

# Ternary chalcogenide $\text{Na}_2\text{MoSe}_4$ is a direct bandgap semiconductor\*

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First-principles computations were performed on a set of hypothetical crystal structures to determine the ground state structure and electronic properties of the sodium molybdenum selenide  $\text{Na}_2\text{MoSe}_4$ . First, we approximate the ground state of the candidate structures through isotropic expansion. Formation enthalpy calculations reveal that the ground state structure of  $\text{Na}_2\text{MoSe}_4$  is a simple orthorhombic *oP* lattice, with space group *Pnma*. We investigate the electronic structure of the *oP* structure as well as the metastable phases, showing semiconducting behaviour in three of them. Finally, we employ the Tran-Blaha modified Becke-Johnson exchange potential to model the electronic band structure of  $\text{Na}_2\text{MoSe}_4$  and show that it is a direct bandgap semiconductor with a fundamental band gap at the  $\Gamma$  point. This is the first detailed analysis of the structure and properties of the semiconducting  $\text{Na}_2\text{MoSe}_4$ , an alkali transition metal chalcogenide that is isostructural to a set of known inorganic and hybrid organic/inorganic analogs whose physics have been until now overlooked, but display high promise for next generation optoelectronics and photovoltaics.

## I. INTRODUCTION

A truly fascinating class of inorganic materials is the cherished family of transition metal chalcogenides (TMC). Its general composition is  $\text{M-X}$ , where  $\text{M}$  = transition metals (e.g. Mo, W, Re) and  $\text{X}$  = chalcogen anion (i.e. S, Se, Te) [1]. This broad family of presents extended structures (bulk) [1–3] and two-dimensional materials, with promising applications in next-generation technologies due to their broad spectra of physical and chemical properties [4–7]. Discoveries in TMC research have led to advances in our understanding of semiconductors, superconductors, topological insulators, ionic and mixed conductors, catalysts and photocatalysts [8–12] and recently an entirely exciting novel class of TMC-perovskites engineered for applications in optoelectronics and solar cells [13, 14].

Although the TMC family is large, its chemical space remains overwhelmingly unexplored when compared to transition metal oxides (TMO), with a ratio of roughly 2500 TMO to 450 TMCs [15]. Moreover, little is currently known about ternary transition metal selenides (TM-Se).

As members of the TMC family, TM-Se materials have increased in interest and have found applications within the realm of catalysis [16]. In 2005, Evans and coworkers demonstrated that solvothermal techniques are appropriate for the synthesis of bulk ternary TM-Se. In their report, they discussed the chemistry of  $\text{Cu}_2\text{MX}_4$ , where  $\text{M} = \text{W}$  or  $\text{Mo}$ ;  $\text{X} = \text{S}$ ,  $\text{Se}$ ,  $\text{Se/S}$ . Among these materials,  $\text{Cu}_2\text{WSe}_4$  was identified as semiconductor [17].

Efforts are continuously made to explore the chemical space of TMCs. However, the discovery of QMX compounds, where  $\text{Q}$  = cation,  $\text{M}$  = transition metal and  $\text{X}$  = chalcogen anion, remains a challenge. This fact was highlighted in a recent computational and experimental investigation of TMCs, reporting the phase diagrams for twenty-four new structures. Although the materials are predicted to be stable, the synthesis of the compounds was rendered unsuccessful. [3].

The recent discovery of low dimensional ternary  $\text{Na-Cu-Se}$  materials by Kanatzidis and coworkers provides new insights into the chemistry of the  $\text{A-TM-Se}$  chemical space, where  $\text{A}$  is an alkali metal. We highlight the discovery of the layered metal  $\text{NaCu}_6\text{Se}_4$  with mixed valency [18], the mixed-valent two-dimensional metal  $\text{NaCu}_4\text{Se}_3$  [19] and the two-dimensional metal  $\text{NaCu}_4\text{Se}_4$ , which presents high hole mobility and giant magnetoresistance [20].

This leads us to think in general terms about the physics and applications of alkali-transition metal chalcogenides,  $\text{AMX}$  ( $\text{A}$  = alkali metal,  $\text{M}$  = transition metal,  $\text{X}$  = chalcogen). In a recent work by Z. Xia *et al*, it is discussed the chemistry of a new alkali-transition metal chalcogenide,  $\text{CsCu}_5\text{Se}_3$ . This ternary selenide is an *almost* direct bandgap semiconductor, with an  $E_g = 1.04\text{eV}$ . The material crystallizes in the *oP* lattice, in space group *Pmma* (No. 51) and it can be achieved via the solvothermal method. Authors also explored similar compounds to complete the family of materials by synthesizing and characterizing  $\text{CsCu}_5\text{S}_3$  and proposing  $\text{CsCu}_5\text{Te}_3$  [21]. On another note, superconductivity has also been observed in narrow-gap semiconductors within the  $\text{AMX}$  family,  $\text{RbBi}_{11/3}\text{Te}_6$  and  $\text{CsBi}_4\text{Te}_6$  [22, 23]. Although a few (3) sodium molybdenum selenides can be found in the Materials Project and the ICSD [15] databases, none are semiconducting (See Supplemental

\* Supplementary information available

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Information).

The combination of computational materials design and experiment will undoubtedly continue to accelerate catalysis and materials science [24]. Particularly, the availability of crystal exploration and prediction tools such as the web-based repository, The Materials Project [25] and crystal structure prediction packages such as CALYPSO [26, 27] and XtalOpt [28, 29] are key for materials discovery and bottom-up design [30–34]. As these techniques become robust, they become accepted by the broader community as an standard for materials design.

The traditional or go-to route in materials design consists of "manual" solid substitution (ionic or interstitial) of a known and analogous material. In this work, our objective is to model and determine the ground state structure for  $\text{Na}_2\text{MoSe}_4$  to then investigate its electronic structure. In this sense, it might be wise to take the stable and clearly chemically similar  $\text{Na}_2\text{MoO}_4$  (face centered cubic (*cF*), substitute our chalcogen anion and find the ground state. Another lesser obvious possibility would be to part from the relative  $\text{Rb}_2\text{WS}_4$  (primitive orthorhombic (*oP*)) by substituting  $\text{K}^+$  for  $\text{Na}^+$ ,  $\text{W}^+$  for  $\text{Mo}^+$ ,  $\text{S}^-$  for  $\text{Se}^-$  and find the ground state. Which would be a better choice? One route (i.e. case the structure's symmetry) will lead us to the ground state and the other must lead towards instability. From the *a priori* postulate of statistical thermodynamics, before we calculate anything, the probability of these structures being the energetically favored is exactly the same for both routes.

