

RATIONALLY INATTENTIVE HETEROGENOUS AGENTS

ETIENNE BRIAND[†]

Université du Québec à Montréal

FIRST DRAFT: OCTOBER 2024

THIS VERSION: OCTOBER 2025

Abstract

We solve business cycle models with rationally inattentive heterogeneous agents and compare their predictions with the data. When the wage rate is determined competitively, households' cross-sectional expectation accuracy matches survey evidence, but rational inattention is a weak propagation mechanism that fails to generate persistence in macroeconomic quantities. The opposite occurs when wages are set by households with labor market power. This conundrum arises because neither model can simultaneously generate losses from intra- and intertemporal decisions of similar magnitude *and* persistence in the growth rate of labor income. Moreover, conducting the same policy experiment in both models leads to starkly different conclusions. We discuss modifications to the models' microfoundations, such as wages set by unions and market power on the side of firms, as possible ways to jointly match micro and macro evidence, eliminating the need to compromise between models that perform well in only one dimension.

Keywords: information choice, rational inattention, business cycles, heterogeneous agents, monetary policy.

JEL Codes: D83, E21, E31, E32, E71

[†] For comments and useful suggestions, we thank Jaroslav Borovička, Alain Guay, Kristoffer Nimark, Pavel Ševčík, participants at the Bank of Canada Montreal Workshop, 7th Behavioral Macroeconomics Workshop Heterogeneity and Expectations in Macroeconomics and Finance, and seminar at the CERGE-EI.

e-mail: etiennebriand1@gmail.com

web: etiennebriand.github.io.

1. Introduction

Recent work in macroeconomics has placed emphasis on the importance of building models that can match key micro moments from the data, typically focusing on wealth heterogeneity and marginal propensity to consume (MPC). This literature reshaped our understanding of the propagation mechanisms behind aggregate fluctuations and the transmission of economic policies (see [Violante 2021](#) for a survey). In the heterogeneous agents framework, indirect effects, such changes in disposable income, have an important role in the propagation of macro shocks. As a result, this class of models is often viewed as more intuitive and realistic than the representative household paradigm.

Most heterogeneous agents models still suffer from two main weaknesses when confronted with data. First, like many of their representative agent counterparts, they usually assume perfect information, which is well known to contradict evidence from survey data showing significant belief heterogeneity among firms and households (see [Carroll 2003](#), [Weber et al. 2022](#)). This assumption also implies the absence of systematic correlations between individual characteristics and expectations (see [Mittman et al. 2022](#) for empirical evidence supporting such a link), a problematic omission given that the core philosophy of these models is to match empirical heterogeneity. Second, the presence of a realistic share of high-MPC agents tends to produce shock responses that are peaked on impact, failing to generate the hump-shaped dynamics (e.g., positive autocorrelation in growth rates) for macro variables consistently documented in VAR studies (see [Christiano, Eichenbaum, and Evans 2005](#), [Altig et al. 2011](#)). This is a known limitation of the framework, already noted by [Kaplan, Moll, and Violante \(2018\)](#), but where progress has been lacking.

In this paper, we tackle both issues—degenerate distribution of expectations implied by perfect information and macro variables lacking persistence—simultaneously by introducing inattention ([Sims 1998, 2003](#)) into dynamic stochastic general equilibrium (DSGE) models with heterogeneous households. Rational inattention (RI) is a theory of information acquisition that endogenously leads to imperfect expectations and sluggish responses to shocks. Specifically, decision-makers allocate their attention by solving a maximization problem based on a tradeoff; acquiring more precise signals increases both expected payoffs and the attention costs incurred from information processing. In line with the RI-DSGE literature, we assume that this mechanism is the sole source of inertia in prices and quantities. Previous work (see [Maćkowiak and Wiederholt 2015, 2023](#)) showed that the theoretical impulse response functions and expectations implied by calibrated inattentive Ricardian economies with ex-ante identical agents can match those obtained from time-series

and survey data. Whether inattention can deliver similar results in a setting with heterogeneous households remains an open question to which we aim to provide answers.

For our analysis, we adopt a stylized view of heterogeneity based on the long tradition of two-agent models (Campbell and Mankiw 1989, Bilbiie 2008). Households are divided into constant shares of savers (\mathcal{S}) and hand-to-mouth (\mathcal{H}). This dimension of heterogeneity is arguably sufficient to replicate most of the responses to aggregate shocks implied by state-of-the-art heterogeneous agent economies (see Debortoli and Galí 2024 for a discussion¹), and yields an aggregate MPC on impact that is consistent with the data². More importantly, the setup is tractable enough to solve for each type of agent’s optimal signal(s) in a dynamic environment and for the economy’s equilibrium (fixed-point) as a function of the distribution of expectations generated by those signals.

Combining rational inattention and heterogeneous households, yields a framework in which each household faces a type-specific attention problem that determines their optimal signal(s). Differences in attention allocation arise endogenously across types. There are three main channels that generate this heterogeneity in optimal signals, and thus expectations. First, losses from suboptimal actions depend on the marginal utility of consumption. If the steady-state consumption of one household type is lower than that of another, that type has a stronger incentive to pay attention. Second, savers make an additional decision regarding consumption, which generally broadens their relevant span of information. Third, the extent to which each action directly affects utility may differ across types (i.e., how much labor income contributes to total income). As a result, the signals designed by savers and hand-to-mouth households and the implied accuracy of their forecasts are endogenous outcomes of our models, jointly determined with macroeconomic dynamics.

We quantitatively evaluate our models using data on macroeconomic variables and households’ measured expectations from the *Survey of Consumer Expectations* (SCE). From this survey, we document new empirical evidence, which we refer to as micro moments on expectations. In particular, we find that forecast errors of hand-to-mouth households are larger than those of other households (e.g., between 0.56 and 1.73 percentage points for inflation). Our results provide additional empirical evidence of an interaction between heterogeneity and expectations formation. Mitman et al. (2022) termed this nexus the *attention channel*. Our finding itself represents a novel dimension of heterogeneity, namely cross-sectional forecast accuracy between hand-to-mouth households and savers, against which two-agent models can

¹All models considered in their analysis suffer from the two weaknesses mentioned above.

²This result holds if the share of hand-to-mouth households is close to the desired MPC.

be evaluated.

We compare two baseline models. The model with a competitive labor market fits the micro moments on expectations well but fails to generate sufficient persistence in output growth to match the data. In that setting, losses from intra and intertemporal suboptimal actions are of similar magnitude, which leads savers to pay more attention than hand-to-mouth households and to form more accurate forecasts. However, rational inattention is a weak propagation mechanism with competing effects. The more inattentive savers are, the more delayed their consumption response to shocks is, inducing positive autocorrelation in its growth rate. At the same time, greater inattention generates more inertia in labor supply decisions, requiring more volatile wages for the labor market to clear. This second effect induces negative autocorrelation in the consumption growth rate of hand-to-mouth households. In our calibration, these two effects almost cancel out, leading to near-zero autocorrelation in growth of aggregate output. This result contrasts with a Ricardian version of the economy, where inattention can generate any desired degree of output persistence.

Next, we evaluate a version of the model in which households have some market power, allowing them to set the wage for their differentiated variety of labor. This change in labor market structure resolves the problem of weak persistence in output growth and induces empirically plausible dynamics for the real wage. Inattention now generates hump-shaped labor income responses through sluggish wage adjustments, leading to positive autocorrelation in the consumption growth of hand-to-mouth households. However, with labor market power the stakes of setting wages suboptimally are much higher for households, as firms substitute toward varieties with lower wages. Because of these high stakes, both types of households design signals that are informative for this decision. Since this mechanism is quantitatively strong and its effects are almost symmetrical for both hand-to-mouth households and savers, the differences in forecast accuracy across the two types of households become too small relative to what we observe in the data.

The baseline models differ not only in their fit to micro and macro moments but also in their predictions regarding propagation and policy experiments. For example, under competitive wages, varying the share of hand-to-mouth households—which can be interpreted as a policy that lowers barriers to financial markets—leads to significant changes in aggregate dynamics. In the extreme case where this share is set to zero, the output response to an expansionary monetary policy shock falls by one-third, and its unconditional first-order serial correlation increases from -0.06 to 0.19. By contrast, in the model with market power, the effect is negligible for both outcomes.

Our results reveal a novel challenge: embedding baseline two-agent business cycle

models with inattention does not allow us to simultaneously match the persistence of macro data and cross-sectional expectations from survey evidence. At the heart of the issue is the labor market, which must be structured to generate hump-shaped adjustments in labor income while avoiding stakes so high that attention becomes skewed toward intratemporal decisions. Moreover, depending on one’s preferred philosophy—emphasizing the matching of micro moments, as in the heterogeneous-agent literature, or macro data, as in the representative-consumer literature—conclusions about policy experiments based on the chosen models differ starkly. To tackle this challenge and eliminate the need to arbitrarily choose between models that fail along one dimension, one would like to set up a model in which wages adjust sluggishly, inducing inertial responses of households’ labor income to shocks while removing the wage-setting decision from households’ attention problem, thereby preserving savers’ higher incentives to pay attention. Plausible examples of such environments include models with union-negotiated wages subject to adjustment frictions or models with monopsonistic labor markets.

Finally, we compare inattention with other propagation mechanisms known in the literature as ways to address either of the aforementioned weaknesses present in heterogeneous agent models (i.e., the lack of persistence and the assumption of perfect information). Overall, the rational inattention framework provides a clean way to assess how information frictions, which are essential to match measured expectations, alter the model’s dynamics while remaining impervious to the Lucas critique. Computational challenges limit its applicability, but insights from smaller models, such as those solved in this paper, provide useful starting points for setting up the microfoundations and information structures of more complex models that aim to address the issues we highlighted.

Related Litterature. [Maćkowiak and Wiederholt \(2015, 2023\)](#) solve benchmark Ricardian business cycle models with rationally inattentive agents. They show that rational inattention is a strong propagation mechanism, capable of simultaneously matching the impulse responses of macro shocks and producing empirically plausible measures of aggregate information rigidity. We show that in models where a fraction of households are non-Ricardian, achieving a good fit of both macro and micro moments for expectations is challenging when assuming standard microfoundations for the labor market.

[Song and Stern \(2020\)](#) solve an RI-DSGE model with firms that are heterogeneous in terms of attention costs. Similarly as in our models, when the fraction of firms of different types changes, so do the real effects of monetary policy shocks. However, the differences in attention allocation across firm types are explained exclusively by

differences in information processing capacity, not by their characteristics.

Auclert, Rognlie, and Straub (2020) address the persistence issue in heterogeneous agent New Keynesian (HANK) models. They highlight the failure of using habit formation in consumption to generate inertial responses while simultaneously preserving an empirically realistic MPC. They show that embedding sticky information *à la* Mankiw and Reis (2007) can preserve micro jumps (high MPC) and aggregate persistence. However, their proposed departure from perfect information ignores micro moments on expectations. As a result, changes in the distribution of households leave out the attention channel. For example, an increase in the share of hand-to-mouth households does not reduce the average accuracy of forecasts as suggested by out empirical evidence.

Gallegos (2024) studies a two-agent model with an exogenously specified imperfect information structure. In this setting, amplification arising from high-MPC households is dampened by slow learning about the fundamentals, but the model predictions are qualitatively unchanged. We show that information frictions, depending on the microfoundations, can in fact substantially change aggregate dynamics, almost nullifying the relevance of heterogeneity.

Mitman et al. (2022) study the interaction between heterogeneity and endogenous expectation formation in a neoclassical model *à la* Krusell and Smith (1998). They specify the set of available signals and calibrate their (monetary) costs to match data on cross-sectional expectations. In a policy experiment, they show that the predictions of their model with endogenous attention and those of a perfect information benchmark may be contradictory. The attention channel is also relevant for counterfactuals in our framework. However, we do not initially set it up to match micro moments on expectations; we treat it as an endogenous outcome, which allows us to discriminate between different specifications.

Layout. The remainder of the paper is structured as follows. Section 2 presents empirical evidence on the relationship between hand-to-mouth status and the quality of households’ forecasts. Section 3 outlines the model’s economic environment. Section 4 states the attention problems faced by decision-makers within the model. Section 5 defines the equilibrium of the inattentive economy. Section 6 describes the dynamics and the fits of baseline inattentive economies. Section 7 presents extensions. Section 8 compares rational inattention with other frameworks. Section 9 concludes.

2. Expectations Heterogeneity

A large body of empirical research shows that expectations are sluggish at the macro level (see Coibion and Gorodnichenko 2015, Bordalo et al. 2020) and dispersed at the micro level (see Mankiw, Reis, and Wolfers 2003, Weber et al. 2022). However, there is considerably less evidence about whether and how expectations correlate with individual characteristics (see Mitman et al. (2022)’s first footnote for a review of the existing literature).

In this section, we investigate the existence of such correlation, which we refer to as *micro moments on expectations*. We do so by partitioning households into two groups, hand-to-mouth and savers, and assessing cross-sectional differences in forecast accuracy. This approach is consistent with the stylized definition of heterogeneity in our theoretical framework and, to the best of our knowledge, is conducted at a broader level than comparable studies.

We use microdata from the SCE³, a large panel survey held by the New-York Fed that collects monthly expectations from heterogeneous households. Typically, identifying hand-to-mouth households requires detailed microdata on finances, which the SCE lacks⁴. However, by merging the SCE with its supplemental surveys on households’ spending and credit we can leverage the answers to questions on consumption behavior and liquidity constraints to determine whether or not a household is assigned the hand-to-mouth label. We also use the finance survey to obtain descriptive statistics for the households within each type. Our sample spans the period from 2013M6 to 2024M4, which ends in the latest period for which we can compare expectations to their actual outcomes.

2.1. Measuring Expectations Relative Accuracy. We focus on forecasts of variables that are present in baseline business cycles models, namely inflation and the nominal interest rate. The SCE collects households’ 12 months ahead point forecasts for inflation, and subjective probabilities of the interest rate on savings account being higher. We measure accuracy using the absolute value of forecast errors.

For inflation, measuring accuracy is straightforward, as both point forecasts and realized outcomes are observable. Our definition of realized inflation is the year-over-year growth rate of the Consumer Price Index (CPI), expressed in percentage points. For robustness, we also consider alternative measures based on the Personal Consumption Expenditures (PCE) price index and the CPI core, results are qualitatively the same.

³The SCE releases are available at <https://www.newyorkfed.org/microeconomics/sce>.

⁴Specifically, the SCE does not collect data on credit limits.

For the interest rate, the main challenge is that the true underlying probability is unobservable⁵. To address this, we estimate a Bayesian Vector Autoregression (BVAR) using monthly macroeconomic data from 1960M1 to 2025M1 and infer the probability of higher interest rates from the model’s posterior distribution.

2.2. Identifying Hand-to-Mouth. At the crux of our analysis is the identification of hand-to-mouth households in the SCE. To do so, we leverage responses to three specific questions that, in our view, systematically elicit different answers from hand-to-mouth households compared to others (the savers).

Liquidity Constraint. In the credit survey, households are asked about their ability to respond to an unforeseen need for liquidity,

Q: *What do you think is the percent chance that you could come up with 2,000\$ if an unexpected need arose within the next month?* ____

A low probability of being able to come up with the money simultaneously reflects a household’s lack of liquid wealth and/or limited access to credit markets. A drawback of this proxy is that it may fail to capture households with zero liquid wealth who currently behave as hand-to-mouth, but would take on debt to respond to an unexpected need for liquidity. In practice, we identify as hand-to-mouth those households who report a probability below 30%.

Negative Income Shock. In the survey on spending habits, the households are asked the following multiple-choice question,

Q: *Suppose next year you were to find your household with 10% less household income than you currently expect. What would you do?*

1. Cut spending by the whole amount
2. Not cut spending at all, but cut my savings by the whole amount
3. Not cut spending at all, but increase my debt by borrowing the whole amount
4. Cut spending by some and cut savings by some
5. Cut spending by some and increase debt by some.

⁵For unemployment, [Mitman et al. \(2022\)](#) approximate the true probability using the average forecast from the *Survey of Professional Forecasters*, but no equivalent exists for interest rates.

6. Cut savings by some and increase debt by some.
7. Cut spending by some, cut savings by some and increase debt some.

In theory, a household selecting option 1 must be hand-to-mouth, as this is the only response consistent with an inability to smooth consumption intertemporally following a negative income shock.⁶ However, because the income loss is relatively large, the same caveat mentioned for the liquidity constraint proxy applies; households who behave as hand-to-mouth not due to liquidity constraints, but to avoid borrowing costs, might be missed. How relevant this concern is depends on whether those households consider a 10% income drop significant enough to take on debt. An additional source of misclassification may arise because the question does not clearly specify whether the shock is permanent or temporary; any household may choose option 1 when interpreting the income shock as permanent.

Default Probability. Lastly, in the SCE main survey households are asked about their perceived probability of defaulting on any of their current debt,

Q: *What do you think is the percent chance that, over the next 3 months, you will NOT be able to make one of your debt payments (that is, the minimum required payments on credit and retail cards, auto loans, student loans, mortgages, or any other debt you may have)?* ____

This question can only identify a subset of hand-to-mouth households, those whom [Lusardi, Schneider, and Tufano \(2011\)](#) would classify as *financially fragile*. Specifically, it captures households that are near or at their borrowing limit. In practice, we classify a household as hand-to-mouth if it reports a perceived probability of defaulting greater than 70%. Results based on this proxy offer insight into whether meaningful differences in the accuracy of expectations exist among hand-to-mouth households and serve as a robustness check.

Descriptive Statistics. For our three proxies, we compute the share of households identified as hand-to-mouth relative to the total number of households. We denote this fraction ϕ . We also compute ratios of medians and standard deviations of the absolute value of forecast errors. For each proxy, we discard outliers, defined as households whose inflation expectations are above the 95th percentile of the distribution across all households.

⁶Interestingly, an equivalent question is asked for a hypothetical 10% positive income shock, which we expect to move households away from hand-to-mouth behavior. Indeed, nearly all respondents choose an option implying consumption smoothing.

Table 1: Descriptive Statistics

	Negative Income Shock	Liquidity Constraint	Default Probability
ϕ (HtM share)	0.47	0.24	0.04
<i>Inflation forecast errors</i>			
Median ratio	0.83	0.57	0.65
S.D. ratio	0.88	0.74	0.79
<i>Interest rate forecast errors</i>			
Median ratio	0.95	0.85	0.97
S.D. ratio	0.98	0.97	1.03

Notes: Forecast errors measured in absolute values. Ratios are savers over hand-to-mouth.

Liquidity constraints yield the most realistic share of hand-to-mouth households compared to empirical estimates based on financial data typically reported in the literature (see [Kaplan, Violante, and Weidner 2014](#), [Kaplan and Violante 2014](#)). The negative income shock appears to classify too many households as hand-to-mouth, which may reflect different interpretations of the survey question by respondents. As expected, default probability captures only a small share of total households, a subset of the hand-to-mouth. Overall, [Table 1](#) indicates that expectations of hand-to-mouth households are unconditionally less accurate and more dispersed than those of other households for both inflation and the probability of interest rates increasing.

