The Gamma Distribution

Properties, Proofs and Applications

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Introduction

The gamma distribution is part of the two-parameters family of continuous probability distributions. Indeed, it may be parameterized with two different parameterizations [1] :

Parameterization 1:

$$\text{Shape: } \alpha > 0 \qquad \text{Rate: } \lambda > 0 \tag{1}$$

Parameterization 2:

$$\text{Shape: } k > 0 \qquad \text{Scale: } \theta > 0 \tag{2}$$

Explain why there are two parameterizations and why the λ is sometimes replaced with a β.

That being said, in this document, as the two parameterizations only exist for the sake of convenience and are identical in their results, only the parameterization 1 presented in Equation 1 will be considered and used for proofs.

Properties to prove

Support: $x \in (0, \infty)$

Probability density function:

$$f(x) = \frac{\lambda^\alpha}{\Gamma(\alpha)} x^{\alpha-1} e^{-\lambda x} \tag{3}$$

Cumulative distribution function:

$$F(x) = \frac{1}{\Gamma(\alpha)} \gamma(\alpha, \lambda x) \tag{4}$$

Expected value, also known as the theoretical mean:

$$\mu = E(x) = \frac{\alpha}{\lambda} \tag{5}$$

There is no simple closed form equation for the median of a gamma distribution.

Mode:

$$\text{Mode} = \frac{(\alpha - 1)}{\lambda} \text{ for } \alpha \geq 1 \tag{6}$$

Variance:

$$\text{Var}(x) = \frac{\alpha}{\lambda^2} \tag{7}$$

Skewness:

$$\text{Skewness} = \frac{2}{\sqrt{\alpha}} \tag{8}$$

Excess kurtosis:

$$\text{Kurtosis} = \frac{6}{\alpha} \tag{9}$$

Entropy:

$$\text{Entropy} = \alpha + \ln \lambda + \ln \Gamma(\alpha) + (1 - \alpha)\psi(\alpha) \tag{10}$$

Moment generating function:

$$M(t) = \left(1 - \frac{t}{\lambda}\right)^{-\alpha} \text{ for } t < \lambda \tag{11}$$

Characteristic function:

$$\text{CF} = \left(1 - \frac{it}{\lambda}\right)^{-\alpha} \tag{12}$$

Methods of moments:

$$\begin{aligned} \alpha &= \frac{E(X)^2}{\text{Var}(X)} \\ \lambda &= \frac{E(X)}{\text{Var}(X)} \end{aligned} \tag{13}$$

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References

[1] Wikipedia. *Gamma Distribution*. In: *Wikipedia*. 2022-02-23. URL: https://en.wikipedia.org/w/index.php?title=Gamma_distribution&oldid=1073512326 (visited on 03/02/2022).