RATIONALLY INATTENTIVE HETEROGENOUS AGENTS

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Abstract

We solve business cycle models with rationally inattentive heterogeneous agents and compare their predictions with the data. When the wage rate is determined competitively, households' cross-sectional expectations match survey evidence, but rational inattention is a weak propagation mechanism that fails to generate persistence in macroeconomic quantities. The opposite occurs when wages are set by households with monopoly power; a catch-22 for standard labor market structures. We discuss modifications to the models' microfoundations as ways out of this conundrum. Other heterogeneous agent models have not sought to jointly match these macro and micro moments, which have implications for both policy experiments and inequality.

Keywords: information choice, rational inattention, business cycles, heterogenous agents, monetary policy.

JEL Codes: D83, E21, E31, E32, E71

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1. Introduction

Recent work in macroeconomics has placed emphasis on the importance of building models that can match key micro moments from the data, typically focusing on wealth heterogeneity and marginal propensity to consume (MPC). A major contribution of this literature has been its role in reshaping our understanding of the propagation mechanisms behind aggregate fluctuations and the transmission of economic policies (see Violante 2021 for a survey). In particular, the heterogeneous agent framework provides greater scope than the representative agent approach for indirect effects, such as changes in disposable income, to play a central role in the propagation of macro shocks. As a result, the perspective of the economy depicted by this class of models is often viewed as more intuitive and realistic than that implied by the infinitely lived representative consumer paradigm.

Most heterogeneous agent models still suffer from two main weaknesses when confronted with data. First, like many of their representative agent counterparts, they usually assume perfect information, which is well known to contradict evidence from survey data showing significant belief heterogeneity among firms and households (see Carroll 2003, Weber et al. 2022). Where it truly hurts, however, is that it also implies no systematic correlations between individual characteristics and expectations (see Mitman et al. 2022 for empirical evidence supporting such a linkage). This assumption is a problematic omission given that the core philosophy behind these models is to match empirical heterogeneity. Second, the presence of a realistic share of high-MPC agents tends to produce shock responses that are peaked on impact, failing to generate the hump-shaped dynamics (e.g., positive autocorrelation in growth rates) for macro variables consistently documented in VAR studies (see Christiano, Eichenbaum, and Evans 2005, Altig et al. 2011). This is a known limitation of the framework, already noted by Kaplan, Moll, and Violante (2018), but where progress has been lacking.

Improving heterogeneous agent models along these two dimensions is not without challenges. Moving away from perfect information leads to an equilibrium that is function of a higher-order expectations distribution, which can quickly become intractable as model complexity increases. Furthermore, even when perfect information is considered a reasonable assumption, the conventional methods for generating inertial responses in macroeconomic variables, such as incorporating habit formation in consumption (see Auclert et al. 2021) or introducing wage stickiness, either prove problematic or require unrealistic assumptions within heterogenous agent models.

In this paper, we tackle both issues simultaneously by introducing inattention (Sims 1998, 2003) into dynamic stochastic general equilibrium (DSGE) models with heterogeneous households. Rational inattention (RI) is a theory of information acquisition that endogenously leads to imperfect expectations and sluggish responses to shocks. Specifically, decision-makers allocate their attention by solving a maximization problem based on a tradeoff; acquiring more informative signals increases both expected payoffs and attention costs. In line with the RI-DSGE literature, we assume that this mechanism is the sole source of inertia in prices and quantities. Previous work (see Maćkowiak and Wiederholt 2015, 2023) shows that the theoretical impulse response functions and expectations implied by calibrated inattentive Ricardian economies can match those obtained from time-series and survey data. Whether inattention can deliver similar results in a setting with heterogeneous households remains an open question to which we aim to provide answers.

For our analysis, we adopt a stylized view of heterogeneity based on the long tradition of two-agent models (Campbell and Mankiw 1989, Bilbiie 2008). Households are divided into constant shares of savers ($\mathcal{S}$) and hand-to-mouth ($\mathcal{H}$). This dimension of heterogeneity is arguably sufficient to replicate most of the responses to aggregate shocks implied by state-of-the-art heterogeneous agent economies (see Debortoli and Galí 2024 for a discussion[1]), and yields an aggregate MPC on impact that is consistent with the data[2]. More importantly for us, the set-up is tractable enough to solve for each type of agent's optimal signal(s) in a dynamic environment as well as the economy's fixed point.

Combining rational inattention and heterogenous households, yields a framework in which each household faces a type-specific attention problems from which are determined their optimal signal(s). Consequently, differences in attention allocation between types endogenously arise. We recount three main channels. First, losses from suboptimal actions depend on the marginal utility of consumption; if hand-to-mouth households' consumption is lower, they are incentivized to pay more attention. Second, savers make an additional decision, which generally broadens their relevant span of information. Third, the extent to which each action directly affects utility may differ across types. As a result, both the choice of signals by savers and hand-to-mouth households and the accuracy of their forecasts are endogenous outcomes of our models, jointly determined with macroeconomic dynamics.

Our theoretical framework allows us to evaluate the fit of our models using macroeconomic variables *and* households' measured expectations. For the latter, we rely on the *Survey of Consumer Expectations* (SCE) and document new empirical findings, which we refer to as micro moments about expectations. In particular, we find a systematic relationship between a household's forecast accuracy of macro

[1]All models considered in their analysis suffer from the two weaknesses mentioned above.

[2]This result holds if the share of hand-to-mouth households is close to the desired MPC.

variables and its hand-to-mouth status. Our results provide supplemental empirical evidence of an interaction between heterogeneity and expectations formation. Mitman et al. (2022) termed this nexus the *attention channel*. In and of itself, our finding represents a novel dimension of heterogeneity, cross-sectional forecast accuracy between hand-to-mouth households and savers, to which two-agent models can be confronted.

We start our theoretical analysis by comparing the results from two baseline models. We first consider a version in which the labor market is perfectly competitive. We find that, while the model fits micro moments in expectations well, it fails to generate sufficient persistence in output growth to match the data, particularly in response to demand shocks. In that setting, rational inattention is a weak propagation mechanism with competing effects. The more inattentive savers are, the more delayed their consumption response to shocks, which induces positive autocorrelation in its growth rate. However, greater inertia in their labor supply decisions requires a more volatile wage rate for the labor market to clear, which induces negative autocorrelation in the consumption growth rate of hand-to-mouth households. This contrasts with a Ricardian version of this economy, where inattention can generate any desired degree of output persistence.

Next, we evaluate a version of the model in which households have some monopoly power, allowing them to set wages. This minimal change fully resolves the problem of persistence in output growth, and induces empirically consistent dynamics for the real wage. This reversal is due to inattention generating hump-shaped labor income responses, through slughish wage adjustment, that, in turn, produces positive autocorrelation in the consumption growth of hand-to-mouth households. However, households' monopoly power leads to mistakes from suboptimal wage-setting decisions that are an order of magnitude larger than labor supply mistakes in the competitive-wage version. As a result, differences in forecast accuracy between hand-to-mouth households and savers become minimal, a catch-22.

Our results reveal a novel challenge; embedding baseline two-agent business cycle models with inattention does not allow us to simultaneously match macro and micro moments that includes cross-sectional expectations. At the crux of the issue is the labor market, which needs to be set up in a way that generates hump-shaped adjustments in labor income, while avoiding stakes so high that attention becomes overly skewed toward intratemporal decisions. More broadly, we are left with the message that ignoring micro moments about expectations may lead to erroneous conclusions about a model's fit, which in turn make the model ill-suited for policy experiments.

We propose plausible assumptions that build on the insights gained from the analysis of our baseline two-agent economies to resolve the conundrum. In particular, either assuming that wages are sticky due to infrequent contract renegotiations not carried out by the households themselves or that a sufficiently large fraction of hand-to-mouth households operate in a monopsonistic labor market allows the model to match macro and micro moments simultaneously.

Finally, we compare inattention with other propagation mechanisms known in the literature as ways to address either of the aforementioned weaknesses present in heterogeneous agent models (i.e., the lack of persistence and the assumption of perfect information). Overall, the rational inattention framework provides a clean way to assess how information frictions, which are essential to match measured expectations, alter the model's dynamics while remaining impervious to the Lucas critique. Computational challenges limit its applicability, but insights from smaller models, such as those solved in this paper, provide useful starting points for setting up the microfoundations and information structures of more complex models that aim to address the issues we highlighted.

Related Litterature. Maćkowiak and Wiederholt (2015, 2023) solve benchmark Ricardian business cycles models with rationally inattentive agents. They show that RI is a strong propagation mechanism that can replace adjustment frictions, and that models with inattention have different implications for dynamics and policy experiments than that of perfect information counterparts.

Song and Stern (2020) solve an RI-DSGE with firm that are heterogenous in terms of attention costs. When the fraction of firms that are more attentive than other changes so does the real effects of monetary policy shocks.

Auclert, Rognlie, and Straub (2020) adress the persistence issue in heterogenous agent models by embeding sticky information *à la* Mankiw and Reis (2007) into a HANK[3]. This departure from perfect information allows them to match conditional impulse response functions and estimate the model.

Gallegos (2024) studies a two-agent model with an exogenously specified imperfect information structure. In this setting, amplification that arise from high-MPC households in dampened by slow learning about the fundamentals.

Mitman et al. (2022) study the interaction between heterogeneity and endogenous expectation formation in a neoclassical model *à la* Krusell and Smith (1998). They calibrate the signals' costs to match the data on expectations. In a policy experiment, they show that the predictions of their model with endogenous attention and those a perfect information benchmark may be contradictory.

[3]Heterogenous Agent New Keynesian.

Layout. The remainder of the paper is structured as follows. Section 2 presents empirical evidence on the relationship between hand-to-mouth status and the quality of households' forecasts. Section 3 outlines the model's economic environment. Section 4 states the attention problems faced by decision-makers within the model. Section 5 characterizes the equilibrium of the inattentive economy. Section 6 describes the dynamics and the fits of baseline inattentive economies. Section 7 presents extensions. Section 8 compares rational inattention with other frameworks. Section 9 concludes.

2. Expectations Heterogeneity

A large body of empirical research shows that expectations are sluggish at the macro level (see Coibion and Gorodnichenko 2015, Bordalo et al. 2020) and dispersed at the micro level (see Mankiw, Reis, and Wolfers 2003, Weber et al. 2022). However, there is considerably less evidence about whether and how expectations correlate with individual characteristics (see Mitman et al. (2022)'s first footnote for a review of the existing litterature).

In this section, we investigate the existence of such correlation, which we refer to as *micro moments about expectations*. We do so by partitioning households into two groups, hand-to-mouth and savers, and assessing cross-sectional differences in forecast accuracy. This approach is consistent with the stylized definition of heterogeneity in our theoretical framework and, to the best of our knowledge, is conducted at a broader level than comparable studies.

We use microdata from the SCE[4], a large panel survey held by the New-York Fed that collects monthly expectations from heterogeneous households. Typically, identifying hand-to-mouth households requires detailed microdata on finances, which the SCE lacks[5]. However, by merging the SCE with its supplemental surveys on households' spending and credit we can leverage the answers to questions on consumption behavior and liquidity constraintsto determine whether or not a household is assigned the hand-to-mouth label. We also use the finance survey to obtain descriptive statistics for the households within each types. Our sample spans the period from 2013M6 to 2024M4, which ends in the latest period for which we can compare expectations to their actual outcomes.

2.1. Measuring Expectations Relative Accuracy. We focus on forecasts of variables that are present in baseline business cycles models, namely inflation and

[4]The SCE releases are available at https://www.newyorkfed.org/microeconomics/sce.

[5]In particular, the SCE does not collect data on credit limits.

the nominal interest rate. The SCE collects households' 12 months ahead point forecasts for inflation, and subjective probabilities of the interest rate on savings account being higher. We measure accuracy using the absolute value of forecast errors.

For inflation, measuring accuracy is straightforward, as both point forecasts and realized outcomes are observable. Our definition of tealized inflation is the year-over-year growth rate of the Consumer Price Index (CPI), expressed in percentage points. For robustness, we also consider alternative measures based on the Personal Consumption Expenditures (PCE) price index and the CPI core, results are qualitatively the same.

For the interest rate, the main challenge is that the true underlying probability is unobservable[6]. To address this, we estimate a Bayesian Vector Autoregression (BVAR) using monthly macroeconomic data from 1960M1 to 2025M1 and infer the probability of higher interest rates from the model's posterior distribution.

2.2. Identifying Hand-to-Mouth. At the crux of our analysis is the identification of hand-to-mouth households in the SCE. To do so, we leverage responses to three specific questions that, in our view, systematically elicit different answers from hand-to-mouth households compared to others (the savers).

Liquidity Constraint. In the credit survey, households are asked about their ability to respond to an unforeseen need for liquidity,

Q: *What do you think is the percent chance that you could come up with 2,000$ if an unexpected need arose within the next month?* ____

A low probability of being able to come up with the money simultaneously reflects a household's lack of liquid wealth and/or limited access to credit markets. A drawback of this proxy is that it may fail to capture households with zero liquid wealth who currently behave as hand-to-mouth, but would take on debt to respond to an unexpected need for liquidity. In practice, we identify as hand-to-mouth those households who report a probability below 30%.

Negative Income Shock. In the survey on spending habits, the households are asked the following multiple-choice question,

Q: *Suppose next year you were to find your household with* 10% *less household income than you currently expect. What would you do?*

[6]For unemployment, Mitman et al. (2022) approximate the true probability using the average forecast from the *Survey of Professional Forecasters*, but no equivalent exists for interest rates.

1. Cut spending by the whole amount
2. Not cut spending at all, but cut my savings by the whole amount
3. Not cut spending at all, but increase my debt by borrowing the whole amount
4. Cut spending by some and cut savings by some
5. Cut spending by some and increase debt by some.
6. Cut savings by some and increase debt by some.
7. Cut spending by some, cut savings by some and increase debt some.

In theory, a household selecting option 1 must be hand-to-mouth, as this is the only response consistent with an inability to smooth consumption intertemporally following a negative income shock.[7] However, because the income loss is relatively large, the same caveat mentioned for the liquidity constraint proxy applies; households who behave as hand-to-mouth not due to liquidity constraints, but to avoid borrowing costs, might be missed. How relevant this concern is depends on whether those households consider a 10% income drop significant enough to take on debt. An additional source of misclassification may arise because the question does not clearly specify whether the shock is permanent or temporary; any household may choose option 1 when interpreting the income shock as permanent.

Default Probability. Lastly, in the SCE main survey households are asked about their perceived probability of defaulting on any of their current debt,

Q: *What do you think is the percent chance that, over the next 3 months, you will NOT be able to make one of your debt payments (that is, the minimum required payments on credit and retail cards, auto loans, student loans, mortgages, or any other debt you may have?* ____

This question can only identify a subset of hand-to-mouth households, those whom Lusardi, Schneider, and Tufano (2011) would classify as *financially fragile*. Specifically, it captures households that are near or at their borrowing limit. In practice, we classify a household as hand-to-mouth if it reports a perceived probability of defaulting greater than 70%. Results based on this proxy offer insight into whether meaningful differences in the accuracy of expectations exist among hand-to-mouth households and serve as a robustness check.

[7] Interestingly, an equivalent question is asked for a hypothetical 10% positive income shock, which we expect to move households away from hand-to-mouth behavior. Indeed, nearly all respondents choose an option implying consumption smoothing.

Descriptive Statistics. For our three proxies, we compute the share of households identified as hand-to-mouth relative to the total number of households. We denote this fraction ϕ. We also compute ratios of medians and standard deviations of the absolute value of forecast errors. For each proxy, we discard outliers, defined as households whose inflation expectations are above the 95th percentile of the distribution across all households.

Table 1: Descriptive Statistics

	Negative Income Shock	Liquidity Constraint	Default Probability
ϕ (HtM share)	0.47	0.24	0.04
Inflation forecast errors			
Median ratio	0.83	0.57	0.65
S.D. ratio	0.88	0.74	0.79
Interest rate forecast errors			
Median ratio	0.95	0.85	0.97
S.D. ratio	0.98	0.97	1.03

Notes: Forecast errors measured in absolute values. Ratios hand-to-mouth over savers.

Liquidity constraints yield the most realistic share of hand-to-mouth households compared to empirical estimates based on financial data typically reported in the litterature (see Kaplan, Violante, and Weidner 2014, Kaplan and Violante 2014). The negative income shock appears to classify too many households as hand-to-mouth, which may reflect different interpretations of the survey question by respondents. As expected, default probability captures only a small share of total households, a subset of the hand-to-mouth. Overall, Table 1 indicates that expectations of hand-to-mouth households are unconditionally less accurate and more dispersed than those of other households for both inflation and the probability of interest rates increasing.

Table 2: Median Liquid Assets

	Negative Income Shock	Liquidity Constraint	Default Probability
Hand-to-mouth	3,450	-4,500	-20,000
Savers	10,500	15,000	8,000

Notes: Liquid assets calculated as the current value of savings accounts (excluding retirement accounts) minus outstanding debt (excluding housing).

Table 2 presents median liquid assets for the hand-to-mouth and the remaining households for each of the proxy. The reported values confirm that our proxies successfully identify households with lower liquid assets as hand-to-mouth.

2.3. Relative Accuracy of Hand-to-mouth Expectations. Unconditional differences in expectations across our household partitions could simply reflect composition effects. The SCE collects additional information that allows us to conduct an analysis controlling for individual characteristics that may influence forecast quality. This approach allows us to assess whether two otherwise identical households form expectations differently based on their classification as hand-to-mouth. It aligns with the standard assumption in heterogeneous agent models, whether *à la* Aiyagari or two-agent, that households are identical before entering the economy[8].

Table 3 reports regressions of the absolute value of inflation forecast errors on a binary variable indicating whether a household is classified as hand-to-mouth according to our three proxies. Controlling for education and numeracy, hand-to-mouth households make significantly less accurate forecasts than other households which should not be attributable to differences in information-processing ability.

These empirical results are a first contribution of the paper; novel micro moments about expectation that can be used to evaluate or calibrate information structures in heterogeneous agent models. In the remainder of the paper, we turn to theoretical business cycle models with rationally inattentive agents, and test whether they can replicate the cross-sectional differences in expectations accuracy reported in Table 3, while also generating empirically consistent dynamics for macroeconomic variables.

3. Economic Environment

In this section, we describe the features of a baseline two-agent economy, excluding attention problems. Time is discrete, and periods correspond to quarters. Aggregate fluctuations are driven by two exogenous shocks; aggregate technology and innovations to the monetary policy rule. We focus on these shocks for two reasons. First, prior work shows that Ricardian models with inattention can generate aggregate dynamics that are consistent with the data for these shocks. Second, the presence of a demand and a supply shock ensures that households' optimal actions need not be correlated with inflation. In turn, optimal attention allocations may or may not reveal information about inflation, a micro moment we use to evaluate the model's fit.