Table 2: Median Liquid Assets

	Negative Income Shock	Liquidity Constraint	Default Probability
Hand-to-mouth	3,450	-4,500	-20,000
Savers	10,500	15,000	8,000

Notes: Liquid assets calculated as the current value of savings accounts (excluding retirement accounts) minus outstanding debt (excluding housing).

[Table 2](#) presents median liquid assets for the hand-to-mouth and the remaining households for each of the proxy. The reported values confirm that our proxies successfully identify households with lower liquid assets as hand-to-mouth.

2.3. Relative Accuracy of Hand-to-mouth Expectations. Unconditional differences in expectations across our household partitions could simply reflect composition effects. The SCE collects additional information that allows us to conduct an analysis controlling for individual characteristics that may influence forecast quality. This approach allows us to assess whether two otherwise identical households form

expectations differently based on their classification as hand-to-mouth. It aligns with the standard assumption in heterogeneous agent models, whether *à la* Aiyagari or two-agent, that households are identical before entering the economy⁷.

Table 3 reports regressions of the absolute value of inflation forecast errors on a binary variable indicating whether a household is classified as hand-to-mouth according to our three proxies. Controlling for education and numeracy, hand-to-mouth households make significantly less accurate forecasts than other households which should not be attributable to differences in information-processing ability.

Table 3: Relative Inflation Expectations Accuracy of Hand-to-mouth Households

	Negative Income Shock	Liquidity Constraint	Default Probability
	-	-	-
Hand-to-mouth	0.563*** (0.030)	1.734*** (0.038)	1.731*** (0.071)
High School	-	-	-
Some College	-0.732*** (0.038)	-0.751*** (0.039)	-0.862** (0.035)
College	-1.848*** (0.038)	-1.764*** (0.040)	-2.178*** (0.035)
Low Numeracy	-	-	-
High Numeracy	-2.087*** (0.033)	-2.061*** (0.035)	-2.474*** (0.030)
Unemployed	-	-	-
Part-time employed	0.041 (0.048)	0.107** (0.050)	-0.013 (0.044)
Full-time employed	-0.508*** (0.033)	-0.399*** (0.034)	-0.552*** (0.030)
Observations	109,879	112,972	156,160
F Statistic	1783.44	2306.78	2981.38
R^2	0.112	0.133	0.125
Time Fixed Effects	yes	yes	yes

Notes: Estimates from a regression of the absolute value of inflation forecast errors on the household hand-to-mouth status identified with different proxies. Estimates are relative to households that do not qualify as hand-to-mouth. Robust standard errors in parentheses, * $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$. Sample: 2013M8-2024M4.

These empirical results are a first contribution of the paper; novel micro moments

⁷In incomplete market models *à la* Aiyagari, heterogeneity arises from realizations of idiosyncratic shocks, whereas in two-agent models an initial draw before $t = 0$ partitions households into savers and hand-to-mouth.

about expectation that can be used to evaluate or calibrate information structures in heterogeneous agent models. In the remainder of the paper, we turn to theoretical business cycle models with rationally inattentive agents, and test whether they can replicate the cross-sectional differences in expectations accuracy reported in [Table 3](#), while also generating empirically consistent dynamics for macroeconomic variables.

3. Economic Environment

In this section, we describe the features of a baseline two-agent economy with a competitive labor market, excluding attention problems. Time is discrete, and periods correspond to quarters. Aggregate fluctuations are driven by two exogenous shocks; aggregate technology and innovations to the monetary policy rule. We focus on these shocks for two reasons. First, prior work shows that Ricardian models with inattention can generate aggregate dynamics that are consistent with the data for these shocks. Second, the presence of a demand and a supply shock ensures that households' optimal actions need not be correlated with inflation. In turn, optimal attention allocations may or may not reveal information about inflation, a micro moment we use to evaluate the model's fit.

The model features a continuum of firms producing differentiated goods, a Central Bank setting the nominal interest rate, and a continuum of households composed of savers and hand-to-mouth. This framework resembles [Bilbiie \(2008, 2020\)](#), and [Debortoli and Galí \(2024\)](#), except that all sources of New Keynesian adjustment frictions are discarded.

In [Appendix E](#), we describe the modifications to the economic environment and attention problems implied by assuming that each household possesses some market power, allowing them to set a wage rate for the differentiated labor services they supply.

3.1. Households. There is a continuum of households indexed by $j \in [0, 1]$, each of which belongs to one of two types $h \in \{\mathcal{H}, \mathcal{S}\}$. Each household keeps its type permanently and seeks to maximize its expected discounted sum of period utilities. The discount factor is $\beta \in (0, 1)$, and the period utility function is

$$U(C_{jt}, L_{jt}) = \frac{C_{jt}^{1-\gamma} - 1}{1-\gamma} - \varphi^h \frac{L_{jt}^{1+\psi}}{1+\psi} \quad (1)$$

where

$$C_{jt} = \left(\int_0^1 C_{ijt}^{\frac{\theta-1}{\theta}} di \right)^{\frac{\theta}{\theta-1}}. \quad (2)$$

Here, C_{jt} is a composite consumption index, C_{ijt} is the consumed quantity of variety i and L_{jt} denotes the labor supplied. The parameter γ is the inverse of the intertemporal elasticity of substitution, φ^h scales labor disutility⁸, ψ is the inverse of the Frisch elasticity and $\theta > 1$ is the preference parameter for the elasticity of substitution between consumption varieties.

Household $j \in [0, \phi]$ is a hand-to-mouth, $h = \mathcal{H}$, and its flow budget constraint is given by

$$\int_0^1 P_{it} C_{ijt} di = W_t L_{jt} - T_t^{\mathcal{H}}. \quad (3)$$

Household $j \in [\phi, 1]$ is a saver, $h = \mathcal{S}$, and its flow budget constraint is

$$\int_0^1 P_{it} C_{ijt} di + B_{jt} = R_{t-1} B_{j,t-1} + W_t L_{jt} + D_t^{\mathcal{S}} - T_t^{\mathcal{S}}. \quad (4)$$

Here P_{it} is the price of variety i , W_t is the nominal wage rate, $T_t^{\mathcal{H}}$ and $T_t^{\mathcal{S}}$ are nominal lump-sum taxes, B_{jt} is household j 's nominal bond holdings, R_t is the gross nominal interest rate paid on period $t-1$ nominal bonds, and $D_t^{\mathcal{S}}$ are dividends accrued from firms ownership by the savers.

We make the assumption that $B_{jt} > 0$ always holds for any household $j \in [\phi, 1]$. This allows us to write down [Equation \(4\)](#) in terms of logged variables⁹ and also effectively rules out Ponzi schemes.

Households of the same type are ex-ante identical, but ex-post they are not under imperfect information. As detailed below, each household in the inattentive economy receives noisy signal(s) about the state of the world, and idiosyncratic noise realizations lead them to take different actions.

In each period t , every household chooses a consumption vector $\{C_{ijt}\}_{i \in [0,1]}$ and labor supply L_{jt} . If household j is a saver, it also chooses its nominal bonds holdings B_{jt} . All decisions are made taking as given exogenous shocks, the vector of prices for consumption varieties, the wage rate, the nominal interest rate and all aggregate quantities.

⁸Households preferences only differ in terms of labor disutility, φ^h , which lets us derive a non-stochastic steady-state that is symmetrical in hours worked. This parameter has no influence on the linearized dynamics.

⁹In turn, it becomes straightforward to approximate an household utility flow in terms of log-deviations from the non-stochastic steady-state.

3.2. Firms. There is a continuum $i \in [0, 1]$ of firms. Firm i produces a differentiated variety of the consumption good using the production function

$$Y_{it} = e^{a_t} e^{a_{it}} L_{it}^\alpha. \quad (5)$$

Here, L_{it} is the quantity of labor used by firm i for production, while a_t and a_{it} are mean-zero stochastic processes for aggregate and firm-specific technology, respectively. The parameter $\alpha \in (0, 1]$ is the elasticity of output with respect to labor, which determines the degree of returns to scale.

Firm i seeks to maximize the discounted¹⁰ sum of its period nominal profits (or dividends) given by

$$D_{it} = (1 + \tau_p) P_{it} Y_{it} - W_t L_{it} \quad (6)$$

where τ_p is a production subsidy.

In each period t , firm i sets a price P_{it} for its variety and demands quantity L_{it} of effective labor taking as given exogenous shocks, the vector of prices set by other firms, aggregate demand, the wage rate, the nominal interest rate and all aggregate quantities. Each firm commits to supplying any quantity of its consumption variety demanded, at the price it sets.

3.3. Government. The government consist of a monetary and a fiscal authority. The Central Bank sets the nominal rate according to a Taylor rule

$$\frac{R_t}{R} = \left(\frac{R_{t-1}}{R} \right)^{\rho_r} \left[\left(\frac{\Pi_t}{\Pi} \right)^{\phi_\pi} \left(\frac{Y_t}{Y_t^*} \right)^{\phi_{y^*}} \right]^{1-\rho_r} e^{v_t} \quad (7)$$

where $\Pi_t := (P_t/P_{t-1})$ denotes the inflation rate, P_t is a price index, (Y_t/Y_t^*) is the output gap defined as the ratio between actual output and its value that would prevail under perfect information and v_t is a monetary policy shock. Variables without index refer to steady-state values. The parameters ρ_r , ϕ_π and ϕ_{y^*} control the degree of inertia and the strength of the response of the monetary policy.

The government budget constraint in period t is

$$T_t + B_t = R_{t-1} B_{t-1} + \tau_p \int_0^1 P_{it} Y_{it} di. \quad (8)$$

Following common practice, monetary policy is active and fiscal policy is passive in the sense of [Leeper \(1991\)](#). To finance interest on nominal bonds and subsidy

¹⁰Formally, firms use a stochastic discount factor that is a function of the consumption flows of households of type \mathcal{S} , to value profits across periods. The definition of the stochastic discount factor's functional form can be found in [Appendix D.1](#).

payments, the government can either collect lump-sum taxes or issue new bonds. Given the presence of hand-to-mouth households, the method of government financing matters. In the following, we will assume that lump-sum taxes for hand-to-mouth households are fixed,

$$T_t^{\mathcal{H}} = T^{\mathcal{H}} \quad \forall t \geq 0. \quad (9)$$

This assumptions allows us to abstract from considerations regarding how the government finances its expenditures in response to shocks, and guarantees that our results are not driven by unrealistic fluctuations in real taxes. We also solved the model under empirically plausible fiscal rules and found no significant changes in our findings.

Lastly, the production subsidy is set to correct distortions arising from market power in the non-stochastic steady-state such that

$$\tau_p = \frac{\tilde{\theta}}{\tilde{\theta} - 1} - 1 \quad (10)$$

where $\tilde{\theta} > 1$ is the model implied elasticity of substitution between varieties of goods, which is strictly smaller than the preference parameter θ whenever households are inattentive.

3.4. Shocks. There are three types of exogenous processes: monetary policy shocks, aggregate technology, and firm-specific technology. The latter ensures that the model's elasticity of substitution, $\tilde{\theta}$, is consistent with optimal attention allocation for any attention costs.

Monetary policy shocks are *i.i.d.* Gaussian innovations and the technological shocks follow independent stationnary Gaussian first-order autoregressive processes. The cross-sectional mean of firm-specific technology averages to zero in the cross-section.

The vector,

$$\boldsymbol{\varepsilon}_t = (\varepsilon_t^v, \varepsilon_t^a, \{\varepsilon_{it}^a\}_{i \in [0,1]})' \quad (11)$$

collects all of the exogenous innovations affecting the economy at time t .

3.5. Aggregation. Average composite consumption and labor supply among household types are defined as

$$C_t^{\mathcal{H}} = \int_0^\phi C_{jt} dj \quad , \quad C_t^{\mathcal{S}} = \int_\phi^1 C_{jt} dj \quad (12)$$

and

$$L_t^{\mathcal{H}} = \int_0^\phi L_{jt} dj, \quad L_t^{\mathcal{S}} = \int_\phi^1 L_{jt} dj. \quad (13)$$

Aggregate composite consumption and labor supply are defined by the weighted sums

$$C_t = \phi C_t^{\mathcal{H}} + (1 - \phi) C_t^{\mathcal{S}}, \quad L_t^s = \phi L_t^{\mathcal{H}} + (1 - \phi) L_t^{\mathcal{S}}. \quad (14)$$

Aggregate demand for consumption variety i is defined analogously.

Aggregate output, labor demand and dividends are obtained by integrating over the continuum of firms

$$Y_t = \int_0^1 Y_{it} di, \quad L_t^d = \int_0^1 L_{it}^d di, \quad D_t = \int_0^1 D_{it} di. \quad (15)$$

Aggregate bonds, dividends, and taxes are given by

$$B_t = (1 - \phi) \int_\phi^1 B_{jt} dj, \quad D_t = (1 - \phi) D_t^{\mathcal{S}}, \quad T_t = \phi T_t^{\mathcal{H}} + (1 - \phi) T_t^{\mathcal{S}} \quad (16)$$

Lastly, we assume that the price index can always be written as

$$1 = \int_0^1 d_p(\hat{P}_{it}) di \quad (17)$$

where d_p is some twice continuously differentiable function. Notice that this functional nests the index that would prevail under perfect information¹¹ and yields an identical expression once log-linearized.

3.6. Notation. The relative price of consumption variety i and the relative consumption of variety i by household j are denoted

$$\hat{P}_{it} = \frac{P_{it}}{P_t}, \quad \hat{C}_{ijt} = \frac{C_{ijt}}{C_{jt}}. \quad (18)$$

The real wage rate is given by

$$\tilde{W}_t = \frac{W_t}{P_t}, \quad (19)$$

and aggregate real fiscal variables and dividends are given by

¹¹For example, when households have perfect information, $1 = \int_0^1 \hat{P}_{it}^{1-\tilde{\theta}} di$, and $\tilde{\theta} = \theta$.

$$\tilde{B}_t = \frac{B_t}{P_t}, \quad \tilde{T}_t = \frac{T_t}{P_t}, \quad \tilde{D}_t = \frac{D_t}{P_t}. \quad (20)$$

Household real bonds holdings, real taxes and real dividends for each type of households are defined analogously.

4. Attention Problems

Decision-makers subject to rational inattention face a tradeoff where processing more information increases their expected payoff, but requires more attention, which is costly. This section describes the maximization problems faced by decision-makers in the inattentive economy.

The attention problem is a linear quadratic Gaussian (LQG) dynamic rational inattention problem. The objective is quadratic in the state and the decision-maker's actions, the state follows linear dynamics with Gaussian innovations, and the information cost is linear in Shannon's mutual information.

To express the objectives from [Section 3](#) as LQG-RI problems, we compute log-quadratic approximations of the expected discounted sum of profits and utilities around the non-stochastic steady-state. More details are provided below.

An inattentive decision-maker, indexed by ι , maximizes by choosing a costly attention strategy consisting of signal(s) about the state of the economy. Formally, the decision-maker solves:

$$\max_{\mathbf{\Gamma}_\iota, \mathbf{\Sigma}_\iota} \left\{ \sum_{t=0}^{\infty} \beta^t E_{\iota,-1} \left[\frac{1}{2} (\mathbf{x}_{\iota t} - \mathbf{x}_{\iota t}^*)' \mathbf{H}_{\mathbf{x}_\iota} (\mathbf{x}_{\iota t} - \mathbf{x}_{\iota t}^*) \right] - \lambda_\iota \sum_{t=0}^{\infty} \beta^t I(\boldsymbol{\xi}_{\iota t}; \mathbf{S}_{\iota t} | \mathcal{I}_{\iota t-1}) \right\} \quad (21)$$

subject to

$$\mathbf{x}_{\iota t}^* = \mathbf{G} \boldsymbol{\xi}_{\iota t} \quad (22)$$

$$\boldsymbol{\xi}_{\iota t+1} = \mathbf{F} \boldsymbol{\xi}_{\iota t} + \boldsymbol{\mu}_{\iota t+1} \quad (23)$$

$$\mathbf{x}_{\iota t} = E[\mathbf{x}_{\iota t}^* | \mathcal{I}_{\iota t}] \quad (24)$$

$$\mathcal{I}_{\iota t} = \mathcal{I}_{\iota, -1} \cup \{\mathbf{S}_{\iota 0}, \dots, \mathbf{S}_{\iota t}\} \quad (25)$$

$$\mathbf{S}_{\iota t} = \mathbf{\Gamma}_\iota \boldsymbol{\xi}_{\iota t} + \boldsymbol{\nu}_{\iota t} \quad (26)$$

$$I(\boldsymbol{\xi}_{it}; \boldsymbol{S}_{it} | \mathcal{I}_{it-1}) = H(\boldsymbol{\xi}_{it} | \mathcal{I}_{it-1}) - H(\boldsymbol{\xi}_{it} | \mathcal{I}_{it}). \quad (27)$$

with

$$\mathcal{I}_{it-1} \mid \boldsymbol{\Gamma}_i, \boldsymbol{\Sigma}_{\nu_i} \quad (28)$$

The vector \boldsymbol{x}_{it} contains the decision-maker's actions and the vector \boldsymbol{x}_{it}^* contains the actions they would take under perfect information. The first term appearing in Equation (21) is the part of per-period payoff (i.e. expected losses incurred from suboptimal actions) that is affected by the decision-maker actions and has a quadratic form with weighting matrix $\boldsymbol{H}_{\boldsymbol{x}_i}$.

The second term in Equation (21) is a known quantity representing the discounted sum of information costs. The per-period information cost consists of the product between the marginal cost of attention, λ_i , and the per-period information flow.

Equation (22) defines a linear mapping between the current state, $\boldsymbol{\xi}_{it}$, and the vector of optimal actions. Given that structural shocks are Gaussian by assumption, we know that there exists at least one representation for the state vector for which this equality holds exactly¹².

Equation (23) describes the evolution of the state vector between two consecutive periods where $\boldsymbol{\mu}_{it+1}$ follows a white noise vector process with covariance $\boldsymbol{\Sigma}_{\mu_i}$ and \boldsymbol{F} is matrix with eigenvalues that may lie on the unit circle. Thus, the state-space for $\boldsymbol{\xi}_{it}$ and \boldsymbol{x}_{it}^* , described by Equation (22) and Equation (23), is linear with Gaussian innovations, but stationnarity is not imposed¹³.

The decision-maker's information set in the current period is described by Equation (25). It consists of the initial information, $\mathcal{I}_{i,-1}$, and all signals received up to and including the current period.

Equation (26) describes the signal received in period t . This equation posits that the signal loads on the period t state vector according to the matrix $\boldsymbol{\Gamma}_i$ plus $\boldsymbol{\nu}_{it}$, a white noise vector process with diagonal covariance matrix $\boldsymbol{\Sigma}_{\nu_i}$ ¹⁴.

Equation (27) measures the per-period information flow as the difference in the conditional entropy, $H(\cdot \mid \cdot)$, about the state before and after observing the signal in

¹²However, the relevant state vector in the presence of information frictions may be infinite-dimensional, and solving this problem numerically requires some level of approximation.