In brief, we employ a materials prediction method combined with Kohn-Sham density functional theory to model and characterize the sodium molybdenum selenide structure,  $\text{Na}_2\text{MoSe}_4$ . We generate a pool of candidate structures, set criteria to quickly screen our candidates based on symmetry and stoichiometry, and determine the energetically favored candidate through first-principles thermodynamics. Finally, we focus on the electronic structure of the ground state structure.

## II. CRYSTAL STRUCTURE PREDICTION

To address the issue of structure determination, we followed the crystal structure prediction method of Hautier *et al* [35] based on data mined ionic substitutions. It has been demonstrated that the method of data mined ionic substitutions can generate likely crystal structures at a fraction of the computational cost of evolutionary algorithms [3], due to the fact that the substitutions are generated posterior to the analysis of existing crystal structures listed in the ICSD. For the reader's convenience, we briefly outline the data mining + ionic substitution (DM+IS) methodology:

(i) Systems of ions  $X_i$  ( $i = 1, 2, 3, \dots, n$ ) are represented as a component vectors of  $n$  elements,

$$\mathbf{X} = (X_1, X_2, X_3, \dots, X_n). \quad (1)$$

(ii) Once a given number of candidate structures is generated, the probability function  $P_n$  for two compounds existing in the same crystal structure is expressed as

$$P_n(\mathbf{X}, \mathbf{X}') = P_n(X_1, \dots, X'_1, \dots, X'_n). \quad (2)$$

(iii)  $p_n$  is approximated by using the feature function  $f(\mathbf{X}, \mathbf{X}')$ ,

$$P_n(\mathbf{X}, \mathbf{X}') \approx \frac{\exp[\sum_i \lambda_i f_i(\mathbf{X}, \mathbf{X}')] }{\Xi} \quad (3)$$

(iv) In equation (3),  $\Xi$  is analogous to a partition function that ensures normalization of  $P_n$ , and  $\lambda_i$  is the weight corresponding to  $f_i(\mathbf{X}, \mathbf{X}')$ . It is noteworthy that only binary feature functions  $f_i(\mathbf{X}, \mathbf{X}')$  are assigned to pairs of ions ( $\alpha, \beta$ ),

$$f_k^{\alpha, \beta}(\mathbf{X}, \mathbf{X}') = \begin{cases} 1, & X_k = \alpha, X'_k = \beta. \\ 0, & \text{else.} \end{cases} \quad (4)$$

(v) The likelihood of the binary  $\alpha$  to  $\beta$  substitution is determined by  $\lambda_i$ , obtained from ionic compounds in the ICSD. For a detailed description of DM+IS method and its reach, we refer the reader to the paper by Ceder and coworkers [35].

Here, our DM+IS search generated over sixty-five hypothetical sodium molybdenum selenide structures, of which twenty-one present our hypothesized stoichiometry. The list of  $\text{Na}_2\text{MoSe}_4$  candidates is found in Table S1 and it has been made public in the Materials Project Database [25]. The four candidates with highest likelihood of existing in their predicted space groups were analyzed in this work.

## III. FIRST-PRINCIPLES COMPUTATIONS

Our first-principles computations based on Kohn-Sham Density Functional Theory (DFT) were carried out primarily using the plane-wave pseudo-potential method implemented in Quantum ESPRESSO [36, 37]. We employ the General Gradient Approximation (GGA) functional with corrected-for-solids Perdew-Burke-Ernzerhof (PBEsol) parametrization for the exchange correlation potential [38], as it has been shown that the PBEsol is better suited to approximate lattice constants and surface energies when compared to PBE and LDA [39–41]. The ground state structures were determined by varying the volume isotropically, to control symmetry while fully relaxing atomic positions under a tolerance of  $13.605 \times 10^{-5}$  eV/atom for total energy and net forces of  $0.025$  eV·Å<sup>-1</sup> per atom. These computations were performed using ultrasoft pseudopotentials (USP) with a plane-wave kinetic energy cutoff of 40 Ry (544 eV), charge density cutoff  $f$  of 320 Ry (4,354 eV) and a convergence threshold of  $1 \times 10^{-8}$  eV for self-consistency. Monkhorst-Pack  $\Gamma$ -centered integration grids were used to sample the first Brillouin Zone [42]. For structural

optimization computations of the  $cF$ ,  $oF$ ,  $oP$  and  $mC$  phases, integration grids of  $6 \times 6 \times 4$ ,  $8 \times 6 \times 5$ ,  $6 \times 8 \times 4$ ,  $5 \times 5 \times 4$  were used. To model the electronic structure we then increased the density of the  $k$ -points integration grids to  $24 \times 28 \times 20$  ( $cF$ ),  $32 \times 24 \times 20$  ( $oF$ ),  $24 \times 32 \times 18$  ( $oP$ ) and  $30 \times 30 \times 24$  ( $mC$ ). The USP used in all pw-DFT computations were generated with the following valence configurations: Na ( $2s^1$ ,  $3s^2$ ,  $2p^2$ ), Mo ( $4s^1$ ,  $5s^2$ ,  $5p^2$ ,  $4d^5$ ) and Se ( $4s^1$ ,  $4p^2$ ,  $3d^3$ ) [43].

It is well known GGA fails to predict the fundamental bandgap of semiconductors. Therefore, we correct the electronic band structure of the favored phase by performing meta GGA (MGGA) calculations using the Trahn-Blaha modified Becke-Johnson (TB-mBJ) exchange potential [44]. In order to obtain optimal results, our TB-mBJ computations were carried out in the framework of the Augmented Planewave plus Local Orbital (APW+lo) method as implemented in the all-electron code Wien2k[45].