The model features a continuum of firms producing differentiated goods, a competitive labor market, a Central Bank setting the nominal interest rate, and a continuum of households composed of savers and hand-to-mouth. This framework re-

[8]In incomplete market models *à la* Aiyagari, heterogeneity arises from realizations of idiosyncratic shocks, whereas in two-agent models an initial draw before $t = 0$ partitions households into savers and hand-to-mouth.

Table 3: Relative Inflation Expectations Accuracy of Hand-to-mouth Households

	Negative Income Shock	Liquidity Constraint	Default Probability
	-	-	-
Hand-to-mouth	0.563^{***}	1.734^{***}	1.731^{***}
	(0.030)	(0.038)	(0.071)
High School	-	-	-
Some College	-0.732^{***}	-0.751^{***}	-0.862^{**}
	(0.038)	(0.039)	(0.035)
College	-1.848^{***}	-1.764^{***}	-2.178^{***}
	(0.038)	(0.040)	(0.035)
Low Numeracy	-	-	-
High Numeracy	-2.087^{***}	-2.061^{***}	-2.474^{***}
	(0.033)	(0.035)	(0.030)
Unemployed	-	-	-
Part-time employed	0.041	0.107^{**}	-0.013
	(0.048)	(0.050)	(0.044)
Full-time employed	-0.508^{***}	-0.399^{***}	-0.552^{***}
	(0.033)	(0.034)	(0.030)
Observations	109,879	112,972	156,160
F Statistic	1783.44	2306.78	2981.38
R^2	0.112	0.133	0.125
Time Fixed Effects	yes	yes	yes

Notes: Estimates from a regression of the absolute value of inflation errors on the household hand-to-mouth status identified with different proxies. Estimates are relative to households that do not qualify as hand-to-mouth. Robust standard errors in parentheses, * $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$. Sample: 2013M8-2025M1.

sembles Bilbiie (2008, 2020), and Debortoli and Galí (2024), except that all sources of New Keynesian adjustment frictions are discarded.

In Appendix E, we describe the modifications required to solve an alternative version of the baseline model in which households possess some monopoly power allowing them to set a wage rate for the differentiated labor services they supply.

3.1. Households. There is a continuum of households indexed by $j \in [0,1]$, each of which belongs to one of two types $h \in \{\mathcal{H}, \mathcal{S}\}$. Each household keeps its type permanently and seeks to maximize its expected discounted sum of period utilities. The discount factor is $\beta \in (0,1)$, and the period utility function is

$$U(C_{jt}, L_{jt}) = \frac{C_{jt}^{1-\gamma} - 1}{1-\gamma} - \varphi^h \frac{L_{jt}^{1+\psi}}{1+\psi} \tag{1}$$

where

$$C_{jt} = \left(\int_0^1 C_{ijt}^{\frac{\theta-1}{\theta}} di \right)^{\frac{\theta}{\theta-1}}. \tag{2}$$

Here, C_{jt} is a composite consumption index, C_{ijt} is the consumed quantity of variety i and L_{jt} denotes the labor supplied. The parameter γ is the inverse of the intertemporal elasticity of substitution, φ^h scales labor disutility[9], ψ is the inverse of the Frisch elasticity and $\theta > 1$ is the preference parameter for the elasticity of substitution between consumption varieties.

Household $j \in [0, \phi]$ is a hand-to-mouth, $h = \mathcal{H}$, and its flow budget constraint is given by

$$\int_0^1 P_{it} C_{ijt} di = W_t L_{jt} - T_t^{\mathcal{H}}. \tag{3}$$

Household $j \in [\phi, 1]$ is a saver, $h = \mathcal{S}$, and its flow budget constraint is

$$\int_0^1 P_{it} C_{ijt} di + B_{jt} = R_{t-1} B_{jt-1} + W_t L_{jt} + D_t^{\mathcal{S}} - T_t^{\mathcal{S}}. \tag{4}$$

Here P_{it} is the price of variety i, W_t is the nominal wage rate, $T_t^{\mathcal{H}}$ and $T_t^{\mathcal{S}}$ are nominal lump-sum taxes, B_{jt} is household j's nominal bond holdings, R_t is the gross nominal interest rate paid on period $t-1$ nominal bonds, and $D_t^{\mathcal{S}}$ are dividends accrued from firms ownership by the savers.

[9]Households preferences only differ in terms of labor disutility, φ^h, which lets us derive a non-stochastic steady-state that is symetrical in hours worked. This parameter has no influence on the linearized dynamics.

We make the assumption that $B_{jt} > 0$ always holds for any household $j \in [\phi, 1]$. This allows us to write down Equation (4) in terms of logged variables[10] and also effectively rules out Ponzi schemes.

Households of the same type are ex-ante identical, but ex-post they are not under imperfect information. As detailed below, each household in the inattentive economy receives noisy signal(s) about the state of the world, and idiosyncratic noise realizations lead them to take different actions.

In each period t, every household chooses a consumption vector $\{C_{ijt}\}_{i\in[0,1]}$ and labor supply L_{jt}. If household j is a saver, it also chooses its nominal bonds holdings B_{jt}. All decisions are made taking as given exogenous shocks, the vector of prices for consumption varieties, the wage rate, the nominal interest rate and all aggregate quantities.

3.2. Firms. There is a continuum $i \in [0, 1]$ of firms. Firm i produces a differentiated variety of the consumption good using the production function

$$Y_{it} = e^{a_t} e^{a_{it}} L_{it}^{\alpha}. \tag{5}$$

Here, L_{it} is the quantity of labor used by firm i for production, while a_t and a_{it} are mean-zero stochastic processes for aggregate and firm-specific technology, respectively. The parameter $\alpha \in (0, 1]$ is the elasticity of output with respect to labor, which determines the degree of returns to scale.

Firm i seeks to maximize the discounted[11] sum of its period nominal profits (or dividends) given by

$$D_{it} = (1 + \tau_p) P_{it} Y_{it} - W_t L_{it} \tag{6}$$

where τ_p is a production subsidy.

In each period t, firm i sets a price P_{it} for its variety and demands quantity L_{it} of effective labor taking as given exogenous shocks, the vector of prices set by other firms, aggregate demand, the wage rate, the nominal interest rate and all aggregate quantities. Each firm commits to supplying any quantity of its consumption variety demanded, at the price it sets.

[10]In turn, it becomes straightforward to approximate an household utility flow in terms of log-deviations from the non-stochastic steady-state.

[11]Formally, firms use a stochastic discount factor that is a function of the consumption flows of households of type $\mathcal{S}$, to value profits across periods. The defintion of the stochastic discount factor's functional form can be found in Appendix D.1.

3.3. Government. The government consist of a monetary and a fiscal authority. The Central Bank sets the nominal rate according to a Taylor rule

$$\frac{R_t}{R} = \left(\frac{R_{t-1}}{R}\right)^{\rho_r} \left[\left(\frac{\Pi_t}{\Pi}\right)^{\phi_\pi} \left(\frac{Y_t}{Y_t^*}\right)^{\phi_{y^*}}\right]^{1-\rho_r} e^{v_t} \tag{7}$$

where $\Pi_t := (P_t/P_{t-1})$ denotes the inflation rate, P_t is a price index, (Y_t/Y_t^*) is the output gap defined as the ratio between actual output and its value that would prevail under perfect information and v_t is a monetary policy shock. Variables without index refer to steady-state values. The parameters ρ_r, ϕ_π and ϕ_{y^*} control the degree of inertia and the strength of the response of the monetary policy.

The governement budget constraint in period t is

$$T_t + B_t = R_{t-1}B_{t-1} + \tau_p \int_0^1 P_{it}Y_{it}di. \tag{8}$$

Following common practice, monetary policy is active and fiscal policy is passive in the sense of Leeper (1991). To finance interest on nominal bonds and subsidy payments, the government can either collect lump-sum taxes or issue new bonds. Given the presence of hand-to-mouth households, the method of government financing matters. In the following, we will assume that lump-sum taxes for hand-to-mouth households are fixed,

$$T_t^{\mathcal{H}} = T^{\mathcal{H}} \;\; \forall t \geq 0. \tag{9}$$

This assumptions allows us to abstract from considerations regarding how the government finances its expenditures in response to shocks, and guarantees that our results are not driven by unrealistic fluctuations in real taxes. We also solved the model under empirically plausible fiscal rules and found no significant changes in our findings.

Lastly, the production subsidy is set to correct distortions arising from market power in the non-stochastic steady-state such that

$$\tau_p = \frac{\tilde{\theta}}{\tilde{\theta} - 1} - 1 \tag{10}$$

where $\tilde{\theta} > 1$ is the model implied elasticity of substitution between varieties of goods, which is strictly smaller than the preference parameter θ whenever households are inattentive.

3.4. Shocks. There are three types of exogenous processes: monetary policy shocks, aggregate technology, and firm-specific technology. The latter ensures that the model's elasticity of substitution, $\tilde{\theta}$, is consistent with optimal attention allocation for any attention costs.

Monetary policy shocks are *i.i.d.* Gaussian innovations and the technological shocks follow independent stationnary Gaussian first-order autoregressive processes. The cross-sectional mean of firm-specific technology averages to zero in the cross-section.

The vector,

$$\boldsymbol{\varepsilon_t} = \left(\varepsilon_t^v, \varepsilon_t^a, \{\varepsilon_{it}^a\}_{i\in[0,1]}\right)' \tag{11}$$

collects all of the exogenous innovations affecting the economy at time t.

3.5. Aggregation. Average composite consumption and labor supply among household types are defined as

$$C_t^{\mathcal{H}} = \int_0^\phi C_{jt}dj \quad , \quad C_t^{\mathcal{S}} = \int_\phi^1 C_{jt}dj \tag{12}$$

and

$$L_t^{\mathcal{H}} = \int_0^\phi L_{jt}dj \quad , \quad L_t^{\mathcal{S}} = \int_\phi^1 L_{jt}dj. \tag{13}$$

Aggregate composite consumption and labor supply are defined by the weighted sums

$$C_t = \phi C_t^{\mathcal{H}} + (1-\phi)C_t^{\mathcal{S}} \quad , \quad L_t^s = \phi L_t^{\mathcal{H}} + (1-\phi)L_t^{\mathcal{S}}. \tag{14}$$

Aggregate demand for consumption variety i is defined analogously.

Aggregate output, labor demand and dividends are obtained by integrating over the continuum of firms

$$Y_t = \int_0^1 Y_{it}di, \quad L_t^d = \int_0^1 L_{it}^d di, \quad D_t = \int_0^1 D_{it}di. \tag{15}$$

Aggregate bonds, dividends, and taxes are given by

$$B_t = (1-\phi)\int_\phi^1 B_{jt}dj, \quad D_t = (1-\phi)D_t^S, \quad T_t = \phi T_t^{\mathcal{H}} + (1-\phi)T_t^{\mathcal{S}} \tag{16}$$

Lastly, we assume that the price index can always be written as

$$1 = \int_0^1 d_p\left(\hat{P}_{it}\right) di \tag{17}$$

where d_p is some twice continuously differentiabe function. Notice that this functional nests the index that would prevail under perfect information [12] and yields an identical expression once log-linearized.

3.6. Notation. The relative price of consumption variety i and the relative consumption of variety i by household j are denoted

$$\hat{P}_{it} = \frac{P_{it}}{P_t}, \quad \hat{C}_{ijt} = \frac{C_{ijt}}{C_{jt}}. \tag{18}$$

The real wage rate is given by

$$\tilde{W}_t = \frac{W_t}{P_t}, \tag{19}$$

and aggregate real fiscal variables and dividends are given by

$$\tilde{B}_t = \frac{B_t}{P_t}, \quad \tilde{T}_t = \frac{T_t}{P_t}, \quad \tilde{D}_t = \frac{D_t}{P_t}. \tag{20}$$

Household real bonds holdings, real taxes and real dividends for each type of households are defined analogously.

3.7. Non-Stochastic Steady-State. The non-stochastic steady-state is an equilibrium of the economy in the absence of shocks, with the property that real quantities, relative prices, the nominal rate, and inflation remain constant over time. In the following, variables without time-subscript denotes steady-state values.

In a non-stochastic steady-state where hours worked are symmetric across household types, $L = L^{\mathcal{H}} = L^{\mathcal{S}}$, the first-order condition of a hand-to-mouth household is

$$\tilde{W} = \varphi^{\mathcal{H}} L^{\psi} (C^{\mathcal{H}})^{\gamma}, \tag{21}$$

with

$$C^{\mathcal{H}} = \tilde{W} L - T^{\mathcal{H}}. \tag{22}$$

The first-order conditions of an optimizing household are

[12]For example, when households have perfect information, $1 = \int_0^1 \hat{P}_{it}^{1-\tilde{\theta}} di$, and $\tilde{\theta} = \theta$.

$$\tilde{W} = \varphi^{\mathcal{S}} L^{\psi} (C^{\mathcal{S}})^{\gamma}, \tag{23}$$

and

$$\frac{R}{\Pi} = \frac{1}{\beta}. \tag{24}$$

The optimality condition for relative consumption for both types of household types is

$$\hat{C}^{h}_{ij} = \hat{P}_i^{-\theta}. \tag{25}$$

Lastly, firm i's first order condition is

$$\hat{P}_i = \frac{\tilde{W}}{\alpha} (\hat{P}_i^{-\theta} C)^{\frac{1-\alpha}{\alpha}}. \tag{26}$$

Equation (26) implies that all firms set the same price. Equation (25) then implies that each household consumes the same relative quantity of each consumption variety. Thus, all firms produce the same output, and since all firms have the same productivity, they also have the same labor input. Aggregate labor determines aggregate output, which in turn determines the market-clearing real wage and aggregate dividends. Equation (21), Equation (22) and Equation (23) determine the consumption levels.

The Euler equation of optimizing households, Equation (24), determines the real interest rate, but not R and Π individually. Thus, we will assume a constant price level such that $\Pi = 1$ and $R = \beta^{-1}$ and posit an initial value, P_{-1}, for the price level.

Given initial values for nominal bonds held by savers, $B_{-1} = B_{j,-1}\ \forall j \in [\phi, 1]$, fiscal variables are uniquely determined in the non-stochastic steady-state. The reason is that real bond holdings, B_{-1}/P_{-1}, is a quantity that must remain constant, and this can only be the case if the government runs a balanced budget in real terms. Thus, real lump-sum taxes must equate the sum of real interest and subsidy payments.

Given values for L, B and $T^{\mathcal{H}}$, all non-stochastic steady-state variables can be computed. Appendix C combines some of these relationships into expressions used in the approximation of the firms and households' objectives.

4. Attention Problems

Decision-makers subject to rational inattention face a tradeoff where processing more information increases their expected payoff, but requires more attention, which is costly. This section describes the maximization problems faced by decision-makers in the inattentive economy.

The attention problem is a linear quadratic Gaussian (LQG) dynamic rational inattention problem. The objective is quadratic in the state and the decision-maker's actions, the state follows linear dynamics with Gaussian innovations, and the information cost is linear in Shannon's mutual information.

To express the objectives from Section 3 as LQG-RI problems, we compute log-quadratic approximations of the expected discounted sum of profits and utilities around the non-stochastic steady-state. More details are provided below.

An inattentive decision-maker, indexed by ι, maximizes by choosing a costly attention strategy consisting of signal(s) about the state of the economy. Formally, the decision-maker solves:

$$\max_{\boldsymbol{\Gamma}_\iota, \boldsymbol{\Sigma}_{\boldsymbol{\nu}\iota}} \left\{ \sum_{t=0}^{\infty} \beta^t E_{\iota,-1}\left[\frac{1}{2}(\boldsymbol{x}_{\iota t} - \boldsymbol{x}^*_{\iota t})' \boldsymbol{H}_{\boldsymbol{x}_\iota} (\boldsymbol{x}_{\iota t} - \boldsymbol{x}^*_{\iota t})\right] - \lambda_\iota \sum_{t=0}^{\infty} \beta^t I(\boldsymbol{\xi}_{\iota t}; \boldsymbol{S}_{\iota t} | \mathcal{I}_{\iota t-1}) \right\} \tag{27}$$

subject to

$$\boldsymbol{x}^*_{\iota t} = \boldsymbol{G}\boldsymbol{\xi}_{\iota t} \tag{28}$$

$$\boldsymbol{\xi}_{\iota t+1} = \boldsymbol{F}\boldsymbol{\xi}_{\iota t} + \boldsymbol{\mu}_{\iota t+1} \tag{29}$$

$$\boldsymbol{x}_{\iota t} = E[\boldsymbol{x}^*_{\iota t} | \mathcal{I}_{\iota t}] \tag{30}$$

$$\mathcal{I}_{\iota t} = \mathcal{I}_{\iota,-1} \cup \{\boldsymbol{S}_{\iota 0}, ..., \boldsymbol{S}_{\iota t}\} \tag{31}$$

$$\boldsymbol{S}_{\iota t} = \boldsymbol{\Gamma}_{\boldsymbol{\iota}}\boldsymbol{\xi}_{\iota t} + \boldsymbol{\nu}_{\iota t} \tag{32}$$

$$I(\boldsymbol{\xi}_{\iota t}; \boldsymbol{S}_{\iota t} | \mathcal{I}_{\iota t-1}) = H(\boldsymbol{\xi}_{\iota t} | \mathcal{I}_{\iota t-1}) - H(\boldsymbol{\xi}_{\iota t} | \mathcal{I}_{\iota t}). \tag{33}$$

with

$$\mathcal{I}_{\iota-1} \mid \boldsymbol{\Gamma}_{\boldsymbol{\iota}}, \boldsymbol{\Sigma}_{\boldsymbol{\nu}_{\boldsymbol{\iota}}} \tag{34}$$

The vector $\boldsymbol{x}_{\iota t}$ contains the decision-maker's actions and the vector $\boldsymbol{x}^*_{\iota t}$ contains the actions they would take under perfect information. The first term appearing in Equation (27) is the part of per-period payoff (i.e. expected losses incurred from suboptimal actions) that is affected by the decision-maker actions and has a quadratic form with weighting matrix $\boldsymbol{H}_{\boldsymbol{x}_\iota}$.

The second term in Equation (27) is a known quantity representing the discounted sum of information costs. The per-period information cost consists of the product between the marginal cost of attention, λ_ι, and the per-period information flow.

Equation (28) defines a linear mapping between the current state, $\boldsymbol{\xi}_{\iota t}$, and the vector of optimal actions. Given that structural shocks are Gaussian by assumption, we know that there exists at least one representation for the state vector for which this equality holds exactly[13].