¹³For the maximization problem to be well-defined, all we need is the conditional second moments to be finite which does not require stationnarity. We allow for unit roots dynamics.

¹⁴It can be shown that a signal loading on the current state plus a vector of *i.i.d.* Gaussian noise, as described in Equation (26), is of the optimal form given the decision-maker problem defined by Equations (21) to (27). For a formal proof, see Maćkowiak, Matějka, and Wiederholt (2018).

period t . This quantity represents Shannon's mutual information and quantifies the per-period uncertainty reduction.

[Equation \(24\)](#) describes how the decision-maker optimally selects \mathbf{x}_{it} according to his information set. Given the linear Gaussian structure, the decision-maker applies the Kalman filter to optimally infer \mathbf{x}_{it}^* from any sequence of noisy signals.

[Equation \(28\)](#) states that the initial information set is not entirely exogenous, but at least some of its characteristics depend on the attention strategy chosen by the decision-maker. The exact relationship is described below.

The decision-maker optimizes freely over the matrices $\mathbf{\Gamma}_\iota$ and $\mathbf{\Sigma}_{\nu_\iota}$ (i.e., the rank of these matrices which determines the total number of signals received is endogenous) to maximize the difference between the expected discounted sum of per-period payoffs and information costs. The fundamental tradeoff is that receiving more informative signals raises both the expected payoff and information costs. All decisions regarding the attention strategy that maximize [Equation \(21\)](#) are made in period -1 , with [Equations \(24\) to \(28\)](#) taken as given.

I make the standard assumption that the initial information set, $\mathcal{I}_{\iota,-1}$, depends on the chosen matrices $\mathbf{\Gamma}_\iota$ and $\mathbf{\Sigma}_{\nu_\iota}$. Specifically, given its chosen attention strategy the decision-maker receives a long sequence of signals that places him at the steady-state of the Kalman filter in period -1 . This assumption ensures that the problems of choosing $\{\mathbf{\Gamma}_{it}, \mathbf{\Sigma}_{\nu_{it}}\}_{t=0}^\infty$ sequentially or $\mathbf{\Gamma}_\iota$ and $\mathbf{\Sigma}_{\nu_\iota}$ once and for all in period -1 are equivalent.

All noise terms in the signals are idiosyncratic, meaning that realizations of ν_{it} are independent across firms and households, and sum to zero in the cross-section. We use this property when aggregating individual decisions.

In the following, we present and describe the problems of each agent in the inattentive economy. To keep things brief, we abstract from cross-sectional efficiency decisions (i.e., households' consumption mix), as these can be shown to be optimally independent from those affecting aggregate dynamics. Lastly, we assume that paying attention to idiosyncratic shocks is a separate activity. Our results are independent of this assumption, but it is practical as it allows us to remain agnostic about the stochastic process governing firm-specific productivity.

4.1. Firms. The step-by-step approximation to the expected discounted sum of firm $i \in [0, 1]$'s period profits can be found in [Appendix D.1](#). The resulting matrix featured in firm i 's period payoff is

$$\mathbf{H}_{\mathbf{x}_i} = -(C^S)^{-\gamma} Y \left[\frac{\tilde{\theta}(\tilde{\theta} + \alpha(1 - \tilde{\theta}))}{\alpha} \right]. \quad (29)$$

Firm i 's vector of choice variables is

$$\mathbf{x}_{it} = (p_{it})' \quad (30)$$

and its vector of optimal actions in response to aggregate shocks is

$$\mathbf{x}_{it}^* = \left(p_t + \frac{\frac{1-\alpha}{\alpha}}{1+\frac{1-\alpha}{\alpha}\bar{\theta}} c_t + \frac{1}{1+\frac{1-\alpha}{\alpha}\bar{\theta}} \tilde{w}_t - \frac{\frac{1}{\alpha}}{1+\frac{1-\alpha}{\alpha}\bar{\theta}} a_t \right)' . \quad (31)$$

4.2. Hand-to-mouth. The step-by-step approximation to the expected discounted sum of hand-to-mouth household $j \in [0, \phi]$'s period utilities can be found in [Appendix D.2](#). The resulting matrix featured in household j 's period payoff is

$$\mathbf{H}_{x_j}^{\mathcal{H}} = -(C^{\mathcal{H}})^{1-\gamma} [\omega_W^{\mathcal{H}} (\omega_W^{\mathcal{H}} \gamma + \psi)] . \quad (32)$$

Household j 's vector of choice variables is

$$\mathbf{x}_{jt} = (l_{jt})' , \quad (33)$$

its vector of optimal actions is

$$\mathbf{x}_{jt}^* = \left(\frac{\tilde{w}_t - \gamma c_{jt}^*}{\psi} \right) , \quad (34)$$

with

$$c_{jt}^* = \omega_w^{\mathcal{H}} (\tilde{w}_t + l_{jt}^*) \quad (35)$$

4.3. Savers. The step-by-step approximation to the expected discounted sum of saver $j \in [\phi, 1]$'s period utilities can be found in [Appendix D.3](#). An additional step is taken to obtain an expression that reduces to a pure tracking problem,¹⁵ with details provided in [Appendix D.4](#). The resulting matrix featured in household j 's period payoff is

$$\mathbf{H}_{x_j}^{\mathcal{S}} = -(C^{\mathcal{S}})^{1-\gamma} \begin{bmatrix} \gamma & 0 \\ 0 & \omega_w^{\mathcal{S}} \psi \end{bmatrix} . \quad (36)$$

Household j 's vector of choice variables is

$$\mathbf{x}_{jt} = (c_{jt}, l_{jt})' \quad (37)$$

and its vector of optimal actions is

¹⁵In a pure tracking problem, the objective is independent of the relationship between today's and tomorrow's mistakes. In other words, the state is purely exogenous, and all that matters for the decision-maker is to keep \mathbf{x}_{jt} as close as possible to \mathbf{x}_{jt}^* .

$$\mathbf{x}_{jt}^* = \begin{pmatrix} E_{jt} \left[-\frac{1}{\gamma}(r_t - \pi_{t+1}) + c_{jt+1}^* \right] \\ \frac{\tilde{w}_t - \gamma c_{jt}^*}{\psi} \end{pmatrix} \quad (38)$$

with c_{jt}^* and c_{jt+1}^* defined accordingly to Equation (105).

The change of variables in Appendix D.4 reformulates the savers' problem in terms of consumption and labor decisions. This transformation eliminates the intertemporal interaction in the loss function that arises when households instead choose their labor supply and bond holdings, as in Appendix D.3. The intuition for why, under this formulation, errors today do not carry over to subsequent periods is that households can always adjust their labor supply to satisfy their intratemporal optimality condition for any chosen level of consumption.

5. Equilibrium under Rational Inattention

An equilibrium of the inattentive economy, consisting of Section 3's environment and Section 4's attention problems, is a fixed-point where each decision-maker's signals are optimal, given the signals selected by all others.

In this section, we first define an equilibrium for a linear approximation of the economy's dynamics around the non-stochastic steady-state¹⁶ where variables are expressed in terms of log-deviations (i.e., $x_t := \frac{X_t - X}{X}$) from this point. We then outline a numerical procedure to solve for the economy's fixed-point.

Definition 1. *Given initial bonds holdings $b_{j,-1} = 0$ for every household $j \in [0, \phi]$, and for any sequence of exogenous innovations, $\{\varepsilon_t\}_{t=0}^\infty$, sequence of noise, $\{\nu_{it}\}_{t=0}^\infty$, for every firm $i \in [0, 1]$ and sequence of noise, $\{\nu_{jt}\}_{t=0}^\infty$, for every household $j \in [0, 1]$ a Sequential Rational Inattention Competitive Equilibrium (SRICE) is:*

- An allocation, $\Omega_i := \{\mathbf{S}_{it}, y_{it}, l_{it}^d, p_{it}\}_{t=0}^\infty$ for every firm $i \in [0, 1]$.
- An allocation, $\Omega_j^H := \{\mathbf{S}_{jt}, c_{jt}, l_{jt}^s, \{\hat{c}_{ijt}\}_{i \in [0, 1]}\}_{t=0}^\infty$ for every hand-to-mouth $j \in [0, \phi]$.
- An allocation, $\Omega_j^S := \{\mathbf{S}_{jt}, c_{jt}, l_{jt}^s, b_{jt}^d, \{\hat{c}_{ijt}\}_{i \in [0, 1]}\}_{t=0}^\infty$ for every saver $j \in [\phi, 1]$.
- An allocation, $\Omega_G := \{b_t^s, t_t\}_{t=0}^\infty$ for the government.

¹⁶We cannot define a global equilibrium as Section 4's attention problems are only valid for linear Gaussian dynamics.

- A set of prices $\{p_t, w_t, r_t\}_{t=0}^{\infty}$.
- Stationnary distributions of households and firms over idiosyncratic innovations and noise.

such that

1. Given $\left\{ \Omega_{j \in [0, \phi]}^{\mathcal{H}}, \Omega_{j \in [\phi, 1]}^{\mathcal{S}}, p_t, r_t, w_t \right\}_{t=0}^{\infty}$, a firm's allocation, Ω_i , solves *Section 4.1's* attention problem $\forall i \in [0, 1]$.
2. Given $\left\{ \Omega_{i \in [0, 1]}, \Omega_{j \in [\phi, 1]}^{\mathcal{S}}, p_t, r_t, w_t \right\}_{t=0}^{\infty}$, a hand-to-mouth's allocation, $\Omega_j^{\mathcal{H}}$, solves *Section 4.2's* attention problem $\forall j \in [0, \phi]$.
3. Given $\left\{ \Omega_{i \in [0, 1]}, \Omega_{j \in [0, \phi]}^{\mathcal{H}}, p_t, r_t, w_t \right\}_{t=0}^{\infty}$, a saver's allocation, $\Omega_j^{\mathcal{S}}$, solves *Section 4.3's* attention problem $\forall j \in [\phi, 1]$.
4. Monetary policy satisfies the Taylor rule, fiscal policy satisfy its specified rules, and given interest and subsidies payments, the government budget is satisfied.
5. All markets clear $\forall t \geq 0$:
 - (a) $y_t = c_t$
 - (b) $l_t^s = l_t^d$
 - (c) $b_t^s = b_t^d$
 - (d) $y_{it} = c_{it} \forall i \in [0, 1]$
6. Aggregate price index is given by $\forall t \geq 0$:
 - (a) $p_t = \int_0^1 p_{it} di$
7. Aggregate quantities are given by $\forall t \geq 0$:
 - (a) $y_t = \int_0^1 y_{it} di$
 - (b) $c_t = \phi(C^{\mathcal{H}}/C) \int_0^{\phi} c_{jt} dj + (1 - \phi)(C^{\mathcal{S}}/C) \int_{\phi}^1 c_{jt} dj$
 - (c) $l_t^s = \phi(L^{\mathcal{H}}/L) \int_0^{\phi} l_{jt}^s dj + (1 - \phi)(L^{\mathcal{S}}/L) \int_{\phi}^1 l_{jt}^s dj$
 - (d) $l_t^d = \int_0^1 l_{it}^d di$
 - (e) $b_t^s = (1 - \phi)(B^{\mathcal{S}}/B) \int_{\phi}^1 b_{jt}^d dj$
 - (f) $c_{it} = \phi(C_i^{\mathcal{H}}) \int_0^{\phi} c_{ijt} dj + (1 - \phi)(C_i^{\mathcal{S}}) \int_{\phi}^1 c_{ijt} dj$

5.1. Computing the Aggregate Equilibrium. We now describe an iterative procedure for solving the SRICE numerically. First, notice that an equilibrium (of aggregate prices and quantities) in the inattentive economy can always be summarized by a finite set of stochastic processes for exogenous and endogenous variables. In this case, the dynamics of the price level, the real wage, and the consumption of both household types, along with the exogenous processes for aggregate technology and monetary policy shocks, are sufficient to determine the responses of all other aggregate variables. We can solve for the endogenous processes with arbitrary precision as follows;

Algorithm. First, we guess the impulse responses over T periods for p_t , \tilde{w}_t , c_t^H , and c_t^S . This is equivalent to guessing a moving-average representation, $MA(T)$, for each of these variables.

Second, given these guesses, we compute the profit-maximizing price for a firm. We then approximate the resulting process with a finite-order VARMA¹⁷, which yields a state-space representation of the same form as [Equations \(22\) to \(23\)](#).

Third, we solve the firm's attention problem in [Section 4.1](#). Given the optimal signal(s), we compute the stochastic processes for the aggregate price level and labor demand implied by aggregating individual actions.

Fourth, we compute the optimal labor supply for each type of household and the optimal consumption for savers. We approximate the dynamics of utility-maximizing actions with finite VARMA processes and obtain state-space representations for each of the attention problem.

Fifth, we solve the [Section 4.2](#) and [Section 4.3](#)'s attention problems. Given the optimal signal(s), we compute the stochastic processes for consumption and labor supply for each household type implied by aggregating individual actions.

Sixth, we use the dividend equation, labor demand and labor supply to compute a process for the real wage.

Seventh, we compare the $MA(T)$ representations of the stochastic processes for p_t , \tilde{w}_t , c_t^H , and c_t^S with their initial guesses. If any process differs by more than a prespecified tolerance, we update them via linear extrapolation, and repeat the procedure from step two until convergence. Once convergence is achieved, we verify that labor demand equals labor supply; if so, we have found a fixed point.

When solving the attention problems, we iterate on the first-order condition using [Afrouzi and Yang \(2021\)](#)'s algorithm. To approximate the $MA(T)$ representations of stochastic processes as finite VARMA, we apply the frequency-domain projection

¹⁷That is, we approximate firms' optimal price decisions with a reduced-form VAR(1) with two exogenous shocks.

method of Han, Tan, and Wu (2022). Non-stationary processes are first differenced before the projection step, and an additional row is appended to the state-space representation to restore unit root dynamics.

6. Dynamics under Rational Inattention

With the models set up and the numerical algorithm defined, we are now ready to analyze the dynamics of the inattentive economies.

6.1. Evaluation Strategy. We follow a two-step procedure. In the first step, we calibrate all structural and policy parameters except the marginal costs of attention. In the second step, we solve the models over a grid of values for marginal costs of attention. Our main objective is to isolate the role of household inattention as a propagation mechanism for the dynamic response of output to shocks. We proceed as follows: for every value of households’ marginal cost of attention, we choose the firm’s marginal cost of attention that allows us to match the persistence of inflation in the data (0.62). We retain the household inattention parameter that bring us the closest the persistence of output growth (0.3) in the data. Our procedure differs from a standard minimization approach, which may lead to different inflation persistence across models.

Note that because our model contains only a subset of all the possible shocks affecting the economy, we are implicitly assuming that the conditional responses to technological and monetary policy shocks, taken together, generate persistence in inflation and output growth that is not too far from what would arise if all sources of fluctuations were considered. We believe that to be generally true.¹⁸

We evaluate the fit of the models based on how well they generate persistence in the growth rate of output, and match other macro and micro moments discussed below. A priori, there may not exist any combination of attention parameters that can induce persistence in the growth rate of output.

When using data to calibrate parameters, we rely on empirical evidence from the United States covering the period 1969Q1–2019Q4. However, we employ the full available SCE sample to characterize micro moments on expectations as it provides more variation in inflation and other macroeconomic variables to identifying patterns in cross-sectional expectations. Restricting the analysis to the overlapping period, would not qualitatively nor significantly change our estimates.

¹⁸In a previous version of the paper, we jointly estimated the impulse responses to both of these shocks and found this to hold. Results hinge on the estimation methods and identifying assumptions.

Calibrated Parameters. The left panel of [Table 4](#) summarizes the calibrated parameters of our model. We set the inverse of the elasticity of intertemporal substitution, γ , to 1.5 which allows for some interaction with steady-state marginal utility of consumption in the households’ attention problems. We set the inverse of the Frisch elasticity, ψ , to 1.0. We assume a discount factor of $\beta = 0.99$, which corresponds to an annual return of 4% on government bonds. Aggregate technology, A , is used to normalize steady-state output to $Y = 1$. The labor share is set to $\alpha = 0.66$. The price and wage elasticity, $\tilde{\theta}$, and $\tilde{\eta}$ are set to 4, as in [Maćkowiak and Wiederholt \(2015\)](#).

We assume a Taylor rule with interest rate persistence, ρ_r , of 0.9, and reaction coefficients to inflation and the output gap, ϕ_π and ϕ_{y^*} , of 1.5 and 0.125, respectively.

We estimate the stochastic process for technology using the [Fernald \(2014\)](#) measure of Total Factor Productivity adjusted for utilization. Specifically, we linearly detrend the cumulated series and estimate a first-order autoregressive process. The resulting estimates are a persistence parameter, ρ_a , of 0.95 and a standard deviation of shocks, $100\sigma_a$, of 0.8. To estimate the standard deviation of monetary policy shocks, we invert a Taylor rule, using the parameters set above, on time series of interest rates, inflation, and the output gap over our sample. This yields a value for $100\sigma_v$ of 0.2. As long as the monetary policy shocks account for a small portion of the volatility in the nominal interest rate, this procedure should provide accurate estimates, as argued in [Carvalho, Nechio, and Tristao \(2021\)](#).

The share of hand-to-mouth households, ϕ , is set to 0.28, the average value over our sample. We estimate the fraction of hand-mouth in each available waves of the *Survey of Consumer Finances* (SCF) using [Kaplan and Violante \(2014\)](#)’s method. To determine the consumption ratio, $\frac{C^S}{C^H}$, we infer a proxy for consumption expenditure for each type of household, and take the average ratio over the sample, yielding a value of 1.4. We obtain the desired ratio by targeting the non-stochastic steady-state value of taxes \tilde{T}^H . Details on the estimation of the share of hand-to-mouth and computation of the consumption proxies are provided in [Appendix F](#).

Note that given our calibration, we do not encounter the knife-edge case where wealth and substitution effects perfectly offset each other (e.g., this occurs when $\psi = \gamma = \omega_w^H = 1$, as in [Bilbiie 2008](#)) that would lead to a trivial attention problem for the hand-to-mouth.

6.2. Models’ Fit. [Table 5](#) presents unconditional moments from the data and the baseline models. [Figure 1](#) shows the impulse response functions of output and inflation to a positive technological shock and an expansionary monetary policy shock.