#### IV. CANDIDATE STRUCTURES

We consider the following four candidate structures of  $\text{Na}_2\text{MoSe}_4$ : a face centered cubic ( $cF$ ) phase with space group  $Fd-3m$  (227), a face centered orthorhombic ( $oF$ ) with space group  $Fddd$  (70), a primitive orthorhombic ( $oP$ ) with space group  $Pnma$  (62) and a base centered monoclinic ( $mC$ ) with space group  $C2/m$  (12). The unit cells are sketched in Figure 1 (a)-(d). The structural details for the evaluated systems are shown in Table 1, which includes the raw (generated) DM+IS and the computed (equilibrium) parameters. From the obtained results it is clear that the equilibrium volume changes significantly with respect to the DM+IS volume, with the exception of the  $oP$  structure. For example,  $\Delta V^{oP} = 2\%$  versus  $\Delta V^{cF} = 26\%$ . Since we have unit cells with different number of atoms, to formally asses which of the candidates is the favored ground state phase of  $\text{Na}_2\text{MoSe}_4$ , we proceed to calculate the molar formation enthalpy  $\Delta H_{f(AMX)}^s$ . It reads

$$\Delta H_{f(AMX)}^s = U_{(AMX)}^s - \sum_i N_i \mu_i^s, \quad (5)$$

where  $U_{(AMX)}^s$  is the total energy of disodium tetraseleniummolybdate in solid phase,  $N_i$  is the number of atoms of a constituent  $i$  with chemical potential  $\mu_i$  in solid phase. This is,  $2\mu_{Na}^s$ ,  $\mu_{Mo}^s$  and  $4\mu_{Se}^s$ . The chemical potentials  $\mu$  for the constituent species were calculated based on DFT energies of their equilibrium crystal structures under the same relaxation criteria and convergence thresholds as the  $\text{Na}_2\text{MoSe}_4$  candidates. Additionally, the energetics analysis was cross-checked, fully relaxing all candidate structures at the PBE level. No significant changes on the lattice parameters were found. The values for  $\Delta H_{f(AMX)}^s$  are shown in Table 2. Note that  $oP$  is the favored phase of the selenide with  $\Delta H_f^s$  values

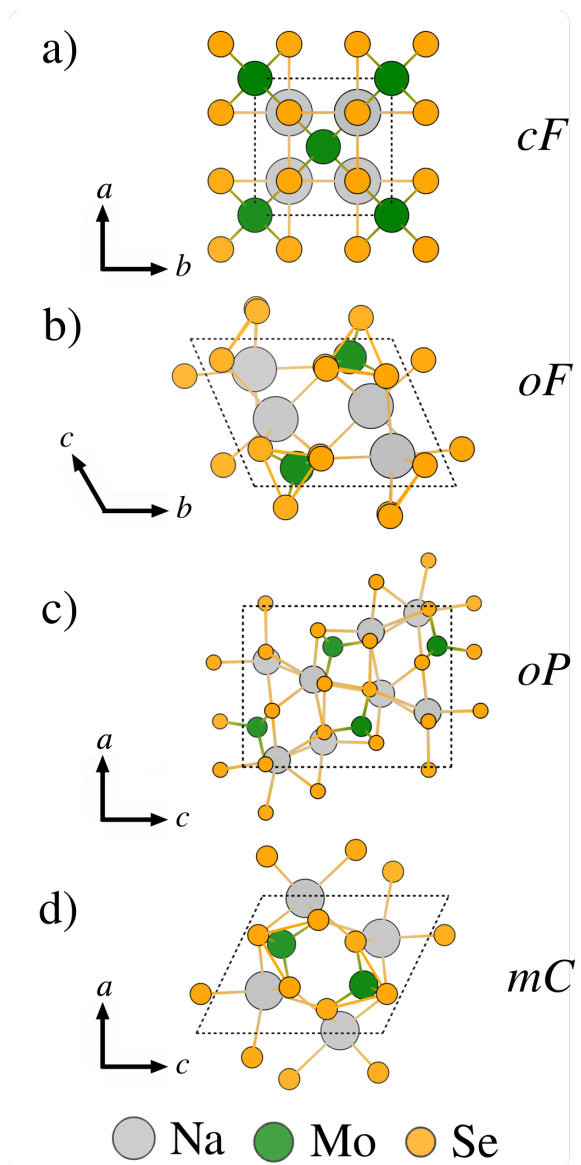


FIG. 1. Sketch of the candidate structure unit cells under this study for  $\text{Na}_2\text{MoSe}_4$ . The candidates are labeled by their lattice type (a) face centered cubic  $cF$ , (b) face centered orthorhombic  $oF$ , (c) simple orthorhombic  $oP$  and (d) base centered monoclinic  $mC$ . The colors corresponding to Na, Mo and Se are grey, green and orange respectively.

of -5.14 (PBEsol) and -5.11 (PBE) eV per formula unit (eV/f.u.). We find these relative formation enthalpies to be in good agreement, as the relative error between the PBE/PBEsol calculations is less than 2% for all phases with the exception of the least stable  $cF$ , in which a larger discrepancy is observed. We encourage further studies to calculate the phonon band structure for the meta-stable phases, as we recognize the importance of these calculations to assess its dynamical stability. Here, our scope is to determine the most energetically stable structure for the already synthesized  $\text{Na}_2\text{MoSe}_4$  compound[46, 47], and calculate its electronic properties. Once determined

TABLE I. Predicted and optimized lattice parameters for the  $\text{Na}_2\text{MoSe}_4$  candidate structures computed at the GGA-(PBEsol) level of theory.