Equation (29) describes the evolution of the state vector between two consecutive periods where $\boldsymbol{\mu}_{\iota t+1}$ follows a white noise vector process with covariance $\boldsymbol{\Sigma}_{\boldsymbol{\mu}_\iota}$ and $\boldsymbol{F}$ is matrix with eigenvalues that may lie on the unit circle. Thus, the state-space for $\boldsymbol{\xi}_{\iota t}$ and $\boldsymbol{x}^*_{\iota t}$, described by Equation (28) and Equation (29), is linear with Gaussian innovations, but stationnarity is not imposed[14].

The decision-maker's information set in the current period is decscribed by Equation (31). It consists of the initial information, $\mathcal{I}_{\iota,-1}$, and all signals received up to and including the current period.

Equation (32) describes the signal received in period t. This equation posits that the signal loads on the period t state vector according to the matrix $\boldsymbol{\Gamma}_\iota$ plus $\boldsymbol{\nu}_{\iota t}$, a white noise vector process with diagonal covariance matrix $\boldsymbol{\Sigma}_{\boldsymbol{\nu}_\iota}$[15].

Equation (33) measures the per-period information flow as the difference in the conditional entropy, $H(\cdot \mid \cdot)$, about the state before and after observing the signal in period t. This quantity represents Shannon's mutual information and quantifies the per-period uncertainty reduction.

Equation (30) describes how the decision-maker optimally selects $\boldsymbol{x}_{\iota t}$ according to his information set. Given the linear Gaussian structure, the decision-maker applies the Kalman filter to optimally infer $\boldsymbol{x}^*_{\iota t}$ from any sequence of noisy signals.

Equation (34) states that the initial information set is not entierly exogenous, but at least some of its characteristics depend on the attention strategy chosen by the decision-maker. The exact relationship is described below.

The decision-maker optimizes freely over the matrices $\boldsymbol{\Gamma}_\iota$ and $\boldsymbol{\Sigma}_{\boldsymbol{\nu}_\iota}$ (i.e., the rank of these matrices which determines the total number of signals received is endogenous)

[13]However, the relevant state vector in the presence of information frictions may be infinite-dimensional, and solving this problem numerically requires some level of approximation.

[14]For the maximization problem to be well-defined, all we need is the conditional second moments to be finite which does not require stationnarity. We allow for unit roots dynamics.

[15]It can be shown that a signal loading on the current state plus a vector of *i.i.d.* Gaussian noise, as described in Equation (32), is of the optimal form given the decision-maker problem defined by Equations (27) to (33). For a formal proof, see Maćkowiak, Matějka, and Wiederholt (2018).

to maximize the difference between the expected discounted sum of per-period payoffs and information costs. The fundamental tradeoff is that receiving more informative signals raises both the expected payoff and information costs. All decisions regarding the attention strategy that maximize Equation (27) are made in period -1, with Equations (30) to (34) taken as given.

I make the standard assumption that the initial information set, $\mathcal{I}_{\iota,-1}$, depends on the chosen matrices $\boldsymbol{\Gamma}_{\boldsymbol{\iota}}$ and $\boldsymbol{\Sigma}_{\boldsymbol{\nu}_{\boldsymbol{\iota}}}$. Specifically, given its chosen attention strategy the decision-maker receives a long sequence of signals that places him at the steady-state of the Kalman filter in period -1. This assumption ensures that the problems of choosing $\{\boldsymbol{\Gamma}_{\iota t}, \boldsymbol{\Sigma}_{\boldsymbol{\nu}_{\iota t}}\}_{t=0}^{\infty}$ sequentially or $\boldsymbol{\Gamma}_{\boldsymbol{\iota}}$ and $\boldsymbol{\Sigma}_{\boldsymbol{\nu}_{\boldsymbol{\iota}}}$ once and for all in period -1 are equivalent.

All noise terms in the signals are idiosyncratic, meaning that realizations of $\boldsymbol{\nu}_{\iota t}$ are independent across firms and households, and sum to zero in the cross-section. We use this property when aggregating individual decisions.

In the following, we present and describe the problems of each agent in the inattentive economy. To keep things brief, we abstract from cross-sectional efficiency decisions (i.e., households' consumption mix), as these can be shown to be optimally independent from those affecting aggregate dynamics. Lastly, we assume that paying attention to idiosyncratic shocks is a separate activity. Our results are independent of this assumption, but it is practical as it allows us to remain agnostic about the stochastic process governing firm-specific productivity.

4.1. Firms. The step-by-step approximation to the expected discounted sum of firm $i \in [0,1]$'s period profits can be found in Appendix D.1. The resulting matrix featured in firm i's period payoff is

$$\boldsymbol{H}_{\boldsymbol{x}_i} = -(C^{\mathcal{S}})^{-\gamma} Y \left[\frac{\tilde{\theta}(\tilde{\theta}+\alpha(1-\tilde{\theta}))}{\alpha} \right]. \tag{35}$$

Firm i's vector of choice variables is

$$\boldsymbol{x}_{it} = \big(p_{it}\big)' \tag{36}$$

and its vector of optimal actions in response to aggregate shocks is

$$\boldsymbol{x}^*_{it} = \left(p_t + \frac{\frac{1-\alpha}{\alpha}}{1+\frac{1-\alpha}{\alpha}\tilde{\theta}} c_t + \frac{1}{1+\frac{1-\alpha}{\alpha}\tilde{\theta}} \tilde{w}_t - \frac{\frac{1}{\alpha}}{1+\frac{1-\alpha}{\alpha}\tilde{\theta}} a_t \right)'. \tag{37}$$

4.2. Hand-to-mouth. The step-by-step approximation to the expected discounted sum of hand-to-mouth household $j \in [0,\phi]$'s period utilities can be found in Appendix D.2. The resulting matrix featured in household j's period payoff is

$$\boldsymbol{H}^{\mathcal{H}}_{\boldsymbol{x_j}} = -(C^{\mathcal{H}})^{1-\gamma}\left[\omega^{\mathcal{H}}_{W}(\omega^{\mathcal{H}}_{W}\gamma+\psi)\right]. \tag{38}$$

Household j's vector of choice variables is

$$\boldsymbol{x}_{jt} = \begin{pmatrix} l_{jt} \end{pmatrix}', \tag{39}$$

its vector of optimal actions is

$$\boldsymbol{x}^*_{jt} = \begin{pmatrix} \frac{\tilde{w}_t-\gamma c^*_{jt}}{\psi} \end{pmatrix}, \tag{40}$$

with

$$c^*_{jt} = \omega^{\mathcal{H}}_{w}(\tilde{w}_t + l^*_{jt}) \tag{41}$$

4.3. Savers. The step-by-step approximation to the expected discounted sum of saver $j \in [\phi, 1]$'s period utilities can be found in Appendix D.3. An additional step is taken to obtain an expression that reduces to a pure tracking problem,[16] with details provided in Appendix D.4. The resulting matrix featured in household j's period payoff is

$$\boldsymbol{H}^{\mathcal{S}}_{\boldsymbol{x_j}} = -(C^{\mathcal{S}})^{1-\gamma}\begin{bmatrix}\gamma & 0\\ 0 & \omega^{\mathcal{S}}_{w}\psi\end{bmatrix}. \tag{42}$$

Household j's vector of choice variables is

$$\boldsymbol{x}_{jt} = \begin{pmatrix} c_{jt}, & l_{jt} \end{pmatrix}' \tag{43}$$

and its vector of optimal actions is

$$\boldsymbol{x}^*_{jt} = \begin{pmatrix} E_{jt}\left[-\frac{1}{\gamma}(r_t-\pi_{t+1}) + c^*_{jt+1}\right] \\ \frac{\tilde{w}_t-\gamma c^*_{jt}}{\psi} \end{pmatrix} \tag{44}$$

with c^*_{jt} and c^*_{jt+1} defined accordingly to Equation (106).

The change of variables in Appendix D.4 reformulates the savers' problem in terms of consumption and labor decisions. This transformation eliminates the intertemporal interaction in the loss function that arises when households instead choose their labor supply and bond holdings, as in Appendix D.3. The intuition for

[16] In a pure tracking problem, the objective is independent of the relationship between today's and tomorrow's mistakes. In other words, the state is purely exogenous, and all that matters for the decision-maker is to keep $\boldsymbol{x}_{jt}$ as close as possible to $\boldsymbol{x}^*_{jt}$.

why, under this formulation, errors today do not carry over to subsequent periods is that households can always adjust their labor supply to satisfy their intratemporal optimality condition for any chosen level of consumption.

5. Equilibrium under Rational Inattention

An equilibrium of the inattentive economy, consisting of Section 3's environment and Section 4's attention problems, is a fixed-point where each decision-maker's signals are optimal, given the signals selected by all others.

In this section, we first define an equilibrium for a linear approximation of the economy's dynamics aroundthe non-stochastic steady-state[17] where variables are expressed in terms of log-deviations (i.e., $x_t := \frac{X_t - X}{X}$) from this point. We then outline a numerical procedure to solve for the economy's fixed-point.

Definition 1. *For any sequence of exogenous innovations,* $\{\boldsymbol{\varepsilon}_t\}_{t=0}^{\infty}$*, sequence of noise,* $\{\boldsymbol{\nu}_{it}\}_{t=0}^{\infty}$*, for every firm* $i \in [0,1]$ *and sequence of noise,* $\{\boldsymbol{\nu}_{jt}\}_{t=0}^{\infty}$*, for every household* $j \in [0,1]$ *a Sequential Rational Inattention Competitive Equilibrium (SRICE) is:*

- *An allocation,* $\Omega_i := \left\{\boldsymbol{S}_{it}, y_{it}, l_{it}^d, p_{it}\right\}_{t=0}^{\infty}$ *for every firm* $i \in [0,1]$.
- *An allocation,* $\Omega_j^{\mathcal{H}} := \left\{\boldsymbol{S}_{jt}, c_{jt}, l_{jt}^s, \{\hat{c}_{ijt}\}_{i\in[0,1]}\right\}_{t=0}^{\infty}$ *for every hand-to-mouth* $j \in [0,\phi]$.
- *An allocation,* $\Omega_j^{\mathcal{S}} := \left\{\boldsymbol{S}_{jt}, c_{jt}, l_{jt}^s, b_{jt}^d, \{\hat{c}_{ijt}\}_{i\in[0,1]}\right\}_{t=0}^{\infty}$ *for every saver* $j \in [\phi, 1]$.
- *An allocation,* $\Omega_G := \{b_t^s, t_t\}_{t=0}^{\infty}$ *for the governement.*
- *A set of prices* $\{p_t, w_t, r_t\}_{t=0}^{\infty}$.
- *Stationnary distributions of households and firms over idiosyncratic innovations and noise.*

such that

1. *Given* $\left\{\Omega_{j\in[0,\phi]}^{\mathcal{H}}, \Omega_{j\in[\phi,1]}^{\mathcal{S}}, p_t, r_t, w_t\right\}_{t=0}^{\infty}$*, a firm's allocation,* Ω_i*, solves Section 4.1's attention problem* $\forall i \in [0,1]$.

[17] We cannot define a global equilibirum as Section 4's attention problems are only valid for linear Gaussian dynamics.

2. Given $\left\{\Omega_{i\in[0,1]}, \Omega^{\mathcal{S}}_{j\in[\phi,1]}, p_t, r_t, w_t\right\}_{t=0}^{\infty}$*, a hand-to-mouth's allocation,* $\Omega^{\mathcal{H}}_j$*, solves* *Section 4.2's attention problem* $\forall j \in [0, \phi]$.

3. Given $\left\{\Omega_{i\in[0,1]}, \Omega^{\mathcal{H}}_{j\in[0,\phi]}, p_t, r_t, w_t\right\}_{t=0}^{\infty}$*, a saver's allocation,* $\Omega^{\mathcal{S}}_j$*, solves Section 4.3's attention problem* $\forall j \in [\phi, 1]$.

4. Monetary policy satisfies the Taylor rule, fiscal policy satisfy its specified rules, and given interest and subsidies payments, the governement budget is satisfied.

5. All markets clear $\forall t \geq 0$ *:*

   (a) $y_t = c_t$
   (b) $l^s_t = l^d_t$
   (c) $b^s_t = b^d_t$
   (d) $y_{it} = c_{it}\ \forall i \in [0, 1]$

6. Aggregate price index is given by $\forall t \geq 0$ *:*

   (a) $p_t = \int_0^1 p_{it} di$

7. Aggregate quantities are given by $\forall t \geq 0$ *:*

   (a) $y_t = \int_0^1 y_{it} di$
   (b) $c_t = \phi(C^{\mathcal{H}}/C) \int_0^{\phi} c_{jt} dj + (1-\phi)(C^{\mathcal{S}}/C) \int_{\phi}^1 c_{jt} dj$
   (c) $l^s_t = \phi(L^{\mathcal{H}}/L) \int_0^{\phi} l^s_{jt} dj + (1-\phi)(L^{\mathcal{S}}/L) \int_{\phi}^1 l^s_{jt} dj$
   (d) $l^d_t = \int_0^1 l^d_{it} di$
   (e) $b^s_t = (1-\phi)(B^{\mathcal{S}}/B) \int_{\phi}^1 b^d_{jt} dj$
   (f) $c_{it} = \phi(C^{\mathcal{H}}_i) \int_0^{\phi} c_{ijt} dj + (1-\phi)(C^{\mathcal{S}}_i) \int_{\phi}^1 c_{ijt} dj$

5.1. Computing the Aggregate Equilibrium. We now describe an iterative procedure for solving the SRICE numerically. First, notice that an equilibrium (of aggregate prices and quantities) in the inattentive economy can always be summarized by a finite set of stochastic processes for exogenous and endogenous variables. In this case, the dynamics of the price level, the real wage, and the consumption of both household types, along with the exogenous processes for aggregate technology and monetary policy shocks, are sufficient to determine the responses of all other aggregate variables. We can solve for the endogenous processes with arbitrary precision as follows;

Algorithm. First, we guess the impulse responses over T periods for p_t, $\tilde{w}_t$, $c_t^{\mathcal{H}}$, and $c_t^{\mathcal{S}}$. This is equivalent to guessing the moving-average representation, $MA(T)$, of these variables.

Second, given these guesses, we compute the profit-maximizing price for a firm and approximate the resulting process with a finite-order VARMA[18], which yields a state-space representation of the same form as Equations (28) to (29).

Third, we solve the firm's attention problem in Section 4.1 and compute the resulting stochastic processes for the aggregate price level and labor demand, given optimal signals for the firms and initial guesses.

Fourth, we compute optimal labor supply for each type of household and the optimal consumption for savers, given the stochastic processes for the price level and initial guesses. We obtain a state-space representation for their attention problems by approximating the dynamics of utility-maximizing actions with finite VARMA processes.

Fifth, we solve the Section 4.2 and Section 4.3's attention problems, and compute the resulting stochastic processes for consumption and labor supply for each household type, given households' optimal signals and guesses.

Sixth, we use the dividend equation, labor demand and labor supply to compute a process for the real wage.

Seventh, we compare the $MA(T)$ representations of the stochastic processes for p_t, $\tilde{w}_t$, $c_t^{\mathcal{H}}$, and $c_t^{\mathcal{S}}$ with their initial guesses. If any process differs by more than a prespecified tolerance, we update them via linear extrapolation, and repeat the procedure from step two until convergence. Once convergence is achieved, we verify that labor demand equals labor supply; if so, we have found a fixed point.

When solving the attention problems, we iterate on the first-order condition using Afrouzi and Yang (2021)'s algorithm. To approximate the $MA(T)$ representations of stochastic processes as finite VARMA, we apply the frequency-domain projection method of Han, Tan, and Wu (2022). Non-stationary processes are first differenced before the projection step, and an additional row is appended to the state-space representation to restore unit root dynamics.

6. Dynamics under Rational Inattention

With the models set up and the numerical algorithm defined, we are now ready to analyze the dynamics of the inattentive economies.

[18] That is, we approximate firms' optimal price decisions with a reduced-form VAR(1) with two exogenous shocks.

6.1. Evaluation Strategy. We follow a two-step procedure. In the first step, we calibrate all structural and policy parameters except the marginal costs of attention. In the second step, we solve the models over a grid of values for marginal costs of attention. Our objective is to isolate the role of households' inattention as a propagation mechanism for the dynamic response of output to shocks. We proceed as follows: for every value of households' marginal cost of attention, we choose the firm's marginal cost of attention that allows us to match the persistence of inflation in the data (0.62). We retain the household inattention parameter that bring us the closest the persistence of output (0.3) in the data. Our procedure differs from a standard minimization approach, which may lead to different inflation persistence across models.

Note that because our model contains only a subset of all the possible shocks affecting the economy, we are implicitly assuming that the conditional responses to technological and monetary policy shocks, taken together, generate persistence in inflation and output growth that is not too far from what would arise if all sources of fluctuations were considered. We believe that to be generally true.[19]

We evaluate the fit of the models based on how well they generate persistence in the growth rate of output, and match other macro and micro moments discussed below. A priori, there may not exist any combination of attention parameters that can induce persistence in the growth rate of output.

When using data to calibrate parameters, we rely on empirical evidence from the Unites States covering the period 1969Q1–2019Q4. However, we employ the full available SCE sample to characterize micro moments as it provides more variation in inflation and other macroeconomic variables to identifying patterns in cross-sectional expectations. Restricting the analysis to the overlapping period, would not qualitatively nor significantly change our estimates.

Calibrated Parameters. The left panel of Table 4 summarizes the calibrated parameters of our model. We set the inverse of the elasticity of intertemporal substitution, γ, to 1.5 which allows for some interaction with steady-state marginal utility of consumption in the households' attention problems. We set the inverse of the Frisch elasticity, ψ, to 1.0. We assume a discount factor of $\beta = 0.99$, which corresponds to an annual return of 4% on government bonds. Steady-state labor is set to $\frac{1}{3}$. Aggregate technology, A, is used to normalize steady-state output to $Y = 1$. The labor share is set to $\alpha = 0.66$. The price and wage elasticity, $\tilde{\theta}$, and $\tilde{\eta}$ are set to 4, as in Maćkowiak and Wiederholt (2015).

[19]In a previous version of the paper, we jointly estimated the impulse responses to both of these shocks and found this to hold. Results hinge on the estimation methods and identifying assumptions.

We assume a Taylor rule with interest rate persistence, ρ_r, of 0.9, and reaction coefficients to inflation and the output gap, ϕ_π and ϕ_{y^*}, of 1.5 and 0.125, respectively.

We estimate the stochastic process for technology using the Fernald (2014) measure of Total Factor Productivity adjusted for utilization. Specifically, we linearly detrend the cumulated series and estimate a first-order autoregressive process. The resulting estimates are a persistence parameter, ρ_a, of 0.95 and a standard deviation of shocks, $100\sigma_a$, of 0.8. To estimate the standard deviation of monetary policy shocks, we invert a Taylor rule, using the parameters set above, on time series of interest rates, inflation, and the output gap over our sample. This yields a value for $100\sigma_v$ of 0.2. As long as the monetary policy shocks account for a small portion of the volatility in the nominal interest rate, this procedure should provide accurate estimates, as argued in Carvalho, Nechio, and Tristao (2021).

