Computational Economics

Lecture 6: Aiyagari Model

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Introduction

From this lecture onward, we apply the computational tools from Lectures 1–5 to a range of economic models. We will build toward solving Heterogeneous Agent New Keynesian (HANK) models, which arguably represent the current frontier of business cycle research.

In this lecture, we study the Aiyagari model. The model abstracts from aggregate shocks but features an endogenous distribution of agents over the state, a core object in HANK models.

Overview

The model's main features are:

- Heterogeneous agents.
- A single vehicle for borrowing and lending
  (can be extended to include assets of different classes).
- A borrowing limit
  (generally tighter than the natural borrowing limit).

The model allows us to study several topics:

- Precautionary savings from liquidity constraints.
- Risk sharing and asset pricing.
- Shape of the wealth distribution.

The Economy

Households

A unit mass of ex-ante identical households subject to idiosyncratic income shocks facing a common borrowing constraint.

Each household $i \in [0,1]$ solves the following problem:

$$\max_{\{c_{it}\}_{t=0}^{\infty}} E_t \sum_{t=0}^{\infty} \beta^t u(c_{it}) \tag{1}$$

subject to

$$a_{it+1} + c_{it} = wz_{it} + (1+r)a_{it} \text{ , and } a_{it} \geq B. \tag{2}$$

where c_{it} is consumption, a_{it} is assets, z_{it} is an idiosyncratic income shock, w_t is the wage rate, r_t is the net interest rate, $B \leq 0$ is the borrowing limit.

Note: Whenever $B < 0$, we assume that a financial intermediary collects savings and uses them to finance loans and firms' capital.

Households (cont'd)

We solve the household problem using dynamic programming:

$$V(a,z) = \max_{\{c,a'\}} \left\{ u(c) + \beta E\left[V(a',z') \right] \right\}$$

subject to

$$a' + c = wz + (1+r)a, \qquad a' \geq B.$$

The associated policy function for savings is denoted by

$$a' = g(a,z).$$

Firms

The firms' side is summarized by a profit-maximizing representative firm that produces using capital and labor as inputs.

In every period, the representative firm solves

$$\max_{\{K_t, L_t\}} Z_t K_t^{\alpha} L_t^{1-\alpha} - w_t L_t - (r_t + \delta) K_t \tag{3}$$

where K_t is aggregate capital, L_t is aggregate labor, Z_t is productivity and δ is the capital's depreciation rate.

Note: $(r_t + \delta)$ follows from non-arbitrage between investing in an asset that returns $(1 + r_t)$ or $(R_t + 1 - \delta)$.

Firms (cont'd)

The representative firm's FOCs are given by:

$$\alpha Z_t K_t^{\alpha-1} L_t^{1-\alpha} = (r_t + \delta) \tag{4}$$

$$(1-\alpha) Z_t K_t^{\alpha} L_t^{\alpha} = w_t. \tag{5}$$

From Equation (4), we can express r_t as a function of K_t:

$$r_t = \alpha Z_t \left(\frac{L_t}{K_t}\right)^{1-\alpha} - \delta. \tag{6}$$

Combining Equation (5) and Equation (6) yields an expression for w_t as a function of r_t:

$$w_t(r_t) = Z_t(1-\alpha)(\alpha Z_t/(r_t+\delta))^{\alpha/(1-\alpha)}. \tag{7}$$

Equilibrium

We are looking to construct a stationary equilibrium with respect to aggregate variables: individual households move across the wealth distribution, but aggregate variables remain constant.

As a result, we can drop time indices for the following variables: Z, r, w, L, with

$$L = \int_0^1 z_{it}\, di.$$

Equilibrium

A Stationary Rational Expectations Equilibrium (SREE) is:

- A value function $V(a, z)$ and its associated policy function $g(a, z)$.
- A distribution of households over assets (and idiosyncratic productivity).
- An aggregate level of capital K.
- A set of prices $\{r, w\}$.

such that

1. $V(a_i, z_i)$ and $g(a_i, z_i)$ solve the households' problem taking prices as given.
2. The representative firm maximizes taking prices as given.
3. Aggregate capital is given by: $K = \int_0^1 a_i di$.
4. The distribution over assets and idiosyncratic productivity $\Psi(a, z)$ is stationary: $\Psi(a', z') = \int g(a, z)\, Q(z, z')\, d\Psi(a, z)$

Solving the Model

Solving the Model

Solving for the SREE can be done by iterating over the following steps:

0. Guess a value for aggregate capital K.
1. Compute the prices r, w implied by K.
2. Solve for the policy function $g(a, z)$ taking r, w as given.
3. Compute aggregate capital implied by individuals savings $K' = \int_0^1 a_i \, di$.
4. Compare K and K'. If they differ by more than a prespecified criterion, update the guess for K and return to step 1.

Solving the Model (cont'd)

Solving the model involves one "outside" loop to find the equilibrium capital level, the assets distribution, and the associated prices, and one "inner" loop to solve the households' problem.

We solve the households problem using the FOC with the Endogenous Grid Method (EGM). EGM exploits the Euler equation to avoid maximizing over discretized controls as in VFI:

Solving the Model (cont'd)

EGM method:

1. Fix a grid for next period assets, a' and guess a policy function $g(a', z')$ for savings.
2. For each (a', z'), use the budget constraint to compute next period consumption $c' = (1+r)a' + wz' - g(a', z')$.
3. For each c', compute the marginal utility of consumption $u'(c')$.
4. From the Euler equation, compute the current period marginal utility of consumption $u'(c) = \beta E[(1+r)u'(c')]$.
5. From the marginal utility of consumption, recover current consumption $c = [u'(c)]^{-1}$.
6. From the budget constraint, recover current assets $a = (a' - c + wz)/(1+r)$. This defines an endogenous grid for assets.
7. Interpolate the savings policy function implied by the mapping from a to the fixed grid for a'.
8. Update the policy function, iterate until convergence.

Note: The endogenous grid may violate the borrowing constraint, but the policy function never does.