We begin by commenting on the model RI-I, in which the wage rate is determined competitively. In terms of macro moments, the model provides a poor fit. Adding

Table 4: Calibrated and Estimated Parameters

Panel A: Calibrated Parameters			Panel B: Inattention Parameters		
Parameter		Value	Parameter		Value
β	Discount factor	0.99	<i>RI-I: competitive-wage</i>		
γ	EIS	1.5	λ_j	Firms marginal cost of attention	485.0
ψ	Inverse Frisch	1.0	λ_j	Households marginal cost of attention	0.8
α	Labor share	0.66	<i>RI-II: monopolistic-wage</i>		
$\tilde{\theta}$	Price elasticity of demand	4.0	λ_j	Firms marginal cost of attention	33.0
$\tilde{\eta}$	Wage elasticity of demand	4.0	λ_j	Households marginal cost of attention	5.8
ϕ	HtM share	0.28	<i>RI-F: inattentive firms, attentive households</i>		
$\frac{C^S}{C^H}$	Steady-state consumption ratio	1.4	λ_j	Firms marginal cost of attention	360.0
ρ_r	Taylor rule inertia	0.9			
ϕ_π	Taylor rule coefficient (inflation)	1.5			
ϕ_{y^*}	Taylor rule coefficient (output gap)	0.125			
ρ_a	Persistence of aggregate technology	0.95			
$100\sigma_a$	100× S.D. of aggregate technology shocks	0.8			
$100\sigma_v$	100× S.D. of monetary policy shock	0.2			

Notes: Marginal attention costs scaled up by a factor of 100,000.

inattention on the side of households is a quantitatively weak propagation mechanism, providing only a slight improvement in output growth persistence compared to the model RI-F, wherein households are perfectly attentive. We trace most of this shortcoming back to the real wage, which absorb most of the effects from shocks and is, in turn, too volatile and insufficiently persistent compared to the data.

We can decompose the mechanics leading to these dynamics and its consequences on output as follows. First, while savers adjust consumption with a delay due to inattention, increasing the persistence of output growth, they also insufficiently adjust their labor supply. Consequently, following any shock, the real wage must fluctuate even more than in a already problematic perfect-information economy for the labor market to clear. The disposable income of hand-to-mouth agents also becomes more volatile, which has a negative effect on the autocorrelation of output growth. As a result, household inattention has a non-monotonic effect on the persistence of output growth in this model, reaching a peak at households' marginal attention costs presented in Table 4.

On the other hand, household inattention generates inflation expectations that are consistent with the data. The model-implied coefficient of hand-to-mouth status on the absolute value of one-year-ahead inflation forecast errors is 0.42. This value is smaller than the one reported in Table 3 for our preferred proxy, but given the parsimonious economic environment, which cannot capture all the incentives households face to pay attention, we consider it meaningful. We can understand this result as follows. The competitive labor market provides little incentive for hand-to-mouth to monitor the economy, only small fluctuations in their labor supply are necessary to

Table 5: Unconditional Moments I

	Data	RI-I	RI-II	RI-F	PI
<i>Targeted Moments</i>					
ρ_π	0.62	0.62	0.62	0.62	0.023
$\rho_{\Delta y}$	0.3	-0.06	0.3	-0.14	-0.025
<i>Untargeted Moments</i>					
$\rho_{\Delta w}$	0.48	-0.20	0.63	-0.18	-0.025
$\sigma_\pi/\sigma_{\Delta y}$	1.06	0.33	1.15	0.37	1.17
$\sigma_{\Delta w}/\sigma_{\Delta y}$	1.10	3.72	0.80	3.07	2.81
$\beta_{\pi,\mathcal{H}}$	1.73	0.42	-0.07	-	-

Notes: Models' moments computed from the equilibrium MA representations. RI-I is the model with competitive-wage. RI-II is the model with monopolistic-wage. RI-F is the model with firms subject to inattention, and households with perfect information.

equalize their marginal rate of substitution to the real wage. The savers must track the real interest rate and adjust labor supply by a larger margin, leading them to pay greater attention. This result remains robust to changes in preference parameters, including the inverse of the EIS and Frisch elasticity.

We now comment on the baseline with monopolistic wages. In terms of fitting the macro moments, the model provides a sharp improvement compared to all other versions considered. Not only can it generate the desired persistence for output growth, but it also yields values for untargeted moments that are close to their empirical counterparts. In this setting, households' inattention is a strong propagation mechanism, as it also induce persistence in the growth rate of labor income, which then generates hump-shaped consumption responses for hand-to-mouth agents.

These improvements, however, come at a cost in terms of micro moments on expectations. This failure is due to the stakes associated with wage-setting decisions being so high that they constitute the main incentive for why households pay attention to the state of the economy. In this setting, it even leads hand-to-mouth agents to form expectations that are more accurate than those of the savers, for whom the additional decisions provide only marginal incentives to pay more attention.

We have three main takeaways from the impulse response functions presented in [Figure 1](#). The first is that the failure of model RI-I to generate output growth persistence mainly resides in the response conditional on monetary innovations. This is to be expected, as the real rate responds to this shock monotonically and therefore induces negative autocorrelation under perfect information for the growth rate of

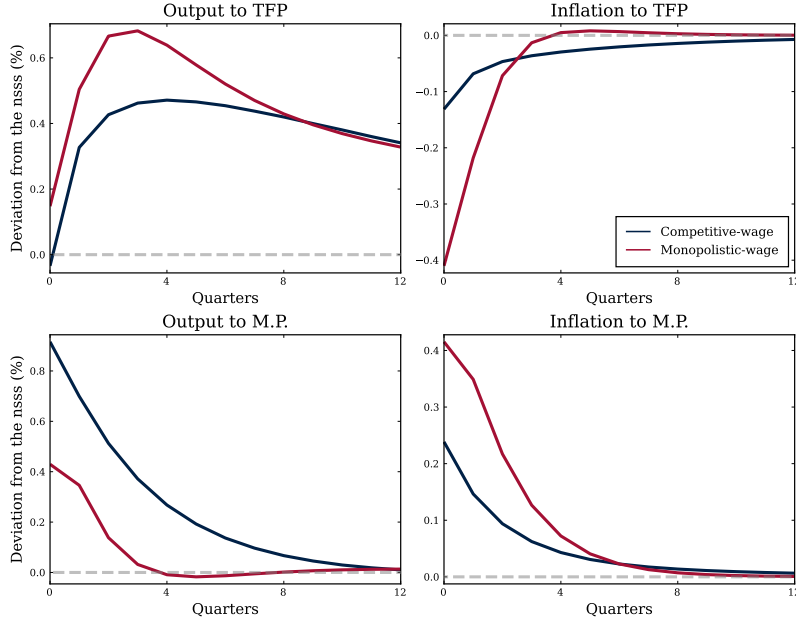


Figure 1: Impulse responses of inflation and output under rational inattention.

consumption, making it harder to reverse. Secondly, for the same level of persistence, inflation is less volatile in model RI-I than in RI-II. This occurs despite the higher volatility of the real wage, as firms need to be (too) inattentive, which ultimately reduces the volatility of inflation even more so than in model RI-II. Lastly, the model with competitive wage setting exhibits a weaker output response to supply shocks compared to the model with labor market power, while its response to demand shocks is relatively stronger.

6.3. The Relevance of Heterogeneity. A common exercise with heterogeneous agent models is to assess how much their dynamics differ from those of representative agent counterparts¹⁹. In our framework, this experiment specifically determines the extent to which the presence of hand-to-mouth households, with an MPC of one, alters the dynamics. We thus proceed to solve both baseline versions setting $\phi = 0.0$, without any changes to the other parameters. This exercise can be viewed as the extreme case of a policy that eliminates all barriers to financial access, resulting in fully Ricardian economies.

¹⁹An RI-DSGE is not, strictly speaking, a representative-agent model, since a continuum of households is needed for noise to sum to zero in the cross-section. Yet the average dynamics under the assumption that all households are savers are effectively the same as those implied by a representative inattentive household whose signal noise realizations are zero in every periods.

Table 6: Unconditional Moments II

	Data	RI-I		RI-II	
		TA	RA	TA	RA
ρ_π	0.62	0.62	0.75	0.62	0.69
$\rho_{\Delta y}$	0.3	-0.06	0.19	0.3	0.34
$\rho_{\Delta w}$	0.48	-0.20	-0.12	0.63	0.59
$\sigma_\pi/\sigma_{\Delta y}$	1.06	0.33	0.29	1.15	1.10
$\sigma_{\Delta w}/\sigma_{\Delta y}$	1.10	3.72	4.05	0.80	0.85

Notes: Models' moments computed from the equilibrium MA representations. TA for two-agent model. RA for Ricardian-agent model.

Table 6 presents the unconditional macro moments, and Figure 2 shows the impulse responses implied by the baseline two-agent economies and their Ricardian counterfactuals. Both confirm our previous comments on the strength and mechanisms by which households' inattention induces persistence in the growth rates of quantities. The predictions of model RI-II with monopolistic wages are barely affected when hand-to-mouth households are not present. In other words, there is barely any tension between the effects of the responses of each type of household on aggregate output; they both pull in the same direction. On the other hand, the Ricardian economy with competitive wages now displays positive output growth autocorrelation. This reversal occurs because labor income is irrelevant for savers' consumption decisions, and therefore the wage rate, which remains too volatile, does not hinder the capacity of inattention to induce sluggishness.

This experiment shows that for rational inattention, or any form of information friction, to act as a strong propagation mechanism in the presence of hand-to-mouth households (even when the growth rate of savers' consumption is persistent), it must generate autocorrelation in the growth rate of labor income, which depends on the model's microfoundations. Assuming labor market on the side of households achieves this goal, but it does so at the cost of producing cross-sectional expectations inconsistent with the data, casting doubt on the realism of the source of persistence in the model.

6.4. Inequality Dynamics. We conclude this section by commenting on the average dynamic responses of different variables for both types of households. Figure 3 shows the impulse response functions of consumption, hours worked, and labor income for savers and hand-to-mouth households in the two baseline economies. The first row shows the dynamics implied by a positive TFP shock, and the second row

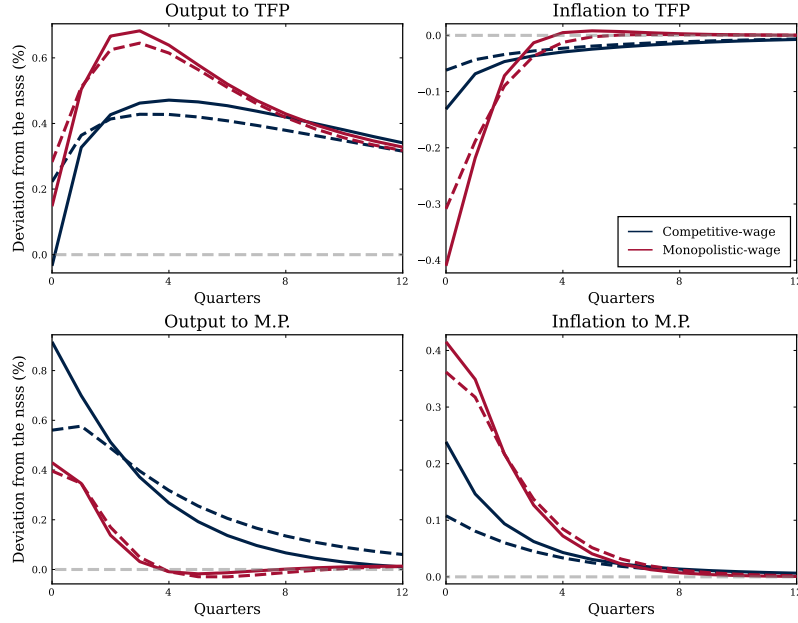


Figure 2: Impulse responses in two-agent and representative-agent economies.

Notes: Solid lines (—) TA models. Dashed lines (---) Ricardian models.

shows those implied by an expansionary monetary policy shock.

It is worthwhile to start by recalling the importance of the households' intratemporal optimality condition. In particular, together with the budget constraint, it pins down optimal hand-to-mouth consumption. Under perfect information, households make decisions such that their marginal rate of substitution equates the real wage. Under rational inattention, this condition does not hold at equality, but it still dictates the target households aim to achieve when deciding how much labor to supply or when setting wages.

Savers' consumption is always strictly determined by their beliefs about the path of the real interest rate, and, through general equilibrium effects, this affects the optimal actions of the hand-to-mouth households. In particular, the wage rate (whether set or determined in competitive equilibrium) is a function of how much labor is required to produce total output.

In the competitive-wage economy, hand-to-mouth consumption fluctuates almost one-to-one with the wage rate required to clear the labor market, and their labor supply response is almost flat. As mentioned earlier, this behavior keeps them close to equalizing their marginal rate of substitution with the real wage and is therefore near-optimal. This is not the case for savers, who should optimally also adjust labor

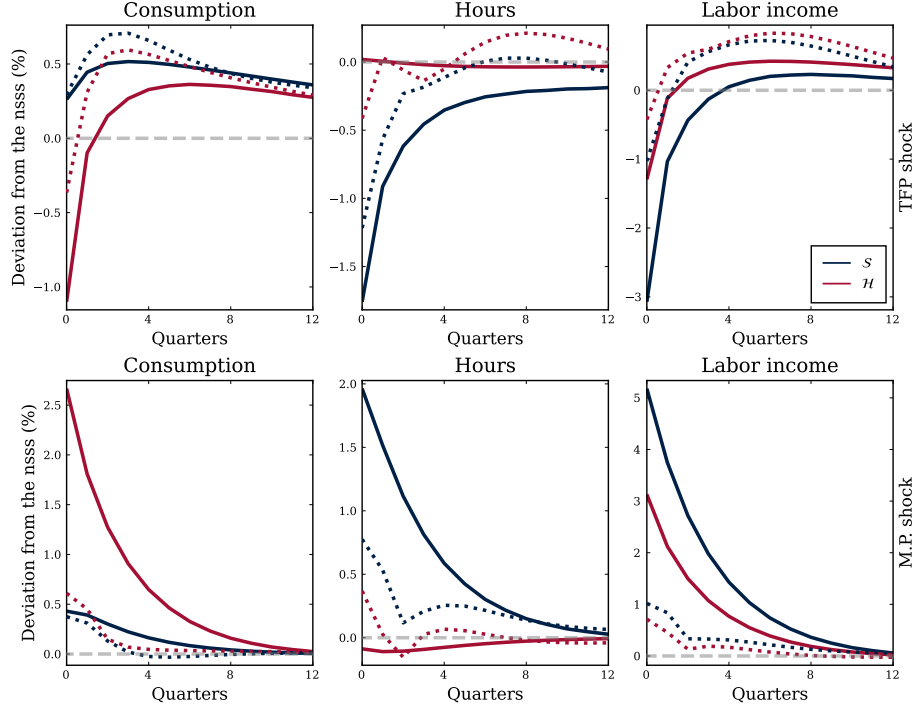


Figure 3: Impulse responses of consumption, hours, and labor income.

Notes: Solid lines (—) RI-I model's dynamics. Dotted lines (···) RI-II model's dynamics.

supply because of their consumption responding to how they perceive the path of the real interest rate. As a result, hours worked and labor income are more volatile for savers than for hand-to-mouth households—an arguably unrealistic prediction given empirical evidence (see [Heathcote, Perri, and Violante 2020](#)) but unavoidable in this setting.

In this model, hand-to-mouth consumption fluctuates several percentage points more than that of savers in response to both types of shocks. In other words, the effects of labor income on hand-to-mouth consumption are larger than those of the real interest rate on savers' consumption. This pattern mirrors that of the perfect-information counterpart and implies that consumption inequality—measured as the difference between savers and hand-to-mouth households—increases following positive supply shocks but decreases in response to positive demand shocks.

On the other hand, the model where households set their wage rate predicts significant fluctuations for hours worked by hand-to-mouth households, meaning that an attention strategy implying a nearly flat labor supply curve is not attainable.

Because households provide labor services that are imperfect substitutes and wages exhibit inertia due to information frictions, labor income is less volatile, and hours are distributed more evenly across types than in the RI-I model.

Hand-to-mouth consumption inherits the properties of labor income²⁰, which reduces inequality fluctuations. As a result, demand shocks are less amplified than in the economy with competitive wages; the dampening of supply shocks is also mitigated. The counterfactual prediction about whose income rises most in response to shocks is likewise attenuated.

More generally, these observations illustrate why the response of output growth to a shock depends on the persistence of labor income growth (even when savers' inattention induces persistence in their own consumption). When such persistence exists, it translates into persistent hand-to-mouth consumption; when it does not, any remaining aggregate-level persistence arises despite labor income peaking on impact.

Robustness. We also solve the models with attentive firms under Calvo pricing (see [Appendix G](#)), which leaves our results unchanged. This shows that the interaction between the source of inflation persistence and household (in)attention is not critical.

7. Ways out of the Conundrum

In the previous section, we showed that baseline two-agent inattentive economies fail to simultaneously match macro and micro moments. Consequently, when choosing between models, researchers must compromise between one that generates a hump-shaped output response to monetary policy shocks and one in which savers form expectations that are relatively more accurate than those of hand-to-mouth households.

Building on these insights, we propose plausible modifications to the microfoundations that can resolve this conundrum.

7.1. Labor Unions. The most straightforward way out of the conundrum is to maintain the assumption that labor market power lies on the side of households, but that wage-setting decisions are delegated to representative unions subject to adjustment frictions. We believe that slow wage adjustment being characterized

²⁰In our calibration, this does not produce pronounced hump-shaped responses for labor income, although persistence could be increased arbitrarily by raising inattention.

by infrequent contract renegotiations, not directly carried out by households, is a realistic assumption.

In this case, hand-to-mouth need not pay attention to the economy, while savers face a single consumption–savings decision that requires them to track the real interest rate. As a result, savers’ forecast are always the most accurate, and the real wage is hump-shaped with a volatility similar to that of output growth. The relevance of heterogeneity is minimal in that setting; the consequences of setting $\phi = 0$ resemble those of the RI-II model.

This version of the model allows us to match both micro and macro moments simultaneously. In particular, the source of wage stickiness—whether Calvo frictions or inattention—turns out to be largely irrelevant. Similarly, whether real wages differ across household types, as in the inattentive economy from [Section 6](#) where each household sets its wage based on its own marginal rate of substitution, is unnecessary for generating persistence in the growth rate of labor income; assuming a single union or a union for each household type does not significantly affect the dynamics.

However, the microfoundations underlying the labor supplied by each household when unions set wages can be considered weak²¹; for details, see [Appendix G](#). Furthermore, moving away from inattention as the source of wage inertia raises a *Lucas critique* issue that was not previously relevant: how should the frequency of wage adjustment respond to policies or changes in the economic environment?

7.2. Firms with Monopsony Power.

[Currently being updated.]

In [Section 6](#), we showed that cross-sectional expectations accuracy can match the data when households take the wage rate as given, which lowers the stakes and thus the incentives to pay attention. We suggest taking inspiration from a number of recent papers (see [Alpanda 2025](#) for a review) arguing that labor markets are often better described as monopsonistic, with market power lying with firms rather than households.

In this setting, firms set wages as a markdown relative to their marginal product of labor, while households supply labor so that their marginal rate of substitution equals the real wage. Firms being subject to inattention generates hump-shaped wage dynamics. Households taking the wage as given implies low stakes for losses incurred from suboptimal actions which leads to more accurate forecasts for the savers.

²¹We make the standard assumption that each household supplies a continuum of labor services.

8. Inattention Compared to Other Frameworks

[*Currently being updated.*]

To the best of our knowledge, this paper is the first to aim at matching both the persistence of macro variables and micro moments of expectations in a heterogeneous-agent model. Previous work has typically addressed only one of these issues at a time. In this section, we compare the approaches taken in these papers to ours and discuss how they might perform if they attempted to match both sets of moments simultaneously, as we do.