The share of hand-to-mouth households, ϕ, is set to 0.28, the average value over our sample. We estimate the fraction of hand-mouth in each available waves of the *Survey of Consumer Finances* (SCF) using Kaplan and Violante (2014)'s method. To determine the consumption ratio, $\frac{C^{\mathcal{S}}}{C^{\mathcal{H}}}$, we infer a proxy for consumption expenditure for each type of household, and take the average ratio over the sample, yielding a value of 1.4. We obtain the desired ratio by targeting the non-stochastic steady-state value of taxes $T^{\mathcal{H}}$. Details on the estimation of the share of hand-to-mouth and computation of the consumption proxies are provided in Appendix F.

Table 4: Calibrated and Estimated Parameters

Panel A: Calibrated Parameters

Parameter		Value
β	Discount factor	0.99
γ	EIS	1.5
ψ	Inverse Frisch	1.0
α	Labor share	0.66
$\tilde{\theta}$	Price elasticity of demand	4.0
ϕ	HtM share	0.28
$\frac{C^{\mathcal{S}}}{C^{\mathcal{H}}}$	Steady-state consumption ratio	1.4
ρ_r	Taylor rule inertia	0.9
ϕ_π	Taylor rule coefficient (inflation)	1.5
ϕ_{y^*}	Taylor rule coefficient (output gap)	0.125
ρ_a	Persistence of aggregate technology	0.9
$100\sigma_a$	100× S.D. of aggregate technology shocks	0.8
$100\sigma_v$	100× S.D. of monetary policy shock	0.2

Panel B: Inattention Parameters

Parameter		Value
RI-I: competitive-wage		
λ_j	Firms marginal cost of attention	485 (0.15)
λ_j	Households marginal cost of attention	0.8 ($\mathcal{H}$: 1.20 ; $\mathcal{S}$: 2.10)
RI-II: monopolistic-wage		
λ_j	Firms marginal cost of attention	33.0 (0.66)
λ_j	Householdss marginal cost of attention	5.8 ($\mathcal{H}$: 1.20 ; $\mathcal{S}$: 0.75)
RI-F: inattentive firms, attentive households		
λ_j	Firms marginal cost of attention	360 (0.180)

Notes: Marginal attention costs scaled up by 100,000. In parentheses, the implied information flows per period in *bits*.

Note that given our calibration, we do not encounter the knife-edge case where wealth and substitution effects perfectly offset each other (e.g., this occurs when $\psi = \gamma = \omega_w^{\mathcal{H}} = 1$, as in Bilbiie 2008) that would lead to a trivial attention problem for the hand-to-mouth.

6.2. Models' Fit. Table 5 presents the unconditional macro moments from the data and the baseline models. Figure 1 shows the impulse response functions of output and inflation to a positive technological shock and an expansionary monetary policy shock.

Table 5: Unconditional Moments I

	Data	RI-I	RI-II	RI-F	PI
Targeted Moments					
ρ_π	0.62	0.62	0.62	0.62	0.023
$\rho_{\Delta y}$	0.3	-0.06	0.3	-0.14	-0.025
Untargeted Moments					
$\rho_{\Delta w}$	0.48	-0.20	0.63	-0.18	-0.025
$\sigma_\pi/\sigma_{\Delta y}$	1.06	0.33	1.15	0.37	1.17
$\sigma_{\Delta w}/\sigma_{\Delta y}$	1.10	3.72	0.80	3.07	2.81
$\beta_{\pi,\mathcal{H}}$	1.73	0.42	-0.07	-	-

Notes: Models' moments computed from the equilibrium MA representations. RI-I is the model with competitive-wage. RI-II is the model with monoplistic-wage. RI-F is the model with firms subject to inattention, and households with perfect information.

We begin by commenting on the model RI-I, in which the wage rate is determined competitively. In terms of macro moments, the model provides a poor fit. Adding inattention on the side of households is a quantitatively weak propagation mechanism, providing only a slight improvement in output growth persistence compared to the model RI-F, wherein households are perfectly attentive. We trace most of this shortcoming back to the real wage, which absorb most of the effects from shocks and is, in turn, too volatile and insufficiently persistent compared to the data.

We can decompose the mechanics leading to these dynamics and its consequences on output as follows. First, while savers adjust consumption with a delay due to inattention, increasing the persistence of output growth, they also insufficiently adjust their labor supply. Consequently, following any shock, the real wage must fluctuate even more than in a already problematic perfect-information economy for the labor market to clear. The disposable income of hand-to-mouth agents also becomes more volatile, which has a negative effect on the autocorrelation of output growth. As a result, household inattention has a non-monotonic effect on the persistence of output growth in this model, reaching a peak at households' marginal attention costs presented in Table 4.

On the other hand, household inattention yields expectations for inflation that are consistent with the data. The model-implied coefficient of hand-to-mouth status on the absolute value of one-year-ahead inflation forecast errors is 0.42. This value is smaller than the one reported in Table 3 for our preferred proxy. However, given the parsimonious economic environment, which cannot capture all the incentives households face to pay attention, this is a meaningful difference. We can understand this result as follows. The competitive labor market provides little incentive for hand-to-mouth to monitor the economy, only small fluctuations in their labor supply are necessary to equalize their marginal rate of substitution to the real wage. The savers must track the real interest rate and adjust labor supply by a larger margin, leading them to pay greater attention. This result remains robust to changes in preference parameters, including the inverse of the EIS and Frisch elasticity.

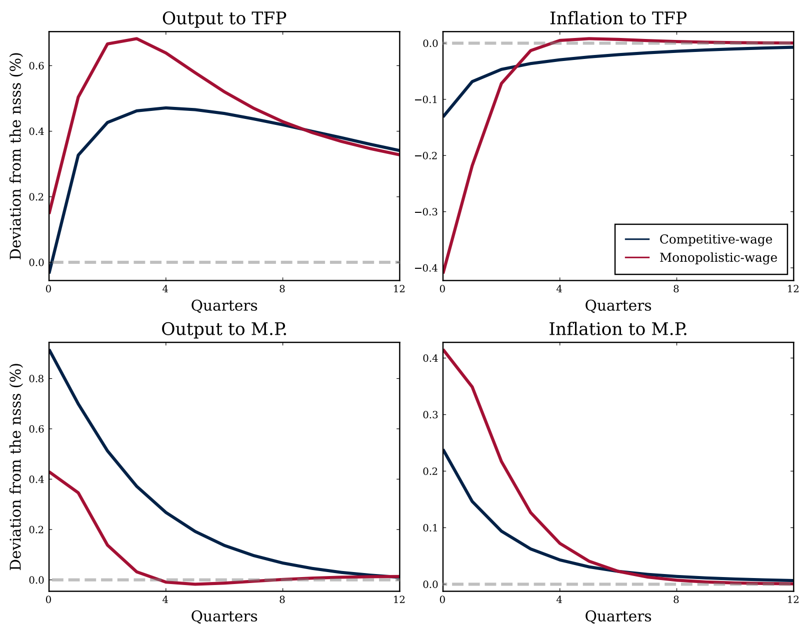

Figure 1: Impulse responses of inflation and output under rational inattention.

We now comment on the baseline with monopolistic wages. In terms of fitting the macro moments, the model provides a sharp improvement compared to all other versions considered. Not only can it generate the desired persistence for output growth, but it also yields values for untargeted moments that are close to their empirical counterparts. In this setting, households' inattention is a strong propagation mechanism, as it also induce persistence in the growth rate of labor income, which then generates hump-shaped consumption responses for hand-to-mouth agents.

These improvements, however, come at a cost in terms of micro moments related to expectations. This failure is due to the stakes associated with wage-setting decisions being so high that they constitute the main incentive for why households pay attention to the state of the economy. In this setting, it even leads hand-to-mouth agents to form expectations that are more accurate than those of the savers, for whom the additional decisions provide only marginal incentives to pay attention.

We have two main takeaways from the impulse response functions presented in Figure 1. The first is that the failure of model RI-I to generate output growth persistence mainly resides in the response conditional on monetary innovations. This is to be expected, as the real rate responds to this shock monotonically and therefore induces negative autocorrelation under perfect information for the growth rate of consumption, making it harder to reverse. Secondly, for the same level of persistence, inflation is less volatile in model RI-I than in RI-II. This occurs despite the higher volatility of the real wage, as firms need to be (too) inattentive, which ultimately reduces the volatility of inflation even more so than in model RI-II.

6.3. The Relevance of Heterogeneity. A common exercise with heterogeneous agent models is to assess how much their dynamics differ from those of representative agent counterparts[20]. In our framework, this experiment specifically determines the extent to which the presence of hand-to-mouth households, with an MPC of one, alters the dynamics. We thus proceed to solve both baseline versions setting $\phi = 0.0$, without any changes to the other parameters.

Table 6: Unconditional Moments II

		RI-I		RI-II	
	Data	TA	RA	TA	RA
ρ_π	0.62	0.62	0.75	0.62	0.69
$\rho_{\Delta y}$	0.3	-0.06	0.19	0.3	0.34
$\rho_{\Delta w}$	0.48	-0.20	-0.12	0.63	0.59
$\sigma_\pi/\sigma_{\Delta y}$	1.06	0.33	0.29	1.15	1.10
$\sigma_{\Delta w}/\sigma_{\Delta y}$	1.10	3.72	4.05	0.80	0.85

Notes: Models' moments computed from the equilibrium MA representations. TA for two-agent model. RA for representative-agent model.

Table 6 presents the unconditional macro moments, and Figure 2 shows the im-

[20]An RI-DSGE is never, per se, a representative-agent model, as a continuum of households is required for noise to sum to zero in the cross-section. However, the average dynamics obtained when assuming that all households are savers are, for all purposes, the same.

pulse responses implied by the baseline two-agent economies and their Ricardian counterfactuals. Both confirm our previous comments on the strength and mechanisms by which households' inattention induces persistence in the growth rates of quantities. The predictions of model RI-II with monopolistic wages are barely affected when hand-to-mouth households are not present. In other words, there is no tension between the effects of the responses of each type of household on aggregate output; they both pull in the same direction. On the other hand, the Ricardian economy with competitive wages now displays positive output growth autocorrelation. This reversal occurs because labor income is irrelevant for savers' consumption decisions, and therefore the wage rate, which remains too volatile, does not hinder the capacity of inattention to induce sluggishness.

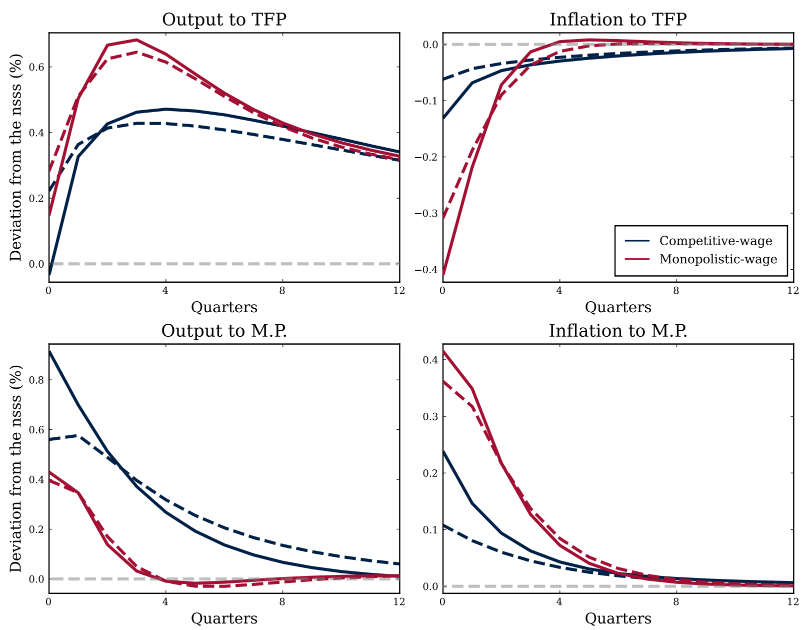

Figure 2: Impulse responses in two-agent and representative-agent economies.

Notes: Solid lines (—) TA models. Dashed lines (–––)Ricardian models.

This experiment tell us that for rational inattention, or any form of information friction, to act as a strong propagation mechanism in the presence of hand-to-mouth households, it must be able to induce autocorrelation in the growth rate of labor income which depends on the model's microfoundations.

6.4. Inequality Dynamics. We conclude this section by commenting on the average dynamic responses of different variables for both types of households. Figure 3 shows the impulse response functions of consumption, hours worked, and labor income for savers and hand-to-mouth households in the two baseline economies.

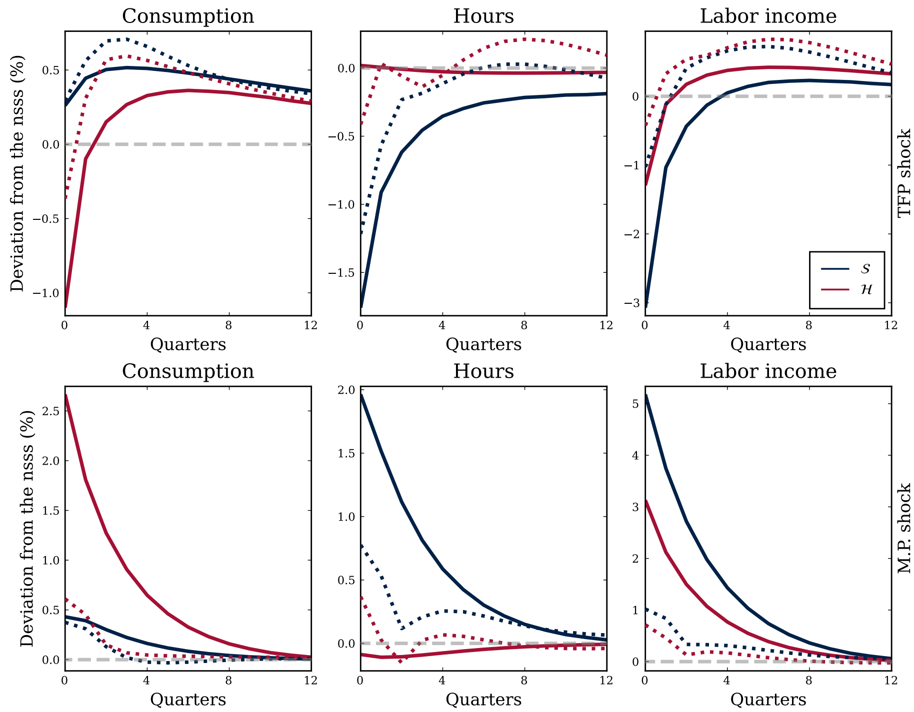

Figure 3: Impulse responses of consumption, hours, and labor income.

Notes: Solid lines (—) RI-I model's dynamics. Dotted lines (· · ·) RI-II model's dynamics.

It is worthwhile to start by remembering that, at the core of both models, is the households' intratemporal optimality condition, which states that the real wage should equate the marginal rate of substitution. Under rational inattention, this condition does not hold at equality, but still dictates what households aim to achieve when deciding how much labor to supply or when setting wages.

Savers' consumption is always strictly determined by their perception of the path of the real interest rate, and, through general equilibrium effects, is what kickstarts the response of hand-to-mouth households. In particular, the wage rate (whether set or determined in competitive equilibrium) is a function of how much labor is required to produce total output.

In the competitive-wage economy, hand-to-mouth consumption fluctuates significantly with the wage rate required to clear the labor market, but their labor supply is relatively constant. As mentioned earlier, this behavior leaves them close to equalizing their marginal rate of substitution with the real wage and is thus near-optimal. This is not the case for savers, who must also adjust labor supply due to consumption following the real interest rate path. As a result, hours worked (and labor income) fluctuate more for savers than for hand-to-mouth households; an arguably unrealistic prediction given empirical evidence (see Heathcote, Perri, and Violante 2020) but unavoidable in this setting.

Hand-to-mouth consumption fluctuates by several percentage points more than that of savers, as the real wage varies more than the real interest rate. Consumption inequality, defined as the difference between savers and hand-to-mouth, rises significantly in the face of positive supply shocks but decreases with positive demand shocks.

In the economy where households set their wage rate, hours for hand-to-mouth households show variations due to imperfect substitutability, and when combined with sluggish wage adjustment, this dampens the variation of their labor income and consumption. For our calibration, it leads to mild hump-shaped responses to monetary policy shocks, but these can be made arbitrarily more persistent by increasing inattention. Consumption inequality is also reduced, and the counterfactual prediction regarding whose income rises the most in the face of shocks is mitigated.

To sum up this section, for any propagation mechanism to generate strong persistence in output growth in a two-agent model, it must also induce persistence in the growth rate of labor income. When it occurs, the effect works through hand-to-mouth consumption; when it does not, any persistence that remains arises in spite of labor income. To check the robustness of our conclusions, we also solve the models with attentive firms subject to Calvo pricing, which does not alter our results. In the next section, we consider the case where wages are sticky due to adjustment frictions. For details on Calvo prices and wages, see Appendix G.

7. Extensions: Ways out of the Conundrum

In the previous section, we showed that the baseline two-agent inattentive economies fail to simultaneously match macro and micro moments. In particular, the two models with inattentive households can either generate hump-shaped output response to monetary policy shocks *or* yield savers' expectations that are relatively more accurate than those of hand-to-mouth households, but not both. We build on the insights that we gained to propose realistic modifications to the microfoundations that can resolve this conundrum. A brief description of each new model is provided; details are left to the online appendix.

7.1. Labor Unions. The most straightforward way out of the conundrum is to maintain the assumption that labor markets are monopolistic, but that wages are set by representative unions subject to adjustment frictions. We believe it is a plausible assumption that slow wage adjustment is characterized by infrequent contract renegotiations that are not directly carried out by households.

In this case, hand-to-mouth need not pay attention to the economy, while savers face a single consumption–savings decision, which requires tracking the real interest rate. As a result, savers' forecast accuracy cannot be lower than that of hand-to-mouth households and, the real wage is prevented from being overly volatile

This versionof the model allows us to match both micro and macro moments simultaneously. In particular, it turns out that the source of wage stickiness, whether Calvo or inattention, does not matter. However, the microfoundations behind the labor supplied by each household when unions set wages can be deemed weak[21]; for details, see Appendix G. Lastly, the inattentive economy from Section 6 implies average real wages that differ between household types, as each sets its wage rate based on its own marginal rate of substitution. It also happens to be unnecessary for inducing persistence in the growth rate of labor income, and whether there is one union or a union for each type of household does not matter significantly.