| System                    | <b>a</b> (Å) | <b>b</b> (Å) | <b>c</b> (Å) | V (Å <sup>3</sup> ) | $\alpha$ | $\beta$ | $\gamma$ |
|---------------------------|--------------|--------------|--------------|---------------------|----------|---------|----------|
| <b>cF</b> : $Fd-3m$ (227) |              |              |              |                     |          |         |          |
| DM+IS                     | 6.44         | 6.44         | 9.11         | 188.89              | 45       | 45      | 90       |
| This work                 | 8.14         | 8.14         | 11.51        | 238.76              | 45       | 45      | 90       |
| <b>oF</b> : $Fddd$ (70)   |              |              |              |                     |          |         |          |
| DM+IS                     | 6.31         | 8.39         | 10.85        | 223.77              | 130      | 149     | 56       |
| This work                 | 7.71         | 10.24        | 13.26        | 273.38              | 130      | 149     | 56       |
| <b>oP</b> : $Pnma$ (62)   |              |              |              |                     |          |         |          |
| DM+IS                     | 9.32         | 6.84         | 12.08        | 770.49              | 90       | 90      | 90       |
| This work                 | 9.12         | 6.69         | 11.81        | 753.92              | 90       | 90      | 90       |
| <b>mC</b> : $C2/m$ (12)   |              |              |              |                     |          |         |          |
| DM+IS                     | 6.88         | 6.08         | 7.54         | 254.92              | 90       | 67      | 63       |
| This work                 | 7.78         | 6.88         | 8.52         | 368.58              | 90       | 67      | 63       |

TABLE II. Formation enthalpies  $\Delta H_f$  for  $\text{Na}_2\text{MoSe}_4$  candidate structures calculated from GGA(PBEsol) and GGA(PBE) computations.  $\Delta H_f$  is shown in eV per formula unit (eV/f.u.) and eV/atom.

| System    | $\Delta H_f$ (eV/f.u.) | $\Delta H_f$ (eV/atom) |
|-----------|------------------------|------------------------|
| <b>cF</b> |                        |                        |
| PBEsol    | -4.39                  | -0.63                  |
| PBE       | -4.02                  | -0.57                  |
| <b>oF</b> |                        |                        |
| PBEsol    | -4.90                  | -0.70                  |
| PBE       | -4.83                  | -0.69                  |
| <b>oP</b> |                        |                        |
| PBEsol    | -5.14                  | -0.73                  |
| PBE       | -5.11                  | -0.73                  |
| <b>mC</b> |                        |                        |
| PBEsol    | -5.03                  | -0.72                  |
| PBE       | -4.91                  | -0.70                  |

the most stable phase, from this point forward, we focus our attention only to the *oP* phase with *Pnma* space group. To further cross check the ground state of the *Pnma* candidate, we take the initial (DM+IS) structure and optimize it at the APW+lo/PBEsol level of theory. The volume  $V$  is varied isotropically. The equilibrium volume is obtained using the Birch-Murnaghan equation of state (BM-EOS), given by

$$E(V) = E(V_0) + \frac{B_0 V}{B'_0} \left[ 1 + \frac{(V_0/V)^{B'_0}}{B'_0 - 1} \right] \quad (6)$$

where  $V_0$  is the predicted equilibrium volume, and  $B_0$ ,  $B'_0$  are the bulk modulus and its pressure derivative respectively. The ground lattice parameter is  $a = 9.21$  Å, which is in fine agreement with our pw(pseudo-potential) computed lattice constant  $a = 9.12$ , with an absolute error  $e_a = \leq 1\%$ . At  $V_0$ , the bulk modulus for  $\text{Na}_2\text{MoSe}_4$  is  $B_0 = 56.07$  GPa with a numerical derivative of  $B'_0 = 4.41$ .

Due to the fact that the hypothetical crystal structures were generated from chemical analogs, it is reasonable to infer that the true equilibrium structure of our material (*oP*) should result isostructural to other ma-

terials governed by same chemical principles (e.g. valency, electronegativity, position within the periodic table etc.). Furthermore, it is said that compounds governed by the same chemical principles as  $\text{Na}_2\text{MoSe}_4$  will also be favored in the *oP* lattice. This is in fact the reasoning and strength behind probabilistic models for crystal structure prediction. Therefore, the probability  $P$  of a material to exist in a specific crystal system (e.g. *oP* (*Pnma*)) will be influenced by the size of the known isostructural family[35, 48–50]. To elucidate, we match our evaluated structures to their analogs. First, we found our least stable candidate *cF*( $Fd-3m$ ) to be isostructural to the stable phase of  $\text{Na}_2\text{MoO}_4$  [ICSD No. 44523]. The metastable phase of  $\text{Na}_2\text{MoO}_4$  [ICSD No. 151971] is characterized by the *Fddd* space group and isostructural to our *oF* candidate. The *mC* candidate is isostructural to the molecular crystal  $\text{K}_2\text{MoO}_4$  [ICSD No. 16154]. Lastly, our most stable *oP* phase is isostructural to (or based on)  $\text{K}_2\text{MoS}_4$  [ICSD No. 409563]. Notably, the following isostructures have been synthesized:  $\text{Na}_2\text{MoSe}_4$  with *oP*(*Pnma*) symmetry:  $\text{K}_2\text{MoS}_4$ ,  $\text{Cs}_2\text{MoSe}_4$ ,  $\text{Rb}_2\text{MoSe}_4$ ,  $\text{Cs}_2\text{MoS}_4$ ,  $\text{Rb}_2\text{MoS}_4$ ,  $\text{K}_2\text{WSe}_4$  and  $\text{Rb}_2\text{WS}_4$  [51–55] and could potentially be intrinsic semiconductors. Additionally, hybrid organic/inorganic members of the family  $\text{R}_2\text{MX}_4$  (e.g.  $\text{R} = \text{CH}_3\text{NH}_3$ ;  $\text{M} = \text{Mo}, \text{W}$ ;  $\text{X} = \text{S}, \text{Se}$ ) have also been reported [56, 57]. Note, throughout the  $\text{A}_2/\text{R}_2\text{MX}_4$  family,  $\text{X}$  is either  $\text{S}$  or  $\text{Se}$ . To the best of our knowledge, no detailed theoretical studies of these materials are available in the literature. Given the electronic structure of  $\text{Na}_2\text{MoSe}_4$  (see section VI), it results of interest to the broader community to understand the physics and chemistry of the *oP*  $\text{A}/\text{R}_2\text{MX}_4$  compounds and their potential applications. Hence, we intend for our work to serve as a primer for future theoretical and experimental studies on the structural and electronic properties of both the inorganic and hybrid organic/inorganic analogs of  $\text{Na}_2\text{MoSe}_4$ .