8.1. Habits Formation. One of the most common sources of slow adjustment of quantities in the representative agent literature is habit formation in consumption, as detailed in [Appendix G](#). We find it to have similar effects as inattentive savers’ consumption–savings choices, which are essential for the model to generate persistence in the growth rate of output even when prices and wages are sluggish. The mechanism is clearly not suited to induce differences in cross-sectional expectations. Habit formation can also be viewed as *ad-hoc*, especially in the context of baseline two-agent models, where it can only be introduced in savers’ preferences, given that hand-to-mouth households solve a static maximization problem²². It is also hard to discipline, since the degree of habit formation may be set arbitrarily high to match desired moments irrespective of the intratemporal decision. By contrast, inertia induced by rational inattention is summarized by a single parameter that governs the speed of adjustment for all decision variables.

Moreover, habit formation has an undesirable property for the aggregate MPC in heterogeneous agents settings, as highlighted by [Auclert et al. \(2021\)](#). It mechanically induces slow adjustment to all shocks—both aggregate and idiosyncratic—which is not the case under inattention. For example, if households face idiosyncratic shocks that are larger than aggregate shocks, hand-to-mouth households will maintain a high MPC in response to these idiosyncratic shocks while reacting slowly to aggregate fluctuations; this arises from their endogenously allocating more attention to the idiosyncratic component.

If the goal is estimation, for instance using Bayesian techniques as in [Bilbiie, Primiceri, and Tambalotti \(2023\)](#), and the model is structured so that the functional form of preferences is homogeneous across types, habit formation can be employed to introduce persistence. However, it comes with all of the caveats outlined above.

²²If households can switch types, as in [Bilbiie, Primiceri, and Tambalotti 2023](#), hand-to-mouth preferences can include habit formation.

8.2. Exogenous Information Structures. We have in mind frameworks that resemble Gallegos (2024) or Auclert et al. (2021). Gallegos (2024) solves a two-agent model with imperfect information, in which signals are exogenously specified as the aggregate stochastic fundamental plus a Gaussian noise term. One could also assume a more complex structure with multiple signals that are linear combinations of endogenous variables, as in Nimark (2014), but this has yet to be pursued in heterogeneous agent models. Auclert et al. (2021) assumes sticky information with respect to aggregate variables. In theory, such information structures are flexible enough to match any cross-sectional expectations patterns from the data and can induce persistence in macro variables, especially when combined with New Keynesian adjustment frictions. The contributions of imperfect information, however, can be hard to disentangle from other sources of slow adjustment in the model.

Some caveats are worth mentioning. First, the choice of signals is completely arbitrary. While it may induce the desired moments for expectations, it is by no means evidence that these are the signals agents would select if solving a maximization problem. Second, there is a *Lucas critique*–type argument that arises if the model is used for counterfactual experiments. Changing policy parameters or stochastic processes in this environment disregards the attention channel altogether. Lastly, the lack of microfoundation for the information structure means that it can affect the dynamics in implausible ways. For example, we showed that it is unlikely to be the source of inertia in wages in models where attention is endogenous and households are the wage-setters.

This type of information structure has the advantage of being more tractable than the one we propose. If one specifies signals that are plausible, especially those that load on endogenous variables such as inflation and output, these models can still yield interesting results, particularly if they can be fully estimated.

9. Conclusions.

We solve business cycle models with heterogeneous households: hand-to-mouth consumers and savers. Models with standard labor market structures fail to simultaneously generate hump-shaped responses of macro variables *and* cross-sectional differences in expectation accuracy that match the data. These findings have two main implications. First, compared to non-Ricardian economies, inattention acts as a strong propagation mechanism in the presence of heterogeneous households only if it induces persistence in the growth rate of labor income. Second, inattention does not naturally generate the same cross-sectional expectation patterns as those observed in the data. In particular, the microfoundations of the labor market, that

is, the variables over which the decision makers exert influence, are crucial in this setting, as they define the incentives to acquire more or less precise signals.

We propose plausible modifications to our baseline models that allow us to obtain both the persistence in the growth of labor income and cross-sectional differences in expectation accuracy. Our results provide insights for future research on how to simultaneously introduce imperfect information and induce persistence in heterogeneous-agent models in a way that is consistent with the data. In our analysis, we consider simple economic environments that abstract from a number of sources of propagation that could be embedded in this class of models, including cyclical exposure to shocks, precautionary savings, and fiscal policy. Future work could incorporate these channels and assess whether they make it harder or easier to match the persistence of macro variables and micro moments of expectations.

Our models feature limited heterogeneity in terms of hand-to-mouth status. However, we believe that this environment still provides a reasonable approximation to more general settings in which agents can move across the wealth distribution. The reason is that under rational inattention, if individual states are sufficiently persistent (in heterogeneous-agent models, the process for idiosyncratic productivity is typically highly persistent), agents are likely to disregard the contingency associated with switching types when allocating their attention.

Finally, we want to stress that, although the models analyzed here are simpler than state-of-the-art heterogeneous-agent economies, the latter remain subject to the Lucas critique. For policy experiments—the primary use of these models—frameworks with endogenous information structures are preferable because they can yield conclusions that diverge from HANK economies due to the attention channel playing a role in shaping responses to policy experiments.

References

- Afrouzi, Hassan, and Choongryul Yang. 2021. “Dynamic rational inattention and the Phillips curve.”
- Alpanda, Sami. 2025. “Monopsony in Labor Markets: Not Important in the Aggregate.” *Economic Modelling* 152: 107274.
- Altig, David, Lawrence J Christiano, Martin Eichenbaum, and Jesper Linde. 2011. “Firm-specific capital, nominal rigidities and the business cycle.” *Review of Economic Dynamics* 14 (2): 225–247.
- Auclert, Adrien, Bence Bardóczy, Matthew Rognlie, and Ludwig Straub. 2021. “Using the sequence-space Jacobian to solve and estimate heterogeneous-agent models.” *Econometrica* 89 (5): 2375–2408.
- Auclert, Adrien, Matthew Rognlie, and Ludwig Straub. 2020. “Micro Jumps, Macro Humps: Monetary Policy and Business Cycles in an Estimated HANK Model.” NBER Working Paper 26647, National Bureau of Economic Research.
- Bilbiie, Florin O. 2008. “Limited asset markets participation, monetary policy and (inverted) aggregate demand logic.” *Journal of Economic Theory* 140 (1): 162–196.
- Bilbiie, Florin O. 2020. “The New Keynesian Cross.” *Journal of Monetary Economics* 114: 90–108.
- Bilbiie, Florin O., Giorgio E. Primiceri, and Andrea Tambalotti. 2023. “Inequality and Business Cycles.” Working Paper w31729, National Bureau of Economic Research.
- Bordalo, Pedro, Nicola Gennaioli, Yueran Ma, and Andrei Shleifer. 2020. “Overreaction in macroeconomic expectations.” *American Economic Review* 110 (9): 2748–2782.
- Campbell, John Y., and N. Gregory Mankiw. 1989. “Consumption, income, and interest rates: Reinterpreting the time series evidence.” *NBER Macroeconomics Annual* 4: 185–216.
- Carroll, Christopher D. 2003. “Macroeconomic Expectations of Households and Professional Forecasters.” *The Quarterly Journal of Economics* 118 (1): 269–298.

- Carvalho, Carlos, Fernanda Nechio, and Tiago Tristao. 2021. "Taylor rule estimation by OLS." *Journal of Monetary Economics* 124: 140–154.
- Christiano, Lawrence J., Martin Eichenbaum, and Charles L. Evans. 2005. "Nominal rigidities and the dynamic effects of a shock to monetary policy." *Journal of Political Economy* 113 (1): 1–45.
- Coibion, Olivier, and Yuriy Gorodnichenko. 2015. "Information rigidity and the expectations formation process: A simple framework and new facts." *American Economic Review* 105 (8): 2644–2678.
- Debortoli, Davide, and Jordi Galí. 2024. "Heterogeneity and aggregate fluctuations: insights from TANK models." Technical Report w32557, National Bureau of Economic Research.
- Fernald, John. 2014. "A Quarterly, Utilization-Adjusted Series on Total Factor Productivity." Technical report, Federal Reserve Bank of San Francisco.
- Gallegos, Jose E. 2024. "HANK beyond FIRE: Amplification, Forward Guidance, and Belief Shocks." Unpublished manuscript or working paper.
- Galí, Jordi. 2015. *Monetary Policy, Inflation, and the Business Cycle: An Introduction to the New Keynesian Framework and Its Applications.*: Princeton University Press.
- Han, Zhao, Fei Tan, and Jieran Wu. 2022. "Analytic policy function iteration." *Journal of Economic Theory* 200: 105395.
- Heathcote, Jonathan, Fabrizio Perri, and Giovanni L. Violante. 2020. "The rise of US earnings inequality: Does the cycle drive the trend?" *Review of Economic Dynamics* 37: S181–S204.
- Kaplan, Greg, Benjamin Moll, and Giovanni L. Violante. 2018. "Monetary policy according to HANK." *American Economic Review* 108 (3): 697–743.
- Kaplan, Greg, and Giovanni L. Violante. 2014. "A Model of the Consumption Response to Fiscal Stimulus Payments." *Econometrica* 82 (4): 1199–1239.
- Kaplan, Greg, Giovanni L. Violante, and Justin Weidner. 2014. "The wealthy hand-to-mouth."
- Krusell, Per, and Anthony A. Smith. 1998. "Income and wealth heterogeneity in the macroeconomy." *Journal of Political Economy* 106 (5): 867–896.

- Leeper, Eric M. 1991. “Equilibria under ‘Active’ and ‘Passive’ Monetary and Fiscal Policies.” *Journal of Monetary Economics* 27 (1): 129–147.
- Lusardi, Annamaria, Daniel J. Schneider, and Peter Tufano. 2011. “Financially Fragile Households: Evidence and Implications.” Working Paper w17072, National Bureau of Economic Research.
- Maćkowiak, Bartosz, and Mirko Wiederholt. 2015. “Business cycle dynamics under rational inattention.” *The Review of Economic Studies* 82 (4): 1502–1532.
- Maćkowiak, Bartosz, and Mirko Wiederholt. 2023. “Rational inattention and the business cycle effects of productivity and news shocks.”
- Mankiw, N. Gregory, and Ricardo Reis. 2007. “Sticky information in general equilibrium.” *Journal of the European Economic Association* 5 (2-3): 603–613.
- Mankiw, N Gregory, Ricardo Reis, and Justin Wolfers. 2003. “Disagreement about inflation expectations.” *NBER macroeconomics annual* 18: 209–248.
- Maćkowiak, Bartosz, Filip Matějka, and Mirko Wiederholt. 2018. “Dynamic rational inattention: Analytical results.” *Journal of Economic Theory* 176: 650–692.
- McCracken, Michael W., and Serena Ng. 2016. “FRED-MD: A monthly database for macroeconomic research.” *Journal of Business & Economic Statistics* 34 (4): 574–589.
- Mitman, Kurt, Broer Tobias, Kohlhas Alexandre, and Schlafmann Kathrin. 2022. “Expectation and Wealth Heterogeneity in the Macroeconomy.” Working Paper.
- Nimark, Kristoffer P. 2014. “Man-bites-dog business cycles.” *American Economic Review* 104 (8): 2320–2367.
- Sims, Christopher A. 1998. “Stickiness.” In *Carnegie-rochester conference series on public policy*, vol. 49: 317–356, Elsevier.
- Sims, Christopher A. 2003. “Implications of rational inattention.” *Journal of monetary Economics* 50 (3): 665–690.
- Song, Wuwei, and Scott Stern. 2020. “Firm inattention and the efficacy of monetary policy: A text-based approach.” *Available at SSRN 3900641*.
- Telyukova, Irina A. 2013. “Household need for liquidity and the credit card debt puzzle.” *Review of Economic Studies* 80 (3): 1148–1177.

- Violante, Giovanni. 2021. “What have we learned from HANK models, thus far?” In *Beyond the Pandemic: The Future of Monetary Policy*, 267.
- Weber, Michael, Francesco d’Acunto, Yuriy Gorodnichenko, and Olivier Coibion. 2022. “The Subjective Inflation Expectations of Households and Firms: Measurement, Determinants, and Implications.” *Journal of Economic Perspectives* 36 (3): 157–184.

Appendix A. Expectations Accuracy

A.1. Forecasting BVAR The model is estimated on the period 1960M1-2025M1 with Bayesian techniques (Minnesota priors on the VAR coefficients and inverse-Wishart priors on the error covariance matrix) and includes 12 lags of the CPI, Industrial Production, the Unemployment Rate, the market yield on 1-year U.S. Treasury securities, and the first three factors from [McCracken and Ng \(2016\)](#). We estimate the posterior distribution with 100,000 draws which we then use to forecast the interest rate one-year ahead. From these forecasts, we proxy the true probability of interest rates being higher 12M from now, which we compare with the probability reported by each households to measure expectations accuracy.

A.2. Expectations Relative Accuracy [Table 7](#) presents the results of panel regressions on the accuracy of households' expectations and the hand-to-mouth status including the same set of controls as in [Section 2](#) plus binary variables for the income bin to which each household belongs.

Appendix B. Perfect Information Economy

In this section, we detail the equilibrium of an economy without frictions (neither informational nor adjustment costs). The firms' and households' optimality conditions are the same that characterize their optimal actions in the inattentive economy. The non-stochastic steady state is also the same.

A perfect information economy has the following property

Definition 2. *Every decision-maker has rational expectations and knows the complete history of shocks up to, and including the current period.*

B.1. Optimality Conditions. Optimality in the frictionless perfect information economy follows from the maximization problems of firms and households. The same conditions characterize optimal actions in the inattentive economy.

Hand-to-mouth $j \in [0, \phi]$ solves a static problem that yields the following first-order condition for labor supply

$$\varphi^{\mathcal{H}} L_{jt}^{\psi} C_{jt}^{\gamma} = \tilde{W}_t. \quad (39)$$

Saver $j \in [\phi, 1]$ solves a dynamic problem, whose first-order conditions imply an Euler equation given by

$$C_{jt}^{-\gamma} = \beta E_{jt} \left[C_{jt+1}^{-\gamma} \left(\frac{R_t}{\Pi_{t+1}} \right) \right] \quad (40)$$

Table 7: Relative Expectations Accuracy of Hand-to-mouth Households

	Negative Income Shock		Liquidity Constraint		Default Probability	
	(1)	(2)	(3)	(4)	(5)	(6)
Hand-to-mouth	0.530*** (0.030)	0.015*** (0.001)	1.450*** (0.039)	0.021*** (0.002)	1.478*** (0.071)	-0.017*** (0.003)
High School	-	-	-	-	-	-
Some College	-0.636*** (0.038)	-0.011*** (0.002)	-0.681*** (0.039)	-0.011*** (0.002)	-0.748** (0.034)	-0.008** (0.001)
College	-1.484*** (0.039)	-0.045*** (0.002)	-1.450*** (0.041)	-0.041*** (0.002)	-1.772*** (0.036)	-0.051*** (0.001)
Low Numeracy	-	-	-	-	-	-
High Numeracy	-1.861*** (0.034)	-0.027*** (0.001)	-0.021*** (0.035)	-2.110*** (0.001)	-2.237*** (0.031)	-0.028*** (0.001)
Unemployed	-	-	-	-	-	-
Part-time employed	0.079 (0.049)	0.014 (0.003)	0.127** (0.050)	-0.014*** (0.002)	0.034 (0.044)	-0.014 (0.002)
Full-time employed	-0.148*** (0.034)	-0.002 (0.001)	-0.130*** (0.035)	-0.001 (0.001)	-0.188*** (0.031)	-0.004*** (0.001)
Income < 50K	-	-	-	-	-	-
Income ∈ [50, 100]	-1.036*** (0.037)	-0.007*** (0.002)	-0.777*** (0.039)	-0.003** (0.001)	-1.050*** (0.034)	-0.008*** (0.001)
Income > 100K	-1.444*** (0.042)	-0.021*** (0.002)	-1.153*** (0.045)	-0.015** (0.002)	-1.504*** (0.039)	-0.022*** (0.002)
Observations	109,879	109,879	112,972	112,972	156,160	156,160
F Statistic	1521.92	346.71	1831.60	323.94	2471.39	529.33
R ²	0.123	0.424	0.139	0.447	0.134	0.400
Time Fixed Effects	yes	yes	yes	yes	yes	yes

Notes: Columns (1), (3), and (5) show estimates from a regression of the absolute value of inflation forecast errors on the household hand-to-mouth status identified with different proxies. Columns (2), (4), and (6) show estimates from a regression of the absolute value of the forecasts errors of the probability of interest rates going up in 12M on the household hand-to-mouth status identified with different proxies. Robust standard errors in parentheses, * $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$. Sample: 2013M8-2024M4.

and a labor supply condition

$$\varphi^S L_{jt}^\psi C_{jt}^\gamma = \tilde{W}_t. \quad (41)$$

Firm i solves a static problem that yields the following first-order condition for optimal pricing

$$\hat{P}_{it} = \frac{\tilde{W}_t}{\alpha e^{a_t} e^{a_{it}}} \left(\frac{\hat{P}_{it}^{-\theta} C_t}{e^{a_t} e^{a_{it}}} \right)^{\frac{1-\alpha}{\alpha}}. \quad (42)$$

B.2. Non-Stochastic Steady-State. The non-stochastic steady-state is an equilibrium of the economy in the absence of shocks, with the property that real quantities, relative prices, the nominal rate, and inflation remain constant over time. In the following, variables without time-subscript denotes steady-state values.

Evaluating the first-order conditions in a non-stochastic steady-state where hours worked are symmetric across household types, $L = L^H = L^S$, we obtain

$$\tilde{W} = \varphi^H L^\psi (C^H)^\gamma, \quad (43)$$

with

$$C^H = \tilde{W} L - T^H. \quad (44)$$

for the hand-to-mouth household. The first-order conditions of a saver at the non-stochastic steady-state are

$$\tilde{W} = \varphi^S L^\psi (C^S)^\gamma, \quad (45)$$

and

$$\frac{R}{\Pi} = \frac{1}{\beta}. \quad (46)$$

The optimality condition for relative consumption for both types of household types is

$$\hat{C}_{ij}^h = \hat{P}_i^{-\theta}. \quad (47)$$

Lastly, firm i 's first order condition is

$$\hat{P}_i = \frac{\tilde{W}}{\alpha} (\hat{P}_i^{-\theta} C)^{\frac{1-\alpha}{\alpha}}. \quad (48)$$

Equation (48) implies that all firms set the same price. Equation (47) then implies that each household consumes the same relative quantity of each consumption variety. Thus, all firms produce the same output, and since all firms have the same productivity, they also have the same labor input. Aggregate labor determines aggregate output, which in turn determines the market-clearing real wage and aggregate dividends. Equation (43), Equation (44) and Equation (45) determine the consumption levels.

The Euler equation of optimizing households, Equation (46), determines the real interest rate, but not R and Π individually. Thus, we will assume a constant price level such that $\Pi = 1$ and $R = \beta^{-1}$ and posit an initial value, P_{-1} , for the price level.