7.2. Firms with Monopsony Power. [***Currently being updated.***]

In Section 6, we showed that cross-sectional expectations can match the data when households take the wage rate as given, which lowers the stakes and incentives to pay attention. We may replicate such an environment and also limit the problematic wage volatility by taking inspiration from a number of recent papers arguing that labor markets are often better described as monopsonistic, with market power lying with firms rather than households. In this setting, firms set wages as a markdown relative to their marginal product of labor, while households seek to supply labor so that their marginal rate of substitution equals the real wage.

To set up the model, we assume a Cobb-Douglas production function with two types of labor, and assume that workers are also distinguished by the labor market in which they operate. Specifically, a fraction of households of each type may operate in monopsonistic markets.

Households operating in monopolistic markets face the same problem as in the baseline economy. Households operating in monopsonistic markets supply homogeneous labor which is transformed into firm-specific inputs. There are several possible mechanisms by which this can be micofounded (see online appendix), but it is sufficient to know that all result with a CES relationship between aggregate labor supply

[21] We make the standard assumption that each household supply a continuum of labor services.

in the monopsonistic market, denoted by the superscript $\mathcal{F}$, and firm-specific labor inputs given by

$$L_t^{\mathcal{F}} = \left(\int_0^{\varpi} \left\{ L_{it}^{\mathcal{F}} \right\}^{\frac{\varrho-1}{\varrho}} di \right)^{\frac{\varrho}{\varrho-1}} \tag{45}$$

where ϖ is the share of firms with monopsony power, and ϱ is the labor supply elasticity faced by individual firms. To fully specify the model, we also need to set the shares of hand-to-mouth households and savers operating in monopsonistic markets.

A note of caution on this way out; Alpanda (2024) estimates a medium-scale DSGE model and finds that the share of total labor operating in monopsonistic markets is relatively small. His results are obtained in a setting that assumes adjustment frictions, perfect information, and a representative household.

8. Inattention Compared to Other Frameworks

[***Currently being updated.***]

To the best of our knowledge, this paper is the first that aim to match both the persistence of macro variables and micro moments about expectations in heterogeneous agent models. Previously, other attempts addressed only one of these issues at a time. In this section, we compare these alternative approaches to ours and discuss how they might perform if they attempted to match both simultaneously.

8.1. Habits Formation. One of the most common sources of slow adjustment of quantities in the representative agent literature is habit formation in consumption, as detailed in Appendix G. We find it to have similar effects to that of inattentive savers choosing consumption-savings, which we show to be essential for the model to generate persistence in the growth rate of output even under prices and wages that are sluggish. The mechanism is obviously not suited to induce any differences in cross-sectional expectations.Habit formation can also be viewed as *ad-hoc*, especially in the context of baseline two-agent models, where it can only be introduced in savers' preferences, given that hand-to-mouth households solve a static maximization problem[22].

Moreover, an undesirable property of habit formation in heterogeneous agent settings, highlighted by Auclert et al. (2021), is its effect on the MPC. The mechanism mechanically induces slow adjustment to all shocks, which is not the case for inattention. For example, in a more complex environment, if idiosyncratic shocks are

[22]If households can switch types as in Bilbiie, Primiceri, and Tambalotti 2023, hand-to-mouth preferences can include habit formation.

large, inattentive hand-to-mouth households will preserve a high MPC in response to these shocks while reacting slowly to aggregate fluctuations.

8.2. Exogenous Information Structures. We have in mind frameworks that resemble Gallegos (2024) or Auclert et al. (2021). The former solves a two-agent model with imperfect information, in which signals are exogenously specified as the aggregate stochastic fundamental plus a Gaussian noise term. One could also assume a more complex structure with multiple signals that are linear combinations of endogenous variables, as in Nimark (2014). The latter assumes sticky information with respect to aggregate variables. In theory, such information structures are flexible enough to match any cross-sectional expectations patterns from the data and could induce persistence in macro variables, especially when combined with New Keynesian adjustment frictions. However, the contributions of imperfect information can be hard to disentangle from other sources of slow adjustment in the model.

Some caveats are worth mentioning. First, the choice of signals is completely arbitrary. While it may induce the desired moments for expectations, it is by no means evidence that these are the signals agents would select if solving a maximization problem. Second, and more problematic, is a *Lucas critique* type argument that arises if the model is used for counterfactual experiments. Changing policy parameters or stochastic processes in this environment while keeping the signals fixed is bound to yield results that cannot be taken seriously.

This type of information structure has the advantage of being more tractable than the one we propose. If one specifies signals that are plausible, especially those that load on endogenous variables such as inflation and output, these models can still yield interesting results, particularly if they can be fully estimated.

9. Conclusions.

We solve RI-DSGE with heterogeneous households: hand-to-mouth and savers. Models with standard business cycle labor market structures fail to simultaneously generate hump-shaped responses for macro variables and cross-sectional differences in expectations that match the data. These findings have two main implications. First, in comparison to non-Ricardian economies, inattention is a strong propagation mechanism in the presence of heterogeneous households only if it induces persistence in the growth rate of labor income. Second, inattention does not naturally induce the same cross-sectional expectations patterns as those observed in the data. In particular, the microfoundations, who are the decision-makers exerting influence on which variables, are questions that matter in this setting.

We propose plausible modifications to our baselines that resolve this conundrum. Our results provide insights for future research on how to introduce imperfect information and improve persistence in heterogeneous agent models in a way that is consistent with the data. We conducted our analysis in simple economic environments that abstract from a number of sources of propagation that can be embedded in this class of models, namely cyclical exposure to shocks, precautionary savings, and fiscal policy. In the future, it will be important to include these channels to assess how much they modify our results, in particular whether they make it harder or easier to match moments related to cross-sectional expectations.

Lastly, we want to emphasize that, while one may feel that the models analyzed in this paper are "too" simple compared to state-of-the-art heterogeneous agent economies, the latter are not impervious to the Lucas critique. Hence, for policy experiments, which are one of the main purposes of DSGE models, we believe that a comprehensive toolbox should also include models with endogenous information structures. If their conclusions differ from those of perfect information economies, it is worth taking them into consideration.

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Appendix A. Expectations Accuracy

A.1. Forecasting BVAR The model is estimated on the period 1960M1-2025M1 with Bayesian techniques (Minnesota priors on the VAR coefficients and inverse-Wishart priors on the error covariance matrix) and includes 12 lags of the CPI, Industrial Production, the Unemployment Rate, the market yield on 1-year U.S. Treasury securities, and the first three factors from McCracken and Ng (2016). We estimate the posterior distribution with 100,000 draws which we then use to forecast the interest rate one-year ahead. From these forecasts, we proxy the true probability of interest rates being higher 12M from now, which we compare with the proability reported by each households to measure expectations accuracy.

A.2. Exectations Relative Accuracy Table 7 presents the results of panel regressions on the accuracy of households' expectations and the hand-to-mouth status including the same set of controls as in Section 2 plus binary variables for the income bin to which each household belongs.

Appendix B. Perfect Information Economy

A perfect information economy has the following property

Definition 2. *Every decision-maker has rational expectations and knows the complete history of shocks up to, and including the current period.*

B.1. Equilibrium under Perfect Information. An equilibrium, in a neighborhood of the non-stochastic steady-state, of the frictionless perfect information economy is defined as follows:

Definition 3. *For any sequence of exogenous innovations, $\{\boldsymbol{\varepsilon_t}\}_{t=0}^{\infty}$, a Sequential Perfect Information Competitive Equilibrium (SPICE) is:*

- *An allocation, $\Omega_i := \left\{y_{it}, l_{it}^d, p_{it}\right\}_{t=0}^{\infty}$ for every firm $i \in [0,1]$.*
- *An allocation, $\Omega_j^{\mathcal{H}} := \left\{c_{jt}, l_{jt}^s, \{\hat{c}_{jit})\}_{i\in[0,1]}\right\}_{t=0}^{\infty}$ for every hand-to-mouth $j \in [0,\phi]$.*
- *An allocation, $\Omega_j^{\mathcal{S}} := \left\{c_{jt}, l_{jt}^s, b_{jt}^d, \{\hat{c}_{ijt}\}_{i\in[0,1]}\right\}_{t=0}^{\infty}$ for every saver $j \in [\phi, 1]$.*
- *An allocation, $\Omega_G := \{b_t^s, t_t\}_{t=0}^{\infty}$ for the governement.*
- *A set of prices $\{p_t, w_t, r_t\}_{t=0}^{\infty}$.*

Table 7: Relative Expectations Accuracy of Hand-to-mouth Households

	Negative Income Shock		Liquidity Constraint		Default Probability	
	(1)	(2)	(3)	(4)	(5)	(6)
Hand-to-mouth	0.530***	0.015***	1.450***	0.021***	1.478***	−0.017***
	(0.030)	(0.001)	(0.039)	(0.002)	(0.071)	(0.003)
High School	-	-	-	-	-	-
Some College	−0.636***	−0.011***	−0.681***	−0.011***	−0.748**	−0.008**
	(0.038)	(0.002)	(0.039)	(0.002)	(0.034)	(0.001)
College	−1.484***	−0.045***	−1.450***	−0.041***	−1.772***	−0.051***
	(0.039)	(0.002)	(0.041)	(0.002)	(0.036)	(0.001)
Low Numeracy	-	-	-	-	-	-
High Numeracy	−1.861***	−0.027***	−0.021***	−2.110***	−2.237***	−0.028***
	(0.034)	(0.001)	(0.035)	(0.001)	(0.031)	(0.001)
Unemployed	-	-	-	-	-	-
Part-time employed	0.079	0.014	0.127**	−0.014***	0.034	−0.014
	(0.049)	(0.003)	(0.050)	(0.002)	(0.044)	(0.002)
Full-time employed	−0.148***	−0.002	−0.130***	−0.001	−0.188***	−0.004***
	(0.034)	(0.001)	(0.035)	(0.001)	(0.031)	(0.001)
Income $<$ 50K	-	-	-	-	-	-
Income $\in [50, 100]$	−1.036***	−0.007***	−0.777***	−0.003**	−1.050***	−0.008***
	(0.037)	(0.002)	(0.039)	(0.001)	(0.034)	(0.001)
Income $>$ 100K	−1.444***	−0.021***	−1.153***	−0.015**	−1.504***	−0.022***
	(0.042)	(0.002)	(0.045)	(0.002)	(0.039)	(0.002)
Observations	109,879	109,879	112,972	112,972	156,160	156,160
F Statistic	1521.92	346.71	1831.60	323.94	2471.39	529.33
R^2	0.123	0.424	0.139	0.447	0.134	0.400
Time Fixed Effects	yes	yes	yes	yes	yes	yes

Notes: Columns (1), (3), and (5) show estimates from a regression of the absolute value of inflation forecast errors on the household hand-to-mouth status identified with different proxies. Columns (2), (4), and (6) show estimates from a regression of the absolute value of the forecasts errors of the probability of interest rates going up in 12M on the household hand-to-mouth status identified with different proxies. Robust standard errors in parentheses, * $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$. Sample: 2013M8-2025M1.

- *Stationnary distributions of households and firms over idiosyncratic innovations.*

such that

1. *Given* $\left\{\Omega^{\mathcal{H}}_{j\in[0,\phi]}, \Omega^{\mathcal{S}}_{j\in[\phi,1]}, p_t, w_t\right\}_{t=0}^{\infty}$, *a firm's allocation,* Ω_i, *maximizes its discounted sum of profits.*

2. *Given* $\left\{\Omega_{i\in[0,1]}, \Omega^{\mathcal{S}}_{j\in[\phi,1]}, p_t, r_t, w_t, l_t\right\}_{t=0}^{\infty}$, *a hand-to-mouth's allocation,* $\Omega^{\mathcal{H}}_{j\in[0,\phi]}$, *maximizes its discounted sum of period utilities.*

3. *Given* $\left\{\Omega_{i\in[0,1]}, \Omega^{\mathcal{H}}_{j\in[0,\phi]}, p_t, r_t, w_t, l_t\right\}_{t=0}^{\infty}$, *an saver's allocation,* $\Omega^{\mathcal{S}}$, *maximizes its discounted sum of period utilities.*

4. *Monetary policy satisfies the Taylor rule, fiscal policy satisfy its specified rules and given interest and subsidies payments, the governement budget constraint holds at equality.*

5. *All markets clear* $\forall t \geq 0$ *:*

   (a) $y_t = c_t$
   (b) $l^s_t = l^d_t$
   (c) $b^s_t = b^d_t$
   (d) $y_{it} = c_{it}\ \forall i \in [0,1]$

6. *Aggregate price index is given by* $\forall t \geq 0$ *:*

   (a) $p_t = \int_0^1 p_{it} di$

7. *Aggregate quantities are given by* $\forall t \geq 0$ *:*

   (a) $y_t = \int_0^1 y_{it} di$
   (b) $c_t = \phi(C^{\mathcal{H}}/C) \int_0^{\phi} c_{jt} dj + (1-\phi)(C^{\mathcal{S}}/C) \int_{\phi}^1 c_{jt} dj$
   (c) $l^s_t = \phi(L^{\mathcal{H}}/L) \int_0^{\phi} l^s_{jt} dj + (1-\phi)(L^{\mathcal{S}}/L) \int_{\phi}^1 l^s_{jt} dj$
   (d) $l^d_t = \int_0^1 l^d_{it} di$
   (e) $b^s_t = (1-\phi)(B_j/B) \int_{\phi}^1 b^d_{jt} dj$
   (f) $c_{it} = \phi(C^{\mathcal{H}}_i) \int_0^{\phi} c_{ijt} dj + (1-\phi)(C^{\mathcal{S}}_i) \int_{\phi}^1 c_{ijt} dj$

B.2. Optimality Conditions. Optimality in the frictionless perfect information economy follows from the maximization problems of firms and households. The same conditions characterize optimal actions in the inattentive economy.

Hand-to-mouth $j \in [0, \phi]$ solves a static problem that yields the following first-order condition for labor supply

$$\varphi^{\mathcal{H}} L_{jt}^{\psi} C_{jt}^{\gamma} = \tilde{W}_t. \tag{46}$$

Saver $j \in [\phi, 1]$ solves a dynamic problem, whose first-order conditions imply an Euler equation given by

$$C_{jt}^{-\gamma} = \beta E_{jt}\left[C_{jt+1}^{-\gamma}\left(\frac{R_t}{\Pi_{t+1}}\right)\right] \tag{47}$$

and a labor supply condition

$$\varphi^{\mathcal{S}} L_{jt}^{\psi} C_{jt}^{\gamma} = \tilde{W}_t. \tag{48}$$

Firm i solves a static problem that yields the following first-order condition for optimal pricing

$$\hat{P}_{it} = \frac{\tilde{W}_t}{\alpha e^{a_t} e^{a_{it}}}\left(\frac{\hat{P}_{it}^{-\theta} C_t}{e^{a_t} e^{a_{it}}}\right)^{\frac{1-\alpha}{\alpha}}. \tag{49}$$

B.3. Linearized Equilibrium We solve the perfect information numerically by linearizing its equilibrium conditions aroiund its non-stochastic steady-state. Given that idiosyncratic technology shocks sum to zero in the cross-section, we drop indexes i for firms. Given that households of type h are identica ex-ante and also ex-post, so we drop the indexes j. Equations (50) to (60) are the economy's equilibrium conditions

$$c_t^{\mathcal{S}} = E_t[c_{t+1}^{\mathcal{S}}] - \frac{1}{\gamma} E_t[r_t - \pi_{t+1}] \tag{50}$$

$$\tilde{w}_t = \psi l_t^{\mathcal{S}} + \gamma c_t^{\mathcal{S}} \tag{51}$$

$$\tilde{w}_t = \psi l_t^{\mathcal{H}} + \gamma c_t^{\mathcal{H}} \tag{52}$$

$$c_t^{\mathcal{H}} = \omega_w^{\mathcal{H}}(\tilde{w}_t + l_t^{\mathcal{H}}) \tag{53}$$

$$\tilde{w}_t = \tfrac{\alpha-1}{\alpha} y_t + \frac{1}{\alpha} a_t \tag{54}$$

$$r_t = \rho_r r_{t-1} + (1-\phi_r)(\phi_\pi \pi_t) + v_t \tag{55}$$

$$c_t = \phi(C^{\mathcal{H}}/C) c_t^{\mathcal{H}} + (1-\phi)(C^{\mathcal{S}}/C) c_t^{\mathcal{S}} \tag{56}$$

$$l_t = \phi(L^{\mathcal{H}}/L) l_t^{h} + (1-\phi)(L^{\mathcal{S}}/L) l_t^{\mathcal{S}} \tag{57}$$

$$y_t = a_t + \alpha l_t \tag{58}$$

$$y_t = c_t \tag{59}$$

$$a_t = \rho_a a_{t-1} + \varepsilon_t^a. \tag{60}$$

Equation (50) and Equation (51) are the savers Euler equation and labor supply conditions. Equation (52) and Equation (53) are the hand-to-mouth's labor supply conditon and budget constraint. Equation (54) is the firms' optimal pricing condition. Equation (55) is the Taylor rule. Equation (56) and Equation (57) defined aggregate consumption and labor. Equation (58) is aggregate output and Equation (59) is the equilibrium condition on the goods market. Equation (60) is the exogenous process for technology.

Definition 4. *Given the same initial bonds holdings for savers and the following non-explosive sequence of real bond holdings*

$$\lim_{s\to\infty} E_t[\beta^{s+1}(\tilde{b}_{t+s+1} - \tilde{b}_{t+s})] = 0, \tag{61}$$

an equilibrium of the perfect information economy is a solution to the system of equations that collects Equations (50) to (60).

Appendix C. Non-Stochastic Steady-State

The following non-stochastic steady-state relationships, which hold for all economies analyzed, are useful for approximating the objective functions of households and firms.

The combination of firm i's optimal pricing condition and production function yields

$$
\begin{aligned}
\hat{P}_i &= \tilde{W}\frac{1}{\alpha}(\hat{P}_i^{-\theta}C_i^{\frac{1}{\alpha}-1}) \\
\hat{P}_i &= \tilde{W}\frac{C_i^{\frac{1}{\alpha}}}{\alpha C_i} \\
\hat{P}_i &= \tilde{W}\frac{L_i}{\alpha C_i} \\
\alpha C &= \tilde{W}L. \qquad (62)
\end{aligned}
$$

We used the fact that firms are symmetrical in the non-stochastic steady state to eliminate the indexes i in the last line.

Rearranging the labor supply condition for both household types yields

$$
\begin{aligned}
\varphi^h L_j^{\psi} &= (C_j^h)^{-\gamma}\tilde{W} \\
\varphi^h L_j^{1+\psi} &= (C_j^h)^{-\gamma}\tilde{W}L_j \\
\varphi^h L_j^{1+\psi} &= (C_j^h)^{1-\gamma}\frac{\tilde{W}L_j}{C_j^h} \\
\varphi^h L^{1+\psi} &= (C^h)^{1-\gamma}\omega_w^h. \qquad (63)
\end{aligned}
$$

Again, we used the symmetry between households of the same type h to eliminate the indexes j in the last line.

