Report on paper entitled "Monitoring Structural Breaks in Dynamic Regression Models with Bayesian Sequential Probability Test"

The paper deals with online change detection in a Bayesian context. It is supposed that observations arrive sequentially and the objective is to detect, as soon as possible, a change (if it occurs) in the distribution, ensuring a lot rate of false alarms. In the Bayesian context, the change time is not deterministic but random. In this paper, a geometric prior distribution is considered for this change time. Shiryaev (1963) was the first to treat this problem and to propose an optimal stopping rule in the case where the distributions before and after the change are fully known. The objective of this paper is to consider a more general case, in which the distribution before the change is known but the distribution after the change is a parameterized distribution with an unknown parameter θ.

The subject of this paper is obviously of interest but to my point of view, a major concern is the positioning relatively to the existing literature. Indeed, since the first works of Shiryaev, online Bayesian changepoint detection has led to an important number of papers (see for example [4]). It would even seem that the problem of unknown distribution after the change has been considered by some authors ([2] and [3], notably). I think that it is important to compare your approach with the existing ones.

Moreover, to my opinion, the manuscript is not always very clear. For example, in Sections 1 and 2.1, the authors insist on the regression model (the objective of this paper according to the title) and show that the change considered is equivalent to a change in the mean of a Gaussian distribution (equation (2.2)). But, if I well understand, in Section 2.2 and in the sequel, it seems that the proposed solution is more general since they consider f_0 and $f_1(\theta)$ not necessarily Gaussian ? Another example concerns the choice of the detection threshold $\pi^\star$ which is of crucial importance. Nothing is said in the core of the manuscript and this issue is only approached in the Appendix. How the threshold is obtained should be put forward. In the same way, the Section 5 should appear before the simulation and the real data treatment sections, as well as Figure 13.

An online change detection rule can be viewed as a sequential hypothesis testing procedure. Generally, the users want to control a kind of type I error related to the rate of false alarm (it can be the probability of false alarm on a given interval or the mean time before the first false alarm, for example). In the procedure proposed in this paper, if I well understand, we have to choose a value for parameter c in equation (2.10). But how this parameter can be chosen if we want to fix a particular type I error ? By simulation ? May be the authors could address this problem ?

Some minor comments:

- There is no verb in the first sentence of Lemma 2.1

- In Lemma 2.2, τ_t is not the stopping time. The stopping time is, as it is written in (2.16):

$$\tau = \inf\{t : \tau_t = 1\}$$

- In the simulation and real-data treatment sections, the models, the parameter, the prior distribution should be clearly explicit in each case. It is not always the case.

- In Table 2, why not consider the GLR (Generalized Likelihood Ratio) rule (see for example [1]), which seems to be a competitor of your rule more adapted than the CUSUM ?

- In Section 4, $\pi = 0.5$, which means than the change occurs, with probability 1/2, before the beginning of the monitoring ? Why such a choice ?

- In the proof of Lemma 2.1, the quantity M is not clearly defined, and there is a problem with brackets in the bounding of $|M|$ (and also in the equation that follows).

References

[1] Tze Leung Lai. Information bounds and quick detection of parameter changes in stochastic systems. *IEEE Transactions on Information Theory*, 44(7):2917–2929, 1998.

[2] Tze Leung Lai and Haipeng Xing. Sequential change-point detection when the pre-and post-change parameters are unknown. *Sequential analysis*, 29(2):162–175, 2010.

[3] Alexander G Tartakovsky. Asymptotic optimality of mixture rules for detecting changes in general stochastic models. *IEEE Transactions on Information Theory*, 65(3):1413–1429, 2018.

[4] Alexander G Tartakovsky and George V Moustakides. State-of-the-art in bayesian change-point detection. *Sequential Analysis*, 29(2):125–145, 2010.