Given initial values for nominal bonds held by savers, $B_{-1} = B_{j,-1} \forall j \in [\phi, 1]$, fiscal variables are uniquely determined in the non-stochastic steady-state. The reason is that real bond holdings, B_{-1}/P_{-1} , is a quantity that must remain constant, and this can only be the case if the government runs a balanced budget in real terms. Thus, real lump-sum taxes must equate the sum of real interest and subsidy payments.

Given values for L , B and $\tilde{T}^{\mathcal{H}}$, all non-stochastic steady-state variables can be computed. Appendix C combines some of these relationships into expressions used in the approximation of the firms and households' objectives around that point.

B.3. Linearized Equilibrium We solve the perfect information numerically by linearizing its equilibrium conditions around its non-stochastic steady-state. Given that idiosyncratic technology shocks sum to zero in the cross-section, we drop indexes i for firms. Given that households of type h are ex-ante and ex-post identical, we also drop the indexes j . Equations (49) to (59) are the economy's equilibrium conditions

$$c_t^{\mathcal{S}} = E_t[c_{t+1}^{\mathcal{S}}] - \frac{1}{\gamma} E_t[r_t - \pi_{t+1}] \quad (49)$$

$$\tilde{w}_t = \psi l_t^{\mathcal{S}} + \gamma c_t^{\mathcal{S}} \quad (50)$$

$$\tilde{w}_t = \psi l_t^{\mathcal{H}} + \gamma c_t^{\mathcal{H}} \quad (51)$$

$$c_t^{\mathcal{H}} = \omega_w^{\mathcal{H}} (\tilde{w}_t + l_t^{\mathcal{H}}) \quad (52)$$

$$\tilde{w}_t = \frac{\alpha-1}{\alpha} y_t + \frac{1}{\alpha} a_t \quad (53)$$

$$r_t = \rho_r r_{t-1} + (1 - \phi_r)(\phi_\pi \pi_t) + v_t \quad (54)$$

$$c_t = \phi(C^H/C)c_t^H + (1 - \phi)(C^S/C)c_t^S \quad (55)$$

$$l_t = \phi(L^H/L)l_t^H + (1 - \phi)(L^S/L)l_t^S \quad (56)$$

$$y_t = a_t + \alpha l_t \quad (57)$$

$$y_t = c_t \quad (58)$$

$$a_t = \rho_a a_{t-1} + \varepsilon_t^a. \quad (59)$$

Equation (49) and Equation (50) are the savers Euler equation and labor supply conditions. Equation (51) and Equation (52) are the hand-to-mouth's labor supply condition and budget constraint. Equation (53) is the firms' optimal pricing condition. Equation (54) is the Taylor rule. Equation (55) and Equation (56) defined aggregate consumption and labor. Equation (57) is aggregate output and Equation (58) is the equilibrium condition on the goods market. Equation (59) is the exogenous process for technology.

Definition 3. *Given an initial value and the following non-explosive sequence for the savers' real bond holdings*

$$\lim_{s \rightarrow \infty} E_t[\beta^{s+1}(\tilde{b}_{t+s+1} - \tilde{b}_{t+s})] = 0, \quad (60)$$

an equilibrium of the perfect information economy is a solution to the system of equations that collects Equations (49) to (59).

Appendix C. Non-Stochastic Steady-State

The following non-stochastic steady-state relationships, which hold for all economies analyzed, are useful for approximating the objective functions of households and firms (i.e. sum of expected dividends and utilities).

The combination of firm i 's optimal pricing condition and production function yields

$$\begin{aligned}
\hat{P}_i &= \tilde{W} \frac{1}{\alpha} (\hat{P}_i^{-\theta} C_i^{\frac{1}{\alpha}-1}) \\
\hat{P}_i &= \tilde{W} \frac{C_i^{\frac{1}{\alpha}}}{\alpha C_i} \\
\hat{P}_i &= \tilde{W} \frac{L_i}{\alpha C_i} \\
\alpha C &= \tilde{W} L.
\end{aligned} \tag{61}$$

We used the fact that firms are symmetrical in the non-stochastic steady state to eliminate the indexes i in the last line.

Rearranging the labor supply condition for both household types yields

$$\begin{aligned}
\varphi^h L_j^\psi &= (C_j^h)^{-\gamma} \tilde{W} \\
\varphi^h L_j^{1+\psi} &= (C_j^h)^{-\gamma} \tilde{W} L_j \\
\varphi^h L_j^{1+\psi} &= (C_j^h)^{1-\gamma} \frac{\tilde{W} L_j}{C_j^h} \\
\varphi^h L^{1+\psi} &= (C^h)^{1-\gamma} \omega_w^h.
\end{aligned} \tag{62}$$

Again, we used the symmetry between households of the same type h to eliminate the indexes j in the last line.

Appendix D. Expected Losses from Suboptimal Actions

D.1. Firms First, we guess that model-implied demand for consumption variety i is²³

$$C_{it} = \left(\frac{P_{it}}{P_t} \right)^{-\tilde{\theta}} C_t. \tag{63}$$

Second, we substitute the demand function into the expression for period nominal profits

$$D_{it} = (1 + \tau_p) P_{it} \hat{P}_{it}^{-\tilde{\theta}} C_t - W_t L_{it} \tag{64}$$

²³We can prove that optimal attention allocation yields a demand function of that form.

Third, we replace the labor input using the production function

$$D_{it} = (1 + \tau_p)P_{it}\hat{P}_{it}^{-\theta}C_t - W_t\left(\frac{\hat{P}_{it}^{-\theta}C_t}{e^{a_t}e^{a_{it}}}\right)^{\frac{1}{\alpha}} \quad (65)$$

Next, we assume that, in period -1 , households who own the firms value profits at time t using the following discount factor

$$Q_{-1,t} = \beta^t \Lambda(\{C_{jt}\}_{j \in [\phi, 1]}) \frac{1}{P_t} \quad (66)$$

where the functional $\Lambda(\cdot)$ is twice continuously differentiable and satisfies

$$\Lambda(\{C_{jt}^S\}_{j \in [\phi, 1]}) = (C^S)^{-\gamma} \quad (67)$$

in the non-stochastic steady-state.

We multiply [Equation \(65\)](#), the period nominal profits, by the stochastic discount factor (exempt of β^t) which yields

$$\Lambda(\{C_{jt}^S\}_{j \in [\phi, 1]}) \left[(1 + \tau_p)\hat{P}_{it}^{1-\theta}C_t - \left(\frac{\hat{P}_{it}^{-\theta}C_t}{e^{a_t}e^{a_{it}}}\right)^{\frac{1}{\alpha}} \tilde{W}_t \right]. \quad (68)$$

We denote [Equation \(68\)](#) the real period profits function.

We rewrite this expression in terms of log-deviations around the non-stochastic steady-state

$$\Lambda(\{C_{jt}^S\}_{j \in [\phi, 1]}) \left[\frac{\theta}{\theta - 1} Y \left(\int_0^1 e^{(1-\theta)\hat{p}_{it} + c_{jt}} dj \right) - \alpha Y e^{\frac{\theta}{\alpha}\hat{p}_{it} - \frac{1}{\alpha}(a_t + a_{it}) + \tilde{w}_t} \left(\int_0^1 e^{(1-\theta)\hat{p}_{it} + c_{jt}} dj \right) \right] \quad (69)$$

Let \mathbf{x}_t denote the variables appearing in firm i 's real period profit function that the firm can affect and $\boldsymbol{\zeta}_t$ the vector of variables that are taken as given

$$\mathbf{x}_{it} = (\hat{p}_{it})' \quad (70)$$

$$\boldsymbol{\zeta}_{it} = (a_t, a_{it}, \tilde{w}_t, \{c_{jt}\}_{j \in [0, 1]})' \quad (71)$$

We define \mathcal{F}_i as the functional obtained from multiplying the period real profit function, [Equation \(69\)](#), by β^t and summing over all t from zero to infinity.

We let $\tilde{\mathcal{F}}_i$ denote the second-order approximation of that functional around the non-stochastic steady-state

$$E_{i,-1} \left[\tilde{\mathcal{F}}_i(\mathbf{x}_{i0}, \boldsymbol{\zeta}_{i0}, \mathbf{x}_{i1}, \boldsymbol{\zeta}_{i1}, \dots) \right] \quad (72)$$

$$\approx \mathcal{F}_i(\mathbf{0}, \mathbf{0}, \dots) + E_{i,-1} \left[\sum_{t=0}^{\infty} \beta^t \left(\mathbf{h}'_{\mathbf{x}_i} \mathbf{x}_{it} + \mathbf{h}'_{\boldsymbol{\zeta}_i} \boldsymbol{\zeta}_{it} + \frac{1}{2} \mathbf{x}_{it}' \mathbf{H}_{\mathbf{x}_i} \mathbf{x}_{it} + \mathbf{x}_{it}' \mathbf{H}_{\mathbf{x}_i \boldsymbol{\zeta}_i} \boldsymbol{\zeta}_{it} + \frac{1}{2} \boldsymbol{\zeta}_{it}' \mathbf{H}_{\boldsymbol{\zeta}_i} \boldsymbol{\zeta}_{it} \right) \right]$$

where the vectors $\mathbf{h}_{\mathbf{x}_i}$ and $\mathbf{h}_{\boldsymbol{\zeta}_i}$ are first derivatives with respect to \mathbf{x}_{it} and $\boldsymbol{\zeta}_{it}$ evaluated at the non-stochastic steady-state respectively. Similarly, $\mathbf{H}_{\mathbf{x}_i}$, $\mathbf{H}_{\boldsymbol{\zeta}_i}$ and $\mathbf{H}_{\mathbf{x}_i \boldsymbol{\zeta}_i}$ are matrices of second order derivatives evaluated at the non-stochastic steady-state.

We can show that under some regularity conditions Equation (72) converges to a finite element in \mathcal{R} along with each of its components²⁴.

The process defining firm's i vector of optimal actions, noted \mathbf{x}_{it}^* , is defined by the following requirement

$$\mathbf{h}_{\mathbf{x}_i} + \mathbf{H}_{\mathbf{x}_i} \mathbf{x}_{it}^* + \mathbf{H}_{\mathbf{x}_i \boldsymbol{\zeta}_i} \boldsymbol{\zeta}_{it} = 0. \quad (73)$$

The requirement above implies the same equation obtained by log-linearizing the firms' optimal pricing condition.

Next, we define firm i 's objective function as the losses incurred from suboptimal actions which reads

$$E_{i,-1} \left[\tilde{\mathcal{F}}_i(\mathbf{x}_{i0}, \boldsymbol{\zeta}_{i0}, \mathbf{x}_{i1}, \boldsymbol{\zeta}_{i1}, \dots) \right] - E_{i,-1} \left[\tilde{\mathcal{F}}_i(\mathbf{x}_{i0}^*, \boldsymbol{\zeta}_{i0}, \mathbf{x}_{i1}^*, \boldsymbol{\zeta}_{i1}, \dots) \right] \quad (74)$$

$$= E_{i,-1} \left[\sum_{t=0}^{\infty} \beta^t \left(\mathbf{h}_{\mathbf{x}_i}(\mathbf{x}_{it} - \mathbf{x}_{it}^*) + \frac{1}{2} \mathbf{x}_{it}' \mathbf{H}_{\mathbf{x}_i} \mathbf{x}_{it} - \frac{1}{2} \mathbf{x}_{it}^* \mathbf{H}_{\mathbf{x}_i} \mathbf{x}_{it}^* + (\mathbf{x}_{it} - \mathbf{x}_{it}^*) \mathbf{H}_{\mathbf{x}_i \boldsymbol{\zeta}_i} \boldsymbol{\zeta}_{it} \right) \right]$$

Lastly, we use Equation (73) to substitute for the term $\mathbf{H}_{\mathbf{x}_i \boldsymbol{\zeta}_i} \boldsymbol{\zeta}_{it}$ in Equation (74). After rearranging we obtain

$$E_{i,-1} \left[\tilde{\mathcal{F}}_i(\mathbf{x}_{i0}, \boldsymbol{\zeta}_{i0}, \mathbf{x}_{i1}, \boldsymbol{\zeta}_{i1}, \dots) \right] - E_{i,-1} \left[\tilde{\mathcal{F}}_i(\mathbf{x}_{i0}^*, \boldsymbol{\zeta}_{i0}, \mathbf{x}_{i1}^*, \boldsymbol{\zeta}_{i1}, \dots) \right]$$

$$= \frac{1}{2} (\mathbf{x}_{it} - \mathbf{x}_{it}^*) \mathbf{H}_{\mathbf{x}_i} (\mathbf{x}_{it} - \mathbf{x}_{it}^*) \quad (75)$$

²⁴See Maćkowiak and Wiederholt (2015) for the formal proof. The same result holds for households' approximations of period utilities defined below, provided certain initial conditions are satisfied for the savers.

where

$$\mathbf{H}_{\mathbf{x}_i} = -(C^S)^{-\gamma} Y \left[\frac{\theta(\theta + \alpha(1 - \theta))}{\alpha} \right] \quad (76)$$

and

$$\mathbf{x}_{it}^* = \left(p_t + \frac{\frac{1-\alpha}{\alpha}}{1+\frac{1-\alpha}{\alpha}\bar{\theta}} c_t + \frac{1}{1+\frac{1-\alpha}{\alpha}\bar{\theta}} \tilde{w}_t - \frac{\frac{1}{\alpha}}{1+\frac{1-\alpha}{\alpha}\bar{\theta}} (a_t + a_{it}) \right)'. \quad (77)$$

Notice that we used $\hat{p}_{it} - \hat{p}_{it}^* = p_{it} - p_{it}^*$ so that firms choose p_{it} , their price in level, instead of \hat{p}_{it} , their relative price. The only matrix of second-order derivatives needed to formulate firm i 's attention problem in [Section 4.1](#) is $\mathbf{H}_{\mathbf{x}_i}$.

D.2. Hand-to-Mouth First, we substitute the consumption aggregator into the flow budget constraint to get

$$C_{jt} \left(\int_0^1 P_{it} \hat{C}_{ijt} di \right) = W_t L_{jt} - T^{\mathcal{H}}. \quad (78)$$

Second, we isolate composite consumption and divide both the numerator and denominator by the price index

$$C_{jt} = \frac{\tilde{W}_t L_{jt} - \tilde{T}^{\mathcal{H}}}{\int_{[0,1)} \hat{P}_{it} \hat{C}_{ijt} di + \hat{P}_{1t} \left(1 - \int_{[0,1)} \hat{C}_{ijt}^{\frac{\theta-1}{\theta}} di \right)^{\frac{\theta}{\theta-1}} di}. \quad (79)$$

Notice that we partially relax mathematical rigor by treating the integral as a finite sum and di as a weight²⁵.

Third, we substitute the expression for consumption in the period utility function

²⁵Throughout this section, we use this approach to ensure there is always a free variable when working with equality conditions. Alternatively, we could assume a finite number of households and firms, as in [Maćkowiak and Wiederholt \(2015\)](#), but this would make the relationship between aggregate and individual variables dependent on the size of the economy.

$$\begin{aligned}
U(C_{jt}, L_{jt}) = & \frac{1}{1-\gamma} \left(\frac{\tilde{W}_t L_{jt} - \tilde{T}^{\mathcal{H}}}{\int_{[0,1)} \hat{P}_{it} \hat{C}_{ijt} di + \hat{P}_{1t} \left(1 - \int_{[0,1)} \hat{C}_{ijt}^{\frac{\theta-1}{\theta}} di \right)^{\frac{\theta}{\theta-1}}} \right)^{1-\gamma} \\
& - \frac{1}{1-\gamma} \\
& - \varphi^{\mathcal{H}} \frac{L_{jt}^{1+\psi}}{1+\psi}.
\end{aligned} \tag{80}$$

Next, we rewrite this expression in terms of log-deviations around the non-stochastic steady-state

$$\begin{aligned}
U(C_{jt}, L_{jt}) = & \frac{(C^{\mathcal{H}})^{1-\gamma}}{1-\gamma} \left(\frac{\omega_w^{\mathcal{H}} e^{\tilde{w}_t + l_{jt}} + \omega_t^{\mathcal{H}}}{\int_{[0,1)} e^{\hat{p}_{it} + \hat{c}_{ijt}} di + e^{\hat{p}_{1t}} \left(1 - \int_{[0,1)} e^{\frac{\theta-1}{\theta} \hat{c}_{ijt}} di \right)^{\frac{\theta}{\theta-1}}} \right)^{1-\gamma} \\
& - \frac{1}{1-\gamma} \\
& - \frac{(C^{\mathcal{H}})^{1-\gamma}}{1+\psi} \omega_w^{\mathcal{H}} e^{(1+\psi)l_{jt}}
\end{aligned} \tag{81}$$

where $\omega_w^{\mathcal{H}} = \frac{\tilde{W}L}{C^{\mathcal{H}}}$ and $\omega_t^{\mathcal{H}} = \frac{\tilde{T}^{\mathcal{H}}}{C^{\mathcal{H}}}$ are non-stochastic steady-state ratios.

Let \mathbf{x}_t denote the variables appearing in the period utility function that the hand-to-mouth households can affect and $\boldsymbol{\zeta}_t$ the vector of variables that are taken as given

$$\mathbf{x}_t = (l_{jt}, \{\hat{c}_{ijt}\}_{i \in [0,1)})' \tag{82}$$

$$\boldsymbol{\zeta}_t = (\tilde{w}_t, \{\hat{p}_t(i)\}_{i \in [0,1)})' \tag{83}$$

We define $\mathcal{F}^{\mathcal{H}}$ as the functional resulting from multiplying the period utility function, [Equation \(81\)](#), by β^t and summing over all t from zero to infinity.

We let $\tilde{\mathcal{F}}^{\mathcal{H}}$ denote the second-order approximation of that functional around the non-stochastic steady-state

$$E_{j,-1} \left[\tilde{\mathcal{F}}^{\mathcal{H}}(\mathbf{x}_0, \boldsymbol{\zeta}_0, \mathbf{x}_1, \boldsymbol{\zeta}_1, \dots) \right] \quad (84)$$

$$= \mathcal{F}^{\mathcal{H}}(\mathbf{0}, \mathbf{0}, \dots) + E_{j,-1} \left[\sum_{t=0}^{\infty} \beta^t \left(\mathbf{h}'_{\mathbf{x}} \mathbf{x}_t + \mathbf{h}'_{\boldsymbol{\zeta}} \boldsymbol{\zeta}_t + \frac{1}{2} \mathbf{x}_t' \mathbf{H}_{\mathbf{x}} \mathbf{x}_t + \mathbf{x}_t' \mathbf{H}_{\mathbf{x}\boldsymbol{\zeta}} \boldsymbol{\zeta}_t + \frac{1}{2} \boldsymbol{\zeta}_t' \mathbf{H}_{\boldsymbol{\zeta}} \boldsymbol{\zeta}_t \right) \right] \quad (85)$$

where the vectors $\mathbf{h}_{\mathbf{x}}$ and $\mathbf{h}_{\boldsymbol{\zeta}}$ are first derivatives with respect to \mathbf{x}_t and $\boldsymbol{\zeta}_t$ evaluated at the non-stochastic steady-state respectively. Similarly, $\mathbf{H}_{\mathbf{x}}$, $\mathbf{H}_{\boldsymbol{\zeta}}$ and $\mathbf{H}_{\mathbf{x}\boldsymbol{\zeta}}$ are the matrices of second order derivatives evaluated at the non-stochastic steady-state.