Appendix D. Expected Losses from Suboptimal Actions

D.1. Firms First, we guess that model-implied demand for consumption variety i is[23]

$$
C_{it} = \left(\frac{P_{it}}{P_t}\right)^{-\tilde{\theta}} C_t. \qquad (64)
$$

Second, we substitute the demand function into the expression for period nominal profits

[23]We can prove that optimal attention allocation yields a demand function of that form.

$$D_{it} = (1+\tau_p) P_{it} \hat{P}_{it}^{-\tilde{\theta}} C_t - W_t L_{it} \tag{65}$$

Third, we replace the labor input using the production function

$$D_{it} = (1+\tau_p) P_{it} \hat{P}_{it}^{-\tilde{\theta}} C_t - W_t \left(\frac{\hat{P}_{it}^{-\theta} C_t}{e^{a_t} e^{a_{it}}} \right)^{\frac{1}{\alpha}} \tag{66}$$

Next, we assume that, in period -1, households who own the firms value profits at time t using the following discount factor

$$Q_{-1,t} = \beta^t \Lambda(\{C_{jt}\}_{j\in[\phi,1]}) \frac{1}{P_t} \tag{67}$$

where the functional $\Lambda(\cdot)$ is twice continuously differentiable and satisfies

$$\Lambda(\{C_{jt}^{\mathcal{S}}\}_{j\in[\phi,1]}) = (C^{\mathcal{S}})^{-\gamma} \tag{68}$$

in the non-stochastic steady-state.

We multiply Equation (66), the period nominal profits, by the stochastic discount factor (exempt of β^t) which yields

$$\Lambda(\{C_{jt}^{\mathcal{S}}\}_{j\in[\phi,1]}) \left[(1+\tau_p) \hat{P}_{it}^{1-\theta} C_t - \left(\frac{\hat{P}_{it}^{-\theta} C_t}{e^{a_t} e^{a_{it}}} \right)^{\frac{1}{\alpha}} \tilde{W}_t \right]. \tag{69}$$

We denote Equation (69) the real period profits function.

We rewrite this expression in terms of log-deviations around the non-stocastic steady-state

$$\Lambda(\{C_{jt}^{\mathcal{S}}\}_{j\in[\phi,1]}) \left[\frac{\theta}{\theta-1} Y \left(\int_0^1 e^{(1-\theta)\hat{p}_{it} + c_{jt}} \, dj \right) - \alpha Y e^{\frac{\theta}{\alpha}\hat{p}_{it} - \frac{1}{\alpha}(a_t + a_{it}) + \tilde{w}_t} \left(\int_0^1 e^{(1-\theta)\hat{p}_{it} + c_{jt}} \, dj \right) \right] \tag{70}$$

Let $\boldsymbol{x}_t$ denote the variables appearing in firm i's real period profit function that the firm can affect and $\boldsymbol{\zeta}_t$ the vector of variables that are taken as given

$$\boldsymbol{x}_{it} = (\hat{p}_{it})' \tag{71}$$

$$\boldsymbol{\zeta}_{it} = \left(a_t, a_{it}, \tilde{w}_t, \{c_{jt}\}_{j\in[0,1]} \right)' \tag{72}$$

We define $\mathcal{F}_i$ as the functional obtained from multiplying the period real profit function, Equation (70), by β^t and summing over all t from zero to infinity.

We let $\tilde{\mathcal{F}}_i$ denote the second-order approximation of that functional around the non-stochastic steady-state

$$\begin{aligned}
&E_{i,-1}\left[\tilde{\mathcal{F}}_i\left(\boldsymbol{x}_{i0}, \boldsymbol{\zeta}_{i0}, \boldsymbol{x}_{i1}, \boldsymbol{\zeta}_{i1}, \cdots\right)\right] \qquad (73)\\
\approx&\mathcal{F}_i\left(\mathbf{0}, \mathbf{0}, \cdots\right) + E_{i,-1}\left[\sum_{t=0}^{\infty}\beta^t\left(\boldsymbol{h}_{\boldsymbol{x_i}}'\boldsymbol{x}_{it} + \boldsymbol{h}_{\boldsymbol{\zeta_i}}'\boldsymbol{\zeta}_{it} + \frac{1}{2}\boldsymbol{x}_{it}'\boldsymbol{H}_{\boldsymbol{x}_i}\boldsymbol{x}_{it} + \boldsymbol{x}_{it}'\boldsymbol{H}_{\boldsymbol{x_i}\boldsymbol{\zeta_i}}\boldsymbol{\zeta}_{it} + \frac{1}{2}\boldsymbol{\zeta}_{it}'\boldsymbol{H}_{\boldsymbol{\zeta}_i}\boldsymbol{\zeta}_{it}\right)\right]
\end{aligned}$$

where the vectors $\boldsymbol{h}_{\boldsymbol{x}_i}$ and $\boldsymbol{h}_{\boldsymbol{\zeta}_i}$ are first derivatives with respect to $\boldsymbol{x}_{it}$ and $\boldsymbol{\zeta}_{it}$ evaluated at the non-stochastic steady-state respectively. Similarly, $\boldsymbol{H}_{\boldsymbol{x_i}}$, $\boldsymbol{H}_{\boldsymbol{\zeta_i}}$ and $\boldsymbol{H}_{\boldsymbol{x_i}\boldsymbol{\zeta_i}}$ are matrices of second order derivatives evaluated at the non-stochastic steady-state.

We can show that under some regularity conditions Equation (73) converges to a finite element in $\mathcal{R}$ along with each of its components[24].

The process defining firm's i vector of optimal actions, noted $\boldsymbol{x}_{it}^*$, is defined by the following requirement

$$\boldsymbol{h}_{\boldsymbol{x_i}} + \boldsymbol{H}_{\boldsymbol{x_i}}\boldsymbol{x}_{it}^* + \boldsymbol{H}_{\boldsymbol{x_i}\boldsymbol{\zeta_i}}\boldsymbol{\zeta}_{it} = 0. \qquad (74)$$

The requirement above implies the same equation obtained by log-linearizing the firms' optimal pricing condition.

Next, we define firm i's objective function as the losses incurred from suboptimal actions which reads

$$\begin{aligned}
&E_{i,-1}\left[\tilde{\mathcal{F}}_i\left(\boldsymbol{x}_{i0}, \boldsymbol{\zeta}_{i0}, \boldsymbol{x}_{i1}, \boldsymbol{\zeta}_{i1}, \cdots\right)\right] - E_{i,-1}\left[\tilde{\mathcal{F}}_i\left(\boldsymbol{x}_{i0}^*, \boldsymbol{\zeta}_{i0}, \boldsymbol{x}_{i1}^*, \boldsymbol{\zeta}_{i1}, \cdots\right)\right] \qquad (75)\\
=&E_{i,-1}\left[\sum_{t=0}^{\infty}\beta^t\left(\boldsymbol{h}_{\boldsymbol{x_i}}(\boldsymbol{x}_{it} - \boldsymbol{x}_{it}^*) + \frac{1}{2}\boldsymbol{x}_{it}\boldsymbol{H}_{\boldsymbol{x_i}}\boldsymbol{x}_{it} - \frac{1}{2}\boldsymbol{x}_{it}^*\boldsymbol{H}_{\boldsymbol{x_i}}\boldsymbol{x^*}_{it} + (\boldsymbol{x}_{it} - \boldsymbol{x}_{it}^*)\boldsymbol{H}_{\boldsymbol{x_i}\boldsymbol{\zeta_i}}\boldsymbol{\zeta}_{it}\right)\right]
\end{aligned}$$

Lastly, we use Equation (74) to substitute for the term $\boldsymbol{H}_{\boldsymbol{x_i}\boldsymbol{\zeta_i}}\boldsymbol{\zeta}_{it}$ in Equation (75). After rearranging we obtain

[24] See Maćkowiak and Wiederholt (2015) for the formal proof. The same result holds for households' approximations of period utilities defined below, provided certain initial conditions are satisfied for the savers.

$$
\begin{aligned}
&E_{i,-1}\left[\tilde{\mathcal{F}}_i\left(\boldsymbol{x}_{i0},\boldsymbol{\zeta}_{i0},\boldsymbol{x}_{i1},\boldsymbol{\zeta}_{i1},\cdots\right)\right]-E_{i-1}\left[\tilde{\mathcal{F}}_i\left(\boldsymbol{x}^*_{i0},\boldsymbol{\zeta}_{i0},\boldsymbol{x}^*_{i1},\boldsymbol{\zeta}_{i1},\cdots\right)\right]\\
=&\frac{1}{2}(\boldsymbol{x}_{it}-\boldsymbol{x}^*_{it})\boldsymbol{H}_{\boldsymbol{x_i}}(\boldsymbol{x}_{it}-\boldsymbol{x}^*_{it})
\end{aligned}
\tag{76}
$$

where

$$
\boldsymbol{H}_{\boldsymbol{x_i}}=-(C^{\mathcal{S}})^{-\gamma}Y\left[\frac{\theta(\theta+\alpha(1-\theta))}{\alpha}\right]\tag{77}
$$

and

$$
\boldsymbol{x}^*_{it}=\left(p_t+\frac{\frac{1-\alpha}{\alpha}}{1+\frac{1-\alpha}{\alpha}\tilde{\theta}}c_t+\frac{1}{1+\frac{1-\alpha}{\alpha}\tilde{\theta}}\tilde{w}_t-\frac{\frac{1}{\alpha}}{1+\frac{1-\alpha}{\alpha}\tilde{\theta}}(a_t+a_{it})\right)'.\tag{78}
$$

Notice that we used $\hat{p}_{it}-\hat{p}^*_{it}=p_{it}-p^*_{it}$ so that firms choose p_{it}, their price in level, instead of $\hat{p}_{it}$, their relative price. The only matrix of second-order derivatives needed to formulate firm i's attention problem in Section 4.1 is $\boldsymbol{H}_{\boldsymbol{x_i}}$.

D.2. Hand-to-Mouth First, we substitute the consumption aggregator into the flow budget constraint to get

$$
C_{jt}\bigg(\int_0^1 P_{it}\hat{C}_{ijt}di\bigg)=W_tL_{jt}-T^{\mathcal{H}}.\tag{79}
$$

Second, we isolate composite consumption and divide both the numerator and denominator by the price index

$$
C_{jt}=\frac{\tilde{W}_tL_{jt}-\tilde{T}^{\mathcal{H}}}{\int_{[0,1)}\hat{P}_{it}\hat{C}_{ijt}di+\hat{P}_{1t}\bigg(1-\int_{[0,1)}\hat{C}_{ijt}^{\frac{\theta-1}{\theta}}di\bigg)^{\frac{\theta}{\theta-1}}di}.\tag{80}
$$

Notice that we partially relax mathematical rigor by treating the integral as a finite sum and di as a weight[25].

Third, we substitute the expression for consumption in the period utility function

[25]Throughout this section, we use this approach to ensure there is always a free variable when working with equality conditions. Alternatively, we could assume a finite number of households and firms, as in Maćkowiak and Wiederholt (2015), but this would make the relationship between aggregate and individual variables dependent on the size of the economy.

$$
\begin{aligned}
U(C_{jt}, L_{jt}) = & \frac{1}{1-\gamma}\left(\frac{\tilde{W}_t L_{jt} - \tilde{T}^{\mathcal{H}}}{\int_{[0,1)} \hat{P}_{it}\hat{C}_{ijt} di + \hat{P}_{1t}\left(1 - \int_{[0,1)} \hat{C}_{ijt}^{\frac{\theta-1}{\theta}} di\right)^{\frac{\theta}{\theta-1}} di}\right)^{1-\gamma} \\
& - \frac{1}{1-\gamma} \\
& - \varphi^{\mathcal{H}} \frac{L_{jt}^{1+\psi}}{1+\psi}.
\end{aligned} \tag{81}
$$

Next, we rewrite this expression in terms of log-deviations around the non-stocastic steady-state

$$
\begin{aligned}
U(C_{jt}, L_{jt}) = & \frac{(C^{\mathcal{H}})^{1-\gamma}}{1-\gamma}\left(\frac{\omega_w^{\mathcal{H}} e^{\tilde{w}_t + l_{jt}} + \omega_t^{\mathcal{H}}}{\int_{[0,1)} e^{\hat{p}_{it}+\hat{c}_{ijt}} di + e^{\hat{p}_{1t}}\left(1 - \int_{[0,1)} e^{\frac{\theta-1}{\theta}\hat{c}_{ijt}} di\right)^{\frac{\theta}{\theta-1}} di}\right)^{1-\gamma} \\
& - \frac{1}{1-\gamma} \\
& - \frac{(C^{\mathcal{H}})^{1-\gamma}}{1+\psi} \omega_w^{\mathcal{H}} e^{(1+\psi) l_{jt}}
\end{aligned} \tag{82}
$$

where $\omega_w^{\mathcal{H}} = \frac{\tilde{W}L}{C^{\mathcal{H}}}$ and $\omega_t^{\mathcal{H}} = \frac{\tilde{T}^{\mathcal{H}}}{C^{\mathcal{H}}}$ are non-stochastic steady-state ratios.

Let $\boldsymbol{x}_t$ denote the variables appearing in the period utility function that the hand-to-mouth households can affect and $\boldsymbol{\zeta}_t$ the vector of variables that are taken as given

$$
\boldsymbol{x}_t = \left(l_{jt}, \{\hat{c}_{ijt}\}_{i\in[0,1)}\right)' \tag{83}
$$

$$
\boldsymbol{\zeta}_t = \left(\tilde{w}_t, \{\hat{p}_t(i)\}_{i\in[0,1]}\right)' \tag{84}
$$

We define $\mathcal{F}^{\mathcal{H}}$ as the functional resulting from multiplying the period utility function, Equation (82), by β^t and summing over all t from zero to infinity.

We let $\tilde{\mathcal{F}}^{\mathcal{H}}$ denote the second-order approximation of that functional around the non-stochastic steady-state

$$E_{j,-1}\left[\tilde{\mathcal{F}}^{\mathcal{H}}\left(\boldsymbol{x}_0,\boldsymbol{\zeta}_0,\boldsymbol{x}_1,\boldsymbol{\zeta}_1\cdots\right)\right] \tag{85}$$

$$=\mathcal{F}^{\mathcal{H}}\left(\mathbf{0},\mathbf{0},\cdots\right)+E_{j,-1}\left[\sum_{t=0}^{\infty}\beta^t\left(\boldsymbol{h}_{\boldsymbol{x}}'\boldsymbol{x}_t+\boldsymbol{h}_{\boldsymbol{\zeta}}'\boldsymbol{\zeta}_t+\frac{1}{2}\boldsymbol{x}_t{}'\boldsymbol{H}_{\boldsymbol{x}}\boldsymbol{x}_t+\boldsymbol{x}_t{}'\boldsymbol{H}_{\boldsymbol{x}\boldsymbol{\zeta}}\boldsymbol{\zeta}_t+\frac{1}{2}\boldsymbol{\zeta}_t{}'\boldsymbol{H}_{\boldsymbol{\zeta}}\boldsymbol{\zeta}_t\right)\right] \tag{86}$$

where the vectors $\boldsymbol{h}_{\boldsymbol{x}}$ and $\boldsymbol{h}_{\boldsymbol{\zeta}}$ are first derivatives with respect to $\boldsymbol{x}_t$ and $\boldsymbol{\zeta}_t$ evaluated at the non-stochastic steady-state respectively. Similarly, $\boldsymbol{H}_{\boldsymbol{x}}$, $\boldsymbol{H}_{\boldsymbol{\zeta}}$ and $\boldsymbol{H}_{\boldsymbol{x}\boldsymbol{\zeta}}$ are the matrices of second order derivatives evaluated at the non-stochastic steady-state.

Under some regularity conditions Equation (85) and each of its elements converge to a finite element in $\mathcal{R}$.

The process defining the vector of optimal actions for household j, noted $\boldsymbol{x}_t^*$, is defined by the following requirement

$$\boldsymbol{h}_{\boldsymbol{x}}+\boldsymbol{H}_{\boldsymbol{x}}\boldsymbol{x}_t^*+\boldsymbol{H}_{\boldsymbol{x}\boldsymbol{\zeta}}\boldsymbol{\zeta}_t=0. \tag{87}$$

Next, we defined household j's objective function as the losses incurred from suboptimal actions which reads

$$E_{j,-1}\left[\tilde{\mathcal{F}}^{\mathcal{H}}\left(\boldsymbol{x}_0,\boldsymbol{\zeta}_0,\boldsymbol{x}_1,\boldsymbol{\zeta}_1,\cdots\right)\right]-E_{j,-1}\left[\tilde{\mathcal{F}}^{\mathcal{H}}\left(\boldsymbol{x}_0^*,\boldsymbol{\zeta}_0,\boldsymbol{x}_1^*,\boldsymbol{\zeta}_1,\cdots\right)\right] \tag{88}$$

$$=E_{j,-1}\left[\sum_{t=0}^{\infty}\beta^t\left(\boldsymbol{h}_{\boldsymbol{x}}(\boldsymbol{x}_t-\boldsymbol{x}_t^*)+\frac{1}{2}\boldsymbol{x}_t\boldsymbol{H}_{\boldsymbol{x}}\boldsymbol{x}_t-\frac{1}{2}\boldsymbol{x}_t^*\boldsymbol{H}_{\boldsymbol{x}}\boldsymbol{x}_t^*+(\boldsymbol{x}_t-\boldsymbol{x}_t^*)\boldsymbol{H}_{\boldsymbol{x}\boldsymbol{\zeta}}\boldsymbol{\zeta}_t\right)\right]$$

Lastly, we use Equation (87) to substitute for $\boldsymbol{H}_{\boldsymbol{x}\boldsymbol{\zeta}}\boldsymbol{\zeta}_t$ in Equation (88). After rearranging we obtain

$$E_{j,-1}\left[\tilde{\mathcal{F}}^{\mathcal{H}}\left(\boldsymbol{x}_0,\boldsymbol{\zeta}_0,\boldsymbol{x}_1,\boldsymbol{\zeta}_1,\cdots\right)\right]-E_{j,-1}\left[\tilde{\mathcal{F}}^{\mathcal{H}}\left(\boldsymbol{x}_0^*,\boldsymbol{\zeta}_0,\boldsymbol{x}_1^*,\boldsymbol{\zeta}_1,\cdots\right)\right] \tag{89}$$

$$=E_{j,-1}\sum_{t=0}^{\infty}\frac{1}{2}(\boldsymbol{x}_t-\boldsymbol{x}_t^*)\boldsymbol{H}_{\boldsymbol{x}}(\boldsymbol{x}_t-\boldsymbol{x}_t^*)$$

where

$$\boldsymbol{H}_x=-(C^{\mathcal{H}})^{1-\gamma}\begin{bmatrix}\omega_w^{\mathcal{H}}(\omega_w^{\mathcal{H}}\gamma+\psi) & 0 & \cdots & 0\\ 0 & \frac{2}{\theta}di & \cdots & \frac{1}{\theta}di\\ \vdots & \vdots & \ddots & \vdots\\ 0 & \frac{1}{\theta}di & \cdots & \frac{2}{\theta}di\end{bmatrix}, \tag{90}$$

$$\boldsymbol{x}_t^* = \begin{pmatrix} \frac{\tilde{w}_t - \gamma c_{jt}^*}{\psi} \\ -\theta(p_{it} - p_t) \\ \vdots \end{pmatrix}, \tag{91}$$

and

$$c_{jt}^* = \omega_w^{\mathcal{H}}(\tilde{w}_t + l_{jt}^*) \tag{92}$$

The only matrix of second-order derivatives needed to formulate the household j's attention problem in Section 4.2 is $\boldsymbol{H_x}$ given by Equation (90).