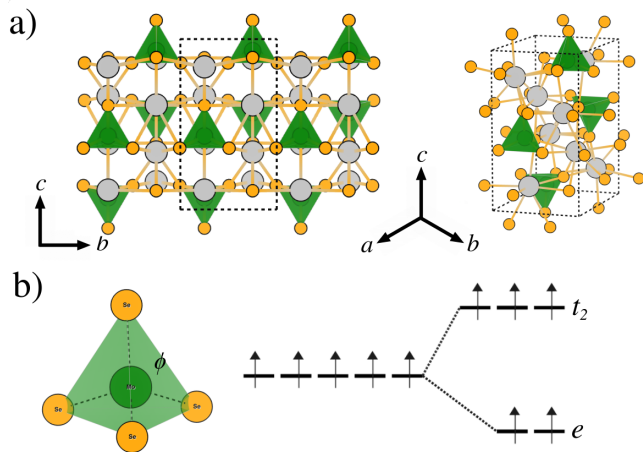


FIG. 2. Illustration of (a) isometric and front views of  $oP$  ( $Pnma$ ) phase of  $\text{Na}_2\text{MoSe}_4$  and (b) the  $[\text{MoSe}_4]^-$  tetrahedron with a schematic illustration of the tetrahedral crystal field that splits the Mo  $4d_5$  orbitals into two groups, namely  $t_2$  and  $e$ . As seen in (a)  $\text{Na}^+$  ions are intercalated through the  $\text{MoSe}_4$  tetrahedral layers.

## V. STRUCTURE AND SYMMETRY OF ORTHORHOMBIC SODIUM MOLYBDENUM SELENIDE

The disodium tetrasedimolybdate, which can be written as  $\text{Na}_2[\text{MoSe}_4]$ , is a molecular crystal favored to grow in the simple orthorhombic ( $oP$ ) crystal system with space group  $Pnma$  (No. 62), as shown in Figure 2. It has inversion symmetry, and it can be defined by eight symmetry operations. In Figure 2-a, an illustration of the frontal and isometric views are shown. The  $\text{Na}_2\text{MoSe}_4$  structure is three-dimensional, (i.e. there are no van der Waals layers) with two equivalent  $\text{Na}^{1+}$  sites. Note that  $\text{Mo}^{6+}$  is in tetrahedral coordination  $\tau_4$  (i.e.  $\tau_4 = 1$ ), bonded to four  $\text{Se}^{2-}$  atoms at each vertex. There is no Mo-Na bond, and the Na cations are intercalated throughout the lattice. This promotes the one-dimensional (directional) growth of the  $[\text{MoSe}_4]$  tetrahedrons. Additionally, the electronic charge of Na induces a distortion in the tetrahedron (distortion index  $t'_4 = 0.006$ ) yielding a slightly elongated Mo-Se bond ( $d = 2.33 \text{ \AA}$ ) versus the other three ( $d = 2.30 \text{ \AA}$ ), and a broadening of Se-Mo-Se bond angle  $\phi$  from  $\phi = 109.55^\circ$  to  $\phi = 115^\circ$  as shown in Figure 2-b. The average Mo-Se bond length is  $\bar{l} = 2.31 \text{ \AA}$ . This distortion is caused by electronic charge effects of a  $\text{Na}^{1+}$  atom in proximity to one Se vertex ( $r = 2.98 \text{ \AA}$ ). The distortion can be also measured by the tetrahedron edges, conformed by Se-Se inter-atomic distances. The tetrahedron edge lengths are  $l = 3.778, 3.576$  and  $3.793 \text{ \AA}$  respectively. The omitted length values are redundant in nature. A spread of Na-Se bond distances can be found in the two Na sites, with values ranging between  $2.98 - 3.48 \text{ \AA}$ . In one  $\text{Na}^{1+}$  site,  $\text{Na}^{1+}$  is bonded in a 8-coordinate geometry to eight  $\text{Se}^{2-}$

atoms. In the other, our sodium cation is bonded in a 9-coordinate geometry to 9 Se anions. There are three nonequivalent  $\text{Se}^{2-}$  sites. In the first Se site,  $\text{Se}^{2-}$  is bonded in a single coordinate geometry to four  $\text{Na}^{1+}$  and one  $\text{Mo}^{6+}$  atom. In the second site,  $\text{Se}^{2-}$  is bonded in a 6-coordinate geometry to five  $\text{Na}^{1+}$  and one  $\text{Mo}^{6+}$  atom. In the third Se site,  $\text{Se}^{2-}$  is bonded in a 5-coordinate geometry to four  $\text{Na}^{1+}$  and one  $\text{Mo}^{6+}$  atom. Our structural analysis is in agreement with the available information for its reported analogs [51–55]. Note, the intercalation of Na atoms could favor an energetically low-cost substitution (or displacement) of cations, enabling the tunability of these materials' properties.

## VI. ELECTRONIC STRUCTURE

The electronic structure of all candidates was investigated. We show in Figure 3 the electronic band structure of the candidate phases of  $\text{Na}_2\text{MoSe}_4$  and their corresponding total density of states (DOS). Conventional  $k$  paths were used [58, 59]. In all plots, the Fermi energy  $E_F$  is set to zero. From the band structure plots, it is clear that one of four phases is metallic;  $cF$  (Figure 3(a)), where the Density of States (DOS) plot shows the typical electronic structure of a metal. The remaining three (less symmetric) are intrinsic semiconductors. The  $oF$  phase is a direct bandgap semiconductor (Figure 3 (b)), with a bandgap  $E_g = 0.62 \text{ eV}$  at  $\Gamma$ , while the  $mC$  phase is an indirect bandgap semiconductor with  $E_g = 0.45 \text{ eV}$  running from  $A - \Gamma$  (Figure 3(d)). Total DOS is shown depicting the corresponding semiconducting characteristics. Lastly, our GGA calculations show that  $oP$   $\text{Na}_2\text{MoSe}_4$  has a fundamental bandgap of  $E_g(\text{GGA}) = 0.24 \text{ eV}$  at  $\Gamma$  (Figure 3 (c)). Since the  $oP$  structure is energetically favored, we focus only on this phase from here in and shall refer to this phase simply as " $\text{Na}_2\text{MoSe}_4$ ".