Under some regularity conditions [Equation \(84\)](#) and each of its elements converge to a finite element in \mathcal{R} .

The process defining the vector of optimal actions for household j , noted \mathbf{x}_t^* , is defined by the following requirement

$$\mathbf{h}_{\mathbf{x}} + \mathbf{H}_{\mathbf{x}} \mathbf{x}_t^* + \mathbf{H}_{\mathbf{x}\boldsymbol{\zeta}} \boldsymbol{\zeta}_t = 0. \quad (86)$$

Next, we defined household j 's objective function as the losses incurred from suboptimal actions which reads

$$\begin{aligned} & E_{j,-1} \left[\tilde{\mathcal{F}}^{\mathcal{H}}(\mathbf{x}_0, \boldsymbol{\zeta}_0, \mathbf{x}_1, \boldsymbol{\zeta}_1, \dots) \right] - E_{j,-1} \left[\tilde{\mathcal{F}}^{\mathcal{H}}(\mathbf{x}_0^*, \boldsymbol{\zeta}_0, \mathbf{x}_1^*, \boldsymbol{\zeta}_1, \dots) \right] \\ &= E_{j,-1} \left[\sum_{t=0}^{\infty} \beta^t \left(\mathbf{h}_{\mathbf{x}}(\mathbf{x}_t - \mathbf{x}_t^*) + \frac{1}{2} \mathbf{x}_t' \mathbf{H}_{\mathbf{x}} \mathbf{x}_t - \frac{1}{2} \mathbf{x}_t^* \mathbf{H}_{\mathbf{x}} \mathbf{x}_t^* + (\mathbf{x}_t - \mathbf{x}_t^*) \mathbf{H}_{\mathbf{x}\boldsymbol{\zeta}} \boldsymbol{\zeta}_t \right) \right] \end{aligned} \quad (87)$$

Lastly, we use [Equation \(86\)](#) to substitute for $\mathbf{H}_{\mathbf{x}\boldsymbol{\zeta}} \boldsymbol{\zeta}_t$ in [Equation \(87\)](#). After rearranging we obtain

$$\begin{aligned} & E_{j,-1} \left[\tilde{\mathcal{F}}^{\mathcal{H}}(\mathbf{x}_0, \boldsymbol{\zeta}_0, \mathbf{x}_1, \boldsymbol{\zeta}_1, \dots) \right] - E_{j,-1} \left[\tilde{\mathcal{F}}^{\mathcal{H}}(\mathbf{x}_0^*, \boldsymbol{\zeta}_0, \mathbf{x}_1^*, \boldsymbol{\zeta}_1, \dots) \right] \\ &= E_{j,-1} \sum_{t=0}^{\infty} \frac{1}{2} (\mathbf{x}_t - \mathbf{x}_t^*) \mathbf{H}_{\mathbf{x}} (\mathbf{x}_t - \mathbf{x}_t^*) \end{aligned} \quad (88)$$

where

$$\mathbf{H}_{\mathbf{x}} = -(C^{\mathcal{H}})^{1-\gamma} \begin{bmatrix} \omega_w^{\mathcal{H}}(\omega_w^{\mathcal{H}}\gamma + \psi) & 0 & \dots & 0 \\ 0 & \frac{2}{\theta}di & \dots & \frac{1}{\theta}di \\ \vdots & \vdots & \ddots & \vdots \\ 0 & \frac{1}{\theta}di & \dots & \frac{2}{\theta}di \end{bmatrix}, \quad (89)$$

$$\mathbf{x}_t^* = \begin{pmatrix} \frac{\tilde{w}_t - \gamma c_{jt}^*}{\psi} \\ -\theta(p_{it} - p_t) \\ \vdots \end{pmatrix}, \quad (90)$$

and

$$c_{jt}^* = \omega_w^{\mathcal{H}}(\tilde{w}_t + l_{jt}^*) \quad (91)$$

The only matrix of second-order derivatives needed to formulate the household j 's attention problem in [Section 4.2](#) is \mathbf{H}_x given by [Equation \(89\)](#).

D.3. Savers The first few steps are identical to those employed in the derivation of the hand-to-mouth's objective, except that in the period budget constraint other variables than labor income and taxes appear. In particular, bonds from period $t-1$ are a variable that saver get to choose.

We start by expressing consumption as a function of real variables and substitute in the period utility function. We get

$$U(C_{jt}, L_{jt}) = \frac{1}{1-\gamma} \left(\frac{\frac{R_{t-1}}{\Pi_t} \tilde{B}_{jt-1} - \tilde{B}_{jt} + \tilde{W}_t L_{jt} + \tilde{D}_t^S - \tilde{T}_t^S}{\int_{[0,1)} \hat{P}_{it} \hat{C}_{ijt} di + \hat{P}_{1t} \left(1 - \int_{[0,1)} \hat{C}_{ijt}^{\frac{\theta-1}{\theta}} di \right)^{\frac{\theta}{\theta-1}} di} \right)^{1-\gamma} \\ - \frac{1}{1-\gamma} - \varphi^S \frac{L_{jt}^{1+\psi}}{1+\psi}. \quad (92)$$

Next, we rewrite the period utility expression in terms of log-deviations from the non-stochastic steady-state

$$U(C_{jt}, L_{jt}) = \frac{(C^S)^{1-\gamma}}{1-\gamma} \left(\frac{\frac{\omega_b^S}{\beta} e^{r_{t-1} - \pi_t + \tilde{b}_{jt-1}} - \omega_b^S e^{\tilde{b}_{jt}} + \omega_w^S e^{\tilde{w}_t + l_{jt}} + \omega_d^S e^{\tilde{d}_t^S} - \omega_t^S e^{\tilde{t}_t^S}}{\int_{[0,1)} e^{\hat{p}_{it} + \hat{c}_{ijt}} di + e^{\hat{p}_{1t}} \left(1 - \int_{[0,1)} e^{\frac{\theta-1}{\theta} \hat{c}_{ijt}} di \right)^{\frac{\theta}{\theta-1}} di} \right)^{1-\gamma} \\ - \frac{1}{1-\gamma} \\ - \frac{(C^S)^{1-\gamma}}{1+\psi} \omega_w^S e^{(1+\psi)l_{jt}} \quad (93)$$

where $\omega_b^S, \omega_d^S, \omega_w^S, \omega_t^S$ are the following non-stochastic steady-state ratios

$$(\omega_w^S, \omega_b^S, \omega_d^S, \omega_t^S) = \left(\frac{\tilde{W}L}{C^S}, \frac{\tilde{B}^S}{C^S}, \frac{\tilde{D}^S}{C^S}, \frac{\tilde{T}^S}{C^S} \right). \quad (94)$$

Let \mathbf{x}_t denote the variables appearing in the period utility function that saver j can affect, and let the vector $\boldsymbol{\zeta}_t$ denote the variables that are taken as given such that

$$\mathbf{x}_t = \left(\tilde{b}_{jt}, l_{jt}, \{\hat{c}_{ijt}\}_{i \in [0,1]} \right)', \quad (95)$$

and

$$\boldsymbol{\zeta}_t = \left(r_{t-1}, \pi_t, \tilde{w}_t, \tilde{d}_t^S, \tilde{t}_t^S, \{\hat{p}_{it}\}_{i \in [0,1]} \right)'. \quad (96)$$

Additionally, we define the vector \mathbf{x}_{-1} , of the same length as \mathbf{x}_t , containing the variable $\tilde{b}_{j,-1}$, the only variable not present in either \mathbf{x}_t or $\boldsymbol{\zeta}_t$

$$\mathbf{x}_{-1} = \left(\tilde{b}_{j,-1}, 0, \dots \right)'. \quad (97)$$

We define \mathcal{F}^S as the functional resulting from multiplying the period utility function, [Equation \(93\)](#), by β^t and summing over all t from zero to infinity.

Letting $\tilde{\mathcal{F}}^S$ denote the second-order Taylor approximat on of this functional evaluated at the non-stochastic steady-state, we get

$$\begin{aligned} & E_{j,-1} \left[\tilde{\mathcal{F}}^S(\mathbf{x}_{-1}, \mathbf{x}_0, \boldsymbol{\zeta}_0, \mathbf{x}_1, \boldsymbol{\zeta}_1, \dots) \right] \\ &= \left[\begin{array}{c} \mathcal{F}^S(\mathbf{0}, \mathbf{0}, \mathbf{0}, \dots) \\ \mathbf{h}'_{\mathbf{x}} \mathbf{x}_t + \mathbf{h}'_{\boldsymbol{\zeta}} \boldsymbol{\zeta}_t \\ + \sum_{t=0}^{\infty} \beta^t \left(\begin{array}{c} + \frac{1}{2} \mathbf{x}'_t \mathbf{H}_{\mathbf{x},-1} \mathbf{x}_{t-1} + \frac{1}{2} \mathbf{x}'_t \mathbf{H}_{\mathbf{x}} \mathbf{x}_t + \frac{1}{2} \mathbf{x}'_t \mathbf{H}_{\mathbf{x},1} \mathbf{x}_{t+1} \\ + \frac{1}{2} \mathbf{x}'_t \mathbf{H}_{\mathbf{x}\boldsymbol{\zeta}} \boldsymbol{\zeta}_t + \frac{1}{2} \mathbf{x}'_t \mathbf{H}_{\mathbf{x}\boldsymbol{\zeta},1} \boldsymbol{\zeta}_{t+1} \\ + \frac{1}{2} \boldsymbol{\zeta}'_t \mathbf{H}_{\boldsymbol{\zeta}} \boldsymbol{\zeta}_t + \frac{1}{2} \boldsymbol{\zeta}'_t \mathbf{H}_{\boldsymbol{\zeta}\mathbf{x},-1} \mathbf{x}_{t-1} + \frac{1}{2} \boldsymbol{\zeta}'_t \mathbf{H}_{\boldsymbol{\zeta}\mathbf{x}} \mathbf{x}_t \end{array} \right) \\ + \beta^{-1} \left(\mathbf{h}'_{-1} \mathbf{x}_{-1} + \frac{1}{2} \mathbf{x}'_{-1} \mathbf{H}_{-1} \mathbf{x}_{-1} + \frac{1}{2} \mathbf{x}'_{-1} \mathbf{H}_{-1\mathbf{x}} \mathbf{x}_0 + \frac{1}{2} \mathbf{x}'_{-1} \mathbf{H}_{-1\boldsymbol{\zeta}} \boldsymbol{\zeta}_0 \right) \end{array} \right] \end{aligned} \quad (98)$$

We can show that under regularity conditions, each of the elements in [Equation \(98\)](#) converge to finite elements in \mathcal{R} ²⁶.

The process defining the vector of optimal actions for savers j , noted \mathbf{x}_t^* , is defined by the following requirement

²⁶The formal proof can be found in the appendix of [Maćkowiak and Wiederholt \(2015\)](#).

$$E_{jt}[\mathbf{h}_x + \mathbf{H}_{x,-1}\mathbf{x}_{t-1}^* + \mathbf{H}_x\mathbf{x}_t^* + \mathbf{H}_{x,1}\mathbf{x}_{t+1}^* + \mathbf{H}_{x\zeta}\zeta_t + \mathbf{H}_{x\zeta,1}\zeta_{t+1}] = 0. \quad (99)$$

We can rearrange Equation (99) to obtain the following expression

$$\begin{aligned} & E_{jt}[(\mathbf{x}_t - \mathbf{x}_t^*)'(\mathbf{h}_x + \mathbf{H}_{x\zeta}\zeta_t + \mathbf{H}_{x\zeta,1}\zeta_{t+1})] \\ &= -E_{jt}[(\mathbf{x}_t - \mathbf{x}_t^*)'(\mathbf{H}_{x,-1}\mathbf{x}_{t-1}^* + \mathbf{H}_x\mathbf{x}_t^* + \mathbf{H}_{x,1}\mathbf{x}_{t+1}^*)]. \end{aligned} \quad (100)$$

Next, using Equation (100) along with the assumption that $\mathbf{x}_{-1}^* = \mathbf{x}_{-1}$, some rearrangement yields the following expression quantifying the losses household j incurs due to suboptimal actions

$$\begin{aligned} & E_{j,-1} \left[\tilde{\mathcal{F}}^S(\mathbf{x}_0, \zeta_0, \mathbf{x}_1, \zeta_1, \dots) \right] - E_{j,-1} \left[\tilde{\mathcal{F}}^S(\mathbf{x}_0^*, \zeta_0, \mathbf{x}_1^*, \zeta_1, \dots) \right] \\ &= E_{j,-1} \sum_{t=0}^{\infty} \left[\frac{1}{2}(\mathbf{x}_t - \mathbf{x}_t^*)\mathbf{H}_x(\mathbf{x}_t - \mathbf{x}_t^*) + (\mathbf{x}_t - \mathbf{x}_t^*)\mathbf{H}_{x,1}(\mathbf{x}_{t+1} - \mathbf{x}_{t+1}^*) \right] \end{aligned} \quad (101)$$

where

$$\mathbf{H}_x = -(C^S)^{1-\gamma} \begin{bmatrix} \gamma\omega_b^2(1 + \frac{1}{\beta}) & -\gamma\omega_b\omega_w & 0 & \dots & 0 \\ -\gamma\omega_b\omega_w & \omega_w(\omega_w\gamma + \psi) & 0 & \dots & 0 \\ 0 & 0 & \frac{2}{\theta}di & \dots & \frac{1}{\theta}di \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \frac{1}{\theta}di & \dots & \frac{2}{\theta}di \end{bmatrix}, \quad (102)$$

$$\mathbf{H}_{x,1} = (C^S)^{1-\gamma} \begin{bmatrix} \gamma\omega_b^2 & -\gamma\omega_b\omega_w & 0 & \dots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & \dots & \dots & \dots & 0 \end{bmatrix}. \quad (103)$$

and

$$\mathbf{x}_t^* = \begin{pmatrix} \frac{1}{\beta}(r_{t-1} - \pi_t + \tilde{b}_{jt-1}^*) + \frac{\omega_w^S}{\omega_b^S}(\tilde{w}_t + l_{jt}^*) + \frac{\omega_d^S}{\omega_b^S}\tilde{d}_t^S - \frac{\omega_t^S}{\omega_b^S}\tilde{t}_t^S - \frac{1}{\omega_b^S}c_{jt}^* \\ \frac{\tilde{w}_t - \gamma c_{jt}^*}{\psi} \\ -\theta(p_{it} - p_t) \\ \vdots \end{pmatrix} \quad (104)$$

with

$$c_{jt}^* = E_{jt} \left[-\frac{1}{\gamma}(r_t - \pi_{t+1}) + c_{jt+1}^* \right] \quad \text{and} \quad \omega_b^S > 0. \quad (105)$$

D.4. Change of Variable. Equation (101) does not have the standard form of the objective in a dynamic attention rational inattention problem because of the intertemporal interaction term. We therefore perform a change of variable that allows us to write down the savers attention problem as a pure tracking problem.

We specifically focus on the 2 by 2 upper left elements of \mathbf{H}_x and $\mathbf{H}_{x,1}$. The remaining terms relative to cross-sectional efficiency are unaffected by the following manipulations.

Equation (101) then reads

$$E_{j,-1} \sum_{t=0}^{\infty} \left[\frac{1}{2} (\bar{x}_t - \bar{x}_t^*) \bar{\mathbf{H}}_x (\bar{x}_t - \bar{x}_t^*) + (\bar{x}_t - \bar{x}_t^*) \bar{\mathbf{H}}_{x,1} (\bar{x}_{t+1} - \bar{x}_{t+1}^*) \right] \quad (106)$$

with

$$\bar{x}_t = (\tilde{b}_{jt}, l_{jt})', \quad (107)$$

$$\bar{\mathbf{H}}_x = -(C^S)^{1-\gamma} \begin{bmatrix} \gamma \omega_b^2 (1 + \frac{1}{\beta}) & -\gamma \omega_b \omega_w \\ -\gamma \omega_b \omega_w & \omega_w (\omega_w \gamma + \psi) \end{bmatrix}, \quad (108)$$

and

$$\bar{\mathbf{H}}_{x,1} = (C^S)^{1-\gamma} \begin{bmatrix} \gamma \omega_b^2 & -\gamma \omega_b \omega_w \\ 0 & 0 \end{bmatrix}. \quad (109)$$

Substituting Equations (107) to (109) in Equation (106), we obtain

$$-(C^S)^{1-\gamma} E_{j,-1} \sum_{t=0}^{\infty} \beta^t \left[-\frac{1}{2} \begin{pmatrix} \gamma \omega_b^2 (1 + \frac{1}{\beta}) (\tilde{b}_{jt} - \tilde{b}_{jt}^*)^2 \\ -2\gamma \omega_b \omega_w (\tilde{b}_{jt} - \tilde{b}_{jt}^*) (l_{jt} - l_{jt}^*) \\ +\omega_w (\gamma \omega_w + \psi) (l_{jt} - l_{jt}^*)^2 \\ +\gamma \omega_b^2 (\tilde{b}_{jt} - \tilde{b}_{jt}^*) (\tilde{b}_{jt+1} - \tilde{b}_{jt+1}^*) \\ -\gamma \omega_b \omega_w (\tilde{b}_{jt} - \tilde{b}_{jt}^*) (l_{jt+1} - l_{jt+1}^*) \end{pmatrix} \right]. \quad (110)$$

Next, we subtract the linearized budget constraints evaluated at \bar{x}_t and \bar{x}_t^* , we get

$$\omega_b (\tilde{b}_{jt} - \tilde{b}_{jt}^*) = \frac{\omega_b}{\beta} (\tilde{b}_{jt-1} - \tilde{b}_{jt-1}^*) - (c_{jt} - c_{jt}^*) + \omega_w (l_{jt} - l_{jt}^*). \quad (111)$$