D.3. Savers The first few steps are identical to those employed in the derivation of the hand-to-mouth's objective, except that in the period budget constraint other variables than labor income and taxes appear. In particular, bonds from period $t-1$ are a variable that saver get to choose.

We start by expressing consumption as a function of real variables and substitute in the period utility function. We get

$$\begin{aligned} U(C_{jt}, L_{jt}) = \frac{1}{1-\gamma} & \left(\frac{\frac{R_{t-1}}{\Pi_t}\tilde{B}_{jt-1} - \tilde{B}_{jt} + \tilde{W}_t L_{jt} + \tilde{D}_t^{\mathcal{S}} - \tilde{T}_t^{\mathcal{S}}}{\int_{[0,1)} \hat{P}_{it}\hat{C}_{ijt} di + \hat{P}_{1t}\left(1 - \int_{[0,1)} \hat{C}_{ijt}^{\frac{\theta-1}{\theta}} di\right)^{\frac{\theta}{\theta-1}} di} \right)^{1-\gamma} \\ & - \frac{1}{1-\gamma} - \varphi^{\mathcal{S}} \frac{L_{jt}^{1+\psi}}{1+\psi}. \end{aligned} \tag{93}$$

Next, we rewrite the period utility expression in terms of log-deviations from the non-stochastic steady-state

$$\begin{aligned} U(C_{jt}, L_{jt}) = \frac{(C^{\mathcal{S}})^{1-\gamma}}{1-\gamma} & \left(\frac{\frac{\omega_b^{\mathcal{S}}}{\beta} e^{r_{t-1} - \pi_t + \tilde{b}_{jt-1}} - \omega_b^{\mathcal{S}} e^{\tilde{b}_{jt}} + \omega_w^{\mathcal{S}} e^{\tilde{w}_t + l_{jt}} + \omega_d^{\mathcal{S}} e^{\tilde{d}_t^{\mathcal{S}}} - \omega_t^{\mathcal{S}} e^{\tilde{t}_t^{\mathcal{S}}}}{\int_{[0,1)} e^{\hat{p}_{it} + \hat{c}_{ijt}} di + e^{\hat{p}_{1t}} \left(1 - \int_{[0,1)} e^{\frac{\theta-1}{\theta}\hat{c}_{ijt}} di\right)^{\frac{\theta}{\theta-1}} di} \right)^{1-\gamma} \\ & - \frac{1}{1-\gamma} \\ & - \frac{(C^{\mathcal{S}})^{1-\gamma}}{1+\psi} \omega_w^{\mathcal{S}} e^{(1+\psi) l_{jt}} \end{aligned} \tag{94}$$

where $\omega_b^{\mathcal{S}}, \omega_d^{\mathcal{S}}, \omega_w^{\mathcal{S}}$, $\omega_t^{\mathcal{S}}$ are the following non-stochastic steady-state ratios

$$\left(\omega_w^{\mathcal{S}}, \omega_b^{\mathcal{S}}, \omega_d^{\mathcal{S}}, \omega_t^{\mathcal{S}}\right) = \left(\frac{\tilde{W}L}{C^{\mathcal{S}}}, \frac{\tilde{B}^{\mathcal{S}}}{C^{\mathcal{S}}}, \frac{\tilde{D}^{\mathcal{S}}}{C^{\mathcal{S}}}, \frac{\tilde{T}^{\mathcal{S}}}{C^{\mathcal{S}}}\right). \tag{95}$$

Let $\boldsymbol{x}_t$ denote the variables appearing in the period utility function that saver j can affect, and let the vector $\boldsymbol{\zeta}_t$ denote the variables that are taken as given such that

$$\boldsymbol{x}_t = \left(\tilde{b}_{jt}, l_{jt}, \{\hat{c}_{ijt}\}_{i\in[0,1)}\right)', \tag{96}$$

and

$$\boldsymbol{\zeta}_t = \left(r_{t-1}, \pi_t, \tilde{w}_t, \tilde{d}_t^{\mathcal{S}}, \tilde{t}_t^{\mathcal{S}}, \{\hat{p}_{it}\}_{i\in[0,1]}\right)'. \tag{97}$$

Additionally, we define the vector $\boldsymbol{x}_{-1}$, of the same length as $\boldsymbol{x}_t$, containing the variable $\tilde{b}_{j,-1}$, the only variable not present in either $\boldsymbol{x}_t$ or $\boldsymbol{\zeta}_t$

$$\boldsymbol{x}_{-1} = \left(\tilde{b}_{j,-1}, 0, \cdots\right)'. \tag{98}$$

We define $\mathcal{F}^{\mathcal{S}}$ as the functional resulting from multiplying the period utility function, Equation (94), by β^t and summing over all t from zero to infinity.

Letting $\tilde{\mathcal{F}}^{\mathcal{S}}$ denote the second-order Taylor approximaton of this functional evaluated at the non-stochastic steady-state, we get

$$\begin{aligned} & E_{j,-1}\left[\tilde{\mathcal{F}}^{\mathcal{S}}\left(\boldsymbol{x}_{-1}, \boldsymbol{x}_0, \boldsymbol{\zeta}_0, \boldsymbol{x}_1, \boldsymbol{\zeta}_1, \cdots\right)\right] \\ = & \left[\begin{array}{c} \mathcal{F}^{\mathcal{S}}\left(\mathbf{0}, \mathbf{0}, \mathbf{0}, \cdots\right) \\ +\sum_{t=0}^{\infty}\beta^t \left(\begin{array}{c} \boldsymbol{h}_{\boldsymbol{x}}'\boldsymbol{x}_t + \boldsymbol{h}_{\zeta}'\boldsymbol{\zeta}_t \\ +\frac{1}{2}\boldsymbol{x}_t'\boldsymbol{H}_{\boldsymbol{x},-1}\boldsymbol{x}_{t-1} + \frac{1}{2}\boldsymbol{x}_t'\boldsymbol{H}_{\boldsymbol{x}}\boldsymbol{x}_t + \frac{1}{2}\boldsymbol{x}_t'\boldsymbol{H}_{\boldsymbol{x},1}\boldsymbol{x}_{t+1} \\ +\frac{1}{2}\boldsymbol{x}_t'\boldsymbol{H}_{\boldsymbol{x}\zeta}\boldsymbol{\zeta}_t + \frac{1}{2}\boldsymbol{x}_t'\boldsymbol{H}_{\boldsymbol{x}\zeta,1}\boldsymbol{\zeta}_{t+1} \\ +\frac{1}{2}\boldsymbol{\zeta}_t'\boldsymbol{H}_{\zeta}\boldsymbol{\zeta}_t + \frac{1}{2}\boldsymbol{\zeta}_t'\boldsymbol{H}_{\zeta\boldsymbol{x},-1}\boldsymbol{x}_{t-1} + \frac{1}{2}\boldsymbol{\zeta}_t'\boldsymbol{H}_{\zeta\boldsymbol{x}}\boldsymbol{x}_t \end{array}\right) \\ +\beta^{-1}\left(\boldsymbol{h}_{-1}'\boldsymbol{x}_{-1} + \frac{1}{2}\boldsymbol{x}_{-1}'\boldsymbol{H}_{-1}\boldsymbol{x}_{-1} + \frac{1}{2}\boldsymbol{x}_{-1}'\boldsymbol{H}_{-1\boldsymbol{x}}\boldsymbol{x}_0 + \frac{1}{2}\boldsymbol{x}_{-1}'\boldsymbol{H}_{-1\zeta}\boldsymbol{\zeta}_0\right) \end{array}\right] \end{aligned} \tag{99}$$

We can show that under regularity conditions, each of the elements in Equation (99) converge to finite elements in $\mathcal{R}$[26].

The process defining the vector of optimal actions for savers j, noted $\boldsymbol{x}_t^*$, is defined by the following requirement

[26] The formal proof can be found in the appendix of Maćkowiak and Wiederholt (2015).

$$E_{jt}[\boldsymbol{h}_{\boldsymbol{x}} + \boldsymbol{H}_{\boldsymbol{x},-1}\boldsymbol{x}^*_{t-1} + \boldsymbol{H}_{\boldsymbol{x}}\boldsymbol{x}^*_t + \boldsymbol{H}_{\boldsymbol{x},1}\boldsymbol{x}^*_{t+1} + \boldsymbol{H}_{\boldsymbol{x}\boldsymbol{\zeta}}\boldsymbol{\zeta}_t + \boldsymbol{H}_{\boldsymbol{x}\boldsymbol{\zeta},1}\boldsymbol{\zeta}_{t+1}] = 0. \tag{100}$$

We can rearrange Equation (100) to obtain the following expression

$$\begin{aligned} &E_{jt}[(\boldsymbol{x}_t - \boldsymbol{x}^*_t)'(\boldsymbol{h}_{\boldsymbol{x}} + \boldsymbol{H}_{\boldsymbol{x}\boldsymbol{\zeta}}\boldsymbol{\zeta}_t + \boldsymbol{H}_{\boldsymbol{x}\boldsymbol{\zeta},1}\boldsymbol{\zeta}_{t+1})] \\ =& - E_{jt}[(\boldsymbol{x}_t - \boldsymbol{x}^*_t)'(\boldsymbol{H}_{\boldsymbol{x},-1}\boldsymbol{x}^*_{t-1} + \boldsymbol{H}_{\boldsymbol{x}}\boldsymbol{x}^*_t + \boldsymbol{H}_{\boldsymbol{x},1}\boldsymbol{x}^*_{t+1})]. \end{aligned} \tag{101}$$

Next, using Equation (101) along with the assumption that $\boldsymbol{x}^*_{-1} = \boldsymbol{x}_{-1}$, some rearrangement yields the following expression quantifying the losses household j incurs due to suboptimal actions

$$\begin{aligned} &E_{j,-1}\left[\tilde{\mathcal{F}}^{\mathcal{S}}(\boldsymbol{x}_0, \boldsymbol{\zeta}_0, \boldsymbol{x}_1, \boldsymbol{\zeta}_1, \cdots)\right] - E_{j,-1}\left[\tilde{\mathcal{F}}^{\mathcal{S}}(\boldsymbol{x}^*_0, \boldsymbol{\zeta}_0, \boldsymbol{x}^*_1, \boldsymbol{\zeta}_1, \cdots)\right] \\ =& E_{j,-1}\sum_{t=0}^{\infty}\left[\frac{1}{2}(\boldsymbol{x}_t - \boldsymbol{x}^*_t)\boldsymbol{H}_{\boldsymbol{x}}(\boldsymbol{x}_t - \boldsymbol{x}^*_t) + (\boldsymbol{x}_t - \boldsymbol{x}^*_t)\boldsymbol{H}_{\boldsymbol{x},1}(\boldsymbol{x}_{t+1} - \boldsymbol{x}^*_{t+1})\right] \end{aligned} \tag{102}$$

where

$$\boldsymbol{H}_{\boldsymbol{x}} = -(C^{\mathcal{S}})^{1-\gamma}\begin{bmatrix} \gamma\omega_b^2(1+\frac{1}{\beta}) & -\gamma\omega_b\omega_w & 0 & \cdots & 0 \\ -\gamma\omega_b\omega_b & \omega_w(\omega_w\gamma+\psi) & 0 & \cdots & 0 \\ 0 & 0 & \frac{2}{\theta}di & \cdots & \frac{1}{\theta}di \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \frac{1}{\theta}di & \cdots & \frac{2}{\theta}di \end{bmatrix}, \tag{103}$$

$$\boldsymbol{H}_{\boldsymbol{x},1} = (C^{\mathcal{S}})^{1-\gamma}\begin{bmatrix} \gamma\omega_b^2 & -\gamma\omega_b\omega_w & 0 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & \cdots & \cdots & \cdots & 0 \end{bmatrix}. \tag{104}$$

and

$$\boldsymbol{x}^*_t = \begin{pmatrix} \frac{1}{\beta}(r_{t-1} - \pi_t + \tilde{b}^*_{jt-1}) + \frac{\omega_w^{\mathcal{S}}}{\omega_b^{\mathcal{S}}}(\tilde{w}_t + l^*_{jt}) + \frac{\omega_d^{\mathcal{S}}}{\omega_b^{\mathcal{S}}}\tilde{d}^{\mathcal{S}}_t - \frac{\omega_t^{\mathcal{S}}}{\omega_b^{\mathcal{S}}}\tilde{t}^{\mathcal{S}}_t - \frac{1}{\omega_b^{\mathcal{S}}}c^*_{jt} \\ \frac{\tilde{w}_t - \gamma c^*_{jt}}{\psi} \\ -\theta(p_{it} - p_t) \\ \vdots \end{pmatrix} \tag{105}$$

with

$$c^*_{jt} = E_{jt}\left[-\frac{1}{\gamma}(r_t - \pi_{t+1}) + c^*_{jt+1}\right] \text{ and } \omega_b^{\mathcal{S}} > 0. \tag{106}$$

D.4. Change of Variable. Equation (102) does not have the standard form of the objective in a dynamic attention rational inattention problem because of the intertemporal interaction term. We therefore perform a change of variable that allows us to write down the savers attention problem as a pure tracking problem.

We specifically focus on the 2 by 2 upper left elements of $\boldsymbol{H_x}$ and $\boldsymbol{H}_{\boldsymbol{x},1}$. The remaining terms relative to cross-sectional efficiency are unaffected by the following manipulations.

Equation (102) then reads

$$E_{j,-1}\sum_{t=0}^{\infty}\left[\frac{1}{2}(\bar{\boldsymbol{x}}_t-\bar{\boldsymbol{x}}_t^*)\bar{\boldsymbol{H}}_{\boldsymbol{x}}(\bar{\boldsymbol{x}}_t-\bar{\boldsymbol{x}}_t^*)+(\bar{\boldsymbol{x}}_t-\bar{\boldsymbol{x}}_t^*)\bar{\boldsymbol{H}}_{\boldsymbol{x},1}(\bar{\boldsymbol{x}}_{t+1}-\bar{\boldsymbol{x}}_{t+1}^*)\right] \tag{107}$$

with

$$\bar{\boldsymbol{x}}_t=\left(\tilde{b}_{jt},l_{jt}\right)', \tag{108}$$

$$\bar{\boldsymbol{H}}_{\boldsymbol{x}}=-(C^{\mathcal{S}})^{1-\gamma}\begin{bmatrix}\gamma\omega_b^2(1+\frac{1}{\beta}) & -\gamma\omega_b\omega_w\\ -\gamma\omega_b\omega_w & \omega_w(\omega_w\gamma+\psi)\end{bmatrix}, \tag{109}$$

and

$$\bar{\boldsymbol{H}}_{\boldsymbol{x},1}=(C^{\mathcal{S}})^{1-\gamma}\begin{bmatrix}\gamma\omega_b^2 & -\gamma\omega_b\omega_w\\ 0 & 0\end{bmatrix}. \tag{110}$$

Substituing Equations (108) to (110) in Equation (107), we obtain

$$-(C^{\mathcal{S}})^{1-\gamma}E_{j,-1}\sum_{t=0}^{\infty}\beta^t\begin{bmatrix}-\frac{1}{2}\begin{pmatrix}\gamma\omega_b^2(1+\frac{1}{\beta})(\tilde{b}_{jt}-\tilde{b}_{jt}^*)^2\\ -2\gamma\omega_b\omega_w(\tilde{b}_{jt}-\tilde{b}_{jt}^*)(l_{jt}-\bar{l}_{jt}^*)\\ +\omega_w(\gamma\omega_w+\psi)(l_{jt}-l_{jt}^*)^2\end{pmatrix}\\ +\gamma\omega_b^2(\tilde{b}_{jt}-\tilde{b}_{jt}^*)(\tilde{b}_{jt+1}-\tilde{b}_{jt+1}^*)\\ -\gamma\omega_b\omega_w(\tilde{b}_{jt}-\tilde{b}_{jt}^*)(l_{jt+1}-l_{jt+1}^*)\end{bmatrix}. \tag{111}$$

Next, we substract the linearized budget constraints evaluated at $\bar{\boldsymbol{x}}_t$ and $\bar{\boldsymbol{x}}_t^*$, we get

$$\omega_b(\tilde{b}_{jt}-\tilde{b}_{jt}^*)=\frac{\omega_b}{\beta}(\tilde{b}_{jt-1}-\tilde{b}_{jt-1}^*)-(c_{jt}-c_{jt}^*)+\omega_w(l_{jt}-l_{jt}^*). \tag{112}$$

We define the right-hand-side of Equation (112) as a new variable, Δ_t, proportional to mistakes in real bond holdings

$$\Delta_t = \omega_b(\tilde{b}_{jt} - \tilde{b}^*_{jt}). \tag{113}$$

Morevover, we can decompose Δ_t into two components, one reflecting mistakes in consumption and another specific to errors in labor supply such that

$$\Delta^c_t = \frac{1}{\beta}\Delta^c_{t-1} - (c_{jt} - c^*_{jt}) \tag{114}$$

and

$$\Delta^l_t = \frac{1}{\beta}\Delta^l_{t-1} + \omega_w(l_{jt} - l^*_{jt}) \tag{115}$$

with $\Delta^c_{-1} = 0$ and $\Delta^l_{-1} = 0$. By assumptions, we also have $(\tilde{b}_{j,-1} - \tilde{b}^*_{j,-1}) = 0$. Therefore, mistakes in real bond holdings are given by

$$\Delta_t = \Delta^c_t + \Delta^l_t. \tag{116}$$

Substituing Equation (113) and Equation (116) in Equation (111) yields

$$-(C^{\mathcal{S}})1 - \gamma E_{j,-1} \sum_{t=0}^{\infty} \beta^t \begin{bmatrix} -\frac{1}{2}\begin{pmatrix} \gamma(1+\frac{1}{\beta})(\Delta^c_t + \Delta^l_t)^2 \\ -2\gamma\omega_w(\Delta^c_t + \Delta^l_t)(l_{jt} - l^*_{jt}) \\ +\gamma\omega_w^2(l_{jt} - l^*_{jt})^2 + \psi\omega_w(l_{jt} - l^*_{jt})^2 \end{pmatrix} \\ +\gamma(\Delta^c_t + \Delta^l_t)(\Delta^c_{t+1} + \Delta^l_{t+1}) \\ -\gamma\omega_w(\Delta^c_t + \Delta^l_t)(l_{jt+1} - l^*_{jt+1}) \end{bmatrix}. \tag{117}$$

