Computational Economics

Lecture 1: An Intro to Dynamic Stochastic General Equilibrium Models

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Motivation

The Lucas critique (1976) warns that policy evaluation cannot rely on historical empirical relationships. This argument changed how to (not) conduct macroeconomic research and led to the emergence of DSGE models.

DSGEs provide a disciplined (and simplified) framework to study real-world phenomena.

Importantly, these models are microfounded: aggregate dynamics are consistent with individuals optimizing given constraints and expectations.

Motivation

DSGE models typically cannot be solved using pen and paper. This class will cover methods to tackle this issue, allowing us to use these models to study:

- Dynamic responses to shocks.
- Propagation mechanisms.
- Counterfactuals and policy experiments.
- Historical decomposition.

A Simple Model

Overview

As an intro to D(S)GE models, we study an optimal consumption–saving problem. The model is effectively a deterministic RBC framework in which labor input is set exogenously to one.

This environment remains the backbone of state-of-the-art macroeconomic models.

Note: In the absence of uncertainty, one could drop the "S" from DSGE.

Economic Environment

- Representative household period utility: $U(C_t) = \frac{C_t^{1-\gamma}-1}{1-\gamma}$
- Discount factor: $0 < \beta < 1$.
- Aggregate production function: $Y_t = K_{t-1}^{\alpha}$
- Law of motion for capital: $K_t = (1-\delta)K_{t-1} + I_t$
- Aggregate resource constraint: $Y_t = C_t + I_t$

Social Planner's Problem

Social Planner's Problem

We solve the consumption–saving problem from the social planner's perspective. In a frictionless environment like the one above, the solution coincides with the decentralized equilibrium, where households and firms maximize taking prices as given.

The social planner maximizes the discounted sum of period utility, taking as given the aggregate production function, aggregate resource constraint and the capital's law of motion.

Social Planner's Problem (cont'd)

Formally the social planner's problem solves:

$$\max_{\{C_t,K_t,I_t\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \beta^t E_t U(C_t) \tag{1}$$

subject to

$$Y_t = K_{t-1}^{\alpha} \tag{2}$$

$$K_t = (1-\delta)K_{t-1} + I_t \tag{3}$$

$$Y_t = C_t + I_t \tag{4}$$

$$K_{-1} \text{ given.} \tag{5}$$

Note: As mentionned earlier, there is no source of uncertainty in this model, and thus the use of the expectation operator, $E_t(\cdot)$ is not required.

Social Planner's Problem (cont'd)

Combining the constraints, we get:

$$\max_{\{C_t, K_t\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \beta^t \frac{C_t^{1-\gamma} - 1}{1-\gamma} \tag{6}$$

subject to

$$K_{t-1}^{\alpha} = C_t + K_t - (1-\delta)K_{t-1} \tag{7}$$

$$K_{-1} \text{ given.} \tag{8}$$

Note: $C_t = K_{t-1}^{\alpha} - K_t + (1-\delta)K_{t-1}$, thus it would be possible to replace in the objective and solve for an unconstrained problem.

Social Planner's Problem (cont'd)

We can write this problem as a Lagrangian:

$$\mathcal{L} = \sum_{t=0}^{\infty} \beta^t \left\{ \frac{C_t^{1-\gamma} - 1}{1-\gamma} + \lambda_t \left[K_{t-1}^{\alpha} - C_t - K_t + (1-\delta) K_{t-1} \right] \right\}. \quad (9)$$

Optimality Conditions

Taking the derivatives of $\mathcal{L}$ wrt to the controls, C_t and K_t, we get the FOCs:

$$C_t^{-\gamma} = \lambda_t \tag{10}$$

$$\lambda_t = \beta\lambda_{t+1}\left[\alpha K_t^{\alpha-1} + (1-\delta)\right]. \tag{11}$$

The KKT are given by:

$$\lambda_t\left[K_{t-1}^{\alpha} - C_t - K_t + (1-\delta)K_{t-1}\right] = 0 \tag{12}$$

$$\lambda_t \geq 0, \left[K_{t-1}^{\alpha} - C_t - K_t + (1-\delta)K_{t-1}\right] \geq 0. \tag{13}$$

Optimality Conditions (cont'd)

The aggregate resource constraint always holds at equality, thus $\lambda_t > 0 \, \forall t$ and with $K_{-1} > 0$, we never encouter corner solutions.

Optimality implies:

$$C_t^{-\gamma} = \beta C_{t+1}^{-\gamma}[\alpha K_t^{\alpha-1} + (1-\delta)] \qquad \text{[Euler Eq.]}$$

with

$$K_{t-1}^{\alpha} = C_t + K_t - (1-\delta)K_{t-1} \qquad \text{[Aggr. resource constr.]}$$

Equilibrium

Definition

Formally, the definition of an equilibrium consists of a list of objects and conditions that these objects must satisfy.

Definition: Given an initial stock of capital K_{-1}, a sequential equilibirum to the consumption-saving problem is:

- An allocation $\Omega := \{C_t, K_t\}_{t=0}^{\infty}$ for the social planner.

such that:

1. The allocation Ω maximizes the discounted of sum of period utilities.
2. The aggregate resource constraint holds at equality
$$K_{t-1}^{\alpha} = C_t + K_t - (1-\delta)K_{t-1}.$$

Characterization

The characterization of an equilibrium consists of listing all of the mathematical conditions necessary compute the social planner's allocation Ω.

In the present case, the Euler equation and the aggregate resource constraint are sufficient.

Steady-state

The Steady-State

In a steady-state, all endogenous variables are constant over time and satisfy the model's equilibrium conditions.

We can compute the values for these variables by dropping the time index in the FOC and constraints:

$$K = \left(\frac{\beta^{-1} - (1 - \delta)}{\alpha} \right)^{\frac{1}{\alpha - 1}} \tag{14}$$

$$I = \delta K \tag{15}$$

$$C = K^{\alpha} - I \tag{16}$$

$$\lambda = C^{-\gamma} \tag{17}$$

Dynamics

Solving for the Transition Dynamics

Our goal is solve for the dynamics associated with the pair of non-linear first-order difference equations formed by the Euler eq. and the resource constraint (obviously for a given $K_{-1} \neq K$).

We can show that this economy has exactly one stable manifold, meaning that for a given K_{-1}, consumption and capital converge to their steady-state values only if C_t is chosen optimally $\forall t$.

Shooting Algorithm

In other words, given K_{-1}, there exists a unique path $\{C_t^*, K_t^*\}_{t=0}^T$ that satisfies optimality and converges to the steady state $\{C, K\}$.

The challenge is that C_0^* is not pinned down by the eq. cond.

We can solve for this path using a shooting algorithm:

1. Guess a value for $C_0 \in [0, K_{-1}^{\alpha}]$.
2. Given (C_0, K_{-1}), use the Euler equation and the aggregate resource constraint to compute (C_1, K_0). Iterate forward up to $t = T$.
3. If $K_T < K$, set the guess for C_0 as the lower bound of the interval; otherwise, set it as the upper bound.
4. Repeat steps 1–3 until convergence (i.e., $K_T = K$ and $C_T = C$).

This procedure transforms a two–point boundary value problem into an initial–condition problem.

Shooting Algorithm (cont'd)

The shooting algorithm is not practical: (i) it solves for a single path and (ii) it becomes computationally burdensome (infeasible) without perfect foresight.

It is therefore natural for us to seek a solution in the form of a policy function that:

- Maps the state K_{t-1} to the control C_t for all admissible values of the state.
- Accounts for uncertainty about future outcomes.