In order to better understand the electronic band structure of  $\text{Na}_2\text{MoSe}_4$ , we adopt an alternative  $k$  path from the work of Xia *et al* [21] on  $oP$   $\text{CsCu}_5\text{Se}_3$ . At this point, we analyze nature of the  $\text{Na}_2\text{MoSe}_4$  band structure. Let us recall that the Mo is in coordination  $\tau_4$  with Se atoms at each vertex. Given the electronic nature of our species, it is inferred that the valence bands that contribute to the Fermi level  $E_F$  are composed of Mo  $d$  orbitals, with mild hybridization from the Se  $p$  orbitals. Therefore, it can be said that the electronic (semiconducting) properties of the material arise mainly from the  $[\text{MoSe}_4]$  sub-unit and that the effects of Na  $s, p$  electrons are negligible. However, although Na doesn't seem have influence on the band structure of the selenide, it is reasonable to believe that the displacement of Na throughout the lattice (diffusion) or its substitution to another ion (or molecule) could have an effect on the value of  $E_g$ .

We further investigate the  $\tau_4$  sub-unit by treating the Se atoms as point charges around the Mo central ion, to study their bonding strength in terms of Coulombic interactions. In this sense, Crystal Field Theory (CFT) qual-



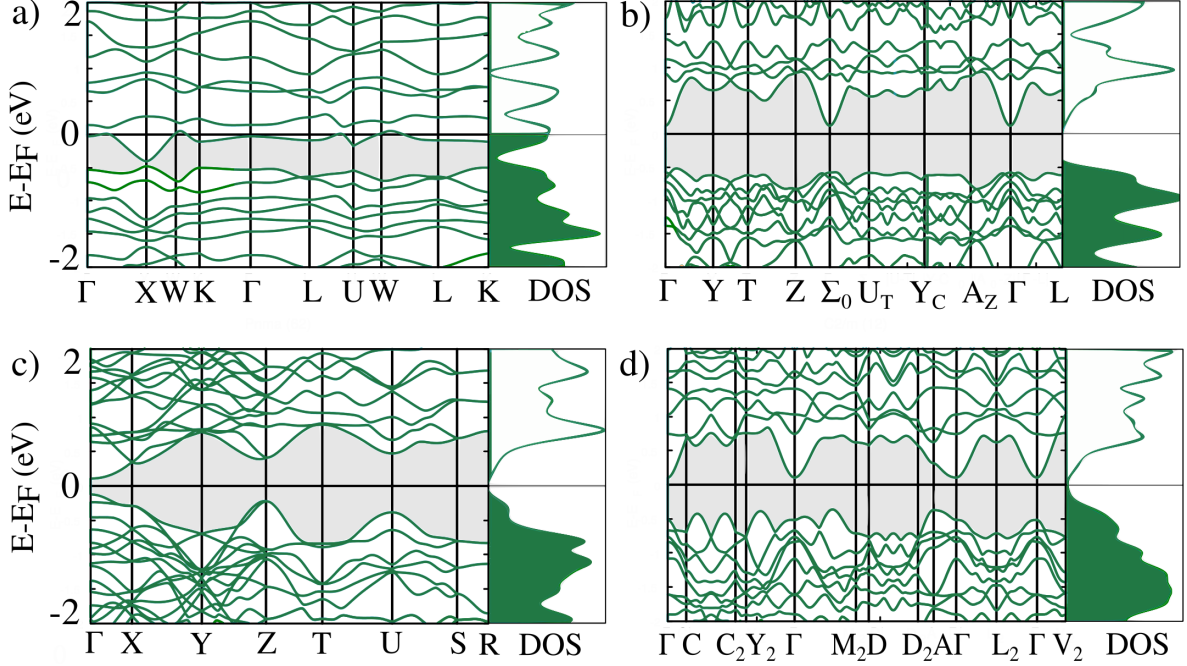


FIG. 3. The electronic band structure and total density of states is shown for the (a) *cF*, (b) *oF*, (c) *oP* and (d) *mC* candidate phases of  $\text{Na}_2\text{MoSe}_4$ . The Fermi energy  $E_F$  is set to 0.

itatively predicts how the electrons in the Mo  $4d$  orbitals respond to the effective electrostatic potential imposed by its neighboring Se atoms. In  $t_4$ , the initial five-fold degeneracy of the  $4d$  orbitals will break. This will give rise to the the orbital groups  $t_2$  orbitals ( $d_{xy}, d_{yz}, d_{yz}$ ) and the  $e$  orbitals ( $d_{x^2-y^2}, d_{z^2}$ ). A schematic illustration of an idealized  $\tau_4$  CF is shown in the right hand side of Figure 2(b). Let us recall the electronic configuration for Mo,  $[Kr]4d_5 5s_1$ , to see that there is only one (unpaired) electron occupying each  $d$  orbital. This conformation is energetically much more stable than if we were to occupy the orbitals in such a way that we have paired electrons. As a consequence, this along with weak overlap between the Mo  $d$  and Se  $p$  orbitals leads to high-spin behaviour. In essence,  $\text{Na}_2\text{MoSe}_4$  is paramagnetic with a potentially large Fermi velocity  $v_F$ . Also note, that the qualitative CFT analysis indicates that spin orbit coupling (SOC) effects are weak [60] in  $\text{Na}_2\text{MoSe}_4$ . In figure 4, we elucidate on this by plotting the GGA and GGA+SOC band structures along the  $\Gamma-X-U-\Gamma-Y-T-\Gamma-Z-S-R$   $k$ -path. The FBZ is shown in Figure 4 (b). Here, we refer to the band structure simply as GGA as PBE and PBEsol, having the same form of correlation-exchange, yield the same band structure. For completeness, we also plot the band structure by introducing an empirical Hubbard potential (GGA+ $U$  ;  $U = U_{eff} = 3.5$  eV [31]) to test for Coulomb effects in Mo  $d$  electrons (see Supplementary Information). As expected, no change was observed in  $E_g$  upon calculating the band structure with SOC or  $U_{eff}$ .

