# The Submodular Santa Claus Problem in the Restricted Assignment Case

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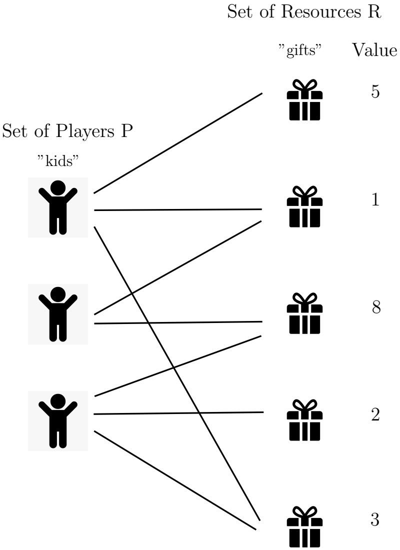
# The (Linear) Santa Claus Problem




- A set of resources $R$ and a set of players $P$. Each resource $j$ has a value $p_j$.
- Assignment restrictions given by some bipartite graph.
- **Goal:** Find an assignment $\sigma : R \mapsto P$ respecting the restrictions such that

$$\min_{i \in P} \sum_{j \in \sigma^{-1}(i)} p_j$$

is maximized, i.e. make the least happy kid as happy as possible!

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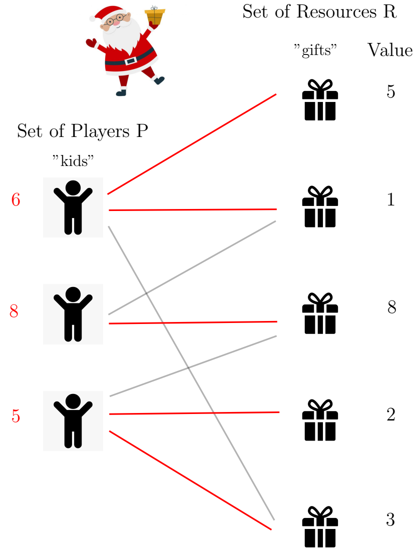


Santa is only as happy as the least happy kid.

# The **Submodular** Santa Claus Problem

An equivalent formulation of the **linear** Santa Claus:

Given a **linear** function $f : 2^R \mapsto \mathbb{R}_+$, find an assignment (with assignment restrictions) $\sigma : R \mapsto P$ such that

$$\min_{i \in P} \quad f\left(\sigma^{-1}(i)\right)$$

is maximized.

What happens if $f$ becomes a monotone **submodular** function?

**Submodular** Santa Claus problem (with assignment restrictions).

# Previous results for Santa Claus

In the **linear** case, very well studied problem.

- Introduced by Bansal and Srividenko (STOC'06) who gives an $O(\log\log(m)/\log\log\log(m))$-approximation algorithm (with $m = |P|$).
- Numerous improvements over the years on the approximation guarantee, the technique and/or the running time. The current best approximation is a $(4+\epsilon)$-approximation in polynomial time (Davies, Rothvoss, and Zhang SODA'20, Cheng and Mao ICALP'19).

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In the **submodular** case, a more general result by Goemans, Harvey, Iwata, and Mirrokni (SODA'09) implies a $O(n^{1/2+\epsilon})$-approximation in polynomial time (where $n = |R|$) in the restricted assignment case.

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**Our result:**

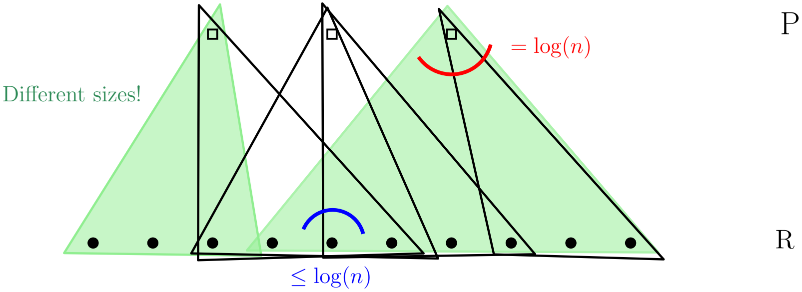
## Theorem

*There exists a $O(\log\log(n))$-approximation algorithm running in polynomial time for the Submodular Santa Claus in the Restricted Assignment case.*

# A **new** bipartite hypergraph matching problem for the **submodular** case

Given a regular and **non-uniform** bipartite hypergraph, find for each vertex $i \in P$ one hyperedge $C_i$ such that:

1. $i \in C_i$, and player $i$ is assigned a good fraction of resources in $C_i$.
2. No resource is taken more than $\log\log(n)$ times.



# Open questions

- $O(1)$-approximation ?
- We extended the Lovász Local Lemma technique to the submodular case. What about the local search technique?

Thank you for your attention!