Next, we use Equation (115) to substitute for the term $(l_{jt+1} - l^*_{jt+1})$, and the first term that features $(l_{jt} - l^*_{jt})$ in Equation (117), we obtain

$$-(C^{\mathcal{S}})^{1-\gamma} E_{j,-1} \sum_{t=0}^{\infty} \beta^t \begin{bmatrix} -\frac{1}{2}\begin{pmatrix} \gamma(1+\frac{1}{\beta})(\Delta^c_t + \Delta^l_t)^2 \\ -2\gamma(\Delta^c_t + \Delta^l_t)(\Delta^l_t - \frac{1}{\beta}\Delta^l_{t-1}) \\ +\gamma(\Delta^l_t - \frac{1}{\beta}\Delta^l_{t-1})^2 + \psi\omega_w(l_{jt} - l^*_{jt})^2 \end{pmatrix} \\ +\gamma(\Delta^c_t + \Delta^l_t)(\Delta^c_{t+1} + \Delta^l_{t+1}) \\ -\gamma(\Delta^c_t + \Delta^l_t)(\Delta^l_{t+1} - \frac{1}{\beta}\Delta^l_t) \end{bmatrix}. \tag{118}$$

Rearranging, we obtain

$$-(C^{\mathcal{S}})^{1-\gamma}E_{j,-1}\sum_{t=0}^{\infty}\beta^t\begin{bmatrix}-\frac{\gamma}{2}(1+\frac{1}{\beta})(\Delta_t^c)^2+\gamma\Delta_t^c\Delta_{t+1}^c\\ +\gamma\left[\Delta_t^l\Delta_{t+1}^c-\frac{1}{\beta}\Delta_{t-1}^l\Delta_t^c\right]\\ +\frac{\gamma}{2\beta}\left[(\Delta_t^l)^2-\frac{1}{\beta}(\Delta_{t-1}^l)^2\right]\\ -\frac{\omega_w\psi}{2}(l_{jt}-l_{jt}^*)^2\end{bmatrix}. \tag{119}$$

We combine and rewrite the first two terms, substituting in Equation (119) yields

$$-(C^{\mathcal{S}})^{1-\gamma}E_{j,-1}\sum_{t=0}^{\infty}\beta^t\begin{bmatrix}-\frac{\gamma}{2}(c_{jt}-c_{jt}^*)^2\\ +\frac{\gamma}{2\beta}\left[(\Delta_t^c)^2-\frac{1}{\beta}(\Delta_{t-1}^c)^2\right]\\ +\frac{\gamma}{2\beta}\left[(\Delta_t^l)^2-\frac{1}{\beta}(\Delta_{t-1}^l)^2\right]\\ +\gamma\left[\Delta_t^c(c_{jt+1}-c_{jt+1}^*)-\frac{1}{\beta}\Delta_{t-1}^c(c_{jt}-c_{jt}^*)\right]\\ -\frac{\omega_w\psi}{2}(l_{jt}-l_{jt}^*)^2\end{bmatrix}. \tag{120}$$

Summing terms appearing in consecutive periods, and using $\lim_{T\to\infty}\beta^T E_{j,-1}\left[(\Delta_T^c)^2\right]$ $=\lim_{T\to\infty}\beta^T E_{j,-1}\left[(\Delta_T^l)^2\right]=\lim_{T\to\infty}\beta^T E_{j,-1}\left[\Delta_T^c\Delta_{T+1}^l\right]=\lim_{T\to\infty}\beta^T E_{j,-1}\left[\Delta_T^c(c_{jT+1}-c_{jT+1}^*)\right]$ together with $\Delta_{-1}^c=\Delta_{-1}^l=0$ yields

$$-(C^{\mathcal{S}})^{1-\gamma}E_{j,-1}\sum_{t=0}^{\infty}\beta^t\left[\frac{\gamma}{2}(c_{jt}-c_{jt}^*)^2+\frac{\omega_w\psi}{2}(l_{jt}-l_{jt}^*)^2\right]. \tag{121}$$

Under matricial form Equation (121) writes

$$E_{j,-1}\sum_{t=0}^{\infty}\beta^t\left[\frac{1}{2}(\tilde{\boldsymbol{x}}_t-\tilde{\boldsymbol{x}}_t^*)'\tilde{\boldsymbol{H}}_{\boldsymbol{x}}(\tilde{\boldsymbol{x}}_t-\tilde{\boldsymbol{x}}_t^*)\right]. \tag{122}$$

where

$$\tilde{\boldsymbol{H}}_{\boldsymbol{x}}=-(C^{\mathcal{S}})^{1-\gamma}\begin{bmatrix}\gamma & 0\\ 0 & \omega_w\psi\end{bmatrix}, \tag{123}$$

$$\tilde{\boldsymbol{x}}_t=(c_{jt},l_{jt})', \tag{124}$$

and

$$\tilde{\boldsymbol{x}}_t^*=\begin{pmatrix}E_{jt}\left[-\frac{1}{\gamma}(r_t-\pi_{t+1})+c_{jt+1}^*\right]\\ \frac{\tilde{w}_t-\gamma c_{jt}^*}{\psi}\end{pmatrix} \tag{125}$$

with c_{jt}^* and c_{jt+1}^* defined accordingly to Equation (106).

Appendix E. Households with Monopoly Power

In this section, we describe the modifications to the economic environment and the attention problems that arise when we assume that households have some monopoly power, allowing them to set wages for their differentiated labor services.

E.1. Economic Environment. Firms aggregate the differentiated labor services into a single productive input using a CES aggregator

$$L_{it} = \left(\int_0^1 L_{ijt}^{\frac{\eta-1}{\eta}} dj \right)^{\frac{\eta}{\eta-1}} \tag{126}$$

where η is the elasticity of substitution between labor types.

As a result, firm i's optimal demand for labor of type j is given by

$$L_{ijt} = \left(\frac{W_{jt}}{W_t} \right)^{-\eta} L_{it}. \tag{127}$$

Inattention by firms implies $\tilde{\eta} < \eta$. We assume an aggregate wage index of the form

$$1 = \int_0^1 d_w \left(\hat{W}_{jt} \right) dj \tag{128}$$

where d_w is some twice continuously differentiabe function.

Note that outside the non-stochastic steady state, the average wage rates (and thus labor supplied) set by the two household types will differ due to the marginal rates of substitution not being equalized. The linearized aggregate wage rate faced by firms is given by

$$\tilde{w}_t = \phi \tilde{w}_t^{\mathcal{H}} + (1-\phi)\tilde{w}_t^{\mathcal{S}}. \tag{129}$$

The non-stochastic steady state of this economy is identical to that described in Section 3.7, except for the inclusion of Equation (127), which has no effect in a symmetric equilibrium. To ensure that the model's wage elasticity is consistent with the economic environment, we may assume that households are subject to idiosyncratic productivity shocks, which have no effect on the aggregate dynamics.

E.2. Expected Losses from to Suboptimal Actions. The firm's attention problem remains the same once we abstract from cross-sectional efficiency (i.e., labor mix decisions). The matrices in the period payoff functions and the optimal actions of the wage-setting households are defined below. As before, we present only the elements relevant to aggregate dynamics..

Hand-to-mouth. The matrix appearing in household j of type $\mathcal{H}$'s period payoff function is

$$\boldsymbol{H}^{\mathcal{H}}_{\boldsymbol{x_j}} = -(C^{\mathcal{H}})^{1-\gamma}\left[\tilde{\eta}\omega^{\mathcal{H}}_w(1+\tilde{\eta}(\gamma\omega^{\mathcal{H}}_w+\psi))\right]. \tag{130}$$

Household j of type $\mathcal{H}$'s vector of choice variables and optimal actions are respectively

$$\tilde{\boldsymbol{x}}_{jt} = \left(\tilde{w}_{jt}\right)' \tag{131}$$

and

$$\tilde{\boldsymbol{x}}^*_{jt} = \left(\tfrac{\gamma\omega^{\mathcal{H}}_w+\psi}{1+\tilde{\eta}\psi+\gamma\omega^{\mathcal{H}}_w(\tilde{\eta}-1)}\left[\psi\left(\tilde{\eta}\tilde{w}_t+l_t\right)\right]\right). \tag{132}$$

Savers. The matrix appearing in household j of type $\mathcal{S}$'s period payoff is

$$\boldsymbol{H}^{\mathcal{S}}_{\boldsymbol{x_j}} = -(C^{\mathcal{S}})^{1-\gamma}\begin{bmatrix}\gamma & 0\\ 0 & \tilde{\eta}\omega^{\mathcal{S}}_w(1+\tilde{\eta}\psi)\end{bmatrix}. \tag{133}$$

Household j of type $\mathcal{S}$'s vector of choice variables and optimal actions are respectively

$$\boldsymbol{x}_{jt} = \begin{pmatrix}c_{jt}, & \tilde{w}_{jt}\end{pmatrix}' \tag{134}$$

and

$$\boldsymbol{x}^*_{jt} = \begin{pmatrix} E_{jt}\left[-\frac{1}{\gamma}(r_t-\pi_{t+1})+c^*_{jt+1}\right]\\ (1+\tilde{\eta}\psi)^{-1}\left[\gamma c^*_{jt}+\psi\left(\tilde{\eta}\tilde{w}_t+l_t\right)\right]\end{pmatrix} \tag{135}$$

with c^*_{jt} and c^*_{jt+1} defined in accordance with Equation (106).

Lastly, the equilibrium around the non-stochastic steady-state, as well as the numerical procedure used to solve for it, are analogous to those described in the main text for the economy with a perfectly competitive labor market.

Appendix F. Calibration

F.1. Estimating the Share of Hand-to-Mouth We estimate the share of hand-to-mouth in the economy using the SCF, a triennial survey conducted by the Federal Reserve Board that collects detailed information on household finances in the United

States. It covers a representative sample of over 4,500 households and provides data on income, assets and debts.

First, we define the conditions under which a household classifies as hand-to-mouth;

Definition 5. *A household is considered hand-to-mouth if it either has*

- *zero liquid wealth*
- *or its credit limit is binding.*

This definition is standard in the literature, and is derived from the endogenous behavior of households in hetereogenous agent incomplete market models. As shown in Kaplan and Violante (2014), a household at a kink in its budget constraint, either at zero liquid wealth or the credit limit, exhibits a strong propensity to consume, which typically translates into allocating all of its period income to consumption and debt repayments, but none to savings. Hand-to-mouth households in our simplified framework display the exact same behavior.

Kaplan, Violante, and Weidner (2014) propose identifying hand-to-mouth households in the data using the following inequality conditions;

1. $0 \leq m_t(i) \leq \frac{w_t(i)}{2f}$
2. $m_t(i) \geq 0$ and $m_t(i) \leq \frac{w_t(i)}{2f} - \underline{m}_t(i)$

where $m_t(i)$ represents net liquid wealth, $w_t(i)$ denotes monthly income, and $f \geq 1$ is the frequency at which a household receives payments during a month[27].The first inequality identifies households with zero liquid wealth, while the second identifies those at their credit limit.

We estimate monthly income by dividing reported annual income by 12. We measure net liquid wealth based on the composition of liquid assets and debt defined in Table 8. We follow Kaplan and Violante (2014) and inflate the value of transactional accounts (LIQ) by a factor of 1.05 to account for cash holdings that are not reported in the SCF. In the model, borrowing should be considered as unsecured credit, so debt is measured using revolving credit card debt (i.e., credit card balances that are not repaid in full at every payment) which avoids including as debt purchases made through credit cards in between regular payments (see e.g., Telyukova (2013)). We infer revolving credit card debt by multiplying credit card balances after the last

[27]Income is measured monthly because, when $f = 1$, it represents the lowest plausible payment frequency. For more details on the estimator, see Kaplan, Violante, and Weidner (2014).

payment (CCBAL) with an indicator function that takes the value one if respondents answered either “sometimes” or “almost never” to the question “How often do you pay your credit card balance in full?”. To ensure consistency with our model, we exclude households with zero wage income, those with a respondent under 22 or over 79 years old, and those above the 95th income percentile.

Table 8: Assets and Debt in the SCF

Category	Mnemonic
Liquid Assets	
All types of transaction account	LIQ
Directly held pooled investment funds	NMMF
Directly held stocks	STOCKS
Liquid Debt	
Credit card balances after last payment	CCBAL

Notes: This is a conservative definition of liquid wealth, the same as in Kaplan and Violante (2014). One could extend it to include assets with a lesser degree of liquidity.

F.2. Consumption Ratio We use our estimates of hand-to-mouth households in Appendix F.1 to calibrate the non-stochastic steady-state consumption ratio. First, we measure wage income and net liquid assets within each group. For both variables, we take the median. Wage income is directly reported in the survey, while net liquid assets is computed for each household by taking the difference between liquid assets and debts defined in Table 8.

We follow the strategy of Maćkowiak and Wiederholt (2015) to estimate consumption expenditures. First, apply the tax rate schedule[28] to median total income within each group. Next, we compute the total savings required to maintain median liquid net worth constant at an annual inflation rate of 2.5%. Consumption expenditures for each group is obtained by subtracting median savings from after-tax median income, and the ratio follows directly.

Appendix G. New Keynesian Frictions

Throughout the paper, we also consider models in which some of the slow adjustment in prices or quantities are due to standard adjustement frictions instead of imperfect

[28] Specifically, we apply the tax rate for ”married filing jointly”. Tax schedule for a given year can be found at https://www.irs.gov/pub/irs-prior.

information. In this section, we provide an overview of the microfoundations behind the following frictions: (i) sticky prices and (ii) sticky wages *à la* Calvo, and (ii) habit formation in consumption.

G.1. Sticky Prices. With probability $1-\zeta_p$, firms are free to reset their price P_{it}. Conditional on resetting, they choose the optimal price P_{it}^* that maximizes the discounted sum of expected dividends.

Optimal price-setting gives rise to a New Keynesian Phillips curve, which, when linearized, is

$$\pi_t = \kappa_p s_t + \beta E_t[\pi_{t+1}] \tag{136}$$

with

$$\kappa_p = \frac{(1-\zeta_p)(1-\beta\zeta_p)}{\zeta_p}\left(\frac{\alpha}{\alpha+(1-\alpha)\theta}\right) \tag{137}$$

and

$$s_t = \tilde{w}_t + \frac{1-\alpha}{\alpha} y_t - \frac{1}{\alpha} a_t. \tag{138}$$

Equation (136) is the slope of the Phillips curve and Equation (138) is the real marginal cost. A complete derivation can be found in Galí (2015).

G.2. Sticky Wages. We assume that every household supply a continuum of differentiated labor services with wage elasticity of demand η. The wages for all types of labor is set by a union that chooses it to maximize the average utility of its members, given the demand it faces and subject to Calvo frictions. For each of labor services, the union can reset the wage rate at which it remunerated with unconditional probability $1-\zeta_w$.

We allow each household type to be represented by its own union. This structure gives rise to a New Keynesian Phillips curve for the wage index of each household type, of the form

$$\tilde{w}_t^h = \frac{1}{1+\beta}\tilde{w}_{t-1}^h + \frac{\beta}{1+\beta}E_t\tilde{w}_{t+1}^h + \kappa_w^h\left(mrs_t^h - \tilde{w}_t^h\right) + \frac{1}{1+\beta}\pi_t + \frac{\beta}{1+\beta}E_t\pi_{t+1} \tag{139}$$

with

$$\kappa_w^h = \frac{(1-\zeta_w)(1-\beta\zeta_w)}{\zeta_w(1+\psi\eta)} \tag{140}$$

and

$$mrs_t^h = \psi l_t^h + \gamma c_t^h. \tag{141}$$

Equation (140) and Equation (141) are respectively the slope of the New Keynesian Philips curve for wages and the marginal rate of substitution between labor supply and consumption for household of type h.

Lastly, a linearization of the wage index implies

$$\tilde{w}_t = \phi \tilde{w}_t^{\mathcal{H}} + (1-\phi)\tilde{w}_t^{\mathcal{S}}. \tag{142}$$

One can also assume that a single unions represent all households as in Debortoli and Galí (2024). In that case, the relevant marginal rate of subsitution is a weighted average of the savers and hand-to-mouth.

G.3. Habit Formation. We modify the savers preferences such that utility depends not only on today's consumption but also on its value in the previous period

$$U(C_{jt}, C_{jt-1}, L_{jt}) = \frac{(C_{jt} - hC_{jt-1})^{1-\gamma} - 1}{1-\gamma} - \varphi^{\mathcal{S}} \frac{L_{jt}^{1+\psi}}{1+\psi}. \tag{143}$$

where $h \in [0,1]$ denotes the degree of habit formation.

A linearization of the Euler equation resulting from utility maximization for optimizing households yields the following two equations,

$$\lambda_t^{\mathcal{S}} = E_t[\lambda_{t+1}^{\mathcal{S}} + r_t - \pi_{t+1}] \tag{144}$$

and

$$\lambda_t^{\mathcal{S}} = \frac{\gamma h}{(1-h)(1-\beta h)} c_{t-1}^{\mathcal{S}} - \frac{\gamma(1+\beta h^2)}{(1-h)(1-\beta h)} c_t^{\mathcal{S}} + \frac{\gamma \beta h}{(1-h)(1-\beta h)} E_t[c_{t+1}^{\mathcal{S}}]. \tag{145}$$

where $\lambda_t^{\mathcal{S}}$ is the Lagrange multiplier on the household's flow budget constraint.

G.4. Linearized Optimality Conditions. To include the adjustment frictions in our baseline economic environement, summarized by Equations (50) to (60), we proceed as follows. For Calvo prices, replace the firms' optimal pricing equations with the New Keynesian Phillips curve, Equations (136) to (138). For Calvo wages, replace the households labor supply equations with the New Keynesian Phillips curve for wages with Equations (139) to (142). For habit formation in consumption, substitute the savers' Euler equation with Equations (144) to (145). Modify the Taylor rule to account for the presence of an output gap due to frictions such that

$$r_t = \phi_r r_{t-1} + (1 - \phi_r)[\phi_\pi \pi_t + \phi_{y^*}(y_t - y_t^*)] \tag{146}$$

where y_t^* represents the natural level of output (i.e., the level of economic activity that would prevail in the frictionless, perfect information economy).

Note that it is possible to assume that some decisions are subject to adjustment frictions and others to inattention.