Note, in Figure 4(a) that the effect of SOC,  $\Delta_{SOC}$ , is

negligible and does not open/widen  $E_g$ . Note also, the band structure of  $\text{Na}_2\text{MoSe}_4$  is highly isotropic. Note also, that the conduction band minimum (CBM) and valence band maximum (VBM) are mainly composed of Mo  $d$  and Se  $p$ . The dispersion of  $E(k)$  near  $\Gamma$  is linear in regions  $U \rightarrow \Gamma$ ,  $\Gamma \rightarrow Y$ , with clear parabolic topology at the VBM and CBM. In this region,  $U \rightarrow \Gamma \rightarrow Y$ , the electron and hole effective masses  $m_h^*, m_e^*$  and are expected to be constant. Therefore, there could potentially be high mobility transport within  $\text{Na}_2\text{MoSe}_4$ .

In Figure 5, the projected DOS (pDOS) is shown from  $-2 \leq E_F \leq 2$  eV for the atomic contributions, Mo  $d$ , Se  $p$  and Na  $s$  electrons. Clearly the Na  $s$  states do not contribute near the Fermi level, and the hybridized  $d/p$  orbitals; Mo  $d$  states which (note  $0 \leq E_F \leq 2$  eV) and the Se  $p$  states (note  $-2 \leq E_F \leq 0$  eV) are the ones dominating the contributions around the Fermi level. We then decompose the contribution of summed Mo  $d$  into its constituent atomic orbitals in Figure 6 ;  $d_{z^2}, d_{zx}, d_{zy}, d_{x^2-y^2}, d_{xy}$ . Note that higher pDOS is observed above the Fermi level, due to the unoccupied states in the shells.

Although ground state GGA calculations can provide a reasonable first approximation to model the electronic structure of a material, it fails to predict the fundamental value of  $E_g$ . To be precise, GGA underestimates the bandgap. Hence, a precise approach such as the use of MGGA, hybrid functional (e.g. HSE06 [61]) or the many-body perturbation theory method *GW* is desired. To achieve a predictive level, we have opted for the MGGA,

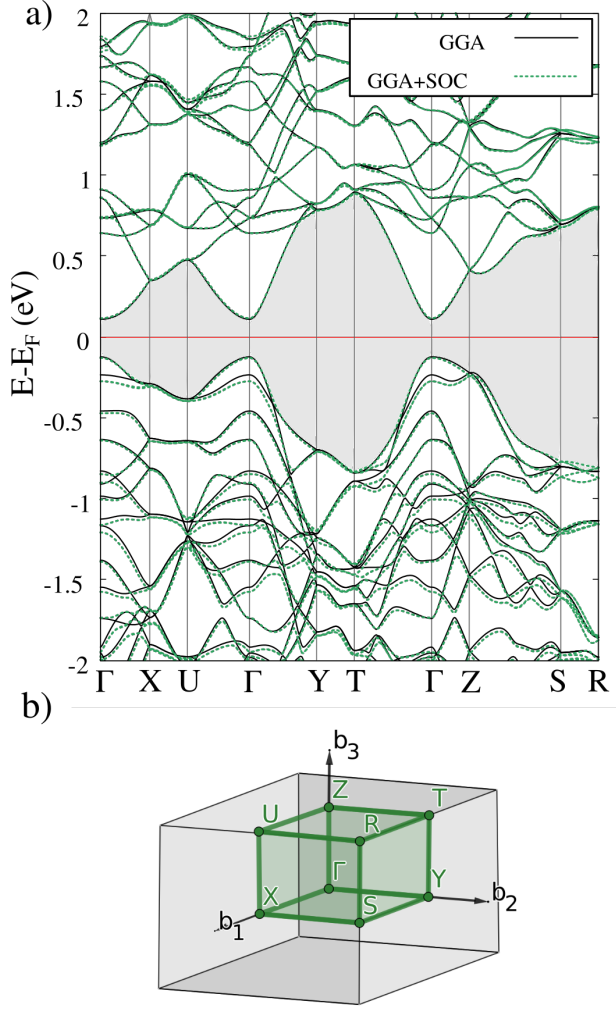


FIG. 4. Electronic band structure is shown in (a) modeled at the GGA (black - solid) and GGA+SOC (green - dashed) levels. The irreducible Brillouin zone and  $k$ -path for the  $oP$  lattice is shown in (b).

Trahn-Blaha modified version of the Becke-Johnson potential (TB-mBJ), designed and proven to yield robust results and comparable to HSE06,  $GW$  and experimental results, when used with a APW+lo basis set at a fraction of the computational cost [44, 62–67]. In the present scenario, the use of TB-mBJ is considered adequate as no further corrections for correlation or spin-orbit coupling are needed.

The TB-mBJ exchange potential reads

$$\mathbf{v}_{x,\sigma}^{TB-mBJ}(\mathbf{r}) = c\mathbf{v}_{x,\sigma}^{BR}(\mathbf{r}) + (3c-2)\frac{1}{\pi}\sqrt{\frac{5}{12}}\sqrt{\frac{2t_{\sigma}(\mathbf{r})}{\rho_{\sigma}(\mathbf{r})}} \quad (7)$$

where  $\rho_{\sigma}$  is the electronic density,  $t_{\sigma}$  is the kinetic energy density and  $\mathbf{v}_{x,\sigma}^{BR}(\mathbf{r})$  is the Becke-Roussel potential[68]. Additionally, since it was shown that correlation does not play an important role in the band structure of  $\text{Na}_2\text{MoSe}_4$ , not affecting the band topology or chang-

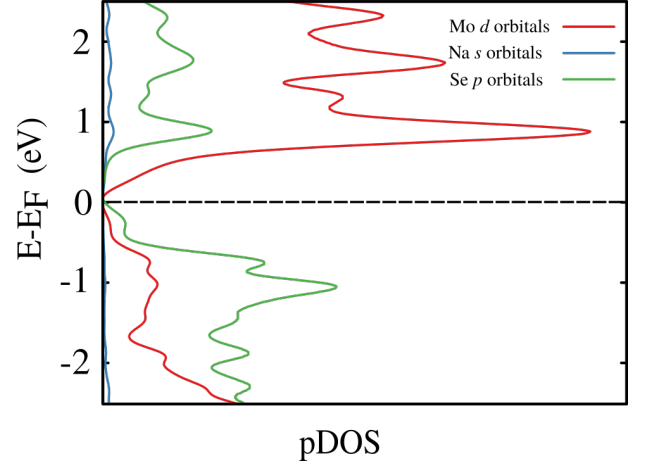


FIG. 5. Projected density of states is shown for the valence orbitals of each species.

