LJ force

$$\frac{d}{dr}(r^{-12} - r^{-6})\frac{dr}{dx} = (-12\,r^{-13} + 6r^{-7})\frac{d}{dx}(x^2 + y^2)^{1/2}$$

$$= (-12\,r^{-13} + 6r^{-7})\frac{1}{2}(x^2 + y^2)^{-1/2}\frac{d}{dx}(x^2 + y^2)$$

$$= (-12\,r^{-13} + 6r^{-7})\frac{1}{2r}2x = (-12\,r^{-14} + 6r^{-8})x$$

$$= (6r^{-8} - 12\,r^{-14})x$$

$$\bar{\boldsymbol{F}}_{\boldsymbol{ij}} = \boldsymbol{\nabla V}(\boldsymbol{r}_{\boldsymbol{ij}}) = (6r^{-8} - 12\,r^{-14})\bar{r}$$

$$= 6r^{-8}(1 - 2\,r^{-6})\bar{r}$$

$$= 6(r^2)^{-4}(1 - 2(r^2)^{-3})\bar{r}$$

LJ equilibrium distance

$$\frac{d}{dr}(r^{-12} - r^{-6}) = (-12\, r^{-13} + 6r^{-7})$$
$$= 6(1 - 2\, r^{-6})r^{-7}$$
$$= 6\left(1 - \frac{2}{r^6}\right)r^{-7} = 0$$
$$\Leftrightarrow r = 2^{\frac{1}{6}} \cong 1.122462048309373$$