The Gamma Distribution

Properties, Proofs and Applications

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The gamma distribution is part of the two-parameters family of continuous probability distributions. Indeed, it may be parameterized with two different configurations: [1]

Configuration 1

Shape: $k > 0$ Scale: $\theta > 0$

Support: $x \in (0, \infty)$

Probability density function:

$$f(x) = \frac{1}{\Gamma(k)\theta^k} x^{k-1} e^{-\frac{x}{\theta}} \tag{1}$$

Cumulative distribution function:

$$F(x) = \frac{1}{\Gamma(k)} \gamma\left(k, \frac{x}{\theta}\right) \tag{2}$$

Expected value, also known as the theoretical mean:

$$\mu = E(x) = k\theta \tag{3}$$

There is no simple closed form equation for the median of a gamma distribution.

Mode:

$$\text{Mode} = (k-1)\theta \text{ for } k \geq 1 \tag{4}$$

Variance:

$$\text{Var}(x) = k\theta^2 \tag{5}$$

Skewness:

$$\text{Skewness} = \frac{2}{\sqrt{k}} \tag{6}$$

Excess kurtosis:

$$\text{Kurtosis} = \frac{6}{k} \tag{7}$$

Entropy:

$$\text{Entropy} = k + \ln\theta + \ln\Gamma(k) + (1-k)\psi(k) \tag{8}$$

Moment generating function:

$$M(t) = (1-\theta t)^{-k} \text{ for } t < \frac{1}{\theta} \tag{9}$$

Characteristic function:

$$\text{CF} = (1-\theta it)^{-k} \tag{10}$$

Methods of moments:

$$\begin{aligned} k &= \frac{E(X)^2}{\text{Var}(X)} \\ \theta &= \frac{\text{Var}(X)}{E(X)} \end{aligned} \tag{11}$$

Configuration 2

Shape: $\alpha > 0$ Rate: $\beta > 0$

Support: $x \in (0, \infty)$

Probability density function:

$$f(x) = \frac{\beta^\alpha}{\Gamma(\alpha)} x^{\alpha-1} e^{-\beta x} \tag{12}$$

Cumulative distribution function:

$$F(x) = \frac{1}{\Gamma(\alpha)} \gamma(\alpha, \beta x) \tag{13}$$

Expected value, also known as the theoretical mean:

$$\mu = E(x) = \frac{\alpha}{\beta} \tag{14}$$

There is no simple closed form equation for the median of a gamma distribution.

Mode:

$$\text{Mode} = \frac{(\alpha - 1)}{\beta} \text{ for } \alpha \geq 1 \tag{15}$$

Variance:

$$\text{Var}(x) = \frac{\alpha}{\beta^2} \tag{16}$$

Skewness:

$$\text{Skewness} = \frac{2}{\sqrt{\alpha}} \tag{17}$$

Excess kurtosis:

$$\text{Kurtosis} = \frac{6}{\alpha} \tag{18}$$

Entropy:

$$\text{Entropy} = \alpha + \ln \beta + \ln \Gamma(\alpha) + (1 - \alpha)\psi(\alpha) \tag{19}$$

Moment generating function:

$$M(t) = \left(1 - \frac{t}{\beta}\right)^{-\alpha} \text{ for } t < \beta \tag{20}$$

Characteristic function:

$$\text{CF} = \left(1 - \frac{it}{\beta}\right)^{-\alpha} \tag{21}$$

Methods of moments:

$$\begin{aligned} \alpha &= \frac{E(X)^2}{\text{Var}(X)} \\ \beta &= \frac{\text{Var}(X)}{E(X)} \end{aligned} \tag{22}$$

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References

[1] Wikipedia. *Gamma Distribution*. In: *Wikipedia*. 2022-02-23. URL: https://en.wikipedia.org/w/index.php?title=Gamma_distribution&oldid=1073512326 (visited on 03/02/2022).