We define the right-hand-side of Equation (111) as a new variable, Δ_t , proportional to mistakes in real bond holdings

$$\Delta_t = \omega_b(\tilde{b}_{jt} - \tilde{b}_{jt}^*). \quad (112)$$

Moreover, we can decompose Δ_t into two components, one reflecting mistakes in consumption and another specific to errors in labor supply such that

$$\Delta_t^c = \frac{1}{\beta} \Delta_{t-1}^c - (c_{jt} - c_{jt}^*) \quad (113)$$

and

$$\Delta_t^l = \frac{1}{\beta} \Delta_{t-1}^l + \omega_w(l_{jt} - l_{jt}^*) \quad (114)$$

with $\Delta_{-1}^c = 0$ and $\Delta_{-1}^l = 0$. By assumptions, we also have $(\tilde{b}_{j,-1} - \tilde{b}_{j,-1}^*) = 0$. Therefore, mistakes in real bond holdings are given by

$$\Delta_t = \Delta_t^c + \Delta_t^l. \quad (115)$$

Substituting Equation (112) and Equation (115) in Equation (110) yields

$$-(C^S)1 - \gamma E_{j,-1} \sum_{t=0}^{\infty} \beta^t \left[-\frac{1}{2} \begin{pmatrix} \gamma(1 + \frac{1}{\beta})(\Delta_t^c + \Delta_t^l)^2 \\ -2\gamma\omega_w(\Delta_t^c + \Delta_t^l)(l_{jt} - l_{jt}^*) \\ +\gamma\omega_w^2(l_{jt} - l_{jt}^*)^2 + \psi\omega_w(l_{jt} - l_{jt}^*)^2 \\ +\gamma(\Delta_t^c + \Delta_t^l)(\Delta_{t+1}^c + \Delta_{t+1}^l) \\ -\gamma\omega_w(\Delta_t^c + \Delta_t^l)(l_{jt+1} - l_{jt+1}^*) \end{pmatrix} \right]. \quad (116)$$

Next, we use Equation (114) to substitute for the term $(l_{jt+1} - l_{jt+1}^*)$, and the first term that features $(l_{jt} - l_{jt}^*)$ in Equation (116), we obtain

$$-(C^S)^{1-\gamma} E_{j,-1} \sum_{t=0}^{\infty} \beta^t \left[-\frac{1}{2} \begin{pmatrix} \gamma(1 + \frac{1}{\beta})(\Delta_t^c + \Delta_t^l)^2 \\ -2\gamma(\Delta_t^c + \Delta_t^l)(\Delta_t^l - \frac{1}{\beta}\Delta_{t-1}^l) \\ +\gamma(\Delta_t^l - \frac{1}{\beta}\Delta_{t-1}^l)^2 + \psi\omega_w(l_{jt} - l_{jt}^*)^2 \\ +\gamma(\Delta_t^c + \Delta_t^l)(\Delta_{t+1}^c + \Delta_{t+1}^l) \\ -\gamma(\Delta_t^c + \Delta_t^l)(\Delta_{t+1}^l - \frac{1}{\beta}\Delta_t^l) \end{pmatrix} \right]. \quad (117)$$

Rearranging, we obtain

$$-(C^S)^{1-\gamma} E_{j,-1} \sum_{t=0}^{\infty} \beta^t \begin{bmatrix} -\frac{\gamma}{2}(1 + \frac{1}{\beta})(\Delta_t^c)^2 + \gamma \Delta_t^c \Delta_{t+1}^c \\ +\gamma \left[\Delta_t^l \Delta_{t+1}^c - \frac{1}{\beta} \Delta_{t-1}^l \Delta_t^c \right] \\ +\frac{\gamma}{2\beta} \left[(\Delta_t^l)^2 - \frac{1}{\beta} (\Delta_{t-1}^l)^2 \right] \\ -\frac{\omega_w \psi}{2} (l_{jt} - l_{jt}^*)^2 \end{bmatrix}. \quad (118)$$

We combine and rewrite the first two terms, substituting in Equation (118) yields

$$-(C^S)^{1-\gamma} E_{j,-1} \sum_{t=0}^{\infty} \beta^t \begin{bmatrix} -\frac{\gamma}{2}(c_{jt} - c_{jt}^*)^2 \\ +\frac{\gamma}{2\beta} \left[(\Delta_t^c)^2 - \frac{1}{\beta} (\Delta_{t-1}^c)^2 \right] \\ +\frac{\gamma}{2\beta} \left[(\Delta_t^l)^2 - \frac{1}{\beta} (\Delta_{t-1}^l)^2 \right] \\ +\gamma \left[\Delta_t^c (c_{jt+1} - c_{jt+1}^*) - \frac{1}{\beta} \Delta_{t-1}^c (c_{jt} - c_{jt}^*) \right] \\ -\frac{\omega_w \psi}{2} (l_{jt} - l_{jt}^*)^2 \end{bmatrix}. \quad (119)$$

Summing terms appearing in consecutive periods, and using $\lim_{T \rightarrow \infty} \beta^T E_{j,-1} [(\Delta_T^c)^2] = \lim_{T \rightarrow \infty} \beta^T E_{j,-1} [(\Delta_T^l)^2] = \lim_{T \rightarrow \infty} \beta^T E_{j,-1} [\Delta_T^c \Delta_{T+1}^l] = \lim_{T \rightarrow \infty} \beta^T E_{j,-1} [\Delta_T^c (c_{jT+1} - c_{jT+1}^*)]$ together with $\Delta_{-1}^c = \Delta_{-1}^l = 0$ yields

$$-(C^S)^{1-\gamma} E_{j,-1} \sum_{t=0}^{\infty} \beta^t \left[\frac{\gamma}{2} (c_{jt} - c_{jt}^*)^2 + \frac{\omega_w \psi}{2} (l_{jt} - l_{jt}^*)^2 \right]. \quad (120)$$

Under matricial form Equation (120) writes

$$E_{j,-1} \sum_{t=0}^{\infty} \beta^t \left[\frac{1}{2} (\tilde{\mathbf{x}}_t - \tilde{\mathbf{x}}_t^*)' \tilde{\mathbf{H}}_{\mathbf{x}} (\tilde{\mathbf{x}}_t - \tilde{\mathbf{x}}_t^*) \right]. \quad (121)$$

where

$$\tilde{\mathbf{H}}_{\mathbf{x}} = -(C^S)^{1-\gamma} \begin{bmatrix} \gamma & 0 \\ 0 & \omega_w \psi \end{bmatrix}, \quad (122)$$

$$\tilde{\mathbf{x}}_t = (c_{jt}, l_{jt})', \quad (123)$$

and

$$\tilde{\mathbf{x}}_t^* = \begin{pmatrix} E_{jt} \left[-\frac{1}{\gamma} (r_t - \pi_{t+1}) + c_{jt+1}^* \right] \\ \frac{\tilde{w}_t - \gamma c_{jt}^*}{\psi} \end{pmatrix} \quad (124)$$

with c_{jt}^* and c_{jt+1}^* defined accordingly to Equation (105).

Appendix E. Households with Labor Market Power

In this section, we describe the modifications to the economic environment and the attention problems that arise when we assume that households have some market power, allowing them to set wages for their differentiated labor services.

E.1. Economic Environment. Firms aggregate the differentiated labor services into a single productive input using a CES aggregator

$$L_{it} = \left(\int_0^1 L_{ijt}^{\frac{\eta-1}{\eta}} dj \right)^{\frac{\eta}{\eta-1}} \quad (125)$$

where η is the elasticity of substitution between labor types.

As a result, firm i 's optimal demand for labor of type j is given by

$$L_{ijt} = \left(\frac{W_{jt}}{W_t} \right)^{-\eta} L_{it}. \quad (126)$$

Inattention by firms implies $\tilde{\eta} < \eta$. We assume an aggregate wage index of the form

$$1 = \int_0^1 d_w \left(\hat{W}_{jt} \right) dj \quad (127)$$

where d_w is some twice continuously differentiable function.

Note that outside the non-stochastic steady state, the average wage rates (and thus labor supplied) set by the two household types will differ due to the marginal rates of substitution not being equalized. The linearized aggregate wage rate faced by firms is given by

$$\tilde{w}_t = \phi \tilde{w}_t^H + (1 - \phi) \tilde{w}_t^S. \quad (128)$$

The non-stochastic steady state of this economy is identical to that described in [Appendix B.2](#), except for the inclusion of [Equation \(126\)](#), which has no effect in a symmetric equilibrium. To ensure that the model's wage elasticity is consistent with the economic environment, we may assume that households are subject to idiosyncratic productivity shocks, which have no effect on the aggregate dynamics.

E.2. Expected Losses from to Suboptimal Actions. The firm's attention problem remains the same once we abstract from cross-sectional efficiency (i.e., labor mix decisions). The matrices in the period payoff functions and the optimal actions of the wage-setting households are defined below. As before, we present only the elements relevant to aggregate dynamics..

Hand-to-mouth. The matrix appearing in household j of type \mathcal{H} 's period payoff function is

$$\mathbf{H}_{x_j}^{\mathcal{H}} = -(C^{\mathcal{H}})^{1-\gamma} [\tilde{\eta}\omega_w^{\mathcal{H}}(1 + \tilde{\eta}(\gamma\omega_w^{\mathcal{H}} + \psi))]. \quad (129)$$

Household j of type \mathcal{H} 's vector of choice variables and optimal actions are respectively

$$\tilde{\mathbf{x}}_{jt} = (\tilde{w}_{jt})' \quad (130)$$

and

$$\tilde{\mathbf{x}}_{jt}^* = \left(\frac{\gamma\omega_w^{\mathcal{H}} + \psi}{1 + \tilde{\eta}\psi + \gamma\omega_w^{\mathcal{H}}(\tilde{\eta}-1)} [\psi(\tilde{\eta}\tilde{w}_t + l_t)] \right). \quad (131)$$

Savers. The matrix appearing in household j of type \mathcal{S} 's period payoff is

$$\mathbf{H}_{x_j}^{\mathcal{S}} = -(C^{\mathcal{S}})^{1-\gamma} \begin{bmatrix} \gamma & 0 \\ 0 & \tilde{\eta}\omega_w^{\mathcal{S}}(1 + \tilde{\eta}\psi) \end{bmatrix}. \quad (132)$$

Household j of type \mathcal{S} 's vector of choice variables and optimal actions are respectively

$$\mathbf{x}_{jt} = (c_{jt}, \tilde{w}_{jt})' \quad (133)$$

and

$$\mathbf{x}_{jt}^* = \begin{pmatrix} E_{jt} \left[-\frac{1}{\gamma}(r_t - \pi_{t+1}) + c_{jt+1}^* \right] \\ (1 + \tilde{\eta}\psi)^{-1} [\gamma c_{jt}^* + \psi(\tilde{\eta}\tilde{w}_t + l_t)] \end{pmatrix} \quad (134)$$

with c_{jt}^* and c_{jt+1}^* defined in accordance with [Equation \(105\)](#).

Lastly, the equilibrium around the non-stochastic steady-state, as well as the numerical procedure used to solve for it, are analogous to those described in the main text for the economy with a perfectly competitive labor market.

Appendix F. Calibration

F.1. Estimating the Share of Hand-to-Mouth We estimate the share of hand-to-mouth in the economy using the SCF, a triennial survey conducted by the Federal Reserve Board that collects detailed information on household finances in the United

States. It covers a representative sample of over 4,500 households and provides data on income, assets and debts.

First, we define the conditions under which a household classifies as hand-to-mouth;

Definition 4. *A household is considered hand-to-mouth if it either has*

- *zero liquid wealth*
- *or its credit limit is binding.*

This definition is standard in the literature, and is derived from the endogenous behavior of households in heterogeneous agent incomplete market models. As shown in [Kaplan and Violante \(2014\)](#), a household at a kink in its budget constraint, either at zero liquid wealth or the credit limit, exhibits a strong propensity to consume, which typically translates into allocating all of its period income to consumption and debt repayments, but none to savings. Hand-to-mouth households in our simplified framework display the exact same behavior.

[Kaplan, Violante, and Weidner \(2014\)](#) propose identifying hand-to-mouth households in the data using the following inequality conditions;

1. $0 \leq m_t(i) \leq \frac{w_t(i)}{2f}$
2. $m_t(i) \geq 0$ and $m_t(i) \leq \frac{w_t(i)}{2f} - \underline{m}_t(i)$

where $m_t(i)$ represents net liquid wealth, $w_t(i)$ denotes monthly income, and $f \geq 1$ is the frequency at which a household receives payments during a month²⁷. The first inequality identifies households with zero liquid wealth, while the second identifies those at their credit limit.

We estimate monthly income by dividing reported annual income by 12. We measure net liquid wealth based on the composition of liquid assets and debt defined in [Table 8](#). We follow [Kaplan and Violante \(2014\)](#) and inflate the value of transactional accounts (LIQ) by a factor of 1.05 to account for cash holdings that are not reported in the SCF. In the model, borrowing should be considered as unsecured credit, so debt is measured using revolving credit card debt (i.e., credit card balances that are not repaid in full at every payment) which avoids including as debt purchases made through credit cards in between regular payments (see e.g., [Telyukova \(2013\)](#)). We infer revolving credit card debt by multiplying credit card balances after the last

²⁷Income is measured monthly because, when $f = 1$, it represents the lowest plausible payment frequency. For more details on the estimator, see [Kaplan, Violante, and Weidner \(2014\)](#).

payment (CCBAL) with an indicator function that takes the value one if respondents answered either “sometimes” or “almost never” to the question “How often do you pay your credit card balance in full?”. To ensure consistency with our model, we exclude households with zero wage income, those with a respondent under 22 or over 79 years old, and those above the 95th income percentile.

Table 8: Assets and Debt in the SCF

Category	Mnemonic
<i>Liquid Assets</i>	
All types of transaction account	LIQ
Directly held pooled investment funds	NMMF
Directly held stocks	STOCKS
<i>Liquid Debt</i>	
Credit card balances after last payment	CCBAL

Notes: This is a conservative definition of liquid wealth, the same as in [Kaplan and Violante \(2014\)](#). One could extend it to include assets with a lesser degree of liquidity.

F.2. Consumption Ratio We use our estimates of hand-to-mouth households in [Appendix F.1](#) to calibrate the non-stochastic steady-state consumption ratio. First, we measure wage income and net liquid assets within each group. For both variables, we take the median. Wage income is directly reported in the survey, while net liquid assets is computed for each household by taking the difference between liquid assets and debts defined in [Table 8](#).

We follow the strategy of [Maćkowiak and Wiederholt \(2015\)](#) to estimate consumption expenditures. First, apply the tax rate schedule²⁸ to median total income within each group. Next, we compute the total savings required to maintain median liquid net worth constant at an annual inflation rate of 2.5%. Consumption expenditures for each group is obtained by subtracting median savings from after-tax median income, and the ratio follows directly.

Appendix G. New Keynesian Frictions

Throughout the paper, we also reference models in which some of the slow adjustment in prices or quantities are due to adjustment frictions instead of imperfect

²⁸Specifically, we apply the tax rate for “married filing jointly”. Tax schedule for a given year can be found at <https://www.irs.gov/pub/irs-prior>.

information. In this section, we provide an overview of the microfoundations behind the following frictions: (i) sticky prices and (ii) sticky wages *à la* Calvo, and (ii) habit formation in consumption.

G.1. Sticky Prices. With probability $1 - \zeta_p$, firms are free to reset their price P_{it} . Conditional on resetting, they choose the optimal price P_{it}^* that maximizes the discounted sum of expected dividends.

Optimal price-setting gives rise to a New Keynesian Phillips curve, which, when linearized, is

$$\pi_t = \kappa_p s_t + \beta E_t[\pi_{t+1}] \quad (135)$$

with

$$\kappa_p = \frac{(1 - \zeta_p)(1 - \beta\zeta_p)}{\zeta_p} \left(\frac{\alpha}{\alpha + (1 - \alpha)\theta} \right) \quad (136)$$

and

$$s_t = \tilde{w}_t + \frac{1 - \alpha}{\alpha} y_t - \frac{1}{\alpha} a_t. \quad (137)$$

[Equation \(135\)](#) is the slope of the Phillips curve and [Equation \(137\)](#) is the real marginal cost. A complete derivation can be found in [Galí \(2015\)](#).

G.2. Sticky Wages. We assume that every household supply a continuum of differentiated labor services with wage elasticity of demand η . The wages for all types of labor is set by a union that chooses it to maximize the average utility of its members, given the demand it faces and subject to Calvo frictions. For each of labor services, the union can reset the wage rate at which it remunerated with unconditional probability $1 - \zeta_w$.

We allow each household type to be represented by its own union. This structure gives rise to a New Keynesian Phillips curve for the wage index of each household type, of the form

$$\tilde{w}_t^h = \frac{1}{1 + \beta} \tilde{w}_{t-1}^h + \frac{\beta}{1 + \beta} E_t \tilde{w}_{t+1}^h + \kappa_w^h (mrs_t^h - \tilde{w}_t^h) + \frac{1}{1 + \beta} \pi_t + \frac{\beta}{1 + \beta} E_t \pi_{t+1} \quad (138)$$

with

$$\kappa_w^h = \frac{(1 - \zeta_w)(1 - \beta\zeta_w)}{\zeta_w(1 + \psi\eta)} \quad (139)$$

and

$$mrs_t^h = \psi l_t^h + \gamma c_t^h. \quad (140)$$

Equation (139) and Equation (140) are respectively the slope of the New Keynesian Philips curve for wages and the marginal rate of substitution between labor supply and consumption for household of type h .

Lastly, a linearization of the wage index implies

$$\tilde{w}_t = \phi \tilde{w}_t^H + (1 - \phi) \tilde{w}_t^S. \quad (141)$$

One can also assume that a single unions represent all households as in [Debortoli and Galí \(2024\)](#). In that case, the relevant marginal rate of substitution is a weighted average of the savers and hand-to-mouth.

G.3. Habit Formation. We modify the savers preferences such that utility depends not only on today's consumption but also on its value in the previous period

$$U(C_{jt}, C_{jt-1}, L_{jt}) = \frac{(C_{jt} - hC_{jt-1})^{1-\gamma} - 1}{1-\gamma} - \varphi^S \frac{L_{jt}^{1+\psi}}{1+\psi}. \quad (142)$$

where $h \in [0, 1]$ denotes the degree of habit formation.

A linearization of the Euler equation resulting from utility maximization for optimizing households yields the following two equations,

$$\lambda_t^S = E_t[\lambda_{t+1}^S + r_t - \pi_{t+1}] \quad (143)$$

and

$$\lambda_t^S = \frac{\gamma h}{(1-h)(1-\beta h)} c_{t-1}^S - \frac{\gamma(1+\beta h^2)}{(1-h)(1-\beta h)} c_t^S + \frac{\gamma \beta h}{(1-h)(1-\beta h)} E_t[c_{t+1}^S]. \quad (144)$$

where λ_t^S is the Lagrange multiplier on the household's flow budget constraint.

G.4. Linearized Optimality Conditions. To include the adjustment frictions in the economic environment, summarized by [Equations \(49\) to \(59\)](#), we proceed as follows. For Calvo prices, replace the firms' optimal pricing equations with the New Keynesian Phillips curve, [Equations \(135\) to \(137\)](#). For Calvo wages, replace the households labor supply equations with the New Keynesian Phillips curve for wages with [Equations \(138\) to \(141\)](#). For habit formation in consumption, substitute the savers' Euler equation with [Equations \(143\) to \(144\)](#). Modify the Taylor rule to account for the presence of an output gap due to frictions such that

$$r_t = \phi_r r_{t-1} + (1 - \phi_r)[\phi_\pi \pi_t + \phi_y(y_t - y_t^*)] \quad (145)$$

where y_t^* represents the natural level of output (i.e., the level of economic activity that would prevail in the frictionless, perfect information economy).

Note that, while typically not done in the rational inattention literature, it is possible to assume that some decisions are subject to adjustment frictions and others to inattention.