python library, to perform constrained spherical deconvolution on the Fibercup phantom
- The notebook will take you through the steps of estimating the fibre response function and running the constrained spherical deconvolution

## Different Software for dMRI analysis

Not an exhaustive list!















### Conclusions

- Analysis techniques unlock biological insight from dMRI
  - o e.g. microstructural information and fibre orientations for tractography
- The diffusion tensor quantifies anisotropy in the diffusion signal, but lacks microstructural specificity
- Compartment models, such as ball-and-stick, characterise different tissue environments with biophysical models
- Constrained spherical deconvolution is a method for estimating the fibre orientation distribution function

### Acknowledgements





Engineering and Physical Sciences Research Council







Professor Danny Alexander and the CU-MONDAI group at UCL Hawkes Institute

#### contact: