

THE GAMMA DISTRIBUTION  
PROPERTIES, PROOFS AND APPLICATIONS

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The gamma distribution is part of the two-parameters family of continuous probability distributions. Indeed, it may be parameterized with two different configurations:<sup>[1]</sup>

## Configuration 1

Shape:  $k > 0$

Scale:  $\theta > 0$

Support:  $x \in (0, \infty)$

Probability density function:

$$f(x) = \frac{1}{\Gamma(k)\theta^k} x^{k-1} e^{-\frac{x}{\theta}} \quad (1)$$

Cumulative distribution function:

$$F(x) = \frac{1}{\Gamma(k)} \gamma\left(k, \frac{x}{\theta}\right) \quad (2)$$

Expected value, also known as the theoretical mean:

$$\mu = E(x) = k\theta \quad (3)$$

There is no simple closed form equation for the median of a gamma distribution.

Mode:

$$\text{Mode} = (k - 1)\theta \text{ for } k \geq 1 \quad (4)$$

Variance:

$$\text{Var}(x) = k\theta^2 \quad (5)$$

Skewness:

$$\text{Skewness} = \frac{2}{\sqrt{k}} \quad (6)$$

Excess kurtosis:

$$\text{Kurtosis} = \frac{6}{k} \quad (7)$$

Entropy:

$$\text{Entropy} = k + \ln \theta + \ln \Gamma(k) + (1 - k)\psi(k) \quad (8)$$

Moment generating function:

$$M(t) = (1 - \theta t)^{-k} \text{ for } t < \frac{1}{\theta} \quad (9)$$

Characteristic function:

$$\text{CF} = (1 - \theta it)^{-k} \quad (10)$$

Methods of moments:

$$\begin{aligned} k &= \frac{E(X)^2}{\text{Var}(X)} \\ \theta &= \frac{\text{Var}(X)}{E(X)} \end{aligned} \quad (11)$$

## Configuration 2

Shape:  $\alpha > 0$

Rate:  $\beta > 0$

Support:  $x \in (0, \infty)$

Probability density function:

$$f(x) = \frac{\beta^\alpha}{\Gamma(\alpha)} x^{\alpha-1} e^{-\beta x} \quad (12)$$

Cumulative distribution function:

$$F(x) = \frac{1}{\Gamma(\alpha)} \gamma(\alpha, \beta x) \quad (13)$$

Expected value, also known as the theoretical mean:

$$\mu = E(x) = \frac{\alpha}{\beta} \quad (14)$$

There is no simple closed form equation for the median of a gamma distribution.

Mode:

$$\text{Mode} = \frac{(\alpha - 1)}{\beta} \text{ for } \alpha \geq 1 \quad (15)$$

Variance:

$$\text{Var}(x) = \frac{\alpha}{\beta^2} \quad (16)$$

Skewness:

$$\text{Skewness} = \frac{2}{\sqrt{\alpha}} \quad (17)$$

Excess kurtosis:

$$\text{Kurtosis} = \frac{6}{\alpha} \quad (18)$$

Entropy:

$$\text{Entropy} = \alpha + \ln \beta + \ln \Gamma(\alpha) + (1 - \alpha)\psi(\alpha) \quad (19)$$

**INCOMPLETE from this point  $\Rightarrow$**

Moment generating function:

$$M(t) = \left(1 - \frac{t}{\beta}\right)^{-\alpha} \text{ for } t < \beta \quad (20)$$

Characteristic function:

$$\text{CF} = \left(1 - \frac{it}{\beta}\right)^{-\alpha} \quad (21)$$

Methods of moments:

$$\begin{aligned} \alpha &= \frac{E(X)^2}{\text{Var}(X)} \\ \beta &= \frac{\text{Var}(X)}{E(X)} \end{aligned} \quad (22)$$

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## References

- [1] Wikipedia. *Gamma Distribution*. In: *Wikipedia*. 2022-02-23. URL: [https://en.wikipedia.org/w/index.php?title=Gamma\\_distribution&oldid=1073512326](https://en.wikipedia.org/w/index.php?title=Gamma_distribution&oldid=1073512326) (visited on 03/02/2022).