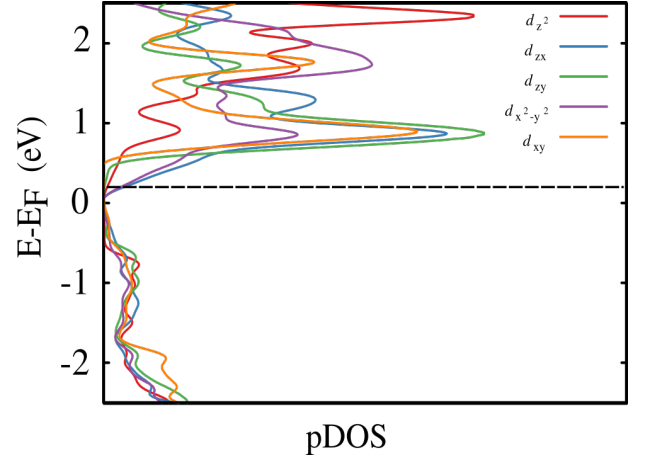


FIG. 6. Projected density of states is decomposed to show individual Mo  $d$  orbital contributions.

ing  $E_g$ , a MGGA approach that corrects exchange is appropriate. Our TB-mBJ calculation of the band structure yields the same band topology and a bandgap of  $E_g^{TB-mBJ} = 0.53$  eV at the  $\Gamma$  point. This is shown in figure 6. One would expect to obtain of  $E_g = 0.53 \pm 0.02$  eV from HSE06, as TB-mBJ has been shown to perform at the same level of accuracy as HSE06 when treating semiconductors, both with a standard deviation of the relative error  $\sigma = 22\%$  within the APW+lo framework. Additionally, Tran and Blaha demonstrate that as the  $E_g$  increases, TB-mBJ yields better performance than HSE06 when compared to the experimental data [69]. Thus, one can calculate the bandgap  $E_g$  of large and complex systems at a predictive level. Therefore, a MGGA TB-mBJ should faithfully reproduce the physics of the inorganic  $\text{A}_2\text{MX}_4$  and hybrid organic/inorganic  $\text{R}_2\text{MX}_4$ , with 84 atoms in the unit cell. This has recently been achieved and for complex semiconducting materials structures such as inorganic organic/inorganic perovskites [70, 71], serving as motivation for complete

description of the aforementioned *oP* compounds. Al-

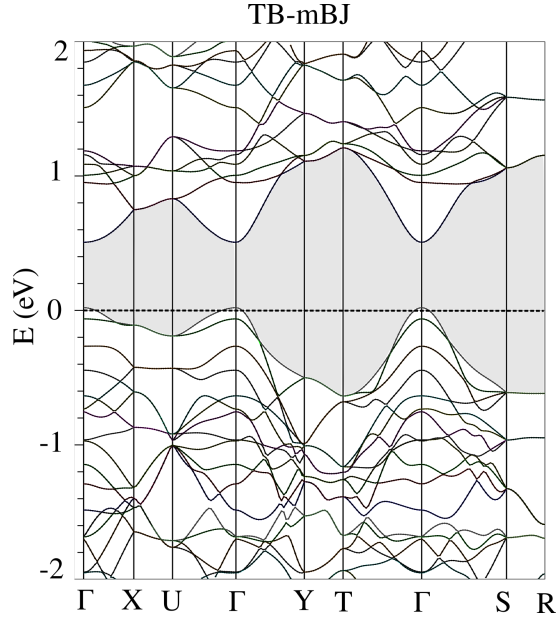


FIG. 7. Electronic band structure of  $\text{Na}_2\text{MoSe}_4$  corrected by TB-mBJ. Here, the VBM is set to zero. Note that the band topology is preserved when compared to GGA.

though the bandgap isn't yet "ideal" for applications in traditional optoelectronic devices or photovoltaics,  $\text{Na}_2\text{MoSe}_4$  with an intrinsic direct bandgap of 0.53 eV holds potential for applications in infrared optoelectronics and high-speed electronic heterostructures and devices. Keeping in mind, the bandgap is tunable via ionic displacement and/or substitution. Hence, it is of interest to investigate (i) the electronic structure of the  $\text{A}_2\text{MX}_4$  ( $\text{R}_2\text{MX}_4$ ) and (ii) the effects of Na-displacement throughout the lattice. This can be achieved, for example, through nudge-elastic band DFT calculations[72].

## VII. SUMMARY AND CONCLUSIONS

In summary, the ground state structure and electronic properties of the  $\text{Na}_2\text{MoSe}_4$  structures were theoretically investigated. First-principles computations were performed on four candidate structures generated through

ionic substitution. Namely, face centered cubic, face centered orthorhombic, simple orthorhombic and base centered monoclinic. The stability analysis reveals that the simple orthorhombic is the favored phase, with space group  $Pnma$ . Additionally, the electronic band structure depicts semiconducting behaviour for all candidates with an exception of the cubic phase. Finally, the band structure of orthorhombic  $\text{Na}_2\text{MoSe}_4$  is modeled through MGGA computations based on the Trahn-Blaha modified Becke-Johnson exchange potential, yielding a direct fundamental bandgap of 0.53 eV at  $\Gamma$ . Thus, this bandgap is suitable  $\text{Na}_2\text{MoSe}_4$  for applications in infrared optoelectronics and high speed electronics. The valence and conduction bands near the Fermi level are composed of Mo *d* and Se *p* orbitals, although it is expected that substitution or diffusion of the alkali-metal cation could pose an effect on the band structure. Additionally, due to its structure, it is expected that  $\text{Na}_2\text{MoSe}_4$  could be classified as a mixed conductor, applicable in Na-ion batteries. Our analysis indicates that the general physics of  $\text{Na}_2\text{MoSe}_4$  is transferable to *oP* chemical analogs. By disclosing the crystal and electronic band structure of disodium molybdenum selenide, we also shine light onto the potential of related compounds. Lastly, this work is a major first step towards a fundamental understanding and application of alkali-metal transition metal chalcogenides and hybrid organic/inorganic transition metal chalcogenide semiconductors.

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